Experimental Investigations Of Thermal Pulsatile Boundary Layer Flow

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EXPERIMENTAL INVESTIGATIONS OF THERMAL PULSATILE BOUNDARY LAYER FLOW

BY

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BS, Mechanical Engineering, University of New Hampshire, New Hampshire, 2013

DISSERTATION

Submitted to the University of New Hampshire in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy in
Mechanical Engineering

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Original approval signatures are on file with the University of New Hampshire Graduate School.
To my mother, Kate Conway, for a lifetime of unconditional love, support, and encouragement.
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TABLE OF CONTENTS

ACKNOWLEDGEMENTS ................................................................. iv
LIST OF TABLES ................................................................. ix
LIST OF FIGURES ................................................................. x
ABSTRACT ................................................................. xix

CHAPTER

1. INTRODUCTION ................................................................. 1
   1.1 Contextual Overview ....................................................... 2
       1.1.1 Equilibrium Boundary Layer Flow ............................... 3
       1.1.2 Non-Equilibrium Wall-bounded Flows ............................ 5
       1.1.3 Thermal Transport in Non-Equilibrium Boundary Layer Flow ....... 6
       1.1.4 Measurement Needs of Thermal Transport in Non-Equilibrium Boundary Layer Flow ........................................ 8
   1.2 Organization of the Dissertation .......................................... 9

2. THEORETICAL BACKGROUND ................................................. 11
   2.1 Governing Equations of Fluid Dynamics .................................. 11
   2.2 Reynolds Averaged Navier Stokes Equations ............................. 12
   2.3 Triple Decomposed Long Time Averaged Navier Stokes Equations ......... 13
   2.4 Phase Averaged Navier Stokes Equation ................................... 14
   2.5 Boundary Layer Equations .................................................. 15
   2.6 RANS Boundary Layer Equations .......................................... 16
   2.7 Pulsatile Boundary Layer Flow ............................................. 17
   2.8 Triple Decomposed Long Time Averaged Pulsatile BL Equations ............ 18
   2.9 Phase Averaged Pulsatile BL Equations .................................... 18
   2.10 Normalizations ............................................................. 19
       2.10.1 Inner-normalization ................................................. 19
4.3 Pulsatile Flow Generator .................................................. 57
   4.3.1 Rotor-Stator Assembly: V0 ........................................... 57
   4.3.2 Pulsatile Flow Generator: V1 ....................................... 58
   4.3.3 Pulsatile Flow Generator: V2 ....................................... 59

4.4 Validation Tests .................................................................. 61
   4.4.1 ZPG turbulent boundary layer flow over an isothermal wall plate ...... 61
   4.4.2 ZPG boundary layer encountering a sharp step in wall temperature .... 65
   4.4.3 ZPG turbulent boundary layer flow around a wall mounted hemisphere body .................................................. 67

5. PREVIOUS WORK ON PULSATILE FLOW .................................. 72
   5.1 Pulsatile Forcing Effects on the Momentum Field .............................. 73
      5.1.1 Long-time averaged mean turbulence ................................... 73
   5.2 Phase-averaged turbulence ...................................................... 75
      5.2.1 Turbulent Structure ........................................................ 78
   5.3 Pulsatile Forcing Effects on the Heat Transfer .................................. 80
   5.4 Experimental Pulsatile Thermal Flow ......................................... 82

6. TIME AVERAGE PULSATILE BOUNDARY LAYER FLOW .................. 85
   6.1 Zero Pressure Gradient Flow .................................................. 88
   6.2 Pulsatile Boundary Layer Flow .............................................. 93
      6.2.1 Integral Parameters ....................................................... 93
      6.2.2 Wall Fluxes ............................................................... 97
      6.2.3 Outer Normalization .................................................... 98
      6.2.4 Inner Normalization .................................................... 102
      6.2.5 Buoyant Driven Transport ........................................... 106
      6.2.6 $w^+$ Domains ......................................................... 108
   6.3 Summary ........................................................................... 110

7. PHASE AVERAGE PULSATILE BOUNDARY LAYER FLOW: FORCED CONVECTION ........................................ 111
   7.1 Integral Parameters ............................................................ 111
   7.2 Outer Normalization ........................................................... 116
   7.3 Inner Normalization ............................................................. 119
   7.4 Near Wall Dynamics ............................................................ 123
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>The length of the convective plates. Plate 1 is at the upstream end and plate 12 is at the downstream end of the wind tunnel.</td>
</tr>
<tr>
<td>4.2</td>
<td>Parameters for measured temperature validation profiles.</td>
</tr>
<tr>
<td>5.1</td>
<td>Table identifying previous studies performed to investigate both thermal and non-equilibrium wall bounded flow dynamics with a working fluid of air along with data of current investigation.</td>
</tr>
<tr>
<td>6.1</td>
<td>Parameters for Thermal Field Experiments in the NEAT Pulsatile Campaign</td>
</tr>
<tr>
<td>6.2</td>
<td>Parameters for ZPG Thermal Field Experiments in the NEAT Pulsatile Campaign</td>
</tr>
<tr>
<td>6.3</td>
<td>Parameters for Momentum Field Experiments in the NEAT Pulsatile Campaign</td>
</tr>
<tr>
<td>6.4</td>
<td>Parameters for ZPG Momentum Field Experiments in the NEAT Pulsatile Campaign</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Mean temperature profile in wall units. Solid line represent Eq. 1.3, open triangles, squares and circles represent FPG, ZPG and APG flow, respectively. Figure adopted from Bradshaw and Huang [1995].</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Reciprocating channel flow (RCF) inner normalized data plotted with the an inner normalized near wall approximation with data from Pond [2015].</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>Important parameters used in the computational model for determining the wall heat flux off of a near wall gradient in non-equilibrium flow, where ( y_{low} ) is the bottom point in the experimental data and ( y_{fit} ) is the top point of the near wall master fit profile.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3</td>
<td>True wall heat flux values along with computed wall heat flux values for each of the profiles in the oscillatory DNS data where points beneath ( y^+ = 1 ) are removed.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>( Pr_T ) profiles as a function of the distance from the wall in a steady channel flow, with temperature as a passive scalar. Figure courtesy of Samir Sid.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Details of the type E thermocouple probe.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2</td>
<td>Wiring overview of DAQ system for thermal measurements.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3</td>
<td>Mock setup of IR camera positioned above working section of tunnel, imaging down onto thermal wall plate for validation purposes.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4</td>
<td>Schematic of Pitot-Static tube with pressure tapped holes labeled.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>Setup of 2 camera high speed PIV system.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.6</td>
<td>Raw counts acquired during a single snap shot in a PIV experiment.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.7</td>
<td>Vector fields produced from each camera, left figure displays the vector field from Camera 2 and the right images displays the high resolved vector field from Camera 1.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.8</td>
<td>Details the operation of the PIV system within the NEAT facility during the experimental campaign to acquire pulsatile velocity fields.</td>
</tr>
</tbody>
</table>
3.9 Details of the 4 time series recorded during thermal profiling experiment; top panel: paddle encoder angular output, 2nd panel: $u_\infty$ as recorded by a pitot-static tube, 3rd panel: temperature at specific wall normal location from profiling thermocouple, bottom panel: pulse counter (z-index) from encoder.

3.10 Details of the method of stitching the data from the separate FOV's together, $Cam_1 =$ near wall camera, $Cam_2 =$ far field camera. Where overlap start is the start of the stitching between datasets and overlap end is the end of the stitching between datasets.

3.11 Displays a representative $|FFT|$ from freestream values of the velocity field measurements.

4.1 Schematic of the experimental facility. Air enters the facility from right-to-left. The primary components include: (1) freestream heaters, (2) seeding manifold, (3) turbulence management section, (4) contraction, (5) thermal wall-plate located on the bottom wall of the test-section, (6) rotor-stator assembly, (7) diffuser, (8) centrifugal fan.

4.2 (a) Probability density function (PDF) of freestream temperature measured in the wind tunnel over a two hour period. The vertical green line represents the setpoint temperature and the vertical red line represents the measured mean temperature with a confidence interval of 99%. (b) Schematic of the seeding manifold used for particle image velocimetry (PIV).

4.3 Multi-plane view of the working section of the tunnel with physical dimensions.

4.4 Schematic of removable windows which allow for optical measurements and installation of temperature/velocity probes. Design was developed by Dr. Michael Allard.

4.5 Solid model of prototype thermal wall plate with two convective plates.

4.6 (a) Solid model schematic of the thermal wall-plate. (b) Detailed schematic of a section of the thermal wall-plate: right tilted 45° lines denotes the heated wall plate, left tilted 45° lines shows the surrounding layer of calcium silicate insulation and the hash marked section represents the co-polymer acetal frame which positions all components. Three evenly spaced embedded thermocouples are located along line AA.

4.7 (a) Image of roughing pass for the fabrication of the super-ellipse leading edge nose of the wall plate (b) final image of leading edge frame component for the wall plate.

4.8 A diagram detailing the wall plate heater controller circuit. (1) designates the embedded thermocouples, which are fed to an amplifier (2), whose signal is then fed to a LabVIEW program (3), which determines if the heaters should be in an off or on position and sends a final signal to an SCR circuit (4), which communicates to the thermal wallplate heaters.

4.9 Schematic of rotor-stator design, left shows the rotor, shown in the middle is the stator, and the right plot depicts a freestream velocity time series taken with a pitot-static tube for a quarter revolution of the rotor-stator mechanism.
4.10 Schematic of pulsatile wave generator with a paddle design. The left image displays the y-z plane with flow going into the paper, the right figure shows a cutaway image of the x-y plane on the tunnel centerline (B-B). The paddle height $H_p$ can be adjusted to change the fluctuating area and therefore the pulsatile amplitude.

4.11 Schematic of updated pulsatile wave generator with a paddle design. The left image displays the y-z plane with flow going into the paper, the right figure shows a cutaway image of the x-y plane on the tunnel centerline (A-A). The paddle height $H_p$ can be adjusted to change the fluctuating area and therefore the pulsatile amplitude.

4.12 Freestream pulsatile velocity signals at a variety of freestream speeds and freestream forcing frequencies plotted vs once cycle ($\gamma = 0 - 360^\circ$).

4.13 Fabricated paddle system displaying the prototype on the right and the finalized design utilized in the PBL campaign on the left.

4.14 (a) Representative ensemble-averaged IR image of plate #5 for $T_{set} = 40^\circ$C. (b) Spanwise profile of streamwise averaged temperature from figure (a).

4.15 Wall-normal profiles of mean streamwise velocity at $Re_\theta = 1527$ with thermal wall plate on ($\times$) and off ($\circ$) plotted in (a) outer-coordinates and (b) inner-coordinates. The solid lines denote the data from Wu and Moin [2010] at a similar $Re$.

4.16 Wall-normal profiles of mean temperature plotted in (a) outer-coordinates and (b) inner-coordinates. Shading denotes a $\pm 5\%$ variance in computed $u_\tau$ value. The dotted line denote the data from Arya et al. Araya and Castillo [2012] at $Re_\theta = 2290$.

4.17 (a) Representative ensemble-averaged IR image of temperature step. The top-plate is unheated where $T=25^\circ$C and the bottom plate is set to $T=40^\circ$C. The flow is from top-to-bottom. (b) The streamwise profile of spanwise averaged temperature.

4.18 Wall-normal profiles of mean temperature after thermal step. Plates heated corresponding to profiles is as follows; #9 $\times$, #8-9 $\heartsuit$, #7-9 $\blacklozenge$, #6-9 $\star$, #5-9 $\blacksquare$, #4-9 $+$, #3-9 $\blacklozenge$, #2-9 $\bullet$, #1-9 $\blacklozenge$. Plotted in (a) inner-coordinates using $St$ (Eq.2.45), and (b) modified inner-coordinates using $St_T$ (Eq.4.3), subsequent profiles are offset by $\Theta = 6$ for visual clarity. The dotted line denotes the law of the wall turbulent profile.

4.19 Spatial temperature distributions located downstream of a hemisphere for four values of $Re_D$. The streamwise and spanwise positions have been normalized by the hemisphere diameter $D = 3\text{cm}$.

4.20 Two plane view of cartoon depiction for resulting flow field from hemisphere perturbation. The XZ plane provides a birds eye view of the developing vortex wake, and the YZ plane provides a view of vortex wake downstream of the hemisphere and the resulting wall temperature.
4.21 Spanwise temperature profiles taken from the center of the IR images at a downstream position of
1 controller, X; 2 controllers, ▶; 3 controllers. ♦ .................................................. 71

5.1 Wall-normal profiles of the mean streamwise velocity normalized by inner scales. Symbols
 correspond to (ω+ = 0.007), (ω+ = 0.014), (ω+ = 0.02). The green triangle
corresponds to ZPG boundary layer profile at approximately same Reynolds number. This
figure corresponds to figure 7.10 of Ebadi et al. [2015] ............................................ 74

5.2 Impact of w+ regimes on near wall velocity field through wall shear stress. Top plot denotes the
modification of the sinusoidal wall shear stress relative to the mean wall shear stress and the
bottom plot denotes the near wall phase shift relative to the freestream. Plot used with
permission from Dr. Alireza Ebadi Ebadi [2016] with data from Brereton and Mankbadi
[1995]. ................................................................. 77

5.3 Profiles of (u) as a function of phase angle. The left figure is for a marginal low frequency flow
(ω+ = 0.001) and the right figure is for a marginal intermediate frequency flow
(ω+ = 0.009). The top profiles (left axis) are for the decelerating period and the bottom
profiles (left axis) are for the acceleration period. Symbols in the decelerating period:
(○ = 0°; △ = 45°; □ = 90°; × = 135°). Symbols in the accelerating period: (○ = 180°;
△ = 225°; □ = 270°; × = 315°). This figure corresponds to figure 10 of Tu and Ramaprian
[1983]. ................................................................. 78

5.4 Profiles of (u) as a function of phase with the lowest profile starting from the first accelerating
phase and each offset profile increasing by T/8: (left) l+a = 7 – high frequency regime,
(middle) l+a = 14 – intermediate frequency regime, (right) l+a = 35 – low frequency regime.
The x’s denote steady channel flow data. The vertical dashed line denotes the position 2l+a.
This figure corresponds to figure 13 of Scotti and Poimelli [2001]. .......................... 79

5.5 Profiles of (left) (u) and (right) (T) as a function of phase with 0tτ/8 denoting the first
accelerating phase. (top) low frequency flow region, (middle) intermediate frequency flow
region, (bottom) high frequency flow regime. These figures are taken from Wang and Lu
[2004]. ................................................................. 83

5.6 Parameter overview for existing literature data on pipe and boundary layer pulsatile thermal flow
along with additional data presented in this text. .................................................. 84

6.1 Time average ZPG outer normalized mean profiles for the thermal and momentum field, markers
denoted in Tables 6.2 and 6.4. Dashed lines denote the respective Blasius and Polhausen
solution for each field, and dash-dot lines denote turbulent profile from DNS data of Araya
and Castillo [2012]. ................................................................. 88

6.2 Time averaged ZPG outer normalized rms fluctuating component of the momentum and thermal
field. ................................................................. 89
6.3 Time averaged ZPG inner normalized mean profiles of the momentum and thermal field plotted with DNS data from Araya et al., $Re_\theta = 2290$ [Araya and Castillo, 2012]. Experimental profiles are shaded with ±5% deviation. .................................................. 91

6.4 Time averaged boundary layer thickness verse $Re$ for PBL and ZPG tests for both momentum ($\delta$) and thermal ($\delta^*_T$) fields, where dash lines denote textbook approximations. ......................... 93

6.5 Time averaged thermal boundary layer thickness verse time average boundary layer thickness for PBL and ZPG tests, with a dashed line that denotes an empirical correlation. ......................... 94

6.6 Time averaged displacement thickness verse $Re$ for PBL and ZPG tests for both momentum ($\delta^*$) and thermal ($\delta^*_T$) fields. ............................................................... 95

6.7 Time averaged shape factor verse $Re$ for PBL and ZPG tests for both momentum ($H$) and thermal ($H^*_T$) fields. ............................................................... 96

6.8 Time averaged wall heat flux ($\overline{q}_w$) and $\overline{S}_t$ for PBL and ZPG tests. ............................................................... 97

6.9 Time averaged friction velocity ($\overline{\tau}_w$) and $\overline{C}_f$ for PBL and ZPG tests. ............................................................... 98

6.10 Time averaged PBL outer normalized mean profiles of the momentum field, where each figure contains a frequency sweep at a single freestream velocity marker notation can be found in Table. 6.3 ................................................. 99

6.11 Time averaged PBL outer normalized mean profiles of the thermal field, where each figure contains a frequency sweep at a single freestream velocity marker notation can be found in Table. 6.1 ................................................. 100

6.12 Time averaged PBL outer normalized mean profiles evaluated against a $1/7^{th}$ power law fit for both fields, where each figure contains a frequency sweep at a single freestream velocity marker notation can be found in Table. 6.1 ................................................. 101

6.13 Time averaged PBL inner normalized mean profiles of the momentum field, where each figure contains a frequency sweep at a single freestream velocity marker notation can be found in Table. 6.3 ................................................. 103

6.14 Time averaged PBL inner normalized mean profiles of the thermal field, where each figure contains a frequency sweep at a single freestream velocity marker notation can be found in Table. 6.1 ................................................. 104

6.15 Turbulent $Pr$ computed as a function of $y^+$ for the three cases under investigation. .......................... 105

6.16 Parameter space of the time average data sets in Richardson number vs Reynolds number. Line denoting critical Richardson number between forced and mixed convection is shown as a dashed horizontal line. ................................................. 107

xiv
6.17 Data sets plotted as a function of $Re$ vs $w^+$ with outlines denoting the relative pre-established $w^+$ domain each data set has been placed into.

6.18 Time averaged PBL inner normalized mean profiles of the thermal field, where each figure contains data sets representative of the varied $w^+$ domains covered through the study, marker notation can be found in Table 6.1

7.1 Phase averaged outer boundary layer conditions ($\langle u_\infty \rangle$, $\langle \theta_\infty \rangle$) verse phase ($\gamma$).

7.2 Oscillatory component of phase averaged boundary layer thickness ($\widetilde{\delta}$, $\widetilde{\delta_T}$) verse phase ($\gamma$).

7.3 Angular value of maximum correlation for phase lag between $\widetilde{\delta}$ and $\widetilde{\delta_T}$, where a positive lag indicates the momentum field leading and a negative lag indicates the thermal field leading.

7.4 Oscillatory component of phase averaged boundary layer displacement thickness ($\widetilde{\delta^*}$, $\widetilde{\delta^*_T}$) verse phase ($\gamma$).

7.5 Oscillatory component of phase averaged boundary layer shape factor ($\widetilde{H}$, $\widetilde{H_T}$) verse phase ($\gamma$).

7.6 Angular value of maximum correlation for phase lag between $\widetilde{H}$ and $\widetilde{H_T}$, where a positive lag indicates the momentum field is leading and a negative lag indicates the thermal field is leading.

7.7 Outer normalized phase–average velocity profiles plotted $\approx$ every 45° with the corresponding freestream velocities shown in the bottom panel and the time average profile overlaid on each individual plot, shown in colored markers. Bottom panel displays the phase of each marker.

7.8 Outer normalized phase average temperature profiles plotted $\approx$ every 45° with the corresponding freestream velocities shown in the bottom panel and the time average profile overlaid on each individual plot, shown in colored markers. Bottom panel displays the phase of each marker.

7.9 Inner normalized phase average velocity profiles plotted $\approx$ every 45° with the corresponding freestream velocities shown in the bottom panel, shown in colored markers. Bottom panel displays the phase of each marker.

7.10 Inner normalized phase average temperature profiles plotted $\approx$ every 45° with the corresponding freestream velocities shown in the bottom panel, shown in colored markers. Bottom panel displays the phase of each marker.

7.11 Freestream velocity amplitude percentage plotted on top of flow separation criteria of $\frac{\ell^+}{\sqrt{2}}$ displayed in dashed (–) for the respective data set colors.
7.12 Oscillatory component of phase averaged boundary layer wall stress/flux (\(\tilde{\tau}_w, \tilde{q}_w\)) verse phase (\(\gamma\)). .......................................................... 123

7.13 Respective oscillatory component magnitude (top), and phase shift (bottom) vs \(w^+\) for the momentum field (left panel) and thermal field (right panel). Phase shift is normalized by 45° which represents the resultant phase shift of stokes flow. ........................................ 124

7.14 Respective oscillatory component magnitude (top), and phase shift (bottom) vs \(w^+\) of the near wall momentum field vs near wall thermal field. Phase shift is normalized by 45° which represents the resultant phase shift of stokes flow. ........................................ 125

7.15 Contour plot of outer normalized velocity field (\(\langle \tilde{u} \rangle\)) arranged in wall normal distance (\(\eta\)) verse time evolution of pulsatile forcing (\(\gamma/\omega\)). ................................................ 127

7.16 Contour plot of outer normalized temperature field (\(\langle \tilde{\phi} \rangle\)) arranged in wall normal distance (\(\eta_T\)) verse time evolution of pulsatile forcing (\(\gamma/\omega\)). ................................................ 128

7.17 Contour plot of outer normalized temperature rms field (\(\langle \tilde{\phi}^2 \rangle\)) arranged in wall normal distance (\(\eta_T\)) verse time evolution of pulsatile forcing (\(\gamma/\omega\)). ................................................ 129

7.18 Contour plot of outer normalized velocity field gradient in phase based time (\(\omega d\langle \tilde{u} \rangle / d\gamma\)) arranged in wall normal distance (\(\eta\)) verse time evolution of pulsatile forcing (\(\gamma/\omega\)). ................................................ 130

7.19 Contour plot of outer normalized temperature field gradient in phase based time (\(\omega d\langle \tilde{\phi} \rangle / d\gamma\)) arranged in wall normal distance (\(\eta\)) verse time evolution of pulsatile forcing (\(\gamma/\omega\)). ............ 131

7.20 Outer normalized thermal phase average field contours overlaid with black shaded regions denoting Richardson number above critical threshold. ........................................ 132

8.1 Oscillatory component of phase averaged boundary layer thickness (\(\tilde{\delta}, \tilde{\delta}_T\)) verse phase (\(\gamma\)). .............. 134

8.2 Oscillatory component of phase averaged boundary layer displacement thickness (\(\tilde{\delta}^*, \tilde{\delta}_T^*\)) verse phase (\(\gamma\)). ................................................................. 135

8.3 Oscillatory component of boundary layer shape factor (\(\tilde{H}, \tilde{H}_T\)) verse phase (\(\gamma\)). ......................... 136

8.4 Outer normalized phase average velocity profiles plotted \(\approx\) every 45° with the corresponding freestream velocities shown in the bottom panel and the time average profile overlaid on each individual plot, shown in colored markers. Bottom panel displays the phase of each marker. ................................................................. 138

8.5 Outer normalized phase average temperature profiles plotted \(\approx\) every 45° with the corresponding freestream velocities shown in the bottom panel and the time average profile overlaid on each individual plot, shown in colored markers. Bottom panel displays the phase of each marker. ................................................................. 139
8.6 Inner normalized phase average velocity profiles plotted \(\approx\) every 45° with the corresponding freestream velocities shown in the bottom panel and the time average profile overlaid on each individual plot, shown in colored markers. Bottom panel displays the phase of each marker.

8.7 Inner normalized phase average temperature profiles plotted \(\approx\) every 45° with the corresponding freestream velocities shown in the bottom panel and the time average profile overlaid on each individual plot, shown in colored markers. Bottom panel displays the phase of each marker.

8.8 Oscillatory component of phase averaged boundary layer wall flux \((\tilde{\tau}_w, \tilde{q}_w)\) verse phase \((\gamma)\).

8.9 Respective oscillatory component magnitude (top), and phase shift (bottom) vs \(w^+\) for the momentum field (left panel) and thermal field (right panel). Phase shift is normalized by 45° which represents the resultant phase shift of stokes flow.

8.10 Respective oscillatory component magnitude (top), and phase shift (bottom) vs \(w^+\) of the near wall momentum field vs near wall thermal field. Phase shift is normalized by 45° which represents the resultant phase shift of stokes flow.

8.11 Contour plot of outer normalized velocity field \((\langle \tilde{u}\rangle)\) arranged in wall normal distance \((\eta)\) verse time evolution of pulsatile forcing \((\gamma/f)\).

8.12 Contour plot of outer normalized temperature field \((\langle \tilde{\phi}\rangle)\) arranged in wall normal distance \((\eta_T)\) verse time evolution of pulsatile forcing \((\gamma/f)\).

8.13 Contour plot of outer normalized rms temperature field \((\langle \tilde{\phi}\rangle^2\rangle)\) arranged in wall normal distance \((\eta_T)\) verse time evolution of pulsatile forcing \((\gamma/f)\).

8.14 Contour plot of phase based temporal gradient of outer normalized temperature field \((\frac{d\langle \tilde{\phi}\rangle}{d\gamma})\) arranged in wall normal distance \((\eta_T)\) verse time evolution of pulsatile forcing \((\gamma/f)\).

8.15 Joint contour plot of \(Ri, Gr\) and \(Pr_T\) arranged in wall normal distance \((\eta_T)\) verse time evolution of pulsatile forcing \((\gamma/f)\).

8.16 Ratio of thermal event time scale to forcing time scale for mixed convective flow.

8.17 Ratio of buoyant time scale to forcing time scale for mixed convective flow.

9.1 Respective oscillatory component magnitude (top), and phase shift (bottom) vs \(w^+\) for the momentum field (left panel) and thermal field (right panel). Phase shift is normalized by 45° which represents the resultant phase shift of stokes flow. Black dashed line denotes a trend from Brereton and Mankbadi [1995]. Red dashed lines denote the various \(w^+\) regimes where:

i=Quasi-steady, ii=Low frequency, iii=Intermediate frequency, iv=High Frequency [Brereton and Mankbadi, 1995]
9.2 Respective oscillatory component magnitude (top), and phase shift (bottom) vs $w^+$ for the momentu
m field (left panel) relative to the thermal field (right panel). Phase shift is normalized by $45^\circ$ which represents the resultant phase shift of stokes flow. Red dashed lines denote the various $w^+$ regimes where; $i=$Quasi-steady, $ii=$Low frequency, $iii=$Intermediate frequency, $iv=$High Frequency Brereton and Mankbadi [1995].
ABSTRACT
Experimental Investigations Of Thermal Pulsatile Boundary Layer Flow
by
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University of New Hampshire, May, 2019

The need to reliably analyze, predict, and control the transport of mass, momentum, and energy in turbulent boundary layers is critically important across a broad spectrum of technological applications and scientific disciplines. While there has been extensive—and continuing—research investigating laboratory-scale canonical wall-bounded flows, beyond the scope of these well-studied flows there exists a broad range of application relevant flows that are far less studied. The theme of this dissertation research is to study a non-canonical flow, specifically to study coupled momentum and thermal transport in pulsatile boundary layer flows.

The primary contributions of the present work are (1) the design, fabrication, and validation of a unique flow facility to study non-equilibrium boundary layer flow and (2) to use this facility to study momentum and thermal transport in pulsatile boundary layer flow over a heated wall. In pulsatile boundary layer flow, the freestream velocity has a time-steady mean component and an unsteady oscillatory component superimposed on the mean component. Pulsatile boundary layer flows are therefore periodic and unidirectional characterized by both their frequency and amplitude, and are fundamentally important in many aerodynamic, industrial, and natural flow systems. The experimental data will be analyzed through time-average and phase-average frameworks to understand the underlying flow physics of pulsatile boundary layer dynamics. Results demonstrate a varied response between the momentum and thermal field, indicating a break down in the so-called Reynolds analogy assumption. The importance of buoyant transport is also demonstrated, even at low Richardson number flows.
CHAPTER 1
INTRODUCTION

The broad objective of this thesis is to advance the understanding of non-equilibrium boundary layer flow. The practical importance of the research is that in many engineering and natural flow systems, boundary layers are often influenced by non-equilibrium forcing effects such as pressure gradients, wall curvature, strong three-dimensionality, wall roughness, or dynamic walls. These non-equilibrium forcing effects can result in a significant modification of the transport of mass, momentum, and energy and corresponding surface fluxes (e.g., wall shear stress or wall heat flux) with a net integrated influence on fluid drag and/or heat transfer. Presently, the modeling and prediction of non-equilibrium boundary layer flow is difficult owing to both a knowledge gap in understanding the transport dynamics and a lack of experimental data for developing and validating models. It follows that the study of non-equilibrium boundary layer flow is important as the knowledge gained will increase the fundamental understanding of these types of flows and the data acquired can be used to develop and validate engineering models.

The primary contributions of the present work are (1) the design, build, and validation of a unique flow facility to study non-equilibrium boundary layer flow and (2) to use this facility to study momentum and thermal transport in pulsatile boundary layer flow over a heated wall. In pulsatile boundary layer flow, the freestream velocity has a time-steady mean component and an unsteady oscillatory component superimposed on the mean component. Pulsatile boundary layer flows are therefore periodic and unidirectional characterized by both their frequency and amplitude, and are fundamentally important in many aerodynamic, industrial, and natural flow systems. The aim of the work is to better understand the unsteady pressure forcing effects on both the momentum and thermal fields.
1.1 Contextual Overview

The concept of the boundary layer was introduced by Prandtl [1905] in a seminal presentation and accompanying paper at the Third International Mathematics conference in Heidelberg Germany. The underlying principle of the boundary layer concept is that fluid particles immediately adjacent to a material surface stick to the surface such that the relative velocity between the fluid and the material surface is zero. The influence of this so-called no-slip boundary condition (a consequence of friction between the fluid and the material surface) is confined to a relatively thin region of the fluid adjacent to the material surface (i.e, the boundary layer). Outside of the boundary layer, the effects of friction are negligible and the flow is inviscid.

Since boundary layers form whenever a fluid flows tangentially over a material surface, boundary layers are ubiquitous and pervasive in engineered and natural fluid flow processes. To illustrate this point, boundary layers form on a surface whenever a material body moves through a fluid (e.g., on automobiles, trains, airplanes, ships, human/animal/insect locomotion) and when a fluid flows over or through a material body (e.g, jet engines, pumps, human vascular/respiratory systems, internal combustion engines). If a temperature difference exists between the no-slip material surface boundary and the fluid, a thermal boundary layer will form as well [Kays and Crawford, 1980]. Similar to the no-slip boundary condition, the fluid in contact with the material surface will have the same temperature as the material surface. The temperature change between the material surface temperature and the fluid temperature far from the material body occurs entirely within the thermal boundary layer. The size of the thermal boundary layer relative to the momentum boundary layer will depend on the Prandtl number, \( Pr = \nu / \alpha \), where \( \nu \) and \( \alpha \) are the kinematic viscosity and thermal diffusivity of the fluid, respectively.

By definition, boundary layers are very thin compared to a characteristic geometric dimension of the no-slip surface (e.g., the length of a solid wall or the diameter of a cylindrical body) over which the fluid flows. In spite of its relative thinness, the dynamics internal to the boundary layer determine the rate at which mass, momentum, and energy are transferred between the fluid and the
surface boundary. With increasing Reynolds number the boundary layer transitions from a laminar flow to a turbulent flow due to the growth of flow instabilities. If we recall the definition of the Reynolds number

\[ Re = \frac{uL}{\nu} \]  

(1.1)

where \( u \) and \( L \) are characteristic velocity and length scales, it follows that high Reynolds numbers can be created either by large length scale, high flow speeds, or through small viscosity. In the majority of engineering systems as well as geophysical flows, the dynamical flow state of the boundary layer is turbulent. It follows that the need to reliably analyze, predict, and control the transport of mass, momentum, and energy in turbulent boundary layers is critically important across a broad spectrum of technological applications and scientific disciplines [Fox et al., 2012].

Owing to the practical importance of turbulent boundary layers, there has been extensive–and continuing–research to better understand the underlying physics of turbulent boundary transport [Clauser, 1956, Bradshaw and Huang, 1995, Sreenivasan and Antonia, 1997, Marusic et al., 2010, Garai and Kleissl, 2011, Ebner, 2014, Ebadi, 2016]. The extensive body of research includes experimental, theoretical, and numerical studies, where the overwhelming majority of these studies focused on laboratory-scale canonical momentum wall-bounded flows such as fully-developed pipe and channel flow or zero pressure gradient (ZPG) boundary layer flow. It is fair to say that much has been learned about the dynamics of these canonical flows [Sreenivasan and Antonia, 1997], although there is much still to learn [Sreenivasan, 1999, Jimenez, 2012]. Significantly less, however, is known about flows that fall outside the domain of these well-studied flows.

### 1.1.1 Equilibrium Boundary Layer Flow

Clauser [1956] defined an equilibrium boundary layer as a flow in which the driving force (i.e., the pressure gradient) and the resisting force (i.e., the wall shear stress) are in balance. The Clauser equilibrium condition is satisfied when \( \beta = \delta^* \frac{(dp/dx)}{\tau_w} \) is constant, where \( \delta^* \) is the displacement thickness, \( \tau_w \) is the wall shear stress and \( dp/dx \) is the streamwise pressure gradient.
Song et al. [2000] defined an equilibrium boundary layer as a flow where the shear stress distribution is balanced by the wall shear stress. Zero pressure gradient (ZPG) boundary layer (BL) and fully-developed channel/pipe flows are the most common canonical equilibrium wall-bounded flows.

In canonical equilibrium wall-bounded flows, the time scale over which the mean field varies is large compared to local turbulent time scales. The turbulent field rapidly adjusts to mean field variations, and the flow exhibits universal behaviors when scaled by local parameters [Townsend, 1976]. One very important universal behavior is the logarithmic dependence of the time averaged mean velocity profile, in the so-called logarithmic region,

\[
\pi^+ = \frac{1}{\kappa} \log(y^+) + C_1, \tag{1.2}
\]

where \( u \) is the streamwise velocity, \( \langle \rangle \) denotes a long time average, the superscript \( + \) denotes normalization by the friction velocity \( u_\tau = \sqrt{\tau_w/\rho} \) and kinematic viscosity \( \nu \), where \( \tau_w \) is the shear stress at the wall and \( \rho \) the fluid density; \( 1/\kappa \) (typically \( \kappa \) is called the von Kármán coefficient) is the slope; \( y^+ \) is the wall-normal coordinate; and \( C_1 \) is the intercept at \( y^+ = 1 \). Equation 1.2 is referred to as the law of the wall with coefficients (at sufficiently high Reynolds number) \( \kappa \approx 0.384 \) and \( C_1 \approx 5 \), varying for a given canonical flow type [Nagib and Chauhan, 2008].

The temperature distribution in the boundary layer, using similar dimensional scaling arguments used to derive Eq. 1.2, yields the law of the wall for the mean temperature distribution,

\[
\Theta^+ = \frac{T_w - T(y)}{T_\tau} = \frac{1}{\kappa_T} \log(y^+) + C_2(Pr), \tag{1.3}
\]

where \( T_w \) is the wall temperature, \( T(y) \) is the temperature in the boundary layer, the superscript \( + \) denotes normalization by the friction temperature \( T_\tau = q''_w/(\rho c_p u_\tau) \), where \( q''_w \) is wall heat flux and \( c_p \) is specific heat. The dimensionless coefficients \( 1/\kappa_T \) is the slope and \( C_2 \), which is a function of \( Pr \), is the intercept at \( y^+ = 1 \). In general at sufficiently high Reynolds number, \( \kappa_T \approx 0.48 \) but also varies for a given canonical flow type [Kader and Yaglom, 1972].
1.1.2 Non-Equilibrium Wall-bounded Flows

Non-equilibrium boundary layer flows are flows that do not satisfy the set of conditions defined by Clauser [1956] or by Song et al. [2000] as described above. In non-equilibrium boundary layers, the time scales over which the mean field varies are comparable (or smaller) to local turbulent time scales, and the flow field cannot be characterized solely in terms of local parameters [Townsend, 1976]. Such rapid changes in the mean momentum field typically result from pressure gradients, wall curvature, strong three-dimensionality, wall roughness, or dynamic walls. For non-equilibrium boundary layers in which an equilibrium boundary layer flow experiences a localized non-equilibrium forcing perturbation (e.g., flow over an obstacle/cavity or flow subjected to an adverse pressure gradient), there is a considerable body of research investigating the redistribution of the momentum field when the equilibrium state is disturbed [Antonia et al., 1977, Bradshaw, 1972, Bandyopadhyay and Ahmed, 1993, Castro and Epik, 1998]. In general, these studies show that: (a) in a small local region near the perturbation the log-layer is obliterated (i.e., the law of the wall given by Eq. 1.2 does not hold), (b) downstream of the perturbation, in the so-called recovery region, an internal stress equilibrium layer grows and the boundary layer recovers towards equilibrium. However, the experimental measurements of Jovic and Driver [1994, 1995] showed that the velocity profile in the recovery region of a flow behind a backward-facing step falls below the universal log-law. This finding was confirmed by DNS of Le et al. [1997]. It is now established that in mild to strong adverse pressure gradient (APG) wall-bounded flows, the mean velocity profile is shifted downward from the log law, the wake region is amplified, and the extent of the logarithmic region is shrunk [Aubertine and Eaton, 2005, Monty et al., 2011]. Periodically forced wall-bounded flow is also a relatively well-studied non-equilibrium flow. These flows are generally classified as either pulsatile, when the flow rate oscillates in time around a nonzero mean, or reciprocating, when the cycle-averaged flow rate is zero [Liberto and Ciofalo, 2011]. Akhavan et al. [1991] empirically predicted and experimentally showed the y-intercept of Eq. 1.2 is strongly modified in a periodic wall-bounded flow.
1.1.3 Thermal Transport in Non-Equilibrium Boundary Layer Flow

While the effects of non-equilibrium forcing on the momentum field have been considerably studied, heat transfer in non-equilibrium boundary layers has received far less attention despite the importance of such flows in engineering and geophysics. For example, flow through turbo-machinery, flow in the combustion chamber of a reciprocating engine, or flow within the atmospheric surface layer (e.g., flow from sea to land or vice versa). Of the limited studies, the overwhelming majority have focused on quantifying the change in the heat transfer coefficient compared to an equilibrium wall-bounded flow counterpart (e.g., heat transfer in APG flow compared to ZPG flow). Nevertheless, despite somewhat limited data, it is a well-accepted fact that the law of the wall for temperature is more affected by mean field variations than the velocity field [Blackwell et al., 1972, Kader and Yaglom, 1991, Bradshaw and Huang, 1995, Kong et al., 2001, Houra and Nagano, 2006, Wang, 2008]. For example, in non-equilibrium boundary layer flow subjected to a pressure gradient, the constants in Eq. 1.3 vary significantly with pressure gradient while the constants in Eq. 1.2 vary little (see figure 1.1). This difference in sensitivity is unexpected given that Eqs. 1.2-1.3 were derived from analogous dimensional scaling arguments, and has brought into question the validity of the law of the wall [Bradshaw and Huang, 1995]. Moreover, the high sensitivity of the temperature field to pressure gradient flows is remarkable since the pressure gradient does not appear in the transport equation for temperature.

The consensus, although not entirely well-understood, is that while the law of the wall for velocity is fairly resilient, the law of the wall for temperature is very strongly affected by upstream disturbances [Bradshaw and Huang, 1995]. The implication is that the scaling used to derive the law of the wall fails to describe the behaviors of the mean dynamics (especially for temperature) when there are large gradients in the mean flow direction. Consequently, to capture non-equilibrium effects on mean field dynamics, the present state of the research is to reformulate the law of the wall through the use of new scaling laws [Durbin and Belcher, 1992, George and Castillo, 1993, Cruz and Freire, 1998, 2002, Wang, 2008] or use single-point closure models.
Extrapolating the results discussed above to strong non-equilibrium forcing, in which mean field perturbations vary rapidly in magnitude both spatially and temporally (e.g., in-cylinder engine flows or other reciprocating machinery), a logical conclusion with respect to the law of the wall is that Eqs. 1.2-1.3 will either (a) not hold or (b) the slope and intercept will vary strongly in space and time. In the context of the present study, an open question is how does the temperature field respond to unsteady pressure forcing in pulsatile boundary layer flow? Aside from its importance related to technological applications, the question is fundamentally important giving the apparent robustness of the mean velocity profile and the sensitivity of the temperature field to mean field disturbances.
1.1.4 Measurement Needs of Thermal Transport in Non-Equilibrium Boundary Layer Flow

The potential for computational fluid dynamics (CFD) Reynolds Averaged Navier-Stokes (RANS) simulations to accurately predict boundary layer transport depends on the specifics of the closure model and wall functions used. In general, the development of turbulent closure models and wall functions are informed, refined, and validated by experimental data. One obstacle for formulating new engineering heat transfer models that better capture the physics of non-equilibrium flows (i.e., flows with complex dynamics) is the lack of robust experimental data needed to both develop and validate models. This is an especially acute problem with respect to heat transfer in non-equilibrium wall-bounded flow. The measurement need is evident by the fact that the experimental measurement of Perry et al. [1966] and Blackwell et al. [1972] remain the primary datasets utilized in the validation of DNS of thermal boundary layers [Araya and Castillo, 2012]

The lack of experimental studies is not surprising given that controlling thermal boundary conditions is non-trivial, and the simultaneous measurement of temperature and velocity fluctuations in turbulent boundary layers with heat transfer is very difficult. In addition, direct measurement of the wall-heat flux, which is the primary scaling variable to study thermal boundary layers, is challenging. One aim of the present work is to close this knowledge gap by developing a unique thermal boundary layer flow facility that can be used to study thermal transport in canonical ZPG boundary layer flow and in non-equilibrium boundary layer flow. The key component of this facility is a thermal wall plate designed to control the thermal boundary conditions under the condition of non-equilibrium flow forcing. Owing to the strong spatial variations of the flow associated with non-equilibrium boundary layers, this is a nontrivial but important objective needed to better understand thermal transport in these types of flows. This flow facility, called the Non-Equilibrium And Thermal (NEAT) boundary layer wind tunnel, is described in Chapter 4.
1.2 Organization of the Dissertation

The work of this dissertation can be divided into two primary contributions: (1) the design, fabrication, and validation of a feedback controlled thermal wall plate to investigate thermal transport in non-equilibrium boundary layers, (2) an experimental study to measure and quantify the velocity field and temperature field in pulsatile boundary layer flow over a heated wall.

The dissertation is organized into the following chapters:

- **Chapter 2: Theoretical Background**
  The governing equations describing turbulent transport, near wall dynamics, and the affect of unsteady forcing on the boundary layer dynamics are described. The component of interest is the variation in the near wall dynamics in the momentum and thermal fields and their influence on boundary layer self similarity.

- **Chapter 3: Experimental Techniques**
  The experimental techniques used to measure the time evolution of the momentum and thermal fields are described. Next, the post-processing procedures used on the data sets is described.

- **Chapter 4: NEAT Boundary Layer Wind Tunnel**
  The experimental facility, the NEAT tunnel, is described in detail along with the validation experiments performed to ensure that the facility produces the desired flow behaviors.

- **Chapter 5: Previous Work on Pulsatile Boundary Layer**
  Chapter 5 provides a literature review of previous experimental and computational work done on studies of pulsatile boundary layers.

- **Chapter 6: Time Average Pulsatile Boundary Layer**
  Experimental results are presented in a time-average framework and variations due to $Re$ and forcing frequency are discussed.
• Chapter 7: Phase Average Pulsatile Boundary Layer: Forced Convection

Forced convective pulsatile flows (i.e. buoyancy effects are negligible) are quantified through phase average descriptions of the flow field. The analysis focuses on the phase based varied response of the momentum and thermal fields.

• Chapter 8: Phase Average Pulsatile Boundary Layer: Mixed Convection

Mixed convective pulsatile flows (i.e. buoyancy is non-negligible) are quantified through a phase average framework. The analysis focuses on the varied responses of the momentum and thermal fields and the phase based influence of buoyant transport.

• Chapter 9: Conclusions

The significant results of the current study are summarized and the conclusions based on these results are described along with an outline of future research directions.
CHAPTER 2
THEORETICAL BACKGROUND

2.1 Governing Equations of Fluid Dynamics

The equations governing the incompressible flow of Newtonian fluids written in their most general form consists of the continuity equation:

$$\frac{\partial u_i}{\partial x_i} = 0,$$

(2.1)

where $u_i$ and $x_i$ are the velocity and spatial vector components;

the Navier-Stokes equation:

$$\rho \left( \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = - \frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_j^2},$$

(2.2)

where $\rho$ is density, $p$ is pressure, $g$ is gravitational acceleration, and $\mu$ is the dynamic viscosity;

and the thermal transport equation:

$$\rho c_p \left( \frac{\partial \theta}{\partial t} + u_i \frac{\partial \theta}{\partial x_i} \right) = \lambda \frac{\partial^2 \theta}{\partial x_i^2} + \phi_v + \dot{q},$$

(2.3)

where $c_p$ is the specific heat, $\theta$ is temperature, $\lambda$ is the thermal conductivity of the fluid, $\dot{q}$ is the heat generation and $\phi_v$ is the heat dissipation due to viscous effects. Note that for the flows studied in this dissertation both $\dot{q}$ and $\phi_v$ are negligible. In addition to the governing equations, appropriate initial/boundary conditions are needed to solve the system of equations.
2.2 Reynolds Averaged Navier Stokes Equations

The solution of the governing equations listed above is not feasible when the flow is turbulent, even for the most simplest of flows. Given this limitation, one common analytical/modeling approach is to first decompose each flow variable in Eqs. 2.1–2.3 into a mean and a fluctuating component:

\[ a_i(x, y, z, t) = \bar{a}_i(x, y, z) + a'_i(x, y, z, t), \]  

(2.4)

where \( a_i \) denotes a generic variable, \( \bar{a}_i \) denotes a time average of \( a_i \), and prime denotes the fluctuation of \( a_i \) about its average. Next, taking the average of Eqs. 2.1-2.3 yields the so-called Reynolds Averaged Navier Stokes (RANS) equations that describe the mean dynamics of a turbulent flow. The mean continuity equation is

\[ \frac{\partial \bar{u}_i}{\partial x_j} = 0. \]  

(2.5)

The mean momentum equation is

\[ \frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho} \left( \frac{\partial \bar{P}}{\partial x_i} + \mu \frac{\partial^2 \bar{u}_i}{\partial x_j^2} - \frac{\partial (\rho \bar{u}_i' \bar{u}_j')}{\partial x_j} \right). \]  

(2.6)

The mean thermal transport equation is

\[ \frac{\partial \bar{\theta}}{\partial t} + \bar{u}_i \frac{\partial \bar{\theta}}{\partial x_i} = \alpha \frac{\partial^2 \bar{\theta}}{\partial x_i^2} - \frac{\partial (\rho c_p \bar{u}_i' \bar{\theta}')}{\partial x_i}, \]  

(2.7)

where \( \alpha = \lambda/\rho c_p \) is the thermal diffusivity. Equations (2.6) and (2.7) include two new terms which result from the averaging process, the turbulent or Reynolds stress \( (\rho \bar{u}_i' \bar{u}_j') \) and the turbulent heat flux \( (\rho c_p \bar{u}_i' \bar{\theta}') \). The former represents the turbulent momentum flux (i.e., the redistribution of mean momentum by turbulent motions) and the latter represents the turbulent heat flux (i.e., the redistribution of heat by turbulent motions).
2.3 Triple Decomposed Long Time Averaged Navier Stokes Equations

In flow with an imposed sinusoidal nature, it is convenient to use the triple decomposition of Hussain and Reynolds [1970] of an instantaneous flow variable given by

\[ a_i(x, y, z, t) = \bar{a}_i(x, y, z) + \tilde{a}_i(x, y, z, t) + a'_i(x, y, z, t), \tag{2.8} \]

where \( \bar{a}_i \) is the long time-averaged mean (or cycle-averaged mean), \( \tilde{a}_i \) is the oscillatory component, and \( a'_i \) is the fluctuation.

The long-time averaged (or cycle averaged mean) form of Eqs. 2.1-2.3 with flow variables that have been decomposed following Eq 2.8 results in the long-time averaged (cycle-averaged) mean continuity equation

\[ \frac{\partial \bar{u}_i}{\partial x_j} = 0, \tag{2.9} \]

the long-time averaged (cycle-averaged) mean momentum equation

\[ \frac{\bar{u}_j \partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho} \left( \frac{\partial \bar{P}}{\partial x_i} + \mu \frac{\partial^2 \bar{u}_i}{\partial x_j^2} - \frac{\partial (\rho \bar{u}_i \bar{u}_j')}{\partial x_j} - \frac{\partial (\rho \tilde{u}_i \tilde{u}_j)}{\partial x_j} \right), \tag{2.10} \]

and the long-time averaged (cycle-averaged) mean thermal transport equation

\[ \frac{\bar{u}_i \partial \bar{\theta}}{\partial x_i} = \alpha \frac{\partial^2 \bar{\theta}}{\partial x_i^2} - \frac{\partial (\bar{u}_i \bar{\theta}')}{\partial x_i} - \frac{\partial (\bar{u}_i \tilde{\theta})}{\partial x_i}. \tag{2.11} \]

Note that on average the oscillatory motion and the background turbulence are uncorrelated such that these correlation terms have been omitted in the above equations [Hussain and Reynolds, 1970]. It follows that the oscillatory flow directly interacts with the time-averaged mean flow via the long-time averaged oscillatory component of the Reynolds stress \( \bar{u}_i \bar{u}_j \) as given by Eq. 2.10 [Tardu et al., 1994]. Similarly, the oscillatory flow directly interacts with the time-averaged mean thermal transport equation via the long-time averaged oscillatory component of the turbulent heat flux \( \bar{u}_i \bar{\theta} \) as given by Eq. 2.11. In addition, the oscillatory flow can indirectly interact with the
time-averaged mean flow through influence on the fluctuating Reynolds stress or the fluctuating turbulent heat flux.

### 2.4 Phase Averaged Navier Stokes Equation

In oscillatory flow, it is also convenient to acquire phase averaged measurements. The phase averaged decomposition of a generic flow variable is

\[ a_i(x, y, z, t) = \langle a_i(x, y, z, t) \rangle + a'_i(x, y, z, t), \]  

where the phase average is defined as

\[ \langle a_i(x, y, z, t) \rangle = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} a_i(x, y, z, t + nT), \]  

where \( \omega = \frac{2\pi}{T} \) is the period of the cycle and \( \omega \) the angular frequency. It follows that

\[ \langle a_i(x, y, z, t) \rangle = \overline{a}_i(x, y, z) + \tilde{a}_i(x, y, z, t), \]

where \( \langle \cdot \rangle \) denotes the phase average and, as described earlier, an overbar denotes the long time average (cycle average) and a prime denotes the fluctuation about the average. The phase averaged forms of Eqs. 2.1-2.3 are given by the phase averaged continuity equation

\[ \frac{\partial \langle u_i \rangle}{\partial x_j} = 0, \]

the phase averaged momentum equation

\[ \frac{\partial \langle u_i \rangle}{\partial t} + \langle u_j \rangle \frac{\partial \langle u_i \rangle}{\partial x_j} = -\frac{1}{\rho} \left( \frac{\partial \langle p \rangle}{\partial x_i} + \mu \frac{\partial^2 \langle u_i \rangle}{\partial x_j^2} - \frac{\partial \langle \rho (u'_i u'_j) \rangle}{\partial x_j} \right), \]  

and the phase averaged thermal transport equation
∂⟨θ⟩∂t + ⟨ui⟩∂⟨θ⟩∂xi = α ∂2⟨θ⟩∂x2i − ∂⟨(u′θ′)⟩∂xi,  \[(2.17)\]

Note that the governing equations for the oscillatory component of the flow can be obtained by subtracting the long time averaged governing equations from the phase averaged governing equations. In this thesis, the oscillatory component of a flow variable is acquired by subtracting the measured long time averaged mean of the flow variable from the measured phase averaged mean of the same flow variable:

\[\tilde{a}_i(x, y, z, t) = \langle a_i(x, y, z, t) \rangle - \bar{A}_i(x, y, z). \quad (2.18)\]

2.5 Boundary Layer Equations

The equations governing the incompressible flow of Newtonian fluids within boundary layers are simplified versions of the generalized governing equations of the coupled momentum and thermal field based on the following set of assumptions [Schlichting, 1979]:

1. two-dimensional flow in the mean (i.e., ∂/∂z = 0, where z denotes the spanwise direction in the chosen coordinate system),

2. the boundary layer thickness \(\delta\) is the characteristic length scale in the wall-normal direction (y-direction in the chosen coordinate system),

3. \(L\) is the the characteristic length scale in the streamwise direction (x-direction in the chosen coordinate system),

4. \(L \gg \delta\) (i.e., the boundary layer is thin), and

5. \(\bar{u}_\infty\) is the characteristic velocity in the streamwise direction.

Under these assumptions and with \(u_i = \hat{u}i + \hat{v}j + \hat{w}k\), where \(\hat{i}, \hat{j}, \hat{k}\) denote unit vectors in the \(x, y, z\) directions, respectively, an order of magnitude analysis shows that the continuity equation reduces to
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (2.19)
\]

The momentum equation reduces to

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dP}{dx} + \nu \frac{\partial^2 u}{\partial y^2}, \quad (2.20)
\]

The thermal transport equation reduces to

\[
\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \alpha \frac{\partial^2 \theta}{\partial y^2}. \quad (2.21)
\]

### 2.6 RANS Boundary Layer Equations

The RANS boundary layer equations to leading order [Monkewitz et al., 2007] are the mean continuity equation:

\[
\frac{\partial \pi}{\partial x} + \frac{\partial \pi}{\partial y} = 0, \quad (2.22)
\]

the mean momentum equation:

\[
\frac{\partial \pi}{\partial t} + u \frac{\partial \pi}{\partial x} + V \frac{\partial \pi}{\partial y} = -\frac{1}{\rho} \frac{dP}{dx} + \nu \frac{\partial^2 \pi}{\partial y^2} - \frac{\partial (u'v')}{\partial y}, \quad (2.23)
\]

and mean thermal transport equation:

\[
\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + V \frac{\partial \theta}{\partial y} = \alpha \frac{\partial^2 \theta}{\partial y^2} - \frac{\partial (\theta'\theta')}{\partial y}, \quad (2.24)
\]

where \( \nu = \mu/\rho \) is the kinematic viscosity of the fluid. Note that in steady ZPG boundary layer flow over a bounding wall with fixed thermal boundary conditions \( \partial \pi/\partial t = \partial \theta/\partial t = dP/dx = 0 \).

In this thesis, ZPG boundary layer flow is used as a baseline comparison for pulsatile boundary layer (PBL) flow.
2.7 Pulsatile Boundary Layer Flow

Pulsatile boundary layer flow is defined here as a boundary layer flow in which the freestream velocity has a time-steady mean component and an unsteady oscillatory component superimposed on the mean component. Pulsatile boundary layer flows are therefore periodic and unidirectional characterized by both their frequency and amplitude. The flow over a half-period first accelerates to maximum velocity then decelerates to the mean velocity. During the next half-period, the flow first decelerates to minimum velocity then accelerates to the mean velocity. This process is then repeated indefinitely. The flow at a given wall-normal position may lead/lag the freestream depending on flow forcing. Pulsatile boundary layer flows are fairly well-studied owing to their fundamental importance in many aerodynamic and industrial flow systems. Typical problem arise in the study of the response of lifting surfaces to unsteady winds, airplane wing flutter, turbo machinery and internal combustion engines [Patel et al., 1975].

The freestream velocity of a pulsatile boundary flow for a purely time dependent oscillation can be expressed by

\[ u_\infty(t) = u_o + N \sin(\omega t), \]  

where \( u_o \) is the mean freestream velocity and \( N \) and \( \omega \) are the amplitude and angular frequency of the oscillatory component of the freestream velocity, respectively. Since \( u_\infty \neq f(x) \) there is no mean pressure gradient and the inviscid flow outside of the pulsatile boundary layer is governed by

\[ \frac{du_\infty}{dt} = -\frac{1}{\rho} \frac{dp}{dx}. \]  

(2.26)

The momentum equation for the PBL flow can then be written as

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{du_\infty}{dt} + \nu \frac{\partial^2 u}{\partial y^2}. \]  

(2.27)

The thermal transport equation for PBL flow is the same as for the general boundary layer flow as given by Eq. 2.21.
2.8 Triple Decomposed Long Time Averaged Pulsatile BL Equations

The long-time averaged (or cycle averaged mean) form of Eqs. 2.31 to 2.34 is given by the long-time averaged continuity equation:

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0,
\]

(2.28)

the long-time averaged mean momentum equation:

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = \nu \frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\partial (\bar{u} \bar{v}')}{\partial y} - \frac{\partial (\bar{u} \bar{v})}{\partial y},
\]

(2.29)

and the long-time averaged mean thermal transport equation:

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = \alpha \frac{\partial^2 \bar{\theta}}{\partial y^2} - \frac{\partial (\bar{v} \bar{\theta}')}{\partial y} - \frac{\partial (\bar{v} \bar{\theta})}{\partial y}.
\]

(2.30)

The direct influence of the oscillatory flow field on the long-time averaged (cycle averaged) mean flow is from the wall-normal gradient of the oscillatory component of the Reynolds shear stress (momentum equation) and from the wall-normal gradient of the oscillatory component of the turbulent heat flux (thermal transport equation). As described previously, an indirect influence of the oscillatory flow field on the long-time averaged mean flow field is possible if the fluctuating component of the Reynolds shear stress or turbulent heat flux is modified by the oscillatory flow.

2.9 Phase Averaged Pulsatile BL Equations

The phase averaged form of Eqs. 2.31 to 2.34 is given by the phase-averaged continuity equation

\[
\frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y} = 0.
\]

(2.31)

The phase-averaged mean momentum equation:

\[
\frac{\partial \langle u \rangle}{\partial t} + \langle u \rangle \frac{\partial \langle u \rangle}{\partial x} + \langle v \rangle \frac{\partial \langle u \rangle}{\partial y} = -\frac{1}{\rho} \frac{\partial \langle p \rangle}{\partial x} + \nu \frac{\partial^2 \langle u \rangle}{\partial y^2} - \frac{\partial \langle (u'v') \rangle}{\partial y},
\]

(2.32)

which can be written as
The phase averaged thermal transport equation is

\[
\frac{\partial \langle \theta \rangle}{\partial t} + \langle u \rangle \frac{\partial \langle \theta \rangle}{\partial x} + \langle v \rangle \frac{\partial \langle \theta \rangle}{\partial y} = \alpha \frac{\partial^2 \langle \theta \rangle}{\partial y^2} - \frac{\partial \langle (v'\theta') \rangle}{\partial y}.
\]  \tag{2.34}

2.10 Normalizations

The boundary layer equations are typically made non-dimensional by inner- or outer-normalization. The benefit of these normalizations is that they allow direct comparison between different flows and provide insight into the underlying dynamics and their scaling behaviors. The normalization process is demonstrated here for the phase-averaged pulsatile boundary layer equations, though the process is the same regardless of the system of equations.

2.10.1 Inner-normalization

Inner normalization uses \( u_\tau, \nu/u_\tau, \theta_\tau \) and \( \omega^{-1} \) for velocity, length, temperature, and time scaling. The friction velocity \( u_\tau = \sqrt{\tau_w/\rho} \) where the wall shear stress \( \tau_w = \mu du/dy \mid_{y=0} \). The friction temperature \( \theta_\tau = q''_w/(\rho c_p u_\tau) \), where the wall heat flux \( q''_w = \lambda d\theta/dy \mid_{y=0} \). The inner normalized form of the phase-averaged PBL equation is given by the following set of equations. The inner-normalized continuity equation is

\[
\frac{\partial \langle u^+ \rangle}{\partial x^+} + \frac{\partial \langle v^+ \rangle}{\partial y^+} = 0,
\]  \tag{2.35}

where \( u^+ = u/u_\tau, x^+ = xu_\tau/\nu, \) and \( y^+ = yu_\tau/\nu \). The inner-normalized momentum equation is

\[
\omega^+ \frac{\partial \langle (u^+ - u^\infty) \rangle}{\partial t^+} + \langle u^+ \rangle \frac{\partial \langle u^+ \rangle}{\partial x^+} + \langle v^+ \rangle \frac{\partial \langle u^+ \rangle}{\partial y^+} = \frac{\partial^2 \langle u^+ \rangle}{\partial y^2} + \frac{\partial \langle (u'^+v'^+) \rangle}{\partial y^+},
\]  \tag{2.36}

where \( t^+ = \omega t \) and the inner-normalized angular frequency \( \omega^+ = \omega \nu/u_\tau^2 \). For the normalized thermal transport equation, it is convenient (for scaling purposes) to reference the temperature to
the wall temperature. This new temperature variable is defined as \( \phi = \theta_w - \theta \), where \( \theta_w \) denotes the wall temperature. The inner-normalized thermal transport equation is

\[
\omega^+ \frac{\partial (\phi^+)}{\partial t^+} + \langle u^+ \rangle \frac{\partial \langle \phi^+ \rangle}{\partial x^+} + \langle v^+ \rangle \frac{\partial \langle \phi^+ \rangle}{\partial y^+} = \frac{1}{Pr} \frac{\partial^2 \langle \phi^+ \rangle}{\partial y^+ 2} - \frac{\partial \langle (v'\phi') \rangle}{\partial y^+},
\]

where the Prandtl number \( Pr = \nu / \alpha \).

### 2.10.2 Outer-normalization

Outer normalization uses \( u_\infty, \delta, \theta_\infty \) and \( w^{-1} \) for velocity, length, temperature, and time scaling, where \( \delta \) is some measure of the wall-normal thickness of the boundary layer. The outer-normalized form of the phase-averaged PBL equations are given by the following set of equations. The outer-normalized continuity equation is

\[
\frac{\partial \langle \tilde{u} \rangle}{\partial \tilde{x}} + \frac{\partial \langle \tilde{v} \rangle}{\partial \tilde{y}} = 0,
\]

where \( \tilde{u} = u / u_\infty, \tilde{x} = x / \delta, \) and \( \tilde{y} = y / \delta \). The outer-normalized momentum equation is

\[
\tilde{\omega} \frac{\partial \langle (\tilde{u} - \tilde{u}_\infty) \rangle}{\partial \tilde{t}} + \langle \tilde{u} \rangle \frac{\partial \langle \tilde{u} \rangle}{\partial \tilde{x}} + \langle \tilde{v} \rangle \frac{\partial \langle \tilde{u} \rangle}{\partial \tilde{y}} = \frac{1}{Re_\delta} \frac{\partial^2 \langle \tilde{u} \rangle}{\partial \tilde{y}^2} - \frac{\partial \langle (\tilde{u}'\tilde{v}') \rangle}{\partial \tilde{y}},
\]

where \( \tilde{t} = \omega t \) and the coefficient parameters are \( \tilde{\omega} = \omega \delta / u_\infty \) and \( Re_\delta = u_\infty \delta / \nu \). Similar to the inner-normalized thermal transport equation, it is convenient to reference temperature to the wall temperature: \( \phi = \theta_w - \theta \). The outer-normalized phase averaged thermal transport equation is

\[
\tilde{\omega} \frac{\partial \langle \tilde{\phi} \rangle}{\partial \tilde{t}} + \langle \tilde{u} \rangle \frac{\partial \langle \tilde{\phi} \rangle}{\partial \tilde{x}} + \langle \tilde{v} \rangle \frac{\partial \langle \tilde{\phi} \rangle}{\partial \tilde{y}} = \frac{1}{PrRe_\delta} \frac{\partial^2 \langle \tilde{\phi} \rangle}{\partial \tilde{y}^2} - \frac{\partial \langle ((\tilde{v}'\tilde{\phi}') \rangle}{\partial \tilde{y}},
\]

where \( \tilde{\phi} = (\theta_w - \theta) / (\theta_w - \theta_\infty) \). Note that in PBL flow, the velocity scaling parameter can either by the long-time averaged freestream velocity \( (\tilde{u}_\infty) \) or the phase-averaged freestream velocity \( \langle u_\infty \rangle \). The difference between these two values will depend on the specific phase angle and the amplitude of the oscillatory velocity. The “suitable” measure of the boundary layer thickness used for the length scaling parameter also has choices as described in the next section.
2.11 Boundary Layer Thickness

The boundary layer thickness is a length measurement in the direction normal to a no-slip surface that characterizes the penetration depth of the no-slip boundary condition within the flow. Acknowledging that vorticity is finite within the boundary layer and zero outside of the boundary layer, the boundary layer thickness is the length measured from the no-slip surface to this vorticity/no-vorticity interface. Since vorticity is a difficult measurement, other suitable comparable measures of the boundary layer thickness are used:

1. $\delta$ is the wall normal location where $u(\delta) = 0.99u_\infty$,

2. $\delta_T$ is the wall normal location where $\bar{\theta}_w - \bar{\theta}(\delta_T) = 0.99(\bar{\theta}_w - \bar{\theta}_\infty)$,

3. $\delta^* = \int_0^\infty \left(1 - \frac{u}{u_\infty}\right) dy$ is the displacement thickness,

4. $\delta^*_T = \int_0^1 \left(1 - \tilde{\phi}\right) dy$ is the thermal displacement thickness,

5. $\theta^* = \int_0^1 \tilde{u} \left(1 - \frac{u}{u_\infty}\right) dy$ is the momentum thickness, and

6. $\theta^*_T = \int_0^1 \tilde{\phi} \left(1 - \tilde{\phi}\right) dy$ is the thermal analogous version of the momentum thickness.

2.12 Surface Fluxes

The surface flux of momentum and heat are critical flow measurement parameters because they are (1) fundamental to fluid physics as they are used as scaling parameters for evaluating wall-bounded flows, (2) indicator of near-wall dynamics and the primary metric by which non-equilibrium forcing effects are assessed, (3) necessary to evaluate the similarity (dissimilarity) between the transport of momentum and the transport of heat in a given flow, (4) used as validation metrics for engineering models.
2.12.1 Wall Shear Stress

The no-slip boundary condition imposes a shear stress on the fluid layer attached to the wall with an equal and opposite shear stress on the wall. This shear stress $\tau_w$ is called the wall shear stress and is expressed by

$$\tau_w = \mu \frac{du}{dy} \bigg|_{y=0}$$

where $y = 0$ denotes the wall. In this thesis, the integral method of Mehdi and White [2011] and Mehdi et al. [2014] will be used to measure the phase-averaged and long-time averaged wall shear stress from particle image velocimetry (PIV) measurements. Note that in PBL flow, the phase-averaged wall shear stress will have both an oscillatory and a long-time averaged component. The mathematical formulation with application to an unsteady flow is reviewed here as it is not explicitly described in Mehdi and White [2011] or Mehdi et al. [2014].

For a steady state, two-dimensional, wall-bounded turbulent flow, the $x$-momentum equation reduces to

$$0 = \frac{\partial}{\partial y} \left[ u'v' - \nu \frac{\partial u}{\partial y} \right] + I_x + \frac{1}{\rho} \frac{dP}{dx},$$

where

$$I_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} + \frac{\partial}{\partial x} \left( u'^2 - v'^2 \right).$$

Integrating Eq 2.42 thrice and applying proper boundary conditions results in the following expression for the wall shear stress:

$$\tau_w = \mu \frac{du}{dy} \bigg|_{y=0}$$
\[
\tau_w = \frac{2\rho}{y_f^2} \left[ \nu \int_0^{y_f} u dy - \int_0^{y_f} (y_t - y) \overline{u'v'} dy - \frac{1}{2\rho} \int_0^{y_f} (y_t - y)^2 \frac{\partial \tau}{\partial y} dy \right], \tag{2.44}
\]

where

\[
\frac{\tau}{\rho} = \nu \frac{\partial u}{\partial y} - \overline{u'v'} \tag{2.45}
\]

and the expression is evaluated to arbitrary integration height \( y_t \).

2.12.2 Wall Heat Flux

The wall heat flux is defined as the local thermal power per unit area transferred between a fluid and a bounding wall. Owing to its immense value in engineering application there has been considerable interest in determining and evaluating different methodologies to measure wall heat flux [Bose and Park, 2018, Li et al., 2014]. The wall heat flux is expressed as

\[
q''_w = -\lambda \frac{d\theta}{dy} \bigg|_{y=0}. \tag{2.46}
\]

The primary non-dimensional form of the wall heat flux used to parameterize thermal transport in wall-bounded flow is the Stanton number

\[
St = \frac{q''_w}{\rho c_p u_{\infty} (\theta_w - \theta_{\infty})}. \tag{2.47}
\]

One common approach in evaluating the wall heat flux is to assume similarity between the thermal field and the momentum field such that the wall heat flux can be determined by measurement of the wall shear stress following Colburn’s formulation [Kays and Crawford, 1980] given by

\[
St = Pr^{-2/3}(1/2)C_f \tag{2.48}
\]

where \( C_f \) is skin friction coefficient.
Since one aim of the present study is to evaluate if there are differences between the momentum field and the thermal field in response to non-equilibrium forcing effects, the use of Colburn’s method cannot be used. While the integral method of Ebadi et al. [2015] would be suitable, in the present study the temperature field and velocity are not measured simultaneously so profiles of turbulent heat flux needed for the integral method are not available. Instead, in this thesis we employ a near-wall fit to the wall-normal profiles of mean temperature.

The near-wall inner normalized temperature profile has been shown in the literature [Araya and Castillo, 2012] to be very-well approximated by

$$\phi^+ = y^+ Pr$$

where $\phi = \theta_w - \theta$ is the temperature referenced to the wall temperature. The applicability of this fit to oscillatory flow is informed by comparing the fit to the oscillatory channel flow DNS data of Pond [2015]. Since this dataset is for reversing oscillatory flow it provides, in principle, a conservative estimate of the uncertainty since the flow does not reverse in pulsatile boundary layer flow.
Fig. 2.1 shows the wall-normal profiles of mean temperature at phase increments of $\pi/16$ for oscillatory channel flow over a complete period. The line color matches the freestream velocity as shown in the lower plot. The near-wall fit given by Eq. 2.49 shows very good agreement with the DNS data, even when the flow is reversing.

The procedure to determine the wall heat flux in this thesis is now described. The wall-normal mean temperature profile is formulated in inner variables where $q_w$ is a free parameter. The free parameter $q_w$ is then determined that minimizes the least-squares regression fit of the data when fitted to Eq. 2.49 up to $y^+ = 15$. This “best fit value” is then taken as the measure of the wall heat flux.
Fig. 2.2 outlines the important parameters used in the computational model, where $y_{low}$ is the bottom point in the experimental data and $y_{fit}$ is the top point of the near wall master fit profile. Note that in practice, the uncertainty of the wall-normal position of the data and the closest data point to the wall strongly influences the uncertainty in $q_w$ using this fitting method. The effect of these values on the overall uncertainty is modeled using the oscillatory DNS channel flow data of Pond [2015]. Fig. 2.2 outlines the features of this model in graphical form. Statistical analysis of the model output estimates that with values of $y_{fit}^+ \leq 10$ and with $y_{low}^+ \leq 10$ the error in the wall heat flux was less than 10%. This is demonstrated in Fig. 2.3 which shows the model computed wall heat flux to the actual wall heat flux for the DNS reciprocating channel flow of Pond [2015] for $y_{low}^+ = 1$ and $y_{fit}^+ = 10$. Here it is evident that the fit does a very good job of reproducing the wall heat flux with the largest error, as expected, to be near the phases corresponding to flow reversal.
Figure 2.3. True wall heat flux values along with computed wall heat flux values for each of the profiles in the oscillatory DNS data where points beneath $y^+ = 1$ are removed.

### 2.13 Analogy Between Transport of Momentum and Heat

The mechanisms governing the transport of momentum and heat (when considered a passive scalar) are effectively the same. This similarity can be observed in the equations governing the boundary layer transport of momentum and heat given by Eqs. 2.33 and 2.34. In fact, in a ZPG boundary layer flow, where $dp/dx = 0$, the equations are exactly similar for $Pr = 1$. The underlying principle of Reynolds analogy is that if the transport dynamics of momentum and heat are similar, then the surface flux should also be similar. Then in principle, the surface flux of heat can be inferred from the surface flux of momentum or vice versa.

Within an equilibrium turbulent boundary layer flow, transport is dominated by the turbulent fluxes and since the same turbulent eddies are mixing the momentum and temperature field it is reasonable to assume that the turbulent flux of momentum and temperature are proportional. The
Reynolds analogy assumes that the turbulent flux of momentum is related to the turbulent flux of heat through the turbulent Prandtl number. This relation is briefly described here since it is important to understanding the difference (if any) in the velocity field and temperature field in pulsatile boundary layer flow.

First, the turbulent fluxes are modeled by a gradient transport model (i.e., a mixing length model)

\[ \overline{u'v'} = \nu_T \frac{d\overline{u}}{dy} \quad \text{and} \quad \overline{v'\theta'} = \alpha_T \frac{d\overline{\theta}}{dy} \quad (2.50) \]

where \( \nu_T \) and \( \alpha_T \) are the eddy viscosity and eddy thermal diffusivity, respectively. Then, based on the observation that in very near-wall region the stress and the wall heat flux are constant (i.e., the constant stress region) and equal to their wall values it can then be written that

\[ \tau_w = (\mu + \rho \nu_T) \frac{d\overline{u}}{dy}, \quad \text{and} \]

\[ \overline{q''_w} = - (\lambda + \rho c_p \alpha_T) \frac{d\overline{\theta}}{dy} \quad (2.52) \]

Taking the ratio of these two equations yields

\[ - \frac{\tau_w}{\overline{q''_w}} = \frac{\rho (\nu + \nu_T)}{\rho c_p (\alpha + \alpha_T)} \frac{d\overline{u}}{d\overline{\theta}} \quad (2.53) \]

If \( Pr = \nu/\alpha = 1 \) and \( Pr_T = \nu_T/\alpha_T = 1 \) then the ratio simplifies to

\[ - \frac{\tau_w}{\overline{q''_w}} = \frac{d\overline{u}}{d\overline{\theta}} \quad (2.54) \]

which when separating variables and integrating \( \int_0^\infty dy \) yields

\[ - \frac{\tau_w}{\overline{q''_w}} = \frac{\overline{u}_\infty}{c_p (\overline{\theta}_w - \overline{\theta}_\infty)} \quad (2.55) \]

Recalling the definition of the Stanton number and the friction velocity, the above relation can be rearranged to show that
\[ St = \frac{C_f}{2}, \]  

(2.56)

which yields the relation between the wall shear stress and the wall heat flux for the special case when \( Pr = 1 \) and \( Pr_T = 1 \). The more general relationship is known as the Colburn formulation introduced previously.

The critical element that holds together the analogy relation between the wall shear stress and the wall heat flux in a turbulent flow is that that the \( Pr_T(y) = \text{constant} \). This is because in a turbulent flow the wall-normal transport of momentum and heat is dominated by the turbulent fluxes (i.e., the eddy viscosity and eddy diffusivity are much larger than their molecular counterparts). Importantly, to assess the validity of Reynolds analogy, DNS allows for the \( Pr_T \) to be computed directly:

\[ Pr_T = \frac{\nu_T}{\alpha_T} = \frac{u'v' \left( \frac{\partial \theta}{\partial y} \right)}{v'\theta' \left( \frac{\partial u}{\partial y} \right)}. \]  

(2.57)

The assumption is relatively valid in equilibrium boundary layers. For example, Fig. 2.4 shows wall-normal profiles of \( Pr_T \) in steady-state turbulent channel flow at two \( Re_T = u_T h/\nu \). Evident is that \( Pr_T \) is approximately constant across the channel height, and that the degree of constancy is greater for the higher \( Re_T \). The assumption, however, is known to breakdown in non-equilibrium flow [Blackwell et al., 1972, Bradshaw and Huang, 1995, Araya and Castillo, 2012]. One element of the present work is to evaluate if the wall shear stress and the wall heat flux are proportional in pulsatile boundary layer flow.
Figure 2.4. $Pr_T$ profiles as a function of the distance from the wall in a steady channel flow, with temperature as a passive scalar. Figure courtesy of Samir Sid.
CHAPTER 3
EXPERIMENTAL TECHNIQUES

The focus of this work involves the precise description of the velocity and temperature fields in a developing boundary layer flow in both time and space. For the flow under investigation, a broad range of spatial and temporal scales exist, the examination of which requires careful and detailed measurements. The following sections detail the experimental techniques and post-analysis procedures used within this study.

3.1 Thermal Measurements

Measurement of the temperature field were acquired through a number of methods with both single point measurement probes and surface field measurements described below.

3.1.1 Thermocouple

Thermocouples were used as a method of feedback and control to maintain the fixed thermal boundary conditions and as a method of experimental evaluation to acquire wall-normal profiles of temperature within the boundary layer. Thermocouples employ the Seebeck effect which occurs when the thermocouple probe, consisting of two dissimilar bonded metals, is exposed to a differing temperature than the thermocouple leads (i.e. temperature difference between two junctions), resulting in a voltage potential. The voltage potential correlates to the temperature difference. When one junction is held at a fixed temperature, the voltage provides a measure of the temperature at the measurement junction. Fig. 3.1 displays a representation of a type E thermocouple as was used in the present study to measure the temperature within the thermal boundary layer. The sensitivity of thermocouples is within the range of 50-100mV/°C. The measured voltage potential from the probe requires a reference voltage potential which corresponds to the temperature at the DAQ,
this is known as the cold junction temperature. The cold junction voltage is used for cold junction compensation which adjusts the measured probe voltage (probe temperature) to remove any drift to changes in temperature of the DAQ.

A thermistor chip from analog devices (TMP36) is used to measure the cold junction temperature. The thermistor relates a change in resistance to a change in temperature, the output of which is fed through a low pass filter (2Hz cut off frequency) prior to being sampled, the details of this circuitry can be observed in Fig. 3.2. Probe thermocouple signals are amplified and converted to a temperature value using a National Instruments DAQ board, NI-6215.

The temporal and spatial scales that can be resolved with a thermocouple are dependent upon the probe size and the DAQ board utilized. Through a first order transfer function, the time response of a thermocouple system can be approximately determined from:

$$\tau = \frac{\rho c_p V}{h A_s}$$  \hspace{1cm} (3.1)

in which $\rho$ is the average density, $c_p$ is the specific heat, $V$ is the volume, $h$ is the convective heat transfer coefficient, and $A_s$ is the surface area where all are respective to the thermocouple probe. The type E micro Campbell Scientific thermocouple, which has a diameter of 0.0127mm, employed for profiling has an approximate time response of 150Hz.

Figure 3.1. Details of the type E thermocouple probe.
Figure 3.2. Wiring overview of DAQ system for thermal measurements.

3.1.2 Profile Characteristics

For each temperature dataset acquired during the pulsatile thermal boundary layer measurement, the thermocouple probe was traversed across the boundary layer in discrete steps. At each discrete step, corresponding to a fixed wall-normal position, the probe was held stationary and a time series was collected that will be used to quantify the thermal field behavior at that location. The first measurements was at $y = 0.43$ mm and the last measurements was at $y = 60.43$ mm, where $y = 0$ denotes the wall. Temperature data was collected at forty discrete wall-normal positions between the first measurement point and the last measurement point. These data points were chosen to approximately match the wall-normal locations corresponding to the PIV data set used to measure velocity. The sampling points were spaced linearly from the wall as both the near wall and far field were of interest and each required high spatial resolution. Each point was sampled for 180 seconds to provide high confidence in the time averaged mean values. The sampling time resulted in at least 7 forcing cycles based on the lowest forcing frequency studied. Each position was sampled at 3kHz which was set by the requirement of fully resolving the quadrature encoder pulse.
signal at the highest forcing frequency studied. For further discussion see Section: Uncertainty Analysis.

### 3.1.3 IR-Camera

Surface temperature measurements of the thermal wall plate were acquired by an Infra-Red FLIR SC645 camera in order to evaluate the spatial uniformity. The IR camera utilizes a microbolometer 14-bit sensor with an array size of 640 x 480 pixels. Images can be acquired at up to 50Hz. The camera comes with a factory calibration with a prescribed accuracy of ±2% of the measurement value. This IR camera is used to validate that the thermal wall plate develops and maintains a uniform wall temperature in both the spanwise and streamwise directions.

![Mock setup of IR camera positioned above working section of tunnel, imaging down onto thermal wall plate for validation purposes.](image)

**Figure 3.3.** Mock setup of IR camera positioned above working section of tunnel, imaging down onto thermal wall plate for validation purposes.

### 3.2 Velocity Measurements

Velocity measurements within the NEAT tunnel were acquired by two separate methods: micro pitot-static tube, and high-speed Particle Image Velocimetry (PIV).

#### 3.2.1 Pitot-Static Tube

At the measurement end of the pitot-static tube there is a 90 degree bend which orients the measurement end of the tube into the flow direction (-x). After the bend the probe reduces from 3.1mm to approximately 1mm before coming to a rounded end with a pressure tap at the measurement.
port. The 90 degree bend is > 10 diameters away from the measurement location and therefore causes little to no interference with the measurement. The measurement port of the pitot-static tube induces flow stagnation due to its parallel orientation to the normal flow direction. A second pressure tap on the upper surface of the tube, downstream of the measurement port, is exposed to the static pressure of the flow as it moves parallel to the tube. A diagram depicting the structure of a pitot-static tube is given in Fig. 3.4. Both the stagnation \((P_1)\) and static pressure \((P_2)\) are measured using a pressure anemometer which measures the differential pressure through the use of a diaphragm. The output differential pressure can be converted into a velocity value as described by reducing Bernoullis’ equation down to:

\[
V_\infty = \sqrt{\frac{2 \times (P_1 - P_2)}{\rho}}
\]  

(3.2)

The output of the anemometer is fed into the NI-DAQ-6215 DAQ depicted in Fig. 3.2. Based on the length of the tubing running between the diaphragm and the probe body, the pitot-static tube used has a time response of 0.0027s [Iberall, 1950].

The pitot-static tube measurements was used to monitor \(u_\infty(t)\) during the temperature profiles and assisted in the post-processing method of phase averaging. The pitot-static tube also served as an independent velocity measurement when particle image velocimetry was employed as described below.
3.2.2 Particle Image Velocimetry

Particle image velocimetry (PIV) is a non-intrusive experimental method to measure instantaneous planar velocity fields. Laser light formed into a sheet is scattered by tracer particles seeded into the flow and recorded by a camera position orthogonal to the laser sheet. For this study the laser sheet is oriented in the streamwise-wallnormal plane. Upstream of the measurement location the tracer particles are introduced dispersively throughout the flow field. These particles are chosen to have Stokes ($St_p$) and Froude ($Fr$) $<< 1$ such that their movement will accurately follow the motion of the flow field. The $St_p$ represents the ratio of the inertia of a tracer particle relative to the inertia of the flow field and the $Fr$ represents the ratio of the settling velocity of the particle to the advection velocity. For the particles used as tracer particles in this study $St_p$ is $3 \times 10^{-8} \leq St_p \leq 2 \times 10^{-6}$ and $Fr$ is $1 \times 10^{-6} \leq Fr \leq 7 \times 10^{-6}$, with a nominal particle diameter of $1 \mu m$, where the ranges represent the range of time and velocity scales throughout the boundary layer and throughout the experimental conditions under test.

The tracer particle scatter laser light from the laser sheet towards a camera oriented perpendicular to the laser sheet. When the camera and the laser sheet are synced at high frame rates
 (>1kHz), subsequent images are acquired which capture the rapid small scale motion of the tracer particles. Due to the known frame rate of the system (Δt), adjacent images can be processed to determine the displacement field (Δx, Δy) which leads to the computation of the velocity field.

![Figure 3.5. Setup of 2 camera high speed PIV system](image)

The camera and laser setup, located at the downstream measurement location used in this study, is detailed in Fig. 3.5. The laser light is produced by a Photonics DM-series dual cavity Nd:YLF laser capable of 30mJ per pulse. The optics train consist of a spherical focusing optics, a series of 90° turning mirrors, and a cylindrical lens to create the laser sheet. The laser sheet is directed into the tunnel through a window in the top wall of the tunnel made up of BK7 high quality optics glass. The laser sheet is aligned in the center of the tunnel and is focused on the tunnel floor resulting in a sheet thickness of ≈ 1mm.

Images are acquired using two 12-bit Photron FASTCAM SA4 CMOS cameras. The CMOS array size of each camera is 1024 pixels × 1024 pixels. The two cameras are placed on opposite sides of the tunnel and image the same plane but with different field-of-views (FOV). This is done to achieve high spatial resolution in the near wall region while still resolving the entire boundary layer. Camera 1 has a FOV ranging from 2mm ≤ y ≤ 26mm and camera 2 has a FOV ranging from 5mm ≤ y ≤ 54mm, where y = 0 is the bottom wall. Two instantaneous images of the raw counts acquired by the two cameras during a pulsatile boundary layer experiment with PIV are displayed in Fig. 3.6.
PIV images pairs were acquired on a 40Hz loop with a frequency between subsequent images of 3.6kHz. The image pair looping procedure is used to increase the number of independent measures acquired. The obtained spatial resolution of Camera 2 is $0.4\mu m$ per pixel and $0.2\mu m$ per pixel for Camera 1. The images were then analyzed using LaVision PIV software, DaVis 8.3.1. The resulting vector fields are shown in Fig. 3.7.

Fig. 3.8 displays the PIV system in operation. The analysis procedure and details on the post-processing methods are detailed below in Sec. 3.4.
Figure 3.7. Vector fields produced from each camera, left figure displays the vector field from Camera 2 and the right images displays the high resolved vector field from Camera 1.

Figure 3.8. Details the operation of the PIV system within the NEAT facility during the experimental campaign to acquire pulsatile velocity fields
3.3 Thermal Field Post Processing

To filter out high frequency noise, the temperature data is passed through a 6th order Butterworth filter with a cut-off frequency at 40Hz, set to match the sampling frequency of the PIV velocity measurements. Fig. 3.9 displays a portion of a sample temperature time series at a single wall normal position. The top panel represents the angular position of the encoder mounted to the paddle producing the pulsatile waves. A full paddle cycle (360 degrees) corresponds to two full cycles in the velocity forcing. The second panel displays the velocity time series from the pitot-static tube, the third panel displays the temperature time series from the profiling thermocouple and the bottom panel displays the pulse counter (z-index) on the encoder which is used to verify the angular position output.
3.3.1 Phase Averaging

The thermal field is phase averaged based on the output from a quadrature encoder mounted onto the paddle system which produces the sinusoidal varying pressure gradient. This is done as the point-wise thermal measurement technique could not be used to reliably discretize the individual wave pulses required for phase averaging. The exact thermal field response is unknown and therefore true angular output, seen in Fig. 3.9 of the encoder is utilized to bin the temperature signal into discrete phases. \( \langle \theta \rangle \) is determined through a rms value of all the temperature values placed
within a discrete phase bin. Through the use of the phase average velocity each phase average temperature value is shifted such that the minimum \( \tilde{u} \) velocity corresponds to \( \phi = 0 \).

### 3.4 Velocity Field Post Processing

The PIV data is passed through a standard deviation filter which has been designed to remove spurious vectors from the instantaneous PIV vector fields [Westerweel, 1994]. A time series of velocity values is computed for each test, where each value in the time series corresponds to the spatial average of subsequent velocity vector fields within one experiment. These time series are strongly sinusoidal due to the nature of the flow, therefore basic long time average standard deviation filters do not work as the variance can not be properly determined. A model time series is fit to the computed time series based upon; the mean value of the time series, the pulsatile frequency (as determined below in Section: Phase Averaging), and the amplitude of the time series. Variance is then computed between the model time series and experimental time series such that the sinusoidal nature of the flow is removed based upon a long time average of the sinusoidal nature. Vector fields corresponding to data points outside of 3 standard deviations are removed and this process is repeated three times.

#### 3.4.1 Profile Stitching

To acquire fully resolved velocity measurements of the boundary layer the data from Camera 1 and Camera 2 are stitched together. The stitching method utilizes a single streamwise velocity profile \( u(y) \) from each data set.

A wall normal matching region is defined between the two profiles to evaluate dataset collapse which is then used to define an overlap region which is used to perform the stitching between datasets. The extent of the collapse in velocity values between the two datasets is examined in the matching region for time averaged streamwise velocity values, \( \overline{u} \), as described below. The wall normal location of the highest point for the matching region is defined as top vector position in the Camera 1 dataset. The bottom most point in the matching region is defined as the bottom point in the Camera 2 dataset, where spurious edge vectors are excluded from the choice of both positions.
The matching region, the $\bar{u}$ velocity values from each dataset are interpolated onto a uniform grid for comparison. The mean error between $\bar{u}_{Cam1}(match)$ and $\bar{u}_{Cam2}(match)$ is computed. The wall location from Camera 2 data is allowed to vary by $\pm$ the height of one interrogation area ($\approx 0.32\text{mm}$) as the higher resolution data of Camera 1 more precisely depicts the $y = 0$ location. The wall location shift corresponds to the lowest mean error in the matching region. Within the matching region the wall normal location corresponding to the lowest error between the two datasets is taken as the middle overlap point, $y_{mid}$. The overlap region is then defined around this value, where the start is defined as 5 points below $y_{mid}$ in Camera 1 data, $u_{Cam1}(mid - 5)$, and the end is defined as 5 points above $y_{mid}$ in Camera 2 data, $u_{Cam2}(mid + 5)$. Smoothing functions are used such that as the wall normal position increases through the overlap region the weight of Camera 1 goes from (1-0) and Camera 2 goes (0-1), smoothly transitioning between the two datasets. The value of the stitched data at $y_{OverlapStart}$ and $y_{OverlapEnd}$ then undergoes a forward smoothing ($y_i = (\frac{1}{3})(y_{i-1} + y_i + y_{i+1})$) to reduce any non-physical discontinuities or jumps in the profile. Fig. 3.10 displays an example of the implementation of this stitching method, where the overlap region between the two datasets is defined. The near wall error in Camera 2 and far field error of Camera 1 are clear, where the resulting stitched profile smoothly transitions between the datasets and discards the regions of high error.
3.4.2 Phase Averaging

The velocity field is phase averaged based on the pulsatile forcing frequency determined from the magnitude of an FFT of the free-stream velocity, $u_\infty(t)$. An example plot of $FFT$ magnitude from one of the experiments is displayed in Fig. 3.11.
Figure 3.11. Displays a representative $|FFT|$ from freestream values of the velocity field measurements.

The peak value of this trace is the forcing frequency and the velocity time series is then phase averaged based upon this frequency and the known sampling frequency. The phase average traces of each test are shifted such that the minimum $\tilde{u}$ velocity corresponds to $\phi = 0$.

3.5 Uncertainty Analysis

Measurement uncertainty was quantified through examination of the standard deviation of the respective measurements. The uncertainty was determined from;

$$\Delta X = \sigma \sqrt{\frac{1}{N_{eff}}}$$  \hspace{1cm} (3.3)

where $X$ is the variable of interest, $\sigma$ is the standard deviation of the respective variable, and $N_{eff}$ is the number of independent measures [Bendat and Piersol, 2000]. The number of independent measures can be determined from the turn-over time of the tunnel.

$$N_{eff} = \frac{T_s}{L/u_\infty}$$  \hspace{1cm} (3.4)
In which \( L \) is the streamwise distance to the measurement location, \( T_s \) is the sampling duration, and \( \overline{u_\infty} \) is the time average mean freestream velocity. Based on the sampling time of the data sets and the lowest \( \overline{u_\infty} \) under investigation, the lowest number of independent samples is 65. This is used to determine that for the velocity fields under investigation a ±2% measurement uncertainty range should be incorporated. For the temperature fields under investigation a ±5% error is determined from Eq. 3.4, which approximately matches the listed error of type E thermocouples. For the determination of the measurement uncertainty in the wall shear stress the analysis of Mehdi and White [2011] is used to determine the measurement uncertainty is ±5%. The error in the determination of the wall heat flux comes from the analysis of measurement uncertainty of the data of Pond [2015], the details of which can be seen provided in Appendix A. The measurement uncertainty of the wall heat flux computation is ±8%. For the cross correlations used to determine angular lag of time series, the confidence in these correlations being significantly different from zero was established from the use of a Chi-Square distribution significance test [Bendat and Preisol, 2000]. These above detailed errors are carried through the analysis and displayed as shaded contours or ± error bars on singular measurements.
CHAPTER 4
NEAT BOUNDARY LAYER WIND TUNNEL

4.1 Experimental Facility

The NEAT boundary layer wind tunnel is an open-circuit in-draft type designed to study heat transfer in non-equilibrium boundary layers. Figure 4.1 shows the design schematic for the facility. The key design components are: the thermal wall plate which is used to control the lower-wall thermal boundary conditions seen by the flow, and the pulsatile flow generator which creates sinusoidal pressure gradients within the tunnel which is the main non-equilibrium condition under investigation. The coordinate system for the tunnel is such that \( x \) denotes streamwise distance, \( y \) denotes wall normal distance, and \( z \) denotes spanwise distance.

4.1.1 General Description

The inlet section to the tunnel consists of a feed-back controlled resistive heater bank, a seeding manifold, a turbulent management section and a 4:1 contraction. The test-section of the tunnel nominally measures \( 303\text{mm} \times 111\text{mm} \) cross-section and 2.75m in length and is made of plexiglass to allow optical access. Removable windows in the top wall and bottom wall of the tunnel sit at
45, 136.3, and 251.8 cm from the test section inlet which allow for the introduction of probes, laser light, IR measurements, and wall plate control wiring into the tunnel test section.

Air enters the tunnel through a series of systems which set the outer boundary conditions and condition the flow prior to entering the tunnel test section. A feedback controlled thermal wall-plate sits on the floor of the test section and is used to control the lower-wall temperature seen by the flow. The upper wall of the test-section is angled at $0.23^\circ$ to closely maintain a ZPG condition along the length of the test-section. Downstream of the test-section is a pulsatile flow generator used to produce a sinusoidal pressure gradient, creating a non-equilibrium flow condition. The rear diffuser transitions the flow area from a rectangular to a circular cross-section where it connects to a belt driven centrifugal fan. A frequency controller is used to control and maintain the fan speed and corresponding flow speed in the test-section. The freestream velocity in the test-section can vary between 1 to 12 m/sec. The entire tunnel sits on a custom frame which levels and isolates the tunnel test-section. In the following subsections, the components of the facility are described in detail.

### 4.1.2 Inlet Section

Air entering the test-section first passes through an OMEGA Engineering air duct heater (model CABB-1211/208). The 3-phase 208V heater consists of nine sheathed finned chrome steel resistive heaters that provide 12 kW of power (with a power density of $4.03\text{W/cm}^2$). The cross-sectional area of the heater is $390.5\text{mm} \times 358.8\text{mm}$ with an open area of $859.4\text{cm}^2$ (blockage of about 39%). The three legs of the AC power are connected to the heater through a Watlow Din-A-Mite C silicone controller rectifier (SCR) power controller. A type J thermocouple placed in the freestream 1 m downstream of the test-section inlet provides feedback for a proportional, integral, and derivative (PID) controller used as the input for the SCR power controller. The SCR power controller, configured for zero-voltage crossover firing (as opposed to phase angle crossover firing) to reduce electrical noise, is used to regulate the duty cycle of the voltage (either 0% or 100%) to the heaters thereby controlling the freestream temperature. A SCR power controller was chosen...
Figure 4.2. (a) Probability density function (PDF) of freestream temperature measured in the wind tunnel over a two hour period. The vertical green line represents the setpoint temperature and the vertical red line represents the measured mean temperature with a confidence interval of 99%. (b) Schematic of the seeding manifold used for particle image velocimetry (PIV).

(over a mechanical relay or solid state relay) as it is more suitable for handling the large current (33.4A) needed for the heater and has a fast response time of 5.56ms, which not only allows for a tightly controlled freestream temperature but also prolongs the life of the heating elements by reducing the thermal fatigue.

The PID controller gains are optimized for varying freestream velocity and temperature conditions, as one set of gain values does not work well over a large operating range. This is achieved using the autotune feature of the SCR power controller-PID feedback system. The ability of the system to hold a freestream set point temperature is shown in Fig. 4.2(a). After exiting the heater, the air flow passes through a honeycomb-type seeding manifold to introduce (if desired) tracer particles into the flow to be used for particle image velocimetry (PIV). The challenge with seeding an open-circuit wind tunnel is that the flow tracers must be uniformly distributed into the flow at the source. This is much more difficult than seeding a closed-circuit wind tunnel where the seed can be allowed to circulate through the facility to produce a uniform seed concentration.

The custom designed seeding manifold is shown in Fig. 4.2(b). The inlet air flows across the manifold through 248 PVC tubes of length 38.1mm and diameter 21.34mm. Four slots of 25.4mm length and 6.35mm width separated by 90° (center-to-center) are cut into each PVC tube. The plenum volume of the seeding manifold is filled with a dense fog of nominal 1µm diameter oil.
droplets [Shakerin and Miller, 1995] through up to twenty-three 21.34mm diameter pipes mounted around the perimeter of the seeder connected to a ROSCO 1700 fogger. The fog is drawn into the PVC inlet air piping through the four slots owing to the pressure difference between the fog in the plenum and the air in the PVC tubes. The open area percentage of the manifold (56.5%) is comparable to the open area of the freestream heater (61%). The air then passes through a turbulence management section containing 4 screens of decreasing wire mesh size and then a honeycomb section. The screens reduce axial turbulence while the honeycomb reduces lateral turbulence. The air flow then proceeds through a 4 : 1 contraction to speed up the flow and to further reduce turbulent intensities.

4.1.3 Test Section

![Multi-plane view of the working section of the tunnel with physical dimensions.](image)

To investigate turbulent boundary layers, a 3mm diameter trip rod extending the spanwise extent of the test-section is placed on the rear of the leading-edge nose, just upstream of the first convective plate. The rod induces transition to turbulence and fixes (on average) the starting location of a developing turbulent boundary layer. A range of trip diameters as well as shape were tested to determine the smallest diameter which would lead to a steady (on average) transition. An undersized trip does not induce enough energy and the flow can relax back to a laminar state, while too large of a trip produces near wall effects which do not decay in an appropriate range [Marusic et al., 2015]. Wall normal velocity profiles taken at the first measurement location (45cm) verified the appropriate trip diameter by comparing against velocity profiles from literature. This can be done as the recovery length does not depend on the order of the statistic measured [Marusic et al., 2015].
Three windows of size 254mm \times 102mm sit in the top wall of the test-section located at approximately 45, 136.3, and 251.8cm from the test-section inlet. The windows are used for introduction of laser light, for infrared (IR) imaging and for inserting various measurement probes. The window body, seen in Fig. 4.4, contains 4 leveling screws such that the window can be set flush with the inside wall of the tunnel. Window inserts made from either BK7 glass or clear poly-carbonate can be installed depending on the requirement. The optical quality of the glass inserts are better than the plexiglass walls and necessary for IR imaging or the introduction of laser light. The tunnel frame, which supports the working section of the tunnel, has leveling adjustments in the x and z direction such that the tunnel floor sits parallel to the adjacent optics table. This then orients the x-y plane of the tunnel to any optical equipment mounted to the optics table. Custom optics mounting boards line the streamwise length on either side of the tunnel for ease of experimental setup. The entire frame sits on damping feet which isolate the tunnel from any ambient vibrations in the room.
4.2 Thermal Wallplate

4.2.1 Prototype Wallplate

The thermal wall plate covers the bottom floor of the NEAT test section and controls the bottom wall boundary condition by setting $T_{wall}$. The design is modeled after the work of Blackwell et al. [1972]. The design consists of convective plates which are heated and maintained at a set temperature. The main design condition is that the system must be able to maintain a set wall temperature independent of the above flow state. Other design constraints were to minimize the height of the system so as to reduce blockage effects, minimize conductive heat loss from the convective plates so as to ensure all heat was going into convective transfer, and to design a system which could be manufactured within the UNH Kingsbury Hall machine shop within a suitable time frame and installed with ease into the NEAT tunnel.

For prototype development a different open circuit down draft wind tunnel was used as it had a smaller development length and had a greater ease of access to the working section. This was desireable for the ability to perform quick changes and modifications within the wallplate system. The working section of the tunnel is $0.45m \times 0.45m \times 0.66m$ (z by y by x). To account for the streamwise variation in heat flux (coming from a developing boundary layer) the prototype design consisted of two convective plates with increasing streamwise length based on downstream location. This also assisted in ease of manufacturing as the parts could then fit within the working area of the UNH Kingsbury Hall machine shop.

Fig. 4.5 shows the solid model design of the prototype thermal wall plate. The convective plates are aluminum 6061 plates which are 9.5mm thick. For heating the Al plate, there is an array of Kapton polyimide-fllm resistive resistive heaters (affixed to the bottom of the aluminum plate) with a heating density of $1.5W/cm^2$. In between the heaters there are a series of holes which contain J-type thermocouples, which then sit 2.5mm beneath the top surface of the plate and act in a feed back loop to control the convective plate temperature. Each Al plate sits into
Figure 4.5. Solid model of prototype thermal wall plate with two convective plates.

A custom machined section of insulation made from calcium-silicate. Each of the insulated wall plates then sit into a frame manufactured from Delrin which sets the spacing of each plate, and was chosen for its low thermal conductivity and machinability. Due to a desire to produce high quality physics grade boundary layers, a leading edge nose was used to smoothly transition the flow onto the thermal wall plate. The nose utilizes a super-ellipse design to prevent flow separation which was determined according to Eqn. 1 of Narasimha and Prasad [1994],

\[
\left(\frac{a - x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1
\]  

(4.1)

where \(x\) is the streamwise position, \(y\) is the wall normal position, \(a\) is the length of the nose, \(2b\) is the height of the nose. To accurately recreate the desired profile, a three-axis controlled CNC machine was used to smoothly cut the profile into the Delrin. Fig. 4.7 shows the fabrication process applied to the final design.

The prototype design was used for testing and developing wall temperature control schemes. A variety of measurements were made with the plate including: ZPG wall temperature distribution, and wall temperature distribution due to wake behind a hemisphere. The ZPG conditions validated the base design and demonstrated that ZPG thermal boundary layer developed as expected when compared to the literature. The wake study gave insight into the performance of the design in
complex flow states. It was found that future designs should utilize spanwise oriented bottom heaters as streamwise oriented heaters require individual zone control, where multiple controllers would be used to control the temperature of a single plate. The final design for installation into the NEAT tunnel incorporates many lessons learned from the prototype plate such as manufacturing limits, material limits, and ease of operation and repair.

4.2.2 Finalized Wallplate

![Diagram of thermal wall plate](image)

**Figure 4.6.** (a) Solid model schematic of the thermal wall-plate. (b) Detailed schematic of a section of the thermal wall-plate: right tilted 45° lines denote the heated wall plate, left tilted 45° lines show the surrounding layer of calcium silicate insulation and the hash marked section represents the co-polymer acetal frame which positions all components. Three evenly spaced embedded thermocouples are located along line AA.

The final thermal wall plate design comprises many of the features of the prototype system, yet all parameters will be re-iterated here for purposes of clarification. The thermal wall plate (shown in Fig. 4.6) comprises of twelve sections each independently heated and controlled. Each section consists of a 9.5mm thick aluminum 6061 plate, Kapton polyimide-film resistive resistive
Table 4.1. The length of the convective plates. Plate 1 is at the upstream end and plate 12 is at the downstream end of the wind tunnel.

<table>
<thead>
<tr>
<th>Plate number</th>
<th>Length (mm)</th>
<th>Plate number</th>
<th>Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2</td>
<td>75</td>
<td>3, 4, 5</td>
<td>125</td>
</tr>
<tr>
<td>6, 7, 8</td>
<td>175</td>
<td>9, 10</td>
<td>275</td>
</tr>
<tr>
<td>11</td>
<td>325</td>
<td>12</td>
<td>425</td>
</tr>
</tbody>
</table>

Heaters (affixed to the bottom of the aluminum plate) with a heating density of $1.5 W/cm^2$, and a 5mm thick calcium silicate holder used for thermal isolation of the aluminum plate (the thermal conductivity of the calcium silicate is four-orders of magnitude less the aluminum 6061). Three evenly spaced J-type thermocouples arranged spanwise in the center of each plate and embedded 2.5mm beneath the top surface of each aluminum plate are used to monitor wall temperature for feedback control of wall heating. To increase ease of assembly and repair, a thermocouple holder was designed to maintain the position of each thermocouple throughout assembly and operation. To ensure each thermocouple had adequate contact with the convective plate, each thermocouple hole was filled with a silver grease compound which is thermally conductive and electrically insulative. The section components sit in a Delrin (acetal) frame, which incorporates a method of leveling each convective plate individually such that the overall surface roughness of the assembly can be reduced. The streamwise (flow direction) length of each section increases with downstream position such that the convective heat transfer from plate-to-plate does not vary by more than a suitably chosen threshold of 15%. This threshold is set by examining the smallest feasibly manufacturable streamwise plate length of the most upstream plate, as this is where the heat flux changes most rapidly. The length of the plates are summarized in Table 4.1.

For ease of manufacturing the streamwise length of the plates stayed constant for 2-3 adjacent plates. The entire wall plate system was designed in a manner such that it could be assembled/disassembled by only one person. Interlocking frame sections are custom fit to within $-0.127 +0.0$ mm of the tunnel width such that the system can be assembled plate by plate and slid into the tunnel from the inlet. Routed wires for heater power and thermocouple output are fed out through the bottom windows which match the position of the top windows. Detailed descriptions and draw-
ings of the individual wall-plate components as well as description of the assembly and installation process can be found in App. B.

Figure 4.7. (a) Image of roughing pass for the fabrication of the super-ellipse leading edge nose of the wall plate (b) final image of leading edge frame component for the wall plate

The temperature of each section of the thermal wall-plate is monitored and maintained by its own feedback controller as illustrated in Fig. 4.8. LabVIEW is used to define and implement the controller settings. Each controller consists of a 10A SCR and an NPN transistor. The average temperature of the three embedded thermocouples in each convective plate serves as the feedback parameter to direct the SCR to block/pass the 110 VAC that powers the resistive heaters (effectively the SCR serves as a switch to quickly turn the heaters on/off). The time constant of the controller is significantly smaller than the time constant of any given convective plate so the controller can monitor and adjust the heat-input to the convective plate much faster than the plate can lose (gain) heat to (from) the flow. The operating plate temperature range is 20°C to 65°C which is set by the working temperature of the materials used. The controller is able to set and maintain a temperature of the system to within 0.5°C as determined from the variance of the embedded thermocouples during testing. Details on the controller design and operation can be found in Ebadi [2016]. Importantly, since the temperature of each section of the thermal wall plate is independently controlled, the design allows for the application of a wide-range of thermal boundary conditions: e.g., isothermal, streamwise temperature gradient, discrete temperature steps, among others.
4.3 Pulsatile Flow Generator

Pulsatile flows are periodic and unidirectional characterized by both their frequency and amplitude. The flow over a half-period first accelerates to maximum velocity then decelerates to the mean velocity. During the next half-period, the flow first decelerates to minimum velocity then accelerates to the mean velocity. In a pulsatile boundary layer flow, the acceleration/deceleration of the freestream velocity produces non-equilibrium flow behaviors. In particular, when the oscillation period is comparable to (or smaller than) the turbulence relaxation time (order $100\nu/u_\tau^2$), [Peters et al., 1993]), there will be a phase difference between the oscillating strain and stress field [Weng et al., 2016]. Detailed below are a variety of methods for implementing pulsatile flow in an experimental setup. Each system, which has been tested in the NEAT tunnel, is described with the intent to provide sufficient information to be useful for future development of pulsatile flow generating systems.

4.3.1 Rotor-Stator Assembly:V0

A rotor-stator assembly located downstream of the test-section was the first design used to produce a pulsatile freestream velocity in the tunnel. The design of the rotor-stator, shown in
Fig. 4.9. Schematic of rotor-stator design, left shows the rotor, shown in the middle is the stator, and the right plot depicts a freestream velocity time series taken with a pitot-static tube for a quarter revolution of the rotor-stator mechanism.

Fig. 4.9, is modeled after the design of Al-Asmi and Castro [1993]. It consists of a rotor with 4 uniformly spaced holes, and a stator with 4 matching slots. The open area to the incoming flow is continuously modulated which produces a smooth sinusoidal freestream velocity signal as shown in Fig. 4.9. The rotor frequency is controlled via a DC motor with variable speed settings from 1 – 100Hz. Twelve air-bleed slots arranged on the stator control the ratio of fluctuating area to open area. The slots ensure the existence of a mean flow and by changing the ratio of fluctuating area to mean through area the pulsatile flow amplitude can be adjusted. In the present design, this area ratio can be adjusted from 40% to 100%. The rotor-stator assembly was mounted downstream of the test section to reduce the effect of induced rotational flow within the test-section. The assembly was designed such that it can be easily removed if a pulsatile flow is not desired.

4.3.2 Pulsatile Flow Generator: V1

The rotor-stator mechanism went through a re-design process due to the desire for a design with a lower blockage ratio which would then allow for testing of higher $Re$ flows, as seen in Fig. 4.10. Karlsson [1958] used a vertical array of spanwise orientated paddles were rotated to create a pulsatile pressure wave. The rotating paddles create a similar sinusoidal change in through area yet drastically reduce the blockage ratio, relative to design V0. A similar design was developed to fit within the rotor-stator mechanism box which uses 1 paddle that mounts into bearings on each
spanwise wall. The paddle spans the entire width of the tunnel and connects to a DC gear motor that sits outside the tunnel and operates in a feedback loop with an Arduino. The paddle height $H_p$ can be varied by installing different paddles which provides the ability to vary the fluctuating area ratio. The blockage ratio is reduced to 5% allowing for higher $Re$ flows to be studied, yet pulsatile amplitude due to design V1 remained low at $\approx 10\%$.

4.3.3 Pulsatile Flow Generator: V2

Figure 4.11. Schematic of updated pulsatile wave generator with a paddle design. The left image displays the y-z plane with flow going into the paper, the right figure shows a cutaway image of the x-y plane on the tunnel centerline (A-A). The paddle height $H_p$ can be adjusted to change the fluctuating area and therefore the pulsatile amplitude.
In an effort to increase the parameter space accessible with the pulsatile system an updated design was developed to yield a larger cross section. By maintaining the average blockage ratio and increasing the % closed blockage ratio in a cross section that has twice the original open area the system can operate at amplitudes ($\tilde{U}_∞/U_∞$) of over 60%. Fig. 4.12 displays a selection of freestream pulsatile velocity signals over one full pulsatile forcing cycle or one half turn of the paddle system.

Figure 4.12. Freestream pulsatile velocity signals at a variety of freestream speeds and freestream forcing frequencies plotted vs once cycle ($\gamma = 0 – 360^\circ$).

The updated pulsatile system was designed to fit behind the rectangle to round diffuser where the tunnel has a 0.305m inch diameter. A 0.305m long clear acrylic section of tubing was installed with rubber damper fittings on either end to increase the isolation of the working section of the tunnel from vibrations of the driving fan. Inside the tubing a circular paddle was designed with a closed blockage ratio of 80%. The circular paddle has two center mounted axles which protrude out of the tubing through exterior mounted bearings. All components were designed to a tight fit to reduce any pressure drop leakage. One of the axles is attached to a DC gear motor through a flex coupler, which is controlled by an Arduino. On the opposite side, the other axle is mounted to a 1024 rotary encoder through another flex couple for precise measurements of the paddle position. A detailed drawing, in Fig. 4.11, depicts a solid model of the design. An image of the prototype and final fabricated design, as produced in the UNH Kingsbury machine shop, shown in Fig. 4.13.

This system is designed such that any frequency range can be obtained by changing the selected gear motor used to drive the system. The pulsatile paddle system is currently capable of operating
Figure 4.13. Fabricated paddle system displaying the prototype on the right and the finalized design utilized in the PBL campaign on the left.

from 0.01Hz to 1Hz. At each paddle frequency, the tunnel is able to produce freestream speeds ranging from 1m/sec to 5m/sec, this results in a $w^+$ domain of $1 \times 10^{-4}$ to $1 \times 10^{-2}$. With this parameter space this facility will have the capabilities of reaching the intermediate frequency forcing condition.

4.4 Validation Tests

The NEAT tunnel is validated for three test cases: nominal ZPG turbulent boundary layer flow (1) over an isothermal wall plate, (2) encountering a sharp step in wall temperature, and (3) around a wall mounted hemisphere body. Wall-normal profiles of temperature and velocity across the boundary layer are acquired for case (1) and (2). For case (3), IR imaging is used to show that the wall plate can maintain a constant temperature even when the flow above the wall is unsteady and strongly three-dimensional.

4.4.1 ZPG turbulent boundary layer flow over an isothermal wall plate

The freestream temperature of the air in the tunnel is set to 25°C and each section of the thermal wall plate is set to 40°C. Measurements are acquired at five different momentum thickness
Reynolds numbers, $Re_\theta$, as listed in Table 4.2. The freestream velocity is steady with magnitudes varying between $2 - 10$ m/sec. The FLIR SC645 IR camera is positioned above plate #5 located 0.8m downstream of the trip and the FOV is $85\text{mm} \times 64\text{mm}$ in the spanwise and streamwise directions, respectively. The temperature of the convective plate was evaluated by ensemble-averaging 100 IR images acquired at 16Hz to reduce pixel-noise. The effect of averaging over shorter or longer periods showed no statistical difference between the results.

Fig. 4.14(a) shows a representative ensembled-averaged IR image of the thermal wall plate. The colorbar represents $T/T_{set}$ where $T_{set}$ is $40^\circ C$. The range of the color bar magnitude is $\pm 5\%$ of the set-point temperature. Spatially averaging the corresponding image in the streamwise direction results in the averaged spanwise temperature profile shown in Fig. 4.14(b). Note that the spanwise variation is less than the measurement uncertainty of the IR temperature measurement which is
±2% ≈ ±0.8°C. In brief, the IR images show that the thermal wall plate can be held to a constant temperature within a tolerance that is less than the measurement uncertainty of the camera. The embedded thermocouples, located just downstream of Fig. 4.14(a) and at spanwise locations of 0 and ±75mm, indicate the spanwise gradient to be much smaller and that the tolerance of the wall plate temperature is ±0.5°C.

Figure 4.15. Wall-normal profiles of mean streamwise velocity at $Re_\theta = 1527$ with thermal wall plate on (×) and off (○) plotted in (a) outer-coordinates and (b) inner-coordinates. The solid lines denote the data from Wu and Moin [2010] at a similar $Re$.

It is first shown that the momentum boundary layer behaves as expected, through the agreement with the DNS data, and the collapse of the two mean velocity profiles demonstrates that temperature behaves like a passive scalar. The wall-normal mean velocity profile was obtained by ensemble-averaging over 30,000 instantaneous PIV vector fields and then spatially averaging over the streamwise ($x-$) direction. The corresponding mean velocity profiles from camera 1 and camera 2 were then stitched together following the method of Shea et al. [2014] to obtain a profile that spans the entire wall-normal height of the boundary layer. The outer normalized mean velocity profiles when the thermal plate was at 40°C and when the thermal plate was unheated is shown in Fig. 4.15(a) (all other conditions being the same), where $U_\infty$ and $\delta$ are the freestream velocity and momentum boundary layer thickness, respectively. The same profiles plotted in so-called inner coordinates are shown in Fig. 4.15(b) (see Eq. 1.2). Here the wall shear stress was computed from
Figure 4.16. Wall-normal profiles of mean temperature plotted in (a) outer-coordinates and (b) inner-coordinates. Shading denotes a $\pm 5\%$ variance in computed $u_\tau$ value. The dotted line denote the data from Arya et al. Araya and Castillo [2012] at $Re_\theta = 2290$.

The PIV data using the integral method of Mehdi and White [2011]. The agreement with the DNS data and the apparent logarithmic region of the velocity profile demonstrates the flow over the plate is consistent with a canonical ZPG boundary layer.

Table 4.2. Parameters for measured temperature validation profiles.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$T_w(\degree C)$</th>
<th>$U_\infty(\text{ms}^{-1})$</th>
<th>$Re_\theta$</th>
<th>$\delta_T(mm)$</th>
<th>$u_\tau(\text{ms}^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>■</td>
<td>40</td>
<td>1.95</td>
<td>568</td>
<td>38</td>
<td>0.10</td>
</tr>
<tr>
<td>✗</td>
<td>40</td>
<td>2.87</td>
<td>825</td>
<td>34</td>
<td>0.14</td>
</tr>
<tr>
<td>▲</td>
<td>40</td>
<td>3.90</td>
<td>1147</td>
<td>34</td>
<td>0.17</td>
</tr>
<tr>
<td>◆</td>
<td>40</td>
<td>5.04</td>
<td>1454</td>
<td>33</td>
<td>0.22</td>
</tr>
<tr>
<td>•</td>
<td>40</td>
<td>9.15</td>
<td>2415</td>
<td>29</td>
<td>0.37</td>
</tr>
</tbody>
</table>

The wall-normal mean temperature profile was obtained by time averaging the thermocouple signal at a given $y$-position over a 100 seconds record length. Fig. 4.16(a) and Fig. 4.16(b) show the mean temperature profiles plotted in outer and inner-coordinates, respectively at five different values of $Re_\theta$ as described in Table 4.2.

Collectively, the two plots demonstrate that the measured temperature profile is consistent with a ZPG boundary layer over an isothermal wall with heat transfer.
4.4.2 ZPG boundary layer encountering a sharp step in wall temperature

![Figure 4.17](image.png)

Figure 4.17. (a) Representative ensemble-averaged IR image of temperature step. The top-plate is unheated where $T=25^\circ C$ and the bottom plate is set to $T=40^\circ C$. The flow is from top-to-bottom. (b) The streamwise profile of spanwise averaged temperature.

In this configuration, a ZPG turbulent boundary layer initially develops over an unheated portion of the wall-plate and then encounters a sharp one-dimensional step in wall temperature at some distance $x_T$ downstream of the boundary layer trip. Effectively, a thermal boundary develops internal to an existing turbulent momentum boundary layer. The configuration is relevant to a flow where a boundary layer encounters a change in surface conditions such as when the atmospheric boundary layer flows from sea to land or when the flow over an engineered surface flows from a cold region to a hot region. Owing to its importance in both geophysical flows and engineering systems, this particular flow configuration has been fairly well-studied in the literature [Antonia et al., 1977, Hoffmann and Perry, 1979, Moretti and Kays, 1965] and is therefore an appropriate test case for validating the NEAT boundary layer facility.

In the present study, the length of the unheated portion of the wall plate is varied by systematically changing the start location of the temperature step relative to the trip position. A sting-mounted type J fine-wire (0.65mm probe diameter) thermocouple attached to a Velmex BiSlide traverse was placed 1.26m downstream of the trip (i.e., in the middle of plate #9). The freestream velocity was set to 4m/sec and wall-normal temperature profiles were acquired for varying unheated starting lengths. The temperature of the convective plate where the temperature step occurred and those downstream of the step were set to $40^\circ C$ while the convective plates upstream of
the step were unheated. In terms of the convective plate numbers, the first profile was acquired when only plate #9 was heated, the second profile was acquired when plates #8 and #9 were heated and so forth until the last profile was acquired when plates #1-9 were heated. Note that in this configuration, the thickness of the momentum boundary layer within which the thermal boundary layer begins to grow varies from profile to profile.

Fig. 4.17(a) shows a representative ensembled-averaged IR image of the temperature step. The flow is from top-to-bottom and the colorbar represents the surface temperature of the plate. The top-plate is unheated where $T = 25^\circ C$ and the bottom plate is set to $T = 40^\circ C$. Fig. 4.17(b) shows the mean streamwise temperature profile obtained by spatially averaging in the spanwise direction. The thermal jump is considered a sharp thermal interface with a thermal gradient of $450^\circ C$ per meter ($0.45^\circ C/mm$), which is greater than that investigated by Moretti and Kays [1965]. Note that the temperature of the insulating region separating the plates cannot be accurately determined from the IR images owing to the difference in emissivity between the insulated region and the convective plate. The embedded thermocouples show the temperature difference between the two plates to be $15^\circ C$.

Figure 4.18. Wall-normal profiles of mean temperature after thermal step. Plates heated corresponding to profiles is as follows; #9 ×, #8-9 ▲, #7-9 ◊, #6-9 ★, #5-9 ■, #4-9 +, #3-9 ♦, #2-9 ●, #1-9 ▲. Plotted in (a) inner-coordinates using $St$ (Eq.2.45), and (b) modified inner-coordinates using $St_T$ (Eq.4.3), subsequent profiles are offset by $\Theta^+ = 6$ for visual clarity. The dotted line denotes the law of the wall turbulent profile.
The wall-normal (y-direction) mean temperature profile was obtained by time averaging the thermocouple signal at a given $y$-position over a record length of 100 seconds ($\approx 1000\delta/u_\tau$). Fig. 4.18 shows the mean temperature profiles plotted in outer and inner coordinates for the nine different unheated starting lengths. Owing to the difficulty of evaluating $\delta_T$ for short thermal development lengths (i.e., large $x_T$ with fixed profile measurement location), the so-called thermal displacement thickness given by

$$\delta_T^* = \int_0^\infty \frac{T(y) - T_\infty}{T_w - T_\infty} dy,$$

(4.2)

was used for outer normalization Antonia et al. [1977]. The wall heat flux values used for inner-normalization is determined from the correlation

$$St_T(x) = St \left[ 1 - \left( \frac{x_T}{x} \right)^{9/10} \right]^{-1/9},$$

(4.3)

developed by Reynolds et al. [1958] as as an approximate solution for the heat transfer downstream of a wall temperature step in a ZPG boundary layer flow, where $St$ is given by Eq. 2.45 and $x_T$ is the unheated starting length. The profiles (each shifted by $\Theta^+ = 6$) show approximate collapse on the expected turbulent profile (dashed lines in the figure) that improves with decreasing $x_T$. These datasets demonstrate that the flow over the plate encountering a wall temperature step is consistent with previous studies [Reynolds et al., 1958, Hoffmann and Perry, 1979], validating the ability of the thermal wall plate to produce temperature step wall boundary conditions.

### 4.4.3 ZPG turbulent boundary layer flow around a wall mounted hemisphere body

When a boundary layer flow encounters a wall-mounted hemisphere body, the pressure gradients around the hemisphere lead to the formation of a necklace vortex at the hemisphere base with legs that extend downstream [Simpson, 2001]. Flow separation at the top of the hemisphere produces vortex loops in the near-wake region [Savory and Toy, 1986, Hansen and Cermak, 1975]. The vorticity in far wake structure is comprised of both the necklace vortex and the vortex loops
Figure 4.19. Spatial temperature distributions located downstream of a hemisphere for four values of $Re_D$. The streamwise and spanwise positions have been normalized by the hemisphere diameter $D = 3\text{cm}$.

(see Fig. 4.20). The influence of these types of fluid-structure interactions on heat transfer are important across a broad range of problems from flow over glaciers to atmospheric re-entry vehicles.

In the present study, a wall mounted hemisphere body of diameter $D = 3\text{cm}$ was placed on the leading edge nose just upstream of the first convective wall plate. The boundary layer upstream of the hemisphere is laminar and the boundary layer is thin compared to the hemisphere with $\delta/D \approx 0.1$. The set point temperature of the convective wall plate was $60^\circ\text{C}$. The inlet air was unheated with $T_\infty = 23^\circ\text{C}$. IR images of the thermal wall plate were acquired downstream of the hemisphere with an imaged area from $4.5 \lesssim X/D \lesssim 8.2$ (i.e., near the trailing edge of the first conductive wall plate) and from $-3.0 \lesssim Z/D \lesssim 2.5$, where $Z/D = 0$ denotes the centerline of the hemisphere in the streamwise direction. Ensemble-averaged IR images of the spatial distribution of the wall plate temperature at four values of $Re_D$ are shown in Fig. 4.19. The colormap corresponds to the plate temperature normalized by the average temperature of the plate.

The spatial distribution of temperature in the IR images show a distinct pattern that becomes more pronounced with increasing $Re_D$. The spatial pattern of temperature indicates that wake vor-
ticity carries cold freestream fluid towards the wall near the plane of symmetry (i.e. near $Z/D = 0$ in the figure). The tapered pattern of minimum temperature (maximum heat transfer) is consistent with the study of Chyu and Natarajan [1996] where local mass (heat) transfer measurements were acquired using naphthalene sublimation that showed a similar tapered shape and that the maximum heat transfer occurred near the reattachment point (which is likely upstream the measurement field-of-view near $X/D \approx 2.5$.) The high temperature lopes in the pattern likely result from the spanwise motion of high temperature fluid along the wall (see Fig. 4.20 plane view). Moreover, the center of the lope is likely a signature of the vortex structure driving the spanwise motion.

**Figure 4.20.** Two plane view of cartoon depiction for resulting flow field from hemisphere perturbation. The XZ plane provides a birds eye view of the developing vortex wake, and the YZ plane provides a view of vortex wake downstream of the hemisphere and the resulting wall temperature.

While the pattern observed in Fig. 4.19 is interesting and informative of the flow structure above the plate, it is evident that the wall-plate temperature is not constant as desired but varies by approximately $\pm 5\%$ across the convective plate. The difficulty of course is that large spatial variations in the convective heat transfer coefficient exist across the convective plate owing to the strong three dimensionality of the flow behind the hemisphere. Since the design purpose was
to set and hold fixed the convective wall plate temperature at a desired temperature, the design needed to be modified. This was accomplished by first replacing each of the two center thin film resistive heaters by two heaters of half the original size, now giving 4 separate heaters in the middle section of the convective plate. Next, the number of controllers was increased from one to two, one for each half of the convective plate. Then the number of controllers was increased from two to three controllers, where the center controller was used for the center two resistive heaters and the remaining two controllers were used for the left and right section of the plate. The mean spanwise temperature profile obtained by spatially averaging in the streamwise direction is shown in Fig. 4.21 for each control scheme at $Re_D = 2.4 \times 10^4$.

The profiles for two and three controllers shows that the spatial variation is further reduced by increasing the number of independent controllers. By using three controllers, the spatial variation of temperature is within $\pm 1\%$ which is less than the measurement uncertainty of the IR temperature measurement. Given this lack of resolution it is difficult to assess the apparent asymmetry in the profiles but a reasonable assumption is that the asymmetry is likely due to somewhat different responses of the left and right controller. In brief, these results demonstrate the ability of the thermal wall plate design to be easily adjusted to maintain a nearly uniform wall temperature in a highly three-dimensional flow.
Figure 4.21. Spanwise temperature profiles taken from the center of the IR images at a downstream position of 1 controller, X; 2 controllers, ▶: 3 controllers, ♦
CHAPTER 5
PREVIOUS WORK ON PULSATILE FLOW

Oscillatory wall-bounded turbulent flow, often referred to as Stokes boundary layer flow, is observed in flow through turbo-machinery, flow through internal combustion engines, flow around lifting surfaces, wave boundary flow, and biofluid flows. These flows are generally classified as reciprocating (or purely oscillatory) when the cycle-averaged flow rate is zero or pulsatile when the flow rate oscillates in time around a non-zero mean. The focus of this thesis is on pulsatile turbulent boundary layer flow. The specific objective is to measure, quantify, and interrupt the effects of unsteady forcing on the transport of momentum and heat for varying freestream velocity, forcing frequency, and forcing amplitude.

Owing to its broad technological importance, pulsatile flows have been widely studied over the past 60 years. The majority of these studies have focused on evaluating the effects of periodic forcing on the long-time averaged (cycle mean) and phase averaged flow variables. These studies include the investigation of pulsatile turbulent pipe/channel flow (Lu 1973, Binder and Kuney 1982, Tu & Ramaprian1983, Ramaprian & Tu 1983, Mao & Hanratty 1986, Tardu et al. 1994, Scotti & Piomelli 2001, Xu & Avila 2018 and Sundstroma & Cervantes 2018) and the investigation of pulsatile boundary layer flow over a flat plate (Karlsson 1959, Cousteix et al. 1977, Patel 1977, Brereton 1990, and Ebadi 2015).

The number of studies that have investigated the effects of periodic forcing on thermal transport is far less than those that have investigated the effects on momentum transport. Moreover, the majority of these studies focused on bulk flow effects by comparing the change in the heat transfer coefficient for periodically forced flows compared to their steady flow canonical counterparts. These studies include Feiler & Yeager 1962, Feiler 1964, Miller 1969, Wang & Zhang 2005, Zohir 2012, and Patroa et al. 2015. It follows that one aim of the present study is to acquire detailed
measurements to advance the knowledge of how unsteady forcing affects the transport of heat in pulsatile boundary layer flow.

The governing equations for pulsatile flow and variants of averages of these equations have been presented in Chapter 2. In this chapter, we summarize the current state of the knowledge on pulsatile wall-bounded turbulent flow. In order to make this chapter self-contained, several equations and flow parameters presented in Chapter 2 are reproduced in this chapter.

5.1 Pulsatile Forcing Effects on the Momentum Field

5.1.1 Long-time averaged mean turbulence

The freestream velocity in developing boundary layer flow or the centerline velocity in pipe or channel flow for a purely time dependent oscillation can be written as

\[ \langle u_\infty(t) \rangle = \bar{u}_\infty + \tilde{u}_\infty(t), \text{ with } \]

\[ \tilde{u}_\infty(t) = \bar{u}_\infty A \sin(\omega t) \]

where the amplitude of the oscillatory component \( A \) is expressed as a fraction of \( u_\infty \), \( \omega \) is the forcing frequency and \( \langle \cdot \rangle \) and \( \bar{\cdot} \) represent the phase-average and long-time (cycle averaged), respectively. For convenience of data presentation, the freestream in boundary layer flow and center-line velocity in pipe or channel flow will be represented as \( u_\infty \) and the boundary layer thickness and channel half-height or pipe radius will be represented as \( \delta \).

It is well-accepted fact that the long-time averaged mean turbulence in pulsatile wall-bounded flow is independent of forcing frequency or forcing amplitude and is the same as its steady flow counterpart at the same Reynolds number, with the exception of a slight difference in \( \overline{\nu'} \). This finding remarkably holds true even for large forcing amplitudes (\( \approx 0.7u_\infty \)) and near-wall flow reversal over a portion of the cycle [Tardu et al., 1994]. This finding in terms of the long-time averaged mean momentum equation
Figure 5.1. Wall-normal profiles of the mean streamwise velocity normalized by inner scales. Symbols correspond to $\lll(\omega^+ = 0.007)$, $\triangle(\omega^+ = 0.014)$, $\Delta(\omega^+ = 0.02)$. The green triangle corresponds to ZPG boundary layer profile at approximately same Reynolds number. This figure corresponds to figure 7.10 of Ebadi et al. [2015].

\[
\bar{u} \frac{\partial \bar{u}}{\partial x} + \nabla \cdot \bar{u} = \nu \frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\partial (\bar{u}' \bar{v}')}{\partial y} - \frac{\partial (\bar{v} \bar{v})}{\partial y},
\]  

(5.2)

implies that $\bar{u} \bar{v} = 0$ and that the fluctuating Reynolds stress is unaffected by the oscillatory motion. Brereton et al. [1990] succinctly summarized that the insensitivity of the long-time averaged mean turbulence to periodic freestream forcing is consistent with the view of turbulence as a broadband phenomenon. Such that excitation at a single frequency is unlikely to have a noticeable effect upon a measured averaged across a broad spectrum of scales of motion. The insensitivity of the long-time averaged mean flow to periodic forcing is illustrated by the mean velocity profiles acquired by Ebadi et al. [2015] in the same flow facility as used in the present study in the absence of a heated wall as shown in Fig. 5.1.
5.2 Phase-averaged turbulence

Despite the lack of influence on the long-time averaged mean turbulence, periodic forcing does influence the phase-averaged turbulence. It has been shown that the dynamically important scaling parameter is the inner-normalized Stokes length \( l_s^+ \) (or equivalently the inner-normalized forcing frequency as shown below) with the oscillation amplitude having only a weak influence:

The physical reasoning for the importance of \( l_s^+ \) is that since the Stokes length \( l_s \) defines the length over which the oscillatory vorticity diffuses away from the wall (in a laminar flow), the inner normalized Stokes length provides a measure of how far the oscillatory component interacts with the turbulent flow away from the wall. To account for the effects of turbulence on the diffusion of vorticity, Scotti and Poimelli [2001] defined a turbulent Stokes length based on a mixing length model given by

\[
l_t^+ = l_s^+ \left[ \frac{\kappa l_s^+}{2} + \sqrt{1 + \left( \frac{\kappa l_s^+}{2} \right)^2} \right]
\]

where \( \kappa \) is the von Karman coefficient. In accord with previous flow regime classifications by Ramaprian and Tu [1983], Brereton and Mankbadi [1995] based on the magnitude of \( \omega^+ \), Scotti and Poimelli [2001] classified pulsatile flow into four regimes bases on \( l_t^+ \). These four regimes and a brief description are provided below. In these descriptions, the approximate range of values of \( \omega^+ \) for each flow regime are based on the values provided by Ramaprian and Tu [1983], Brereton and Mankbadi [1995].

1. **Quasi-steady** \([\lim \omega^+ \to 0 \text{ or } l_t^+ \gg \delta^+]\) In this flow regime, the oscillation period is much larger than the turbulence relaxation time, the turbulence rapidly adjusts to the unsteady forcing and the flow at each instant in time is in a state of quasi-steady equilibrium. There is no phase difference between turbulence quantities and the turbulence is the same as that in steady boundary layer flow when scaled by the instantaneous values of the driving force.
2. **Low frequency** $[0.003 \lesssim \omega^+ \lesssim 0.006 \text{ or } l^+_t \approx \delta^+]$ In this flow regime, the entire flow is affected by the unsteadiness and the flow is not in equilibrium. Specifically, the turbulent production and dissipation begin to move out of phase with respect to each other. The acceleration and deceleration phases are not symmetrical as the turbulent production of kinetic energy is inhibited during acceleration. If the forcing amplitude is large enough the flow may relaminarize.

3. **Intermediate frequency** $[0.006 \lesssim \omega^+ \lesssim 0.02 \text{ or } l^+_t \approx \delta^+/2]$ In this flow regime, the outer flow ($y^+ > \delta^+/2$) is unaffected by the unsteadiness and turbulence advects as frozen plug flow. Below this region ($y^+ < \delta^+/2$), turbulence quantities are out of phase and the flow is not in equilibrium.

4. **High frequency** $[0.02 \lesssim \omega^+ \text{ or } \lim l^+_t \lesssim 20]$ In this high frequency flow regime, the Stokes length $l^+_s$ is confined to the viscous sublayer, the diffusion of oscillating vorticity is purely viscously driven and the oscillatory flow is decoupled from the turbulent mean. The modulation of the turbulent qualities by the oscillatory flow should relax to zero and the amplitude and phase-lag of the wall shear stress should approach their values in a Stokes laminar boundary layer flow.

The four frequency regimes are illustrated in Fig. 5.2. The top plot shows the ratio of the modulation amplitude of the wall shear stress to the modulation amplitude of the freestream velocity. The bottom figure shows the phase lag between the oscillatory wall shear stress and the oscillatory strain. The reader is referred to Brereton and Mankbadi [1995] and Scotti and Poimelli [2001] for a more detailed discussion of the different flow regimes.

The effect of periodic forcing on the phase-averaged turbulence is non-intuitive given that the long-time averaged turbulence is insensitive to the periodic forcing. This is demonstrated here by viewing plots of the phase-averaged profiles as a function of phase angle and flow regime. The first set of plots is reproduced from Tu and Ramaprian [1983] as shown in Fig. 5.3. The left figure is marginally within the low frequency regime and the right figure is marginally within the interme-
Figure 5.2. Impact of $w^+$ regimes on near wall velocity field through wall shear stress. Top plot denotes the modification of the sinusoidal wall shear stress relative to the mean wall shear stress and the bottom plot denotes the near wall phase shift relative to the freestream. Plot used with permission from Dr. Alireza Ebadi Ebadi [2016] with data from Brereton and Mankbadi [1995].

The phase-averaged mean velocity profiles reproduced from Scotti and Poimelli [2001] figure 13 shown in Fig. 5.4 demonstrate similar complexity. The left column corresponds to the high frequency regime, the middle column to the intermediate frequency regime, and the right column to the low frequency regime. The lowest profile (solid line) corresponds to the first phase ($T/8$) of the accelerating portion of the cycle and each offset profile increases by $T/8$. The vertical dashed line denotes the position $2l_1^+$. The $\times$’s denote the steady channel flow at the same $Re$. For the high frequency regime, the flow away from the wall is effectively "frozen" and is only shifted up and down by the modulation of the core velocity. Flow reversal near the wall is observed during
Figure 5.3. Profiles of $\langle u \rangle$ as a function of phase angle. The left figure is for a marginal low frequency flow ($\omega^+ = 0.001$) and the right figure is for a marginal intermediate frequency flow ($\omega^+ = 0.009$). The top profiles (left axis) are for the decelerating period and the bottom profiles (left axis) are for the acceleration period. Symbols in the decelerating period: ($\circ = 0^\circ; \triangle = 45^\circ; \square = 90^\circ; \times = 135^\circ$. Symbols in the accelerating period: ($\circ = 180^\circ; \triangle = 225^\circ; \square = 270^\circ; \times = 315^\circ$. This figure corresponds to figure 10 of Tu and Ramaprian [1983].

The early phases of the acceleration portion of the cycle and the late phases of the deceleration portion of the cycle. For the intermediate frequency regime, there is a strong coupling between the oscillatory inner layer and the outer turbulent layer and the flow is strongly modified as a function of phase. For example, a logarithmic layer is observed for only a portion of the cycle when the centerline velocity is at its maximum. At the lowest frequency, the entire flow is affected by the oscillation (i.e., the dashed line is beyond the channel half-height). Note that flow reversal can be evaluated by the Binder and Kueny [1981] criterion which states that flow reversal occurs when $A > l_s^+ / \sqrt{2} = 1 / \sqrt{\omega^+}$. This relation shows that flow reversal is very likely in the high-frequency regime.

5.2.1 Turbulent Structure

The impact of periodic forcing on the phase-averaged turbulence is strongly suggestive that the turbulent structure is strongly influenced by the periodic forcing despite a negligible effect on the long time-mean flow. In fact, the turbulent structure in pulsatile wall-bounded turbulent flow differs from its steady counterpart [Scotti and Poimelli, 2001]. The most apparent difference is during the accelerating period which has a flow structure more similar to transitional flow. It is not until close to the decelerating period where the turbulent structure more resembled the turbulent structure
Figure 5.4. Profiles of $\langle u \rangle$ as a function of phase with the lowest profile starting from the first accelerating phase and each offset profile increasing by $T/8$: (left) $l_+^* = 7$ – high frequency regime, (middle) $l_+^* = 14$ – intermediate frequency regime, (right) $l_+^* = 35$ – low frequency regime. The $x'$s denote steady channel flow data. The vertical dashed line denotes the position $2l_+^*$. This figure corresponds to figure 13 of Scotti and Poimelli [2001].
of full-developed turbulent wall-bounded flow. This difference in turbulent structure between the accelerating and decelerating period is similar to that observed in reciprocating flows.

5.3 Pulsatile Forcing Effects on the Heat Transfer

While the effects of pulsatile forcing on the momentum field have been fairly well-documented and there is broad experimental/numerical agreement between various studies, the same cannot be said regarding the effects of periodic forcing on heat transfer. In fact, there are wide disparities in the experimental results and conclusions between the various studies investigating the subject.

Specifically, several studies have reported a large increase in heat transfer while several other studies have reported little or no change in the heat transfer with periodic forcing. For example, Feiler and Yeageb [1962] reported that heat transfer from a heated plate increased by as much as 65% with periodic forcing of the freestream. Wang and Zhang [2005] report that heat transfer in a pulsating pipe flow increases by approximately 70% compared to steady flow. Zohir [2012] reported an increase in the heat transfer by a factor of $10 \times$ in a shell and tube heat exchanger with pulsation compared to the non-pulsating flow case.

Contrary to these results, Miller [1970] found only a small increase ($3 - 5\%$) in the heat transfer coefficient for boundary layer flow over a heated plate with a pulsatile freestream compared to a steady freestream. He used large eddy simulation (LES) to study heat transfer in pulsatile channel flow and found that the turbulent heat transfer coefficient scales with $Pr$ as predicted for steady channel flow. Patroa et al. [2015] found that the heat transfer coefficient in a pulsating pipe flow both increased and decreased (both on the order of 20%) compared to steady pipe flow. The obvious conclusion from these varied and contradictory results is that it is presently not clear what effects, if any, the periodic forcing of the freestream/centerline velocity has on the wall heat flux. Surprisingly, our understanding of the problem has not changed since Havemann and Narayan-Rao [1954] published a Nature article titled “Heat Transfer in Pulsating Flow” in which the author reviewed experimental work on the effect of pulsations on heat transfer within a pipe. In particular, Havemann and Narayan-Rao [1954] reported that some studies showed an increase in the
Nusselt number $Nu$ by as much as 70% while other studies reported a decrease in $Nu$ by as much as 30%.

The long-view suggests that given the combination of (a) that the long-time averaged mean turbulence is insensitive to pulsatile freestream/centerline velocity forcing and (b) that the transport of momentum and heat are similar in a turbulent boundary layer flow (i.e., the so-called Reynolds analog), that the long-time averaged (cycle averaged) heat transfer should also be insensitive to pulsatile forcing. The only scenario in which heat transfer is affected by pulsatile forcing is if and only if the transport of momentum and heat in pulsatile wall-bounded flow are dissimilar, i.e., Reynolds analogy does not hold.

The potential for the breakdown of Reynolds analogy in pulsatile flows can be linked to the pioneering work of Lighthill [1954]. In this work, Lighthill investigated the skin friction and heat transfer in a laminar boundary layer around a cylinder when the upstream flow is periodically varied about its mean. A telling direct quote from Lighthill (page 2) is as follows “It is found that the fluctuations in the skin friction and heat transfer behave very differently, in defiance of ‘Reynolds’ analogy’.” Specifically, Lighthill found that the skin-friction is always phase advanced relative to the outer flow, while the situation is very different for the wall heat flux. In favorable pressure gradient, the wall heat flux phase lags the freestream owing to the effects of thermal inertia. In zero pressure gradient, the wall heat flux is in phase with the freestream and is slightly phase advanced in an adverse pressure gradient. Importantly, the wall heat flux and the wall shear stress are almost always out of phase with respect to each other with the wall heat flux lagging the wall shear stress.

An important question is if, and how, the findings of Lighthill translate to a turbulent boundary layer. Again, taking the long-view, one may suspect that in the quasi-steady flow regime the momentum and the heat transfer scale similarly since the transport is dominated by turbulent fluxes. Conversely, since the oscillatory flow in the high frequency flow region behaves like a near-wall laminar flow (following Stoke’s solution), the wall skin friction and the wall heat flux may be out of phase and scale differently in time. Of course, to complicate the matter it is uncertain how a
phase difference (if any) between the skin friction and the wall heat flux manifests itself in the long
time averaged (cycle averaged) surface fluxes.

The study that sheds light on the above question is the LES studies of Wang and Lu [2004] in
which the flow cases investigated by Scotti and Poimelli [2001] were repeated with heat transfer.
The phase averaged mean velocity profiles and the phase averaged mean temperature profiles for
low, intermediate, and high forcing frequency are are shown in Fig. 5.5. The top row corresponds
to the low frequency flow region, the middle row to the intermediate frequency flow region, and
the bottom row to the high frequency flow region. It is evident in the figures, that the influence of
the oscillatory flow on the phase-averaged mean temperature profiles decreases significantly with
increasing forcing frequency. Perhaps most surprising is that the temperature profile is apparently
insensitive to the near-wall flow reversal that occurs at the highest forcing frequency. While not
shown, the temperature fluctuations and turbulent heat flux show similar dependencies on the forc-
ing frequency. In view of these results, one would suspect that any increase (or decrease) in the
heat transfer with pulsatile freestream/centerline velocity forcing would be limited to the low and
intermediate flow regimes, with the influence being greater in the latter.

The uncertainties in the results of Wang and Lu [2004] are (i) the effects of the sub-grid scale
model (is backscatter important in these flows) (ii) the relatively low Reynolds number of $Re_\tau = 350$
and (iii) is it appropriate to model temperature as a passive scalar (i.e., is an the potential
influence of buoyancy important?). One goal of the present study is to address these questions by
providing experimental data similar to that presented here.

5.4  Experimental Pulsatile Thermal Flow

The table below outlines the experimental work conducted thus far on thermal pulsatile flow and
demonstrates the range of parameter space investigated. Due to the applicability of the work to heat
exchangers a large body of the work is done in pipe flow with investigations focused on integral
parameters. What work does exist on boundary layer flow has been done at low $Re$. This parameter
space is shown visually in Fig. 5.6, with the datasets contributed by this work outlined as well.
Figure 5.5. Profiles of (left) $\langle u \rangle$ and (right) $\langle T \rangle$ as a function of phase with $0t_p/8$ denoting the first accelerating phase. (top) low frequency flow region, (middle) intermediate frequency flow region, (bottom) high frequency flow regime. These figures are taken from Wang and Lu [2004].
Table 5.1. Table identifying previous studies performed to investigate both thermal and non-equilibrium wall bounded flow dynamics with a working fluid of air along with data of current investigation.

<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
<th>Flow Type</th>
<th>$Re$</th>
<th>$T_{wall} - T_{\infty}$</th>
<th>$f$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feiler [1964]</td>
<td>1964</td>
<td>BL</td>
<td>$2.5 - 9.9 \times 10^4$</td>
<td>$30K$</td>
<td>100</td>
</tr>
<tr>
<td>Jackson and Purdy [1965]</td>
<td>1965</td>
<td>Pipe</td>
<td>$0.21 - 7.5 \times 10^4$</td>
<td>$15K$</td>
<td>222</td>
</tr>
<tr>
<td>Kearney et al. [2001]</td>
<td>2001</td>
<td>BL</td>
<td>$0.89 - 2.7 \times 10^4$</td>
<td>$117K$</td>
<td>2.5-5</td>
</tr>
<tr>
<td>Habib et al. [2002]</td>
<td>2002</td>
<td>Pipe</td>
<td>$0.78 - 1.987 \times 10^3$</td>
<td>$-$</td>
<td>0.5 - 15</td>
</tr>
<tr>
<td>Elshafei et al. [2008]</td>
<td>2008</td>
<td>Pipe</td>
<td>$3.7 \times 10^4$</td>
<td>$500K$</td>
<td>6-68</td>
</tr>
<tr>
<td>Li et al. [2014]</td>
<td>2014</td>
<td>Pipe</td>
<td>$0.5 - 4.5 \times 10^3$</td>
<td>$36K$</td>
<td>15-98</td>
</tr>
<tr>
<td>Biles</td>
<td>2019</td>
<td>BL</td>
<td>$0.9 - 4 \times 10^5$</td>
<td>$25K$</td>
<td>.1 - 1.0</td>
</tr>
</tbody>
</table>

Figure 5.6. Parameter overview for existing literature data on pipe and boundary layer pulsatile thermal flow along with additional data presented in this text.
CHAPTER 6

TIME AVERAGE PULSATILE BOUNDARY LAYER FLOW

The time average results of PBL flow are investigated through comparison of integral parameters, followed by outer and inner normalized profiles whose dynamics which will be compared against representative cases of ZPG flow. For all experiments conducted the following experimental parameters were held constant.

- Wall temperature \((\theta_w) = 50^\circ C\)
- Freestream temperature \((\theta_\infty) = 25^\circ C\)
- Streamwise measurement location \((x_m) = 1.28m\)

The thermal and momentum fields were measured in separate experiments due to the physical setup requirements of each measurement system. In the experiments, the freestream velocity of the wind tunnel was fixed at approximately 1, 4, and 5.5\(m/sec\). For each freestream velocity, five oscillatory forcing frequencies, \(f\), were investigated: 0.144, 0.366, 0.690, and 1.069\(Hz\). Steady-state ZPG boundary layer flow was used for baseline comparison to evaluate the effects of periodic forcing on the transport of momentum and temperature within the boundary layer. For initial analysis the set freestream velocity is used to group the experiments into low \((\sim 1m/sec)\), medium \((\sim 4m/sec)\) and high \((\sim 5.5m/sec)\) speed cases. The experimental conditions for each test are provided in Tables 6.1 – 6.4.
Table 6.1. Parameters for Thermal Field Experiments in the NEAT Pulsatile Campaign

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$\bar{u}_\infty$ (m/sec)</th>
<th>$\tilde{u}_\infty$ (%)</th>
<th>$f$ (Hz)</th>
<th>$\delta_T$ (m)</th>
<th>$q_{iw}$ (W/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>■</td>
<td>1.33</td>
<td>0.59</td>
<td>0.144</td>
<td>0.049</td>
<td>248</td>
</tr>
<tr>
<td>x</td>
<td>4.20</td>
<td>0.51</td>
<td>0.144</td>
<td>0.047</td>
<td>256</td>
</tr>
<tr>
<td>•</td>
<td>5.64</td>
<td>0.54</td>
<td>0.144</td>
<td>0.047</td>
<td>380</td>
</tr>
<tr>
<td>■</td>
<td>1.20</td>
<td>0.57</td>
<td>0.366</td>
<td>0.0538</td>
<td>233</td>
</tr>
<tr>
<td>x</td>
<td>4.15</td>
<td>0.60</td>
<td>0.366</td>
<td>0.048</td>
<td>266</td>
</tr>
<tr>
<td>•</td>
<td>5.63</td>
<td>0.58</td>
<td>0.366</td>
<td>0.047</td>
<td>385</td>
</tr>
<tr>
<td>■</td>
<td>1.02</td>
<td>0.48</td>
<td>0.690</td>
<td>0.054</td>
<td>194</td>
</tr>
<tr>
<td>x</td>
<td>3.96</td>
<td>0.56</td>
<td>0.690</td>
<td>0.047</td>
<td>352</td>
</tr>
<tr>
<td>•</td>
<td>5.47</td>
<td>0.46</td>
<td>0.690</td>
<td>0.050</td>
<td>350</td>
</tr>
<tr>
<td>■</td>
<td>0.96</td>
<td>0.31</td>
<td>1.069</td>
<td>0.054</td>
<td>160</td>
</tr>
<tr>
<td>x</td>
<td>3.67</td>
<td>0.47</td>
<td>1.069</td>
<td>0.050</td>
<td>254</td>
</tr>
<tr>
<td>•</td>
<td>5.15</td>
<td>0.51</td>
<td>1.069</td>
<td>0.047</td>
<td>362</td>
</tr>
</tbody>
</table>

Table 6.2. Parameters for ZPG Thermal Field Experiments in the NEAT Pulsatile Campaign

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$\bar{u}_\infty$ (m/sec)</th>
<th>$\delta_T$ (m)</th>
<th>$q_{iw}$ (W/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>■</td>
<td>0.978</td>
<td>0.051</td>
<td>219</td>
</tr>
<tr>
<td>x</td>
<td>2.79</td>
<td>0.042</td>
<td>206</td>
</tr>
<tr>
<td>•</td>
<td>3.9</td>
<td>0.038</td>
<td>331</td>
</tr>
</tbody>
</table>
### Table 6.3. Parameters for Momentum Field Experiments in the NEAT Pulsatile Campaign

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$\pi_\infty$(m/sec)</th>
<th>$\tilde{u}_\infty$(%)</th>
<th>$f$ (Hz)</th>
<th>$\delta$(m)</th>
<th>$u_\tau$(m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>■</td>
<td>1.15</td>
<td>0.59</td>
<td>0.132</td>
<td>0.0433</td>
<td>0.057</td>
</tr>
<tr>
<td>x</td>
<td>3.95</td>
<td>0.51</td>
<td>0.139</td>
<td>0.037</td>
<td>0.192</td>
</tr>
<tr>
<td>•</td>
<td>5.38</td>
<td>0.54</td>
<td>0.132</td>
<td>0.035</td>
<td>0.255</td>
</tr>
<tr>
<td>■</td>
<td>0.98</td>
<td>0.60</td>
<td>0.344</td>
<td>0.043</td>
<td>0.051</td>
</tr>
<tr>
<td>x</td>
<td>3.79</td>
<td>0.60</td>
<td>0.330</td>
<td>0.040</td>
<td>0.18</td>
</tr>
<tr>
<td>•</td>
<td>5.02</td>
<td>0.58</td>
<td>0.344</td>
<td>0.039</td>
<td>0.221</td>
</tr>
<tr>
<td>■</td>
<td>0.84</td>
<td>0.56</td>
<td>0.681</td>
<td>0.042</td>
<td>0.052</td>
</tr>
<tr>
<td>x</td>
<td>3.62</td>
<td>0.56</td>
<td>0.681</td>
<td>0.041</td>
<td>0.166</td>
</tr>
<tr>
<td>•</td>
<td>5.04</td>
<td>0.46</td>
<td>0.681</td>
<td>0.039</td>
<td>0.220</td>
</tr>
<tr>
<td>■</td>
<td>0.82</td>
<td>0.47</td>
<td>1.062</td>
<td>0.0415</td>
<td>0.045</td>
</tr>
<tr>
<td>x</td>
<td>3.42</td>
<td>0.47</td>
<td>1.062</td>
<td>0.042</td>
<td>0.152</td>
</tr>
<tr>
<td>•</td>
<td>4.8</td>
<td>0.51</td>
<td>1.062</td>
<td>0.040</td>
<td>0.218</td>
</tr>
</tbody>
</table>

### Table 6.4. Parameters for ZPG Momentum Field Experiments in the NEAT Pulsatile Campaign

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$\pi_\infty$(m/sec)</th>
<th>$\delta$(m)</th>
<th>$u_\tau$(m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>■</td>
<td>0.86</td>
<td>0.042</td>
<td>0.059</td>
</tr>
<tr>
<td>x</td>
<td>2.75</td>
<td>0.038</td>
<td>0.137</td>
</tr>
<tr>
<td>•</td>
<td>3.92</td>
<td>0.035</td>
<td>0.192</td>
</tr>
</tbody>
</table>
6.1 Zero Pressure Gradient Flow

Figure 6.1. Time average ZPG outer normalized mean profiles for the thermal and momentum field, markers denoted in Tables 6.2 and 6.4. Dashed lines denote the respective Blasius and Polhausen solution for each field, and dash-dot lines denote turbulent profile from DNS data of Araya and Castillo [2012].

In Fig. 6.1 the outer normalized ZPG profiles for velocity, shown on top, and temperature, shown on bottom, are presented with markers denoted in Tables 6.2 and 6.4. In both figures the respective
laminar boundary layer solution is plotted as dashed curves for physical reference and a turbulent profile is plotted as dash-dots from the DNS data of Araya and Castillo [2012]. Measurement uncertainties are displayed as shaded areas surrounding the respective profile. The measurement uncertainties bars are determined as detailed in Chapter 3 and the respective uncertainties will be carried throughout this analysis displayed as either shaded regions or uncertainty bars for single parameters. The medium and high speed cases show clear collapse in both fields with a characteristic turbulent profile. The low speed profile shows an increase in the near wall gradient in both the thermal and momentum fields. This is reverse of expected due to the fact that for a low $Re (~ 10^4)$ one would expect the profiles to shift toward their respective laminar solutions.

![Diagram](image)

**Figure 6.2.** Time averaged ZPG outer normalized rms fluctuating component of the momentum and thermal field.
Fig. 6.2 plots the rms fluctuations of the two fields. The medium and high speed cases again collapse and show the respective trends of progression towards a near zero value at the boundary layer thickness, representative of the freestream value. The low speed cases presents a separate story where in the momentum field the fluctuations are higher throughout the field with an off wall peak occurring at $\eta \approx 0.02$. The fluctuations then remain relatively constant above $\eta \approx 0.5$. The rms of the thermal field at low speeds displays the same off wall fluctuating peak yet then proceeds into a smoother decrease with progression towards the freestream. Both of these low speed fluctuating profiles suggest that a distinctly separate set of transport dynamics is occurring than that of anticipated forced convective ZPG flow demonstrated in the medium and high speed cases.
Fig. 6.3 displays the inner normalized form of the ZPG thermal flow for the three speed cases under investigation. DNS data at $Re_\theta = 2290$ from Araya and Castillo [2012] is plotted as a dashed line for both $u^+$ and $T^+$. The $\overline{u}_\tau$ values for the above normalization were determined through use of the integral method of Mehdi and White [2011]. The thermal inner normalized profiles were determined from using the developed near wall gradient method which is described in Chapter 2. The medium and high speed cases collapse with the DNS data where the medium speed thermal profiles display the correct shape yet are offset, indicative of a slightly lower $Re$.
case. Both fields denote self similar development in $Re$ and $y^+$. For both fields, the low speed profile produces behavior which is reminiscent of the deviation seen in rough wall boundary layer flows [Ebner, 2014] or adverse pressure gradient flow [Perry et al., 1966] where the profile sits well below the expected law of the wall behavior. In these profiles the slope of the log region is significantly different from that of law of the wall formulations. A collapse is observed towards lower wall normal locations where the flow appears to remain viscously dominated ($\approx y^+ = 10$). This suggests the presence of another forcing term which skews the balance between viscous, inertial and turbulent forcing.
6.2 Pulsatile Boundary Layer Flow

The following figures are time averaged results of the experiments conducted in pulsatile boundary layer conditions which are outlined in Tables 6.1 and 6.3.

6.2.1 Integral Parameters

Understanding the scaling of the developing pulsatile boundary layer through the two flow fields under investigation begins with an examination of the integral parameters which are indicative of how the boundary layer is evolving throughout the forcing cycle. The determination of these integral parameters is described in Chapter 2. The markers are denoted in the legend along with the speed case (low, med, high) and frequency (1 → 5) where increasing frequency number indicates increasing frequency. The values of the frequency and freestream velocity can be referenced in Tables 6.1 and 6.2, these notations will be kept constant throughout this analysis.

*Figure 6.4.* Time averaged boundary layer thickness verse $Re$ for PBL and ZPG tests for both momentum ($\delta$) and thermal ($\delta_T$) fields, where dash lines denote textbook approximations.

- *Boundary Layer Thickness ($\delta, \delta_T$) verse $Re$*
Fig. 6.4 shows $\delta_{99}$ for the thermal and momentum fields plotted vs time average $Re$. Two models lines are plotted on both figures which identify the standard textbook developing boundary layer thickness for a turbulent and laminar flow respectively [Fox et al., 2012].

$$\delta_{turb} = 0.382x/Re^{1/5}$$

$$\delta_{lam} = 4.9x/Re^{1/2}$$

(6.1)

Figure 6.5. Time averaged thermal boundary layer thickness verse time average boundary layer thickness for PBL and ZPG tests, with a dashed line that denotes an empirical correlation.

For the low speed case the boundary layer thickness is slightly reduced compared to $\delta_{turb}$ and with slight variation of the development length, ZPG $\delta$ would collapse onto $\delta_{turb}$. This is suggestive of a virtual origin in relation to the true origin location of the boundary layer trip. The pulsatile $\delta$ shows clear variation with frequency yet this trend can be removed due to the variation in the outer set boundary condition of $u_\infty$. The difference can also be observed in the thermal field suggesting both fields have a development length based on a virtual origin. The medium and high speed cases PBL $\delta_T$ fall in line with $\delta_{T,turb}$. From Fig. 6.5 the boundary
layer thickness of the two fields are compared along with a dotted line denoting the empirical relationship. With increasing $Re$ a collapse is observed suggesting the anticipated relationship between the two fields holds yet at an offset value.

**Figure 6.6.** Time averaged displacement thickness verse $Re$ for PBL and ZPG tests for both momentum ($\delta^*$) and thermal ($\delta^*_T$) fields.

- **Boundary Layer Displacement Thickness ($\delta^*$, $\delta^*_T$) verse $Re$**

  Fig. 6.6 shows the displacement thickness as a function of $Re$. The growth of the displacement thickness is representative of a reduction in the near wall gradient and when compared with $\delta$ can lead to an understanding of the near wall gradient with respect to the gradient across the entire boundary layer. The momentum and thermal displacement thickness for the medium and high speed cases sit close to the respective turbulent development lines with significant deviation occurring at the low speed case. With increasing frequency, a decrease in $\delta^*$ is observed yet an increase in $\delta^*_T$ is observed. The thermal field behavior can be partially accounted for by the increase in $Re$. Examination of the ZPG $\bar{\phi}(\eta_T)$ profiles in Fig. 6.1 demonstrate an increase in near wall gradient indicated by $\delta^*_T$.  

95
**Boundary Layer Shape Factor** \((\overline{H}, \overline{H}_T)\) verse \(Re\)

Fig. 6.7 plots the shape factor for both the momentum and thermal field, for which the approximate literature standard value of 1.3 is plotted for \(H\) [Vincenti et al., 2013]. The computed \(H\) values provide confidence that the momentum and thermal boundary layer are developing in a self similar fashion and retain a turbulent or turbulent "like" shape through the variety of flow conditions investigated.

The medium and high speed case match existing forced convective boundary layer flow data with little to no influence of periodic forcing. The low speed case retains a turbulent profile shape as seen by Fig. 6.7 yet has compressed the near wall gradient. For the low speed case, higher frequency pulsations reduce the near wall temperature gradient towards their expected values but have little affect on the value of \(\delta^*\). This suggest that the physical mechanisms which increase the near wall gradient at low speeds is modified by the higher frequency pulsatile forcings.
6.2.2 Wall Fluxes

Figure 6.8. Time averaged wall heat flux ($\bar{q}_w$) and $\bar{S}_t$ for PBL and ZPG tests.

- **Boundary Layer Heat Flux ($\bar{q}_w$, $\bar{S}_t$) verse Re**

  Fig. 6.8 plots the computed wall heat flux and respective normalized parameter. As $\bar{q}_w$ increases with increasing $Re$. When the heat flux is normalized and expressed as $\bar{S}_t$ the medium and high speed cases collapse with expected correlation as shown by the dashed line in the figure [Bergman et al., 2011]. The low speed case shows a significantly higher $\bar{S}_t$ than expected.

- **Boundary Layer Shear Stress ($\bar{\tau}_w$, $\bar{C}_f$) verse Re**

  The wall shear stress, shown in Fig. 6.9, as a function of $Re$ is computed using the integral method. The wall shear stress increases with increasing $Re$, $\bar{C}_f$ values display a reduction in the PBL values when compared to black markers of the ZPG data.

Comparing PBL to ZPG-BL a significant increase in $\bar{S}_t$ is not observed by periodic forcing of the freestream. A significant frequency based trend is not observable for $\bar{q}_w''$ outside of the low
speed case. From examination of the integral parameters, an inverse trend was noted between frequency and near wall gradient within the low speed thermal case. This is shown clearly in Fig. 6.8, yet the $S_t$ show no significant modification. Fig. 6.9 shows a decrease in $C_f$ between the ZPG and PBL cases respectively.

6.2.3 Outer Normalization

Fig. 6.10 is representative of the data presentation that will be used for PBL. Specifically, each subplot within a figure contains all the pulsatile frequencies at a single fixed long-time averaged freestream velocity (which is labeled in the bottom corner) Lastly, measurement uncertainties are shown as shaded regions surrounding the individual profiles.

- Outer Normalized Velocity Profiles ($\bar{u}$) verse $\eta$

Fig. 6.10 plots the time average outer normalized mean velocity verse the outer normalized wall normal distance, $\eta = y/\delta$. The profiles collapse between the ZPG and PBL cases for both the high and medium speed cases yet significant deviation is observed for low speed
Figure 6.10. Time averaged PBL outer normalized mean profiles of the momentum field, where each figure contains a frequency sweep at a single freestream velocity marker notation can be found in Table. 6.3

PBL profiles. As the pulsatile frequency increases the profiles began to collapse in the outer region while still deviating in the near wall region, although this is somewhat based upon construction due to the normalization but remain different in the near-wall region.
Figure 6.11. Time averaged PBL outer normalized mean profiles of the thermal field, where each figure contains a frequency sweep at a single freestream velocity marker notation can be found in Table 6.1

- Outer Normalized Thermal Profiles ($\bar{\phi}$) verse $\eta$

The trends noted in Fig. 6.10 are similar to those observed in Fig. 6.11 in which the thermal profiles are plotted in outer-normalized coordinates. A collapse is observed at the medium and high speed cases with minimal changes in the set $\bar{u}_\infty$. Significant deviations exist in the
low speed case yet the effect of frequency trends differently compared to Fig. 6.10. With increasing forcing frequency the profiles move away from the ZPG profile.

![Graphs showing normalized profiles](image)

**Figure 6.12.** Time averaged PBL outer normalized mean profiles evaluated against a $1/7^{th}$ power law fit for both fields, where each figure contains a frequency sweep at a single freestream velocity marker notation can be found in Table. 6.1

- **Outer Normalized Profiles Compared to $1/7^{th}$ Power Law**

  A common approximation for a turbulent boundary layer profile is the $1/7^{th}$ power law fit, where the outer normalized parameter ($\overline{u}, \overline{\phi}$) becomes a function of their respective outer normalized wall normal position to the $1/7^{th}$ power. The difference between this fit and each profile is plotted in Fig. 6.12 such that the profile shape can be compared between flow speeds and between the two fields. The largest deviation as expected is observed in the low speed case and a similar trend in the deviation is observed in both the velocity and temperature profile. The remaining medium and high speed profiles show similar trends with the largest
deviations coming from the near wall. The significant negative near wall section of the low speed profiles demonstrate the large near wall gradient. In the outer region these low speed profiles return slowly before collapsing in the far field.

Outer normalize profiles demonstrate consistent collapse at the medium and high speed cases with significant deviation away from ZPG profiles occurring at the low speeds cases. Relative difference profiles comparing the profiles to a $1/7^{th}$ power law fit demonstrate the near wall variation in the low speed cases. From Chen et al. 1977 a similar overshoot is noted in outer normalized profiles with the increase of the buoyancy parameter; $Gr \cdot Re^{-5/2}$, where $Gr$ is the Grashof number which compares buoyant to viscous forces Chen et al. [1977]. This overshoot coming from an increase in the near wall gradient could potentially come from the time average effects of buoyancy seen as a favorable pressure gradient.

### 6.2.4 Inner Normalization

Determination of the inner normalized plus units for the time average profiles follows the same procedure as described for the ZPG profiles. Measurement uncertainties are denoted as shaded regions that envelope the data.

- **Inner Normalized Profiles ($\pi^+$) verse $y^+$**

  Fig. 6.13 plots the inner normalized velocity profiles along with a DNS profile from Araya and Castillo [2012], $Re_\theta = 2290$. As seen in previous normalizations, the medium and high speed cases collapse and the low speed case does not. The near wall region of the low speed profile collapses. Specifically, the log region exhibits a different slope from the expected law of the wall.

- **Inner Normalized Profiles ($\phi^+$) verse $y^+$**

  Fig. 6.14 displays a wider spread in the plotted dynamics amongst the data sets. An evident collapse is not observed in the low or medium speed cases, and the higher $Re$ case of the high speed case is in better agreement with the DNS data. The medium speed case displays the
correct shape yet indicates a wide spread in the y-offset and the low speed case displays a collapse to an entirely different set of controlling dynamics.

- **Turbulent Prandtl number** ($Pr_T$) verse $y^+$

  Fig. 6.14 shows $Pr_T$ approximated by:
\[ \epsilon = \frac{u_T^2}{\frac{du}{dy}} \]
\[ \epsilon_T = \frac{\frac{q_w}{(\rho C_p)}}{\frac{d\theta}{dy}} \]
\[ Pr_T = \frac{\epsilon}{\epsilon_T} \] (6.2)

Figure 6.14. Time averaged PBL inner normalized mean profiles of the thermal field, where each figure contains a frequency sweep at a single freestream velocity marker notation can be found in Table. 6.1
Figure 6.15. Turbulent $Pr$ computed as a function of $y^+$ for the three cases under investigation.

where the Reynolds stress and the turbulent heat flux are approximated through respective wall flux parameters [Petukhov and Polyakov, 1988]. Reynolds analogy requires that $Pr_T = 1$, yet Fig. 6.2 indicates that $Pr_T$ varies significantly with respect to $Re$ and $w^+$. The high speed and medium speed cases (lower $w^+$) resemble literature based $Pr_T$ plots, such as Fig. 2.4, yet the variation observed in the low speed case suggest a break-down in Reynolds-analogy. For the medium speed and high speed cases $Pr_T$ is high in the near wall region representing an increase in eddy diffusivity, this is consistent with the ZPG case as well. The
ZPG profile collapses to 1 at $y^+ = 100$ agreeing with the data of Blackwell et al. [1972], whose near wall $Pr_T$ reached $\approx 2.25$. A strong variation due to induced forcing frequency is observed within the medium speed case. The low speed case presents a different set of dynamics where the entire profile remains below 1 in the the near wall region with noisy outer value reaching above 1. This indicates that the thermal eddy diffusity is dominant throughout the boundary layer thickness.

### 6.2.5 Buoyant Driven Transport

Within thermal boundary layer flow the inclusion of buoyant driven transport is observed in the time-averaged governing equation as:

$$
\frac{\partial \tau}{\partial x} + \frac{\partial \tau}{\partial y} = \nu \frac{\partial^2 \tau}{\partial y^2} - \frac{\partial (\overline{u'v'})}{\partial y} - \frac{\partial (\overline{\tilde{u}\tilde{v}})}{\partial y} - (\rho g \beta_T) (\overline{\theta_w - \theta_\infty}),
$$

where Eqn. 6.3 includes the Boussinesq approximation. The importance of buoyant driven transport within the flow cases under investigation is evaluated through $Ri$ number based thresholds. The $Ri$ space can be split into three zones;

- $Ri >> 1$: Flow is dominated by buoyant forces, free convective flow.
- $Ri \approx 1$: Buoyant and shear forces play important roles in transport, mixed convective flow.
- $Ri << 1$: Flow is dominated by shear forces, forced convective flow.

A literature based critical value of 0.25 is drawn as a straight dotted line to assist in separating the forced and mixed convective flow zones Galperin et al. [2007].
Figure 6.16. Parameter space of the time average data sets in Richardson number vs Reynolds number. Line denoting critical Richardson number between forced and mixed convection is shown as a dashed horizontal line.

Fig. 6.16 separates the flow cases into the category of mixed convective flow and forced convective flow. These distinctive flow cases will be used for the following analysis contained in Chapter 7 and Chapter 8.
6.2.6 $w^+$ Domains

To analyze the data sets in the context of the varied forcing domains established in varying $w^+$, the data sets have been grouped into different regimes based on $\omega^+$ as described in Chapter 5. Fig. 6.17 shows how the data sets are grouped as represented by the dashed boxes.

![Figure 6.17](image)

**Figure 6.17.** Data sets plotted as a function of $Re$ vs $w^+$ with outlines denoting the relative pre-established $w^+$ domain each data set has been placed into.

Inner normalized profiles are re-plotted in Fig. 6.18 based on the $\omega^+$ groupings shown in Fig. 6.17, as labeled in each panel of the figure. A clear progression in increased modification is evident with increasing $w^+$ in both the thermal and momentum field.
Figure 6.18. Time averaged PBL inner normalized mean profiles of the thermal field, where each figure contains data sets representative of the varied $\omega^+$ domains covered through the study, marker notation can be found in Table 6.1.
6.3 Summary

- Periodic forcing of the freestream did not significantly increase or decrease the near wall transport of momentum or heat.

- The medium and high speed (quasi-steady and low frequency in $w^+$) flow cases demonstrate similar behavior to steady boundary layer dynamics.

- Increases in $w^+$, corresponds to a forcing time scale that approaches the near wall transport time scale, results in significant changes to the boundary layer integral scales relative to steady boundary layer flow.

- Reynolds analogy does not strictly hold for the flow cases under investigation as $Pr_T$ is a function off wall-normal position and not approximately equal to 1.

- The low speed case falls into the category of mixed convection flow based on $R_i$ threshold.
CHAPTER 7
PHASE AVERAGE PULSATILE BOUNDARY LAYER FLOW: FORCED CONVECTION

The phase evolution of the medium and high speed data sets are presented within this chapter with a focus on the varied phase based differences in the phase response between the thermal and momentum field. From the time-averaged results, these data sets were shown to exhibit the dynamics similar to steady, incompressible boundary layer flow. The data is presented as a function of $\tilde{u}_\infty$ phase ($\gamma$) ranging from $0 - 360^\circ$, where markers and symbols correspond to that of the tables provided in Chapter 6 and all units match Tables 6.1 – 6.4. Data sets are grouped according to the $w^+$ scaling zones described within Ch 6.

7.1 Integral Parameters

The oscillatory ($\tilde{\cdot}$) component of the boundary layer integral parameters are investigated. Integral parameters are given in physical units of $m$ and plotted vs phase angle in degrees. Angular phase lead/lag plots are displayed for integral parameters, which determine the phase difference between the momentum integral parameter and thermal integral parameter through the use of a cross correlation. These plots are presented such that a positive angular value denotes the momentum integral parameter as leading and a negative angular lag denotes the thermal integral parameter as leading.

- Outer phase averaged boundary conditions ($\langle u_\infty \rangle$, $\langle \theta_\infty \rangle$) verse $\gamma$

To provide context for the data presentation to be used the outer boundary condition is presented in Fig. 7.1. $\langle u_\infty \rangle$ displays a smooth sinusoidal variation that decrease in amplitude with increasing frequency and becomes slightly skewed at the higher forcing frequencies. $\langle \theta_\infty \rangle$ displays fluctuations that are contained within the same magnitude as the uncertainty of the measurement and therefore shows no phase dependence.
Figure 7.1. Phase averaged outer boundary layer conditions ($\langle u_\infty \rangle, \langle \theta_\infty \rangle$) verse phase ($\gamma$).

- **Boundary Layer Thickness ($\tilde{\delta}, \tilde{\delta}_T$) verse $\gamma$**

Fig. 7.2 shows $\tilde{\delta}(\gamma)$ and $\tilde{\delta}_T(\gamma)$ for the two cases. A distinct decrease in thickness during the accelerating portion of the cycle is observed in the profiles. The phase angle where a distinct decrease is first observed increases with increasing frequency. The magnitude of the momentum integral decrease becomes suppressed with increasing $Re$ in the momentum field yet remains the same order across $Re$ in the thermal field. A tighter collapse of the data sets is observed for the profiles in the decelerating portion of the cycle. The thermal boundary layer thickness displays a strongly anti-symmetric shape across the cycle while the momentum boundary layer thickness is largely symmetric over the cycle. Fig. 7.3 shows that $\tilde{\delta}$ lags $\tilde{\delta}_T$ for all the data sets. A trend is not observed within the framework of $Re$ or $w^+$ yet values trend to remain at about $45^\circ$.

- **Boundary Layer Displacement Thickness ($\tilde{\delta}^*, \tilde{\delta}_T^*$) verse $\gamma$**

Fig. 7.4 shows the oscillatory component of the boundary layer displacement thickness plotted as a function of phase. The momentum displacement thickness varies strongly across the cycle while the thermal displacement thickness varies very little. Maximum values of
Figure 7.2. Oscillatory component of phase averaged boundary layer thickness ($\tilde{\delta}$, $\tilde{\delta}_T$) verse phase ($\gamma$).

Figure 7.3. Angular value of maximum correlation for phase lag between $\tilde{\delta}$ and $\tilde{\delta}_T$, where a positive lag indicates the momentum field leading and a negative lag indicates the thermal field leading.
the displacement thickness correspond to the phase angle of minimum velocity as this point has the largest boundary layer thickness. The dip in $\tilde{\delta}_T$ coincides with the location of the decrease of $\tilde{\delta}_T$ and indicates a rapid thinning of the thermal boundary layer thickness that occurs during the accelerating phases. As noted previously, the momentum integral variable demonstrates strong symmetry following $\tilde{u}_\infty$ which is not observed in the thermal field, with strong asymmetry occurring in the thermal field at lower $Re$ between the two half-periods of the cycle.

**Figure 7.4.** Oscillatory component of phase averaged boundary layer displacement thickness ($\tilde{\delta}_*, \tilde{\delta}_T$) verse phase ($\gamma$).

- **Boundary Layer Shape Factor ($\tilde{H}, \tilde{H}_T$) verse $\gamma$**

  Fig. 7.5 shows the oscillatory component of the boundary layer shape factor ($\tilde{\delta}_* / \tilde{\theta}$) plotted vs phase angle. The momentum shape factor shows sinusoidally varying profiles. At the higher $w^+$ forcing (low speed) case, the profiles approach the laminar boundary layer thickness value of 2.6 [Schlichting, 1979], which is indicative of a boundary layer under strong adverse pressure gradients. Under acceleration $\tilde{H}$ becomes mildly negative which is indicative of a boundary layer under favorable pressure gradient flow. This behavior is well understood and has been documented within the literature [Clauser, 1957]. The increase in $\tilde{H}_T$ aligns with the
zone of reduction observed in $\delta_T, \delta_T^*$. Angular lead plots displayed in Fig. 7.6 shows that $\tilde{H}$ leads $\tilde{H}_T$ for all cases yet the angular lead is reduced with decreasing frequency. When the angular lead is examined vs $w^+$ a clear increase in angular lead is observed with increasing $w^+$.

Figure 7.5. Oscillatory component of phase averaged boundary layer shape factor ($\tilde{H}, \tilde{H}_T$) verse phase ($\gamma$).

In summary the momentum integral parameters are symmetric about the two-half periods of a cycle. The thermal integral parameters response differs significantly between the accelerating and decelerating forcing conditions. A strong modification in all thermal integral parameters coincide with a frequency and $Re$ dependent event during the accelerating phase. For all integral parameters, high frequency and lower $Re$ led to a stronger modification across the cycle which is indicative of the field modification felt by increasing values of $w^+$. 
Figure 7.6. Angular value of maximum correlation for phase lag between $\tilde{H}$ and $\tilde{H_T}$, where a positive lag indicates the momentum field is leading and a negative lag indicates the thermal field is leading.

7.2 Outer Normalization

For a given variable, profiles are plotted at eight discrete phase angles starting at $0^\circ$ and separated by $45^\circ$. For a given phase angle, the legend for symbols is provided on the bottom panel of the plot. The respective time average profile is displayed on each plot based on the markers identified in Chapter 6.

- **Outer normalized velocity profile** ($\langle \tilde{u} \rangle$) verse $\langle \eta \rangle$

  Fig. 7.7 plots phase averaged outer normalized streamwise velocity profiles. Increased forcing frequency increases the variation observed within the profiles. The apparent increase in the measurement near the wall is not unexpected given the measurement uncertainty is highest near the wall.

- **Outer normalized temperature profile** ($\langle \tilde{\phi} \rangle$) verse $\langle \eta_T \rangle$
Figure 7.7. Outer normalized phase–average velocity profiles plotted $\approx$ every 45° with the corresponding freestream velocities shown in the bottom panel and the time average profile overlaid on each individual plot, shown in colored markers. Bottom panel displays the phase of each marker.
Figure 7.8. Outer normalized phase average temperature profiles plotted ≈ every 45° with the corresponding freestream velocities shown in the bottom panel and the time average profile overlaid on each individual plot, shown in colored markers. Bottom panel displays the phase of each marker.
Fig. 7.8 shows the phase–averaged outer normalized temperature profiles. A tight collapse is observed for all data-sets investigated with stronger variation occurring at the initial stages of the accelerating phases.

In comparing Fig. 7.7 and Fig. 7.8 the oscillatory velocity is modified by the forcing frequency whereas the oscillatory temperature shows only a weak sensitivity to forcing frequency. This is consistent with the study of Wang and Zhang [2005] where their LES model results showed that the momentum field is more sensitive to periodic forcing than the temperature field.

### 7.3 Inner Normalization

The data presentation format for the inner normalized profiles is the same as for the outer normalized profiles.

- **Inner normalized velocity profile** ($\langle u^+ \rangle$) verse $\langle y^+ \rangle$

  Fig. 7.9 plots the variation in the phase averaged inner normalized velocity profiles for the medium and high speed cases. Significant modification is observed through all data sets.

- **Inner normalized temperature profile** ($\langle \phi^+ \rangle$) verse $\langle y^+ \rangle$

  Fig. 7.10 plots the phase averaged inner normalized temperature profiles. An increase in $Re$ is noted to tighten up the collapse within the temperature profiles.

- **Freestream velocity amplitude** ($\frac{u^*}{u_\infty}$) with reversal criteria ($\frac{U}{\sqrt{2}}$)

  Fig. 7.11 plots the freestream velocity amplitude shown in the respective data set markers as a function of phase. Also plotted is the phase evolution of the separation criteria which specifies that when the freestream amplitude exceeds $\frac{U}{\sqrt{2}}$, flow separation along with reversal is likely to occur. Due to the difficulty of measuring the near-wall velocity, separation was not measured yet as evident from the plot, all cases except the lowest frequency flow separation is likely to occur.
Figure 7.9. Inner normalized phase average velocity profiles plotted $\approx$ every 45° with the corresponding freestream velocities shown in the bottom panel, shown in colored markers. Bottom panel displays the phase of each marker.
Figure 7.10. Inner normalized phase average temperature profiles plotted $\approx$ every $45^\circ$ with the corresponding freestream velocities shown in the bottom panel, shown in colored markers. Bottom panel displays the phase of each marker.
Figure 7.11. Freestream velocity amplitude percentage plotted on top of flow separation criteria of $\frac{U_s^+}{U_\infty\sqrt{2}}$ displayed in dashed (–) for the respective data set colors.
7.4 Near Wall Dynamics

Figure 7.12. Oscillatory component of phase averaged boundary layer wall stress/flux ($\tilde{\tau}_w$, $\tilde{q}_w$) verse phase ($\gamma$).

- Oscillatory component of phase averaged boundary layer wall stress/flux ($\tilde{\tau}_w$, $\tilde{q}_w$) verse phase ($\gamma$)

Fig. 7.12 displays the oscillatory component of the wall shear stress and wall heat flux for both fields. As noted in the integral parameters the momentum field wall shear stress, $\tilde{\tau}_w$, shows a sinusoidally varying profile which is consistent with why the profiles of long-time averaged variables in a PBL are similar to the profiles in ZPG boundary layer. The thermal wall flux shows differing behavior based on both the phase of the pulsatile wave and the respective frequency and $Re$. The sharp increase in the $\tilde{q}_w$ profile corresponds to the location of the modification is consistent with the phase angle where the integral parameters sharply vary.

- Magnitude and Phase shift of freestream verse near wall region

Fig. 7.13 represents a comparison between the effects of periodic forcing on the near wall vs the freestream, where the left plots represent the momentum field and the right plots represent the thermal field. The top plots represent the magnitude of the oscillations at the wall verse
Figure 7.13. Respective oscillatory component magnitude (top), and phase shift (bottom) vs $w^+$ for the momentum field (left panel) and thermal field (right panel). Phase shift is normalized by $45^\circ$ which represents the resultant phase shift of stokes flow.

the oscillations in the freestream. The magnitudes in the two fields are of the same order with neither containing a specific $w^+$ trend. The bottom plots represent the phase difference between the near wall and the freestream and is normalized by the Stokes phase lag which is $45^\circ$. The suggestion of inverse trends is present in the phase plots with increasing $w^+$ leading to a phase lead of the momentum near wall and leading to a phase lag of the thermal near wall. Dec et al. [1992] noted that the near wall region lagged freestream pulsations by up to $1/4$ of a cycle or $2 \times \gamma_{\text{stokes}}$.

**Magnitude and Phase shift of near wall region**

Fig. 7.14 depicts the magnitude and phase relationship between the wall shear stress and wall heat flux, where the top plot compares the magnitude of the oscillatory component of the two surface fluxes and the bottom plot compares the phase difference between them normalized by Stokes phase lag, $45^\circ$. A clear separation is evident in the magnitude plot where data sets
Figure 7.14. Respective oscillatory component magnitude (top), and phase shift (bottom) vs $w^+$ of the near wall momentum field vs near wall thermal field. Phase shift is normalized by 45° which represents the resultant phase shift of stokes flow.

with higher $Re$ sit about 1 and data sets with lower $Re$ sit below 1. This states that with the increase in $Re$ the shear stress oscillatory component is larger than the heat flux yet the reverse is true at lower $Re$. Both data sets trend in opposite directions with increasing $w^+$. A net shift toward positive phase lag is seen in the bottom plot indicating that the wall shear stress leads the wall heat flux.

In summary, the near wall dynamics indicate similar trends to the integral parameters where the momentum field remains largely sinusoidal in the variation of it’s oscillatory components where the thermal field is not symmetric between the accelerating and decelerating portion of the cycle. $\tilde{q}_{w''}$ shows a increased sensitivity to changes during the accelerating phase as shown in the integral parameters of the cycle. When the freestream and near wall oscillations are compared, the oscillatory velocity and oscillatory temperature are on similar order of magnitude yet the that the
near-wall phase leads the freestream while the thermal field shows that the wall heat flux phase lags the freestream. In comparing the wall shear stress to the wall heat flux a strong \( Re \) dependence is observed along with an indication that with increasing \( w^+ \) the wall shear stress leads the wall heat flux.

### 7.5 Phase Based Coherent Structures

To investigate the phase evolution of the momentum field and thermal field, phase contours of \( \langle \tilde{u} \rangle \), \( \langle \tilde{\phi} \rangle \), and \( \langle \tilde{\phi}' \rangle \) are plotted in Fig. 7.15, 7.16, 7.17. From the integral parameters and near wall dynamics analysis it was observed that the thermal field had a significantly varied response during the accelerating portion of the cycle compared to the decelerating portion of the cycle, which will be interpreted in the context of these figures. The figures presented here show intensity based color contours of outer normalized variables. The contours are plotted in outer normalized wall normal location \((\eta, \eta_T)\) vs \( \gamma/\omega \) such that they are displayed in the quantities of distance and time and make comparison between plots straight forward.

In examining figures 7.15 and 7.16 a distinctive flow structure is observed during the accelerating phases. This structure can be seen as a region of high velocity, low temperature fluid (light colored) penetrating towards the wall in the accelerating phases. Examination of the time occurrence of this structure shows that it occurs at \( t \approx 0.5 \text{sec} \) for the medium speed case and \( t \approx 0.3 \text{sec} \) for the high speed case.

Fig. 7.17 indicates that there exists high near wall phase averaged temperature fluctuations until the above indicated flow structure brings high momentum low temperature fluid towards the wall. A near wall suppression of the fluctuations then results. The timing of which is consistent with what is indicated above for the medium and high speed case. To examine this flow structure further, the gradient of the two fields, velocity and temperature are computed as a function of \( \gamma/\omega \).

Fig. 7.18 and 7.19 display the phase average derivative of velocity and temperature respectively. Interestingly, Fig. 7.18 retains a strongly symmetric contour for all forcing frequencies. Examination of Fig. 7.19 shows a strong footprint of the so called acceleration flow structure. The
Figure 7.15. Contour plot of outer normalized velocity field \(\langle \vec{u} \rangle\) arranged in wall normal distance \(\eta\) verse time evolution of pulsatile forcing \((\gamma/\omega)\).

A feature is represented by a strong positive gradient symmetrically followed by a strong negative gradient of slightly weaker strength. The positive gradient is indicative of a decrease in temperature which is then followed by a rapid rise in temperature after which a weaker signature of the same structure can be detected. This feature can be described by a sweep of high momentum low temperature fluid followed by an ejection of high temperature fluid which can be seen through the negative gradient spike and the reduction in the near wall fluctuating component. This feature can be paired with the near wall dynamics to understand the rapid rise in wall heat flux due to the sweep of cold fluid, which compresses the near wall gradient observed in Fig. 7.12 and 7.4. From an integral scale viewpoint this structure can be used to explain the reduction in \(\langle \delta_T \rangle\).
To investigate if buoyant forces play a role in this sweep and ejection feature the $Ri$ is computed for the thermal field. Fig. 7.20 plots the outer normalized temperature field as shown in Fig. 7.16 with an overlaid contour of areas that surpass the critical threshold of $Ri = 0.25$. What is observed is a wall region where buoyancy is non-negligible that advects off wall during and post the signature of the sweep event. This indicates that buoyancy does play a key role in the development of this flow structure and assists in the ejection of hot fluid from the wall in so-called buoyant plumes.
Figure 7.17. Contour plot of outer normalized temperature rms field ($\langle \tilde{T} \langle \rangle$) arranged in wall normal distance ($\eta_T$) verse time evolution of pulsatile forcing ($\gamma/\omega$).
Figure 7.18. Contour plot of outer normalized velocity field gradient in phase based time ($\omega d(\langle \hat{u} \rangle)/d\gamma$) arranged in wall normal distance ($\eta$) verse time evolution of pulsatile forcing ($\gamma/\omega$).
Figure 7.19. Contour plot of outer normalized temperature field gradient in phase based time ($\omega d(\bar{\phi})/d\gamma$) arranged in wall normal distance ($\eta$) verse time evolution of pulsatile forcing ($\gamma/\omega$).
Figure 7.20. Outer normalized thermal phase average field contours overlaid with black shaded regions denoting Richardson number above critical threshold.
7.6  Summary

- The momentum field is largely symmetric between the accelerating and decelerating portions of the cycle.

- The thermal field is largely anti-symmetric with a varied response between the accelerating and decelerating portion of the cycle.

- The ratios between the freestream and near wall oscillations are of similar order between the momentum and thermal fields.

- The momentum field however shows that the near-wall phase leads the freestream while for the thermal field, the near-wall phase lags the freestream.

- The footprint of a sweep and ejection event during the accelerating portion of the cycle is strongly suggestive that buoyancy is important during some phases of the cycle.
CHAPTER 8

PHASE AVERAGE PULSATILE BOUNDARY LAYER FLOW: MIXED CONVECTION

The low speed case as determined in Chapter 6 has significant buoyant effects which modify the controlling dynamics. An analysis is performed here to further understand the effects of mixed convection on boundary layer transport. The data is presented as a function of phase ($\gamma$) ranging from $0 - 360^o$, where markers and symbols match that of the tables displayed in Chapter 6, and all units match the base unit system ($m, m/sec$).

8.1 Integral Parameters

The evaluation of the phase evolution of both momentum and thermal integral parameters provides an understanding of how the flow responds to the periodic forcing of the freestream.

Figure 8.1. Oscillatory component of phase averaged boundary layer thickness ($\tilde{\delta}, \tilde{\delta}_T$) verse phase ($\gamma$).

- Boundary Layer Thickness ($\tilde{\delta}, \tilde{\delta}_T$) verse $\gamma$
Fig. 8.1 plots the momentum and thermal boundary layer thickness as a function of phase. A distinctive anti-symmetric shape is seen in both profiles. In the decelerating region a strong drop in $\tilde{\delta}$ occurs. This drop is observed in $F_3$, $F_4$, and $F_5$. The thermal boundary layer thickness show a sharp drop during the decelerating portion of the cycle only for $F_2$.

![Graphs showing momentum and thermal boundary layer thickness](image)

**Figure 8.2.** Oscillatory component of phase averaged boundary layer displacement thickness ($\tilde{\delta}^*$, $\tilde{\delta}_T^*$) verse phase ($\gamma$).

- **Boundary Layer Displacement Thickness ($\tilde{\delta}^*$, $\tilde{\delta}_T^*$) verse $\gamma$**

  Fig. 8.2 plots the evolution of the boundary layer thickness in each field as a function of phase evolution of the momentum and thermal displacement thickness. A stronger sinusoidal feature is observed in $\tilde{\delta}^*$ than $\tilde{\delta}$, which suggest that the inner region and near wall gradient remains largely controlled by the freestream oscillations. The skewness of the profile could be attributed to the skewness occurring in the forcing condition as the accelerating portion of the freestream forcing is drawn out. The thermal displacement shows muted sensitivity to bth phase angle and forcing frequency.

- **Boundary Layer Displacement Thickness ($\tilde{H}$, $\tilde{H}_T$) verse $\gamma$**

  Fig. 8.3 plots the phase average evolution of the oscillatory component of the momentum and thermal shape factor. $\tilde{H}$ shows the anticipated modification with a reduction in the accelerating region followed by an increase in the decelerating region. A signature of the decelerating modification of $\tilde{\delta}$ is seen in $F_3$ and $F_4$. Although the same feature is not shown in $\tilde{\delta}^*$ this
suggest that significant off wall gradient exists within the flow field. $\tilde{H}_T$ follows $\tilde{\delta}_T$ with the exception of $F_2$, where a significant spike over the course of the accelerating region is observed.

**8.2 Outer Normalization**

Data presentation of the outer normalized oscillatory profiles is as follows: For a given variable, profiles are plotted at eight discrete phase angles starting at $0^\circ$ and separated by $45^\circ$. For a given phase, angle, the legend for the symbols is provided on the bottom panel of the plot.

- **Outer normalized velocity profiles ($\langle \tilde{u} \rangle$) verse $\langle \eta \rangle$**

  Fig. 8.4 plots profiles of the outer normalized phase-averaged velocity. The profiles modulate symmetrically in $F_2$ about the time average profile. Where as an increased modulation is observed in $F_4$ and $F_5$, specifically the early phases of the acceleration portion of the cycle shows a lower velocity near the wall and an increased gradient in velocity away from the wall. $F_3$ displays a differing type of dynamics where early acceleration profiles and early decelerating profiles display a high gradient in the freestream.

- **Outer normalized temperature profiles ($\langle \tilde{\phi} \rangle$) verse $\langle \eta \rangle$**
Fig. 8.5 plots the outer normalized phase averaged temperature profiles. The high gradient away from the wall observed in $F_3$ represent a similar feature within $\langle \tilde{\phi} \rangle F_3$ as well. A spike is observed in $F_3$ which indicates a portion of cold fluid with high momentum entering into the boundary layer. $F_2$ displays profiles in the early accelerating portion which carry more heat higher into the boundary layer which could come from the observed buoyancy effects.
Figure 8.4. Outer normalized phase average velocity profiles plotted \( \approx \) every \( 45^\circ \) with the corresponding freestream velocities shown in the bottom panel and the time average profile overlaid on each individual plot, shown in colored markers. Bottom panel displays the phase of each marker.
Figure 8.5. Outer normalized phase average temperature profiles plotted ≈ every 45° with the corresponding freestream velocities shown in the bottom panel and the time average profile overlaid on each individual plot, shown in colored markers. Bottom panel displays the phase of each marker.
8.3 Inner Normalization

Inner normalized profiles are displayed in the following figures which are determined from computation of $u_\tau$ and $q_w$ as outlined in Chapter 2. Profiles are plotted for $\approx$ every $45^\circ$. Interestingly as noted in Chapter 7 a stronger collapse for the respective phase profiles plotted is noted for $\langle u^+ \rangle$ than $\langle \phi^+ \rangle$ which as noted previously runs counter to Wang and Zhang [2005]. Distinctive connecting dynamics between the fields are not evident in this framework and it is clear that neither field is dominated by the dynamics present in the near wall region. From Fig. 8.7 it is observed that profiles in the late acceleration early deceleration period have high near wall gradient and reach higher ultimate $\langle \phi^+ \rangle$ values noting lower temperature flow.
Figure 8.6. Inner normalized phase average velocity profiles plotted \(\approx\) every 45° with the corresponding freestream velocities shown in the bottom panel and the time average profile overlaid on each individual plot, shown in colored markers. Bottom panel displays the phase of each marker.
Figure 8.7. Inner normalized phase average temperature profiles plotted every 45° with the corresponding freestream velocities shown in the bottom panel and the time average profile overlaid on each individual plot, shown in colored markers. Bottom panel displays the phase of each marker.

8.4 Near Wall Dynamics

Near wall dynamics are analyzed in the framework of the oscillatory component of the near wall shear stress and heat flux. Values are shown as a function of phase (γ). Phase positions are analyzed
in the framework of lead lag relationships between the near wall and freestream for the two fields under investigations followed by a joint investigation between the near wall dynamics of the two fields.

![Graph showing near wall and freestream dynamics](image)

**Figure 8.8.** Oscillatory component of phase averaged boundary layer wall flux ($\tilde{\tau}_w$, $\tilde{q}_w$) verse phase ($\gamma$).

- **Oscillatory component of phase averaged boundary layer wall stress/flux ($\tilde{\tau}_w$, $\tilde{q}_w$) verse phase ($\gamma$)**

  Fig. 8.8 plots the oscillatory component of the phase averaged wall shear stress and wall heat flux, where $\tilde{\tau}_w$ displays a skewed sinusoidal profile which is representative of the skewed freestream forcing. The wall heat flux presents two varied set of dynamics; in $F_2$ and $F_3$ a spike is observed which dominates the form of the heat flux during the decelerating portion of the cycle, and in $F_4$ and $F_5$ a relatively smooth modulation is observed which peaks during the accelerating portion of the cycle.

- **Magnitude and Phase shift of freestream verse near wall region**

  Fig. 8.9 displays the relative magnitude and phase shifts associated between the near wall flow modification and freestream modification. The velocity field is shown in the left panels and displays the expected increasing phase shift towards $\gamma_{stokes}$ as well as an increase in the relative near wall modulation. The right panels, displaying the thermal field, tell a different story where two separate set of controlling dynamics appear to be present. An increase in
Figure 8.9. Respective oscillatory component magnitude (top), and phase shift (bottom) vs $w^+$ for the momentum field (left panel) and thermal field (right panel). Phase shift is normalized by $45^\circ$ which represents the resultant phase shift of stokes flow.

A phase shift and in relative magnitude is observed which is then followed by a decrease in both values. Less weight is put into these values as the near wall modification profile does not take on the sinusoidal modification as observed in $F_2$ and $F_3$.

- **Magnitude and Phase shift of near wall region momentum field vs near wall thermal field**

Fig. 8.10 compares the magnitude and phase shifts of the wall shear stress and the wall heat flux. The magnitude of the wall shear stress modulations are significantly greater than those of the wall heat flux for all frequencies. Phase shifts indicate similar trends as those observed in Fig. 8.9 with the wall shear stress modifications significantly leading the wall heat flux.
Figure 8.10. Respective oscillatory component magnitude (top), and phase shift (bottom) vs $w^+$ of the near wall momentum field vs near wall thermal field. Phase shift is normalized by $45^\circ$ which represents the resultant phase shift of stokes flow.

8.5 Phase Based Coherent Structures

To investigate the phase evolution of the momentum field and the thermal field, phase contours of $\langle \tilde{u} \rangle$, $\langle \tilde{\phi} \rangle$, and $\langle \tilde{\phi}' \rangle$ are plotted in Fig. 8.11 through 8.14.

Fig. 8.11 through 8.14 are displayed in the form of wall normal position ($\eta, \eta_T$) verse phase based time ($\gamma/f$). Comparing figures 8.13 and 8.14 the sweep and ejection signature observed in Ch 7 is evident from the coupled zones of gradient in Fig. 8.14 along with the reduction in near wall fluctuations in Fig. 8.13, the thermal event occurs at $t \approx 1.8sec$. These same signatures are not evident at the higher forcing frequencies, suggesting that the formation of these sweep and
ejection events did not have time to develop. Interestingly the contours of $\langle \tilde{\phi} \rangle$ are significantly higher than that of the forced convection and persist further into the boundary layer.

Figure 8.11. Contour plot of outer normalized velocity field ($\langle \tilde{\phi} \rangle$) arranged in wall normal distance ($\eta$) verse time evolution of pulsatile forcing ($\gamma/f$).

To better understand the dynamically important forces developing in the flow field the dimensionless parameters: $Ri$, $Gr$, and $Pr_T$ are all plotted in Fig. 8.15. $Ri$ remains high across most phases of the cycle and demonstrates that, as anticipated, buoyancy plays an important role in the
boundary layer transport. Phases with high $Ri$ correspond to phases with high $Pr_T$. This suggests that buoyancy increases the turbulent diffusion in the boundary layer, which manifests itself in the time-average by producing skewed profiles as observed in Chapter 6 Fig. 6.12. Interestingly, $Gr$ is high in some spatial regions of buoyant flow, it is a poor predictor of the spatial distribution of the buoyant force, this comes from the integrated view of $Gr$ which bases the balance of forces off of
Figure 8.13. Contour plot of outer normalized rms temperature field $\langle \tilde{\phi} \rangle$ arranged in wall normal distance ($\eta_T$) versus time evolution of pulsatile forcing ($\gamma/f$).

thermal properties of the working fluid and does not involve direct shear measurements based on the definition used in this study. When these spatial distribution are interpreted in the context of the outer forcing it becomes clear that with higher forcing frequencies, the buoyant forces dominate a wider portion of the overall cycle. The reason may be that the buoyant time scales are smaller than the forcing time scales yet the sweep and ejection motion time scale becomes larger and therefore
Figure 8.14. Contour plot of phase based temporal gradient of outer normalized temperature field \( (f \frac{d\langle \hat{\phi} \rangle}{d\gamma}) \) arranged in wall normal distance \( (\eta_T) \) verse time evolution of pulsatile forcing \( (\gamma/f) \).

the dominant mode of transport shifts to dominated by buoyancy. The following section works to better understand the time scale evolution.
8.6 Temporal Scaling

Understanding the relevant forcing dynamics and dominant modes of transport can be viewed from a matter of time scales within pulsatile flow. When the forcing time scale becomes smaller than the time scales which control the flow dynamics an influence in the dynamics is observed. Fig. 8.16 plots the ratio of the time scale of the sweep and ejection event to the time scale of the freestream forcing. $F_4$ and $F_5$ are observed to have a forcing time scale which is smaller than the determined sweep/ejection time scale, this removes a dominant mode of heat transfer with the lack of sweeping high momentum cold fluid to the wall. Fig. 8.17 plots the ratio of a buoyant time scale to the forcing time scale. The buoyant time scale is determined as:

$$\frac{\langle u_T^2 \rangle}{\left( \langle q_w \rangle \right) (\rho C_p \gamma g \Delta T)}$$  

(8.1)
where \( \beta_T \) is the thermal expansion coefficient, and \( g \) is the gravitational acceleration. This time scale is based upon the work of Petukhov and Polyakov [1988] in which the free parameters in buoyant flow were used to determine respective controlling scales. This figure shows that ratio between the two time scales becomes order 1 through the phase evolution of the flow and therefore the buoyancy is able to play a role in transport yet becomes suppressed and indicates why the high frequency forcing has a lower \( S_t \) then the respective ZPG flow without forcing and is a function of phase angle.

**Figure 8.16.** Ratio of thermal event time scale to forcing time scale for mixed convective flow
Figure 8.17. Ratio of buoyant time scale to forcing time scale for mixed convective flow.
8.7 Summary

- Mixed convection is highly frequency dependent due to the wide variety of dynamically important time scales.

- For all cases investigated the magnitude of the oscillatory near wall stress dominated the magnitude of the oscillatory wall heat flux.

- Observed sweep and ejection structures are time scale sensitive and do not have time to develop in the high frequency forcing.

- Buoyant time scales remain smaller or on the same order as the forcing frequency for all low-speed cases investigated and therefore play a dominant role.
CHAPTER 9
SUMMARY AND CONCLUSIONS

The present work has focused on the following question:

What are the effects of periodic forcing of the freestream velocity on the thermal boundary layer structure and thermal transport?

The answer to which has seen little work over the last few decades and remains largely an open question [Li et al., 2014, Dec et al., 1992]. The implication to the lack of physical understanding is that approximations and assumptions are extrapolate from simple lab scale studies to industry relevant flows about the similarities between momentum and thermal boundary layer flows due to common assumption that the thermal field acts as a passive scalar. The approach to providing an answer to this question primarily focused on experimental studies aimed to investigate the similarities or difference between momentum and thermal boundary layer flow in the industry relevant flow containing heat in a thermal boundary layer with an imposed sinusoidal pressure forcing. A purpose built facility, described in Chapter 4, was constructed for the careful examination of the so-called pulsatile boundary layer flow, details of the specific experimental conditions can be seen in Chapter 6.

9.1 Time Average

A primary finding of the present study is that periodic forcing of the freestream does not yield a significant increase (or decrease) in the cycle-averaged wall shear stress or wall heat flux compared to steady ZPG boundary layer flow. While the former is fairly well-accepted, the latter results are new and important. Examination of outer and inner normalized time average profiles showed that with the increase of a parameter called \( w^+ \) (compared forcing time scale to near wall transport time scale) a deviation can be seen in the profiles. Amazingly, though at high \( Re \) the time average
profiles collapse stating that whatever effect the forcing has is averaged out over a cycle, which demonstrates the robustness of the boundary layer dynamics in both fields. Offsets through the time average framework came from the introduction of the buoyant forcing term which strongly influenced the low Re, low f cases and had the effect of increasing near wall transport in both fields.

9.2 Phase Average

9.2.1 Forced Convection

The phase evolution of the phase-averaged flow variables differed between the momentum field and the thermal field. The momentum field retained a largely symmetric shape, between the accelerating and decelerating portion of the cycle. This is a strong indication for why a time average description of the boundary layer flow differs very little from its steady-state counterpart which has been shown in multiple studies [Brereton et al., 1990, Ebadi, 2016]. On the other hand, the thermal boundary layer flow is largely asymmetric between the accelerating and decelerating portion of the cycle. This increased and varied sensitivity is believed to come from the existence of a sweep and ejection structure which is formed during the accelerating portion of the cycle. This flow structure is observed to coincide with the modification of the integral and near wall parameters in the thermal field and has also been observed in previous studies [Dec et al., 1992]. It was also observed that a contributing factor to the strength of this structure was the emerging importance of buoyant forces.

9.2.2 Mixed Convection

Mixed convection is highly dependent on the forcing frequency due to the wide variety of dynamically important time scales. Off wall gradients and transport become important and sometimes dominant based on forcing frequency and phase. The thermal field again demonstrates an increased sensitivity to periodic forcing. With the progression towards higher forcing frequencies, the importance of the buoyant and sweep-ejection time scale were highlighted. At high forcing frequencies the sweep and ejection structure was not present which supports the notion of a time-
scale criteria for sweep and ejection structures to emerge. It was shown that the buoyant time scales remained dominant throughout the mixed convection cases.

9.3 Forcing Magnitude and Phase Shift

Brereton and Mankbadi [1995] depicted the evolution of the influence of a pulsatile freestream through a timescale based argument on $w^+$. The proceeding analysis works to compare this data set to the previous work and repeat the analysis for the thermal field to draw conclusions between the two fields response in both magnitude and phase as seen in Fig. 9.1. The top panels denote the magnitude of the near wall modulation relative to the outer forcing modulation, and the bottom panels denote the phase shift between the near wall field transport parameter and the freestream, where positive denotes the near wall leading the freestream. Within the momentum field the near wall leads the freestream with increasing $w^+$, trending towards a value of 1 which would denote $\gamma_{stokes}$ of 45°, where the dashed lines represents the trend seen in Brereton and Mankbadi [1995].

The thermal field, displayed on the right panels, shows a decrease in oscillatory magnitudes with increasing $w^+$, opposite the response of the momentum field. Two distinctly separate stories exist within the phase shift of the wall heat flux relative to the freestream forcing. For forced convective flow, the near wall flux lags the freestream and suggests at an increasing phase lag until the trend is significantly altered by the emerging importance of buoyancy. For mixed convective flow, a significantly elevated lag with the near wall leading the freestream is observed.

Fig. 9.2 shows a comparison between the magnitude and phase of the oscillatory component of the wall shear stress and wall heat flux. The top panel represents the comparison of the oscillatory magnitudes and the bottom panel represents the phase shift of the oscillatory wall shear stress compared to the oscillatory wall heat flux. Within the context of the oscillatory magnitude comparison, two separate stories exits that are suggestive of a trend to much greater wall stress magnitudes yet the change in thermal flow regimes makes it challenging to draw definitive conclusions. For quasi-steady and low frequency flow, the oscillatory magnitudes are in balance with the thermal
Figure 9.1. Respective oscillatory component magnitude (top), and phase shift (bottom) vs \( w^+ \) for the momentum field (left panel) and thermal field (right panel). Phase shift is normalized by 45° which represents the resultant phase shift of stokes flow. Black dashed line denotes a trend from Brereton and Mankbadi [1995]. Red dashed lines denote the various \( w^+ \) regimes where; i=Quasi-steady, ii=Low frequency, iii=Intermediate frequency, iv=High Frequency [Brereton and Mankbadi, 1995]

component being slightly larger. For the higher \( w^+ \) mixed convection cases the oscillatory shear stress is larger and grows in relative magnitude with higher \( w^+ \).

For the lower \( w^+ \) regimes, the lower panel shows that the wall shear stress and wall heat flux remain closely in phase, with the wall shear stress slightly leading the wall heat flux. From comparison of the two bottom panels of Fig. 9.1 this would be anticipated as the wall shear leads the freestream and the wall flux lags. The inclusion of buoyant flow presents a discontinuous trend in phase yet higher \( w^+ \) values in buoyant flow present a trend leading to in-phase flow which may represent a shifted version of the trend observed at low forcing frequencies. Ultimately it can be stated that the relationship between the near and far field momentum field develop as anticipated from previous studies [Brereton and Mankbadi, 1995], yet the thermal field shows an oscillatory
suppression from increasing $w^+$ and does not show a clear phase shift trend associated with increasing $w^+$ domain.

In context of drawing a direct relationship between the near wall transport of the two fields it is clear to see that the data presented in the thesis clearly shows that the momentum boundary layer and thermal boundary layer responded differently to periodic forcing and Reynolds analogy would not hold in PBL flow. In the time averaged framework at high $Re$ significant difference between the two fields near wall transport were not observed and it could be said that Reynolds analogy may still hold here, yet the phase average frame work details how this would fail. 1) The magnitude of the near wall pulsatile effect would be overestimated based upon the magnitude
comparison. 2) The peak in the wall heat flux occurs at a different phase than the peak in the wall shear stress having implications for any dynamic boundary. It is also clear that the characterization of temperature as a passive scalar breaks down even without examining the underlying phase based evolution of the fields.

### 9.4 Overview

Investigation in these flows has shown that specifically, even in low $Ri$ flows buoyancy is non-negligible during certain phases of the cycle and is dynamically important. Ultimately, the thermal field in PBL flow cannot be treated as a passive scalar. For true understanding and computation of the underlying physics, the coupling of the momentum and thermal flow field must be examined.


\section*{9.5 Future Work}

The future work leading from this campaign can be split into a number of investigations:

1. \textit{Investigations in pulsatile flow}:

   The NEAT tunnel allows for the investigation of PBL over a broad range of $w^+$ and $Re$. These studies can include various $w^+$ forcing profiles through the control of the imposed wave shape and therefore forcing gradient.

2. \textit{Investigations in convective flow}:

   Due to the quality of the experimental setup a wide variety of experiments can be conducted to improve our understanding of mixed-convection boundary layer transport. Spatially varying convective cases can also be coupled to this work due to the flexibility of the facility design.

3. \textit{Investigations in pulsatile convective flow}:

   An open question still remains as to the modification of the rms field and specifically the modification of the turbulent heat transfer term $v\Theta$. Resolution of this terms would require simultaneous measurements of the velocity and temperature field yet this could be achieved through the added capability of dual single point measurements such as a hot/cold wire system. Finely resolved $vT$ would be of great benefit for understanding the flow dynamics in pulsatile and in mixed convective flow.
BIBLIOGRAPHY


Srba Jovic and David M. Driver. Backward-facing step measurements at low reynolds number, \( \text{re}(\text{sub} \ h) = 5000 \). NASA, 1:1–30, 1994.


164


APPENDIX A

COMPUTATION OF WALL HEAT FLUX
1 Near Wall Gradient Method For Determining Wall Heat Flux

1.1 Intro
The measurement of wall heat flux is challenging in an experimental data set due to the required near wall resolution. When coupled with non-equilibrium flow behavior the required resolution becomes greater and commonly utilized near wall fits and models break down. Therefore the development of a method for determining near wall heat flux in non-equilibrium flow is desirable.

To serve as a starting point for the development of a computational method the DNS RCF data of UVM will be utilized. This data which represents fully reversing heated flow which will serve as a “worst” case test. From comparing model heat flux values to true values computed in the DNS the acceptable range of applicability for the model can be bounded in a non-equilibrium parameter such as the Clauser parameter $\beta$.

1.2 Outline
The following outlines the approach:

**Procedure for fit:**

*known:* $u_\tau, T_{wall}, T_\infty$

$y^+ = \frac{yu_\tau}{\nu}$

$T^+ = \frac{T_{wall} - T}{T_e}$

$T_e = \frac{q_w}{\rho c_p u_\tau}$

**Normalize**

1. Compute $y^+$ with know $u_\tau$ value
2. Compute $T^+$ with initial $q_w$ guess

**Extrapolate**

1. Create near wall $y^+ = T^+ \ast Pr$ profile
2. Modify the value of $n$ until profile collapses with near wall region of experimental $T^+$ profile
3. append near wall extrapolated data onto experimental profile

**Overlap**

1. Determine the number of position beneath “fit height” ($y_{fit}^+$)
2. Interpolate points from mean master profile onto $y$ positions of experimental data beneath fit height
3. Compute error between two profiles
4. Weight error to put more weight into experimental points and less into extrapolated points

**Iterate**

1. Increase guess in $q_w$
2. Cycle through the following procedure until error is minimized:
• Normalize
• Extrapolate
• Overlap

1.3 Definitions of Terms

In evaluating the model performance there are a number of parameters that are tuned based on set error thresholds:

- \( y_{\text{low}}^+ \): The bottom point of the data being evaluated
- \( y_{\text{fit}}^+ \): The top point of the near wall fit created from the master profile which is submitted to the model
- \( \phi \): phase
- \( \delta^* \): displacement thickness
- \( \beta = \delta^* \frac{\partial P}{\partial x} / \tau_w - \tau_w = u_{\tau}^2 / \rho - T^+ = \text{inner normalized temperature} \)

2 Load RCF Data

DNS Reciprocating channel flow data from UVM [Pond 2015]

```
Out[5]:  Re    nu  Pr  alpha  rho  cp  k  mu
       0  2000 0.0005 0.7 0.000714 1 1 0.000714 0.0005
```
2.1 Plot Temperature Profiles
3 Normalize Data

Define Terms:
4 Create Mean Master Profile

Create a vector of $y^+$ points all spaced by $1y^+$ from 1 to $y_{fik}$, which all profiles will then be interpolated onto such that all DNS data sets can be averaged together to determine the master profile.

4.1 Plot $T_{\text{nearwall}}$ on top of all DNS Data

The plot below shows a zoomed in view of all the data which was averaged together as well as the near wall $T_{\text{master}}$ shown as red circles.
5 Compute Qw

Using the DNS OCF data with no points beneath \( y^+ = y_{low}^+ \) create a matching code that utilizes the wall heat flux value contained within the inner normalization as a tuning parameter for overlapping and matching the near wall temperature profile.

Outline:

- Determine how to append near wall data - Run example computations as model test
- Determine \( y_{fit}^+ \) - Determine error associated with \( y_{low}^+ \)
- Determine error associated with \( u_\tau \)
- Determine effect of addition of noise

5.1 Examine near wall Appended Data

- Examine how the appending of the near wall data is changes the data set
- Work to append near wall data that stems from physical rational
- Data is appended by computing linear fit between \( y^+ \) and \( T^+ \cdot Pr \)
- Based on this fit near wall points are then added in between the bottom point and \( y=0 \)

![Graph 1](image1)

![Graph 2](image2)

Above profile has points beneath \( y^+=20 \) removed
5.2 Example Wall Heat Flux Computation

Display an example computation of the wall heat flux to give more understanding for when error surface are examined.

- \( y_{low} = 1 \)
- \( y_{fit} = 20 \)
- Computed Richardson Deriv used avg of 5 near wall points.
5.2.1 Reduce points to examine robustness of Richardson Deriv

Display an example computation of the wall heat flux to give more understanding for when error surface are examined.

- \( y_{\text{low}}^+ = 8 \)
- \( y_{\text{fit}}^+ = 20 \)
- Computed Richardson Deriv used avg of 5 near wall points
5.3 Determine $y^{+}_{fit}$

With input data of DNS down to $y^{+}_{low} = 1$ vary $y^{+}_{fit}$ until a reasonable error level is reached, then determine the acceptable $y^{+}_{fit}$ through a surface map of phase vs $y^{+}_{fit}$ vs error.

The above plot shows contours of error relative to the phase of the profile under investigation and the maximum height that the profile was fit to. Error greater than 10% is not shown to result as a threshold. From this plot there is always an occurrence of high area at the flow reversal. At approximately $y^{+} = 8 = 10$ there appears a minimum in the error plot. $y^{+} = 10$ is taken as the value of the appropriate fit height.
The above plot shows the error between the fit Master profile which is submitted to the model and the minimized error profile plot determined in the model. The regions of highest error occur in the location of the steepest gradient in the freestream velocity of highest values of $dp/dx$ which also corresponds to the area of reversal. The maximum error occurs slightly after the point of reversal and is not affected by location of Fit height.
5.4 Test Robustness of $y_{f_{fit}}$

The number of near wall points is reduced back to $y^+ = 5 = y_{low}^+$ and the above code is re-run to verify the robustness of the determined fit height.

With the bottom position of the data being cut to $y^+ = 5$ a top fit value of approximately 8 is still the best performing across all phases.
5.5 Determine error as a function of $y_{low}^+$

Vary $y_{low}^+$ from 1 to 20 and examine resulting error with $y_{fit}^+$ value determined above.

Plot displays error profiles which matches with physical understanding and showcases expected trends. For experimental values under investigation phase 0-2 represent the observed $\beta$ values and will taken as indications of developed near wall error.
5.6 Error in Utau

- Add in 10 percent error into $u_\tau$ measurement to determine sensitivity of model value.

- A significant deviation in model predicted heat flux is not observed due to the computed error in the $u_\tau$ values.
B.1 Components

A subsection of the drawings developed for the production of the thermal wall plate are shown below. The drawings detail the size and shape of the wall plate components, yet other features such as hole patterns or underneath cut-outs can be obtained by request. Following each set of drawings included figures show the final fabricated components. The convective plates are not pictured as they are simple rectangular plates which can be fully realized from the plate drawings.
**Full Wallplate** Components are denoted with sequential letters A through F, where drawings detail the acetal frame, insulation and convective plate.
Components A

See drawing AssembledFirstSection for additional details

See drawing AssembledFirstSection2 for additional details

 dimensions are in inches

tolerances:

fractional

angular: mach bend

two place decimal

three place decimal

Do not scale drawing

Eric Desjardins

UNLESS OTHERWISE SPECIFIED:

SCALE: 1:2 WEIGHT:

REV'DWG. NO.

SIZE

TITLE:

NAME DATE

COMMENTS:

Q.A.

MFG APPR.

ENG APPR.

CHECKED

DRAWN

FINISH

MATERIAL

INTERPRET GEOMETRIC TOLERANCING PER:

DIMENSIONS ARE IN INCHES

TOLERANCES:

FRACTIONAL

ANGULAR: MACH BEND

TWO PLACE DECIMAL

THREE PLACE DECIMAL

DO NOT SCALE DRAWING

185
Components B
Components C

This center line is based on pocket including tabs.

Dimensions are in millimeters.

From Lab #5

C_size

MFG

APPV'D

CHK'D

DRAWN

UNLESS OTHERWISE SPECIFIED:

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LINEAR:

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Q.A.

MFG

APPV'D

CHK'D

DRAWN

DIMENSIONS ARE IN MILLIMETERS

SHEET 1 OF 1

SCALE: 1:5

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MATERIAL:

DIMENSIONS ARE IN INCHES

TOLERANCES:

FRACTIONAL

ANGULAR: MACH

BEND

TWO PLACE DECIMAL

THREE PLACE DECIMAL

(X3) S111InsulC_V4

Insulation C

A

DO NOT SCALE DRAWING

(2019)
Components D

Wall Plate, D

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- DIMENSIONS ARE IN MILLIMETERS
- SURFACE FINISH:
- TOLERANCES:
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  - ANGULAR:

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10.82677
4.33032
11.02260
6.48106
1.77150
5.31440
5.21655
11.40130
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(X2) S111InsulD_V3

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TOLERANCES:
- FRACTIONAL
- ANGULAR: MACH
- BEND
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- THREE PLACE DECIMAL

A
DO NOT SCALE DRAWING
Components F

Section F

Size Pockets

4/7/15 D. Biles

WEIGHT: A2

SHEET 1 OF 1

SCALE: 1:5

DWG NO.

TITLE:

REVISION

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Section F

Size Pockets

4/7/15 D. Biles

WEIGHT: A2

SHEET 1 OF 1

SCALE: 1:5

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Section F

Size Pockets

4/7/15 D. Biles

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SHEET 1 OF 1

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Section F

Size Pockets

4/7/15 D. Biles

WEIGHT: A2

SHEET 1 OF 1

SCALE: 1:5

DWG NO.

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REVISION

DO NOT SCALE DRAWING

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DEBUR AND BREAK SHARP EDGES

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15.7500

1.1811
(X1) S111PlateF_V3

Dimensions are in inches.

Interpret geometric tolerancing per:

- 10-32 Tapped
- .37500 16.91413

Tolerances:

- FRACTIONAL
- TWO PLACE DECIMAL
- THREE PLACE DECIMAL

Material:

- UNLESS OTHERWISE SPECIFIED:
- AS SHOWN
- DO NOT SCALE DRAWING


Comments:

- Q.A.
Components G

Depth = -0.5906

Wall Plate, G

DIMENSIONS ARE IN MILLIMETERS
SURFACE FINISH:
TOLERANCES:
   LINEAR:
   ANGULAR:
Q.A
MFG
APPV'D
CHK'D
DRAWN
B.2 Assembly Procedure

The following outlines the procedure for assembling the thermal wall plate into the NEAT tunnel, the reverse method could be established for removal of the thermal wall plate. Installation requires full access to the working section of the tunnel, therefore the inlet section assembly must be removed and the thermal wall plate assembly platform shown in the figure below must be setup in front of the tunnel. Wooden dowels are then installed along the inside of the working section to allow the thermal wall plate to slide down the working section and allow for proper routing of the control and heating wires.
Starting from the rear most section individual sections are assembled on the assembly platform and then slid into the tunnel to allow for the assembly of the next section on the platform. Careful note of the bottom window location must be made to insure that wires are routed to acceptable exit location. The figure below shows the assembly of thermocouples into the convective plate. The thermocouples are placed into a custom thermocouple holder which is then screwed into the convective plate thermocouple holes which is filled with a thermally conductive electrically insulation grease.
Next the insulation is installed over the convective plate as shown in the figure below.
This entire assembly is then installed into the correct frame position, the below figure shows the appropriate method of routing all heating and control wiring.
This entire section is assembled on the assembly platform as shown in the figure below. Leveling screws located in the 4 corners of each plate are used to pull the convective plate flush with the frame and should be adjusted prior to moving onto the next section, note some convective plates span two sections and frame mount screws should be applied prior to leveling.
Beneath the platform all wires must be routed using the provided thin carbon fiber cover sections so that wires are routed to appropriate bottom windows.
This section is then slid into the tunnel on the wooden dowels and the process is repeated for each subsequent section as shown in the figure below.
Once all sections have been installed and wires have been routed, the wooden dowels can be simultaneously removed to lower the wall plate onto the tunnel floor. The below figure displays the image of the final assembly.