A Study of Random Walking Triangles

Phenomenology of a Toy Model Inspired by the Spontaneous Reduction of the Spectral Dimension in Quantum Gravity

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Outline

Quantum Gravity
Background

- Theories of **Quantum Gravity** seek to create a complete quantum theory of gravity.
- Some examples include String Theory, Asymptotic Safety, Horava-Lifshitz Gravity, Causal Set Theory, Causal Dynamical Triangulations, and Loop Quantum Gravity.
- Quantum mechanics is often characterized by discrete observations, and gravity has been well described by the geometry of space-time.
- It is thus natural that some theories of quantum gravity should result in discrete spacetime. Such models include the second half of the list above.
- The model we will consider is a discrete model of quantum gravity.

Inspiration
Background

- It has been observed that the Spectral Dimension changes at small scales in many models of Quantum Gravity.
- The Spectral Dimension usually agrees with the typical sense of dimension.
- A common transition was from 4 to 2, so we decided to consider the possibility of constructing 4D space from 2D triangles in a Lorentz Invariant manner.
- Key Question: If we start with 2D surfaces, with the spectral and typical dimensions agreeing, can we get a reasonable spacetime at large scales?
- Note that all the above models break Lorentz Invariance; String Theory and Asymptotic Safety do not show this property.

Spectral Dimension
Background

- The **spectral dimension**, d_s, characterizes the return probability of particles undergoing a random walk:

$$P_n \sim n^{-d_s/2}$$

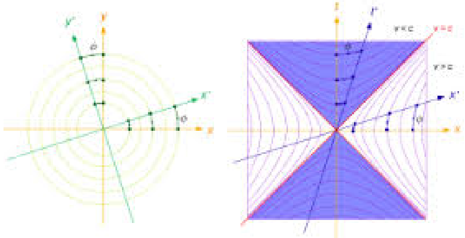
where P_n is the return probability, and n is the number of iterations.

- From the equation above, the spectral dimension can be found as a function of iteration using:

$$d_s = -2\frac{\partial \log(P_n)}{\partial \log(n)}$$

- In quantum gravity, the spectral dimension is sometimes used when the typical sense of dimension is not well defined.

Lorentz Transformations
Background

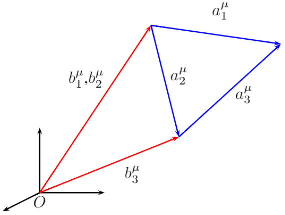


- **Lorentz Transformations**, which are the most familiar application of special relativity, are generalized rotations between time and space, as shown above.
- The Lorentz Invariant surfaces are hyperbolae. *This means that most vectors from a Lorentz Invariant distribution will correspond to nearly light-speed particles in a particular frame.*

Logical Development

Developing the Model

- Our goal was to find the simplest approach, so we chose triangles as the simplest 2D surfaces.
- Filling space with a uniform lattice would break Lorentz Invariance.
- The alternative: a space constructed from a Lorentz Invariant distribution of triangles glued together and oriented randomly.
- If so, then when a particle reaches the edge of a triangle, the next triangle must be selected from a Lorentz Invariant distribution.
- It is very difficult, if not impossible, to simply lay down such a space in its entirety.
- So I generated sequences of triangles where new triangles spawned from the sides of old triangles in a Lorentz Invariant manner.



- Each side was represented with a vector b_i^μ for the origin and a vector a_i^μ for the side itself, as shown above.
- The proper lengths were randomly selected from a Gaussian distribution centered at one Planck Length.
- We then solved for the exact components of the other sides with the basic constraints that the vectors meet to form a triangle.

The following is the process used to generate new triangles:

- A side was randomly selected from a pre-existing triangle, and would serve as the first side for the next triangle.
- The proper lengths of the other two sides were then randomly selected from the same Gaussian distribution centered at 1 Planck length.
- The angle between the spatial components was also selected randomly within the range that would allow for a triangle to be formed.
- The time and spatial radius of the seconds side were then solved for, and those for the third side were then found as the difference between the other two sides.

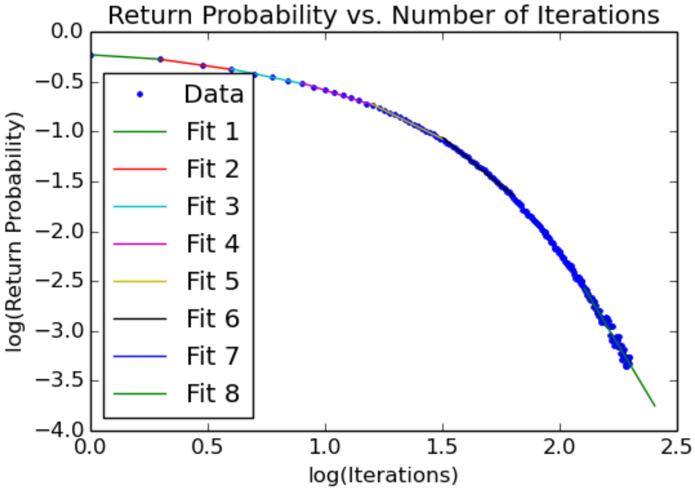
Strategy

Walking the Triangles

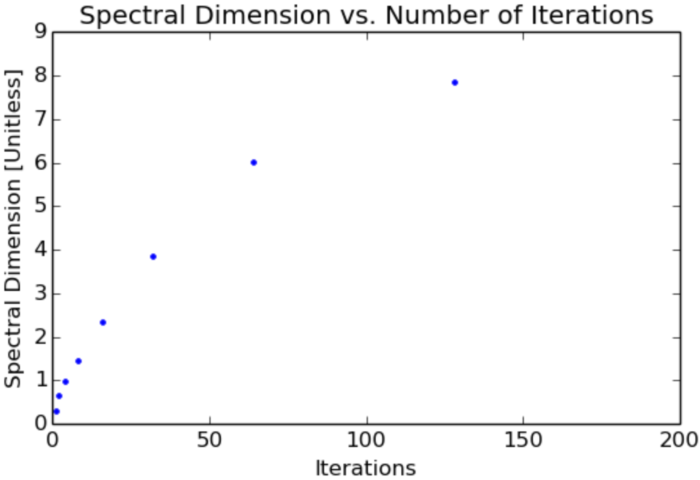
- By performing many runs of sequences of up to 200 triangles, I calculated the return probability, where triangles whose centers were within on proper Planck Length were considered to have returned.
- From the return probabilities, I calculated the spectral dimension.
- In addition, we were interested in whether the process was a true random walk, in which each step has no dependence in the previous steps. In other words: do the triangles look the same after many generations?
- Further investigation might explore the effects on translational invariance.

The Return Probabilities

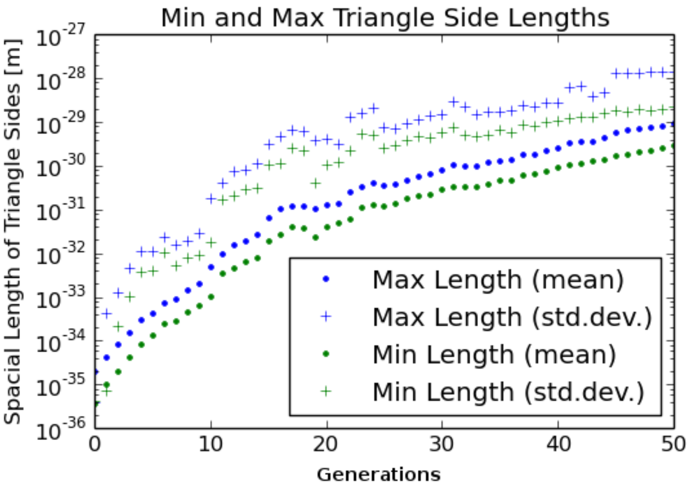
Walking the Triangles



The Spectral Dimensions
Walking the Triangles



The Spatial Radii
Walking the Triangles



Conclusions

- The spectral dimension appears to grow without bound, certainly well past 4.
- This is the result of a steady growth in the spatial lengths of the sides as the corresponding speed becomes arbitrarily close to c.
- This is, we suspect, the result of a random walk up the Lorentz Invariant hyperbola.
- If so, the implication is that our requirement that the distribution of triangles be Lorentz Invariant is inconsistent with a uniform 4D space constructed from 2D triangles.
- Thus, I have shown that 4D space constructed with connected 2D triangles with causal paths and the same Gaussian distributed proper lengths for all sides cannot be both uniform and Lorentz invariant.

Questions?