A combined model of statistical downscaling and latent process multivariable spatial modeling of precipitation extremes

Meng Zhao
University of New Hampshire, Durham

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A COMBINED MODEL OF STATISTICAL DOWNSCALING AND LATENT PROCESS
MULTIVARIABLE SPATIAL MODELING OF PRECIPITATION EXTREMES

BY

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DISSERTATION

Submitted to the University of New Hampshire
in Partial Fulfillment of
the Requirements for the Degree of

Doctor of Philosophy
in
Statistics

December 2018
This dissertation was examined and approved in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Statistics by:

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On September 13, 2018
DEDICATION

To my wife, Yuanyu Cao and my parents, Xuejun Zhao and Yanping Liu
ACKNOWLEDGEMENTS

First and foremost, I would like to give a special thank to my advisor Dr. Ernst Linder for his guidance and support. He introduced me to Bayesian and Spatial statistics, and extreme value theory, which are the key parts of my dissertation.

And then I would like to thank my committee members here, Dr. Jennifer Jacobs, Dr. Philip Ramsey, Dr. Kim Ann Kaufeld and Dr. Beth Ziniti for their help.

The work I did is funded by Department of Mathematics and Statistics. For this, I would like to thank Dr. Rita Hibschweiler. This work is also partially funded by Los Alamos National Laboratory. I would like to thank Dr. Nathan Urban and Dr. Kim Ann Kaufeld. Dr. Kaufeld was my mentor when I worked at Los Alamos National Laboratory last summer. I want to thank her for taking the time to discuss my dissertation and to provide me useful suggestions and valuable comments. I also like to thank Graduate School for the travel grand.

At end, I would like to thank my parents who support me to pursue my dream and I would like to thank my wife, Helen. She is always there for me, support me and understand me.
# TABLE OF CONTENTS

DEDICATION .................................................................................................................. iii

ACKNOWLEDGEMENTS .............................................................................................. iv

TABLE OF CONTENTS .................................................................................................. v

LIST OF TABLES ........................................................................................................... vii

LIST OF FIGURES ......................................................................................................... viii

ABSTRACT ..................................................................................................................... x

INTRODUCTION ............................................................................................................. 1

DATA ............................................................................................................................... 6

CHAPTER I ....................................................................................................................... 12

Single-Location Bayesian Estimation of Generalized Pareto Distribution (GPD) .......... 12
  Introduction .................................................................................................................. 12
  A Brief Review of Extreme Value Theory ................................................................. 13
  Bayesian Model and Monte Carlo Estimation ........................................................... 17
    Bayesian Inference .................................................................................................... 17
    MCMC ...................................................................................................................... 18
  Model Likelihood and Prior ....................................................................................... 20
  Simulation .................................................................................................................... 21
  Application .................................................................................................................. 24
  Conclusion .................................................................................................................. 29

CHAPTER II ................................................................................................................... 30

Multiple-Location Bayesian Estimation of GPD with a Spatial Latent Process .......... 30
  Introduction .................................................................................................................. 30
  Model ............................................................................................................................ 33
    The First Layer (Data - Likelihood) ......................................................................... 33
    Process Layer ............................................................................................................ 34
    Prior Layer ............................................................................................................... 38
  Application and Results ............................................................................................. 38
  Conclusion .................................................................................................................. 42

CHAPTER III .................................................................................................................. 44

Spatial Combining of Multiple Climate Model Outputs and Downscaling for Projections of Future Extreme Precipitation ................................................................. 44
  Introduction .................................................................................................................. 44
  Overview of Some Statistical Downscaling Methods ................................................ 46
    Delta method .............................................................................................................. 46
    Quantile Matching Method ...................................................................................... 46
    Bayesian Hierarchical Model ................................................................................... 48
  Review of Combining Climate model outputs ......................................................... 49
  Proposed Statistical Model and Data .......................................................................... 50
  Results ......................................................................................................................... 53
LIST OF TABLES

Table 1: RCMs Driven by GCMs ........................................................................................................... 7
Table 2: GCMs and RCMs ..................................................................................................................... 8
Table 3: List of Weather Stations .......................................................................................................... 9
Table 4: $\xi$ simulation results with $N=50$ .......................................................................................... 20
Table 5: $\sigma$ simulation results with $N=50^*$ ..................................................................................... 21
Table 6: $\xi$ simulation results with $N=175$ ......................................................................................... 22
Table 7: $\sigma$ simulation results with $N=175$ ..................................................................................... 23
Table 8: 25-year return level at Durham ............................................................................................... 29
Table 9: Prior Distribution ..................................................................................................................... 38
Table 10: Average Reduction of 95% Credible Intervals for Return Levels ........................................... 43
Table 11: Bayesian Spatial Hierarchical Model of Statistical Downscaling of Extremes and
          Combining Multiple Climate Model Output ....................................................................................... 54
Table 12: The Estimation Results for Hyper Parameters in Downscaling Model in New
          Hampshire ........................................................................................................................................ 56
Table 13: The Estimation Results for Hyper Parameters in Downscaling Model in New
          Hampshire ........................................................................................................................................ 57
LIST OF FIGURES

Figure 1: Return level $z_p$ as the upper quantile with the exceedance probability $p$ ........................................... 2
Figure 2: Climate Model ........................................................................................................................................ 3
Figure 3: NARCCAP Grid ....................................................................................................................................... 4
Figure 4: Upper Rio Grande Watershed .................................................................................................................. 6
Figure 5: Grid Points of Six RCMs in New Hampshire (left) and upper Rio Grande watershed (right). ................................................................. 8
WRFG and MM5I share common grid points ........................................................................................................ 8
Figure 6: Locations for Center Points for the Grid Cells of CRCM CCSM in New Hampshire (left) and in upper Rio Grande watershed (right). ........................................... 9
Figure 7: Locations for the weather stations in New Hampshire (left) and upper Rio Grande watershed (right) listed in the Table 3 ............................................................................. 11
Figure 8: GEV with $\alpha = 0, \sigma = 3$, and $\xi = 0.3$ and $-0.3$ for Frechet and Weibull Distribution .......................................................................................................................... 14
Figure 9: Top 10 Daily Precipitations .................................................................................................................... 14
Figure 10: GPD Scale Parameter Estimates for daily precipitation in New Hampshire (top) and upper Rio Grande watershed (bottom) ........................................................................................................ 25
$\sigma y$ and $\sigma x$ are estimated from weather stations(left) and climate model CRCM CCSM outputs at nearest grid point (right). MLE estimations with 95% confidence intervals are in blue and Bayesian estimates with 95% credible intervals are in red. Red points and black triangle are the mean and the median of the posterior sampling, respectively ........................................... 25
Figure 11: GPD Shape Parameter Estimates for daily precipitation in New Hampshire (top) and upper Rio Grande watershed (bottom) ........................................................................................................ 26
Figure 12: 25, 50 and 100 return levels at weather stations in New Hampshire .......................................................... 28
Figure 13: 25, 50, 100 year return levels at weather stations in upper Rio Grande watershed ......................................... 28
Figure 14: Semivariograms for $\sigma$, $\xi$ and $\omega$ in New Hampshire .......................................................................... 35
Figure 15: Semivariograms for $\sigma$, $\xi$ and $\omega$ in the upper Rio Grande watershed ............................................... 35
Figure 16: Comparison Between Spatial and Non-spatial Models for Scale (top) and Shape (bottom) Parameters obtained from Weather Stations (left) and CRCM CCSM model output (right) in NH .................................................................................................................. 39
Figure 17: Comparison Between Spatial and Non-spatial Models for Scale (top) and Shape (bottom) Parameters obtained from Weather Stations (left) and CRCM CCSM model output (right) in URGW .................................................................................................................. 40
Figure 18: 25, 50 and 100 Years Returns Level for Weather Stations in New Hampshire ............................................. 41
Figure 19: 25, 50 and 100 Years Return Levels for Weather Station in upper Rio Grande watershed ......................................................... 41
Figure 20: Example of Weather Stations and RCM Grid ......................................................................................... 51
Points are 27 weather stations in New Hampshire .................................................................................................. 51
Figure 21: Spatially Interpolated Mean 25-year Return Levels (mm), $\beta_0$. The points are the locations of prediction locations (391 on the left and 1087 on the right) ........................................................................... 53
Figure 22: Four Weather Stations in New Hampshire (left) and upper Rio Grande watershed (right) ................................................................. 55
Figure 23: Boxplot of the Posterior Draws of 25-year Return Levels ........................................................................ 55
Figure 24: Spatial Interpolations of Relative Weight, $\beta_i(s)$, of Each Climate Model on Downscaled Return Level................................................................. 59

Figure 25: $y$, mean of 25-year return level based on current climate model outputs. (New Hampshire on the left and upper Rio Grande watershed on the right) ........................................ 61

Figure 26: $y - y$, the difference of downscaled return levels based on future climate model outputs from current climate model outputs................................................................. 61

Figure 27: The difference of the return levels of future climate model outputs from the return levels of current climate model outputs in New Hampshire (a) and upper Rio Grande watershed (b)........................................................................................................ 62

Figure 28: Lower (left) and Upper (right) Bound of 95% Credible Intervals of Downscaled 25-year Return Level........................................................................................................ 63

Figure 29: Lower and Upper Bound of 95% Credible Intervals of Downscaled 25-year Return Level. ......................................................................................................................... 64
ABSTRACT

A COMBINED MODEL OF STATISTICAL DOWNSCALING AND LATENT PROCESS
MULTIVARIABLE SPATIAL MODELING OF PRECIPITATION EXTREMES

By
Meng Zhao
University of New Hampshire

Future projections of extreme precipitation can help engineers and scientists with infrastructure design projects and risk assessment studies. Extreme events are usually represented as return levels which are equivalent to upper percentiles of an extreme value distribution, such as the Generalized Pareto distribution, which is used for exceedances above a certain threshold. My dissertation focus is on uncertainty quantification related to estimation of future return levels for precipitation at the local (weather station) to regional level. Variance reduction is achieved through spatial modeling and optimally combining suites of climate model outputs. The main contribution is a unified statistical model that combines the variance reduction methods with a latent model statistical downscaling technique. The dissertation is presented in three chapters: (I) Single-Location Bayesian Estimation of Generalized Pareto Distribution (GPD); (II) Multiple-Location Bayesian Estimation of GPD with a Spatial Latent Process. (III) Spatial Combining of Multiple Climate Model Outputs and Downscaling for Projections of Future Extreme Precipitation.
INTRODUCTION

On August 27, 2011, Hurricane Irene created devastation in New England. In Vermont, most of the rivers and streams were flooding and two towns, Killington and Pittsfield, were completely isolated for two weeks. It has been reported that since 1984 extreme downpours and snowfall events in New England have increased by 85% (Rogers et al. 2014).

One year later, Hurricane Sandy affected multiple states on the east coast. Sandy caused more than 70 billion dollars worth of damage and over 6 million people lost power for days. Live Science reported: "It has an average probability of happening only once every 700 years." In 2017, Hurricanes Harvey, Irma and Maria devastated Texas and Florida and the Caribbean Islands within about a month. Days to weeks after these storms there were tens of thousands of people still homeless or living in the dark. A study from University of Wisconsin's Space Science and Engineering Center reports that Hurricane Harvey is a 1-in-1000 years flood event.

What this report referred to is related to the hydrology term, "return level". By definition, a return level $z_p$ is exceeded by the annual maximum in any particular year with probability $p$ and is associated with an average return period $1/p$. Another way to understand this concept is that the return level $z_p$ is expected to be exceeded on average once every $1/p$ years (Coles 2001). For example, "100 years return level" means the upper quantile $z_p$ of a random variable $Z$ of the annual maximum has exceedance probability $P(Z > z_p) = p = 0.01$ (See Figure 1).

Even though humans are not able to stop all natural forces, scientists study the physical systems that produce extreme weather events and future climate. For the short term, meteorologists are
focusing on weather forecast. How to accurately predict the path of extreme weather events ahead of time is the key for preparation of extreme weather events like hurricanes. For the long term, it is impossible to predict when and where extreme weather events will happen in the future. Instead, based on historical observations from weather station and climate projections, scientists try to estimate future weather scenarios. In terms of statistics, the statistical distribution of extreme precipitation and temperature is the area of interest. Estimating the probability of occurrences and the extent of future extreme precipitation not just aids scientists and engineers with infrastructure design projects and impact assessment, it can also be used in agriculture and ecosystem research.

Two types of data are commonly used to predict the extreme distribution of future precipitation: (1) Daily precipitation, which is available over the past 30 - 50 years from land-based weather stations and (2) output of climate models which are based on well-documented physical processes to simulate the transfer of energy and materials through the climate system. Scientists use mathematical equations to simulate the interactions of the important drivers of climate, including atmosphere, oceans, land surface and anthropogenic drivers such as greenhouse gas emissions. Climate model outputs are from either General Circulation Models (GCMs - also

![Figure 1: Return level $z_p$ as the upper quantile with the exceedance probability $p$.](image-url)
referred to as Global Climate Models) or from Regional Climate Models (RCMs). Climate models are run under several different greenhouse gas emission scenarios at coarse spatial resolutions of more than 100 km for GCMs and in the tens of kilometers for RCMs. To simulate a GCM, scientists divide the planet into a 3-dimensional grid (See Figure 2) and each of the grid cells has one simulation value at each time step (hourly or 3-hourly). On the other hand, RCMs are focused on subregions, for example a continent like North America (See Figure 3), and describe local climate changes which are influenced by local topographical features, such as mountains. GCMs are not able to account for these local topographies as they use a coarse spatial resolution. It is much more computationally intensive to run RCMs, so they are usually run over a limited area (UK Climate Projections) and a limited time horizon

![Figure 2: Climate Model (Image source: NOAA)](image)

The development of RCMs, embedded within boundary conditions obtained from the GCMs is called Dynamical Downscaling. Because of the computational cost, RCMs are limited in the number of emission scenarios and the future time periods for which projections are available. For example, there are several CO2 emission scenarios released by the IPCC (Intergovernmental
Panel on Climate Change). The Coupled Model Intercomparison Project (CMIP) has collected output from an idealized scenario of global warming, with atmospheric CO$_2$ increasing at the rate of 1% per year until it doubles at about Year 70 ([https://cmip.llnl.gov](https://cmip.llnl.gov)). Phase three of CMIP (CMIP3) was previously used and the current one is CMIP5.

![Figure 3: NARCCAP Grid](image)

In contrast, statistical downscaling refers to establishing a statistical relationship between coarse resolution climate data and finer resolution data. Daily precipitation at weather stations are historical observations while GCM and RCM output data are from simulations of both the past and future. In the following, I will link historical climate model outputs and the observations from local weather stations. Future climate model outputs are then used to predict future local weather conditions by assuming that statistical relationships from the past remain true in the future. This is the so-called stationary assumption of downscaling. Compared with dynamic downscaling, statistical methods are more flexible, much less computationally demanding, and can be easily interpolated to any location. Another advantage of statistical downscaling is uncertainty quantification. Dynamical downscaling only provides the value of point estimation. However, for decision making like infrastructure design, uncertainty bounds are essential.
To respond to the needs of decision makers to plan for climate change, there are a variety of climate model outputs under different greenhouse gas emission scenarios. Many statistical downscaling methods have been suggested to predict future local weather variables, such as precipitation and temperature. Here, I propose a Bayesian spatial model for estimating distributions of extremes and for performing statistical downscaling. Because I use a Bayesian framework for modeling, I will be able to propagate the variation of the entire statistical process. This will enable uncertainty quantification of the future extreme precipitation, something that is currently absent from the literature, but is crucially required for infrastructure design, planning and impact assessment.

Thus, my focus is on uncertainty quantification of future extreme distributions and return levels for precipitation at the local (weather station) to regional level. This is presented in three chapters: (I) Single-Location Bayesian Estimation of Generalized Pareto Distribution (GPD); (II) Multiple-Location Bayesian Estimation of GPD with a Spatial Latent Process. (III) Spatial Combining of Multiple Climate Model Outputs and Downscaling for Projections of Future Extreme Precipitation.
DATA

I will apply the downscaling methodology to two regions: New Hampshire (NH) and the upper Rio Grande watershed (URGW). The latter is located in the southern part of Colorado and the northern part of New Mexico (See Figure 4). These two regions are quite different in terms of geography and climate.

New Hampshire experiences a humid continental climate with precipitation showing little to no seasonality. The northern part contains the White Mountains which have high elevation, severe winds and snow in the winter. Southeastern New Hampshire is along the coast; the climate is moderated by the Atlantic Ocean and has higher humidity. Most parts of the upper Rio Grande watershed are at higher elevations and experience dry air most of the year. Unlike

Figure 4: Upper Rio Grande Watershed

The region of interests in my dissertation is the northern part of the Rio Grande watershed which includes the area northern than Albuquerque. (Shaded area above red line)
New Hampshire, precipitation in New Mexico and Colorado exhibits seasonal variation. In the summer ("monsoon season"), many places experience daily short-duration convective rainfall events. In the winter, the mountainous areas have snow.

For both regions, I use two sources of data: (1) Daily precipitation in millimeter from weather stations, which is collected by the National Climatic Data Center (NOAA), and (2) regional climate model outputs from the North American Regional Climate Change Assessment Program (NARCCAP).

<table>
<thead>
<tr>
<th>RCMs</th>
<th>Driving Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CCSM</td>
</tr>
<tr>
<td>CRCM</td>
<td>✔</td>
</tr>
<tr>
<td>ECP2</td>
<td>✔</td>
</tr>
<tr>
<td>HRM3</td>
<td>✔</td>
</tr>
<tr>
<td>MM5I</td>
<td>✔</td>
</tr>
<tr>
<td>RCM3</td>
<td>✔</td>
</tr>
<tr>
<td>WRFG</td>
<td>✔</td>
</tr>
</tbody>
</table>

NARCCAP data covers continental US and most of Canada and Mexico (Figure 3). It provides regional climate models (RCMs) at a spatial resolution of 50 km from both the past (1968-2000) and the future (2038-2070) using the SRES A2 emissions scenario, a relative higher emissions scenario (Mearns, L.O., et al., 2017). There are 6 RCMs which were developed from 4 different GCMs (see Table 1). Each of the six RCMs were driven by boundary conditions taken from a pair of GCMs selected from a set of four in a balanced incomplete block design framework resulting in a total of 12 RCM runs. Each RCM has its own grid except MM5I and WRFG share a common grid (Figure 5). For example, CRCM, MM5I and WRFG are dynamical downscaling products from CCSM. For details of each model see Table 2. In Chapters 1 and 2, I used CRCM.
CCSM as example to demonstrate the results. The center points of grid cells of CRCM CCSM covered in New Hampshire and upper Rio Grande watershed are shown in Figure 6.

Table 2: GCMs and RCMs

<table>
<thead>
<tr>
<th>General Circulation Models or Global Climate Models (GCMs)</th>
<th>Regional Climate Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCSM</td>
<td>Community Climate System Model (NCAR)</td>
</tr>
<tr>
<td>CGCM3</td>
<td>Third Generation Coupled Global Climate Model (Canada)</td>
</tr>
<tr>
<td>GFDL</td>
<td>Geophysical Fluid Dynamics Laboratory</td>
</tr>
<tr>
<td>HadCM3</td>
<td>Hadley Centre Coupled Model (UK)</td>
</tr>
<tr>
<td>CRCM</td>
<td>Canadian Regional Climate Model</td>
</tr>
<tr>
<td>ECPC</td>
<td>Experimental Climate Prediction Center Regional Spectral Model</td>
</tr>
</tbody>
</table>

Figure 5: Grid Points of Six RCMs in New Hampshire (left) and upper Rio Grande watershed (right). WRFG and MM5I share common grid points.

Observations from weather stations are collected all over the US. They include weather measurements such as temperature, relative humidity, precipitation, wind speed, wind direction, and atmospheric pressure. Over 210 million weather observations are collected daily in the US (http://www.noaa.gov/resource-collections/weather-observations).
To match such observations with climate model outputs from NARRCAP, I used data from weather stations for the years between 1968 and 2000. The observations are recorded hourly and

Table 3: List of Weather Stations  
(a) Names of the Weather Stations in NH

<table>
<thead>
<tr>
<th></th>
<th>New Hampshire</th>
<th></th>
<th>New Hampshire</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NASHUA 2 NNW</td>
<td>2</td>
<td>FITZWILLIAM 2 W</td>
</tr>
<tr>
<td>3</td>
<td>MILFORD</td>
<td>4</td>
<td>SOUTH LYNDEBORO</td>
</tr>
<tr>
<td>5</td>
<td>KEENE</td>
<td>6</td>
<td>MASSABESIC LAKE</td>
</tr>
<tr>
<td>7</td>
<td>EPPING</td>
<td>8</td>
<td>WEARE</td>
</tr>
<tr>
<td>9</td>
<td>MARLOW</td>
<td>10</td>
<td>DURHAM</td>
</tr>
<tr>
<td>11</td>
<td>CONCORD MUNICIPAL AIRPORT</td>
<td>12</td>
<td>BRADFORD</td>
</tr>
<tr>
<td>13</td>
<td>MOUNT SUNAPEE</td>
<td>14</td>
<td>NEWPORT</td>
</tr>
<tr>
<td>15</td>
<td>LAKEPORT 2</td>
<td>16</td>
<td>GRAFTON</td>
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<tr>
<td>17</td>
<td>HANOVER</td>
<td>18</td>
<td>PLYMOUTH</td>
</tr>
<tr>
<td>19</td>
<td>BENTON 5 SW</td>
<td>20</td>
<td>MOUNT WASHINGTON</td>
</tr>
<tr>
<td>21</td>
<td>PINKHAM NOTCH</td>
<td>22</td>
<td>BERLIN</td>
</tr>
<tr>
<td>23</td>
<td>LANCASTER</td>
<td>24</td>
<td>ERROL</td>
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<tr>
<td>25</td>
<td>DIXVILLE NOTCH</td>
<td>26</td>
<td>COLEBROOK</td>
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<td>27</td>
<td>FIRST CONNECTICUT LAKE</td>
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to the nearest 1/10th of a millimeter. Within this 32-year window, most of the stations in New Hampshire and upper Rio Grande watershed have missing records.

(b) Names of the Weather Stations in New Mexico

<table>
<thead>
<tr>
<th></th>
<th>New Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>KELLY RANCH</td>
</tr>
<tr>
<td>2</td>
<td>AUGUSTINE 2 E</td>
</tr>
<tr>
<td>3</td>
<td>SOCORRO</td>
</tr>
<tr>
<td>4</td>
<td>RAMON 8 SW</td>
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<td>5</td>
<td>GRAN QUIVIRA NATIONAL MON</td>
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<td>6</td>
<td>CANTON</td>
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<td>7</td>
<td>FORT SUMNER 5 S</td>
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<td>BERNARDO</td>
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<td>FORT SUMNER</td>
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<td>12</td>
<td>MOUNTAINAIR</td>
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<td>13</td>
<td>SUMNER LAKE</td>
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<td>PEDERNAL 9 E</td>
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<td>HOUSE</td>
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<td>16</td>
<td>LOS LUNAS 3 SSW</td>
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<td>ESTANCIA 4 N</td>
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<td>18</td>
<td>CLINES CORNERS 7 SE</td>
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<tr>
<td>19</td>
<td>SANTA ROSA</td>
</tr>
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<td>20</td>
<td>ALBUQUERQUE</td>
</tr>
<tr>
<td>21</td>
<td>LAGUNA</td>
</tr>
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<td>22</td>
<td>GRANTS MILAN</td>
</tr>
<tr>
<td>23</td>
<td>STANLEY 2 NNE</td>
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<tr>
<td>24</td>
<td>DILIA</td>
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<td>25</td>
<td>SANDIA PARK</td>
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<td>26</td>
<td>GOLDEN</td>
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<td>27</td>
<td>PECOS NATIONAL MONUMENT</td>
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<td>28</td>
<td>GLORIETA</td>
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<td>29</td>
<td>LAS VEGAS MUNICIPAL AIRPORT</td>
</tr>
<tr>
<td>30</td>
<td>JEMEZ SPRINGS</td>
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<td>31</td>
<td>TORREON NAVAJO MISSION</td>
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<td>32</td>
<td>LOS ALAMOS</td>
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<tr>
<td>33</td>
<td>WOLF CANYON</td>
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<td>34</td>
<td>JOHNSON RANCH</td>
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<td>CUBA</td>
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<td>ESPANOLA</td>
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<td>CANJION R S</td>
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<td>EL VADO DAM</td>
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<td>TRES PIEDRAS</td>
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<td>RED RIVER</td>
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<td>CERRO</td>
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<td>BRAZOS LODGE</td>
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<td>TIERRA AMARILLA 4 N</td>
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<td>CHAMA</td>
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Some stations only have 2 to 3 years of daily precipitation. To balance the number of stations and the number of observations in each station, I only considered the weather stations that have more than 10000 out of 11685 daily records, which amounts to fewer than 15% of missing values. As a result I will use records from 27 stations in New Hampshire and 58 stations in the
upper Rio Grande watershed. Names of the weather stations are listed in Table 3 and corresponding positions are provided in Figure 7.

(c) Names of the Weather Stations in Colorado

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Weather Stations in New Hampshire  
Weather Stations in Upper Rio Grande Watershed

Figure 7: Locations for the weather stations in New Hampshire (left) and upper Rio Grande watershed (right) listed in the Table 3
CHAPTER I

Single-Location Bayesian Estimation of Generalized Pareto Distribution (GPD)

Introduction

Predicting extreme weather conditions can help us control damages and even save lives. However, extreme weather conditions, like hurricanes, are quite uncommon. With limited records and few weather stations, estimating the probability of extreme meteorological events is difficult as the rareness of the event reduces the statistical precision. The goal of the first part of my dissertation is to estimate parameters of extreme distributions for a range of values and to quantify uncertainties.

The Generalized extreme value (GEV) distribution and the generalized Pareto distribution (GPD) are commonly used statistical models for estimating the probability of extreme events. These distributions arise in extreme value theory, which enables one to extrapolate tail behaviors of daily precipitation even when there is a lack of observations. To improve the estimating accuracy and quantify uncertainties, a reparametrized Bayesian model will be proposed to estimate extreme precipitation.

In section 2, I will introduce GEV and GPD and review recent research about estimating the parameters of the GPD. In section 3, I will present the reparametrized Bayesian model for estimating GPD, which can facilitate Monte Carlo sampling by addressing the varying range of the parameter space. In addition, I will talk about Hamiltonian Monte Carlo (HMC) as my sampling method to perform the Bayesian model. In section 4, I will discuss a simulation that
compares my method with maximum likelihood estimation (MLE). In section 5, I apply my method to observations of daily precipitation from weather stations and climate model outputs in New Hampshire and the upper Rio Grande watershed. Section 5 provides some conclusions.

A Brief Review of Extreme Value Theory

**Theorem 1:** (The main result of classical extreme value theory)

Assume that $X = \{X_1, X_2, ..., X_n\}$ is an i.i.d random variable and $X_i \sim F$, a probability distribution, and $M_n = \max \{X_1, X_2, ..., X_n\}$ is the maximum over a block of size $n$. If there exist sequences of constants $a_n \geq 0$ and $b_n$ such that $P\left(\frac{M_n - b_n}{a_n} \leq z\right) \rightarrow G(z)$ (convergence in distribution) as $n \rightarrow \infty$, then the limiting distribution has the following form,

$$G(z) = \exp \left[- \left(1 + \xi \left(\frac{z-\mu}{\sigma}\right)\right)^{-1/\xi}\right], \{z: 1 + \xi \left(\frac{z-\mu}{\sigma}\right) > 0\}$$

(1)

$G(z)$ is called the Generalized Extreme Value (GEV) distribution with the location parameter $\mu$, scale parameter $\sigma$ and shape parameter $\xi$. GEV is the generalization of three different distributions: Gumbel ($\xi = 0$), Frechet ($\xi > 0$) and Weibull ($\xi < 0$). The sign of $\xi$ describes the tail behavior of the distribution $g(z)$ (See Figure 8). The Weibull distribution has a finite upper bound. The Frechet distribution decays polynomially and the Gumbel distribution decays exponentially. The result in Equation (1) also holds (with some modification) for stationary time series that are not necessarily i.i.d. (Coles 2001, Chapter 3). In practice, we divide independent observations $x_1, x_2, ...$ into $m$ multiple blocks of the same length $n$ and then use block maxima $M_{n,1}, ..., M_{n,m}$ to fit the GEV distribution. For example, the GEV is used to fit yearly maxima of daily precipitations for multiple years. However, using only block maxima for extreme value analysis is a wasteful approach if other data on extremes are available (Coles 2001). Yearly
Figure 8: GEV with $u = 0$, $\sigma = 3$, and $\xi = 0.3$ and $-0.3$ for Frechet and Weibull Distribution

Figure 9: Top 10 Daily Precipitations. 
Red Points are yearly maxima of daily precipitations and black points are the top 10 values in each year. Green lines are 98.5% quantiles of daily precipitations between 1968 and 2000.

maxima provide limited data and we lose important information. In Figure 9, the maximum precipitation at Mount Washington in 1990 and in Los Alamos in 1980 are below some other years' fourth or fifth largest precipitation. A similar theory has been developed for the joint distribution of the $k$ largest values in a block ($k > 2$). The question then remains how to choose $k$. 
A more practical approach is to consider exceedances over a high threshold. This results in the Generalized Pareto Distribution with a high threshold value. Pickands et al. (1975) showed that the random variable \( Y = X - u, X > 0 \) as \( u \to \infty \) has the Generalized Pareto Distribution (GPD), \( H(y|\sigma, \xi) \), given by:

\[
H(y) = 1 - P(X > u + y | X > u) = \begin{cases} 
1 - \left(1 + \frac{\xi y}{\sigma}\right)^{-\frac{1}{\xi}}, & (1 + \frac{\xi y}{\sigma}) > 0, y > 0 \\
1 - \exp\left(-\frac{y}{\sigma}\right), & \xi = 0, y > 0 
\end{cases}
\]

and density function,

\[
h(y) = \begin{cases} 
\sigma^{-1} \left(1 + \frac{\xi y}{\sigma}\right)^{-\frac{1}{\xi} - 1} \exp\left(-\frac{y}{\sigma}\right), & (1 + \frac{\xi y}{\sigma}) > 0, \\
\sigma^{-1} \exp\left(-\frac{y}{\sigma}\right), & \xi = 0, 
\end{cases}
\]

Here, \( u \) is the threshold. \( \sigma > 0 \) and \( \xi \) are scale parameter and shape parameter, respectively. For the generalized Pareto distribution, the m-observation return level is

\[
x_m = \begin{cases} 
\left(u + \frac{\sigma}{\xi}\right) \left[(m\zeta_u)^{\xi} - 1\right], & \xi \neq 0, \\
u + \sigma \log(m\zeta_u), & \xi = 0, 
\end{cases}
\]

where \( \zeta_u = P(X > u) \) and \( m \) is the number of the observations.

The estimation procedure for the scale parameter \( \sigma \) and the shape parameter \( \xi \) has been addressed by many authors.

Maximum likelihood estimation (MLE) and its corresponding asymptotic properties were obtained by Smith (1984). Hosking and Wallis (1987) discussed the performance of MLE, method of moments (MOM) and probability-weighted moments (PWM) estimation. Bayesian estimation was also proposed by Castellanos and Cabras (2007); Bermudez and Turkman (2003) and Diebold et al. (2005).

However, because of the support of \( y \), there are limitations for these methods. Smith 1984 showed that MLE are obtainable and asymptotic properties hold only when \( \xi > -0.5 \). MLE is
the most practical method. It allows inclusion of covariates in the model, but requires large samples to out-perform the other methods. The computation of MOM and PWM are relatively simple but estimators only exist for $\xi < 0.5$ and are not always consistent with the observed data (Hosking and Wallis 1987). Castillo and Hadi (1997) proposed the elemental percentile method (EPM). This method holds for any value of $\xi$ but performance can be inferior when other methods exist (Hosking and Wallis 1987).

Bayesian methods for estimating GPD have been discussed in recent years. There are multiple reasons for using Bayesian methods. Prior information can be provided by experts in the domain of an application. For example, shape parameters tend to be between -0.5 and 0.5 in the case of extreme rainfall (Cooley 2010). Spatial extremes, which I will talk about in Chapter 2, can reduce the variation of the parameter estimation. This work requires modeling in the Bayesian framework. Compared with other estimation method, Bayesian estimation enable quantification of uncertainties via credible intervals and propagates the variance of statistical downscaling which will be the topic of Chapter 3 of this dissertation.

Diebolt et. al. (2005) proposed a quasi-conjugate Bayes method for $\xi > 0$. Castellanos and Cabras (2007) extended the restriction for the shape parameter and considered $\xi > -0.5$ which in practice holds for most applications. Bermudez and Turkman (2003) proposed to estimate the GPD in two different situations: $\xi > 0$ and $\xi < 0$. When $\xi < 0$, Bermudez and Turkman (2003) used restricted likelihood and a transformed parameter $\delta = -\frac{\sigma}{\xi}$ bounded by the maximum value of the observed data. However, the sign of the shape parameter $\xi$ is not known a-priori.

Bermudez and Turkman (2003) pointed out if a model with $\xi > 0$ is used to estimate a GPD with negative $\xi$, the posterior distribution of $\xi$ will be concentrated over a narrow range near zero. Without knowing the tail behaviors of the sample, the result can be misleading when the true
shape parameter is close to zero. In iterative estimation, the method could take twice the computation time if the first guess is wrong. The combination of Gibbs sampling and Metropolis-Hasting (MH) algorithm was used in the above papers.

In this chapter, I will propose a reparametrized Bayesian model without any restriction or prior information on the parameters. I consider Hamiltonian Monte Carlo (HMC) sampling for my estimation.

**Bayesian Model and Monte Carlo Estimation**

In equation (3), the domain of the GPD density involves the data \( y \) and both parameters \( \sigma \) and \( \xi \). This creates difficulties in MLE or Bayesian method of parameters estimation, especially when \( \xi < 0 \). Without prior information, there exist no model which can consistently estimate GPD for the entire range of possible values. In this section, I propose to reparametrize GPD density in (Equation (3)) and use the software Stan (Stan Development Team 2016) to implement Hamiltonian Monte Carlo sampling method.

**Bayesian Inference**

Bayesian inference is the process of fitting a probability model to a set of data and summarizing the result by a probability distribution on the parameters of the model. (Gelman et al., 2004, pp. 1). Unlike frequentist statistics, Bayesian statistics treats unknown parameters as random variables. By applying Bayes’ rule (Equation (5)), we obtain the posterior distribution of the parameter as follows

\[
f(\theta|Y) = \frac{f(\theta)f(Y|\theta)}{\int f(\theta)f(Y|\theta)d\theta} \propto f(\theta)f(Y|\theta)
\]  

(5)

\( Y \) represents data with density or probability function, \( f(Y|\theta) \) and \( \theta \) is the parameter, possibly a vector. Here \( f(Y|\theta) \) is called the likelihood which means it is viewed as a function of \( \theta \), and \( f(\theta) \) is the probability density function of the prior distribution, which represents any prior
information on the parameter $\theta$. The quantity of interest is the density function of the posterior distribution, $f(\theta | Y)$, which enables drawing inference on the parameter $\theta$. $\int f(\theta) f(Y | \theta) d\theta$ is the normalizing constant of the posterior distribution, hence does not depend on $\theta$. Therefore the posterior distribution $f(\theta | Y)$ is proportional to the product of the likelihood and the prior distribution.

**MCMC**

In complicated Bayesian models such as high-dimensional parameter models and hierarchical models, the normalizing constant of the posterior distribution does not have an analytical solution. Bayesian inference then relies on iterative sampling algorithms, such as Markov Chain Monte Carlo (MCMC) simulation to obtain samples from the posterior distribution. The Metropolis-Hastings (MH) algorithm (Hastings 1970) and the Gibbs sampler (Smith and Gelfand, 1992), or a combination thereof are commonly used methods to implement MCMC. Given a current value $\theta^t$, the MH algorithm starts with proposing a new value $\theta^*$, based on some jumping distribution $q(\theta^* | \theta^t)$. The probability to accept the new proposed value is determined by the acceptance probability:

$$
\alpha(\theta^t, \theta^*) = \min \left( 1, \frac{p(\theta^* | y) q(\theta^t | \theta^*)}{p(\theta^t | y) q(\theta^* | \theta^t)} \right)
$$

(6)

If the proposed value is accepted, then $\theta^{t+1} = \theta^*$; otherwise, the chain does not move, i.e. $\theta^{t+1} = \theta^t$. If the jumping distribution $q(\cdot)$ is a symmetric distribution, then $q(\theta^t | \theta^*) = q(\theta^* | \theta^t)$ and $\alpha(\theta^t, \theta^*) = \min \left( 1, \frac{p(\theta^* | y)}{p(\theta^t | y)} \right)$ which does not depend on the normalizing constant in the posterior distribution. This was the original version of the Metropolis algorithm (Metropolis et al. 1953).
The idea in Gibbs sampling is to generate posterior samples by sweeping through the parameter space by using each scalar parameter (or vector “block” of parameters) to sample from its conditional distribution with the remaining parameter variables fixed at their current values. (http://www.mit.edu/~ilkery/papers/GibbsSampling.pdf)

The Gibbs sampler is a commonly used MCMC method due to its computational advantage. However, it requires the user to integrate out the analytical forms of the full conditional probability distribution for each parameter which does not always exist.

Iteration of MH algorithm or Gibbs sampler or both (hybrids) result in a random walk Markov chain. When sampling from the posterior distribution, a random walk Markov chain sometimes is not able to efficiently explore the posterior distribution. A poorly chosen jumping distribution could make the Markov chain spend a lot of time to circle around and not efficiently explore the parameter space or could waste time by rejecting many proposed parameter values. In addition, the Markov chain can be stuck near the boundary of a pathological region and unable to explore the entire parameter space resulting in biased sampling. (Betancourt 2017)

An alternative sampling method is the Hybrid or Hamiltonian Monte Carlo sampling. Instead of using a jumping distribution to propose new values, HMC uses Hamiltonian dynamics with a leapfrog method (Neal 2011) to propose a new value and uses similar acceptance procedures as the HM algorithm. The Stan software implements Hybrid or Hamiltonian Monte Carlo methods. (Stan Development Team 2016). The Gelman-Rubin statistic (Rhat) was used to evaluate model convergence (Gelman and Rubin, 1992). In Stan, it is required to bound the parameter space properly so that the Monte Carlo sampling can explore the entire parameter space. Applying this to my research, I need to simplify the domain of the GPD density parameters.
Model Likelihood and Prior

To simplify the parameter space in (3), I reparametrize the GPD density and assume \( \omega = \frac{\xi}{\sigma} \). Then the density will be

\[
h(y) = \begin{cases} 
\sigma^{-1}(1 + \omega y)^{-\frac{1}{\omega} - 1}, & \omega > -\frac{1}{y_{n:n}}, \sigma > 0, \\
\sigma^{-1} \exp\left(-\frac{y}{\sigma}\right), & \omega = 0, \sigma > 0,
\end{cases}
\]

where \( y_{n:n} \) is the largest of \( y = \{y_1, y_2, ..., y_n\} \).

In the reparametrized GPD (5), since both parameters are bounded separately, non-informative priors can be used to explore the parameter space in Stan. Of course, with more information, informative priors could be used. For \( \sigma \), the prior can be any positive distribution, for example,

| Table 4: \( \xi \) simulation results with N=50* |
|-----------------|-----------------|-----------------|-----------------|
| MLE: \( \hat{\xi} \) | Bayesian: \( \tilde{\xi} \) |
| \( \xi \) | \( \xi_{bias} \) | \( \xi_{rmse} \) | \( \xi_{95\% CI} \) | \( \xi_{bias} \) | \( \xi_{rmse} \) | \( \xi_{95\% CI} \) |
| 10 | -0.0270 | 0.2961 | 0.927 | 0.1544 | 0.3497 | 0.929 |
| 30 | -0.0431 | 0.2906 | 0.919 | 0.1376 | 0.3366 | 0.928 |
| 60 | -0.0509 | 0.3001 | 0.907 | 0.1313 | 0.3419 | 0.933 |
| 10 | -0.0363 | 0.2230 | 0.922 | 0.1137 | 0.2598 | 0.941 |
| 0.5 | -0.0460 | 0.2342 | 0.904 | 0.1045 | 0.2650 | 0.942 |
| 30 | -0.0540 | 0.2306 | 0.903 | 0.0959 | 0.2566 | 0.926 |
| 60 | -0.0617 | 0.1863 | 0.892 | 0.0647 | 0.1919 | 0.942 |
| 10 | -0.0555 | 0.1833 | 0.892 | 0.0716 | 0.1938 | 0.94 |
| 0.05 | -0.0600 | 0.1848 | 0.887 | 0.0669 | 0.1930 | 0.939 |
| 60 | -0.0606 | 0.1759 | 0.899 | 0.0616 | 0.1799 | 0.934 |
| 10 | -0.0644 | 0.1813 | 0.881 | 0.0609 | 0.1825 | 0.943 |
| -0.05 | -0.0629 | 0.1820 | 0.88 | 0.0641 | 0.1856 | 0.935 |
| 60 | -0.0853 | 0.1822 | 0.883 | 0.0431 | 0.1582 | 0.939 |
| 10 | -0.0935 | 0.1871 | 0.872 | 0.0386 | 0.1510 | 0.942 |
| 0.4 | 30 | -0.0968 | 0.1870 | 0.866 | 0.0391 | 0.1489 | 0.956 |
| 60 | - | - | - | 0.0152 | 0.1618 | 0.943 |
| -0.8 | 30 | - | - | 0.0084 | 0.1565 | 0.949 |
| 60 | - | - | - | 0.0015 | 0.1586 | 0.949 |

*Standard MLE based inference is available only when \( \xi > -0.5 \) (Smith 1984)
lognormal or half Cauchy (positive side). Both distributions have a large range. And for \( \omega \), we could consider all range of values, for example Normal(0,100). In the simulation section, I used a truncated positive Cauchy(0, 5) and Normal(0, 100) as prior for \( \sigma \) and \( \omega \) (\( \omega > -\frac{1}{\gamma_{n:n}} \)).

**Simulation**

In this section, I consider a simulation study with varying range of parameters and sample sizes. I compare the proposed Bayesian model with maximum likelihood estimation (MLE). In practice, it is unusual to have a large number of observations over a high threshold. In this simulation, I choose sample sizes of \( N = \{50, 175\} \). However, in theory, large sample comparisons are always of interest. Therefore, \( N = 500 \) is also included in the simulation (See Appendix A). Since the results are generally robust with respect to \( \sigma \). See e.g. Castellanos and Cabras (2006). \( \xi \) is the

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parameter that decides tail behaviors of samples, I choose three different values for \( \sigma \) and six different values for \( \xi \) as follows: \( \sigma = \{10,30,60\} \); \( \xi = \{-0.8, -0.4, -0.05, 0.05, 0.5, 1\} \).

I simulated 1000 data samples from the GPD with all combinations of \( N, \sigma \) and \( \xi \). The simulation reports bias, root mean squared error (RMSE) and 95% confidence (credible) interval coverage. The results of this simulation are provided in Table 4, 5, 6, 7 which correspond to shape and scale parameters with sample sizes \( N = \{50, 175\} \).

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For both maximum likelihood and the proposed Bayesian model, estimation of shape and scale parameters improve when the sample size \( N \) increases. MLE tends to underestimate the shape parameter \( \xi \) and the Bayesian method tends to overestimate it. It is the opposite for the scale parameter \( \sigma \). The Bayesian estimates are consistently better in the case of the scale parameter.
However, for the shape parameter, results depend on the sign of $\xi$. When $\xi > 0$, Bias and RMSE of MLE ($\hat{\xi}_{bias}$ and $\hat{\xi}_{rmse}$) are slightly smaller than those of the Bayesian model ($\hat{\xi}_{bias}$ and $\hat{\xi}_{rmse}$). However, when $\xi$ is negative, Bayesian estimation performs better. The smaller $\xi$ (negative) the better the Bayesian estimation is. When $\xi < 0$, especially $\xi$ close to -0.5 or less than -0.5, Bayesian estimation out-performs MLE. Besides point estimation, uncertainty bounds (intervals) are also important criteria to decide which model performs better. Credible

<table>
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<th>$\hat{\sigma}_{rmse}$</th>
<th>$\hat{\sigma}_{95% CI}$</th>
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<td>0.0123</td>
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<tr>
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<td>-</td>
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<td>-</td>
<td>0.3942</td>
<td>4.8179</td>
<td>0.963</td>
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</table>

intervals, in terms of interval coverage, in the Bayesian model are almost always better and are close to 0.95 no matter the value of $\xi$. On the other hand, confidence intervals in MLE works well only for positive $\xi$. 
Application

In this section, I apply both MLE and the proposed Bayesian model to estimate the GPD for daily precipitation in New Hampshire and the upper Rio Grande watershed. To estimate the GPD model, choosing a threshold is difficult. It is a trade-off between variance and bias. If the threshold is too low, then the observations over the threshold are not extreme enough and the estimation will result in bias. On the other hand, if the threshold is too high, there are not enough observations to fit the GPD and the variance will increase. There are two ways to choose a threshold (Coles 2001) (1) mean residual life plot and (2) assessment of stability. Both methods have their limitations. The interpretation of a mean residual life plot is subjective and more than one hundred plots need to be examined because of multiple locations in this study. The assessment of stability method requires fitting multiple GPDs with a range of thresholds $u$. When $u$ are greater than the true threshold $u_0$, the estimation of the shape parameter $\xi$ should be approximately constant.

The estimated shape and scale parameters from the proposed Bayesian model are similar to the MLE results. The Rhat value (Gelman-Rubin statistic) of each parameter in the Bayesian model are less than 1.01, which implies convergence of the iterative Markov chain Monte Carlo estimation procedure.

Figure 10 shows the results for estimating the scale parameters $\sigma_y$ from weather stations and $\sigma_x$ from the regional climate model output (CRCM-ccsm) in New Hampshire and in the upper Rio Grande watershed.
New Hampshire

(a) $\sigma_{y,NH}$

(b) $\sigma_{x,NH}$

Upper Rio Grande Watershed

(c) $\sigma_{y,URGW}$

(d) $\sigma_{x,URGW}$

Figure 10: GPD Scale Parameter Estimates for daily precipitation in New Hampshire (top) and Upper Rio Grande watershed (bottom).

$\sigma_y$ and $\sigma_x$ are estimated from weather stations (left) and from climate model CRCM CCSM outputs at nearest grid point (right). MLE estimations with 95% confidence intervals are in blue and Bayesian estimates with 95% credible intervals are in red. Red points and black triangle are the mean and the median of the posterior sampling, respectively.

The shape parameters $\xi_y$ and $\xi_x$ are displayed in Figure 11. The indices of weather stations and grid points of CRCM-ccsm are mapped in Figure 6 and 7 from Introduction chapter. They are, generally ordered from West to East and from South to North.

For the scale parameters, $\sigma_y$ are greater than $\sigma_x$. Data from weather stations has higher variance than data from climate model output. Weather stations are local observations and can be quite different from location to location. However, climate model outputs are spatially smoother and each grid point of climate model represents the average over 50 kilometers resolution space.
Figure 11: GPD Shape Parameter Estimates for daily precipitation in New Hampshire (top) and upper Rio Grande watershed (bottom).

\( \xi_y \) and \( \xi_x \) are estimated from weather stations (left) and from climate model CRCM CCSM outputs at nearest grid point (right). MLE estimations with 95% confidence intervals are in blue and Bayesian estimates with 95% credible intervals are in red. Red points and black triangle are the mean and the median of the posterior sampling, respectively.

Among weather stations, three locations, Mount Washington and Pinkham Notch in New Hampshire and Wolf Creek Pass in upper Rio Grande watershed, have higher values of scale parameters than nearby weather stations. All three locations are in the mountains and have high elevations. As we expected, the variation of the extreme daily precipitation in these three locations is larger than at the locations in the lower elevations.
In Figure 11 we notice that the shape parameter $\xi$ can vary considerably for different locations. More shape parameters from climate model outputs are less than zero compared with those from weather stations. Again, climate model outputs represent averages over 50 kilometer grids, and unlike those from weather stations, they are less extreme. It explains that there are more negative values in $\xi_s$ than $\xi_y$.

Compared with the shape parameter $\xi_y$ estimated from observations in New Hampshire, the majority of $\xi_y$ from the upper Rio Grande watershed are below zero. That's because the precipitation in the upper Rio Grande watershed is not as extreme as in New Hampshire. Hence, it is not a good idea to consider uniform or constant $\xi$ over large spatial areas when fitting the GPD with a spatial latent process. More details will be discussed in the next chapter.

Based on thresholds and the estimations of shape and scale parameters, we obtained 25, 50 and 100 years return level by using Equation (4). Figure 12 and 13 shows the 25, 50 and 100 years return levels for weather stations in New Hampshire and upper Rio Grande watershed. Besides the point estimates, credible intervals are also included in these two figures.

The values of the return levels in this chapter are, in general, less than the results from current public sources, such as Hydrometeorological Design Studies Center, Precipitation Frequency Data Server (PFDS). For example, in Durham, New Hampshire, the 25-year return level from PEDS is 6.51 inches and the estimation based on our proposed models is only 5.41 inches. There are two main reasons these two are different. First, this dissertation was focused on extreme precipitation for daily observations and Generalized Pareto distribution (GPD) was used to estimate the return levels. On the other hand, PFDS used either annual maximum or 24 hours partial duration. It has been shown that 24 hours partial duration series on average produce return levels that are around 10 – 15% larger than those obtained from calendar-based daily observations.
Figure 12: 25, 50 and 100 return levels at weather stations in New Hampshire

Figure 13: 25, 50, 100 year return levels at weather stations in upper Rio Grande watershed
Table 8: 25-year return level at Durham

<table>
<thead>
<tr>
<th>Thresholds of GPD</th>
<th>98.50%</th>
<th>99.00%</th>
<th>99.50%</th>
<th>99.60%</th>
<th>99.70%</th>
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<tr>
<td>25 Year Return Level</td>
<td>5.7 inch</td>
<td>5.96 inch</td>
<td>6.34 inch</td>
<td>6.47 inch</td>
<td>6.67 inch</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>270</td>
<td>180</td>
<td>91</td>
<td>73</td>
<td>55</td>
</tr>
</tbody>
</table>

25-year return level with different threshold estimated by transformed GPD. Number of observations are the number of daily precipitation over threshold used to fit reparametrized GPD.

 exceedances over threshold. Second, in this dissertation, I only considered the daily precipitation between years 1968 and 2000 to match the NARCCAP climate model outputs. Extreme precipitation happened more often recent years. In Table 8, I used the GPD with different thresholds to estimate the 25-year return level at Durham, New Hampshire, for daily precipitation between the year of 1968 and 2018. As I mentioned early in this chapter, when the threshold of the GPD is large enough, extreme precipitation over threshold is equivalent to annual maximum. The 25-year return level based on the proposed model is equivalent with the one using annual maximum from PFDS when the threshold is large enough. (See table 8)

Conclusion

The purpose of this chapter was to present the model that is able to estimate the parameters of the GPD without any restrictions on the shape parameter $\xi$. The proposed Bayesian model does not require informative prior on either $\sigma$ or $\xi$.

In the simulation study, the results of the proposed Bayesian model are similar to the results of MLE when $\xi > 0$ and the proposed Bayesian model performs better than MLE when $\xi < 0$.

While Bayesian modeling for extremes is not a new estimation method the proposed reparametrization of the GPD with non-informative priors is novel and facilitates the numerical estimation in a Bayesian framework for modeling extreme precipitation with a spatial latent process. I will discuss this further in the next chapter.
CHAPTER II

Multiple-Location Bayesian Estimation of GPD with a Spatial Latent Process

Introduction

Predicting extreme weather conditions can help us control damages and even save lives. Estimating the probability of occurrence and the extent of extreme weather events aids engineers and scientists with infrastructure design projects. However, extreme weather events, like hurricanes Harvey and Irma, are quite uncommon. With limited records and few weather stations, estimating the probability of extreme meteorological events is difficult as the rareness of the events reduces the statistical precision. The goal of this chapter is to improve the precision of forecasting extremes and return levels which is currently an active research area.

A statistical consequence of the lack of data is that inference on the tail of an extreme value distribution tends to be highly uncertain, and the uncertainty increases sharply as one moves further into the tail. In applications, this can lead to alarmingly wide confidence intervals. (Davison et al. 2012)

To improve the statistical precision, spatial models have been proposed to analyze daily rainfall and temperature extremes over a geographical region. In order to estimate the spatial structure for extreme data, we need the formulation of a multivariate distribution of extremes (GEV or GPD), which is not well established. Davison et al. (2012) discussed three main methods for spatial extreme models: (1) copulas; (2) spatial max-stable processes; (3) spatial latent variables. In spatial models, Gaussian-based geostatistical models are widely used due to the availability of...
a closed-formed multivariate distribution. Unlike most other distributions including GEV and GPD, both the joint and marginal Gaussian distributions are easily obtained by specifying the mean and spatial covariance structure. Extremal copula is a way to estimate the joint extreme distribution. The extremal Gaussian and t copula are proposed by Hüsler and Reiss (1989) and, Demarta and McNeil, (2007). Both models are only available to model bivariate extreme distribution and both are symmetric. Multivariate skewed t distributions are more appropriate for modeling the joint extreme behavior. Padoan et al. (2011) and Morris et al. (2017) proposed skewed t distributions to model the dependence structure, which allowed them to model multivariate spatial extremes with dimension greater than two. A max-stable process, another way to model spatial extreme, is defined through a spectral representation. A Commonly used spectral representation is proposed by Schlather (2002). Max-stable processes are mostly used for pairwise models, because only the pairwise marginal distributions are known for most models., Even if the analytical form of the full joint distribution were available, it would be computationally infeasible to obtain the density function from it unless the dimension was small. (Davison et al. 2012)

Spatial latent process models, without considering the spatial structure for the data layer, assume the parameters in the models change over space. Unlike copulas and max-stable processes, spatial latent process models are not able to model joint or spatial behavior of extreme data. They have two drawbacks (Davison et al. 2012). First, the marginal distribution of the extremes is not of standard extreme value form. Hence the max-stable property cannot be used here. Second, extreme data at adjacent locations are conditionally independent in the limit, which is not realistic for daily temperature that is well structured and has high spatial correlations among adjacent locations. However, a spatial latent process model is still a good choice for
precipitation. Unlike temperature, precipitation tends to show less correlations among nearby locations. Our goal here is to obtain return levels, which solely depends on the latent process variables of the GEV and GPD distributions. Return levels are not weather patterns. They are not day-to-day observations at a given location and time. Return levels are climatological quantities based on the probability of rare meteorological events, and their spatial dependence must be modeled outside of the framework provided by the multivariate extreme models such as copulas and max-stable process. Here, we want to focus on how the distribution of precipitation varies over space rather than on the multivariate structure of particular extreme precipitation events. (Cooley et al. 2007). In addition, spatial latent variable methods are more flexible and allow us to form different spatial covariance structure for each parameter of either the GEV and GPD. Coles and Casson (1998) and Casson and Coles (1999) are the first papers that considered modeling spatial variation in parameters of the extreme distribution. The simulation study in the latter paper showed that there is a substantial increase in precision of short-term return levels due to the pooling of information from neighboring locations. The three parameters of the GEV in Casson and Coles (1999) are assumed to have Gaussian distributions and to be mutually independent. Sang and Gelfand (2010) relaxed this assumption and modeled the location and scale parameters of the GEV jointly.

For statistical models with nonlinear parameters like the GPD, frequentist estimation methods need to be based on the likelihood function which is the joint probability of the data. However, it is impossible to write this function in analytical form when a Gaussian spatial model for the parameters is combined with non-Gaussian data. The development of computational methods has enabled statisticians to develop estimation procedures in a conditionally specified Bayesian
hierarchical model. This means that we can assume a spatial Gaussian model for the parameter of the GPD, thus linking non-Gaussian data with an underlying spatial field.

Bayesian spatial hierarchical modeling with MCMC posterior sampling is one of the solutions to deal with the spatial GPD model (Cooley et al 2007 and 2010). Without considering the spatial structure for the data layer, these papers assumed the shape and scale parameters of the GPD vary spatially in a latent process. Using draws from the posterior distributions of the shape and scale parameters, we easily obtain return levels and corresponding credible intervals.

This chapter contains 4 sections. Section 1 is a review of recent literature on spatial extreme models. In Section 2, I will discuss spatial extreme models for georeferenced data and lattice data separately. Section 3 describes an application. I will apply the proposed models to the same data set as in Section 1.4 and discuss the results. Section 4 provides conclusions.

**Model**

To improve the statistical precision, I propose a Bayesian spatial hierarchical model for extreme daily precipitation by assuming a Gaussian spatial process prior for the parameters of the statistical model for the extremes.

There are three layers in the Bayesian spatial hierarchical model. The first layer models the extreme precipitation at each location. The second layer provides the structure of the spatial latent process prior. The third layer gives the prior distributions for the parameters used in the process layer, as well as priors of additional parameters used in the model.

**The First Layer (Data - Likelihood)**

In Chapter I, we proposed a reparameterization of the GPD model - see equation (8). In the modified GPD model the shape parameter $\xi$ is allowed to take on any value in $\mathbb{R}$. See the simulation in Table 4, 5, 6 and 7 in Chapter 1. Positive Cauchy (0, 5) and Normal (0, 100) were
used as non-informative priors for estimating shape parameters (\(\xi\)) and transformed parameters (\(\omega\)) of GPD. This flexibility gives us the liberty to choose different spatial covariance structures in the Gaussian latent process for \(\omega\) and \(\log (\sigma)\). To build the spatial latent process model for extreme precipitation at each location, the reparametrized GPD will be used here, which is similar to the reparametrized GPD density in equation (8) except the parameters vary over space.

\[
\begin{align*}
\hat{h}(y) &= \begin{cases} 
\frac{1}{\sigma(s)} (1 + \omega(s)y) \frac{1}{\sigma(s) \omega(s)}^{-1}, & \omega(s) > -\frac{1}{y_{n.n.}} \text{ and } \sigma(s) > 0 \\
\frac{1}{\sigma(s)} \exp \left(-\frac{y}{\sigma(s)}\right), & \omega(s) = 0, \sigma(s) > 0
\end{cases}
\end{align*}
\]  
(8)

Process Layer

There are two different sources of data in this dissertation, observations from weather stations and climate model outputs (CRCM ccsm). The first is geo-referenced data and the second is lattice data, which requires us to use different techniques to model these two different spatial structures.

(a) Weather Station Data

In Chapter 1, we obtained the posterior distributions of the parameters of the GPD at each location. By using the mean of draws from the posterior distribution of the parameters: \(\omega(s)\) and \(\sigma(s)\) at each location, we fit isotropic empirical semivariograms to NH and URGW regions and results are shown in Figure 14 and Figure 15. In New Hampshire, the scale parameter has spatial correlation at 15 km and the spatial range parameter for the shape parameters \(\xi(s)\) is approximately 8 km, which means the spatial correlation of the scale parameter extends to longer distances. In upper Rio Grande watershed, both parameters have spatial correlation over longer distances (between 35 km and 40 km). Observations at nearby locations are expected to have similar extreme behavior in distribution. Pooling information from nearby observations increases the estimation precision. Examination of the empirical semivariograms supports the
Figure 14: Semivariograms for $\sigma$, $\xi$ and $\omega$ in New Hampshire. The unit of distance on the X-axis is 10km. The Y-axis is scaled by the sample variance.

Figure 15: Semivariograms for $\sigma$, $\xi$ and $\omega$ in the upper Rio Grande watershed.
consideration of spatial latent process models. Because of different geography and climate in New Hampshire and the upper Rio Grande watershed, different spatial patterns are expected. Cooley et al. (2007) and Pan (2016) discussed the difficulty of estimating a spatially changing shape parameter $\xi(s)$. Pan (2016) used a constant value for $\xi$ over the entire region of New England. Cooley et al. (2007) considered a single value for $\xi$ for a mountainous subregion and one for a plain region due to the special geography of Colorado.

We propose a Gaussian process with an exponential covariance model for both $\omega(s)$ and $\log(\sigma(s))$, which is similar to Cooley et al. (2007) for $\log(\sigma(s))$. We use MCMC sampling to obtain the posterior distributions of $\omega(s)$ and $\sigma(s)$ and the posterior distributions of $\xi(s)$ ($\xi(s) = \omega(s) \ast \sigma(s)$). Letting $\theta = \{\log(\sigma), \omega\}$, we then assume the spatial process priors

$\theta(s) \sim N(\mu_\theta, \Sigma_\theta),$

with entries of the covariance matrix $\Sigma_\theta$ denoted as $k_{ij}$, and

$$k_{ij} = \begin{cases} 
\beta_{\theta,0} \ast \exp(-\beta_{\theta,1} \ast \|x_i - x_j\|), & i \neq j \\
\beta_{\theta,0} + \tau_\theta, & i = j 
\end{cases}$$

Here, all three parameters, $\beta_{\theta,0}$, $\beta_{\theta,1}$ and $\tau_\theta$, are required to be positive. For now, the mean of the $\theta(s)$ is assumed to be a constant. We will relax this assumption in the next chapter by connecting the extreme parameters from weather stations with those from climate model outputs.

(b) Climate model outputs

Climate model outputs from NARRCAP are average values over 50 kilometer grids, which means there is one value over an approximately 50 by 50 km grid area. In spatial analysis, this type of data is called lattice or areal data. Banerjee, Carlin and Gelfand (2015) provide a good description of the characteristics of lattice data and the methodology to model this type of data. The use of Conditional Autoregressive (CAR) and Intrinsic Autoregressive (IAR) models for
lattice data has seen a dramatic increase due to advances in computation. CAR was first introduced by Besag (1974). Sang and Gelfand (2010) used IAR models for the location and scale parameters of the GEV and assumed the shape parameter to be constant. Cooley et al. (2010) pointed out that the shape parameter $\xi$ in extreme precipitation studies usually has estimated values between 0 and 0.2. They used the idea from Martins and Stedinger (2000) to add a beta density function to have a support for $\xi$ between $-0.5$ and $0.5$ resulting in a restricted likelihood. After putting the constraint on $\xi$ in the likelihood function, Cooley et al. (2010) used Gaussian CAR and IAR priors for all three parameters.

Since climate model outputs are lattice data, we propose to model $\omega(s)$ and $\sigma(s)$ as Gaussian CAR or IAR models. In the CAR model, the precision matrix, not the covariance matrix, is directly obtainable from the data thus resulting in the Gaussian process.

$$\theta(s) \sim N(\mu_0, Q_0^{-1})$$

The precision matrix is $Q_\theta = \tau^2(D - \psi W)$. $W$ is the adjacency matrix with $w_{ii} = 0$, and $w_{ij} = 1$ if $i$ is a neighbor of $j$, otherwise $w_{ij} = 0$. The eigenvalue decomposition of $Q_\theta$ can be written as $= \tau^2 F A F'$. Both $D$ and $A$ are diagonal matrices with $d_i$ and $\lambda_i$ as the entries on the diagonal. $d_i$ is the number of the neighbors for location $i$ and $\lambda_i$ is the eigenvalue of $(D - \psi W)$. The vectors of $F$ are eigenvectors of $(D - \psi W)$. Hence $F$ is an orthogonal matrix, i.e. $F^{-1} = F'$ and $F F' = I$. Because of this property, the CAR model provides a computationally efficient procedure to obtain the covariance $\Sigma_\theta$ and the parameter $\theta(s)$

$$\Sigma_\theta = Q_\theta^{-1} = \frac{1}{\tau^2} F A^{-1} F' ; \quad \theta(s) = \mu_0 + \Sigma_\theta^{1/2} \ast z$$

where $z \sim N(0, 1)$ and $\Sigma_\theta = \Sigma_\theta(\Sigma_\theta^{-1})' = \left(\frac{1}{\tau^2} F A^{-1/2} F' \right) \left(\frac{1}{\tau^2} F A^{-1/2} F' \right)' = \frac{1}{\tau^2} F A^{-1} F'$, hence $\Sigma_\theta^{1/2} = \frac{1}{\tau^2} F A^{-1/2} F'$. 


In a CAR model, $\psi$ is the parameter to control the spatial dependence. When $\psi = 0$, it implies spatial independence, and when $\psi = 1$, it collapses to an Intrinsic Autoregressive (IAR) model. We propose to use IAR to fit our spatial latent process model, due to computational feasibility and difficulty to estimate dependence parameter $\psi$. (Cooley et al. 2010)

Prior Layer

In this layer, we choose prior distributions for parameters $\beta_0, \beta_1, \tau$ and $\mu$. Cooley et al. (2010) put a restriction on the likelihood and then used non-informative priors for the CAR and IAR models. Cooley et al. (2007) suggested a rather strict uniform prior with small range for parameters $\beta_0, \beta_1$ in the exponential covariance case. In our Bayesian hierarchical models, we do not have much information on these parameters. The non-informative priors are used for both the Gaussian process models with exponential covariance and the IAR model. Table 8 shows the prior distributions that are used in the model.

Table 9: Prior Distribution

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<th>Range</th>
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<tr>
<td>$\beta_0$</td>
<td>$(0, \infty)$</td>
<td>Positive Cauchy (0, 5)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>$(0, \infty)$</td>
<td>Positive Cauchy (0, 5)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$(0, \infty)$</td>
<td>Positive Cauchy (0, 5)</td>
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</table>

Application and Results

For consistency with Chapter 1 we consider two regions with different geography and climate: NH and URGW. We use Stan to perform MCMC with 10000 iterations and 5000 burn-in steps. Draws from the posterior distributions of the scale and the shape parameters of the GPD are obtained. Figures 16 and 17 show the results of the proposed Bayesian spatial latent process extreme models and non-spatial extreme models. For both weather station observations and climate model outputs (CRCM ccsm), spatial latent process models improve the precision
significantly. For both NH and URGW, the lengths of the 95% credible intervals have around 30% average reduction for scale parameters and around 50% average reduction for shape parameters in the spatial latent process model compared to the non-spatial model. Compared with weather station, CRCM CCSM has more reduction for both scale and shape parameters. In the spatial model, posterior estimates (i.e. interval centers) are “shrunk” towards the overall average. That is because nearby locations are used to pool more information for the estimation.

**New Hampshire**

![Graphs showing comparison between spatial and non-spatial models](image)

*Figure 16: Comparison Between Spatial and Non-spatial Models for Scale (top) and Shape (bottom) Parameters obtained from Weather Stations (left) and CRCM CCSM model output (right) in NH.*

The purpose of this research was to quantify the uncertainties of return levels and to try to reduce them. Return levels are obtained from the draws of the posterior distributions of the parameters of the GPD.
Upper Rio Grande Watershed

Figure 17: Comparison Between Spatial and Non-spatial Models for Scale (top) and Shape (bottom) Parameters obtained from Weather Stations (left) and CRCM CCSM model output (right) in URGW.

We use the same thresholds as in Chapter 1 \( P(s > u(s)) = \zeta = 0.015 \). Figure 18 and 19 show the 25, 50, and 100 years return levels of weather stations in New Hampshire and upper Rio Grande watershed. For the weather stations, the uncertainties of the return levels at some locations are more reduced than others with spatial modeling. Larger return levels tend to have wider intervals. In the 25-year return level plot, when return levels are close or less than 100 mm, the corresponding length of the intervals are less than 20 mm. However, when return levels are
Figure 18: 25, 50 and 100 Years Returns Level for Weather Stations in New Hampshire

Figure 19: 25, 50 and 100 Years Return Levels for Weather Station in upper Rio Grande watershed
larger than 200 mm, the reduction of the uncertainty is not significant. For instance, there are two weather stations (20 and 21) located in the White Mountain and Pinkham Notch region with higher elevations. Both locations have the largest intervals among all weather stations in New Hampshire. The results for climate model output are shown in Table 10. Compared with weather stations, most of climate models on average have more than 50% of reductions when using spatial model except for HRM3 gfdl and HRM3 hadcm3.

Conclusion

In this chapter, we considered Bayesian spatial latent process models to improve the estimation precision of GPD models. Gaussian spatial process priors were used to model the spatial structure of the parameters in the transformed GPD. For both weather stations and climate model outputs, our models show significant reductions of the uncertainties in the extreme distribution parameters and return levels. Unlike Laflamme et al. (2016) and Pan (2016), the Bayesian hierarchical spatial latent process model pools the information from nearby locations for both shape and scale parameters in GPD and is thus able to reduce the uncertainty, i.e. the length of the credible intervals, for the return levels.

However, there is still room to improve the estimation precision. Return levels from climate model outputs are often used by scientists and engineers. However, the return levels from regional climate model outputs represent 50 by 50 km grid averages, which is not appropriate for building bridges or roads at a local level. Observations from weather stations provide the local return levels. However, there are two issues here. First, we only have return levels where weather stations are located, not everywhere. Second, after applying the spatial process models to improve the estimation precision, the credible intervals are still too large to be useful for
scientists and engineers. In the next chapter, we propose a statistical downscaling method to deal with this issue.

<table>
<thead>
<tr>
<th>Climate Models</th>
<th>New Hampshire</th>
<th>Upper Rio Grande Watershed</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25 Year</td>
<td>50 Year</td>
<td>100 Year</td>
</tr>
<tr>
<td>CRCM ccsm</td>
<td>66.74%</td>
<td>68.43%</td>
<td>69.83%</td>
</tr>
<tr>
<td>CRCM cgcm3</td>
<td>54.87%</td>
<td>58.47%</td>
<td>61.19%</td>
</tr>
<tr>
<td>ECP2 gfdl</td>
<td>69.36%</td>
<td>71.64%</td>
<td>73.40%</td>
</tr>
<tr>
<td>ECP2 hadcm3</td>
<td>68.82%</td>
<td>70.92%</td>
<td>72.61%</td>
</tr>
<tr>
<td>HRM3 gfdl</td>
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<td>42.17%</td>
</tr>
<tr>
<td>HRM3 hadcm3</td>
<td>39.02%</td>
<td>42.90%</td>
<td>45.73%</td>
</tr>
<tr>
<td>MM5I ccsm</td>
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<td>64.83%</td>
<td>67.12%</td>
</tr>
<tr>
<td>MM5I hadcm3</td>
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<td>58.66%</td>
</tr>
<tr>
<td>RCM3 cgcm3</td>
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<td>63.73%</td>
<td>65.79%</td>
</tr>
<tr>
<td>RCM3 gfdl</td>
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<td>62.63%</td>
<td>65.32%</td>
</tr>
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<td>56.45%</td>
<td>59.89%</td>
</tr>
<tr>
<td>WRFG cgcm3</td>
<td>65.49%</td>
<td>58.65%</td>
<td>71.10%</td>
</tr>
</tbody>
</table>
CHAPTER III

Spatial Combining of Multiple Climate Model Outputs and Downscaling for Projections of Future Extreme Precipitation

Introduction

Two types of data are commonly used to predict the distributions of future extreme precipitation and the return levels: (1) Over the past 50 - 100 years, daily observations from local weather stations have been collected in various regions and by various agencies, such as the U.S. National Climatic Data Center (NCDC) and (2) climate model outputs from either Global Climate Models (GCM) or from Regional Climate Models (RCMs). NCDC data are historical observations while GCM and RCM output data are from simulations of both the past and future. Climate models are physical process simulations that are run under several different greenhouse gas emission scenarios. Recently, through the use of Dynamical Downscaling, it has become possible to obtain data from the North American Regional Climate Change Assessment Program (NARCCAP) and its successor, Coordinated Regional Climate Downscaling Experiment (CORDEX) North America of higher spatial resolution, which represents an improvement from GCM grids of 200 kilometers resolution to RCM grids of 20 - 50 kilometer resolution. Although finer resolution is desirable, dynamical downscaling requires intensive computation. This creates practical limitations to the number of emissions scenarios and the future time periods for which projections are available. NARCCAP data, provides regional climate model
(RCM) output at a 50 kilometers spatial resolution over North America from both the past (1969-2000) and the future (2038-2070) using a mid-level emissions scenario (see www.narccap.ucar.edu)

In contrast, Statistical Downscaling refers to establishing a statistical relationship between coarse resolution climate data at the level of grid cells and weather station measurements of climate variables at spatial points, hence a finer resolution data. Statistical downscaling is a two-step process consisting of 1) the development of a statistical relationships between local climate variables and large-scale predictors, and 2) the application of such relationships to the output of large-scale output to simulate local climate characteristics in the future (Hoar and Nychka 2008). Statistical downscaling will provide the distribution of the climate variables not actual observation. Maraun and Widmann (2018) provide an up-to-date review of statistical downscaling principles and methods.

Procedures for estimating uncertainty and variability that exist in the downscaling process are not well established in the climate literature. The aim of this paper is to propose a spatial statistical downscaling model to reduce estimation variance, which allows me to obtain more accurate predictions than is currently possible with an approach of individual station downscaling and subsequent regional summarization, such as in Laflamme et al. (2016). In addition, there are multiple climate models with different emission scenarios, spatial resolutions and internal physical processes, which creates uncertainties due to the lack of agreement. Another motivation of this chapter is to optimally combine multiple climate model outputs. This chapter consists of four sections. In Section 1 and 2, I will briefly review some of the recent developments of statistical downscaling and the combining of ensembles of climate model outputs. In section 3, I will lay out the downscaling model. In Section 4, I will discuss results
from applying the proposed model to the two regions: New Hampshire and upper Rio Grande watershed. The last section will provide some conclusions.

**Overview of Some Statistical Downscaling Methods**

Many varieties of statistical downscaling methods are available. See Maraun and Widmann (2018) for a detailed account. Among them, the most popular methods are the Delta method, Quantile matching and Bayesian downscaling.

**Delta method**

The Delta method produces a smoothed surface of changes in large-scale climate model outputs and then applies this interpolated surface to the local weather condition (Ramirez-Villegas and Jarvis, 2010). The Delta method would correct bias from large scale climate model outputs to local observations and apply the same bias correction to the future climate model outputs in order to estimate the distribution of future local observations. However, this method only corrects the mean but not the variance.

**Quantile Matching Method**

One of the more popular methods of statistical downscaling is called ‘Quantile Matching’ (QM). It proposes to match the quantiles of the underlying distribution of climate model outputs and corresponding observations from weather stations. Michelangeli et al., (2009) used a variant of QM, called Empirical CDF Mapping, and McGinnis et al. (2014) proposed using Kernel Density estimation within QM. In these models, usually letting $F_X(x)$ and $F_Y(y)$ be, respectively, the cumulative distribution function of $X$, the variable from climate model outputs, and $Y$, the variable from weather station observations, then a value $y^*$ matches the corresponding value $x^*$ if

$$F_Y(y^*) = F_X(x^*)$$
hence the translation $T$ that maps $x^*$ to $y^*$ is defined by

$$y^* = T(x^*) = F_{y}^{-1}\left(F_{x}(x^*)\right)$$

However, Michelangeli et. al. (2009) pointed out that the QM method does not take into account the information on the distribution of the future climate simulations. Michelangeli et al. introduced a probabilistic downscaling method, CDF-transform, which assumes that the relationship $T(*)$ will remain valid in the future. Then the CDF of the local data for the future period is

$$F_{Y_f}(x) = F_{Y_c}\left(F_{X_c}^{-1}\left(F_{X_f}(x)\right)\right)$$

where $F_{Y_c}$ and $F_{X_c}$ are the empirical CDFs of local observations and climate model outputs in the past (current), respectively, and $F_{X_f}$ is the empirical CDFs of climate model outputs in the future.

Kallache et al. (2011) considered probabilistic downscaling of extreme precipitation and proposed the XCDF-t technique, which is in essence a parametric version of the Michelangeli et al. (2009) CDF transform. Instead of using empirical CDFs, the Generalized Pareto Distribution (GPD) was used to model the distribution of extreme precipitation. However, Kallache et al. (2011) constrained their analysis to the GPD with positive shape parameters. In most cases, shape parameters are positive for precipitation data (Reiss and Thomas 1997; Katz et al. 2002), but Maraun et al. (2011) and Fowler et al. (2010) pointed out that occasionally shape parameters can be negative, especially for climate model outputs. In our data, the multiple draws of shape parameters cross zero; see Figures 13 and 14 in Chapter 2. The negative shape parameter ($\xi$) in $F_{X_c}$ creates instability concerns in QM downscaling (Kallache et al. 2011 and Pan 2016). When $\xi < 0$, $F_{X_f}^{-1}(u)$ can be out of the domain of $F_{X_c}$, and when $F_{X_c}\left(F_{X_f}^{-1}(u)\right)$ returns a 1, $F_{Y_c}^{-1}$ will be $\infty$, which creates a problem in repeated automatic model fitting such as in Bayesian MCMC.
estimation (see Chapter 1). Instead of matching the estimated GPD, Laflamme et al. (2016) predicted a specified return level via quantile matching, which was an example of individual station downscaling. Mannshardt-Shamseldin et al. (2010) also proposed to downscale the return levels. Mannshardt-Shamseldin et al. (2010) developed the regression relationship between return levels estimated from the daily observations and from climate model outputs. Besides climate model outputs, spatial components such as latitude, longitude and elevation are also included as covariates in the regression model. However, Mannshardt-Shamseldin et al. (2010) considered to use only two runs (current and future) of the National Center for Atmospheric Research’s Community Climate System Model (CCSM) and reanalysis data from the National Centers for Environmental Prediction (NCEP) in separate downscaling models. In this chapter, the full suite of twelve regional climate model outputs with different spatial grids from NARCCAP were applied in my downscaling model.

**Bayesian Hierarchical Model**

Berrocal et al. (2010) proposed a spatial model within the Bayesian framework, which regresses the observed data on a numerical model output using spatially-varying coefficients in an application to ozone concentration. The Bayesian downscaling model could quantify the uncertainties in estimating the coefficients as well as spatially vary the relationship between climate model outputs and local observations. In the Bayesian framework, we could easily consider downscaling the sampling distribution of the return levels and generating the future GPD and the distribution of return levels. In this chapter, I will incorporate the ideas of Mannshardt-Shamseldin et al. (2010) and Berrocal et al. (2010) to use the Bayesian method to downscale the return levels.
Review of Combining Climate model outputs

Improvements of computing power and speed have greatly assisted in the development of many different climate model outputs in the last 20 years. Uncertainty due to climate modeling is usually illustrated using overlay trace plots of various climate model outputs. For local impact assessment and infrastructure design, it is not obvious how to utilize the information provided say in an example of probabilistic risk assessment. From a statistical standpoint, the best approach for making decision is to use as much information as possible.

Climate model outputs produce different simulations even under the same emission scenario. Especially for precipitation projections, climate model outputs show lack of agreement. Finding an optimal way to combine available climate model outputs is desperately needed. Tebaldi and Knutti (2007) discuss recent developments of the multi-model ensemble in probabilistic climate projections. The reliability ensemble average (REA), proposed by Giorgi and Mearns (2002) rewarded climate models which agree with the ensemble ‘consensus’ and discount the climate models that appear as outliers. Nychka and Tebaldi (2003) showed that REA is equivalent to estimate consensus signals of the different climate models. Tebaldi et al. (2004, 2005) and Smith et al. (2009) used the idea of REA and treated the unknown consensus signals as random variables and provided them with prior distributions. They assumed that climate model outputs are Gaussian processes centered at unknown consensus signals. The results of the ensemble multiple climate model outputs, the consensus signals, are the weighted average of the climate models. It performs well for large scale (continent level) average temperature and precipitation changes, however, averages are sensitive to ‘extreme’ observations (Tebaldi and Knutti 2007).

Bayesian spatial models have been considered recently to deal with spatial variations among RCMs not sharing the same grid cells (Banerjee at al. 2008 and Kang et al. 2012). Bayesian
methods (Robertson et al. 2004) or weighted averages (Krishnamurti et al. 2000) use the historical relationship between forecasts and observations. In this chapter, I consider extreme precipitation. Averaging extreme precipitation from different climate models will lose information on extreme events. I propose a Bayesian spatial hierarchical model to link the distribution of the parameter of interests (return levels) from observations with multiple climate model outputs. In this case, I am able to develop a single model that incorporates statistical downscaling and combining ensembles of climate model output.

**Proposed Statistical Model and Data**

In Mannshardt-Shamseldin et al. (2010), the downscaling model includes large-scale spatial trends through polynomial functions of latitude and longitude as well as elevation. The model only considered climate models that have a common spatial grid. Here, I propose a Bayesian spatial hierarchical model to downscale the sampling distribution of return levels (RL), similar to Mannshardt-Shamseldin et al. (2010). It incorporates the change of the threshold, scale and shape parameters of the GPD over the years in the climate model outputs. In contrast to Mannshardt-Shamseldin et al (2010) who considered spatial trend and elevation as covariates in regression, I use the idea of Berrocal et al (2010) to add another layer on the regression coefficients in the Bayesian hierarchical model, which guarantees that the weights of each climate model change over space. In this case, the model facilitates spatial interpolation of the downscaling coefficients and return levels beyond the original weather station locations. I will produce predictions of return levels at 391 and 1087 locations in NH and URGW, respectively. The prediction locations in each area are gridded and roughly 5 kilometers apart from each other. There are two stages in my Bayesian hierarchical model. The results of the spatial modeling of Chapter 2 which are the posterior draws of the shape and scale parameters of the generalized Pareto distribution of
exceedences over threshold, $\xi$, $\sigma$, respectively, and the corresponding return levels
$R_L \ (RL = g(u, \xi, \sigma), u$ is threshold), represent the first stage model. Instead of downscaling the extreme observations, I propose, in the second stage, to downscale the distribution of the parameters by subsampling the sorted posterior draws of the return levels from weather stations $RL_Y$ and from climate model outputs $RL_X$. Here we denote $RL_Y(s)$ to be the return levels from a weather station located at $s$ which is assumed to be inside grid cell $B$ and $RL_X(B)$ to represent the return levels from climate model outputs over grid cell $B$ (See Figure 20). Then the downscaling model will be a regression model:

$$RL_Y(s) \sim N(\beta_0(s) + \mathbf{x}\beta, \tau^2_0)$$

$$\mathbf{x}\beta = \beta_1(s)RL_{X_1}(B) + \beta_2(s)RL_{X_2}(B) + \cdots + \beta_{12}(s)RL_{X_{12}}(B)$$

Here, I centered the return levels from climate model outputs $RL_{X_i}(B)$. Therefore, $\beta_0(s)$ is the mean of the posterior draws of $RL_Y$ at location $s$. There are 27 and 52 weather stations located in NH and in URGW for predicting (interpolating to) 391 and 1087 grid ‘locations’, respectively. Elevations and coordinates are not sufficient as hyper priors in the Bayesian hierarchical

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**Figure 20:** Example of Weather Stations and RCM Grid
Points are 27 weather stations in New Hampshire.

*Grid cells are grids from regional climate model, ECP2.*
downscaling model for estimating the variation of $\beta_0$ or average of the return levels, $RL_Y$, for the entire region of NH or URGW. I interpolated $RL_Y$, same as $\beta_0(s)$, using a Multilevel B-spline approximation using the R package ‘MBA’ (See Figure 21). The coefficients, $\beta_i(s)$, depend on the elevations of weather stations at locations $s$, $E(s)$, and the difference of longitude, $Lon_{diff}(s)$, and latitude, $Lat_{diff}(s)$, between locations $s$ and the center point of the grid cell $B$. To preserve relative distance between two locations, longitude and latitude were projected to units of kilometers. These three variables help us distinguish the difference among the return levels that are located within one grid cell $B$. I also centered these three variables for computational purposes. Since we assumed a spatial latent process while estimating the return levels, here it is reasonable to assume independence of $RL_Y(s)$ conditional on the latent process.

Table 11 shows the Bayesian hierarchical downscaling model. The downscaling model enables us to combine multiple climate models with different spatial grids and possibly different spatial resolution. The coefficients $\beta_i(s)$ are the weights from the $i$th climate model outputs at the locations $s$. Since $\beta_i(s)$ only depends on geographic variables, they can be obtained at any location in the future. To improve the power of computation, I applied QR decomposition for $X\beta$ in the downscaling model (See Table 11). Hence return levels can be estimated at any location by using future climate model outputs. The motivation of this chapter is to propagate the variation of the entire statistical process and the combining of the ensemble of the multiple climate model outputs. This will enable uncertainty quantification of future projections of extreme precipitation, which is the reason I use a Bayesian spatial hierarchical model for estimating extreme distributions and for performing statistical downscaling. However, due to the computational intensity, I used a two-stage model. I obtained posterior draws of the return levels from the shape and scale parameters in the GPD. Then I used a subset of 100 sorted draws from
the first stage to perform statistical downscaling using quantile matching to connect the
distribution of 25-year return levels at weather stations with the distribution of 25-year return
levels at corresponding center points of the climate models.

Figure 21: Spatially Interpolated Mean 25-year Return Levels (mm), $\beta_0$. The points are the locations of
prediction locations (391 on the left and 1087 on the right).

Results

From the results in Chapter 2, I applied the Bayesian hierarchical spatial downscaling model to
New Hampshire and the upper Rio Grande watershed and calculated 25-year return levels. To
validate the downscaling model, I chose 4 weather stations each from the two study regions, NH
and URGW (see Figure 22) to compare the estimated distribution of 25-year return levels from
Chapter 2 and predicted (downscaled) 25-year return levels based on the downscaling model.
(See Figure 23)
Table 11: Bayesian Spatial Hierarchical Model of Statistical Downscaling of Extremes and Combining Multiple Climate Model Output

### Stage One:

#### Data Model/ likelihood:

GPD density \( f(y) = \sigma(s)^{-1} \left( 1 + \frac{\xi(s)y}{\sigma(s)} \right)^{-\frac{1}{\xi}} \), \( 1 + \frac{\xi(s)y}{\sigma(s)} > 0 \)

Let \( \omega(s) = \frac{\xi(s)}{\sigma(s)} \), then \( \tilde{f}(y) = \sigma(s)^{-1} (1 + \omega(s)y)^{-\frac{1}{\omega(s)}}, \omega(s) > -\frac{1}{\max(y)} \)

Threshold \( u \): \( P(y > u(s)) = 0.015 \)

#### Latent Process:

- \( \theta(s) = \omega(s) \) or \( \log(\sigma(s)) \), spatial latent process: \( \theta(s) \sim N(\mu, \Sigma) \)

For Weather station data \( (Y) \): \( \Sigma_Y \) is spatial exponential covariance

\[
\Sigma_Y = \begin{bmatrix} k_{ij} \end{bmatrix}, k_{ij} = \begin{cases} \beta_{\theta,0} \cdot \exp(-\beta_{\theta,1} \cdot \|x_i - x_j\|), & i \neq j \\ \beta_{\theta,0} + \tau_{\theta}, & i = j \end{cases}
\]

For Climate model data \( (X) \): \( \Sigma_X \) is Intrinsic Autoregressive (IAR)

\[
\Sigma_X = (\tau_{\theta} (D - W))^{-1}
\]

- \( W \): spatial adjacency indicator matrix
- \( D \): diagonal matrix with number of neighbors for location \( i \) on diagonal

#### Prior:

- \( \mu \sim N(0, 100) \);
- \( \beta_{\theta,0} \sim \text{Cauchy}^* (0, 5) \);
- \( \beta_{\theta,1} \sim \text{Cauchy}^* (0, 5) \);
- \( \tau_{\theta} \sim \text{Cauchy}^* (0, 5) \)

#### Posterior:

- \( \sigma_Y(s), \xi_Y(s) \) and \( \sigma_X(B), \xi_X(B) \)
- \( RL_Y(s) = g(u_Y(s), \sigma_Y(s), \xi_Y(s)) \);
- \( RL_X(B) = g(u_X(s), \sigma_X(B), \xi_X(B)) \)

### Stage Two:

#### Model/ likelihood:

- \( RL_Y(s) \sim N(\mu(s), \tau_0^2) \)
- \( RL_X(B) = \beta_0(s) + \sum^N_{i=1} \beta_i(s) RL_{X_i}(B) = \beta_0(s) + X\beta \)

Use QR decomposition: \( X = Q \cdot R \), then \( X\beta = Q \cdot \theta, \theta = R \cdot \beta \)

- \( Q \) is unitary matrix and \( R \) is lower triangle matrix

#### Latent Process:

- \( \theta_i(s) \sim N(\alpha_{0i} + \alpha_{1i} \cdot E(s) + \alpha_{2i} \cdot Lon_{diff}(s) + \alpha_{3i} \cdot Lat_{diff}(s), \tau_{i}^2) \), \( i = 1, 2, ..., 12 \)

#### Prior:

- \( \alpha_{ki} \sim N(0, 10) \) \( k = 0, 1, 2, 3 \);
- \( \tau_0 \sim \text{lognormal}(0, 5) \);
- \( \tau_i \sim \text{lognormal}(0, 5) \)

\text{Cauchy}^*: \text{half (positive) Cauchy}
Figure 22: Four Weather Stations in New Hampshire (left) and upper Rio Grande watershed (right).

Predicted return levels have similar distributions as estimated return levels based on the boxplot of the four stations in NH and URGW.

Figure 23: Boxplot of the Posterior Draws of 25-year Return Levels.
Table 12 and 13 show the mean and standard deviation of posterior draws of $\alpha_{ki}$. In the
downscaling model, $\alpha_{ki}$ shows the effect of elevation ($k=1$), $Lon_{diff}$ ($k=2$) and $Lat_{diff}$ ($k=3$) on
the weight of the $i$th climate models, $\beta_i(s)$. For example, $\alpha_{1i}$ describes the amount of
corrections of elevation incorporated by the physical process of each climate model and their
dynamical downscaling. The center points of grids for each climate model provide the average
value for the entire 50 by 50 kilometer grids. $\alpha_{2i}$ and $\alpha_{3i}$ provide corrections of distance and
direction when locations are away from the center points.

Table 12: The Estimation Results for Hyper Parameters in Downscaling Model in New Hampshire

<table>
<thead>
<tr>
<th>Climate Model/Parameters</th>
<th>$\alpha_0$ (intercept)</th>
<th>$\alpha_1$ (elevation)</th>
<th>$\alpha_2$ (Lon-diff)</th>
<th>$\alpha_3$ (Lat-diff)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRCM ccm</td>
<td>9.9623 (1.0034)</td>
<td>11.9014 (2.5441)</td>
<td>-0.0877 (0.0718)</td>
<td>-0.1179 (0.0803)</td>
</tr>
<tr>
<td>CRCM cgcm3</td>
<td>0.2163 (0.1436)</td>
<td>-3.1574 (0.4100)</td>
<td>-0.0066 (0.100)</td>
<td>0.0066 (0.0123)</td>
</tr>
<tr>
<td>ECP2 gfdl</td>
<td>0.3495 (0.1572)</td>
<td>0.2286 (0.4650)</td>
<td>0.0158 (0.0107)</td>
<td>-0.0060 (0.0167)</td>
</tr>
<tr>
<td>ECP2 hadcm3</td>
<td>-0.1937 (0.0670)</td>
<td>-0.7600 (0.1880)</td>
<td>-0.0039 (0.0041)</td>
<td>0.0014 (0.0063)</td>
</tr>
<tr>
<td>HRM3 gfdl</td>
<td>0.2169 (0.0322)</td>
<td>0.1716 (0.0960)</td>
<td>-0.0002 (0.0020)</td>
<td>-0.0020 (0.0023)</td>
</tr>
<tr>
<td>HRM3 hadcm3</td>
<td>0.0240 (0.0186)</td>
<td>-0.0408 (0.0520)</td>
<td>-0.0005 (0.0009)</td>
<td>-0.0011 (0.0013)</td>
</tr>
<tr>
<td>MM5I ccm</td>
<td>0.0239 (0.0134)</td>
<td>0.0315 (0.0390)</td>
<td>0.0009 (0.0009)</td>
<td>0.0008 (0.0011)</td>
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<tr>
<td>MM5I hadcm</td>
<td>0.0155 (0.0084)</td>
<td>0.0320 (0.0290)</td>
<td>-0.0008 (0.0008)</td>
<td>-0.0001 (0.0008)</td>
</tr>
<tr>
<td>RCM3 cgcm3</td>
<td>-0.0011 (0.0051)</td>
<td>-0.0195 (0.0160)</td>
<td>-0.0003 (0.0003)</td>
<td>0.0008 (0.0005)</td>
</tr>
<tr>
<td>RCM3 gfdl</td>
<td>0.0047 (0.0050)</td>
<td>-0.0437 (0.0150)</td>
<td>-0.0004 (0.0003)</td>
<td>-0.0003 (0.0004)</td>
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<tr>
<td>WRFG ccm</td>
<td>0.0104 (0.0044)</td>
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<td>-0.0001 (0.0003)</td>
<td>0.0003 (0.0003)</td>
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<tr>
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<td>0.0076 (0.0030)</td>
<td>-0.0118 (0.0110)</td>
<td>0.0002 (0.0003)</td>
<td>0.0001 (0.0003)</td>
</tr>
</tbody>
</table>

The numbers in the table represent the mean and the standard deviation of the posterior draws. The units
for elevation and distance are kilometers.

In this layer of the model, I centered the variables elevation as well as $Lon_{diff}$ and $Lat_{diff}$. 56
Hence, $\alpha_{0i}$ are the average values of the weights of $ith$ climate model, $\beta_i$, over the entire space of NH and URGW. From both Table 12 and Table 13, we can tell $\alpha_1$ is greater than $\alpha_2$ and $\alpha_3$ in general. But we cannot conclude that elevation has more control on $\beta_i$ than distance and direction. An explanation for this would be that the values of $Lon_{diff}$ and $Lat_{diff}$ are larger than those of elevation.

For both NH and URGW, the absolute value of the coefficients of elevation for CRCM-ccsm, $\alpha_{1i}$, is larger than the coefficients of elevation for other climate models, $\alpha_{1i}$. Negative values of $\alpha_{1i}$ mean increase of elevations results in decrease of the spatial coefficients of return. 

<table>
<thead>
<tr>
<th>Climate Model/Parameters</th>
<th>$\alpha_0$ intercept</th>
<th>$\alpha_1$ elevation</th>
<th>$\alpha_2$ Lon-diff</th>
<th>$\alpha_3$ Lat-diff</th>
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<tr>
<td>CRCM ccsm</td>
<td>5.5215 (0.3285)</td>
<td>-4.1454 (0.9145)</td>
<td>-0.0490 (0.00260)</td>
<td>-0.0019 (0.0340)</td>
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<tr>
<td>CRCM cgcm3</td>
<td>0.0615 (0.0299)</td>
<td>-0.3938 (0.0874)</td>
<td>-0.0027 (0.0027)</td>
<td>0.0009 (0.0029)</td>
</tr>
<tr>
<td>ECP2 gfdl</td>
<td>0.1168 (0.0558)</td>
<td>0.1499 (0.1588)</td>
<td>-0.0010 (0.0046)</td>
<td>0.0068 (0.0060)</td>
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<tr>
<td>ECP2 hadcm3</td>
<td>-0.0088 (0.0158)</td>
<td>-0.1278 (0.0431)</td>
<td>-0.0004 (0.0013)</td>
<td>-0.0005 (0.0018)</td>
</tr>
<tr>
<td>HRM3 gfdl</td>
<td>0.0771 (0.0092)</td>
<td>-0.0591 (0.0240)</td>
<td>-0.0005 (0.0006)</td>
<td>-0.0006 (0.0006)</td>
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<td>HRM3 hadcm3</td>
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<td>-0.0192 (0.0090)</td>
<td>0.0000 (0.0002)</td>
<td>-0.0004 (0.0002)</td>
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<td>MM5I ccsm</td>
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<td>0.0001 (0.0001)</td>
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<td>0.0034 (0.0018)</td>
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<td>RCM3 gfdl</td>
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<tr>
<td>WRFG ccsm</td>
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<td>0.0001 (0.0001)</td>
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<tr>
<td>WRFG cgcm3</td>
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<td>0.0000 (0.0001)</td>
<td>0.0001 (0.0001)</td>
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</table>

The numbers in the table represent the mean and the standard deviation of the posterior draws. The units for elevation and distance are kilometers.
levels, $\beta_i$. This indicates, for example, CRCM cccm, in New Hampshire includes more elevation correction in its physical process or dynamical downscaling process than the other 11 climate models, vice versa for CRCM cccm in the upper Rio Grande watershed. After I have the predictions of $\beta_i(s)$ at 391 and 1087 locations based on the downscaling model, I interpolate the results to the entire region of NH and URGW, respectively, using a Multilevel B-spline approximation using the R package ‘MBA’ (See Figure 24).

The coefficient plots for New Hampshire and the upper Rio Grande watershed describe the relationship of 25-year return levels between weather stations and climate models. As we expect, they are all different. In both regions, climate models, CRCM and ECP2, dominate the downscaling process. In New Hampshire, most of the climate models have different relationships in Mount Washington from surrounding locations. Unlike New Hampshire, the weights of each climate models, $\beta_i(s)$ at the upper Rio Grande watershed show less smoothness.

In general, return levels from all 12 climate models in New Hampshire and the upper Rio Grande watershed have both positive and negative relationship with the return levels at local scales. In any locations, all 12 climate models contribute in different ways to the prediction of return levels at local scales. With estimated $\beta_i(s)$, we calculate the mean of 25-year return levels based on current climate model outputs, $\hat{y}$, in NH and URGW (See Figure 25). Compared with the results from Figure 19 and 23, downscaled return levels at weather stations have lower values. There are two possible reasons. One, after we applied hierarchical spatial downscaling model, the high return levels, such as the one at Mount Washington, can be averaged down by nearby locations. Second, data source for elevations are different. I used elevation data from NCDC for 27 and 52 weather stations. Elevations for prediction locations, 391 in New Hampshire and 1087 in upper
Figure 24: Spatial Interpolations of Relative Weight, $\beta_i(s)$, of Each Climate Model on Downscaled Return Level.
Rio Grande watershed, are from Google API. These two data sources do not always agree with each other. Since elevation is the hyper parameters in the downscaling model, the return levels could be different.

Using the same estimated $\beta_i(s)$, we also calculate the mean of 25-year return level based on future climate model outputs, $\bar{y}$, in NH and URGW. Figure 26 shows the difference between $\bar{y}$ and $\bar{y}$, $(\bar{y} - \bar{y})$. There is little difference between $\bar{y}$ and $\bar{y}$ in New Hampshire. In upper Rio Grande watershed, in some area $\bar{y}$ are greater than $\bar{y}$ and other areas are opposite. That’s because, among 12 climate model outputs, the future climate model outputs return levels are not necessarily larger than the current climate model outputs return levels. Each climate models at different grid cells also can have both positive and negative difference of return levels. (See Figure 27).

In this chapter, besides predicting return levels at locations other than weather stations, another motivation is to contribute to uncertainty quantification by providing credible intervals (i.e. Bayesian analog to confidence intervals) of predicted return levels. In Figures 28 and 29, upper and lower bounds of 95% credible intervals of 25 years return levels in New Hampshire and upper Rio Grande watershed based on both current and future climate models are provided.

In New Hampshire, most of the uncertainty bounds (width of credible intervals) are less than 40 mm. In the upper Rio Grande watershed, there are less than 20 mm.

**Conclusion**

In this chapter, similar to Mannshardt-Shamseldin et al (2010), I downscaled the return levels instead of the distribution of extreme precipitation. Drawing from ideas of Berrocal et al (2010), I considered a full suite of regional climate models with different grids and I let the downscaling coefficients $\beta(s)$ change over space.
Figure 25: $\hat{y}$, mean of 25-year return level based on current climate model outputs. (New Hampshire on the left and upper Rio Grande watershed on the right)

Figure 26: $\bar{y} - \hat{y}$, the difference of downscaled return levels based on future climate model outputs from current climate model outputs.

I applied a Bayesian hierarchical model to include all 12 regional climate model outputs from NARCCAP and propagated the variance in the entire process. The regression setting of the
Figure 27: The difference of the return levels of future climate model outputs from the return levels of current climate model outputs in New Hampshire (a) and upper Rio Grande watershed (b)

proposed model not only gives me the flexibility to consider all the climate models for the statistical downscaling, but also enables me to draw inference on the parameters and interpret the
Figure 28: Lower (left) and Upper (right) Bound of 95% Credible Intervals of Downscaled 25-year Return Level

It is based on current regional climate model outputs in New Hampshire (a) and upper Rio Grande watershed (b)

results. I investigated the relationship of return levels between climate model outputs and weather stations.
Figure 29: Lower and Upper Bound of 95\% Credible Intervals of Downscaled 25-year Return Level. It is based on future regional climate model outputs in New Hampshire (a) and upper Rio Grande watershed (b).

In addition, I calculated the 95\% credible intervals of 25-year return levels at any location in New Hampshire and the upper Rio Grande watershed based on both current and future regional climate model outputs. However, there is still room to improve the downscaling model.
Before I applied the proposed Bayesian hierarchical downscaling model (See Table 11), I needed to spatially interpolate the intercept, $\beta_0$, due to the limited number of weather stations in NH and URGW. Since this procedure was outside of the Bayesian framework, I was not able to propagate the variation of the entire downscaling process.
CONCLUSION

The goal of this dissertation is to estimate return levels at regional scales and project these into the future based on climate model information. An important aspect of my work is to account for the variability in the estimation procedure, thus enabling improved uncertainty quantification which helps scientists and engineers with infrastructure design projects and impact assessment. I focused on two different geological and climatological regions, New Hampshire and the upper Rio Grande watershed and used more than 30 years of observed precipitation from weather stations and simulated precipitation from NARCCAP for this research.

The return level is a function of the shape and scale parameters of the extreme value distribution, either GPD or GEV. In Chapter 1, I discussed some difficulties in estimating the shape and scale parameters of the GPD. As a remedy, I proposed a parameter transformation of the GPD model and used Bayesian methods for estimation. The proposed model maintains the accurate estimation for a wide range of values for scale and shape parameters (See Table 4-7 in Chapter 1). Later, I applied the proposed model to data in New Hampshire and the upper Rio Grande watershed. Figures 10 and 11 again show the proposed model has similar point estimations and credible (confidence) intervals as the common maximum likelihood estimation (MLE) procedure. I tested the proposed model formulation in simulations. In both, the simulations and the data applications non-informative prior distributions were used for the unknown parameters.

In Chapter 2, I considered a spatial latent process to help increase precision in the estimation of the shape and scale parameters of the GPD, compared to ‘individual-station’ estimation. Unlike Cooley et al (2007), who used informative priors, I used non-informative priors for the range and
sill parameters of the exponential spatial covariance function that defined the Gaussian latent process for weather station data. For climate model outputs, I used an Intrinsic Spatial Autoregression (IAR) lattice-type model. For most of the climate model outputs (See Table 10), the width of credible intervals of 25-year return levels is reduced by more than 50%. For weather stations (See Figure 18), the reduction of the widths of credible intervals is about 35%.

Chapter 3 is the core of this dissertation. I proposed a Bayesian spatial hierarchical model for statistical downscaling. Drawing from ideas of Mannshardt-Shamseldin et al. (2010) and Berrocal et al (2010), we are not only able to estimate the return levels where weather stations are located, but enable spatial interpolation to any choice of spatial resolution within a region of interest. The regression framework with the Bayesian hierarchical model provides considerable flexibility to obtain posterior distributions of any quantities of interest, in particular of return levels. Hence, I am able to draw statistical inference at any location, for example, provide error bounds or perform significance testing in a probabilistic risk assessment. In addition, I am able to incorporate the full suite of 12 regional climate model outputs from NARCCAP and integrate an optimal weighting scheme for linearly combining model outputs into my model.

Besides NARCCAP, more recently available data from CORDEX-North America, with finer spatial resolution could also be considered as regional climate models in the downscaling model. However, there are some limitations in this downscaling model. My intention was to build a model within the Bayesian framework to propagate the variation of the entire statistical process. However, due to the computational cost, I have only been able to propose a two-stage model. In the second stage, I have to spatially interpolate $\beta_0$ outside of the Bayesian framework due to limited information at the prediction locations.
There are several ways that the performance of the downscaling model could be improved in future work.

More climate model outputs from either RCMs or GCMs with different spatial resolution could be included. Improvement in computation by utilizing more powerful hardware or cloud computing could allow us to apply the downscaling model to larger regions.

Instead of the two-stage model, the optimal solution would be combining estimation of return levels (Chapter 1 and 2) and downscaling (Chapter 3) in a single Bayesian framework. This would enable us to propagate the statistical variation in the entire process.

I proposed a model to downscale the return level (extreme precipitation), which is a summary calculated from daily data. In recent years, wide-spread damage, not only from the very largest floods, but also from mid-sized and short-duration flash floods have become a concern. Hence scientists and engineers have considered downscaling daily precipitation, or even hourly precipitation to plan for adapting infrastructure design to be resilient to all types of storms and rare events, and to reduce future damage. The implementation of my downscaling model to daily or hourly precipitation, as opposed to only extremes, can be considered for future research.
REFERENCES


## APPENDICES

### APPENDIX A

Table A1: $\xi$ simulation results with $N=500$

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<th>$\xi$</th>
<th>$\sigma$</th>
<th>$\bar{\xi}_{bias}$</th>
<th>$\bar{\xi}_{rmse}$</th>
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<th>$\bar{\eta}_{bias}$</th>
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Table A2: $\sigma$ simulation results with N=500

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APPENDIX B

Stan Code Downscaling Model
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  int<lower=0> M1; // number of original station
  int<lower=0> N; // number of sampling from previous posterior N=100
  int<lower=0> k1; // number of the covariates
  int<lower=0> k2; // number of the climate models+1
  int<lower=0> M2; // number of prediction locations
  matrix[N,M1] y; // weather station return levels
  matrix[N,k2] x[M1]; // current climate mode outputs return levels
  matrix[M1,k1] cov[k2]; // covariates
  matrix[N,k2] x_F[M1]; // future climate mode outputs return levels
  matrix[N,k2] new_x[M2];
  matrix[N,k2] new_x_F[M2];
  matrix[M2,k1] new_cov[k2];
  vector[M1] beta0;
  vector[M2] new_beta0;
} transformed data{
  matrix[N,k2] Q_ast[M1];
  matrix[k2,k2] R_ast[M1];
  matrix[k2,k2] R_ast_inverse[M1];
  matrix[N,k2] new_Q_ast[M2];
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  matrix[N,k2] Q_ast_F[M1];
  matrix[k2,k2] R_ast_F[M1];
  matrix[k2,k2] R_ast_inverse_F[M1];
  matrix[N,k2] new_Q_ast_F[M2];
  matrix[k2,k2] new_R_ast_F[M2];
  for(i in 1:M1) {
    QAst[i]=qr_Q(x[i])[1:k2]*sqrt(N-1);
    RAst[i]=qr_R(x[i])[1:k2]/sqrt(N-1);
    RAst_inverse[i]=inverse(RAst[i]);
    QAst_F[i]=qr_Q(x_F[i])[1:k2]*sqrt(N-1);
    RAst_F[i]=qr_R(x_F[i])[1:k2]/sqrt(N-1);
    RAst_inverse_F[i]=inverse(RAst_F[i]);
  }
}
for(i in 1:M2)
{
  new_Q_ast[i]=qr_Q(new_x[i])[1:k2]*sqrt(N-1);
  new_R_ast[i]=qr_R(new_x[i])[1:k2,]/sqrt(N-1);
  new_R_ast_inverse[i]=inverse(new_R_ast[i]);
  new_Q_ast_F[i]=qr_Q(new_x_F[i])[1:k2]*sqrt(N-1);
  new_R_ast_F[i]=qr_R(new_x_F[i])[1:k2,]/sqrt(N-1);
  new_R_ast_inverse_F[i]=inverse(new_R_ast_F[i]);
}

parameters {
  real<lower=0> tau;
  vector<lower=0>[k2] tau1;
  matrix[k1,k2] alpha;
  matrix[M1,k2] z1;
}

transformed parameters{
  matrix[k2,M1] theta;
  for(j in 1:k2)
    theta[j,] = (cov[j] * alpha[j] + tau1[j] * z1[j]);
}

model {
  for(j in 1:k2)
  {
    z1[,j] ~ normal(0,1);
    alpha[,j] ~ normal(0,10);
  }
  tau ~ lognormal(0,5);
  tau1 ~ lognormal(0,5);
  for(i in 1:M1)
    y[,i] ~ normal(Q_ast[i]*theta[,i]+beta0[i], tau);
}

generated quantities{
  matrix[k2,M1] beta;
  matrix[k2,M1] beta_F;
  matrix[N,M1] y_hat; // current
  matrix[k2,M2] new_theta;
  matrix[k2,M2] new_beta;
  matrix[k2,M2] new_beta_F;
  matrix[N,M2] new_y_hat; // current
  matrix[M2,k2] new_z1;
matrix[N,M1] y_tild; // future
matrix[N,M2] new_y_tild; // future
for(i in 1:M1)
{
    beta[,] = R_ast_inverse[i] * theta[,]i;
    beta_F[,] = R_ast_inverse_F[i] * theta[,]i;
    for(j in 1:N)
    {
        y_hat[j,i] = normal_rng((Q_ast[i] * theta[,]i + beta0[,]i)[j], tau); // original place, current data
        y_tild[j,i] = normal_rng((Q_ast_F[i] * theta[,]i + beta0[,]i)[j], tau); // original place, future data
    }
}
for(j in 1:k2)
{
    for(i in 1:M2)
    {
        new_z1[i,j] = normal_rng(0,1);}
    }
new_theta[,] = (new_cov[,] * alpha[,] + tau[,] * new_z[,]1);'};
for(i in 1:M2)
{
    new_beta[,] = new_R_ast_inverse[i] * new_theta[,]i;
    new_beta_F[,] = new_R_ast_inverse_F[i] * new_theta[,]i;
    for(j in 1:N)
    {
        new_y_hat[j,i] = normal_rng((new_Q_ast[i] * new_theta[,]i + new_beta0[,]i)[j], tau);
        new_y_tild[j,i] = normal_rng((new_Q_ast_F[i] * new_theta[,]i + new_beta0[,]i)[j], tau);
    }
}