CWD := "C:\PhD\2015\00 Papers\Paper 2 – Amazon River\04 – Outputs Mathcad" $sigma := 3$ $mini := 1$ $maxi := 2$

1

$im01$:= READBMP("LT52250601986196XXX03_LogRatioB3B4.bmp")   WRITEBMP("im1986_.tif", 255 · zoom(canny($im01$, $sigma$, $mini$, $maxi$), 1, 1)) = 0

$im02$:= READBMP("LT52250601987183CUB00_LogRatioB3B4.bmp")   WRITEBMP("im1987_.tif", 255 · zoom(canny($im02$, $sigma$, $mini$, $maxi$), 1, 1)) = 0

$im03$:= READBMP("LT52250601996240CUB00_LogRatioB3B4.bmp")   WRITEBMP("im1996_.tif", 255 · zoom(canny($im03$, $sigma$, $mini$, $maxi$), 1, 1)) = 0

$im04$:= READBMP("LT52250602000251CUB00_LogRatioB3B4.bmp")   WRITEBMP("im2000_.tif", 255 · zoom(canny($im04$, $sigma$, $mini$, $maxi$), 1, 1)) = 0

$im05$:= READBMP("LT52250602006299CUB00_LogRatioB3B4.bmp")   WRITEBMP("im2006_.tif", 255 · zoom(canny($im05$, $sigma$, $mini$, $maxi$), 1, 1)) = 0

$im06$:= READBMP("LT52250602008145CUB00_LogRatioB3B4.bmp")   WRITEBMP("im2008_.tif", 255 · zoom(canny($im06$, $sigma$, $mini$, $maxi$), 1, 1)) = 0

$im07$:= READBMP("LC82250602013270LGN00_LogRatioB3B4.bmp")   WRITEBMP("im2013_.tif", 255 · zoom(canny($im07$, $sigma$, $mini$, $maxi$), 1, 1)) = 0

$im08$:= READBMP("LC82250602014257LGN00_LogRatioB3B4.bmp")   WRITEBMP("im2014_.tif", 255 · zoom(canny($im08$, $sigma$, $mini$, $maxi$), 1, 1)) = 0

$im09$:= READBMP("LE72250602015236CUB00_LogRatioB3B4.bmp")   WRITEBMP("im2015_.tif", 255 · zoom(canny($im09$, $sigma$, $mini$, $maxi$), 1, 1)) = 0

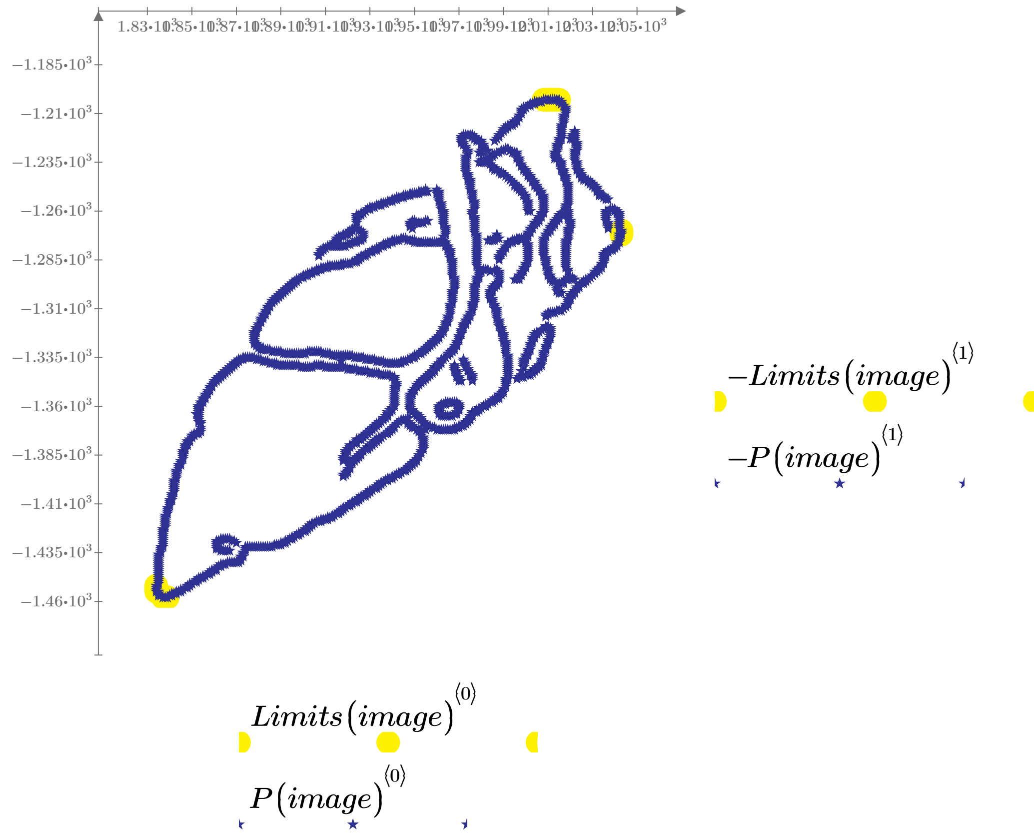
CWD = "C:\PhD\2015\00 Papers\Paper 2 – Amazon River\04 – Outputs Mathcad"   WRITEBMP("im2015_.tif", 255 · zoom(canny($im09$, 2, 1, 2.2), 1, 1)) = 0

CWD := "C:\PhD\2015\00 Papers\Paper 2 – Amazon River\04 – Outputs Mathcad\Canny v2"

```
P(im) := | im ← READBMP(im)
         | for i ∈ 0 .. rows(im) − 1
         |   | for j ∈ 0 .. cols(im) − 1
         |   |   | if im_{i,j} = 255
         |   |   |   | Out_{cont,0} ← j
         |   |   |   | Out_{cont,1} ← i
         |   |   |   | cont ← cont + 1
         | Out
```

$image$:= "im2014_Canny.tif"

```
Limits(im) := | P ← P(im)
              | cont ← 0
              | [minX  minY  maxX  maxY] ← [min(P^⟨0⟩)  min(P^⟨1⟩)  max(P^⟨0⟩)  max(P^⟨1⟩)]
              | for i ∈ 0..rows(P) − 1
              |   if (P^⟨0⟩)_i = minX
              |     [Out_{cont,0}  Out_{cont,1}] ← [P_{i,0}  P_{i,1}]
              |     cont ← cont + 1
              |   else
              |     if (P^⟨0⟩)_i = maxX
              |       [Out_{cont,0}  Out_{cont,1}] ← [P_{i,0}  P_{i,1}]
              |       cont ← cont + 1
              |     else
              |       if (P^⟨1⟩)_i = minY
              |         [Out_{cont,0}  Out_{cont,1}] ← [P_{i,0}  P_{i,1}]
              |         cont ← cont + 1
              |       else
              |         if (P^⟨1⟩)_i = maxY
              |           [Out_{cont,0}  Out_{cont,1}] ← [P_{i,0}  P_{i,1}]
              |           cont ← cont + 1
              |         else
              |           continue
              | Out
              | [Cx  Cy] ← [mean(Out^⟨0⟩)  mean(Out^⟨1⟩)]
              | for k ∈ 0..rows(Out) − 1
              |   if (Out_{k,0} − Cx > 0 ∧ Out_{k,1} − Cy > 0) ∨ (Out_{k,0} − Cx < 0 ∧ Out_{k,1} − Cy > 0)
              |     comp ← 0
              |     multi ← 1
              |   else
              |     if (Out_{k,0} − Cx < 0 ∧ Out_{k,1} − Cy < 0) ∨ (Out_{k,0} − Cx > 0 ∧ Out_{k,1} − Cy < 0)
              |       comp ← 2 π
              |       multi ← −1
              |   ang_k ← multi · acos( (Out_{k,0} − Cx) / sqrt((Out_{k,0} − Cx)^2 + (Out_{k,1} − Cy)^2) ) + comp
              | Out ← csort(augment(Out, ang), 2)
              | stack(submatrix(Out, 0, rows(Out) − 1, 0, 1), [Out_{0,0}  Out_{0,1}])
```



$$f(x, y, im) := \left\| \begin{array}{l} Limits \leftarrow Limits(im) \\ X \leftarrow Limits^{\langle 0 \rangle} \\ Y \leftarrow Limits^{\langle 1 \rangle} \\ \text{for } i \in 0 .. \text{rows}(Limits) - 1 \\ \quad \| L_i \leftarrow -X_i^{\,2} \\ \text{for } i \in 0 .. \text{rows}(Limits) - 1 \\ \quad \| \begin{bmatrix} A_{i,0} & A_{i,1} & A_{i,2} & A_{i,3} & A_{i,4} \end{bmatrix} \leftarrow \begin{bmatrix} X_i \cdot Y_i & Y_i^{\,2} & X_i & Y_i & 1 \end{bmatrix} \\ a \leftarrow \left(A^{\mathrm{T}} \cdot A\right)^{-1} \cdot \left(A^{\mathrm{T}} \cdot L\right) \\ x^2 + a_0 \cdot x \cdot y + a_1 \cdot y^2 + a_2 \cdot x + a_3 \cdot y + a_4 \end{array} \right.$$

$$EllipseSphere(im) := \left\| \begin{array}{l} Limits \leftarrow Limits(im) \\ X \leftarrow Limits^{\langle 0 \rangle} \\ Y \leftarrow Limits^{\langle 1 \rangle} \\ \text{for } i \in 0 .. \text{rows}(Limits) - 1 \\ \quad \left\| \begin{array}{l} L_i \leftarrow -X_i^{\,2} \\ \begin{bmatrix} A_{i,0} & A_{i,1} & A_{i,2} & A_{i,3} & A_{i,4} \end{bmatrix} \leftarrow \begin{bmatrix} X_i \cdot Y_i & Y_i^{\,2} & X_i & Y_i & 1 \end{bmatrix} \end{array} \right. \\ a \leftarrow \left(A^{\mathrm{T}} \cdot A\right)^{-1} \cdot \left(A^{\mathrm{T}} \cdot L\right) \\ testaElipse \leftarrow 4 \cdot 1 \cdot a_1 - a_0^{\,2} \\ \text{if } testaElipse > 0 \\ \quad \| a \leftarrow a \\ \text{else} \\ \quad \left\| \begin{array}{l} \text{for } i \in 0 .. \text{rows}(Limits) - 1 \\ \quad \left\| \begin{array}{l} L_i \leftarrow -\left(X_i^{\,2} + Y_i^{\,2}\right) \\ \begin{bmatrix} A_{i,0} & A_{i,1} & A_{i,2} \end{bmatrix} \leftarrow \begin{bmatrix} X_i & Y_i & 1 \end{bmatrix} \end{array} \right. \\ a \leftarrow \text{stack}\left(\begin{bmatrix} 0 & 1 \end{bmatrix}^{\mathrm{T}}, \left(A^{\mathrm{T}} \cdot A\right)^{-1} \cdot \left(A^{\mathrm{T}} \cdot L\right) \right) \end{array} \right. \\ a \end{array} \right.$$

$$EllipseSphere(image) = \begin{bmatrix} 1.172 \\ 0.706 \\ -5.438 \cdot 10^3 \\ -4.154 \cdot 10^3 \\ 8.031 \cdot 10^6 \end{bmatrix}$$

Solution: http://math.stackexchange.com/questions/280937/finding-the-angle-of-rotation-of-an-ellipse-from-its-general-equation-and-the-ot
http://mathworld.wolfram.com/Ellipse.html

$$
\begin{aligned}
EllipseParam(im) := \; & El \leftarrow EllipseSphere(im) \\
& \begin{bmatrix} A & B & C & D & F & G \end{bmatrix} \leftarrow \begin{bmatrix} 1 & \dfrac{El_0}{2} & El_1 & \dfrac{El_2}{2} & \dfrac{El_3}{2} & El_4 \end{bmatrix} \\
& \text{if } B = 0 \wedge A < C \\
& \quad \theta 2 \leftarrow 0 \\
& \text{else} \\
& \quad \text{if } B = 0 \wedge A > C \\
& \quad\quad \theta 2 \leftarrow 0 \\
& \quad \text{else} \\
& \quad\quad \text{if } B \neq 0 \wedge A < C \\
& \quad\quad\quad \theta 2 \leftarrow \frac{1}{2} \cdot \operatorname{atan}\left(\frac{2 \cdot B}{A - C}\right) \\
& \quad\quad \text{else} \\
& \quad\quad\quad \text{if } B \neq 0 \wedge A > C \\
& \quad\quad\quad\quad \theta 2 \leftarrow \frac{\pi}{2} + \frac{1}{2} \cdot \operatorname{atan}\left(\frac{2 \cdot B}{A - C}\right) \\
& x0 \leftarrow \frac{C \cdot D - B \cdot F}{B^2 - A \cdot C} \\
& y0 \leftarrow \frac{A \cdot F - B \cdot D}{B^2 - A \cdot C} \\
& a \leftarrow \sqrt{2 \cdot \frac{\left(A \cdot F^2 + C \cdot D^2 + G \cdot B^2 - 2 \cdot B \cdot D \cdot F - A \cdot C \cdot G\right)}{\left(B^2 - A \cdot C\right) \cdot \left(\sqrt{(A - C)^2 + 4 \cdot B^2} - (A + C)\right)}} \\
& b \leftarrow \sqrt{2 \cdot \frac{\left(A \cdot F^2 + C \cdot D^2 + G \cdot B^2 - 2 \cdot B \cdot D \cdot F - A \cdot C \cdot G\right)}{\left(B^2 - A \cdot C\right) \cdot \left(-\sqrt{(A - C)^2 + 4 \cdot B^2} - (A + C)\right)}} \\
& \begin{bmatrix} \theta 2 \\ x0 \\ y0 \\ a \\ b \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
EllipseX(im) := \; & EP \leftarrow EllipseParam(im) \\
& \begin{bmatrix} \theta & x0 & y0 & a & b \end{bmatrix} \leftarrow \begin{bmatrix} EP_0 & EP_1 & EP_2 & EP_3 & EP_4 \end{bmatrix} \\
& \text{for } t \in 0 \,..\, 36 \cdot 5 \\
& \quad Out_t \leftarrow a \cdot \cos\left(t \cdot \frac{\pi}{18 \cdot 5}\right) \cdot \cos(\theta) + b \cdot \sin\left(t \cdot \frac{\pi}{18 \cdot 5}\right) \cdot -\sin(\theta) + x0 \\
& Out
\end{aligned}
$$

$$
\begin{aligned}
EllipseY(im) := \; & EP \leftarrow EllipseParam(im) \\
& \begin{bmatrix} \theta & x0 & y0 & a & b \end{bmatrix} \leftarrow \begin{bmatrix} EP_0 & EP_1 & EP_2 & EP_3 & EP_4 \end{bmatrix} \\
& \text{for } t \in 0 \,..\, 36 \cdot 5 \\
& \quad Out_t \leftarrow a \cdot \cos\left(t \cdot \frac{\pi}{18 \cdot 5}\right) \cdot (\sin(\theta)) + b \cdot \sin\left(t \cdot \frac{\pi}{18 \cdot 5}\right) \cdot \cos(\theta) + y0 \\
& Out
\end{aligned}
$$

$$
f\left(EllipseX(image)_{20}, EllipseY(image)_{20}, image\right) = -2.794 \cdot 10^{-9}
$$

$$
f\left(EllipseX(image)_{1}, EllipseY(image)_{1}, image\right) = -3.725 \cdot 10^{-9}
$$

$$\frac{d}{dx}\left(x^2+a_0\cdot x\cdot y+a_1\cdot y^2+a_2\cdot x+a_3\cdot y+a_4\right)\rightarrow 2\cdot x+y\cdot a_0+a_2$$

$$\frac{d}{dy}\left(x^2+a_0\cdot x\cdot y+a_1\cdot y^2+a_2\cdot x+a_3\cdot y+a_4\right)\rightarrow x\cdot a_0+2\cdot y\cdot a_1+a_3$$

Finding the points with maximum distance to the first approximation ellipse. Those points will be translated to the next point on gradient direction and a new ellipse will be built surrounding all points.

$$k\cdot\left(2\cdot x+y\cdot a_0+a_2\right)=X-x$$

$$k\cdot\left(x\cdot a_0+2\cdot y\cdot a_1+a_3\right)=Y-y$$

$$(k\cdot 2+1)\cdot x+\left(k\cdot a_0\right)\cdot y=X-k\cdot a_2$$

$$\left(k\cdot a_0\right)\cdot x+\left(k\cdot 2\cdot a_1+1\right)\cdot y=Y-k\cdot a_3$$

$$\left(-k\cdot a_0\right)\cdot\left(k\cdot a_0\right)\cdot y+\left(k\cdot 2\cdot a_1+1\right)\cdot(k\cdot 2+1)\cdot y=\left(k\cdot a_2-X\right)\cdot\left(k\cdot a_0\right)+(k\cdot 2+1)\cdot\left(Y-k\cdot a_3\right)$$

$$\left(k\cdot 2\cdot a_1+1\right)\cdot(k\cdot 2+1)\cdot x-\left(k\cdot a_0\right)^2\cdot x=\left(X-k\cdot a_2\right)\cdot\left(k\cdot 2\cdot a_1+1\right)+\left(-\left(k\cdot a_0\right)\right)\cdot\left(Y-k\cdot a_3\right)$$

$$y=\frac{\left(k\cdot a_2-X\right)\cdot\left(k\cdot a_0\right)+(k\cdot 2+1)\cdot\left(Y-k\cdot a_3\right)}{\left(k\cdot 2\cdot a_1+1\right)\cdot(k\cdot 2+1)-\left(k\cdot a_0\right)^2}$$

$$x=\frac{\left(X-k\cdot a_2\right)\cdot\left(k\cdot 2\cdot a_1+1\right)+\left(-\left(k\cdot a_0\right)\right)\cdot\left(Y-k\cdot a_3\right)}{\left(k\cdot 2\cdot a_1+1\right)\cdot(k\cdot 2+1)-\left(k\cdot a_0\right)^2}$$

$$f1(k,a,X,Y):=\left(\frac{\left(X-k\cdot a_2\right)\cdot\left(k\cdot 2\cdot a_1+1\right)+\left(-\left(k\cdot a_0\right)\right)\cdot\left(Y-k\cdot a_3\right)}{\left(k\cdot 2\cdot a_1+1\right)\cdot(k\cdot 2+1)-\left(k\cdot a_0\right)^2}\right)^2+a_0\cdot\left(\frac{\left(X-k\cdot a_2\right)\cdot\left(k\cdot 2\cdot a_1+1\right)+\left(-\left(k\cdot a_0\right)\right)\cdot\left(Y-k\cdot a_3\right)}{\left(k\cdot 2\cdot a_1+1\right)\cdot(k\cdot 2+1)-\left(k\cdot a_0\right)^2}\right)\cdot\left(\frac{\left(k\cdot a_2-X\right)\cdot\left(k\cdot a_0\right)+(k\cdot 2+1)\cdot\left(Y-k\cdot a_3\right)}{\left(k\cdot 2\cdot a_1+1\right)\cdot(k\cdot 2+1)-\left(k\cdot a_0\right)^2}\right)$$

$$f2(k,a,X,Y):=a_1\cdot\left(\frac{\left(k\cdot a_2-X\right)\cdot\left(k\cdot a_0\right)+(k\cdot 2+1)\cdot\left(Y-k\cdot a_3\right)}{\left(k\cdot 2\cdot a_1+1\right)\cdot(k\cdot 2+1)-\left(k\cdot a_0\right)^2}\right)^2+a_2\cdot\left(\frac{\left(X-k\cdot a_2\right)\cdot\left(k\cdot 2\cdot a_1+1\right)+\left(-\left(k\cdot a_0\right)\right)\cdot\left(Y-k\cdot a_3\right)}{\left(k\cdot 2\cdot a_1+1\right)\cdot(k\cdot 2+1)-\left(k\cdot a_0\right)^2}\right)+a_3\cdot\left(\frac{\left(k\cdot a_2-X\right)\cdot\left(k\cdot a_0\right)+(k\cdot 2+1)\cdot\left(Y-k\cdot a_3\right)}{\left(k\cdot 2\cdot a_1+1\right)\cdot(k\cdot 2+1)-\left(k\cdot a_0\right)^2}\right)+a_4$$

$$f(k,a,X,Y):=f1(k,a,X,Y)+f2(k,a,X,Y)$$

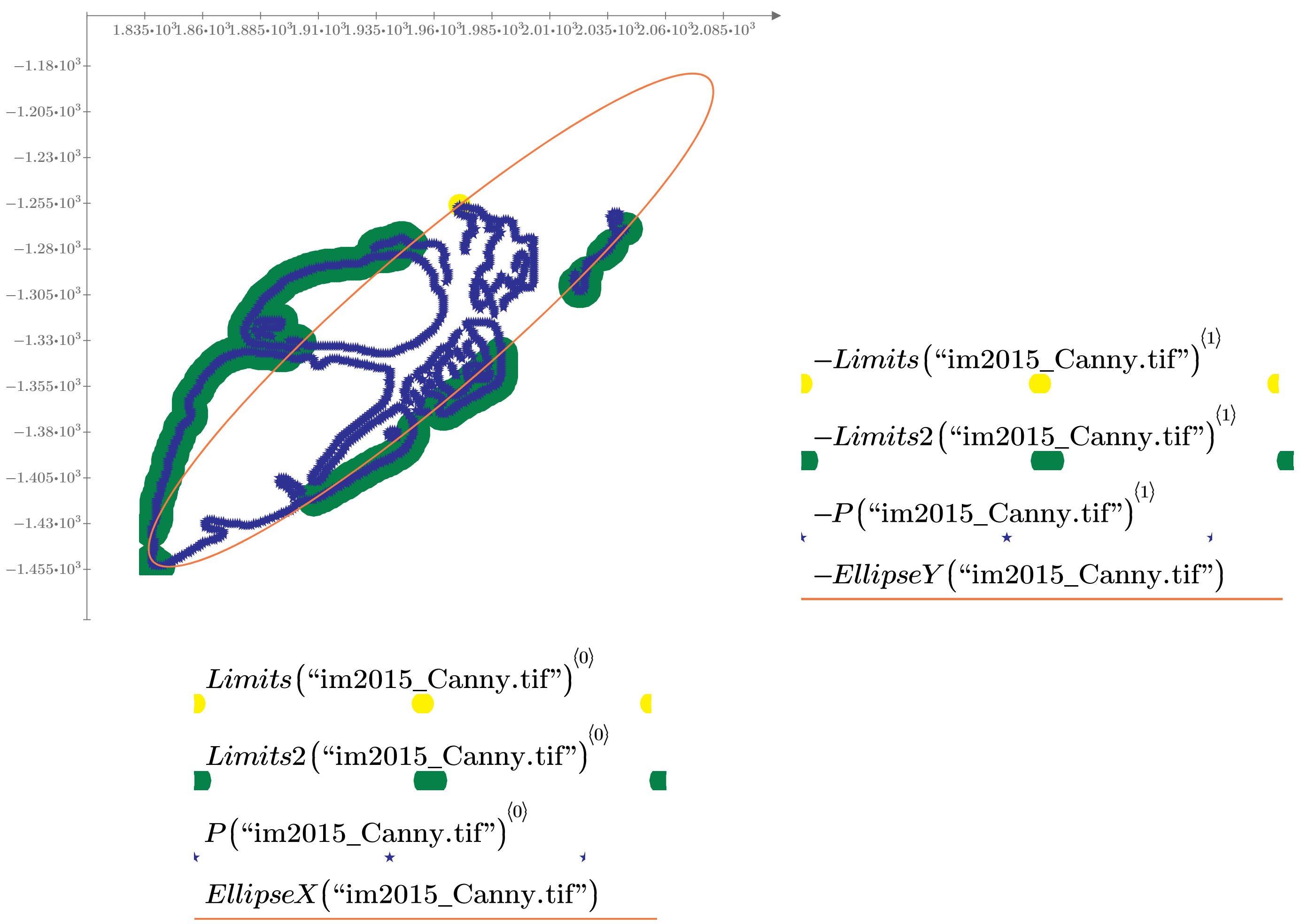
$$Df(k,a,X,Y):=\frac{d}{dk}f(k,a,X,Y)$$

$$
\begin{aligned}
Limits2(im) :=\ & a \leftarrow EllipseSphere(im)\\
& P \leftarrow P(im)\\
& \text{for } i \in 0+0\,..\,1\cdot(\text{rows}(P)-1)+0\\
& \quad x \leftarrow P_{i,0}\\
& \quad y \leftarrow P_{i,1}\\
& \quad Ax \leftarrow 1\\
& \quad Bx \leftarrow a_0 \cdot y + a_2\\
& \quad Cx \leftarrow a_1 \cdot y^2 + a_3 \cdot y + a_4\\
& \quad Ay \leftarrow a_1\\
& \quad By \leftarrow a_0 \cdot x + a_3\\
& \quad Cy \leftarrow x^2 + a_2 \cdot x + a_4\\
& \quad x1 \leftarrow \frac{-Bx + \sqrt{Bx^2 - 4\cdot Ax \cdot Cx}}{2\ Ax}\\
& \quad x2 \leftarrow \frac{-Bx - \sqrt{Bx^2 - 4\cdot Ax \cdot Cx}}{2\ Ax}\\
& \quad y1 \leftarrow \frac{-By + \sqrt{By^2 - 4\cdot Ay \cdot Cy}}{2\ Ay}\\
& \quad y2 \leftarrow \frac{-By - \sqrt{By^2 - 4\cdot Ay \cdot Cy}}{2\ Ay}\\
& \quad \text{if } (\text{Im}(x1)=0 \wedge \text{Im}(x2)=0) \vee (\text{Im}(y1)=0 \wedge \text{Im}(y2)=0)\\
& \quad\quad contx \leftarrow 0\\
& \quad\quad conty \leftarrow 0\\
& \quad\quad \text{try}\\
& \quad\quad\quad \text{if } (\text{Im}(x1)=0 \wedge \text{Im}(x2)=0) \wedge (x1<x \wedge x2<x \vee x1>x \wedge x2>x)\\
& \quad\quad\quad\quad \begin{bmatrix} Out_{cont,0} & Out_{cont,1} \end{bmatrix} \leftarrow \begin{bmatrix} P_{i,0} & P_{i,1} \end{bmatrix}\\
& \quad\quad\quad\quad contx \leftarrow 1\\
& \quad\quad \text{on error}\\
& \quad\quad\quad contx \leftarrow 0\\
& \quad\quad \text{try}\\
& \quad\quad\quad \text{if } (\text{Im}(y1)=0 \wedge \text{Im}(y2)=0) \wedge (y1<y \wedge y2<y \vee y1>y \wedge y2>y)\\
& \quad\quad\quad\quad \begin{bmatrix} Out_{cont,0} & Out_{cont,1} \end{bmatrix} \leftarrow \begin{bmatrix} P_{i,0} & P_{i,1} \end{bmatrix}\\
& \quad\quad\quad\quad conty \leftarrow 1\\
& \quad\quad \text{on error}\\
& \quad\quad\quad conty \leftarrow 0\\
& \quad\quad cont \leftarrow cont + \max(contx, conty)\\
& \quad \text{else}\\
& \quad\quad cont \leftarrow cont
\end{aligned}
$$

$$
\begin{aligned}
EllipseSphere2(im) :=\ & Limits \leftarrow Limits2(im)\\
& X \leftarrow Limits^{\langle 0\rangle}\\
& Y \leftarrow Limits^{\langle 1\rangle}\\
& \text{for } i \in 0\,..\,\text{rows}(Limits)-1\\
& \quad L_i \leftarrow -{X_i}^2\\
& \quad \begin{bmatrix} A_{i,0} & A_{i,1} & A_{i,2} & A_{i,3} & A_{i,4} \end{bmatrix} \leftarrow \begin{bmatrix} X_i \cdot Y_i & {Y_i}^2 & X_i & Y_i & 1 \end{bmatrix}\\
& a \leftarrow (A^{\mathrm{T}} \cdot A)^{-1} \cdot (A^{\mathrm{T}} \cdot L)\\
& testaElipse \leftarrow 4 \cdot 1 \cdot a_1 - {a_0}^2\\
& \text{if } testaElipse > 0\\
& \quad a \leftarrow a\\
& \text{else}\\
& \quad \text{for } i \in 0\,..\,\text{rows}(Limits)-1\\
& \quad\quad L_i \leftarrow -({X_i}^2 + {Y_i}^2)\\
& \quad\quad \begin{bmatrix} A_{i,0} & A_{i,1} & A_{i,2} \end{bmatrix} \leftarrow \begin{bmatrix} X_i & Y_i & 1 \end{bmatrix}\\
& \quad a \leftarrow \text{stack}\left(\begin{bmatrix} 0 & 1 \end{bmatrix}^{\mathrm{T}}, (A^{\mathrm{T}} \cdot A)^{-1} \cdot (A^{\mathrm{T}} \cdot L)\right)\\
& a
\end{aligned}
$$

$Limits2(\text{“im2015\_Canny\_1.tif”})$

Out


1.835·10^3 1.86·10^3 1.885·10^3 1.91·10^3 1.935·10^3 1.96·10^3 1.985·10^3 2.01·10^3 2.035·10^3 2.06·10^3 2.085·10^3
−1.18·10^3
−1.205·10^3
−1.23·10^3
−1.255·10^3
−1.28·10^3
−1.305·10^3
−1.33·10^3
−1.355·10^3
−1.38·10^3
−1.405·10^3
−1.43·10^3
−1.455·10^3
Limits(“im2015_Canny.tif”)⟨0⟩
Limits2(“im2015_Canny.tif”)⟨0⟩
P(“im2015_Canny.tif”)⟨0⟩
EllipseX(“im2015_Canny.tif”)
−Limits(“im2015_Canny.tif”)⟨1⟩
−Limits2(“im2015_Canny.tif”)⟨1⟩
−P(“im2015_Canny.tif”)⟨1⟩
−EllipseY(“im2015_Canny.tif”)

$$EllipseParam2(im) := \left\| \begin{array}{l} El \leftarrow EllipseSphere2(im) \\ \begin{bmatrix} A & B & C & D & F & G \end{bmatrix} \leftarrow \begin{bmatrix} 1 & \frac{El_0}{2} & El_1 & \frac{El_2}{2} & \frac{El_3}{2} & El_4 \end{bmatrix} \\ \text{if } B = 0 \wedge A < C \\ \quad \| \theta 2 \leftarrow 0 \\ \text{else} \\ \quad \text{if } B = 0 \wedge A > C \\ \quad\quad \| \theta 2 \leftarrow 0 \\ \quad \text{else} \\ \quad\quad \text{if } B \neq 0 \wedge A < C \\ \quad\quad\quad \| \theta 2 \leftarrow \frac{1}{2} \cdot \operatorname{atan}\left(\frac{2 \cdot B}{A - C}\right) \\ \quad\quad \text{else} \\ \quad\quad\quad \text{if } B \neq 0 \wedge A > C \\ \quad\quad\quad\quad \| \theta 2 \leftarrow \frac{\pi}{2} + \frac{1}{2} \cdot \operatorname{atan}\left(\frac{2 \cdot B}{A - C}\right) \\ x0 \leftarrow \frac{C \cdot D - B \cdot F}{B^2 - A \cdot C} \\ y0 \leftarrow \frac{A \cdot F - B \cdot D}{B^2 - A \cdot C} \\ a \leftarrow \sqrt{2 \cdot \frac{\left(A \cdot F^2 + C \cdot D^2 + G \cdot B^2 - 2 \cdot B \cdot D \cdot F - A \cdot C \cdot G\right)}{\left(B^2 - A \cdot C\right) \cdot \left(\sqrt{(A - C)^2 + 4 \cdot B^2} - (A + C)\right)}} \\ b \leftarrow \sqrt{2 \cdot \frac{\left(A \cdot F^2 + C \cdot D^2 + G \cdot B^2 - 2 \cdot B \cdot D \cdot F - A \cdot C \cdot G\right)}{\left(B^2 - A \cdot C\right) \cdot \left(-\sqrt{(A - C)^2 + 4 \cdot B^2} - (A + C)\right)}} \\ \begin{bmatrix} \theta 2 \\ x0 \\ y0 \\ a \\ b \end{bmatrix} \end{array} \right.$$

$$EllipseX2(im) := \left\| \begin{array}{l} EP \leftarrow EllipseParam2(im) \\ \begin{bmatrix} \theta & x0 & y0 & a & b \end{bmatrix} \leftarrow \begin{bmatrix} EP_0 & EP_1 & EP_2 & EP_3 & EP_4 \end{bmatrix} \\ \text{for } t \in 0 .. 36 \cdot 5 \\ \quad \| Out_t \leftarrow a \cdot \cos\left(t \cdot \frac{\pi}{18 \cdot 5}\right) \cdot \cos(\theta) + b \cdot \sin\left(t \cdot \frac{\pi}{18 \cdot 5}\right) \cdot -\sin(\theta) + x0 \\ Out \end{array} \right.$$

$$EllipseY2(im) := \left\| \begin{array}{l} EP \leftarrow EllipseParam2(im) \\ \begin{bmatrix} \theta & x0 & y0 & a & b \end{bmatrix} \leftarrow \begin{bmatrix} EP_0 & EP_1 & EP_2 & EP_3 & EP_4 \end{bmatrix} \\ \text{for } t \in 0 .. 36 \cdot 5 \\ \quad \| Out_t \leftarrow a \cdot \cos\left(t \cdot \frac{\pi}{18 \cdot 5}\right) \cdot (\sin(\theta)) + b \cdot \sin\left(t \cdot \frac{\pi}{18 \cdot 5}\right) \cdot \cos(\theta) + y0 \\ Out \end{array} \right.$$

$$
\begin{aligned}
&TestaPontos(Set, im) := \\
&\quad [X\ \ Y] \leftarrow \left[Set^{\langle 0\rangle}\ \ Set^{\langle 1\rangle}\right] \\
&\quad \text{for } i \in 0..\operatorname{rows}(Set)-1 \\
&\qquad L_i \leftarrow -X_i^{\,2} \\
&\qquad \left[A_{i,0}\ \ A_{i,1}\ \ A_{i,2}\ \ A_{i,3}\ \ A_{i,4}\right] \leftarrow \left[X_i \cdot Y_i\ \ Y_i^{\,2}\ \ X_i\ \ Y_i\ \ 1\right] \\
&\quad a \leftarrow \left(A^{\mathrm{T}} \cdot A\right)^{-1} \cdot \left(A^{\mathrm{T}} \cdot L\right) \\
&\quad testaElipse \leftarrow 4 \cdot 1 \cdot a_1 - a_0^{\,2} \\
&\quad \text{if } testaElipse > 0 \\
&\qquad a \leftarrow a \\
&\quad \text{else} \\
&\qquad \text{for } i \in 0..\operatorname{rows}(Set)-1 \\
&\qquad\quad L_i \leftarrow -\left(X_i^{\,2} + Y_i^{\,2}\right) \\
&\qquad\quad \left[A_{i,0}\ \ A_{i,1}\ \ A_{i,2}\right] \leftarrow \left[X_i\ \ Y_i\ \ 1\right] \\
&\qquad a \leftarrow \operatorname{stack}\left(\left[0\ \ 1\right]^{\mathrm{T}}, \left(A^{\mathrm{T}} \cdot A\right)^{-1} \cdot \left(A^{\mathrm{T}} \cdot L\right)\right) \\
&\quad P \leftarrow P(im) \\
&\quad testaElipse \leftarrow 4 \cdot 1 \cdot a_1 - a_0^{\,2} \\
&\quad \text{if } testaElipse > 0 \\
&\qquad \text{for } i \in 0..\operatorname{rows}(P)-1 \\
&\qquad\quad [x\ \ y] \leftarrow \left[P_{i,0}\ \ P_{i,1}\right] \\
&\qquad\quad Ax \leftarrow 1 \\
&\qquad\quad Bx \leftarrow a_0 \cdot y + a_2 \\
&\qquad\quad Cx \leftarrow a_1 \cdot y^2 + a_3 \cdot y + a_4 \\
&\qquad\quad Ay \leftarrow a_1 \\
&\qquad\quad By \leftarrow a_0 \cdot x + a_3 \\
&\qquad\quad Cy \leftarrow x^2 + a_2 \cdot x + a_4 \\
&\qquad\quad x1 \leftarrow \frac{-Bx + \sqrt{Bx^2 - 4 \cdot Ax \cdot Cx}}{2\ Ax} \\
&\qquad\quad x2 \leftarrow \frac{-Bx - \sqrt{Bx^2 - 4 \cdot Ax \cdot Cx}}{2\ Ax} \\
&\qquad\quad y1 \leftarrow \frac{-By + \sqrt{By^2 - 4 \cdot Ay \cdot Cy}}{2\ Ay} \\
&\qquad\quad y2 \leftarrow \frac{-By - \sqrt{By^2 - 4 \cdot Ay \cdot Cy}}{2\ Ay} \\
&\qquad\quad \text{if } \left(\operatorname{Im}(x1) = 0 \wedge \operatorname{Im}(x2) = 0\right) \\
&\qquad\qquad \text{if } \left(x1 < x \wedge x2 < x \vee x1 > x \wedge x2 > x\right) \\
&\qquad\qquad\quad cont \leftarrow cont + 1
\end{aligned}
$$

$$
\begin{aligned}
&QuaisPontos(Set, im) := \\
&\quad [X\ \ Y] \leftarrow \left[Set^{\langle 0\rangle}\ \ Set^{\langle 1\rangle}\right] \\
&\quad \text{for } i \in 0..\operatorname{rows}(Set)-1 \\
&\qquad L_i \leftarrow -X_i^{\,2} \\
&\qquad \left[A_{i,0}\ \ A_{i,1}\ \ A_{i,2}\ \ A_{i,3}\ \ A_{i,4}\right] \leftarrow \left[X_i \cdot Y_i\ \ Y_i^{\,2}\ \ X_i\ \ Y_i\ \ 1\right] \\
&\quad a \leftarrow \left(A^{\mathrm{T}} \cdot A\right)^{-1} \cdot \left(A^{\mathrm{T}} \cdot L\right) \\
&\quad testaElipse \leftarrow 4 \cdot 1 \cdot a_1 - a_0^{\,2} \\
&\quad \text{if } testaElipse > 0 \\
&\qquad a \leftarrow a \\
&\quad \text{else} \\
&\qquad \text{for } i \in 0..\operatorname{rows}(Set)-1 \\
&\qquad\quad L_i \leftarrow -\left(X_i^{\,2} + Y_i^{\,2}\right) \\
&\qquad\quad \left[A_{i,0}\ \ A_{i,1}\ \ A_{i,2}\right] \leftarrow \left[X_i\ \ Y_i\ \ 1\right] \\
&\qquad a \leftarrow \operatorname{stack}\left(\left[0\ \ 1\right]^{\mathrm{T}}, \left(A^{\mathrm{T}} \cdot A\right)^{-1} \cdot \left(A^{\mathrm{T}} \cdot L\right)\right) \\
&\quad P \leftarrow P(im) \\
&\quad testaElipse \leftarrow 4 \cdot 1 \cdot a_1 - a_0^{\,2} \\
&\quad \text{if } testaElipse > 0 \\
&\qquad \text{for } i \in 0..\operatorname{rows}(P)-1 \\
&\qquad\quad [x\ \ y] \leftarrow \left[P_{i,0}\ \ P_{i,1}\right] \\
&\qquad\quad Ax \leftarrow 1 \\
&\qquad\quad Bx \leftarrow a_0 \cdot y + a_2 \\
&\qquad\quad Cx \leftarrow a_1 \cdot y^2 + a_3 \cdot y + a_4 \\
&\qquad\quad Ay \leftarrow a_1 \\
&\qquad\quad By \leftarrow a_0 \cdot x + a_3 \\
&\qquad\quad Cy \leftarrow x^2 + a_2 \cdot x + a_4 \\
&\qquad\quad x1 \leftarrow \frac{-Bx + \sqrt{Bx^2 - 4 \cdot Ax \cdot Cx}}{2\ Ax} \\
&\qquad\quad x2 \leftarrow \frac{-Bx - \sqrt{Bx^2 - 4 \cdot Ax \cdot Cx}}{2\ Ax} \\
&\qquad\quad y1 \leftarrow \frac{-By + \sqrt{By^2 - 4 \cdot Ay \cdot Cy}}{2\ Ay} \\
&\qquad\quad y2 \leftarrow \frac{-By - \sqrt{By^2 - 4 \cdot Ay \cdot Cy}}{2\ Ay} \\
&\qquad\quad \text{if } \left(\operatorname{Im}(x1) = 0 \wedge \operatorname{Im}(x2) = 0\right) \\
&\qquad\qquad \text{if } \left(x1 < x \wedge x2 < x \vee x1 > x \wedge x2 > x\right)
\end{aligned}
$$

```
        else
            cont ← cont
    else
        if (Im(y1) = 0 ∧ Im(y2) = 0)
            if (y1 < y ∧ y2 < y ∨ y1 > y ∧ y2 > y)
                cont ← cont + 1
            else
                cont ← cont
        else
            cont ← cont
cont
```

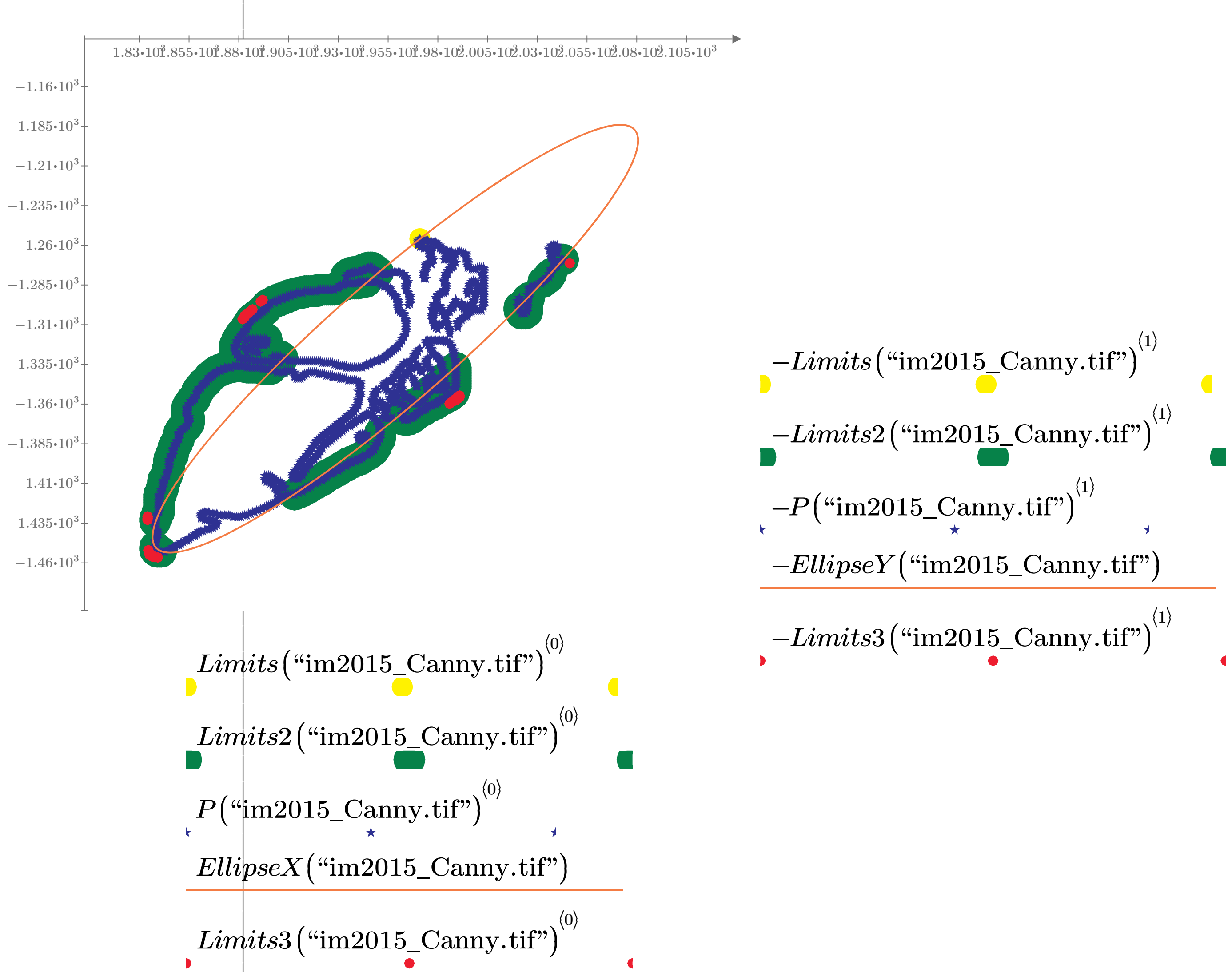
```
Ellipse1(im) :=
    Limits ← Limits(im)
    X ← Limits^⟨0⟩
    Y ← Limits^⟨1⟩
    for i ∈ 0 .. rows(Limits) − 1
        L_i ← −X_i^2
    for i ∈ 0 .. rows(Limits) − 1
        [A_{i,0}  A_{i,1}  A_{i,2}  A_{i,3}  A_{i,4}] ← [X_i · Y_i   Y_i^2   X_i   Y_i   1]
    a ← (A^T · A)^−1 · (A^T · L)
```

```
            [Out_{cont,0}  Out_{cont,1}] ← [P_{i,0}  P_{i,1}]
            cont ← cont + 1
        else
            cont ← cont
    else
        if (Im(y1) = 0 ∧ Im(y2) = 0)
            if (y1 < y ∧ y2 < y ∨ y1 > y ∧ y2 > y)
                [Out_{cont,0}  Out_{cont,1}] ← [P_{i,0}  P_{i,1}]
                cont ← cont + 1
            else
                cont ← cont
        else
            cont ← cont
Out
```

$$
\begin{aligned}
teste :=\ & Limits2 \leftarrow Limits2(image)\\
& a \leftarrow Ellipse1(image)\\
& Dist0 \leftarrow 141215\\
& \text{for } h \in 0 .. \text{rows}(Limits2)-1\\
& \quad X \leftarrow Limits2_{h,0}\\
& \quad Y \leftarrow Limits2_{h,1}\\
& \quad k0 \leftarrow 0\\
& \quad \text{for } i \in 0 .. 100\\
& \quad\quad k1 \leftarrow k0\\
& \quad\quad k0 \leftarrow k0 - \frac{f(k0,a,X,Y)}{Df(k0,a,X,Y)}\\
& \quad\quad \text{if } \left|f(k0,a,X,Y)\right| > \left|f(k1,a,X,Y)\right| \vee \text{Im}(k0) \neq 0\\
& \quad\quad\quad k0 \leftarrow k1\\
& \quad\quad\quad \text{break}\\
& \quad k \leftarrow k0\\
& \quad x \leftarrow \frac{(X-k\cdot a_2)\cdot(k\cdot 2\cdot a_1+1)+(-(k\cdot a_0))\cdot(Y-k\cdot a_3)}{(k\cdot 2\cdot a_1+1)\cdot(k\cdot 2+1)-(k\cdot a_0)^2}\\
& \quad y \leftarrow \frac{(k\cdot a_2-X)\cdot(k\cdot a_0)+(k\cdot 2+1)\cdot(Y-k\cdot a_3)}{(k\cdot 2\cdot a_1+1)\cdot(k\cdot 2+1)-(k\cdot a_0)^2}\\
& \quad Dist \leftarrow \sqrt{(X-x)^2+(Y-y)^2}\\
& \quad \text{if } (X-x \geq 0 \wedge Y-y \geq 0) \vee (X-x \leq 0 \wedge Y-y \geq 0)\\
& \quad\quad \theta \leftarrow \text{acos}\left(\frac{X\cdot x+Y\cdot y}{\sqrt{(x^2+y^2)\cdot(X^2+Y^2)}}\right)\cdot\frac{180}{\pi}\\
& \quad \text{else}\\
& \quad\quad \theta \leftarrow 360-\text{acos}\left(\frac{X\cdot x+Y\cdot y}{\sqrt{(x^2+y^2)\cdot(X^2+Y^2)}}\right)\cdot\frac{180}{\pi}\\
& \quad \begin{bmatrix} Out_{h,0} & Out_{h,1} \end{bmatrix} \leftarrow \begin{bmatrix} X & Y \end{bmatrix}\\
& Out
\end{aligned}
$$

$$
\begin{aligned}
belong(x,y) :=\ & \text{for } i \in 0 .. \text{rows}(y)-1\\
& \quad \text{if } y_i - x = 0\\
& \quad\quad Out \leftarrow 1\\
& \quad\quad \text{break}\\
& \quad \text{else}\\
& \quad\quad Out \leftarrow 0\\
& Out
\end{aligned}
$$

```
Limits3(im) := | degree ← 5
               | Limits ← Limits(im)
               | Limits2 ← Limits2(im)
               | a ← Ellipse1(im)
               | Dist0 ← 141215
               | for h ∈ 0 .. rows(Limits2) − 1
               |   | X ← Limits2_{h,0}
               |   | Y ← Limits2_{h,1}
               |   | k0 ← 0
               |   | contou ← 0
               |   | for i ∈ 0 .. 100
               |   |   | k1 ← k0
               |   |   | k0 ← k0 − f(k0, a, X, Y) / Df(k0, a, X, Y)
               |   |   | contou ← contou + 1
               |   |   | if |f(k0, a, X, Y)| − |f(k1, a, X, Y)| > 10^{−1} ∨ Im(k0) ≠ 0
               |   |   |   | k0 ← k1
               |   |   |   | contou ← contou − 1
               |   |   |   | break
               |   | conto_h ← contou · 0 + k0
               |   | k ← k0
               |   | x ← ((X − k·a_2)·(k·2·a_1 + 1) + (−(k·a_0))·(Y − k·a_3)) / ((k·2·a_1 + 1)·(k·2 + 1) − (k·a_0)^2)
               |   | y ← ((k·a_2 − X)·(k·a_0) + (k·2 + 1)·(Y − k·a_3)) / ((k·2·a_1 + 1)·(k·2 + 1) − (k·a_0)^2)
               |   | Dist ← sqrt((X − x)^2 + (Y − y)^2)
               |   | if (X − x ≥ 0 ∧ Y − y ≥ 0) ∨ (X − x ≤ 0 ∧ Y − y ≥ 0)
               |   |   | θ ← acos( ((X − x)·1 + (Y − y)·0) / sqrt(((X − x)^2 + (Y − y)^2)·(1^2 + 0^2)) ) · 180/π
               |   | else
               |   |   | θ ← 360 − acos( ((X − x)·1 + (Y − y)·0) / sqrt(((X − x)^2 + (Y − y)^2)·(1^2 + 0^2)) ) · 180/π
               |   | [Out_{h,0}  Out_{h,1}  Out_{h,2}  Out_{h,3}] ← [X  Y  Dist  θ]
               | Out ← csort(Out, 3)
               | for ii ∈ 0 .. rows(Out) − 1
```



$\text{if } \left(belong\left(Out_{ii,0}, Limits^{\langle 0\rangle}\right) \wedge belong\left(Out_{ii,1}, Limits^{\langle 1\rangle}\right)\right) = 1$

$SetMaxMin_{contei,0} \leftarrow Out_{ii,0}$

$SetMaxMin_{contei,1} \leftarrow Out_{ii,1}$

$SetMaxMin_{contei,2} \leftarrow Out_{ii,2}$

$SetMaxMin_{contei,3} \leftarrow Out_{ii,3}$

$contei \leftarrow contei + 1$

$contei \leftarrow 0$

$\text{for } ii \in 0 .. \text{rows}(Out) - 1$

$\text{if } \left(belong\left(Out_{ii,0}, Limits^{\langle 0\rangle}\right) \wedge belong\left(Out_{ii,1}, Limits^{\langle 1\rangle}\right)\right) = 0$

$SetExtra_{contei,0} \leftarrow Out_{ii,0}$

$SetExtra_{contei,1} \leftarrow Out_{ii,1}$

$SetExtra_{contei,2} \leftarrow Out_{ii,2}$

$SetExtra_{contei,3} \leftarrow Out_{ii,3}$

$contei \leftarrow contei + 1$

$Sai \leftarrow \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$

$\text{for } mm \in 0 .. 3$

$c1 \leftarrow 0$

$\text{for } m \in 0 .. \text{rows}(SetExtra) - 1$

$\text{if } (mm) \cdot 90 \le SetExtra_{m,3} < (mm+1) \cdot 90$

$\begin{bmatrix} g1_{c1,0} & g1_{c1,1} & g1_{c1,2} & g1_{c1,3} \end{bmatrix} \leftarrow \begin{bmatrix} SetExtra_{m,0} & SetExtra_{m,1} & SetExtra_{m,2} & SetExtra_{m,3} \end{bmatrix}$

$c1 \leftarrow c1 + 1$

$\text{if } c1 > 0$

$g1 \leftarrow \text{csort}(g1, 2)$

$g1 \leftarrow \text{submatrix}(g1, \text{rows}(g1) - 1 - min(8, \text{rows}(g1) - 1), \text{rows}(g1) - 1, 0, 3)$

$Sai \leftarrow \text{stack}(Sai, g1)$

$Sai \leftarrow \text{stack}(Sai, SetMaxMin)$

$Out3 \leftarrow \text{submatrix}(Sai, 1, \text{rows}(Sai) - 1, 0, 3)$

$Out3 \leftarrow \text{csort}(Out3, 3)$

$maxDist \leftarrow \max\left(Out3^{\langle 2\rangle}\right)$

$\text{for } pp \in 0 .. 100$

$\text{for } n \in 0 .. \text{rows}(Out3) - 1$

$Out4_{n,0} \leftarrow Out3_{n,0} + pp \cdot 1 \cdot \cos\left(Out3_{n,3} \cdot \frac{\pi}{180}\right)$

$Out4_{n,1} \leftarrow Out3_{n,1} + pp \cdot 1 \cdot \sin\left(Out3_{n,3} \cdot \frac{\pi}{180}\right)$

try

$Limits \leftarrow Out4$

$X \leftarrow Limits^{\langle 0\rangle}$

$$belong\left(522, Limits(\text{“im2015\_Canny.tif”})^{\langle 0\rangle}\right) = 0$$

$X \leftarrow Limits$

$Y \leftarrow Limits^{\langle 1 \rangle}$

for $i \in 0 .. \text{rows}(Limits) - 1$

$\quad L_i \leftarrow -X_i^{\,2}$

for $i \in 0 .. \text{rows}(Limits) - 1$

$\quad \begin{bmatrix} A_{i,0} & A_{i,1} & A_{i,2} & A_{i,3} & A_{i,4} \end{bmatrix} \leftarrow \begin{bmatrix} X_i \cdot Y_i & Y_i^{\,2} & X_i & Y_i & 1 \end{bmatrix}$

$El \leftarrow \left(A^{\mathrm{T}} \cdot A\right)^{-1} \cdot \left(A^{\mathrm{T}} \cdot L\right)$

$\begin{bmatrix} A & B & C & D & F & G \end{bmatrix} \leftarrow \begin{bmatrix} 1 & \dfrac{El_0}{2} & El_1 & \dfrac{El_2}{2} & \dfrac{El_3}{2} & El_4 \end{bmatrix}$

if $B = 0 \wedge A < C$

$\quad \theta 2 \leftarrow 0$

else

$\quad$ if $B = 0 \wedge A > C$

$\quad\quad \theta 2 \leftarrow 0$

$\quad$ else

$\quad\quad$ if $B \neq 0 \wedge A < C$

$\quad\quad\quad \theta 2 \leftarrow \dfrac{1}{2} \cdot \text{atan}\left(\dfrac{2 \cdot B}{A - C}\right)$

$\quad\quad$ else

$\quad\quad\quad$ if $B \neq 0 \wedge A > C$

$\quad\quad\quad\quad \theta 2 \leftarrow \dfrac{\pi}{2} + \dfrac{1}{2} \cdot \text{atan}\left(\dfrac{2 \cdot B}{A - C}\right)$

$x0 \leftarrow \dfrac{C \cdot D - B \cdot F}{B^2 - A \cdot C}$

$y0 \leftarrow \dfrac{A \cdot F - B \cdot D}{B^2 - A \cdot C}$

$a \leftarrow \sqrt{2 \cdot \dfrac{\left(A \cdot F^2 + C \cdot D^2 + G \cdot B^2 - 2 \cdot B \cdot D \cdot F - A \cdot C \cdot G\right)}{\left(B^2 - A \cdot C\right) \cdot \left(\sqrt{(A - C)^2 + 4 \cdot B^2} - (A + C)\right)}}$

$b \leftarrow \sqrt{2 \cdot \dfrac{\left(A \cdot F^2 + C \cdot D^2 + G \cdot B^2 - 2 \cdot B \cdot D \cdot F - A \cdot C \cdot G\right)}{\left(B^2 - A \cdot C\right) \cdot \left(-\sqrt{(A - C)^2 + 4 \cdot B^2} - (A + C)\right)}}$

$El \leftarrow \begin{bmatrix} \theta 2 \\ x0 \\ y0 \\ a \\ b \end{bmatrix}$

if $\text{Im}(\theta 2) = \text{Im}(x0) = \text{Im}(y0) = \text{Im}(a) = \text{Im}(b) = 0$

$\quad Ok \leftarrow pp$

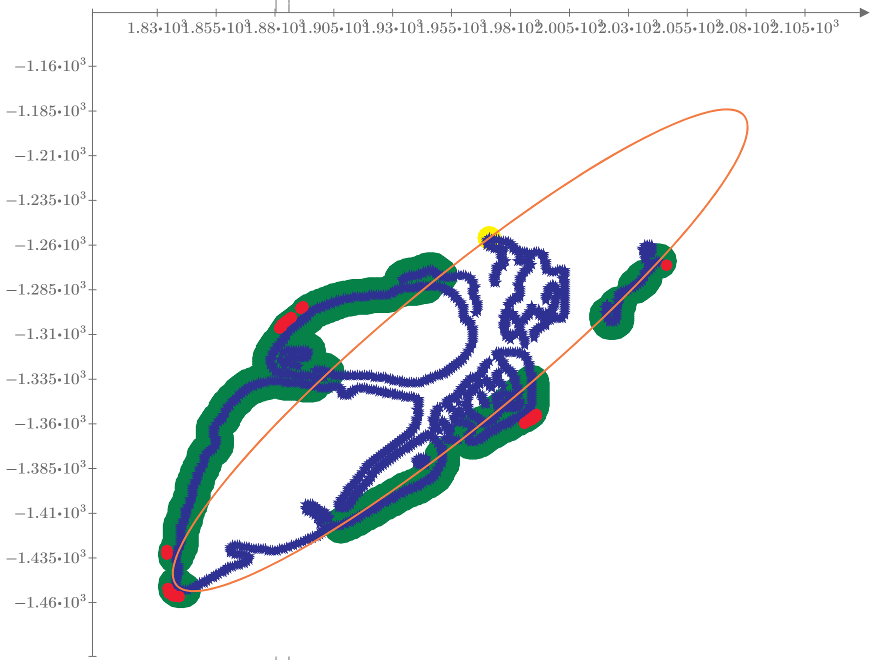
$\quad$ if $TestaPontos(Out4, im) = 0$

$\quad\quad Out4 \leftarrow Out4$

$\quad\quad$ break

else

$TestaPontos(Limits2(\text{“im2015\_Canny.tif”}), \text{“im2015\_Canny.tif”}) = 352$



$-Limits(\text{“im2015\_Canny.tif”})^{\langle 1 \rangle}$

$-Limits2(\text{“im2015\_Canny.tif”})^{\langle 1 \rangle}$

$-P(\text{“im2015\_Canny.tif”})^{\langle 1 \rangle}$

$-EllipseY(\text{“im2015\_Canny.tif”})$

$-Limits3(\text{“im2015\_Canny.tif”})^{\langle 1 \rangle}$

$Limits(\text{“im2015\_Canny.tif”})^{\langle 0 \rangle}$

$Limits2(\text{“im2015\_Canny.tif”})^{\langle 0 \rangle}$

$P(\text{“im2015\_Canny.tif”})^{\langle 0 \rangle}$

$EllipseX(\text{“im2015\_Canny.tif”})$

$Limits3(\text{“im2015\_Canny.tif”})^{\langle 0 \rangle}$

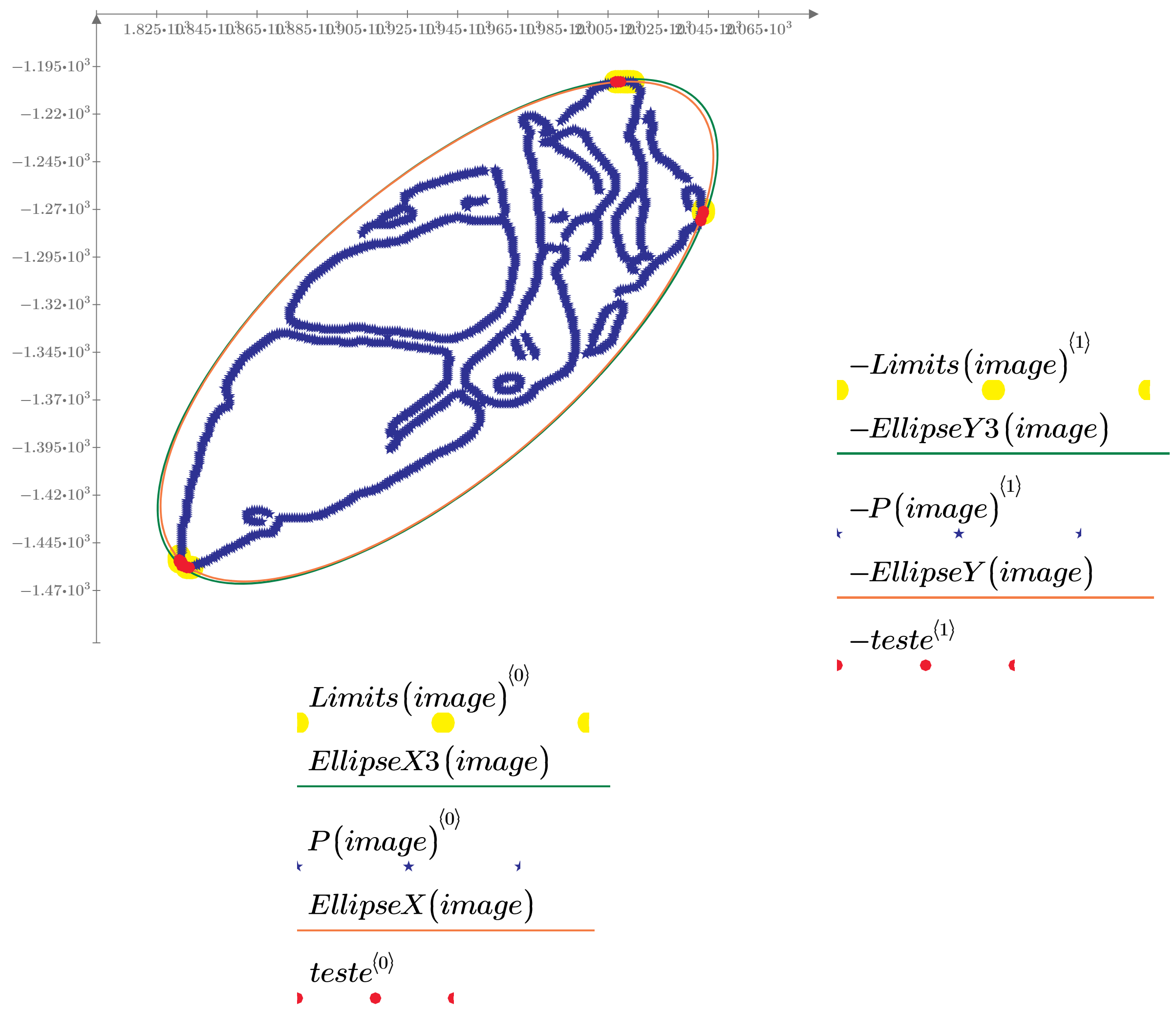
```
else
    contasaida ← contasaida + 1
    if contasaida > 5
        Out4 ← Out3
        break
    else
        continue
on error
    continue

Out4
```

$$QuaisPontos\left(Limits2\left(\text{"im2015\_Canny.tif"}\right), \text{"im2015\_Canny.tif"}\right) = \begin{bmatrix} 2.037 \cdot 10^3 & 1.26 \cdot 10^3 \\ 2.038 \cdot 10^3 & 1.26 \cdot 10^3 \\ 2.039 \cdot 10^3 & 1.26 \cdot 10^3 \\ 2.04 \cdot 10^3 & 1.26 \cdot 10^3 \\ 2.037 \cdot 10^3 & 1.261 \cdot 10^3 \\ 2.04 \cdot 10^3 & 1.261 \cdot 10^3 \\ 2.036 \cdot 10^3 & 1.262 \cdot 10^3 \\ 2.04 \cdot 10^3 & 1.262 \cdot 10^3 \\ 2.036 \cdot 10^3 & 1.263 \cdot 10^3 \\ 2.04 \cdot 10^3 & 1.263 \cdot 10^3 \\ 2.037 \cdot 10^3 & 1.264 \cdot 10^3 \\ 2.04 \cdot 10^3 & 1.264 \cdot 10^3 \\ & \vdots \end{bmatrix}$$

```
EllipseParam3(im) := ‖ Limits ← Limits3(im)
                     ‖ X ← Limits^⟨0⟩
                     ‖ Y ← Limits^⟨1⟩
                     ‖ for i ∈ 0 .. rows(Limits) − 1
                     ‖   ‖ L_i ← −X_i^2
                     ‖ for i ∈ 0 .. rows(Limits) − 1
                     ‖   ‖ [A_{i,0} A_{i,1} A_{i,2} A_{i,3} A_{i,4}] ← [X_i·Y_i  Y_i^2  X_i  Y_i  1]
                     ‖ El ← (A^T·A)^−1·(A^T·L)
                     ‖ [A B C D F G] ← [1  El_0/2  El_1  El_2/2  El_3/2  El_4]
                     ‖ if B = 0 ∧ A < C
                     ‖   ‖ θ2 ← 0
                     ‖ else
                     ‖   ‖ if B = 0 ∧ A > C
                     ‖   ‖   ‖ θ2 ← 0
                     ‖   ‖ else
                     ‖   ‖   ‖ if B ≠ 0 ∧ A < C
                     ‖   ‖   ‖   ‖ θ2 ← (1/2)·atan(2·B/(A−C))
                     ‖   ‖   ‖ else
                     ‖   ‖   ‖   ‖ if B ≠ 0 ∧ A > C
                     ‖   ‖   ‖   ‖   ‖ θ2 ← π/2 + (1/2)·atan(2·B/(A−C))
                     ‖ if θ2 < 0
                     ‖   ‖ θ2 ← θ2 + 2·π
                     ‖ x0 ← (C·D − B·F)/(B^2 − A·C)
                     ‖ y0 ← (A·F − B·D)/(B^2 − A·C)
                     ‖ a ← sqrt(2·(A·F^2 + C·D^2 + G·B^2 − 2·B·D·F − A·C·G)/((B^2 − A·C)·(sqrt((A−C)^2 + 4·B^2) − (A+C))))
                     ‖ b ← sqrt(2·(A·F^2 + C·D^2 + G·B^2 − 2·B·D·F − A·C·G)/((B^2 − A·C)·(−sqrt((A−C)^2 + 4·B^2) − (A+C))))
                     ‖ [θ2; x0; y0; a; b]
```

```
EllipseX3(im) := ‖ EP ← EllipseParam3(im)
                 ‖ [θ x0 y0 a b] ← [EP_0 EP_1 EP_2 EP_3 EP_4]
                 ‖ for t ∈ 0 .. 360
                 ‖   ‖ Out_t ← a·cos(t·π/180)·cos(θ) + b·sin(t·π/180)·−sin(θ) + x0
                 ‖ Out
```

```
EllipseY3(im) := ‖ EP ← EllipseParam3(im)
                 ‖ [θ x0 y0 a b] ← [EP_0 EP_1 EP_2 EP_3 EP_4]
                 ‖ for t ∈ 0 .. 360
                 ‖   ‖ Out_t ← a·cos(t·π/180)·(sin(θ)) + b·sin(t·π/180)·cos(θ) + y0
                 ‖ Out
```



$$\lfloor b \rfloor$$

$$ImageEllipse(im) := \left\| \begin{array}{l} EP \leftarrow EllipseParam3(im) \\ [\theta\ \ x0\ \ y0\ \ a\ \ b] \leftarrow [EP_0\ \ EP_1\ \ EP_2\ \ EP_3\ \ EP_4] \\ \text{for } t \in 0..3600 \\ \quad \left\| \begin{array}{l} Out_{t,0} \leftarrow \text{trunc}\left(a \cdot \cos\left(t \cdot \frac{\pi}{1800}\right) \cdot \cos(\theta) + b \cdot \sin\left(t \cdot \frac{\pi}{1800}\right) \cdot -\sin(\theta) + x0\right) \\ Out_{t,1} \leftarrow \text{trunc}\left(a \cdot \cos\left(t \cdot \frac{\pi}{1800}\right) \cdot (\sin(\theta)) + b \cdot \sin\left(t \cdot \frac{\pi}{1800}\right) \cdot \cos(\theta) + y0\right) \end{array} \right. \\ \text{for } t \in 0..\text{rows}(Out) - 2 \\ \quad \left\| \begin{array}{l} \text{if } Out_{t,0} = Out_{t+1,0} \wedge Out_{t,1} = Out_{t+1,1} \\ \quad \| \text{continue} \\ \text{else} \\ \quad \left\| \begin{array}{l} Out2_{count,0} \leftarrow Out_{t,0} \\ Out2_{count,1} \leftarrow Out_{t,1} \\ count \leftarrow count + 1 \end{array} \right. \end{array} \right. \\ Out2 \\ im01 \leftarrow im01 \cdot 0 \\ \text{for } i \in 0..\text{rows}(Out2) - 1 \\ \quad \| im01_{Out2_{i,1}, Out2_{i,0}} \leftarrow 255 \\ im01 \end{array} \right.$$

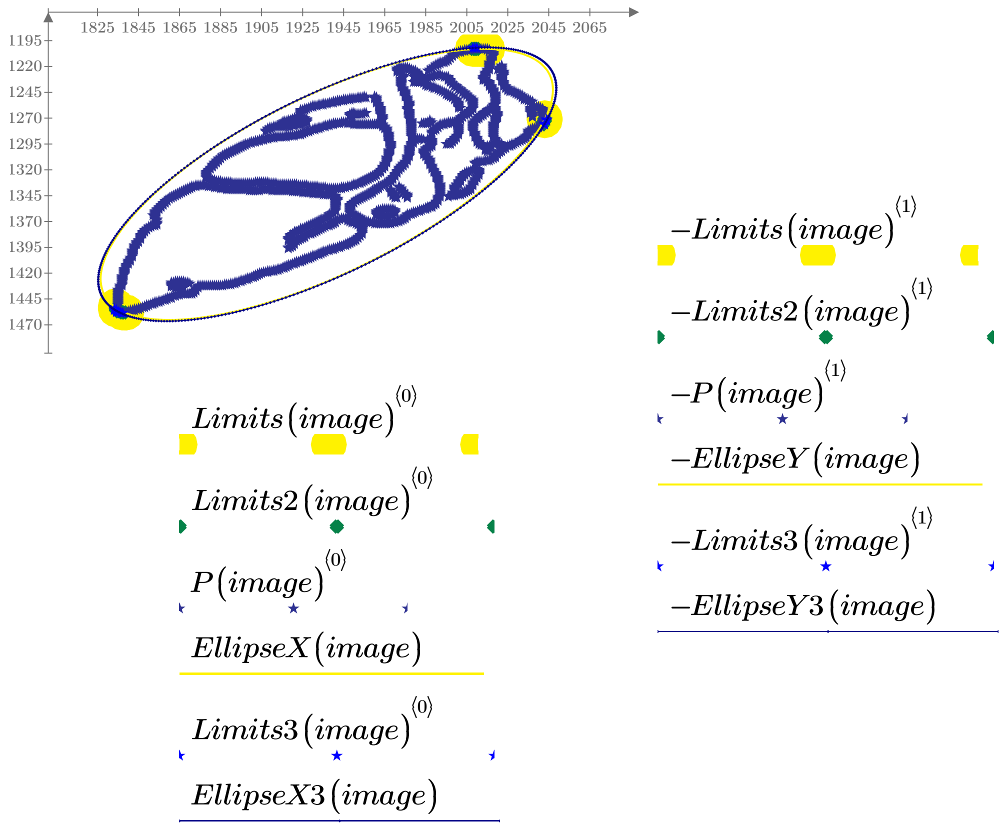
$$PrintEllipses(im1, im2) := \left\| \begin{array}{l} E1 \leftarrow ImageEllipse(im1) \\ ET \leftarrow E1 \\ \text{WRITEBMP}(\text{concat}(\mathbf{CWD}, \text{“\textbackslash”}, \text{substr}(im1, 0, 14), \text{“\_Ellipse.tif”}), ET) \end{array} \right.$$

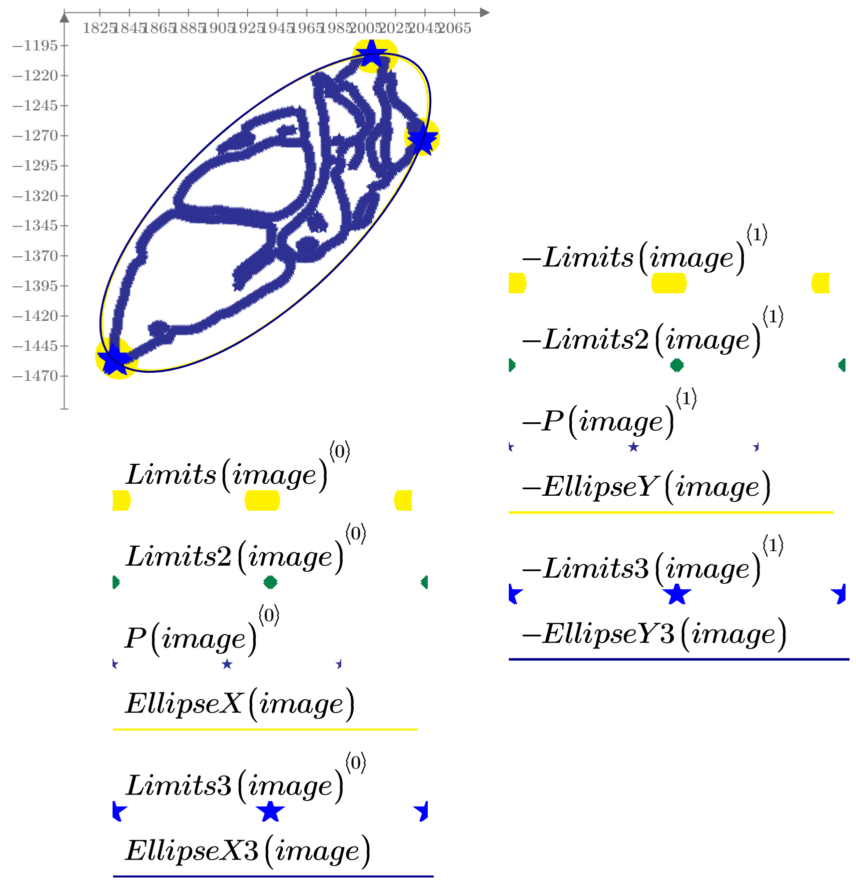
$\mathbf{CWD}$ = “C:\PhD\2015\00 Papers\Paper 2 – Amazon River\04 – Outputs Mathcad\Canny v2”

$$PrintEllipsesTFW(im1, im2) := \| \text{WRITETEXT}(\text{concat}(\mathbf{CWD}, \text{“\textbackslash”}, \text{substr}(im1, 0, 14), \text{“\_Ellipse.tfw”}), \text{READEXCEL}(\text{“HeaderTFW.xlsx”}))$$

$$Results := \begin{bmatrix} PrintEllipses(\text{“im1986\_Canny.tif”}, \text{“im1986\_Canny.tif”}) & PrintEllipsesTFW(\text{“im1986\_Canny.tif”}, \text{“im1986\_Canny.tif”}) \\ PrintEllipses(\text{“im1987\_Canny.tif”}, \text{“im1987\_Canny.tif”}) & PrintEllipsesTFW(\text{“im1987\_Canny.tif”}, \text{“im1987\_Canny.tif”}) \\ PrintEllipses(\text{“im1996\_Canny.tif”}, \text{“im1996\_Canny.tif”}) & PrintEllipsesTFW(\text{“im1996\_Canny.tif”}, \text{“im1996\_Canny.tif”}) \\ PrintEllipses(\text{“im2000\_Canny.tif”}, \text{“im2000\_Canny.tif”}) & PrintEllipsesTFW(\text{“im2000\_Canny.tif”}, \text{“im2000\_Canny.tif”}) \\ PrintEllipses(\text{“im2006\_Canny.tif”}, \text{“im2006\_Canny.tif”}) & PrintEllipsesTFW(\text{“im2006\_Canny.tif”}, \text{“im2006\_Canny.tif”}) \\ PrintEllipses(\text{“im2008\_Canny.tif”}, \text{“im2008\_Canny.tif”}) & PrintEllipsesTFW(\text{“im2008\_Canny.tif”}, \text{“im2008\_Canny.tif”}) \\ PrintEllipses(\text{“im2013\_Canny.tif”}, \text{“im2013\_Canny.tif”}) & PrintEllipsesTFW(\text{“im2013\_Canny.tif”}, \text{“im2013\_Canny.tif”}) \\ PrintEllipses(\text{“im2014\_Canny.tif”}, \text{“im2014\_Canny.tif”}) & PrintEllipsesTFW(\text{“im2014\_Canny.tif”}, \text{“im2014\_Canny.tif”}) \\ PrintEllipses(\text{“im2015\_Canny.tif”}, \text{“im2015\_Canny.tif”}) & PrintEllipsesTFW(\text{“im2015\_Canny.tif”}, \text{“im2015\_Canny.tif”}) \end{bmatrix}$$

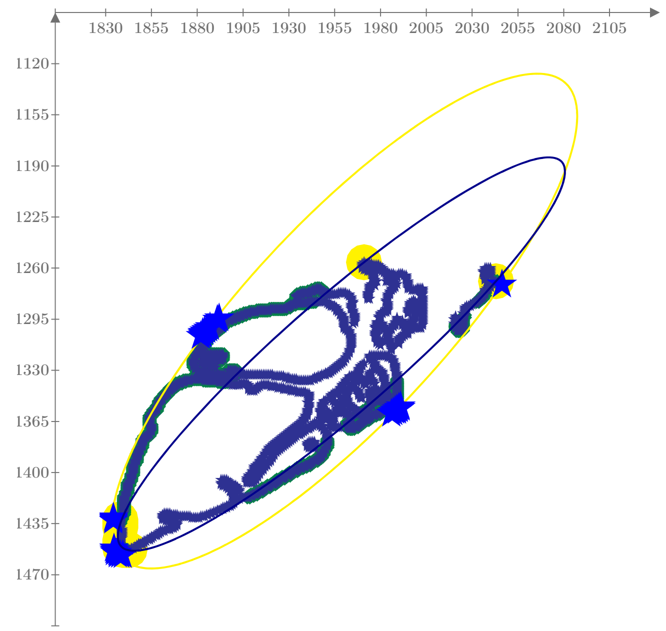
$$image = \text{“im2014\_Canny.tif”}$$





$$imagens := \begin{bmatrix} \text{“im1986\_Canny.tif”} \\ \text{“im1987\_Canny.tif”} \\ \text{“im1996\_Canny.tif”} \\ \text{“im2000\_Canny.tif”} \\ \text{“im2006\_Canny.tif”} \\ \text{“im2008\_Canny.tif”} \\ \text{“im2013\_Canny.tif”} \\ \text{“im2014\_Canny.tif”} \\ \text{“im2015\_Canny.tif”} \end{bmatrix}$$

$indice := 2 \qquad image2 := imagens_{indice} \qquad image2 = \text{“im1996\_Canny.tif”}$



$Limits(\text{“im2015\_Canny.tif”})^{\langle 0 \rangle}$, $-Limits(\text{“im2015\_Canny.tif”})^{\langle 1 \rangle}$

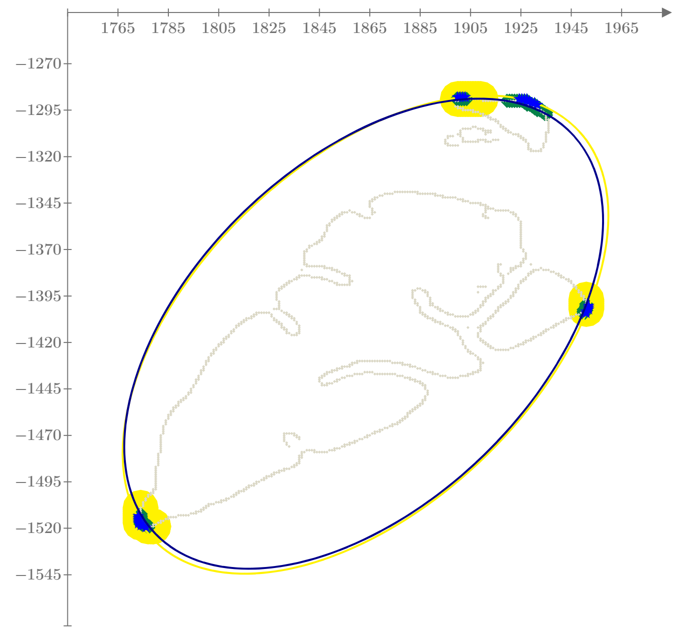
$Limits2(\text{“im2015\_Canny.tif”})^{\langle 0 \rangle}$, $-Limits2(\text{“im2015\_Canny.tif”})^{\langle 1 \rangle}$

$P(\text{“im2015\_Canny.tif”})^{\langle 0 \rangle}$, $-P(\text{“im2015\_Canny.tif”})^{\langle 1 \rangle}$

$EllipseX3(\text{“im2015\_Canny.tif”})$, $-EllipseY3(\text{“im2015\_Canny.tif”})$

$Limits3(\text{“im2015\_Canny.tif”})^{\langle 0 \rangle}$, $-Limits3(\text{“im2015\_Canny.tif”})^{\langle 1 \rangle}$

$EllipseX(\text{“im2015\_Canny.tif”})$, $-EllipseY(\text{“im2015\_Canny.tif”})$



$Limits(image2)^{\langle 0 \rangle}$, $-Limits(image2)^{\langle 1 \rangle}$

$Limits2(image2)^{\langle 0 \rangle}$, $-Limits2(image2)^{\langle 1 \rangle}$

$P(image2)^{\langle 0 \rangle}$, $-P(image2)^{\langle 1 \rangle}$

$EllipseX3(image2)$, $-EllipseY3(image2)$

$Limits3(image2)^{\langle 0 \rangle}$, $-Limits3(image2)^{\langle 1 \rangle}$

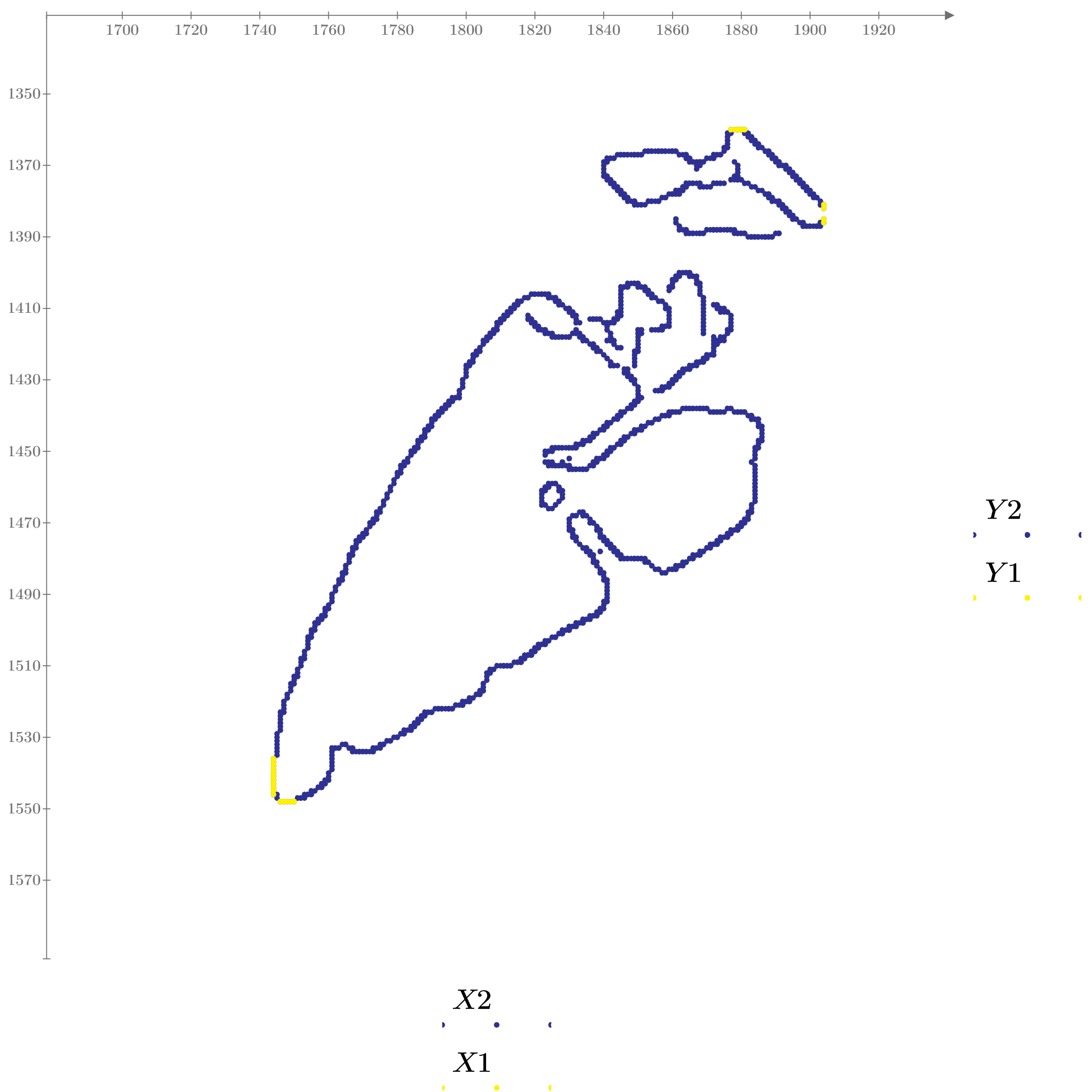
$EllipseX(image2)$, $-EllipseY(image2)$

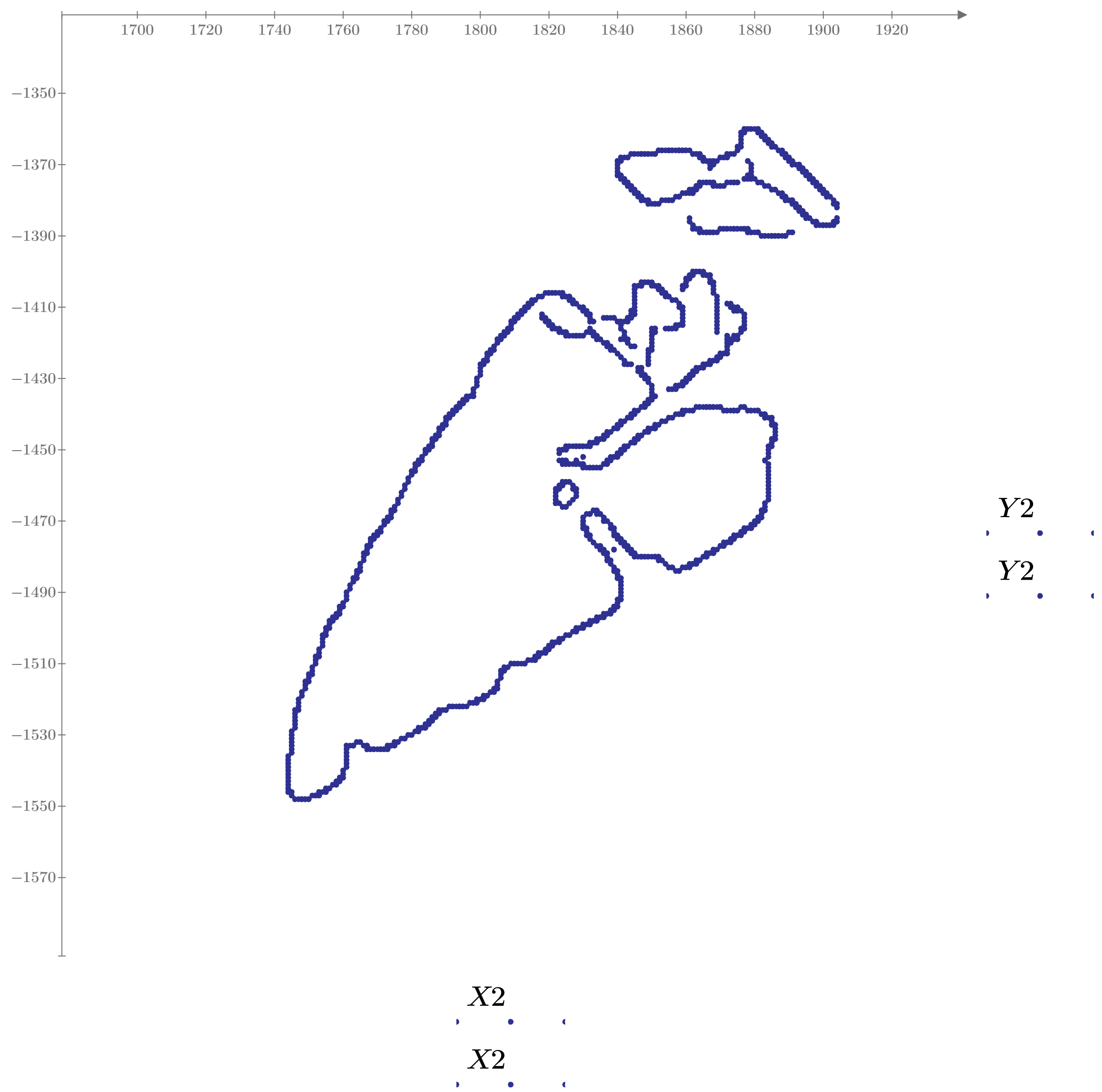
$xmin := 1700$ $\quad xmax := 1920$ $\quad image := \text{"im1986\_Canny.tif"}$ $\quad image = \text{"im1986\_Canny.tif"}$

$ymin := 1350$ $\quad ymax := 1570$

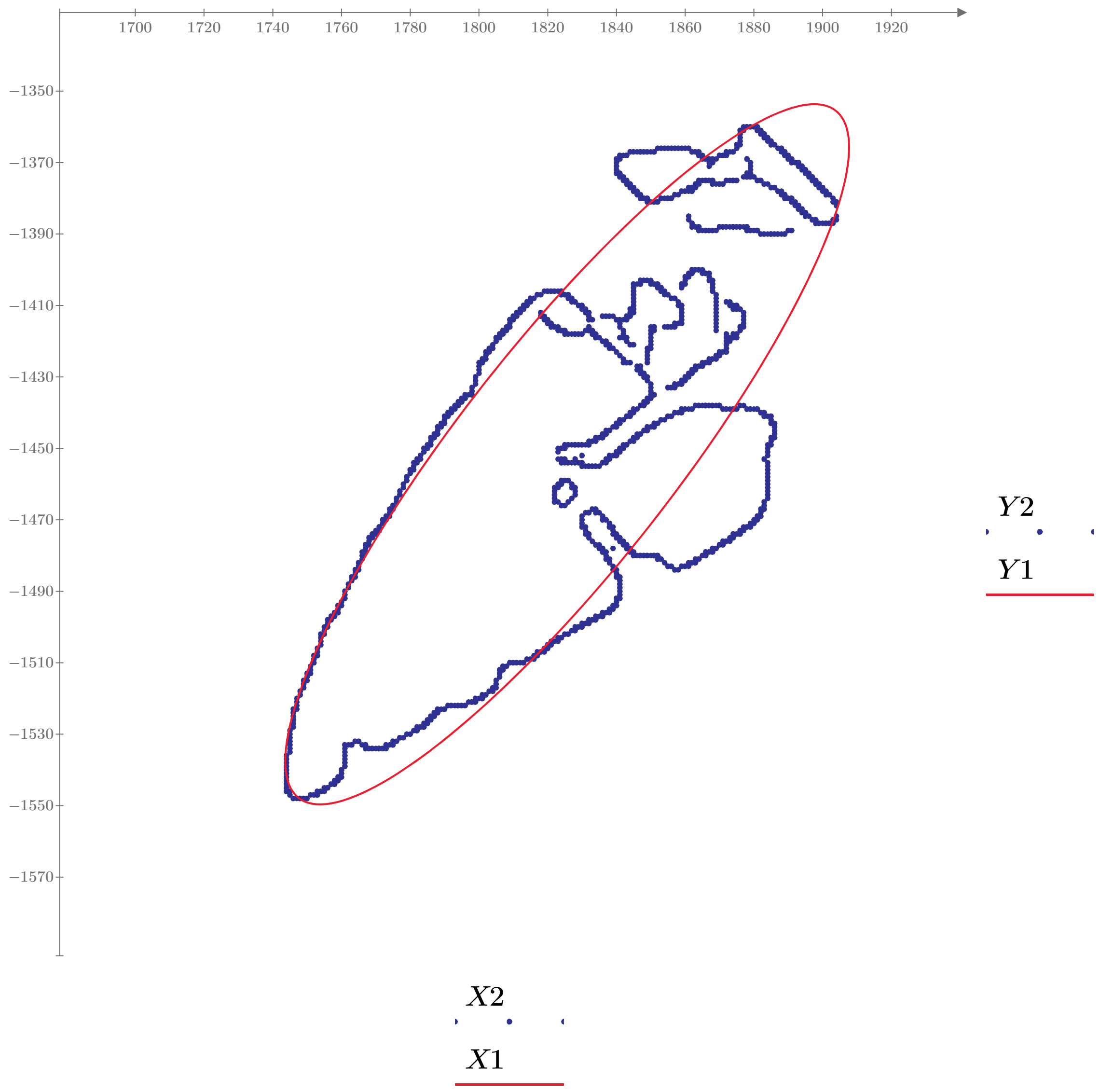
$X1 := Limits(image)^{\langle 0 \rangle}$ $\quad Y1 := -Limits(image)^{\langle 1 \rangle}$

$X2 := P(image)^{\langle 0 \rangle}$ $\quad Y2 := -P(image)^{\langle 1 \rangle}$



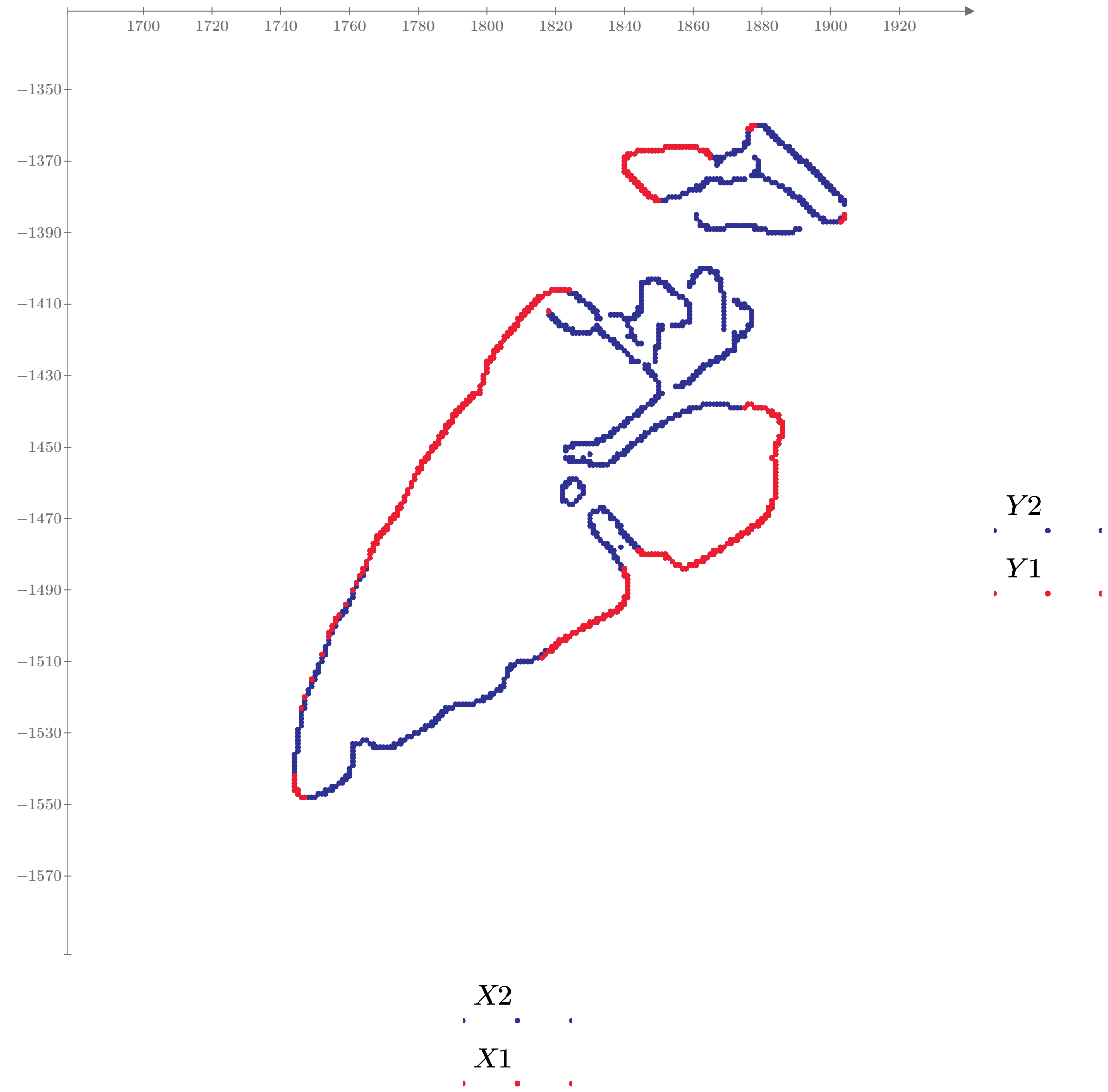


$X1 := EllipseX(image)$ $\qquad$ $Y1 := -EllipseY(image)$

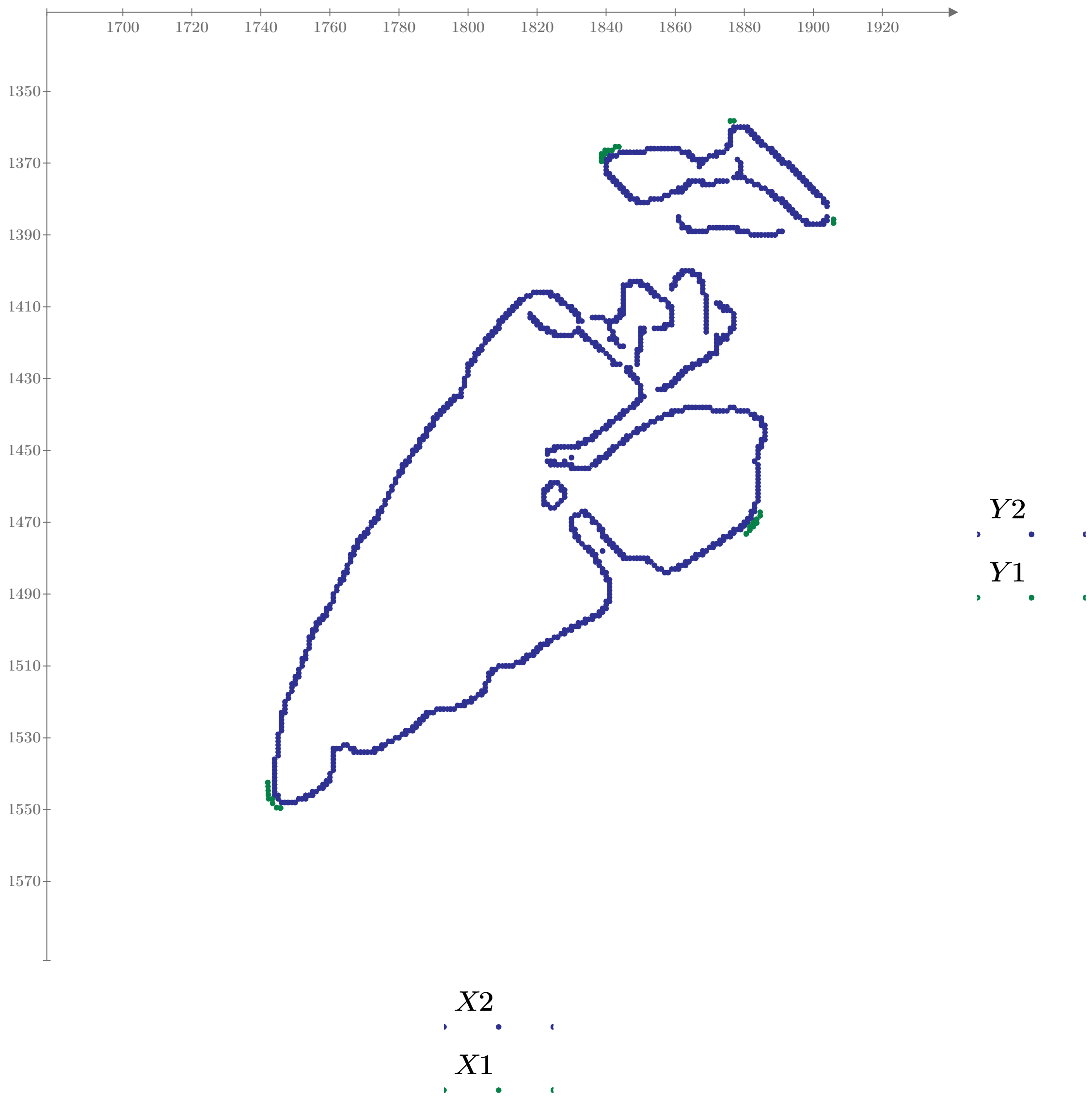


EllipseX

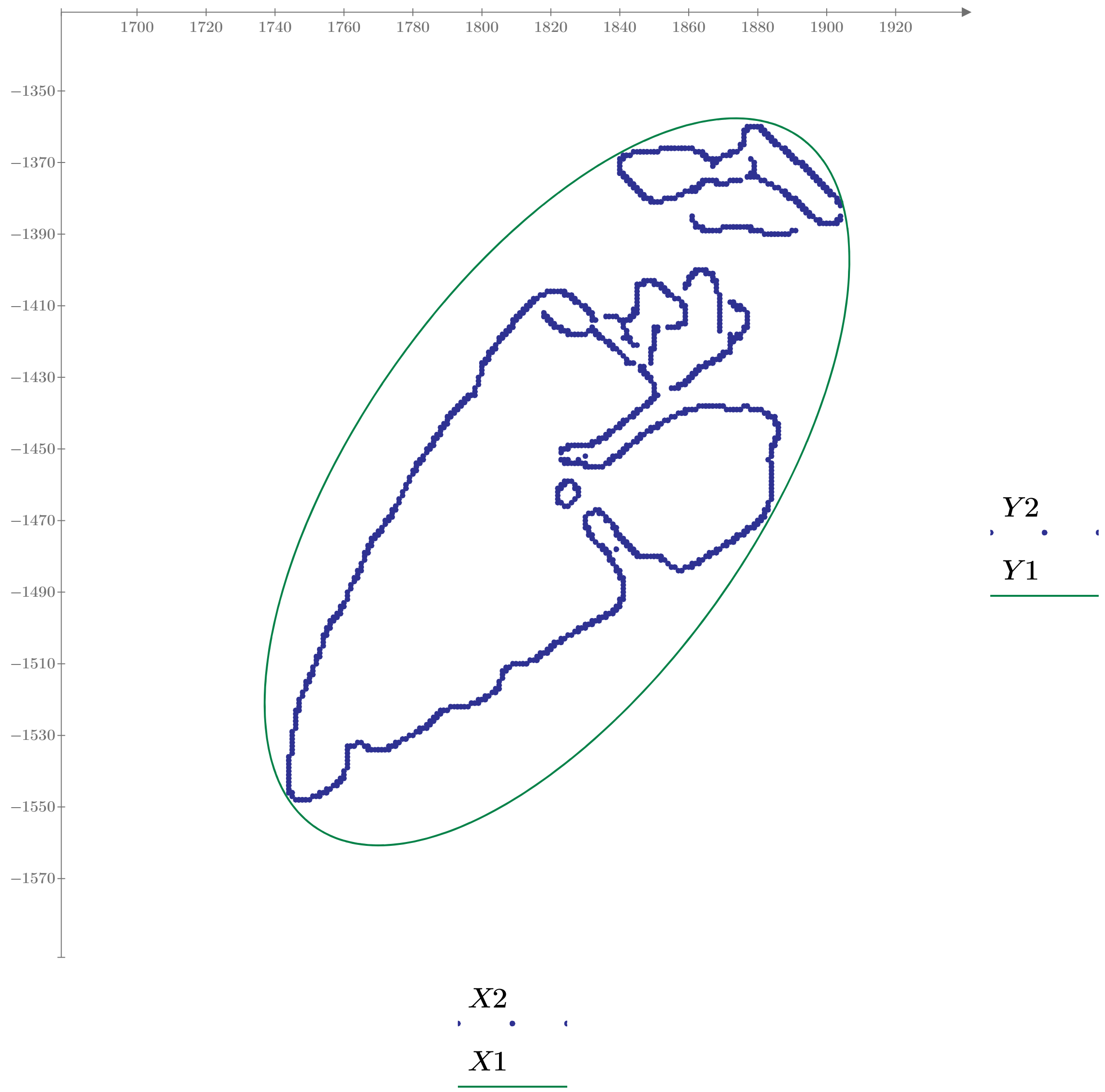
$X1 := Limits2(image)^{\langle 0 \rangle}$ $\qquad$ $Y1 := -Limits2(image)^{\langle 1 \rangle}$

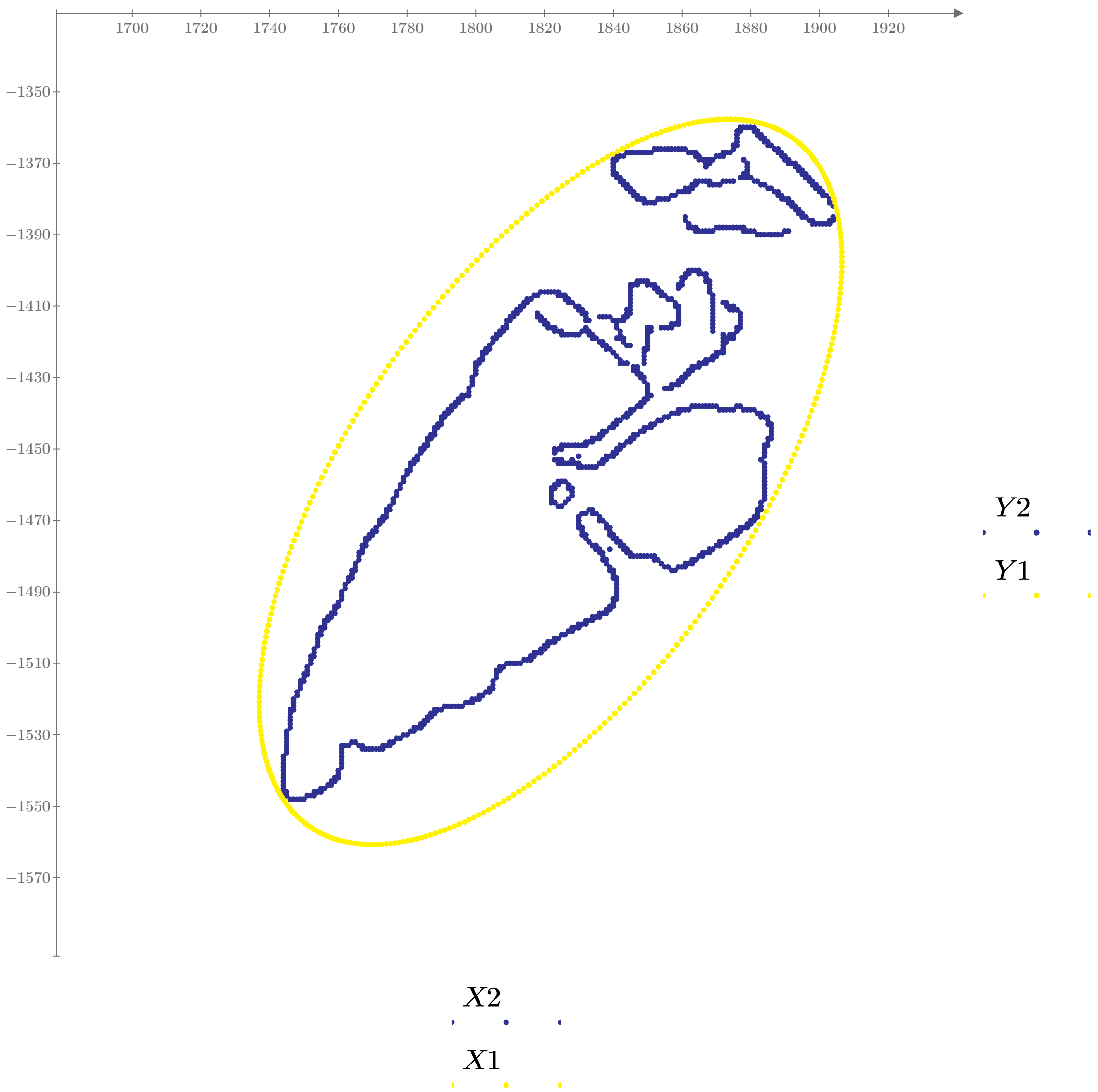


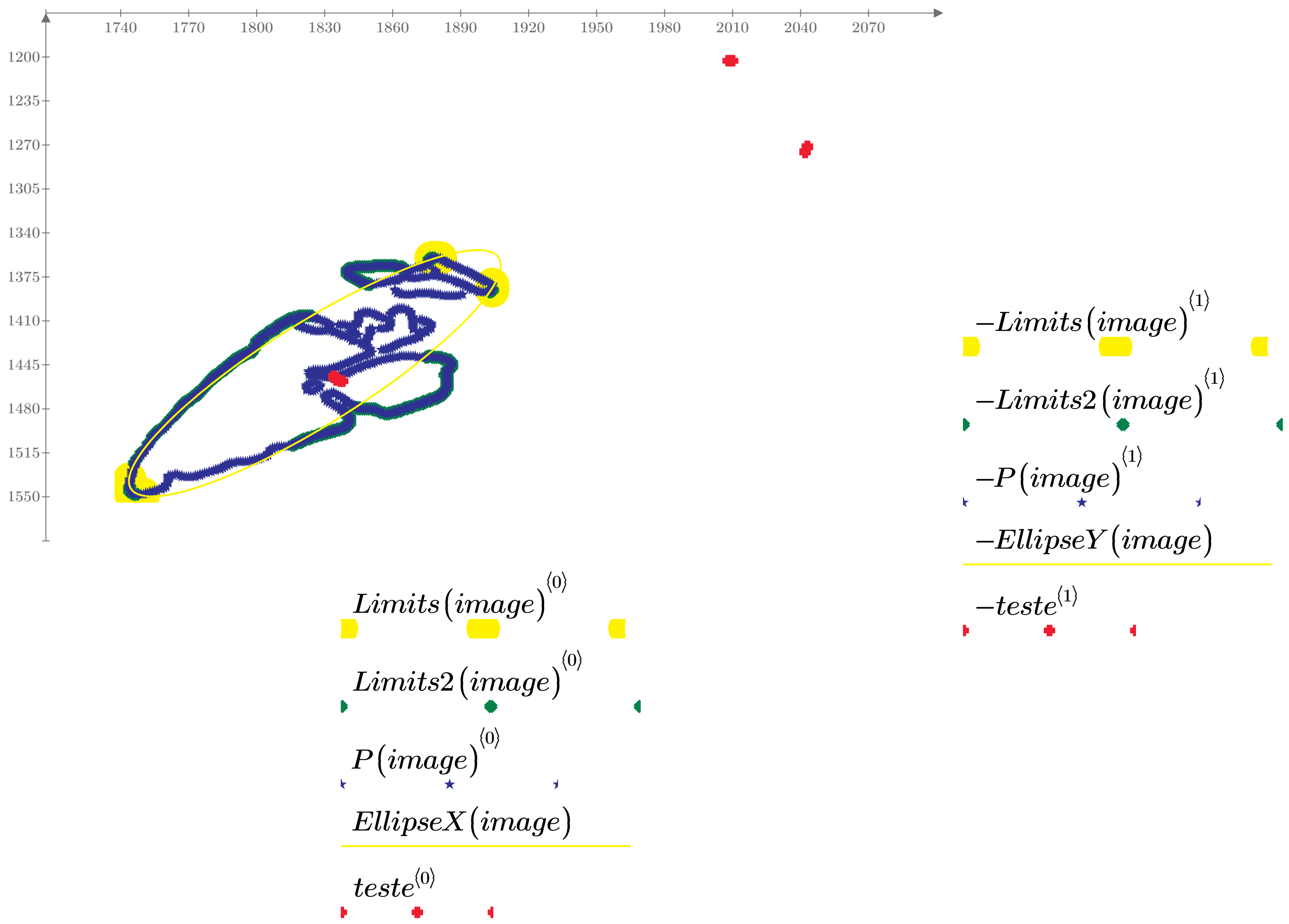
$$X1 := Limits3(image)^{\langle 0 \rangle} \qquad Y1 := -Limits3(image)^{\langle 1 \rangle}$$



$$X1 := EllipseX3(image) \qquad Y1 := -EllipseY3(image)$$




1700
1720
1740
1760
1780
1800
1820
1840
1860
1880
1900
1920
−1350
−1370
−1390
−1410
−1430
−1450
−1470
−1490
−1510
−1530
−1550
−1570
$Y2$
$Y1$
$X2$
$X1$



$$EllipseQuad(im) := \left\| \begin{array}{l} Limits \leftarrow im \\ X \leftarrow Limits^{\langle 0 \rangle} \\ Y \leftarrow Limits^{\langle 1 \rangle} \\ \text{for } i \in 0..\text{rows}(Limits)-1 \\ \quad \left\| L_i \leftarrow -X_i^{\,2} \right. \\ \text{for } i \in 0..\text{rows}(Limits)-1 \\ \quad \left\| \begin{bmatrix} A_{i,0} & A_{i,1} & A_{i,2} & A_{i,3} & A_{i,4} \end{bmatrix} \leftarrow \begin{bmatrix} X_i \cdot Y_i & Y_i^{\,2} & X_i & Y_i & 1 \end{bmatrix} \right. \\ El \leftarrow \left(A^{\mathrm{T}} \cdot A\right)^{-1} \cdot \left(A^{\mathrm{T}} \cdot L\right) \end{array} \right.$$

$$\text{rows}(imagens) = 9$$

$$imagens = \begin{bmatrix} \text{“im1986\_Canny.tif”} \\ \text{“im1987\_Canny.tif”} \\ \text{“im1996\_Canny.tif”} \\ \text{“im2000\_Canny.tif”} \\ \text{“im2006\_Canny.tif”} \\ \text{“im2008\_Canny.tif”} \\ \text{“im2013\_Canny.tif”} \end{bmatrix}$$

$E86 := EllipseParam3\left(imagens_0\right)$ $E87 := EllipseParam3\left(imagens_1\right)$ $E96 := EllipseParam3\left(imagens_2\right)$

$E00 := EllipseParam3\left(imagens_3\right)$ $E06 := EllipseParam3\left(imagens_4\right)$ $E08 := EllipseParam3\left(imagens_5\right)$

"im2014_Canny.tif"
"im2015_Canny.tif"

$E13 := EllipseParam3\left(imagens_6\right)$ $E14 := EllipseParam3\left(imagens_7\right)$ $E15 := EllipseParam3\left(imagens_8\right)$

$$E86 = \begin{bmatrix} 2.212 \\ 1.822 \cdot 10^3 \\ 1.459 \cdot 10^3 \\ 119.286 \\ 57.172 \end{bmatrix} \quad E87 = \begin{bmatrix} 2.297 \\ 1.847 \cdot 10^3 \\ 1.442 \cdot 10^3 \\ 147.136 \\ 66.53 \end{bmatrix} \quad E96 = \begin{bmatrix} 2.091 \\ 1.863 \cdot 10^3 \\ 1.416 \cdot 10^3 \\ 141.72 \\ 75.947 \end{bmatrix} \quad E00 = \begin{bmatrix} 2.196 \\ 1.866 \cdot 10^3 \\ 1.404 \cdot 10^3 \\ 132.216 \\ 70.912 \end{bmatrix} \quad E06 = \begin{bmatrix} 2.174 \\ 1.908 \cdot 10^3 \\ 1.369 \cdot 10^3 \\ 167.476 \\ 77.534 \end{bmatrix} \quad E08 = \begin{bmatrix} 2.064 \\ 1.901 \cdot 10^3 \\ 1.386 \cdot 10^3 \\ 148.585 \\ 88.686 \end{bmatrix}$$

$$E13 = \begin{bmatrix} 5.515 \\ 1.933 \cdot 10^3 \\ 1.363 \cdot 10^3 \\ 148.37 \\ 80.956 \end{bmatrix} \quad E14 = \begin{bmatrix} 2.236 \\ 1.937 \cdot 10^3 \\ 1.334 \cdot 10^3 \\ 160.262 \\ 65.725 \end{bmatrix} \quad E15 = \begin{bmatrix} 2.187 \\ 1.96 \cdot 10^3 \\ 1.296 \cdot 10^3 \\ 203.306 \\ 59.326 \end{bmatrix}$$

$$X0 := \begin{bmatrix} E86_1 \\ E87_1 \\ E96_1 \\ E00_1 \\ E06_1 \\ E08_1 \\ E13_1 \\ E14_1 \\ E15_1 \end{bmatrix} \cdot 30 \quad Time := \begin{bmatrix} 0 \\ 352 \\ 3696 \\ 5168 \\ 7405 \\ 7984 \\ 9936 \\ 10288 \\ 10632 \end{bmatrix} \cdot \frac{1}{365} \quad Y0 := \begin{bmatrix} E86_2 \\ E87_2 \\ E96_2 \\ E00_2 \\ E06_2 \\ E08_2 \\ E13_2 \\ E14_2 \\ E15_2 \end{bmatrix} \cdot 30 \quad \theta 0 := \begin{bmatrix} E86_0 \\ E87_0 \\ E96_0 \\ E00_0 \\ E06_0 \\ E08_0 \\ E13_0 \\ E14_0 \\ E15_0 \end{bmatrix} \cdot \frac{1}{deg} \quad a0 := \begin{bmatrix} E86_3 \\ E87_3 \\ E96_3 \\ E00_3 \\ E06_3 \\ E08_3 \\ E13_3 \\ E14_3 \\ E15_3 \end{bmatrix} \cdot 30 \quad b0 := \begin{bmatrix} E86_4 \\ E87_4 \\ E96_4 \\ E00_4 \\ E06_4 \\ E08_4 \\ E13_4 \\ E14_4 \\ E15_4 \end{bmatrix} \cdot 30$$

$$\theta 0 = \begin{bmatrix} 126.731 \\ 131.61 \\ 119.785 \\ 125.795 \\ 124.57 \\ 118.251 \\ 315.963 \\ 128.124 \\ 125.303 \end{bmatrix} \quad Years := \begin{bmatrix} 0 \\ 352 \\ 3696 \\ 5168 \\ 7405 \\ 7984 \\ 9936 \\ 10288 \\ 10632 \end{bmatrix} \cdot \frac{1}{365} + \begin{bmatrix} 1986 \\ 1986 \\ 1986 \\ 1986 \\ 1986 \\ 1986 \\ 1986 \\ 1986 \\ 1986 \end{bmatrix}$$

$$\Delta X0 := \begin{bmatrix} X0_0 - min(X0) \\ X0_1 - min(X0) \\ X0_2 - min(X0) \\ X0_3 - min(X0) \\ X0_4 - min(X0) \\ X0_5 - min(X0) \\ X0_6 - min(X0) \end{bmatrix} \quad \Delta Y0 := \begin{bmatrix} Y0_0 - min(Y0) \\ Y0_1 - min(Y0) \\ Y0_2 - min(Y0) \\ Y0_3 - min(Y0) \\ Y0_4 - min(Y0) \\ Y0_5 - min(Y0) \\ Y0_6 - min(Y0) \end{bmatrix} \quad \Delta\theta 0 := \begin{bmatrix} \theta 0_0 - min(\theta 0) \\ \theta 0_1 - min(\theta 0) \\ \theta 0_2 - min(\theta 0) \\ \theta 0_3 - min(\theta 0) \\ \theta 0_4 - min(\theta 0) \\ \theta 0_5 - min(\theta 0) \\ \theta 0_6 - min(\theta 0) \end{bmatrix} \quad \Delta a0 := \begin{bmatrix} a0_0 - min(a0) \\ a0_1 - min(a0) \\ a0_2 - min(a0) \\ a0_3 - min(a0) \\ a0_4 - min(a0) \\ a0_5 - min(a0) \\ a0_6 - min(a0) \end{bmatrix} \quad \Delta b0 := \begin{bmatrix} b0_0 - min(b0) \\ b0_1 - min(b0) \\ b0_2 - min(b0) \\ b0_3 - min(b0) \\ b0_4 - min(b0) \\ b0_5 - min(b0) \\ b0_6 - min(b0) \end{bmatrix}$$

$\text{corr}(Time, X0)^2 = 0.936$

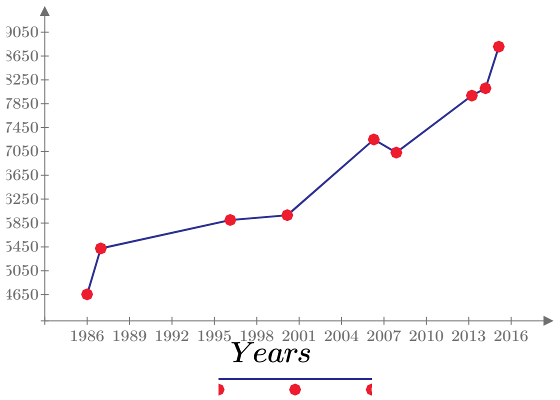
$\text{corr}(Time, Y0)^2 = 0.883$

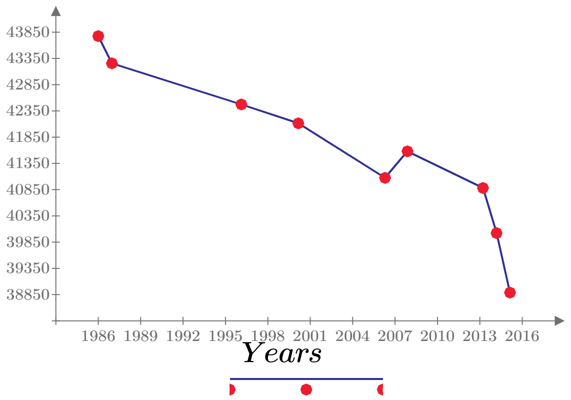
$\text{corr}(Time, \theta 0)^2 = 0.107$ 0.13

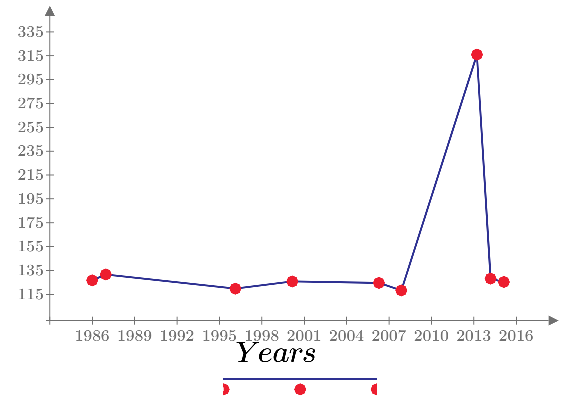
$$\begin{bmatrix} X0_7 - min(X0) \\ X0_8 - min(X0) \end{bmatrix} \quad \begin{bmatrix} Y0_7 - min(Y0) \\ Y0_8 - min(Y0) \end{bmatrix} \quad \begin{bmatrix} \theta0_7 - min(\theta0) \\ \theta0_8 - min(\theta0) \end{bmatrix} \quad \begin{bmatrix} a0_7 - min(a0) \\ a0_8 - min(a0) \end{bmatrix} \quad \begin{bmatrix} b0_7 - min(b0) \\ b0_8 - min(b0) \end{bmatrix}$$

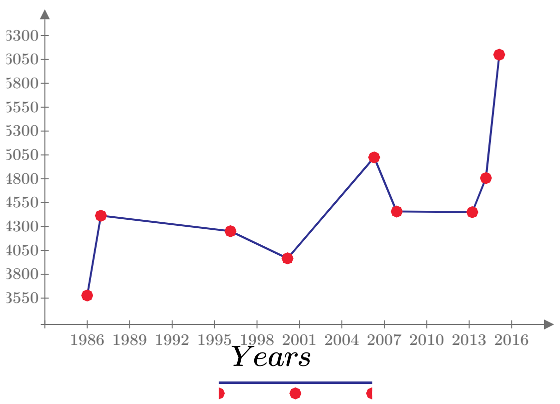
$$\mathrm{corr}(Time, a0)^2 = 0.47$$

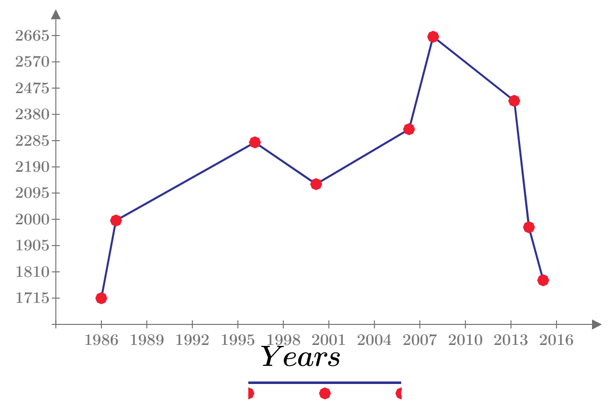
$$\mathrm{corr}(Time, b0)^2 = 0.092$$











$$a0 = \begin{bmatrix} 3.579 \cdot 10^3 \\ 4.414 \cdot 10^3 \\ 4.252 \cdot 10^3 \\ 3.966 \cdot 10^3 \\ 5.024 \cdot 10^3 \\ 4.458 \cdot 10^3 \\ 4.451 \cdot 10^3 \\ 4.808 \cdot 10^3 \\ 6.099 \cdot 10^3 \end{bmatrix}$$

$$Time = \begin{bmatrix} 0 \\ 0.964 \\ 10.126 \\ 14.159 \\ 20.288 \\ 21.874 \\ 27.222 \\ 28.186 \\ 29.129 \end{bmatrix} \ years$$

Regression to predict a ellipse at 2013, 2014 and 2015

$$X1 := \begin{bmatrix} E86_1 \\ E87_1 \\ E96_1 \\ E00_1 \\ E06_1 \\ E08_1 \end{bmatrix} \cdot 30 \quad Time1 := \begin{bmatrix} 0 \\ 352 \\ 3696 \\ 5168 \\ 7405 \\ 7984 \end{bmatrix} \cdot \frac{1}{365} \quad Y1 := \begin{bmatrix} E86_2 \\ E87_2 \\ E96_2 \\ E00_2 \\ E06_2 \\ E08_2 \end{bmatrix} \cdot 30 \quad \theta1 := \begin{bmatrix} E86_0 \\ E87_0 \\ E96_0 \\ E00_0 \\ E06_0 \\ E08_0 \end{bmatrix} \cdot \frac{1}{deg} \quad a1 := \begin{bmatrix} E86_3 \\ E87_3 \\ E96_3 \\ E00_3 \\ E06_3 \\ E08_3 \end{bmatrix} \cdot 30 \quad b1 := \begin{bmatrix} E86_4 \\ E87_4 \\ E96_4 \\ E00_4 \\ E06_4 \\ E08_4 \end{bmatrix} \cdot 30 \quad A := \begin{bmatrix} Time1_0 & 1 \\ Time1_1 & 1 \\ Time1_2 & 1 \\ Time1_3 & 1 \\ Time1_4 & 1 \\ Time1_5 & 1 \end{bmatrix}$$

$$\operatorname{corr}(Time1, X1)^2 = 0.903$$

$$\operatorname{corr}(Time1, Y1)^2 = 0.938$$

$$\operatorname{corr}(Time1, \theta1)^2 = 0.464$$

$$\operatorname{corr}(Time1, a1)^2 = 0.389$$

$$\operatorname{corr}(Time1, b1)^2 = 0.785$$

$$Years1 := \begin{bmatrix} 0 \\ 352 \\ 3696 \\ 5168 \\ 7405 \\ 7984 \end{bmatrix} \cdot \frac{1}{365} + \begin{bmatrix} 1986 \\ 1986 \\ 1986 \\ 1986 \\ 1986 \\ 1986 \end{bmatrix}$$

10632

$$a2 := min(\operatorname{mean}(a1) + 2 \cdot \operatorname{stdev}(a1), \max(a1)) = 5024.289$$

$$b2 := min(\operatorname{mean}(b1) + 2 \cdot \operatorname{stdev}(b1), \max(b1)) = 2660.586$$

$$\theta2\_min := (min(\operatorname{mean}(\theta1) + 2 \cdot \operatorname{stdev}(\theta1), \max(\theta1))) \cdot deg = 2.297$$

$$\theta2\_max := (\max(\operatorname{mean}(\theta1) - 2 \cdot \operatorname{stdev}(\theta1), min(\theta1))) \cdot deg = 2.064$$

$$a0_8 = 6.099 \cdot 10^3$$

$$b0_8 = 1.78 \cdot 10^3$$

$$X1coef := \left(A^T \cdot A\right)^{-1} \cdot \left(A^T \cdot X1\right) = \begin{bmatrix} 99.398 \\ 54923.265 \end{bmatrix}$$

$$Y1coef := \left(A^T \cdot A\right)^{-1} \cdot \left(A^T \cdot Y1\right) = \begin{bmatrix} -105.084 \\ 43560.082 \end{bmatrix}$$

$$Time = \begin{bmatrix} 0 \\ 352 \\ 3696 \\ 5168 \\ 7405 \\ 7984 \\ 9936 \\ 10288 \\ 10632 \end{bmatrix} \frac{1}{365}$$

$$X2\_15 := Time_8 \cdot X1coef_0 + X1coef_1 = 57818.596 \qquad X0_8 = 58807.686 \qquad X2\_15 - X0_8 = -989.09$$

$$Y2\_15 := Time_8 \cdot Y1coef_0 + Y1coef_1 = 40499.102 \qquad Y0_8 = 38888.498 \qquad Y2\_15 - Y0_8 = 1.611 \cdot 10^3$$

$$X2\_14 := Time_7 \cdot X1coef_0 + X1coef_1 = 57724.917 \qquad X0_7 = 58110.375 \qquad X2\_14 - X0_7 = -385.458$$

$$Y2\_14 := Time_7 \cdot Y1coef_0 + Y1coef_1 = 40598.14 \qquad Y0_7 = 40021.009 \qquad Y2\_14 - Y0_7 = 577.131$$

$$X2\_13 := Time_6 \cdot X1coef_0 + X1coef_1 = 57629.06 \qquad X0_6 = 57986.679 \qquad X2\_13 - X0_6 = -357.62$$

$$Y2\_13 := Time_6 \cdot Y1coef_0 + Y1coef_1 = 40699.482 \qquad Y0_6 = 40882.958 \qquad Y2\_13 - Y0_6 = -183.477$$

$X2\_15 \quad Y2\_15 \quad a2 \quad b2$

$$
\begin{aligned}
&ImageEllipse15\_min := \\
&\quad [\theta\ \ x0\ \ y0\ \ a\ \ b] \leftarrow \left[\theta2\_min\ \ \frac{X2\_15}{30}\ \ \frac{Y2\_15}{30}\ \ \frac{a2}{30}\ \ \frac{b2}{30}\right] \\
&\quad \text{for } t \in 0..360\cdot 60 \\
&\quad\quad Out_{t,0} \leftarrow \mathrm{trunc}\left(a\cdot\cos\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot\cos(\theta)+b\cdot\sin\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot -\sin(\theta)+x0\right) \\
&\quad\quad Out_{t,1} \leftarrow \mathrm{trunc}\left(a\cdot\cos\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot(\sin(\theta))+b\cdot\sin\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot\cos(\theta)+y0\right) \\
&\quad \text{for } t \in 0..\mathrm{rows}(Out)-2 \\
&\quad\quad \text{if } Out_{t,0} = Out_{t+1,0} \wedge Out_{t,1} = Out_{t+1,1} \\
&\quad\quad\quad \text{continue} \\
&\quad\quad \text{else} \\
&\quad\quad\quad Out2_{count,0} \leftarrow Out_{t,0} \\
&\quad\quad\quad Out2_{count,1} \leftarrow Out_{t,1} \\
&\quad\quad\quad count \leftarrow count+1 \\
&\quad Out2 \\
&\quad im01 \leftarrow im01\cdot 0 \\
&\quad \text{for } i \in 0..\mathrm{rows}(Out2)-1 \\
&\quad\quad im01_{Out2_{i,1},Out2_{i,0}} \leftarrow 255 \\
&\quad im01
\end{aligned}
$$

$$
\begin{aligned}
&ImageEllipse14\_min := \\
&\quad [\theta\ \ x0\ \ y0\ \ a\ \ b] \leftarrow \left[\theta2\_min\ \ \frac{X2\_14}{30}\ \ \frac{Y2\_14}{30}\ \ \frac{a2}{30}\ \ \frac{b2}{30}\right] \\
&\quad \text{for } t \in 0..360\cdot 60 \\
&\quad\quad Out_{t,0} \leftarrow \mathrm{trunc}\left(a\cdot\cos\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot\cos(\theta)+b\cdot\sin\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot -\sin(\theta)+x0\right) \\
&\quad\quad Out_{t,1} \leftarrow \mathrm{trunc}\left(a\cdot\cos\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot(\sin(\theta))+b\cdot\sin\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot\cos(\theta)+y0\right) \\
&\quad \text{for } t \in 0..\mathrm{rows}(Out)-2 \\
&\quad\quad \text{if } Out_{t,0} = Out_{t+1,0} \wedge Out_{t,1} = Out_{t+1,1} \\
&\quad\quad\quad \text{continue} \\
&\quad\quad \text{else} \\
&\quad\quad\quad Out2_{count,0} \leftarrow Out_{t,0} \\
&\quad\quad\quad Out2_{count,1} \leftarrow Out_{t,1} \\
&\quad\quad\quad count \leftarrow count+1 \\
&\quad Out2 \\
&\quad im01 \leftarrow im01\cdot 0 \\
&\quad \text{for } i \in 0..\mathrm{rows}(Out2)-1 \\
&\quad\quad im01_{Out2_{i,1},Out2_{i,0}} \leftarrow 255 \\
&\quad im01
\end{aligned}
$$

$$
\begin{aligned}
&ImageEllipse15\_max := \\
&\quad [\theta\ \ x0\ \ y0\ \ a\ \ b] \leftarrow \left[\theta2\_max\ \ \frac{X2\_15}{30}\ \ \frac{Y2\_15}{30}\ \ \frac{a2}{30}\ \ \frac{b2}{30}\right] \\
&\quad \text{for } t \in 0..360\cdot 60 \\
&\quad\quad Out_{t,0} \leftarrow \mathrm{trunc}\left(a\cdot\cos\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot\cos(\theta)+b\cdot\sin\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot -\sin(\theta)+x0\right) \\
&\quad\quad Out_{t,1} \leftarrow \mathrm{trunc}\left(a\cdot\cos\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot(\sin(\theta))+b\cdot\sin\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot\cos(\theta)+y0\right) \\
&\quad \text{for } t \in 0..\mathrm{rows}(Out)-2 \\
&\quad\quad \text{if } Out_{t,0} = Out_{t+1,0} \wedge Out_{t,1} = Out_{t+1,1} \\
&\quad\quad\quad \text{continue} \\
&\quad\quad \text{else} \\
&\quad\quad\quad Out2_{count,0} \leftarrow Out_{t,0} \\
&\quad\quad\quad Out2_{count,1} \leftarrow Out_{t,1} \\
&\quad\quad\quad count \leftarrow count+1 \\
&\quad Out2 \\
&\quad im01 \leftarrow im01\cdot 0 \\
&\quad \text{for } i \in 0..\mathrm{rows}(Out2)-1 \\
&\quad\quad im01_{Out2_{i,1},Out2_{i,0}} \leftarrow 255 \\
&\quad im01
\end{aligned}
$$

$$
\begin{aligned}
&ImageEllipse14\_max := \\
&\quad [\theta\ \ x0\ \ y0\ \ a\ \ b] \leftarrow \left[\theta2\_max\ \ \frac{X2\_14}{30}\ \ \frac{Y2\_14}{30}\ \ \frac{a2}{30}\ \ \frac{b2}{30}\right] \\
&\quad \text{for } t \in 0..360\cdot 60 \\
&\quad\quad Out_{t,0} \leftarrow \mathrm{trunc}\left(a\cdot\cos\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot\cos(\theta)+b\cdot\sin\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot -\sin(\theta)+x0\right) \\
&\quad\quad Out_{t,1} \leftarrow \mathrm{trunc}\left(a\cdot\cos\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot(\sin(\theta))+b\cdot\sin\left(t\cdot\frac{\pi}{180\cdot 60}\right)\cdot\cos(\theta)+y0\right) \\
&\quad \text{for } t \in 0..\mathrm{rows}(Out)-2 \\
&\quad\quad \text{if } Out_{t,0} = Out_{t+1,0} \wedge Out_{t,1} = Out_{t+1,1} \\
&\quad\quad\quad \text{continue} \\
&\quad\quad \text{else} \\
&\quad\quad\quad Out2_{count,0} \leftarrow Out_{t,0} \\
&\quad\quad\quad Out2_{count,1} \leftarrow Out_{t,1} \\
&\quad\quad\quad count \leftarrow count+1 \\
&\quad Out2 \\
&\quad im01 \leftarrow im01\cdot 0 \\
&\quad \text{for } i \in 0..\mathrm{rows}(Out2)-1 \\
&\quad\quad im01_{Out2_{i,1},Out2_{i,0}} \leftarrow 255 \\
&\quad im01
\end{aligned}
$$

$$
\begin{aligned}
&ImageEllipse13\_max := \\
&\quad [\theta\ \ x0\ \ y0\ \ a\ \ b] \leftarrow \left[\theta2\_max\ \ \frac{X2\_13}{30}\ \ \frac{Y2\_13}{30}\ \ \frac{a2}{30}\ \ \frac{b2}{30}\right]
\end{aligned}
$$

```
ImageEllipse13_min := | [θ x0 y0 a b] ← [θ2_min  X2_13/30  Y2_13/30  a2/30  b2/30]
                      | for t ∈ 0..360·60
                      |   | Out_{t,0} ← trunc(a·cos(t·π/(180·60))·cos(θ) + b·sin(t·π/(180·60))·−sin(θ) + x0)
                      |   | Out_{t,1} ← trunc(a·cos(t·π/(180·60))·(sin(θ)) + b·sin(t·π/(180·60))·cos(θ) + y0)
                      | for t ∈ 0..rows(Out) − 2
                      |   | if Out_{t,0} = Out_{t+1,0} ∧ Out_{t,1} = Out_{t+1,1}
                      |   |   | continue
                      |   | else
                      |   |   | Out2_{count,0} ← Out_{t,0}
                      |   |   | Out2_{count,1} ← Out_{t,1}
                      |   |   | count ← count + 1
                      | Out2
                      | im01 ← im01·0
                      | for i ∈ 0..rows(Out2) − 1
                      |   | im01_{Out2_{i,1}, Out2_{i,0}} ← 255
                      | im01
```

```
                      30          30       30   30 ]
                      | for t ∈ 0..360·60
                      |   | Out_{t,0} ← trunc(a·cos(t·π/(180·60))·cos(θ) + b·sin(t·π/(180·60))·−sin(θ) + x0)
                      |   | Out_{t,1} ← trunc(a·cos(t·π/(180·60))·(sin(θ)) + b·sin(t·π/(180·60))·cos(θ) + y0)
                      | for t ∈ 0..rows(Out) − 2
                      |   | if Out_{t,0} = Out_{t+1,0} ∧ Out_{t,1} = Out_{t+1,1}
                      |   |   | continue
                      |   | else
                      |   |   | Out2_{count,0} ← Out_{t,0}
                      |   |   | Out2_{count,1} ← Out_{t,1}
                      |   |   | count ← count + 1
                      | Out2
                      | im01 ← im01·0
                      | for i ∈ 0..rows(Out2) − 1
                      |   | im01_{Out2_{i,1}, Out2_{i,0}} ← 255
                      | im01
```

```
PrintEllipses15_min := | E1 ← ImageEllipse15_min
                       | ET ← E1
                       | WRITEBMP(concat(CWD, "\", "EllipseEst15_min.tif"), ET)

PrintEllipses15_max := | E1 ← ImageEllipse15_max
                       | ET ← E1
                       | WRITEBMP(concat(CWD, "\", "EllipseEst15_max.tif"), ET)

PrintEllipsesTFW15_min := | WRITETEXT(concat(CWD, "\", "EllipseEst15_min.tfw"), READEXCEL("HeaderTFW.xlsx"))

PrintEllipsesTFW15_max := | WRITETEXT(concat(CWD, "\", "EllipseEst15_max.tfw"), READEXCEL("HeaderTFW.xlsx"))

PrintEllipses14_min := | E1 ← ImageEllipse14_min
                       | ET ← E1
                       | WRITEBMP(concat(CWD, "\", "EllipseEst14_min.tif"), ET)

PrintEllipses14_max := | E1 ← ImageEllipse14_max
                       | ET ← E1
                       | WRITEBMP(concat(CWD, "\", "EllipseEst14_max.tif"), ET)

PrintEllipsesTFW14_min := | WRITETEXT(concat(CWD, "\", "EllipseEst14_min.tfw"), READEXCEL("HeaderTFW.xlsx"))

PrintEllipsesTFW14_max := | WRITETEXT(concat(CWD, "\", "EllipseEst14_max.tfw"), READEXCEL("HeaderTFW.xlsx"))

CWD = "C:\PhD\2015\00 Papers\Paper 2 – Amazon River\04 – Outputs Mathcad\Canny v2"
```

$$PrintEllipses13\_min := \left\| \begin{array}{l} E1 \leftarrow ImageEllipse13\_min \\ ET \leftarrow E1 \\ \mathrm{WRITEBMP}\left(\mathrm{concat}\left(\mathbf{CWD}, \text{“\textbackslash”}, \text{“EllipseEst13\_min.tif”}\right), ET\right) \end{array} \right.$$

$$PrintEllipses13\_max := \left\| \begin{array}{l} E1 \leftarrow ImageEllipse13\_max \\ ET \leftarrow E1 \\ \mathrm{WRITEBMP}\left(\mathrm{concat}\left(\mathbf{CWD}, \text{“\textbackslash”}, \text{“EllipseEst13\_max.tif”}\right), ET\right) \end{array} \right.$$

$$PrintEllipsesTFW13\_min := \left\| \mathrm{WRITETEXT}\left(\mathrm{concat}\left(\mathbf{CWD}, \text{“\textbackslash”}, \text{“EllipseEst13\_min.tfw”}\right), \mathrm{READEXCEL}\left(\text{“HeaderTFW.xlsx”}\right)\right) \right.$$

$$PrintEllipsesTFW13\_max := \left\| \mathrm{WRITETEXT}\left(\mathrm{concat}\left(\mathbf{CWD}, \text{“\textbackslash”}, \text{“EllipseEst13\_max.tfw”}\right), \mathrm{READEXCEL}\left(\text{“HeaderTFW.xlsx”}\right)\right) \right.$$

$$ImageEllipse15\_real := \left\| \begin{array}{l} [\theta \;\; x0 \;\; y0 \;\; a \;\; b] \leftarrow \left[E15_0 \;\; E15_1 \;\; E15_2 \;\; E15_3 \;\; E15_4\right] \\ \text{for } t \in 0..360 \cdot 60 \\ \quad \left\| \begin{array}{l} Out_{t,0} \leftarrow a \cdot \cos\left(t \cdot \dfrac{\pi}{180 \cdot 60}\right) \cdot \cos(\theta) + b \cdot \sin\left(t \cdot \dfrac{\pi}{180 \cdot 60}\right) \cdot -\sin(\theta) + x0 \\ Out_{t,0} \leftarrow \mathrm{trunc}\left(Out_{t,0}\right) \\ Out_{t,1} \leftarrow a \cdot \cos\left(t \cdot \dfrac{\pi}{180 \cdot 60}\right) \cdot (\sin(\theta)) + b \cdot \sin\left(t \cdot \dfrac{\pi}{180 \cdot 60}\right) \cdot \cos(\theta) + y0 \\ Out_{t,1} \leftarrow \mathrm{trunc}\left(Out_{t,1}\right) \end{array} \right. \\ \text{for } t \in 0..\mathrm{rows}(Out) - 2 \\ \quad \left\| \begin{array}{l} \text{if } Out_{t,0} = Out_{t+1,0} \wedge Out_{t,1} = Out_{t+1,1} \\ \quad \left\| \text{continue} \right. \\ \text{else} \\ \quad \left\| \begin{array}{l} Out2_{count,0} \leftarrow Out_{t,0} \\ Out2_{count,1} \leftarrow Out_{t,1} \\ count \leftarrow count + 1 \end{array} \right. \end{array} \right. \\ Out2 \\ im01 \leftarrow im01 \cdot 0 \\ \text{for } i \in 0..\mathrm{rows}(Out2) - 1 \\ \quad \left\| im01_{Out2_{i,1}, Out2_{i,0}} \leftarrow 255 \right. \\ im01 \end{array} \right.$$

$$PrintEllipses15\_real := \left\| \begin{array}{l} E1 \leftarrow ImageEllipse15\_real \\ ET \leftarrow E1 \\ \mathrm{WRITEBMP}\left(\mathrm{concat}\left(\mathbf{CWD}, \text{“\textbackslash”}, \text{“EllipseEst15\_real.tif”}\right), ET\right) \end{array} \right.$$

$$PrintEllipsesTFW15\_real := \left\| \mathrm{WRITETEXT}\left(\mathrm{concat}\left(\mathbf{CWD}, \text{“\textbackslash”}, \text{“EllipseEst15\_real.tfw”}\right), \mathrm{READEXCEL}\left(\text{“HeaderTFW.xlsx”}\right)\right) \right.$$

$$ImageEllipse14\_real := \left\| \begin{array}{l} [\theta\ \ x0\ \ y0\ \ a\ \ b] \leftarrow [E14_0\ \ E14_1\ \ E14_2\ \ E14_3\ \ E14_4] \\ \text{for } t \in 0..360 \cdot 60 \\ \quad \left\| \begin{array}{l} Out_{t,0} \leftarrow a \cdot \cos\left(t \cdot \frac{\pi}{180 \cdot 60}\right) \cdot \cos(\theta) + b \cdot \sin\left(t \cdot \frac{\pi}{180 \cdot 60}\right) \cdot -\sin(\theta) + x0 \\ Out_{t,0} \leftarrow \text{trunc}\left(Out_{t,0}\right) \\ Out_{t,1} \leftarrow a \cdot \cos\left(t \cdot \frac{\pi}{180 \cdot 60}\right) \cdot (\sin(\theta)) + b \cdot \sin\left(t \cdot \frac{\pi}{180 \cdot 60}\right) \cdot \cos(\theta) + y0 \\ Out_{t,1} \leftarrow \text{trunc}\left(Out_{t,1}\right) \end{array} \right. \\ \text{for } t \in 0..\text{rows}(Out) - 2 \\ \quad \left\| \begin{array}{l} \text{if } Out_{t,0} = Out_{t+1,0} \wedge Out_{t,1} = Out_{t+1,1} \\ \quad \| \text{ continue} \\ \text{else} \\ \quad \left\| \begin{array}{l} Out2_{count,0} \leftarrow Out_{t,0} \\ Out2_{count,1} \leftarrow Out_{t,1} \\ count \leftarrow count + 1 \end{array} \right. \end{array} \right. \\ Out2 \\ im01 \leftarrow im01 \cdot 0 \\ \text{for } i \in 0..\text{rows}(Out2) - 1 \\ \quad \| \ im01_{Out2_{i,1}, Out2_{i,0}} \leftarrow 255 \\ im01 \end{array} \right.$$

$$ImageEllipse13\_real := \left\| \begin{array}{l} [\theta\ \ x0\ \ y0\ \ a\ \ b] \leftarrow [E13_0\ \ E13_1\ \ E13_2\ \ E13_3\ \ E13_4] \\ \text{for } t \in 0..360 \cdot 60 \\ \quad \left\| \begin{array}{l} Out_{t,0} \leftarrow a \cdot \cos\left(t \cdot \frac{\pi}{180 \cdot 60}\right) \cdot \cos(\theta) + b \cdot \sin\left(t \cdot \frac{\pi}{180 \cdot 60}\right) \cdot -\sin(\theta) + x0 \\ Out_{t,0} \leftarrow \text{trunc}\left(Out_{t,0}\right) \\ Out_{t,1} \leftarrow a \cdot \cos\left(t \cdot \frac{\pi}{180 \cdot 60}\right) \cdot (\sin(\theta)) + b \cdot \sin\left(t \cdot \frac{\pi}{180 \cdot 60}\right) \cdot \cos(\theta) + y0 \\ Out_{t,1} \leftarrow \text{trunc}\left(Out_{t,1}\right) \end{array} \right. \\ \text{for } t \in 0..\text{rows}(Out) - 2 \\ \quad \left\| \begin{array}{l} \text{if } Out_{t,0} = Out_{t+1,0} \wedge Out_{t,1} = Out_{t+1,1} \\ \quad \| \text{ continue} \\ \text{else} \\ \quad \left\| \begin{array}{l} Out2_{count,0} \leftarrow Out_{t,0} \\ Out2_{count,1} \leftarrow Out_{t,1} \\ count \leftarrow count + 1 \end{array} \right. \end{array} \right. \\ Out2 \\ im01 \leftarrow im01 \cdot 0 \\ \text{for } i \in 0..\text{rows}(Out2) - 1 \\ \quad \| \ im01_{Out2_{i,1}, Out2_{i,0}} \leftarrow 255 \\ im01 \end{array} \right.$$

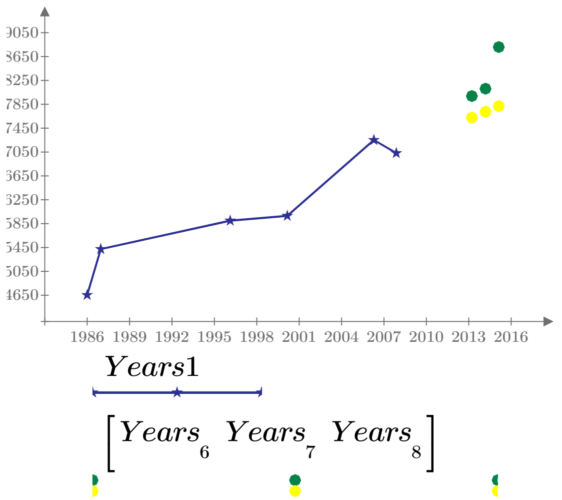
$$PrintEllipses14\_real := \left\| \begin{array}{l} E1 \leftarrow ImageEllipse14\_real \\ ET \leftarrow E1 \\ \text{WRITEBMP}(\text{concat}(\mathbf{CWD}, \text{“\textbackslash”}, \text{“EllipseEst14\_real.tif”}), ET) \end{array} \right.$$

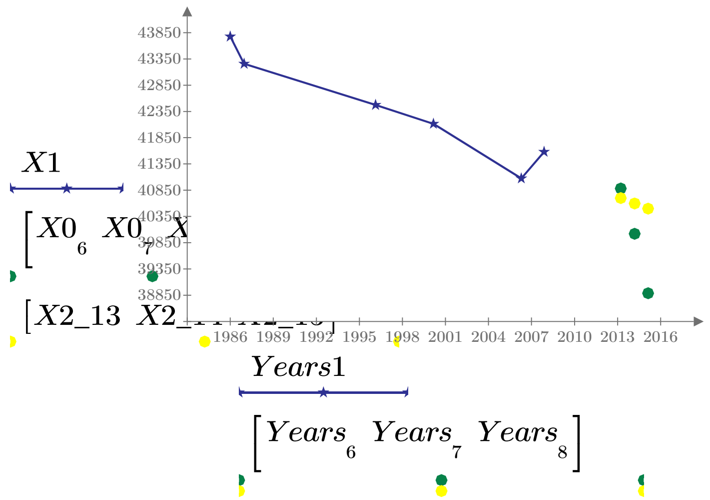
$$PrintEllipsesTFW14\_real := \| \ \text{WRITETEXT}(\text{concat}(\mathbf{CWD}, \text{“\textbackslash”}, \text{“EllipseEst14\_real.tfw”}), \text{READEXCEL}(\text{“HeaderTFW.xlsx”}))$$

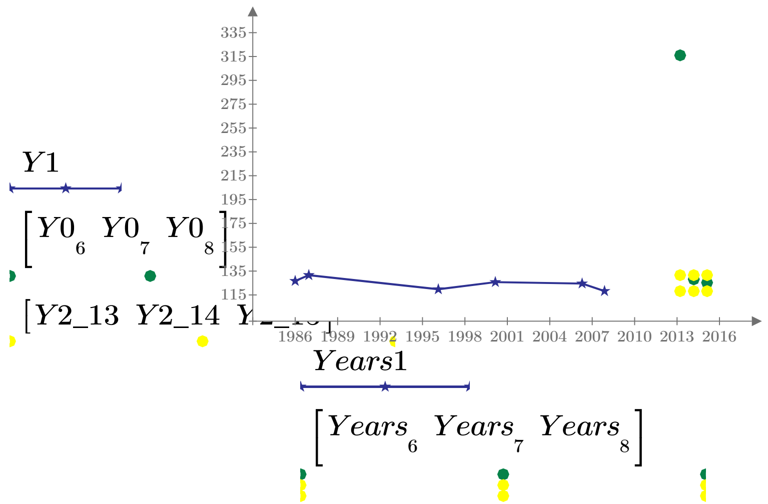
$$PrintEllipses13\_real := \left\| \begin{array}{l} E1 \leftarrow ImageEllipse13\_real \\ ET \leftarrow E1 \\ \text{WRITEBMP}(\text{concat}(\mathbf{CWD}, \text{“\textbackslash”}, \text{“EllipseEst13\_real.tif”}), ET) \end{array} \right.$$

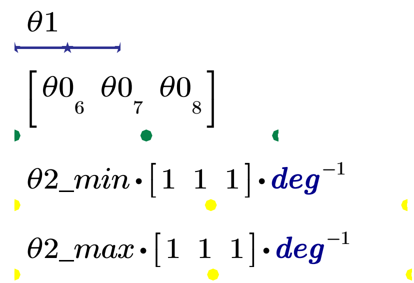
$$PrintEllipsesTFW13\_real := \| \ \text{WRITETEXT}(\text{concat}(\mathbf{CWD}, \text{“\textbackslash”}, \text{“EllipseEst13\_real.tfw”}), \text{READEXCEL}(\text{“HeaderTFW.xlsx”}))$$

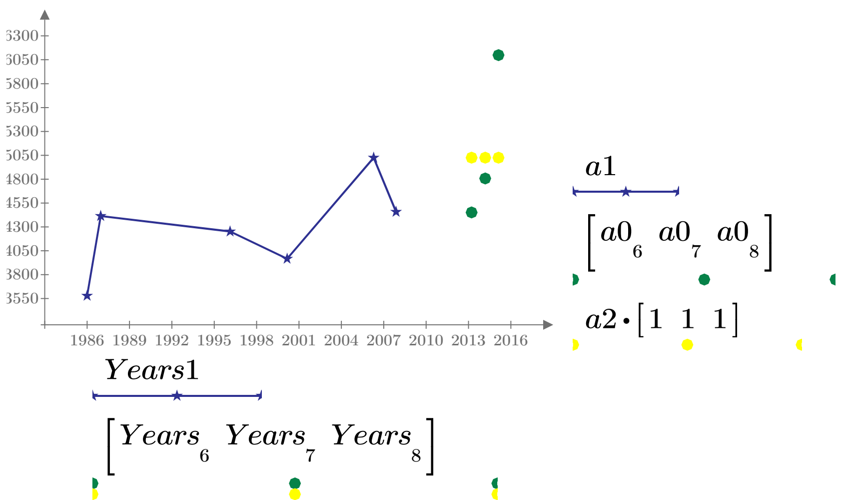
2013, 2014, 2015

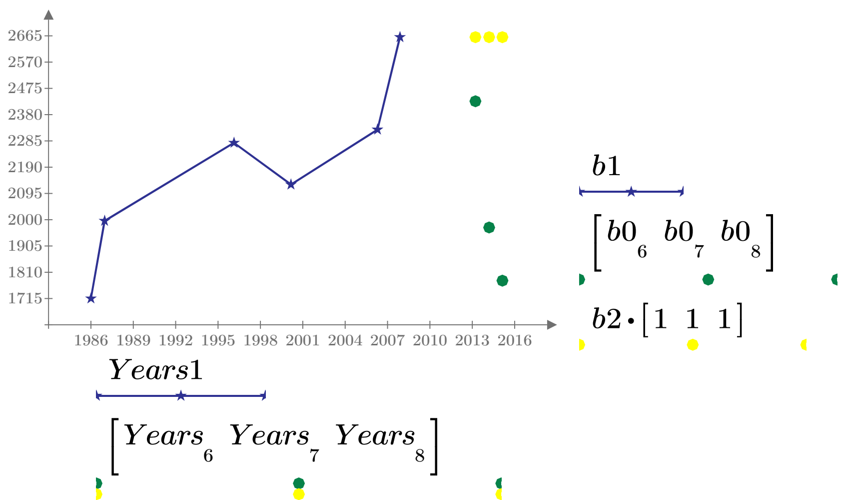












$$Time := \begin{bmatrix} 0 \\ 352 \\ 3696 \\ 5168 \\ 7405 \\ 7984 \\ 9936 \\ 10288 \\ 10632 \end{bmatrix} \frac{1}{365} \quad E1 := \begin{bmatrix} E86_1 \\ E87_1 \\ E96_1 \\ E00_1 \\ E06_1 \\ E08_1 \\ E13_1 \\ E14_1 \\ E15_1 \end{bmatrix} \cdot 30 \quad N1 := \begin{bmatrix} E86_2 \\ E87_2 \\ E96_2 \\ E00_2 \\ E06_2 \\ E08_2 \\ E13_2 \\ E14_2 \\ E15_2 \end{bmatrix} \cdot -30$$

$$La := \frac{\begin{bmatrix} E86_1 - min(E1) \cdot 30^{-1} \\ E87_1 - min(E1) \cdot 30^{-1} \\ E96_1 - min(E1) \cdot 30^{-1} \\ E00_1 - min(E1) \cdot 30^{-1} \\ E06_1 - min(E1) \cdot 30^{-1} \\ E08_1 - min(E1) \cdot 30^{-1} \end{bmatrix} \cdot 30}{1000} \quad Lb := \frac{\begin{bmatrix} E86_2 - min(N1) \cdot -30^{-1} \\ E87_2 - min(N1) \cdot -30^{-1} \\ E96_2 - min(N1) \cdot -30^{-1} \\ E00_2 - min(N1) \cdot -30^{-1} \\ E06_2 - min(N1) \cdot -30^{-1} \\ E08_2 - min(N1) \cdot -30^{-1} \end{bmatrix} \cdot -30}{1000}$$

$$E1 := \frac{\begin{bmatrix} E86_1 - min(E1) \cdot 30^{-1} \\ E87_1 - min(E1) \cdot 30^{-1} \\ E96_1 - min(E1) \cdot 30^{-1} \\ E00_1 - min(E1) \cdot 30^{-1} \\ E06_1 - min(E1) \cdot 30^{-1} \\ E08_1 - min(E1) \cdot 30^{-1} \\ E13_1 - min(E1) \cdot 30^{-1} \\ E14_1 - min(E1) \cdot 30^{-1} \\ E15_1 - min(E1) \cdot 30^{-1} \end{bmatrix} \cdot 30}{1000} \quad N1 := \frac{\begin{bmatrix} E86_2 - min(N1) \cdot -30^{-1} \\ E87_2 - min(N1) \cdot -30^{-1} \\ E96_2 - min(N1) \cdot -30^{-1} \\ E00_2 - min(N1) \cdot -30^{-1} \\ E06_2 - min(N1) \cdot -30^{-1} \\ E08_2 - min(N1) \cdot -30^{-1} \\ E13_2 - min(N1) \cdot -30^{-1} \\ E14_2 - min(N1) \cdot -30^{-1} \\ E15_2 - min(N1) \cdot -30^{-1} \end{bmatrix} \cdot -30}{1000}$$

$$Time' := \text{augment}\left(Time, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right) \quad Aa := \text{augment}\left(\begin{bmatrix} 0 \\ 352 \\ 3696 \\ 5168 \\ 7405 \\ 7984 \end{bmatrix} \cdot \frac{1}{365}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right)$$

$$Xcoef := \left(Aa^{\mathrm{T}} \cdot Aa \right)^{-1} \cdot Aa^{\mathrm{T}} \cdot La = \begin{bmatrix} 0.099 \\ 0.269 \end{bmatrix}$$

$$Ycoef := \left(Aa^{\mathrm{T}} \cdot Aa \right)^{-1} \cdot Aa^{\mathrm{T}} \cdot Lb = \begin{bmatrix} 0.105 \\ 0.215 \end{bmatrix}$$

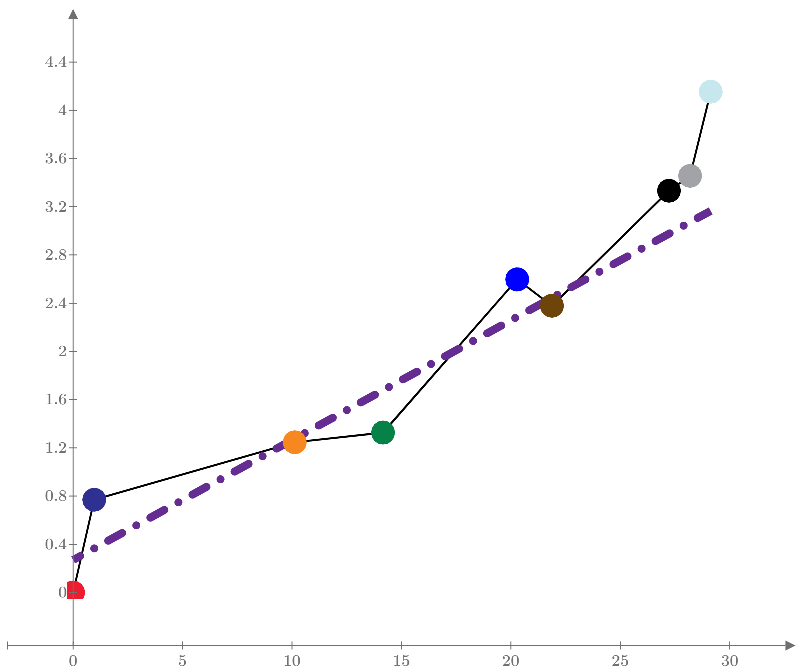
$$EstX := Time' \cdot Xcoef = \begin{bmatrix} 0.269 \\ 0.365 \\ 1.276 \\ 1.677 \\ 2.286 \\ 2.444 \\ 2.975 \\ 3.071 \\ 3.165 \end{bmatrix} \quad EstY := Time' \cdot Ycoef = \begin{bmatrix} 0.215 \\ 0.317 \\ 1.279 \\ 1.703 \\ 2.347 \\ 2.514 \\ 3.076 \\ 3.177 \\ 3.276 \end{bmatrix}$$

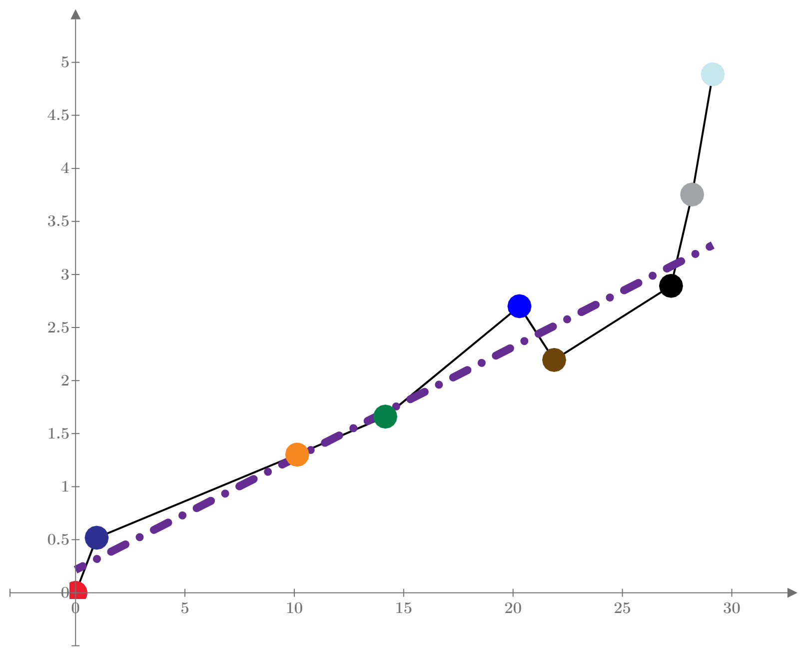
$$\frac{EstX_5 - EstX_0}{Time_5 - Time_0} = 0.099$$

$$\operatorname{corr}\left(Aa^{\langle 0 \rangle}, La\right)^2 = 0.903$$

$$\frac{EstY_5 - EstY_0}{Time_5 - Time_0} = 0.105$$

$$\operatorname{corr}\left(Aa^{\langle 0 \rangle}, Lb\right)^2 = 0.938$$





$$E15 = \begin{bmatrix} 2.187 \\ 1960.256 \\ 1296.283 \\ 203.306 \\ 59.326 \end{bmatrix}$$

$$\begin{bmatrix} \theta 2\_min & \frac{X2\_15}{30} & \frac{Y2\_15}{30} & \frac{a2}{30} & \frac{b2}{30} \end{bmatrix} = \begin{bmatrix} 2.297 & 1927.287 & 1349.97 & 167.476 & 88.686 \end{bmatrix}$$

$$\begin{bmatrix} \theta 2\_max & \frac{X2\_15}{30} & \frac{Y2\_15}{30} & \frac{a2}{30} & \frac{b2}{30} \end{bmatrix} = \begin{bmatrix} 2.064 & 1927.287 & 1349.97 & 167.476 & 88.686 \end{bmatrix}$$

$$\left((1960.256 - 1927.287)^2 + (1296.283 - 1349.97)^2\right)^{.5} \cdot 30 = 1890.059$$

$$\frac{2.187 - 2.297}{deg} = -6.303$$

$$\frac{2.187 - 2.064}{deg} = 7.047$$

$$E14 = \begin{bmatrix} 2.236 \\ 1937.013 \\ 1334.034 \\ 160.262 \\ 65.725 \end{bmatrix}$$

$$\begin{bmatrix} \theta 2\_min & \frac{X2\_14}{30} & \frac{Y2\_14}{30} & \frac{a2}{30} & \frac{b2}{30} \end{bmatrix} = \begin{bmatrix} 2.297 & 1924.164 & 1353.271 & 167.476 & 88.686 \end{bmatrix}$$

$$\begin{bmatrix} \theta 2\_max & \frac{X2\_14}{30} & \frac{Y2\_14}{30} & \frac{a2}{30} & \frac{b2}{30} \end{bmatrix} = \begin{bmatrix} 2.064 & 1924.164 & 1353.271 & 167.476 & 88.686 \end{bmatrix}$$

$$\left((1937.013 - 1924.164)^2 + \left(1334.033 - 1353.271\right)^2\right)^{.5} \cdot 30 = 694.03$$

$$\frac{2.236 - 2.297}{deg} = -3.495$$

$$\frac{2.236 - 2.064}{deg} = 9.855$$

deg deg

$$E13 = \begin{bmatrix} 5.515 \\ 1932.889 \\ 1362.765 \\ 148.37 \\ 80.956 \end{bmatrix}$$

$$\begin{bmatrix} \theta2\_min & \frac{X2\_13}{30} & \frac{Y2\_13}{30} & \frac{a2}{30} & \frac{b2}{30} \end{bmatrix} = \begin{bmatrix} 2.297 & 1920.969 & 1356.649 & 167.476 & 88.686 \end{bmatrix}$$

$$\begin{bmatrix} \theta2\_max & \frac{X2\_13}{30} & \frac{Y2\_13}{30} & \frac{a2}{30} & \frac{b2}{30} \end{bmatrix} = \begin{bmatrix} 2.064 & 1920.969 & 1356.649 & 167.476 & 88.686 \end{bmatrix}$$

$$\left((1932.889 - 1920.969)^2 + (1362.765 - 1356.649)^2\right)^{.5} \cdot 30 = 401.924$$

$$\frac{5.515 - 2.297}{deg} = 184.378 \qquad 4.378$$

$$\frac{5.515 - 2.064}{deg} = 197.728 \qquad 7.728$$