CONVECTIVE ELECTRIC FIELD MEASUREMENTS IN AN AURORAL PLASMA

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AURORAL PLASMA

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UNIVERSITY OF NEW HAMPSHIRE, PH.D., 1978

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Lawrence Joseph Zanetti, Jr.
CONVECTIVE ELECTRIC FIELD MEASUREMENTS
IN AN AURORAL PLASMA

by

LAWRENCE JOSEPH ZANETTI, JR.
B.S., University of Colorado, 1974
M.S., University of New Hampshire, 1976

A DISSERTATION

Submitted to the University of New Hampshire
in Partial Fulfillment of
the Requirements for the Degree of

Doctor of Philosophy
in
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September, 1978
This thesis has been examined and approved.

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August 3, 1978

Date
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ABSTRACT

CONVECTIVE ELECTRIC FIELD MEASUREMENTS
IN AN AURORAL PLASMA

by

LAWRENCE JOSEPH ZANETTI, JR.

Reported are measurements of the convective electric field in the ionospheric F region. NASA flights 18:1005 and 18:1004 were launched from Andoya Rocket Range in January and February 1977, sampling respectively quiet and break-up auroral conditions. On board were multiple detectors sampling the 0-5 eV ion spectrum. A least squares fit of an altered Maxwellian to the data results in the ion bulk flow velocity, temperature and density and the rocket potential. In addition one detector was a low resolution mass spectrometer capable of separating O+ from NO+. The electric field results are compared with the dual probe experiment, also on board. The first flight traversed a quiet pre-midnight arc to the north. Ground magnetometer readings showed a positive bay, indicating an eastward electrojet current. The stability of this arc and the electric field reversal encountered has allowed the initial stages of high latitude magnetospheric modeling. Conductivities were evaluated given the accompanying high energy electron data. From these values and the electric
fields, the horizontal current systems have been calculated. The divergence of the northward current may be attributed to field aligned current. Discrepencies have arisen comparing \( J = 0 \) to the measured thermal ion and suprathermal electron field aligned currents indicating an undetected current carrier.
CHAPTER I

PROLOGUE

Magnetospheric physics is awaiting a more complete understanding of our upper environment. Conflicting theories of particle origin, electrodynamic processes, magnetohydrodynamics, fluid flow and wave particle interactions abound. The magnetosphere is defined to be the earth's magnetic environment. The earth's contribution, which is approximately a magnetic dipole field, interacts with the magnetic field carried by the solar wind to produce the resulting magnetosphere (Hill, 1977). The solar wind is the sun's coronal plasma, consisting mostly of protons and electrons, which flows outward toward the earth. The magnetosphere is compressed on the sunward side and extended on the anti-sunward side.

There are several particle and field interactions in the system. Specifically, those between the sun and the solar wind, the solar wind and the magnetosphere, and the magnetosphere and the near earth are of contemporary
interest. A more complete discussion of the technical magnetospheric structure and history has been reserved for Chapter 5.

Although much of the solar wind-magnetosphere structure has been mapped by the advent of rocket and satellite measurements since the late 1950's (Burch, 1977), there are basic questions as yet unanswered. A major problem of concern to both magnetospheric physics and astrophysics is the energy exchange between particles and electromagnetic fields. This exchange results in the acceleration of particles to higher than expected energies. Particles of solar wind origin (~400 km/sec or 800 eV per charge) appear in the ionosphere with energies upwards of 20-40 keV per charge (Arnoldy, 1974). Possibly the same mechanism is responsible for the acceleration found in solar flare particles.

Another problem is the origin of the plasma populating the magnetosphere. Some is of solar wind origin; some comes from the ionosphere. The solar wind portion poses the problem of plasma transfer. A result of these particles in the ionosphere is the aurora. Aurora are the result of high energy electrons precipitating from the outer magnetosphere and exciting upper atmospheric molecules.

One of the most dynamic events of our magnetosphere is the magnetic substorm (Akasofu, 1977). A magnetic substorm results when a slow accumulation of magnetic energy on the antisunward side of the earth (magnetotail) explosively
collapses, transferring energy to the plasma. This energetic plasma results in auroral substorms at high latitudes (~68°) near the earth (~200 km). This energy is not insignificant. Some $10^{11}$ to $10^{12}$ watts is deposited in the atmosphere continuously for typically one hour by magnetic substorms. During normal sunspot activity substorms occur on the average of every four hours.

Aside from furthering knowledge of our own environment and the fascinating science involved, a naturally existing plasma physics laboratory is available. The densities are low (~1 - 10 particles/cc), and the scale is large (10's -> 100's of earth radii (R)). For example, UNH - Minnesota Echo series (Winckler, 1975) studies magnetospheric characteristics resulting from the injection of 40 keV electron beams from rocket packages. These experiments require a knowledge of the environment.

To explain plasma particle motion and energy the forces present must be known. This implies a measurement of fields. The magnetic field of the earth has been known and studied for ages, along with mounting theoretical work. Extending this study to increased height was immediate and natural; consequently magnetic fields of the ionosphere have been well studied and mapped. The next step is the evaluation of the electric fields.
The purpose of this project is to participate in a fraction of this evaluation. The determination of ionospheric electric fields at altitudes of ~200 km is the primary objective of this work. In the process improvements in calibration and data reduction were made.

The motion of ions in the magnetically dominated E and F regions of the ionosphere (above 160 km) are due to electric fields, curvature and gradient magnetic fields and gravity fields. Consider the motion of charged particles in a magnetic field with a generalized force perpendicular to \( \vec{B} \). Perpendicular is defined as perpendicular to the \( \vec{B} \) magnetic field vector; parallel implies along this vector, i.e. geomagnetic coordinates. The magnetic field cannot alter motion in its own vector direction, the Lorentz force being \( \vec{v} \times \vec{B} \). Any force present in this direction results in direct acceleration.

The result of a perpendicular force \( (F_\perp) \) is a drift given by

\[
\vec{v} = \frac{c}{q} \left( \frac{F_\perp \times \vec{B}}{B^2} \right)
\]

The details are given in Appendix A. Table 1.1 gives a summary of the major drifts and estimates of their magnitudes for ionospheric conditions. It is evident that the dominant motion in the ionosphere - lower magnetosphere is the E cross B drift. Note also that this drift is independent of \( q \) and thus creates no currents.
Table 1.1

| DRIFT                  | $v[\text{drift}]$                                                                 | $|v[\text{drift}]| \text{(m/s)}$ |
|------------------------|----------------------------------------------------------------------------------|-----------------------------------|
| $E \times B$           | $c(\vec{E} \times \vec{B})/B^2$                                                 | $1 \times 10^3$                   |
| gravitational          | $c(m/g)(\vec{g} \times \vec{B})/B^2$                                            | $1 \times 10^{-12}$               |
| gradient               | $-c(v_\perp r_\perp /2) \left( \nabla \times \vec{B} \right) /B^2$             | $5 \times 10^{-13}$               |
| curvature              | $cm/(gR_e^2)(v_\parallel^2+v_\perp^2/2)$                                        | $5 \times 10^{-1}$                |
|                        | $(\vec{R} \times \vec{B})/B^2$                                                 |                                   |

This motion may be understood by considering Figure 1.1. An ion with a velocity in the $+y$ direction will begin to gyrate around $\vec{B}$ in a left hand sense. As its position attains a larger $y$ value, the electric field has contributed to its velocity. Since the Larmor radius goes as the perpendicular velocity, a wide curve is traced. The opposite is true closer to the $x$-$z$ plane, the result being a net gain in the $+x$ or $\vec{E} \times \vec{B}$ direction. It is precisely this drift which has been measured.

NASA flights 18:1005 and 18:1004 were launched in Winter 1977 from Andoya, Norway. The University of Minnesota, co-investigators of the project, packaged three-axis fluxgate magnetometers, AC and DC electric field instruments and transverse, axial, and proton magnetometers. The contribution by the University of New Hampshire consisted of photometers, measuring the intensity of 5577Å metastable atomic $O^+$ (white-green, 0.7s lifetime) and 4278Å molecular $N_2^+$ (blue-uv, $\mu$s lifetime) transitions, high energy
Figure 1.1 E Cross B Drift
(0-20 keV) electron and ion detectors (DESA), a high count rate (high energy electrons) detector attempting to observe flux modulations (OCTOSPHERE), high energy (>40 keV electrons and >120 keV protons) geiger counters, low energy ion detectors (EFM's) and a low energy ion mass spectrometer (MAG-EFM). The last two experiments are the chief concern of this thesis. It is the concern of these flights to decipher the fields, current systems, energy deposition and possible wave measurements utilizing all instruments.

Measurement of the E x B drift is achieved by energy selection using an electrostatic curved plate analyser (EFM). The MAG-EFM unit also has the capability of separating O\(^+\) from NO\(^+\), analysing the first 90% of the time. The initial steps in this work were the design and calibration of the MAG-EFM. There is a question concerning the major constituents of the E and F regions. The data analysis necessitates knowledge of the ion mass. Previously the mass has been assumed from some atmospheric model.

In the region where the E fields were measured, the effects of ion collisions are negligible (Evans, 1971). Ions execute cyclotron motion around the magnetic field vector with a gyrofrequency of 190 rad/sec whereas the collision frequency is 4.1/sec. At 130 km the gyrofrequency of ions equals the collision frequency with neutrals. Above this altitude the motion of charged particles is dominated by fields.
Various other methods of measuring electric fields or drifts are available (see Morgan, 1976). The following is a short discussion of the two experiments operating in conjunction with these flights and a third widely used ion drift method. These techniques are dual probe antennae, Doppler radar measurements, and the retarded potential analyzer.

The dual probe antennae (Cahill, 1978b) was flown aboard flights 18:1005 and 18:1004 by the University of Minnesota. The antennae consist of booms, 3 meters tip to tip, with spherical probes on either end. When immersed in the ionospheric plasma the two probes will sustain different potentials due to electric fields ($\Delta$potential/distance), plasma anisotropies and craft motion ($\vec{v} \times \vec{B}$). The craft motion is known and may be subtracted, and the data analyzed over a spin period will remove a constant offset due to anisotropies. The electric field remains and these results will be compared to the thesis results.

The ion drift velocity can also be measured from the ground with the STARE (Scandinavian Twin Auroral Radar Experiment) radar facility (Greenwald, 1978). Radar signals are reflected from ionospheric irregularities with the return signals Doppler shifted. These irregularities are generally unstable waves whose phase velocities in the 100-120 km altitude range are approximately equal to the electron drift velocity (Sudan, 1973). This drift is due to the $E \times B$ drift. Though not shown here, these results
have compared well with Minnesota's double probe experiment (Cahill, 1978b).

A second widely used drift detector making in situ measurements is the retarded potential analyzer (Hanson, 1973). This was not a coincident experiment. This experiment has been included on the Atmospheric Explorer and OGO-6 satellites. The design incorporates a four segment planar collector whose look direction is forward and normal to the craft when it is not spinning. Each segment is attached to an independent variable-gain electrometer. Ions enter through a swept negative potential grid. The electrometer currents are fed in pairs to a differential amplifier and this output is telemetered to the ground. If a flow is encountered head-on, all segments will record the same output and no difference is registered. If the flow is skewed to the craft normal, the sections will record differently. Theoretical relationships are known between the skew and current differences. Since the vehicle orientation with respect to its velocity vector is known, the skewed resultant must be formed from an additional perpendicular component. This component is attributed to the E cross B drift. The variable gain is attractive as ion densities may vary over as much as three orders of magnitude ($10^3 \rightarrow 10^6/cc$). Spatial resolution is somewhat sacrificed due to the necessary collection times. Directional resolution was limited only by attitude information. No absolute ion densities are measured.
All techniques suffer involved data analysis, a consequence of low electric field intensities. Ionospheric fields are on the order of millivolts/meter to upwards of 100 mv/m on occasion. Correlation between such differing methods is however building confidence in measurements of electric fields. What succeeds this introduction is a presentation of the electrostatic analyser ion drift method. Available comparisons have been made. The overall confidence in the presented results has led to the initial stages of modeling of auroral phenomena.
CHAPTER II

INSTRUMENTATION

The detection of the drift of ions is accomplished by electrostatic analyzers. They are so named because ions flow between two curved plate electrodes which envelope a static electric field. If particles are at the proper energy they will pass the entrance and exit holes and will be detected with a Galileo Model 4025 Channeltron Electron Multiplier. This analyser (EFM) has been tested and flown previously with results (Morgan, 1978), as has the method (Green and Whalen, 1974).

The energy/ΔV factor is about 5.25 eV/volt. Since the energies to be investigated are 0-5 eV, the outer plate was swept from 0 to +.5 volt while the inner plate progresses to -.5 volts in 32 steps. An energy spectrum of ions is obtained every 25.6 ms. The rocket spins at 5 rps or 46 degrees/spectrum. The acceptance cone is about 2 degrees by 7 degrees full angle. The EFM is pictured in Figure 2.1.
Figure 2.1 EFM Electrostatic Ion Spectrometer
A new instrument (MAG-EFM) was designed utilizing the above section for initial low energy selection with a low resolution magnetic mass spectrometer added. This second section post-accelerates the ions by floating at a negative potential, thus separating them according to mass. The ions would have an energy of ~100 volts. For packaging and weight a radius of curvature of ~1 cm was decided. The resulting field necessary to pass O\(^+\) (16 amu) would then be 6000 gauss. The magnetic section was calibrated to pass O\(^+\) 90% of the time or ~4 seconds. The remaining 10% of the time the voltage was changed to pass NO\(^+\), thus measuring the two major constituents of the E and F regions. Accompanying this instrument were two standard EFM packages covering pitch angle ranges of 0-120 degrees. The MAG-EFM is depicted in Figure 2.2.

The design of the energy selecting analyser utilizes the influence of the Lorentz forces on charged particles. That is:

\[
\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})/c]
\]

The first stage is two parallel curved electrodes swept through different potentials. To find the dependence on the fields for a particle's travel solve Laplace's equation in cylindrical coordinates. In this case there is no dependence, and the gap is considered small enough compared to the width of the plates such that the z-dependence can be ignored.
Energy - Mass Ion Spectrometer

Figure 2.2 MAG-EFM Energy-Mass Ion Spectrometer
\[ \nabla^2 \phi = 0 \rightarrow \rho \frac{\partial \phi}{\partial r} \frac{\partial \phi}{\partial r} = 0 \]
\[ \frac{\partial}{\partial r} (\rho \frac{\partial \phi}{\partial r}) = 0 \]
integrating once
\[ \rho \frac{\partial \phi}{\partial r} = C \]
\[ \frac{\partial \phi}{\partial r} = c/r \]
integrating again
\[ \phi = c \ln \rho + D \]

but since \( E = -\nabla \phi \) the constant \( D \) is not contributing to any
electric field. Knowing the boundary conditions at the
plates

\[ \phi = v_0 \text{ at outer plate (} r_o \text{)} \]
\[ \phi = v_i \text{ at inner plate (} r_i \text{)} \]

\[ v_o = c \ln (r_o) \]
\[ -v_i = c \ln (r_i) \text{ subtract} \]
\[ v_o - v_i = c \ln (r_o/r_i) \]
\[ \Delta v = c = \frac{\Delta v}{\ln (r_o/r_i)} \]
\[ \phi = (\frac{\Delta v}{\ln (r_o/r_i)}) \ln \rho \]

For the electric field

\[ \bar{E}(\rho) = -\nabla \phi = -\Delta v/\ln (r_o/r_i) \hat{\rho} \]

For ions the \( \bar{E} \) field is directed inward and goes as \( 1/\rho \).
This is exact.
Approximating the separation of the plates as small compared to the radii (for EFM's $r_{\text{mean}} \sim 2$ cm, plate separation 2 mm), letting $r_o = r_i + d$, $d$ = plate separation

$$
\bar{E} = -\Delta V/\{ln[(r_i+d)/r_i]\} \hat{\rho} = -\Delta V/[\rho \ln(1+d/r_i)] \hat{\rho}
$$

$$
ln(1+x) = x - 1/2x^2 + 1/3x^3 \ldots \text{ if } -1 < x < 1
$$

so

$$
\bar{E} \approx \Delta V/[\rho (d/r_i)] \hat{\rho}
$$

$$
\bar{E} = -\Delta V r_i/(d \rho) \hat{\rho}
$$

specifically if $\rho = r_i + d/2$, which is the center line path

$$
\bar{E} = -\Delta V r_i/\{d(r_i + 1/2d)\} \hat{\rho} \propto -\Delta V[1-d/(2r_i)]/d \hat{\rho}
$$

with the binomial expansion. The first term ($\bar{E} = -\Delta V/d \hat{\rho}$) is the field for a parallel plate capacitor.

The force a charged particle feels in an electric field is $q\bar{E}$ which balances the centrifugal force such that the particle can proceed between the plates as

$$
\frac{mv^2}{\rho} + qE = 0
$$

$$
\frac{mv^2}{\rho} = -qE
$$

$$
\frac{mv^2}{\rho} = q(\Delta V/d)
$$

$$
1/2mv^2 = E = \text{energy} = q \Delta V \rho/(2d)
$$

where we take $\rho = r_{\text{mean}} = r_m$ so

$$
E = q \Delta V r_m/(2d) \quad \text{(cgs)}
$$
and everything on the left hand side is fixed but the potential difference on the plates.

Also note that the energy selected is independent of the mass of the ion so any ion with the correct energy to satisfy the above condition will pass through the analyser.

The energy resolution is measured by $\text{FWHM}/\langle E \rangle$. Full width at half maximum is defined as $\delta E$. Looking at a particle that begins travelling the centerline between the plates ($r_m$), what is the deviation in energy allowable before one plate is contacted? Twice this value is $\delta E$.

The deviation is

$$dE = A dr$$

from equation 2.1 where $A = q \Delta V/(2d)$

$$\text{FWHM}/\langle E \rangle = \frac{\delta E}{E} = 2dE/E$$

$$\text{FWHM}/\langle E \rangle = 2A dr/(A r_m)$$

$$\text{FWHM}/\langle E \rangle = 2dr/r_m$$

For standard EFM as well as the energy selector of the MAG-EFM, $dr = 1\text{mm}$ and $r = 20\text{mm}$

$$\text{FWHM}/\langle E \rangle = 10\%$$

For the MAG-EFM only the length of the path of flight was altered, the energy resolution is not affected.
Analysis of the mass selector proceeds as follows. Charged particles traversing a magnetic field experience a force perpendicular to both their velocity and the field as
\[ \mathbf{F} = \frac{q(\mathbf{v} \times \mathbf{B})}{c}. \]
This will balance a centrifugal force resulting in cyclotron motion.

\[
\begin{align*}
\mathbf{F}_{\text{cent}} + \mathbf{F}_{\text{magnetic}} &= 0 \\
\mathbf{F}_{\text{cent}} &= -\mathbf{F}_{\text{magnetic}} \\
\frac{mv^2}{r} \hat{\mathbf{r}} &= -\frac{q(\mathbf{v} \times \mathbf{B})}{c}
\end{align*}
\]

Since all directions are perpendicular

\[
(\mathbf{v} \times \mathbf{B}) \rightarrow \mathbf{vB}
\]

\[
\frac{mv^2}{r^2} = qvB/r
\]

and if non-relativistic

\[
E = \frac{mv^2}{2} \quad \text{and} \quad v = \left(\frac{2E}{m}\right)^{1/2}
\]

\[
E = qBR \left(\frac{2E}{m}\right)^{1/2}/(2c)
\]

\[
E = \frac{q^2B^2r^2}{2mc^2}
\]

Mass is now a factor in the passage of the ions through the analyser. The achieved rigidity (BR) was 6863 gauss-cm. The energy needed to pass 0+ is

\[
E(\text{eV}) = \frac{1}{2}\left(4.8 \times 10^{-10}\text{esu}\right)^2 (6863\text{ gauss-cm})^2/
\]

\[
[ (3 \times 10^{10})\text{cm/s} (16\text{amu}) (1.66 \times 10^{-24}\text{gm/amu})
\]

\[
(1.6 \times 10^{-12}\text{ erg/eV})
\]

\[
E(\text{eV}) = 141.9 \text{ eV (O_{1b})}
\]

\[
E(\text{eV}) = 75.7 \text{ eV (NO}_{3\text{o}})
\]
Thus a post-accelerating voltage of 142 volts is necessary to mass select $O^+$. 

Measured resolution of the magnetic section is about 20%. Since differentiating between $O^+$ and $NO^+$ is all that is necessary, this resolution was found satisfactory. Counting rates are a problem, and reduction of such is not desirable. The design satisfying all packaging and performance requirements was two magnets about 2 cm in diameter facing the gap with a return to complete the circuit. Refer to Figure 2.2. With the new magnetic materials such as Alnico and Cunife larger fields are accessible with less material than has previously been possible.

Weight being a consideration, a $M$-$O$-metal shield was first employed as a return for the magnetic flux. $M$-$O$-metal has a maximum permeability of about 100,000 whereas soft iron (transformer core iron) has a permeability of 6,000. The attenuation of the leakage field outside the analyzer is much better in $M$-$O$-metal. However, proper consideration of flux and saturation was ignored leading to the failure of this return. $M$-$O$-metal saturates at 6,000, soft-iron at 16,000 gauss. Gauss is a measure of field line density so flux consideration [$\phi = B(area)$] led to essentially equal cross-sectional areas for both the magnet and the return. This mass of $M$-$O$-metal is prohibitively expensive. The soft iron was used as a return with possible $M$-$O$-metal shielding outside the casing to further reduce stray fields. Stray
fields are not to be taken lightly as there are sensitive magnetometers measuring fluctuations as low as a few gamma (\( = 10^{-5}\) gauss) in the ambient magnetic field as well as 3-axis magnetometers within 2 ft of this package. Although the majority of field lines are contained in the soft iron (soft means magnetically soft - no hysteresis), there is always some degree of leakage field. To further reduce this field the magnet was totally enclosed with soft iron. This was done with the magnets centered in 2 circular cups. Entrance and exit holes allow passage of the particles. The magnet design proceeded from Maxwell's equation

\[
\vec{\nabla} \times \vec{H} = 4\pi \vec{J}/c + 1/c \frac{d\vec{D}}{dt}
\]

There are no current loops and no changing D fields

\[
\vec{\nabla} \times \vec{H} = 0
\]

integrating over this surface bounded by the magnet circuit

\[
\int_s (\vec{\nabla} \times \vec{H}) \cdot \hat{n} \, ds = 0
\]

with Stokes law

\[
\int_s (\vec{\nabla} \times \vec{H}) \cdot \hat{n} \, ds = \oint_c \vec{H} \cdot d\vec{l} \\
\oint_c \vec{H} \cdot d\vec{l} = 0
\]

Since the interest is in the field of the gap from \(a\rightarrow b\) of Figure 2.3, separate this part
Figure 2.3 Magnet Assembly
\[ \int_b^c \mathbf{H} \cdot d\mathbf{l} = - \int_b^c \mathbf{H} \cdot d\mathbf{l} - \int_c^d \mathbf{H} \cdot d\mathbf{l} - \int_d^a \mathbf{H} \cdot d\mathbf{l} \]

\( \mathbf{H} \) is always parallel to \( d\mathbf{l} \) and

\[ \int_b^c \mathbf{H} \cdot d\mathbf{l} = \int_d^a \mathbf{H} \cdot d\mathbf{l} = -2 \int_b^c \mathbf{H} \cdot d\mathbf{l} - \int_c^d \mathbf{H} \cdot d\mathbf{l} \]

In the gap \( \mathbf{H}_g = \mathbf{H}_3 \); from \( b \rightarrow c \) (permanent magnet)

\[ - \int_b^c \mathbf{H} \cdot d\mathbf{l} = -H_m l_{bc} \]

from \( c \rightarrow d \) using the fact that magnetic flux is continuous across any boundary, or \( \phi_t = \phi_m = B_m A_m \)

\[ - \int_c^d \mathbf{H} \cdot d\mathbf{l} = -H_b l_{cd} = -\phi_t l_t / \mu_i A_i = B_m A_m l_t / \mu_i A_i \]

where

- \( B_m \) = B field in magnet in gauss
- \( A_m \) = cross-section area of magnet
- \( l_t \) = length iron
- \( \mu_i \) = permeability of iron
- \( A_i \) = cross-section of iron

\[ \phi = \text{constant} = B_m A_m = B_3 A_3 \] and \( \mathcal{B} = \mu H \)

\[ \phi = \mu H A_m ; \quad H = \phi / \mu A_m \]

so \( H \) in gap is

\[ H_g = B_m A_m / A_3 \]

\[ B_m l_g = -2 (H_m l_m) - (B_m A_m l_t / A_i \mu_i) \]

\[ B_m (l_g + A_m l_t / A_i \mu_i) = -2 H_m l_m \]

2.3
Using the approximation that the length of the gap is not too small, the second term on the left hand side of equation 2.3 can be neglected since $\mu_i$ is very large. The term in parenthesis is often referred to as the reluctance of the circuit (reluctant to passing flux). Refer to Appendix B for justification of this neglect.

Notice that to satisfy the equation 2.3, which is exact, $\overline{H}$ ($\overline{H}$ in the magnet) must be negative since all other quantities are positive. Thus the $\overline{H}$ field in the magnet is a demagnetizing force opposing $\overline{B}$ working to randomize the alignment of the permanent magnet atoms' magnetic moments. This is important for finding the operating point of the circuit. The circuit operates in the second quadrant of a hysteresis curve ($\overline{B}$ pos., $\overline{H}$ neg.) shown in Figure 2.4 for Hyflux Alnico 5-7.

As previously mentioned flux considerations require sufficient cross-sectional area to pass the flux density; that is, flux is continuous.

$$B_m A_m = \phi = B_i A_i = B_j A_j$$

The flux density in the magnet is 6,000 gauss whereas a safe operating flux density (~75% of saturation value) for the soft iron is 10,000 gauss so

$$A_i / A_m = 6,000/10,000 = 3/5$$
Figure 2.4 Alnico 5-7 Hysteresis Curve


\[ A_i = \frac{3}{5} A_m \]

at least to insure proper flux return.

To return to equation 2.3 with the reluctance of the return neglected

\[ B_m l_3 = -21_m H_m \]
\[ B_m/H_m = -21_m/l_3 \quad \text{this is called Permeance Coefficient} \]

This is a straight line with a slope of negative \( 21_m/l_3 \) and was determined solely from the geometry of the circuit, lengths and cross-sections, and gives a ratio of \( B \) to \( H \) in the magnet. This must be solved simultaneously with the characteristics of the magnetic material, i.e., the characteristic hysteresis curves.

The intersection of the two curves, the relation of \( B \rightarrow H \) caused by the geometry external to the magnet (the circuit allowing the flux to return) coupled with the microscopic properties internal to the magnet (magnetic moment alignment) determines the operation of the circuit. Further specifics of this magnet design are included in Appendix B.

The third section is a charged particle detector. The detector is a Channeltron Electron Multiplier\(^\circ\), which is a small curved glass tube beginning with an open cone, shown in Figure 2.5. The inside is coated with a semi-conductor. Semi-conductors usually have a valence of 4 or 5, providing this many loosely bound electrons per atom. Impact of the ion on the cone releases secondary electrons from the
Figure 2.5 Galileo Model 4025c Channeltron Electron Multiplier
coating, which are accelerated down the tube \[ \Delta v (\text{cone} \rightarrow \text{tail}) = +2800 \text{V} \]. These electrons impact continuously, resulting in a cascade. The amplification is about \(10^6\) and a neg pulse \(-100\) ns long is generated.

The signal is coupled through a pulse transformer to the preamp. The preamp reacts to the negative pulse producing a 10 volt square pulse. The output is fed to the rocket telemetry, counted and transmitted continuously to receivers.

The geometry factor was obtained by lab calibration using a simulated omni-directional ion beam. The geometry factor is a measure of the particles actually counted when the instrument is viewing an isotropic source. The geometry factor is necessary in the data analysis to find a Maxwellian distribution responsible for the observed counts. The counts were normalized to a geometry factor of \(9.6 \times 10^{-6}\) cm\(^2\) ster eV/eV for the non-magnetic version EFM units. Individual calibration techniques and results are described in Appendix C. The magnetic separation, also discussed in Appendix C, resulted in some reduction in counting rates.

In summary, the successful mass separation has settled doubts regarding mass assumptions in the data analysis. Small modifications including higher resolution and swept mass separation would result in this instrument being a general energy-mass spectrometer for low energy ions.
CHAPTER III

DATA REDUCTION

3.1 Introduction

The object of the data reduction is to determine the bulk flow velocity vector. Additional results include ion temperature, ion density and rocket potential. The MAG-EFM unit also gives information on the relative densities of ions of mass 16 amu and 30 amu.

This is accomplished by first assigning each data point a vector direction ($\Theta$, $\phi$) and an energy (0-5 eV). Data points were accumulated for two seconds to give adequate representation of the distribution. These associated variables were displayed in various ways to help estimate the final answers. An iterative least-squares fitting routine was applied to this collection, resulting in the energy distribution function parameters.
3.2 Particle Aspect

The initial step in this process is to locate the vector representing the rocket spin axis in some convenient coordinate system. Generally for local ionospheric studies the geographic or geomagnetic system is used. The geographic system is the horizon system with the celestial sphere centered at the locale. The equator of this sphere is coincident with the local horizon, and three-axis Cartesian coordinates would be geographic north, west and zenith. The geomagnetic system has the z-axis parallel to $+\vec{B}$. Completing the three axes would be geomagnetic north, parallel to the geographic horizontal $\vec{B}$ component, and geomagnetic east. Formulae for locating the spin axis vector in a geographic coordinate system is described by Kitner (thesis, U of Minn.), see Figure 3.1.

The analysis begins by fixing the rocket spin axis in geomagnetic coordinates. Two independent rocket-based reference angles are needed. The available references are the moon and the magnetic field, both fixed in the above coordinate systems. The location of the moon is given in the American Ephemeris and Nautical Almanac for the equatorial system. This is transformed using spherical trigonometry to the desired local system. The moon is assumed stationary during the eight minute flight. The two angles are achieved using an on-board lunar sensor (Bayshore Systems Model LS-12a) and three-axis magnetometers (U of Minn). The lunar sensor is an opaque glass plate upon which
L' = HA(L) = Lunar hour angle
LZ = Lunar declination
R' = HA(R) = Rocket hour angle
RZ = Rocket declination
B' = HA(B) = Magnetic hour angle
BZ = Magnetic declination

ASPECT SYSTEM PROJECTED ON CELESTIAL SPHERE

Figure 3.1 Geographic Rocket Aspect Angles from Kintner (1973)
is cleared two curves, cotangent and its mirror image. Centered behind the glass is a light sensitive diode. The plane of the curves is parallel to the spin axis. Two pulses result when moon light enters these slits. The ratio of the short time interval to the completion of the spin is linear with the angle between the spin axis and moon vector. The Develco Model 9200 three-axis fluxgate magnetometer consists of three coils giving the instantaneous \( \vec{B} \) field component present on each axis. The vector addition gives the magnitude and direction of \( \vec{B} \) relative to the rocket coordinate system.

Magnetometers will give the magnetic field vector in the rocket or the angle between the rocket spin axis and the \( \vec{B} \) field. Thus the spin axis vector is somewhere on a cone whose half-angle is the above mentioned angle. Identically the lunar sensor provides the angle between the spin axis and the moon vector defining a second cone of possible spin axis vectors. The intersection of the two cones defines two possible vectors. From the initial trajectory usually one can be discarded. The triangles whose vertices are formed from the termination of the above vectors on the celestial sphere are solved with spherical trigonometry (Smart, 1937). A description of the formulae used is given in Appendix D.

Since the rocket rises spinning in a right-handed sense, the geomagnetic coordinate system is temporarily rotated 180 degrees. Referring to Figure 3.2, \( \vec{B} \) is the magnetic field vector, \( \vec{W} \) is the spin axis of the rocket, \( \vec{Z} \)
Figure 3.2 Geomagnetic Spin Axis Aspect Angles
is -\text{B} and \text{L} points to the moon. Geomagnetic north and east are shown. Arc \text{ZR} is the supplement of the angle given by the magnetometer. Arc \text{LB} is provided by the lunar sensor. Two angles determine the spin axis; zenith distance (arc \text{ZR}) and azimuth (angle \text{NZR}). Angle \text{LNZ} is known from the moon’s location, and angle \text{LZR} is given by

\[
\text{LZR}=\cos^{-1}\left\{\frac{\cos (\text{arc LB}) - \cos (\text{arc LZ})\cos (\text{arc ZR})}{\sin (\text{arc LZ})\sin (\text{ZB})}\right\}
\]

To achieve the final particle velocity vectors the absolute aspect of each detector is required. A time reference is needed to orient the plane perpendicular to the spin axis. The magnetometer routine gives the time reference of the x-axis magnetometer zero going positive. At this time the x-axis magnetometer is exactly perpendicular to the plane formed by the \text{B} and \text{R} vectors. This information with the spin rate gives a rocket based azimuth (\phi'(t) in Figure 3.3) for all detectors. Figure 3.3 is identical to Figure 3.2 but includes the location of a specific detector (\text{D}) with respect to the rocket. \phi'(t) is the supplement of the included angle \text{ZRD}. The zenith distance of detector \text{D} is

\[
\text{arc ZD}=\cos^{-1}\left[\frac{\cos (\text{ZB})\cos (\text{DR}) + \sin (\text{ZB})\sin (\text{DR})\cos (\text{ZRD})}{\sin (\text{DR})}\right]
\]

The azimuth of \text{D} (angle \text{NZD}) is obtained by finding angle \text{NZD}
Figure 3.3 Geomagnetic Detector Aspect Angles
\[ \text{angle } \text{RZD} = \cos^{-1} \left( \frac{\cos(\text{RD}) - \cos(\text{ZR}) \cos(\text{ZD})}{\sin(\text{ZR}) \sin(\text{ZD})} \right) \]

\[ \text{angle } \text{NZD} = \text{angle } \text{NZR} + \text{angle } \text{RZD} \]

Detector look directions in the coordinate system pictured in Figure 3.3 are now known. Desired are the particle velocity vectors (opposite to the detector look direction) in the standard geomagnetic coordinate system. Immediately the magnitude of arc ZD is the particle zenith distance or pitch angle, angle between the velocity vector and \( +\vec{B} \). The azimuth is the opposite of the azimuth of \( \vec{D} \) or angle \( \text{NZD} + 180 \) degrees. A further change is necessary so that azimuth is measured from north through east. A brief description of the above Fortran program VECTOR.FOR is in Appendix D.

3.3 Initial Data Analysis

The answers from the process of least squares fitting thousands of data points that are a function of many variables are at times mysterious. It is essential to view raw data in as many ways as possible. For this project the energy spectra are a function of six independent variables; energy, particle pitch angle and azimuth and three rocket ram velocities. The parameters needed to determine the electric fields are the magnitude and direction of the \( \vec{E} \) cross \( \vec{B} \) drift vector. The magnitude of this velocity (translated to energy) is nearly impossible to view since
the dominant contribution to the energy peak of the ion spectra is due to the drop through the -1.5 V rocket potential. The energy contribution due the E cross B drift is about a quarter of an eV. The energy contribution due to the relative velocity between the plasma and the rocket (ram velocity) is also 1/4 eV.

A mapping program called PSUMAP (PSUMAP(1969)) gathers a 2-dimensional grid of values, interpolates between adjoining points and produces a grey-scale line-printer output of the grid. A 3-dimensional plot of the above is also available. The rocket-based velocity vector can be viewed by plotting counts vs. pitch angle vs. azimuth (Figure 3.4). In this figure the dimension front to rear is linear in pitch angle, 0° -> 180°. From left to right is plotted azimuth, 0° -> 360°, measured from north through east. The vertical axis is counts. Essentially what is displayed is the observable velocities or the coverage in space of both instruments. Thus the flow direction can be visually located by a peak and compared with the fit answers. This specific time was on the downleg of flight 18:1005 at flight time equal to 275 seconds. As is shown in the chapter on results, the least squares fit predicted this time to have a generally south to south-southwest electric field. \( \overrightarrow{B} \) is toward the earth, \( \overrightarrow{B} \) cross \( \overrightarrow{E} \) results in an east drift. Again the relative rocket-plasma motion is still in the original counts and must be mentally subtracted. The ram at this time is south as the rocket was fired nearly
Figure 3.4 Plot of Pitch Angle vs. Azimuth vs. Counts
straight north. The result is an east flow combined with a comparably equal south ram velocity or a southeast peak (≈135° azimuth). This result can be seen in Figure 3.4. These pitch angle - azimuth plots are most useful for locating the azimuth of the flow, the front to back dimension giving the pitch angle coverage available to each analyzer. Figures as 3.4 are also useful when both analyzers are active to be certain they both register the same azimuth.

The same package was used to plot counts vs. energy vs. pitch angle (Figure 3.5). In this figure are plotted a series of energy spectra ranging in pitch angle. Consider the energy dimension as a radius arm originating at the center back. From this point along the back to the left is 0° pitch angle. The radius arm pivots about its origin in a right hand sense with increasing pitch angle. A radius arm starting at center back pointing right would be antiparallel to \( \vec{B} \) or 180°. The vertical axis is again counts. This plot shows the smooth peak in energy as should be expected. It also shows the flow mostly perpendicular to \( \vec{B} \), with ram velocity contributing a component greater than 90° pitch angle for this flight time. Figure 3.5 is especially useful for detecting noise in the energy sweep. These aids have proved invaluable for initial analysis and parameter confidence.
Flight 18:1005
Pitch Angle vs Energy vs counts

Figure 3.5 Plot of Pitch Angle vs. Energy vs. Counts
### 3.4 Least Squares Fit

The data resulting from the EFM energy sweeps are the number of ions accepted at a given energy step. The energy range is 0-5 eV accumulated during 32 steps in 25.6 ms. The original thermal ion distribution is assumed Maxwellian

\[ f(\tilde{v}') = n \left( \frac{m}{2\pi kT} \right)^{3/2} \exp\left( -\frac{\tilde{v}'^2}{v_+^2} \right) \]

where the prime indicates the independent variable is outside the rocket Debye sheath.

This must be converted to a directional energy flux since the detectors view a limited portion of the 4\pi steradians. The thermal population experiences nonrelativistic speeds no larger than about 1 km/sec so

\[ \tilde{v}' = \left( \frac{2E'}{m} \right)^{1/2} \]

\[ J(\tilde{E}') = \left( \frac{2E'}{m} \right) f(\tilde{E}') \]

\[ J(\tilde{E}') = (2n/m) \left[ m/(2\pi kT) \right]^{3/2} E' \exp\left\{ -\left[ \left( \frac{2E'}{m} \right)^{1/2} \tilde{v}' \right]^2/(2kT/m) \right\} \]

The bulk is drifting due to \( E \) cross \( B \) (\( \tilde{v}_e \)) and of course has a ram velocity (\( \tilde{v}_r \)) due to the relative rocket - plasma motion.

\[ J(\tilde{E}') = (2n/m) \left[ m/(2\pi kT) \right]^{3/2} E' \exp\left\{ -\left[ \left( \frac{2E'}{m} \right)^{1/2} \tilde{v}' - \tilde{v}_e - \tilde{v}_r \right]^2/(2kT/m) \right\} \]

with the units [\#/\( \text{cm}^2 \text{ ster sec eV} \)].
The rocket's Debye sheath results in about a 1.5 volt negative potential compared to the ambient plasma. By the Liouville Theorem the quantity conserved is

\[ J(\bar{E})/E = J(\bar{E'})/E' \]

\[ E' = E - U \]

where \( U \) is the energy gained by passing through the potential drop. \( J(\bar{E}) \) is the energy flux arriving at the analyzer with \( E \) being the energy of selection. Compensating for the directionality and limited effective area (geometry factor) of the instrument, energy step, and time spent at each energy step, the final fitting equation results.

\[
COUNTS = J(\bar{E}) \cdot E \cdot \text{G.F.} \cdot \Delta t
\]

\[
COUNTS = (2n/m) \left( \frac{m}{2\pi kT} \right)^{3/2} E^2 \cdot \text{G.F.} \cdot \Delta t \cdot e^{-\left( \frac{(2(E-U)/m)^{1/2}v - v_r}{2kT/m} \right)^2}
\]

\[ \text{G.F.}= \text{geometry factor} \]
\[ E= \text{selection energy} \]
\[ \Delta t= \text{accumulation time} \]
\[ n= \text{density} \]
\[ m= \text{mass} \]
\[ k= \text{Boltzmann's constant} \]
\[ T= \text{temperature} \]
\[ U= \text{rocket potential} \]
\[ \vec{v}= \text{particle direction} \]
\[ \vec{v}_r= \text{velocity due to rocket motion} \]
\[ \vec{v}_o= \text{drift velocity} \]
\[ v_\text{th} = \text{thermal velocity} = 2kT/m \ [\text{cm/sec}] \]

with appropriate conversion factors to compensate for unit differences.

The unknown parameters to be determined are \( \bar{v}, U, T, \) and \( n \). The data are related to a Cartesian coordinate system based on geomagnetic coordinates (\( \hat{x} \) (North), \( \hat{y} \) (East), \( \hat{z} \) (along \(-\hat{B}\))). This system simplifies the vector combination in the numerator of the exponent.

The data is fitted to the above function by iteratively adjusting the six parameters (3 drifts, \( T, U, N \)) and finding the least of the sum of the squares of the differences of the data points to their corresponding calculated function points. The method has been developed by Marquardt (1963).

The following is explained in more depth in Appendix E. The least squares function is defined as

\[ \Phi = \sum_{i=1}^{\infty} (f_i - y_i)^2 / \sigma_i^2 \]

where \( f \) = calculated point
\( y \) = data point for \( n \) data points.
\( \sigma_i \) = weighting factor (assumed equal in this analysis)

The usual minimization (not Marquardt) is accomplished by taking the \( \partial \Phi / \partial p_k \), \( p_k \) is the \( k \)th parameter. If fitting to a power series \( f = a + bx + cx^2 + \ldots \) the basis vector is \( \vec{x} = (x^0, x^1, x^2, \text{etc}) \); \( f \) is linear in the parameters \( (a, b, c, \ldots) \) and \( \partial \Phi / \partial p_k \) results in an equation of the form

\[ \vec{A} \cdot \vec{p} = \vec{g} \]
\( \tilde{p} (p_1, p_2, p_3, \ldots) \) is solved for exactly by inverting \( \tilde{A} \).

For non-linear problems such an analytical solution for \( \tilde{p} \) does not exist. Two methods are generally used, the Taylor method and the gradient method. Both iteratively find not the set of parameters but the set of parameter changes that approach the minimum of \( \Phi \). The Taylor method does a Taylor expansion to first order of \( \Phi \) about the original parameters and maximizes the difference

\[
\Phi(\tilde{p} + \Delta \tilde{p}) - \Phi(\tilde{p})
\]

with respect to \( \Delta \tilde{p} \)

This produces an equation of the form

\[
\tilde{E} \cdot \Delta \tilde{p} = \tilde{h}
\]

The gradient produces \( \Delta \tilde{p} (\Delta p_1, \Delta p_2, \ldots, \Delta p_j) \) from the negative gradient of the surface \( \phi(p) \). Specified length steps are taken until \( |\Delta \tilde{p}| \) is small enough.

The Marquardt method uses a combination of the two. At times of transient slopes of the \( \Phi \) surface, the Taylor expansion to first order may err representing the surface. The gradient method will always head toward a minimum, however, this method only determines the direction; the step size must be specified. The step cannot be very large, or there will be problems tracking the negative gradient. The gradient method is slow but accurate. In the vicinity of the minimum of \( \Phi \) the slopes are shallow, highly correlated parameters often producing troughs. The Taylor method is accurate, and the step size can be large (i.e.
faster). The program must calculate \( f_i \cdot \Phi(\vec{p}) \cdot \Phi(\vec{p} + \Delta \vec{p}) \), etc. for thousands of data points for each fit. There are a possible few hundred fits per flight, and efficient algorithms are essential.

The worst problem encountered has been noisy data, which distorts the \( \Phi \) surface. Exponentials are extremely non-linear and very difficult to fit. The altered Maxwellian has highly correlated parameters which causes conflicting negative gradient directions, necessitating the bounding of parameters. These limits as well as the initial parameter guesses were chosen by viewing the raw data spectra in the convenient manners discussed above.

Care must be taken if the flow is in between two detector scans. If the statistics of one analyser are not good, the fit will converge on the better distorting the density and temperature. Figure 3.6 is a comparison of parameter generated values to the data. Figure 3.7 displays the reproduction of data from the fit parameters for a spin period when both detectors were active.

Statistical analysis is also provided as shown in Table 3.1. The correlation matrix is from a calculation of relationships between any two parameters. Also provided from a comparison of the collection of data points to the calculated points weighted by the function variation with respect to each parameter is the standard deviation (SIGMA \( \sigma \)). The confidence levels can be quoted as

\[
P_{\%} \pm T \cdot \sigma(P_{\%})
\]
Flight 18:1005
T = 255 sec

- Distribution of ions vs Energy
- Least Squares Fit

Temp. = 1630° K
Potential = 1.57 v
Density = 0.64 x 10^4 /cc

Figure 3.6 Fit Generated Energy Spectrum vs. Data
Figure 3.7 Spin Comparison of Energy Spectra vs. Fit
E FIELDS 5-270 FINAL FIT

CORRELATION MATRIX CORMAT(I,J)

\begin{align*}
VC N & \quad 1.0000 \quad -0.5751 \quad -0.8458 \quad 0.3385 \quad 0.1762 \quad 0.8233 \\
V0 E & \quad -0.5751 \quad 1.0000 \quad 0.3535 \quad -0.3226 \quad -0.1826 \quad -0.5293 \\
V0 B & \quad -0.8458 \quad 0.3535 \quad 1.0000 \quad -0.2246 \quad -0.1020 \quad -0.7372 \\
TEMP & \quad 0.3385 \quad -0.3226 \quad -0.2246 \quad 1.0000 \quad -0.2150 \quad 0.5714 \\
U & \quad 0.1762 \quad -0.1826 \quad -0.1020 \quad -0.2150 \quad 1.0000 \quad -0.0414 \\
DENS & \quad 0.8233 \quad -0.5293 \quad -0.7372 \quad 0.6714 \quad -0.0414 \quad 1.0000 \\
\end{align*}

CONFIDENCE LEVEL= 95.00 T = 1.96

CONFIDENCE LEVEL= 50.00 T = 0.67

PARAMETERS

\begin{align*}
V0 N & \quad -2.80301603E-01 \quad 3.29053584E-02 \quad 8.52 \quad 95. \\
V0 E & \quad 7.63578437E-01 \quad 1.63536740E-02 \quad 48.72 \quad 95. \\
V0 B & \quad -3.40734188E-01 \quad 3.45371207E-02 \quad 9.87 \quad 95. \\
TEMP & \quad 1.36769916E+00 \quad 2.84457759E-02 \quad 48.07 \quad 95. \\
U & \quad 1.72357216E+00 \quad 1.8544973E-03 \quad 929.43 \quad 95. \\
DENS & \quad 2.90212906E+01 \quad 4.19723217E-02 \quad 7.03 \quad 95. \\
\end{align*}

Table 3.1 GLSWS Statistical Analysis
where $T = 1.96$ for level 95. These are the parameter errors to be quoted, that is, with 95% confidence $p_k$ is between $p_k + T\sigma$ and $p_k - T\sigma$.

Note the low correlation between velocities and the rocket potential. Referring to Figure 3.6, these are the two quantities that determine the location of the peak in energy. This provides confidence that the velocity contribution may be separated from the rocket potential contribution.
CHAPTER IV

DRIFT DETECTOR RESULTS

4.1 Electric Field Vectors

Reported here are the $\vec{E}$ field results of the two auroral sounding rockets. The products of the fitting routine are $\vec{V}$[drift], rocket potential, ion temperature and ion density. As stated before the electric field can be obtained from the drift velocity.

$$\vec{V} = c (\vec{E} \times \vec{B}) / B^2 \quad \leftarrow \times \vec{B}$$

$$\vec{V} \times \vec{B} = c ((\vec{E} \times \vec{B}) \times \vec{B}) / B^2$$

$$\vec{V} \times \vec{B} = cB(\vec{E} \cdot \vec{B}) / B^2 - cE(B^2) / B^2$$

we are referring to electric fields perpendicular to $\vec{B}$ so the first term is zero. Therefore

$$\vec{E} = -(\vec{V} \times \vec{B}) / c$$

To evaluate this

$$|\vec{V}| = 10^8 \ |\vec{E}(v/cm) | /|\vec{B}(\text{gauss})| \ [\text{cm/sec}]$$

at 200 km above Andoya, Norway $|\vec{B}| = 0.48$ gauss
\[ |\vec{v}| = 2.08 \times 10^8 |\vec{E}(v/cm)| \]
\[ |\vec{v}(cm/sec)| = 2.08 \times 10^3 |\vec{E}(mv/m)| \]
\[ |\vec{v}(m/sec)| = 20.8 |\vec{E}(mv/m)| \]
or \[ |\vec{E}| = |\vec{v}(m/sec)|/20.8 \]

The results of the two flights are shown in Figures 4.1 and 4.2. The vertical axis represents flight time. The vectors represent the electric field vectors derived from 2 sec fits to the data (-2000 data points). The scale is 1 inch = 50 mv/m electric field magnitude. The direction is relative to geomagnetic coordinates.

The first figure displays the results from flight 18:1005. This flight traversed a quiet arc in the evening sector. This sector typically has stable diffuse and discrete arc aurora. The direction of the electric field reverses at about 200 seconds. Before reversal the field was \(-25\) mv/m north while after it was twice that value south. The field did not decrease appreciably during the arc as is often noticed (Kintner, 1974, Evans, 1977). The accompanying set of vectors is from the University of Minnesota's dual probe experiment. These results also agreed well with the STARE radar (Cahill, 1978b). Radar measurements had inconclusive results through the arc \(\rightarrow\) 200 sec but do show the noted reversal to be spatial.

Figure 4.2 shows the corresponding information for flight 18:1004. This was near magnetic local midnight and during a break-up phase. The field was quickly changing but generally in the west direction, at \(-50\) mv/m, as was also
Figure 4.1 Electric Field Vectors (Flight 18:1005)
Figure 4.2 Electric Field Vectors (Flight 18:1004)
noted by STARE radar and the University of Minnesota. Ground magnetometers registered a westward electrojet current.

4.2 Mass analysis

Both flight packages contained a MAG-EFM instrument redundant to the 90 EFM unit. As was stated, this analyzer provided a relative measurement of NO⁺ and O⁺ in the ionosphere. Apparently the magnetic section of flight 18:1004 was detuned during the payload's vibration test. Spectra resulted but counting rates did not compare with the duplicate 90 degree EFM unit. Discussed are the results of flight 18:1005.

In the region of the spectral peak, the data were integrated over energy. The peak values of a spin were taken for both mass selection times. Assuming nitric oxide and atomic oxygen ions to be the two major constituents above 200 km, the fluxes of ions were compared during these different times. Figure 4.3 is a plot of the fraction of NO⁺ to the total ion count vs. flight time. The line is a fit to these data points excluding those around the arc center (140s.-155s.). The fit shows the general altitude decrease characteristic of NO⁺. Admittedly the ions of mass 30 amu could well have been O⁺ as this instrument could not resolve a difference of 2 amu. As will be discussed below, the increased fraction near the arc is most likely NO⁺. As seen in Figure 4.4a (Barth, 1973), O⁺ is the dominant ion
Fractional Density \( \frac{\text{NO}^+}{\text{Total}} \)

Flight 18:1005

MAG-EFM Mass Composition

Figure 4.3 Mass Analysis Results (Flight 18:1005)
concentration. This is a profile of the daytime ion proportions. However Figure 4.4b (Barth, 1973), indicating the decay of the F-region, shows that a launch of 20:18 LT (flight 18:1005) retains a good fraction of the daytime ionization. Rees and Walker (1968) show NO⁺ dominant to equal NO⁺-O⁺ fractions for altitude ranges 180 km-250 km for auroral arc conditions.

To justify the higher NO⁺ values through the arc (see Figure 4.3), Gerard and Barth (1977) discuss the steady state NO⁺/O₂⁺ ratio of about 4 during sporadic auroral energy deposition. Figure 4.5a shows the growth and decay of NO⁺ and O⁺ for 110 km. Figure 4.5b indicates similar growth for 150 km. The increased nitric oxide production is mainly a result of the increase in the density of various nitrogen species. The dissociative recombination

\[
\text{NO}^+ + e^- \rightarrow \text{N}(2D,4S) + \text{O}
\]

produces more 2D states than 4S states (1.6 times). The N(2D) with molecular oxygen is the main producer of nitric oxide molecules.

\[
\text{N}(2D) + \text{O}_2 \rightarrow \text{NO} + \text{O}
\]

Since N(4S) is the dominant destruction mechanism,

\[
\text{N}(4S) + \text{NO} \rightarrow \text{N}_2 + \text{O}
\]

the decay rate of NO molecules is reduced leaving the ionosphere richer in NO. Subsequent electron precipitation
Figure 4.4 Ionospheric Composition and Nighttime Decay from Bauer (1973)
Figure 4.5  NO\(^+\) Production and Decay from Gerard and Barth (1977)
results in increased NO$^\ast$.

As stated previously the primary impetus for this section of the research was not ionospheric chemistry but to observe the effects on the fitting routines. Table 4.1 is the compilation of fit parameters using two methods. The first fits the data to a Maxwellian whose mass is assumed to consist entirely of O$^\ast$. The second fits to the sum of two Maxwellsians with the composition ratio determined by Figure 4.3.

Table 4.1

<table>
<thead>
<tr>
<th>METHOD</th>
<th>AMU 30=0.35</th>
<th>ALL O$^\ast$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v#N$ [m/s]</td>
<td>-256 ±60</td>
<td>-289 ±64</td>
</tr>
<tr>
<td>$v#E$ [m/s]</td>
<td>825 ±34</td>
<td>919 ±34</td>
</tr>
<tr>
<td>$v#B$ [m/s]</td>
<td>-257 ±66</td>
<td>-361 ±86</td>
</tr>
<tr>
<td>TEMP [K]</td>
<td>1520 ±66</td>
<td>1670 ±72</td>
</tr>
<tr>
<td>$U [V]$</td>
<td>1.72 ±0.004</td>
<td>1.72 ±0.006</td>
</tr>
<tr>
<td>DENS [#/cc]</td>
<td>3.20x10$^3$ ±800</td>
<td>3.07x10$^3$ ±200</td>
</tr>
</tbody>
</table>

Generally about a 10%-15% error between the two methods is common which is as good as the error in the individual parameter results. The conclusion is that mass analysis is not essential to the determination of electric fields for this technique.
4.3 Supplemental Results

The bulk flow along the field line along with the ion density results in the field aligned current shown in Figure 4.6 for flight 18:1005. This has all been current out of the ionosphere. Comparison to typical ion flows parallel to \( \mathbf{B} \) seen in Figure 4.7 of flight 18:1004 shows unusual field aligned currents near the flow reversal region or polar cap boundary. Additional results of ion temperature are shown in Figures 4.8 and 4.9. Figure 4.8 of 18:1005 shows heating in the vicinity of the arc and preceding the boundary. Figure 4.9 of 18:1004 displays more uniform though higher temperatures than normal ambient ionospheric conditions. This result is not unusual for break-up phase aurora.

Further analysis was done in an attempt to establish the parallel flow velocity. A twelve parameter fit to two drifting Maxwellians was employed. The statistical analysis accompanying these answers was poor, and this attempt was abandoned. The conclusion is that the bulk thermal background of ions are moving together with one velocity.

Statistical analysis was good for all answers presented, that is, the interval estimates of the parameters was 20% or better. There is some apparent general disagreement in magnitude between the dual probe measurements and this experiment. The error in combining the two flow parameters is about 28% maximum. No knowledge of the error in the dual probe is available at this time, however, the above errors could account for the discrepancy.
Figure 4.6 Field Aligned Current (Flight 18:1005)
Figure 4.7 Field Aligned Current (Flight 18:1004)
Figure 4.8 Ion Temperature (Flight 18:1005)
Figure 4.9 Ion Temperature (Flight 18:1004)
Another cause of discrepancy is integration time. Both dual probe measurements and STARE radar integrated over 20 seconds, whereas these results were taken every two seconds. If the direction changes quickly, as seen in either Figure 4.1 or 4.2, the average over several vectors will reduce the magnitude. This error will be found in all methods, since dual probes must deal with velocities relative to the rocket (measuring $E'$ in a moving reference frame) and radar also deals with drift velocities.

The method and instrument have been proven reliable as seen by duplicate experiments in this campaign. Further improvements have been considered for future campaigns. An auroral flight is scheduled for Winter 79-80 that will have an ion drift detector with four analyzers positioned at $40^\circ$-$70^\circ$-$110^\circ$-$140^\circ$ to the spin axis. There is no substitute for large quantities of data in short amounts of time. Essentially these four analyzers will fill in the pitch angle coverage in one spin providing improved counting statistics and better temperature and density measurements.

Counting rates have been a problem given the variable densities possible in auroral flights (3 orders of magnitude) and fixed analyzer geometry factors. Extensive lab testing has indicated the channeltron-preamp to have a dynamic range of two orders of magnitude. Improved preamp design hopefully will increase this range, as channeltrons should have megahertz counting capabilities.
Results tend to justify the instruments ability and accuracy. Of course the goal of any project is the understanding of scientific principles. Having had the good fortune of nature, flight 18:1005 has delivered a classic pre-midnight auroral oval - polar cap environment. Figure 4.10 is the Andoya all sky photograph of this discrete arc. Although the intensity of light was low (<10KR), the arc was steady. Figure 4.11 is an electron spectrogram indicating the passage of an arc from 135 seconds to 160 seconds flight time. Electron energy is the ordinate. A region of high electron energies bordered by lower energies as seen in this figure usually accompanies auroral displays. They are referred to as inverted V events.
Figure 4.10 Andoya All Sky Photograph (Flight 18:1005)  
19:23 UT Jan. 23, 1977
Figure 4.11 Flight 18:1005 0-20 keV Electron Spectrogram
CHAPTER V

INTERPRETATION

5.1 Introduction To Magnetospheric Structure

A discussion of the above results would be incomplete without an overview of magnetospheric dynamics. Flight 18:1005 traversed a stable quiet auroral arc and has presented evidence of entering the polar cap region (~68°-90°). This flight typifies the classic evening horizontal ionospheric current systems, Birkeland field-aligned currents and auroral displays.

Since the postulation of the solar wind in the 1930's by Chapman and the succeeding discovery 30 years later, magnetospheric structure has been learned. Continuous experimental investigation of current systems and fields is slowly eroding the incompleteness and dispute surrounding a unifying theory.

As explained by Radcliffe (1972) the solar wind is the sun's coronal plasma, populated mostly by protons and electrons, which is controlled by pressure gradients and gravity. The gravitational potential goes as 1/r whereas
the pressure gradients are controlled by temperature and do not fall off as quickly. Hence the plasma is accelerated outwards. As is shown in Appendix F, magnetic flux accompanying a system of charged particles can be interpreted as being "frozen" to the streaming plasma if infinite conductivity is assumed. The magnetic field from the sun arriving near the earth has a strength of about 5 gamma.

As an introduction to magnetospheric structure consider the earth's magnetic dipole field. Superimposing a northward directed interplanetary field on this dipole results in the "closed" configuration of Figure 5.1b. The term "closed" refers to the fact that there is no connection between earth and interplanetary field lines. If the interplanetary field is instead directed southward, Figure 5.1a would result forming a circular region at each pole marking the boundary between "open" field lines (those extending into the solar wind) and "closed" ones (those clearly joining the north and south poles). If to this static situation is imposed the relative velocity between the earth and the solar wind, Figure 5.1c will result. The magnetic pressure of the magnetosphere balances the particle pressure of the solar wind resulting in an aerodynamic shape. Another manifestation of flux being constant with a given set of particles allows individual field lines to successively occupy positions 1, 2, 3, etc. of Figure 5.1c.
Figure 5.1 Magnetosphere Construction from Ratcliffe (1972)
The exact connection between the solar wind and the magnetosphere is in dispute. The controversy is whether to depict the magnetosphere as "open" or "closed". The differences between the closed and open magnetospheres are displayed in Figure 5.2, a north-south, noon-midnight plane view. If "open", the method by which the magnetic fields merge and particles and fields exchange energy is questionable. However, since the flux is frozen in the solar wind plasma and particles are free to move along field lines, the transfer of particles, momentum and energy is immediate in the open model. Motion across the high latitude polar cap is due to the solar wind motion. Any electric field configuration resulting from this motion is mapped directly along magnetic field lines. Considering the time scale of plasma motion, the $\mathbf{B}$ field is assumed static. Therefore

$$\nabla \times \mathbf{E} = 0$$

or $\mathbf{E}$ may be written

$$\mathbf{E} = - \nabla \phi$$

Charged particles feel only forces perpendicular to $\mathbf{B}$, thus conductivity along $\mathbf{B}$ is infinite.

$$\mathbf{E} \cdot \mathbf{B} = 0$$
$$\nabla \phi \cdot \mathbf{B} = 0$$
Figure 5.2 Closed vs Open Magnetospheres from a) Hill (1977) and b) Akasofu (1977)
Since \( \vec{B} \) is not equal to zero the field lines are also equipotentials (Stern, 1976).

There is mounting evidence (Hill, 1977, Akasofu, 1977 and Review) for the open configuration. Solar flare particles have been detected in the earth's ionosphere. Akasofu has shown in Figure 5.3 a connection between the interplanetary \( B_x \) component (north-south) and the size of the auroral oval. This figure begins with a view of the polar cap from above. It expands and shrinks according to the value of the north-south interplanetary field component \( B_x \). Akasofu also notes that the polar cap never shrinks to a point implying that the magnetosphere is permanently open. Also correlated are the \( B_y \) component (east-west) and the polar cap convection asymmetries, shown in Figure 5.3. These patterns are the flow patterns across the polar cap showing symmetry for \( B_y = 0 \), dusk favored flows for \( -B_y \), etc. Recent work by Potemra (1978) (Figure 5.4) has shown similar asymmetry associated with the interplanetary field component \( B_y \).

The antisunward section of the magnetosphere called the magnetotail may be viewed with equivalent current systems (Akasofu, Review) as Figure 5.5. Two solenoidal currents surround the tail joining in the equatorial plane and are responsible for the observed magnetic field structure. The tail may stretch to 1000's of earth radii. The magnetotail maps to the polar cap latitudes (~68°-90°). The auroral oval has been statistically shown to coincide with the polar cap boundary (Feldstein, 1973, Akasofu, 1977, Kamide, 1977,
Figure 5.3  Interplanetary Magnetic Field and the Auroral Oval from Akasofu (review)
Figure 5.4 Interplanetary Magnetic Field and Polar Cap Asymmetries from Potemra (1978)
Figure 5.5 Current System Interpretation of the Magnetosphere from Akasofu (review)
Iijima, 1977). This is a band of -5° latitudinal width encircling the north and south poles, usually shifted off the pole in the antisunward direction by about 3°.

Figure 5.6 is an idealized picture of the magnetosphere and associated current systems (Hulqvist, 1978). The magnetosheath separates the magnetosphere from the solar wind. The magnetopause is the outer boundary of the magnetosphere containing the currents necessary to account for the structure. The tail current was discussed above. The ring current arises from the solar wind plasma gyrating about $\mathbf{B}$ as it approaches the earth’s magnetic field. The direction of the motion depends on charge, protons diverted to the dusk side and electrons diverted to the dawn side. The resultant field from this current cancels the interplanetary field outside the magnetosphere and strengthens the field inside. The cleft or cusp region is the area of field line connection. The plasma sheet contains the tail plasma and lies on the boundary between the magnetotail and closed field lines. The magnetotail is generally accepted to contain open field lines.

There, of course, must be energy and momentum transfer in both the open and closed cases to cover the dissipation in the ionosphere. Such transfer in the closed model is accomplished by gradient or curvature drifts and diffusion caused by wave-particle interactions in the magnetosheath. Viscous momentum transfer creates the equatorial flow patterns depicted in Figure 5.7, proposed by Axford and
Figure 5.6 Idealized Magnetosphere and Current Structure from Hulqvist (review)
Figure 5.7 Axford and Hines (1961) Convection Patterns
Hines (1961). If there is diffusion, the conductivity must not be infinite at the magnetosheath, and "frozen-in-flux" is violated. There is a problem predicting the large potential drop across the polar cap with only equatorial flow patterns. The average change in potential observed is about 50 kV with one measurement of 225 kV (Hill, 1977). The maximum available from diffusion estimates is 10 kV.

For the open field model, the field lines are directly connected to the sun's field as shown in Figure 5.2b. The boundary is not as distinct in this case, but the particle and energy transfer is easy. Magnetic merging theories play a crucial role in this model. The open model purposes magnetic field merging in the dayside magnetosheath as well as across the tail equatorial plane. If the sunward merging region is taken wide (~15 Re), the potential drop due to the motion of the solar wind is about 200 kV across the polar cap. This region would map into the cleft region of the ionosphere as seen in Figure 5.6.

5.2 Ionospheric Current Systems

Study of the polar cap boundary region investigates a region of varied influence. The boundary is generally accepted to be the boundary of open and closed field lines, containing most of the currents aligned with the magnetic field and consequently the auroral displays. The motion of ionized particles near the lower altitudes in the ionosphere
is dominated by collisions with neutrals. Table 5.1 (Evans, 1971) summarizes the collision and gyrofrequencies as a function of altitude for electrons and ions.

Table 5.1

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>100 km</th>
<th>200 km</th>
<th>300 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ion-neutral</td>
<td>5800</td>
<td>4.1</td>
<td>0.4</td>
</tr>
<tr>
<td>collision freq.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electron-neutral</td>
<td>92,000</td>
<td>130</td>
<td>11</td>
</tr>
<tr>
<td>collision freq.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ion</td>
<td>160</td>
<td>190</td>
<td>210</td>
</tr>
<tr>
<td>gyrofrequency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electron gyrofrequency</td>
<td>$8.4 \times 10^6$</td>
<td>$8.0 \times 10^6$</td>
<td>$8.0 \times 10^6$</td>
</tr>
</tbody>
</table>

Consider low latitude current systems. At low altitudes collisions begin to dominate and particles begin to rotate with the earth, termed corotation. Since this motion is at times perpendicular to $\vec{B}$ there is an induced electric field $\mathbf{E} = (-\mathbf{V} \times \vec{B})$. This field will then generate currents that will close according to the most favorable terms in the conductivity tensor. Conductivity in the horizontal plane of the earth is comparatively large near the equator as is the velocity. The current due to this atmospheric dynamo has been named the equatorial electrojet.
Across the polar cap boundary into the high latitude region, the solar wind motion or corresponding electric field vector is mapped directly to the ionosphere. The motion of particles in the distant magnetosphere is due to the circulatory convection discussed by Axford and Hines (1961) and shown in Figure 5.7. Similar patterns exist in the open model. Anti-sunward convection is maintained at the edges of the magnetosphere with sunward convection along the middle. Figure 5.8 displays the mapping of this motion to the polar cap. This convection pattern is idealized in Figure 5.9 (Stern, 1976). This figure looks down on the polar cap viewing the antisunward flow across the cap and the return flow outside the cap border. Since ions encounter equal collision frequency versus gyrofrequency before electrons, the electrons continue to execute this motion down to altitudes of 90-110 km. Conventional current flows in the opposite direction. These are the electrojet currents.

Accounting for the effects of corotation will result in ionospheric current systems shown in Figure 5.10a by Siguiria (1965) and improved by Langel (1974) (Figure 5.10b). This general pattern was also shown by Hanson (1978). Note the similarity to the convection flows of Figure 5.4. In Figure 5.10a the heavy currents on the tail side are the auroral electrojets. The overlap of the eastward electrojet with the westward electrojet in the dusk sector is the Harang Discontinuity.
Figure 5.8 Equatorial Convection Mapping to the Polar Cap from Ratcliffe (1972)
Figure 5.9  Idealized Polar Cap Convection Patterns from Stern (1976)
Figure 5.10 Observed Polar Cap Convection from a) Sigiura (1965) with corrections b) Langel (1974)
It has been the dusk sector of the eastward electrojet at about 21:18 MLT that has been investigated by Flight 18:1005. Referring back to Figure 4.1 the electric field was measured from low to high latitude across the quiet arc from 135-155 seconds. The field was generally directed poleward south of the reversal region. An $\mathbf{E} \times \mathbf{B}$ drift from this type of field direction would give plasma flow to the west or an eastward electrojet current. The flight before 200 seconds was in the sunward convection region. Ground magnetometers (Figure 5.11) show a positive northward magnetic perturbation as well as negative z going to positive indicating that an eastward horizontal current sheet had passed overhead. The next section will pinpoint this electrojet current. If the polar cap was entered the plasma flow should reverse from sunward to anti-sunward convection. At this local time this would be generally an eastward flow resulting from an equatorward electric field. Figure 4.1 indicates such a field after 200 seconds flight time. Andoya rocket range is at 67 degrees magnetic invariant latitude, with the rocket flight traversing two more degrees. The Kp index was low (-1>-2) indicating normal oval conditions. Figure 5.10, Kamide(1977), or Iijima(1978), indicate these latitudes are proper locations for crossing into the polar cap. Generally high equatorward electric fields in the cap may indicate crossing into the westward intrusion of the strong westward electrojet, however, this would require a local time more toward
Figure 5.11 Andoya Ground Magnetometer (Flight 18:1005)
midnight. Gurnett (1972) shows polar observations from Injun 5 (Figure 5.12) showing similar flow reversals at this latitude and time. Note also that magnitudes are similar. The magnitudes reported in this work are upwards of 1-1.5 km/sec, which are some of the larger values observed. Gurnett states that increasing flow magnitudes often accompany this boundary crossing. Additional support that the westward electrojet was not penetrated is the low conductivities poleward of the reversal region. Electrojets generally lie below auroral arcs due to the increased ionization.

There is no doubt that this flight has proven valuable documentation of this reversal region. The inherent fast accumulation of rocket data has provided fine structure analysis of this pre-midnight classic arc. The evidence of this polar cap crossing secures this experiment with respect to known overall ionospheric position and structure. A continuous problem with single craft experiments analyzing dynamic processes is the distinction between spatial and temporal events. In this case gross features can be equated to those provided by satellite measurements. Further analysis should be made using the high resolution measurements presented here.
Figure 5.12 Flow Reversal Data from Gurnett (1972)
5.3 Field Aligned Currents

As first noted by Birkeland (1908), the ionosphere does not have a two-dimensional horizontal current system. Birkeland currents or currents along the magnetic field connect the ionosphere to the distant tail. The high energy electron contribution to these currents are responsible directly for day and night auroral displays. Auroral light is the result of high energy electrons bombarding the atmosphere ionizing nitrogen and oxygen into energetic states. The relaxation of these ions as well as recombination gives off the various wavelengths of light observed. This field aligned current energy is indirectly responsible for the horizontal current system by creating the night side ionosphere. This process creates the conductivity which allows the horizontal flow. Field aligned precipitating electron currents have been exhaustively studied (Choy, 1971, Arnoldy, 1974, Whalen and McDiarmid, 1972, Evans, 1977, Iijima and Potemra, 1978, Kamide and Rostoker [Review], Zmuda and Armstrong, 1974) and correlated with auroral arcs and polar cap boundaries. The majority of these authors have observed electron fluxes into the ionosphere from the plasma sheet or tail boundary whose energy is commensurate with the auroral intensity. It has been well established that evening sector Birkeland currents are outward poleward of the oval and inward at the equatorward edge. The outward currents correspond to discrete, higher intensity aurora while equatorward auroral
events are largely diffuse and barely visible. The first are more widely studied due to the reluctance of project scientists to fire rockets into black skies.

Rostoker and Bostrom (1976) have suggested mechanisms for field aligned current systems accompanying the horizontal electrojet currents. Mapping the electric field corresponding to the flow of Figure 5.9 into the tail Figure 5.13 (Bostrom, 1975) will result. This field crossed into B will cause neutral sheet particles to drift to the outer edges of the tail. This flow, which is perpendicular to B, allows the Lorentz force to separate the charges. This sets up currents in the tail opposite to the mapped E field thus constituting a generator. Birkeland currents now flow out of the ionosphere on the poleward edge of the auroral oval in the evening sector. Current back in is on the equatorward edge as shown in Figure 5.14 (Bostrom, 1975).

Akasofu (1977) has suggested that the dawn to dusk neutral sheet current of Figure 5.5 or 5.6 be disrupted by a current instability. This purposeal requires current completion through the ionospheric electrojets from dawn to dusk. This justifies the net quiet time inward current imbalance in the dawn sector (Iijima, 1978), Sigiura and Potemra, 1976) feeding the strong westward electrojet.

Kamide and Rostoker (1977) have investigated the complex connection of Birkeland currents horizontally. The observations were mainly from Triad magnetometer data and DMSP satellite auroral imagery and electron data. This work
Figure 5.13 Tail Field Aligned Current Generator from Rostoker (1976)
Figure 5.14 Field Aligned Currents from Bostrom (1975)
has also found larger inward current in the morning sector. Also investigated are the variety of Hall and Pederson currents.

Hall currents are the currents produced by the $\mathbf{E} \times \mathbf{B}$ drift of electrons without ions discussed in Section 5.1. Pedersen currents are conventional currents in the direction of an electric field due to particle collisions. Both of these conductivities were calculated according to Cahill, Arnoldy, Taylor (1978), Evans (1977), and Rees (1973) from the energy flux of electrons into the oval. These are presented in Figures 5.15 and 5.16 respectively. The conductivity increased through the arc. This is expected from the increased energy deposition.

The horizontal current system is given by

$$\mathbf{I} = \bar{\Sigma} \cdot \mathbf{E}$$

where

$$\bar{\Sigma} = \begin{bmatrix} \Sigma_p & \Sigma_h \\ -\Sigma_h & \Sigma_p \end{bmatrix}$$

is the horizontal conductivity tensor employing height-integrated Hall and Pederson conductivities. The currents of the present coordinate system of geomagnetic north, east, and along $\mathbf{B}$ are

$$I_N = \Sigma_p E_N + \Sigma_h E_E$$

$$I_E = -\Sigma_h E_N + \Sigma_p E_E$$

The Hall current would of course exist mostly at the 110 km altitude because of ion collisions. In modeling auroral
Figure 5.15 Hall Conductivity (Flight 18:1005)
Figure 5.16 Pedersen Conductivity (Flight 18:1005)
events the general procedure has been to extend the arc to infinity in the east and west direction. This is reasonable considering the long structure of most pre-midnight quiet arcs. The east-west electrojet currents would then be considered continuous in the range of analysis. Neither flight went east or west more than 50 km.

Current is analysed by assuming the local space charge buildups are dissipated quickly in the time scale of the arc fluctuations. Thus the total current is divergence free

\[ \nabla \cdot \vec{J} = 0 \]

Assuming infinite extent in the east-west direction, the electrojet currents are divergence free in themselves. These currents are displayed in Figure 5.17 showing an eastward electrojet or sunward particle convection equatorward of the reversal region. Poleward of this boundary there is westward current. Information from a proton magnetometer on board showed a discontinuity of 60 directly beneath this current sheet. Assuming an infinite sheet is responsible for this perturbation, a linear current density of 0.095 amp/m is required. The average current density across the high conductivity region was 0.26 amp/m from Figure 5.17. Calculated current from electric fields and conductivities is high by a factor of ~2.5. Approximately this same disagreement will appear below in the field aligned current calculation.
Figure 5.17  East Current (Flight 18:1005)
Figure 5.18 shows the northward current systems due mostly to the Pedersen currents. These are not divergence free especially near the reversal region as well as the arc and must be completed with field aligned currents. Figure 5.19 shows the current due to suprathermal electrons and thermal ions. From the divergence in the horizontal north-south direction attributed to field aligned currents

$$\partial I_n / \partial B + \partial I_n / \partial N = 0$$

assuming $E_n$ and $E_f$ are height independent. This follows from the assumption of magnetic field lines being equipotentials. Integrating then produces

$$I_{fa} = - \int_{h}^{b} \partial I_n / \partial N = - \partial / \partial N [ \int_{h}^{b} \sigma_p E_n + \int_{h}^{b} \sigma_n E_f ] dB$$

Height integrated conductivities are defined as

$$\Sigma_p = \int_{h}^{b} \sigma_p dz$$

where $dz = -dB$

thus

$$I_{fa} = \partial / \partial N \Sigma_p E_n + \partial / \partial N \Sigma_f E_f$$

Figure 5.19b is a plot of this calculation generated from

$$\partial / \partial N \Sigma_p E_n = \partial / \partial N \Sigma_f E_f = \Delta I_n / \Delta x_n$$

The general character of the measured currents is correct except for the positive current before this arc. The current attained from $\vec{V} \cdot \vec{J} = 0$ through the arc disagrees with the total field aligned current by about a factor of three considering the scale differences of Figure 5.19. Evans (1977) as well as Cahill (1978) have had similar
Figure 5.18 North Current (Flight 18:1005)
Figure 5.19 Comparison of Divergence of North Current with Measured Field-Aligned Current
discrepancies. All current problems could be resolved by attributing the missing components to thermal electrons. This is generally accepted and reasonable given their high mobility.

Possibly an alternative, although this would presuppose local parallel potential drops, may be the invalidity of the time invariant charge density assumption. The electric fields are mapped from the measured height (~200 km near the arc) down through the arc to the high conductivity regions (110-130 km). If there is local charge build up, these fields will change as will the currents. Parallel electric fields are not usually measured, however, Mozer(1970) found a parallel field of up to 20 mv/m. An adjustment to the southern edge to lower the potential with space charge (lower by 2/3) would require a steady downward parallel electric field of 2 mv/m from 200 km->120 km. The most likely conclusion is still the unmeasured current carriers since other findings (Cahill,1978) seem to correlate local magnetometer readings well with $\nabla \cdot J = 0$. Further analysis was done at various sections of the ionosphere. The conductivity was integrated in layers shown from 110-200 km in Figures 5.20-5.24. These figures show the east and north currents. Note that much of the eastward electrojet is carried higher than 110 km the altitude of maximum Hall conductivity. Maximum Pedersen conductivity is at 130 km and is carrying some of this current. Kamide's Triad analysis has also noted a westward electrojet dependent on
Figure 5.20 Currents, North and East, at 110 km.
Figure 5.21  Currents, North and East, at 130 km.
Figure 5.22 Currents, North and East, at 150 km.
Figure 5.24 Currents, North and East, at 180 km.
Figure 5.24 Currents, North and East, at 220 km.
both conductivities for the morning sector.

This has been a classic quiet auroral oval event similar to that encountered by Evans et al. and Maynard et al. (1977). Both cases have shown similar results with the outward current observed first hand and consistent with overall current systems. Inward currents were not observed. Consistency with theories as Rostoker's (1976) is laying solid foundations for ambient magnetospheric processes. It is only with confidence of this ground work that more explosive energy-exchange mechanisms can be investigated.

The major point of this thesis is the validity of the technique as evidenced from general agreement with the dual probe and STARE radar experiments. The by-products of temperature, density, rocket potential and field aligned, current are valuable parameters. Specific to this flight the basic spatial structure has been determined. Some problems still lie with absolute quantitative agreement both in E-field magnitude and \( \nabla \cdot \mathbf{J} = 0 \). The dominant measured current flux is due to low energy (<500 eV) electrons (Arnoldy, 1977) and some from the thermal ion background. This electron contribution is substantiated by Peterson (1977) in wide statistical AE-C and AE-D satellite coverage. These particles most likely have only received energy in the distant tail, perhaps, as Rostoker's braking of the magnetosheath movement of neutral sheet particles.
The high energy auroral producing electron precipitation most likely has experienced an additional wave-particle interaction in the upper ionosphere (<10,000 km). This area is an entirely different concern but not separate. As proper data reduction required logical sequence from instrument calibration and response through advantageous raw count representation to final parameter fits, so the study of complicated wave-particle energy exchange study requires firm understanding of steady state phenomena. Flight 18:1005 has provided a small piece of this vast quantity of information. Current systems, both ionospheric and field aligned have been established. Fields have been mapped and high energy particles measured, accounting for the ionospheric environment that has supported a quiet discrete arc. Instrumentation as flown here should be a basis for observing environmental conditions in any attempt of study of the higher altitude interaction region.
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APPENDIX A

Guiding Center Motion

Guiding center motion in the magnetosphere is dominated by the ambient magnetic field $\vec{B}$ and various forces. The equation of motion for a perpendicular force is (Chen, 1974)

$$\frac{mdv_x}{dt} = F_x + q(v_x \times \vec{B})/c \quad \text{(cgs)}$$

Assume $F_x = F = F_x$, $\vec{B} \rightarrow = B_z$, symmetry considerations implying no difference in the perpendicular plane. The above in components is

$$\frac{dv_x}{dt} = \frac{F}{m} + qv_y B/mc$$
$$\frac{dv_y}{dt} = -\frac{F}{mc}$$

If $F$ is static, differentiating produces

$$\frac{d^2v_x}{dt^2} = \frac{qB}{mc} \frac{dv_x}{dt} = -(qB/mc)^2 v_x$$
$$\frac{d^2v_y}{dt^2} = -(qb/mc) v_x = -(qb/mc) (F/m + (qb/mc) v_y)$$
$$= -(qb/mc)^2 (cF/qB + v_y)$$

The first equation shows $v_x$ to be sinusoidal with frequency $\omega_c = qB/(mc)$, known as the cyclotron frequency. The second equation can also be written sinusoidally by adding a constant to the velocity.

$$\frac{d^2}{dt^2} (v_y + cF/qB) = -(qB/mc)^2 (v_y + cF/qB)$$
The y motion is also sinusoidal but drifting with velocity $c\mathbf{F}/(q\mathbf{B})$. Recall that this additional y motion is perpendicular to both $\mathbf{F}$ and $\mathbf{B}$. More generally

$$\mathbf{v}_{\text{drift}} = \left(\frac{c}{q}\right) (\mathbf{F} \times \mathbf{B}/B^2)$$

Replacing $\mathbf{F}$ with various forces the specific drifts can be identified

1) Electric field

$$\mathbf{F} = q\mathbf{E} \quad \mathbf{v}_{E,\mathbf{B}} = c\left(\mathbf{E} \times \mathbf{B}/B^2\right)$$

2) Gravitational field

$$\mathbf{F} = m\mathbf{g} \quad \mathbf{v}_{\mathbf{g}} = \left(\frac{m}{q}\right) \left(\mathbf{g} \times \mathbf{B}/B^2\right)$$

3) Gradient of $\mathbf{B}$ perpendicular to $\mathbf{B}$

The force is the Lorentz force on $\mathbf{B}$, if the cyclotron motion is small compared to the change in $\mathbf{B}$, express $\mathbf{B}$ with a Taylor expansion

$$\mathbf{B} = \mathbf{B}_0 + (\mathbf{r} \cdot \mathbf{v}) \mathbf{B}_x \ldots$$

$$\mathbf{F}_c = \left(\frac{q}{c}\right) (\mathbf{v} \times \mathbf{B}_x)$$

results in the standard cyclotron motion

$$\mathbf{F} = \left(\frac{q}{c}\right) \left(\mathbf{v} \times (\mathbf{r} \cdot \mathbf{v}) \mathbf{B}\right) \quad \mathbf{v} \times \mathbf{B} = \text{negative for ions}$$

if $\mathbf{v} \neq 0$ in one direction,

$$\mathbf{F} \cdot \mathbf{v} \to r_n [\cos (\omega t)]$$

$$\mathbf{v} \to \mathbf{v} [\cos (\omega t)]$$

$$\langle \cos^2 (\omega t) \rangle = 1/2$$

$$\mathbf{v}_{\text{grad}} = 1/2 \mathbf{v} \times \mathbf{r} \cdot \mathbf{v} \times \mathbf{B}/B^2$$

4) Curved $\mathbf{B}$ field
There is a centrifugal force if moving parallel to $\mathbf{B}$.

$$\vec{F} = m\vec{v}_F R_c / R_c^2$$

$$\vec{v}_{curv.} = (cmv_f^2/gR_c^2)(\vec{R}_c x \vec{B}/B^2)$$

To solve Maxwell's equations in vacuum, there must be a gradient accompanying a curvature. This results in $v^2 \rightarrow v^2 + v_L^2/2$.

Estimating these drifts for ionospheric conditions,

$$\bar{E} = 5 \times 10^{-2} \text{ V/m} = 5/3 \times 10^{-6} \text{ statvolts/cm}$$

$$B_o = 0.5 \text{ gauss}$$

$$r_L = 3 \times 10^2 \text{ cm}$$

$$\omega_c = 300 \text{ hz}$$

$$v_L = 9 \times 10^4 \text{ cm/s}$$

$$\nabla B = \partial/\partial \mathbf{r} (B_o R_c / r^3) \mid = -3B_o / R_c^3$$

$$R_c = 6.4 \times 10^8 \text{ cm}$$

the following drifts result:

$$\bar{v}_{E-B} = (3 \times 10^{10}) (5/3 \times 10^{-6}) / 0.5$$

$$= 1 \times 10^5 \text{ cm/s}$$

$$\bar{v}_{J} = (16 (1.6 \times 10^{-24} \text{ gm}) (1 \times 10^3 \text{ cm/s}^2)$$

$$/ (4.8 \times 10^{-10}) (0.5 \text{ gauss})$$

$$= 1 \times 10^{-10} \text{ cm/s}$$

$$\bar{v}_{g} = 1 / s (9 \times 10^4) (3 \times 10^2) (-3) (6.4 \times 10^8)^{-3}$$

$$= 1.5 \times 10^{-19} \text{ cm/s}$$

$$\bar{v}_{curv} = (3 \times 10^{10}) (16) (1.6 \times 10^{-24}) (1 \times 10^5)^2$$

$$/ (4.8 \times 10^{-10}) (6.4 \times 10^8) (0.5)$$

$$= 5 \times 10^{-2} \text{ cm/s}$$
APPENDIX B

Permanent Magnet Design

The reluctance of the soft iron return must be calculated to justify its neglect. As seen in Figure 2.2, the return was a capped hollow cylinder with the magnets centered on each cap. The magnet poles face the gap.

Mapping the cylindrical geometry to rectangular geometry, the reluctance can be easily calculated. The cross-sectional area is limited to that adjacent to the magnet. The geometry is shown in Figure B.1. Transform from the z plane (x+iy) to the w plane (u+iv) with

\[ w = \ln(z) \]
\[ w = \ln(x+iy) = \ln \phi \ e^{i\theta} = \ln(\phi) + i\theta \quad 0 < \theta < 2\pi \]

Equate real and imaginary parts.

\[ u+iv = \ln(\phi) + i\theta \]
\[ u = \ln(\phi) \]
\[ v = \theta \]
\[ \phi = \text{constant} = 1 \text{ cm} \]

All thicknesses are 1/4" = 0.64 cm so the cross-sectional area in the w plane is

\[ A = (2\pi)(0.64 \text{ cm}) \]

and the reluctance is given by
Figure B.1 Conformal Mapping of Magnet Return
\[ R = \frac{A_m l}{(A_i \mu_i)} \]

where  
\( A_m = \) area of magnet  
\( A_i = \) area of iron  
\( l = \) length of section  
\( \mu_i = \) permeability of iron = 2,700

The reluctance for one cap is  
\[ R(\rho) = \frac{(1 \text{ cm})^2}{[2 \pi (0.64 \text{ cm}) \mu_i]} \quad u = Cu \]

transforming back  
\[ u = \ln(\rho) \]
\[ R(\rho) = C[\ln(\rho)]_{a}^{b} /\mu_i \]
\[ a = 1 \text{ cm} \]
\[ b = 2 \text{ cm} \]
\[ R(b) = C\ln(2/1) (1/\mu_i) \]
\[ R(b) = C\ln(2) (1/\mu_i) \]

There is a cylinder and 2 caps so the total reluctance is  
\[ R = 2R(b) + R_{cyl}. \]
\[ R = \left\{ \frac{2\ln(2)}{[2 (0.64)]} + \pi (1 \text{ cm})^2 (2.79)/[2 \pi (2) (0.64)] \right\} /\mu_i \]
\[ R = 8.05 \times 10^{-4} \text{ cm} \]

The reluctance of the air gap is  
\[ R = 0.25 \text{ cm} \]

so the reluctance of the return can be ignored.
An engineering description of magnet design is presented in manuals published by Indiana General, producers of Alnico and related products. These manuals speak of reluctance factors and leakage factors, reluctance factors dealing with the reluctance of the circuit and leakage dealing with the loss of flux.

In this design a factor of 1.25 was used to lengthen the magnet. This large a factor was not totally necessary. The reluctance of the circuit was only about 0.003 above 1, and the leakage is not as damaging in this case. This is evidenced by the rigidity being ~6,800 gauss-cm as opposed to the designed 6,000 gauss-cm. Leakage results in not all of the field being contained in the gap causing some decrease in the field at the center. Since particles travelling toward the gap start experiencing the field before entering, the leakage is somewhat cancelled.

The magnets were purchased from an Indiana General distributor:

PEEHAL NORTHEAST
Billerica, Massachusetts

3/4" diam ± .015
1/2" length ± 0.10 Alnico 5-7

for Alnico 5-7

\[ l_m = - \left( \frac{1}{2} \right) \frac{B_m}{H_m} \]
\[ = - (0.1"/2) (6000 \text{ gauss}/-720 \text{ oersted}) (1.25) \]
\[ l_m = 0.51" \]
gauss and oersted are the same units in cgs \((\mu_0 = 1)\). The soft iron was purchased from

**STEEL SALES CORPORATION**  
3348 S. Pulaski Road  
Chicago, Illinois 60623

2 1/2 inch round stock  
17 lbs/ft $91.95/100 lbs

Product: Armco Electromagnetic Iron

An important procedure is the magnetizing of the completed circuit. **THIS MUST BE DONE AS A WHOLE.**

If the magnets are magnetized out of the circuit, e.g., inserting them into an electromagnet and producing \(H\) in the magnet larger than saturation, the field is lessened. Saturating the magnets as in Figure B.2a and removing them naked implies returning them to an air return circuit which has a very high reluctance (second factor on left of equation 2.3). The permeance coefficient \((-l_m/l_3)\) gets very small and \(B_m/H_m\) is a line of small slope (Figure B.2b).

Note -- \(H_m\) is larger here than \(\bar{H}_m\) at the operating point of the good return, so more magnetic moments are randomly oriented, the demagnetizing force \((-\bar{H}_m\) is larger. Energy is lost, and one cannot return up the hysteresis curve. Instead, a smaller hysteresis loop is followed whose slope is equal to the slope at \(\bar{H} = 0\) (total \(\bar{H} = 0 \int \bar{H} \cdot d\bar{l} = 0\) (Figure B.2c). Putting the magnet back together does not return the circuit to its original point. For Alnico 5-7 the 2nd quadrant is like Figure B.2d. Quite a bit (one to
Figure B.2 Permanent Magnet Design
two orders of magnitude) is lost when the circuit is broken or magnetization is done outside the circuit.
APPENDIX C

Calibration

Previous calibrations (Morgan, 1976) of the geometry factor have been made with an electron gun of narrow beam and a wobbling table to make the beam appear isotropic (Arnoldy, 1975). The analyser is mounted on the table and moved in $\Theta$, $\phi$, $y$, $x$. Each movement, respectively, was 10-fold from the previous. The electron gun was of cylindrical symmetry. An axial filament at the accelerating potential provided electrons which moved toward the grounded cylinder. This detector does not have infinite energy resolution, so a sweep through accelerating potential accepted all possible electrons.

Since the MAG-EFM unit selected various ion masses, the first attempt was to run the gun in the ion mode and bleed nitrogen gas near the filament. The beam produced was of a small part nitrogen and its composition was not constant.

The first successful magnetic passage of the magnet-channeltron assembly was with a $^{90}\text{Sr}$ source and a NaI crystal. $^{90}\text{Sr}$ gives a continuous -decay spectrum of 0.55-2.27 MeV electrons. For electrons traversing a magnetic field, 1.4 MeV electrons have a rigidity of 6142 gauss-cm.
Following this an ion gun shown in Figure C.1 was made operative producing an approximate 60-70% N+ beam from 0-1 keV. Ions are produced in the collision chamber in which an electron beam crosses the chosen gas. The ions are extracted by a grid outside the chamber that is held at a lower potential. A focusing grid follows which is at a higher potential and finally the end of the gun is at ground. The filament (tungsten) is kept below the collision chamber a constant 75-100 volts. The best ionization occurs at electron energies at least five times the ionization potential of the gas. N2 gas has an ionization potential of 15.5 eV, He gas ~24.6 eV (Locht and Shopman, 1973).

From mass spectrometer data the largest peak seen in air is N+. N+ is hardly ever seen. The ratio of N+/N+ is approximately 23 (Nier, 1973).

With the entire assembly stationary, the EFM plate voltages were observed during a mass analysis. The energy needed to pass O+ ions was calculated. 100 eV ions were used.

Following this the platform was wobbled in θ, ϕ to produce a solid angle to the beam. A standard was also wobbled and a comparison produced the geometry factor. The goal was approximately the same factor as previous EFM analysers. However, only half was achieved according to the calibration. All operations were done at 2x10^-5 Torr.
Figure C.1 Ion Gun
A rigidity vs. energy calibration was performed using different gasses. Water, always present in a vacuum system, was also used. Figure C.2 is the mass calibration curve. This substantiates the calculation for the passage of O\(^+\) at the energy of 145 eV. Table C.1 is a summary of the data.

Table C.1

<table>
<thead>
<tr>
<th>ION</th>
<th>MASS</th>
<th>PASSAGE ENERGY (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>He(^+)</td>
<td>4</td>
<td>570 ±10</td>
</tr>
<tr>
<td>H(_2)O(^+)</td>
<td>18</td>
<td>128 ± 2</td>
</tr>
<tr>
<td>N(^+)</td>
<td>28</td>
<td>82 ± 2</td>
</tr>
</tbody>
</table>

The entire assembly was calibrated for O\(^+\) passage. The energy required was calculated from the passage of N\(^+\).

Table C.2 is a composite of data for individual units designated EFM-3 (magnet version) ASR-6 (18.1005) and ASR-5 (18.1004) (Auroral Sounding Rocket).
Figure C.2 MAG-EFM Magnetic Calibration (N28, H₂018, He4)
Table C.2

<table>
<thead>
<tr>
<th>Unit</th>
<th>18.1005</th>
<th>18.1004</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ion Source</td>
<td>N⁺</td>
<td>N⁺</td>
</tr>
<tr>
<td>Electrostatic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plates</td>
<td>15.85 ± 1</td>
<td>15.25 ± 1</td>
</tr>
<tr>
<td>Energy Factor</td>
<td>5.41 ± 0.013</td>
<td>5.42 ± 0.013</td>
</tr>
<tr>
<td>eV(N⁺)</td>
<td>85.75 ± 0.58</td>
<td>62.50 ± 0.59</td>
</tr>
<tr>
<td>eV(O⁺)</td>
<td>150.1 ± 1.02</td>
<td>144.4 ± 1.03</td>
</tr>
<tr>
<td>eV(NO⁺)</td>
<td>80.03 ± 0.54</td>
<td>77.00 ± 0.55</td>
</tr>
<tr>
<td>Magnet Voltage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>O⁺(16)</td>
<td>153.3</td>
<td>149.3</td>
</tr>
<tr>
<td>NO⁺(30)</td>
<td>84.0</td>
<td>79.2</td>
</tr>
</tbody>
</table>

All geometry factor calibrations were related to a calculated standard. The standard was an assembly with an entrance and exit followed by a channeltron. The geometrical factor is

\[(\text{solid angle})(\text{rear area}) = \mathcal{G}A\]

where

\[
G.F. = \left[ (-0.020/2)(2.54 \text{ cm}/\text{m}) \right]^2 \left(4\pi\right) \left[ (0.078/2) \right]^n
(2.54 \text{ cm}/\text{m})^2 /\left[4\pi \left[ (1.438)(2.54 \text{ cm}/\text{m}) \right]^2\right]
\]

\[G.F. = 4.684 \times 10^{-6} \text{ cm}^2 \text{ ster}\]
Theoretical comparison of the standard EFM geometry factor are made by Reidler (1970). Consider the look angles of the curved plates, \( \alpha \) (in the plane of the curve), \( \beta \) (perpendicular to the plane of the curve). The angles are given by

\[
\alpha = \sin(\sqrt{2} \psi) \frac{d}{\sqrt{2}} \cos(\sqrt{2} \psi) r_m
\]

\[\beta = h/L\]

\( \psi \) = total angular curve of plates
\( d \) = plate separation
\( r \) = mean radius of curvature
\( d = 2 \text{ mm} \)
\( r = 2 \text{ cm} \)

**PATH LENGTH** \( L = 4.73 \text{ cm} \)

**REAR EXIT HOLE** \( = .078" \text{ diameter} \)

\( E/<E> = .087 \)

\( \psi = \pi/2 \)

G.F. = \( 7.69 \times 10^{-6} \text{ cm}^2 \text{ st} \text{er ev/ev} \)

The above angles are for windowless analysers. EFMs have a collimator in front. A. D. Johnston (1972) quotes a disagreement of about a factor of 2 less than above considering better trajectories but still no collimator.

Table C.3 is a summary of the individual calibration conditions and results. Calculations of expected fluxes prompted the enlargement of the analyser fronts to 0.081" diameter. Theoretical calculations above consider geometry factor as a function of entrance and exit areas. The
increased geometry factor was calculated as such. The calibration was accomplished with a standard as a basis, channeltron efficiency thus being eliminated. Calibration of a Double Electrostatic Analyser (DESA) by similar methods resulted in an average channeltron efficiency of 78%. This was accomplished by comparing with a geometry factor measured by Choy, 1971). Also in this paper is a report on CEM efficiencies, displaying 78% as a reasonable result. The consequence is an average standard EFM geometry factor of 9.5x10^-6 cm² ster eV/eV. Theory (Reidler, 1970) would calculate 1.4x10^-5 cm² ster eV/EV, again with a disagreement factor of 1/2 by Johnstone (1972).

Table C.3

Geometry factors (cm² ster eV/eV)

<table>
<thead>
<tr>
<th>UNIT #</th>
<th>1(EPFM)</th>
<th>2(EPFM)</th>
<th>3(MAG-EPFM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18:1005</td>
<td>6.23</td>
<td>7.01</td>
<td>3.42</td>
</tr>
<tr>
<td>18:1004</td>
<td>5.78</td>
<td>6.72</td>
<td>3.77</td>
</tr>
</tbody>
</table>
Calibration and Instrument Specifications:

All units compared to standard unit \((4.68 \times 10^{-6} \text{ cm}^2 \text{ster})\)

with ion gun as beam source (67% \(N^+_2\))

wobbler only moves in \(\theta, \phi\).

\[
\begin{align*}
\theta &= 15.0 \text{ degrees} \\
\phi &= 14.7 \text{ degrees} \\
&= 6.6 \times 10^{-2} \text{ ster}
\end{align*}
\]

Standard analysers:
- entrance 0.059" diameter
- exit 0.078" diameter
- radius 2 cm
- plate separation 2 mm
- plates @ \(\pm 10.0\) V.

Suggestions for further calibration include recommendation of this technique. There is a distinct advantage to restriction of wobbler motion to \(\theta, \phi\) both in time (factor of 100) and consistency. Proper alignment of the cylindrical electron gun could produce a measured geometry factor for the standard. The ion gun could be focused properly to give a wide uniform beam. Mapping of
the beam topology could be achieved with the standard. All instruments of use thus far could be calibrated under equal conditions.
APPENDIX D

Spherical Trigonometry

The following is a proof of the cos and sin formulae of spherical trigonometry. The proof is achieved by attaching a tangent plane to the sphere at the vertex of the included angle. Project the vertices onto the plane, solve the triangle with plane trigonometry and transfer back. Notice that for arcs larger than 180°, for example, arc a > 180° and arc b + arc c > 180°, no intersection is possible. The vertex origin lines may be projected onto a parallel plane on the other side of the sphere, and that proof follows for the cosine formula.

Care must be exercised when taking the inverse functions to retrieve angles since the arc sin repeats from 0° -> 180° as does the arc cos from 90° -> 270°. Since the arc cos is unambiguous from 0° -> 180°, it has been used throughout. Refer to Figure D.1

\[ DE^2 = AD^2 + AE^2 - 2(AD)(AE)\cos(\angle A) \]
\[ DE^2 = (OA)^2\tan^2(c) + (OA)^2\tan^2(b) + 2(OA)^2\tan(c)\tan(b)\cos(\angle A) \]
\[ DE^2 = OD^2 + OE^2 - 2(OD)(OE)\cos(\angle D) \]
\[ DE^2 = (OD)^2\sec^2(c) + (OA)^2\sec^2(b) + 2(OA)^2\sec(c)\sec(b)\cos(a) + \tan^2(c) \]
\[ \tan^2(c) + \tan^2(b) - 2\tan(c)\tan(b)\cos(\angle A) \]
Figure D.1 Spherical Trigonometry Proof
\[
\sec^2(c) + \sec^2(b) - 2\sec(c)\sec(b)\cos(A) = 1 + \tan^2(c) + \tan^2(b) - 2\sec(c)\sec(b)\cos(A)
\]
\[
\cos(a) = \frac{2[(1 + \tan(c)\tan(b)\cos(A)]/2\sec(b)\sec(c)}{2sec(b)sec(c)}
\]

\[
\cos(a) = \cos(b)\cos(c) + \sin(b)\sin(c)\cos(A)
\]

\[
cosine\ \text{formula}
\]
\[
\cos(A) = \frac{(\cos(a) - \cos(b)\cos(c))}{\sin(b)\sin(c)}
\]

\[
sine\ \text{formula}
\]
\[
\sin(A)/\sin(a) = \sin(B)/\sin(b) = \sin(C)/\sin(c)
\]

The subroutines in VECTOR.FOR [1014, 14142] start with two directors, VECTR4 and VECTR5, since the lunar sensor on 18:1005 did not work on PCM output. FM-PM pulses provided by University of Minnesota produced the lunar sensor angle as a function of flight time.

EVENT processes the lunar sensor pulses returning LANGLE, the angle between the moon and the spin axis. EVENT checks for special calibration pulses occurring at -20°, 80°, and 110° degrees.

LUNCHK checked the smoothness of the returned angles and compensated as necessary. A perfectly working lunar sensor would make this routine obsolete.

ROLFIX was written by Peter Lewis and returns the magnetometer-spin axis angle (BANGLE), time of x-axis positive zero crossing (TREF) and the roll period averaged for 20 sec (ROLP). Initial estimates of the roll period as well as magnetometer calibrations must be provided.
POINTS puts these two angles together returning RANGLE(4) which contains the FLIGHT TIME, ZENITH DISTANCE (radians), AZIMUTH (North = 0 radians) and time reference in the 180 degree reversed geomagnetic system (N, W, -B).

ASSTOO returns the particle velocities in the geomagnetic system (N, E, B) for a given detector. The rocket based coordinates (θ off spin axis, φ from 0° in a r.h.s.) for a detector must be defined in the SPECS. Flights 18:1005 and 18:1004 used UNH raceway as zero (0). The last parameter sent defines which detector is to be analysed. Notice that the statement ARG = must be specific to a rocket, eg. the x-axis magnetometer was -42.7 degrees from the UNH raceway 0 degree reference.

MAGCHK is a smoothing check of the magnetic angle BANGLE. TAPE45.FOR and INTRA45.FOR typify VECTOR.FOR's use for flights 18:1005 and 18:1004. TAPE45.FOR wrote the merged tapes X00812 and X00813 from Goddard Format tapes X00939 and X00940 respectively. All programs may be accessed by running Backup on the UNH DEC-10 system on internal tape 000914.
APPENDIX E

Least Squares Fitting

General Least Square with Statistics (GLSWS) is an iterative data fitting Fortran program. The maximum neighborhood method of Marquardt (Marquardt, 1963) is used. The following (Streater, private communication) is an algebraic view of fitting linear and non-linear functions. For functions linear in its parameters the problem of fitting reduces to minimizing

$$\Phi = \sum_{i=1}^{n} (f_i - y_i)^2$$

where $f$ is the calculated function, $y_i$ is the corresponding data point and all weights are equivalent. Since $\Phi$ is the definition of the square of the difference of two vectors $\vec{y} = y_1, y_2, \ldots, y_n$ and $\vec{f} = f_1, f_2, \ldots, f_n$, minimizing $\Phi = \vec{y} - \vec{f}$ is equivalent. If $\vec{f}(\vec{x}, \vec{p})$ is linear in $\vec{p} = p_1, p_2, \ldots, p_m$ with the independent variables defined as $\vec{x}$, then $\vec{f}$ may be defined as

$$\vec{f} = \sum_{m=1}^{k} \vec{f}_m \vec{p}_m$$

where $\vec{f}_m$ is one of the $k$ sets of $n$-dimensional basis vectors $\vec{f}_m = f_1, f_2, \ldots, f_n$. 
Figure 3.1 is a surface of 2 of the n dimensions representing the n calculated data points. The vector \( \bar{y} \) is not necessarily on the surface, however, \( \bar{f} \) is a function of \( \bar{f}_m \) and therefore does lie on the surface. To travel the shortest distance from \( \bar{y} \) to \( \bar{f}(\phi) \) we travel a line perpendicular to all \( \bar{f}_m \) or

\[
\begin{align*}
\bar{f}_1 \cdot \phi &= 0 \\
\bar{f}_2 \cdot \phi &= 0 \\
\vdots & \vdots \\
\bar{f}_k \cdot \phi &= 0 \\
\bar{f}_k \cdot \bar{f} &= \bar{f}_k \cdot \bar{y}
\end{align*}
\]

where \( \bar{f} = \sum_{m=1}^{k} \bar{f}_m p_m \)

\[
\begin{align*}
p_1 (\bar{f}_1 \cdot \bar{f}_1) &= \bar{f}_1 \cdot \bar{y} \\
p_2 (\bar{f}_2 \cdot \bar{f}_1) &= \bar{f}_2 \cdot \bar{y} \\
\vdots & \vdots \\
p_k (\bar{f}_k \cdot \bar{f}_1) &= \bar{f}_k \cdot \bar{y}
\end{align*}
\]

\[
\begin{align*}
p_1 (\bar{f}_k \cdot \bar{f}_1) &= \bar{f}_1 \cdot \bar{y} \\
p_2 (\bar{f}_k \cdot \bar{f}_2) &= \bar{f}_2 \cdot \bar{y} \\
\vdots & \vdots \\
p_k (\bar{f}_k \cdot \bar{f}_k) &= \bar{f}_k \cdot \bar{y}
\end{align*}
\]

\[
\bar{p} \cdot \bar{F} = \bar{F} \bar{y}
\]

\[
\bar{p} = \bar{F} \bar{y} \cdot \bar{F}^{-1}
\]

and \( \bar{p} \) is determined exactly.
2 of n dimensional data space

Figure E.1 Linear Least Squares Fitting
The altered Maxwellian as described in section 3.4 is non-linear in the parameters. Now \( \mathbf{F} \) cannot be written as \( \sum_{m=1}^{k} \mathbf{f}_m \mathbf{p}_m \) and the method breaks down. The method of Marquardt essentially determines which direction to head in parameter space by extremizing the difference between least squares calculated with initial parameters and least squares calculated with new parameters.

Consider the space depicted in Figure E.2. This figure has two of \( k \) dimensions of parameter space. The functional form of the surface is the least squares function \( \Phi = \sum_{i=1}^{N} \left[ y_i - f_i(\mathbf{x}, \mathbf{b}) \right]^2 \) for various parameters. The surface shown is of course idealized for clarity. Extremizing \( F = \Phi(\mathbf{p}_{\text{initial}}) - \Phi(\mathbf{p}_{\text{initial}} + \Delta \mathbf{p}) \) in the correct direction (i.e., the negative gradient in \( p \)-space) determines \( \Delta \mathbf{p} \) to best advantage. Two methods are involved: the Taylor method which expands \( \Phi(\mathbf{p}_{\text{initial}} + \Delta \mathbf{p}) \) to first order to find the correction vector and the gradient method which has a set step size. The direction of \( \Delta \mathbf{p} \) is determined by the negative gradient. The step size must be controlled when utilizing the Taylor method to keep the first order approximation valid. This is done with a constraint on \( \Delta \mathbf{p}^2 \)

The Taylor method extremizes

\[
F(\Delta \mathbf{p}) = | \Phi(\mathbf{p}_{\text{initial}} + \Delta \mathbf{p}) - \Phi(\mathbf{p}_{\text{initial}}) |
\]

subject to the step size constraint

\[
G(\Delta \mathbf{p}) = 0 = \sum_{m=1}^{k} \Delta \mathbf{p}_m^2 - k \text{ where } k = |\mathbf{p}_m|^2
\]
Figure E.2 Non-linear Least Squares Fitting

\[ \phi(P_1, P_2, \ldots) \]

\[ \phi(\tilde{P}_{\text{initial}}) \]

\[ \phi(\tilde{P}_{\text{initial}} + \Delta \tilde{P}) \]
or extremizing

\[ \mathcal{F} = F + \lambda G \]

where \( \lambda \) is a Lagrange multiplier

so solve

\[ \frac{\partial \mathcal{F}}{\partial \Delta p_j} = 0 \quad j = 1 \rightarrow k \quad k \text{ equations} \]

\[ \frac{\partial \mathcal{F}}{\partial \lambda} = G(\Delta \lambda) = 0 \quad 1 \text{ equation} \]

k+1 equations -- k+1 unknowns (k \( \Delta p \)'s, 1 lambda)

To get a set of equations linear in \( \Delta \lambda \) expand the calculated points at the new parameter estimates.

\[
\begin{align*}
\tilde{f}_i (\bar{p} + \Delta \tilde{p}) &= \tilde{f}_i (\bar{p}) + \sum_{m} \left( \frac{\partial \tilde{f}_i}{\partial \lambda_m} \right) \Delta \lambda_m \\
\tilde{\phi}(\bar{p} + \Delta \tilde{p}) &= \sum_{i} \left( \tilde{f}_i + \sum_{m} \left( \frac{\partial \tilde{f}_i}{\partial \lambda_m} \right) \Delta \lambda_m - y_i \right)^2 / \sigma_i^2
\end{align*}
\]

weights have been considered equal and set to 1.

\[
\begin{align*}
F(\Delta \lambda) &= \tilde{\phi}(\bar{p} + \Delta \tilde{p}) - \tilde{\phi}(\bar{p}) \\
F(\Delta \lambda) &= \sum_{i} \left[ (\tilde{f}_i + \sum_{m} \left( \frac{\partial \tilde{f}_i}{\partial \lambda_m} \right) \Delta \lambda_m - y_i \right)^2 - (\tilde{f}_i - y_i)^2 \\
F(\Delta \lambda) &= \sum_{i} \left[ (2(\tilde{f}_i - y_i) - \sum_{m} \left( \frac{\partial \tilde{f}_i}{\partial \lambda_m} \right) \Delta \lambda_m \right] \\
&\quad + \sum_{n} \sum_{m} \left( \frac{\partial \tilde{f}_i}{\partial \lambda_m} \right) \left( \frac{\partial \tilde{f}_i}{\partial \lambda_n} \right) \Delta \lambda_m \Delta \lambda_n
\end{align*}
\]

since only the cross terms are left. Solving the extremizing equations:

\[
\begin{align*}
\frac{\partial \mathcal{F}}{\partial \Delta p_j} &= \frac{\partial F}{\partial \Delta p_j} + \frac{\partial G}{\partial \Delta p_j} \\
&= \sum_{i} \left[ 2(\tilde{f}_i - y_i) \frac{\partial \tilde{f}_i}{\partial p_j} \\
&\quad + 2 \sum_{m} \left( \frac{\partial \tilde{f}_i}{\partial \lambda_m} \right) \left( \frac{\partial \tilde{f}_i}{\partial \lambda_n} \right) \Delta \lambda_m \Delta \lambda_n \right] = 0
\end{align*}
\]
this gives k sets of equations

\[ \sum \sum (\frac{\partial f_i}{\partial P_m}) (\frac{\partial f_i}{\partial P_j}) \Delta P_m + \Delta P_i = \sum (f_i - y_i) \frac{\partial f_i}{\partial P_i} \]

or

\[ (\lambda^2 + \lambda \Gamma) \Delta \bar{p} = \bar{g} \]

where \( \lambda \) has elements

\[ a_m = \sum (\frac{\partial f_i}{\partial P_k}) (\frac{\partial f_i}{\partial P_n}) \Delta P_k \]

\( \bar{g} \) has elements

\[ g_n = \sum (f_i - y_i) (\frac{\partial f_i}{\partial P_n}) \]

Invert \( \lambda^2 + \lambda \Gamma \) and multiply through, \( \Delta \bar{p} \) being determined.

Since parameters have different dimensions and different orders of magnitude, computational accuracy could change from one parameter to the next. This is alleviated with the equations in terms of normalized dimensionless parameters defined by

\[ P_i = b_i l_i \]

where \( b_i \) is the original parameter, \( l_i \) is a scale length with units of \( [b_i]^{-1} \)

\[ l_i = \left[ \sum (\frac{\partial f_i}{\partial b_i})^2 \right]^{1/2} \]

\[ \frac{\partial f_i}{\partial P_i} = \left( \frac{\partial f_i}{\partial b_i} \right) \left( \frac{\partial b_i}{\partial P_i} \right) \]

where
\[ \frac{\partial b_i}{\partial p_j} = 1/l_i p_j \left( \frac{\partial}{\partial p_j} \left( 1/l_i \right) \right) \]

The second term is second order and discarded and

\[ \frac{\partial f_i}{\partial p_j} = \left( \frac{\partial f_i}{\partial b_j} \right) \left( 1/l_i \right) \)

is the definition of \[ \frac{\partial f_i}{\partial p_j} \] encountered above.

The gradient method sets the step size and \( \Delta p \) is determined by

\[ \Delta p = -\left( \frac{\partial \bar{\Sigma}}{\partial b_1}, \frac{\partial \bar{\Sigma}}{\partial b_2}, \ldots, \frac{\partial \bar{\Sigma}}{\partial b_k} \right)^T \]

Referring back to the solution for \( \Delta p \)

\[ (\hat{\Lambda} + \lambda \bar{\Xi}) \cdot \Delta p = \bar{g} \]

the importance of \( \lambda \) can be seen. In the limit of \( \lambda \ll \) the solution goes to

\[ \hat{\Lambda} \cdot \Delta p = \bar{g} \]

which is Taylor's iterative method. In the other limit, \( \lambda \gg 1 \) the equation reduces to

\[ \lambda \bar{\Xi} \cdot \Delta p = \bar{g} \]

recalling that

\[ g_j = \frac{\partial}{\partial \Sigma} \left( f_i - y_i \right) \left( \frac{\partial f_i}{\partial p_j} \right) = 1/2 \left( \frac{\partial \bar{\Sigma}}{\partial p_j} \right) \]

the gradient method is this limit.

Consequently the value of \( \lambda \) determines how much of each method to use. \( \lambda \) is determined by calculating \( \bar{\Xi} \) at the original place. Find \( \Delta p \) with a new solution of \( \bar{\Xi}(\lambda/m) \) where \( m > 1 \). Calculate \( \bar{\Xi}(\lambda/m) \) and be assured
\( \bar{f}(\lambda/\nu) < \bar{f}(\lambda) \). This being true, the minimization of \( \bar{f} \)
is proceeding properly, and more Taylor method is necessary, 
\( \lambda = \lambda/\nu \) for the next iteration. If \( \bar{f}(\lambda/\nu) > \bar{f}(\lambda) \),
then increase \( \lambda \) since the gradient method will assure
smaller \( \bar{f} \) each time. This method is slower as a result of
restricted step size of \( \Delta p \).

The pitfall of this method, as with any other least
squares, is of course badly contoured parameter surfaces
containing local minima. This can be alleviated somewhat by
good initial estimates and upper and lower bounds on the
parameters.

GLSW.FOR \([1014, 14142]\) contains the processing routines
FIT, MATINV, etc. written by Marquardt to do the iterative
least squares fitting and statistical analysis described
above.

FITZ.FOR \([1014, 14142]\) contains the user written
subroutines necessary to catalyse GLSW.FOR. They include
FUN to define the function that will be fitted. Input is
basically the independent variable array \( X(48) \) with \( YC \) the
calculated value. Also the numerical partial derivatives of
the function with respect to each parameter \([P(48)]\) must be
supplied \([DFDP(48)]\).

PARAM reads in the parameters into \( P(48,5) \). \( P(I,5) \) is
the name, \( P(I,1) \) is the initial estimate, \( P(I,4) \) implies
fixed (1) or variable (0). \( NP \) is the number of parameters,
for example these fits had six, 3 drifts, temperature,
rocket potential and density.
OUTPUT is optional output to plotter, lineprinter, etc.
LIMITP is also an optional parameter limiting subroutine.

RDATA reads the raw data source and supplies one data point with its accompanying independent variables each time it is called. M is the routine controller: M = -1 first time, M = 0 reading data, M = 1 the data set is ended. The routine must return to the beginning of the set and prepare to read the same set again.

Specifically, flights 18:1004 and 18:1005 used subroutine RDATA in FITZ.FOR which read a merged tape of the form of Table E.1.

Each block contained 32 words of word 1 (Flighttime), 32 words of word 2 (EFM 1 energy), etc. up to word 14. This tape (X00812, 18:1004; X00813, 18:1005) was written by TAPE45.FOR [1014,14142] from standard GODDARD format rocket tapes (X00939,X00940).
<table>
<thead>
<tr>
<th>WORD</th>
<th>DESCRIPTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Flighttime</td>
</tr>
<tr>
<td></td>
<td><strong>EPM 1 (30 degrees)</strong></td>
</tr>
<tr>
<td>2</td>
<td>Energy</td>
</tr>
<tr>
<td>3</td>
<td>Counts</td>
</tr>
<tr>
<td>4</td>
<td>Theta-Geomagnetic pitch angle</td>
</tr>
<tr>
<td>5</td>
<td>Phi-Geomagnetic azimuth</td>
</tr>
<tr>
<td></td>
<td><strong>EPM 2 (90 degrees)</strong></td>
</tr>
<tr>
<td>6</td>
<td>Energy</td>
</tr>
<tr>
<td>7</td>
<td>Counts</td>
</tr>
<tr>
<td>8</td>
<td>Theta</td>
</tr>
<tr>
<td>9</td>
<td>Phi</td>
</tr>
<tr>
<td></td>
<td><strong>EPM 3 (90 degrees) magnetic unit</strong></td>
</tr>
<tr>
<td>10</td>
<td>Energy</td>
</tr>
<tr>
<td>11</td>
<td>Counts</td>
</tr>
<tr>
<td>12</td>
<td>Theta</td>
</tr>
<tr>
<td>13</td>
<td>Phi</td>
</tr>
<tr>
<td>14</td>
<td>Magnet voltage monitor</td>
</tr>
</tbody>
</table>
APPENDIX F

Frozen Field Lines

The strength of the magnetic induction at a point can be represented with the aid of field lines. The assumption of infinite conductivity leads to the colloquial "frozen-in flux." As will be shown, the field lines accompanying a group of particles are dragged along with the bulk velocity.

This can be interpreted either by Maxwell's equations (Jackson, 1962) or by considering local current systems (Ratcliffe, 1972).

Starting with Ohm's law and Maxwell's equations

\[ \mathbf{J} = \sigma (\mathbf{E} + (\mathbf{v} \times \mathbf{B})/c) \]
\[ \nabla \times \mathbf{B} = (4\pi/c)\mathbf{J} \quad \text{and} \quad \nabla \times \mathbf{E} = -(1/c) \partial \mathbf{B}/\partial t \]
\[ \nabla \cdot (\mathbf{v} \times \mathbf{B}) = (4\pi/c) \mathbf{v} \times \mathbf{J} \]
\[ \nabla \times (\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = (4\pi/c) \mathbf{v} \times \mathbf{J} \]
\[ - \nabla^2 \mathbf{B} = (4\pi\sigma/c) (\nabla \times \mathbf{E} + \nabla \times (\mathbf{v} \times \mathbf{B})/c) \]
\[ \nabla \times \mathbf{B} = (4\pi\sigma/c^2) [ \nabla \times \mathbf{E} + \mathbf{v} \times (\nabla \times \mathbf{B}) ] \]
\[ \nabla \times \mathbf{E} = \nabla \times \mathbf{B} = (c^2/4\pi\sigma) \nabla \times \mathbf{B} \]

If now the conductivity \( \sigma \) is infinite, (see Table 5.1 for collisions vs. gyrofrequency) the above equation is reduced.

\[ \partial \mathbf{B}/\partial t = \nabla \times (\mathbf{v} \times \mathbf{B}) \quad \text{F.1} \]

The magnetic induction is equal to flux times area, and
Stokes theorem may be invoked. There are both time changes in flux due to changing induction and changing shape of the area \( \overrightarrow{da} \) moving in time with velocity \( \overrightarrow{v} \) (i.e. volume \((\overrightarrow{da})(\overrightarrow{v}dt \times \overrightarrow{dl})\), \(\overrightarrow{dl}\) borders \(\overrightarrow{da}\)). The magnetic field is divergence free so differing flux from changes in \(\overrightarrow{da}\) correspond to opposite changes through the sides of the volume (see Figure F.1).

\[
\frac{d\Phi}{dt} = \frac{d}{dt} \int \overrightarrow{B} \cdot \hat{n}da
\]
\[
= \int \frac{\partial \overrightarrow{B}}{\partial t} \cdot \hat{n}da + \int \frac{\partial}{\partial t} (\overrightarrow{B} \cdot \hat{n}da)
\]
\[
= \int \frac{\partial \overrightarrow{B}}{\partial t} \cdot \hat{n}da + \int \overrightarrow{B} \cdot \frac{d}{dt} (\overrightarrow{v}dt \times \overrightarrow{dl})
\]
\[
\frac{\partial \overrightarrow{B}}{\partial t} \cdot \frac{d}{dt} (\overrightarrow{v}dt \times \overrightarrow{dl}) = \frac{\partial \overrightarrow{B}}{\partial t} \cdot (\overrightarrow{B} \times \overrightarrow{v}dt) \cdot \overrightarrow{dl}
\]
\[
\frac{\partial \overrightarrow{B}}{\partial t} \cdot (\overrightarrow{B} \times \overrightarrow{v}dt) \cdot \overrightarrow{dl} = \int \nabla \times \frac{d}{dt} (\overrightarrow{B} \times \overrightarrow{v}dt) \cdot \hat{n}da
\]

Equation F.1 states that

\[
\frac{\partial \overrightarrow{B}}{\partial t} = \nabla \times (\overrightarrow{v} \times \overrightarrow{B}) = -\nabla \times (\overrightarrow{B} \times \overrightarrow{v})
\]

therefore

\[
\frac{d\Phi}{dt} = \frac{\partial \overrightarrow{B}}{\partial t} \cdot \hat{n}da - \frac{\partial \overrightarrow{B}}{\partial t} \cdot \hat{n}da = 0
\]

and the flux is frozen in time with a group of particles moving with velocity \(\overrightarrow{v}\).

For the alternate justification Ratcliffe (1972) considers a circuit enclosing a given amount of flux. The emf of the circuit is given by \(-\frac{d\Phi}{dt}\) where \(\Phi\) is the flux defined above. The flux is partly an external flux \(\Phi_e\) and the balance due to the current of the circuit. The circuit will respond to external changes as
Figure F.1 Moving Flux Surface
\[ L \frac{di}{dt} + Ri = -\frac{d\phi}{dt} \]

Again, if resistance is negligible, then

\[ L \frac{di}{dt} = -\frac{d\phi}{dt} \]

or more generally, if the shape may change constituting a change in \( L \),

\[ \frac{d}{dt}(Li) = -\frac{d\phi}{dt} \]

The inductive flux is defined as \( Li \) so that the internal flux compensates exactly for external changes. This of course assumes that the circuit can change much faster than any flux changes. This is essentially a superconductive inductor with plasma particles making up the current. Any flux associated with a group of plasma will remain so, and the field lines are frozen in place.