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ESSAYS ON MACROECONOMICS AND MARKET FAILURES

BY

BAYARMAA DALKHJAV

BA., Economics, National University of Mongolia, 2000

MA., Development Studies, Sophia University, 2007

MA., Economics, University of New Hampshire, 2018

DISSERTATION

Submitted to the University of New Hampshire
in Partial Fulfillment of
the Requirements for the Degree of

Doctor of Philosophy
in
Economics

May, 2024

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Bayarmaa Dalkhjav

This dissertation has been examined and approved in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics by:

Dissertation Chair

Dr. Loris Rubini, Associate Professor of Economics
University of New Hampshire

Dr. Karen Conway, Professor of Economics
University of New Hampshire

Dr. Yin Germaschewski, Associate Professor of Economics
University of New Hampshire

Dr. Jaroslav Horvath, Associate Professor of Economics
University of New Hampshire

Dr. German Cubas, Associate Professor of Economics
University of Houston

On April 11, 2024

Original approval signatures are on file with the University of New Hampshire Graduate School.

DEDICATION

To my parents, husband, daughter, son, and extended family. Your sacrifices and belief in me have given me the strength to pursue my dreams.

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Table of Contents

	Page No.
Dedication	iv
Acknowledgements	v
List of Tables	ix
List of Figures	x
Abstract	xi
Chapter 1: Hidden Information as a Source of Misallocation: An Application to the Opioid Crisis	1
1.1 Introduction	2
1.2 The opioid crisis	7
1.2.1 Opioid Misuse is More Prevalent among Low-Skilled Workers	7
1.2.2 Opioid Misuse is Associated with Higher Rates of Absenteeism	8
1.2.3 Opioid Misuse Leads to Lower Productivity	9
1.3 The Model	10
1.3.1 No Health Status	11
1.3.2 Introducing Health Status	14
1.4 Solving the Model Under Full Information	17
1.4.1 Solution	18
1.4.2 Decentralized Equilibrium	24
1.5 Market Equilibrium under Hidden information	27
1.5.1 A Competitive Equilibrium	31
1.6 Separating Equilibrium	32
1.7 Application to the Opioid Crisis	34
1.8 Results	37
1.9 Conclusion	43
Chapter 2: Trade, Innovation, and Pollution	45
2.1 Introduction	46
2.2 Setup of the model	48
2.2.1 Preference and Demand	48
2.2.2 Production	49
2.2.2.1 Pricing rule	50
2.2.2.2 Entry and innovation decisions	51

2.2.2.3	Pollution emission	53
2.2.3	Labor Market Equilibrium	54
2.2.4	Equilibrium in an economy	54
2.3	Calibration	55
2.4	Results	56
2.5	Conclusion	57
2.6	Appendix	59
Chapter 3: Are Sovereign Wealth Funds a Good Idea in the Presence of Corruption?		61
3.1	Introduction	62
3.2	Fiscal rules in Mongolia	65
3.3	Corruption in Mongolia	66
3.4	The Model	66
3.4.1	Households	67
3.4.2	Production	67
3.4.3	Interest rate in a small open economy	68
3.4.4	Commodity sector	69
3.4.5	Fiscal policy	69
3.4.5.1	Fiscal policy without a SWF	69
3.4.5.2	Fiscal policy with a SWF	70
3.4.6	Expropriations	72
3.4.7	Feasibility	73
3.5	Data and parameterization	74
3.5.1	Data	74
3.5.2	Calibrated parameters	75
3.5.3	Bayesian estimation	76
3.6	Results	76
3.6.1	Performance of the model	76
3.6.2	Welfare analysis	77
3.6.3	Effects of shocks to the commodity price	78
3.7	Conclusion	80
Bibliography		82
Appendices		86
Appendix Chapter A: Chapter 1: Equilibrium Characterization and Proofs		87
A.1	Equilibrium Characterization and Proofs	87
A.1.1	Equilibrium Characterization	87
A.1.2	Proof of Proposition 4	89
Appendix Chapter B: Chapter 2: Entry and Innovation Decisions		92

B.1 Firm's decision 92

List of Tables

	Page No.
1.1 Calibrated parameters and targets matched in the model	37
1.2 Main Results	38
2.1 Parameters and targets	55
2.2 Key Findings	57
3.1 Calibrated parameters	75
3.2 Priors and posteriors of estimated parameters	76
3.3 Moments	77
3.4 Welfare Impacts	78

List of Figures

	Page No.
1.1 Opioid misuse rates across educational attainment	8
1.2 Distribution of missing workdays	9
1.3 Optimal assignment function	23
1.4 Calibration of ρ function	35
1.5 Fit of the Lorenz curve	36
1.6 Distortions in Wages of Unhealthy Workers	40
1.7 Distortions in Wages of Healthy Workers	40
1.8 Distortions in the assignment	41
1.9 Distortions in the number of workers hired	42
1.10 Distortions in Profits	42
2.1 Types of firms in the state space	53
3.1 Cyclical Components	74
3.2 Responses to a copper price shock	79
3.3 Responses to a copper price shock (cont.)	80

ABSTRACT

ESSAYS ON MACROECONOMICS AND MARKET FAILURES

by

Bayarmaa Dalkhjav

University of New Hampshire, May, 2024

What are the macroeconomic consequences of market failure in the long and short runs? How do asymmetric information, externality, and lack of the rule of law contribute to resource misallocation, output, and the business cycle in an economy at the aggregate level? My research focuses on these research questions. Specifically, I explore the macroeconomic effects of asymmetric information, the external impacts of innovation decisions as a part of firm's dynamic decisions, and the implications of corruption on the fluctuation of the business cycle.

In the first chapter, titled "Hidden Information as a Source of Misallocation: An Application to the Opioid Crisis", we develop a general equilibrium model in which essential information about employee productivity is hidden from employers, resulting in a suboptimal allocation of resources. The health of employees is unverifiable by employers, and employees with poor health are less productive than their healthier counterparts. We utilize this framework to examine the output losses associated with the opioid crisis. Workers with opioid use disorder exhibit higher absenteeism and reduced productivity, directly contributing to output losses. Furthermore, since employers cannot distinguish addicts from non-addicts, wages deviate from marginal productivity, leading to a suboptimal allocation of resources. Calibrating the model to the US, we find that opioid misuse reduces output by \$133 billion and the misallocation channel accounts for 17.6% of this.

The second chapter of my dissertation, titled "Trade, Innovation, and Pollution" presents a model examining the impact of reduced trade costs on pollution, particularly in scenarios with an extensive margin of innovation. While conventional wisdom suggests that international trade leads to increased pollution due to higher output from polluting firms, recent empirical evidence contradicts this notion. Our model reveals that some new exporters tend to adopt cleaner technologies, unintentionally becoming environmentally friendly producers. Calibrated with data from the Chilean economy, our findings indicate a 4.4% reduction in pollution and a 0.85% decrease in pollution intensity between 1995 and 2007 as trade costs decline. Moreover, there is an increase in pollution without technology adoption, that is, when all or none of the firms innovate. An extensive margin of adopting new technology causes a reduction in total pollution and its intensity.

For the third chapter, my research question is: "Are Sovereign Wealth Funds a Good Idea in the Presence of Corruption?". Commodity-exporting countries are highly vulnerable to commodity price shocks, and many governments establish sovereign wealth funds (SWFs) to smooth consumption and accumulate revenue during periods of high commodity prices, relative to the reference price. While SWFs are typically an important source of government revenue in such countries, their accumulation can also create opportunities for corruption that may undermine the benefits of stabilizing consumption. Mongolia is one such country where corruption is a significant problem (IMF, 2021). I analyze how corruption can diminish the effectiveness of SWFs in mitigating business cycle fluctuations in resource-rich countries, using data from Mongolia and developing a theoretical model.

CHAPTER 1

Hidden Information as a Source of Misallocation: An Application to the Opioid Crisis

by

Bayarmaa Dalkhjav & Loris Rubini

1.1 Introduction

One key source of misallocation thoroughly studied in microeconomics is asymmetric information, but this is rarely used in macroeconomic settings. Similar to other sources of misallocation studied in macroeconomics, such as taxation, hidden information has the potential of reducing the output per unit of input. The problem is particularly relevant in the context of a prevailing opioid crisis that has affected US labor markets since the early 2000s (Krueger, 2017). Microeconomic studies have identified several consequences of this crisis, including increases in absenteeism and reductions in worker productivity. In addition to these consequences, the fact that employers cannot observe the productivity of their workers implies that wages do not equal marginal product, resulting in a reduced amount of output per worker. To study the losses associated with this resource misallocation, a general equilibrium model is needed. This paper develops such a model.

The model builds on Garicano and Rossi-Hansberg (2004) and Caicedo et al. (2019). Production requires a manager paired with workers. Individuals differ in their ability to solve a problem, and the ones with high ability self-select into a management position. The remaining individuals are workers. Workers solve problems of varying difficulty that arrive randomly. The ability to solve these problems differs across workers, and when workers cannot solve a problem, they transfer it to the manager. This implies that the manager must take into account the number of problems she is likely to have to deal with when hiring workers since each problem requires time. When managers know the problem-solving ability of their workers, the manager optimally sets wages equal to marginal productivity and knows how many workers to hire so that their problem-solving time is efficiently used. Thus, the inability of a manager to predict the productivity of her workers will result in a sub-optimal allocation.

We add health status to the model. Managers cannot observe the health status of their employees. Unhealthy workers are less productive than healthy ones. Thus, managers cannot

set wages equal to marginal productivity, and when deciding the number of workers to hire, they must hire a smaller number of workers than if all were healthy to account for the extra time that will be needed to address the problems left unsolved by unhealthy workers.¹

Health affects productivity in two ways. First, being unhealthy reduces a worker's ability to solve problems or complete tasks within a given time. Consequently, managers must spend more time per worker. Second, unhealthy workers have higher rates of absenteeism compared to their healthy counterparts, resulting in reduced output.

This model is especially well suited to study the opioid crisis in the U.S. One often neglected consequence of such a crisis is the uncertainty employers face when hiring an individual with opioid use disorder (OUD). These workers tend to be absent more often and even when present, less productive than individuals without OUD. According to data provided by the National Survey on Drug Use and Health (NSDUH) in 2018, the average number of days absent from work in a month is 3.4 days among individuals with OUD and 1.1 days among individuals without OUD. If managers knew who they were hiring, they would adjust wages to reflect the lower productivity. OUD workers have incentives to hide their condition, and managers are forced to hire workers understanding that a fraction of them suffer from substance use disorder. With hidden information, both types of workers receive the same wage, which implies that workers with OUD receive compensation that exceeds their marginal productivity, and workers without OUD receive compensation that is lower than their marginal productivity.

We calibrate the parameters in our model to match key targets, including the proportion of individuals with OUD across different educational levels, the productivity loss associated with addiction, and the rates of absenteeism among workers with OUD. The NSDUH finds that the share of opioid abusers in the working-age population was 3.8% in 2018-2019 on average. Also, individuals with OUD had absenteeism rates approximately 7 percentage

¹We assume the manager's health status does not affect their productivity. This greatly simplifies the analysis at a very low cost, since the quantitative consequences are very small given that opioid addiction is highly concentrated among low-education individuals, that rarely occupy management positions.

points higher than healthy workers. Additionally, according to the National Drug Intelligence Center (NDIC), OUD workers are on average between 17 and 18 percent less productive than healthy workers when both are present.

Our findings suggest that opioid misuse results in a 0.74% reduction in total output, equivalent to \$133 billion in 2015. These losses arise from a combination of reduced worker productivity, including both absenteeism and lower productivity when workers are present, as well as resource misallocation due to hidden information. Specifically, the former contributes to 82.4% of the overall losses, while the latter accounts for the remaining 17.6%, or \$23.4 billion.

One important aspect of our results is that we complement the losses from other channels. The opioid crisis has often been related to considerable socio-economic losses, including individuals leaving the labor force, costs imposed on the healthcare system, crimes induced by opioid misuse, and costs of incarceration. The misallocation channel is orthogonal to these, so that losses can be added to unveil the cost of opioid abuse.

The total cost of opioid misuse in society has been examined in various studies, resulting in a wide range of estimates. Some studies primarily focus on healthcare costs (White et al., 2005; McAdam-Marx et al., 2010; Kirson et al., 2017). For instance, White et al. (2005) finds that the average annual direct healthcare costs for individuals with opioid abuse are more than eight times higher compared to those without opioid abuse. Additionally, the average drug costs for opioid abusers are more than five times higher than for non-abusers. McAdam-Marx et al. (2010) estimates that total costs are significantly higher for opioid abusers (\$14,537) than matched controls (\$8,663). When controlling for baseline characteristics, adjusted costs continue to be higher for opioid abusers (\$23,556 versus \$8,436). Kirson et al. (2017) finds similar results via a matching exercise.

Some studies expand the scope to include additional expenses, such as foregone earnings and costs related to the criminal justice system. In estimating criminal justice costs, Birnbaum et al. (2011) consider total expenditures on police protection, legal and adjudication

processes, correctional facilities, and property losses due to crimes. Each of these factors is multiplied by the proportion associated with opioid abuse. Lost workplace productivity costs are determined based on various factors, including premature death, incarcerations, excess medically related absenteeism costs, and disability costs. Using this approach, [Birnbaum et al. \(2011\)](#) estimate the total societal costs associated with opioid misuse at \$55 billion in 2007. Of this total, approximately 45% can be attributed to workplace costs, while health-care costs accounted for approximately 46%. In a more recent study, [Florence et al. \(2016\)](#) adjusts the criminal justice costs to 2013 and uses the Cost of Injury Reports application and the concept of "lost productive hours" (the average time spent on employment and household production multiplied by the percentage reduction in productivity attributable to drug dependence) to estimate the lost productivity costs. They found that the total costs associated with opioid abuse reached \$78 billion in 2013. 73% of this total cost was attributed to nonfatal consequences, such as healthcare costs, criminal justice costs, and lost productivity costs due to addiction and incarceration. The remaining 27% of the cost was associated with fatality costs, primarily resulting from the loss of potential earnings.

Recent studies incorporate the value of a statistical life to calculate costs related to fatalities resulting from overdoses, leading to a significant increase in the societal cost of opioids. The Council of Economic Advisers (CEA) reports the cost as \$504 billion in 2015, which is equivalent to 2.8% of nominal GDP in that year ([CEA, 2017](#)). Additionally, the Society of Actuaries estimate the cost at \$179 billion in 2018 ([Davenport et al., 2019](#)). Most recently, [Florence et al. \(2021\)](#) unveils that the societal cost of opioid misuse in 2017 amounts to \$1.02 trillion. This estimate incorporates costs associated with the reduced quality of life resulting from opioid use disorder as well as the loss of life due to fatal opioid overdoses. We see our findings as an additional factor to the losses identified in these previous studies.

If we consider the cost estimated by the CEA as a lower bound, incorporating worker-related costs associated with lower productivity and hidden information increases these costs by 26.4%, equivalent to approximately \$133 billion in 2015. Out of this total, \$23.4 billion

can be attributed to misallocation. If we extrapolate using the nominal annual GDP for 2022, the overall loss would escalate to \$901 billion. It is important to note that economic and health studies have generally overlooked these additional losses. Consequently, our findings reveal that these studies considerably underestimate the true extent of losses, as they fail to capture the impact of employer-employee mismatch.

Our research brings two areas of literature together. The first studies the macroeconomic consequences of misallocation from different sources, such as progressive taxation (Restuccia and Rogerson, 2008), housing regulation (Hsieh and Moretti, 2019), size contingent policies (Garicano et al., 2016), and childcare policy (Escobar Salcedo et al., 2020). More generally, there is a large literature that detects misallocation and examines its causes and consequences (for a review, see Restuccia and Rogerson, 2013, 2017). To the best of our knowledge, there is no attempt to estimate the magnitude of the effects of hidden information in the labor market in a general equilibrium framework.

The second strand of literature is focused on empirical studies examining the impacts of the opioid crisis. Given that the opioid crisis has become a major public health problem, there exists a substantial and expanding body of literature on the subject (see Maclean et al., 2020). In addition to the studies mentioned above that quantify the economic costs associated with the opioid crisis, numerous other studies have documented its effects on a wide range of outcome variables, such as the labor market, costs to families, and crime. In particular, the higher rates of opioid prescribing decrease labor force participation rates (Krueger, 2017; Aliprantis et al., 2019; Harris et al., 2020; Deiana and Giua, 2018). Moreover, some studies found that the opioid crisis leads to reduced labor market engagement and to rising rates of disability applications and enrollment (Park and Powell, 2021). Additional opioid exposure is negatively associated with measures of entrepreneurship and small business formation (Rietveld and Patel, 2021). As a social problem, the opioid crisis imposes costs on both the well-being of children living with an adult with OUD (Bullinger and Wing, 2019; Buckles et al., 2023; Gihleb et al., 2020) and increases crime (Maclean et al., 2022).

The rest of the paper is structured as follows. Section 2 describes the key features of opioid misuse that are critical to our analysis. Section 3 builds the model while Sections 4 and 5 provide solutions under the scenarios of full-information and hidden-information, respectively. Section 6 calibrates the model. Section 7 presents the results obtained from the models. Finally, Section 8 concludes.

1.2 The opioid crisis

Our empirical application treats individuals with OUD as unhealthy workers in the model. While the theoretical model is independent of the fact that we apply it to opioid addiction, it is useful to take into consideration the key effects of addiction to opioids that affect productivity for the development of such a model. This section outlines what these effects are.

The characteristics associated with opioid misuse can be summarized as follows: i) a higher prevalence among individuals with lower educational attainment, ii) a higher rate of absenteeism, and iii) lower productivity compared to healthy workers. In this section, we present empirical evidence supporting these observations.

It is important to mention that we do not take into consideration opioid misusers that do not participate in the labor force. Most empirical studies examining the impact of the opioid crisis on the labor market focus on individuals out of the labor force, which are the majority of opioid users. For example, [Krueger \(2017\)](#) and [Aliprantis et al. \(2019\)](#) find that labor force participation declined more in areas of the U.S. with a high rate of opioid prescriptions using variations at the county level.

1.2.1 Opioid Misuse is More Prevalent among Low-Skilled Workers

Opioid misuse across educational attainments is provided by the Substance Abuse and Mental Health Services Administration (SAMHSA)'s restricted online data analysis system (RDAS). Figure 1.1 shows that the opioid misuse rates seem to be comparable to educational attain-

ments below a college degree. Furthermore, [Aliprantis et al. \(2019\)](#) examines various groupings and finds that labor market outcomes for individuals with some college were closer to those of high school graduates than to those of individuals with a college degree. Following this, we classify individuals completing some colleges as individuals without a college degree. Then, the opioid misuse rate is 4.3% among individuals without a college degree and 2.7% among college graduates in 2018-2019. It suggests that the higher the education level is, the lower the misuse rate of the group is.

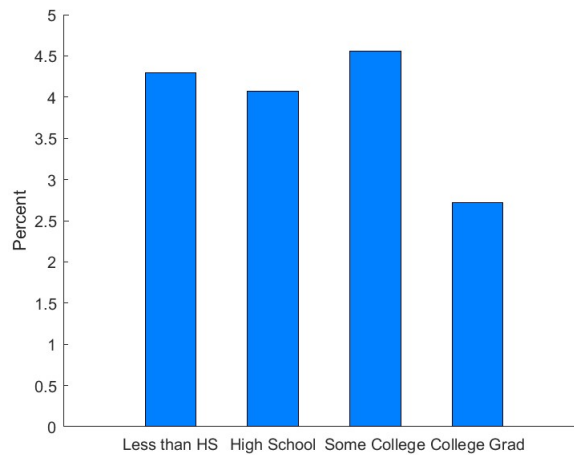


Figure 1.1: Opioid misuse rates across educational attainment

1.2.2 Opioid Misuse is Associated with Higher Rates of Absenteeism

The recent data from NSDUH in 2018 indicates that only about 40% of individuals with OUD had no absence in the last month whereas about 70% of individuals without OUD did not miss any workday. Also, those workers are absent more often, for example, around 40% of them missed more than two workdays.

The average number of days absent from work in the last 30 days is 3.4 days among opioid misusers and 1.1 days among other workers. Using these numbers, we calculate the probability of showing up for work as 0.89 among workers with OUD and 0.96 among other workers. Assuming that the absence rate of healthy worker is a benchmark, workers with OUD tend to be absent more by 7 percentage points than other workers.

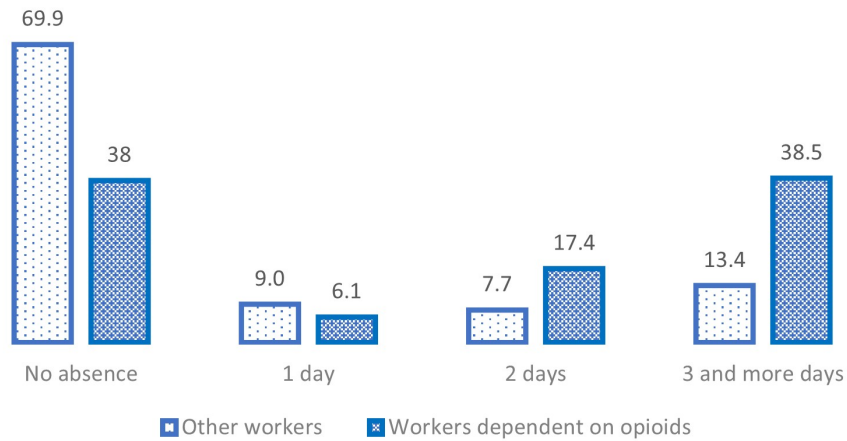


Figure 1.2: Distribution of missing workdays

A similar result has been found by [Van Hasselt et al. \(2015\)](#) showing that the probability of absence from work is higher among addicted workers by 7 percentage points than other workers using the National Survey on Drug Use and Health (NSDUH) in 2008–2012. They estimate a logistic regression model controlling confounding factors such as physical and mental health, the uses of other substances, workplace characteristics, and occupation type, including demographic characteristics.

1.2.3 Opioid Misuse Leads to Lower Productivity

Being addicted to opioids reduces worker productivity. According to a document by the National Drug Intelligence Agency ([NDIC, 2011](#)), productivity declines because of incapacitation or lack of motivation. To estimate the reduction in productivity attributable to illicit drug abuse, NDIC employs the NSDUH data that provides information on income, hours worked, and indicators for drug abuse and dependence. First, they create an index that allows them to estimate the impacts of illicit drugs on productivity in percentage change. Then, they regress it on drug use controlling other factors such as age, marital status, education, and alcohol abuse or dependence. The estimation results suggest a 17 percent reduction

in productivity attributable to drug abuse for males and an 18 percent for females.

[Maclean et al. \(2020\)](#) discusses that opioids could improve or deteriorate labor market outcomes. Specifically, opioids have the potential to enhance worker productivity by effectively managing chronic pain. However, labor market outcomes may deteriorate if addiction or other problems associated with prescription opioids, such as dizziness, nausea, and sedation, diminish worker productivity. Existing research mostly suggests that, overall, the increased utilization of opioids negatively impacts labor market outcomes by reducing labor productivity.

1.3 The Model

The model builds on [Garicano and Rossi-Hansberg \(2004, 2006\)](#) and [Caicedo et al. \(2019\)](#), in which managers match with workers to produce a final good. Production takes place in layers of workers and managers. Workers in the lowest layers produce by solving problems, that differ in their complexity and arrive probabilistically. Every so often, they face a problem they cannot solve. When this happens, the problem is transferred to the layer above, where managers are able to solve more complex problems. If they can solve the problem at hand, production takes place. Otherwise, the problem is sent to the layer above if there is one. Production fails when the executives in top-layers cannot solve the problem.

The choice of framework is not trivial. A crucial aspect to generate misallocation from hidden information about worker productivity is some type of supermodularity between the productivity of the worker and the manager. Other models in which productivity per employee is not relevant would not generate any misallocation, since managers would concern themselves with hiring the right amount of “efficiency units” of labor, regardless of how many workers are employed. This is the case in models that follow a [Hopenhayn \(1992\)](#) type structure, or a model of span of control where workers do not differ in productivity, as in [Lucas \(1978\)](#).

We modify this framework by introducing workers that differ in productivity in ways

that employers cannot observe. For simplicity, we assume this does not affect managers. Changing this assumption would have very little importance from a quantitative point of view, because there are very few opioid misusers with education levels high enough to be managers. The complications in terms of solving the model would be considerable, because some workers or managers would be revealing their health status. To see this, an individual with a relatively high education could become a manager if healthy. If not, he might become a worker instead, thereby revealing his health status. Likewise, a manager with an addiction would be less productive than a manager without one, so they could be offering different wages to workers even when they both have the same education.

1.3.1 No Health Status

We start by describing the model with no health conditions. The model is static. There is a continuum of individuals endowed with one unit of time and a skill level $z \geq 0$. The skill level is distributed by a continuous cumulative distribution function (CDF) $G(z)$ with density $g(z)$ on $[z_0, \infty)$. Production requires a manager paired with at least one worker (we assume only one managerial layer). During production, a problem is presented to each worker. These problems differ in complexity. Tasks with more complex problems are also more valuable, meaning that the output of a production unit that solves a difficult problem is higher than the output of a production unit that solves a simple problem.

Without loss of generality, denote by y both the complexity of a problem and the output associated with solving it. The problem distribution is defined by a CDF $F(y)$ with density $f(y)$ on $[y_0, \infty)$.

Total output depends on the skills of both workers and managers. Each worker faces one problem to solve. If the skill of the worker exceeds the complexity of the problem, production takes place with no managerial time needed. If the complexity of the problem exceeds the worker's skills, the worker sends the problem to the manager. If the manager's skills exceeds this complexity level, production takes place. Otherwise, production is zero.

More specifically, denote by z_w the skill of a worker that is hired by a manager of skill level z_m . If $z_w \geq y$, the worker solves the problem, and output is equal to y . If $z_w < y \leq z_m$, the worker sends the problem to the manager, who spends $\kappa \leq 1$ units of time reviewing it, and produces y units of output. Finally, if $z_w < z_m < y$, the worker passes on the problem to the manager, who spends κ units of time, but is unable to solve it, and thus produces an output equal to 0.

Individuals choose their profession (whether to become a worker or a manager) by maximizing their income. A type z individual solves

$$U(z) = \max\{R(z), w(z)\}$$

where $R(z)$ is income as a manager and $w(z)$ is income as a worker.

The production function of a manager with skill z_m paired with a worker with skill z_w is

$$\int_{y_0}^{z_w} yf(y)dy + \int_{z_w}^{z_m} yf(y)dy = \int_{y_0}^{z_m} yf(y)dy$$

The higher the skill of a manager, the higher the expected output per worker, which does not depend on the skill of the worker.

The number of workers a manager hires depends on the skill level of the workers. Let's consider a manager that hires a worker of skill z_1 . The probability that the worker will not be able to solve the problem at hand is $1 - F(z_1)$, and each requires κ units of managerial time. Since managers have one unit of time endowment, the maximum number of workers with skill z_1 that a manager can hire, n , is obtained by:

$$n = \frac{1}{\kappa(1 - F(z_1))} \tag{1.1}$$

Notice that the larger the z_1 , the larger the $n(z_1)$.

The number of workers depends only on the skills of those workers, not on the skill of the

manager. On the other hand, output per worker does not depend on the skill of the worker, only that of the manager matters.

The problem of a manager is to choose the skill of their workers. Following arguments in [Garicano and Rossi-Hansberg \(2006\)](#), each manager type chooses one and only one worker type. Given wage function, a manager's problem is

$$R(z_m) = \max_{\{z\}} \left[\int_0^{z_m} yf(y)dy \right] n(z) - w(z)n(z)$$

s.t.

$$\kappa(1 - F(z))n(z) = 1$$

The first order condition of this problem is:

$$w'(z) = \left(\int_0^{z_m} yf(y)dy - w(z) \right) \frac{f(z)}{1 - F(z)} \quad (1.2)$$

Solving the differential Equation (1.2) together with the border condition $w(z^*) = R(z^*)$ gives us the continuous wage function, which must satisfy the condition $w'(z^*) < R'(z^*)$ to guarantee the existence of equilibrium. Using the wage function, one could find the rent function as well.

Proposition 1 *There exists a threshold $\kappa^* > 0$ such that if $\kappa \in [0, \kappa^*]$ there exists a unique competitive equilibrium of this economy. In equilibrium the set of managers and the set of workers are connected, the equilibrium exhibits positive sorting, and the earnings function is strictly convex. Furthermore, the equilibrium allocation is efficient.*

Proof: See [Antràs et al. \(2006\)](#)

[Antràs et al. \(2006\)](#) proves that this competitive equilibrium is unique, efficient, and Pareto optimal.

Proposition 2 *Each manager type hires only one worker type, and no two manager types hire the same worker type.*

Proof: See [Antràs et al. \(2005\)](#)

Proposition 1 guarantees the existence of equilibrium that works as in [Lucas \(1978\)](#), that is, there is a threshold skill level determining who becomes a manager and who becomes a worker: high skill individuals become managers, low skill ones workers. Proposition 2 says that each manager type hires only one type of workers, and all those worker types are hired by the same manager type.

1.3.2 Introducing Health Status

We further divide individuals at each skill level into two types: healthy and unhealthy. Let $\rho(z) \in [0, 1]$ be the fraction of unhealthy individuals of skill z . Importantly, this condition is known to the individual suffering from it, but not to other individuals.

A health condition reduces a worker's productivity. At this stage, it is convenient to introduce the concept of individual ability, distinct from skills. We refer to the probability of solving any problem as "ability". For example, a healthy worker with skill level z can solve a problem with probability $F(z)$. That is, a healthy worker of type z is capable of solving a problem y if and only if $y \leq z$, and the probability of receiving such a problem is $F(z)$.

This implies that the ability of a healthy worker is only determined by their skill level. However, for an unhealthy worker, their ability is influenced not only by their skill level but also by the severity of their illness, which can deteriorate their overall ability to work effectively.

When we refer to skills, we are considering observable elements such as education level, which can be easily identified from a resume or other sources. On the other hand, ability is not directly observed since health is unobservable and it specifically emphasizes the loss in productivity resulting from addiction.

Sickness affects individuals by lowering a worker's ability to solve a problem, that is, an unhealthy worker has a lower probability of solving a given problem than a healthy one of the same skill level. Second, a worker with health issues is absent more often due to sickness.

We make the simplifying assumption that addiction does not affect manager productivity. This comes at a relatively low cost, because very few individuals with education levels consistent with that of managers are dependent on opioids. The reason why this assumption simplifies the analysis is that, without it, it could be the case that healthy individuals with skill level \hat{z} become managers, but unhealthy individuals with the same skill level become workers, thus revealing their health status.

Healthy workers are more productive. Sickness prevents workers from concentrating at work, in which case the worker is unable to solve the problem regardless of its complexity. Thus, unhealthy workers with skills z may not be able to solve problems even when $y < z$. Let $\gamma \in [0, 1]$ be the probability of concentrating at work. Then the effective probability of solving a problem for an unhealthy worker of skill level z is:

$$\gamma \cdot F(z) + (1 - \gamma) \cdot 0 = \gamma F(z) \tag{1.3}$$

If the problem is not solved by the worker, either because he cannot concentrate or it is too complex, the manager deals with it. The probability that the manager will need to review the problem when hiring an unhealthy employee of skills z is

$$\gamma \cdot (1 - F(z)) + (1 - \gamma) \cdot 1 = 1 - \gamma F(z) \tag{1.4}$$

Healthy workers show up to work more often. Equations (1.3) and (1.4) are conditional on workers showing up to work. We introduce a further source of losses in unhealthy workers being absent more often than healthy ones. For simplicity, we assume healthy workers are always present. Unhealthy ones are present at work with probability $\beta \in [0, 1]$, implying that the probability of being absent is $1 - \beta$. If a worker does not show up, a manager has

to solve the problem.

Absenteeism both lowers the expected output from an unhealthy worker and increases the number of problems that must be addressed by the manager. The probability that an unhealthy worker with skills z solves the problem at hand is

$$\beta \cdot \gamma F(z) + (1 - \beta) \cdot 0 = \beta \gamma F(z) \quad (1.5)$$

The probability that the manager will have to review the problem is

$$\beta \cdot (1 - \gamma F(z)) + (1 - \beta) \cdot 1 = 1 - \beta \gamma F(z) \quad (1.6)$$

Notice that sickness both lowers the productivity of a worker and increases the time cost of a manager, implying that she can hire a lower number of workers, and therefore produce less. Next, we compute the expected output of a manager hiring healthy workers, and compare this with the expected output of hiring unhealthy workers.

If a manager with skills z_m pairs with a healthy worker with skills z , the expected output per worker is $\int_{y_0}^{z_m} y f(y) dy$, and the time cost associated with this output is $\kappa(1 - F(z))$. Since the manager has one unit of time available to review problems, she will hire workers until she exhausts the unit of time. If hiring healthy workers, she can at most hire $n_h(z)$ workers, where $n_h(z)$ satisfies $n_h(z)\kappa(1 - F(z)) = 1$. The expected number of problems solved is

$$n_h(z)F(z_m) = \frac{F(z_m)}{\kappa(1 - F(z))} \quad (1.7)$$

If she hires unhealthy workers with skills z , equation (1.6) suggests that the number of workers $n_u(z)$ satisfies $n_u(z)\kappa(1 - \beta\gamma F(z)) = 1$. The expected number of problems solved is

$$n_u(z)F(z_m) = \frac{F(z_m)}{\kappa(1 - \beta\gamma F(z))} \quad (1.8)$$

Since $\beta\gamma < 1, n_u(z) < n_h(z)$, so a manager would hire more workers if they are healthy, producing more output.

1.4 Solving the Model Under Full Information

We solve the model under two different assumptions. The first one, which we call the social planner problem, assumes perfect information on who is unhealthy. The second one, market equilibrium, assumes that managers cannot tell healthy from unhealthy workers, and as such must pay them both the same wage, even when their productivities differ.

Under full information, the social planner knows the health status of each individual and their skill level. The planner organizes these individuals to maximize total production. To do this, the planner first assigns individuals to be workers or managers. Second, the planner pairs workers with managers.

The planner problem is to maximize total output given the labor endowment, that is, the planner solves

$$\begin{aligned} \max_{z^*, \varphi_u(z), \varphi_h(z)} & \int_{z_0}^{z^*} \int_{y_0}^{\varphi_h(s)} y f(y) dy g(s) (1 - \rho(s)) ds + \\ & \int_{z_0}^{z^*} \int_{y_0}^{\varphi_u(s)} y f(y) dy g(s) \rho(s) ds \end{aligned} \quad (1.9)$$

subject to

$$\begin{aligned} G(\max\{\varphi_u(z), \varphi_h(z)\}) - G(\min\{\varphi_u(z_0), \varphi_h(z_0)\}) &= \kappa \int_{z_0}^z (1 - \beta\gamma F(s)) g(s) \rho(s) ds + \\ & \kappa \int_{z_0}^z (1 - F(s)) g(s) (1 - \rho(s)) ds, \quad \text{for all } z \in [z_0, z^*] \end{aligned} \quad (1.10)$$

The objective function is total output. The choice variables are a threshold z^* that determines who becomes a manager and who a worker; a function $\varphi_u(z)$ that pairs managers of skill $\varphi_u(z)$ with unhealthy workers of skill z ; and a function $\varphi_h(z)$ that pairs managers of

skill $\varphi_h(z)$ with healthy workers of skill z .

For any manager of skills $\varphi_h(z)$, the expected output of hiring a single healthy worker with skill z is $\int_{y_0}^{\varphi_h(z)} yf(y)dy$. Since there are $g(z)(1-\rho(z))$ healthy, z -type workers, total expected output by managers of skills $\varphi_h(z)$ hiring healthy workers is $\int_{y_0}^{\varphi_h(z)} yf(y)dyg(z)(1-\rho(z))ds$. Adding across all manager types paired healthy workers constitutes the first line in equation (1.9). Similarly, the second line relates to unhealthy workers.

The constraint is the time constraint faced by managers. Each manager has one unit of time available. The left hand side of constraint (1.10) is the managerial time available for each skill type. For each $z \leq z^*$, there is at least one manager type that matches with a z worker.

Individuals with skill $z \leq z^*$ become workers, and $z^* = \min\{\varphi_u(z_0), \varphi_h(z_0)\}$, so $G(\min\{\varphi_u(z_0), \varphi_h(z_0)\})$ are not available as managers. The highest skill managers hiring a z -type worker is $\max\{\varphi_u(z), \varphi_h(z)\}$. Thus, for any $z \geq z^*$, the managerial time available is $G(\max\{\varphi_u(z), \varphi_h(z)\}) - G(\min\{\varphi_u(z_0), \varphi_h(z_0)\})$.

The right hand side is the amount of problems expected not to be solved by these workers, and κ is the time needed to address each of these unsolved problems. Constraint (1.10) is the analogous for healthy workers.

1.4.1 Solution

We start by identifying the threshold z^* that separates workers from managers. To do that we focus on the time constraint. Total managerial time is equal to $1 - G(z^*)$, and this time must be enough to solve all the problems workers are not able to solve. Thus, z^* solves

$$1 - G(z^*) = \kappa \int_{z_0}^{z^*} (1 - F(s))g(s)(1 - \rho(s))ds + \kappa \int_{z_0}^{z^*} (1 - \beta\gamma F(s))g(s)\rho(s)ds \quad (1.11)$$

We next solve for the matching functions $\varphi_u(z)$ and $\varphi_h(z)$. As in [Antràs et al. \(2006\)](#), the optimal mapping exhibits positive sorting. This also holds in the present model. We

show this in the next proposition.

Proposition 3 *In an optimal allocation, the mapping function is strictly increasing on the worker's ability.*

Proof: The proof works by contradiction, so that if the assignment is not strictly increasing, the total output generated is lower than if it were.

Consider two managers with different skill levels, z_{m1} and z_{m2} , such that $z_{m1} < z_{m2}$. Each worker assigned to z_{m1} manager produces the expected value of output $Y_{m1} = \int_{y_0}^{z_{m1}} yf(y)dy$. Similarly, $Y_{m2} = \int_{y_0}^{z_{m2}} yf(y)dy$. Note that $Y_{m1} < Y_{m2}$.

Next consider the number of workers each of these managers hire. Let x_1 be the proportion of unsolved problems by workers paired with a manager type z_{m1} , and define x_2 analogously. Notice that the ability of a worker (not the skill) determines x_1 and x_2 . The number of workers assigned to a manager is independent of the manager's skills and such that

$$n_1 x_1 \kappa = 1$$

$$n_2 x_2 \kappa = 1$$

Toward a contradiction, suppose that the assignment is not increasing on worker's ability so that $x_1 < x_2$. This implies $n_1 > n_2$.

The total output produced by this assignment is

$$Y_- = Y_{m1}n_1 + Y_{m2}n_2$$

Next consider the alternative, in which higher skilled managers are paired with higher skilled workers. The output produced by the alternative is

$$Y_+ = Y_{m1}n_2 + Y_{m2}n_1$$

The difference in output is

$$Y_- - Y_+ = Y_{m1}n_1 + Y_{m2}n_2 - Y_{m1}n_2 - Y_{m2}n_1 = \underbrace{(Y_{m1} - Y_{m2})}_{<0} \underbrace{(n_1 - n_2)}_{>0} < 0$$

The output under the non-increasing match is lower than that under the increasing match, generating a contradiction. \square

Corollary 1 *For a given worker skill, unhealthy workers are assigned to lower skilled managers than healthy workers.*

Proof: This follows from Proposition 3, since the ability of an unhealthy worker is lower than that of a healthy worker of the same skill type.

Corollary 2 *A manager is indifferent between hiring a healthy worker with skills z_h and an unhealthy worker with skills z_u if there exist z_u and z_h such that*

$$F(z_h) = \beta\gamma F(z_u)$$

Proof: The optimal pairing depends on the skill of the manager and the ability of the worker, and an unhealthy worker of skill z_u has the same ability as a healthy worker with skill z_h .

Corollary 2 establishes that some managers are indifferent between hiring a relatively low skilled healthy worker or a relatively high skilled unhealthy worker, as long as their ability to solve problems are the same.

This corollary also implies an important result: for the lowest skilled managers, there might not exist a healthy worker with low enough skills to match the ability of the lowest skilled unhealthy workers. Similarly, there are no unhealthy workers with skills high enough to match the ability of the highest skilled healthy workers.

As a consequence, there exist two additional relevant regions. Among the top skilled healthy workers (close to the threshold z^*) there is no unhealthy worker with the same ability as a healthy worker. And among low skilled workers (close to z_0) there is no healthy worker with the same ability as a z_0 unhealthy worker. Thus, managers on both ends will not be indifferent between a relatively high skilled unhealthy worker and a lower skilled unhealthy one. This is useful to describe the equilibrium, which we do next.

We start by imposing the following boundary conditions: $\lim_{z \rightarrow z^*} \varphi_h(z) = \infty$ and $\varphi_u(z_0) = z^*$. The former condition guarantees that the highest-ability workers are assigned to the highest-skilled managers. The latter shows the assignment of the lowest-ability workers to the lowest-skilled managers.

Starting with the upper boundary condition and applying a positive sorting to the resource constraint, we find an upper part of the mapping function for the highest ability workers in which all the workers are the most skilled and healthy.

High productivity workers. Let z_1 be the skill level of a healthy worker with the same ability as the highest-skilled unhealthy worker. The following equation determines z_1 :

$$F(z_1) = \beta\gamma F(z^*) \tag{1.12}$$

All healthy workers with skills $z \in (z_1, z^*]$ have higher ability than any unhealthy worker. Thus, our first relevant range is $(z_1, z^*]$ for healthy workers. This allows us to identify the mappings $\varphi_h(z)$ for all $z \in (z_1, z^*]$ through the resource constraint, that must satisfy

$$1 - G(\varphi_h(z)) = \kappa \int_z^{z^*} (1 - F(s))g(s)(1 - \rho(s))ds \quad \forall z \in (z_1, z^*]$$

Low productivity workers. Analogously to the top skill range, there is a bottom skill range where managers only match unhealthy workers, because there is no healthy worker with the same ability as the unhealthy workers in this range. Let z_2 describe this range, for

all $z \in [z_0, z_2)$, where z_2 is defined by

$$F(z_0) = \beta\gamma F(z_2)$$

To find the matching function in this range, $\varphi_u(z)$ must satisfy

$$G(\varphi_u(z)) - G(z^*) = \kappa \int_{z_0}^z (1 - \beta\gamma F(s))g(s)\rho(s)ds \quad \forall z \in (z_0, z_2]$$

Middle productivity workers. For any healthy worker of skills $z \in (z_2, z_1)$, there is an unhealthy worker with a higher skill level who has the same ability to solve problems. More precisely, the manager is indifferent between pairing with a healthy worker with skills z_h and pairing with an unhealthy worker with skills z_u , where $F(z_h) = \beta\gamma F(z_u)$.

Once again, we rely on the constraint (1.10) to find the optimal mapping. Take an unhealthy worker with skills $z \in (z_1, z_2)$. He is mapped to a manager of skills $\varphi_u(z) \in (z^*, \varphi_u(z^*))$. The mapping $\varphi_u(z)$ is then described by

$$G(\varphi_h(z)) - G(\varphi_u(z_2)) = \kappa \left[\int_{z_2}^z (1 - \beta\gamma F(s))g(s)\rho(s)ds + \int_{z_0}^{h(z)} (1 - F(s))g(s)(1 - \rho(s))ds \right]$$

where $h(z)$ is implicitly defined by

$$F(h(z)) = \beta\gamma F(z)$$

And

$$\varphi_h(h(z)) = \varphi(z)$$

Figure 1.3 summarizes the solution for the optimal assignment function that we have shown above.

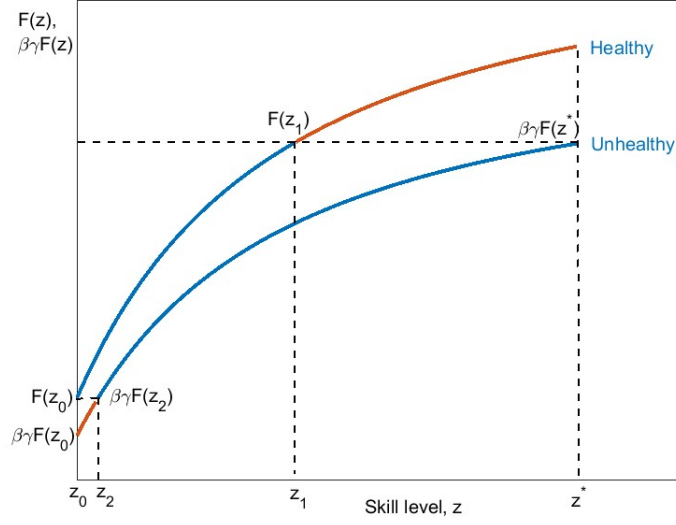


Figure 1.3: Optimal assignment function

Given the mappings $\varphi_l(z)$ and $\varphi_u(z)$ and the thresholds z^* , z_1 and z_2 , it is straightforward to compute total output:

$$Y_{FI} = Y_1 + Y_2 + Y_3 \quad (1.13)$$

$$Y_1 = \int_{z_1}^{z^*} \int_{y_0}^{\varphi_h(z^*)} y f(y) dy g(s) (1 - \rho(s)) ds \quad (1.14)$$

$$Y_2 = \int_{z_2}^{z^*} \int_{y_0}^{\varphi_u(z^*)} y f(y) dy g(s) \rho(s) ds + \int_{z_0}^{z_1} \int_{y_0}^{\varphi_h(z_1)} y f(y) dy g(s) (1 - \rho(s)) ds \quad (1.15)$$

$$Y_3 = \int_{z_0}^{z_2} \int_{y_0}^{\varphi_u(z_2)} y f(y) dy g(s) \rho(s) ds \quad (1.16)$$

where Y_{FI} is the total output with full information, Y_1 is the output produced by the highest skilled managers paired with the highest skilled, healthy workers, Y_2 is the output of both healthy and unhealthy workers paired with middle skilled managers, and Y_3 is the production of the lowest skilled, unhealthy workers paired with the lowest skilled managers.

1.4.2 Decentralized Equilibrium

To find the competitive equilibrium, we follow [Caicedo et al. \(2019\)](#) in solving for the Pareto optimal allocation, and then deriving a wage function that supports that allocation in equilibrium.

Let the payment to an individual of skills z be $w_w(z)$ if worker, and $w_m(z)$ if manager. This system of payments must be profit maximizing, since the compensation to managers maximizes profits in a decentralized equilibrium. These profits are:

$$\Pi(z_w, z_m) = \frac{1}{\kappa(1 - F(z_w))} \int_{y_0}^{z_m} yf(y)dy - w_m(z_m) - \frac{1}{\kappa(1 - F(z_w))} w_w(z_w) \quad (1.17)$$

where $z_m = \varphi(z_w)$, for all $z_w \in [z_0, z^*]$. Note that we have used the fact that the measure of employees hired is $\frac{1}{\kappa(1 - F(z_w))}$ (see Equation 1.1).

We next find the payment function that maximizes profits when the pair of managers and workers is optimal. In other words, the payment function that would make z_w and $z_m = \varphi(z_w)$ optimal choices. To do this, it is convenient to work with output per worker. Since the production function has constant returns to scale, maximizing output per worker yields the same payment structure as maximizing total output. This yields the following maximization problem

$$\max_{z_m, z_w} \int_{y_0}^{z_m} yf(y)dy - \kappa(1 - F(z_w))w_m(z_m) - w_w(z_w) \quad (1.18)$$

With first order conditions:

$$w'_w(z_w) = \kappa f(z_w)w_m(z_m) \quad (1.19)$$

$$w'_m(z_m) = \frac{1}{\kappa(1 - F(z_w))} z_m f(z_m) \quad (1.20)$$

A wage function must satisfy conditions (1.19) and (1.20) to support the social planner

solution as a decentralized equilibrium, for any pair z_w and $z_m = \varphi(z_w)$. These are differential equations of order one. Consequently, a solution requires two border conditions, one per equation. The first is that profits are zero for the pairing z_0, z^* , that is,

$$\Pi(z_0, z^*) = \frac{1}{\kappa(1 - F(z_0))} \int_{y_0}^{z^*} yf(y)dy - w_m(z^*) - \frac{1}{\kappa(1 - F(z_0))} w_w(z_0)$$

The second is that a worker of skills z^* is indifferent between becoming a manager or a worker, that is,

$$w_w(z^*) = w_m(z^*)$$

Note that while we do not impose the zero profit condition for pairings other than z_0, z^* , this holds for all pairings. To see this, note that one of the first order conditions is $\frac{\partial \Pi(z_w, z_m)}{\partial z_m} = 0$, so that profits do not change as manager type changes. Thus, if $\Pi(z_0, z^*) = 0$, then $\Pi(z_w, z_m) = 0$ for all z_m .

Next, re-introduce health. There are now two types of profits: those involving healthy workers, and those involving unhealthy workers:

$$\Pi_h(z_h, \varphi_h(z_h)) = \frac{1}{\kappa(1 - F(z_h))} \int_{y_0}^{\varphi_h(z_h)} yf(y)dy - w_m(z_m) - \frac{1}{\kappa(1 - F(z_h))} w_h(z_h) \quad (1.21)$$

$$\Pi_u(z_u, \varphi_u(z_u)) = \frac{1}{\kappa(1 - \beta\gamma F(z_u))} \int_{y_0}^{\varphi_u(z_u)} yf(y)dy - w_n(z_n) - \frac{1}{\kappa(1 - \beta\gamma F(z_u))} w(z_u) \quad (1.22)$$

The solution involves four payment functions: one for healthy individuals ($w_h(z)$), one for unhealthy ($w_u(z)$), one for managers hiring healthy workers ($w_m(z)$) and one for managers hiring unhealthy workers ($w_n(z)$).

The first order conditions to maximize profits per worker yield

$$w'_h(z_h) = \kappa f(z_h) w_m(z_m) \quad (1.23)$$

$$w'_u(z_u) = \beta \gamma \kappa f(z_u) w_n(z_n) \quad (1.24)$$

$$w'_m(z_m) = \frac{1}{\kappa(1 - F(z_h))} z_m f(z_m) \quad (1.25)$$

$$w'_n(z_n) = \frac{1}{\kappa(1 - \beta \gamma F(z_u))} z_n f(z_n) \quad (1.26)$$

These equations provide the slopes of the payment functions. Notice that all these slopes are positive, meaning that the higher the skills, the larger the payment given health and occupational status. To identify the actual functions, we need one fixed point per function. These points satisfy value matching conditions that guarantee that the payment function is continuous and increasing in ability, irrespective of health or occupational status.

A procedure to identify these functions is as follows. Start by assuming $w_m(z)$ and $w_n(z)$ are known. The border conditions to describe $w_h(z)$ is

$$w_h(z^*) = w_m(z^*) \quad (1.27)$$

Equation (1.27) must hold to guarantee that a healthy individual with skills z^* is indifferent between becoming a manager or a worker. It is a value matching condition between healthy workers and manager.

Similarly, a manager hiring a healthy worker with skills z_1 would also be willing to hire an unhealthy worker with skills z^* , since they both have the same ability. Thus, the border condition to determine the function $w_u(z)$ is

$$w_u(z^*) = w_h(z_1) \quad (1.28)$$

Next determine the functions $w_m(z)$ and $w_n(z)$. Start with $w_n(z)$. The payments to managers must exhaust all resources that result from the pairing. Thus, the border condition to

determine $w_n(z)$ is

$$\Pi_u(z_0, z^*) = 0 \tag{1.29}$$

Finally, the function $w_m(z)$ is determined by the fact that a manager is indifferent between hiring a healthy worker with skills z_0 or an unhealthy worker with skills z_2 . That is,

$$w_m(\varphi_h(z_0)) = w_m(\varphi_u(z_2)) \tag{1.30}$$

The following proposition states that given these payment functions, the competitive equilibrium is optimal.

Proposition 4 *A competitive equilibrium where equations (1.23) through (1.30) are satisfied is optimal.*

Proof: See Appendix [A.1.2](#).

Thus, any change in welfare when comparing the decentralized equilibrium under hidden information with the Pareto optimal is due to misallocation, and not due to the decentralized equilibrium being sub-optimal.

1.5 Market Equilibrium under Hidden information

Under hidden information, managers are able to observe a worker's skill level z , but not his health status. Since employers cannot distinguish between healthy and unhealthy workers, they choose a skill level and number of workers to hire by maximizing expected profits. In this section, we build a pooling equilibrium such that managers offer a single contract $w(z)$ workers with skill level z , independently of health status.

If a manager hires n workers with skill z , then $\rho(z)n$ workers are unhealthy and each of those workers might ask her to review a problem with probability $1 - \beta\gamma F(z)$. This takes

$\kappa[1 - \beta\gamma F(z)]\rho(z)n$ units of her time. But healthy workers require less managerial time, consuming $\kappa[1 - F(z)](1 - \rho(z))n$ units of time. Hence, this manager faces the following time constraint:

$$\kappa[(1 - \rho(z))(1 - F(z)) + \rho(z)(1 - \beta\gamma F(z))]n = 1$$

Consider the output of a manager of skills z_m hiring a worker with skills z . The expected output from this match is $\int_{y_0}^{z_m} yf(y)dy$, regardless of the health status of the worker. However, an unhealthy worker takes up more of the manager's time, so the manager can hire fewer workers if they are unhealthy, reducing total output.

A manager with skill z_m maximizes her rent by choosing the skill level of workers, z , and the number of workers, n , given her time constraint. The manager solves:

$$R(z_m) = \max_{z, n} \left[\int_{y_0}^{z_m} yf(y)dy - w(z) \right] n \quad (1.31)$$

subject to

$$\kappa[1 - (1 - \rho(z))F(z) - \rho(z)\beta\gamma F(z)]n = 1 \quad (1.32)$$

where $R(z_m)$ is the profit of a manager. She pays the wage determined by labor market equilibrium. Obtaining the number of workers from the constraint and substituting it into the objective function, the optimization problem becomes an unconstrained problem:

$$R(z_m) = \max_z \frac{\int_{y_0}^{z_m} yf(y)dy - w(z)}{\kappa[1 - (1 - \rho(z))F(z) - \rho(z)\beta\gamma F(z)]} \quad (1.33)$$

The first order condition of this problem is:

$$w'(z) = \kappa R(z_m) \left[(1 - \rho(z) + \rho(z)\beta\gamma)f(z) - (1 - \beta\gamma)\rho'(z)F(z) \right] \quad (1.34)$$

Appendix [A.1.1](#) shows the solution to this problem.

The labor market-clearing condition is:

$$\int_{z_0}^z g(s)ds = \int_{\varphi(z_0)}^{\varphi(z)} n(\varphi^{-1}(s))g(s)ds \quad \text{for all } z \leq z^* \quad (1.35)$$

where the left-hand side is the supply of workers up to skill z , and the right-hand side is the demand for workers by managers up to skill $\varphi(z)$. Market clearing is guaranteed when supply equals demand for each skill level of workers $z < z^*$.

To find an equilibrium assignment function, totally differentiate equation (1.35) with respect to z_p and solve for $\varphi'(z)$ to obtain:

$$\varphi'(z) = \frac{1}{n(\varphi^{-1}(\varphi(z)))} \frac{g(z)}{g(\varphi(z))} \quad (1.36)$$

Using (1.36) and the boundary condition $\varphi(z_0) = z^*$, we find:

$$G(\varphi(z)) = G(z^*) + \int_{z_0}^z \kappa \left[1 - (1 - \rho(z)) F(z) - \rho(z)\beta\gamma F(z) \right] g(z)dz \quad \text{all } z \in [z_0, z^*] \quad (1.37)$$

which is an equilibrium assignment function:

$$\varphi(z) = G^{-1} \left(G(z^*) + \int_{z_0}^z \kappa \left[1 - (1 - \rho(z)) F(z) - \rho(z)\beta\gamma F(z) \right] g(z)dz \right) \quad \text{all } z \in [z_0, z^*] \quad (1.38)$$

Using equation (32) and the upper boundary condition $\varphi(z^*) \rightarrow \infty$, we solve the following equation for the threshold z^* :

$$1 = G(z^*) + \int_{z_0}^{z^*} \kappa \left[1 - (1 - \rho(z)) F(z) - \rho(z)\beta\gamma F(z) \right] g(z)dz \quad (1.39)$$

As in the full information case, the mapping of workers to managers is strictly increasing, in the sense that the higher the skills of the manager, the higher the skills of the worker. We show the proof for this positive sorting in the Appendix A.1.1. However, it is easy to see

that $\varphi'(z) > 0$ from equation (1.36).

The equation (1.39) is exactly the same as the resource constraint in the social problem, which is represented by equation 1.11. This indicates that the threshold to become a manager is identical in both the full information and hidden information equilibria.

Proposition 5 *The threshold to become a manager, z^* , is the same under the full information and hidden information equilibria.*

Proof: Recall equation 1.11 in the full information model:

$$1 - G(z^*) = \kappa \int_{z_0}^{z^*} (1 - F(s))g(s)(1 - \rho(s))ds + \kappa \int_{z_0}^{z^*} (1 - \beta\gamma F(s))g(s)\rho(s)ds$$

A social planner chooses the threshold for managerial positions such that the total managerial time in the economy equals the total time required to review all the problems left unsolved by all workers. In the full information equilibrium, this equation can be found by adding up the resource constraints in three ranges for high, middle and low productivity workers, respectively:

$$\begin{aligned} 1 - G(\varphi_h(z_1)) &= \kappa \int_{z_1}^{z^*} (1 - F(s))g(s)(1 - \rho(s))ds \\ G(\varphi_h(z_1)) - G(\varphi_u(z_2)) &= \kappa \left[\int_{z_2}^{z^*} (1 - \beta\gamma F(s))g(s)\rho(s)ds + \int_{z_0}^{z_1} (1 - F(s))g(s)(1 - \rho(s))ds \right] \\ G(\varphi_u(z_2)) - G(z^*) &= \kappa \int_{z_0}^{z_2} (1 - \beta\gamma F(s))g(s)\rho(s)ds \end{aligned}$$

Therefore, the aggregate resource constraint in the full information equilibrium is exactly the same as the equation (1.39) above. In the hidden information equilibrium, the time constraint is satisfied for each manager. However, the equation (1.39) means that the highest-skilled workers z^* will be hired by the highest-skilled manager, where $\varphi_h(z^*) \rightarrow \infty$ and

$G(\varphi_h(z^*)) = 1$. Therefore, we obtain:

$$1 = G(z^*) + \int_{z_0}^{z^*} \kappa \left[1 - (1 - \rho(z)) F(z) - \rho(z) \beta \gamma F(z) \right] g(z) dz$$

Thus, the boundary condition in the hidden information equilibrium is equivalent to the resource constraint in the economy with full information. Therefore, the solutions for the threshold z^* are the same under both full information and hidden information equilibria.

Although the hidden information does not change occupational choices, it leads to distortions in the assignment functions, the earnings functions and the number of workers hired by each manager.

Given an equilibrium assignment function together with the condition $w(z^*) = R(z^*)$, we now are able to find the wage function, $w(z)$, and rent function, $R(\varphi(z))$. Appendix [A.1.1](#) shows the solutions explicitly.

1.5.1 A Competitive Equilibrium

A competitive equilibrium in this economy is a wage function $w(z)$, a rent function $R(z)$, an assignment function $\varphi(z)$, and occupational choice decisions of individuals summarized by z^* such that (1) managers maximize their rents; (2) individuals maximize their utility; and (3) labor markets clear for every skill level.

Appendix A shows the detailed steps for finding all the elements of an equilibrium. Given these, it is straightforward to compute the equilibrium output under hidden information.

$$Y_{HI} = \int_{z_0}^{z^*} \left(\int_{y_0}^{\varphi(z^*)} y f(y) dy \right) g(z) dz \quad (1.40)$$

The output loss due to information hidden from employers by comparing outputs of the

social planner's and the competitive equilibrium is:

$$\text{Output Loss} = \frac{Y_{FI} - Y_{HI}}{Y_{FI}} \quad (1.41)$$

The reason for misallocation due to hidden information is that the optimal assignment functions and the number of workers assigned to managers differ between the market equilibrium and the social planner problem.

1.6 Separating Equilibrium

In this section, we examine the existence of an equilibrium where workers choose to reveal their identity. The approach focuses on the difference in behavior observed between healthy and unhealthy workers. Mainly, the unhealthy workers are more likely to miss days at work. Thus, by penalizing absences, one can support a separating equilibrium.

Let's start with the problem faced by the highest-skilled managers. These managers, when hiring workers with skill level z^* , know that there are no unhealthy workers with the same ability as healthy workers, given that $F(z^*) > \beta\gamma F(z^*)$, where β represents the probability of being present at work, and $1 - \gamma$ represents the loss of productivity. Therefore, they try to hire only the highest-skilled healthy workers. One potential contract that these managers could offer to workers z^* is the following:

$$w(z^*) = \begin{cases} w_h(z^*) & \text{if a worker } z^* \text{ always shows up} \\ w_{min} & \text{if a worker } z^* \text{ doesn't show up sometime} \end{cases} \quad (1.42)$$

This contract is achievable since a manager can observe both skill level and an absence. In addition, healthy workers accept this offer since they have no problem with absenteeism. However, unhealthy worker z^* would not accept it if and only if his expected wage of accepting

this contract is lower than a wage that other managers offer to him. That is:

$$\beta w_h(z^*) + (1 - \beta)w_{min} < w_u(z^*) \quad (1.43)$$

Thus, the highest skilled managers must choose w_{min} such that the condition (1.43) holds. There is no such contract that supports the separating equilibrium if this condition doesn't hold. That is:

$$\beta \geq \frac{w_h(z^*) - w_{min}}{w_u(z^*) - w_{min}}$$

In the middle range, managers know that some higher skilled unhealthy workers have the same ability as low skilled healthy ones. Therefore, they try to hire either unhealthy but higher skilled workers or healthy but lower skilled workers. Therefore, those managers could offer a contract:

$$w(z_1) = \begin{cases} w_h(z_1) & \text{if a worker } z_1 \text{ and always shows up} \\ w_u(z^*) & \text{if a worker } z^* \text{ doesn't show up sometime} \\ w_{min} & \text{if a worker } z_1 \text{ doesn't show up sometime} \end{cases} \quad (1.44)$$

where $w_u(z^*) = w_h(z_1)$ since an unhealthy worker z^* possesses the same ability level as a healthy worker z_1 . For simplicity, we assume that w_{min} is the same across all contracts.

An unhealthy worker z^* definitely accepts this contract since his expected wage of working with the highest skilled managers is lower than this wage offer $w_u(z^*)$ which is the highest expected wage they could receive. In order to let an unhealthy worker z_1 not choose this contract, the following condition must hold:

$$\beta w_h(z_1) + (1 - \beta)w_{min} < w_u(z_1) \quad (1.45)$$

For the lowest skilled managers who are hiring workers with the skill level z_2 , they know that all workers are unhealthy, and thus offer only a wage contract $w_u(z)$ where $z_u \in [z_0, z_2]$.

Proposition 6 *The separating equilibrium exists iff the following conditions hold:*

$$\beta < \frac{w_h(z^*) - w_{min}}{w_u(z^*) - w_{min}} \quad (1.46)$$

and

$$\beta < \frac{w_h(z_1) - w_{min}}{w_u(z_1) - w_{min}} \quad (1.47)$$

1.7 Application to the Opioid Crisis

This section calibrates the model to study the reduction in output from the opioid crisis. Henceforth, unhealthy individuals are OUD individuals, and healthy individuals are everyone else.

The key elements of the calibration relate to the effects of opioids on the share of opioid misusers by skills, the productivity loss associated with addiction, and the absenteeism rates among those workers. We describe the calibration process next.

The share of individuals misusing opioids is a function of their skills. To calibrate this function, we use data on opioid misuse rates in the past year across educational attainment in the Substance Abuse and Mental Health Services Administration (SAMHSA) restricted online data analysis system (RDAS) in 2018-2019. As mentioned before, we classify individuals completing some college as individuals without a college degree. The opioid misuse rate is 4.3% among individuals without a college degree and 2.7% among college graduates. The higher the education level is, the lower the misuse rate of the group is.

In this data, 67.6% of individuals have less than a college degree. Therefore, it is reasonable to assume that the bottom 67.6% of the skill distribution reflects the characteristics of individuals without a college degree, and their probability of opioid misuse is 4.3%. Similarly,

individuals in the next 15.4% of the skill distribution show the characteristics of college graduates, so we assign a probability of 2.7%. Individuals in the top 17% of the skill distribution are managers with zero probability of opioid misuse.²

Empirically, a Pareto distribution can closely match the data, so we assume the following functional form for $\rho(z)$.

$$\rho(z) = \lambda K^\lambda z^{-\lambda-1} \tag{1.48}$$

The parameters λ and K are calibrated to minimize the sum of squared differences between the data and the model. We compare the model distribution with that in the data in Figure 1.4.

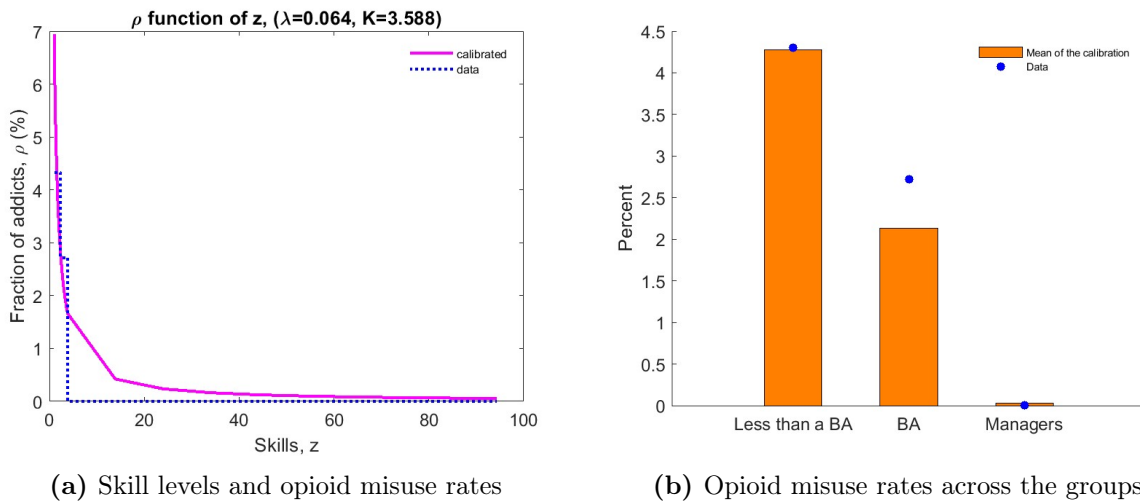


Figure 1.4: Calibration of ρ function

Another important component of the calibration is the parameters for the skill and the problem distributions. Both skill and problem distributions are essential because they determine the distribution of productivity and earnings. It is convenient for our purpose to

²Ideally, one should use only misusers in the labor force, but this data is hard to come by. Currie and Schnell (2018) argue that the majority of working age opioid abusers are working, based in part on the fact that “nearly 85% of opioids prescribed for working age people are paid for by private health insurance, which is overwhelmingly employer provided.”

assume that $G(\cdot)$ and $F(\cdot)$ are Pareto distributions as follows:

$$G(z) = 1 - \left(\frac{z_0}{z}\right)^\theta; \quad F(y) = 1 - \left(\frac{y_0}{y}\right)^\theta$$

Normalizing the location parameter of skill distribution at $z_0 = 1$, we calibrate the location parameter y_0 for the problem distribution. If $y_0 < 1$, the lowest skilled workers z_0 can solve some easy problems and produce some output without a manager's help. The larger the curvature parameter θ is, the more the hard problems and the highly skilled individuals are available in an economy. Using comparative analyses, [Caicedo et al. \(2019\)](#) shows how θ , y_0 , and κ affect the assignment and wage functions and thus the earnings distribution (see [Caicedo et al. \(2019\)](#) for details). We follow them and calibrate these parameters jointly to match the Lorenz curve in the U.S. in 2010 for our model using yearly nominal wages of full-time workers for the whole population in the 2010 census data from IPUMS-USA.

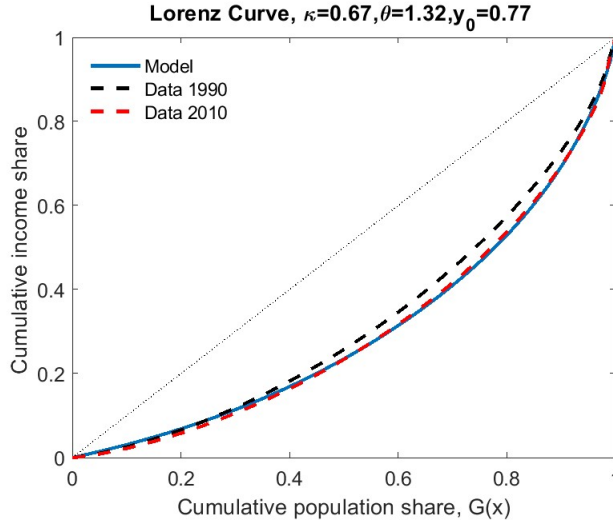


Figure 1.5: Fit of the Lorenz curve

Figure 1.5 displays the Lorenz curves for the wage distribution in the 1990 and 2010 U.S. data, along with the model-based curve using the calibrated parameters. The model-based wage distribution plots $\int_{z_0}^x w(z)dG(z)$ against $G(x)$ for $x \in [0, z^*]$ and $\int_{z_0}^{z^*} w(z)dG(z) + \int_{z^*}^x R(z)dG(z)$ against $G(x)$ for $x \in [z^*, \infty)$.

The calibration of θ , y_0 and κ is as follows. Since the allocation we observe is one

under hidden information, we run the model for a competitive equilibrium under the hidden information. The model provides us with the earnings function and its distribution used to calculate the Lorenz curve in the model. Then, minimizing the sum of the squared differences between the data and the model, parameters of interest are found. Figure 1.5 shows the fit.

Table 1.1: Calibrated parameters and targets matched in the model

Parameter	Target	Value
z_0	Normalize	1
y_0	Lorenz curve 2010	0.77
θ	Lorenz curve 2010	1.32
κ	Lorenz curve 2010	0.67
λ	Data, NSDUH, 2018-2019	0.06
K	Data, NSDUH, 2018-2019	3.59
γ	NDIC, 2011	0.83
β	NSDUH, 2018	0.93

The remaining parameters are set directly according to the data or previous papers. First, the absence rate is from NSDUH in 2018 based on questions about the number of absences in the last 30 days and their reasons. We calculate the absence rate of workers with OUD relative to that of other workers. This calculation suggests that the probability of showing up for work is 0.93 for workers dependent on opioids (see Section 2.2 for details). The loss of productivity is 17-18% which comes from the report by NDIC (2011), so we set the productivity of an unhealthy worker as 17% lower than that of a healthy worker. The parameter values and targets are summarized in Table 3.1.

1.8 Results

To quantify the impact of hidden information on output, we compare the total output in equilibrium with hidden information and two benchmarks. The first benchmark represents a scenario where no workers are affected by health issues, which we refer to as potential output. The second benchmark considers the presence of unhealthy workers but assumes that the social planner possesses complete knowledge of who is unhealthy and utilizes this information to match workers with managers. We refer to this scenario as the full-information output.

The first comparison involves quantifying the total output loss resulting from the presence of unhealthy workers by calculating the difference between the potential output and the output under hidden information. The second comparison focuses on determining the portion of this loss that can be attributed to hidden information by comparing the full-information output with the output under hidden information.

	Output Loss (%)		Loss due to Hidden Information
	Relative to Full Information	Relative to Potential Output	
	0.13	0.74	17.6
<i>Decomposition</i>			
Absenteeism only	0.02	0.24	8.3
Productivity only	0.08	0.54	14.8

Table 1.2: Main Results

The opioid crisis contributes to a GDP loss of 0.74%, with 17.6% of this loss attributed to misallocation resulting from hidden information (see Table 1.2). To assess the impact of absenteeism on output, we run the model assuming that the opioid crisis does not affect the ability to solve problems. In this scenario, the overall output loss in the economy amounts to 0.24% when addicted workers attend work with a probability of 0.93. Furthermore, the output loss associated with misallocation due to hidden information accounts for half a percent.

The CEA estimated the socio-economic cost of the opioid crisis as \$504 billion in 2015, equivalent to 2.8% of the nominal GDP for that year (CEA, 2017). Our result indicates an additional loss of approximately 0.74%, amounting to around \$133 billion in 2015. Out of this, 17.6% or \$23.4 billion is associated with resource misallocation due to hidden information.

In the model, absenteeism impacts output in the following manner. Tasks are assigned regardless of a worker’s absence, resulting in the need for managers to address the problems assigned to absent workers. As the number of unsolved problems from the first layer increases, managers are unable to address all of them due to their time constraints. Consequently, it

becomes more efficient to reduce the number of workers to be hired, similar to cutting production lines. This reduction in the number of problems to draw or final goods within the economy leads to a decline in output.

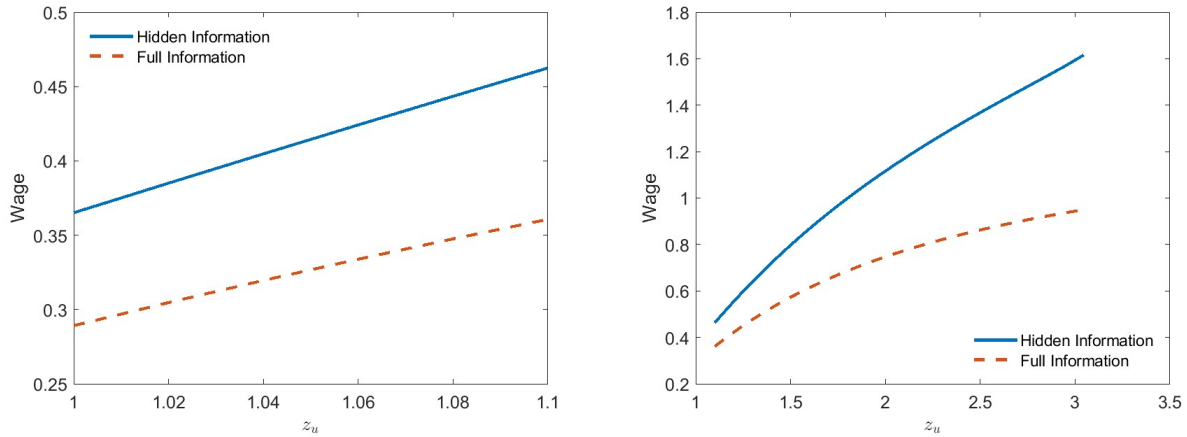
The loss of productivity reduces output as unhealthy workers request managers to review problems that they would have been capable of solving had they been healthy. In essence, they consume managers' time with easier and less valuable problems.

Hidden information leads to resource misallocation by distorting three key outcomes: i) wages and rents; ii) assignments; and iii) the number of workers to be hired by managers. Wage distortion occurs because the hidden information prevents wages from aligning with workers' actual productivity. As a result, healthy workers receive lower wages than their actual productivity, while unhealthy workers benefit from higher wages.

In our calibrated model, we have normalized z_0 , the lowest skill level, to one. The threshold to become a manager, denoted as z^* , is found as 3.05, indicating that 77.05% of the population become workers, while the remaining 22.95% become managers. This proportion of workers is higher than both the calculation by [Caicedo et al. \(2019\)](#), which suggested 63%, and their observation of approximately 50% for the entire U.S. economy. The difference might be attributed to our assumption of limiting the production process to only two layers. As shown in the [Proposition 5](#), note that the threshold to become a manager is the same under both full and hidden information equilibria.

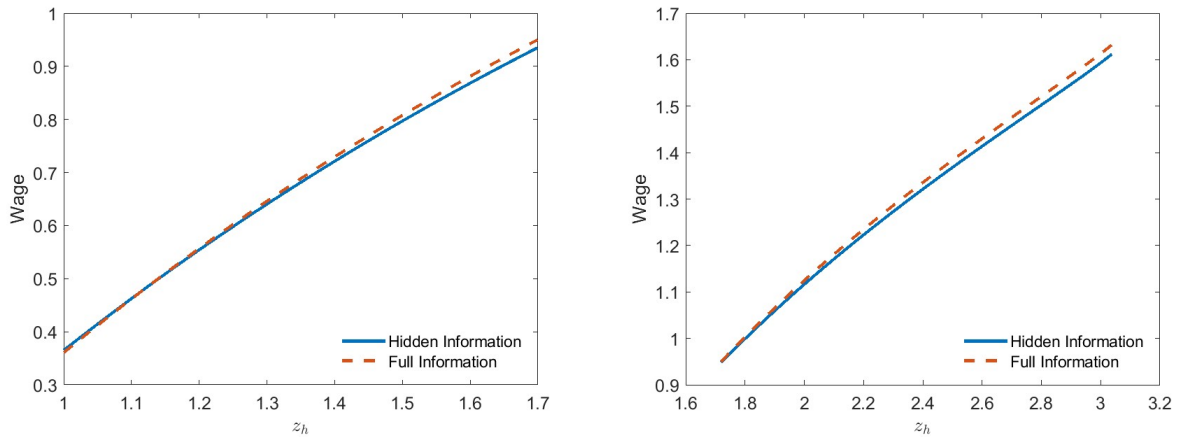
Healthy workers at the threshold z^* are capable of solving 83.6% of any problems they encounter, whereas unhealthy workers with the same skill level z^* are able to solve 64.5% of them. Therefore, those unhealthy workers have the same ability as healthy workers with skill level $z_1 = 1.7$ since $F(z_1) = \beta\gamma F(z^*) = 0.645$. Another relevant threshold under the full information equilibrium is z_2 for skill level of unhealthy workers where those unhealthy workers have the same ability as the lowest skilled healthy workers. According to our calibrated model, $z_2 = 1.1$ since $F(z_0) = \beta\gamma F(z_2) = 0.29$. In other words, the lowest skilled healthy workers z_0 are able to solve 29% of all of the problems. However, the unhealthy

lowest skilled workers are capable of solving 22% of any problems since $\beta\gamma F(z_0) = 0.22$.



(a) The lowest ability unhealthy workers (b) The highest ability unhealthy workers
Figure 1.6: Distortions in Wages of Unhealthy Workers

For the lowest-ability unhealthy workers, the mean wage is higher by 27.1% under hidden information than that under full information in Panel (a). The highest-ability unhealthy workers earn around 49.6% higher wages at the mean in hidden information equilibrium than in full information equilibrium in Panel (b).



(a) The lowest ability healthy workers (b) The highest ability healthy workers
Figure 1.7: Distortions in Wages of Healthy Workers

The highest-ability healthy workers receive lower wages by 1.23% in the hidden information equilibrium compared to the full information equilibrium. Since the share of unhealthy workers is small (on average 3.8%) the impact on healthy workers is relatively smaller in

terms of effect size.

It is worth noting that the lowest-skilled healthy workers may benefit from hidden information, as shown in Panel (a). This is because of changes in demand that occur between the hidden information and full information settings. Consider healthy workers with skill z_0 . Under full information, the managers hiring these workers include managers with skills $\varphi_h(z_0) = \varphi_u(z_2) > z^*$. Under hidden information, managers with a skill level of z^* hire the lowest-skilled workers, both healthy and unhealthy. Given that the density function of z is decreasing (Pareto), the mass of individuals demanding these workers is larger under hidden information than under full information.³

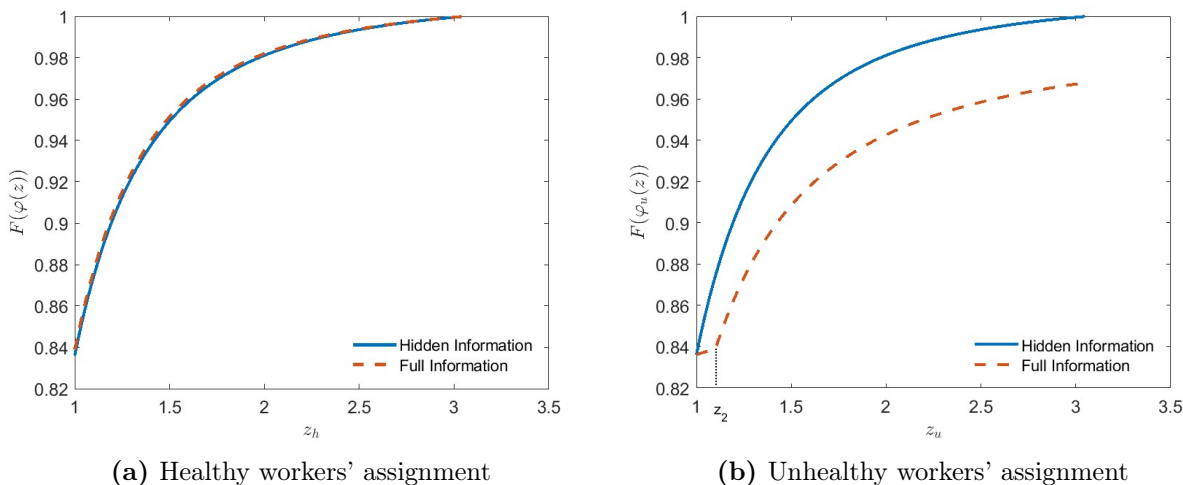
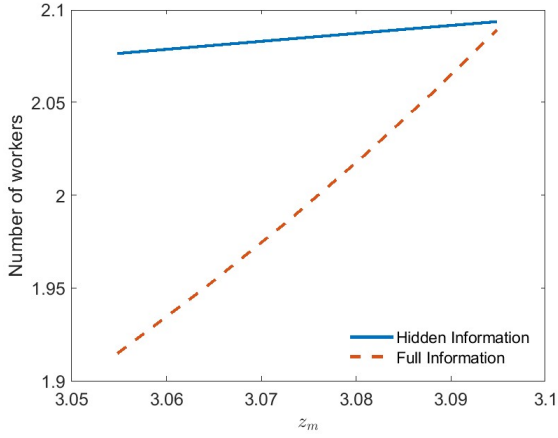


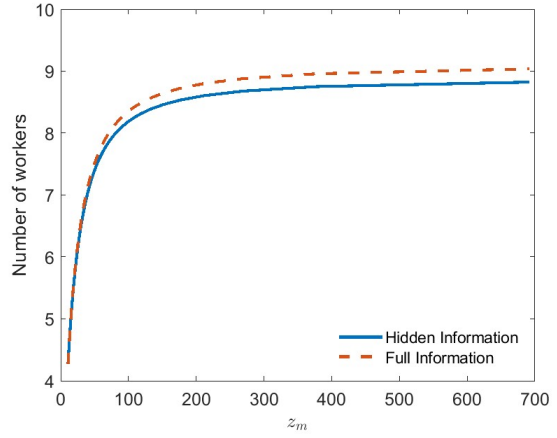
Figure 1.8: Distortions in the assignment

Next, Figure 1.8 compares the worker assignment functions $\varphi_u(z)$ and $\varphi_h(z)$ with the assignment under full information. Clearly, most of the differences lie in comparing $\varphi_u(z)$ with the full information case. There is a clear break in the assignment function around $z_2=1.10$. For unhealthy workers with $z < z_2$, demand comes only from managers with skills $z < \varphi_u(z_2)$. Demand increases considerably past this threshold, explaining the kink.

³One could argue that competition for these workers is tougher under hidden information, since there are more unhealthy workers with $z = z_0$ (their competition under hidden information) than with $z = z_2$ (their competition under full information). While this can be the case, panel (a) of Figure 1.7 shows that the demand channel dominates.



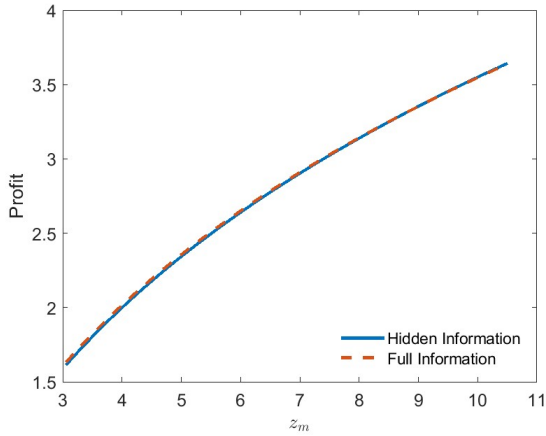
(a) The lowest skilled managers' hiring



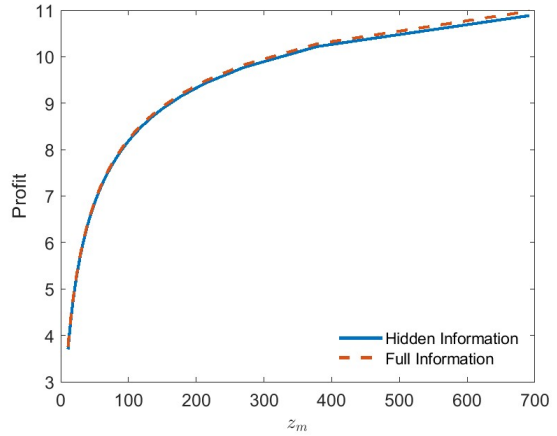
(b) The highest skilled managers' hiring

Figure 1.9: Distortions in the number of workers hired

The lowest skilled managers hire more workers in the hidden information equilibrium than the full information equilibrium because they are able to hire healthy workers whom they couldn't hire under full information. As a result, these healthy workers ask fewer questions and enable their managers to hire more workers with the same level of skill. However, top managers hire fewer workers, leading to a decrease in the production of more valuable final goods.



(a) The lower skilled managers' profits



(b) The highest skilled managers' profits

Figure 1.10: Distortions in Profits

The average profit of the lowest-skilled managers declines by 1.18%, while that of middle managers reduces by 0.34%. On average, the top manager's profits decline by 0.6%, but the

average loss they face (difference in profits between hidden and full information) is 1.9 times larger than that of the lowest-skilled managers.

The loss of 0.74% of GDP is substantial. To put matters in perspective, this amplifies the existing estimated losses by about 26.4%. The Council of Economic Advisers estimate the opioid-related losses associated with increases in workers out of the labor force, increases in healthcare costs, increases in criminal justice costs, and reductions in the labor force to be 2.8% of GDP in 2015 (CEA, 2017) due to *direct* effects, that is, health care costs, criminal justice costs, and workers out of the labor force. The drivers of the losses dealt with in this paper are orthogonal to these causes, and therefore should be directly added to the CEA estimates. Combining the CEA estimates with ours, the cost of the opioid crisis is 3.54% of GDP.

If we extrapolate based on the nominal annual GDP for 2022, the total loss could surge to \$901 billion. It's worth noting that many economic and health studies have largely ignored these extra losses. As a result, our research reveals that these studies considerably underestimate the true extent of losses, as they fail to account for the impact of employer-employee mismatch.

1.9 Conclusion

We develop a macroeconomic framework in which information hidden to employers leads to the misallocation of resources in the economy. Within this framework, we examine the impacts of hidden information on wages, firm sizes, and firm profits in the context of the opioid crisis. Lastly, we quantify the output loss caused by this resource misallocation.

The channels studied presently have not been thus far related to the opioid crisis, and they are substantial, increasing existing estimates by 26.4%, yielding a total loss to opioids of 3.54% of GDP, out of which 3.7% can be attributed to the misallocation created by hidden information. This estimate justifies that that up to 3.54% of our total production could be spent on addressing the problems arising from this crisis.

Our model successfully captures the differences in allocations between full and hidden information scenarios. In contrast, both the span of control model proposed by Lucas (1978) and standard heterogeneous firm models (e.g., Hopenhayn (1992)) fail to generate such distortions due to asymmetric information. This is because the allocation in those models is determined solely by average productivity, which remains the same in both full and hidden information scenarios.

The lowest skilled managers hire more workers in the hidden information equilibrium than the full information equilibrium because they are able to hire healthy workers whom they couldn't hire under full information. As a result, these healthy workers ask fewer questions and enable their managers to hire more workers with the same level of skill. However, top managers hire fewer workers, leading to a decrease in the production of more valuable final goods.

Finally, there are a number of potential applications of this model. It could be interesting to measure welfare loss due to specific health and disease conditions that have a significant impact on employers in terms of productivity-related costs due to absenteeism and on-the-job productivity losses.

CHAPTER 2

Trade, Innovation, and Pollution

by

Bayarmaa Dalkhjav & Ziba Karjoo

2.1 Introduction

There is a common notion that increased production for exports results in higher pollution levels. However, recent empirical evidence suggests otherwise, indicating that trade liberalization can actually benefit the environment by reducing pollution. For instance, [Karjoo and Rubini \(2023\)](#) have found that new exporters tend to generate lower levels of pollution. To explain this phenomenon, we developed a model in this paper where some new exporters upgrade their machinery to begin exporting. As these new machines are cleaner than the old ones, these firms create a positive externality by reducing pollution with no intention of doing so.

We build upon the framework established by [Melitz \(2003\)](#) by incorporating factors related to pollution and innovation. In our model, pollution is a byproduct of production processes, with the intensity of pollution varying based on the technological state of individual firms. We introduce heterogeneity among firms in terms of innovation costs, representing the expenses associated with upgrading technology. Specifically, we define two technological states—high and low—where firms with higher technological capabilities exhibit lower levels of pollution intensity.

In our model, firms face a sunk cost of innovation along with entry costs to both domestic and export markets. Upon drawing its productivity level and sunk cost to innovate from respective distributions, firms decide whether to produce, export, and invest in innovation. Consequently, four types of firms potentially exist: (i) domestic producers with low technology, (ii) domestic producers with high technology, (iii) exporters with low technology, and (iv) exporters with high technology.

The key mechanism is as follows. The reductions in trade costs incentivize the adoption of new technology in two ways: (i) lowering the opportunity cost of adoption and (ii) increasing the benefit from exporting. Thus, firms upgrade their machines when they start exporting. Those new machines are typically cleaner than old ones. Therefore, new exporters become

cleaner producers than non-exporters, even though it was not their intention. As a firm adopts new technology to start exporting, it has a positive “external” impact on reducing pollution.

We calibrate our model to the Chilean economy using firm-level data from the manufacturing sector. The key parameters of the model are calibrated in two steps. In the first step, we calibrate parameters related to the firm distribution: the cost of entering the export market, the variable trade cost, and the technology parameters. In the second step, we calibrate the change in variable trade costs between 1995 and 2007 to match the observed change in the export and output ratio. Additionally, we calibrate the elasticity of pollution with respect to technology state to capture the difference in pollution intensity between new exporters and incumbent domestic producers.

The key finding of our study reveals that a decrease in trade costs, consistent with the change in trade volume in Chile from 1995 to 2007, results in a 4.4 percent decline in pollution even though production increased. Despite the fact that firms expand their output and increase pollution due to reduced trade costs, the adoption of advanced technologies on a larger scale contributes to the reduction of pollution as a positive externality.

Our paper contributes to the theoretical literature that explains a large technique effect observed in many empirical studies. For example, [Cherniwchan et al. \(2017\)](#) shows that trade liberalization leads the dirtiest firms to exit and the cleanest firms to produce more, driving industry-level emissions downward. On the other hand, [Shapiro and Walker \(2018\)](#) concludes that the rise in the implicit pollution tax faced by manufacturers between 1990 and 2008 accounts for most of the emissions reductions, rather than changes in productivity and trade. Moreover, [Antweiler et al. \(2001\)](#) develop a two-sector model for a small open economy, enabling the decomposition of trade’s impact on pollution into scale, technique, and composition effects from the model itself. They conclude that the impacts of trade on pollution emissions depend on comparative advantages of countries.

The rest of the paper proceeds as follows. Section 2 introduces the model incorporating

innovation and pollution within a general equilibrium framework. In Section 3, the model is calibrated to the specifics of the Chilean economy. Section 4 presents the findings of the analysis, while Section 5 provides concluding remarks.

2.2 Setup of the model

The model, which is static, is based on the framework established by Melitz (2003). There are two symmetric countries that produce a continuum of tradable differentiated goods. We use a (*) to denote a foreign country.

2.2.1 Preference and Demand

The preference of a representative consumer is given a C.E.S. utility function over a continuum of goods indexed by ω :

$$U = \left[\int_{\omega \in \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\omega \in \Omega^*} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} \quad (2.1)$$

where Ω and Ω^* are the sets of goods produced in the domestic and foreign countries, respectively.

Each consumer has one unit of labor and supplies it inelastically. Given prices $p(\omega)$, a wage w , and profits $\pi(\omega)$ of firm ω , the budget constraint is:

$$\int_{\omega \in \Omega} p(\omega)q(\omega)d\omega + \tau \int_{\omega \in \Omega^*} p(\omega)q(\omega)d\omega = w + \frac{\int_{\omega \in \Omega} \pi(\omega)d\omega}{L} = I \quad (2.2)$$

where L is a population in an economy. The wage w is a numeraire and set $w = 1$. The right hand side of equation (2) is the income of each consumer, noted as I . Exports and imports are subjects of variable trade cost, τ . Thus, consumers buy a foreign good ω with price $\tau p(\omega)$.

Consumers maximize their utility (1) subject to the budget constraint (2). The demand

of a variety ω by each consumer is:

$$q(\omega) = \begin{cases} p(\omega)^{-\sigma} P^{\sigma-1} I & \text{if } \omega \in \Omega \\ [\tau p(\omega)]^{-\sigma} P^{\sigma-1} I & \text{if } \omega \in \Omega^* \end{cases} \quad (2.3)$$

where P is aggregate price:

$$P = \left(\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega + \tau^{1-\sigma} \int_{\omega \in \Omega^*} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \quad (2.4)$$

2.2.2 Production

A variety ω is produced by a single firm in monopolistic competition industry and is associated with a productivity parameter $z(\omega)$. z is drawn from a productivity distribution $G(z)$ after a firm pays a fixed entry cost κ_e in units of labor. Firms can choose to export by incurring a fixed entry cost to the export market, κ_x in units of labor. We assume that firms cannot produce with a probability of δ , regardless of how productive they are. There is no fixed cost of operation and thus no endogenous exit.

There are two states of technology, A_L and A_H . Technology A_L is available without any additional cost. However, a firm has to pay a sunk cost of innovation κ_I to have an access to higher technology A_H . The sunk cost κ_I is independently drawn from an uniform distribution $H(\kappa_I)$, where $\kappa_I \in [\underline{\kappa}_I, \bar{\kappa}_I]$ or $[0, 1]$. This heterogeneity in sunk cost of innovation guarantees that even a small firm with low z could have access to a high technology by drawing a low cost to innovate. This is consistent with the evidence that some small firms have a potential to grow fast. On the other side of spectrum, some high productive firms could not incur the cost of innovation and stay producing with old technology.

Decisions of a firm ω are fully determined by three state variables: productivity z , innovation cost κ_I , and its state of technology A . In equilibrium, the two firms with the same state variables make the same decisions. Thus, it is convenient to use the space of z , κ_I and

A , instead of ω .

Any firm (z, κ_I, A) makes a decision on pricing, quantities exported and sold domestically, whether to export, and to innovate. All these decisions are simultaneous. Consequently, it is convenient to split a firm's problem into a pricing rule and entry decision. The first involves how much to produce and the price given their current productivity, and the second involves whether to innovate, whether to export, and whether to produce or not.

2.2.2.1 Pricing rule

A variety producer (z, A, κ_I) solves the following problem:

$$\max_{p, Q, \ell} pQ - W \ell \quad (2.5)$$

subject to

$$Q = zA\ell \quad (2.6)$$

where

$$Q = \frac{p^{-\sigma}}{P^{1-\sigma}} I$$

Q is the demand for a variety that a firm faces. $p(z, A, \kappa_I)$, $\ell(z, A, \kappa_I)$ is a demand for labor by a firm. $A = \{A_L, A_H\}$, and I is aggregate expenditure of consumers.

Let $X(z, A, \kappa_I) = 1$ denote the decision to export, $X(z, A, \kappa_I) = 0$ otherwise. Using the optimal price and demand for labor, a profit function of a firm (z, A, κ_I) is:

$$\pi(z, A, \kappa_I) = \begin{cases} \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} \left(\frac{W}{zAP}\right)^{1-\sigma} R & \text{if } X = 0 \\ \left(1 + N\tau^{1-\sigma}\right) \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} \left(\frac{W}{zAP}\right)^{1-\sigma} R & \text{if } X = 1 \end{cases} \quad (2.7)$$

Each firm can assess potential profits associated with different states, based on its respective draws of productivity levels and sunk costs to innovate, from corresponding distributions (see Equation 2.7). In the next section, we discuss how firms decide whether to enter the

export market or innovate, using those profit functions.

2.2.2.2 Entry and innovation decisions

Firms decide whether to produce, to export, to innovate and both to export and innovate.

The value function is:

$$V(z, A, \kappa_I) = \max \left\{ 0, V_d(z, A, \kappa_I), V_I(z, A, \kappa_I), V_x(z, A, \kappa_I), V_{xI}(z, A, \kappa_I), \right\} \quad (2.8)$$

The value function of a firm that does not produce is equal to 0. The value function of a firm that produce goods only for domestic market and does not innovate is $V_d(z, A, \kappa_I)$ obtained below:

$$V_d(z, A, \kappa_I) = \frac{1}{\delta} \pi_d(z, A_L, \kappa_I) \quad (2.9)$$

where $\pi_d(z, A_L, \kappa_I)$, obtained from Equation 2.7, denotes the profit of a nontradable goods producer without innovation.

Similarly, if a firm serves only the domestic market yet chooses to innovate, the value function is:

$$V_I(z, A, \kappa_I) = \frac{1}{\delta} \pi_I(z, A_H) - \kappa_I \quad (2.10)$$

A firm that exports but does not innovate has the following value function:

$$V_x(z, A, \kappa_I) = \frac{1}{\delta} \pi_x(z, A_L) - \kappa_x = \frac{1}{\delta} \left(1 + N\tau^{1-\sigma} \right) \frac{(\sigma - 1)^{\sigma-1}}{\sigma^\sigma} \left(\frac{W}{zA_L P} \right)^{1-\sigma} I - \kappa_x \quad (2.11)$$

A firm that both exports and innovates has the value function:

$$V_{xI} = \frac{1}{\delta} \pi_{xI}(z, A_H) - \kappa_I - \kappa_x = \frac{1}{\delta} \left(1 + N\tau^{1-\sigma} \right) \frac{(\sigma - 1)^{\sigma-1}}{\sigma^\sigma} \left(\frac{W}{zA_H P} \right)^{1-\sigma} I - \kappa_I - \kappa_x \quad (2.12)$$

For any given (z, A, κ_I) , a profit level is the highest for exporting and innovating, while it is the lowest for a domestic producer without innovation. Once firms make decisions of whether to enter the export market and to innovate by maximizing its value functions across all states, the following four types of firms potentially exist:

Type I: Exporters with innovation

Type II: Exporters with old technology

Type III: Producers for the domestic market with innovation

Type IV: Producers for the domestic market without innovation

Each firm chooses the state that offers the highest value. Specifically, a firm (z, A, κ_I) will choose to export with innovation if:

$$V_{xI}(z, A, \kappa_I) \geq \max\{V_d(z, A, \kappa_I), V_I(z, A, \kappa_I), V_x(z, A, \kappa_I)\}$$

A firm (z, A, κ_I) will export without innovation if:

$$V_x(z, A, \kappa_I) \geq \max\{V_d(z, A, \kappa_I), V_I(z, A, \kappa_I), V_{xI}(z, A, \kappa_I)\}$$

A firm (z, A, κ_I) will innovate and serve only in domestic market if:

$$V_I(z, A, \kappa_I) \geq \max\{V_d(z, A, \kappa_I), V_x(z, A, \kappa_I), V_{xI}(z, A, \kappa_I)\}$$

Otherwise, a firm neither exports nor innovates, but produces for a domestic market.

Figure 2.1 depicts the thresholds and regions by state. While other possibilities exist for these regions, we currently focus our analysis on this particular scenario.

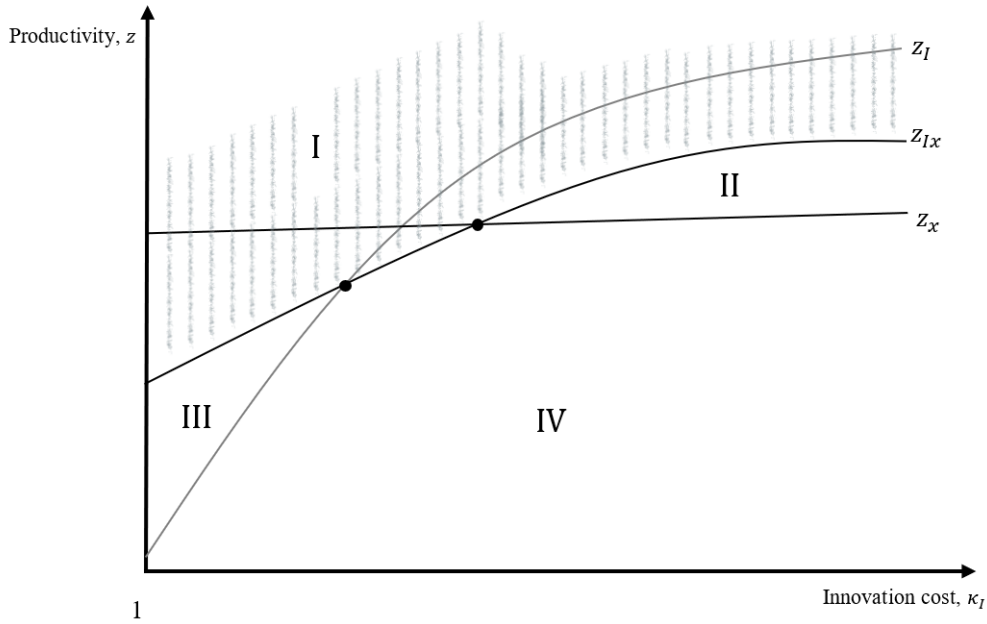


Figure 2.1: Types of firms in the state space

As expected, there are some firms with low productivity but also with low innovation cost that serve the domestic market with new technology by innovating (see area III in Figure 2.1). Another interesting type of firm is those in area II, which have sufficiently high productivity to serve both domestic and export markets; however, their innovation costs are too high to allow for innovation.

2.2.2.3 Pollution emission

Firms emit pollution as an unintended consequence of producing goods. The amount of pollution that a firm emits depends on its technology state, productivity and production. Specifically, we assume that a firm (z, A, κ_I) produces pollution emissions with the following technology:

$$e(z, A, \kappa_I) = \underbrace{m(A)}_{\text{Pollution Intensity}} Q(z, A, \kappa_I)$$

where

$$m(A) = \left(\frac{1}{A}\right)^\alpha$$

where α represents the elasticity of pollution with respect to the technology of a firm. Pollution intensity is defined as units of pollution emitted per unit of output and a decreasing function of technology. That is, we assume that innovation reduces the pollution intensity.

Summing pollution emissions over all the firms in an economy, we can calculate the total pollution emissions as follows:

$$E = \int_{\kappa_I} \int_z e(z, A, \kappa_I) g(z) h(\kappa_I) dz d\kappa_I$$

2.2.3 Labor Market Equilibrium

The labor used in entry cost and other sunk cost to innovate and export must be reflected in labor market. Let M be the mass of firms. The mass of workers is L . Then the labor market clearing is:

$$L = M \left(\delta\kappa_e + s_x\kappa_x + \int_{\kappa_I} \int_z \kappa_I dG(z) dH(\kappa_I) + \int_{\kappa_I} \int_z (\kappa_x + \kappa_I) dG(z) dH(\kappa_I) + \int_{\kappa_I} \int_z \ell(z, A, \kappa_I) dG(z) dH(\kappa_I) \right) \quad (2.13)$$

The last term is labor used in production.

2.2.4 Equilibrium in an economy

An equilibrium consists of a price index P , aggregate profit Π , a mass of firms M , firms' decision rules $\mathbb{I}(A, z, \kappa_I)$, $X(A, z, \kappa_I)$, $X\&\mathbb{I}(A, z, \kappa_I)$, $\ell(A, z, \kappa_I)$ and $p(A, z, \kappa_I)$ and consumer's decision rule $q(A, z, \kappa_I)$ such that

- The decision rules are optimal for firms and consumers
- All the markets clear

2.3 Calibration

We calibrate our model to the Chilean economy. Some parameters are set directly according to previous literature. In particular, σ is set to 2, consistent with [Ruhl et al. \(2008\)](#); [Rubini \(2014\)](#). The mass of workers L is normalized to 1.

The remaining parameters in the model are calibrated in two steps. In the first step, we calibrate the firm distribution parameter θ , the entry cost to export market κ_x , the initial variable trade cost τ_0 , and the advanced technology parameter A_H . For the distribution of firms by size, we follow the literature in trade and choose a Pareto distribution as follows:

$$g(z) = \theta z^{-\theta-1}$$

We calibrate θ to match the slope of the firm size distribution for large manufacturing firms, as they follow a Pareto distribution, while small firms do not. The parameter κ_x is set so that the share of exporters is 29% in Chile in 2007. The variable trade cost τ is calibrated to match the ratio of tradable output to total output, 37%. We target the share of process innovators for all manufacturing firms in Chile to pin down the parameter A_H .

Table 2.1: Parameters and targets

Parameter	Target	Value
σ	Ruhl et al. (2008) ; Rubini (2014)	2
κ_e	Normalized	1
A_L	Normalized	1
θ	Firm size distribution	2.1
κ_x	Share of exporters=21.6%	0.58
τ_0	Export/Sales=28.0% in 1995	1.545
A_H	Share of process innovators, 31.2% in 1993-2007	1.45
α	Pollution intensity starter/domestic only =0.4	7.64
τ_1	Export/Sales=34.4% in 2007	1.335
Distributions		
κ_I	Uniform distribution	[0; 1]
z	Pareto distribution	[1; ∞)

The evidence suggests that those new exporters are cleaner than non-exporters ([Karjoo](#)

and Rubini, 2023). Specifically, the average pollution intensity of new exporters is less than that of non-exporters by 60%. Several firms started exporting between 1995 and 2007, even though there was no change in tariffs. Moreover, the ratio of tradable output to total output rose from 28% to 34.4% in this period. Therefore, non-tariff barriers could decline and lead to a reduction in the variable trade cost.

In the second step, we calibrate the variable cost in 2007 to match the export and output ratio. Declining variable cost increases the expected profits that provide firms close to the margin to export or innovate with opportunities to do so. Some new exporters also decide to invest in innovation that reduces the pollution intensity as an unintended consequence. We calculate the average pollution intensities for new exporters and non-exporters and then calibrate the elasticity of pollution concerning pollution intensity, α , to match the difference in pollution intensity between the two groups, which is 60%. Table 1 shows all the calibrated parameters and their targets.

2.4 Results

The reduction in trade costs or barriers induces some non-exporters to become an exporter. Among new exporters, those who have an advantage of innovation cost will upgrade their technology and contribute to reducing pollution.

Table 2 reports our main results. The variable trade cost in Chile declined by 13.6% between 1995 and 2007. This decline in trade cost leads to 4.4 and 0.85 percent decrease in pollution and pollution intensity, respectively.

To see the impact of innovation on the pollution, we calculate the total pollution and the pollution intensity in two additional scenarios: (i) there is no innovation in the manufacturing sector, and (ii) all firms invest in innovation (call it “full innovation” scenario). “No innovation” means that $\kappa_I = \infty$ for all firms. “Full innovation” means that $\kappa_I = 0$ for all firms.

In the “no innovation” scenario, pollution emissions are the highest and increase with

Table 2.2: Key Findings

	τ_0	τ_1	Change (%)
Total pollution			
Benchmark	0.09	0.086	-4.44
No innovation	0.551	0.569	3.27
Full innovation	0.067	0.069	2.99
Pollution intensity			
Benchmark	0.706	0.7	-0.85
No innovation	1.0	1.0	-
Full innovation	0.057	0.057	-

trade liberalization. Conversely, in the “full innovation” scenario, where all firms adopt new technology, pollution is significantly reduced compared to the first scenario. However, pollution emissions still increase with trade liberalization, suggesting that an extensive margin of adopting new technologies contributes to pollution reduction. In both the no innovation and full innovation scenarios, pollution increases because firms expand output without changing technologies. However, when there is an extensive margin of technology adoption, pollution levels decrease due to a decline in pollution intensity.

2.5 Conclusion

This paper focuses on an often-ignored consequence of international trade. While it is commonly believed that international trade increases pollution by increasing the output of polluting firms, recent empirical findings suggest that firms that enter the export market reduce their emissions. We theorize that this is an unintended outcome: firms that start exporting use the opportunity to renew their machines, and newer machines are typically cleaner. This implies that international trade is less polluting than we previously thought. We develop a theoretical model to account for this and calibrates it to find out the net effects of international trade on pollution.

The key mechanism is as follows. The reductions in trade costs, such as a decline in tariff, incentivize the adoption of new technology in two ways: (i) lowering the opportunity cost of adoption and ii) increasing the benefit from exporting. Thus, firms upgrade their

machines when they start exporting. Those new machines are typically cleaner than old ones. Therefore, new exporters become cleaner producers than non-exporters, even though it was not their intention. As a firm adopts new technology to start exporting, it has a positive “external” impact on reducing pollution.

Our main finding is that the decline in trade cost leads to a 4.4 and 0.85 percent decrease in pollution and pollution intensity, respectively. Although the firms expand output and increase pollution due to a reduction in trade cost, an extensive margin of technology adoption contributes to reducing pollution and pollution intensity as a positive externality.

2.6 Appendix

- Innovate without exporting if

$$\frac{1}{\delta}\pi_I(z, A_H) - \kappa_I \geq \frac{1}{\delta}\pi_d(z, A_L) \quad (2.14)$$

Thus,

$$z_I(\kappa_I) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1} \left(\frac{W}{P}\right) \left[A_H^{\sigma-1} - A_L^{\sigma-1}\right]^{\frac{1}{1-\sigma}} \left(\frac{R}{\delta\kappa_I}\right)^{\frac{1}{1-\sigma}}$$

- Export without innovating if

$$\frac{1}{\delta}\pi_x(z, A_L) - \kappa_x \geq \frac{1}{\delta}\pi_d(z, A_L) \quad (2.15)$$

Thus,

$$z_x = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1} \frac{1}{A_L} \left(\frac{W}{P}\right) \left(\frac{R}{\delta\kappa_x}\right)^{\frac{1}{1-\sigma}} \tau N^{\frac{1}{1-\sigma}}$$

- Export and innovate if

$$\frac{1}{\delta}\pi_{xI}(z, A_H) - \kappa_x - \kappa_I \geq \frac{1}{\delta}\pi_d(z, A_L) \quad (2.16)$$

Then,

$$z_{xI}(\kappa_I) = \sigma^{\frac{\sigma}{\sigma-1}} \frac{1}{\sigma-1} \left(\frac{W}{P}\right) \left(\frac{1}{\delta(\kappa_x + \kappa_I)}\right)^{\frac{1}{1-\sigma}} \left[A_H^{\sigma-1}(1 + N\tau^{1-\sigma}) - A_L^{\sigma-1}\right]^{\frac{1}{1-\sigma}} R^{\frac{1}{1-\sigma}}$$

$$v_e = \frac{1}{\delta}\bar{\pi} - \kappa_e = 0 \quad (2.17)$$

where

$$\bar{\pi} = \frac{\Pi}{M} = \frac{1}{M} \int_{\kappa_I} \int_z \pi(z, A, \kappa_I) M g(z) dz h(\kappa_I) d\kappa_I$$

After some arrangements, we obtain:

$$\int_{\kappa_I} \int_z \pi(z, A, \kappa_I) g(z) dz h(\kappa_I) d\kappa_I = \delta \kappa_e$$

Solve for P . In the stationary equilibrium, the following condition holds:

$$M_e = \delta M$$

Thus,

$$L_e = \kappa_e M_e = \delta M \kappa_e = \Pi \tag{2.18}$$

CHAPTER 3

Are Sovereign Wealth Funds a Good Idea in the Presence of Corruption?

by

Bayarmaa Dalkhjav

3.1 Introduction

In countries reliant on commodity exports, government revenue tends to be volatile, leading to instability in government expenditure and macroeconomic indicators. To address this instability, governments often establish Sovereign Wealth Funds (SWFs), which are investment funds. While designed to mitigate revenue and export fluctuations, the substantial government revenues under SWF management can also heighten the risk of corruption. The research question addressed in this paper is: Are SWFs still beneficial in countries where corruption is prevalent?

To address these questions, I augment a RBC model with a commodity sector, a SWF, and expropriation, capturing the key features of a resource-rich economy. Firstly, the economy possesses an exogenous endowment of a commodity at each period, which is entirely exported. The demand for this commodity comes from the rest of the world and is perfectly elastic at a given price. The commodity sector is subject to price shocks originating in the world market, which are an important driver of business cycles in small emerging market economies (Fernández et al., 2018).

The second feature of my model is a SWF, that functions as a financial buffer, helping governments in managing their revenues more effectively and stabilizing their budgets during economic turmoil. By accumulating funds during periods of high revenue or export earnings, SWFs can provide stability against economic downturns or sudden fluctuations in revenue streams. As of 2021, 70 countries had at least one sovereign wealth fund, with a total of 161 SWFs worldwide managing over \$12 trillion in assets, according to data provided by the Sovereign Wealth Fund Institute. The largest SWFs include those of Norway, China, and the UAE. In my model, the SWF plays a key role in implementing countercyclical fiscal policies through fiscal rules.

The third feature is expropriation. A politician has the ability to expropriate government revenue from the commodity sector by maximizing its profit, subject to a specific technology

with inputs of institutional quality and labor. To model this, I build on [Germaschewski et al. \(2021\)](#). Additionally, I assume that they inefficiently spend the expropriated revenue by disposing of it into the sea. This assumption allows me to examine the worst-case scenario, suggesting that my results represent a lower bound of the welfare loss. Furthermore, there is a shock to the efficiency of expropriation, introducing uncertainty to the process. Thus, while SWFs improve stability, they also reduce the amount of goods available for consumption or investment.

My finding highlights that establishing a SWF improves welfare by stabilizing government revenues and acting as a financial buffer against commodity price fluctuations. Overall, SWFs contribute to reduced volatility in government revenues and consumption, supporting economic stability and welfare. However, corruption undermines the welfare benefits of SWF, reducing its attractiveness. While an economy with an SWF and low levels of corruption experiences a slightly higher welfare loss than one with no corruption, it still performs better than an economy without an SWF. Unfortunately, a high level of corruption leads to a higher welfare loss compared to a scenario without an SWF.

This paper contributes to the literature examining the effects of institutional quality on the procyclicality of fiscal policy in developing or commodity-exporting countries. Resource-rich economies often face more volatile business cycles due to fluctuations in commodity prices. [Fernández et al. \(2018\)](#) provide evidence on small emerging countries, including Brazil, Chile, Colombia, and Peru, indicating that commodity prices are a key driver of business cycles. Furthermore, procyclical macroeconomic policies exacerbate this situation by increasing the sensitivity of business cycles to changes in commodity prices ([Frankel, 2011](#)). Additionally, [Frankel et al. \(2013\)](#) demonstrates that about a third of developing countries have successfully transitioned to countercyclical policies, with the quality of institutions playing a key role.

Furthermore, extensive research has explored the negative impacts of resource wealth on macroeconomic stability, long-term economic growth, and political institutions. These

studies conclude that the quality of institutions plays a critical role in the dynamics of the “resource curse” (Ploeg, 2011; Sachs and Warner, 2001; Arezki et al., 2011). Transforming this curse into a blessing is possible through the establishment of SWFs, albeit with a strong emphasis on the necessity of good governance (Frynas, 2017).

Effective SWF management requires the presence of strong, transparent, and accountable institutions. Unfortunately, in many resource-rich countries, weak law enforcement and widespread corruption hinder efficient revenue management. This inefficiency, in turn, compromises the ability of SWFs to mitigate business cycle fluctuations and to promote long-term economic growth. This paper contributes to the existing literature by providing new evidence on the welfare implication of SWFs under varying qualities of institutions, both good and poor.

Another related area of the literature examines the real business cycle properties in developing countries. Angelopoulos et al. (2011) highlights the significance of weak property rights in shaping the business cycle dynamics of Mexico, viewing productivity shocks as shocks to institutions. However, Germaschewski et al. (2021) endogenizes expropriations to model the behavior of politicians and studies their impacts on business cycles in China.

I apply my model to the case of Mongolia. Firstly, Transparency International’s rankings show a decline in Mongolia’s corruption perception, dropping from 94th in 2012 to 121st in 2023. Secondly, despite this decline, Mongolia established two SWFs, raising questions about their effectiveness in corrupt environments. Would Mongolia benefit from SWFs?

The paper is structured as follows: Section 2 outlines the fiscal rules implemented in Mongolia. Section 3 explains the issue of corruption in Mongolia. Section 4 introduces the model, while Section 5 details the calibration and Bayesian estimation of parameters. Section 6 presents the welfare analysis and examines the effects of shocks to commodity prices and the efficiency of expropriation. Finally, Section 7 offers concluding remarks.

3.2 Fiscal rules in Mongolia

This section explains how the fiscal rule under SWFs works. This helps us to model the fiscal policy. One of the primary objectives of SWFs is to protect and stabilize the budget and economy from excessive volatility in revenues or exports. This is particularly important for countries heavily reliant on revenue from commodities or other volatile sources. SWFs serve as a financial buffer, helping governments to manage their revenues more effectively and stabilize their budgets during periods of economic turmoil. By accumulating funds during periods of high revenue or export earnings, SWFs can provide a source of stability against economic downturns or sudden fluctuations in revenue streams.

In Mongolia, the fiscal rule under SWFs works as follows. The parliament of Mongolia approved the Fiscal Stability Law (FSL) in 2010, which allows the government to establish a SWF such as the Fiscal Stability Fund and formalizes fiscal rules. According to this law, the government accumulates revenue from higher commodity prices than their reference prices, surplus of the structural budget balance, surplus from other government funds, and revenue from financial operations of the Fiscal Stability Fund. The reference prices for key commodities are determined by averaging the previous consecutive 18-20 years' prices, current prices, and forecasts of commodity prices for the next three consecutive years.

The FSL also specifies situations where the government can transfer money from the stability fund to its budget. Examples include: (i) when government revenue declines more than planned, resulting in a budget deficit exceeding 4% of GDP due to unforeseen circumstances; (ii) when structural fiscal revenue does not reach the planned level due to a decline in commodity prices below the reference price or a decrease in commodity quantity by 20%; and (iii) in other unexpected situations, such as natural disasters.

3.3 Corruption in Mongolia

Mongolia, a country where corruption is prevalent, has faced persistent challenges in combating this issue. According to Transparency International, Mongolia was ranked 94th out of 180 countries in 2012 (Transparency International, 2012). The government of Mongolia established SWFs in 2012, as part of a strategic initiative to manage its resources effectively and address challenges associated with the short-term volatility of commodity revenue (World Bank, 2021).

The ranking of Mongolia dropped to 121st in 2023 (Transparency International, 2023), highlighting the ongoing struggle with corruption. Furthermore, the International Monetary Fund (IMF) has highlighted the weaknesses in Mongolia's revenue administration posing significant risks to fiscal sustainability (IMF, 2019). These vulnerabilities could potentially undermine the effectiveness of SWFs and exacerbate existing corruption challenges. Nonetheless, the government's decision to establish SWFs reflects its commitment to diversifying revenue sources and mitigating the impact of economic volatility.

3.4 The Model

The setup of my model is built on a small open economy model presented in Schmitt-Grohé and Uribe (2003) with three key extensions. First, I introduce a commodity sector characterized by an endowment that is entirely exported to the rest of the world. This sector is subject to commodity price shocks originating in the world commodity market. Second, the government establishes a SWF to stabilize its revenue stream and implement countercyclical fiscal policies through a fiscal rule. Finally, I incorporate an expropriation mechanism allowing politicians to steal from the government revenue generated by commodity exports.

3.4.1 Households

A representative household maximizes the expected utility:

$$\max E \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{where } u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \quad (3.1)$$

subject to the budget constraint:

$$c_t + i_t - d_t + \tau_t = y_t - (1 + r_{t-1})d_{t-1} \quad (3.2)$$

where d_t denotes the amount that households borrow from foreign lenders, or foreign debt, r_t denotes the interest rate at which domestic agents borrow in international market, c_t denotes consumption, i_t denotes gross investment, y_t denotes domestic output, and τ_t represents a lump-sum tax or transfer to households by the government.

The stock of capital evolves according to the following law of motion:

$$k_{t+1} = (1 - \delta) k_t + i_t \quad (3.3)$$

where $\delta \in (0, 1)$ denotes the depreciation rate of capital, and k_t denotes physical capital.

3.4.2 Production

Output is produced with a standard neoclassical production technology:

$$y_t = e^{z_t} k_t^\alpha h_{p,t}^{1-\alpha} \quad (3.4)$$

where $\alpha \in (0, 1)$, $h_{p,t}$ denotes labor employed in the final good production, and z_t denotes productivity that follows a first-order autoregressive process:

$$z_{t+1} = \rho_z z_t + \varepsilon_{z,t+1}, \quad \varepsilon_{z,t} \sim N(0, \sigma_z^2) \quad (3.5)$$

where $\rho_z \in (0, 1)$ governs the persistence of productivity, and $\varepsilon_{z,t}$ is an independently and identically distributed (i.i.d) shock.

Since labor is employed in both the production of final goods and the expropriation, we have $h_{p,t} + h_{g,t} = 1$, where $h_{g,t}$ denotes the labor employed in the expropriation sector. When there is no expropriation sector, $h_{p,t} = 1$. However, in the presence of an expropriation sector, politicians hire a fraction of labor to expropriate government revenue from commodity exports.

3.4.3 Interest rate in a small open economy

The interest rate faced by domestic agents, r_t , is given by:

$$r_t = r + p(\tilde{d}_t) \tag{3.6}$$

where \tilde{d}_t is the aggregate level of foreign debt. In equilibrium, aggregate debt equals individual debt, that is, $\tilde{d}_t = d_t$. The interest rate faced by domestic agents is increasing in the aggregate level of foreign debt. Thus, the function $p(\cdot)$ is strictly increasing and represents a country-specific interest rate premium. Following the approach of [Schmitt-Grohé and Uribe \(2003\)](#), the functional form of the risk premium is as follows:

$$p(\tilde{d}_t) = \psi(e^{d_t - \bar{d}} - 1) \tag{3.7}$$

where \bar{d} represents the steady-state level of foreign debt, and ψ represents sensitivity of a country interest rate premium.

In the steady state, the aggregate level of foreign debt held by households is \bar{d} , which implies that the country-specific interest rate premium is zero. Thus, the steady state interest rate faced by domestic agents equals the world interest rate, that is $\bar{r} = r$.

3.4.4 Commodity sector

Following [Medina and Soto \(2016\)](#), I assume that the production of the commodity requires no inputs, and there is an exogenous endowment of natural resources in each period. To guarantee a steady state, the endowment of the commodity good is constant every period. This endowment is completely exported and can be interpreted as the value added by natural resources in the gross production of the commodity.

Foreign agents demand the commodity good which is completely elastic at the price $P_{s,t}$. The price of copper is represented as a first-order autoregressive (AR(1)) process:

$$P_{s,t+1} = (1 - \rho_s)P_s^{ref} + \rho_s P_{s,t} + \varepsilon_{s,t+1}, \quad \varepsilon_{s,t} \sim N(0, \sigma_s^2) \quad (3.8)$$

where $\rho_s \in (0, 1)$ denotes the persistence of the copper price, $\varepsilon_{s,t}$ represents an independently and identically distributed (i.i.d) shock to the copper price, and P_s^{ref} denotes the long-run copper price, referred to as the reference price. Consequently, in the steady state, the price of copper equals P_s^{ref} .

3.4.5 Fiscal policy

A fiscal policy is as a key transmission mechanism through which a copper price shock affects an economy. The primary source of fiscal revenue is copper revenue, represented by $P_{s,t}\chi Y_s$, where χ denotes the government's share of total copper production, and Y_s denotes copper production. Government spending is constant if there is no shock. Let's consider fiscal policy under the following alternative scenarios with and without SWFs.

3.4.5.1 Fiscal policy without a SWF

When there is no SWF, the government collects revenues from copper production, covers its expenditures, and distributes any surplus (manages any deficit) through lump-sum transfers (taxes) to households.

The government balances its budget each period:

$$\bar{g}e^{gt} = P_{s,t}\chi Y_s + \tau_t \quad (3.9)$$

where the first term on the RHS represents the government revenues from copper, τ_t denotes a lump-sum tax or transfer to households, \bar{g} denotes fixed government spending, and g_t denotes a shock to government spending. g_t follows a stationary AR(1) process:

$$g_{t+1} = \rho_g g_t + \varepsilon_{g,t+1}, \quad \varepsilon_{g,t} \sim N(0, \sigma_g^2) \quad (3.10)$$

where $\rho_g \in (0, 1)$ denotes the persistence of the government spending shock, and $\varepsilon_{g,t}$ is independently and identically distributed (i.i.d).

Under this fiscal policy, a shock to the commodity price directly transmits to households through a lump-sum tax or transfer, which reflects the government's deficit or surplus position.

3.4.5.2 Fiscal policy with a SWF

The fiscal rule outlined in the Fiscal Stability Law, which incorporates a Sovereign Wealth Fund (SWF), can be described through the following steps: i) the government determines the reference price for copper, denoted as P_s^{ref} ; ii) based on this reference price, the government balance is determined; and iii) any revenues resulting from prices exceeding the reference price are accumulated in the SWF.

First, in this model, the reference price of copper is exogenously determined by the long-run equilibrium in the world commodity market.

Second, the government balance is defined by:

$$B_t = P_s^{ref} \chi Y_s - \bar{g}e^{gt} \quad (3.11)$$

where B_t denotes the government balance, and the first term on the right-hand side represents the structural revenue of the government, which is determined by the reference price and remains independent of a copper price shock.

The government balances its budget each period:

$$B_t = -\tau_t \quad (3.12)$$

where τ_t is a lump-sum tax on households.

Generally, the purpose of targeting a positive structural balance is to invest in public infrastructure to promote long-run growth in developing or resource-rich countries. However, in this model, I assume that the government does not invest. Instead, it distributes transfers to households, similar to dividends from natural resource endowments. This assumption could be relaxed in future extensions of the model. For now, this scenario can be considered the worst-case, ensuring that my results represent a lower bound of the welfare loss.

If there is no shock to government spending, the structural balance remains fixed at a target and is not subject to a copper price shock. Under this fiscal rule with a SWF, a copper price shock does not affect households or the economy.

Third, the government's excess revenues from copper, denoted as R_t , are computed as follows:

$$R_t = \left[P_{s,t} - P_s^{ref} \right] \chi Y_s \quad (3.13)$$

The SWF accumulates these excess revenues and earns a return of the world interest rate:

$$SWF_t = (1 + r)SWF_{t-1} + R_t \quad (3.14)$$

In this context, the SWF functions as a financial buffer for the economy, absorbing all shocks to copper prices.

In the steady state, there is no excess revenue from copper, as $\bar{P}_s = P_s^{ref}$. Consequently, no revenue is accumulated into the fund, and the SWF in the steady state is zero, indicated by $S\bar{W}F = 0$.

3.4.6 Expropriations

Suppose a representative politician expropriates a share of government revenues from copper⁴ and all proceeds from expropriation are thrown to the sea. The politician's expropriation technology is as follows:

$$s_t = e^{x_t} \Gamma^{1-\eta} h_{g,t}^\eta, \quad \eta \in (0, 1) \quad (3.15)$$

where s_t represents the fraction of government revenues from copper expropriated by politicians. The parameter Γ denotes the quality of institutions, with higher values indicating weaker institutions and easier expropriation. Additionally, x_t denotes a shock to the efficiency of expropriation, which follows a stationary AR(1) process:

$$x_{t+1} = \rho_x x_t + \epsilon_{x,t+1}, \quad \epsilon_{x,t+1} \sim N(0, \sigma_x^2) \quad (3.16)$$

where $\rho_x \in (0, 1)$ denotes the persistence of a shock to the efficiency of expropriation, and $\epsilon_{x,t}$ is independently and identically distributed (i.i.d).

The politicians maximize their profit each period by choosing an optimal fraction of expropriation and labor to employ:

$$\pi_{x,t} = \max_{s_t, h_{g,t}} s_t \left(P_{s,t} \chi Y_s \right) - w_t h_{g,t} \quad (3.17)$$

⁴There are several ways through which politicians attempt to expropriate copper revenues or a SWF (see Appendix).

subject to the expropriation technology

$$s_t = e^{x_t} \Gamma^{1-\eta} h_{g,t}^\eta$$

where R_t is a revenue that politicians try to steal. $h_{g,t}$ is labor employed in this sector.

The optimal fraction to expropriate depends on the quality of institution, wage, and revenues:

$$s_t = e^{\frac{x_t}{1-\eta}} \Gamma \left(\frac{\eta(P_{s,t} \chi Y_s)}{w_t} \right)^{\frac{\eta}{1-\eta}} \quad (3.18)$$

3.4.7 Feasibility

In the scenario where the government does not establish a SWF, the resource constraint of the economy at any t is as follows:

$$c_t + i_t + \bar{g}e^{gt} - d_t = Y_t - (1 + r_{t-1})d_{t-1} + P_{s,t} \chi Y_s \quad (3.19)$$

$$h_{p,t} = 1 \quad (3.20)$$

In the presence of both the SWF and expropriation, the feasibility condition in each period is as follows:

$$c_t + i_t + \bar{g}e^{gt} - d_t = Y_t - (1 + r_{t-1})d_{t-1} + P_s^{ref} \chi Y_s \quad (3.21)$$

$$SWF_t = (1 + r)SWF_{t-1} + \left[(1 - s_t)P_{s,t} - P_s^{ref} \right] \chi Y_s \quad (3.22)$$

$$h_{p,t} + h_{g,t} = 1 \quad (3.23)$$

The comparison between equations (19) and (21) indicates that in equation (19), the right-hand side contains a more volatile term, namely the copper price.

3.5 Data and parameterization

I calibrate several standard parameters and estimate the remaining structural parameters of the model using Bayesian methods and quarterly data from Mongolia. The Bayesian approach is particularly useful in my case because it allows for the identification of several key structural parameters that cannot be observed directly.

3.5.1 Data

My estimation utilizes quarterly data series from Mongolia, including output, consumption, investment, and government expenditure. The data, provided from the National Statistical Office of Mongolia, covers the period from 2005Q1 to 2023Q3. Although the copper price data is available for a longer duration, I restrict it to the 2005Q1 to 2023Q3 period to maintain consistency with other variables in my estimation. All variables are seasonally adjusted and presented in real per capita terms. Additionally, the data series are detrended using the HP filter, with a smoothing parameter set to 1,600 (see Figure 3.1).

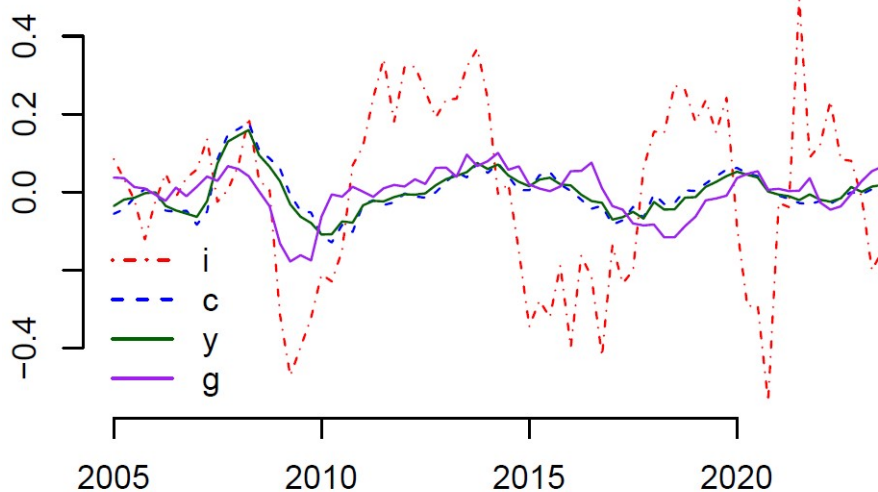


Figure 3.1: Cyclical Components

3.5.2 Calibrated parameters

Table 3.1 summarizes the values chosen for the structural parameters of the model. I assume that the risk aversion coefficient, σ , equals 2, and the quarterly depreciation rate equals 2.5%, in line with the real business cycle literature.

Table 3.1: Calibrated parameters

Parameter	Description	Value
σ	Relative risk aversion coefficient	2
δ	Quarterly capital depreciation rate (%)	2.5
\bar{r}	Real world interest rate (%)	1.01
β	Discount factor	0.99
α	Capital share in production	0.302
\bar{g}	Steady-state government spending	0.12
\bar{d}	Steady-state level of foreign debt	-0.026
ψ	Sensitivity of a country interest rate premium	4.485
χ	Government share in copper revenue	0.3
Y_s	Copper production	0.5
η	Share of labor in expropriation	0.1

The discount factor, β , is pinned down by the world interest rate as follows: $\beta = \frac{1}{1+r}$. The capital share in production, α , is set to the value used in [Mendoza \(1991\)](#). Additionally, the steady-state level of foreign debt, \bar{d} , and the sensitivity of the country's interest-rate premium to deviations of external debt from trend, ψ , are calibrated based on the values suggested by [Schmitt-Grohé and Uribe \(2003\)](#). I assume that the share of labor in expropriation, η , equals 0.1.

The government's share in copper revenue, denoted as χ , is set to 0.3, representing the proportion of copper mines owned by the government. In Mongolia, it is 34%⁵. The steady-state government spending, represented by \bar{g} , and copper production, denoted as Y_s , are calibrated to align with the respective shares of government spending and copper production in GDP.

⁵In Mongolia, 34% of the largest copper mine, Oyu Tolgoi, is owned by the government. Additionally, another copper mine named "Erdenet" is a 100% state-owned enterprise. In future work, I plan to calculate the Mongolian government's share more accurately by using detailed information about all copper mines.

3.5.3 Bayesian estimation

The remaining parameters of the model are estimated using Bayesian methods. They govern the short-run dynamics of the model, except Γ . Namely, the persistence of the four driving forces $\{\rho_z, \rho_g, \rho_s, \rho_x\}$, the standard deviation of their shocks $\{\sigma_z, \sigma_g, \sigma_s, \sigma_x\}$, and the expropriation efficiency $\{\Gamma\}$.

A Beta prior distribution with a mean of 0.5 and a standard deviation of 0.1 is imposed on the persistence parameters for all shocks, following [Germaschewski et al. \(2021\)](#) and [Smets and Wouters \(2007\)](#). The priors for shocks' standard deviations follows an inverse Gamma distribution, while the prior distribution for the quality of institution is Uniform.

Table 3.2: Priors and posteriors of estimated parameters

	Description	Pior	Post.Mongolia
Γ	Expropriation efficiency	U(1.255, 0.719)	0.688
ρ_z	Persistence: Productivity	B(0.5, 0.1)	0.772
ρ_g	Persistence: Gov.spending	B(0.5, 0.1)	0.932
ρ_s	Persistence: Copper price	B(0.5, 0.1)	0.926
ρ_x	Persistence: Expropriation	B(0.5, 0.1)	0.941
σ_z	St.dev: Productivity	IG(0.05, ∞)	0.005
σ_g	St.dev: Gov.spending	IG(0.05, ∞)	0.015
σ_s	St.dev: Copper price	IG(0.05, ∞)	0.016
σ_x	St.dev: Expropriation	IG(0.05, ∞)	0.003

Table 3.2 presents key statistics of the prior and posterior distributions. The reported posterior means are computed from a 4000 MCMC chain from which the first 1800 draws were discarded. The estimation delivers quite persistent processes for shocks to copper price, shocks to a government spending and shocks to efficiency of expropriation, and a relatively large value for the quality of institution for Mongolia.

3.6 Results

3.6.1 Performance of the model

Table 3.3 reports moments from both the data and the model. In the model, consumption and investment exhibit less volatility, while government spending is predicted to have excess

volatility. During the periods covered by my data, there have been significant fluctuations in private investment within Mongolia’s mining sectors. In this model, I have abstracted from this specific investment mechanism. In future work, the model could be extended to better capture these changes in investment, potentially enhancing its performance.

Table 3.3: Moments

	Data	Model
$\sigma(c)/\sigma(y)$	1.11	0.42
$\sigma(i)/\sigma(y)$	4.35	0.78
$\sigma(g)/\sigma(y)$	1.15	1.95
$\rho(y, c)$	0.97	0.71
$\rho(y, i)$	0.16	0.94
$\rho(y, g)$	0.52	0.04

Notes: $\sigma(x)$ denotes the standard deviation of x . $\rho(y, x)$ denotes the contemporaneous correlation between x and y .

The key feature of business cycles in developing countries is that consumption is more volatile than output. To address this issue, existing studies often incorporate financial constraints or stochastic productivity trends (Aguiar and Gopinath, 2007). Building on this approach, I plan to integrate these mechanisms into my model to improve its performance.

3.6.2 Welfare analysis

In this section, I analyze the welfare effects of SWFs and expropriation. First, I simulate consumption under the following scenarios: (i) no SWF; (ii) SWF without corruption; and (iii) SWF with corruption. Second, the lifetime utility under each scenario is computed as follows:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t)$$

Next, I calculate the reduction in consumption required in the steady state to make the agent indifferent between the steady state and the economy with copper price shocks under each

scenario. This calculation follows the methodology outlined by [Otrok \(2001\)](#). The required reduction is denoted by λ in equation (3.24).

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_{ss}(1 + \lambda)) \quad (3.24)$$

where c_{ss} is the steady state consumption; λ is the required reduction. If there is a welfare loss, λ would be negative.

The calculation results are presented in Table 3.4. The welfare loss from business cycles, denoted as λ , is the lowest in an economy with SWF. An economy that establishes SWF with low corruption levels incurs a slightly higher welfare loss, but still performs better than an economy without SWF. Unfortunately, the current level of corruption in Mongolia results in a higher welfare loss compared to a scenario without SWF.

Table 3.4: Welfare Impacts

	Lifetime Utility	λ
SWF	-9.54	-0.133
SWF with low corruption	-9.73	-0.136
SWF with current level of corruption	-9.93	-0.139
SWF with high corruption	-10.40	-0.146
No SWF	-9.59	-0.137

It is worth noting that the welfare losses from commodity price shocks in these scenarios are significantly larger than those observed in standard RBC models. [Lucas \(1987\)](#) and [Otrok \(2001\)](#) find a very small welfare losses due to business cycles. In my model, a primary source of business cycles is the highly volatile price shocks to copper, which result in larger welfare losses. Specifically, while the standard deviation of TFP shocks, σ_z , is 0.05, the standard deviation of copper price shocks, σ_s , is 0.19—nearly four times greater.

3.6.3 Effects of shocks to the commodity price

In this section, I analyze the effects of a positive copper price shock under various scenarios. The size of the shock is its standard deviation. In Figure 3.2 and 3.3, the fiscal policy without

the SWF is evidently procyclical, as the positive price shock leads to increases in government revenues, fiscal surplus, transfers to households, as well as output and wages. As expected, the fiscal policy with SWF is acyclical. A price shock has no impact on consumption, output, wage, interest rate, or fiscal deficit, and therefore, transfers to households remain unaffected.

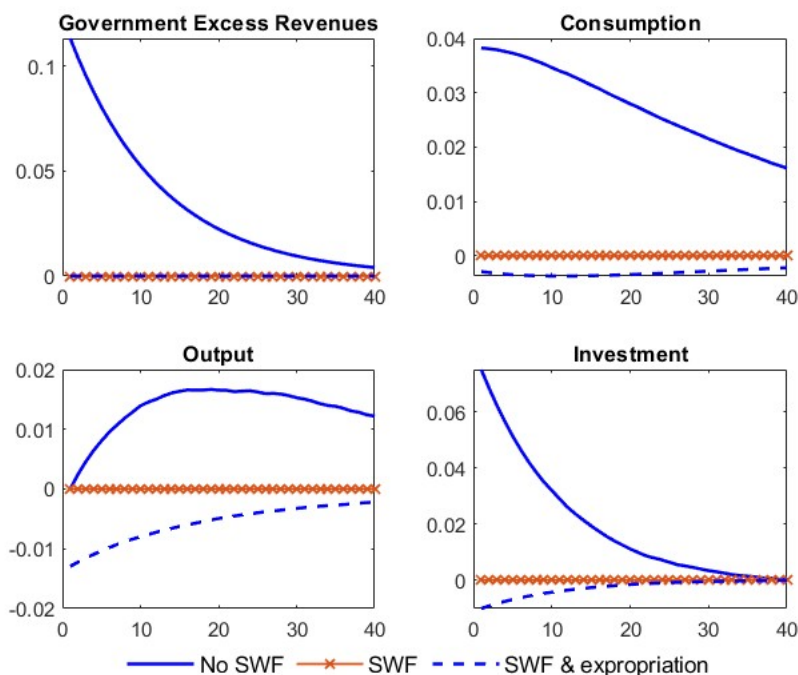


Figure 3.2: Responses to a copper price shock

In the scenario with both the SWF and expropriations, a positive shock to commodity prices increases revenues from copper, which politicians expropriate. This increased opportunity for rent encourages politicians to steal more, leading to an increase in the fraction of expropriation, denoted as s_t . As a result, there is a decrease in labor employed in the production of final goods and an increase in labor in the expropriation sector. The rising demand for labor leads to higher wage rates but decreases production in the final goods sector. Consequently, investment and capital in the final goods sector decrease, ultimately resulting in a reduction in consumption.

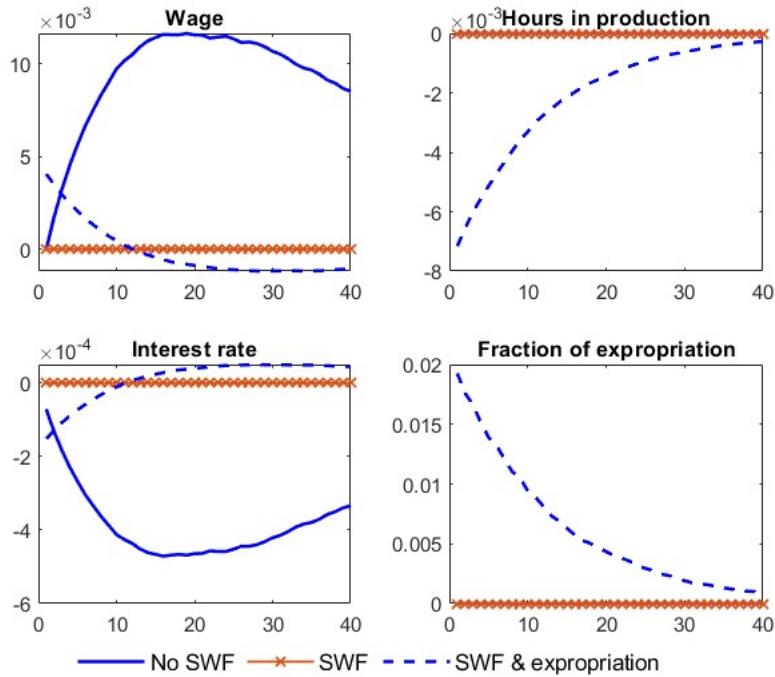


Figure 3.3: Responses to a copper price shock (cont.)

3.7 Conclusion

Commodity prices, including those of oil, gold, and copper, are known for their unpredictable fluctuations. When these prices rise or fall sharply, it can have significant effects on the economies of countries that heavily rely on exporting these resources. As a result, governments in these countries often experience volatile revenue streams. During times of high commodity prices, they may enjoy windfall profits, whereas periods of low prices can lead to dramatic drops in revenue. To manage these fluctuations and stabilize their economies, many resource-rich countries have established SWFs. These state-owned investment funds are designed to accumulate and manage financial assets to ensure fiscal stabilization and benefit the country's future. However, large inflows of revenue into SWFs may inadvertently encourage opportunistic behavior among politicians and increase the risk of corruption, especially in contexts where corruption is already prevalent.

This paper develops a RBC model for a small open economy that incorporates a com-

modity sector, a SWF, and expropriation, with the aim of examining the welfare implications of SWFs and corruption. First, the analysis reveals that the implementation of a SWF enhances welfare by effectively smoothing government revenues and acting as a financial buffer against fluctuations in commodity prices. Second, the corruption diminishes the welfare benefits associated with a SWF. While an economy with an SWF and low levels of corruption experiences a slightly higher welfare loss than one with no corruption, it still outperforms an economy without an SWF. Unfortunately, a high level of corruption results in a greater welfare loss than that observed in a scenario without a SWF.

It is worth mentioning that these are preliminary results. In future research, I could relax certain assumptions, such as allowing the government to make investments. To better match the observed volatility of consumption and investment, various mechanisms could be introduced, such as financial constraints, exogenous shocks to investment, and factors influencing the quality of institutions.

Bibliography

- Aguiar, M. and Gopinath, G. (2007). Emerging market business cycles: The cycle is the trend. *Journal of political Economy*, 115(1):69–102.
- Aliprantis, D., Fee, K., and Schweitzer, M. (2019). Opioids and the labor market.
- Angelopoulos, K., Economides, G., and Vassilatos, V. (2011). Do institutions matter for economic fluctuations? weak property rights in a business cycle model for mexico. *Review of Economic Dynamics*, 14(3):511–531.
- Antràs, P., Garicano, L., and Rossi-Hansberg, E. (2005). Offshoring in a knowledge economy. boston: Harvard institute of economic research. Technical report, Discussion Paper.
- Antràs, P., Garicano, L., and Rossi-Hansberg, E. (2006). Offshoring in a knowledge economy. *The Quarterly Journal of Economics*, 121(1):31–77.
- Antweiler, W., Copeland, B. R., and Taylor, M. S. (2001). Is free trade good for the environment? *American economic review*, 91(4):877–908.
- Arezki, M. R., Kazimov, M. K., and Hamilton, M. K. (2011). *Resource windfalls, macroeconomic stability and growth: the role of political institutions*. International Monetary Fund.
- Birnbaum, H. G., White, A. G., Schiller, M., Waldman, T., Cleveland, J. M., and Roland, C. L. (2011). Societal costs of prescription opioid abuse, dependence, and misuse in the united states. *Pain medicine*, 12(4):657–667.
- Buckles, K., Evans, W. N., and Lieber, E. M. (2023). The drug crisis and the living arrangements of children. *Journal of Health Economics*, 87:102723.
- Bullinger, L. R. and Wing, C. (2019). How many children live with adults with opioid use disorder? *Children and Youth Services Review*, 104:104381.
- Caicedo, S., Lucas Jr, R. E., and Rossi-Hansberg, E. (2019). Learning, career paths, and the distribution of wages. *American Economic Journal: Macroeconomics*, 11(1):49–88.
- CEA (2017). *The underestimated cost of the opioid crisis*. Executive Office of the President of the United States, Council of Economic Advisers.
- Cherniwchan, J., Copeland, B. R., and Taylor, M. S. (2017). Trade and the environment: New methods, measurements, and results. *Annual Review of Economics*, 9:59–85.
- Currie, J. and Schnell, M. (2018). A closer look at how the opioid epidemic affects employment. *Harvard Business Review*.
- Davenport, S., Weaver, A., and Caverly, M. (2019). Economic impact of non-medical opioid use in the united states: Annual estimates and projections for 2015 through 2019. *Society of Actuaries*.

- Deiana, C. and Giua, L. (2018). The us opidemic: Prescription opioids, labour market conditions and crime.
- Escobar Salcedo, D., Lafortune, J., Rubini, L., and Tessada, J. (2020). Resource misallocation from childcare policies. *Available at SSRN 3611778*.
- Fernández, A., González, A., and Rodriguez, D. (2018). Sharing a ride on the commodities roller coaster: Common factors in business cycles of emerging economies. *Journal of International Economics*, 111:99–121.
- Florence, C., Luo, F., and Rice, K. (2021). The economic burden of opioid use disorder and fatal opioid overdose in the united states, 2017. *Drug and alcohol dependence*, 218:108350.
- Florence, C., Luo, F., Xu, L., and Zhou, C. (2016). The economic burden of prescription opioid overdose, abuse and dependence in the united states, 2013. *Medical care*, 54(10):901.
- Frankel, J. A. (2011). How can commodity exporters make fiscal and monetary policy less procyclical? *HKS Faculty Research Working Paper Series*.
- Frankel, J. A., Vegh, C. A., and Vuletin, G. (2013). On graduation from fiscal procyclicality. *Journal of Development Economics*, 100(1):32–47.
- Frynas, J. G. (2017). Sovereign wealth funds and the resource curse: Resource funds and governance in resource-rich countries.
- Garicano, L., Lelarge, C., and Van Reenen, J. (2016). Firm size distortions and the productivity distribution: Evidence from france. *American Economic Review*, 106(11):3439–79.
- Garicano, L. and Rossi-Hansberg, E. (2004). Inequality and the organization of knowledge. *American Economic Review*, 94(2):197–202.
- Garicano, L. and Rossi-Hansberg, E. (2006). Organization and inequality in a knowledge economy. *The Quarterly journal of economics*, 121(4):1383–1435.
- Germaschewski, Y., Horvath, J., and Rubini, L. (2021). Property rights, expropriations, and business cycles in china. *Journal of Economic Dynamics and Control*, 125:104100.
- Gihleb, R., Giuntella, O., and Zhang, N. (2020). Prescription drug monitoring programs and neonatal outcomes. *Regional Science and Urban Economics*, 81:103497.
- Harris, M. C., Kessler, L. M., Murray, M. N., and Glenn, B. (2020). Prescription opioids and labor market pains the effect of schedule ii opioids on labor force participation and unemployment. *Journal of Human Resources*, 55(4):1319–1364.
- Hopenhayn, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica: Journal of the Econometric Society*, pages 1127–1150.
- Hsieh, C.-T. and Moretti, E. (2019). Housing constraints and spatial misallocation. *American Economic Journal: Macroeconomics*, 11(2):1–39.

- IMF (2019). IMF Country Report No. 19/298.
- Karjoo, Z. and Rubini, L. (2023). *Exporters and Energy Efficiency: Evidence from Chilean Firms*. SSRN.
- Kirson, N. Y., Scarpati, L. M., Enloe, C. J., Dincer, A. P., Birnbaum, H. G., and Mayne, T. J. (2017). The economic burden of opioid abuse: updated findings. *Journal of managed care & specialty pharmacy*, 23(4):427–445.
- Krueger, A. B. (2017). Where have all the workers gone? an inquiry into the decline of the us labor force participation rate. *Brookings papers on economic activity*, 2017(2):1.
- Lucas, R. E. (1978). On the size distribution of business firms. *The Bell Journal of Economics*, pages 508–523.
- Lucas, R. E. (1987). Models of business cycles. (*No Title*).
- Maclean, J. C., Mallatt, J., Ruhm, C. J., and Simon, K. (2020). Economic studies on the opioid crisis: A review.
- Maclean, J. C., Mallatt, J., Ruhm, C. J., and Simon, K. (2022). The opioid crisis, health, healthcare, and crime: A review of quasi-experimental economic studies. *The ANNALS of the American Academy of Political and Social Science*, 703(1):15–49.
- McAdam-Marx, C., Roland, C. L., Cleveland, J., and Oderda, G. M. (2010). Costs of opioid abuse and misuse determined from a medicaid database. *Journal of pain & palliative care pharmacotherapy*, 24(1):5–18.
- Medina, J. P. and Soto, C. (2016). Commodity prices and fiscal policy in a commodity exporting economy. *Economic Modelling*, 59:335–351.
- Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *econometrica*, 71(6):1695–1725.
- Mendoza, E. G. (1991). Real business cycles in a small open economy. *The American Economic Review*, pages 797–818.
- NDIC (2011). *The economic impact of illicit drug use on American society*. Washington D.C., United States Department of Justice, National Drug Intelligence Center.
- Otrok, C. (2001). On measuring the welfare cost of business cycles. *Journal of Monetary Economics*, 47(1):61–92.
- Park, S. and Powell, D. (2021). Is the rise in illicit opioids affecting labor supply and disability claiming rates? *Journal of Health Economics*, 76:102430.
- Ploeg, F. v. d. (2011). Natural resources: curse or blessing? *Journal of Economic literature*, 49(2):366–420.

- Restuccia, D. and Rogerson, R. (2008). Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic dynamics*, 11(4):707–720.
- Restuccia, D. and Rogerson, R. (2013). Misallocation and productivity.
- Restuccia, D. and Rogerson, R. (2017). The causes and costs of misallocation. *Journal of Economic Perspectives*, 31(3):151–74.
- Rietveld, C. A. and Patel, P. C. (2021). Prescription opioids and new business establishments. *Small Business Economics*, 57:1175–1199.
- Rubini, L. (2014). Innovation and the trade elasticity. *Journal of Monetary Economics*, 66:32–46.
- Ruhl, K. J. et al. (2008). The international elasticity puzzle. *unpublished paper, NYU*, pages 703–726.
- Sachs, J. D. and Warner, A. M. (2001). The curse of natural resources. *European economic review*, 45(4-6):827–838.
- Schmitt-Grohé, S. and Uribe, M. (2003). Closing small open economy models. *Journal of international Economics*, 61(1):163–185.
- Shapiro, J. S. and Walker, R. (2018). Why is pollution from us manufacturing declining? the roles of environmental regulation, productivity, and trade. *American Economic Review*, 108(12):3814–3854.
- Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American economic review*, 97(3):586–606.
- Transparency International (2012). Corruption Perceptions Index 2012. MArch 22, 2024.
- Transparency International (2023). Corruption Perceptions Index 2023. MArch 22, 2024.
- Van Hasselt, M., Keyes, V., Bray, J., and Miller, T. (2015). Prescription drug abuse and workplace absenteeism: evidence from the 2008–2012 national survey on drug use and health. *Journal of Workplace Behavioral Health*, 30(4):379–392.
- White, A. G., Birnbaum, H. G., Mareva, M. N., Daher, M., Vallow, S., Schein, J., and Katz, N. (2005). Direct costs of opioid abuse in an insured population in the united states. *Journal of Managed Care Pharmacy*, 11(6):469–479.
- World Bank (2021). Mongolia’s proposed sovereign wealth fund in the broader fiscal framework. Technical report, World Bank, Washington, DC. License: CC BY 3.0 IGO.

Appendices

APPENDIX A

Chapter 1: Equilibrium Characterization and Proofs

A.1 Equilibrium Characterization and Proofs

A.1.1 Equilibrium Characterization

The first order conditions to the problem described in equation (1.31) subject to equation (1.32) are as follows:

$$w'(z) = \frac{\int_{y_0}^{z_m} y f(y) dy - w(z)}{\left[1 - (1 - \rho(z)) F(z) - \rho(z) \beta \gamma F(z)\right]} \left[(1 - \rho(z) + \rho(z) \beta \gamma) f(z) - (1 - \beta \gamma) \rho'(z) F(z) \right]$$

To find an equilibrium assignment function, totally differentiate equation (1.35) with respect to z_p to obtain:

$$g(z) = n(\varphi^{-1}(\varphi(z))) g(\varphi(z)) \varphi'(z) \tag{A.1}$$

Solving for $\varphi'(z)$,

$$\varphi'(z) = \frac{1}{n(\varphi^{-1}(\varphi(z)))} \frac{g(z)}{g(\varphi(z))} \tag{A.2}$$

Substituting for $n(\varphi^{-1}(z_m))$, yields the following differential equation:

$$\varphi'(z) = \kappa \left[1 - (1 - \rho(z)) F(z) - \rho(z) \beta \gamma F(z) \right] \frac{g(z)}{g(\varphi(z))} \tag{A.3}$$

where there are two boundary conditions, $\varphi(z_0) = z^*$ and $\lim_{z \rightarrow z^*} \varphi(z) = \infty$. These conditions and the differential equation together determine the equilibrium assignment function. Notice that all terms on the right-hand side are positive, implying that $\varphi'(z) > 0$. It

implies that at the equilibrium, the higher-skilled workers work for higher-skilled managers regardless their health condition.

By solving two differential equations (A.1) and (1.36), we find the wage and assignment functions: $w(z)$ and $\varphi(z)$. Let us solve (1.36) equation first because an equilibrium assignment function $\varphi(z)$ can be found without knowing the wage function. Then we find equilibrium wage and rent functions so that they always maintain this equilibrium assignment.

Using (1.36) and the boundary condition $\varphi(z_0) = z^*$, we find:

$$G(\varphi(z)) = G(z^*) + \int_{z_0}^z \kappa \left[1 - (1 - \rho(z)) F(z) - \rho(z) \beta \gamma F(z) \right] g(z) dz \quad \text{all } z \in [z_0, z^*] \quad (\text{A.4})$$

Using the upper boundary condition $\varphi(z^*) = \infty$, we solve the following equation for the threshold z^* :

$$1 = G(z^*) + \int_{z_0}^{z^*} \kappa \left[1 - (1 - \rho(z)) F(z) - \rho(z) \beta \gamma F(z) \right] g(z) dz \quad (\text{A.5})$$

From (1.37), an equilibrium assignment function is:

$$\varphi(z) = G^{-1} \left(G(z^*) + \int_{z_0}^z \kappa \left[1 - (1 - \rho(z)) F(z) - \rho(z) \beta \gamma F(z) \right] g(z) dz \right) \quad \text{all } z \in [z_0, z^*] \quad (\text{A.6})$$

Since the z^* individual is indifferent between becoming a manager or a worker, $w(z^*) = R(z^*)$. Using an equilibrium assignment function and $w(z^*) = R(z^*)$, equilibrium wage

function is obtained as follows:

$$w(z) = w(\varphi^{-1}(z^*)) + \frac{\int_{z_0}^z \left(\left[(1 - \rho(z) + \rho(z)\beta\gamma)f(z) - (1 - \beta\gamma)\rho'(z)F(z) \right] \int_{y_0}^{\varphi(z)} yf(y)dy \right) dz}{\left[1 - (1 - \rho(z))F(z) - \rho(z)\beta\gamma F(z) \right]} \quad (\text{A.7})$$

This is the solution to the equation 1.34. Given the wage function, the rent function will be:

$$R(z_m) = \frac{\int_{y_0}^{z_m} yf(y)dy - w(\varphi^{-1}(z_m))}{\kappa \left[1 - (1 - \rho(\varphi^{-1}(z_m)))F(\varphi^{-1}(z_m)) - \rho(\varphi^{-1}(z_m))\beta\gamma F(\varphi^{-1}(z_m)) \right]} \quad (\text{A.8})$$

A.1.2 Proof of Proposition 4

Consider the problem faced by an individual with skill level z_m conditional on being a manager:

$$w_m(z_m) = \max\{\pi_h(z_m), \pi_u(z_m)\}$$

where

$$\pi_h(z_m) = \max_z \frac{1}{\kappa(1 - F(z))} \left[\int_{y_0}^{z_m} yf(y)dy - w_h(z) \right] \quad (\text{A.9})$$

$$\pi_u(z_m) = \max_z \frac{1}{\kappa(1 - \beta\gamma F(z))} \left[\int_{y_0}^{z_m} yf(y)dy - w_u(z) \right] \quad (\text{A.10})$$

The first order conditions to solve equations (A.9) and (A.10) are

$$\begin{aligned} \frac{f(z)}{\kappa(1 - F(z))^2} \left[\int_{y_0}^{z_m} yf(y)dy - w_h(z) \right] - \frac{1}{\kappa(1 - F(z))} w'_h(z) &= 0 \\ \frac{\beta\gamma f(z)}{\kappa(1 - \beta\gamma F(z))^2} \left[\int_{y_0}^{z_m} yf(y)dy - w_u(z) \right] - \frac{1}{\kappa(1 - \beta\gamma F(z))} w'_u(z) &= 0 \end{aligned}$$

Both of these equations must hold in equilibrium if both healthy and unhealthy types are hired. Also, if type z_m hires healthy workers, then $w_m(z_m) = \frac{1}{\kappa(1-F(\hat{z}_h))} \left[\int_{y_0}^{z_m} yf(y)dy - w_h(\hat{z}_h) \right]$, where \hat{z}_h is the optimal choice, and if the manager hires unhealthy workers, then $w_m(z_m) = \frac{1}{\kappa(1-F(\hat{z}_u))} \left[\int_{y_0}^{z_m} yf(y)dy - w_h(\hat{z}_u) \right]$. Then the first order conditions to the problem of a manager imply the following:

$$\begin{aligned} w'_h(\hat{z}_h) &= \kappa f(\hat{z}_h) w_m(z_m) \\ w'_u(\hat{z}_u) &= \kappa \beta \gamma f(\hat{z}_u) w_m(z_m) \end{aligned}$$

These conditions are the same as conditions (1.23) and (1.24). Next we show that conditions (1.25) and (1.26) also hold in a decentralized equilibrium. To do that, differentiate equations (A.9) and (A.10) with respect to z_m using the envelope theorem:

$$\begin{aligned} \pi'_h(z_m) &= \frac{1}{\kappa(1-F(\hat{z}_h))} z_m f(z_m) \\ \pi'_u(z_m) &= \frac{1}{\kappa(1-\beta\gamma F(\hat{z}_u))} z_m f(z_m) \end{aligned}$$

given that $w'_m(z_m) = \pi'_h(z_m)$ if manager z_m hires healthy workers and $w'_m(z_m) = \pi'_u(z_m)$ if she hires unhealthy workers, these conditions are the same as conditions (1.25) and (1.26). Thus, all the first order conditions in the decentralized equilibrium are the same as the allocation that decentralizes the social planner solution. To show that the equilibrium is optimal, we only need to show that the border conditions hold.

Start by considering the zero profit condition $\Pi_u(z_0, \varphi_u(z_0)) = 0$. This holds by definition, since the compensation of the manager hiring the least productive workers is equal to output minus worker compensation.

To see that the value matching conditions also hold, note that the problem of any individual is to solve $\max\{w_i(z), w_n(z), w_m(z)\}$ for all z and for $i = h, u$. For each health status, there are marginal workers that are indifferent between becoming a worker or a manager hir-

ing healthy workers, or a manager hiring unhealthy workers. That is, there exists a healthy individual that is indifferent between being a worker, a manager hiring a healthy worker, or a manager hiring an unhealthy worker, which implies that $w_h(z^*) = w_m(z^*)$.

To show this, proceed by contradiction. Suppose this is not the case. Without loss of generality, suppose $w_h(z^*) < w_m(z^*)$. Then a healthy individual of type z^* would strictly prefer to be a manager. By continuity, there exists some $\epsilon > 0$ such that $w_m(z^* - \epsilon) > w_h(z^* - \epsilon)$, so that a healthy worker type $z^* - \epsilon$ would also prefer to be a manager, showing that z^* is not an equilibrium threshold, and reaching a contradiction. A similar argument also holds for the case if $w_h(z^*) > w_m(z^*)$.

Using similar arguments, it is straightforward to show that the remaining border conditions $w_u(z^*) = w_h(z_1)$ and $w_m(\varphi_h(z_0)) = w_n(\varphi_u(z_2))$ must also hold. The first of these guarantees that a manager that hires an unhealthy worker with skills z^* would also hire a healthy worker with skills z_1 , and the second implies that a manager is indifferent between hiring a healthy worker with skills z_0 or an unhealthy one with skills z_2 .

APPENDIX B

Chapter 2: Entry and Innovation Decisions

B.1 Firm's decision

- Innovate without exporting if

$$\frac{1}{\delta}\pi_I(z, A_H) - \kappa_I \geq \frac{1}{\delta}\pi_d(z, A_L) \quad (\text{B.1})$$

Thus,

$$z_I(\kappa_I) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1} \left(\frac{W}{P}\right) \left[A_H^{\sigma-1} - A_L^{\sigma-1}\right]^{\frac{1}{1-\sigma}} \left(\frac{R}{\delta\kappa_I}\right)^{\frac{1}{1-\sigma}}$$

- Export without innovating if

$$\frac{1}{\delta}\pi_x(z, A_L) - \kappa_x \geq \frac{1}{\delta}\pi_d(z, A_L) \quad (\text{B.2})$$

Thus,

$$z_x = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1} \frac{1}{A_L} \left(\frac{W}{P}\right) \left(\frac{R}{\delta\kappa_x}\right)^{\frac{1}{1-\sigma}} \tau N^{\frac{1}{1-\sigma}}$$

- Export and innovate if

$$\frac{1}{\delta}\pi_{xI}(z, A_H) - \kappa_x - \kappa_I \geq \frac{1}{\delta}\pi_d(z, A_L) \quad (\text{B.3})$$

Then,

$$z_{xI}(\kappa_I) = \sigma^{\frac{\sigma}{\sigma-1}} \frac{1}{\sigma-1} \left(\frac{W}{P}\right) \left(\frac{1}{\delta(\kappa_x + \kappa_I)}\right)^{\frac{1}{1-\sigma}} \left[A_H^{\sigma-1}(1 + N\tau^{1-\sigma}) - A_L^{\sigma-1}\right]^{\frac{1}{1-\sigma}} R^{\frac{1}{1-\sigma}}$$

$$v_e = \frac{1}{\delta} \bar{\pi} - \kappa_e = 0 \quad (\text{B.4})$$

where

$$\bar{\pi} = \frac{\Pi}{M} = \frac{1}{M} \int_{\kappa_I} \int_z \pi(z, A, \kappa_I) M g(z) dz h(\kappa_I) d\kappa_I$$

After some arrangements, we obtain:

$$\int_{\kappa_I} \int_z \pi(z, A, \kappa_I) g(z) dz h(\kappa_I) d\kappa_I = \delta \kappa_e$$

Solve for P . In the stationary equilibrium, the following condition holds:

$$M_e = \delta M$$

Thus,

$$L_e = \kappa_e M_e = \delta M \kappa_e = \Pi \quad (\text{B.5})$$