RELATIVE AND ABSOLUTE PRICE EFFICIENCY IN STOCK MARKETS: COULD BEHAVIORAL FINANCE AND EFFICIENT MARKETS BOTH BE CORRECT?

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RELATIVE AND ABSOLUTE PRICE EFFICIENCY IN STOCK MARKETS: COULD BEHAVIORAL FINANCE AND EFFICIENT MARKETS BOTH BE CORRECT?

BY

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DISSERTATION

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in
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ABSTRACT

RELATIVE AND ABSOLUTE PRICE EFFICIENCY IN STOCK MARKETS: COULD BEHAVIORAL FINANCE AND EFFICIENT MARKETS BOTH BE CORRECT?

by

James Ryan Hickey

University of New Hampshire

Prior research on stock prices has found that most variation in stock prices is attributable to variation in expected returns rather than cash flows. Prices exhibit “excess volatility” and are too persistent to be explained in terms of cash flows. This finding has contributed to the divergence between the so-called “behavioral finance” school of research and advocates of “efficient markets.” I examine whether the relation between fundamentals and prices is stronger in relative stock prices than absolute stock prices. I also explore whether the reasons for the excess volatility and persistence problems may be attributable to structural change.

Chapter 1 reviews the literature on stock prices. The extant research overwhelmingly relies on absolute stock price data; few researchers even consider relative stock prices. Research in psychology, often cited by behavioral researchers as reason to think markets are not efficient, provides reasons to suspect that relative stock prices may more closely reflect fundamentals than absolute prices. In addition, research on disaggregated prices suggests that the relationship between relative prices and fundamentals may be clearer in
the data than between fundamentals and absolute prices. In Chapters 2 and 3, I use time-series techniques, adapted from widely accepted approaches in previous literature, to examine this question. Chapter 2 uses short-run tests to examine whether relative prices also exhibit excess volatility. Chapter 3 uses cointegration techniques to examine the persistence problem; but I also examine whether there is less evidence of structural change effects on relative prices than absolute prices. I find evidence consistent with relative prices behaving more “efficiently” than absolute prices.
Chapter 1

Literature Review

ABSTRACT

This chapter examines the literature on stock market efficiency and argues that there is an important open question about market efficiency that the literature has not yet adequately addressed. I provide evidence that, while the behavioral finance literature grounds itself on psychological realism, it has overlooked the implications of the psychology research on relative versus absolute valuations. I argue that this previously-ignored branch of research may help explain why the question of stock market efficiency remains unsettled and controversial, and suggest a way forward for future research.
1.1 Introduction

The efficient markets hypothesis (EMH) defines a capital market as ‘efficient’ if asset prices fully reflect all available information about fundamentals (see Fama (1970) and Fama (1965)). If prices fully reflect information about fundamentals, the prices of the various assets in the market should send accurate signals to investors about which assets are likely to be “good” investments and which are not. Whether financial markets actually are efficient, though, remains unsettled: the 2013 Nobel Prize in Economic Sciences was awarded to Eugene F. Fama, Lars Peter Hansen, and Robert J. Shiller - despite the fact that Fama is a proponent of efficient markets and Shiller is a proponent of behavioral finance. Fama and Shiller agree that these two approaches to financial economics are mutually contradictory. Fama writes that “the behavioral finance literature is largely an attack on market efficiency” (Fama (2014) p. 1477), while Shiller notes that there is a “split between efficient markets enthusiasts...and those who believe in behavioral finance” (Shiller (2014) p. 1487).

This chapter provides a review of the current debate on stock market efficiency. I argue that both camps have missed important evidence already in the economics and psychology literatures, and part of the reason they have overlooked this evidence is because the standard view of “market efficiency” is too simplistic. Most economists, I think, would view the question of whether financial markets are efficient as a simple binary: either they are efficient or they aren’t.

I will argue that there are two different concepts of market efficiency that we should be testing. The first claims that the $ value of a stock or portfolio of stocks (the “absolute price” of the asset) closely reflects the underlying fundamentals. I refer to this hypothesis as “absolute price efficiency.” This notion of efficiency is what all of the extant literature of which I am aware has examined. The second claims that the ratio of the prices of two different stocks (the relative price of one asset in terms of the other) reflects the underlying (relative) fundamentals. I refer to this hypothesis as relative price efficiency. This chapter proceeds as follows:
First, I provide a motivation for what relative prices teach us about financial markets. I show that if all individual stocks are priced efficiently in absolute terms, then it follows that they are also priced efficiently in relative terms. However, the converse is not true. Two stocks or baskets of stocks may be priced inefficiently in absolute terms but efficiently in relative terms. The distinction is important. The concepts of relative and absolute price efficiency are closely aligned with the notions of informational and allocative efficiency. Informational efficiency is obtained when asset prices fully reflect all available information, as mentioned above; tests of EMH examine whether available information can be used to earn above-average risk-adjusted returns. Allocative efficiency is a stronger condition: it is obtained when society’s scarce capital is allocated across companies and investment projects according to their expected returns and riskiness. Some refer to this as “separating the wheat from the chaff.” An alternative but closely related way to define allocative efficiency is in terms of Pareto-optimality. The market’s pricing of various assets implies a specific allocation of available capital; if there does not exist a reallocation of the market’s capital that would make at least one person better off without harming anyone, prices may be said to be allocatively efficient. In financial markets, this translates to the statement that risk is optimally shared among market participants.

In general, informational efficiency is a necessary but not sufficient condition for allocative efficiency - but we will see that most tests of efficiency make an assumption that implies an if-and-only-if relation between informational and allocative efficiency. If stock markets are relative-price efficient, that would be evidence of informational efficiency even if absolute prices are not efficient. Moreover, it would also possible that they are allocatively efficient in a certain specific sense.¹

Second, I will review the debate between EMH and behavioral finance in order to characterize the nature of the divide. Researchers in both fields primarily examine data on absolute prices. In addition, researchers often use techniques that allow them to decompose (absolute) returns into a “cash flow” component and an “expected returns” component. In this context, the expected returns component may be thought of as a catch-all term for factors unrelated to cash flows; this includes both psychological biases (systematic

¹This would signal that the stock market is internally allocatively efficient. I will argue this more explicitly in Section 2
forecast errors) and time-variation in discount rates. Studies of broad market indices generally find that only a small share of price variation can be traced to movements in cash flows. This is the essence of the “excess volatility” problem. On this point, at least, the two schools of thought are in agreement. The divide between behavioral economists and proponents of EMH derives largely from the question of how to interpret these results. Behavioral economists believe that excess volatility stems from the influence of psychological factors and momentum trading. According to this view, these factors lead markets to bid prices away from levels consistent with the fundamental value. EMH proponents argue that the excess volatility stems primarily from a time-varying discount rate or risk premium. According to this view, stock prices reflect their fundamental values, but are more volatile than predicted by the canonical model because the discount rate is more volatile. To reiterate, these conclusions are based on the patterns observed in absolute price data; few researchers in either camp have given serious thought to examining relative prices. Essentially no-one has examined whether relative prices exhibit the same excess volatility as absolute prices.

Third, I discuss studies on disaggregated stock prices. Researchers find that variation in firm-level stock prices is driven largely by cash flows, not expected returns. In other words, disaggregated stock prices do not exhibit nearly as much excess volatility as do aggregate stock prices. This finding represents a paradox for both EMH proponents and behavioral economists. It is a paradox for behavioral economists because these results contradict the predictions of their psychology-based theories. For EMH, the problem is this: if discount rate variation is really driving prices so strongly, why doesn’t that effect appear in the disaggregated data? I will argue that the answer here may have to do with the effects of market-wide shocks on absolute prices. Such “macro” shocks may well be better described as uncertain - in the Knightian sense - rather than as risky. Certain findings from the research on disaggregated stock prices suggests that the relationship between cash flows and prices may appear “clearer” - that is, easier to detect - if we examine relative prices instead of absolute prices. While this dissertation focuses on relative versus absolute price efficiency, a natural secondary hypothesis that I also explore is whether the symptoms of inefficiency are reduced in

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2This is a significant simplification, of course, which will be more fully fleshed out later in this chapter; but that is the basic intuition of what this technique accomplishes.
disaggregated data.

Fourth, I review a branch of psychology research that, I argue, has been overlooked, or at least underappreciated, by behavioral and EMH economists. Research from psychology is frequently cited by behavioral economists as support for the view that markets do not do a particularly good job at setting stock prices. But psychology research finds that people are actually quite accurate at making relative judgments of value, even in contexts where their “absolute” judgments may be suspect. Whether these findings apply to financial markets is an open question. However, if they do, one would expect markets to do a better job setting relative prices than absolute prices. If so, the connection between relative price movements and fundamentals would be stronger - or “closer” than the connection between absolute prices and fundamentals. From this it would follow that markets may do a reasonably good job setting prices that provide accurate signals to investors about which assets are likely to be good investments and which are not. As an example, people might struggle with the question of whether the price of a share of Microsoft stock is at the right level; but they find it much easier to determine whether Microsoft is under- or over-valued relative to Caterpillar or HP.

Thus, the studies of disaggregated data and the psychology research suggest the same thing: it might be possible that we will see a stronger connection between relative prices and fundamentals than between absolute prices and fundamentals. That might be because the connection It would be reasonable to conclude from such a finding that markets may be more relative price efficient than they are absolute price efficient.

The remainder of this paper proceeds as follows: Section 2 explores what “relative price efficiency” and “absolute price efficiency” teach us about markets and how they relate to informational and allocative efficiency. Section 3 discusses the origins of the EMH-Behavioral divide - specifically, the excess volatility problem - and the theories that developed to explain this problem. Section 4 explores recent developments in the EMH literature. I note that research in this area generally focuses on absolute prices. Section 5 discusses developments in behavioral finance. I note that this research, like the EMH research, focuses on absolute prices. Section 6 discusses studies that use disaggregated data, and why they suggest a “clearer” relationship. Section 7 discusses the psychology literature and why it suggests a “closer” relationship. The final section
concludes.

1.2 What Relative and Absolute Stock Prices Tell Us About Markets

This section first presents a series of propositions showing that absolute and relative price efficiency are separable (to a degree). If the prices of all stocks equal their fundamental values (defined as the present discounted value of future cash flows), then the market is allocatively efficient. If this seems like a strong statement to the reader, it should be noted that it is not a controversial idea in the literature; it is almost taken for granted. Cochrane (2011) pointed out that his work, which purported to show that all price variation can be explained by variation in expected returns (not cash flows), was analogous to that of Fama (1970). While that work dealt explicitly with informational efficiency, Fama made a strong assumption that implies an equivalence between informational and allocative efficiency (I discuss this at length in Section 2.3, below). Moreover, Cochrane (2011) bases a number of his arguments on Campbell and Shiller’s (1988) decomposition, which explicitly claimed to be testing “market efficiency” by measuring excess volatility (Cochrane (2011) 1047 - 1048; see also citations therein).

However, even under a more general assumption than that made by Fama and subsequent researchers, allocative efficiency ought to imply informational efficiency (see discussions in subsequent sections); a situation in which the market is allocatively efficient but informationally inefficient would be pathological, if it were even possible. These propositions allow us to develop an argument that there is a link between the concepts of absolute and relative price efficiency are connected to the notions of informational and allocative efficiency. The most important of these arguments shows that if absolute prices are inefficient but relative prices are efficient (according to empirical criteria), then prices are likely to be informationally efficient and allocatively efficient across stocks, but prices are not allocatively efficient across asset classes (e.g. stocks versus bonds).
1.2.1 Propositions: Relative and Absolute Price Efficiency

Proposition 1: If absolute stock prices are all equal to their fundamental value (thus are individually efficient), then relative prices are efficient; but not the converse.

Proof: Let $P_i$ and $P_j$ be the prices of assets $i$ and $j$, with $i \neq j$. Denote the efficient prices of each asset $P_i^*$ and $P_j^*$. Then the efficient relative price of $i$ in terms of $j$ must be:

$$\left(\frac{P_i}{P_j}\right)^* = \frac{P_i^*}{P_j^*} \tag{1.1}$$

Suppose that for all $i$, $P_i = P_i^*$. Then for all $i \neq j$,

$$\left(\frac{P_i}{P_j}\right) = \left(\frac{P_i^*}{P_j^*}\right) \tag{1.2}$$

Thus, absolute price efficiency for disaggregated prices implies relative price efficiency. However, consider the converse. Assume that $\frac{P_i}{P_j} = \left(\frac{P_i^*}{P_j^*}\right)^*$. This does not imply that $P_i = P_i^*$; it is easy to see that for any non-negative number $\omega$,

$$\frac{\omega P_i^*}{\omega P_j^*} = \frac{P_i^*}{P_j^*} \tag{1.3}$$

In the above equation $P_i \neq P_i^*$ unless $\omega = 1$. This demonstrates that it is possible for relative prices to be efficient even if absolute prices are not.

Proposition 2: If a value-weighted market index is not absolute-price efficient, at least one of the individual stocks included in the index is not absolute-price efficient.

Proof: Let $\bar{P}$ be the price of a (value-weighted) index of the stocks of $N$ firms. This is, by definition, a weighted average of the prices of the $N$ stocks. It follows that

$$\bar{P} = I \times \sum_{i=1}^{N} \eta_i P_i \tag{1.4}$$
where $I$ is an indexing constant and $\eta_i$ is the number of shares of firm $i$ outstanding. Note that, as defined above, $\eta_i P_i$ is the number of outstanding shares of firm $i$ times the price of one share in firm $i$; it is therefore the market capitalization of firm $i$. Therefore the index value $\bar{P}$ is the sum of the market capitalizations of the $N$ firms included in the index. Notice that the weight of any one firm in the value of this index varies with the firms overall valuation. For instance, if a firm issued a 2-for-1 stock split, but the price of individual shares simultaneously fell by 50%, the firm’s market valuation would not change: $\eta_i$ would have doubled but $P_i$ would have fallen by half, so the firm’s contribution to the value of the index would be constant.

The value of a broad market index, therefore, tracks the level of total market capitalization one-for-one in percent terms. \(^3\) Obviously the index value and the level of market capitalization are not identically equally to each other, but the index is just a constant ($I$) times the total level of market capitalization $\sum_{i=1}^{N} \eta_i P_i$. The definition of the efficient price of an individual stock, $P^*_i$, necessarily defines the efficient value of the index:

$$\bar{P} = I \times \sum_{i=1}^{N} \eta_i P^*_i$$

(1.5)

If it were true that for all $i$, $P_i = P^*_i$, then $\bar{P} = \bar{P}^*$. The market index would be at its efficient level. By the law of contrapositives, then, $\bar{P} \neq \bar{P}^*$ implies that there exists at least one $i$ for which $P_i \neq P^*_i$. This is precisely the statement made in Proposition 2. Note that we can only say that at least one individual stock is inefficient. It is possible that all the individual stocks are absolute-price inefficient, but it is also possible that some of the individual stocks are absolute price efficient and some are not.

**Proposition 3**: It is possible that although a market price index is absolute-price inefficient, one or more relative prices are efficient.

By Proposition 2, an absolute-price inefficient market index implies that at least one individual stock is absolute-price inefficient; so that $P_i = \omega P^*_i$ with $\omega \neq 1$. Define $i_{IE} \leq N$ such that $P_i$ is absolute-price inefficient if and only if $i \leq i_{IE}$ (In other words, ordering the stocks in the index by whether they are

---

\(^3\) As a simple demonstration of the intuition here, suppose that the prices of all stocks in the index increase by 1% over their initial values $P_i$. But then $\bar{P} = I \times \sum \eta_i (P_i \times 1.01) = I \times 1.01 \sum \eta_i P_i = 1.01 \times I \times \sum \eta_i P_i$. The final term in this equation shows that the index value has increased by 1%; the preceding term shows that market capitalization has increased by 1%.
individually absolute-price efficient or not, with \( i_{IE} \) being the last inefficiently-priced stock). Consider, then, assets \( i, j, \) and \( k \) such that \( i, j \leq i_{IE} \) and \( k > i_{IE} \). If \( P_i = \omega P_i^* \) and \( P_j = \omega P_j^* \) (as before, \( \omega \neq 1 \)) then \( \frac{P_i}{P_j} = \left( \frac{P_i}{P_j} \right)^* \) (see Proposition 1). But \( \frac{P_i}{P_k} \neq \left( \frac{P_i}{P_k} \right)^* \). An analogous statement holds if one replaces the \( i \) in the previous sentence with \( j \). It follows that in this case some relative prices are efficient and some are not.

In the special case \( i_{IE} = N \), there is no such \( k \). In the special case in which all prices deviate from their efficient levels by exactly the same factor \( \omega \), then dividing any one price by any other cancels the \( \omega \) term from both the numerator and denominator; all relative prices would be efficient. Therefore it is possible (at least in theory) that absolute prices are in general not efficient, but relative prices are efficient. There may be good a priori reasons to suspect that deviations from efficiency are generally not all of the same magnitude for all prices. In that more general case, we would not observe that all relative prices are efficient.

To summarize the important points of these propositions: if individual (disaggregated) prices are efficient, then relative prices are efficient. But it is possible that a market aggregate is absolute-price inefficient while some or all relative stock prices are efficient.\(^4\) This concludes the propositions about the connections between absolute and relative price efficiency. I now turn to a discussion of what absolute and relative price efficiency teach us about markets.

### 1.2.2 Propositions: Implications

The preceding discussion shows that there are four possible states of market efficiency, not two:

1. Absolute prices are generally efficient and relative prices are generally efficient.

2. A market index’s value is absolute-price efficient, but disaggregated prices are generally not absolute-price efficient and relative prices are generally not efficient.\(^5\)

3. Absolute prices are generally inefficient but relative prices are generally efficient.

\(^4\)It is theoretically possible that a market aggregate is efficient even though individual prices and relative prices are not. This case, however, is somewhat pathological. It is also not consistent with the literature, which finds better evidence for the efficiency of disaggregated prices than for aggregate prices; see below. I therefore give no further consideration to this case.

\(^5\)As mentioned, this case is both pathological and hard to reconcile with the existing literature, so is not considered further here.
4. Absolute prices are generally inefficient and relative prices are generally inefficient.

Which of these four states the market falls into is in large part an empirical question. But if the reader is unwilling to abandon the standard efficient/not efficient binary, his appraisal may involve an underlying value judgment. This binary labeling works fine if we have reason to think that the “true” state is (1) or (4). In (1), the market is both allocatively and informationally efficient. In state (4), it is neither. States (2) and (3), however correspond to states in which the market’s allocative efficiency and informational efficiency are not the same.

Consider first case (2). That state implies that the market index value is allocatively efficient; it sends an accurate signal about the future prospects of investment in the stock market as a whole. But the stock market itself is not doing a good job of pricing individual stocks because individual stock prices do not send accurate signals about their prospects; individual stock prices are not allocatively efficient and may also be informationally inefficient. Case (3), though, being much more realistic than case (2), is of greater interest. In that case, the value of the market index is not allocatively efficient - it does not send an accurate signal about the future cash flows earned from investment in the stock market as a whole. But the market prices individual stocks in a way that sends accurate signals about the relative prospects of different stocks. This would mean that the market is externally not allocatively efficient, but the market is internally allocatively efficient and must therefore be informationally efficient in relative terms (movements in the relative values of individual stocks reflect changes in the relative prospects of those stocks; thus new relevant information about any two firms must be incorporated into their respective prices as that information becomes publicly available, and that information is incorporated into prices in a way that reflects the new information’s “true” impact on the relative prospects of the two firms). In other words, under this condition, it might be possible to improve upon the current social allocation of capital by moving some funds into or out of the stock market as a whole (that is to say, a Pareto-improvement might be possible); but it is not possible to reliably “beat the market” with a strategy involving shorting one stock and buying another based on any (public) information about the two firms.

11
The above argument demonstrates the reason to test both absolute and relative prices: it turns out that relative and absolute price efficiency tests can inform us about informational and allocative efficiency in a way that absolute-price tests alone cannot. The connection is not a simple one-to-one relation; in other words, it is not the case that "relative efficiency means informational efficiency and absolute efficiency means allocative efficiency," or some such simplistic structure. Rather, Relative prices inform us about aspects of both allocative and informational efficiency. We can only make a well-informed assessment about informational and allocative efficiency by knowing something about both absolute and relative prices. However, thus far I have not provided a formal definition or discussion of informational and allocative efficiency. Because a better understanding of these concepts will aid us in understanding what relative and absolute prices have to teach us about markets, I turn to an examination of this issue below.

1.2.3 Informational versus Allocative Efficiency

So far, I have defined what is meant by relative price efficiency as opposed to absolute price efficiency. I have also provided simple demonstrations of the fact that these concepts are related to each other, but are logically distinct. But what do these notions have to do with informational and allocative efficiency? As mentioned, Fama (1970) defined a capital market as efficient if prices fully reflect all available information about fundamentals. Many tests therefore examine whether it is possible to earn excess returns by using available information; if so, prices must not fully reflect available information, and one would conclude that the market is not efficient. This is what is meant by “informational” efficiency. In contrast, “allocative efficiency may be described as the proposition that the allocation of capital across various assets is Pareto-efficient; risk is optimally shared. This implies that prices must accurately reflect the present discounted value of future cash flows - \( P = P^* \).  These notions may be expressed mathematically as:

\[ P = P^* \]

This is a necessary and sufficient condition because if \( P \neq P^* \), a stock’s price does not reflect the future prospects of investment in it, and the amount of capital allocated to holdings in that stock must be non-optimal; a Pareto-improvement could be obtained by moving a non-zero amount of capital into or out of that asset depending on whether \( P < P^* \) or \( P > P^* \). If \( P = P^* \), a stock’s price accurately reflects the future prospects of investment in it and the amount of capital allocated to that investment is optimal.
\[ \phi^m_t = \phi_t \]  

(1.6)

and

\[ f^m(p_{t+\tau}|\phi^m_t) = f(p_{t+\tau}|\phi_t) \]  

(1.7)

where \( \phi_t \) is the set of relevant and available information at time \( t \), \( \phi^m_t \) is the set of information used by the market to form its forecasts, \( f(\cdot) \) is the “true” joint probability distribution of prices \( p \) at time \( t+\tau \) conditional on available information, and \( f^m(\cdot) \) is the market’s assessment of the joint probability distribution at time \( t+\tau \) conditional on the market’s information set. Since (1.6) states that the market uses all relevant information in constructing its forecasts, it corresponds to informational efficiency. Allocative efficiency corresponds to (1.7) because that condition states that the probability distribution of future returns used in the market’s forecast is the same as the “true” probability distribution. This implies perfect knowledge about the process determining future outcomes - a strong assumption, but one that delivers sharp textable predictions (see Fama (1976) 134 - 137).

However, notice that in this description of efficiency, Fama (1976) and indeed almost all research on these topics up to the present day assumes that there exists one objective model or process governing the evolution of asset prices over time, and that this model is known by participants. This strong assumption leads us to a sharp conclusion: under these assumptions, if the market is informationally efficient, it must be allocatively efficient, and vice versa. In other words, Fama’s assumption implies that (1.6) is true if and only if (1.7) is true.

Suppose instead that we abandon the assumption that there is such a knowable, objective model. Then we have imperfect knowledge about the future. As a result, it would then be possible that the market is informationally efficient but not allocatively efficient. In other words, the market knows what information

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7At this point I am following Fama (1976) and assuming the objective distribution \( f(\cdot) \) exists. This is a strong assumption, and the reader will note that I argue below that we may want to consider relaxing it.
matters and adjusts its forecasts as new information becomes available, but it does not have perfect knowledge about how to interpret that information. If (hypothetically) a market always incorporated all relevant information in its forecast, but its forecasts failed to accurately assess $P^*$, the market would not do a perfect job allocating capital to good investment prospects due to imperfect knowledge. It would seem natural to say that the market is informationally efficient but not allocatively efficient. This might appear in the data as something akin to market state (3): absolute price inefficiency but relative price efficiency.

It is therefore necessary to design a testing strategy that may allow us to distinguish between informational and allocative efficiency. In the preceding section, I showed that absolute- and relative-price efficiency are logically distinct. In addition, however, the use of both absolute- and relative-price data may help us distinguish between informational and allocative efficiency. For example, in Chapter 2, I employ VAR-based tests of absolute- and relative-price efficiency. Using a VAR in this way imposes Fama’s (1976) assumption that the price process can be described with a single “true” or “objective” model. The null hypotheses of these tests are consistent with both informational and allocative efficiency (because Fama’s assumption means that informational efficiency implies allocative efficiency). If the null hypothesis is not rejected, the inference is clear: the model appears to be a reasonably good representation, and the market appears to be both informationally and allocatively efficient. Suppose, though, that the hypothesis is rejected for absolute prices (as it is in most empirical tests). One interpretation of this result is the obvious one: the model is fine, but the market is neither allocatively nor informationally efficient. However, it is also possible that the rejection of the null hypothesis occurred because the VAR is not a good model of the price process. In other words, the VAR is not a good description of the forecasting strategies used by a diverse set of market participants with imperfect knowledge about the process. Given their imperfect knowledge, it is likely that market participants will not rely on a single forecasting strategy, and they will likely find it quite difficult to make a confident assessment of the “true” fundamental value of an asset. But they might find it easier to assess the relative value of different assets; this is an idea with support in the psychology literature, and one to which I will return below.
However, suppose that one runs a test of market efficiency for both absolute and relative prices, and we find that the null hypothesis is rejected for absolute prices but not rejected for relative prices. This would suggest that the market is incorporating new information in a way that causes its forecast of $P^*$ to deviate from the actual value of $P^*$; but these deviations “wash out,” so to speak, in relative prices. This means the market’s forecasts (as represented by the VAR) deviate from the “true” fundamental value by the same factor for all prices. This would support the idea that market participants find it difficult to identify a good estimate of the efficient price level in absolute terms, but that they find it easier to assess the relative values of different assets. In other words, the market chooses a forecasting strategy for prices, applies it consistently across different stocks, and updates those forecasts as new information becomes available. Knowledge is imperfect, but the imperfection of knowledge is less extreme for relative values than for absolute values.

In this scenario, therefore, the reason for the rejection of absolute price efficiency can be attributed to the fact that the VAR is not a good description of the markets forecasting strategy. The results would be consistent with the proposition that participants have more difficulty when attempting to estimate the absolute value of assets $i$ and $j$ individually than when attempting to assess the value of $i$ relative to $j$; the market’s difficulty at assessing absolute values appears in the data as deviations in the market’s forecast (away from $f^m(P^*) = P^*$). But market participants do not have as much difficulty assessing relative values, so the market’s estimate of $\frac{P_i^*}{P_j^*}$ stays fairly close to the “true” value of $\frac{P_i^*}{P_j^*}$. This would be consistent with informational efficiency (conditional on the market’s imperfect knowledge about the price process) because it would suggest that the market does update its forecasts as new relevant information becomes available. The market is not, however, allocatively efficient - absolute prices are not consistent with an optimal allocation of capital across asset classes. However, because in this scenario relative prices appear to be efficient, the markets allocation of capital within the stock market is efficient. Thus, if we reject absolute-price efficiency, but do not reject relative-price efficiency, the data suggests a market that is informationally efficient, internally allocatively efficient, but externally not allocatively efficient. In other words, the market incorporates all

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8This corresponds to the third of the four scenarios mentioned in the previous section.
relevant information in its forecasts, but does not necessarily do a good job determining how much capital should be invested in the market as a whole. However, once the amount of capital available for investment in stocks is decided, the market does a good job allocating that capital across the stocks of various companies.

This is why relative stock prices are important and relevant: they allow us to examine the market’s ability to distribute a given amount of capital among different assets. Asset prices are supposed to be signals that help investors separate the wheat from the chaff. Relative price efficiency would mean that market prices send accurate signals about which assets are likely to generate high cash flows and which are not. But this is a different question than the one examined by most of the literature. Even papers that analyze firm-level data do not directly address the market’s internal allocation of funds.\(^9\)

Therefore, one must distinguish between, respectively, the efficiency of price(s) that measure the level of market capitalization and the efficiency of price(s) measuring the distribution of that capital among various assets. The price of a broad market index measures the level of market capitalization but does not, on its own, imply anything about the distribution of that capital. Relative price efficiency is consistent with efficient (internal) allocation, but does not, on its own, speak directly to the efficiency of capital allocation across different types of assets.

I now turn to the sources of the divide between behavioral and EMH and what we can learn from this literature. We will observe that the origins of this divide stem from analyses of what I have termed absolute price efficiency.

1.3 The Source of the EMH-Behavioral Divide

1.3.1 Early Evidence of Excess Volatility

This section documents a major reason for the divide between EMH and the behavioral approach. Our awareness that stock prices do not appear to behave in a manner consistent with EMH appears to have \(^9\)Vuolteenaho (2002), for instance, is really examining whether firms’ total market capitalizations are related to their underlying fundamentals; see below.
emerged in large part due to a set of studies that used the present value model to uncover evidence of “excess volatility.” While excess volatility is just one of many “anomalies” that can be identified, it is among the most troubling anomalies for efficient markets theory (see generally Siegel (2002) for a full discussion of purported anomalies and the evidence for them). The present value model states that the fundamental value of a stock’s price is the present discounted value of its future cash flows (dividends):

\[ P_t^{*} = \sum_{i=1}^{\infty} \rho^i D_{t+i} \]  

(1.8)

where \( \rho \) is the discount rate and \( D_t \) is the dividend paid at time \( t \). The canonical version assumes a constant discount rate, but that assumption has been relaxed in many studies. Under rational expectations, \( P_t \), the actual price at time \( t \), should equal the mathematical expectation of \( P_t^{*} \). I will explore the implications of this argument below as well as in Chapter 2. \(^{10}\) Chapter 3 will include a discussion of cointegrated vector autoregression results that address the validity of the Gordon growth model, and therefore, the present value model.

Leroy and Porter (1981), Shiller (1981), Campbell and Shiller (1988), and Campbell (1991) all provided important early evidence that stock prices are too volatile to be explained by future dividend flows. According to LeRoy and Porter, “[T]he bounds on price dispersion implied by the efficient markets model are dramatically violated empirically...” (p. 571). Shiller (1981) argued that, if \( P_t = E_t(P_t^{*}) \) as mentioned above, the fundamental value must equal the price plus a forecast error. If so, and again by imposing rational expectations, the variance of \( P^{*} \) must equal the sum of the variances of \( P \) and the forecast error. \(^{11}\) Therefore, a necessary (but not sufficient) condition for market efficiency is:

\[ \sigma(P) \leq \sigma(P^{*}) \]  

(1.9)

\(^{10}\)There are some models that are special cases of the present value model but are known by other names. A well-known example is the so-called “Gordon growth model” (see Gordon (1962) and Gordon and Shapiro (1956)).

\(^{11}\)There is no covariance term because if the covariance was non-zero, the forecast could be improved; rational expectations rules this out.
He then plotted $P$ and $P^*$ (detrended) for the S&P Composite Index (left) and for the Dow Jones Industrial Average (right) and got a similarly dramatic result:

![Figure 1.1: Shiller (1981) Volatility Test](image)

Plainly, the condition (??) was violated by this data. The actual price appears to be far too volatile for its movements to be plausibly explained by movements in cash flows, assuming a constant discount rate. This is the essence of “excess volatility.” Furthermore, Shiller argued that the amount of movement in expected real interest rates that would be necessary to explain the observed price volatility was implausibly large (pp. 433-434). However, among the responses to this was the argument that there are plausible econometric reasons why this result might be spurious. The problem is that stock prices and dividends are nonstationary. In order to address the nonstationarity issue, Campbell and Shiller (1988) examined excess volatility in another way. Using data on the S&P Composite Index, they ran a vector autoregression to generate an estimate of the market’s forecast of the fundamental value of the dividend-price ratio. They then compared it with the actual dividend-price ratio observed in the data. If REH is correct, these ought to be quite closely correlated. However, they found a correlation of only 0.175 - within 1.2 standard deviations of 0. Their findings implied that one-period returns are about four times as variable as they ought to be given the model (p. 673). That’s slightly less than the factor of five to thirteen times found by Shiller (1981) (p. 432), but nevertheless this is a corroboration, not a rejection, of Shiller’s earlier finding. Another important study is Campbell (1991),
which found that the variance of news about future cash flows can account for only one third to one half of the variance of unexpected stock returns (p. 176). The data source was CRSP’s value-weighted NYSE index. Despite differences in the precise variance ratios these studies found, they corroborate each other qualitatively: stock prices are much too volatile to be explained by cash flow variation. Note also that these findings emerged from studies that examined absolute price data on several different broad market indices. None of the studies considered relative prices.

These findings have, if anything, been even further corroborated in more recent years. Shiller (2003) wrote that “[T]here is still every reason to think that, while markets are not totally crazy, they contain quite substantial noise, so substantial that it dominates the movements in the aggregate market” (p. 90). Indeed, evidence has been uncovered that this problem is not unique to equities markets: Giglio and Kelly (2018) find evidence of excess volatility in a wide variety of financial markets, and they argue that time variation in discount rates cannot account for this volatility.\(^{12}\)

### 1.3.2 Further Developments: Decomposition Techniques

However, these results were not the end of the excess volatility literature. Subsequent research attempted to determine what was driving prices, if not cash flows. This led to the development of decomposition techniques that allow the researcher to quantify the degree of excess volatility. They allow the researcher to make a well-supported claim about the percentage of price- or return-variation that can be explained by fundamentals. These approaches vary in their precise formulations, but most appear to rely on essentially the same underlying idea. For instance, Campbell, Lo, and Mackinlay (1997) show that, if one assumes a constant discount rate, the price of a stock at time \(t\) can be written:

\[
P_t = P_{Dt} + B_t
\]

\(^{12}\)Specifically, they find evidence of excess volatility in markets for sovereign and corporate default risk, commodities, interest rates, and currencies.
where $P_{Dt}$ is the present discounted value of expected future dividends (the so-called “fundamental value”) and $B_t$ is a “rational bubble” term: it is the expected discounted value at time $t$ of future price increases. The presence of $B_t$ is (at least theoretically) consistent with both constant expected returns and rational expectations. However, the authors argue that empirical and theoretical reasons exist to rule out the bubble term in practice. (Campbell, Lo, and Mackinlay (1997) 258 - 259). In addition, they explore the insight this decomposition provides for studying returns and the variance of returns, and consider the consequences of allowing for a time-varying discount rate (Campbell, Lo, and Mackinlay 260 - 267).

Cochrane (2011) provides two other approaches to this decomposition. Suppose one performs three regressions in which the dependent variables are discounted long-run returns, discounted dividend growth, and the log dividend-price ratio. The independent variable for each is past values of the log dividend price ratio. Then the following relation holds:

$$1 \approx b_r^{(k)} - b_{d\Delta}^{(k)} + \rho^k b_{dP}^{(k)}$$

where $k$ represents the number of periods that the independent variable is lagged behind the dependent variable. Each of the three $b$s in the above is the coefficient on the lagged dividend-price ratio drawn from one of the three regressions. The first is drawn from the regression of discounted returns; the second from the regression of discounted dividend growth; and the last from the regression of the log dividend-price ratio. These coefficients can be interpreted as the fraction of dividend yield variation attributable to, respectively, expected returns, dividend growth, and a rational bubble. However, that decomposition is equivalent to a variance decomposition. Define the following terms:

- $dp_t$: the log dividend-price ratio at time $t$
- $\rho$: the (assumed constant) discount rate
- $r_{t+k}$: the expected return $k$ periods ahead
- $\Delta d_{t+k}$: the dividend growth rate from $t = 0$ to $t = k$
Then:

\[
\text{var}(d_{Pt}) = \text{cov}(d_{Pt}, \sum_{j=1}^{k} \rho^{j-1} r_{t+j}) - \text{cov}(d_{Pt}, \sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j}) + \rho^{k} \text{cov}(d_{Pt}, d_{Pt+k})
\]

(1.12)

Like several of his predecessors, Cochrane drew data from a CRSP value-weighted market index. He found that “all price-dividend ratio volatility corresponds to variation in expected returns. None corresponds to variation in expected dividend growth, and none to ‘rational bubbles’ ” (Cochrane (2011) 1050; emphasis added). This basic qualitative finding - that all the “action” is in returns, and little or none of it in fundamentals - also appears across studies of a number of other asset classes: Treasuries, bonds, foreign exchange, sovereign debt, and housing. Moreover, at least according to Cochrane, his finding is typical and representative of the larger literature on this topic (Cochrane (2011) 1051 - 52).

While the techniques employed in the research discussed above may appear different on the surface, all are derived from the present value model. Moreover, these papers use absolute-price data, often the price of a broad market index. When economists attempted to quantify the extent to which price variation is due to fundamentals, they decomposed stock prices (or returns, or variances) into two or three parts. Cash flows (that is, dividends) is always one component; one or both of “expected returns” and a “bubble” term may also be included depending on the researcher’s precise goal and approach.\(^{13}\) Despite the diversity in techniques, the core results are not in dispute: stock prices do not appear to be driven primarily by cash flows.

The current divide between behavioral economists and EMH proponents is not, then, over whether dividends matter: it is over why they appear to explain such a small proportion of movements in prices (see the above discussion of Cochrane’s result). Behavioral economists focus on the role of human psychology, structural factors such as limitations on short-selling, and known behavioral regularities such as momentum trading. This claim, of course, means that behavioral researchers tend to believe that fundamentals don’t seem to be important because they aren’t important: markets are not efficient. EMH researchers, however, believe that the fact that all the “action” is in the expected returns component arises because the primary

\(^{13}\)For instance, Campbell and Shiller (1988) used a VAR to compare the actual dividend-price ratio with an estimate of its fundamental value; they found a correlation between these of less than 0.2, suggesting that cash flows can explain less than a fifth of price variations.
force driving asset price movements is time-variation in discount rates or risk premia.

Two points about this argument are worth further discussion. Firstly, in the context of the present value model, the discount factor $\rho$ is really an amalgamation of both intertemporal preference structures and attitudes toward risk. That may sound odd, but it can be demonstrated easily enough. Suppose that market participants become, on average, less (more) willing to bear risk. Then for any given price, investors would require a greater (lesser) stream of future dividends to justify buying the stock at that price. But this logic is easily inverted: if the stream of expected future dividends is held constant, a reduction (increase) in willingness to bear risk would cause the present value of the stock’s future cash flows to the investor - that is, the stock’s fundamental value as defined by $\rho$ - to fall (rise). If so, the only way the fundamental value could fall (rise) while holding constant the future dividend stream is for $\rho$ to fall (rise).

Second, why would discount rate variation appear in the data as “expected returns” variation? Recall equation (??) and consider the three terms on the right-hand side:

1. $\text{cov}[d_{pt}, \sum_{j=1}^{k} \rho^{j-1} r_{t+j}]$
2. $\text{cov}[d_{pt}, \sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j}]$
3. $\rho^k \text{cov}[d_{pt}, d_{pt+k}]$

An important point is that this decomposition - like others previously mentioned - assumes a constant discount rate. So if discount rates are actually varying, where would that variation be ‘picked up’ in the decomposition? The key is that the econometric models used in this context provide estimates of expected future dividend-price ratios ($d_{pt+k}$) and expected future dividends ($d_{t+j}$). Thus terms 2 and 3 - the “cash flows” term and the “rational bubble” term respectively - could not change. The estimation procedure essentially fixes the value of these terms for given $\rho$. Therefore, discount rate variation, being ruled out by assumption, would be attributed by these decompositions to the only “free” variable: $r_{t+j}$, which represents expected returns. This is what is meant when we say that EMH proponents believe that excess volatility is
explained by a time-varying discount rate/risk premium.\textsuperscript{14}

While these techniques corroborated Shiller’s (1981) finding of excess volatility and provided measurements of the extent to which prices are too volatile, they also illuminated a related problem. Excess volatility is a property of period-to-period price movements; in other words, it is a short-run property of the data. But the fact that movements in stock prices are mostly attributed to expected returns also suggested a problem with the long-run properties of the data: the degree of persistence in price movements. The fact that we observe swings in stock prices in the same direction over long periods of time, and away from estimates of the fundamental value, is the long-run analogue to the excess volatility problem. It also presents a problem for the EMH explanation of excess volatility. Since EMH advocates claim that excess volatility reflects a time-varying risk premium, their model of the risk premium must produce the same degree of persistence to be an adequate explanation. Models of a time-varying risk premium struggle to match the persistence in the data to the degree of persistence predicted by their models. I will present a more comprehensive review of the literature on persistence in Chapter 3, which uses cointegration techniques to examine the long-run properties of relative prices. The remainder of this Chapter focuses on excess volatility.

In summary, it is widely agreed that absolute stock prices exhibit excess volatility, and that this excess volatility is largely traceable to variation in “expected returns”. Decomposition techniques have revealed that movements in stock prices appear to be almost entirely driven by changes in expected returns, not by cash flows. Naturally, because so much of the action appears to be in expected returns - which may be attributable to discount rate variation - economists interested in preserving EMH have developed a large body of theories that attempt to explain discount rate variation. The following section explores what we can still learn from an examination of the EMH literature.

\textsuperscript{14}Given the above arguments, the terms “discount rate” and “risk premium” end up being used essentially interchangeably for most of this paper. That choice is at least partially dictated by the literature I am reviewing here.
1.4 Time-Varying Discount Rates and EMH

This section attempts to provide a brief snapshot of the EMH literature on time-varying discount rates. My primary purpose here is to provide the reader with evidence for my claim that the EMH literature focuses on absolute prices. This literature, however, is far too large for a complete exploration here to be feasible. Therefore, I attempt to select a representative set of papers from this literature. The next subsection will explain the reasoning by which I have selected the set of papers I discuss; after that, I will document the fact that those papers generally focus on absolute prices, with a strong tendency to refer to examine broad market indices rather than disaggregated stock prices.

1.4.1 Selecting a Representative Set of EMH Papers

EMH advocates knew very well that it is not enough to simply assert that expected returns vary because risk and appetites for risk vary, then go hunting at random for predictive variables; the researcher must develop a theory of discount rate variation.15 Fortunately, Cochrane (2011) can serve as a guide for this review: he provided an overview of several classes of theories that have been developed to explain discount rate variation. He discusses two broad classes of theories and their subfields: (1) investor-based models, which include macroeconomic models, behavioral theories, and finance theories; and (2) theories of market frictions, including segmented markets, intermediated markets, and liquidity-based theories (pp 1065 - 1072).

Cochrane points out that the finance theories actually require the “deeper” macroeconomic theories in order for them to represent a meaningful explanation of the data (p. 1068). So for our purposes, the finance theories are subsumed in the macroeconomic theories. The behavioral research rejects EMH, as mentioned previously. Finally, the theories of market frictions effectively reject EMH. They postulate market structures that may well prevent the market from operating as efficiently even if investors are ‘fully rational’. A

15In other words, a testable theory of time-varying discount rates must impose restrictions on how discount rates can vary. Without such restrictions, the central claim would be unfalsifiable: any thinkable state of expected return variation would be consistent with variation in the underlying preferences of market participants.
simple example is a model with (binding) limitations on short-selling. In such a market, if an asset became overpriced, investors aware of the overpricing might not be able to take a desired short position, and the asset could remain overpriced. This research is, therefore, not what I mean by “behavioral”; so I do not explore these theories further.\textsuperscript{16}

The point is this: we can avoid explorations of a number of these lines of research without committing a serious oversight. The relevant research for purposes of this section belong to the subfields identified by Cochrane as macroeconomic models. Cochrane provided a list of important papers that use macroeconomic models to explain discount rate variation (p. 1066). In the upcoming subsection, therefore, I discuss the papers named by Cochrane in his list. Their theoretical contributions are well-known, but insufficient attention has been given to their almost singular focus on absolute rather than relative prices.

1.4.2 Macroeconomic Theories of the Discount Rate

In this subsection, I will discuss a number of important papers that examine EMH-consistent theories of the discount rate. The point is primarily to document the kinds of questions the researchers are asking (in other words, what phenomena specifically are they trying to explain?), and what kinds of data they use to test their ideas. Some of the research I discuss will mention the “equity premium.” These papers may address the equity premium puzzle \textit{as well as} variation in discount rates, but the equity premium puzzle is not the focus here. I will document a strong tendency for researchers to consider the implications of their theories for absolute prices, not relative; and their use of absolute stock price data, generally on market-level indices.

Several studies build on canonical intertemporal consumption models, which have the desirable feature of tying discount-rate variation to macroeconomic data. Cochrane’s examples cover the following theoretical developments: nonseparability across types of good or over time; habit formation in the utility function; and recursive preferences over inertemporal consumption lotteries. Studies that serve as examples of these developments are, respectively, Eichenbaum, Hansen, and Singleton (1988) and Yogo (2006); Campbell and

\textsuperscript{16}I do not mean to disparage this research in any way. My focus here is the EMH-behavioral divide, but these theories do not squarely belong to either the EMH or behavioral branches; thus, they are not relevant for this paper.
Cochrane (1999); and Epstein and Zin (1989, 1991), Bansal and Yaron (2004), and Hansen, Heaton, and Li (2008). For Eichenbaum, Hansen, and Singleton (1988), stock prices are neither part of the data nor a serious element of their discussion. Asset returns appear only as an independent variable in their estimation procedure.\textsuperscript{17} Yogo (2006) uses data on industry-level portfolio returns (constructed in accordance with the Fama-French (1993) three-factor model) in order to address two major questions about his model’s implications for expected returns (see Yogo (2006) pp. 547, 555). First, can the model explain the intertemporal variation in the equity premium?\textsuperscript{18} Second, can the model explain cross-sectional variation in returns? (This exploration of cross-sectional variation marks Yogo (2006) as a singular exception within this list of papers).

Campbell and Cochrane (1999) used calibration techniques to simulate data and compared that simulated data to actual data from the S&P 500 index (p. 235). Their focus was on explaining aggregate stock market behavior, including return predictability, high volatility, and countercyclical movements in risk premia (pp. 248 - 249).


Finally, there is a class of models known as “rare disaster” models. Instead of modifying the utility function, these models modify the hypothesized distribution of returns - essentially by adding a fatter “left tail.\textsuperscript{17} Indeed, the only mention of stock returns made in this paper is in a brief footnote, see p. 58. \textsuperscript{18} Note that, given the emphasis on explaining “expected returns,” this really refers to the discount rate.

In more recent years, the rare-disaster framework appears to be growing in popularity. Manela and Moreira (2017) Wachter (2013), and Gabaix (2012) all argue that rare disaster models help explain the patterns seen in stock market price fluctuations - and all examine stock market data at the level of some kind of market aggregate (see Manela and Moreira (2017) 143, 156; Wachter (2013) 1016, 1019; Gabaix (2012) 645 - 646).

The apparent success of these models at accounting for features of stock market data - especially excess volatility - has been well-documented before now. But the sources and type of data they explain has not been a major consideration in previous reviews of the literature. Table I, below, summarizes the data types and sources used in these studies.

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<th>Paper</th>
<th>Data Source/Type</th>
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There is a strong tendency for researchers to focus on the behavior of stock prices measured by the level
of a market index - in other words, the absolute price of a broad market aggregate.\textsuperscript{19} As I argued in Section 1.1, above, this means that these studies say nothing about relative prices. EMH proponents are engaged in searching for an explanation of apparently anomalous behavior - attempting to ‘rescue’ the proposition that the absolute prices of stock market indices are efficient. They do this by seeking explanations for the observed variation in the discount rate. But in seeking to explain the behavior of a relatively narrow class of asset prices, researchers have spawned a remarkably diverse array of theoretical approaches. Few have considered alternative ways to explain the anomalous behavior, such as the market attempting to cope with unanticipated structural change (Knightian uncertainty). Furthermore, few researchers considered alternative ways of looking at stock market data. They continue to examine the time-series properties of absolute prices, and almost always those of broad aggregates. While some of this research considers the implications for the cross-sectional variation in returns (Yogo (2006)), such explorations appear to make up only a small minority of the extant literature. While cross-sectional variation in returns represents an indirect way to study how relative stock prices behave, I know of \textit{no} literature within the milieu of EMH that considers relative stock prices explicitly.

What about the behavioral finance literature? How did behavioral theories develop? Does this literature exhibit the same almost singular focus on absolute prices? The next section explores these questions.

\section*{1.5 Patterns in the Behavioral Finance Literature}

I begin by exploring the central premise of this research and the specific questions behavioral finance researchers have been investigating. Numerous reviews of this literature already exist: a full paper-by-paper summation here would, therefore, be of minimal use.\textsuperscript{20} I mean to draw attention in this section to the phenomena that behavioral finance seeks to explain, the evidence that researchers in this field consider, and the role that research from psychology plays in their research.

\textsuperscript{19}The \textit{returns} on such an index are, of course, simply the percent change in the absolute price of the index, possibly plus a dividend yield; thus returns data is in practice measuring the same thing as price data.

\textsuperscript{20}See Shiller (2014), p. 1504, citing 5 different survey studies, the most recent in 2011.
Shiller’s *Irrational Exuberance* provides a useful snapshot of the state of behavioral finance through 2005. The most important point is that behavioral finance researchers believe that ”[T]he market is not well-anchored by fundamentals” (p.147). While arguments in favor of market efficiency are considered, the data used by Shiller in his arguments against these claims is entirely based on absolute prices, and it is predominantly market-level data. Moreover, disaggregated data, and related studies, are usually considered only to the extent that they serve as examples of inefficient pricing (see pp. 181 - 191 generally). Where mention is made of evidence based on the prices of individual stocks, that evidence appears to be treated as an afterthought. Behavioral finance posits that the explanation for the observed variation in expected returns is to be found in human psychology. If psychological factors drive expected returns, such factors are the underlying cause of excess volatility. Thus, behavioral finance necessarily rests on the premise that markets are inefficient. Researchers in the field rely on absolute-price data, and usually market-level indices or other aggregates. The literature is primarily engaged in offering an explanation for excess volatility (and high persistence).

One chapter of *Irrational Exuberance* is titled ”Psychological Anchors for the Market,” and the contents of this and later chapters demonstrate that ”the Market” in this context means a measure of the level of the market as a whole. Behavioral finance researchers claim that the salient features of the market can be explained with reference to human psychology. *Irrational Exuberance* cites specific and distinct publications from psychology on pages 148, 150, 151, 152, 154, 155, 157, and three studies on page 158. The central thesis of the work is that the absolute price of the market as a whole is not efficient, and that human psychology is the explanation. Where studies in psychology are considered, therefore, the focus is naturally placed on what these studies teach us about the behavior of the market as a whole. They are deployed as suggestive evidence of market inefficiency (in other words, as explanations for excess volatility). The implications of these studies on, say, the cross-sectional variation in returns, or even on the disaggregated data (absolute prices of individual stocks), are barely considered.

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21Evidence that the prices of individual stocks appear less excessively volatile than the market is mentioned in just one sentence out of a 17-page chapter, and its relationship with psychology research is not considered at all (p. 194).
Lest one wonder whether this is really a representative picture of behavioral finance, consider Shefrin’s *A Behavioral Approach to Asset Pricing*. While *Irrational Exuberance* was written to be comprehensible by the general public, Shefrin’s book is aimed at an academic audience. It covers a wide range of topics, including heuristics and representativeness, prospect theory, heterogeneity in risk tolerance and time preferences, and the role of “sentiment.” Shefrin, like Shiller, emphasizes behavioral explanations of aggregate phenomena. Shefrin’s central concern in the book is describing and advocating behavioral explanations of the ”stochastic discount factor” or SDF. “[T]he behavioral SDF decomposes into a fundamental component and a sentiment component, where the sentiment component captures the aggregate error in the market” (p. 3, emphasis added).

Though Shefrin does not totally restrict his analysis to market-level aggregates, it is clear that for purposes of equity markets, his primary focus is on absolute prices. In a section where he discusses the implications of allowing for heterogeneous traders that may make systematic errors, he asks: “[M]ight the errors be self-canceling in the aggregate?” (p. 116, emphasis added). The subsequent sections discuss the necessary and sufficient conditions for market efficiency, but it is plain from context that he means the efficiency of an absolute price (see pp. 117 - 122 generally). He might just as easily have asked whether the errors cancel in relative prices - but that alternative formulation makes no appearance in the section. Later, he shows that “The expected return of any security Z is the sum of three components” (p. 232). But the section leading up to this conclusion, and his discussion of its interpretation, focus on the returns on that singular asset Z. Little mention is made of the implications of the methods he discusses for either cross-sectional variation in returns or for relative prices. Finally, consider his discussion of prospect theory: he investigates “how the presence of prospect theory investors affects prices, relative to a market composed entirely of expected utility maximizing investors, when the market portfolio is risky” (p. 408). Note that this is not a reference to the relative prices of two assets, but to how inserting prospect-theory investors alters the model’s prediction of the absolute price of the market portfolio. Despite a long discussion of the implications of prospect theory for investors buying and selling behaviors, implications for absolute prices are discussed but implications
for cross-sectional returns or relative prices are barely considered (see pp. 419 - 428 generally).

Research in behavioral finance through 2005 was, it appears, sharply focused on explaining the perceived inefficiency of absolute prices. A diverse array of theories from psychology were cited by these researchers as suggestive reasons why financial markets might be inefficient. Has this pattern changed at all in more recent years? All the evidence of which I am aware seems to indicate that there has been no major shift. Shiller (2014) provides a very broad review of research on financial markets, including a review of the history of behavioral economics (Shiller (2014) pp. 1504 - 1507). No mention of behavioral theories that explore the cross-sectional variation in returns, or relative stock prices, is made. Consider also just a few recent publications. Leal (2016) finds that a model with bounded rationality and heterogeneous traders ‘chartists and ‘fundamentalists can help explain excess volatility (p. 1847 1848). The data used in the paper is predominantly returns on the S&P 500 Index. Chen, Lung & Wang (2013) find that “heterogeneous beliefs strongly Granger cause excess volatility using quarterly S&P 500 Index data (p. 633). He & Li (2012) emphasize that a continuous-time, heterogeneous-traders model can replicate some of the stylized facts observed in stock markets - but again, when they refer to prices they mean the absolute prices of a market index (see pp. 975, 986).

Researchers in behavioral finance focus predominantly on explaining the perceived inefficiency of absolute prices, and usually they have in mind a singular “market price” measured by an index. In that sense, behavioral finance is a sort of photo-negative of EMH research: each is engaged in “searching for theories” - in the EMH case, ideas that reconcile the data with efficient markets; in the behavioral case, theories that explain why markets are inefficient. Both camps focus largely on absolute price data, and generally look at a broad market aggregate instead of lots of disaggregated prices. The findings from psychology cited by behavioral economists naturally belong to families of psychology research that speak to the data they are concerned with, and which provide justification for their premise that markets are, indeed, inefficient. It should not be surprising that they have not generally considered whether there might be strands of psychology research that might suggest a different view.

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Nothing in either this section or the previous should be construed as suggesting that there is no literature that studies the cross-sectional variation in returns. Indeed, there is a rich literature that studies the empirical properties of this cross-section - but these analyses are largely based on the CAPM and related multi-factor analysis, often involving the so-called “Fama-French portfolios” and estimates of market alphas and betas (see Cochrane (2011) pp. 1057 - 1064). That research belongs to the category of “finance” theories mentioned previously, which require a deeper economic theory to be well-grounded. This research, broadly speaking, provides evidence that prices are related to fundamentals and risk, and that the direction-of-change of prices with respect to fundamentals and risk is generally as we would expect under the present value model. But these papers do not provide direct evidence that these relations are quantitatively consistent with the present value model. As such they cannot be regarded as offering a sound explanation for the observed problems of excess volatility and persistence.

Thus, within the mainstream of both EMH and behavioral literature, there remain two important but under-explored questions. Is there any research using disaggregated data that directly speaks to the validity of the present value model? Might there be reasons from psychology that financial markets might perform certain tasks rather well? The answer to the first question is that few such studies exist - but their findings represent a puzzle for both EMH and behavioral researchers, because they provide evidence that the relationship between fundamentals and prices is clearer in the disaggregated data than in studies of market-level indexes. To the second question, the answer is a resounding yes - and that furthermore, other research areas within economics are already aware of it. The next sections explore these issues.

### 1.6 Studies of Disaggregated Data

Some of the leading researchers cited previously have noticed the dearth of research that both speaks to the validity of the present value model and uses disaggregated data or the cross-section of returns. Cochrane (2011) cited just two papers, Vuolteenaho (2002) and Cohen, Polk, and Vuolteenaho (2003), calling them “a

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22 This, of course, excludes the aforementioned Fama-French multi-factor models, for reasons previously mentioned
start, with too few followers” (p. 1064). Shiller (2014) also noticed this remarkable void in the literature. He also cited exactly two papers that directly address the question: Vuolteenaho (2002) and Jung and Shiller (2006).

The review of this literature, therefore, must necessarily be brief. Jung and Shiller (2006) found that with continuous CRSP data from 1926, firm-level dividend price ratios were significantly related to future dividend growth rates. Cohen, Polk, and Vuolteenaho (2003) found that 75 to 80 percent of the dispersion in book-to-market ratios is due to dispersion in expected profitability, and only 20 to 25 percent due to dispersion in expected returns. Vuolteenaho (2002) found that firm-level returns are mostly driven by cash-flow news. These papers also found evidence that expected returns news is highly correlated across stocks, but cash flow news is not. To this short list we might append the closely-related work of Yogo (2006), previously mentioned; and Cohen, Polk, and Vuolteenaho (2009). The latter is the only additional paper of which I am aware that uses cross-sectional returns data to examine the present value model. The evidence they present is consistent with the present value model (p. 2778). Moreover, this is the only paper of which I am aware that says anything explicitly about relative stock prices. The authors find that the relative prices of stocks are consistent with their betas (p. 2739). What these papers suggest is that the relationship between prices and fundamentals may be seen more clearly in the disaggregated data. The data sets I use in Chapters 2 and 3 are more aggregated than individual stock prices, but are not market-level indices. The subsequent chapters primarily test relative price efficiency, but I will also be able to see whether this disaggregation effect can be seen in sector-level aggregates.

If there is more recent work in this area, I have not found it. Nevertheless, this small area of the literature should not be discounted - in fact, it is key, because it presents a paradox for both EMH and behavioral finance. For EMH researchers, it poses the question: if expected returns (equivalently, discount rates) strongly drive aggregate prices, why do they appear to play a much smaller role at the level of individual stock prices? Conversely, why do fundamentals (cash flows) appear to be the major driver of firm-level prices, but not

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23 Of course, the previously mentioned caveat about studies of market betas applies here as well
market-level aggregates? For behavioral researchers, the paradox is this: why, if psychological biases drive aggregate prices, do they appear to be of so little importance for firm-level prices?

These findings provide a clue about why we might want to examine relative prices: the relationship between prices and fundamentals may be easier to see in the data. The high cross-asset correlation of expected returns news suggests that aggregate prices may be driven largely by market-wide “macro” shocks. Such market-wide shocks are likely to be related to events such as domestic policy shifts, major economic events, and the development of new technologies. Such overarching societal changes are likely to be uncertain in the Knightian sense: they cannot be fully described in advance by either market participants or by economists. In other words, unanticipated structural change has strong effects at the market level. Taking relative prices may have the effect of canceling out those shocks. In other words, the relationship between cash flows and prices may be clearer when we examine relative prices.

1.6.1 A Clearer Relationship

Even if we assume that the underlying relationship between prices and fundamentals is the same for absolute and for relative prices, we might still expect to see this relationship more clearly in the relative-price data than the absolute price data. The reason for this is the effect of uncertain “macro” shocks on absolute prices. For any two assets \( i \) and \( j \), something that moves both \( P_i \) and \( P_j \) by the same percentage has no effect on the relative price. Therefore, we might expect that an underlying macro trend or shock will have little or no effect on relative prices. That is not to say that ‘macro’ shocks such as monetary policy changes, credit constraints, or news about expected returns could not affect different assets in different ways. The difference between the effect of such a shock on one asset versus another would obviously affect relative prices; nevertheless, the underlying \textit{common} trend would affect a broad market index, but not relative prices. Would that “common trend” be connected to expected returns or cash flows?

Vuolteenaho (2002) and Cohen, Polk, and Vuolteenaho (2003) both found evidence suggesting that dis-

\footnote{Note that “unanticipated” here does not reflect irrationality because changes that are uncertain in the Knightian sense \textit{cannot} be anticipated even by fully rational market participants.}
aggregated (firm-level) prices are driven largely by cash-flow news, but that such cash-flow news is weakly correlated across different stocks. However, expected-returns news is strongly correlated across stocks; thus, a broad market index could appear to be driven largely by expected returns, while disaggregated stock prices would not. This story would be consistent with the idea that macro shocks largely drive changes in expected returns, but because expected returns are strongly correlated across stocks, there would be little effect on relative prices.

A brief example will serve to clarify my meaning. Let \( \bar{P} \) be the price of a market index. Suppose a macroeconomic shock occurs that changes the price of all stocks by a percentage of \( \mu \), on average. The shock, however, changes the price of asset \( i \) by \( \mu + \mu_i \) and that of asset \( j \) by \( \mu + \mu_j \). \( \mu \) is the ‘common’ shock, while \( \mu_i \) and \( \mu_j \) are the “idiosyncratic” effects of the shock on \( i \) and \( j \) respectively. Note that these may be thought of as one-time shocks or as underlying trends that last for some time. One may use the well-known approximation (for reasonably small \( \mu \)) that:

\[
\ln(X(1 + \mu)) \approx \ln(X) + \mu
\]  

(1.13)

By (1.13), the change in the log-price of the market index will correspond to the common shock, while the change in the log-price of specific assets will reflect both the common shock and the idiosyncratic effect of the shock:

\[
\Delta \ln(\bar{P}) = \mu \]  

(1.14)

\[
\Delta \ln(P_i) = \mu + \mu_i \]  

(1.15)

The log relative price, however, behaves quite differently:

\[
\ln(P_i(1 + \mu + \mu_i)/P_j(1 + \mu + \mu_j)) = \ln(P_i(1 + \mu + \mu_i)) - \ln(P_j(1 + \mu + \mu_j))
\]  

(1.16)
But by (??), this is approximately equal to:

\[ \ln(P_i) + \mu + \mu_i - (\ln(P_j) + \mu + \mu_j) = \ln(P_i/P_j) + (\mu_i - \mu_j) \]  

(1.17)

The change in the log relative price will be \( \mu_i - \mu_j \) - the idiosyncratic effects matter, but the common trend does not. If \( \mu \) reflects a change in growth trends due to a policy shift or a major economic event, this demonstrates that such shocks - which the mainstream literature chalks up to variation in expected returns - will have greatly reduced effects on relative prices. In addition, it is clear from (??) that disaggregated price movements will reflect both the idiosyncratic effects and the common shock.

The mainstream literature showed that a large proportion of variation in \( \overline{P} \) is due to variation in expected returns, but the literature on disaggregated prices finds a larger role for cash flows. Thus one may roughly interpret \( \mu \) as an expected returns effect and \( \mu u_i \) as a cash-flow effect. If, as I have argued, these shocks reflect unanticipated structural change - that is, the market coping with Knightian uncertainty - we would expect to see a somewhat reduced role for “expected returns” in the disaggregated data and a more sharply reduced role for expected returns in relative prices. Moreover, if \( \mu \) is not a one-time shock but a persistent trend, this would also suggest higher persistence for aggregates and disaggregated absolute prices than for relative prices (a question that will be addressed further in Chapter 3).

In addition, this implies that relative prices may have a practical advantage (from the perspective of the econometrician) over absolute prices: the more it is the case that movements in absolute prices are tied to uncertain macroeconomic events, the more the role of fundamentals in absolute price data may be ‘obscured’ to some extent by those shocks. That is not true of relative prices: the econometrician may be able to identify a clearer role for fundamentals in relative prices because they ‘cancel out’ the effects of unanticipated structural change - effectively, canceling out the effect of changes in expected returns.

Nevertheless, this is not really an explanation of why prices move as they do. We need a theory of how people determine the values of various assets. Behavioral finance taught us that we might learn much about
the performance of financial markets by considering the evidence from psychology. But researchers in that field read the psychology literature as providing suggestive evidence that markets ought to be inefficient. Could the psychology literature provide an answer to the paradox presented by the disaggregated data? In the following section, I will argue that it does.

1.7 The Psychology of Relative Valuations

There is a significant area of research in psychology that suggests that individuals are better at making relative judgments or valuations than they are at making absolute valuations. Is this finding true for asset prices as well? Does it apply to stock valuation? These are open questions, which have not yet been explored directly. I now turn to a review of this research.

Weber (2004) provides a helpful overview. A common finding seems to be that “people find it easier to make relative comparisons rather than provide absolute judgments” (p. 164). In particular, risky choices are better described and predicted by models that use relative risk measures such as coefficient of variation than by models that use an absolute measure of risk like variance or standard deviation. Dyer and Jia (1997) showed that a relative risk-value model “can explain many decision paradoxes” while retaining some of the desirable properties of classical expected-utility theory (p. 182). Indeed, their measure of relative risk is closely related to the coefficient of variation (p. 172 - 173). Brenner, Griffin, and Koehler (2012) studied decision processes in an experimental asset market. One of their findings was that the study participants could “encode differences in base rate of success” but struggled to reliably understand the validity of other ‘cues’ in the context of “overall judgments or in trial by trial pricing or probability” (p. 177). Participants in the experiment were able to make reasonably good judgments relative to a reliable benchmark, but were less able to arrive at an absolute forecast when lacking a benchmark. In other words, the subjects’ forecasts appeared to be “closer” to an accurate value when they were made in relative terms than when made in absolute terms (in isolation).
Chapman and Johnson (1999) studied the psychological phenomenon known as “anchoring”. They presented evidence that humans make evaluations relative to a benchmark - even when no obvious benchmark is presented.\textsuperscript{25} This behavior is closely related to the work of Kahneman and Tversky on prospect theory (Of course, behavioral economists have worked extensively with prospect theory. Just as with the other psychological research they cite, though, they have apparently not considered its implications for relative prices).\textsuperscript{26} Matthew and Stewart (2009) noted that anchoring and ‘sequential effects,’ a class of phenomena closely related to anchoring, are considered fairly robust findings in the psychophysics literature as applied to human reaction to simple stimuli, such as sound. They provided evidence that these decision-related behaviors seem to be similarly present in human reactions to prices. Hsee (1996), Hsee and Leclerc (1998) and Hsee et al (1999) contribute to and review a body of research suggesting that people evaluate options differently, and exhibit preference reversals, when making comparisons between options (which they call “joint evaluation”) as opposed to separate evaluations. Hsee et al (1999) suggest an explanation, which is that this occurs because the values of some attributes may be difficult to evaluate in isolation, and these attributes will gain greater weight in joint evaluations than in separate evaluations. Joint evaluations, therefore, will often be better, because an evaluation of trade-offs between options becomes more strongly emphasized. In other words, people find it easier to make joint evaluations because they are better able to make a judgments of the relative values of the two options under consideration.

Other subfields within economics seem to be aware of this aspect of human decision making. Consider, first, a strand of the environmental valuation literature. Researchers in this area frequently attempt to estimate people’s willingness to pay for environmental goods and services. Several data-gathering methodologies exist, but what is of interest here is the ongoing discussion in that literature about the usefulness of the contingent valuation method (CVM) versus a well-known alternative, choice experiments (CE). CVM asks people directly what they are willing to pay for a product, while CE asks participants to choose from among

\textsuperscript{25}While much of their research was devoted to understanding the sources of anchoring and how to addresses biases that follow from anchoring, they also found that decision-makers make adjustments along a scale from the anchor to the final answer. The anchoring phenomenon arises as an inherent part of human decision making, and judgments are made relative to that anchor.

\textsuperscript{26}See e.g. Kahneman and Tversky (1979).
a set of alternatives. CVM, then, asks for an ‘absolute’ valuation while CE asks for a ‘relative’ valuation.

CVM appears to have fallen out of favor with at least some researchers in this literature. Researchers have documented serious problems with CVM-based estimates of the value of non-market goods, while also documenting reasons to be more optimistic about choice experiments. Hausman (2012) documents a number of empirical problems with CVM, problems which have persisted despite decades of research, and which he characterizes as more or less insoluble. The problems he discusses include response bias, large differences between willingness to pay and willingness to accept, and the embedding and scope problems are all problems inherent to asking survey respondents to state a dollar-based valuation (an “absolute price”). Jin, Wang, and Ran (2006) note widespread skepticism regarding CVM, “especially in situations where multiple options and several attributes are being considered.” However, they also note that choice experiments have obtained more reliable results (p. 431). Hanley et al (1998) note several advantages of CE over CVM. CE is more amenable to ‘benefit transfers,’ a vitally important policy tool in the environmental context, and CE is better able to determine the marginal value of specific characteristics of the options presented to study participants. Diamond and Hausman (1994) state that CVM is “deeply flawed;” the questions asked in a CVM survey are “hard to answer” and the responses are “suspect” (p. 62). Moreover, they did not believe changes in survey methodologies stood much chance of adequately addressing this problem. Finally, consider the result obtained by Luisetti, Bateman, and Turner (2011): “We reject a null hypothesis of no range bias, suggesting that respondents may perceive attribute levels in a relative rather than absolute sense” (p. 284, emphasis added). Several of these papers note that an important part of the problems that occur in CVM is the information pool available to the survey respondent. This evidence is consistent with the aforementioned findings from psychology: people have an easier time, and are more accurate at, making relative valuations than absolute valuations. However, the environmental valuation field is not alone in this finding. There is also a body of marketing research consistent with these findings.

For instance, Posavac et al (2004) examined the ‘brand positivity effect,’ an empirical regularity in which consumers judge a singular brand under consideration more favorably than is warranted. They found, how-
ever, that this effect emerges to a significant degree from ‘selective processing,’ and it is therefore diminished when consumers are engaged in comparative evaluations as opposed to singular. Posavac et al (2005) found that the brand positivity effect is reduced when consumers are encouraged to consider alternative brands or when they already have knowledge about the category of goods to which the brands belong. Kamen and Toman (1970) studied consumer reactions to changes in the price of gasoline and found that “satisfaction or dissatisfaction is a function of the algebraic difference between what is experienced and what had been expected” (p. 35). In other words, even in a context where consumers were apparently making choices based on an ‘absolute’ price, they made this choice in relative terms, based on a mental benchmark.

All of this evidence suggests that a basic fact about human psychology has been underappreciated by most behavioral economists: humans are much better at making relative valuations than absolute valuations. This suggests that, in the context of financial markets, it would not be surprising if we found that relative prices behave “more” efficiently than absolute prices. If this intuition is correct, the relationship between cash flows and relative prices would be “closer” than the relationship between cash flows and absolute prices.

Notice that the studies of disaggregated stock market data and the research from psychology (as well as related research from certain areas of business and economics) would seem to have similar implications for patterns we should expect to see in stock price data, but for different reasons. The disaggregated data studies suggest that the relationship between cash flows and prices will be “clearer” in relative prices than absolute; and the psychology literature suggests that there might be a “closer” relationship between relative prices and cash flows than between absolute prices and cash flows. Both, therefore, suggest that we might expect to see greater evidence of relative price efficiency than absolute price efficiency. Perhaps these lines of research are trying to tell us that the market may do a better job discriminating between stocks with good prospects and stocks with dubious prospects - “separating the wheat from the chaff” - than it does at setting the level of market capitalization in absolute dollar terms.
1.8 Concluding Remarks

What did we learn from this review? A look at several different bodies of literature provides evidence of a major gap in the literature on stock market efficiency. EMH proponents and behavioral economists alike have largely ignored the implications of their ideas for relative stock prices. Much of the statistical evidence cited by both sides uses time-series data on broad market aggregates. While some literature exists that examines cross-sectional price or returns data, describing this branch of the literature as “underdeveloped” would be, if anything, an understatement. Even this small literature looks at the properties of the cross-section. That provides suggestive, but indirect, evidence of relative price efficiency. A more rigorous and direct test of relative price efficiency would look at the time-series properties of relative prices themselves.

Moreover, literature from psychology, corroborated by evidence from some subfields of economics, tells us that it should not be too surprising if we found better evidence for relative price efficiency than for absolute price efficiency. EMH proponents and behavioral economists alike focused their research on explaining the puzzling properties of broad market indices - hence the proliferation, on both sides, of theoretical explanations for the small role for cash flows observed in the data. Their explanations were, so both sides thought, mutually inconsistent. EMH researchers looked for ways to demonstrate that there were no systematic errors - that there was no reason to think markets wouldn’t work well. Behavioral economists paid a great deal of attention to research from psychology, but they used it to justify claims that the market was inefficient. In their focus on justifying their interpretation of the data, they appear to have missed several hints in that literature that financial markets may actually do a good job in at least one sense - separating the wheat from the chaff. The disaggregated data suggests (indirectly) that the strong role of expected returns may be a result of the market coping with Knightian uncertainty. Where do we go from here?

The gap in the finance literature, plus the hints we see in psychology and in other fields within economics, suggests fertile ground for future research. It also suggests that behavioral economics and EMH may not be quite as incompatible as the leaders of these camps seem to think: psychology research, so often cited
by behavioral economists, actually might give us reason to think markets may be relative-price efficient even if they might not be absolute-price efficient. We have rich knowledge about the time-series properties of absolute prices; we have good data about the cross-section. What is missing from the literature is an examination of the time-series properties of relative stock prices. We should ask: do relative prices move as we’d expect them to, given a simple model such as the present value model? Do cash flows drive movements in relative prices? Does structural change play a reduced role in movements in relative prices, as we might expect from the prior literature?

Chapter 2, which focuses on the short-run properties of relative prices, will primarily address the first and second of these questions, but it will also provide some suggestive evidence about the role of structural change. Chapter 3 will examine the long-run properties of relative prices, specifically the matter of persistence, as well as more formally examining the role of structural change. Such examinations, and hopefully future research along these lines, will help economists build a richer understanding of how financial markets work: what they appear to do well, what they appear not to do well, and how human psychology contributes to the market’s ability to adequately perform its basic social tasks.
Chapter 2

Short-Run Tests of Relative Price Efficiency in Stock Markets

ABSTRACT

This paper contributes to the literature examining stock market ‘efficiency’. I provide evidence that the connection between stock prices and fundamentals, as implied by the present value model, can be more clearly seen in relative prices rather than absolute prices. I also discuss several reasons why relative prices may provide a better lens through which to examine whether financial markets do a good job of allocating capital.
2.1 Introduction

The key question in this thesis is whether the evidence for stock market efficiency is stronger on the basis of relative prices - for example, the price of IBM shares relative to the price of P&G shares - than absolute prices - the price of IBM or P&G shares separately (or alternatively, the value of a market index such as the S&P 500). The empirical evidence that underpins the debate between proponents of the efficient markets hypothesis (EMH) and behavioral finance is based entirely on absolute prices. This study is, to the best of my knowledge, the first to investigate market efficiency on the basis of relative prices. In Chapter 1, I argued that the behavior of relative prices may provide greater insight into whether stock markets perform well their role of determining which companies are the most deserving of society's scarce financial capital.

I use the canonical present value model to examine the efficiency of stock markets. According to the present value model, stock prices should equal the market's forecast of the present discounted value of all future cash flows (i.e., dividends). The seminal work of Shiller (1981) and Campbell and Shiller (1988a,b), and much subsequent research, finds that market indexes of absolute prices (such as the S&P 500 price index or the Dow Jones Industrial Index) are much too volatile and too persistent to be rationalized on the basis of future cash flows alone. Proponents of behavioral finance interpret these findings as evidence that markets are not efficient. They argue that market participants are less than fully rational and prone to momentum trading and price bubbles. According to this view, stock markets often do a poor job in setting prices. By contrast, proponents of the efficient markets hypothesis (EMH) argue that these findings are the result of a time-varying and persistent risk premium. If so, stock markets would be efficient and would set prices correctly on average. The problem for the EMH view, however, is that standard risk premium models are unable to account for the excess volatility and persistence.

In this chapter, I examine market efficiency using Shiller and Campbells short-run volatility tests. In the next chapter, I focus on the longer-run properties of the data using the cointegrated VAR framework.

The analysis in both chapters follows a similar structure. I first look for a connection between stock prices
and dividends using absolute prices and dividends. I then examine this connection using relative prices and relative dividends. I refer to these connections as absolute- and relative-price efficiency, respectively.

In chapter 1, I reviewed two strands of research that are suggestive that the evidence for relative-price efficiency may be stronger. One strand comes from Psychology. Researchers have uncovered considerable evidence that individuals are better at assessing the value of say an orange relative to an apple, than the absolute values of an orange and an apple separately. This central finding suggests that stock markets may do a better job at setting relative than absolute prices. I refer to this as proposition as the idea that the connection is “closer” for relative prices. The other strand of research examines the behavior of disaggregated stock prices. This research finds stronger evidence of a connection between fundamentals and prices using disaggregated data rather than market-level price indexes. I showed that this finding can be viewed as indirect evidence that the connection to cash flows may be “clearer” (easier to see) with relative rather than absolute prices.

In order to establish a baseline for my empirical analysis, I assume a constant discount rate and thus, a constant risk premium. Financial economists have shown that the constant-discount-rate model is unable to account for market fluctuations on the basis of absolute prices. The question I pose in this thesis is whether the constant-discount-rate model can do a better job explaining fluctuations in relative prices.

In this chapter, I adapt Shillers (1981) *ex post* dividend volatility test and Campbell and Shillers (1988a) VAR-based volatility test to the case of relative prices. It would be interesting to examine relative prices for all individual pairs of stocks. But for *J* stocks, there are \( J(J - 1)/2 \) distinct (no reciprocals) individual price pairs. This number grows rapidly with the number of individual stocks. For instance, using all the stocks that compose the S&P 500 would result in approximately 125,000 relative prices. In order to obtain a manageable number of relative prices, I categorize the U.S. stock market into 10 sectors. This gives 45 distinct relative prices.

To preview my results, Shillers (1981) *ex post* volatility test produces results that are much more consistent.

---

with the present value model when relative prices are used rather than absolute prices. The VAR-based test also shows evidence more favorable to the present value model with relative prices than absolute prices. However, in a number of cases we can reject the hypothesis that the data matches the VAR’s prediction of the fundamental value even for relative prices. One plausible interpretation of this result is that it is evidence against market efficiency. However, it is also possible that this result is a reflection of structural change. In the presence of structural change, the VAR will have difficulty estimating the fundamental value. A fuller, more formal examination of the role of structural change follows in Chapter 3, but the VAR provides enough evidence to at least explore the possibility herein.

To do this, I examine whether the VAR rejections are in any way related to the sectors. I find that the number of rejections appears unusually large for the Hi-Tech sector, but is apparently random for all other sectors. I argue that this finding may be evidence of problems with VAR estimates of the fundamental value of the High-Tech sector.

Overall, my findings are consistent with the proposition that the connection between prices and fundamentals is clearer in relative prices than in absolute prices. The results provide us with reasons to suspect that the market, like individuals, does better at determining relative rather than absolute values. This is consistent with the suggestive evidence from the psychology and disaggregated data literatures.

The remainder of the paper is structured as follows. Section 2 adapts Shillers (1981) and Campbell and Shillers (1988a) volatility tests to the case of relative prices. Section 3 discusses the data. Sections 4 and 5 present results for absolute and relative prices, respectively. In section 6, I analyze what we learn from the empirical results. Section 7 concludes the chapter.
2.2 Tests of Absolute- and Relative-Price Efficiency

2.2.1 The Variance Test

My analysis relies on several tests. The first comes from Shiller (1981), which relies on the canonical PV model to characterize market efficiency. According to the present value model, the price of a stock or basket of stocks $P_t$ is the mathematical expectation, conditional on all currently available information, of the present discounted value of all future dividends $P_t^*$:

$$P_t^* = \sum_{j=0}^{\infty} \rho^j [D_{t+j}]$$

(2.1)

where $\rho$ is the discount rate and $D_t$ is the dividend paid at time $t$. Shiller (1981) calls $P_t^*$ the \textit{ex post} rational price. His key insight was to recognize that under the rational expectations hypothesis (REH), $P_t$ is the optimal forecast of $P_t^*$. It follows that $P_t^*$ can be written as $P_t$ plus a forecast error:

$$P_t^* = P_t + \epsilon_t$$

(2.2)

where the forecast error $\epsilon_t$ has zero mean and is uncorrelated with available information. Therefore, the forecast error must be uncorrelated with the forecast itself. If this correlation were non-zero, the forecast itself could be improved. It follows that the variance of the actual price should be less than the variance of the \textit{ex post} rational price (see Shiller (1981) 422):

$$\sigma(P) \leq \sigma(P^*)$$

(2.3)

It is straightforward to show that an analogous argument holds for relative prices as well. Let $P^i_t$ and $P^j_t$ be the prices of two different assets (dropping time subscripts). According to REH plus the present value model, each of these must equal the mathematical expectation of the present discounted value of all future dividends:
\[ P^i = E[P^i] \] (2.4)

\[ P^j = E[P^j] \] (2.5)

where \( E \) is the expectation conditional on available information; Dividing (2.4) by (2.5), we obtain the relative price of \( i \) in terms of \( j \):

\[ \frac{P_i}{P_j} = E_i\left[ \frac{P^i}{P^j} \right] \] (2.6)

One might note that the expectation operator has come “outside” the fraction. While the assumption that different prices are independent and identically distributed would make this mathematically true, we need not make such an assumption to justify this. In this case, (2.6) must be correct because if it were not, at least one of the assets must be underpriced relative to the other. If the ratio of prices were less (greater) than the expectation of \( \frac{P^i}{P^j} \), that would imply that investors would expect that they could, on average, increase the present value of their future cash flows by selling (buying) asset \( j \) and buying (selling) asset \( i \). This would drive the price of asset \( i \) up (down) relative to asset \( j \), and \( \frac{P_i}{P_j} \) would approach \( E_i\left[ \frac{P^i}{P^j} \right] \). This By REH, \( \frac{P_i}{P_j} \) must be an optimal forecast of \( \frac{P^i}{P^j} \). So we can write

\[ \frac{P^*_i}{P^*_j} = \frac{P_i}{P_j} + \epsilon_i' \] (2.7)

where \( \epsilon_i' \) has zero mean and is uncorrelated with available information. This implies the first of several criteria I use in this chapter:

\[ \sigma\left( \frac{P_i}{P_j} \right) \leq \sigma\left( \frac{P^*_i}{P^*_j} \right) \] (2.8)

However, there are several implementation issues that need to be addressed in order to make this into a
workable test. I describe the procedures I follow to implement these tests below.

2.2.2 Implementation of the Variance Test

First, one needs a discount rate. Following Campbell and Shiller (1988), I set the annual discount rate as $\rho = 0.936$; since I have quarterly data while their data is annual (and Shiller (1981) also used annual data), I use a quarterly discount rate equal to the $4^{th}$ root of 0.936, and will use this value for $\rho$ throughout.\(^3\) Second, $p_t^*$ is defined based on perfect foresight of the future time-path of dividends. But then it is necessary to replace the infinity term in the summation (see (??)) with a finite number, and one small enough so that $P_t^*$ can be calculated within the bounds of the data that is available. Following Shiller (1981), I therefore calculate $P^*$ using the present value of 30 years (120 quarters) worth of dividends, which provides a reasonably close estimate of the “true” $P^*$.\(^4\) Finally, two adjustments to the nominal price and dividend data must be made. The data is converted to real prices and dividends using the quarterly CPI. The real prices and dividends thus obtained are also used for the VAR tests, discussed below. For the volatility tests given by (??) and (??), it is also necessary to detrend the data, which I do using a standard Hodrick-Prescott (HP) filter.\(^5\)

The other set of tests I use in this chapter are due to Campbell and Shiller (1988). This analysis was prompted by criticism of Shiller (1981): it was argued that those results might be spurious because dividends and prices are nonstationary.\(^6\) Campbell and Shiller (1988) developed an approach to address this concern. Instead of comparing prices with the present value of cash flows, they compare the actual dividend-price ratio with an estimate of the “fundamental value” of the dividend price ratio, defined below. This method uses a vector autoregression (VAR) to forecast future dividend growth rates and dividend-price ratios. These

\(^3\)Campbell and Shiller (1989) calculate the value of the discount rate using either returns on four - to six-month prime commercial paper or using the growth rate of an aggregate consumption measure. In either case, using the same discount rate across all equity assets would be a reasonable next step: such measures are, of course, constant with respect to the stock or stock portfolio being analyzed.

\(^4\)For any reasonable choice of the discount rate, the value of dividends beyond 30 years in the future is minimal. Suppose that the dividend is $10 per quarter forever with the discount rate given above. The present value of 30 years of dividends is 86% of the present value of the dividends forever.

\(^5\)Shiller (1981) also de-trends his data; he states that his detrending method uses “a factor proportional to the long-run exponential growth path.” That may have been computationally easier at the time, but it is now standard to use the HP filter, especially with data at higher frequencies than annual.

forecasts can then be used to calculate an estimate of the fundamental value. It is assumed that this estimate is a good approximation of the market’s forecast. The key equation is the “dividend-ratio model”, from which they derive testable predictions for the VAR:

\[
\delta_t = \sum_{j=0}^{i-1} \rho^j E_t[r_{t+j} - \Delta d_{t+j}|H_t] + \rho^i E_t[\delta_{t+i}|H_t] + \frac{(c - k)(1 - \rho^i)}{(1 - \rho)} \tag{2.9}
\]

where \( \delta_t \) is the log dividend price ratio at time \( t \), \( r_t \) is the return on an alternative asset at time \( t \), \( \Delta d_t \) is the growth rate of dividends at time \( t \), and \( H_t \) is the set of all available information at time \( t \). Note that the log dividend-price ratio is treated as a stationary process (see Campbell & Shiller (1988), pp. 667 - 669). The above equations are derived from a first-order Taylor expansion around the mean of this stationary process, which is why (2.10) contains the constant term \( \frac{(c - k)(1 - \rho^i)}{(1 - \rho)} \). If we take the limit of (2.10) as \( i \) approaches infinity, we obtain

\[
\delta_t = \sum_{j=0}^{\infty} \rho^j E_t[r_{t+j} - \Delta d_{t+j}] + \frac{(c - k)}{(1 - \rho)} \tag{2.10}
\]

Because \( \delta \) is assumed stationary and it is linearly related to dividend growth rates, (2.10) can be tested using the estimates from a VAR. In the case of a one-lag VAR, the system is:

\[
\begin{bmatrix}
  \delta_t \\
  \Delta d_t
\end{bmatrix} = \begin{bmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{bmatrix} \begin{bmatrix}
  \delta_{t-1} \\
  \Delta d_{t-1}
\end{bmatrix} + \epsilon_t \tag{2.11}
\]

This can be written more compactly as

\[
z_t = Az_{t-1} + \epsilon_t \tag{2.12}
\]

This has the useful property that the expected value of \( z_{t+k} \) is

\[
E_t[z_{t+k}|H_t] = A^k z_t \tag{2.13}
\]
Now, let $e_1 = [1 0]'$ and $e_2 = [0 1]'$. Since $\delta_t = e_1'z_t$ and $\Delta d_t = e_2'z_t$, we can rewrite (2.14) as

$$e_1'z_t = \sum_{j=0}^{i-1} \rho^jA^{i+1}e_2'z_t + \rho^jA^i e_1'z_t + \frac{(c-k)(1-\rho^i)}{(1-\rho)}$$

(2.14)

Since this must hold for arbitrary $z_t$, we must have the following condition for all $i = 1, 2, 3, ..., \infty$, where $i$ represents how many periods ahead the VAR estimates are being used to forecast $\delta_t$ and $\Delta d_t$:

$$e_1'(I - \rho^i A^i) = e_2' A(I - \rho A)^{-1}(I - \rho^i A^i)$$

(2.15)

I run this test for two values of $i$: $i = 1$ and $i = \infty$. For $i = 1$, the test is:

$$1 - \rho a_{11} = a_{21} ; -\rho a_{12} = a_{22}$$

(2.16)

This test can be thought of as a test of one-period “return predictability.” If the null hypothesis is correct, the current price accurately reflects what the VAR predicts will happen to the price and dividend over the next period. Thus, any excess returns over the next period are random, i.e. not predictable. There is an equivalent way to say this: if we reject the null hypothesis, an investor who knows the “true” model - the VAR - could earn excess profits by using the VAR forecast and buying (or selling) accordingly.\(^7\) Rejections of this test suggest that one-period returns are predictable. For purposes of this chapter, an in-depth discussion of return predictability would be off-topic; the reader need only be aware that excess volatility and return predictability are closely related phenomena.\(^8\) A rejection of this hypothesis is therefore evidence of excess volatility (and can therefore be interpreted as evidence against market efficiency).

The case where $i = \infty$ is also of interest. In that case the null hypothesis is:

$$e_1'z_t = e_2' A(I - \rho A)^{-1} z_t$$

(2.17)

\(^7\)Technically, this refers to earning a rate of return over and above the rate of return that exactly compensates the investor for discounting at the rate $\rho$.

\(^8\)This is well-known in the literature on stock market efficiency; see e.g. Timmermann (1993).
The left-hand side is the observed dividend-price ratio at time $t$. The right-hand side is the VAR’s estimate of the present discounted value of dividend growth rates (see equation (??)), based on the VAR’s forecast of future dividend growth rates. But the present discounted value of dividend growth rates is the fundamental value of $\delta$. Therefore, this null hypothesis says that the actual log dividend-price ratio is equal to the VAR’s estimate of the fundamental value. Calling the VAR’s estimate of the fundamental value $\delta^*_t$, then, the null hypothesis says that the dividend-price ratio is equal to its fundamental value. Mathematically, this can be written as

$$\delta_t = \delta^*_t$$ \hspace{1cm} (2.18)

It is straightforward to use these methods to identify a relative-price analogue if one assumes the same discount rate for any two assets. Take (??) to be true for any two equity assets; then we have

$$\delta^1_t = \sum_{j=0}^{i-1} \rho^j E_t[r_{t+j} - \Delta d^1_{t+j}] + \rho^i E_t[\delta^1_{t+i}|H_t] + \frac{(c-k)(1-\rho^i)}{(1-\rho)}$$ \hspace{1cm} (2.19)

$$\delta^2_t = \sum_{j=0}^{i-1} \rho^j E_t[r_{t+j} - \Delta d^2_{t+j}] + \rho^i E_t[\delta^2_{t+i}|H_t] + \frac{(c-k)(1-\rho^i)}{(1-\rho)}$$ \hspace{1cm} (2.20)

Subtracting (??) from (??) will give:

$$\delta^1_t - \delta^2_t = \sum_{j=0}^{i-1} \rho^j E_t[\Delta d^2_{t+j} - \Delta d^1_{t+j}] + \rho^i E_t[\delta^1_{t+i} - \delta^2_{t+i}|H_t]$$ \hspace{1cm} (2.21)

and

$$\delta^1_t - \delta^2_t = \sum_{j=0}^{\infty} \rho^j E_t[\Delta d^2_{t+j} - \Delta d^1_{t+j}]$$ \hspace{1cm} (2.22)

---

9Campbell and Shiller (1988a) use a “prime” symbol - ‘ - instead of an asterisk for this notation; but using an asterisk should make it more clear to the reader that this approach is an econometric alternative to calculating $P^*$ directly from observed dividends.
As in the absolute price case, taking the limit of (2.14) as \( i \) goes to infinity causes the \( \rho^i E_t[\delta_{t+i}^1 - \delta_{t+i}^2 | H_t] \) term to become arbitrarily small; (2.15) then becomes (2.16), which states that the relative dividend-price ratio should equal the present value of relative dividend growth rates. The right-hand side of (2.16) is the “fundamental value” of \( \delta_1^1 - \delta_1^2 \). These equations suggest that, for relative prices, we estimate the following VAR:

\[
\begin{bmatrix}
\delta_1^1 - \delta_1^2 \\
\Delta d_2^1 - \Delta d_1^1
\end{bmatrix}
= \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
\delta_{t-1}^1 - \delta_{t-1}^2 \\
\Delta d_{t-1}^2 - \Delta d_{t-1}^1
\end{bmatrix}
+ \epsilon_t
\] (2.23)

The first equation is asset one relative to asset two; the second is the dividend-growth rate of asset two relative to asset 1. By analogy with the absolute-price version of this procedure, call the vector on the left-hand side of (2.23) \( z_t \) and the 2x2 matrix of coefficients on the right-hand side \( A \). Using the VAR forecast given by (2.18) (it is the same for absolute and relative prices) and (2.16) one can generate testable restrictions on the relative-price VAR by following the same method as used above. Therefore, these restrictions take the same form:

\[
e_1'(I - \rho^i A^i) = e_2'(I - \rho A)^{-1}(I - \rho^i A^i)
\] (2.24)

where again \( i \) represents how many periods ahead the VAR is being used to forecast relative dividend growth, \( e_1 = [1 \ 0]' \) and \( e_2 = [0 \ 1]' \). As with the absolute prices, I examine two cases. For the case \( i = 1 \), the test given by (2.24) is a set of two linear restrictions:

\[
1 - \rho a_{11} = a_{21} ; -\rho a_{12} = a_{22}
\] (2.25)

Note that this is identical to restriction (2.19), except that the \( a_{ii} \) are drawn from the relative-price VARs (see (2.18)). We interpret this as a test of one-period relative return predictability (where a relative return is the rate of return on asset one minus the rate of return on asset 2); and as with the absolute-price version
of the test, a rejection of the hypothesis means returns are “predictable.” This would be evidence of excess volatility in relative prices, for the same reasons that a rejection of (??) suggests excess volatility for absolute prices (see the discussion above).

I also consider the case where \( i = \infty \). In that case the null hypothesis will be:

\[
e_{1}'z_{t} = e_{2}'A(I - \rho A)^{-1}z_{t}
\]

The left-hand side is the observed relative dividend-price ratio at time \( t \). The right-hand side is the VAR’s estimate of the present discounted value of relative dividend growth rates (see equations (??) and (??)), based on the VAR’s forecast of future dividend growth rates. But the present discounted value of dividend growth rates is the fundamental value of \( \delta \). Therefore, this null hypothesis says that the actual log dividend-price ratio is equal to the VAR’s estimate of the fundamental value. Since I refer to the VAR’s estimate of the fundamental value for absolute prices as \( \delta^{*}_{t} \), the relative-price fundamental value is \((\delta^{1} - \delta^{2})^{*}_{t}\). The null hypothesis may be succinctly stated as:

\[
\delta^{1}_{t} - \delta^{2}_{t} = (\delta^{1} - \delta^{2})^{*}_{t}
\]

Finally, for each relative price pair, I determine the correlation between \( \delta^{1}_{t} - \delta^{2}_{t} \) and \((\delta^{1} - \delta^{2})^{*}_{t}\). This measures the share of movements in relative prices attributable to movements in the underlying fundamentals. One may notice that (??) is a sufficient condition - but not a necessary one - for \( \text{corr}(\delta^{1}_{t} - \delta^{2}_{t}, \delta^{1} - \delta^{2})^{*}_{t} = 1 \). Therefore, a rejection of (??) does not absolutely rule out \( \text{corr}(\delta^{1}_{t} - \delta^{2}_{t}, \delta^{1} - \delta^{2})^{*}_{t} \approx 1 \). Campbell and Shiller (1988), applying this test to absolute prices of a market index, find a correlation of roughly 0.17 (p. 673). Some of the extant literature finds that fundamentals explain less than 10% of the movements in prices. A range of 0.1 - 0.2 is a reasonable range to regard as the “standard” range of possible values for this correlation in the extant literature. Thus, even a rejection of (??) could still be consistent with our expectation: a much closer

\[\text{\footnote{Consider any two random variables } X \text{ and } Y \text{. Their correlation coefficient is } \rho_{XY} = \frac{\sigma_{XY}}{\sigma_{X}\sigma_{Y}} \text{. Plainly then it is possible that } \sigma_{X} \neq \sigma_{Y}, \text{ and therefore } X \neq Y, \text{ but } \rho = 1.}\]
relationship between fundamentals and relative prices than that found for absolute prices. To know whether that “closer” relationship exists, we need to examine the correlations, not just the results of the test of (??).

I now turn to the data and the results.

2.3 The Data

My data source is Ken French’s website, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, from which I have downloaded returns data for 10 industry portfolios. This data is monthly, beginning in July 1926 and ending in December 2016, for a total of over 1000 observations per portfolio. Notice that since this data is in returns, there is no actual “price” for them. I therefore set the price of each portfolio at the beginning of my sample equal to 100. The data is returns data in percent terms, but includes returns both with and without dividends. The return without dividends is of course the percent change in the price of the portfolio, so I can calculate the price of the portfolio through time; and since the data includes returns both with and without dividends, I am also able to calculate the dividend paid in each period. This approach - using aggregates of stocks at the level of sectors or industries - has been used before in the literature on stock markets. Moreover, the use of 10 sectors has precedent, as does the use of French’s returns data. Laopoulos (2016) used seventeen industry portfolios in the US stock market, and acquired his returns data from Ken French’s website (see above). Bartram and Wang (2015) use 10 sectors in a study of European markets. Choudhry and Osoble (2015) use 10 sectors in a study of the US stock market. Tse (2015) divided the market into 48 industries in an analysis of US stock markets, and repeated the analysis with 34 industries. Dutt and Mihov (2013) use 28 sectors across a number of countries. Lee, Chen, and Chang (2013) use data on 10 sectors in a study of 10 stock markets in 10 countries in eastern and south Asia.¹¹

Table 2.1 lists the 10 sectors of the US economy covered by the data.

The nondurables sector covers consumer products such as food, textiles, and toys. Durables covers consumer products such as cars, furniture, and household appliances. Manufacturing includes companies in

¹¹See also Wu and Shamsuddin (2014); Moore (2011); and Yang, Tapon, and Sun (2006).
machinery, trucks, and chemicals. Energy covers oil, coal, and gas companies. Hi-Tech includes business equipment, computers, and electronic equipment. Telecomm covers telephone and television transmission. Shops includes wholesale and retail as well as some service providers such as repair shops. Health includes pharmaceuticals, medical devices, and care providers. Utilities includes electricity, gas, and water services. Other includes categories not otherwise covered, including entertainment, finance, hotels, and mining.

One problem with this data is there are several sector-month observations for which the dividend paid is 0, and the Campbell and Shiller (1988) approach requires taking logarithms. In that case the existence of a 0 observation would render several of the sectors unusable (this is not an issue with the Shiller (1981) test). Converting the data to quarterly eliminates the zero-dividend problem. While one possible correction is to construct monthly average dividends and prices over each quarter, doing so introduces several econometric problems. It is far simpler to use end-of-period quarterly data, which does not present the same econometric issues. This general approach is not new, though different papers have used data of different frequencies. Shiller (1981), Campbell and Shiller (1988), and Campbell (1991) use annual end-of-period data, as do Cohen et al (2003) and Vuolteenaho (2002). The papers cited above that divide the market into

Specifically, estimates of variances, covariances, and autocorrelations are biased in time-averaged data. These biases are time-varying in nature, which means there is no easy correction. But the problem is actually even more intractable than that: the three biases represent a sort of “impossible trilemma” for the econometrician. One can use filtering techniques to address any two of the problems, but the third problem will persist (see Wilson, Jones, and Ludstrum (2001) 175, 177 - 178, 189, and citations therein)
sectors all use either monthly or weekly data, but these papers are generally applying a different methodology (usually, regressions in which returns are the dependent variable). As a result, their approaches do not require taking the log of dividends, so 0-observations are not a problem. The use of quarterly data in a VAR analysis of prices and dividends appears to be an innovation, but one that should improve the quality of my results by increasing the sample size significantly.

2.4 Absolute Price Tests

In this section, I replicate the absolute-price tests of Shiller (1981), and Campbell and Shiller (1988) outlined above, for the absolute prices of each sector in my data. These replicated tests form a baseline for comparison in two senses. First, I can compare my results to those of Shiller (1981) and Campbell and Shiller (1988). A priori, one should expect that my results would be fairly similar to theirs - and we will see below that this is what I find. Second, I can compare these results to those of the relative-price analogues performed on the same data.

I now turn to the results of the Shiller (1981) volatility test.

2.4.1 The Volatility Test

First, consider the results of the volatility test, \( \gamma \). Table 2.2 provides the standard deviations of the actual price \( P_t \) and the ex post rational price \( P_t^* \). \(^{13}\)

In every case, the standard deviation of the actual price is larger; and in every case, F-Tests that the variances of \( P \) and \( P^* \) are equal (within each sector) can be rejected at any reasonable significance level (even as low as \( \alpha = 0.01\% \)). While the null hypothesis is that the variances are equal, the test is one-tailed; the alternative hypothesis states that the larger of the two variances is actually larger. \(^{14}\) The results are, to

---

\(^{13}\) As mentioned above, this test is based on real data, detrended using a standard HP filter. \( P^* \) is the present value of the next 30 years of dividends. I use the same value for the annual discount rate as Campbell and Shiller (1988), \( \rho = 0.936 \); since I have quarterly data while their data is annual, I use a quarterly discount rate equal to 0.984, the fourth root of 0.936. I use the same discount rate for the relative price tests.

\(^{14}\) The statistic is distributed F with 242 and 242 degrees of freedom.
Table 2.2: Absolute Price Volatility: Standard Deviations

<table>
<thead>
<tr>
<th>Sector</th>
<th>$\sigma(\bar{P})$</th>
<th>$\sigma(\bar{P}^*)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telecomm</td>
<td>92.45</td>
<td>1.19</td>
</tr>
<tr>
<td>Utilities</td>
<td>134.08</td>
<td>2.39</td>
</tr>
<tr>
<td>Energy</td>
<td>210.33</td>
<td>5.13</td>
</tr>
<tr>
<td>Hi-Tech</td>
<td>595.10</td>
<td>18.46</td>
</tr>
<tr>
<td>Shops</td>
<td>191.46</td>
<td>6.31</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>207.25</td>
<td>6.97</td>
</tr>
<tr>
<td>Health</td>
<td>346.36</td>
<td>12.94</td>
</tr>
<tr>
<td>Other</td>
<td>83.60</td>
<td>3.68</td>
</tr>
<tr>
<td>Nondurables</td>
<td>114.22</td>
<td>5.36</td>
</tr>
<tr>
<td>Durables</td>
<td>266.31</td>
<td>14.74</td>
</tr>
</tbody>
</table>

Table sorted by ratio of $\frac{\sigma(\bar{P})}{\sigma(\bar{P}^*)}$, in decreasing order.

put it mildly, not close. In fact, the standard deviation of the actual price is, in all cases, at least eighteen times the standard deviation of the corresponding ex post rational price; in one case, $\sigma(p)$ was larger than seventy-five times $\sigma(p^*)$. The figure below illustrates a typical result.

Figure 2.1: Volatility Test, Durables Sector

Versions of this graph for the other 9 sectors may be found in Appendix I.

These results are qualitatively consistent with the findings of Shiller (1981) and with previous VAR-based tests: the criterion is “violated dramatically by the data” (Shiller (1981) 423). These results suggest that period-to-period variability in stock prices cannot plausibly be explained as the result of variation in dividends (assuming a constant discount rate). Moreover, it suggests that the “excess volatility” problem
is not noticeably reduced by disaggregating the data from a single market index to 10 sectors. This also implies that the evidence of market efficiency is no better for prices of 10 sector-level portfolios. If there is a disaggregation effect, it can not be clearly identified in this test. It is not hard to believe that a price measure composed of the prices of hundreds of firms (as the sectors in this study are) is insufficiently disaggregated compared to the data used by Cohen et al (2003), Vuolteenaho (2002) and Jung and Shiller (2006). That research uses prices at the level of individual firms. The reader will see that this result is also consistent with the results of the VAR tests herein.

2.4.2 Absolute-Price VAR Tests

Before we turn to the VAR results, a brief review of the summary statistics for $\delta_t$ (the log dividend-price ratio) and $\Delta d_t$ (the dividend growth rate) may help clarify what it is we are seeing in the data. Table 2.3 summarizes the means and standard deviations of $\delta_t$ and $\Delta d_t$ observed for all 10 sectors.

<table>
<thead>
<tr>
<th></th>
<th>Range of Means</th>
<th>Range of Standard Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_t$</td>
<td>[-5.41, -4.36]</td>
<td>[0.35, 0.87]</td>
</tr>
<tr>
<td>Dividend Growth</td>
<td>[0.0006, 0.0092]</td>
<td>[0.084, 0.291]</td>
</tr>
</tbody>
</table>

The most striking feature of this is the fact that the range of standard deviations of $\delta_t$ is much larger than that of $\Delta d_t$. While this is far from a formal test of excess volatility, it is a hint that we should not be surprised to see evidence of excess volatility when we run the formal tests.

I now turn to the results of the VAR-based tests. These tests are necessary for the same reason that Campbell and Shiller (1988) used a VAR test to bolster the findings of Shiller (1981). The simple volatility test may return spurious results if the price process contains a unit root. I run 10 VARs, one for each sector. 15 If the results of these tests agree with the results of the volatility tests, it corroborates the above interpretation of those results: disaggregating from a market index to 10 sectors does not noticeably reduce

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15 Previous research using similar methods - e.g. Campbell and Shiller (1988) and Campbell (1991) - use one-lag VARs, but their data is annual. Mine is quarterly. One might argue that the “proper” comparison of my results to that work would be to use a one-lag VAR; but one might also argue that it is better to use the proper lag length. To address these concerns I ran both lag lengths. Results do not differ meaningfully across the two approaches.
the excess volatility problem (and therefore, also corroborates the conclusion that the evidence for absolute-price efficiency is no better at this level of disaggregation). Prices and dividends are in real terms. Two preliminary tests are conducted: I check stability conditions, and perform several versions of Dickey-Fuller and augmented Dickey-Fuller tests for unit roots on the residuals. Both these tests are needed to help ensure that the VAR is not a spurious regression. The results of these diagnostic tests suggest stability and no unit-root residual problem, so there does not appear to be a spurious regression problem.

The test results summarized in Table 2.4 are a test of one-period return predictability (restriction (??)) and a test of whether the actual dividend price ratio equals the VAR’s estimate of its fundamental value ((??)). The test statistics are distributed $\chi^2$ with 2 degrees of freedom. In the first case, the null hypothesis is no return predictability, and in the second, the null hypothesis states that $\delta$ is equal to its fundamental value $\delta^*$. Both null hypotheses are consistent with market efficiency. Note that both of these tests are based on using the VAR to forecast future outcomes and comparing those forecasts to the actual data (in present-value terms). The difference is the time horizon of the forecast. In the first test the VAR’s forecast is of outcomes one quarter ahead. The second test uses the VAR’s forecast into the infinite future.

<table>
<thead>
<tr>
<th>Sector Name</th>
<th>Test Statistic: One-Period</th>
<th>Test Statistic: Infinite-Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>505.04</td>
<td>109.34</td>
</tr>
<tr>
<td>Health</td>
<td>231.94</td>
<td>88.43</td>
</tr>
<tr>
<td>Other</td>
<td>204.02</td>
<td>97.24</td>
</tr>
<tr>
<td>Durables</td>
<td>161.45</td>
<td>142.33</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>150.51</td>
<td>126.96</td>
</tr>
<tr>
<td>Telecomm</td>
<td>150.06</td>
<td>216.35</td>
</tr>
<tr>
<td>Utilities</td>
<td>109.11</td>
<td>261.69</td>
</tr>
<tr>
<td>Hi-Tech</td>
<td>102.42</td>
<td>58.20</td>
</tr>
<tr>
<td>Nondurables</td>
<td>96.90</td>
<td>90.22</td>
</tr>
<tr>
<td>Shops</td>
<td>87.25</td>
<td>64.81</td>
</tr>
</tbody>
</table>

Sorted by one-period statistic in decreasing order. All statistics are significant at the 1% level.

Given the results of the Shiller (1981) volatility test, these results are not surprising: the movements in the absolute prices of these sector portfolios do not match the predictions of the VAR. Assuming that the VAR’s forecast is a good approximation of the market’s forecast, this means that price fluctuations cannot be
explained by changes in expectations of future cash flows. In other words, price movements cannot plausibly be explained by changes in dividends, under the assumption of a constant discount rate. Disaggregating from a single market index to 10 sectors does not improve the case for absolute price efficiency. It should also be noted that these results form a baseline of comparison with the results for the relative-price tests. However, we can’t draw any conclusions about the “closer” or “clearer” hypothesis from these results alone. If the results had been more positive for the present value model, that might have suggested that perhaps disaggregating reduces the important of market wide or “macro” trends, making it easier for us to spot a relationship between fundamentals and prices. We can not rule out that disaggregating may have such an effect, because it may still be the case that 10 sectors is not sufficiently disaggregated to detect this effect.

In addition, the correlations between the fundamental value of the log dividend-price ratio ($\delta_t^*$) and the actual log dividend price ratio ($\delta_t$) are quite low: they range between -0.03 and 0.32; for 7 of the 10 sectors, the figure is between 0.12 and 0.21. Under the null hypothesis (??), these correlations ought to be close to 1. Furthermore, these results are similar to what Campbell and Shiller (1988) found when they performed this exercise: they found a correlation of 0.175 (p. 672). It also falls approximately in the same range as the accepted range in the extant literature (10% to 20%). This is consistent with the interpretation of the other results discussed in this section: disaggregating into 10 sectors does not improve the outcome of tests of the present value model over test on a single market index. The apparent disconnect between the fundamental value and the actual data can be clearly seen by plotting $\delta_t$ (the actual log dividend-price ratio) and $\delta_t^*$ (the VAR’s estimate of the fundamental value) over time. Figure 2.2, below, is a typical example; versions of this graph for the other 9 sectors may be found in the appendices.

The left axis is for $\delta_t$, the right axis for $\delta_t^*$. The reader’s attention should be drawn to two striking features of these series. First, the scales of the left and right axes make clear that $\delta$ is far more variable than the fundamental value. Second, significant movements in $\delta$ occur even during periods where $\delta_t^*$ shows little movement at all, such as the period 1961 - 1984. The first feature corroborates what we saw in the summary statistics and the Shiller (1981) volatility test: the variability of $\delta$ is too high to be plausibly explained by
variation in dividends (assuming a constant discount rate). The second feature shows the same thing, but in a
different way: it illustrates that movements in $\delta$ are not closely connected to movements in the fundamental
value.

These observations show that this data exhibits similar patterns to those found by previous studies. Hav-
ing quarterly data instead of annual, a longer data set, and slightly more granular data does not noticeably
reduce the excess volatility problem. The VAR results, like the simple volatility test result, do not support
the idea that dis-aggregating the data into 10 sectors makes it any easier to see a connection between prices
and dividends. With this baseline set, we can now turn to a consideration of the results for relative prices.

2.5 Relative Price Tests

2.5.1 The Volatility Test

When we observe the absolute prices of 10 sectors, there are 45 distinct relative prices. Performing the
volatility test in equation (??) requires calculating the actual and ex post rational relative prices. The actual
relative price is the quotient $\frac{P_i}{P_j}$ for any two (nonidentical) sectors. The ex post rational relative price is cal-
culated by dividing $P_i^*$ by $P_j^*$, where the individual $P^*$ are calculated as described previously. The condition
consistent with market efficiency is $\sigma(P_i P_j) \leq \sigma(P^*_i P_j)$. As with the absolute price tests, I perform an F-test on the standard deviations of the actual price and the standard deviation of the ex post rational price. The null hypothesis of the F-test is that the standard deviations are equal, but the test is one-tailed; the alternative hypothesis of the test is that the larger of the two standard deviations is actually larger. Table 2.5 provides the standard deviations of the actual and ex post rational relative prices as well as the ratio of the two; Table 2.6 provides a more concise summary of these results. The table is sorted in decreasing order by $\frac{\sigma(P^*_i)}{\sigma(P_i)}$, which is given in the fourth column. The F-statistic for each relative price is the square of the entry in the fourth column, except for the last four rows. For those entries the F-statistic is the square of the reciprocal. For this value, numbers greater than one are consistent with the volatility criterion for market efficiency, (??); numbers less than one are inconsistent with the criterion.

The efficiency condition $\sigma(P_i P_j) < \sigma(P^*_i P_j)$ is met for 41 of the 45 sectors, with 39 of the differences being statistically significant. This means that I find evidence consistent with relative price efficiency in over 90% of the relative prices examined. In four cases, the results suggest relative price inefficiency, with three of these being significant at the 5% level.

What do we learn from this? It seems safe to say that the criterion for relative prices is not “violated dramatically by the data”, to again use Shiller’s description. Depending on the significance level chosen, only 3 or 4 of the 45 iterations of the test suggest a clear rejection of the market-efficiency hypothesis. At least 38 of the 45 results are clearly not inconsistent with market efficiency. If one wants to interpret the ‘statistically indistinguishable’ results as ‘non-rejection’ of the null hypothesis, one could say that 41 or 42 of the tests fail to reject the hypothesis consistent with market efficiency.

Figure 2.3, below, provides time plots of $P$ and $P^*$ for the relative price between the Durables sector and the Shops sector. The figure shows markedly different behavior than the behavior in Figure 2.1 with absolute prices. Figure 2.3 shows that the variation of $P^*$ is greater than that of $P$.

Versions of this diagram for the 44 other relative prices may be found in the appendices. The reader may

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16 The F statistic has degrees of freedom 242 and 242.
Table 2.5: Relative Price Volatility Test: Restriction (??)

<table>
<thead>
<tr>
<th>Relative Price</th>
<th>( \sigma(P) )</th>
<th>( \sigma(P^*) )</th>
<th>( \frac{\sigma(P^*)}{\sigma(P)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durables/Hi-Tech</td>
<td>6.47</td>
<td>1537.07</td>
<td>237.49***</td>
</tr>
<tr>
<td>Nondurables/Hi-Tech</td>
<td>2.14</td>
<td>61.70</td>
<td>28.85***</td>
</tr>
<tr>
<td>Energy/Hi-Tech</td>
<td>8.48</td>
<td>172.74</td>
<td>20.37***</td>
</tr>
<tr>
<td>Telecom/Utilities</td>
<td>4.69</td>
<td>76.50</td>
<td>16.31***</td>
</tr>
<tr>
<td>Durables/Utilities</td>
<td>31.06</td>
<td>345.90</td>
<td>11.14***</td>
</tr>
<tr>
<td>Manufacturing/Telecomm</td>
<td>17.23</td>
<td>173.60</td>
<td>10.07***</td>
</tr>
<tr>
<td>Manufacturing/Utilities</td>
<td>25.63</td>
<td>225.65</td>
<td>8.77***</td>
</tr>
<tr>
<td>Durables/Telecomm</td>
<td>32.82</td>
<td>251.80</td>
<td>7.67***</td>
</tr>
<tr>
<td>Durables/Health</td>
<td>14.08</td>
<td>92.67</td>
<td>6.58***</td>
</tr>
<tr>
<td>Durables/Other</td>
<td>33.27</td>
<td>199.85</td>
<td>6.01***</td>
</tr>
<tr>
<td>Manufacturing/Hi-Tech</td>
<td>4.98</td>
<td>28.63</td>
<td>5.75***</td>
</tr>
<tr>
<td>Manufacturing/Health</td>
<td>8.67</td>
<td>49.22</td>
<td>5.68***</td>
</tr>
<tr>
<td>Durables/Manufacturing</td>
<td>7.26</td>
<td>39.05</td>
<td>5.38***</td>
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<tr>
<td>Shops/Other</td>
<td>18.65</td>
<td>75.28</td>
<td>4.04***</td>
</tr>
<tr>
<td>Shops/Health</td>
<td>6.76</td>
<td>26.17</td>
<td>3.87***</td>
</tr>
<tr>
<td>Nondurables/Durables</td>
<td>1.31</td>
<td>4.98</td>
<td>3.81***</td>
</tr>
<tr>
<td>Shops/Utilities</td>
<td>34.05</td>
<td>120.54</td>
<td>3.54***</td>
</tr>
<tr>
<td>Nondurables/Other</td>
<td>8.00</td>
<td>27.62</td>
<td>3.45***</td>
</tr>
<tr>
<td>Nondurables/Manufacturing</td>
<td>2.40</td>
<td>7.59</td>
<td>3.16***</td>
</tr>
<tr>
<td>Non-Durables/Telecomm</td>
<td>9.95</td>
<td>30.95</td>
<td>3.11***</td>
</tr>
<tr>
<td>Durables/Shops</td>
<td>18.20</td>
<td>55.31</td>
<td>3.04***</td>
</tr>
<tr>
<td>Nondurables/Health</td>
<td>6.21</td>
<td>18.67</td>
<td>3.01***</td>
</tr>
<tr>
<td>Telecom/Other</td>
<td>5.18</td>
<td>14.44</td>
<td>2.79***</td>
</tr>
<tr>
<td>Energy/Health</td>
<td>7.05</td>
<td>16.68</td>
<td>2.37***</td>
</tr>
<tr>
<td>Hi-Tech/Utilities</td>
<td>89.72</td>
<td>206.10</td>
<td>2.30***</td>
</tr>
<tr>
<td>Nondurables/Shops</td>
<td>3.60</td>
<td>7.90</td>
<td>2.19***</td>
</tr>
<tr>
<td>Energy/Utilities</td>
<td>51.74</td>
<td>113.30</td>
<td>2.19***</td>
</tr>
<tr>
<td>Utilities/Other</td>
<td>5.91</td>
<td>12.92</td>
<td>2.19***</td>
</tr>
<tr>
<td>Nondurables/Utilities</td>
<td>14.45</td>
<td>31.02</td>
<td>2.15***</td>
</tr>
<tr>
<td>Energy/Telecomm</td>
<td>23.73</td>
<td>47.84</td>
<td>2.02***</td>
</tr>
<tr>
<td>Health/Utilities</td>
<td>72.30</td>
<td>142.07</td>
<td>1.97***</td>
</tr>
<tr>
<td>Energy/Other</td>
<td>50.44</td>
<td>88.88</td>
<td>1.76***</td>
</tr>
<tr>
<td>Hi-Tech/Telecomm</td>
<td>97.08</td>
<td>160.42</td>
<td>1.65***</td>
</tr>
<tr>
<td>Hi-Tech/Other</td>
<td>78.81</td>
<td>110.88</td>
<td>1.41***</td>
</tr>
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<td>Hi-Tech/Health</td>
<td>22.14</td>
<td>28.55</td>
<td>1.29***</td>
</tr>
<tr>
<td>Manufacturing/Shops</td>
<td>10.51</td>
<td>13.50</td>
<td>1.28***</td>
</tr>
<tr>
<td>Manufacturing/Other</td>
<td>35.72</td>
<td>44.95</td>
<td>1.26***</td>
</tr>
<tr>
<td>Manufacturing/Energy</td>
<td>14.10</td>
<td>17.41</td>
<td>1.23***</td>
</tr>
<tr>
<td>Health/Utilities</td>
<td>73.26</td>
<td>86.15</td>
<td>1.18**</td>
</tr>
<tr>
<td>Durables/Energy</td>
<td>61.92</td>
<td>69.12</td>
<td>1.12*</td>
</tr>
<tr>
<td>Hi-Tech/Shops</td>
<td>113.68</td>
<td>114.24</td>
<td>1.00</td>
</tr>
<tr>
<td>Nondurables/Energy</td>
<td>15.65</td>
<td>14.07</td>
<td>0.90*</td>
</tr>
<tr>
<td>Telecom/Health</td>
<td>6.75</td>
<td>2.76</td>
<td>0.41***</td>
</tr>
<tr>
<td>Energy/Shops</td>
<td>54.87</td>
<td>11.47</td>
<td>0.21***</td>
</tr>
<tr>
<td>Telecom/Shops</td>
<td>62.94</td>
<td>3.69</td>
<td>0.06***</td>
</tr>
</tbody>
</table>

* = Significant at 10%; ** = Significant at 5%; *** = Significant at 1%.
Table 2.6: Summary of Relative-Price Volatility Test: Significance Level 5%

<table>
<thead>
<tr>
<th>Condition</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\frac{P_i}{P_j}) &lt; \sigma(\frac{P^<em>_i}{P^</em>_j})$; Statistically Different</td>
<td>39</td>
</tr>
<tr>
<td>$\sigma(\frac{P_i}{P_j}) &lt; \sigma(\frac{P^<em>_i}{P^</em>_j})$; Not Statistically Different</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma(\frac{P_i}{P_j}) &gt; \sigma(\frac{P^<em>_i}{P^</em>_j})$; Not Statistically Different</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma(\frac{P_i}{P_j}) &gt; \sigma(\frac{P^<em>_i}{P^</em>_j})$; Statistically Different</td>
<td>3</td>
</tr>
</tbody>
</table>

Figure 2.3: Actual vs Ex Post Rational Relative Price: Durables and Shops Sectors

![Figure 2.3: Actual vs Ex Post Rational Relative Price: Durables and Shops Sectors](image)

notice that in the diagram above, the actual relative price is usually higher than the ex post rational price.

There are two reasons not to read too much into this feature. First, it may reflect the fact that $P^*$ only reflects 30 years of (relative) dividends; the “true” value of $P^*$ could be slightly higher than this estimate. Second, this pattern is not seen in all or even most other relative prices (see Appendix I). This diagram also presages the findings of the VAR analysis: note the appearance of what looks like very close comovement between the actual and the ex post rational relative prices.

A final feature of these results should be noted. With relative prices, it was generally not necessary to de-trend the data (that is, results are not meaningfully different with the de-trended data). The reason for this is straightforward. De-trending is necessary with absolute price data to remove the global upward trend
associated with long-run growth in the value of the stock market.\textsuperscript{17} The fact that no de-trending is necessary for relative prices is not in itself evidence of relative price efficiency. $P$ could be efficient, or not, regardless of whether $P^*$ follows a clear trend. We can only say that for $P$ to be efficient, either $P$ and $P^*$ follow approximately the same trend, or neither follow any trend. However, it is clear that relative prices remove “macro” or market-wide trends. This may be evidence in favor of the idea that if relative prices are efficient, the reason could be that the relationship between prices and fundamentals is “clearer” in relative price data.

This test, while suggestive, may be thought of as a necessary condition for market efficiency, but not a sufficient one. It could be the case that $\sigma(P) < \sigma(P^*)$, yet $P$ deviates from $P^*$ in systematic ways; this would violate market efficiency. In addition, the test employed in this section do not address the problem of potential unit roots (as was the case with the absolute-price volatility test). One might reasonably draw the conclusion that the ‘excess volatility’ problem is not nearly as apparent in relative prices as in price levels, but it cannot be said from this test alone that relative price movements are clearly connected to movements in fundamentals. As with the absolute price version of this test, one might wonder how the presence of a unit root might cause these results to be misleading. To address this concern, we turn to the VAR.

### 2.5.2 Relative-Price VAR Tests

In section 4.2, above, I showed that VAR-based tests of absolute price efficiency provide minimal evidence that movements in prices can be explained by movements in dividends, assuming a constant discount rate. The correlations between movements in the dividend-price ratio and its fundamental value are similar to those found by other studies that use similar methodologies (between 10% and 20%); dis-aggregating the market into 10 sectors does not produce a noticeable increase in the degree of co-movement between the fundamental value and actual prices. In this section, I explore whether VAR tests produce more favorable results for relative price efficiency.

I run 45 VARs (one for each relative price) as described in Equation (??). I run VARs on the relative

\textsuperscript{17}This is true of the tests discussed in section 4.1, above, and was also true of the tests used in Shiller (1981), though his de-trending method differs from my own.
dividend-price ratio and the relative dividend growth rate for each distinct pairing, check stability conditions (as before, this helps confirm that the regression is not spurious), and perform tests of the restrictions in (??) and (??).\textsuperscript{18} For the same reasons as cited above for absolute prices, I also ran the same test with one-lag VARs as a robustness check (the one-lag VAR matches the one-lag structure used in Campbell and Shiller (1988) and Campbell (1991)). The one-lag tests were slightly less favorable to relative price efficiency, so I report those results. As the reader will see, the case for a clearer and/or closer relationship between relative prices and fundamentals is quite strong even with the “weaker” evidence.

Table 2.7 provides the results of the one-period predictability test ((??)) and the infinite-period test ((??)), sorted by the one-period test result in decreasing order. The tests statistics are distributed $\chi^2$ with 2 degrees of freedom. Table 2.8 provides a more concise summary of these tests. Recall that (??) is a test of one-period relative return predictability and (??) is a test of whether the relative dividend-price ratio is equal to the VAR’s estimate of its fundamental value; in both cases the null hypothesis is consistent with efficient markets.

From the results, we can observe that the test of restriction (??) can be rejected at the 5% level for 17 of the 45 relative prices and cannot be rejected for 28. (At a significance level of 1%, we can reject the null hypothesis for only 7 of the 45 relative prices). These results are not consistent with relative price efficiency as a general proposition: we do reject that hypothesis more than one-third of the time. The results of these tests, however, suggest that the presence of one-period relative return predictability is not nearly as clear as the presence of return predictability in absolute prices. This suggests that excess volatility is sharply reduced (but not eliminated) for relative prices. Another feature of these results is worth noting. We see more rejections of restriction (??) than rejections of the volatility test, (??) (see section 5.1, above). This might be a symptom of a unit root, for reasons discussed previously, but we should keep in mind that these are not tests of stationarity or cointegration. Even these results, though, are favorable to the hypothesis that the connection between fundamentals and prices is much more obvious for relative prices than absolute prices.

\textsuperscript{18}As with absolute prices, the correct number of lags for the VAR is 4, according to the information criteria, but in this case that is true for 44 of 45 cases. For the sake of consistency, I use 4 lags for all VARs.
Table 2.7: VAR Tests of Relative Price Efficiency: Restrictions (??) and (??)

<table>
<thead>
<tr>
<th>Relative Price</th>
<th>Test Statistic: One-Period</th>
<th>Test Statistic: Infinite-Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hi-Tech/Other</td>
<td>46.32***</td>
<td>64.31***</td>
</tr>
<tr>
<td>Nondurables/Hi-Tech</td>
<td>12.82***</td>
<td>14.00***</td>
</tr>
<tr>
<td>Nondurables/Telecomm</td>
<td>11.89***</td>
<td>11.08***</td>
</tr>
<tr>
<td>Manufacturing/Hi-Tech</td>
<td>10.71***</td>
<td>10.66***</td>
</tr>
<tr>
<td>Telecomm/Health</td>
<td>10.44***</td>
<td>14.19***</td>
</tr>
<tr>
<td>Energy/Hi-Tech</td>
<td>10.31***</td>
<td>11.35***</td>
</tr>
<tr>
<td>Nondurables/Shops</td>
<td>9.76***</td>
<td>5.69*</td>
</tr>
<tr>
<td>Telecom/Utilities</td>
<td>9.05**</td>
<td>13.48***</td>
</tr>
<tr>
<td>Hi-Tech/Utilities</td>
<td>8.76**</td>
<td>10.11***</td>
</tr>
<tr>
<td>Health/Utilities</td>
<td>8.43**</td>
<td>11.16***</td>
</tr>
<tr>
<td>Hi-Tech/Shops</td>
<td>7.56**</td>
<td>6.76**</td>
</tr>
<tr>
<td>Shops/Health</td>
<td>7.31**</td>
<td>9.50***</td>
</tr>
<tr>
<td>Hi-Tech/Telecomm</td>
<td>7.04**</td>
<td>7.95**</td>
</tr>
<tr>
<td>Energy/Shops</td>
<td>6.91**</td>
<td>8.70**</td>
</tr>
<tr>
<td>Manufacturing/Shops</td>
<td>6.79**</td>
<td>7.39**</td>
</tr>
<tr>
<td>Nondurables/Health</td>
<td>6.60**</td>
<td>8.13**</td>
</tr>
<tr>
<td>Durables/Hi-Tech</td>
<td>6.15**</td>
<td>0.83</td>
</tr>
<tr>
<td>Shops/Utilities</td>
<td>5.96*</td>
<td>7.56**</td>
</tr>
<tr>
<td>Hi-Tech/Health</td>
<td>5.90*</td>
<td>6.57**</td>
</tr>
<tr>
<td>Energy/Telecomm</td>
<td>5.50*</td>
<td>6.60**</td>
</tr>
<tr>
<td>Durables/Shops</td>
<td>5.38*</td>
<td>0.12</td>
</tr>
<tr>
<td>Shops/Other</td>
<td>5.34*</td>
<td>4.93*</td>
</tr>
<tr>
<td>Telecom/Shops</td>
<td>5.15*</td>
<td>5.57*</td>
</tr>
<tr>
<td>Energy/Utilities</td>
<td>4.34</td>
<td>2.17</td>
</tr>
<tr>
<td>Telecomm/Other</td>
<td>4.03</td>
<td>1.55</td>
</tr>
<tr>
<td>Nondurables/Energy</td>
<td>3.92</td>
<td>2.46</td>
</tr>
<tr>
<td>Energy/Health</td>
<td>3.41</td>
<td>5.55*</td>
</tr>
<tr>
<td>Manufacturing/Telecomm</td>
<td>3.33</td>
<td>3.57</td>
</tr>
<tr>
<td>Nondurables/Durables</td>
<td>3.28</td>
<td>0.10</td>
</tr>
<tr>
<td>Manufacturing/Energy</td>
<td>3.26</td>
<td>4.23</td>
</tr>
<tr>
<td>Durables/Other</td>
<td>2.79</td>
<td>0.38</td>
</tr>
<tr>
<td>Durables/Manufacturing</td>
<td>2.33</td>
<td>0.25</td>
</tr>
<tr>
<td>Utilities/Other</td>
<td>1.99</td>
<td>0.72</td>
</tr>
<tr>
<td>Nondurables/Other</td>
<td>1.83</td>
<td>0.11</td>
</tr>
<tr>
<td>Durables/Utilities</td>
<td>1.71</td>
<td>0.37</td>
</tr>
<tr>
<td>Durables/Telecomm</td>
<td>1.38</td>
<td>0.73</td>
</tr>
<tr>
<td>Health/Other</td>
<td>1.34</td>
<td>1.09</td>
</tr>
<tr>
<td>Energy/Other</td>
<td>1.00</td>
<td>0.73</td>
</tr>
<tr>
<td>Manufacturing/Health</td>
<td>1.00</td>
<td>0.08</td>
</tr>
<tr>
<td>Nondurables/Utilities</td>
<td>0.98</td>
<td>1.30</td>
</tr>
<tr>
<td>Manufacturing/Other</td>
<td>0.89</td>
<td>0.08</td>
</tr>
<tr>
<td>Nondurables/Manufacturing</td>
<td>0.57</td>
<td>0.08</td>
</tr>
<tr>
<td>Durables/Health</td>
<td>0.53</td>
<td>0.12</td>
</tr>
<tr>
<td>Manufacturing/Utilities</td>
<td>0.44</td>
<td>0.32</td>
</tr>
<tr>
<td>Durables/Energy</td>
<td>0.10</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Sorted by one-period test statistic in decreasing order. * = significant at 10%, ** = significant at 5%, *** = significant at 1%. 
Table 2.8: Test of Restriction (??) (One-Period Predictability)

<table>
<thead>
<tr>
<th>p-value</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 10%</td>
<td>22</td>
</tr>
<tr>
<td>&lt; 5%</td>
<td>6</td>
</tr>
<tr>
<td>&lt; 1%</td>
<td>7</td>
</tr>
<tr>
<td>&lt; 5%</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 2.9: Test of Restriction (??) (Actual Value Equals Fundamental Value)

<table>
<thead>
<tr>
<th>p-value</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 10%</td>
<td>23</td>
</tr>
<tr>
<td>&lt; 5%</td>
<td>4</td>
</tr>
<tr>
<td>&lt; 1%</td>
<td>10</td>
</tr>
<tr>
<td>&lt; 5%</td>
<td>8</td>
</tr>
</tbody>
</table>

The results of the infinite-period test are in many ways qualitatively similar to the test of one-period return predictability. In this case, sixty percent (27) of the tests fail to reject the hypothesis consistent with market efficiency. As with the one-period tests, these results are not consistent with relative price efficiency as a general proposition; we reject that hypothesis in 40% of cases. Nevertheless, this also suggests that excess volatility is reduced in relative prices. Some relative prices appear to fluctuate in ways that are consistent with movements in the fundamental value, but several do not. In other words, these results suggest that divergences between the fundamental value and the actual price are sharply reduced for relative prices, in comparison to absolute prices. It is also worth noting that the results of this test generally agree with the results of the one-period predictability test. This corroborates our interpretation of the results of both tests; if we observed many rejections of one test but not the other, we would have reason to doubt the validity of the tests and therefore our interpretation of the results.

An interesting question is whether the degree of co-movement between the fundamental value and the actual dividend price ratio is higher for relative prices than absolute. While this question can be more fully explored using cointegration techniques (see Chapter 3), the VAR analysis provides one additional measure that may address this question. To examine this, I check the correlations between \( \delta^1_t - \delta^2_t \) and \( (\delta^1 - \delta^2)_t \). I also check the correlation between their respective first differences. For 44 of the 45 iterations, the correlation in levels was above 0.99, and in changes above 0.98. For the other relative price, the level correlation was

\[^{19}\text{If we use a 1% significance level, this becomes 35 “fail to reject” results - just below 80%}.\]
0.77 and the first-difference correlation was 0.65. Even in the worst case for relative prices, this is a much higher correlation than I found for absolute prices. For absolute prices, I found correlations in roughly the same range as previous studies (0.1 - 0.2); see section 4.2, above. This is the strongest piece of evidence in favor of relative price efficiency. These correlations mean that cash flows alone can explain a very large percentage of movements in relative prices. Nor are these large correlations inconsistent with the results given above: if the series are highly correlated but clearly not equal, we would expect to reject (??) because of the difference in their levels. This suggests that the estimate of \( \delta \) generated by the VAR is biased by some amount, but captures *movements* in the fundamental value very well.\(^{20}\)

Figures 2.4 and 2.5, below, illustrate typical results of the VAR forecast. Figure 2.4 shows a case where the null hypothesis was rejected for both the one-period and infinite-period tests; figure 2.5 shows a case where neither of the hypotheses were rejected.

![Figure 2.4: \( \delta_t \) Versus \( \delta^*_t \): Relative, Energy and Hi-Tech Sectors](image)

Note that in figure 2.5, one of the cases where we did not reject the null hypothesis given by restriction (??), the fundamental value and the actual \( \delta \) sit right on top of each other. This is exactly what the null hypothesis says: that the two are equal. Even in Figure 4, \( \delta \) and \( \delta^* \) co-move quite closely, even though this is one of the cases in which we rejected the null hypothesis. One feature of this diagram that we do not see

\(^{20}\)Consider a simple example. Suppose that for all \( t, \delta^1_t - \delta^2_t = (\delta^1 - \delta^2)_t + C, \) where \( C >> 0, \) we would observe a correlation very close to 1, but it would be obvious that the two are not equal.
in Figure 5 is the way that the two series diverge in the middle of the 1950s, but then their co-movements appear to rapidly re-align. This might be because of a shift in the discount rate at around that time, or perhaps a result of some other kind of structural change that the VAR fails to capture. For instance, a change in tax policy that impacts one sector but not the other might produce a result like this. The VAR would of course not “know” about the change, so would not be able to account for why the relative price grew around that date. A shift in the discount rate, though, does not appear to be consistent with the overall results. If a discount rate shift caused the change in Figure 4, why do we not see that effect in other relative prices? The discount rate enters the calculation of the fundamental value in the same way for all relative prices (see (??) and (??)). The examples given above are not atypical. In many other cases, even if we would reject the null hypothesis, the relationship between the actual $\delta$ and the fundamental value appears much closer for relative prices than absolute. The different results across sectors are, in many cases, probably a function of shifts such as the one seen in Figure 4, but not Figure 5. This is consistent with the idea that, for relative prices, the VAR’s estimate of the fundamental value captures movements in cash flows well, but its level might be offset from the “true” fundamental value for at least part of the sample. This would explain why we see rejections

21 Either way, the explanation is a form of structural change; the role of structural change in price movements is a theme to which I will return in Chapter 3.

22 Versions of this diagram for the other 43 relative prices may be found in appendix 2.
of (??) but high correlations between $\delta$ and $\delta^*$ for relative prices; see the discussion above regarding the correlations.

These results are not as strong as the results of the Shiller (1981) volatility test, but they do show improved evidence for relative price efficiency over absolute price efficiency. However, we still see evidence that some relative prices may be inefficient. The results are markedly less problematic for relative price efficiency than the serious empirical problems found when analogous tests were applied to absolute prices of a broad market index, or the absolute prices of the sectors themselves. What these results, taken together, seem to suggest is that relative stock prices reflect the underlying fundamentals much more than do absolute prices. These results provide some evidence that stock markets may do a good job allocating capital across various assets. The overall results provide a hint that the relationship between prices and fundamentals is both “clearer” and “closer” for relative prices than absolute prices. The fact that relative prices appear to remove “macro” trends is a hint in favor of the idea that the relationship is clearer in the data; the fact that we observed very close co-movement between the relative log dividend-price ratio ($\delta_1 - \delta_2$) and its fundamental value ($\delta^*_1 - \delta^*_2$) is evidence in favor of the idea that the relationship is closer. However, we can also address this question by examining whether any patterns appear in the set of prices that “fail” the efficiency tests. The following section explores the possibility that the rejections/non-rejections of market efficiency are in some way systematically related to the division of the stock market into sectors.

2.6 Which Sectors Appear to be Inefficient?

In this section, I examine whether there are patterns in the results of the above tests, and what we might learn from those patterns. For the one-period test, we had 17 relative prices that failed the efficiency test at $\alpha = 5\%$. Since each relative price is a ratio of one sector’s price to another sector’s price, there are 34 instances in which some individual sector was one of the two in a test for which the result was rejection of relative price efficiency. For the infinite-periods test (restriction (7)), we had 18 rejections. Tables 2.10 and
2.11 display the frequency of rejections by sector.

Table 2.10: Test Rejections by Sector: One-Period

<table>
<thead>
<tr>
<th>Sector Number</th>
<th>Name</th>
<th>Number Rejections</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Hi-Tech</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>Shops</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>Nondurables</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>Telecom</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>Health</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>Utilities</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Manufacturing</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>Energy</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Durables</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>Other</td>
<td>1</td>
</tr>
</tbody>
</table>

Sorted in decreasing order. Note that the sum of column 3 is 34 - corresponding to 17 rejections, because each rejection involves 2 sectors. Significance level 5%.

Table 2.11: Test Rejections by Sector: Infinite-Period Test

<table>
<thead>
<tr>
<th>Sector Number</th>
<th>Name</th>
<th>Number Rejections</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Hi-Tech</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>Telecom</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>Shops</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>Health</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>Utilities</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>Nondurables</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>Energy</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Manufacturing</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>Other</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Durables</td>
<td>0</td>
</tr>
</tbody>
</table>

Sorted in decreasing order. Note that the sum of column 3 is 36 - corresponding to 18 rejections, because each rejection involves 2 sectors. Significance level 5%.

A natural approach to detecting whether there are patterns to these rejections is to test whether the rejections are independent of sector. Using this data, I perform a standard independence test.\textsuperscript{23} With ten sectors and two categories (rejection or non-rejection), the test statistic will be distributed \(\chi^2\) with 9 degrees of freedom. For the one-period tests, the test statistic for independence is 19.0966 (p-value 2.44%). For restriction (7), the infinite-periods test, the independence test statistic is 22.4074 (p-value less than 1%). These results suggest that rejections of both tests are not wholly random with respect to which sectors are

\textsuperscript{23}I have omitted a possible fourth column in these tables, which would list the count of non-rejections. With 45 relative dividend-price ratios, each of which is in effect a pairwise comparison, each of the 10 sectors will be included as part of a relative price 9 times. Thus the entry in each cell in the fourth column would be 9 minus the number in column 3.
involved in price pairs for which we reject the null hypothesis.

One may also notice that when portfolio 5 (the Hi-Tech sector) is one of the two sectors used in the relative price, the null hypothesis is rejected 8 out of 9 possible times, for both versions of the efficiency test. This suggests an obvious secondary hypothesis: is this sector driving the result of the prior independence tests? Suppose that we drop all the tests in which sector 5 is involved, then re-run the independence tests. Tables 2.12 and 2.13 list the count of rejections for relative prices in which the Hi-Tech sector is not involved.

<table>
<thead>
<tr>
<th>Sector Number</th>
<th>Name</th>
<th>Number Rejections</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Telecom</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>Shops</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>Health</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
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<td>3</td>
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<tr>
<td>6</td>
<td>Telecom</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>Utilities</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>Manufacturing</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Energy</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Durables</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>Other</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that the sum of Column 3 is 18, corresponding to 9 rejections. Significance level 5%.

<table>
<thead>
<tr>
<th>Sector Number</th>
<th>Name</th>
<th>Number Rejections</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Telecom</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>Shops</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>Health</td>
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<tr>
<td>9</td>
<td>Utilities</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>Nondurables</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>Energy</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>Manufacturing</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Durables</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>Other</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that the sum of Column 3 is 20, corresponding to 10 rejections. Significance level 5%.

What do the independence tests suggest this time? Since we have dropped a sector, this time the test statistics will be distributed $\chi^2$ with 8 degrees of freedom. For the one-period tests, the independence statistic is 13.3333 (p-value 10.09%). For the infinite-periods tests, the independence statistic is 13.4308

\footnote{Since we consider only the tests for which sector 5 is not involved, $J = 9$ and thus there are 36 relative price pairs; and the entry in the omitted fourth column would equal 8 minus column 3 instead of 9.}
(p-value 9.79%). We can conclude from these results that there is some evidence that rejection of relative price efficiency is more likely when the hi-tech sector is included, and that it is at least plausible that other rejections occurred more or less at random. We can conclude that 1) we are more likely to observe relative price inefficiency for the Hi-Tech sector than for any other sector; and 2) of the other sectors, none appears to be more likely to exhibit relative price inefficiency than the other (again, non-Hi-Tech) sectors. The Hi-Tech sector is apparently the major culprit for relative price inefficiency.

Ultimately, the point of this research is to assess whether there is better evidence for relative price efficiency than absolute price efficiency. The results in this paper suggest that the market does a better job for relative prices than absolute prices. In this chapter, I have tested the present value model, but assumed a constant discount rate. In this as well as previous work, absolute prices fail such tests. Attempts to reconcile these results with market efficiency have generally argued in favor of a time-varying discount rate (or risk premium). But these results are hard to reconcile with that hypothesis. One would have to argue that the discount rate is systematically varying “a lot” for the Hi-Tech sector, but not nearly as much for the other sectors. But the discount rate in the present value model reflects investors’ preferences (their level of risk aversion, or their patience). Even theories that tie the discount rate to an interest rate typically use a risk-free rate - but that would be common across all sectors, so could not be an adequate explanation for the results herein.

The results for all the other sectors suggest that the market does a reasonably good job at forecasting future relative values. One might expect, however, that it is harder for the market to arrive at a good forecast for sectors in which there is a higher degree of uncertainty about the future. By “uncertainty” here, I do not mean risk. I mean the extent to which future outcomes can be described ex ante by a probability distribution; that is to say, Knightian uncertainty. I have provided evidence that the market may have difficulty performing its core task - even for relative values - when one of the sectors involved is the Hi-Tech sector. That is consistent with what we’d expect if Knightian uncertainty is the factor driving the results. There is research
providing evidence that this is the case. The Hi-Tech sector is, almost by definition, the sector of any economy (especially a developed economy like the United States) in which the most rapid change - the highest pace of innovation and the most rapid development of new knowledge - are occurring. It is therefore a sector in which we would expect a greater amount of unanticipated structural change - that is, a high degree of uncertainty about the future.

The relative price results help cast light on the behavioral and EMH views of markets. The alternative explanations those theories offer for the failure of the constant-discount-rate present value model are unsatisfying. Consider first behavioral theories. If asset prices are really driven by psychological factors, why do those factors appear to matter more for prices in one specific sector than all others? Would we have to conclude that investors who trade in hi-tech stocks are less “rational” than other investors? But then, what about traders who invest in multiple sectors? Why would such people fall prey to psychological biases when making investment decisions about this one sector, but not others? Why would they not apply the same, apparently “rational” decision-making process to hi-tech stocks as to all other stocks? These results also present a mirror-image of that problem for EMH: why does the risk premium vary through time more strongly for one particular sector than for others? Perhaps this could be explained by a higher correlation between consumption and the hi-tech sector. However, because Hi-Tech includes the stocks of companies that manufacture computer hardware and software, electronic equipment, and business machinery, this seems unlikely: the prices of those stocks would probably be much more highly correlated with investment spending than with consumption. It is well-known that investment spending has historically been strongly pro-cyclical, but consumption is non-cyclical, or at best only mildly pro-cyclical.

These observations do not imply that we should reject any role for human psychology in modeling or understanding asset prices. Indeed, I have argued that relative prices might behave more efficiently than absolute prices based in part on what we know about human psychology. That is one of the motivations for the hypothesis that relative prices will exhibit a “closer” relationship with fundamentals than absolute prices based in part on what we know about human psychology. That is one of the motivations for the hypothesis that relative prices will exhibit a “closer” relationship with fundamentals than absolute

See e.g. Frydman, Goldberg, and Mangee (2015).
prices. These results would appear to suggest that there is some merit to this intuition. Furthermore, the strong concentration of rejections in precisely the sector of the economy that is arguably the most prone to rapid change suggests that we should consider pursuing a third way forward: we ought to be trying to develop models of asset prices that take the possibility of unanticipated structural change seriously, and which allow human psychology to play a role, without simultaneously implying that investors forecast in ways that are both mechanistic and irrational. These results suggest that fundamentals - specifically dividends - matter more for relative prices than for absolute prices.

2.7 Concluding Remarks

In this paper, I have examined a major question in the literature on financial markets: do movements in asset prices reflect changes in fundamentals? I have addressed this question by asking whether movements in relative prices reflect changes in fundamentals more strongly than do movements in absolute prices. The literature on the stock market suggests that it is hard to spot such a connection in data on broad market indices. I have provided reasons why this connection might be easier to spot if we look at relative stock prices, taking insights from literature on finance, non-market valuation, psychology, and marketing. Furthermore, relative prices can teach us about the ability of financial markets to allocate capital among different assets. If relative prices closely reflect fundamentals, markets are doing a good job shifting capital from assets that represent “poor” opportunities for future earnings and towards assets that represent “good” opportunities. This important function of financial markets is one about which data on broad market indexes has little to say. Straightforward market-efficiency tests based on relative prices provide some suggestive evidence in favor of the claim that asset price movements are related to fundamentals in a manner consistent with the present value model. Moreover, I have presented evidence that suggests that researchers should consider taking the possibility of unanticipated structural change seriously.

Future research on relative-price data could provide further insight on these important questions. The
very high correlations observed in this research between the VAR-based estimate of the fundamental value and the actual data is certainly deserving of further scrutiny. In addition, while I have argued that these results suggest that structural change plays an important role in determining the movements of asset prices, I have not formally examined this hypothesis. Nor does this chapter speak directly to the issue of price persistence.

An additional issue with this approach is its assumption that the dividend-price ratio is stationary while prices and dividends are not. This would imply some form of cointegration; a hypothesis I also have not tested in this chapter. In Chapter 3, I will use a cointegrated VAR approach to address the issues of structural change, persistence, and cointegration. I am hopeful, however, that this chapter gives us some reasons to suspect that markets may be efficient - at least in the relative-price sense - and that there are psychological and behavioral reasons to suspect that markets should be relative-price efficient.
Chapter 3

Stock Market Efficiency: Long-Run Evidence from Relative Prices and Dividends

ABSTRACT

This chapter contributes to the literature examining stock market ‘efficiency’. Using tests based on the present value model, I provide evidence that relative stock prices are more closely connected to dividends than absolute prices. This suggests that the market may do a better job setting relative prices. This chapter focuses on the long-run properties of absolute versus relative stock prices, with special focus on the cointegration properties of the data; persistence; and evidence of structural change.
3.1 Introduction

In this chapter, I provide an empirical investigation of whether stock prices depend on fundamentals as suggested by the present value model. In Chapter 1, I showed that much of the extant research on this issue is based on absolute prices. Shiller (1981), Campbell and Shiller (1988), and many subsequent researchers have found that absolute prices are much too volatile to rationalize on the basis of dividends or earnings alone. This finding has given rise to a longstanding debate between proponents of the efficient markets hypothesis (EMH) and proponents of the behavioral finance school. EMH proponents argue that the excess volatility is due to a time varying discount rate. They believe that stock prices reflect their fundamental values, but because of a time varying discount rate, the fundamental value itself is more volatile than predicted by the canonical present value model. Behavioral finance researchers, on the other hand, argue that investors fall prey to psychological biases and momentum trading. In the behavioral paradigm, excess volatility arises because behavioral factors lead participants to bid stock prices away from fundamental values.

Nearly all of the research underpinning the debate between the EMH and behavioral camps is based on absolute prices. By “absolute price” I mean the monetary cost of a stock, or the value of an index at a particular moment in time. I argued in Chapter 1 that the connection between stock prices and fundamental values may be stronger and easier to detect with relative prices. By “relative price,” I mean the ratio of the price of one asset to the price of a different asset. There are two main reasons why detecting a connection may be easier with relative prices. First, there exists a body of psychology research that shows that individuals are better at making assessments of relative values than absolute values. This suggests that market participants may find it easier to judge the fundamental value of IBM relative to Procter & Gamble than to determine the fundamental value of stock in each company individually. If this were the case, we might expect that the market as a whole would be better at setting relative prices than absolute prices. The relationship between prices and cash flows would be “closer” for relative prices than absolute prices. However, there have been

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1 See e.g. Cochrane (2011) and Fama (2014) and references therein.
2 See Barberis and Thaler (2003) and Shiller (2014) and references therein.
3 See e.g. Weber (2004), Matthews and Stewart (2009), Chapman and Johnson (2009).
no direct tests of this proposition. Second, I argued that the literature studying disaggregated stock prices suggests that the relationship between relative prices and cash flows may be clearer in the data than the relationship between absolute prices and cash flows.⁴

In Chapter 2, I examined the excess volatility puzzle using short-run tests. These tests suggested that there is a much stronger connection between movements in fundamentals and relative prices than between fundamentals and absolute prices. In this chapter I examine the connection between prices and the long-run equilibrium relation implied by the present value model. Researchers have found that with absolute prices, this relationship is difficult to see in the data. I examine whether the connection between prices and the equilibrium relationship is stronger with relative prices. I examine this through two main lenses. First, I examine whether the evidence of the expected cointegrating relation is stronger with relative prices; second, I examine whether deviations from the equilibrium are less persistent with relative prices. As in chapter 2, I assume a constant discount rate.

This chapter proceeds as follows. First, I provide a brief review of past literature on the long-run behavior of stock prices. Since these studies, like those discussed in previous chapters, look at absolute prices, they provide a useful baseline for comparison with the results of the relative-price tests I will conduct. Second, I discuss the theoretical basis of my analysis, including an exploration of what the models considered in chapters 1 and 2 imply for the long-run properties of the data. I also discuss the empirical methodology I use: a cointegrated vector autoregression (CVAR) approach. My empirical investigation examines the absolute and relative price PV models along three dimensions: 1) the cointegration properties of the data; 2) the extent to which the models leave unexplained unit root(s); and 3) the degree of persistence present in deviations from absolute and relative fundamental values. I perform the tests for the absolute prices in my data and compare these results to the analogous tests for relative prices.

It would undoubtedly be desirable to examine relative prices for all individual pairs of stocks. But for \( J \) stocks, there are \( \binom{J}{2} \) distinct (no reciprocals) individual price pairs. This number grows rapidly with the

⁴See e.g. Vuolteenaho (2002), Cohen et al (2003), and Jung and Shiller (2006).
number of individual stocks. For instance, using all the stocks that compose the S&P 500 would result in approximately 125,000 relative prices. However, the CVAR methodology is an iterative one, with several steps that cannot be automated. It would therefore be infeasible to perform a CVAR analysis for all of these relative prices. In order to obtain a manageable number of prices, I categorize the US stock market into 5 sectors. This gives 10 distinct relative prices.

The remainder of the paper is structured as follows: Section 2 provides a review of the extant literature studying the long-run properties of stock price data. Section 3 discusses the theoretical basis of my approach and the empirical methodology employed. Section 4 discusses the data. Sections 5 and 6 present results for absolute and relative prices, respectively. Section 7 concludes the chapter.

### 3.2 Literature Review: Cointegration Studies

This section discusses findings from the extant literature on the cointegration and general long-run properties of stock price data. These studies help us understand whether there is any long-run connection between stock prices and dividends. As with the literature discussed in previous chapters, most long-run studies of stock prices use absolute price data - generally a broad market index - and are built on the present value model:

\[ P_t^* = \sum_{j=0}^{\infty} \rho^j [D_{t+j}] \]  

(3.1)

where \( P_t^* \) is the fundamental value at time \( t \), \( \rho \) is the discount rate (assumed constant here, an assumption frequently relaxed in the literature), and \( D_{t+j} \) is the dividend paid at time \( t+j \). The present value model states that the price \( P_t \) equals the market’s expectation of the fundamental value \( P_t^* \), that is to say

\[ P_t = E_t(P_t^*) \]  

(3.2)

where \( E_t \) is the mathematical expectation. If the present value model is correct, the market’s expectation
should equal the mathematical expectation.

The present value model can be studied empirically using both short-run tests and long-run tests. The biggest problem, of course, is how the economist can observe or estimate the market’s expectation. Campbell and Shiller (1988) used a VAR and REH; they found excess volatility. Campbell and Shiller (1987) examined whether there is a long-run tendency for stock prices to converge to this VAR-based estimate of the fundamental value. Short-run tests (such as those I used in Chapter 2) examine whether price changes from one month or quarter to the next are consistent with period-to-period changes in fundamentals (dividends). A long-run test would examine whether stock prices tend to mean-revert to the equilibrium relationship implied by REH and the present value model. In the context of variables with unit roots, the “equilibrium relation” in (??) takes the form of a cointegrating relation. A vector of time-series variables \( X_t = [X_{1,t}, X_{2,t}, ..., X_{n,t}] \) is said to exhibit cointegration if \( X_t \) is integrated of order \( D \) (we would say “\( X_t \) is I(\( D \))” and there exists a linear combination of the \( X_{i,t} \) that is integrated of order \( D - 1 \) or less.5

3.2.1 Non-CVAR Studies

The literature that examines the behavior of aggregate stock prices (often in the context of a broad US stock index) consistently provides evidence that the present value model is a reasonable description of the price process in the long-run, but with several serious problems over shorter time horizons. An early example of a cointegration study is Campbell and Shiller (1987). They assumed a constant discount rate and examined stock price and dividend data. Their data came from annual observations of the S&P composite index, 1871 - 1986. They use a cointegration test based on the seminal work of Engle and Granger (1987). Suppose that \( X_t = [y_t, Y_t]' \), and \( Y_t \) is a linear function of the present discounted value of expected future values of \( y_t \). Engle and Granger (1987) showed that if these variables are I(1) and the two variables are cointegrated, there exists a vector \( \alpha = (-\theta, 1)' \) such that \( \alpha'X_t \) is stationary relation. \( \alpha \) can be found by estimating an error-correction model (ECM):

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5A simple example: let \( x_t = x_{t-1} + \epsilon_t \) and \( y_t = x_{t-1} + \eta_t \), \( \epsilon_t \) and \( \eta_t \) being white-noise errors. Both series are random walks, and therefore I(1), but \( x_t - y_t = \epsilon_t - \eta_t \) is stationary (I(0)); so \( x_t \) and \( y_t \) are cointegrated.
\[ \Delta Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 y_{t-1} + \beta_3 \Delta Y_{t-1} + \beta_4 \Delta y_{t-1} + \epsilon_t \] 

(3.3)

where \( \epsilon_t \) is a white noise error term. The variable \( \theta \) can be estimated as the ratio of the coefficients on lagged \( Y \) and lagged \( y \), so that \( \theta = \frac{\beta_1}{\beta_2} \). The test for cointegration is a test in which the null hypothesis is that \( \alpha'X_t \) is nonstationary, where \( \alpha \) and \( X_t \) are defined as above. Note that this approach will encounter problems if the \( I(1) \) assumption is incorrect: the presence of an unexplained unit root will invalidate the results. Using this approach, Campbell and Shiller (1987) found weak evidence of cointegration at the 10% significance level. They also found that deviations from the fundamental value exhibit a high degree of persistence. Behavioral researchers interpret this as evidence of momentum trading and “bubble” behavior. The CVAR methodology (see Juselius (2006)) generalizes this approach from a one-equation ECM to a multiple-equation VAR context. The CVAR uses full information and thus provides more efficient estimation; in particular, it provides a more efficient test of long-run relationships.\(^6\) This enables us to see if the weakness of Campbell and Shiller’s (1987) result is due to the use of the single-equation framework. The CVAR is also more powerful in looking at short-run dynamics; the Engle-Granger approach says little about this matter.

The Campbell and Shiller (1987) approach ignores several empirical and theoretical factors that may explain the weakness of their results. Many recent developments in the literature since then have been extensions of the Campbell and Shiller approach that attempt to address one or more of these factors. One such innovation is to allow for a time-varying discount rate. However, the evidence that the empirical problems with the present value model can be addressed through this adjustment is weak. Timmermann (1995) allowed for a time-varying discount rate and noted a series of failures in the literature to statistically detect the cointegrating relationship between dividends and prices predicted by the present value model. However, he noted that prior tests of whether log prices and log dividends are cointegrated were robust to a time-varying discount rate (so long as the discount rate is stationary). He argued that the failures to detect cointegration had more to do with a combination of high persistence in the dividend yield and insufficiently large samples

\(^6\)However, the VAR does also require an assumption about whether the data is \( I(1) \) or \( I(2) \); see below for details.
(in other words, the tests had low power). In his simulations, he found that more than 100 observations was generally enough to overcome this issue. My data set has over 350 observations (see below), so that aspect of the problem should not be an issue here. However, note that Timmermann (1995) was unable to say anything about whether the high persistence in the dividend yield was due to the presence of a second, unexplained unit root. If there is a second unit root, the \( I(1) \) assumption underlying the Engle-Granger test is wrong, and the test will not in general be valid. The CVAR is a more powerful tool for understanding the source(s) of this persistence than the Engle-Granger approach, and will allow us to get a better understanding of whether this persistence comes from an unaccounted-for unit root.

Bohl and Siklos (2004) find that a present value model with a time-varying discount rate fits the data well in the long-run - that is, prices and dividends cointegrate as one would expect under the present value model. However, it cannot sufficiently explain short-run deviations from the equilibrium relation. These deviations were large and highly persistent. They claim that non-fundamental factors, such as speculative bubbles, do better at explaining the short-run dynamics. Their methods are similar to those of Campbell and Shiller (1987), but they allow for asymmetric adjustment rates.

The first major takeaway here is that studies find high persistence whether or not they allow for a time-varying discount rate. These findings also illustrate an additional point about the analysis I will conduct: since I will examine the relation between log prices and log dividends, my specification is robust to a stationary but time-varying discount rate, even though I do not explicitly build in that assumption. These studies suggested that the persistence of deviations might be due to an unexplained unit root, but did not rule out the possibility that the deviations might be “highly persistent but stationary”. A highly persistent but stationary process is one in which the parameter(s) on the autoregressive terms are high but not quite high enough to technically qualify the series as \( I(1) \); the simplest example is an AR(1) process in which the coefficient on the lagged dependent variable is just below 1. Such series are likely to be especially problematic with small samples and/or low-power tests. The CVAR allows us to get a better handle on this question in two ways: first, it can help us figure out whether the weakness of these results is due to the use of less powerful
statistical methods; second, we can examine whether the persistence problem is reduced when looking at
relative instead of absolute prices, which would suggest a closer and/or clearer connection between prices
and dividends than previously thought.

Lee (1996) provided evidence that those deviations from equilibrium were caused by an unexplained
unit root. The reader should note that the presence of an unexplained root does not necessarily mean there is
no cointegration. It means only that the simplest possible hypothesis - that prices and dividends cointegrate
from $I(1)$ to $I(0)$ - is inadequate. Lee (1996) provided evidence that by adding log earnings to the data, the
cointegrating relation could be more clearly identified. In the sample, log dividends, earnings, and stock
prices were cointegrated over the period 1871–1992, but the dividend and earnings components explained
less than half of the variation in prices. The additional source of price variation was both large and persistent;
furthermore, the evidence suggested “the presence of another stochastic trend in prices, driven by a third type
of innovation that is not part of either dividends or earnings” (Lee (1996) p. 344, emphasis added). This
finding is further evidence that deviations from equilibrium are driven by an unexplained unit root; but
because Lee (1996) did not examine relative prices, it is an open question whether relative prices would also
show evidence of an unexplained root.

Engsted (2006) found that US stock prices are cointegrated with dividends, but only if one allows for the
presence of an explosive root in the process in addition to the common unit root (Bohl and Siklos (2004)’s
use of a “bubble component” can be interpreted as allowing for an explosive root). My analysis will provide
further evidence for such an unexplained root, if it exists. However, because I use a CVAR, which is a more
powerful and general tool than single-equation regressions, I may also be able to learn more about where this
unexplained root comes from. One of the key questions in this thesis is whether the persistence of deviations
is reduced with relative prices. The CVAR is better able to answer this question. In particular, examining
both relative and absolute prices using CVAR techniques may help us isolate the source of the root, because
the process of converting from absolute to relative prices may “subtract out” the unit root (see Section 3.2,
below, for a fuller exploration of this hypothesis).
Other research also demonstrates how difficult it has been for researchers to identify a close connection with absolute prices. For example, Kanas (2005) provided evidence of a nonlinear cointegrating relationship consistent with the NPV model in the long-run. The term “nonlinear cointegration” here refers to the fact that if one transforms prices and dividends using the nonparametric “alternating conditional expectations” method, there should be standard linear cointegration between the transformed variables. Even the need to attempt such a transformation shows that the cointegrating relation predicted by the present value model is quite difficult to spot in the data; but as with other work, Kanas (2005) did not consider whether the relation is easier to see in relative prices. This analysis will address that question.

Shirvani, Delcoure, and Wilbratte (2011) allow for seasonal (or more broadly, cyclical) variation and test for seasonal cointegration consistent with the present value model. They argue that this is a way to control for an unidentified unit root that would invalidate the results of e.g. Campbell and Shiller (1987). They find no evidence for seasonal cointegration, and argue that their findings represent evidence against the present value model. Of course, if the uncontrolled unit root is present, but is not “seasonal”, their test would not detect it; as I have argued above, the CVAR approach with relative prices might help us detect, control for, and learn about the unexplained root.

Other methodologies also demonstrate the ways in which researchers have bent over backwards, so to speak, to find cointegration between prices and dividends. Multiple studies have found evidence for cointegration, but only if one allows for heteroskedasticity and/or structural break(s) in the price process or regressors. (See e.g. Strauss and Yigit (2001), McCabe, Leybourne, and Harris (2003)). Gabriel and Martins (2011) provided evidence in favor of cointegration consistent with the present value model, but with structural breaks in the cointegrating vector. Esteve, Navarro-Ibanez, and Prats (2017) find similar evidence of cointegration between log stock prices and log dividends, but only when one allows for structural breaks in the cointegrating relation (their data is annual, and they found evidence of two breaks: one in 1944 and one in 1971).

These findings should not really be surprising. Over such long time periods, we would expect that
changes in institutions and policy, new technologies, and political developments would cause structural change in how market participants interpret historical data in forecasting the future. This highlights another possible advantage of examining relative prices: we might expect that the structural breaks are of similar nature across absolute prices. If so those breaks may be “canceled out” by taking relative prices, similar to the way relative prices may help us isolate an unexplained unit root. If so, we would expect to observe that structural break tests in the CVAR will “flag” more breaks in absolute prices than relative prices. Moreover, the CVAR is able to cope with structural breaks and heteroskedasticity if necessary - but in the relative price CVARs, it may be less of a problem.

In summary, much of the prior non-CVAR literature has demonstrated a host of issues that may lead to weak or negative results for the present value model: heteroskedasticity, structural breaks, unidentified roots, and persistence. The key question for my thesis is whether these problems are reduced or eliminated with relative prices. The psychology literature suggests that this may be the case. The CVAR methodology is also more efficient and powerful than single-equation models and is quite capable of coping with all of these issues. It allows the researcher to adjust for heteroskedasticity and persistence; helps us identify the source of unexplained unit roots; and adjust for structural change.

While these studies do not use the CVAR methodology (Engle-Granger cointegration tests are a commonly-used technique in the literature, generally speaking), there are a few studies that have employed the CVAR.

### 3.2.2 CVAR Studies of Stock Prices

Crowder and Wohar (1998) use a CVAR to study the relation between stock prices and dividends (a 2-variable CVAR) under the assumption of a constant discount rate. This matches the assumption I make, so these results are directly comparable with mine. One of the key first steps in a CVAR analysis is to get a well-specified statistical model. In practice this means arriving at an unrestricted statistical model for which the residuals exhibit small skewness and minimal autocorrelation (the hypothesis tests are generally robust to high kurtosis); see Juselius (2006)). They did not discuss at length how they got a well-specified
model, except for the fact that they used a lag length of 5 (they do not explicitly mention adding dummies, heteroskedasticity, or structural breaks). They found a cointegration rank of 1, which means there is one cointegrating relation. With two variables, this of course implies that the two variables are cointegrated with each other. While they found evidence that the cointegration vector (referred to as $\beta$ in the CVAR context) implied by the present value model is consistent with the data, they found that previous studies imposed restrictions on the error-correction vector ($\alpha$, in the CVAR) that they were able to statistically reject. This is good news for the present value model because it suggests that prior results were a result of mis-specification. Their evidence suggested that dividends are weakly exogenous, which is encouraging for the present value model because it supposes that dividends are the fundamental consideration that drives prices. However, it leaves open several major issues previously discussed: structural change, persistence of deviations, and unexplained roots, among others. My approach will explore these issues. I am also able to test hypotheses about $\alpha$, but my primary focus is on what the values of $\alpha$ teach us about the persistence of relative prices versus absolute prices. However, as mentioned previously, I can do this for both absolute and relative prices.

Durre and Giot (2005) use a CVAR to study the behavior of stock indices for 13 developed countries - but each country’s index is analyzed separately (in other words, their analysis is confined to absolute prices “within-country”; it says nothing about relative prices “between-countries”). For each country, the variables included in the CVAR are stock prices, earnings, and the government bond yield. They were able to show evidence that the cointegrating relation does not require the inclusion of the government bond yield; but they find evidence of a single cointegrating relation (within each country, so 13 relations in total - but note that these are 13 separate CVARs, not one CVAR in which 13 relations are found). However, they do not appear to have found any evidence of an unexplained unit root; nor do they discuss the degree of persistence in the deviations from equilibrium. One can interpret this as suggestive evidence that prices and earnings (or dividends, which are likely to be closely related to each other) are cointegrated with each other, without need for additional controls. This suggests that not including the bond yield in a CVAR estimation does not result in a serious omitted variable bias. This provides some justification for the choice I (as well as Crowder and
Wohar (1998), see above) have made against including a measure of the interest rate.

Nasseh and Strauss (2000) also use a CVAR approach. Their data covers six national-level stock indices: Germany, France, Italy, the Netherlands, Switzerland, and the UK. One of their main contributions was to explore the possibility of a relation between international stock prices. However, this work is not a true analysis of market efficiency (whether defined in absolute or relative terms) for two reasons: (1) their data set does not include dividends for any of the countries; (2) they explore only the relation between the German index and each of the other countries. Their analysis treats German stock prices as one of several fundamental factors that could influence prices in other countries; it does not speak to whether the relative values of, say, the UK and German stock indices are consistent with their (relative) fundamental value. However, they do show that various other fundamentals appear to be related to price movements. This suggests that there is a relation among different absolute prices and fundamentals; this may be indirect evidence that relative prices more closely reflect fundamentals than absolute prices, as I have hypothesized, but that is not absolutely clear and this is not a direct test.

Neither these discussions of the extant literature - CVAR or non-CVAR - should be construed as exhaustive. Studies of the properties of broad stock price indices are ubiquitous in the finance literature. However, it provides a fair summation of the general state of that literature. There is evidence of a long-run relationship between dividends (or earnings) and prices, but with many caveats and adjustments being necessary to detect that relation. There also appears to be an unaccountably high degree of persistence in the price movements around the equilibrium. There is some evidence that there may be an unexplained root driving the process. Finally, the reader may notice that some studies provide evidence for the presence of a structural break. There have been a few studies of absolute stock prices using the CVAR methodology, but no analysis for relative prices. The key question I explore is whether the problems seen in absolute prices may be reduced in relative prices. The CVAR studies that have been done leave many of the questions I intend to explore - chiefly, unexplained roots, structural change, and persistence - underexplored. My contribution to

\[7\] Baek (2016) used a CVAR approach to explore the relation between prices, dividends and investor sentiment - but only as a “first pass”; his primary methodology is a copula. This should not be regarded as a full CVAR study.
the literature is therefore twofold: I will provide an examination of these issues in the more powerful CVAR context; and I will be the first to explore the cointegration structure of both relative and absolute prices.

The next section will discuss the theoretical underpinnings of this study and the empirical methodology of the CVAR. I will argue that consideration of both relative and absolute prices in a CVAR allows for a rich exploration of the possible cointegrating relations and short-run dynamics governing stock price movements. This approach may help us learn more about the reasons for the observed persistence and the nature of the unexplained root.

3.3 Theoretical and Empirical Methodology

3.3.1 Theoretical Foundations

In Chapter 2, I mentioned that Shiller (1981)’s volatility test was criticized because it might return a spurious result if a unit root is present in the price process. Campbell and Shiller (1988) responded to this criticism by proposing an alternate, VAR-based test of what they called the “dividend-ratio model”:

$$
\delta_t = \sum_{j=0}^{\infty} \rho^j E_t(\Delta d_{t+j}) + \frac{(r - k)}{(1 - \rho)}
$$

where \( \delta_t \) is the log of the dividend-price ratio, \( d_{t+j} \) is the log real dividend in period \( t + j \), \( \Delta \) denotes a first difference, \( \rho \) is the (assumed constant) discount rate and \( r \) and \( k \) are constants. The value \( r \) is the assumed-constant expected real one-period return; \( k \) is a constant defined by the equilibrium of the stationary series \( \delta_t \).

This specification has several implications for the expected long-run properties of the data. First, it is assumed that \( \delta_t \) is stationary. While this is a standard assumption in the literature (see above), the fact that prior research found evidence of unaccountably high persistence suggests that it may not be a good
assumption. Given (??) and the stationarity of \( \delta_t \), however, two conclusions must follow. First, if the price process contains a unit root, then \( \ln P_t - \ln D_t \) must be a cointegrating relation. Second, the right side of (??) must be stationary (a stationary series cannot equal a non-stationary series). But the right side of this equation is the discounted sum of (expected) future dividend growth rates, which implies that \( \Delta d_t \) is stationary.\(^8\)

This can be shown by contradiction. For instance, if \( \Delta d_{t+j} \) is a simple random walk, then \( E_t(\Delta d_{t+j}) = \Delta d_t \) for all \( j > 0 \). But then the discounted sum of expected future dividend growth rates is \( \frac{1}{1-p} \Delta d_t \). But that is a constant times a variable that is \( I(1) \), so the series must be \( I(1) \). So we are assuming that dividends are \( I(1) \).

The present value model implies that prices are \( I(1) \) and cointegrate from \( I(1) \) to \( I(0) \) with dividends. In other words, if \( \delta_t = \ln P_t - \ln D_t \) is stationary, but the natural logs of prices and dividends are individually \( I(1) \), then prices and dividends cointegrate in a way that suggests that stock prices do depend on fundamentals as suggested by the present value model.

In addition, recall that a cointegration approach is robust to a time-varying but stationary discount rate (like the assumption that \( \delta \) is stationary, it isn’t obvious that this is a good assumption; for one thing, a non-stationary discount rate might explain the persistence problem). This observation is the basic intuition behind Timmermann (1995) and Bohl and Siklos (2004). While it might appear that (??) implies that dividends should be “weakly exogenous” and prices the “purely responding” variable - that is to say, that dividends move and prices follow - this interpretation is a bit too pedantic. A CVAR approach treats these as testable restrictions (see below), but neither restriction is actually implied by the present value model or (??). Prices are supposed to contain information about the stream of expected future dividends, so it would not be at all surprising if we found evidence that price changes “affect” future dividends. But the proper interpretation of that finding would be that prices \textit{lead} dividends, not that they \textit{cause} dividends. A far more sensible interpretation of such a result would be that the \textit{statistical} model did not include all variables that affect future dividend flows. There are a myriad of variables that might affect future dividends, and therefore current prices, but which cannot be controlled for in an econometric model. A well-functioning market

\[^8\]If \( \Delta d_{t+j} \) is non-stationary, it can be shown that the discounted sum of its future values could not be stationary.
would incorporate the signals from those variables into prices, and this would manifest itself in the form of a result suggesting that prices “affect” future dividends.

Whether a variable is “purely adjusting” or “weakly exogenous” are hypotheses that can be tested in the CVAR as restrictions on $\alpha$ (see below). I previously mentioned that Crowder and Wohar (1998) provided evidence that dividends are weakly exogenous; as the previous paragraph suggests, that result is sufficient, but not strictly necessary. The next subsection provides an overview of the CVAR approach.

### 3.3.2 The Cointegrated VAR: An Overview

The estimation of a Cointegrated VAR is an iterative process. One begins by estimating a general unrestricted model (GUM). In this chapter, I use the I(1) model (following the literature, I assume that prices and dividends are I(1), so this is an appropriate assumption), the initial GUM with $k$ lags takes the form:

$$
\Delta x_t = \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \ldots + \Gamma_{k-1} \Delta x_{t-k+1} + \Pi x_{t-1} + \epsilon_t
$$

(3.5)

where $x_t$ is the time-t vector of the endogenous variables. In the context of the present value model, $x_t = [\ln P_t, \ln D_t]$. While the GUM assumes both variables are endogenous, the hypothesis that a variable is weakly exogenous can be tested as a restriction on the GUM. The $\Gamma^i$ are matrices of short-run coefficients, and $\Pi$ is the matrix of long-run coefficients. However, (3.5) does not necessarily generate a well-specified model, which is to say one for which the errors behave appropriately. Valid statistical inference requires a model for which the errors are normal. The most problematic deviations from normality are serial correlation and skewness. In practice, excess kurtosis and ARCH (autoregressive conditional heteroskedasticity) are not serious concerns. One checks whether the GUM is well-specified by conducting statistical tests of the errors; if the errors of the ‘basic’ model do not meet the criteria - and they rarely, if ever, do - one constructs a well-specified model by obtaining the correct lag length and adding additional controls such as constants, time-dummies, or exogenous variables (if applicable) to the basic specification given by (3.5).
The above representation does not explicitly include a constant, but in practice several different kinds of constants may be employed. A restricted constant allows the equilibria of the cointegrating relation(s) to be non-zero, but does not imply a deterministic trend in $x_t$. An unrestricted constant allows for both a non-zero mean in the cointegrating space and a deterministic trend in $x_t$. Both cases are potentially consistent with the present value model; the right side of (??) could easily be a number other than zero, which would require a restricted constant; and there is no reason why, say, $\ln D_t$ could not be a random walk with a trend. In the context of this chapter, the theory does not strongly imply any restrictions on the constant term, so the type of constant is chosen for purely statistical reasons.\footnote{These are the only cases relevant to this paper, though an additional possibility exists. In the case of a cointegrated trend, the cointegrating relations $\beta x_t$ fluctuate around a trend-line instead of around a constant value.}

The methodology of Juselius (2006) recommends the use of dummy variables over the addition of extra lags; in many applications $k = 2$ is sufficient. Suppose we add a dummy variable $I_t$ to the GUM. It will take one of three forms. The dummy is called a “transitory” dummy if its value over time looks like $I_t = (...0, 0, 0, 1, -1, 0, 0,...)$; such a dummy captures a change at the moment where $t = 1$ followed by a change of equal magnitude but opposite direction in the next time period. Transitory dummies therefore represent momentary “spikes” in $x_t$ with no long-lasting effect. $I_t$ is called a “permanent” or “intervention” dummy if its value over time is $I_t = (...0, 0, 0, 1, 0, 0, 0,...)$; such a dummy captures a permanent jump in the level of at least one of the variables in $x_t$ occurring at the moment where $t = 1$. Such a dummy does nor correspond to a change in the equilibrium around which $x_t$ fluctuates. For instance, a moment in time where prices and dividends both increase by roughly the same percentage (with no reversal soon after) would likely be represented by a permanent dummy. Finally, $I_t$ is called a “shift” dummy if it takes the form $I_t = (...0, 0, 0, 1, 1, 1,...)$, but is restricted to lie within the cointegrating relation. Shift dummies capture a change in the level around which the (otherwise stationary) cointegrating relation fluctuates (in other words, a change in the restricted constant). While a transitory dummy represents a shock that does not have any long-lasting effect, permanent and shift dummies may be thought of as evidence of structural change; they are often found at moments in time corresponding to major policy changes or other economic events.
dummies should be added are detected statistically by inspecting the standardized residuals and identifying large outliers. This is especially useful in eliminating problems with skewness as one can eliminate large outliers by the use of an appropriate dummy.

However, an alternative to the use of dummies is the use of extra lags (increasing the value of $k$, so that there are more matrices of short-run parameters $\Gamma^i$). Neither the methodology of using dummy variables nor the methodology of using additional lags are “wrong”; they are merely different. In other words, the use of dummies and the inclusion of extra lags may be thought of as approximate “substitutes” in the process of the economist obtaining a well-specified model. In this chapter I will employ a hybrid of these methodologies. Maintaining $k = 2$ made it very difficult to reduce serial correlation to acceptable levels; but increasing the number of lags to $k = 4$ reduced this problem sharply. This is not surprising given that my data is quarterly; the additional lags probably capture seasonal effects that cannot be adequately captured by dummies. However, because I am interested in examining whether there is evidence of structural change, I also employ the dummy methodology. As mentioned above, permanent and shift dummies act as indicators of structural change.

Once a well-specified GUM is obtained, one conducts tests to determine the cointegration rank of $\Pi$. Since by assumption $\Delta x_t$ is stationary, but $x_t$ is not, this implies two things: first, that the matrix $\Pi$ is singular (has reduced rank); and second, that $\Pi x_t$ defines a set of stationary linear combinations of non-stationary variables. Part of the CVAR procedure is a test that determines the rank of $\Pi$ that best fits the data, which helps the economist determine the structure of the cointegrating relation(s). The cointegrating relations may be thought of as equilibria around which the variables of $x_t$ fluctuate. Thus, for purposes of studying cointegration properties, it is the $\Pi$ matrix one must analyze. The cointegration rank tells one how many separate unit roots are present in the system and how many cointegrating relations exist to which the system is adjusting. This part of the process allows the economist to examine whether the data are consistent with the present value model’s prediction that prices and dividends are cointegrated. It also informs an assessment of whether the data shows evidence of the “unexplained roots” sometimes detected by previous
analyses. If $\Pi$ is singular, it can be written:

$$\Pi = \alpha \beta'$$

where $\alpha$ and $\beta$ are $p \times r$ matrices ($r \leq p$) and $p$ is the number of variables in $x_t$. Once one has obtained estimates of these matrices, one can conduct tests of restrictions on them. That is how the econometrician can test, for example, whether the hypothesized cointegrating relation is consistent with the data. A test of the rank of $\Pi$ helps determine $r$. $r$ is referred to as the cointegration rank of the system and is equal to the number of different cointegrating relations in the system. $\beta$ may be thought of as the matrix of estimates of the coefficients of the cointegration vector(s) and $\alpha$ as the matrix of adjustment parameters; it is possible to test restrictions on both $\alpha$ and $\beta$, either simultaneously or separately. The present value model predicts that log prices and log dividends cointegrate such that the $\beta$ vector can be written $[1, -1]$, so that is the hypothesis about $\beta$ that I will be testing.

The magnitude of the elements of $\alpha$ represent roughly the proportion of deviations from equilibrium that are eliminated every period. The value of the coefficient(s) of $\alpha$ may be thought of as the rate at which deviations from equilibrium are corrected. The sign of the coefficients in $\alpha$ indicates whether the “adjustments” are error-correcting or error-increasing. A positive coefficient in $\alpha$ should correspond to a negative coefficient in $\beta$, and vice-versa, if there is error-correction. This is correct because if, say, $P_t$ has a positive coefficient in $\beta$ and its value is above equilibrium, we would expect its value to decline over coming periods; that is a negative change, which would imply a negative adjustment parameter. The present value model does not make any specific predictions about the values of $\alpha$, but we should expect to see evidence of error-correction if prices are efficient. In addition, the value of the coefficient(s) in $\alpha$ act as a measure of persistence. Small values of $\alpha$ indicate slow convergence back to equilibrium; that is, low values of $\alpha$ indicate high persistence.

This methodology assumes that $x_t$ is $I(1)$. The model is valid only is $x_t$ is $I(1)$ or less. If, however, $x_t$ is

\[96\]

\[10\]If we saw error-increasing behavior, that might be consistent with a behavioral-finance “bubbles” story.
I(2), the econometrician would be likely to detect the kind of persistence and unexplained roots commonly seen in the literature. If this is the case, the CVAR will provide suggestive evidence of it: tests of the rank \( r \) will suggest the presence of an unaccounted-for unit root, no matter the rank one chooses. There is another measure of persistence available as well. Rewrite the \( k \)-lag system as an AR(1):

\[
\tilde{x}_t = \tilde{\Pi} \tilde{x}_{t-1} + \epsilon_t
\]  

where \( \tilde{x}_t \) is the vector \([x_t, x_{t-1}, \ldots, x_{t-k+1}]\). The eigenvalues of \( \tilde{\Pi} \) are referred to as the roots of the companion matrix. If there is a root with modulus greater than 1, the system is explosive (one interpretation of such a finding is that it is evidence of “bubbles”). Roots with modulus less than 1 are stationary. When one sets the cointegration rank \( r \), one will find \( p - r \) unit roots of the companion matrix. Any additional roots with modulus close to or equal to 1 are evidence of unit root(s) that have not been accounted for by the economist’s specification. If such an unexplained large root appears regardless of the economist’s choice of cointegration rank, it is likely that the data is I(2) instead of I(1).

Exploring both absolute and relative prices in the CVAR may help address the active questions in the literature by giving us a hint of “where” the unexplained unit root(s) lie. For instance, suppose that we detect signs of I(2)-ness and high persistence in the absolute-price CVARs, but not in the relative-price CVARs.

One hypothesis that would explain such results is as follows. Suppose that the absolute-price vector for an asset \( i \), \([p_{it}, d_{it}]'\) is I(2), where \( p \) and \( d \) are log prices and log dividends respectively. In other words, we are assuming that the percent change in prices and dividends \( \Delta x_t \) (the left side of (3.7)) is I(1) (in the simplest case, a random walk), not I(0). What would it mean if the relative-price vector between assets \( i \) and \( j - [p_{it} - p_{jt}, d_{it} - d_{jt}]' \) - is I(1), so that changes in relative prices and dividends are stationary? This would appear to suggest that the relative prices and dividends (the relations \( p_{it} - p_{jt} \) and \( d_{it} - d_{jt} \)) are themselves cointegrating relations (specifically, cointegrating from I(2) to I(1)). In this case, the absolute-to-relative transformation is similar to one used in a host of previous CVAR papers (outside the context of stock prices).
These studies found evidence suggesting that the nominal-to-real transformation is a cointegrating relation from $I(2)$ to $I(1)^{11}$.

If relative prices and relative dividends then cointegrate with each other (from $I(1)$ to $I(0)$), then the relation:

$$(p_{it} - p_{jt}) - (d_{it} - d_{jt})$$

is cointegrating from $I(2)$ to $I(0)$. That would indicate a root held in common across prices and a root held in common across dividends, with each individual price series driven by both of these roots. Notice that the strategy of using the CVAR - even the $I(1)$ version - on both absolute and relative prices may provide greater insight than the use of the CVAR with absolute prices alone.

In this paper, I use the $I(1)$ CVAR for both absolute and relative prices; in the absolute-price models, $x_t$ is $[p_t, d_t]$; in the relative-price models, $p$ and $d$ are replaced by $p_i - p_j$ and $d_i - d_j$ respectively. Other than this change, the analysis of relative prices follows the methods used for absolute prices quite closely. I choose to employ the $I(1)$ model instead of a more general approach that is robust to the inclusion of $I(2)$ variables because previous $I(1)$ CVAR studies have not sufficiently addressed the question of whether an unaccounted-for unit root explains the persistence problem (see above). It remains unclear whether the $I(2)$ approach is necessary.

I now turn to a description of the data used in this paper.

### 3.4 The Data

My data source is Ken French’s website, from which I have downloaded returns data for 5 industry portfolios. This data is monthly, beginning in July 1926 and ending in December 2018. There is no actual “price” for these portfolios, so I set the price of each portfolio at the beginning of my sample equal to 100. The data

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^{11}$To clarify further: suppose $X_t$ is some nominal variable - say an exchange rate - and $P_t$ is the price level; the CVAR studies suggest that $\ln X_t$ and $\ln P_t$ are $I(2)$, but $\ln X_t - \ln P_t$ is $I(1)$.
is returns data in percent terms, but includes returns both with and without dividends. The return without dividends is the percent change in the price of the portfolio, so I can measure the percent change in the price of the portfolio through each period. Since the data includes returns both with and without dividends, I am also able to measure the change in the dividend paid in each period. The below table lists the 5 sectors covered by the data. With 5 sectors, there will be 5 absolute prices and 10 relative prices. I also run an absolute-price CVAR for the market as a whole to examine the secondary hypothesis of whether disaggregating the market gives improved results for absolute prices (see Chapters 1 and 2).

Table 3.1: 5 Sector Portfolios

<table>
<thead>
<tr>
<th>1</th>
<th>Consumer</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Manufacturing</td>
</tr>
<tr>
<td>3</td>
<td>Hi-Tech</td>
</tr>
<tr>
<td>4</td>
<td>Health</td>
</tr>
<tr>
<td>5</td>
<td>Other</td>
</tr>
</tbody>
</table>

These sectors are broadly defined and diverse and, taken together, include all stocks listed on NYSE, NASDAQ, and AMEX. Stocks are filed into each portfolio by industry, according to 4-digit SIC codes.¹²

Since the data provides the percent change in prices and the dividend yield in each period, I am able to calculate the nominal price and dividend paid for each sector portfolio in each period. I convert these values to real prices and dividends by dividing the nominal data by the value of the CPI for each period.¹³

Nevertheless, this data is imperfect. The present value model suggests that the log of prices and the log of dividends are cointegrated. Clearly, then, the endogenous variables in my analysis must be $lnP_t$ and $lnD_t$.

But there are industry portfolios with 0-dividend observations in certain months; this creates an observation with an ‘undefined quantity. This problem can be addressed by converting the data from monthly to (end-of-period) quarterly. While an alternative to this approach is to construct monthly average dividends and prices over each quarter, doing so introduces intractable econometric problems. Estimates of variances, covariances, and autocorrelations are biased in time-averaged data. These biases are time-varying in nature, which means there is no straightforward correction. But the problem is actually even worse than that. The

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¹²See Appendix I for a brief description of the kinds of specific firm types that may be found in each portfolio.

¹³Further details available on request.
three biases cannot all be corrected for simultaneously. One can use filtering techniques to address any two of the problems, but the third problem will persist (see Wilson, Jones, and Ludstrum (2001) 175, 177 - 178, 189). In order to avoid these problems, I opt to use quarterly data; while the sample size is therefore proportionately smaller, it is still over 360 observations. Statistical analysis was conducted using the CATS package in RATS.

3.5 CVAR Results: Structural Change

In this section, I summarize the results of the CVAR analysis for each sector and each distinct relative price. First, I briefly discuss the general methodology employed to develop each GUM after initial estimation. Next, I summarize the estimates for each (well-specified) general unrestricted model (GUM). This includes a summary of what kind of constant terms were used, the dummies, and other breaks employed. Particular attention will be paid to the dummies, which as mentioned previously act as indicators of effects of structural change. I then compare these results for absolute prices against those for relative prices and discuss what these results teach us about the market. Second, I discuss the results for some of the “core hypotheses” of the present value model. These include whether the cointegrating relation observed in the data is \( \beta = [1, -1] \), as hypothesized by the present value model, and the implication that prices should move in response to dividends, which is a hypothesis about the structure of \( \alpha \). Finally, I summarize what the results suggest regarding persistence. (A fuller discussion of what these results teach us is provided in the next section).

3.5.1 Well-Specified Models: Methodology and Structure

As mentioned previously, a well-specified GUM is needed to perform valid statistical tests on the \( \Pi \) matrix. This requires that the errors are normally distributed and are not auto-correlated. CATS uses the Shenton-Bowman procedure to generate estimates of skewness and kurtosis that are adjusted for small sample sizes. The resulting test statistic for univariate normality is a weighted average of the adjusted skewness and kur-
tosis measures, and is distributed $\chi^2$ with 2 degrees of freedom. The CATS residual analysis procedure also provides tests of residual autocorrelation as well as measures of sample skewness and kurtosis. If one is able to obtain a model for which the null hypothesis of residual normality cannot be rejected, it is clear that the skewness is not “too high;” if it were, we would reject normality. However, the tests of restrictions on $\Pi$ that will be used are minimally sensitive to ARCH and to excess kurtosis (see Juselius (2006) 74 - 75 and citations therein). In practice, this means that we must be chiefly concerned with residual skewness and autocorrelation. In other words, non-normality is acceptable if it is due to excess kurtosis. If so, the economist must take steps to ensure that the skewness is not too high. Financial data frequently exhibits high kurtosis, so it is usually quite difficult to get normally-distributed residuals. If my data exhibits excess kurtosis, then, I must adopt a procedure to evaluate whether the skewness is too high.

In my data, excess kurtosis is indeed present. Therefore it was necessary to adopt some kind of rule for evaluating whether the skewness was too high. While skewness and kurtosis statistics are asymptotically normal, convergence may be slow. Using the Shenton-Bowman adjustment for skewness and with sample size $T = 367$ (see Juselius (2006) 75 - 76 for details), one can calculate the contribution of the skewness term to the normality test statistic. In the few papers that have used the CVAR methodology in studies of stock prices (see above), I could not identify a clear procedure for addressing this problem. Juselius (2006) also does not provide any explicit recommendations on these lines except for providing the details of the Shenton-Bowman procedure (see Juselius (2006) 75 - 77). I therefore chose to adopt as restrictive a rule as reasonably possible for judging whether skewness is too high. Given that the 10% critical value for a $\chi^2$ with 2 degrees of freedom is 4.605, I chose to adopt a rule that considered the skewness to be “too high” if the skewness term’s contribution exceeded this value. The cutoff value for this is a skewness of roughly 0.62 (in absolute value). Note that this procedure is more restrictive (requires lower skewness) than using, say, the 5% critical value, which would be higher and therefore allow a larger value for skewness.

The residual analysis procedure in CATS provides 3 tests of residual autocorrelation: the Ljung-Box test and two LM tests. In most cases, I was able to arrive at a model for which the null hypotheses of no autocor-
relation could not be rejected at a significance level of 5%. I provide diagrams of residual autocorrelations in an appendix. While some previous CVAR research often finds that a VAR(2) and proper dummy use are sufficient to eliminate autocorrelation, my models will be VAR(4)s. In all cases, including fewer than 4 lags made it exceedingly difficult to eliminate residual autocorrelation. Lag selection tests also indicated that using 2 or 3 lags resulted in significant loss of explanatory power, even when including seasonal dummies.

3.5.2 GUM Results for Absolute Prices

The table below summarizes the structure of each GUM for absolute prices. The second column is the type of constant term used; the third through sixth columns display the number of each type of dummy or break used in each model.

<table>
<thead>
<tr>
<th>Constant Term</th>
<th>Transitory</th>
<th>Permanent</th>
<th>Shift</th>
<th>Break</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>Restricted</td>
<td>1</td>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>Consumer</td>
<td>Restricted</td>
<td>4</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Unrestricted</td>
<td>1</td>
<td>22</td>
<td>5</td>
</tr>
<tr>
<td>Hi-Tech</td>
<td>Restricted</td>
<td>1</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Health</td>
<td>Restricted</td>
<td>7</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>Other</td>
<td>Restricted</td>
<td>1</td>
<td>19</td>
<td>12</td>
</tr>
</tbody>
</table>

The entries in the three right-most columns may be roughly thought to represent incidents of structural change; there are 115 in total across the 6 absolute prices. On average, then, there are 19.17 structural change events per series. The transitory dummies are not counted in this total because they represent “unusual” events that neither changed the underlying structure nor had a permanent effect on the level of either variable. The “Break” column in the above table gives the count of unrestricted shift dummies. These are different from the “shift” dummies in that the latter are restricted to lie in the cointegration space; the unrestricted breaks are not. They represent a change in the unrestricted constant term. In the cases for which the constant term is restricted, but unrestricted breaks are present, the unrestricted constant is 0 at the beginning of the sample but shifts to a non-zero value at some point.

Each of these dummies has its own economic interpretation. Permanent dummies represent unantici-
pated events that affect the level of \( P_t \) and/or \( D_t \), but have no permanent effect on their growth rates or on the equilibrium of the cointegrating relation. Shift dummies represent points at which the equilibrium of the cointegrating relation changes. Given equation (??), this may occur for several reasons; one notable possibility in this category is a discrete shift in the discount rate. An unrestricted break indicates that the unrestricted constant term in the CVAR has shifted. This indicates a change in the underlying growth rate of dividends and/or prices; but it is also possible that the break is associated with changes over time in the discount rate.

Many of the dummies and breaks are clustered together within small windows of time (see Figure 3.1). The most notable such cluster falls during the period of the Great Depression. For the Consumer sector, 8 of the dummies fall between 1929 Q4 and 1939 Q4. 15 of the dummies used in the model for the “Other” sector fell in the same period, as did 16 of the dummies and breaks used in the model for the Manufacturing sector. For the High-Tech sector, all of the dummies (but none of the breaks) fell between 2000 Q4 and 2012 Q4.

In general, the dates of these dummies mostly correspond to major economic or historical events, such as the onset of the Great Depression, ongoing events in financial markets during the early years of the depression, the stock market crash of 1987, and the onset of the Great Recession. This is evidence in favor
of the proposition that these dummies represent structural change. We would expect that, if structural change matters, discrete breaks are likely to coincide with major economic events. Therefore, from the absolute price GUMs alone, we can conclude that structural change is an important component of movements in prices and dividends. The question is whether the signs of structural change effects are reduced in relative prices. To examine this question, I turn to the results for relative prices.

### 3.5.3 GUM Results for Relative Prices

Table 3.3, below, summarizes the structure of each GUM for relative prices. The constant term column is removed from this table; a restricted constant was used in every case. Consumer/HiTec and Manufacturing/HiTec were run in two separate samples for reasons discussed below. Subsample 1 for Consumer/HiTec ends in 1975 Q1. Subsample 1 for Manufacturing/HiTec ends in 1980 Q1.

<table>
<thead>
<tr>
<th></th>
<th>Transitory</th>
<th>Permanent</th>
<th>Shift</th>
<th>Break</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cons/Manuf</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cons/Health</td>
<td>7</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Cons/Hi-Tech (1)</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Cons/Hi-Tech (2)</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Cons/Other</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Manuf/Health</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Manuf/HiTec (1)</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Manuf/Hi-Tech (2)</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Manuf/Other</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Health/Hi-Tech</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Health/Other</td>
<td>2</td>
<td>5</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Hi-Tech/Other</td>
<td>2</td>
<td>11</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Abbreviations used in the above table: Cons = Consumer, Manuf = Manufacturing.

Notice that there are only 74 permanent and shift dummies and 2 unrestricted breaks across these 12 models. Add the 2 sample splits and we have a total of 78 breaks - markedly fewer than the 115 total in the 6 absolute-price models. A better way to look at this, though, is the number of breaks per price series. Consider, as an example, a permanent dummy that appears in the GUM for the manufacturing sector (absolute prices). If the event captured by this dummy was idiosyncratic to the sector, we would expect
it to have an effect on *all* the relative prices involving the manufacturing sector. Since there are 4 such relative prices, we’d expect 4 dummies to appear - one for each relative price. However, suppose the dummy represents a “macro” event (a shock that has a common effect across sectors). When one takes relative prices, the common effect of the macro events would largely cancel out. There are on average 19.17 dummies per series for absolute prices and 7.8 per relative price. This is about 40% of the number for absolute prices. Since many of the breaks appear around the dates of major macroeconomic events, it appears that many - but not all - of the absolute-price breaks represent similar changes across sectors and are connected to “macro” events. This is evidence in favor of an argument I made in Chapter 1: that the relationship between absolute and relative prices is *clearer* in relative prices because the effects of macro events are canceled out in relative prices.

In the relative price GUMs, the clustering of dummies is still present - but the degree to which the dummies correspond across sectors is reduced. The biggest cluster of dummies still falls during the Great Depression. 33 of the dummies and 1 of the 2 breaks, spread across all 12 models, fall during the 1929 Q4 to 1939 Q4 window. There are 9 dummies at dates roughly around the market crash of 1987, 6 of which are in models involving the “Other” sector. Since the Other sector includes financial firms, these dummies may correspond to the market crash of 1987 as well as events associated with the Savings and Loan crisis, which began in the latter half of the 1980s. In addition, 18 dummies fall between 1999 Q4 and 2002 Q4, but 15 of those 18 were used in models for which one of the two sectors was Hi-Tech. This is consistent with the idea that these dummies represent possibly rational revisions in the market’s forecast: events that should affect the Hi-Tech sector affect relative prices involving the Hi-Tech sector; events affecting the financial sector affect Other, which is where financial stocks are found in my data. In addition, 10 dummies fell between 2008 Q2 and 2009 Q2 - all of them in regressions involving at least one of the HiTec or Other sectors and 7 of which involved specifically Other. These observations are consistent with the idea that dummies in the CVAR methodology should ideally reflect shifts or changes connected to underlying economic events. In the context of relative prices, though, the dummies reflect events that alter the prospects
of one sector in comparison with another. For instance, we might expect to see evidence of breaks for relative prices involving the “Other” sector at around the time of the bank runs of 1930 because those events could plausibly be interpreted as negative news for specifically the financial sector, which is part of “Other”. We see other dummies located in time periods that are consistent with this interpretation; for example, dummies for periods associated with the so-called “Dot Com bubble” are concentrated in the HiTec sector, as we’d expect, and dummies associated with the financial crisis and recession of 2008 - 2009 are clustered in Other, which as just mentioned is the sector in which stocks of financial firms are found.

We can also potentially learn something from the type of dummy or break. Permanent dummies are indicative of an unanticipated event that moved the level of prices and/or dividends, but do not change the underlying cointegrating relation (given by \((??)\)). Such events affect neither the discount rate \(\rho\) nor the expected future dividend growth rate. As mentioned previously, a shift dummy represents a change in the constant around which the cointegrating relation fluctuates - therefore, by \((??)\), a discrete change in the discount rate \(\rho\) or a shift in the market’s forecast of future dividend growth rates. An unrestricted break implies a change in the trend rate of growth of dividends and/or prices, in addition to a change in the restricted constant. It might also reflect a gradual drift in the discount rate. However, most of the dummies and breaks are associated with moments of economic upheaval; it is not at all clear why shifts in \(\rho\) - a parameter that reflects investors’ preferences - would shift at times and in ways that are so closely associated with macroeconomic events. Moreover, because previous work has not found strong evidence that prices and dividends cointegrate with the interest rate (see Durre and Giot (2005), discussed above), it is hard to argue that the shifts reflect \(\rho\) moving in concert with interest rates. Therefore, a more straightforward interpretation is that these breaks represent the market coping with Knightian uncertainty: the events were not fully anticipated, so market participants had to adjust their forecasts in response to those events.

There were 27 shifts or breaks in the relative price models and 27 in the absolute-price models. As mentioned previously, the proper way to compare breaks in absolute prices This suggests that this kind of structural change occurs more frequently for absolute prices than relative prices because there are 4.5 per
absolute series and 2.7 per relative series. That is consistent with the overall dummy counts: more frequent structural change for absolute than relative prices. Because the shifts and breaks are likely to represent the market coping with Knightian uncertainty, this corroborates the interpretation that the market finds it harder to forecast absolute prices than relative prices.

In summary, the evidence related to structural change indicates that structural change plays a greater role in moving absolute prices than relative prices. The clustering of dummies suggests that this structural change is a result of the market coping with Knightian uncertainty (that is to say, unanticipated events). This evidence suggests that the market finds it harder to accurately forecast absolute prices and dividends than relative prices and dividends. This is consistent with the idea that many “macro” events are hard to predict, but when such events occur, the market revises its forecasts in sensible ways. These “macro” events do not always affect the relative prices of different sectors, which indicates that these events often do not cause the market to revise their forecasts of the relative future prospects of different sectors.

If Knightian uncertainty explains a significant part of these price movements, then it is not necessarily “irrational” behavior by market participants that is driving price movements. These price movements may well reflect rational market participants doing their best to construct reliable forecasts while coping with Knightian uncertainty. It would follow, then, that if an economist tests an asset price model, but fails to account for this reality, they could easily get results that appear to suggest “inefficiency” or “irrationality.” But this interpretation of those results is suspect because the economist has the wrong model of rational forecasting. If, when we account for Knightian uncertainty, we get results that are more consistent with rational forecasting behaviors, it means that a model without Knightian uncertainty is the wrong model. More specifically, a VAR or even CVAR - which suggest that future outcomes follow from past outcomes - is not a good characterization of the way real-world markets forecast future events.

In addition, I noted that clustering of dummies was observed for relative prices as well as absolute, but these clusters were not common across sectors (with the exception of the Great Depression). For example, in relative prices, clusters of dummies associated with the so-called “Dot com bubble” were largely concen-
trated in the HiTec sector. This suggests that the events of the time caused the market to update its forecasts about specifically the HiTec sector’s prospects relative to other sectors. We don’t observe evidence of structural change at that time for, say, the Consumer/Manufacturing relative price, probably because the market “knew” that the events of that time were not relevant to the relative prospects of those sectors. This also is consistent with the idea that the market finds it harder to forecast absolute prices - macro events often line up with dummies in absolute prices, which appear more frequently than in relative prices. When those events occur, the market revises its forecasts and absolute prices move in response - but we frequently observe no effect in relative prices.

3.5.4 Other Structural Change Tests

Dummies, however, are not the only way to detect structural change in the CVAR. Recursive estimation of the system also allows one to test whether certain estimated parameters of the model are stable over time. One of the most important considerations is whether $\beta$ is constant over the sample. This test is known as the “Max test of $\beta$ Constancy.” This test is known to be conservative - that is, it rarely rejects the null hypothesis that $\beta$ is constant. The test statistic is estimated for the full system $X(t)$ and for the long-run components of the system, $R(t)$, which is the full system with the dummies and short-run components subtracted out. The value of the statistic is plotted over time. For absolute prices, the manufacturing sector exhibits evidence of non-constancy in the full system $X(t)$ - the test exceeds the 95% critical value for almost the entire first half of the sample. However, the evidence is much weaker for the long-run component, for which there is only a brief spike in 1975 (see Figure 3.2). The test statistic there does not quite exceed the critical value, but is very close. For the other 4 sectors, there is mixed evidence that the short-run components exhibit more structural instability than the long-run component (because the test statistic for $X(t)$ is usually either roughly the same as or larger than the statistic for $R(t)$). See the appendices for graphs of these tests. This structural instability in the short-run parameters is not a major issue for purposes of this research, but could be an interesting topic for future work.

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However, when I performed the same test for relative prices, I found that in two cases (Consumer/HiTec and Manufacturing/HiTec), it was necessary to split the sample. In the case of Consumer/HiTec, this was because the test of $\beta$ constancy could be rejected for roughly the period 1975 - 1991 (see graph in the appendix). I therefore split the sample at 1975 Q1. For Manufacturing/HiTec, the test of $\beta$ constancy could not be rejected - but attempts to obtain a well-specified model in the full sample revealed a specification problem. It was impossible to reduce both autocorrelation and skewness to acceptable levels. Specification changes that reduced one invariably resulted in increases in the other. This itself is a signal of misspecification: “(s)olving one problem frequently reveals a new misspecification problem that was previously hidden” (Juselius (2006) p. 77). Since this issue was difficult to solve, and because the test of $\beta$ constancy didn’t provide a clear signal, I examined additional recursive tests. The fluctuation test of the eigenvalues revealed moderate evidence of structural instability. The test statistic closely approached the 95% critical value during a period between 1980 and 1985 and again during the 2000s. A similar test for the constancy of the trace statistics showed somewhat more robust evidence of instability for roughly the same time periods. I therefore split the sample in Q1 1980.

The fact that I needed to split the sample for two of the relative prices but none of the absolute prices might seem to contradict the argument made above - that there is more evidence of structural change for
absolute than relative prices. While there is some merit to this intuition, it should not be given too much weight. Firstly, the fact that there were fewer dummies used for relative prices may explain the apparent non-constancy of $\beta$. In absolute prices, the use of numerous dummies may have simply captured structural change effects more thoroughly than when fewer dummies were employed in the relative price models. This may have manifested itself as evidence of a non-constant $\beta$ in those two relative prices. However, it is also worth noting that we weren’t able to reject $\beta = [1, -1]$ for the sub-samples at high confidence. If $\beta$ were really not constant, we’d expect to see stronger evidence of that in the estimated values of $\beta$ in the sub-samples. It is therefore likely that the sample-splitting acted as a “substitute,” so to speak, for the use of other structural change indicators.

The natural question that follows from this reasoning is: are price movements consistent with the present value model after we account for Knightian uncertainty? I examine this question in the next section.

### 3.6 CVAR Results: Cointegration and Exogeneity

In this section, I summarize results for tests of the core hypotheses of the present value model. First I conducted a test of the cointegration rank $r$ of the system. I do not discuss the results of these tests at length - in every case we could reject $r = 0$, which means the system is not stationary (full results available on request). This implies that there is some cointegrating relation present.

#### 3.6.1 Tests of Cointegration

The present value model implies that the relation $lnP_t - lnD_t$ is cointegrating from $I(1)$ to $I(0)$ ($lnP$ and $lnD$ are assumed to be individually $I(1)$). This implies a null hypothesis about the $\beta$ vector in the CVAR; namely, that $\beta = [1, -1]$. The null hypothesis in this case is consistent with market efficiency. I therefore conducted a test of this restriction (allowing the restricted constant to take any value). Table 3.4 presents the test statistics for absolute prices, and Table 3.5 presents the test statistics for relative prices. In all cases the test statistic
Table 3.4: Test of $\beta = [1, -1]$, Absolute Prices

<table>
<thead>
<tr>
<th></th>
<th>Unadjusted</th>
<th>Adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>0.266</td>
<td>0.168</td>
</tr>
<tr>
<td>Consumer</td>
<td>0.097</td>
<td>0.058</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.288</td>
<td>0.082</td>
</tr>
<tr>
<td>Health</td>
<td>0.039</td>
<td>0.029</td>
</tr>
<tr>
<td>Hi-Tech</td>
<td>0.099</td>
<td>0.036</td>
</tr>
<tr>
<td>Other</td>
<td>0.221</td>
<td>0.098</td>
</tr>
</tbody>
</table>

Significance levels marked by asterisks as follows: * = 10% ; ** = 5% ; *** = 1%.

Table 3.5: Test of $\beta = [1, -1]$, Relative Prices

<table>
<thead>
<tr>
<th></th>
<th>Unadjusted</th>
<th>Adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cons/Manuf</td>
<td>0.698</td>
<td>0.470</td>
</tr>
<tr>
<td>Cons/Hlth</td>
<td>0.425</td>
<td>0.291</td>
</tr>
<tr>
<td>Cons/HiTec (1)</td>
<td>5.000**</td>
<td>3.257*</td>
</tr>
<tr>
<td>Cons/HiTec (2)</td>
<td>0.176</td>
<td>0.088</td>
</tr>
<tr>
<td>Cons/Other</td>
<td>1.428</td>
<td>1.030</td>
</tr>
<tr>
<td>Manuf/Hlth</td>
<td>0.135</td>
<td>0.085</td>
</tr>
<tr>
<td>Manuf/HiTec (1)</td>
<td>0.180</td>
<td>0.098</td>
</tr>
<tr>
<td>Manuf/HiTec (2)</td>
<td>3.833*</td>
<td>2.520</td>
</tr>
<tr>
<td>Manuf/Other</td>
<td>2.146</td>
<td>1.673</td>
</tr>
<tr>
<td>Hlth/HiTec</td>
<td>4.945**</td>
<td>3.196*</td>
</tr>
<tr>
<td>Hlth/Other</td>
<td>0.015</td>
<td>0.010</td>
</tr>
<tr>
<td>HiTec/Other</td>
<td>1.239</td>
<td>0.865</td>
</tr>
</tbody>
</table>

Cons = Consumer, Manuf = Manufacturing. Significance levels marked by asterisks as follows: * = 10% ; ** = 5% ; *** = 1%.

is distributed $\chi^2$ with one degree of freedom. The “adjusted” column for each table is the test statistic multiplied by the Bartlett correction factor (a finite-sample correction; note that it varies by specification and is only an approximation for models that use dummies and breaks). Because the unadjusted results are based on a large-sample, asymptotic-distribution assumption, the adjusted results should probably be viewed as somewhat more reliable.

The results summarized in Tables 3.4 and 3.5 are broadly consistent with the present value model. We cannot reject $\beta = [1, -1]$ for any of the absolute prices. We cannot reject the hypothesis for 9 of the 12 relative-price regressions; and in the other cases, we fail to reject at the 5% level after finite-sample correction. While we can reject the hypothesis at the 10% confidence level (and after finite-sample adjustment) for 2 of the 12 sectors, note that one of these rejections is for the Consumer/Hi-Tech relative price. That is one of the two cases where we had to split the sample; perhaps this results suggests that we need another
split, but that is hardly clear because the rejection is not at all robust. This can hardly be considered good evidence against the present value model; in fact, the overall results are in favor of the present value model for both absolute and relative prices. In conjunction with the results discussed in the preceding section, this is evidence in favor of the present value model, but with the caveat that the relationship is clearer in the data for relative prices because the effects of Knightian uncertainty are not as strong in relative price data.

### 3.6.2 Tests of Variable Exogeneity

Another way to examine the relation between prices and fundamentals in the CVAR is to test whether prices or dividends are “weakly exogenous”. A variable is weakly exogenous if it does not respond to deviations from equilibrium. Statistically, this is a test that there is a zero-row in $\alpha$. For example, if dividends are weakly exogenous, in a two variable system the $\Pi$ matrix could be written:

$$
\Pi = ([\alpha_p, 0])' \beta'
$$

where $\alpha_p$ is the (non-zero) adjustment parameter for prices. I tested for weak exogeneity jointly with the restriction that $\beta = [1, -1]$. The test statistic is distributed $\chi^2$ with 2 degrees of freedom; if the null hypothesis is rejected, the data suggests that the variable is not weakly exogenous. Table 3.6 summarizes the results of these tests for absolute prices, and Table 3.7 for relative prices.

For absolute prices, we rejected dividends being weakly exogenous for the market as a whole and for the health sector. We did not in any case have a robust rejection of prices being weakly exogenous. For
Table 3.7: Tests of Weak Exogeneity, Relative Prices

<table>
<thead>
<tr>
<th></th>
<th>Prices</th>
<th>Prices (Adjusted)</th>
<th>Dividends</th>
<th>Dividends (Adjusted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cons/Manuf</td>
<td>0.730</td>
<td>0.525</td>
<td>9.817***</td>
<td>7.067***</td>
</tr>
<tr>
<td>Cons/Hlth</td>
<td>0.553</td>
<td>0.403</td>
<td>11.939***</td>
<td>8.700**</td>
</tr>
<tr>
<td>Cons/HiTec (1)</td>
<td>5.553*</td>
<td>3.870</td>
<td>13.627***</td>
<td>9.497***</td>
</tr>
<tr>
<td>Cons/HiTec (2)</td>
<td>0.789</td>
<td>0.439</td>
<td>2.409</td>
<td>1.339</td>
</tr>
<tr>
<td>Cons/Other</td>
<td>3.831</td>
<td>2.917</td>
<td>13.130***</td>
<td>9.997***</td>
</tr>
<tr>
<td>Manuf/Hlth</td>
<td>0.425</td>
<td>0.287</td>
<td>9.302**</td>
<td>6.275**</td>
</tr>
<tr>
<td>Manuf/HiTec (1)</td>
<td>0.830</td>
<td>0.498</td>
<td>16.578***</td>
<td>9.943***</td>
</tr>
<tr>
<td>Manuf/HiTec (2)</td>
<td>14.970***</td>
<td>10.492***</td>
<td>4.925*</td>
<td>3.451</td>
</tr>
<tr>
<td>Manuf/Other</td>
<td>3.706</td>
<td>3.015</td>
<td>16.966***</td>
<td>13.804***</td>
</tr>
<tr>
<td>Hlth/HiTec</td>
<td>5.068*</td>
<td>3.519</td>
<td>7.308**</td>
<td>5.074*</td>
</tr>
<tr>
<td>Hlth/Other</td>
<td>1.539</td>
<td>1.084</td>
<td>8.066**</td>
<td>5.682*</td>
</tr>
<tr>
<td>HiTec/Other</td>
<td>1.555</td>
<td>1.162</td>
<td>10.123***</td>
<td>7.561**</td>
</tr>
</tbody>
</table>

Significance levels marked by asterisks as follows: * = 10% ; ** = 5% ; *** = 1%.

relative prices, we fail to reject the hypothesis that prices are weakly exogenous in every case but one. We also reject that dividends are weakly exogenous at the 5% level in 8 of 12 cases (and at 10% in two others). This might seem to be the opposite of what we would have expected to find: since prices should be related to fundamentals, shouldn’t we expect that prices respond to dividends, not the other way around? However, one must keep in mind what patterns in the data these results reflect. The finding reflects a pattern of prices moving prior to dividends; in the CVAR analysis this makes it appear that dividends are the “adjusting” variable. But this may be occurring because the market is receiving signals about future relative dividends that are not available in my data, and bidding prices up (or down) accordingly. In other words, the finding may reflect the fact that the market’s forecast is superior to the CVAR’s forecast.

In other words, one might think that these results suggest that price movements are not connected to dividends. However, the results also suggest that changes in dividends seem to “follow” price movements, because they co-move in a way that is consistent with the present value model. Therefore, we should be skeptical of the claim that the tests of weak exogeneity suggest that the present value model is wrong. In the previous subsection, I mentioned that the economist may get results that look “bad” for the present value model because the economist has the “wrong model” - one that ignores Knightian uncertainty. A related problem may be present here. It may well be the case that the market is better at forecasting changes in
future dividends than the simple CVAR employed here. In that case, and if the present value model is a good
description of how prices evolve, market prices would move ahead of when the CVAR would expect to see
that movement. This would create the illusion that prices are moving and dividends are responding to those
movements. In addition, the reader should recall that the estimated values of $\alpha$ are usually quite small. This
implies high persistence. But higher persistence means adjustment to equilibrium takes a long time. This
slow adjustment means it may be difficult for the test to detect the fact that the adjustments are taking place
(hence, we couldn’t reject that either prices or dividends are weakly exogenous in several cases).

One possible reason for price movements that don’t appear to be connected to dividends is a time-
varying discount rate. While it might appear that this could explain the results described herein, as I noted
previously, a shift in the discount rate $\rho$ that moves prices would also result in a shift in the equilibrium of
the cointegrating relation. But in the CVAR, shift dummies are able to account for these movements. Even
if the model specifications I have used here failed to fully account for such changes in the discount rate, this
apparent explanation is still unsatisfactory. It fails to explain why dividends would appear to adjust to price
movements in a way that is consistent with no shift in the constant term. The fact that relative dividends
appear to adjust to price movements can be plausibly interpreted as indirect evidence against a time-varying
discount rate. For a time-varying discount rate to explain these results, the model would also have to provide
a cogent explanation for why changes in $\rho$ seem to be connected to future movements in dividends that are,
coincidentally, exactly what we’d expect to see if the present value model is right and the discount rate is
constant (at least, constant during sub-periods between shift dummies). We might put this another way:
these results appear to suggest that relative dividends are adjusting to prices. If the “real” explanation is a
time-varying discount rate, what is the mechanism by which changes in preferences just happen to occur
in advance of changes in dividends, and why do those dividend changes just happen to be consistent with
what we’d expect to observe if the discount rate is constant? Absent such a mechanism, this isn’t really an
explanation at all.

The evidence from the weak exogeneity tests is in accord with the results from the GUM: the data is
consistent with the present value model for both absolute and relative prices once we allow for structural change. But the data is also more consistent with the idea that price movements may be driven by the market coping with Knightian uncertainty than with a time-varying discount rate.

The results for absolute prices are quite different: we cannot reject that either prices or dividends are weakly exogenous in 4 of 6 cases. This is indirect evidence that there is a persistence problem. Failing to reject that either variable is weakly exogenous is a sign that adjustment to equilibrium is very slow, or in other words that deviations from equilibrium persist for a very long time. Fortunately, there are other methods in the CVAR that allow us to measure persistence. I review these measures in the next section.

3.7 CVAR Results: Persistence

The CVAR provides several ways to measure the degree of persistence present in the data. Herein I provide results for two of these measures. First, I review the estimates of $\alpha$. As mentioned previously, estimates of $\alpha$ provide a measure of persistence. Smaller values indicate slower convergence, therefore higher persistence. Second, I review findings regarding large unexplained roots of the companion matrix (see above for details).

3.7.1 Persistence Measures: Adjustment Parameters

Tables 3.8 and 3.9 provide estimates of the (absolute) values of $\alpha$ (after imposing $r = 1$ and $\beta = [1, -1]$, but without imposing restrictions on $\alpha$ itself) for absolute and relative prices, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Price</th>
<th>Dividend</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>0.012</td>
<td>0.033</td>
<td>0.045</td>
</tr>
<tr>
<td>Consumer</td>
<td>0.046</td>
<td>0.065</td>
<td>0.111</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.040</td>
<td>0.013</td>
<td>0.053</td>
</tr>
<tr>
<td>Health</td>
<td>0.002</td>
<td>0.023</td>
<td>0.025</td>
</tr>
<tr>
<td>HiTec</td>
<td>0.029</td>
<td>0.013</td>
<td>0.042</td>
</tr>
<tr>
<td>Other</td>
<td>0.021</td>
<td>0.025</td>
<td>0.046</td>
</tr>
<tr>
<td>Average</td>
<td>0.028</td>
<td>0.028</td>
<td>0.055</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.017</td>
<td>0.022</td>
<td>0.033</td>
</tr>
</tbody>
</table>
Table 3.9: Estimates of $\alpha$, Relative Prices

<table>
<thead>
<tr>
<th></th>
<th>Price</th>
<th>Dividend</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cons/Manuf</td>
<td>0.001</td>
<td>0.048</td>
<td>0.049</td>
</tr>
<tr>
<td>Cons/Hlth</td>
<td>0.004</td>
<td>0.099</td>
<td>0.103</td>
</tr>
<tr>
<td>Cons/Hlth (1)</td>
<td>0.014</td>
<td>0.131</td>
<td>0.145</td>
</tr>
<tr>
<td>Cons/Hlth (2)</td>
<td>0.010</td>
<td>0.055</td>
<td>0.065</td>
</tr>
<tr>
<td>Cons/Other</td>
<td>0.021</td>
<td>0.122</td>
<td>0.143</td>
</tr>
<tr>
<td>Manuf/Hlth</td>
<td>0.005</td>
<td>0.053</td>
<td>0.058</td>
</tr>
<tr>
<td>Manuf/Hlth (1)</td>
<td>0.010</td>
<td>0.103</td>
<td>0.113</td>
</tr>
<tr>
<td>Manuf/Hlth (2)</td>
<td>0.042</td>
<td>0.035</td>
<td>0.077</td>
</tr>
<tr>
<td>Manuf/Other</td>
<td>0.011</td>
<td>0.074</td>
<td>0.085</td>
</tr>
<tr>
<td>Hlth/Hlth</td>
<td>0.003</td>
<td>0.028</td>
<td>0.031</td>
</tr>
<tr>
<td>Hlth/Other</td>
<td>0.018</td>
<td>0.080</td>
<td>0.098</td>
</tr>
<tr>
<td>HiTec/Other</td>
<td>0.006</td>
<td>0.061</td>
<td>0.067</td>
</tr>
<tr>
<td>Average</td>
<td>0.012</td>
<td>0.074</td>
<td>0.086</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.011</td>
<td>0.033</td>
<td>0.036</td>
</tr>
</tbody>
</table>

The sum of the $\alpha$ coefficients provides a good approximation for the proportion of deviations from equilibrium that are expected to disappear from one period to the next. The values of $\alpha$ are small, especially for prices. A coefficient of 0.05 implies a “half-life” of 13 - 14 periods - more than 3 years\textsuperscript{14}. Since a number of the sums are smaller than this, this suggests very slow convergence back to equilibrium.

We can also get a sense of whether the degree of persistence is different for relative than absolute prices. With the means and standard deviations of the $\alpha$ coefficients for prices, dividends, and their sums (see above), one can conduct a standard T-test of whether two sample means are equal. The results of these tests show no detectable difference in means for the price coefficients (Test statistic = 1.85, 6 degrees of freedom). Nor is there a statistically significant difference for the sums (Test statistic = 1.72, 9 degrees of freedom). However, the $\alpha$ coefficients for relative dividends have a larger mean than those for absolute dividends (Difference = 0.046, test statistic = 3.39, 12 degrees of freedom).

That difference (0.046) is not small: it means the rate of convergence back to equilibrium is more than twice that for relative dividends than for absolute dividends. The reason we cannot detect a difference for the sums, even though the difference is of similar magnitude, may be due to small sample size (of estimates - only 6 absolute-price estimates and 12 relative-price estimates).

\textsuperscript{14}In this context, the half-life refers to the amount of time it takes for half of a given deviation to dissipate.
Table 3.10: Large Roots of Companion Matrix, Absolute Prices

<table>
<thead>
<tr>
<th>Roots</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>0.989, 0.885</td>
</tr>
<tr>
<td>Consumer</td>
<td>0.930</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.955, 0.933</td>
</tr>
<tr>
<td>Health</td>
<td>0.994</td>
</tr>
<tr>
<td>HiTec</td>
<td>0.961</td>
</tr>
<tr>
<td>Other</td>
<td>0.990, 0.857</td>
</tr>
</tbody>
</table>

The present value model states that the price of a stock (or portfolio) equals the present discounted value of future dividend streams. But we observed that deviations from this “fundamental value” - represented herein by the equilibrium of the cointegrating relation - last a long time. The generally small values of $\alpha$ imply slow adjustment back towards equilibrium. In other words, prices don’t fluctuate tightly around the fundamental value. They undergo long swings away from the fundamental value and converge back to it quite slowly. High persistence does indeed seem to be a problem for absolute prices, but the problem is noticeably reduced for relative prices. This suggests that the relation between prices and fundamentals may be closer for relative prices than absolute prices.

This approach of looking at the $\alpha$ estimates does not, however, help us address the issue of uncontrolled unit roots. For that, I turn to an examination of the companion matrices.

### 3.7.2 Persistence Measures: Unit Roots

Tables 3.10 and 3.11 report the magnitudes of unexplained roots of the companion matrix larger than 0.85. (Note that this cutoff point is arguably somewhat low: the “half-life” corresponding to a root of 0.85 is between 4 and 5 periods, or just over a year in quarterly data).

These results are broadly consistent with what we observed from the $\alpha$ coefficients. There is clear evidence of unexplained persistence in both absolute and relative prices. Every model contains at least one unexplained root close to 1. Moreover, in some absolute-price models there is a second large root. These are classic signals that the data is $I(2)$ instead of $I(1)$. Unfortunately, because there is at least one unexplained root present for both absolute and relative prices, it is difficult to draw specific conclusions about the nature
<table>
<thead>
<tr>
<th>Roots</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cons/Manuf</td>
<td>0.980</td>
</tr>
<tr>
<td>Cons/Hlth</td>
<td>0.968</td>
</tr>
<tr>
<td>Cons/HiTec (1)</td>
<td>0.982</td>
</tr>
<tr>
<td>Cons/HiTec (2)</td>
<td>0.967</td>
</tr>
<tr>
<td>Cons/Other</td>
<td>0.980</td>
</tr>
<tr>
<td>Manuf/Hlth</td>
<td>0.988</td>
</tr>
<tr>
<td>Manuf/HiTec (1)</td>
<td>0.982</td>
</tr>
<tr>
<td>Manuf/HiTec (2)</td>
<td>0.942</td>
</tr>
<tr>
<td>Manuf/Other</td>
<td>1.000</td>
</tr>
<tr>
<td>Hlth/HiTec</td>
<td>0.989</td>
</tr>
<tr>
<td>Hlth/Other</td>
<td>0.952</td>
</tr>
<tr>
<td>HiTec/Other</td>
<td>0.974</td>
</tr>
</tbody>
</table>

of that root. We can, however, conclude that there seems to be a root that is not held in common across sectors (If the only unaccounted-for root in the absolute price models was common to all sectors, we would be very unlikely to detect it in the relative prices). Three explanations of this persistence are possible.

Two of these three explanations are different versions of the same problem: we are using a poor model of price movements. In the first of these cases, the model offers a poor representation of what “rational forecasting” looks like. If Knightian uncertainty plays a major role, price movements could reflect the market gradually revising its forecasts as it copes with such uncertainty. Slow but continuous revisions of forecasting strategies could easily produce the persistence we observe and would explain why the model predicts far too little persistence in price movements. Second, the model used herein could be wrong because it ignored a time-varying discount rate. If the discount rate varies, and if its movements through time are persistent, price movements would reflect that persistence. For instance, if the discount rate for stocks is related to interest rates, failing to account for interest rates could produce the unexplained persistence we observed. However, our observation that relative dividends don’t appear to exhibit as much persistence, and therefore appear to adjust to equilibrium, suggests a degree of skepticism towards this explanation (in addition, recall that results from the preceding sections are not as consistent with a time-varying discount rate as with Knightian uncertainty. The third explanation is, in short, market inefficiency. In this case, we have a good model of rational forecasting - but the market does not use this forecasting strategy. This
explanation is consistent with the basic story told by behavioral finance models. The persistence we observe is a consequence of “non-fundamental” forecasting strategies (e.g. chartism).

There is more to be said about the unexplained roots in the companion matrices. This evidence suggests that a model that says prices and dividends should be $I(1)$ is missing something - a large root in the companion matrix suggests that the data may be $I(2)$. There is little that we can confidently say about this root - it’s called an “unexplained” root for a reason - except that it isn’t ‘canceled out’ in relative prices. This means that one potential explanation for persistence in absolute prices - that an unexplained unit root comes from a “macro” or market-wide stochastic trend - does not appear to be consistent with the data. A root held in common across absolute prices would be unlikely to appear for relative prices. In other words, such a root would mean absolute prices are $I(2)$, but cointegrate with each other (from $I(2)$ to $I(1)$). But then the $I(1)$ analysis for relative prices would not be likely to exhibit any evidence of an extra root.

These findings say much more about the need for future research than about the validity of the present value model. If the data is $I(2)$, the proper mode of analysis is an $I(2)$ CVAR. The $I(1)$ analysis would be misspecified, almost perforce. Nevertheless, these findings are not wholly invalid or spurious - merely incomplete. They are informative for a researcher working on an $I(2)$ analysis - this analysis told us something about “where to look” for that unexplained root. An $I(2)$ analysis might also be useful in understanding why the short-term parameters (the $\Gamma$ matrices) exhibit some evidence of structural instability.

### 3.8 Conclusion

In this chapter, I argued that a cointegrated VAR may help us understand some of the outstanding issues in research on the present value model. In particular, the CVAR approach is useful in examining the roles played by structural change and persistence. It is also useful because it provides a more general way to examine the cointegration properties of the data than the popular univariate methods, such as the Engle-Granger test. I also advocated for examining relative stock prices, as such an approach may have several
advantages over an approach that examines strictly absolute prices. The results of this analysis are broadly consistent with the present value model’s basic message - that prices are connected to fundamentals - but it suggests that the standard version of the model is likely to be missing something. The evidence suggests that Knightian uncertainty plays an important role, because structural change - unanticipated events - play such a large role in movements of prices and dividends. Once we account for structural change, prices and dividends cointegrate in a way that is consistent with the present value model; however, dividends appear to adjust to this relation more than prices. This as well as the important role of structural change are consistent with the idea that economists have the “wrong model” - forecasting strategies are connected to fundamentals, but our models are not good representations of those strategies. The degree of persistence observed in the data is somewhat more troubling for the present value model, but with some adaptations to the model we may be able to reconcile this persistence with the basic intuition that prices are driven by fundamentals.

The findings suggest a logical next step in the research: $I(2)$ analysis. Such an analysis would aid us in understanding more about the sources of the persistence we observe in the data. A better understanding of the sources of this persistence would help move the debate within the literature between behavioral finance and the efficient markets hypothesis forward. It may even help resolve the dispute, or reconcile the role of psychological factors with rational forecasting by market participants.
References


pp. 15779.


He, X.Z., Li, K., Wei, J. and Zheng, M. *Market stability switches in a continuous-time financial market with heterogeneous beliefs.* Economic Modelling, 26(2009), 1432 - 1442.


Matthews, William J., & Stewart, Neil. *Psychophysics and the judgment of price: Judging complex objects on a non-physical dimension elicits sequential effects like those in perceptual tasks.* Judgment & Decision
Making 4(1) (2009), 64 - 81.


Shirvani, Delcoure, and Wilbratte. Periodic Integration and Cointegration of U.S. Stock Prices, Dividends,


Appendix I: Volatility Test Graphs

Durables: Volatility Test, Absolute Prices

Manufacturing: Volatility Test, Absolute Prices
Appendix 2: Fundamental Value ($\delta$) Graphs
Actual vs Fundamental Value, Relative: Durables/Energy

Actual vs Fundamental Value, Relative: Durables/Hi-Tech

Actual vs Fundamental Value, Relative: Durables/Telecomm
Appendix 3: Further Description of the Sector Portfolios, Chapter 3

A full list of the SIC codes for each industry may be found by downloading the industry definitions available on Ken French’s website. Some examples, however, may illuminate the kinds of specific industries that would be found in each portfolio. The “Consumer” sector, for example, includes 0134 (potato farming), 2047 (dog and cat food manufacturing), and 7623 (Appliance repair and maintenance), among many others. “Manufacturing” includes 2522 (Office furniture manufacturing), 3711 (Automobile manufacturing), and 4953 (Hazardous Waste treatment and disposal). “Hi-Tech” includes 3661 (Telephone apparatus manufacturing), 7373 (computer systems design), and 8732 (Research and Development in biotechnology). “Health” includes 2834 (pharmaceutical preparation manufacturing), 3841 (surgical and medical instrument manufacturing), and 8011 (HMO medical centers). “Other” includes many unrelated industries not classified elsewhere; among other things mines, hotels, entertainment, and finance are found in this sector.
Appendix 4: Auto-correlations, Chapter 3

Figure 1: Autocorrelations: Market

Residual Cross- & Autocorrelations

Figure 2: Autocorrelations: Consumer

Cross- & Autocorrelations of Absolute Residuals
Figure 3: Autocorrelations: Manufacturing

Residual Cross- & Autocorrelations

Lags 1 to 92

Figure 4: Autocorrelations: Hi-Tech

Residual Cross- & Autocorrelations

Lags 1 to 92
Figure 5: Autocorrelations: Health

Residual Cross- & Autocorrelations

Lags 1 to 92

Figure 6: Autocorrelations: Other

Residual Cross- & Autocorrelations

Lags 1 to 92
Figure 7: Autocorrelations: Consumer/Manufacturing

Residual Cross- & Autocorrelations

Lags 1 to 92

Figure 8: Autocorrelations: Consumer/Hi-Tech, Sample 1

Residual Cross- & Autocorrelations

Lags 1 to 48
Figure 9: Autocorrelations: Consumer/Hi-Tech, Sample 2

Residual Cross- & Autocorrelations

Lags 1 to 43

Figure 10: Autocorrelations: Consumer/Health

Residual Cross- & Autocorrelations

Lags 1 to 92
Figure 11: Autocorrelations: Consumer/Other

Residual Cross- & Autocorrelations

Lags 1 to 92

Figure 12: Autocorrelations: Manufacturing/Hi-Tech, Subsample 1

Residual Cross- & Autocorrelations

Lags 1 to 53
Figure 13: Autocorrelations: Manufacturing/Hi-Tech, Subsample 2

Residual Cross- & Autocorrelations

Figure 14: Autocorrelations: Manufacturing/Health

Residual Cross- & Autocorrelations
Figure 15: Autocorrelations: Manufacturing/Other

Residual Cross- & Autocorrelations

Figure 16: Autocorrelations: Hi-Tech/Health

Residual Cross- & Autocorrelations
Appendix 5: Test of Beta Constancy

Figure 19: Test of Constancy: Market

Test of Beta Constancy

Figure 20: Test of Constancy: Consumer

Test of Beta Constancy
Figure 21: Test of Constancy: Hi-Tech

Test of Beta Constancy

Figure 22: Test of Constancy: Health

Test of Beta Constancy
Figure 23: Test of Constancy: Other

Test of Beta Constancy

Figure 24: Test of Constancy: Consumer/Manufacturing

Test of Beta Constancy
Figure 25: Test of Constancy: Consumer/Hi-Tech, Sample 1

Figure 26: Test of Constancy: Consumer/Hi-Tech, Sample 2
Figure 27: Test of Constancy: Consumer/Health

Figure 28: Test of Constancy: Consumer/Other
Figure 29: Test of Constancy: Manufacturing/Hi-Tech, Subsample 1

Test of Beta Constancy

Figure 30: Test of Constancy: Manufacturing/Hi-Tech, Subsample 2

Test of Beta Constancy
Figure 31: Test of Constancy: Manufacturing/Health

Test of Beta Constancy

Figure 32: Test of Constancy: Manufacturing/Other

Test of Beta Constancy
Figure 33: Test of Constancy: Hi-Tech/Health

Test of Beta Constancy

Figure 34: Test of Constancy: Hi-Tech/Other

Test of Beta Constancy
Appendix 6: Roots of Companion Matrix
Figure 37: Companion Matrix Roots: Consumer

Figure 38: Companion Matrix Roots: Manufacturing
Figure 39: Companion Matrix Roots: Hi-Tech

Figure 40: Companion Matrix Roots: Health
Figure 41: Companion Matrix Roots: Other

Figure 42: Companion Matrix Roots: Consumer/Manufacturing
Figure 43: Companion Matrix Roots: Consumer/Hi-Tech, Sample 1

Figure 44: Companion Matrix Roots: Consumer/Hi-Tech, Sample 2
Figure 45: Companion Matrix Roots: Consumer/Health

Figure 46: Companion Matrix Roots: Consumer/Other
Figure 47: Companion Matrix Roots: Manufacturing/Hi-Tech, Subsample 1

Figure 48: Companion Matrix Roots: Manufacturing/Hi-Tech, Subsample 2
Figure 49: Companion Matrix Roots: Manufacturing/Health

Figure 50: Companion Matrix Roots: Manufacturing/Other
Figure 51: Companion Matrix Roots: Hi-Tech/Health

Figure 52: Companion Matrix Roots: Hi-Tech/Other
Appendix 7: Cointegrating Relations

Figure 53: Companion Matrix Roots: Health/Other

Figure 54: Cointegrating Relation: Market
Figure 55: Cointegrating Relation: Consumer

Figure 56: Cointegrating Relation: Manufacturing
Figure 57: Cointegrating Relation: Hi-Tech

![Figure 57](image)

Figure 58: Cointegrating Relation: Health

![Figure 58](image)
Figure 59: Cointegrating Relation: Other

Figure 60: Cointegrating Relation: Consumer/Manufacturing
Figure 61: Cointegrating Relation: Consumer/Hi-Tech, Sample 1

Figure 62: Cointegrating Relation: Consumer/Hi-Tech, Sample 2
Figure 63: Cointegrating Relation: Consumer/Health

![Graph showing Beta1*z(t) and Beta1*r(t) for Consumer/Health over the years 1930 to 2015.]

Figure 64: Cointegrating Relation: Consumer/Other

![Graph showing Beta1*z(t) and Beta1*r(t) for Consumer/Other over the years 1930 to 2015.]

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Figure 65: Cointegrating Relation: Manufacturing/Hi-Tech, Subsample 1

Figure 66: Cointegrating Relation: Manufacturing/Hi-Tech, Subsample 2
Figure 67: Cointegrating Relation: Manufacturing/Health

Figure 68: Cointegrating Relation: Manufacturing/Other
Figure 69: Cointegrating Relation: Hi-Tech/Health

Figure 70: Cointegrating Relation: Hi-Tech/Other
Figure 71: Cointegrating Relation: Health/Other

![Graph of Health/Other cointegrating relation](image)

**Beta1*Z*(t)**

-10 to 10


**Beta1*R*(t)**

-10 to 10