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EXAMINING THE BENEFITS OF INSTRUCTIONAL ASSESSMENT AS EXPERIENCED BY SECONDARY MATHEMATICS TEACHERS

Willem Wallinga
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Abstract
Mathematics teachers use a wide variety of assessment tools and methods to measure student understanding and illuminate potential learning gaps (NCTM, 2014). The most frequent and least formal types occur as observations and interactions that provide immediate feedback on the learning process (NRC, 2003, Wiliam, 2007). These Instructional Assessments emerge within the social environment of classroom activity, and serve a formative function by directly impacting the flow of discussion and motivating appropriate teaching interventions. Research has shown that formative assessment improves student performance, but is often challenging for teachers to master (OECD, 2005). The influence of annual high-stakes testing in the United States over the past decade motivates further examination of this conflict at the secondary level. This dissertation describes two case studies performed with secondary mathematics co-teachers in a large, urban, public school. Interviews and classroom observations were performed over the course of an academic semester, exposing challenges and strategies in the areas of professional self-efficacy and knowledge for teaching. Coding and thematic analysis were used to develop structured narratives for each participant. The results illustrate that Instructional Assessment empowers mathematics teachers in part by providing them with unrestricted access to student learning, and develops their ability to interact creatively in productive and meaningful ways.

Keywords
assessment, creativity, emergent, empowerment, formative, instructional, Mathematics education

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EXAMINING THE BENEFITS OF INSTRUCTIONAL ASSESSMENT AS EXPERIENCED BY SECONDARY MATHEMATICS TEACHERS

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DISSERTATION

Submitted to the University of New Hampshire
in Partial Fulfillment of
the Requirements for the Degree of

Doctor of Philosophy
in
Mathematics Education

December, 2017
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Original approval signatures are on file with the University of New Hampshire Graduate School.
DEDICATION

This is for Emily, Jacob, Isaac, and Danielle.

May you live in a world of infinite possibilities and have the desire to learn about them all.
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ABSTRACT

EXAMINING THE BENEFITS OF INSTRUCTIONAL ASSESSMENT AS EXPERIENCED BY SECONDARY MATHEMATICS TEACHERS

by

Willem Wallinga

University of New Hampshire, December, 2017

Mathematics teachers use a wide variety of assessment tools and methods to measure student understanding and illuminate potential learning gaps (NCTM, 2014). The most frequent and least formal types occur as observations and interactions that provide immediate feedback on the learning process (NRC, 2003, Wiliam, 2007). These Instructional Assessments emerge within the social environment of classroom activity, and serve a formative function by directly impacting the flow of discussion and motivating appropriate teaching interventions. Research has shown that formative assessment improves student performance, but is often challenging for teachers to master (OECD, 2005). The influence of annual high-stakes testing in the United States over the past decade motivates further examination of this conflict at the secondary level. This dissertation describes two case studies performed with secondary mathematics co-teachers in a large, urban, public school. Interviews and classroom observations were performed over the course of an academic semester, exposing challenges and strategies in the areas of professional self-efficacy and knowledge for teaching. Coding and thematic analysis were used to develop structured narratives for each participant. The results illustrate that Instructional Assessment empowers mathematics teachers in part by providing them with unrestricted access to student learning, and develops their ability to interact creatively in productive and meaningful ways.
CHAPTER I

THE PROBLEM AND ITS BACKGROUND

The ability to accurately and consistently assess student knowledge has been, and will continue to be, a great challenge in mathematics education. As efforts to reform educational practices in the United States have advanced in recent decades, there has been a marked change in the variety of ways in which student achievement is judged and reported (Driscoll & Bryant, 1998; National Council of Teachers of Mathematics (NCTM), 2014; Stiggins, 2002; Wiliam, 2007). A wide range of assessment issues concerning frameworks and methodology, as well as equity and the role of technology, are being researched and implemented at all levels; from the individual student to state and national populations. Additional focus on below-average results in recent international assessments has publicized and politicized the issue more than ever before (NCTM, 2014, National Research Council (NRC), 2003; Reynolds, Livingston, & Willson, 2006). In short, at the beginning of the 21st Century, there are few topics in mathematics education that can claim to have as much widespread attention as assessment.

Assessment is a general term, encompassing a diverse set of educational tools and techniques (Webb, 1992). It is therefore important to indicate the specific methods of assessment that are the focus of this dissertation study, why they are of particular interest, and how they contrast from other, similar methods. This chapter begins with a summary of the various functions that assessment serves in mathematics education. A conceptual model is
presented for distinguishing different forms of assessment based on their frequency and formality. The scope of this study is then situated from this model.

**Distinguishing Characteristics of Assessment**

Despite the wide variety of assessment methods in mathematics education, there are only three potential functions that such instruments and activities can serve (Wiliam, 2007). Perhaps the most familiar of these is *summative*, which indicates an attempt to “evaluate students’ level of achievement at the completion of a phase of learning” (NRC, 2003, p. 6). Summative assessments are designed to rate the learner’s understanding of mathematical topics, concepts, and/or processes. They are almost exclusively in the form of pencil-and-paper examinations, and are often standardized to prevent bias and allow for quantitative comparisons (NRC, 2001). Over the past decade, many states have turned to large-scale summative assessments to determine whether students have achieved the necessary skills in order to graduate high school (Reynolds et al., 2006; Wilson, 2007).

The second potential function of assessment is *evaluative*, designed for “evaluating the quality of educational programs or institutions” (Wiliam, 2007, p. 1056). Evaluative assessments exist within individual schools, and extend to state, national, and international levels (Evers & Walberg, 2004; Organization for Economic Cooperation and Development (OECD), 2005). In large-scale cases, representative samples of students are chosen in order to facilitate the data collection process. In past decades, public and political attention in the United States was focused primarily on national and international assessments (Baldi, Jin, Skemer, Green, & Herget, 2007; Gonzales et al., 2008; Lee, Grigg, & Dion, 2007). Most recently, increased attention has been placed on local accountability, with individual schools across the nation being rated on a yearly basis (Stullich, Eisner, & McCrary, 2007).
The third potential function of assessment is *formative*, indicating instruments or activities that “support students’ ongoing learning and help teachers make instructional decisions” (NRC, 2003, p. 6). Formative assessments occur whenever feedback is given to a student or group of students that allows for the adaptation of the learning process to meet identified needs (OECD, 2005). What is important for distinguishing assessments of this type is not the span of time this feedback requires, but that it includes “an implicit or explicit recipe for future action” (Wiliam, 2007, p. 1062). Many researchers refer to formative assessments as assessments *for learning* to highlight this function (Black & Wiliam, 1998a). Assessments that are summative and evaluative are correspondingly referred to assessments *of learning*.

It should be noted that these three functions of assessment are not necessarily mutually exclusive. It is quite possible for a given assessment to serve multiple functions. For example, the primary function of a written classroom exam is usually summative. However, when the results of such an exam are used to inform subsequent instructional decisions, the function is also formative. In fact, according to NCTM (2000), “When done well, assessment that helps teachers make decisions about the content or form of instruction can also be used to judge students’ attainment” (p. 24). Indeed, assessments intended to promote classroom learning can even be used to evaluate a school’s effectiveness (Frederiksen & White, 2004; Shepard, 2000). However, regardless of their potential, in practice, most forms of assessment tend to be associated with a single primary function.

Summative, evaluative, and formative designations provide an initial basis for classifying assessments, but further details are warranted. Describing an assessment instrument or activity by the functions it serves indicates why the assessment exists, but fails to designate where, how,
and by whom the assessment is being given. To do so requires additional descriptors which will prove to be sufficient for the purposes of this study.

**A Conceptual Model for Assessment**

Assessment in mathematics education exists on a dual continuum of *frequency* and *formality*. Frequency represents how often the assessment is typically administered and formality represents both the level of standardization and the number of educators involved in determining the assessment parameters. These variables have been chosen as the main characteristics of the conceptual model because they are inversely related. That is, assessments that are the most formal, such as large-scale standardized tests, are the least frequently given. Conversely, assessments that are the least formal, such as those that coincide with instruction, occur much more frequently. A visual representation of this model is presented in Figure 1.

At the far-right end of this continuum, we find large-scale, international exams such as the Trends in International Mathematics and Science Study (TIMSS), and the Programme for International Student Assessment (PISA). These assessments are given very infrequently, only once every few years. Additionally, they are very formal in nature, with dozens of participating countries choosing representative samples of students to be tested (Baldi et al., 2007; Bybee, 2007; Gonzales et al., 2008). Although tests of this type have received a significant amount of public and political attention in recent decades, they will not be the focus of this study. Large-scale assessments, especially those administered internationally, are almost exclusively intended to be evaluative (NRC, 2003).
Figure 1. *The Assessment Continuum*

Moving left across the continuum, assessments become more frequent and less formal in nature. Starting with the introduction of the No Child Left Behind Act of 2001 (NCLB), states have been required to test students in mathematics and reading in grades 3 through 8 annually, and high school students at least once (Wilson, 2007). These tests have been a great source of controversy over the past decade due to inconsistencies between different states as well as having questionable effects on classroom teaching (NRC, 2003; Wilson, 2004). Furthermore, there are important issues concerning equity, reliability and validity, and teaching to the test (Pedulla et al., 2003; Reynolds et al., 2006; Wilson, 2007). The functions of state-level testing are summative and evaluative, indicating the capabilities of individual students and disaggregated categories of students as well as the performance of schools in reaching statewide standards (NRC, 2001). Much like their national and international counterparts, state-level tests will not be the focus of this study.
Further down the continuum, we cross into the domain of individual schools. This is where assessments occur within the classroom and are often administered by a single teacher. Assessments at this scale exist in many forms, the most formal being written tests and quizzes, which occur much more frequently than large-scale versions. Tests primarily serve the summative function of assessment, and often represent a large percentage of a student’s overall grade, especially at the secondary level. NCTM (2014) states, “Too much weight is placed on results from assessments – particularly large-scale, high-stakes assessments – that emphasize skills and fact recall and fail to give sufficient attention to problem solving and reasoning” (p. 3). The formative uses for tests are not as prevalent, and may not directly impact the class in which the assessment was administered. For example, when a teacher considers the results from a test in order to modify instruction, the effects may not be realized until the next time the course is taught (Wiliam, 2007).

Quizzes, when used properly, primarily serve the formative function of assessment. This is due to their frequency, which can be weekly in typical cases, as well as their limitation to a few specific mathematical topics. Teachers use quizzes to monitor the development of student understanding and can adjust instruction accordingly (NCTM, 2000). However, quizzes are often treated by teachers and students as just another form of test, and so their formative potential may be obscured or lost entirely by their summative appearance. Another similarity in this case is that opportunities for feedback are necessarily delayed by the grading process, and may not be immediately beneficial to instruction. Because of this, classroom tests and quizzes will only play a supporting role in the focus of this study.

At the far-left end of the continuum lies the region of interest – assessments that occur the most frequently and are the least formal in design. Assessments of this type occur within the
social environment of the classroom and almost exclusively serve a formative function. The most common types can be categorized as teacher observations of, or interactions with, individual or multiple students. In all cases, the teacher is solely responsible for performing the assessment and administering feedback. Student responses during these interactions are often brief and instantaneous, and provide the teacher with important information, potentially altering the course of instruction. For the remainder of this dissertation, this region will be referred to as *Instructional Assessment*, to distinguish it from other types of formative assessments that do not take place during instruction. The following section will discuss Instructional Assessment in depth followed by a description of the problem of interest and statement of the research questions.

**Rationale for Research in Instructional Assessment**

When compared to large-scale forms of assessment, those that occur daily within the mathematics classroom have the greatest potential to impact teaching and learning (NCTM, 2014; NRC, 2003). This is where research, teacher training, and curriculum design meet the students head on. “The assessment enterprise, as it exists in schools today, consists overwhelmingly of the assessments that teachers themselves design, score, and act upon in their classrooms every day and every week of the school year” (Wilson, 2004, p. 2). At this level, there are so many unique assessment methods that it is difficult to produce a comprehensive list (Heritage, Kim, Vendlinski, & Herman, 2008). According to NCTM (2000), “Many assessment techniques can be used by mathematics teachers, including open-ended questions, constructed-response tasks, selected-response items, performance tasks, observations, conversations, journals, and portfolios” (p. 23). However, this only begins to hint at the complexity of interactions that occur in a typical mathematics classroom.
For the classroom teacher, assessing to determine what students know and can do occurs more frequently and informally than can be accurately recorded. “Teachers make assessments of their students’ learning every day, by noting the misconceptions or insights that underlie a question, for example, or by observing the way a student makes use of materials provided for a task” (NRC, 2003, p. 13). To assess students successfully, teachers rely largely on their ability to interpret behavioral cues, some as subtle as facial expressions (Bloom, Hastings & Madaus, 1971; Williams & Ivey, 2001), in order to monitor learning and adjust instruction accordingly. These informal and continuous interactions are the quantum foundations of classroom assessment, and they are also the least understood. As Stiggins (2002) states, “To date, as a nation, we have invested almost nothing in assessment for learning” (p. 762). Wiliam (2007) echoes this sentiment: “The research literature … is almost entirely about the formal methods of assessment, particularly tests and examinations” (p. 1053).

The ability to frequently and effectively assess student understanding based on cognitive and behavioral cues during classroom instruction is an important and necessary skill for any mathematics teacher (NCTM, 2000; Wiliam, 2007). Moreover, international research has shown that teachers who can accurately assess mathematical understanding during the course of instruction have a positive effect on summative test performance (Black & Wiliam, 1998a; Morgan & Wilson, 2002; OECD, 2005). Despite these findings, the research and implementation of formative assessment in mathematics classrooms remains largely unrecognized in the United States (Stiggins, 2002; Wiliam, 2007). Of primary concern, daily and long-term objectives developed by teachers and administrators are often influenced by attention to standards and large-scale assessments rather than the actual capabilities of their
students (NRC, 2003). This is remarkable, since Instructional Assessments are the most frequently administered and have the most potential to impact instructional decision-making.

This study proposes to examine the role of the teacher during attempts to implement and sustain methods of Instructional Assessment. This decision is largely based on the recognition that research on formative assessment tends to focus solely on student outcomes. Although this is undeniably an area of great importance, hardly any consideration has been given to the impact felt by teachers through administering such assessments. The uninterrupted flow of classroom instruction places constant demands on teachers, requiring them to frequently draw upon their knowledge of mathematics, as well as their knowledge of individual and collective student understanding. Furthermore, this combined knowledge base is being continuously updated through observation, listening, and questioning during classroom instruction. Cobb (2000), states

> It is generally acknowledged that the classroom is the primary learning environment for teachers as well as for students and researchers. In particular, it appears that teachers reorganize their beliefs and instructional practices as they attempt to make sense of classroom events and incidents. Hence, teachers’ learning, as it occurs in a social context, can become a direct focus of investigation in a teaching experiment. (p. 372)

This supports the need for teachers as participants in research on Instructional Assessment, and serves as an entrance point to the problem at hand.

**Problem Description**

When teachers make instructional decisions, they rely on their knowledge of specific mathematical content and its relation to the general curriculum. Additionally, knowledge of student thinking and experience with the classroom environment help to optimize the potential for learning. The ability to move instruction in new and productive directions is directly based on the teacher’s capability to consider and incorporate each relevant type of knowledge when
making instructional decisions. Inherent in this process is a level of self-efficacy that teachers must possess in order to be willing and effective participants in classroom interactions (Bandura, 1997). Teachers who cannot do so will be constrained in their ability to communicate and educate effectively. However, it is evident that current research cannot say how teachers develop such a repertoire of skills and whether doing so tends to benefit or overburden them personally. “The concept of formative assessment often resonates with teachers, but many protest that it is just not possible to put these ideas into regular practice – that there are just too many barriers” (OECD, 2005, p. 69).

Researchers have identified at least three factors that prevent or limit the use of Instructional Assessment. The first is a perceived loss of classroom control (Black & Wiliam, 2006). In order to maximize access to student thinking, teachers need to avoid conventional teaching methods, and allow their students to contribute, and in some cases, dictate, the flow of classroom discussion. In essence, this is a substantial renegotiation of the standard learning contract (Brousseau, 1984, Perrenoud, 1991). International studies have consistently found that both teachers and students are initially uncomfortable with this proposition (Black et al., 2003; OECD, 2005). Despite efforts to incorporate formative assessment into classroom teaching in recent decades, teachers continue to rely heavily on the results of written tests (Shepard, 2000). This is unfortunate, since summative assessments can only provide a snapshot of student progress in relation to certain standards, and have a limited and delayed impact on instruction.

The second and third factors, time constraints and accountability, are related, and particularly relevant to mathematics education in the United States. The most recent and significant developments in American mathematics education are the federal requirement of state curriculum standards and the use of large-scale summative testing (McMillan, 2008). Students at
all levels are now expected to reach content-specific goals followed by a demonstration of their achieved abilities in a standardized setting in a predetermined amount of time. Teachers working in such a system often report that the weight of these external obligations creates enormous time pressure in the classroom. Instead of focusing on the development of individual student understanding, and allowing for student-driven discussion, teachers may concern themselves with covering a list of standards and teaching to a national or state-wide test (Firestone, Monfils, & Shorr, 2004). The NRC (2003) summarized this polarizing effect in American classrooms.

The pressure on teachers to prepare students for large-scale tests developed for accountability purposes can clearly be very great, yet such tests may not bear a close relationship to what is happening in any given classroom. When this happens, there is often a large gap between the objectives teachers and administrators would naturally develop, and those dictated by the inherently circumscribed nature of the external test. (p. 17)

This effect can be greatly amplified at the secondary level in states where large-scale testing is used in a high-stakes manner, as a prerequisite for graduation.

This study is unique by focusing on how mathematics teachers navigate these factors, and whether or not they experience personal or professional benefits while administering Instructional Assessment. Based on the rationale provided in this chapter, the supporting areas of concern include teachers’ self-efficacy and the ongoing modifications to their knowledge of mathematics and student learning trajectories. In the following sections, the research questions are stated and a theoretical basis is provided for their analysis. Finally, a brief discussion on the potential significance of this study is given.

**Research Questions**

This study will spotlight the possible connections between teachers’ Instructional Assessment practices and their professional self-efficacy, knowledge of mathematical content,
and knowledge of students. The following research question and research sub-questions were developed to guide this line of research:

**How are mathematics teachers empowered through Instructional Assessment methods?**

a. How does the integration of assessment into instruction relate to the professional self-efficacy of mathematics teachers?

b. How do mathematics teachers reinforce or modify their understanding of mathematical content and processes through classroom interactions?

c. How do mathematics teachers make use of observation, listening, and questioning during instruction to construct models of student understanding?

These sub-questions are designed to determine how teachers (a) perceive and report the feasibility of Instructional Assessment methods in their classrooms, (b) experience mathematical concepts through multiple perspectives, and (c) address individual and multiple student (mis)conceptions.

**Theoretical Perspective**

When modeling the teaching and learning of mathematics within a modern classroom setting, two factors must be considered: the knowledge created internally by each individual, and the knowledge created externally through social interactions. In the first case, each student develops his or her own understanding of a mathematical concept by assimilating it with previously learned concepts. From this *constructivist* perspective (Simon, 1997), “knowledge is not passively received from the world, from others, or from authoritative sources. Rather, all knowledge is created as individuals (or groups) adapt to and make sense of their experiential world” (p. 57). In the second case, the teacher and students work together to propose, discuss,
negotiate, and establish mathematical meaning. From this interactionist perspective (Voigt, 1996), “social interaction makes possible that subjective ideas become compatible with culture and with intersubjective knowledge like mathematics” (p. 30). However, neither of these perspectives taken alone provides a wide enough lens for the purposes of this study.

The main motivation for choice of perspective comes from the evolution of classroom teaching techniques in mathematics education over the past 50 years (see Ferrini-Mundy & Graham, 2003 for the definitive historical analysis). Contrasting with the traditional methods of previous decades, teachers are no longer expected to be simply transmitters of knowledge. Rather, classrooms in the 21st Century are intended to be domains of inquiry, discourse, and group investigation, where students and teacher share many educational responsibilities (Nelson, 1997). Teachers are expected to “engage the class in mathematical investigation, orchestrate classroom discourse, and create a learning environment that is mathematically empowering” (Herrera & Owens, 2001, p. 90). This development has transformed the mathematics classroom from a largely autonomous environment to one that is socially active and more adaptable to the needs of individual students. Accordingly, teachers’ methods for assessing student understanding have become more varied and appropriate for inclusion within the classroom.

It is no coincidence that the development of epistemological research in education during this time mirrored the development of curriculum reform in mathematics. Many theorists, including constructivists, realized the necessity of addressing the impact of social interactions on the acquisition of knowledge (Lerman, 1996). Two different epistemological camps resulted from these attempts: social constructivist and socio-cultural (Cobb & Yackel, 1996). However, despite the shared goal of each to incorporate the social domain, fundamental differences remain. For the social constructivist, “although social interaction is seen as an important context for
learning, the focus is on the resulting reorganization of individual cognition” (Simon, 1997, p. 116). Thus, even though the processes of students’ mathematical development are recognized as a catalyst for learning, the products are treated exclusively as individual psychological accomplishments (Cobb, 2000; von Glasersfeld, 1995). In contrast, “epistemologists with a sociocultural orientation see higher mental processes as socially determined” (Simon, 1997, p. 116). Thus, interactive rather than individual contributions are emphasized, and students’ knowledge develops out of the social activities in which they participate (Cobb & Yackel, 1996).

The approach for this study is to consider both individual and group influences as complementarily significant in the development of mathematical knowledge within the classroom. A survey of the research literature has revealed the development of the emergent perspective (Cobb, 1995; Cobb, 2000; Cobb & Yackel, 1996), which incorporates the processes of individual construction and social enculturation in the development of mathematical understanding. From this view,

Students are considered to contribute to the evolving classroom mathematical practices as they reorganize their individual mathematical activities. Conversely, the ways in which they make these reorganizations are constrained by their participation in the evolving classroom practices. A basic assumption of the emergent perspective is, therefore, that neither individual students’ activities nor classroom mathematical practices can be accounted for adequately except in relation to the other. (pp. 309, 310)

However, since the focus of this study is on the mathematics teacher, we will need to consider the role of the teacher in this perspective. Instructional Assessment shares many of the characteristics of non-traditional teaching styles (Shepard, 2000), thus, the above statement from Cobb is equally applicable to research aimed at the evolving assessment practices of teachers.

Cobb and his colleagues developed an interpretive framework for the emergent perspective, which is summarized in Table 1. Using this perspective, an alignment between the individual categories and this study’s research questions will be proposed.
Table 1

*The Emergent Perspective* (Cobb, 2000).

<table>
<thead>
<tr>
<th>Social Perspective</th>
<th>Psychological Perspective</th>
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<tbody>
<tr>
<td>Classroom social norms</td>
<td>Beliefs about our own role, others’ roles, and the general nature of mathematical activity</td>
</tr>
<tr>
<td>Sociomathematical norms</td>
<td>Specific mathematical beliefs and values</td>
</tr>
<tr>
<td>Classroom mathematical practices</td>
<td>Mathematical conceptions and activity</td>
</tr>
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</table>

**Classroom Social Norms & General Beliefs**

Teachers play the primary role in the creation and maintenance of classroom routine by initiating and guiding instruction. Those who avoid traditional teaching methods affect changes in the roles determined by each student, as well as themselves (Brousseau, 1984, Perrenoud, 1991). Classroom social norms dictate how the different roles come together to create classroom structure. Complementary to this are the beliefs each member of the classroom has concerning these roles. This combination is of primary importance for investigating teachers who embed assessment into instruction (Shepard, 2000). Students expect assessment to come at the end of an instructional unit, and therefore may not view the teacher as a minute-to-minute evaluator of classroom activity (Wiliam, 2007). Accordingly, teachers’ beliefs about their own roles as assessors of student understanding are necessarily impacted by their perceived ability and willingness to successfully observe and interact with students in a frequent and informal manner. How these beliefs relate to the use of Instructional Assessment is the focus of the first research sub-question.
**Sociomathematical Norms & Mathematical Beliefs and Values**

The development of sociomathematical norms as a theoretical construct was motivated by the recognition that social norms and classroom roles are not particular to any one subject area. Examples of social norms that are specific to mathematics are interpretations of “a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation and justification” (Cobb, 2000, p. 323, emphasis added). Through consistent negotiation of these norms in the classroom, students are provided opportunities to experience mathematical understanding from multiple perspectives. Additionally, as acknowledged by Cobb, “the process of negotiating sociomathematical norms can give rise to learning opportunities for teachers as well as for students” (p. 323). However, an important distinction must be made, since teachers (at the secondary level) possess a substantial amount of subject-matter knowledge relative to their students. How teachers reorganize their understanding of mathematical solutions, explanations, and justifications during Instructional Assessment is the focus of the second research sub-question.

**Classroom Mathematical Practices & Mathematical Conceptions and Activity**

Teachers’ beliefs about their students’ capabilities are incorporated in personalized models of student understanding (Wiliam, 2007). These beliefs can be rooted in student performance, but are often accompanied by descriptions of classroom behavior and participation. When teachers observe and interact with students, they are presented with multiple layers of information that reveal individual and collective levels of mathematical understanding. Examples of social norms exhibited by students that help to inform this process include “explaining and justifying solutions, attempting to make sense of explanations given by others, indicating agreement and disagreement, and questioning alternatives in situations where a
conflict in interpretations or solutions has become apparent” (Cobb, 2000, p. 322). Teachers who perform Instructional Assessment must be able to immediately respond in ways that guide students toward learning objectives. Understanding the function of observation, listening, and questioning that occurs during Instructional Assessment is the focus of the third research sub-question.

The emergent perspective is a natural lens for understanding the role and perspective of the teacher during classroom activities. Based on the above arguments, it is also appropriate for understanding how teachers make use of Instructional Assessment, and therefore, how they stand to benefit from such practices.

**Study Significance**

The list of responsibilities and expectations taken on by mathematics teachers at all levels has always been challenging. In the current climate of increased academic standards and unprecedented federal accountability, the challenge has become more demanding than ever before. Given recently reported levels of nationwide mathematics teacher shortages (Darling-Hammond, 2000) as well as decreased levels of job satisfaction and increased levels of teacher turnover (Artzt & Curcio, 2008; Smylie, 1999), it seems the time has come to gain insight into the realities of teaching mathematics in the 21st Century. Studying how teachers navigate the daily classroom environment in an attempt to prepare students for summative tests is a crucial step toward bridging the current gap between classroom and large-scale assessment (NRC, 2003). Moreover, the time has come for the mathematics teacher to be the focus of assessment research rather than just a consequential participant.

The results of this study will provide valuable information on how mathematics teachers formalize and incorporate formative assessment methods into planning and instruction.
Important relationships between teachers’ classroom decision-making processes and their professional knowledge and self-efficacy will be highlighted. The implications will be relevant for not only mathematics teachers, but also for administrators and policy makers as they continue to align instructional practices with educational goals. Furthermore, curriculum and large-scale assessment designers will be more informed as to the current nature of classroom activity, allowing them to create more effective learning and assessment instruments and activities. Finally, Instructional Assessment practices will be better represented in the research literature.
CHAPTER II

LITERATURE REVIEW

This chapter summarizes research in mathematics education related to Instructional Assessment as defined in Chapter I. First, the evolution of formative assessment in education literature over the past fifty years will be presented. This analysis will demonstrate the difficulty that formative assessment has had in establishing a foothold in research and in practice. Study results from the past two decades outlining the benefits and limitations of formative assessment will then be presented. Research in the areas of teacher self-efficacy and knowledge for teaching that are pertinent to this study will be included. Following the completion of the literature review, this dissertation study will be situated within the current body of research.

The Origin and Evolution of Formative Assessment

The concept of assessing student understanding in order to determine instructional strategies is as old as the Socratic Method (Gareis, 2007; Greenstein, 2010). However, research in formative assessment has only started to gain prominence in the past decade (Kingston & Nash, 2011). As noted in several historical articles (Shepard, 2000; Webb, 1992; Wiliam, 2007), assessment research in the 20th Century was dominated by studies on traditional testing formats. This originated from a psychometric perspective of social efficiency and scientific measurement that followed the growth of industrialization in the early 1900s (Graue, 1993; Shepard, 2000). Correspondingly, the inclusion of non-traditional assessment within modern curriculum frameworks has been gradual and often incomplete. As outlined in Chapter I, public attention
and educational policy have been dramatically influenced by the implementation of yearly, statewide, standardized tests. The use of these tests for accountability purposes has presented a new and formidable challenge for the use of formative assessment. This section will briefly trace the evolution of formative assessment within the research literature, with particular attention to factors that have hindered practical implementation.

The first appearance of the term "formative," attributed to Scriven (1967), was used to describe the evaluation of educational programs. In a response to Cronbach (1963), Scriven challenged the existing theory of curriculum evaluation by distinguishing between the single objective goal (an assessment of value) and the multiple possible roles (such as professional development or curricular modification) of evaluative data. He proposed the terms "formative" and "summative" to distinguish between roles of assessments that assist in developing programs and roles that assess their value after development, respectively. Despite this distinction, Scriven questioned the relative importance of formative evaluations. This was a direct challenge to Cronbach, who stated, "Evaluation, used to improve the course while it is still fluid, contributes more to improvement of education than evaluation used to appraise a product already placed on the market" (p. 236). Shortly thereafter, Bloom (1968) applied the concept of formative evaluation to the teaching and learning of mathematics. Two key aspects of Bloom’s framework were the use of feedback and corrective procedures during classroom instruction (Guskey, 2005).

Feedback during instruction can serve the interests of both the student and the instructor, depending on how it is viewed (Zimmerman & DiBenedetto, 2008). In the case of Bloom's model (1968), short assessments given during a specific curricular unit are used to identify gaps between each student’s current level of understanding and the level dictated by the unit objectives. The feedback elicited by such a process informs the student on their personal
progress through the unit, but more importantly, informs the teacher on the specific areas in which the student needs to review in order to maximize learning potential. The teacher can then provide corrective procedures to each student in an attempt to close the perceived gap in understanding. Bloom's model eventually developed into a widely-regarded instructional strategy called Mastery Learning. Figure 2 illustrates the Mastery Learning sequence.

Figure 2: Bloom's Mastery Learning Model (1968). From Guskey (2005)

Mastery Learning showed great promise for traditional classrooms, and Bloom (1984) posited that this style of teaching had the potential to achieve learning gains similar to those found through one-on-one tutoring. However, the use of formative assessments in Mastery Learning was mostly relegated to the formative use of summative tests, and little was said about the actual process of feedback during daily student-teacher observations and interactions. Bloom recognized that the amount of individual feedback in a traditional classroom setting was often limited. "Frequently the teacher gets most of the feedback on the clarity of his or her explanations, the effect of reinforcements, and the degree of active involvement in learning from a small number of high achieving students" (p. 12). Mastery Learning did not address the ways in which teachers could change their methods of instruction, and thus, failed to take advantage of the social nature of the classroom for formative purposes of assessment.
The overall goal of Mastery Learning is to reduce the variation in student achievement by individualizing the learning process to each student's level of understanding. By doing so, teachers can focus their attention on students who have fallen behind, and provide advanced, or enriched activities for students who demonstrate proficiency. This notion of differentiated instruction can be challenging to implement, especially for inexperienced teachers. For one, teachers are required to split their attention among students with a variety of achievement gaps: a potentially time-consuming proposition. In fact, as will be shown in the following section, classroom time management has been consistently reported as a primary stumbling block to the practical use of formative assessment, especially in the age of accountability testing. Furthermore, by asking students to interpret assessment feedback and adjust their learning strategies accordingly, the typical roles played by teacher and students within the classroom are less well-defined, and often overlap.

The latter of these obstructions was addressed by Perrenoud (1991). He identified the limitations to social behavior among both teacher and students during the process of instruction. This unspoken set of rules, called the teaching contract, essentially dictates that each member of the classroom performs the minimum necessary tasks to facilitate daily objectives, without upsetting any of the other members. In short, teachers are expected to teach, and students are expected to learn. This is especially true of older students, many of whom have acquired certain habits or identities from years of classroom instruction (Shepard, 2000). Teachers who challenge this contract by asking their students to participate in active learning processes risk alienation and rebellion. Perrenoud saw this as a natural impediment. He stated that "every teacher who wants to practice formative assessment must reconstruct the teaching contract so as to counteract the habits acquired by his pupils” (p. 92). Students are expected to become more active in their
own development, a process which upsets the roles expected in the standard teaching contract. Despite the long-term potential promised by models like Mastery Learning, this initial discomfort may be enough to derail the entire process.

At the dawn of the standards movement, research in the area of formative assessment was entering its third decade, yet the adoption of these methods often went undetected. Two article reviews by Natriello (1987) and Crooks (1988) acknowledged the state of disorganization in the research community. Several different phrases were used, often interchangeably, to label formative methods, including classroom evaluation, informative feedback, and embedded assessments (Harmon, 1988). Webb (1992) detailed the reality of assessment in mathematics instruction during this time period.

Although the literature offers scant evidence of the actual practice of classroom assessment, the dominance of paper-and-pencil and short-answer test forms and the lack of clear expectations of performance suggest that assessment in classrooms is not embedded in instruction. (p. 677)

Webb also highlighted the role conflict experienced by teachers when performing formative assessment, and the associated burden.

Integrating assessment with instruction places strong demands on teachers; not only must they have thorough knowledge of content structure, learning, and teaching, but they are also called upon to adopt new ways of teaching by changing their interactions with students and their use of information. (p. 678)

This echoed Perrenoud (1991) and highlighted the difficulty of translating theory into practice.

One way to illustrate the delayed introduction of formative assessment terminology in the United States is by examining the language used in the three major publications from the standards movement (NCTM 1989; 1995; 2000). From Curriculum and Evaluation Standards (1989), “The advantage of using several kinds of assessments, some of which are embedded in instruction, is that students' evolving understanding can be continuously monitored. The
disadvantage is that such a procedure is perceived as cumbersome” (p. 197). Throughout this document, the notion of incorporating assessment into instruction is shown to be a long-term endeavor. Furthermore, the emphasis seems to be on motivating the individual student rather than developing the knowledge shared by an entire classroom. “Extended observations of students’ efforts and interactions in a variety of mathematical contexts can give teachers the feedback and information necessary to adjust their instructional methods and encourage students' progress in attaining intellectual autonomy” (p. 236). In this entire publication, the word ‘feedback’ appears only six times, and the word ‘formative’ does not appear at all.

From Assessment Standards (NCTM, 1995), the recommendations state, “Teachers use evidence of students’ mathematical understanding, along with other evidence from the instructional process, to modify instruction so that it will better facilitate learning. The teacher is the primary assessor of the mathematics that students know and can do” (pp. 25, 26). A wide variety of practical techniques for integrating assessment into instruction are outlined, ranging across the grade bands. The importance of the teacher in documenting student understanding is elevated above that of written assessments. In this (much shorter) publication, the word ‘feedback’ appears 26 times, and the word ‘formative’ remains absent. However, the representative ingredients for formative assessment begin to emerge. Important connections are made between the ways in which teachers monitor students’ progress and make instructional decisions through classroom discussion and evidence-based observation. “[Teachers] make instructional decisions and adapt their teaching to respond simultaneously to the needs of individuals and of groups” (p. 45).

The first use of formative assessment as an encompassing term in the NCTM standards documents appears in Principles and Standards for School Mathematics (2000). Presented as a
fundamental feature of high-quality mathematics education, the *Assessment Principle* outlines research findings and provides a highly-detailed vision of practical classroom implementation.

To ensure deep, high-quality learning for all students, assessment and instruction must be integrated so that assessment becomes a routine part of the ongoing classroom activity rather than an interruption. Such assessment also provides the information teachers need to make appropriate instructional decisions. In addition to formal assessments, such as tests and quizzes, teachers should be continually gathering information about their students' progress through informal means, such as asking questions during the course of a lesson, conducting interviews with individual students, and giving writing prompts. (p. 22)

Discussion of formative assessment dominates the *Assessment Principle*, and instructors are cautioned against relying on singular methods for collecting evidence of student learning, especially formal testing.

In the most recent NCTM publication, *Principles to Actions* (2014), formative assessment appears throughout, and is represented as one of eight mathematics teaching practices that comprise a framework for improved teaching and learning. “Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning” (p. 10). Note that this description of assessment in action highlights the notion of continuous adjustment during instruction on the part of the teacher. However, despite this representation, the examples provided within the text are exclusively at the elementary and middle-school levels.

Furthermore, the only potential teacher-based benefit of formative assessment is “a reduction in the time that they spend preparing their students for state, provincial, or national assessments” (p. 95). Given the prevalence of standards-based curricula in the United States over the past 25 years, this evidence speaks volumes about the delayed adoption of formative assessment methods.
At the turn of the 21st Century, mathematics education researchers had finally begun working toward comprehensive theories of formative assessment. However, at the same time, changes were made to implement yearly statewide testing in a variety of subject areas throughout the K-12 curriculum. This threatened to further widen the gap between research and practice. Stiggins (2002) notes the disconnect between the goals of summative testing and classroom instruction. "If we wish to pursue seriously the use of assessment for learning ... it is important to recognize the pervasive negative effects of accountability tests and the extent to which externally imposed testing programs prevent and drive out thoughtful classroom practices" (p. 9). Shepard (2000) provides a historical review outlining the traditional incompatibility of assessment and instruction. She shows that the reliance on testing in the 20th Century originated from the prevalence of behavioral theories tied to research in social efficiency. This approach is fundamentally at odds with cognitive and constructivist learning theories. "The best way to understand dissonant current practices ... is to realize that instruction (at least in its ideal form) is drawn from the emergent paradigm, while testing is held over from the past" (p. 4). Shepard, Stiggins, and several other authors (McMillan, 2008; Wiliam, 2007) caution about an overreliance on external testing, especially high-stakes accountability testing, which remains at the forefront of educational policy in the United States.

This review of the origin and evolution of formative assessment demonstrates that despite a wealth of research, practical techniques have been delayed by a variety of internal and external factors. This may be a result of the lack of research outlining results that are not restricted to student outcomes. The main goal of this study is to expose ways in which formative assessment empowers mathematics teachers. The next section will focus on specific research studies that outline the benefits and barriers to enacting formative assessment in mathematics classrooms.
Formative Assessment Research

The articles and studies in this section report on the viability of formative assessment methods in mathematics education. Although the findings seem definitive on many levels, there are important limitations that motivate the need for this dissertation. First, most of these studies come from researchers or organizations outside of the United States. This is not surprising, given the arguments presented at the beginning of this literature review. Second, most of these studies were conducted at the elementary and middle-school grades. There are important considerations involving subject matter knowledge, developmental capabilities among students, and the pressures of yearly high-stakes testing that exist exclusively in secondary classrooms. Finally, most of the results of these studies are student-centered. That is, the goal of these researchers is often to determine how formative assessment methods can be used to improve student performance. Very little is said about the effects on teachers, and that which is said is often student-centered in essence. Despite these limitations, the results provide a baseline for researching formative assessment in secondary mathematics classrooms.

Near the end of the 20th Century, Black & Wiliam (1998a) conducted an extensive meta-analysis of the research literature on formative assessment in mathematics education. In a review of over 250 studies from the previous two decades, they concluded that there existed ample evidence supporting the connection between improved formative assessment methods and higher academic standards. They reported significant quantitative results that were "larger than most of those found for educational interventions" (Black & Wiliam, 1998b, p. 141). This document is widely regarded as the definitive resource for formative assessment research, and set the stage for a significant increase in related studies (Kingston & Nash, 2011; Stiggins, 2002). A
summary of findings from Black & Wiliam is provided, followed by a review of more recent studies.

Efforts to implement formative assessment produce significant learning gains as measured by improved test scores following instructional feedback. That is, teachers who make use of formative assessment have students who perform better on summative assessments. Black and Wiliam (1998b) noted effect sizes ranging from 0.4 to 0.7 standard deviations, equivalent to a student moving from the 50th percentile to the 65th percentile. Moreover, the improvement was seen most dramatically in low-achieving students, including those with learning disabilities. "Improved formative assessment helps low achievers more than other students and so reduces the range of achievement while raising achievement overall" (p. 141). This connection between the use of formative assessment and improved test performance has been the primary driving force of research, and the results have been frequently confirmed.

Wiliam, Lee, Harrison, and Black (2004) conducted a study with 12 mathematics teachers and 12 science teachers in the United Kingdom. Known as the King's Medway Oxford Formative Assessment Project (KMOFAP), the study showed an average effect size of 0.32 standard deviations, with 21 of the 24 participant classes showing some level of improvement. The authors identified the use of frequent questioning, evaluative feedback (in place of judgmental feedback), peer assessment, self-assessment, and the formative use of summative tests as successful teaching strategies that emphasize assessment for learning. Their conclusions noted that "improving formative assessment does produce tangible benefits in terms of externally mandated assessments" (p. 63). However, they conceded that the data was difficult to interpret due to inconsistencies between comparison classes. Additionally, there were only limited descriptions of the benefits for the teachers involved in the study (Harrison, 2005).
Shortly after the KMOFAP study, the Organization for Economic Cooperation and Development (OECD) published a collection of eight international case studies conducted at the secondary level (2005). The studies examined mathematics classrooms, as well as other areas of instruction in an effort to better conceptualize the practice of formative assessment. Six key elements to the successful implementation of formative assessment were identified, and are listed in Table 2. The researchers found that each of the six elements were incorporated into the case study classrooms at some level, emphasizing the consistency of formative assessment methods across multiple languages, cultures, and educational policy systems. The authors emphasize the position of the first element over all others in the presentation of a framework for formative assessment. This process of enculturation is required for students to become active participants in classroom interactions, one of the primary sources of formative assessment.

Table 2

Key Elements of Formative Assessment (OECD, 2005).

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<tr>
<td>1.</td>
<td>Establishment of a classroom culture that encourages interaction and the use of assessment tools.</td>
</tr>
<tr>
<td>2.</td>
<td>Establishment of learning goals, and tracking of individual student progress toward those goals.</td>
</tr>
<tr>
<td>3.</td>
<td>Use of varied instruction methods to meet diverse student needs.</td>
</tr>
<tr>
<td>4.</td>
<td>Use of varied approaches to assessing student understanding.</td>
</tr>
<tr>
<td>5.</td>
<td>Feedback on student performance and adaptation of instruction to meet identified needs.</td>
</tr>
<tr>
<td>6.</td>
<td>Active involvement of students in the learning process.</td>
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The OECD study also highlighted the impact of formative assessment practices on the participating teachers. In particular, the study noted the change in their perception of formative assessment as a viable teaching tool. Prior to the study, the teachers expressed many
reservations, including large class sizes, extensive curriculum requirements, and meeting the needs of all students. Following the study, however, these conceptions were greatly modified.

[The teachers] found that formative assessment, instead of adding logistical challenges to teaching, actually helped them to save time, allowed them to focus on the needs of weaker students, and to incorporate varied teaching methods into their repertoire. In the process, they also found that they were making more fundamental changes in how they thought about their students’ abilities, and about teaching and learning itself (p. 69).

Despite identifying these benefits, the study also indicated that the incorporation of formative assessment into the classroom was not a natural process, nor was it the same in every classroom. Some teachers chose to focus on their best students, with the hopes that they would be more willing participants. Other teachers chose to focus on their weakest students, under the assumption that they would benefit the most. In nearly every case, teachers "grappled" (p. 69) with these and other logistical challenges. Furthermore, school administrators allowed the teachers to deviate from the standard educational policy at the time, indicating that formative assessment methods had not yet been incorporated into instruction.

More recent studies have continued to demonstrate similar results. The most commonly stated benefit is the increase in student performance on summative tests. Closely related are improvements to student confidence, motivation, and empowerment in their own education. Panizzon and Pegg (2006) conducted a two-year study with 25 secondary science and mathematics teachers in Australia. The teachers took part in professional development designed to enhance the use of formative assessment methods in their classrooms. Reported improvements included increased levels of student involvement, interest, engagement, and motivation. Shavelson et al. (2008) cited similar results in a study of 12 secondary-level science teachers in Hawaii.
Very few researchers have examined potential benefits for teachers, and in nearly every case, those benefits are student-centered in essence. For example, Stiggins (2002) states, "Teachers benefit because their students become more motivated to learn." (p. 764). Similarly, the most commonly cited improvement from formative assessment, increased student performance, is often regarded as a benefit for teachers. Surely, teachers endeavor to work with highly motivated students who perform well in testing situations, but these cannot be directly regarded as personal benefits for the teachers. Results that were not student-centered proved to be very rare, a fact that served to motivate this dissertation from its earliest stages. Panizzon and Pegg (2006) noted improvements in the participating teachers' ability to identify learning differences in their students. They comment that the teachers "were confident in discriminating between the different levels at which students appeared to be functioning in their classrooms so that tasks and teaching could be directed at an appropriate level to support improvement in student learning" (p. 430). However, no mention was made concerning how formative assessment led to this type of improvement.

These studies show the many positive, student-centered benefits of formative assessment. In order to understand how teachers benefit, it is necessary to incorporate teacher perceptions. Given the documented list of potential barriers, it is important to consider how teachers view their own competencies within the classroom as a necessary component to facilitating change. The next section will examine the concept of self-efficacy in general and as it pertains to the teaching profession.

**Professional Self-Efficacy**

Self-efficacy, defined as "beliefs in one's capabilities to organize and execute the courses of action required to manage prospective situation" (Bandura, 1997, p. 3), plays a significant role
in everyday classroom activities. Originally founded in social psychology, the study of student self-efficacy has had a long tradition in educational research. More specifically, research on student confidence, motivation, and value-beliefs has shaped student-centered learning reforms and standards-based curriculum frameworks. However, few researchers have considered the professional self-efficacy of teachers, especially in relation to pedagogy and assessment in mathematics education. As stated in Chapter I, this study shifts the focus onto the mathematics teacher, and aims to analyze their professional self-efficacy in the age of unprecedented accountability.

The following section presents the origins and evolution of teacher self-efficacy research. Particular attention will be paid to the development and refinement of survey instruments designed to measure teacher self-efficacy. The primary goal is to classify the components of self-efficacy as they pertain to teaching mathematics so that they may be referenced in the analysis of this study.

**Origins**

The development of models for teacher self-efficacy and the subsequent methods for its measurement derive from two major strands of psychological research (Klassen et al., 2009; Tschannen-Moran & Hoy, 2001). The earliest came from Rotter's social learning theory (1966). According to this theory, behavior is determined largely as a result of general expectations. People who feel that they can control behavioral outcomes will act differently than people who feel that the outcomes are out of their control. This notion of *locus of control* was used by a number of researchers to establish a framework for many educational studies in the 1970s and 1980s (Gibson & Dembo, 1984; Tschannen-Moran & Hoy, 2001). However, this framework was eventually deemed as insufficient for the purposes of measuring self-efficacy. Moreover,
the concept of locus of control is rooted in behavioral psychology, which was largely supplanted by cognitive methods in the latter half of the 20th Century. For these reasons, this literature review will not be focused on works derived from Rotter.

The psychologist Albert Bandura is widely regarded as the originator of self-efficacy theory. In the presentation of his theoretical framework, Bandura (1977) suggested an important distinction between a person's ability to predict the outcome of a certain behavior and his or her confidence in its successful execution. He argued that research aimed at measuring a teacher's locus of control provided information concerning outcome expectations rather than efficacy expectations. Bandura stated that the distinction between the two is necessary because, although they are related, they are not always causal in nature. In particular, a high level of outcome expectation does not guarantee a high level of efficacy expectation. "People who regard outcomes as personally determined but who lack the requisite skills would experience low self-efficacy and view activities with a sense of futility" (p. 204). Furthermore, Bandura posited that the social learning components of self-efficacy could be studied independently and measured with a higher level of precision than locus of control. A summary of the components of Bandura's framework is presented in Table 3.

**Teacher Self-Efficacy**

Bandura's theoretical framework for self-efficacy set the stage for research into the educational self-efficacy of students and teachers. For the purposes of this study, the literature review is limited to studies on teacher self-efficacy rather than student self-efficacy, although the two are often related (Green, Anderson, & Loewen, 1988; Tschannen-Moran & Hoy, 2007). A principal challenge in teacher self-efficacy research has been the development of reliable survey tools designed to measure factors across a variety of instructional contexts. Over a dozen such
measurement tools have been developed following the works of Rotter (1966) and Bandura (1977). Since then, education researchers have tested and refined these tools in a number of academic settings. A review of this development is presented, concluding with a description of the Teacher Self-Efficacy Scale (TSES) that was used in this dissertation study. Studies pertaining to mathematics education will be highlighted wherever possible.

Table 3

*Components of Self-Efficacy. From Bandura (1977).*

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance Accomplishments</td>
<td>The cumulative contribution of past successes and failures</td>
<td>Have the strongest effect of efficacy expectations</td>
</tr>
<tr>
<td>Vicarious Experiences (Modeling)</td>
<td>The effects of witnessing the successes and failures of others in similar circumstances</td>
<td>Predict efficacy expectations only when the observer identifies with the model</td>
</tr>
<tr>
<td>Verbal Persuasion</td>
<td>The influence of social interaction, especially in the forms of encouragement and discouragement</td>
<td>Limited to outcome expectations unless combined with the incorporation of practical methods of improvement</td>
</tr>
<tr>
<td>Emotional Arousal</td>
<td>The perception of physiological changes that occur during stressful situations</td>
<td>Difficult to measure, since people react to anxiety in unpredictable ways</td>
</tr>
</tbody>
</table>

The first attempts to measure teacher self-efficacy came from a study by the Rand Corporation (Armor et al., 1976) aimed at analyzing teacher locus of control (Gibson & Dembo, 1984; Tschannen-Moran & Hoy, 2001). The study participants were reading comprehension teachers working with minority students in an urban setting, and the survey tool consisted of two questions included within an extensive questionnaire. The questions measured the teachers' beliefs about their ability to successfully instruct students despite complex and potentially overwhelming external factors. The first question was stated in general terms, and the second
focused on the individual survey respondent. These items were labeled *general teaching efficacy* (GTE) and *personal teaching efficacy* (PTE), respectively. The sum of the two items was simply called *teacher efficacy* (TE), a metric designed to quantify the teachers' overall beliefs on their ability to successfully impact student learning. The study concluded that increased TE was significantly related to positive student performance (Armor et al., 1976).

The Rand study, and many others that followed were based entirely on Rotter's locus of control (1966), and will therefore only be briefly summarized. Guskey (1981) expanded the original Rand questions into a 30-item survey aimed at measuring responsibility for student achievement (RSA). Rose and Medway (1981) created a 28-item survey focusing on teacher locus of control (TLC). In both cases, responses were organized into areas of interest, and the sub-scores were tested for correlation with GTE, PTE, and TE from the original Rand study, with varying results. Finally, the Webb scale (Ashton et al., 1982) reduced the number of survey items to seven, and focused on issues related to context-specific teacher efficacy (hypothetical situations that occur within a classroom setting). In all three cases, the survey tools that were developed never gained widespread acceptance in the research community (Tschannen-Moran & Hoy, 2001). However, they represent the first attempts to measure teacher self-efficacy, and demonstrate the difficulties involved in defining and quantifying such a construct. Furthermore, they served as points of reference for subsequent studies based primarily on the work of Bandura.

In many ways, each of Bandura's four components translates directly to the social environment of the classroom. Stated generally, teachers who feel successful in their ability to manage classroom instruction are more likely to build on that success toward proficiency. Conversely, efficacy beliefs will decrease for teachers when classroom instruction is perceived
as ineffective. Success or failure can be determined by direct interactions with students (performance accomplishments), comparison to other teachers in similar positions (vicarious experiences), performance-based feedback from administrators, colleagues, and parents (verbal persuasion), and/or response to anxiety and stress (emotional arousal). Of these, the most influential is performance accomplishments (Bandura, 1977, 1997). Research has shown that experienced teachers (for whom an abundance of performance accomplishments exist) rate themselves significantly higher on overall self-efficacy than novice teachers (Tschannen-Moran & Hoy, 2007). Additionally, highly efficacious teachers demonstrate higher levels of motivation, effort, persistence, and resilience (Hoy & Davis, 2006).

Gibson and Dembo (1984) noted that past attempts to measure self-efficacy were focused too heavily on outcome expectancies rather than efficacy expectancies. Following Bandura’s theoretical framework (1977), they developed a 16-item survey that attempted to differentiate and balance the two concepts. The Teacher Efficacy Scale (TES) allowed researchers the ability to distinguish between factors contributing to PTE and GTE for the first time (Bandura, 1997; Tschannen-Moran & Hoy, 2001). Gibson and Dembo applied the TES to study the relationship between instructional self-efficacy and classroom tendencies. They found that teachers with high instructional self-efficacy spent more time on academic activities, offered guidance, and praised success more frequently. Conversely, teachers with low instructional self-efficacy spent less time on academic activities, gave up on students, and criticized failures. The study was conducted with over 200 elementary school teachers, and the results have been generally supported in subsequent research (Tschannen-Moran, Hoy, & Hoy, 1998).

The TES received wide acceptance in the research community, and was the standard instrument of choice for the next decade (Ross, 1994). However, by the close of the 20th
Century, several studies demonstrated inconsistencies in the TES, and efforts were made to further refine the measurement process (Tschannen-Moran & Hoy, 2001). One major reason for this update was the realization that teacher self-efficacy is context-dependent (Hoy & Davis, 2006). A teacher may feel very capable teaching certain subjects to certain students, but may feel differently when the subject or students change in some discernible way. For example, in a study by Ross (1998), teacher self-efficacy was shown to be higher when working with high-ability students, when instructing in the area of expertise, when workload was considered moderate, and when the curriculum was collaboratively designed. Researchers worked to develop measurement tools that purported to highlight areas of specificity within the profession, including science education (Riggs & Enochs, 1990), classroom management (Emmer & Hickmen, 1991), and special education (Allinder, 1995, Coladarci & Breton, 1997). Huinker and Madison (1997) created the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI). However, this scale was designed for pre-service elementary school teachers, and is therefore not appropriate for the current study.

One of the most recently developed survey tools for measuring teacher efficacy is the Teachers' Sense of Efficacy Scale (TSES). Originally created by Tschannen-Moran and Hoy (2001) as The Ohio State Teacher Efficacy Scale, the TSES has gained recognition in both American and international education research (Klassen et al., 2009). The TSES has been noted as the first instrument to successfully balance general teaching skills with practical specificity. As summarized by Hoy and Spero (2005), "[The TSES] assesses a broad range of capabilities that teachers consider important to good teaching, without being so specific as to render it useless for comparisons of teachers across contexts, levels, and subjects" (p. 354). A full description of the TSES and its application to this dissertation study is provided in Chapter III.
Efforts to develop survey tools for measuring the self-efficacy of mathematics teachers have been surprisingly limited. A recent review of 202 domain-specific studies in teacher self-efficacy from 1998 to 2009 revealed only nine focused in mathematics education (Klassen et al., 2011). Similarly, studies examining the self-efficacy of mathematics teachers have been almost exclusively conducted at the elementary level. Hoy and Davis (2006) highlighted the need for subject-specific studies in teacher efficacy at the middle and secondary levels due to the advanced level of subject-matter knowledge required. Despite this limitation, results from several studies conducted in elementary classrooms are presented here for their conclusions on classroom instruction.

Beliefs and attitudes toward mathematics have been shown to have a direct influence on the self-efficacy of mathematics teachers and their instructional practices. Briley (2012) showed that teachers with low self-efficacy are less likely to use innovative or exploratory methods. Conversely, teachers with high self-efficacy are more likely to use inquiry-based methods when teaching mathematical concepts. Swars (2005) found that teachers’ past experiences with mathematics played a strong role in the development of teaching efficacy. Teachers with high self-efficacy were found to be more effective, primarily due to their willingness to embrace new instructional strategies. Both studies were conducted with elementary pre-service teachers, and made use of the MTEBI scale. This connection between self-efficacy and a willingness to experiment with teaching methods has been noted in several studies in mathematics education (see Klassen et al., 2011) and provides an interesting implication to the current dissertation study. Since the processes of Instructional Assessment frequently require the teacher to deviate from the prepared lesson plan, teachers with high self-efficacy should exhibit greater flexibility during instruction.
The connection between teacher self-efficacy at the elementary level and mathematical knowledge has received recent attention. Bates, Latham, and Kim (2011) showed that preservice teachers' mathematics self-efficacy is positively correlated to their level of mathematical ability. Furthermore, teachers who performed above-average on a basic skills test rated their teaching efficacy higher than those who performed below-average. However, Wilkins (2008) concluded that teachers' mathematical content knowledge was negatively correlated with their use of inquiry-based methods of instruction. "Higher levels of teacher content knowledge does not necessarily transfer into the use of instructional practices that help promote students' mathematical understanding beyond basic content knowledge" (p. 157). Thus, elementary teachers with advanced mathematical knowledge consider themselves to be highly capable even though they display less flexibility during instruction. Extending the analysis of this relationship to secondary mathematics classrooms is an important component of the current dissertation study.

One final area of interest related to teacher self-efficacy is role conflict. Researchers have noted the complex and sometimes contradictory roles that teachers are expected to take on. Smylie (1999) describes several studies where teachers reported conflicting expectations between students, parents, and school administrators. In nearly every case, the source of the tension was placed on standards and assessment policies at the time of the study. The result was the perception of a loss of control among the participant teachers. Teachers in one study conducted in the United States stated that "centralized tests reduced their ability to experiment with new ideas and adapt curriculum and instruction to meet the specific needs of their students" (Smylie, 1999, p. 74). Berryhill, Linney, and Fromewick (2009) found that teachers often alter their teaching practices in the race to meet the requirements set forth by educational policy. The
consequences of role conflict should be readily apparent during extended classroom observations. This study is designed to highlight the effects as they pertain to Instructional Assessment.

These studies show the connection between teacher self-efficacy and classroom practices. As a final addition to this review, the many forms of knowledge that teachers bring to the classroom will be considered as they relate to the execution of formative assessment. The next section examines the concept of teacher knowledge in mathematics education.

**Knowledge for Teaching**

The diverse and multilayered types of knowledge that are found in classroom instruction have been continuously developed and refined in education research. As stated in Chapter I, when teachers make instructional decisions, they necessarily rely on their knowledge of the subject matter and their familiarity with the students. Therefore, an instructor's knowledge base plays an important role in Instructional Assessment. For the purposes of this study, a brief review of the literature will be provided with specific attention to the relationship between teacher knowledge and formative assessment.

Many of the current models for teacher knowledge are rooted in the work of Shulman (1986). Prior to his revolutionary framework, teacher's content knowledge (CK) and pedagogical knowledge (PK) were widely considered as mutually exclusive constructs by the research community (Petrou & Goulding, 2011). Shulman proposed that understanding teacher effectiveness often required the consideration of both types of knowledge simultaneously. He described Pedagogical Content Knowledge (PCK) as "That special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional
understanding" (p. 9). The following description by Mishra and Koehler (2006) demonstrates the depth of PCK and its usefulness in understanding everyday teacher-student interactions.

PCK is concerned with the representation and formulation of concepts, pedagogical techniques, knowledge of what makes concepts difficult or easy to learn, knowledge of students’ prior knowledge, and theories of epistemology. It also involves knowledge of teaching strategies that incorporate appropriate conceptual representations in order to address learner difficulties and misconceptions and foster meaningful understanding. It also includes knowledge of what the students bring to the learning situation, knowledge that might be either facilitative or dysfunctional for the particular learning task at hand. This knowledge of students includes their strategies, prior conceptions, misconceptions that they are likely to have about a particular domain, and potential misapplications of prior knowledge. (p. 1027)

Upon its publication, Shulman's conceptualization of PCK became a popular topic of research across a wide array of subject areas. This literature review will only focus on developments that have direct implications for mathematics education.

Fennema and Franke (1992) modified Shulman's framework to represent the interactive and dynamic nature of knowledge for teaching mathematics. Their model included four components: knowledge of mathematical content, knowledge of pedagogy, knowledge of students' cognition, and teacher beliefs. Each of these interact and contribute to an overall body of knowledge unique to every teacher. Moreover, they claim that teacher knowledge often emerges from classroom interactions with students. By doing so, "teachers can change their existing knowledge and create new knowledge" (Petrou & Goulding, 2011, p. 13). Furthermore, Fennema and Franke saw the different aspects of teacher knowledge as being related to each other, and therefore all must be considered to understand mathematics teaching.

When teachers perform Instructional Assessment, they draw upon each of the areas of knowledge described by Fennema and Franke (1992). The type of interaction that prompts the Instructional Assessment will vary, but the questions that must be implicitly answered by the teacher are frequently similar. In a typical case, the teacher observes (or otherwise senses) that a
student (or a group of students) has a misconception concerning a specific mathematical topic or process. This recognition is based on the teacher's knowledge of mathematical content and how it conflicts with the student evidence. The choice of response by the teacher is informed by knowledge of pedagogy and knowledge of students' cognition. Depending on the specific case, many choices of response are possible, including telling, questioning, modeling the problem, addressing the room, and doing nothing. Finally, the teacher's beliefs guide each step of this process, and determine the parameters of Instructional Assessment, including length of time, style of intervention, and potential follow-up assessment.

Another group that developed and refined Shulman's framework for mathematics education was a group of researchers based at the University of Michigan. Data analysis of teachers in action led to the creation of a more detailed vision of teacher knowledge. Hill, Ball, and Schilling (2008) split Shulman's PCK into two subcategories: Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and included Knowledge of Curriculum as a third subcategory. The development of KCS in particular holds important implications for understanding formative assessment in action.

Because KCS is an amalgam of subject matter knowledge and knowledge of students, we expect that teachers might invoke mathematical knowledge or engage in mathematical reasoning in order to interpret students' thinking around these topics. However, they should not solely engage in mathematical reasoning when answering these items - they must also invoke knowledge of students. (p. 378)

In this sense, knowledge of students includes recognizing common student misconceptions, faulty reasoning, and gaps in understanding. A teacher with proficiency in this area will be adept at scaffolding questions and activities on-the-fly in ways that will maximize student learning.

A further refinement of mathematical understanding at the secondary level (MUST) was developed by Kilpatrick, et. al. (2015). The authors distinguish MUST from PCK by focusing
on the types of mathematical understanding that occur in the classroom rather than theoretical pedagogical proficiency. Furthermore, they have chosen to create a more dynamic view of ongoing, evolving mathematical understanding compared to the static, prerequisite nature of mathematical knowledge for teaching. The framework covers fourteen aspects of mathematical knowledge across three perspectives: Mathematical Proficiency, Mathematical Activity, and Mathematical Context of Teaching. A categorical summary is provided in Table 4. These perspectives are tailored specifically to the highly-detailed mathematical knowledge required for teaching secondary mathematics. The researchers claim,

> It is not enough for a teacher to know the mathematics that students are learning. Teachers must also possess a depth and extent of mathematical understanding and the ability to enact that understanding that will equip them to foster their students’ mathematical proficiency. (p. 12)

Table 4

*Mathematical Understanding for Secondary Teaching. From Kirkpatrick, et. al. (2015).*

<table>
<thead>
<tr>
<th>Perspective</th>
<th>Types of Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Proficiency</td>
<td>Conceptual Understanding</td>
</tr>
<tr>
<td></td>
<td>Procedural Fluency</td>
</tr>
<tr>
<td></td>
<td>Strategic Competence</td>
</tr>
<tr>
<td></td>
<td>Adaptive Reasoning</td>
</tr>
<tr>
<td></td>
<td>Productive Disposition</td>
</tr>
<tr>
<td></td>
<td>Historical and Cultural Knowledge</td>
</tr>
<tr>
<td>Mathematical Activity</td>
<td>Mathematical Noticing</td>
</tr>
<tr>
<td></td>
<td>Mathematical Reasoning</td>
</tr>
<tr>
<td></td>
<td>Mathematical Creating</td>
</tr>
<tr>
<td>Mathematical Context of Teaching</td>
<td>Probe Mathematical Ideas</td>
</tr>
<tr>
<td></td>
<td>Access and Understand the Mathematical Thinking of Learners</td>
</tr>
<tr>
<td></td>
<td>Know and Use the Curriculum</td>
</tr>
<tr>
<td></td>
<td>Assess the Mathematical Knowledge of Learners</td>
</tr>
<tr>
<td></td>
<td>Reflect on the Mathematics of Practice</td>
</tr>
</tbody>
</table>

Given the goals of this dissertation study, the MUST framework seems particularly appropriate for purposes of data analysis, as it successfully integrates teacher knowledge with student
understanding as they evolve during instruction. This similarly echoes the framework of the emergent perspective, which emphasizes the shared nature of knowledge construction within the classroom.

**Framing this Study within the Existing Research**

Research into formative assessment practices has shown multiple results-based benefits for students, but has not fully considered the impact felt by teachers. Additionally, studies that have incorporated teacher self-efficacy, knowledge, and beliefs have primarily been conducted outside of the United States, and/or at the elementary level. In order to understand how these factors impact (and are impacted by) Instructional Assessment methods in the secondary classroom, it is necessary to observe teachers in action, over an extended period of instruction. By conducting the study in the United States, important information concerning the complex interplay between classroom activities and the existence of yearly standardized testing can be better understood. This is especially true at the high-school level, where these tests have an additional high-stakes component in many states that determines whether a student is allowed to graduate. The incorporation of student and school accountability into a single test is one of many external factors that impact how teachers approach classroom learning (MacMillan, 2008). This study focuses on the unavoidable struggle that teachers must navigate on a daily basis: the desire to incorporate formative assessment into instruction, and the pressure to complete a predetermined list of standards.
CHAPTER III

RESEARCH DESIGN AND METHODOLOGY

The purpose of this dissertation study is to examine how mathematics teachers’ use of Instructional Assessment relates to their sense of professional self-efficacy, development of mathematical knowledge, and evaluation of individual student understanding. To that end, a combination of qualitative methods and descriptive statistics was used to create case studies of two high school mathematics teachers during the spring semester of one academic year. Classroom observations spanning multiple courses and periodic interviews were performed with the participating teachers. The TSES was administered twice in order to provide reliable insight into each teacher's professional self-efficacy as well as to add a quantitative aspect to the research design.

In this chapter, a rationale for the methodology of this study is presented as it evolved from preliminary methods through data collection and analysis. A summary of all data collection techniques and their relationship to each research question will be provided, followed by a detailed description of the data analysis process. Finally, issues concerning the validity, reliability, and generalization of results will be summarized.

Rationale

Qualitative research methods such as observing and interviewing people in their day-to-day activities present researchers with the ability to examine events in the context that they naturally occur (Brown & Dowling, 1998). There are several factors supporting a qualitative
approach to this study. The first is a natural consequence of the main research question: How are mathematics teachers empowered through Instructional Assessment methods? Since Instructional Assessment is developed within the social nature of the classroom, any attempts to analyze the complex interactions that naturally result within such an environment benefit from immersion and highly detailed observation by the researcher (Brown & Dowling, 1998; Burton, 2003; Teppo, 1998). Forman (2003) states that mathematics education research "should examine not only the achievement outcomes of individuals but also the mathematical activities of students and teachers as they work together in formal educational settings" (p. 337). The research design of this study allowed for the collection of a wealth of evidence both as an observer and through interviews with the participating teachers.

The second factor supporting qualitative methods is in reference to the theoretical foundation of this study. The emergent perspective regards knowledge to be simultaneously a product of individual construction and social negotiation. From this perspective, evidence of a mathematics teacher's knowledge in action as it relates to Instructional Assessment can be gathered through direct observation of classroom activities (Cobb & Yackel, 1996). As Cobb (1995) states, "emergent theorists emphasize the diversity of group members' activity. The position from which they analyze activity is, therefore, that of an observer located inside the group" (p. 123). Integrating interview responses with observation data provided further insight into the participants' repertoire of teaching methods.

The third factor supporting qualitative methods is related to the evolution of the mathematics classroom as a result of the reform movement of the late 21st Century. As outlined in Chapter I, the role of the teacher in the classroom has steadily evolved from that of a simple transmitter of knowledge. Curriculum frameworks clearly indicate that classrooms are intended
to be domains of inquiry, discourse, and group investigation, where students and teacher share educational responsibilities (Herrera & Owens, 2001). Romberg & Collins (2000) state, "because an object of the reform movement is classrooms that promote understanding, field studies of students and teachers interacting in classrooms about important problems leading to students' understanding of mathematics and science are warranted" (p. 84). A qualitative approach is therefore appropriate for studying a 21st Century mathematics classroom.

**Pilot Study and Participant Selection**

In order to test the methodology for this research design and to get some practice with observing mathematics classrooms in action, a short pilot study was conducted during the spring semester of 2009. Two teachers from a small high school in Maine were observed over a period of four weeks. One of the teachers had two years of high-school teaching experience, and the other had forty-five years of middle-school and high-school teaching experience. The school employed block scheduling (90 minute classes) which allowed for prolonged interactions during each class period (Biesinger, Crippen, & Muis, 2008). Typical visitations took place over the course of an entire school day, allowing for observations in multiple classrooms covering a variety of mathematical subjects. Field notes were taken on the actions and statements of the teacher and students, but audio- and/or video-recording devices were not used. Particular attention was paid to instances of student misconceptions and the on-the-fly decisions made by the teacher as to the appropriate response. Each participant was interviewed once at the beginning of the observation period, and once at the end. Each interview was audio-recorded and transcribed. The pilot study helped greatly to hone the note-taking, observation, and interview skills necessary for the creation of case studies.

Following the conclusion of the pilot study, a large high school near Boston was selected
for the dissertation study. The school was chosen for several reasons, including location, block scheduling format, and diverse student population. The Director of the Mathematics Program approved the recruitment of any teachers who expressed interest in participating. After meeting with several prospective teacher participants, two agreed to discuss the details of the study. Initially, their demeanor was friendly, but cautious. It was clear they were not fully comfortable welcoming an outside observer into their classrooms. For this reason, only informal pre-study interviews were conducted in order to avoid the official nature of an audio-recorded discussion. Following these meetings, the two teachers agreed to take part in the study. Several weeks passed in order to secure Institutional Review Board (IRB) approval from the university, as well as approval from the School District before beginning the data collection process. A full description of the research participants is given in Chapter IV.

**Data Collection**

A variety of data collection methods were used in order to harmonize the perspectives and experiences of the participants and the researcher (Stake, 2006). The primary methods, indicative of the "real-time" nature of this study, were direct observation of classroom activities, and interviews, both structured and informal. All observations were audio-recorded and accompanied by handwritten field notes. Each interview was audio-recorded and transcribed for analysis. Copies of planning and instructional materials were provided by the participating teachers, as well as state and district policies from school administrators. Finally, the TSES was administered before and after the observation period to support qualitative data informing professional self-efficacy. A summary of these data collection methods and their use in answering each research sub-question is presented in Table 5, followed by an in-depth discussion.
Table 5

Data Collection Matrix

<table>
<thead>
<tr>
<th>Research Sub-question</th>
<th>Purpose</th>
<th>Data Sources</th>
</tr>
</thead>
</table>
| How does the integration of assessment into instruction relate to the professional self-efficacy of mathematics teachers? | To determine how teachers perceive and report the feasibility of Instructional Assessment methods in their classrooms. | – Teachers’ Sense of Efficacy Scale  
– Direct observation of classroom activities  
– Structured and informal interviews |
| How do mathematics teachers reinforce or modify their understanding of mathematical content and processes through classroom interactions? | To determine the impact on teachers of experiencing mathematical concepts through multiple student perspectives. | – Direct observation of classroom activities  
– Structured and informal interviews |
| How do mathematics teachers make use of observation, listening, and questioning during instruction to construct models of student understanding? | To determine how teachers are prepared to address individual and multiple student (mis)conceptions. | – Direct observation of classroom activities  
– Structured and informal interviews |

This selection of data collection methods was appropriate for interpreting the complex nature of classroom events, and facilitated validation through triangulation. For example, claims made by each participant during the interviews were examined for accuracy during classroom observations. Conversely, specific classroom activities and student-teacher discussions were later referenced during interviews for clarification. The TSES completed this triangulation by providing a measurement of each participant’s self-efficacy that informed both classroom observations and interview questions. This strategy was also well-aligned with the emergent perspective by eliciting data specific to both psychological and social aspects of mathematical instruction. For example, scores reported on the TSES represented each participant’s confidence toward individual classroom responsibilities. This data was balanced by observations of the participants while performing these responsibilities, and was further informed by the reactions
and participation of the students. The overall result was a collection of detailed information from multiple perspectives that allowed for informed analysis.

**Observations**

A schedule for classroom observations was developed in concert with each participating teacher to ensure that data would be collected on instruction days, rather than during tests or special projects. Multiple observations were often conducted during the same day. Each teacher provided a description of the lessons, including any handouts or other instructional documents. The observation position was located in the last row of student desks, which provided a view of the entire room with minimal interference. This position also worked well for the audio-recording quality, and it was not necessary to have either teacher wear or carry an additional audio-recording device. This scenario was consistent for all observations, except two that took place in a computer lab. During those observations, the audio-recorder was moved throughout the room by hand to document instructional conversations. Initially, the students were curious about having a visitor in the classroom, but after a few classes, they seemed more comfortable. They were informed that the purpose of the study was to focus on the activities and instructional strategies of the teachers in reaction to natural classroom events, and not to identify any of the students.

During each observation, each teacher was closely monitored to see how they managed the flow of instruction. Field note entries were recorded to indicate changes in activities, and other potentially noteworthy events. Over the course of the semester, a form of shorthand was developed that arose from observing regular events during instruction. Examples of some frequently used abbreviations are listed on Table 6.
Table 6

Sample Field Note Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>HW\checkmark</td>
<td>The teacher is visually checking each student's homework.</td>
</tr>
<tr>
<td>HWQ</td>
<td>The teacher is answering questions from the homework.</td>
</tr>
<tr>
<td>TWA</td>
<td>The teacher is walking around, observing the class.</td>
</tr>
<tr>
<td>WBD</td>
<td>The teacher is writing on the whiteboard.</td>
</tr>
<tr>
<td>WWG</td>
<td>The teacher is working with a group of students.</td>
</tr>
<tr>
<td>WWS</td>
<td>The teacher is working with a single student.</td>
</tr>
</tbody>
</table>

While field notes were written for these and other supporting events, special attention was given to discovering and recording instances of Instructional Assessment. These events served to be the rich points of the data (Agar, 2006b) since they sometimes represented unanticipated changes in the direction of the discussion. Field notes identifying these rich points were starred for emphasis, and the corresponding transcriptions served as the first component of the qualitative working data for this study.

**Interviews**

In addition to regular informal meetings with each teacher, scheduled interviews were conducted near the beginning and end of the research period. Additional interviews were not possible due to scheduling conflicts, and the first interview did not take place until after several observations had been conducted. This was an unfortunate occurrence, however, the interviews that did occur were extensive, and each teacher's responses and reflections complemented the observation data well. The interviewing technique was designed to be long-form and largely unstructured, rather than based entirely on specific pre-written questions. Although some questions were prepared in each case, each participant was encouraged to provide detailed accounts rather than brief answers or general statements (Riessman, 2008). In some cases,
references were made to specific incidents that occurred during observed classes in order to
glean further insight into the instructional strategies of each teacher. The casual and open nature
of the interviews also served as an important rapport-building strategy throughout the data
collection process.

The approach taken in this respect was similar to that described by Lichtman (2006) as an
*in-depth interview*. General topics of importance were identified prior to the interviews and
specific questions were written for each topic. The list of all pre-written interview questions is
provided in Appendix A. During each interview, these questions served as a common starting
point to initiate discussion. However, follow-up questions were based entirely on participant
responses, and were not pre-written. The goal was to allow the conversational nature of each
interview determine the sequence of topics. This encouraged each participating teacher to tell
their story in their own terms (Riessman, 2008). Each participant was willing to provide detailed
responses, with clarifying background information when necessary, such that the result was more
of a narrative than an interview. Several emotional characteristics related to the stresses of being
a high-school mathematics teacher came through vibrantly in ways that were entirely
unanticipated and always appreciated. The audio-recordings and corresponding transcriptions of
the interviews served as the second component of the qualitative working data for this study.

**Teachers’ Sense of Efficacy Scale**

While the use of qualitative methods was appropriate for the majority of the data
collection process, a quantitative survey was also used to assist in measuring each participating
teacher's professional self-efficacy. The Teachers' Sense of Efficacy Scale (TSES) (Tschannen-
Moran & Hoy, 2001) consists of twenty-four statements regarding teacher effectiveness in a
variety of educational situations. Respondents are asked to rate "how much they can do" on a
Likert scale ranging from 1 (nothing) to 9 (a great deal). The TSES was chosen particularly for its emphasis on identifying capabilities that are considered vital for effective teaching. It was developed by mathematics education researchers and has been tested in several studies in order to establish validity and reliability across diverse teaching conditions (see Tschannen-Moran & Hoy and Klassen et al, 2009 for two recent examples). The main advantage of the TSES centers around sub-scores that are derived from the responses. These sub-scores highlight teacher self-efficacy in the areas of student engagement, instructional strategies, and classroom management, each of which is instrumental to performing Instructional Assessment. The sub-scores provided insight into the self-efficacy of each participating teacher at different points in the semester, and were used for cross-case analysis as well. The TSES was administered to each participating teacher at the beginning and at the end of the data collection period and served as the quantitative working data for this study. A copy of the long form is provided in Appendix B.

Data Analysis

The analytic structure of this study is what Stake (2006) calls an instrumental case-study. The goal, therefore, was to develop a highly detailed account of each participant's classroom experiences in order to better understand Instructional Assessment. The multiple forms of collected data provided a wealth of information in this regard. A framework for analysis was initially developed in order to align results with the three research sub-questions. The observation and interview transcripts were then analyzed using a three-step process of categorizing, coding, and identifying themes (Lichtman, 2006). Additionally, quantitative data from the TSES was used to complement and inform qualitative interpretations in the area of teacher self-efficacy. In keeping with the tradition of case-study methodology, each participating
teacher was considered individually prior to the development of cross-case findings. A summary of these data analysis techniques is presented in this section.

**The Analytic Framework**

The analytic challenge specific to qualitative research studies is to reduce and organize the massive amount of data generated from observations and interviews (Lichtman, 2006). Case studies are particularly complex because this data is then reconstructed to craft a detailed description across a variety of contexts (VanWynsberghe & Khan, 2007). In order to do this in an informative and comprehensive fashion, a categorical framework was devised prior to the development of any codes or themes. This approach was chosen in recognition of the connection between the language used by each participating teacher and the multiple areas within which they operate (Gee, 2005). Initially, three categories of interest were established that related directly to the research sub-questions written for this study: Professional Self-Efficacy (PSE), Reinforcement or Modification of Mathematical Content or Processes (RMMCP), and Models of Student Understanding (MSU). A fourth category, Summative Assessment (SA) was created to reflect the frequent influence of state-testing procedures on daily classroom activities as well as the overall mindset of the participants. These areas were further refined by considering the overall context (or scope) of each action or statement that was analyzed. It was desirable, for example, to distinguish between specific statements referencing individual students (short view) from general statements regarding the entire teaching profession (long view). Table 7 illustrates this framework in detail and is followed by further discussion within each category.

**Professional Self-Efficacy.** In a general sense, self-efficacy is one's perceived ability to perform a certain task (Bandura, 1997). For the purposes of this study, professional self-efficacy represents a teacher's perceived ability to anticipate, implement, and make use of Instructional
Assessment methods. In the short view, PSE is characterized by how teachers view their capabilities when interacting with individual students. Typical actions include answering questions and negotiating mathematical meaning. This is expanded in the medium view to general classroom management issues including meeting student demand and anticipating student difficulties. Finally, in the long view, PSE situates each teacher's feelings, beliefs, and values within the requirements and constraints of the teaching profession. Data categorized under the PSE heading was used to develop codes related to the first research sub-question: How does the integration of assessment into instruction relate to the professional self-efficacy of mathematics teachers?

Table 7

*Analytic Framework*

<table>
<thead>
<tr>
<th>Scope</th>
<th>PSE</th>
<th>RMMCP</th>
<th>MSU</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long View</td>
<td>Teaching Profession</td>
<td>Mathematical Subjects</td>
<td>Year to Year</td>
<td>Teaching Profession</td>
</tr>
<tr>
<td>Medium View</td>
<td>Classroom Management</td>
<td>Mathematical Topics</td>
<td>Single Classrooms</td>
<td>Classroom Management</td>
</tr>
<tr>
<td>Short View</td>
<td>Individual Students</td>
<td>Mathematical Concepts</td>
<td>Individual Students</td>
<td>Individual Students</td>
</tr>
</tbody>
</table>

*Reinforcement or Modification of Mathematical Content or Processes.* As discussed in Chapter II, teacher knowledge is not static; it is constantly being reformed through experiences within the classroom. Interactions with students represent frequent opportunities for teachers to have their knowledge of mathematics challenged and, potentially, changed. This is especially true during Instructional Assessment, where teachers are required to compare students' written work or verbal assertions to their own understanding to determine an appropriate method of intervention. In the short view, RMMCP refers to knowledge adaptations that occur when considering a single mathematical concept. In the medium view, changes concern broader
mathematical topics. Finally, in the long view, reinforcement or modification can occur in
general mathematical subject areas. This may include knowledge of course sequencing, as well
as general curriculum frameworks. Data accumulated under the RMMCP heading was used to
develop codes related to the second research sub-question: How do mathematics teachers
reinforce or modify their understanding of mathematical content and processes through
classroom interactions?

**Models of Student Understanding.** By working closely with students, teachers build
models of student understanding at many levels (Wiliam, 2007). This process of experiential
growth guides teachers through their careers and expands their ability to anticipate and meet
student needs. In the short view, this involves identifying the strengths and weaknesses of
individual students. This is also where teachers navigate learning disabilities and adapt to
different student learning approaches. In the medium view, MSU encapsulates entire classrooms
of students, which teachers are often able to characterize by their collective behavior, demeanor,
and performance. Finally, in the long view, teachers take the lessons they have learned from
teaching academic courses to make improvements in subsequent years. Data accumulated under
the MSU heading was used to develop codes related to the third research sub-question: How do
mathematics teachers make use of observation, listening, and questioning during instruction to
construct models of student understanding?

**Summative Assessment.** The influence of summative assessment, whether in the form
of in-class testing or standardized state-testing played an undeniable role in this study. The
consistent pressure on the participants to prepare students was repeatedly evident during daily
instruction. The SA category was created for the purpose of collecting data that highlighted this
ongoing tension. In the short view, SA applies to data concerning individual students. This was
particularly relevant for interpreting data that referred to students with learning disabilities. The medium view is used to establish the effect of summative assessments on classroom management. This includes major issues such as time pressure and meeting curriculum standards. Finally, in the long view, SA covers general issues concerning the use of summative assessment in the teaching profession. It should be noted that the three views of SA closely parallel those of PSE. This is a natural result of the perceived effect summative assessment has on the teaching process.

Once the analytic framework was established, interview responses and observation episodes were labeled with abbreviations from the four categories of interest. In many cases, only a single label was used. This usually corresponded with responses to interview questions that were specifically worded to highlight one particular area. However, in several instances, multiple labels were deemed appropriate whenever it was determined that data could be interpreted across more than one category. As an example, consider the following interview excerpt.

*Do you ever find yourself not answering students because of time concerns?*

I don't think so. I try not to. Maybe subconsciously I'm doing that, on some level, to try to push the class forward. But, usually, my conscious choice to not do something is, it's, you're at the point where you need to own this skill, and you need to do it. And I'm not thinking for you anymore. Especially if we've already processed of, this is this technique. This is what you really need to be looking at. And it's the doing aspect that's not happening.

When analyzing this participant's response, the main category is MSU (medium view), because the participant is describing the tendencies and capabilities of students during classroom instruction. However, this response also informs PSE (medium view), because it highlights the challenges of classroom management. In this sense, one piece of data can be used twice for different purposes. An immediate consequence of this process was the expansion of the overall
data set. Separate files were created in order to isolate data under each category, setting the stage for further analysis.

**Coding**

In order to reduce and manage qualitative data, researchers often employ some form of coding (Lichtman, 2006). This is the process of identifying patterns within the data that allows for the development of detailed interpretations. In this study, the development of codes started informally at the beginning of data collection and evolved as patterns emerged from the interview and observation data. The significance of these actions and events created recursion in the data collection by informing subsequent observations. That is, the designation of certain classroom episodes as worthy of further attention naturally led to the anticipation and recognition of similarly related occurrences. Similarly, claims made by the participants during formal interviews could be later examined during classroom observations. In this sense, the coding process and eventual thematic analysis emerged directly from the data, and was not based on prior assumptions. All interview data was first categorized according to the framework provided in Table 7. Analysis was then performed in two major areas: Professional Self-Efficacy and Knowledge for Teaching. Codes were developed and refined from appropriate sources within the research literature and were checked by a secondary coder to ensure reliability. A detailed explanation of this process is provided below. A full list of codes for each participant is provided in Chapter IV.

**Professional Self-Efficacy.** Data categorized under PSE and SA was parsed into descriptive segments and coded using two sources. The first of these sources was Bandura’s four components of self-efficacy (1977), as presented in Chapter II. The purpose of these codes was to simply identify the motivation behind each participant’s statements. As will be shown in
Chapter IV, most responses represent a recollection of personal experiences (performance accomplishments). This is because participants often respond to questions by describing their memory of events and practices. However, in some cases, responses represent observations of the experiences of others (vicarious experiences), the effects of social interactions (verbal persuasion), and physiological reactions to stress (emotional arousal). A fifth code was added from Smylie (1999) to represent perceived inconsistencies with job roles (role conflict). In total, these five general codes were helpful in organizing responses related to professional self-efficacy, and were integral to the eventual formation of themes for each participant.

The second source of codes for Professional Self-Efficacy data was taken from the long form of the TSES (Tschannen-Moran & Hoy, 2001). The twenty-four statements that make up the survey represent a rich foundation of instructional challenges experienced by teachers. Each of these statements was translated into a code for labeling interview response segments. Despite this level of detail, several additional codes were added for each participant when references emerged from the data that were unrepresented. The use of emergent coding was particularly well-suited in this respect, since different codes could be created for each participant. In total, these specific teacher self-efficacy codes served as the primary source of evidence toward the development of themes designed to answer the first research sub-question.

As an example, consider the following interview response. The numbers represent each coded segment.

[78] I really appreciate the benefit of having that co-teacher, because I have been able to develop those questions [about student disabilities] around her. [79] I remember, last year, there was a student that had visual processing issues, and we were talking about nets, and the wire frames, and shapes, and polyhedrons. And I didn’t get where they [the student] were falling apart. I didn’t get where their disability was impacting the curriculum. [80] And she was able to articulate that they were seeing pick-up sticks.
In this response, there are three coded segments. Segment 78 was coded as L, PA, ST, representing long view, performance accomplishment, and shared teaching (one of the emergent codes). The participant is recalling his general experience of sharing the classroom with a co-teacher. Segment 79 was coded as S, PA, SC, representing short view, performance accomplishment, and student comprehension (a code derived from the TSES). The participant is describing challenges related to assessing the understanding on an individual student. Segment 80 was coded as S, VE, SC, representing short view, vicarious experience, and student comprehension. The participant resolves the difficulty he reported by describing the experience of his co-teacher in helping him to understand the student’s disability. This multi-level coding style was employed throughout the interview transcripts for both participants to characterize their professional self-efficacy.

**Knowledge for Teaching.** As implied by the design of the analytic framework for this study, each research sub-question was initially designed to be considered separately from the others. However, following the data collection process, it became apparent that the second and third research sub-questions were inextricably intertwined. Data isolated to the participants’ own knowledge of mathematics was sparse, as it was typically referenced in relation to student understanding. In many cases, the participants stated that their mathematical knowledge was either largely determined or not something that they considered worthy of further modification. For this reason, it became apparent that teacher knowledge and student understanding should be analyzed simultaneously.

Data categorized under RMMCP and MSU was parsed into descriptive segments and coded using a single source. The MUST framework (2015) presented in Chapter II provided a foundation of fourteen codes for interpreting knowledge for teaching. This framework was
particularly useful for organizing examples of teacher knowledge as they occurred during the process of instruction and as described by the participants. Similar to the coding process for Professional Self-Efficacy, several emergent codes were added when deemed necessary. In total, these knowledge for teaching codes served as the primary source of evidence toward the development of themes designed to answer the second and third research sub-questions.

As an example, consider the following interview response. The numbers represent each coded segment.

*How do you know that [the students] know what they’re doing wrong?*

[146] Because we’ll circle things on their homework, or on their Do-Now problems, or even in class, [147] and say, ‘that’s one little mistake that you keep making, and that’s something we’ve been talking about. You have to be careful with that.’ [148] And we’ll say, ‘no, that’s PEMDAS, you’ve got to go back to PEMDAS.’ Like, give them a specific strategy.

In this response, there are three coded segments. Segment 146 was coded as M, MK, representing medium view and mathematical knowledge. The participant is referencing methods employed in the classroom for assessing student understanding. Segment 147 was coded as M, KC, representing medium view and knowledge of curriculum. The participant is explaining how a certain misconception has been reoccurring and cautioning the students. Segment 148 was coded S, KC, representing short view and knowledge of curriculum. The participant references a specific mathematical concept as it occurs within the overall student learning trajectory. This multi-level coding style was employed throughout the interview transcripts for both participants to characterize their knowledge for teaching.
Thematic Analysis

The final step in the data analysis process was to establish themes for each participant in the areas of professional self-efficacy and knowledge for teaching, with particular attention to the role of Instructional Assessment. Hundreds of coded responses from both participants were entered into a spreadsheet to facilitate organization and sorting ability. Analysis of these responses generated several themes that were further organized based on two particular domains of interest: student engagement and curriculum management. Themes in these areas represent challenges faced by each participant in maintaining professional self-efficacy and knowledge for teaching. A third, overarching domain includes themes representing strategies employed by each participant to manage these challenges. A representation of this structure is provided in Table 8. As will be noted in the presentation of the case studies in Chapter IV, Instructional Assessment played an important role in the strategies discussed by each participant.

Specific themes were determined by examining the coded data for certain interests and concerns as reported by the participants in each domain of interest. Designated response segments were organized by scope code in an effort to represent multiple levels of context. Whenever a certain scope was not represented in the interview data, supporting evidence from classroom observation transcripts was substituted. This process resulted in the identification of primary and secondary themes for each participant, within each domain of interest, in reference to either professional self-efficacy and knowledge for teaching. Each theme was given an initial definition based on the collective interview and observation data, which was further refined during the writing of this dissertation. The case-study approach ensured that themes were designed to analyze the individual participants relative to their own experiences, and not in comparison or contrast to each other.
Table 8

*Thematic Analysis Matrix*

<table>
<thead>
<tr>
<th>Domains of Interest</th>
<th>Research Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Professional Self-Efficacy</strong></td>
</tr>
<tr>
<td>Student Engagement &amp; KCS</td>
<td>Themes representing challenges faced by each participant</td>
</tr>
<tr>
<td>Curriculum Management &amp; KCT</td>
<td></td>
</tr>
<tr>
<td>Overarching Themes</td>
<td>Themes representing strategies for managing the above challenges</td>
</tr>
</tbody>
</table>

**Study Parameters**

Several aspects of this study were altered from the initial description provided in the dissertation proposal. First, it was intended to recruit one novice teacher and one experienced teacher (as was done in the pilot study). The reasoning for this was twofold. First, researchers have noted that novice and experienced teachers report different sources of professional self-efficacy (Bandura, 1997; Tschannen-Moran & Hoy, 2007). Second, given the relatively recent introduction of formative assessment into standards-based curriculum frameworks (as described in Chapter II), it was hoped that the differences between teachers who had experienced contrasting policies toward classroom assessment (through training or professional development) could be highlighted. Instead, the two participating teachers both had eight years of high-school teaching experience. A third participant who had over twenty-five years of experience was initially secured, but after a few observations, she declined to participate in the study.

Another significant factor regarding the two participants was that they served as co-teachers in the same classroom. Initial conversations were intended to recruit multiple teachers
from separate classrooms, but these attempts were not successful. Both participants were members of the mathematics department; one was a subject-matter instructor, and the other was a special education instructor. This presented a unique challenge for data collection, as both instructors were often engaged with students at the same time. Similarly, interviews had to be tailored to each participant’s area of expertise and personal experience of classroom events. However, due to the high frequency of classroom visits, it was relatively easy to determine the various roles played by each participant over a short period of time. Furthermore, several observations involved only a single participant as lead instructor, while the other was absent from the room. This provided a layer of depth to the data analysis, as each participant’s experience could be viewed individually, or in contrast to the other.

One final change was related to the participants’ academic degrees. In the proposal, it was stated, “the participants will hold a state certification in high school mathematics and will have obtained, at a minimum, a Bachelor of Science degree in mathematics. This requirement is to ensure that each participant has had specialized preparation in mathematical content.” In retrospect, this requirement was needlessly specific, as there are many paths to state certification that do not include a Bachelor of Science degree. Both participants in this study were certified to teach 7 - 12 mathematics in Massachusetts, but neither held a Bachelor of Science degree in mathematics. Despite this, it was evident during observations and interactions that both participants were highly knowledgeable in the mathematics they were required to teach, and appeared very competent in their classroom management skills. Detailed descriptions of each participant, including academic credentials, is given in Chapter IV.

These limitations were unfortunate, but did not affect the overall goals of the research. Even though both teachers spent their entire careers during the current period of required state-
testing, the high-stakes component for high-school students was not introduced to their school district until a few years later. Therefore, they were able to speak firsthand about any changes this policy made to their classroom practices. Also, each teacher was instrumental in helping to craft the curriculum frameworks designed to meet the standards imposed by state testing. This further supports the case that each teacher was experienced in the development, instruction, and assessment of high-school mathematics, regardless of their specific academic degree.

**Researcher Bias**

In many senses, the notion of researcher bias is unavoidable. Every step, from the determination of the research topic to the choice of methodology and interpretation of results is influenced by the personal beliefs and values of the researcher (Mehra, 2001). The most obvious factor in this study is the position of the researcher as a mathematics educator, and former high-school teacher. Because of this, there are inherent sympathies to the demands placed on mathematics teachers, especially those from outside sources such as school districts and state standards. It should be noted that the initial choice to study teachers instead of students arose from a recognition of the unbalanced focus of assessment research. Major advances concerning student outcomes had recently been produced, but significant results as to how these changes might affect teachers in a positive way were not to be found. To the contrary, the results were oppositional, showing that teachers found Instructional Assessment to be time-consuming and difficult to master. The study proposed to examine the interplay between instructional decision-making and student understanding from the teacher's point of view. With this goal in mind, it was undeniably beneficial for the researcher to be identified as a member of the teaching community, rather than an administrator. Despite this potential conflict of interest, objectivity was maintained throughout data collection and analysis.
Validity and Reliability

This study was designed to ensure internal validity and reliability. Internal validity was initially driven by the theoretical perspective of the research, and further established through the triangulation of data sources (Johnson, 1997; Hammersley, 2008). As outlined in Chapter I, the interpretive framework for the emergent perspective allows for analysis of the individual and well as the social environment in which the individual exists. Thus, when interpreting the complex relationships regarding the use of Instructional Assessment, contributions from each participating teacher were considered alongside those of the students with whom they interacted. This notion of considering multiple perspectives was manifested in the data collection approach. By combining interview and observation data, as well as results from the TSES, relationships between what the teachers reported and what actually occurred in the classroom could be used to justify interpretations.

Reliability in qualitative research is often established through a well-crafted coding procedure (Brown & Dowling, 1998). This began with preliminary information gained from the pilot study that was used to design a constructive analytic framework. Primary coding sources were derived from the current research literature so that results could be aligned with recent developments in mathematics education. Multiple emergent codes were created to provide coherence and clarity to the results. Additionally, all coding procedures were checked and verified by a second education researcher. Finally, the structure of thematic analysis and interpretation was organized to highlight the potential benefits of Instructional Assessment as it occurs in a typical secondary-level classroom.
**Generalizability**

In statistical research, the generalizability of results depends largely on the random selection of participants, such that data collected from few can be used to represent many. This is not the case with qualitative research, since attempts are rarely made to utilize a representative sample. However, the questions raised by a qualitative study can still be generalized insofar as the activities being performed by the participants are relatively commonplace. Gobo (2004) refers to this as the representativeness of the case characteristics, rather than of the case itself. Thus, even though the participating teachers, their school district, the curriculum, and many other details of this study are not representative of a larger population, the results of the study are grounded in the nature of Instructional Assessment. Mathematics education research has shown that the elements of formative assessment are found even in classrooms where teachers have not taken part in professional development. Furthermore, highlighting the effects of Instructional Assessment on high-school mathematics teachers in the United States broadens the current research base on assessment, and opens the door for future studies involving larger, or more representative participants.
CHAPTER IV

PRESENTATION OF THE CASES

This chapter will describe the setting and participants for the case studies that were performed for this dissertation. Detailed descriptions regarding the community, school, and classrooms will be provided. To complement the co-teaching aspect of this study, a synopsis of daily classroom activities will be presented prior to the introduction of the individual cases. This information will serve as an important contextual intersection shared by the participating teachers. Following this overview, a full account of each case study participant will be given. The narratives provided herein (Davidson, 2003) will help to situate the interpretations presented in Chapter V (Stake, 2005). Pseudonyms are used to protect the identities of the participants and students throughout this dissertation. The two participant teachers are referred to as Mr. Patrick and Mrs. Kline, and the name of the school is Forest River High School. Students are never referred to by name or pseudonym, but rather by letter, indicating the order in which they are speaking during classroom interactions. All other identifying details, including course topics, grade levels, and daily activities have been presented as they were recorded in the data.

The Community, School, and Classroom

This study was performed in a large city in Massachusetts. Forest River High School is a public secondary school, enrolling over 1500 students from the surrounding area. One noteworthy characteristic of the student population is its ethnic diversity. Census data for the city lists two-thirds of the population as Caucasian, a figure that is very similar to the population
of the entire country. In contrast, at Forest River, only one-third of the students are Caucasian. In fact, the school has been described as one of the most ethnically diverse in the United States (Kolker, 2013). This has been attributed to the fact that there are several private schools and academies nearby that tend to draw the Caucasian population away from the public schools. Forest River is well-respected in the community, and maintains educational program partnerships with local Universities. Additionally, at the time of this study, a large construction project was taking place to renovate nearly every building on the campus. Despite this, the grounds and indoor facilities were kept in good condition.

All observations were performed in the same classroom, except for two that took place in a nearby computer lab. The setup of the classroom was largely traditional in nature. Several rows of student desks were arranged neatly in the center of the room, all facing the front whiteboard. The number of desks exceeded the number of students by at least ten, and it was typical for the last row of desks to be entirely empty. Furthest from the only entrance to the room were the two teachers' desks. Mr. Patrick's desk was along the front wall of the room, immediately adjacent to the whiteboard, and Mrs. Kline's desk was toward the back of the room. Both teachers had desktop computers that were used regularly during instruction to access documents, diagrams, and web content such as pictures and videos that could be projected onto the whiteboard. Mr. Patrick also made frequent use of his computer to set alarms for class activities with predetermined time limits. The prevalence of technology in the classroom extended to the side nearest the entrance. Three student computers were set up, but were not used during the observations that were performed. These were primarily reserved for after-school work and independent student research.
Daily Schedule and Classroom Environment

Given the highly-detailed analyses of these case studies, it is important to describe the context surrounding classroom events. Forest River High School had been employing a block scheduling format for over five years prior to this study. Students attended four, eighty-four minute class periods each day, in addition to shorter periods for homeroom and lunch. Both teachers taught classes during three of the four blocks and used the remaining block for planning and administrative purposes. Each teacher was observed multiple times during two blocks of Geometry and one block of Algebra II. Both courses are considered sophomore-level, although some of the students in each class were juniors or seniors. The observations for this study coincided with the Spring semester, beginning in February, and concluding in May, just prior to the administration of state-level testing throughout high schools in Massachusetts.

On a daily basis, classes began with a series of review problems posted on the overhead for students to try immediately (called “Do Now”). These problems were often unrelated to that day's lesson, and sometimes came from different subject areas (i.e., simplifying algebraic expressions in a geometry class). Mr. Patrick typically made use of the alarm on his computer to ensure that the students worked on the problems for a limited amount of time, typically five minutes. Following this activity, the solutions would be briefly discussed with the entire class. This was usually followed by a routine check of the previous night's homework. Both teachers would walk through the rows of students, examine their progress, respond to any questions, and record grades. The students were then allowed to call out any problem numbers that they wanted to see explained. In each case, time was allotted to work out the details on the whiteboard.

The objectives for each day's lesson were written on the board at the beginning of the day so that students could readily see the schedule upon entering the room. Lectures were generally
well-organized and required the students to be actively engaged. Both teachers would often draw upon the students’ prior knowledge by referring to established concepts prior to expanding or introducing new topics. During these interactions, the students would raise their hands to ask questions and respond to teacher prompts. Students would also work individually or in groups for short periods of time. During these activities, Mr. Patrick would again make use of the alarm clock to ensure that class time was being used efficiently. Disruptions, such as students being inattentive or talkative were unusual, and those that did occur were generally resolved quickly, and without further incident.

**Co-Teaching Model**

The most important characteristic of this study is the co-teaching relationship of the participants. Mr. Patrick served as the subject-matter expert in the room, and was generally regarded to be the lead teacher. He was most frequently the one to begin classes, make announcements, and engage in classroom activities. Mrs. Kline’s role was more supportive in nature, as the special education expert in the room. Due to the complex responsibilities of her position, she was often required to leave the classroom for short periods of time, either alone or with a student, to deal with administrative matters. During these occurrences, Mr. Patrick remained as the sole teacher in the room. However, it should be noted that there were several occasions where Mr. Patrick was required to attend to something outside of the classroom. During these episodes, Mrs. Kline remained as the sole teacher in the room, and there was no discernable difference in the students’ demeanor.

The most common situation to occur during observations was with both participants in the room, actively engaged with students. The instructional approach generally involved one teacher leading the discussion with the other teacher supporting or elaborating on statements,
questions, and examples. Mr. Patrick and Mrs. Kline had been co-teaching in this manner for four years, and the comfort level that they had developed was readily apparent. Their demeanor in between classes was cordial and friendly, and they always displayed mutual respect and shared in decision-making processes.

**State-Testing Information**

The Massachusetts Comprehensive Assessment System (MCAS) was established in 1998 and was designed to meet the requirements set forth by the Education Reform Law of 1993. Initially, students in grades 4, 8, and 10 were assessed in English Language Arts (ELA) and Mathematics. Since that time, additional grades and content areas have been added to the testing program. The No Child Left Behind Act of 2001 (NCLB) established standards for state testing, requiring students in grade 10 to pass the ELA and Mathematics MCAS tests prior to receiving a high school diploma (often referred to as the “high-stakes component). Additionally, schools and districts are held accountable for student progress, and are rated annually based on performance standards.

Data regarding Forest River's standing based on MCAS results is presented to provide further insight into related events in the classroom. At the time of this study, Forest River held an Accountability Status of "Improvement - Year 2" in mathematics. This indicates that the school had previously failed to meet Average Yearly Progress two years prior as outlined by the No Child Left Behind Act. NCLB allows two years for schools to meet target improvement when yearly progress goals are not met, after which corrective measures are enacted if the target is not reached. The results from the previous year showed that Forest River had improved, but below the target that was set. Thus, during this study, the mathematics faculty at Forest River
were facing their final opportunity to satisfy the NCLB requirements before having corrective measures initiated by the state.

The impact of this situation played an undeniable role in everyday classroom activities. Time was often allotted to discuss sample MCAS problems, many of which were unrelated to the topic of instruction for that day. Students were encouraged to recognize regular themes found in MCAS questions such as presentation style, grading rubric, and open-response requirements. Homework assignments were frequently supplemented with additional MCAS-prep materials. These problems, along with material from the daily lessons were often presented as potential MCAS questions. For example, consider the following interaction between Mr. Patrick and several students in a Geometry class, recorded early in the semester.

Mr. Patrick: So, in this number one, as she's kind of putting up her work, how many different shapes are there? And there's a couple different answers I'm looking for [students begin raising their hands]. I saw Student A, then Student B, and then Student C.

Student A: There's a triangle.

Mr. Patrick: There's a triangle. Okay, Student B, do you see triangles?

Student B: Yeah, I see a rectangle.

Mr. Patrick: You see a rectangle? And then, Student C, what do you got?

Student C: Same thing.

Mr. Patrick: Same thing? What is the total outside shape? Because you actually had to find the perimeter, but you're not necessarily using the perimeter of the rectangle or the triangles. What's the actual shape that you're finding the perimeter of?

Student C: A hexagon.
Mr. Patrick: An irregular hexagon. Awesome. So, this is a really, I can't remember, I think this was actually an MCAS problem a couple years ago. They will ask you these layered questions about area and perimeter, and not necessarily just give you one shape. They'll give you a layered shape, like this.

In a similar interaction between Mrs. Kline and students in the same Geometry class, emphasis is placed on the correct formatting of responses on the MCAS. The problem being considered involved analyzing a pie chart derived from a list of data (student ages). This interaction was recorded less than a week before the Mathematics MCAS exam was to be administered.

Mrs. Kline: Because this is a short answer question, we’ll... what specifically would your answer have to say? Student A?

Student A: Half of 360 is 180. And then, when you make the 180 degree on the pie chart, it lands on 17 years old.

Mrs. Kline: Okay, so then let me rephrase my question and get specific. On the MCAS, when you do multiple choice, what does your answer look like?

Student A: A dot.

Mrs. Kline: Okay, so you dot, fill it in, and move forward. When you’re doing short answer, what is your answer supposed to look like?

Students: [several speaking over each other]

Mrs. Kline: It’s like a number with units, basically. It’s literally a box like this, and you write a number and some units, and that’s it. You don’t really give them your explanation at that point. So, our number and units to answer this question is specifically 17 years altogether, done! Because it says, ‘in years’ in the question. If you’re not giving them ‘years,’ it’s kind of like you’re not completing the answer. So, you need to give them that information.
Episodes such as this were a common occurrence during classroom observations, reaffirming the impact of state-testing on daily instruction. Given the prevalence of these types of interactions, it is essential to consider their impact on daily instruction.

**Case Study Outline**

In the following sections, a detailed analysis of each case study participant is presented. The introduction provides relevant background information and general comments on personal demeanor and teaching style. This is followed by detailed description of the coding process in the two main areas of interest for this study: professional self-efficacy and knowledge for teaching. Descriptive statistics for all codes are provided, and the emergent themes are supported by interview responses and classroom episodes. Data from the TSES (Tschannen-Moran & Hoy, 2001) is included to supplement themes regarding professional self-efficacy. Contributing examples of Instructional Assessment are included wherever possible, but a full interpretation of the findings is saved for Chapter V.

**Case Study 1 - Mr. Patrick**

At the time of this study, Mr. Patrick had eight years of fulltime high school teaching experience, seven of which were at Forest River. He holds a Bachelor’s Degree in Mathematics Education from an accredited university in the United States, and a Massachusetts State Teaching License in Secondary Level Mathematics. He is an active member of the faculty at Forest River, having served on curriculum development committees and as advisor for several student organizations. Mr. Patrick stated that he feels comfortable teaching high school students and believes that he will stay at that level for the foreseeable future.

Mr. Patrick’s professional demeanor is very open and honest. He is friendly and relaxed with his students, and he treats them with respect and a genuine interest in their day-to-day lives.
During class observations, he would take time to review classroom policies and discuss issues of concern at the school whenever necessary. These ranged from minor problems or disputes among individual students to school-wide events and activities. In every case, Mr. Patrick demonstrated leadership qualities and respect for his position in the school. His students frequently returned this respect with their behavior and work ethic. Instances of disciplinary action were rare, and the majority of class time was focused on academic matters.

Despite his consistent attention to issues of timing and the sequence of class activities, Mr. Patrick did not seem to adhere to a strict pattern of instruction. Class discussions based around a single concept or problem would frequently change pace or direction based on student responses. Whenever it seemed that students were not following his train of thought, Mr. Patrick would take time to approach the problem from a new perspective. In contrast, when the students seemed comfortable with the material, he would move on without fear of leaving anyone in the class behind. Episodes of Instructional Assessment were very common during each class observation, and Mr. Patrick's willingness to engage students in instructional conversations made him a particularly appropriate participant for this study.

The following sections present detailed findings from the data analysis performed for Mr. Patrick's case study. Coding and thematic analysis is provided in the areas of professional self-efficacy and knowledge for teaching according to the methods described in Chapter III. Interview responses and classroom episodes are included as supporting evidence, with particular attention to the impact of Instructional Assessment.

**Professional Self-Efficacy**

Mr. Patrick faces a variety of challenges over the course of an academic year. His perceived ability to successfully navigate these challenges on a daily basis encompasses his
professional self-efficacy. Codes derived from interview responses revealed important insight into the ongoing tension between attending to student needs and adhering to curriculum standards. Descriptive statistics from the coding process are presented in the tables below, followed by the development of themes for professional self-efficacy. Scores from the TSES are included as a quantitative component of the data analysis. Results in this area will be used to answer the first research sub-question: How does the integration of assessment into instruction relate to the professional self-efficacy of mathematics teachers?

**TSES Data.** Mr. Patrick's overall pre- and post-observation scores on the TSES as well as sub-scores in the areas of Student Engagement, Instructional Strategies, and Classroom Management are listed on Table 9. The results, especially in the pre-observation survey, seem to further support a high level of professional self-efficacy. A score of 7 on the survey indicates that the teacher feels he can do "quite a bit" to navigate the difficulties that are presented on a daily basis. Mr. Patrick's sub-scores in Instructional Strategies and Classroom Management were consistently above this threshold.

Table 9

*TSES Data for Case Study 1*

<table>
<thead>
<tr>
<th></th>
<th>Student Engagement</th>
<th>Instructional Strategies</th>
<th>Classroom Management</th>
<th>Overall Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Observation</td>
<td>7.000</td>
<td>7.500</td>
<td>7.500</td>
<td>7.333</td>
</tr>
<tr>
<td>Post-Observation</td>
<td>5.875</td>
<td>7.375</td>
<td>7.500</td>
<td>6.917</td>
</tr>
</tbody>
</table>

However, in the area of Student Engagement, Mr. Patrick recorded his lowest scores. The pre-observation average was 7.000, and the post-observation average was 5.875. A score of 5 on the survey indicates that the teacher feels they have "some influence" over the difficulties
that are presented on a daily basis. This indicates that Mr. Patrick may have had some doubts concerning his ability to work with individual students, especially toward the end of the semester. This result was at odds with many of his claims during the interviews, but could be explained by the MCAS falling within a week post-observation.

**Codes.** In total, 190 interview response segments categorized under Professional Self-Efficacy (PSE) or Summative Assessment (SA) were coded according to three descriptors: scope, general self-efficacy, and teacher self-efficacy. Scope codes and general self-efficacy codes were useful for organizing interview response segments for thematic analysis across multiple contexts. For example, when a particular theme was underrepresented at a specific scope, additional evidence was obtained from the observation transcripts. Using this method ensured a triangulation of data sources for understanding Mr. Patrick’s professional self-efficacy. Teacher self-efficacy codes provided a final layer of detail regarding specific classroom practices, and were the primary sources for the creation of themes.

The first set of codes were designed to describe the scope, or contextual breadth of Mr. Patrick’s statements. Short-view (S) responses referenced individual students, while medium-view (M) responses described entire classrooms of students, or students in general terms. Long-view (L) responses included general statements about the teaching profession. Scope code frequencies are listed in Table 10.

Table 10

**Scope Codes for Case Study 1 (Professional Self-Efficacy)**

<table>
<thead>
<tr>
<th>Code</th>
<th>Name</th>
<th>Context</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Medium</td>
<td>Classrooms of students (or students in general)</td>
<td>125</td>
</tr>
<tr>
<td>L</td>
<td>Long</td>
<td>The teaching profession</td>
<td>50</td>
</tr>
<tr>
<td>S</td>
<td>Short</td>
<td>Individual students</td>
<td>15</td>
</tr>
</tbody>
</table>
The majority of Mr. Patrick’s interview responses (66%) were coded as medium scope. This is not surprising, since many of the interview questions were stated in general terms related to his teaching. Long-scope responses (26%) were the second-most common. In most cases, these statements represented Mr. Patrick’s feelings and beliefs about the current state of mathematics education. All remaining responses (8%) were coded as short-scope. However, it should be noted that short-scope response segments were often lengthy, since they primarily focused on specific, detailed interactions with individual students.

The second set of codes were selected from Bandura’s four components of self-efficacy (1977), and were intended to describe the general area of self-efficacy in each interview response segment. A fifth code (RC) was added from Smylie (1999) to reflect the notion of role conflict that exists in educational settings. General self-efficacy code frequencies are listed in Table 11.

Table 11

<table>
<thead>
<tr>
<th>Code</th>
<th>Name</th>
<th>Context</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA</td>
<td>Performance Accomplishments</td>
<td>Recollection of personal experiences</td>
<td>166</td>
</tr>
<tr>
<td>VE</td>
<td>Vicarious Experiences</td>
<td>Observation of the experiences of others</td>
<td>13</td>
</tr>
<tr>
<td>RC</td>
<td>Role Conflict</td>
<td>Perceived inconsistencies with job roles</td>
<td>6</td>
</tr>
<tr>
<td>VP</td>
<td>Verbal Persuasion</td>
<td>The effects of social interaction</td>
<td>4</td>
</tr>
<tr>
<td>EA</td>
<td>Emotional Arousal</td>
<td>Physiological reactions to stress</td>
<td>1</td>
</tr>
</tbody>
</table>

The majority of Mr. Patrick's interview responses (87%) were coded as Performance Accomplishments. This makes sense, since he was largely responding to questions based on his own experiences. Vicarious Experiences had the second-highest frequency (7%). This number may have been even lower if not for the unique co-teaching aspect of this study. Despite the high frequency of Performance Accomplishment responses, these general self-efficacy codes
were helpful in isolating statements from Mr. Patrick regarding the experiences of other instructors and issues related to role conflict. This, in turn, was beneficial in creating a complete picture of Mr. Patrick’s professional self-efficacy.

Table 12

*Teacher Self-Efficacy Codes for Case Study 1*

<table>
<thead>
<tr>
<th>Code</th>
<th>Name</th>
<th>Context</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC</td>
<td>Student Comprehension</td>
<td>Gauging student comprehension</td>
<td>44</td>
</tr>
<tr>
<td>CS</td>
<td>Curriculum Standards</td>
<td>Identifying curriculum standards</td>
<td>33</td>
</tr>
<tr>
<td>AL</td>
<td>Adjusting Lessons</td>
<td>Adjusting lessons for individual students</td>
<td>20</td>
</tr>
<tr>
<td>ST</td>
<td>Shared Teaching</td>
<td>Shared teaching responsibilities</td>
<td>19</td>
</tr>
<tr>
<td>BO</td>
<td>Burnout</td>
<td>The prospect of teacher burnout</td>
<td>13</td>
</tr>
<tr>
<td>TP</td>
<td>Time Pressure</td>
<td>Dealing with limited time in the classroom</td>
<td>11</td>
</tr>
<tr>
<td>DQ</td>
<td>Difficult Questions</td>
<td>Responding to difficult student questions</td>
<td>10</td>
</tr>
<tr>
<td>AV</td>
<td>Assessment Variety</td>
<td>Using a variety of assessment methods</td>
<td>7</td>
</tr>
<tr>
<td>DS</td>
<td>Difficult Students</td>
<td>Getting through to difficult students</td>
<td>7</td>
</tr>
<tr>
<td>AS</td>
<td>Alternative Strategies</td>
<td>Implementing alternative strategies</td>
<td>5</td>
</tr>
<tr>
<td>SV</td>
<td>Student Values</td>
<td>Helping students to value learning</td>
<td>4</td>
</tr>
<tr>
<td>CE</td>
<td>Clear Expectations</td>
<td>Making expectations clear about behavior</td>
<td>3</td>
</tr>
<tr>
<td>IU</td>
<td>Improving Understanding</td>
<td>Improving understanding of failing students</td>
<td>3</td>
</tr>
<tr>
<td>SB</td>
<td>Student Beliefs</td>
<td>Getting students to believe they can do well</td>
<td>3</td>
</tr>
<tr>
<td>CQ</td>
<td>Crafting Questions</td>
<td>Crafting good questions for students</td>
<td>2</td>
</tr>
<tr>
<td>CT</td>
<td>Critical Thinking</td>
<td>Getting students to think critically</td>
<td>2</td>
</tr>
<tr>
<td>MS</td>
<td>Motivating Students</td>
<td>Motivating students with low interest</td>
<td>2</td>
</tr>
<tr>
<td>AE</td>
<td>Alternative Explanations</td>
<td>Providing alternative explanations</td>
<td>1</td>
</tr>
<tr>
<td>DB</td>
<td>Disruptive Behavior</td>
<td>Controlling disruptive behavior</td>
<td>1</td>
</tr>
</tbody>
</table>
The third set of codes was taken from the long version of the TSES (Tschannen-Moran & Hoy, 2001). Of the twenty-four codes derived from this document, fifteen were recorded at least once in the data. Four additional codes (Burnout, Curriculum Standards, Shared Teaching, and Time Pressure) were included when interview response segments emerged from the data that were not represented in the framework. All nineteen teacher self-efficacy codes are listed and defined in Table 12, along with their frequencies. Two codes stand out from the rest: Student Comprehension (23%) and Curriculum Standards (17%). This reflects Mr. Patrick's recurring descriptions of student understanding and how it related to the material he was teaching. Adjusting Lessons (11%) and Shared Teaching (10%) were the only other codes to appear in at least ten percent of the interview responses. The general distribution of teacher self-efficacy codes shows that Mr. Patrick provided statements concerning a wide variety of issues affecting classroom instruction. Additionally, several of the more prevalent codes (including Student Comprehension, Adjusting Lessons, Time Pressure, and Difficult Questions) provided important insight into the relationship between professional self-efficacy and performing Instructional Assessment. A full thematic analysis is presented in the next section.

**Themes.** All coded interview response segments from Mr. Patrick were entered into a spreadsheet to facilitate organization and sorting ability. Further analysis yielded several themes representing challenges in the domains of student engagement and curriculum management, as well as overarching themes representing specific strategies employed by Mr. Patrick to navigate these challenges. A list of each theme and its definition is provided in Table 12. In the sections that follow, each theme is described in detail, along with representative examples from the interview and observation data. A full interpretation of this portion of the case study is presented in Chapter V.
**Student Engagement.** Mr. Patrick expressed a high level of confidence when working with students in the classroom. He based this largely on his years of experience and accumulated knowledge of the students in the Forest River school district. When discussing the specific factors that contributed to Mr. Patrick's self-efficacy in this domain, two major themes emerged from the data: Anticipation and Student Background. The main codes represented in these themes include Student Comprehension, Student Beliefs, and Student Values.

Table 13

*Professional Self-Efficacy Themes for Case Study 1*

<table>
<thead>
<tr>
<th>Domain</th>
<th>Theme</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Engagement</td>
<td>Anticipation</td>
<td>Predicting common student errors and areas of difficulty in mathematics</td>
</tr>
<tr>
<td></td>
<td>Student Background</td>
<td>Prior knowledge base and willingness to engage in mathematical discussions</td>
</tr>
<tr>
<td>Curriculum Management</td>
<td>Sequencing</td>
<td>The organization and pacing of mathematical topics</td>
</tr>
<tr>
<td></td>
<td>Consistency</td>
<td>Teaching mathematics courses for multiple years to achieve mastery</td>
</tr>
<tr>
<td>Overarching Themes</td>
<td>Balancing</td>
<td>Attending to student needs while adhering to curriculum standards</td>
</tr>
<tr>
<td></td>
<td>Negotiating</td>
<td>Promoting student autonomy in the classroom</td>
</tr>
</tbody>
</table>

*Anticipation.* The first theme to emerge in the area of student engagement is *anticipation*, which represents instances or descriptions of Mr. Patrick predicting common student errors and areas of difficulty in mathematics. Anticipation plays an important role in the structure of classroom discussions (Smith & Stein, 2011), and can determine choices made by teachers when planning their lessons. NCTM (2014) cites anticipation as the first step toward fostering "productive struggle in learning mathematics" (p. 52). It should be noted, however, that the
same document fails to mention the role of teacher confidence or self-efficacy in the development or application of anticipation in the classroom. One of the primary goals of this study is to generate an understanding of this relationship.

When asked about his ability to assess student understanding during instruction, and whether or not this impacted the direction of the class, Mr. Patrick cited his seven years of experience with Forest River students as being key to the anticipation of student misconceptions. He noted,

At this point, I do have sort of an understanding of where they're going to be confused, and where I might have to slow down, or speed up, and that sort of thing. Rarely, at this point, do I have major, like, whoa, we need to spend a second day on that, type moments. (Interview #1)

He also stated, however, that his ability to understand the viewpoint of the students was aided, in part, by not having multiple decades of experience. That is, he felt that he had not lost touch of the student experience. He stated,

I am a fairly young teacher. I'm not necessarily with 20, 30 years’ experience, like some of my colleagues. I am able to see my students a little bit more, and hear, anticipate how they're going to see things. (Interview #1)

In this sense, Mr. Patrick’s portrayed himself as having developed an ongoing connection with the typical student experience at Forest River.

Despite a high level of general confidence in the area of anticipation, Mr. Patrick admitted that considerable time was required to master the understanding of individual students. He claimed, “It usually takes me, minimum a month, closer to two or even three months, to really, like, I've got you. I know what you're doing and what you're thinking” (Interview #2). This indicates that the development of self-efficacy in anticipation was a continuous process that was being supplemented and revised with each day of instruction. This process was aided by the
inclusion of regular classroom discussions and student-teacher interactions that were a standard teaching approach employed by Mr. Patrick. He claimed,

I try to make sure that it's not necessarily me doing a lot of the talking, it's them. Like, I might be doing some clarification, and I have to step in there, but I'm trying to make sure I'm asking them probing questions, to see what they're thinking. Asking them to explain, support, and back things up, and not just give short answers, that they can give me a complete thought. (Interview #1)

This depiction of Mr. Patrick’s method of questioning students was confirmed during classroom observations.

**Student Background.** The second theme that emerged in the area of student comprehension is student background, which represents Mr. Patrick’s accumulated perspective of students’ prior knowledge bases and their willingness to engage in mathematical discussions. This theme initiated from an interview question centered on the perceived barriers to effective instruction: Mr. Patrick responded, “Sometimes, it's students' prior knowledge. Students have certain skill sets, but also, there are gaps in other areas, and they're sort of just barely grasping onto content” (Interview #1). This presented a daily challenge to Mr. Patrick in navigating the range of abilities among the students in his classroom. This effect was particularly pronounced for students with documented learning disabilities.

From an initial recognition of the limitations in students' prerequisite skills in mathematics, the scope of this theme was expanded to include student beliefs and values concerning mathematical exploration. Mr. Patrick described the transition from middle school to high school as being particularly challenging for student confidence and attitude toward mathematical understanding. He stated,

I think, through middle school and elementary school, there's a lot of hand-holding and spoon-feeding, and I don't see the same maturation. ...they get to high school, and there's suddenly this, like, it's on you. And they have a hard time with that. (Interview #2)
In this response, Mr. Patrick is highlighting the increase in student responsibility that is typically expected in high school. He does not see his students as being at an appropriate level of academic maturity, where the onus is on them to do the work and actively engage in classroom activities.

Mr. Patrick frequently encouraged his students to take ownership of their learning, and not to rely too heavily on the teacher when attempting to answer to a problem. He described the difficulties he had in reaching his current students in this regard. When asked how well he thought students typically responded to his guidance, he said,

Fairly well. I mean, I think, there's initial hesitation. One, because of their own confidence, or lack thereof, with math. I'm finding this group does a lot more of the, ‘I don't know exactly what to do, so I'm going to stop completely.’ It's not like, in years past, we'll see, ‘I don't know what to do, exactly, but I'm still going to try something.’ (Interview #2)

Despite these limitations, Mr. Patrick seemed to embrace the challenges of working with students, both in groups and individually. When asked about identifying and resolving student mistakes, he remarked upon his pleasure in sharing responsibility through classroom discussions. He stated,

…when that happens [identifying student mistakes], I really enjoy that, when I get stumped on things like that, because that's, the conversations that come out of it, I find really valuable for the student’s growth. Because, a lot of times, I'm able, it's literally, for me, like proofreading a paper and how, sometimes, you work with your professor, or someone, and it's like a back-and-forth sharing, in a way. (Interview #2)

As with the theme of anticipation, Mr. Patrick's statements regarding student background point toward classroom discussions and student-teacher interactions as primary sources of motivation. Incorporating these components of Instructional Assessment into his teaching style acted as a catalyst for his own self-efficacy development. Moreover, by working directly with students in
the classroom and building his understanding of student background, Mr. Patrick simultaneously developed his ability (and confidence) to correctly anticipate student difficulties.

**Curriculum Management.** Along with the daily challenges centered around student engagement, Mr. Patrick described specific curricular requirements of the courses he taught at Forest River that influenced classroom instruction. Additionally, several instructional conversations regarding the importance of certain mathematical topics were recorded during the classroom observations, in particular concerning how these topics would be assessed for understanding. When discussing the specific factors that contributed to Mr. Patrick's self-efficacy in the domain of curriculum management, two major themes emerged from the data: Sequencing and Consistency. The main code represented in these themes was Curriculum Standards.

**Sequencing.** The first theme that emerged in the domain of curriculum management was *sequencing*, which represents Mr. Patrick's perspective of the organization and pacing of topics within the curriculum. He expressed a general comfort level with the mathematical content of the courses he had been teaching at Forest River, and attributed this largely to an understanding of the underlying structure involving repetition and review.

My approach to teaching is a lot of cycling, of looping, of moving forward, but at the same time, using some Do Now time, or even homework, to kind of loop back, or even sometimes examples that might look at a review concept, but still address something that we need to talk about. (Interview #1)

The use of physical action words such as cycling, looping, sequencing, and pacing indicate that Mr. Patrick had a well-developed conceptual organization of mathematical topics. He felt strongly that the best way to teach these topics was to move in a non-linear manner, frequently reviewing and re-examining previously learned material. He often reminded students when certain topics had been previously covered, when they would be revisited, and why they were
important for a deeper understanding of mathematics in general. In this sense, Mr. Patrick’s approach to teaching new concepts often began by reintroducing previously learned material.

In the following observation, taken from an Algebra II class, Mr. Patrick relates introductory vocabulary for matrices with the previously learned arithmetic concept of additive inverses.

Mr. Patrick: Let me back up. Additive inverses. You know this as what other word? If I'm talking about two plus a negative two equals zero.

Student A: The zero identity?

Mr. Patrick: Okay, these are the additive inverses, right now, okay? The additive inverses. What do you know these things as? This is what [another student] said, earlier.

Student A: Opposites.

Mr. Patrick: Opposites. So, that's how I actually want to do that, okay? And I know I'm doing a lot of talking right now. I really appreciate those people participating right now. Multiplicative inverse was, was this idea. What word do you know to describe that?

Student B: Reciprocal.

Mr. Patrick: So, this is the reciprocal idea. So, and that's important. If you don't have this written down, you're going to lose it, and you have to use it. So, then, where are the actual reciprocals? Reciprocal? Where are the reciprocals, in our example?

Student B: I don't know, A and B?

Mr. Patrick: A and B. Those are the reciprocals?

Student B: Yes.

Mr. Patrick: These are the multiplicative inverses, or reciprocals. Whatever you want, okay? And you're getting to the point where you're not necessarily going to hear the words opposites or reciprocals, as often. You're going to hear,
more, the additive inverses, multiplicative inverses, that type of stuff. So, the same concept, same sort of idea that you know, it's just a different word in math.

In this teaching episode, Mr. Patrick is not only introducing the concepts of additive inverse and multiplicative inverse, he is also connecting these terms to students' established understanding of opposite and reciprocal. He emphasizes the idea that the students should expect to see these terms more frequently as they progress through the material. In this sense, Mr. Patrick is making connections the different vocabulary terms and explaining how they are sequenced in the curriculum. By connecting to previously learned terms, he is passing on his understanding of the sequencing to the students. This episode also demonstrates methods of Instructional Assessment embedded within the questioning structure. Instead of simply defining the concepts of "additive inverse" and "multiplicative inverse," Mr. Patrick relied on the students to provide the terms "opposite" and "reciprocal." If they had been unable to recall these terms, that would have indicated that further review was necessary. However, when the students readily responded, he determined that it was possible to continue with the main lesson. This could not have been easily accomplished without a deep understanding of sequencing.

**Consistency.** The second theme that emerged in the area of curriculum management was *consistency*, which represents Mr. Patrick's perspective of teaching the same courses for multiple years to achieve mastery. During both interviews, Mr. Patrick mentioned the importance of the course selection process, indicating that it was helpful for him to teach the same courses for several consecutive years. He stated,

Next year, if I have completely new courses, there will be, probably, some more of, like, backup plans being made, of, this is what I want to do, but this is what might happen, if such-and-such goes down in the period. I'll have to have a little bit more flexibility with that. But, as of right now, not so much. (Interview #1)
One reason for this decision was that it helped him to see where students typically had trouble in certain areas. In this sense, it allowed Mr. Patrick to anticipate student errors. He described how this ability to discover “patterns of misconception” was driven by consistent course selection.

It would be difficult, for me, as a teacher, to jump from course to course, through the years, because I’d sort of never get to see a pattern. Like, if I taught, one year, Geometry, and then, another one, Pre-Calc, and the Calculus, and then go back to Algebra I, I don't get to see patterns of misconception. And I think that's really important, for me, to be able to address what those are, and kind of help with those misunderstandings. (Interview #2)

Additionally, teaching the same courses for multiple years also helped Mr. Patrick to develop a deeper understanding of each course’s structure. Having followed this approach for seven years at Forest River, Mr. Patrick was able to better see the interrelationships between multiple courses. He stated,

I think it helps me to just look at a course, for a while, and then move on. Because, I get to see scope and sequence and sort of, like, okay, this is how I taught Geometry, but where is how I taught it a problem for Pre-Calc? (Interview #2)

The themes of sequencing and consistency are also related to the use of Instructional Assessment. Teaching the same courses from year to year helped Mr. Patrick to develop a deep understanding of mathematical relationships. This increased his knowledge base (similar to Ma’s (1999) notion of connectedness) and improved his ability to hold effective mathematical discussions during instruction. He was better equipped to recall similar conversations from recent semesters, and could then modify them where necessary. Switching to different courses after several years expanded these abilities and helped Mr. Patrick to discover common threads across multiple subject areas. Over time, his professional self-efficacy gradually progressed,
motivating further opportunities for student participation to determine the pace and direction of instruction.

**Overarching Themes.** The themes presented thus far represent challenges experienced by Mr. Patrick in the domains of student engagement and curriculum management. Two additional themes emerged from the data representing specific strategies employed by Mr. Patrick to navigate these challenges: Balancing and Negotiating. The following sections provide evidence for these overarching themes, and illuminate some of the ways in which Mr. Patrick managed the daily demands of his position while helping students to master course material. The main codes represented in these themes include Student Comprehension, Curriculum Standards, Adjusting Lessons, and Assessment Variety.

**Balancing.** The first overarching theme is *balancing*, which represents Mr. Patrick's perspective of attending to student needs while adhering to curriculum standards. Balancing is considered to be largely a reactive approach to the realities of teaching. Mr. Patrick initially structured each semester based on course requirements, while allowing for adjustments based on the needs of his students as time went on. He expressed a general level of confidence in his ability to balance student needs and curriculum standards. He attributed this primarily to his years of experience with the students at Forest River High School. When describing his role, he responded,

> I look at it, my role is sort of balancing what students I have in front of me, where are they at, what can they access, how are they accessing it? And then, sort of trying to balance that with, as a department, what do we want? What do we want, what do we value, as a staff in the building? ... I would probably say I'm 90-95% confident that I'm successful in that balancing. (Interview #1)

In this statement, Mr. Patrick notes the dual-nature of his position, and points to the abilities of the students as the guiding force behind instructional decision-making.
One specific strategy employed by Mr. Patrick was the use of Instructional Assessment through active questioning. This was particularly evident when reviewing student solutions to homework or in-class problems. Rather than simply providing the answer to a given problem, or writing the solution steps on the board, Mr. Patrick would have student volunteers present and explain their reasoning. The following observation, taken from an Algebra II class, demonstrates this approach.

Mr. Patrick: We started out with the system, and it was $3x + 2y = 6$, and $3x + 3 = y$. So, what method would be easier here, right now, looking at it? Let’s start there. And what I’m looking, what I’m anticipating, as an answer, right now, is either substitution or elimination.

Student A: Substitution.

Mr. Patrick: Okay. Why are people saying substitution there? I agree with that, in this case.

Student A: Because of the $y$. It already has $y$ equals something, so you can just plug it in there.

Mr. Patrick: [Addressing class] Did you hear that?

Student A: We already have $y$ equals something, so you can just plug it into the other equation.

Mr. Patrick: Brilliant. And, sometimes, you might not actually have that. Like, you had one, I can’t remember which station it was, but it was like, you might actually have something that looks like this [writes a new system on the board]. And I would still probably choose substitution, even if one of your equations looks like this. Like, right now, we have $y$ equals, it’s almost like slope-intercept form, right?

Student A: Mmm-hmm.

Mr. Patrick: But why would this be easy to solve for? What would make that easy?

Student B: You can solve for $y$. 


Mr. Patrick: Yeah, right? We can do that quickly, rearrange there, too. Okay? So, then, he’s doing that right here. So, I know it’s hard to see, but what would it be? If we used the top equation, what would it be? I’m going to ask people to read off the board or read the blue stuff for me, and tell me what the substitution stuff is going to look like.

Student B: Three $x$ plus two times three $x$ plus three equals six.

Mr. Patrick: Okay, and then, after that substitution step, what is he doing?

Student C: Solving for $x$.

Mr. Patrick: How? I need a little bit more. Can you elaborate?

Student C: You distribute.

Mr. Patrick: There’s a distribution here. Do you agree with that, with what [Student C] said? Cool. So, then, getting the two times the three $x$ is the six $x$ there. The two times the three is the plus six there, equals six. And if we solve that, three $x$, hanging out front, what’s next?

Student C: You do, like terms.

Mr. Patrick: Combine like terms? So, where are the like terms? Yeah. That’s where the nine $x$ is coming from. And then what?

Student B: Plus six equals six.

Mr. Patrick: Then, after that, then what?

Student B: Minus six to both sides. Then you get nine $x$ equals zero.

Mr. Patrick: Yeah.

Student B: And then, yeah, divide it, so, $x$ equals zero.

Mr. Patrick: And then, what is he doing here? Even if you can’t see it, now that you’ve found $x$?
Student B: You plug it into that equation.

Mr. Patrick: Yeah, yeah. And does it matter which equation?

Student B: No.

Mr. Patrick: Not really. Just don’t come back here and shove it in here. It’s not helpful over here. So, find y. And then, I love it, he’s got his point written out, and he has his zero coming through there. Questions on that?

In this teaching episode, Mr. Patrick asked over two-dozen questions in under four minutes, and received feedback from multiple students. At each step, he probed for understanding, and adjusted his questioning technique accordingly. At any point, he could have transitioned to a more traditional approach. Instead, through the process of active questioning, Mr. Patrick was able to assess student understanding frequently and informally over multiple, brief intervals. He was better equipped to make effective instructional decisions that would be well-received by the students and served to model expectations for in-class discussions. Thus, the inclusion of Instructional Assessment appears fundamental to balancing student needs with curriculum requirements.

**Negotiating.** The second overarching theme is *negotiating*, which represents Mr. Patrick’s perspective on the importance of promoting student autonomy in the classroom. Negotiating is considered to be largely a proactive attitude toward student engagement and teaching in general. That is, Mr. Patrick had accumulated previous experiences with students as passive learners, and wanted to encourage active participation whenever possible. He mentioned in particular students that he had instructed in multiple courses.

I also like the fact that I see them as sophomores and juniors, because then I can totally call the juniors and seniors in Algebra II out, and be like, we covered that in Geometry. Don't even try to do the, ‘I don't know that’ thing. (Interview #2)
In general, he felt strongly that “knowledge needs to be owned by the student,” even in the case of learning new topics. He elaborated on this philosophy by explaining how he negotiates responsibility with his students. He said,

Once they've seen [the initial teaching and communication], sufficiently, then that ownership is on them. And there comes a point in time when I'm like, ‘I'm not telling you. I need you to find that. I need someone in here to tell me, because I'm not.’

(Interview #2)

By developing and maintaining this approach to teaching and learning, Mr. Patrick made his students investors in their own development.

One particular classroom technique that was useful in promoting student responsibility and self-motivation was called, "Think, Pair, Share." A problem set was given to the students, and they were required to work by themselves for a short period of time. Students would then work in pairs, comparing their progress and fixing any mistakes. During this period, both Mr. Patrick and Mrs. Kline would offer suggestions to students who were having difficulties.

Finally, the class would reconvene, and solutions to each problem would be written on the board by the students, followed by a teacher-led discussion. Any questions that were not completed in class were assigned for homework. This approach to instruction placed significant accountability on the students for both the quality of their in-class notes, as well as their interactions with the other students and, ultimately, the entire classroom.

The following observation, taken from a Geometry class where students were taking part in a Think, Pair, Share, shows the emphasis that Mr. Patrick placed on student autonomy.

Mr. Patrick: For the first third of the time, you are literally working by yourself. You are not talking with your neighbor. You are using your notes. You're having questions with us, if you need it, but I need you checking your notes, first. Okay? After that time is up, and I'll let you know what that time is. I'll set the timer. At that point, you're going to partner off with somebody. And you're able to work with them, ask them questions as
well. Maybe use their notes, if you couldn't find it in your notes, that sort of stuff. But, like, cooperative, helping one another. All right? So, be aware of who you're choosing, and who you're working with, because I'm going to do very little redirection. Because this is reviewing for what you need to do. So, if you don't want to stay focused, you're going to sink your own ship.

In this vignette, Mr. Patrick makes very clear statements regarding his expectations of student behavior, both during individual work and when working with a partner. This reminder was consistently repeated throughout many of the observed classes. The effect was a general improvement of classroom behavior, and an understanding of what was acceptable during classroom activities.

Instructional Assessment played an important role in the development of negotiation in Mr. Patrick’s classroom. Encouraging students to be responsible for their share of the learning process elicited more opportunities for instructor feedback. Mr. Patrick could readily identify which students were struggling and engage in one-on-one discussions to assess for understanding. Following this intervention, appropriate changes could be made by the student.

In the case where several students were struggling, Mr. Patrick reserved the option to address the entire classroom in order to provide more efficient feedback. This negotiation of control does not come naturally, as it restructures the traditional learning contract (Perrenoud, 1991), and requires the instructor to relinquish partial control of the classroom.

**Knowledge for Teaching**

Mr. Patrick’s understanding of secondary mathematical content incorporates a wide range of topics from multiple subject areas. This knowledge base was largely established prior to his years of experience at Fall River, but continues to be modified as he instructs and observes students. Central to this adaptation is an understanding of varied student perspectives, especially
in the area of misconceptions. The combined evolution of Mr. Patrick’s mathematical knowledge along with his ability to assess students’ mathematical knowledge encompasses his knowledge for teaching. Codes derived from interview responses revealed important insight into this relationship. Descriptive statistics from the coding process are presented in the tables below, followed by the development of themes in the area of knowledge for teaching. Results in this area will be used to answer the second and third research sub-questions: How do mathematics teachers reinforce or modify their understanding of mathematical content and processes through classroom interactions? How do mathematics teachers make use of observation, listening, and questioning during instruction to construct models of student understanding?

**Codes.** In total, 357 interview response segments categorized under Reinforcement or Modification of Mathematical Content or Processes (RMMCP) or Models of Student Understanding (MSU) were coded according to two descriptors: scope and knowledge for teaching. Scope codes were useful for organizing interview response segments for thematic analysis across multiple contexts. For example, when a particular theme was not fully represented at a specific scope, additional evidence was obtained from the observation transcripts. Using this method ensured a triangulation of data sources for understanding Mr. Patrick’s knowledge of mathematical content, teaching, and students. Knowledge for teaching codes designated a particular context for specific classroom practices, and were the primary sources for the creation of themes.

The first set of codes were designed to describe the scope, or contextual breadth of Mr. Patrick’s statements. Short-view (S) responses referenced either specific mathematical concepts or individual students. Medium-view (M) responses described either common mathematical topics or entire classrooms of students (or students in general terms). Long-view (L) responses
included general statements about either entire mathematical subject areas, or student trends from year to year. Scope code frequencies are listed in Table 14.

Table 14

*Scope Codes for Case Study 1 (Knowledge for Teaching)*

<table>
<thead>
<tr>
<th>Code</th>
<th>Name</th>
<th>Context</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Medium</td>
<td>Mathematical topics</td>
<td>198</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Classrooms of students (or students in general)</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>Long</td>
<td>Mathematical subjects</td>
<td>81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Student trends from year to year</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>Short</td>
<td>Mathematical concepts</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Individual students</td>
<td></td>
</tr>
</tbody>
</table>

Just over half of Mr. Patrick’s interview responses (55%) were coded as medium scope. Long-scope responses (23%) and short-scope responses (22%) were nearly equally represented. This shows that all three contexts were well-represented in the coded interview responses.

The main set of codes was taken from the framework developed by Kilpatrick, et. al. (2015) on mathematical understanding at the secondary level (MUST). Of the fourteen codes derived from this framework, thirteen were recorded at least once in the data. Four additional codes (Summative Assessment, Student Disabilities, Co-Teaching, and Outside Factors) were included when interview response segments emerged from the data that were not represented in the framework. All seventeen knowledge for teaching codes are listed and defined in Table 15, along with their frequencies. Four codes stand out from the rest: Reflective Practice (30%), Knowledge of Curriculum (15%), Mathematical Knowledge (12%), and Mathematical Thinking (11%). This reflects Mr. Patrick’s repeated emphasis on assessing mathematical understanding in the context of teaching. This provided important insight into the shared development of
instructor and student mathematical knowledge that emerges during Instructional Assessment. A full thematic analysis is presented in the next section.

Table 15

**Knowledge for Teaching Codes for Case Study 1**

<table>
<thead>
<tr>
<th>Code</th>
<th>Name</th>
<th>Context</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>RP</td>
<td>Reflective Practice</td>
<td>Examination of classroom decisions</td>
<td>107</td>
</tr>
<tr>
<td>KC</td>
<td>Knowledge of Curriculum</td>
<td>Identify concepts and assign learning goals</td>
<td>53</td>
</tr>
<tr>
<td>MK</td>
<td>Mathematical Knowledge</td>
<td>Access components of student knowledge</td>
<td>42</td>
</tr>
<tr>
<td>MT</td>
<td>Mathematical Thinking</td>
<td>Interpret student explanations</td>
<td>41</td>
</tr>
<tr>
<td>SC</td>
<td>Strategic Competence</td>
<td>Assess problem solving strategies</td>
<td>17</td>
</tr>
<tr>
<td>SD</td>
<td>Student Disabilities</td>
<td>Incorporating documented disabilities</td>
<td>17</td>
</tr>
<tr>
<td>PD</td>
<td>Productive Disposition</td>
<td>Curiosity, enthusiasm, and perseverance</td>
<td>15</td>
</tr>
<tr>
<td>CT</td>
<td>Co-Teaching</td>
<td>Shared classroom responsibilities</td>
<td>15</td>
</tr>
<tr>
<td>SA</td>
<td>Summative Assessment</td>
<td>Preparing students for annual testing</td>
<td>9</td>
</tr>
<tr>
<td>MN</td>
<td>Mathematical Noticing</td>
<td>Recognizing similarities and differences</td>
<td>7</td>
</tr>
<tr>
<td>PM</td>
<td>Probing Mathematics</td>
<td>Investigating mathematical ideas</td>
<td>7</td>
</tr>
<tr>
<td>CU</td>
<td>Conceptual Understanding</td>
<td>Deriving and understanding connections</td>
<td>6</td>
</tr>
<tr>
<td>PF</td>
<td>Procedural Fluency</td>
<td>Recalling and executing procedures</td>
<td>5</td>
</tr>
<tr>
<td>AR</td>
<td>Adaptive Reasoning</td>
<td>Working in multiple mathematical contexts</td>
<td>5</td>
</tr>
<tr>
<td>OF</td>
<td>Outside Factors</td>
<td>Incorporating factors from outside school</td>
<td>5</td>
</tr>
<tr>
<td>MR</td>
<td>Mathematical Reasoning</td>
<td>Producing and justifying arguments</td>
<td>4</td>
</tr>
<tr>
<td>HC</td>
<td>Historical and Cultural</td>
<td>General development of mathematics</td>
<td>2</td>
</tr>
</tbody>
</table>

**Themes.** All coded interview response segments from Mr. Patrick were entered into a spreadsheet to facilitate organization and sorting ability. Further analysis yielded several themes representing challenges in two domains of Pedagogical Content Knowledge: Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT). Additional
overarching themes representing specific strategies employed by Mr. Patrick to navigate these challenges were also developed. A list of each theme and its definition is provided in Table 16. In the sections that follow, each theme is described in detail, along with representative examples from the interview and observation data. A full interpretation of this portion of this case study is presented in Chapter V.

**Knowledge of Content and Students.** Themes in the domain of KCS represent “content knowledge intertwined with knowledge of how students think about, know, or learn this particular content” (Hill, Ball, & Shilling, 2008, p. 375). The primary emphasis, therefore, is on how the challenges of assessing student understanding impact the implementation of teaching strategies. When discussing the specific factors that contributed to Mr. Patrick's knowledge for teaching in this domain, two major themes emerged from the data: Learning Trajectories and General Proficiency. The main codes represented in these themes include Knowledge of Curriculum and Reflective Practice.

Table 16  
**Knowledge for Teaching Themes for Case Study 1**

<table>
<thead>
<tr>
<th>Domain</th>
<th>Theme</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge of Content and Students</td>
<td>Learning Trajectories</td>
<td>Identifying the influence of mathematical concepts on future learning and broader topics</td>
</tr>
<tr>
<td></td>
<td>General Proficiency</td>
<td>Distinguishing between students’ conceptual and procedural knowledge and misconceptions</td>
</tr>
<tr>
<td>Knowledge of Content and Teaching</td>
<td>Mathematical Disposition</td>
<td>The effect of traditional training in mathematics on instructional perspective</td>
</tr>
<tr>
<td></td>
<td>Curriculum Standards</td>
<td>Familiarity with prerequisite knowledge across grades, courses, and assessments.</td>
</tr>
<tr>
<td>Overarching Themes</td>
<td>Assessment Diversity</td>
<td>Using multiple assessment methods to illuminate student understanding</td>
</tr>
<tr>
<td></td>
<td>Instructional Analogies</td>
<td>Connecting new concepts to students’ foundational and real-world knowledge</td>
</tr>
</tbody>
</table>
Learning Trajectories. The first theme to emerge in the domain of KCS is learning trajectories, which represents Mr. Patrick’s understanding of specific mathematical concepts and their impact on future learning and broader topics. As with other themes in this case study, Mr. Patrick attributed his knowledge of learning trajectories to his years of experience with students from Forest River. He acknowledged that this knowledge was unique to the students he had worked with, and that it wouldn’t necessarily translate to students in other school districts. He explained,

Looking back to, maybe, the first or the second time I did teach it, there was more editing, on the fly, that had to happen, because, from not necessarily knowing the population, and sort of where the gaps are, and that sort of thing. Having taught this, these courses for a while, now, I really have an understanding of where the kids are going to have some issues, and where they’re not. (Interview #1)

This quote demonstrates the full breadth involved in developing knowledge of learning trajectories. Mr. Patrick first identifies that students enter his classroom with certain gaps in their prerequisite knowledge. He recognizes that his instructional practices have advanced as a result of working with this population of students. Lastly, he notes that this process has led to an ability to anticipate future areas of difficulty.

Mr. Patrick’s primary source of evidence for building learning trajectories derives from frequent informal classroom assessments. He was asked to describe all of the methods he uses to assess student understanding, including testing. His initial response pointed toward methods of Instructional Assessment.

Most of my classroom assessments are based on conversations, and sort of the questioning technique that I use. It’s a lot of the talking, and verbal stuff. But it’s also, I guess, some of what they can perform. Like, more, doing group work, and their station work, of what they’re recording, and able to show. (Interview #1)
He elaborated on this process by explaining the interplay between conversations and written work.

Sometimes, if they’re not able to explain, but they can show, I can see that on their paper, as well. Whether that be station work, or individual class work, or even a quick homework quiz that we do. I try to mix it between verbal and written. (Interview #1)

Classroom observations confirmed that Mr. Patrick regularly engaged in instructional conversations involving written student work. These conversations were typically informal in nature, but provided Mr. Patrick with a wealth of information regarding the progress of individual students, as well as the comfort level of the entire class.

Mr. Patrick’s knowledge of learning trajectories has led him to interpret certain curriculum standards based on his appraisal of what students can handle, and what will ultimately serve them best in the long run. For example, when discussing methods for solving quadratic equations in Algebra II, he explained that students frequently have difficulty with completing the square. He stated,

That might be a really great activity. There might be beautiful lessons around that. But I know the students that I have in front of me are going to get totally lost on that process. It’s not going to make any sense to them at all, and they’re really going to struggle with that. (Interview #1)

Over the years, he has chosen to de-emphasize completing the square in favor of spending more time on other methods. He feels that this approach will help students to gain a deeper understanding of solving quadratic equations. He explains,

It’s sort of playing that game of, is it relevant, is it not relevant? We hit the graphing, because we talk about the zeros. We hit the quadratic formula. We hit factoring. And at that point, is completing the square relevant anymore? Is it necessarily important? I think some of these standards aren’t necessarily, everything needs to be covered, because some of them are written that way of, ‘can solve equations many different ways.’ Okay, I’ve got three out of four, that’s great. (Interview #1)
It is unclear how this approach might affect student knowledge of related topics (such as writing quadratic functions in standard form). Despite this, Mr. Patrick has made an evidence-based determination about what the students at Forest River can handle, while continuing to meet the particular curriculum standard.

**General Proficiency.** The second theme to emerge in the domain of KCS is *general proficiency*, which represents Mr. Patrick’s ability to distinguish between conceptual and procedural knowledge, including common student misconceptions. From his experience working with Forest River students, Mr. Patrick had developed a comprehensive knowledge base of typical errors. He lamented that students were often more concerned about finding the correct answer rather than mastering the solution process. He described this ongoing challenge.

> They’re all about, ‘well, what’s the answer?’ I don’t care. And I’m finding a lot of our students take a longer time for that trust. Towards the end, and definitely, like, the last nine weeks or so, they have totally bought into that process of, ‘I’m not really concerned about the answer.’ I mean, ultimately, I am, but I am more concerned that you understand the process of why it’s working, why we’re doing this, and that sort of thing. (Interview #1)

The importance of developing and maintaining this balance is highlighted by the NCTM (2014).

> “Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly” (p. 10).

The theme of general proficiency encompasses the knowledge required to successfully promote mathematical fluency.

Included among the recommendations proposed by the NCTM (2014) for best practices in the classroom are methods of Instructional Assessment, specifically teacher-guided, mathematical discourse with purposeful questioning. “[Teachers] guide their teaching and learning interactions by evidence of student thinking so that they can assess and advance student
reasoning and sense making” (p. 57). This underscores the formative nature of instructional discourse, and provides insight into how Mr. Patrick develops general proficiency knowledge of each individual student. He explained,

> It’s their portfolio of work that I’ve seen, over the semester. Whether it’s submitted, written work, or just hearing the way they interact in class, and how they explain, verbally. (Interview #2)

He elaborated on how information derived from classroom interactions affected the direction of instruction.

> If I’m seeing students make major mathematical mistakes, that’s sort of the red flag for me, to be like, ‘I need to revisit something. This is happening two or three times.’ That’s the pattern I’m looking for. Is there a pattern of misconception that needs to be readdressed? Maybe the way I approached it is leading them the wrong way, because there is a miscommunication in what I’m saying. Maybe what I’m saying is too mathematical, and I haven’t really communicated the common process. (Interview #2)

This explanation shows that Mr. Patrick understands the connection between students’ conceptual knowledge and procedural fluency, and is willing to adjust his instructional approach whenever necessary.

One specific method Mr. Patrick used to bridge the gap between procedural and conceptual understanding was the use of “creative formulas” in Geometry. During a unit on surface area of solids, the instructors would work with the students on a variety of shapes. Following the study of simple shapes, such as cylinders and rectangular prisms, more complex shapes were considered, including pyramids, cones, and triangular prisms. Both instructors took part in the instructional discussions that followed, but Mr. Patrick developed a unique notation to write equations representing the structure of a certain formula, without immediately relying on algebraic notation. In one example, the students were calculating the surface area of a triangular prism. Mr. Patrick noted that the two triangular faces were congruent, but that the three
rectangular faces were not. He labelled the rectangular sections, “I,” “II,” and “III,” and then wrote the following equation on the board: $S = 2Δ + [I] + [II] + [III]$. This provided the students with a starting point to begin creating the algebraic formula.

**Knowledge of Content and Teaching.** Themes in the domain of KCT represent “mathematical knowledge that interacts with the design of instruction” (Hill, Ball, & Shilling, 2008, p. 375). The primary emphasis, therefore, is on how the challenges of implementing teaching strategies impact the assessment of student understanding. When discussing the specific factors that contributed to Mr. Patrick’s knowledge for teaching in this domain, two major themes emerged from the data: Mathematical Disposition and Curriculum Standards. The main codes represented in these themes include Mathematical Thinking and Reflective Practice.

**Mathematical Disposition.** The first theme to emerge in the domain of KCT is mathematical disposition, which represents the effect of Mr. Patrick’s traditional training in mathematics on his instructional perspective. During both interviews, Mr. Patrick referenced his educational background as being very technical in nature. He stated,

I was trained as, essentially, a pure mathematician. My degree is a Bachelor of Science in Education in Math, but I was a Math major. So, I would come at things from a very technical, math sense. (Interview #1)

Mr. Patrick’s identity as a traditional mathematician had an undeniable impact on his approach to classroom instruction. He explained this effect, and contrasted his role with that of Mrs. Kline.

I see where we're different is, my perspective is sort of getting them to move into math, and be sort of an insider in math, and use the terms, and be comfortable with that. I try to see my role as trying to get them into being, like, mathematicians. Granted, I know that most of them probably won't, and that's okay. But be able to at least give them an option of exposing them to, like, how math is. How is it generated? How is it, to be an insider in math? (Interview #2)
This shows that Mr. Patrick held very clear views concerning his role as the subject-matter expert in the classroom.

Mr. Patrick acknowledged that his traditional perspective was often at odds with the ways in which students prefer to learn, requiring him to develop a more flexible teaching approach. His primary instructional strategy for aligning the two knowledge bases was to regularly elicit student explanations in an attempt to better understand their mathematical thinking. He explained,

I come at math, a lot of times, from a traditional training, and I think that’s not necessarily where our students are coming from. So, I appreciate their perspectives, and I also try to help them navigate that, a little bit more. (Interview #2)

He further acknowledged that convincing students to share their perspectives in the classroom was not always seen as a reasonable proposition. This extended from individual students to entire classrooms. His years of experience working with Forest River students made him very adept at recognizing and adapting to different learning styles and personalities. He contrasted his current classes with those from previous years.

I’m finding this group does a lot more of the, ‘I don’t know exactly what to do, so I’m going to stop completely.’ In past years, we’d see, ‘I don’t know what to do exactly, but I’m still going to try something.’ This group literally stops and does nothing. (Interview #2)

Mr. Patrick used the term ‘risk-taking’ to describe students’ willingness to participate during in-class activities. He elaborated on this concept by comparing the two active sections of Geometry.

I see more risk-taking in third block, I feel. I think fourth block, although in many ways, they’re more capable, they’re at, ‘this is what I know.’ So, I think it’s more a fear of being wrong, and not just willing to take that risk. (Interview #2)
This shows that Mr. Patrick recognized the challenges involved with motivating student participation, and was also able to identify the students’ disposition toward mathematical instruction.

Despite these ongoing challenges, it was apparent that Mr. Patrick’s efforts to elicit student perspectives had paid off by increasing his instructional flexibility, while still allowing him to maintain his traditional teaching approach. He described how this knowledge affected in-class guidance, when a student remained confused following an initial intervention. He explained,

It’s sort of like, ‘okay, that way didn’t work. Let me try it from a different perspective.’ And there’s a couple resets. And, because at this point, I have seen many different ways of students processing things, I’ve learned other ways of processing it, at this point. So, I feel pretty comfortable with resetting and trying a different way. If they’re still having trouble with that, I ask more probing [questions] from them, to try to get how they are processing it. (Interview #2)

Using this method, Mr. Patrick was able to fine-tune his responses to individual students during instruction. Over the course of a semester, he was better equipped to recognize specific student learning perspectives and instructional preferences. He stated,

At this point, I know who is going to be more of a verbal processor, and write a sentence, or who is going to be more of a, ‘I’m going to show the math work,’ or who is going to look at this in terms of a picture. I kind of have a feel for that already. (Interview #2)

During classroom observations, Mr. Patrick often chose to initially address the class using standard mathematical terminology, but would then adjust to more informal language based on student feedback. This process became further refined during classroom activities, when Mr. Patrick had the opportunity to work with individual students. This shows his ability to assess student needs and respond or adjust his responses to aid in their learning.
Curriculum Standards. The second theme to emerge in the domain of KCT is curriculum standards, which represents Mr. Patrick’s familiarity with prerequisite knowledge across grades, courses, and assessments. During his time at Forest River, he had served on multiple committees tasked with updating curriculum frameworks and designing common assessments. Much of this work was initiated in response to the school’s loss of accreditation from the New England Association of Schools and Colleges (NEASC) from 2003 to 2005 (Moore, 2006). The school had also experienced a high rate of faculty and administrative turnover during this time period, and Mr. Patrick was one of many new hires given a role in the reaccreditation process. He worked primarily on the Algebra I curriculum, which he also taught during his first few years at Forest River. He explained how participating in this process helped to establish his knowledge base.

I haven't taught Algebra I in four years. But, as far as I'm aware, from the department meetings, the curriculum hasn't changed, in terms of the sequencing. And I helped develop that. When I taught it, I was there with the writing of it. So, I can totally come back at it and I'm like, I know it's in that curriculum, because I helped write it. (Interview #2)

Following the school’s reaccreditation in 2006, Mr. Patrick continued to teach Algebra I, but eventually transitioned to Algebra II and Geometry.

During the years leading up to this study, Mr. Patrick remained active in the curriculum development process. Efforts were made to update requirements across the mathematics department, with the specific goal of aligning Forest River with state-level standards, as well as expectations for college preparation. Mr. Patrick provided a glimpse into the complex decisions that had been recently made.

We've been working pretty hard-core, for the past four years, in the department, I want to say, of working on that alignment, of, what is absolutely needed for tenth graders? What is absolutely needed for those SAT questions? What's absolutely needed for college
entrance? General Ed versus science majors, versus that sort of stuff, and making sure that, these are the hard-core skills that need to be in our courses. (Interview #1)

Included in this process was the development of common course assessments, such as chapter tests, midterms, and final exams. Mr. Patrick described the evolution of his involvement, noting the changes that had occurred in his time at Forest River.

We do start looking at those [assessments] several months before we give them. So, that’s helped me, and those assessments are also aligned to MCAS, SAT, and then the core standards and the state standards, as well. I worked in this building before that was there, and the pressure was always there. Granted, I was a young teacher, so it was part of my learning process. We do that as a department now, and I haven’t felt the pressure in a while. (Interview #1)

This shows that Mr. Patrick felt relatively comfortable with his knowledge of the mathematical content, especially as to how it was assessed at various levels. This knowledge was often communicated to the students during class, when concepts or methods were presented in class that would appear on later assessments.

One area where Mr. Patrick admitted that his knowledge needed to be improved was with upper-level courses. Because he had mostly taught freshmen and sophomores, his knowledge of advanced curriculum was incomplete. More importantly, he wasn’t entirely certain how the concepts he was covering would translate to advanced topics. He stated,

Right now, I see Geometry and I see Algebra II. I can see an immediate transition. I taught Algebra I before teaching Geometry, so I see that flow, from one course to the other. But after Algebra II, I don't know where they go with that. What happens in [Functions and Statistics]? What happens in Pre-Calc? What happens in the Calc classes? I don't see where they go. So, sometimes I have to check in with [colleagues], and be like, what do you need them to know? I've been away from that content for a while, so I still have an understanding, but it's still, you know, what’s important, for our courses, in this school district? (Interview #1)

This shows that Mr. Patrick’s knowledge of curriculum standards was still developing. In addition to his experience on curriculum committees, his ability to contribute to ongoing
department revisions necessarily relied on his in-class experiences with students. Methods of Instructional Assessment also provided important details as to which standards were particularly challenging for students, and may require extended instruction time. A full interpretation of this connection is provided in Chapter V.

**Overarching Themes.** The themes presented thus far represent challenges experienced by Mr. Patrick in the domains of KCS and KCT. Two additional themes emerged from the data representing specific strategies employed by Mr. Patrick to navigate these challenges:

Assessment Diversity and Instructional Analogies. The following sections provide evidence for these overarching themes, and demonstrate some of the ways in which Mr. Patrick balanced instructional design with the assessment of student understanding. The main codes represented in these themes include Reflective Practice, Mathematical Knowledge, and Knowledge of Curriculum.

Assessment Diversity. The first overarching theme is *assessment diversity*, which represents Mr. Patrick's strategy of using multiple assessment methods to illuminate student understanding. As evidenced in previous sections, Mr. Patrick described his classroom assessments as being primarily based on verbal conversations and individual or group work. However, several additional techniques were observed, including homework, homework quizzes, quizzes, tests, projects, posters, and notebook assessments. He described this approach as an intentional strategy for avoiding excessive testing. He said,

> All of our assessments, at least in this room, aren’t necessarily pencil and paper tests. I understand that sometimes students don’t necessarily test well. So, I’m aware of that, and I try to make sure that we have other, alternative assessments, that aren’t traditional pencil and paper tests. (Interview #1)

Throughout classroom observations, informal assessment techniques were regularly employed as part of instruction. Sometimes, these were planned in advance, in the form of individual or
group activities. In other cases, they were used more spontaneously, as a follow-up to a
discussion on homework or classwork problems.

During the observation time period, the mathematics department at Forest River was
implementing a common rubric for calculating student grades. Written assessments, including
exams, tests, and quizzes, accounted for 80% of the overall grade, while the remaining 20% was
reserved for projects, participation, and notebook assessments. With such a heavy weight on
traditional methods, it was initially difficult to comprehend how informal assessments were to be
incorporated. Mr. Patrick explained that certain grades were counted partially or entirely as
quizzes or tests, even though they did not actually fall into either of these categories. He stated,

There are other assessments that I do count in a test or quiz category, that aren’t
necessarily a pencil and paper test. For example, with a homework quiz. Yes, they’re
writing things down, but it’s literally after that discussion time, so it’s sort of both
aspects. The geometry projects that we do, like the one that they’re doing right now,
that’s a quiz grade. They might be a project, like the posters in the back [of the room] are
a test grade. It’s the last test for that polynomials unit. (Interview #1)

Students were made aware of this grading policy, and were reminded that extra efforts on
alternative assessments could significantly affect overall grades.

One particular example of this, recorded during a Geometry class, occurred a few weeks
prior to MCAS testing in English and Language Arts (ELA). The students were informed about
poster projects that they would be working on in class. The final products were to be displayed
in the classroom for the remainder of the semester.

Mrs. Kline: We will be making those during the MCAS ELA. So, when you’re
testing, in the morning, and thinking really hard about English, when you
come to math class, on one or possibly two of those days, I don’t
remember how the schedule works, we’ll be tracing it, okay? So, you
don’t have to worry about taxing your brain all day long.
Mr. Patrick: And I want to say this, as well, because it’s a beautiful thing. You’re in high school. Sophomores, juniors, or soon-to-be juniors. You’re being asked to trace and cut, and then assemble and staple, essentially. And this is going to be a quiz grade.

Student A: Test grade?

Mrs. Kline: Definitely not. Quiz grade.

Mr. Patrick: Quiz grade. So, let me say that again. You’re in high school. We’re asking you to cut, trace, and staple for a quiz grade.

Student B: So, what you’re trying to say is, it’s easy.

Mrs. Kline: Yes, thank you. Thank you for saying it for us. That’s great.

In this vignette, both instructors are attempting to communicate the simplicity of the project. Mr. Patrick is particularly deliberate in emphasizing the disparity between the students’ abilities and the level of work required. This shows that both instructors understood that this project allows students to potentially increase their quiz averages, even though no testing was required. By explaining this to the students, they were hoping to convey the importance of this opportunity, similar to that of extra credit.

Another example of this grading flexibility was station work, where students would be separated into small groups to work on problems that had been set up at various locations around the room. Following the initial setup process, the groups would work at each station for 15 minutes before moving to a different station. After the groups had completed working at all stations, the class would reconvene to discuss the solutions. Written work was then collected from each student for the purposes of recording participation and completion of each problem. As additional motivation, the students were informed that the written work would be worth up to five points on the test, which was to be given one day later. Thus, responses to problems in a
group activity, where the solutions were openly discussed, was counted toward each student’s test grade. This shows another case of incorporating alternative methods into the 80% of each student’s overall grade that was reserved for traditional assessments.

Despite the potential concerns related to this type of grading flexibility, Mr. Patrick felt certain that students were being accurately assessed over the course of a semester. He recognized that relying solely on written tests and quizzes for 80% of each student’s grade had the potential to misrepresent the overall levels of understanding. Moreover, by enacting this grading policy, Mr. Patrick was better able to incorporate more frequent, informal assessments into classroom instruction. He described this process as a way of balancing the grading rubric. He stated,

> It is heavily weighted on an assessment component. And I totally hear that, and I agree that it is, but I know I try to make sure it is balanced, in a way. I don’t know how other teachers, I can’t really speak to how they do it, but I do try to balance it that way. I think I do capture what they know, in that way. There’s adequate ways that students can demonstrate what they know, and I really do think that, over the course of a chapter or nine weeks, there’s many different sort of modalities, of their way to show what they know. I’m pretty confident. (Interview #1)

This shows that Mr. Patrick sees the importance of having diverse assessments in his classes, primarily because he recognizes multiple ways to tap into student understanding outside of pencil and paper tests. This was seen as an effective strategy for balancing instructional design with the assessment of student understanding.

**Instructional Analogies.** The second overarching theme is *instructional analogies*, which represents Mr. Patrick's strategy of connecting new concepts to students’ foundational and real-world knowledge. Mr. Patrick used analogies frequently during classroom conversations to bridge between his traditional mathematical knowledge and his perceived abilities of the
students. By framing concepts in more concrete or recognizable terms, he attempted to get students to buy into the learning process. He explained,

You talk to them about the commutative property and associative property, and transitivity, etcetera, and you lose them. You lose them on all that language and vocab. But if you talk to them about, you know, Johnny is the same height as Mike, and Mike is the same height as Ruth, what can you say about the other two heights? And I think that’s more real for them, anyway, because that’s how they’re going to experience math. (Interview #1)

Mr. Patrick’s main justification for this approach was his accumulated knowledge of students in college preparatory courses. He claimed that it was unlikely for him to be teaching students who would eventually major in mathematics or other technical fields, and so he could be less rigorous with the terminology. He explained,

I’m more okay with their application of the math, rather than their technical understanding of it. We talk a lot about, there are some definitions and properties that they need to know. They need to know certain words, certain language, in the subject for them to be successful later on. So, that idea of, where can I be a little loose with the language and understanding, and where do I need to be really structured with it? (Interview #1)

He admitted that he sometimes still struggles with this process, and that he would continue to have conversations with colleagues to increase his knowledge.

Many of the analogies that were recorded in this study occurred during instructional discussions, not as predesigned application problems. Mr. Patrick would interject examples of real-world connections as the corresponding mathematical concepts were being covered in class. A summary of some of these examples is included in Table 17. In some cases, the process seemed preprogrammed, as if he had used the analogies in previous years. In other cases, the examples seemed more improvisational, as if he was thinking of them in the moment. Regardless of the true origin, Mr. Patrick was often able to weave these analogies seamlessly
into instruction so that they didn’t detract from the daily learning objectives. In most cases, the
students were enthusiastic about contributing to the creation and full exploration of the analogy
by responding to questions from either instructor. After a few minutes of discussion, the class
would return to the problem at hand, with only occasional references to the associated analogy.

Table 17

*Instructional Analogies for Case Study 1*

<table>
<thead>
<tr>
<th>Concept</th>
<th>Instructional Analogies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio</td>
<td>Comparing sizes of different sports teams</td>
</tr>
<tr>
<td>Union/Intersection</td>
<td>Students in different classes and homerooms</td>
</tr>
<tr>
<td>Mutually Exclusive</td>
<td>Membership at a country club</td>
</tr>
<tr>
<td>Matrix Structure</td>
<td>Organization of desks in the classroom or chairs in a theater</td>
</tr>
<tr>
<td>Matrix Data</td>
<td>Graphic design programs such as Adobe Photoshop</td>
</tr>
<tr>
<td>Multiplicative Identity</td>
<td>A person’s identity in contrast to physical changes</td>
</tr>
<tr>
<td>Geometric Shapes</td>
<td>Ice cream cones, doorstops, Olympic rings, etc.</td>
</tr>
<tr>
<td>Arithmetic Average</td>
<td>Balancing a see-saw, pouring equal drinks for friends</td>
</tr>
<tr>
<td>Median</td>
<td>Standing in a long line of people</td>
</tr>
<tr>
<td>Slope</td>
<td>Staircases, hills, mountains, valleys, and cliffs</td>
</tr>
<tr>
<td>Probability Distributions</td>
<td>Student height, age, and hair length</td>
</tr>
<tr>
<td>Permutations</td>
<td>People in a line, personalized license plates, zip codes</td>
</tr>
<tr>
<td>Function Representation</td>
<td>Three states of matter (frozen, liquid, solid)</td>
</tr>
</tbody>
</table>

Mr. Patrick made frequent use of instructional analogies because he believed that it
helped students translate difficult concepts into more familiar terms. This included using
everyday language instead of technical jargon and applying mathematical ideas to the students’
real-world experiences. This is a common strategy among mathematics educators, but Mr.
Patrick accelerated the process by incorporating the analogies into classroom discussions while
topics were being introduced, rather than delaying until they had been formally practiced. However, he explained that getting the students to buy into this process took time. One reason for this delay is that the analogies were a departure from the standard cadence of classroom discussion. Mr. Patrick recognized that students’ ability to incorporate real-world concepts on the fly did not always come naturally.

I find myself using more analogies and sort of examples, and being more concrete. I know, sometimes, my directness can be uncomfortable for some students, and that can cause some sort of hesitance, in terms of an academic sense. But, I’m noticing a lot more, recently, that they’ve gotten more comfortable, and just our style of it. (Interview #2)

Mr. Patrick recognized that student experiences outside of the classroom were often more consistent than their prerequisite mathematics knowledge. This provided him with an avenue for increasing the number of potential participants in classroom discussions by making the mathematics more accessible.

**Case Study 2 - Mrs. Kline**

At the time of this study, Mrs. Kline had eight years of fulltime high school teaching experience, four of which were at Forest River. She holds a Bachelor’s Degree in Studio Art and English, and a Master’s Degree in Special Education from accredited universities in the United States. Prior to her current position, she was an English teacher for three years in Japan and a paraprofessional for students with psychological and emotional disabilities at a separate school district in Massachusetts. Her experiences in mathematics and science classrooms during this time led her to obtain a Massachusetts State Teaching License in Secondary Level Mathematics. Despite her primary training in special education, she views her main role as a mathematics teacher, and cites her time at Forest River as a major influence on that identity. She is an active
member of the faculty at Forest River, serving in both the Special Education and Mathematics
departments.

Mrs. Kline is a passionate instructor, whether in front of an entire class or working with
an individual student. It was readily apparent through classroom observations that she works
consistently to meet student needs and to communicate mathematics in a way that is both
engaging and fun. Her training in special education enables her to assist students with learning
disabilities, and she displays genuine enjoyment when exploring mathematics in the classroom.
She seems comfortable leading class discussions, although she more frequently serves in a
supporting role due to her administrative responsibilities outside of the classroom. Nevertheless,
her enthusiasm and willingness to engage students in mathematical discussions made her a
particularly appropriate participant for this study.

The following sections present detailed findings from the data analysis performed for
Mrs. Kline's case study. Coding and thematic analysis is provided in the areas of professional
self-efficacy and knowledge for teaching according to the methods described in Chapter III.
Interview responses and classroom episodes are included as supporting evidence, with particular
attention to the impact of Instructional Assessment.

**Professional Self-Efficacy**

Mrs. Kline’s position as a special education teacher presented many challenges in and out
of the classroom. Her perceived ability to successfully navigate these challenges on a daily basis
encompasses her professional self-efficacy. Codes derived from interview responses revealed
important insight into ongoing tension between attending to student needs and adhering to
curriculum standards. Descriptive statistics from the coding process are presented in the tables
below, followed by the development of themes for professional self-efficacy. Scores from the
TSES are included as a quantitative component of the data analysis. Results in this area will be used to answer the first research sub-question: How does the integration of assessment into instruction relate to the professional self-efficacy of mathematics teachers?

**TSES Data.** Mrs. Kline's overall pre- and post-observation scores on the TSES as well as sub-scores in the areas of Student Engagement, Instructional Strategies, and Classroom Management are listed on Table 18. The results, especially in the pre-observation survey, seem to further support a high level of professional self-efficacy. A score of 7 on the survey indicates that the teacher feels they can do "quite a bit" to navigate the difficulties that are presented on a daily basis. Mrs. Kline's sub-scores in Instructional Strategies and Classroom Management were consistently above this threshold.

Table 18

**TSES Data for Case Study 2**

<table>
<thead>
<tr>
<th></th>
<th>Student Engagement</th>
<th>Instructional Strategies</th>
<th>Classroom Management</th>
<th>Overall Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Observation</td>
<td>7.000</td>
<td>8.500</td>
<td>7.875</td>
<td>7.7917</td>
</tr>
<tr>
<td>Post-Observation</td>
<td>6.875</td>
<td>8.000</td>
<td>8.375</td>
<td>7.7500</td>
</tr>
</tbody>
</table>

Mrs. Kline’s overall score was nearly identical when comparing pre- and post-observation. The Student Engagement and Instructional Strategies sub-scores decreased slightly, while the Classroom Management sub-score increased slightly. This indicates that Mrs. Kline experienced no significant changes in her perceived ability to perform the necessary tasks of her position. Interview responses identified many challenges, primarily centered around administrative processes and state-testing expectations, but throughout the semester, Mrs. Kline consistently showed that she was willing to make every effort to help students be successful.
**Codes.** In total, 174 interview response segments categorized under professional self-efficacy were coded according to three descriptors: scope, general self-efficacy, and teacher self-efficacy. Scope codes and general self-efficacy codes were useful for organizing interview response segments for thematic analysis across multiple contexts. For example, when a particular theme was underrepresented at a specific scope, additional evidence was obtained from the observation transcripts. Using this method ensured a triangulation of data sources for understanding Mrs. Kline’s professional self-efficacy. Teacher self-efficacy codes provided a final layer of detail regarding specific classroom practices, and were the primary source for the creation of themes.

The first set of codes were designed to describe the scope, or contextual breadth of Mrs. Kline’s statements. Short-view (S) responses referenced individual students, while medium-view (M) responses described entire classrooms of students, or students in general terms. Long-view (L) responses included general statements about the teaching profession. Scope code frequencies are listed in Table 19.

Table 19

*Scope Codes for Case Study 2 (Professional Self-Efficacy)*

<table>
<thead>
<tr>
<th>Code</th>
<th>Name</th>
<th>Context</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Medium</td>
<td>Classrooms of students (or students in general)</td>
<td>124</td>
</tr>
<tr>
<td>L</td>
<td>Long</td>
<td>The teaching profession</td>
<td>35</td>
</tr>
<tr>
<td>S</td>
<td>Short</td>
<td>Individual students</td>
<td>15</td>
</tr>
</tbody>
</table>

The majority of Mrs. Kline's interview responses (71%) were coded at a medium scope. This is not surprising, since many of the interview questions were stated in general terms relating to her teaching. Long-scope statements (20%) were the second-most common. In many cases, these...
responses provided insight into Mrs. Kline's beliefs about the current state of mathematics education. All remaining responses (9%) were coded as short-scope. However, it should be noted that the small-scope response segments were often lengthy, since they primarily focused on specific, detailed interactions with individual students.

The second set of codes were selected from Bandura's four components of self-efficacy (1977), and were intended to describe the general area of self-efficacy in each interview response segment. A fifth code (RC) was added from Smylie (1999) to reflect the notion of role conflict that exists in educational settings. General self-efficacy codes are listed in Table 20.

Table 20

<table>
<thead>
<tr>
<th>Code</th>
<th>Name</th>
<th>Context</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA</td>
<td>Performance Accomplishments</td>
<td>Recollection of personal experiences</td>
<td>147</td>
</tr>
<tr>
<td>VE</td>
<td>Vicarious Experiences</td>
<td>Observation of the experiences of others</td>
<td>21</td>
</tr>
<tr>
<td>RC</td>
<td>Role Conflict</td>
<td>Perceived inconsistencies with job roles</td>
<td>6</td>
</tr>
<tr>
<td>VP</td>
<td>Verbal Persuasion</td>
<td>The effects of social interaction</td>
<td>0</td>
</tr>
<tr>
<td>EA</td>
<td>Emotional Arousal</td>
<td>Physiological reactions to stress</td>
<td>0</td>
</tr>
</tbody>
</table>

The majority of Mrs. Kline's interview responses (84%) were coded as Performance Accomplishments. This makes sense, since she was largely responding to questions based on her own experiences. However, Vicarious Experiences accounted for the second-highest frequency (12%), and often represented some of the more insightful interview responses from Mrs. Kline. Despite the high frequency of Performance Accomplishment responses, these general self-efficacy codes were helpful in isolating statements from Mrs. Kline regarding the experiences of other instructors and issues related to role conflict. This, in turn, was beneficial in creating a complete picture of Mrs. Kline’s professional self-efficacy.
Table 21

*Teacher Self-Efficacy Codes for Case Study 2*

<table>
<thead>
<tr>
<th>Code</th>
<th>Name</th>
<th>Usage</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC</td>
<td>Student Comprehension</td>
<td>Gauging student comprehension</td>
<td>30</td>
</tr>
<tr>
<td>SE</td>
<td>Special Education</td>
<td>Teaching students with learning disabilities</td>
<td>17</td>
</tr>
<tr>
<td>SB</td>
<td>Student Beliefs</td>
<td>Getting students to believe they can do well</td>
<td>16</td>
</tr>
<tr>
<td>TP</td>
<td>Time Pressure</td>
<td>Dealing with limited time in the classroom</td>
<td>14</td>
</tr>
<tr>
<td>CS</td>
<td>Curriculum Standards</td>
<td>Identifying curriculum standards</td>
<td>13</td>
</tr>
<tr>
<td>SV</td>
<td>Student Values</td>
<td>Helping students to value learning</td>
<td>12</td>
</tr>
<tr>
<td>ST</td>
<td>Shared Teaching</td>
<td>Shared teaching responsibilities</td>
<td>11</td>
</tr>
<tr>
<td>DQ</td>
<td>Difficult Questions</td>
<td>Responding to difficult student questions</td>
<td>11</td>
</tr>
<tr>
<td>CQ</td>
<td>Crafting Questions</td>
<td>Crafting good questions for students</td>
<td>10</td>
</tr>
<tr>
<td>BO</td>
<td>Burnout</td>
<td>The prospect of teacher burnout</td>
<td>8</td>
</tr>
<tr>
<td>OF</td>
<td>Outside Factors</td>
<td>Incorporating factors from outside of school</td>
<td>8</td>
</tr>
<tr>
<td>DS</td>
<td>Difficult Students</td>
<td>Getting through to difficult students</td>
<td>5</td>
</tr>
<tr>
<td>PP</td>
<td>Personal Pride</td>
<td>Setting a high level of professional standards</td>
<td>5</td>
</tr>
<tr>
<td>IU</td>
<td>Improving Understanding</td>
<td>Improving understanding of failing students</td>
<td>4</td>
</tr>
<tr>
<td>AL</td>
<td>Adjusting Lessons</td>
<td>Adjusting lessons for individual students</td>
<td>3</td>
</tr>
<tr>
<td>AS</td>
<td>Alternative Strategies</td>
<td>Implementing alternative strategies</td>
<td>3</td>
</tr>
<tr>
<td>MS</td>
<td>Motivating Students</td>
<td>Motivating students with low interest</td>
<td>3</td>
</tr>
<tr>
<td>AV</td>
<td>Assessment Variety</td>
<td>Using a variety of assessment methods</td>
<td>1</td>
</tr>
</tbody>
</table>

The third set of codes were taken from the long version of the TSES (Tschannen-Moran & Hoy, 2001). Of the twenty-four codes derived from the document, eleven were recorded at least once in the data. Seven additional codes (Burnout, Curriculum Standards, Outside Factors, Personal Pride, Special Education, Shared Teaching, and Time Pressure) were included when interview response segments emerged from the data that were not represented in the framework. All eighteen teacher self-efficacy codes are listed and defined in Table 21, along with their
frequencies. Only one code stands out from the rest: Student Comprehension (17%). This reflects Mrs. Kline's recurring descriptions of student understanding and how it related to the material she was teaching. Of the remaining codes, Special Education (10%) and Student Beliefs (9%) had the next-highest frequencies. The general distribution of teacher self-efficacy codes shows that Mrs. Kline provided statements concerning a wide variety of issues affecting classroom instruction. Additionally, several of the more prevalent codes (including Student Comprehension, Time Pressure, Difficult Questions, and Crafting Questions) provided important insight into the relationship between professional self-efficacy and performing Instructional Assessment. A full thematic analysis is presented in the next section.

Themes. All coded interview responses from Mrs. Kline were entered into a spreadsheet to facilitate organization and sorting ability. Further analysis yielded several themes representing challenges in the domains of student engagement and curriculum management, as well as overarching themes representing specific strategies employed by Mrs. Kline to navigate these challenges. A list of each theme and its definition is provided in Table 22. In the sections that follow, each theme is described in detail, along with representative examples from the interview and observation data. A full interpretation of this portion of this case study is presented in Chapter V.

Student Engagement. Mrs. Kline expressed a high level of confidence when working with students in the classroom. She based this largely on her years of experience and interactions with the students in the Forest River school district. When discussing the specific factors that contributed to Mrs. Kline's self-efficacy in this domain, two major themes emerged from the data: Student Disabilities and Student Beliefs and Values. The main codes represented
in these themes include Special Education, Difficult Questions, Student Comprehension, Student Values, and Student Beliefs.

Table 22

*Professional Self-Efficacy Themes for Case Study 2*

<table>
<thead>
<tr>
<th>General Area</th>
<th>Theme</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
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</table>

*Student Disabilities.* The first theme to emerge in the area of student engagement is student disabilities, which represents Mrs. Kline’s perspective of her responsibilities for students with learning disabilities. When asked to describe her role at the school, Mrs. Kline highlighted her dual function as a special educator and a Mathematics Teacher. She then described how this unique role impacted her ability to assess student understanding during instruction.

I consider myself a special education math teacher, and I think the special education part comes first, for me. I mean, I don’t have a math background, so I’m not coming at it with a math-centered focus. I’m looking more for what I know about that student, and how they’re going to approach a problem. (Interview #2)

The main source of information she had for the students in each class with learning disabilities derived from each student’s Individual Education Plan (IEP). These documents comprised detailed information about the specific needs and limitations of each student. Despite the
usefulness of these documents, they also made Mrs. Kline aware of the complexities of her position when attempting to monitor the progress of a significant portion of each class. She elaborated on these challenges, saying,

As a special educator, I have to make sure, not only that students are understanding the ten units of geometry curriculum that we’re getting through in 180 days, but that, for every single student I have in my classroom on an IEP, I have to make sure they’re meeting their goals and objectives. … Within my three different sections of classes, out of the approximately 60 students I teach, I have about 30 students who are on Ed. plans. That means that I have a minimum of 30 goals, with four objectives, that I need to be addressing. So, that’s 120 things that I need to be meeting. (Interview #1)

This shows that Mrs. Kline had structured knowledge of the number of students in her classes currently in need of special education services, and the subsequent requirements designated to her role in the classroom.

The ability to incorporate student learning disability information into mathematical instruction was an essential skill for Mrs. Kline. This represented a significant challenge to her day-to-day activities, and necessarily impacted her professional self-efficacy. She commented on the obstacles she faced when assisting students with learning disabilities.

It’s difficult, having that there, sort of looming. You know, like, is this kid truly getting the service that they need? Are they understanding what I’m asking them to understand? Or should I really be stressing that they’re able to do these tasks, and how I can address them to the objectives later? Like, not focus on the objectives, but focus on, what are they capable of doing, in my classroom, at this current time? (Interview #1)

This shows that the challenges faced by Mrs. Kline extended well beyond administrative duties. She was also required to modify her instruction to the capabilities of multiple students across multiple learning objectives. She conceded that some students with learning disabilities were unable to achieve the required standards for certain objectives, despite repeated attempts. However, she seemed to take personal pride in facing the daily challenges of teaching
mathematics to students with learning disabilities. This attitude was a common theme during the interviews for this study, and was reflected in her highly active and individualized approach to teaching.

In addition to documented information for the students with learning disabilities, Mrs. Kline gathered supporting evidence through methods of Instructional Assessment. By working directly with students and engaging in instructional conversations, she was able to determine best practices at an individual level. When asked about her questioning style, she explained,

> It depends on the student, it depends on the task, and it depends on my understanding and awareness of their previous knowledge. If I feel as though they're comfortable and confident with the material, then I might ask sort of a higher-level question. I won't necessarily ask a very, very high-level question, until I know that we've practiced material, and that there are students in the classroom that confidently could answer the question for me, because I don't want to set them up for failure, and I also don't want to set them up for them not feeling like they can ever answer questions in here. I want them to at least feel like they can get half the question, or give an answer. (Interview #1)

This shows the potential level of detail involved in a typical student-teacher interaction. The information gleaned from these conversations was invaluable for characterizing student learning disabilities in the context of the mathematics classroom, and could be shared with teachers in other subject areas during weekly meetings.

**Student Beliefs and Values.** The second theme that emerged in the area of student comprehension is *student beliefs and values*, which represents Mrs. Kline’s perspective of student opinions on the teaching and learning of mathematics. This theme initially emerged from a description of her own experiences as a student.

> If I thought it was a good idea to stand up in front of the room and lecture for 80 minutes, at 15-year-olds, then I would go to Google, or Teacher Tube, find a video, and put it up on the screen. I can do a better job than that. A video can do that job. That’s not my job. As a student, in the classroom, I hate sitting and listening to a teacher, or an administrator, or someone else, talk for 80 minutes. (Interview #1)
This, and other similar statements, indicate that Mrs. Kline placed a high level of importance on avoiding traditional lecture teaching techniques. While some of the interview response segments were coded under Personal Pride, most were coded under Student Beliefs and/or Student Values. This is because Mrs. Kline frequently phrased her interview responses from the student viewpoint. She explained,

I also feel like, as a student, if I’m not engaged, and I’m not participating, and I’m not asking or answering questions, I won’t necessarily understand what took place in that period, because I’m a very visual and kinesthetic learner. So, I need to have that experience with my learning. If I don’t have that, I’m not, well, I don’t buy in, at all. And I don’t really retain the information. (Interview #1)

This shows that Mrs. Kline incorporated the perceptions of the students into her instructional approach. Through this perspective, Mrs. Kline established herself as a facilitator of student-centered learning.

Included in this mindset is an understanding of how her teaching methods impacted student self-efficacy. She explained her approach to working with struggling students. She said,

There are going to be some [students] that I focus on more often than I should, because I know that they need, like, a pat on the back. They need the reassurance that I’m always going to be there, in that positive way, so that they have somebody that does that for them… Other kids that are math-phobic are completely phobic, and they don’t think they can ever do it, so you need to be building their confidence, in more than just math, but in other ways as well. So that you can really get them to step forward and say, this isn’t as bad as a I thought, and I can do this, and we can push forward with it. (Interview #2)

Supporting learning through consistent recognition of student beliefs and values helped Mrs. Kline to gain the trust of individual students. Furthermore, she was able to adjust her approach to students at all levels of ability, regardless of whether or not they had documented learning disabilities. Her statement even shows that her concerns extended beyond the mathematics classroom to incorporate a detailed understanding of the student perspective. During classroom
observations, she was often observed discussing school events with individual students prior to the start of the class period. This showed the students that she cared about their day to day lives, and provided her with an avenue for motivating classroom activity.

One method of Instructional Assessment that Mrs. Kline employed to gain student trust was requiring multiple hands to be raised before selecting a respondent. She would then call on several of these students to describe individual solution steps. This had the effect of making the solution process a group effort rather than dependent on a single student. In the following observation, taken from a Geometry class, Mrs. Kline uses this method to gauge student responses following a group activity.

Mrs. Kline: I’d like to get at least five people raising their hands about what they think about question number one, okay. So, I’m going to wait until I see five people before I call on anyone. I got one, I got two—answering question number one. I got three people so far, wait, four. Nobody else in this group got question number one? Part A there? Okay good. So, I got five people now. So, I’m going to start from this side and work my way this way.

The conversation that followed over the next two minutes included responses from three of the students who raised their hands. As she moved to the second part of the problem, she praised those students who had contributed and again asked for multiple volunteers.

Mrs. Kline: It’s different ways that we can interpret this question, right? So, everything that people gave me, I love. I love that you said that. It’s nice to hear you disagree. It’s possible, based on the vagueness of this question, to answer ‘yes’ or ‘no,’ right?

Students: Yes

Mrs. Kline: Part B. Can I get some people to raise their hands for Part B? I’m going to wait for five again so that people have a chance to think about it and figure out where I’m starting. So, I got three, so far. Four. Got five, alright. So, I’m starting my way from the back this time.
By consistent recognizing the importance of student beliefs and values, Mrs. Kline was better able to gain the trust of individual students, which helped to improve her professional self-efficacy.

Curriculum Management. Along with the daily challenges centered around student engagement, Mrs. Kline described specific curricular requirements of the courses she taught at Forest River that influenced classroom instruction. Additionally, several instructional conversations regarding the importance of certain mathematical topics were recorded during the classroom observations, in particular concerning how they would be assessed for understanding. When discussing the specific factors that contributed to Mrs. Kline's self-efficacy in the domain of curriculum management, two major themes emerged from the data: Summative Testing and Time Management. The main codes represented in these themes were Curriculum Standards, Special Education, Time Pressure, and Shared Teaching.

Summative Testing. The first theme that emerged in the area of curriculum management was summative testing, which represents Mrs. Kline's experiences of preparing students for annual high-stakes testing. The opinions shared by Mrs. Kline during both interviews showed that she had developed significant negative associations and feelings about summative testing during her time at Forest River. Much of this was centered on working with students with learning disabilities, many of whom had trouble passing the exam. She explained,

I have students who work really hard to understand what I’m delivering to them. To make them sit, for two full days, to take a test about math, that makes them, or breaks them, and means they will graduate or they will not graduate, really infuriates me. It shouldn’t be about that. … I’ve had students that have failed [the MCAS], like, three times. And I, as a special educator, have to proctor that test, over and over and over again. (Interview #1)
It is important to note that, despite these comments, Mrs. Kline was also able to describe positive aspects of preparing students for the MCAS, and a general understanding of the purpose of summative testing. However, it remained very clear that she felt that many of the students under her supervision were being put in a position to fail. She said,

I appreciate the idea of testing to meet standards at a state and national level, because there are far too many teachers, in far too many school districts, that say, ‘oh, that kid gets an A.’ And I appreciate the idea of the MCAS sort of leveling that bar. But when the bar is here for my kids already, and they can barely reach it, it’s really irritating to say, ‘well, you’re not going to graduate, so you can just keep swinging. Good luck trying to grab onto that bar.’ (Interview #1)

This shows that the challenges associated with summative testing had become a significant concern for Mrs. Kline, and impacted her professional self-efficacy.

In addition to the barriers affecting students with learning disabilities, Mrs. Kline also described the effect on classroom instruction. Similar to her previous comments she balanced strong negative associations with practical examples of test preparation. She said,

I hate the MCAS. It irritates me how often we have to discuss the test, period. But, I would say that the work that we’ve done is embedded enough that, it’s not like we’re teaching to the test. It’s more like, right now, we’re talking about polygons, so we’re going to go to the MCAS and find a problem that talks about polygons, pull it out, and use it as an example. More often than not, I think that we use it more as another resource, for pulling example problems. Especially problems that are written in a more complex way, or in a different way, with a different approach, than our textbook is providing, or that we can come up with. And I appreciate that that bank of problems is there for us, and that we can say to them, ‘well, this is how they’re approaching it, so let’s see if we can figure it out.’ (Interview #1)

Here, Mrs. Kline highlights the ways in which MCAS problems appear different than those in the standard curriculum. It was deemed important for the students to become familiar with a variety of formats to improve their chances of performing well on the test. To facilitate this, MCAS preparation was regularly used to complement instruction. Additionally, students had to
complete MCAS practice problems that were separate from their regular homework assignments. The solutions to these problems were then covered during the following class as time allowed.

In the following example, taken from a Geometry class, both teachers communicated detailed expectations of the MCAS exam with the students, and explained how that would impact the upcoming week of classes.

Mr. Patrick: So, to give you a little preview, because it is about six days left until MCAS. We’ve been doing a lot of geometry stuff and that sort of thing. Unfortunately, …about one-third of the test is geometry. The other two-thirds of it is algebra stuff, alright; relating to functions, relating to order of operations, square roots, linear stuff. … So, just so you know that, we’re going to be spending some time over the next couple days, going back, stepping out of geometry and really hitting those algebra concepts hard.

Mrs. Kline: So, the questions that you have been doing for the past couple of nights, have basically been the last three years of MCAS, so that’s where those questions are coming from. So, they’re the most recent versions of the test and that’s what we’re going to focus on because they kind of go in cycles and we want to keep it a little bit more current for you.

Mr. Patrick: We can usually tell you, like, I’ve been looking at these tests, the MCAS ones for a while and I can usually tell you they sort of go in a cycle, like every other year, you’ll either have a stem-and-leaf plot question or a box-and-whiskers, usually one or the other. You typically---the test won’t have both of those questions. So, it depends on what year and it sort of cycles. And there’s other really, really similar questions.

This episode shows the importance that both teachers placed on preparing students for the MCAS, and the amount of time that they were willing to dedicate to in-class and at-home activities. They were willing to forego time spent on topics from the class in which the students were enrolled, in order to focus on other topics that would be emphasized on the exam. Other, similar discussions took place during a few observations, with a general increase in frequency as the testing dates approached. These discussions, examples, and assignments necessarily took
time away from the regular classroom instruction, which is the subject of the next theme for professional self-efficacy.

**Time Management.** The second theme that emerged in the area of curriculum management was *time management*, which represents Mrs. Kline's perspective of her ability to perform classroom instruction efficiently. Interestingly, despite the time required to prepare students for MCAS, Mrs. Kline did not seem overly concerned about operating within the limitations of the daily schedule. When asked about time pressure, she acknowledged an understanding of its existence, but explained why it did not necessarily impact her teaching approach. She stated,

> Within an individual period, I don’t necessarily feel the stressors of, oh my God, I have to get this done in the next five minutes. I don’t necessarily feel that’s important. I think it’s more important to clarify, and allow students to take the time to process and understand what they need to do. And if we postpone something, or do something the next day, then we do it. There are certain days where I feel that pressure, where it’s like, we’re getting ready to do a test, or the MCAS, or a project, or I want to make sure that we’re really using our time efficiently, and effectively, for those kids. (Interview #1)

Mrs. Kline further emphasized that learning objectives were not always accomplished during individual class periods. She discussed her overall approach to time management, noting that while class activities were not strictly scheduled, she had a general awareness of the pacing required to stay on task.

> I would say that there’s about, there are usually three or four main things that we’re trying to hit, on the agenda. And I would say that I’m trying to pace myself with, okay, we have about, we’ve already covered like twenty minutes, or we have about a half-an-hour left. I’m pacing myself with those kind of things, like chunks of time. Not, I’m going to do this for five, I’m going to do that for ten, I’m going to do this for five. I can’t do that. (Interview #2)

Consistent with this description was an admission that classroom activities frequently required more time than initially scheduled or anticipated.
Like, we might say, we’re going to spend five minutes on the Do Now. It never takes five minutes. Ever. Ever, ever, ever. Even if we put five minutes on the clock, it takes them five minutes to sit down and do it, and then it takes us another five minutes to give the answers and discuss it. (Interview #2)

Further complicating matters of time management were the administrative duties Mrs. Kline attended to during the school day as part of her special education role. There were frequent instances where she had to leave the classroom for portions of a class period, either accompanied by an individual student, or for an outside purpose. However, during the observations, she always returned before the completion of the class period. Despite the requirements of her positions, she regarded her time in the classroom as very important. She said,

There are teachers, special ed. teachers in this building, who are out of the classroom like half the time, because they’re off doing other duties. I want to make sure I’m in this classroom as much as humanly possible, for these kids. Because I know that they benefit from two different viewpoints, and I know that they benefit from having somebody else to ask a question of, if they’re not sure they’re getting their question answered. (Interview #1)

She attributed much of this approach to the positive co-teaching relationship she had developed with Mr. Patrick over the previous four years, something that had not always been present in previous classrooms.

We are willing to give and take. And I think that’s the biggest thing that makes us a success, is that we are willing to say, I’ll put in half if you put in half. And on days when he gives 70, and I can only give 30, because I have other duties, the next day I’m doing 70 and he’s doing 30. Because I don’t want to necessarily take away from his time and his stress and his, you know, I want to appreciate the work that he’s doing. But he also appreciates my other job, that I’m not just a math teacher. So, he knows that there are going to be times when [he’s] going to be flying solo. I’ve got to go do something else. (Interview #1)
This shows that the challenges associated with time pressure were alleviated through balanced teaching responsibilities and consistent interactions with students during each class period.

**Overarching Themes.** The themes presented thus far represent challenges experienced by Mrs. Kline in the domains of student engagement and curriculum management. Two additional themes emerged from the data representing specific strategies employed by Mrs. Kline to navigate these challenges: Acknowledgement and Exemplifying. The following sections provide evidence for these overarching themes, and illuminate some of the ways in which Mrs. Kline managed the daily demands of her position while helping students to master course material. The main codes represented in these themes include Student Beliefs, Outside Factors, Improving Understanding, and Student Values.

**Acknowledgement.** The first overarching theme is *acknowledgement*, which represents Mrs. Kline's perception of communicating realistic expectations for student success. Mrs. Kline’s understanding of student abilities primarily derived from years of accumulated experience with the students and mathematics curriculum at Forest River. However, she recognized that as students changed from year to year, she would constantly have to update her methods accordingly.

We don’t do the same thing every year, because the group of kids that comes in here every year is so different. Like, this group of kids is very different than last year’s group. So, we can’t necessarily approach them with the exact same worksheets, graphic organizers, and activities, that we approached with last year. (Interview #1)

Mrs. Kline consistently balanced student comprehension and curriculum requirements by acknowledging and operating within individual and collective student abilities and limitations. In doing so, she attempted to maintain acceptable standards of performance, while simultaneously communicating realistic expectations for her students.
Depending on the student, and I’ve had more than enough before school, lunchtime, after-school conversations with students about, your life is always going to have something in it that’s distracting, difficult, or forces you to work past it. That’s life. Those things are going to be there. And right now, this is your job. You’re getting paid with grades, and your diploma is going to push you forward. … I’m here to help you with that, as your boss, and as your manager, but not if you’re not taking those steps and that initiative, and you’re not pushing forward, I can’t help you with this job. (Interview #2)

This remains an ongoing challenge for Mrs. Kline. As discussed earlier in this chapter, the realities of high-stakes testing were often at odds with her professional assessment of student capabilities, especially for students with learning disabilities.

I’m not asking these kids to go out and be engineers and mathematicians. They’re not going to be. Some of these kids will, because they’re really great in mathematics, but not all of them will. Some of these kids are going to be mechanics, and hairdressers, and admins in offices. They don’t have to have the Pythagorean Theorem applicable in their daily life. They have to understand how to problem-solve, and how to work around those kinds of things. But they don’t have to have these techniques embedded in their brain, so that they can apply them to their jobs. (Interview #1)

Despite these challenges, Mrs. Kline stated that she felt comfortable in her abilities to relate to the student on their level. She mitigated her frustration of the factors outside of her control in part by focusing on a student-centered teaching approach, including methods of Instructional Assessment.

*Exemplifying.* The second overarching theme is *exemplifying*, which represents Mrs. Kline's perspective of serving as a role model for the students. Exemplifying is considered to be largely a proactive attitude toward student engagement and teaching in general. That is, Mrs. Kline had developed a well-defined sense of the way in which she would be perceived by students during instruction, as was earlier described in the theme of student beliefs and values. This theme initially emerged from an interview question about correcting student mistakes. She admitted,
It doesn’t go over well, ever, for one. And when you get to a point where they need to understand, they are wrong, and you are right, they need to know, it’s okay to admit that you’re wrong, and to say, ‘I’m sorry, I made a mistake.’ They need to know it’s okay to do that. (Interview #1)

One way in which Mrs. Kline demonstrated this approach was by working through her own mistakes in front of students with a positive attitude.

I don’t like to have to model that I make mistakes all the time, but, you know, if I have to do that, to get them to relax, and feel comfortable about making mistakes, then I’ll do that. I’ve made enough mistakes in here that they know that I don’t know everything. But if they ask me a question, I’ll have an answer. (Interview #1)

During classroom observations, Mrs. Kline demonstrated this approach by calmly working through her own mistakes, while also remaining empathetic when dealing with student mistakes.

In addition to serving the needs of the students, Mrs. Kline’s approach toward classroom instruction seemed to be fueled by a strong sense of personal pride. During each interview, she made it abundantly clear that she takes her job very seriously, and holds her students and herself to a high standard. She attributed her ability to deal with daily and accumulated stressors to past successes with her students at Forest River.

I think, the thing that keeps me coming back for more is knowing that, from the beginning of the year to the end of the year, or even the beginning of the semester to the end of the semester, in our kids, there is usually an immense amount of growth. Whether it’s maturity-wise, mathematically, academically, socially, there is usually, you would hardly recognize some of the kids, because they really have stepped up, in some way or another. (Interview #2)

During the observations, Mrs. Kline exemplified this attitude through consistent enthusiasm and a positive demeanor with her students. It should be noted, however, that her encouragement was not without expectations. She regularly reminded the students of their responsibilities in and out of the classroom. Her ability to balance this message appropriately for each student is the subject of the next overarching theme.
Knowledge for Teaching

Mrs. Kline’s understanding of secondary mathematical content incorporates a wide range of topics from multiple subject areas. This knowledge base was largely established during her years of experience at Fall River, and continues to be modified as she instructs and observes students. Central to this adaptation is an understanding of varied student perspectives, especially in the area of misconceptions. The combined evolution of Mrs. Kline’s mathematical knowledge along with her ability to assess students’ mathematical knowledge encompasses her knowledge for teaching. Codes derived from interview responses revealed important insight into this relationship. Descriptive statistics from the coding process are presented in the tables below, followed by the development of themes in the area of knowledge for teaching. Results in this area will be used to answer the second and third research sub-questions: How do mathematics teachers reinforce or modify their understanding of mathematical content and processes through classroom interactions? How do mathematics teachers make use of observation, listening, and questioning during instruction to construct models of student understanding?

Codes. In total, 250 interview response segments categorized under Reinforcement or Modification of Mathematical Content or Processes (RMMCP) or Models of Student Understanding (MSU) were coded according to two descriptors: scope and knowledge for teaching. Scope codes were useful for organizing interview response segments for thematic analysis across multiple contexts. For example, when a particular theme was underrepresented at a specific scope, additional evidence was obtained from the observation transcripts. Using this method ensured a triangulation of data sources for understanding Mrs. Kline’s knowledge of mathematical content, teaching, and students. Knowledge for teaching codes designated a
particular context for specific classroom practices, and were the primary sources for the creation of themes.

The first set of codes were designed to describe the scope, or contextual breadth of Mrs. Kline’s statements. Short-view (S) responses referenced either specific mathematical concepts or individual students. Medium-view (M) responses described either common mathematical topics or entire classrooms of students (or students in general terms). Long-view (L) responses included general statements about either entire mathematical subject areas, or student trends from year to year. Scope code frequencies are listed in Table 23.

Table 23

<table>
<thead>
<tr>
<th>Code</th>
<th>Name</th>
<th>Context</th>
<th>Freq</th>
</tr>
</thead>
</table>
| M    | Medium | Mathematical topics
Classrooms of students (or students in general)                   | 198  |
| L    | Long  | Mathematical subjects
Student trends from year to year                                     | 11   |
| S    | Short | Mathematical concepts
Individual students                                                      | 41   |

A majority of Mrs. Kline’s interview responses (79%) were coded as medium scope, showing that her primary focus was on classroom management. Short-scope responses (16%) and long-scope responses (4%) were not as well-represented. However, a sufficient amount of data was recorded to develop themes in the domain of knowledge for teaching.

The main set of codes was taken from the framework developed by Kilpatrick, et. al. (2015) on mathematical understanding at the secondary level (MUST). Of the fourteen codes derived from this framework, ten were recorded at least once in the data. Four additional codes
(Summative Assessment, Student Disabilities, Co-Teaching, and Outside Factors) were included when interview response segments emerged from the data that were not represented in the framework. All fourteen knowledge for teaching codes are listed and defined in Table 24, along with their frequencies.

Table 24

Knowledge for Teaching Codes for Case Study 2

<table>
<thead>
<tr>
<th>Code</th>
<th>Name</th>
<th>Context</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>RP</td>
<td>Reflective Practice</td>
<td>Examination of classroom decisions</td>
<td>72</td>
</tr>
<tr>
<td>MT</td>
<td>Mathematical Thinking</td>
<td>Interpret student explanations</td>
<td>42</td>
</tr>
<tr>
<td>MK</td>
<td>Mathematical Knowledge</td>
<td>Access components of student knowledge</td>
<td>28</td>
</tr>
<tr>
<td>PD</td>
<td>Productive Disposition</td>
<td>Curiosity, enthusiasm, and perseverance</td>
<td>19</td>
</tr>
<tr>
<td>KC</td>
<td>Knowledge of Curriculum</td>
<td>Identify concepts and assign learning goals</td>
<td>18</td>
</tr>
<tr>
<td>SD</td>
<td>Student Disabilities</td>
<td>Incorporating documented disabilities</td>
<td>18</td>
</tr>
<tr>
<td>SA</td>
<td>Summative Assessment</td>
<td>Preparing students for annual testing</td>
<td>15</td>
</tr>
<tr>
<td>OF</td>
<td>Outside Factors</td>
<td>Incorporating factors from outside school</td>
<td>10</td>
</tr>
<tr>
<td>SC</td>
<td>Strategic Competence</td>
<td>Assess problem solving strategies</td>
<td>8</td>
</tr>
<tr>
<td>PM</td>
<td>Probing Mathematics</td>
<td>Investigating mathematical ideas</td>
<td>8</td>
</tr>
<tr>
<td>CT</td>
<td>Co-Teaching</td>
<td>Shared classroom responsibilities</td>
<td>5</td>
</tr>
<tr>
<td>PF</td>
<td>Procedural Fluency</td>
<td>Recalling and executing procedures</td>
<td>3</td>
</tr>
<tr>
<td>AR</td>
<td>Adaptive Reasoning</td>
<td>Working in multiple mathematical contexts</td>
<td>3</td>
</tr>
<tr>
<td>CU</td>
<td>Conceptual Understanding</td>
<td>Deriving and understanding connections</td>
<td>1</td>
</tr>
</tbody>
</table>

One code stands out from the rest: Reflective Practice (29%). This shows that Mrs. Kline tended to frame her interview responses as self-assessments of her many roles as a mathematics teacher. Mathematical Thinking (17%) and Mathematical Knowledge (11%) are the only other codes to appear in at least ten percent of the interview responses. This provided important insight into the
shared development of instructor and student mathematical knowledge that emerges during Instructional Assessment. A full thematic analysis is presented in the next section.

**Themes.** All coded interview response segments from Mrs. Kline were entered into a spreadsheet to facilitate organization and sorting ability. Further analysis yielded several themes representing challenges in two domains of Pedagogical Content Knowledge: Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT). Additional overarching themes representing specific strategies employed by Mrs. Kline to navigate these challenges were also developed. A list of each theme and its definition is provided in Table 25. In the sections that follow, each theme is described in detail, along with representative examples from the interview and observation data. A full interpretation of this portion of this case study is presented in Chapter V.

Table 25

**Knowledge for Teaching Themes for Case Study 2**

<table>
<thead>
<tr>
<th>Domain</th>
<th>Theme</th>
<th>Definition</th>
</tr>
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**Knowledge of Content and Students.** Themes in the domain of KCS represent “content knowledge intertwined with knowledge of how students think about, know, or learn this
particular content” (Hill, Ball, & Shilling, 2008, p. 375). The primary emphasis, therefore, is on the challenges of assessing student understanding, rather than instructional design. When discussing the specific factors that contributed to Mrs. Kline's knowledge for teaching in this domain, two major themes emerged from the data: Foundations and Student Condition. The main codes represented in these themes include Mathematical Knowledge, Mathematical Thinking, Student Disabilities, Outside Factors, and Knowledge of Curriculum.

*Foundational Learning.* The first theme to emerge in the domain of KCS is *foundational learning,* which represents Mrs. Kline’s ability to identify and address common gaps in students’ mathematical understanding. During her time at Forest River, Mrs. Kline had become accustomed to the students’ typical learning habits. She explained that most student errors arise from simple mistakes involving prerequisite material or procedural solution methods that have not been fully established.

I would say, with this group of kids, most of the time, we’re looking for sign errors, or [order of operations]. They’re still making those mistakes. They’ve been doing it since eighth grade, and they’re still making those mistakes. (Interview #2)

This recognition played a significant role in determining Mrs. Kline’s instructional approach. First, when introducing material for the first time, she made frequent connections with previously established concepts. This helped her to emphasize the types of prerequisite knowledge that the students needed to access. Second, when reviewing student work, she was able to focus on the structure supporting each problem, rather than the specific concepts that were being presented. This helped her to address student errors using first principles.

Mrs. Kline made use of a variety of formative assessment strategies to identify foundational learning gaps and provide feedback to each student in a timely manner. Daily
homework reviews and in-class work provided ample opportunities to examine student thinking. She described these techniques and explained why it was useful to repeatedly perform them.

The more often we collect things, the more we can see the little mistakes that they’re making. So, if we try to collect things more frequently at the beginning of the year, like Do-Now problems or their homework problems, or if we give them homework quizzes, then it shows us different ways where they might be hitting a homer, or completely striking out. (Interview #2)

Typical feedback from these informal assessments included immediate verbal interventions and handwritten comments on submitted work. Mrs. Kline made sure to inform the students of the types of mistakes they were making, and made efforts to identify appropriate remedies.

We can say, ‘they followed directions here, or they understood what they needed to do, but they’re making calculation errors. Or, they have no idea what they’re doing, and we really need to backtrack.’ Then I come back the next day, and I hand them out, and I have notes on the side. I can come back to those individual students and say, ‘I noticed you did this, this, and this. If you could try these two particular problems today, and focus on the mistakes that you made, see if you can push forward with that. (Interview #2)

This shows that Mrs. Kline was engaging in evaluative feedback, designed to help students identify and resolve their own errors (Wiliam, 2007).

Another effective technique used by Mrs. Kline to identify foundational learning gaps occurred during instructional discussions. Students would take turns verbally responding to instructor questions so that the solution to a problem could be determined incrementally. However, in some cases, when a mistake was made, Mrs. Kline would not intervene immediately. She would write the work exactly as the student had stated it, and move on to the next step of the problem. The goal was to have one of the students identify the mistake, rather than making it the responsibility of the instructor. Mrs. Kline commented on her use of this technique.
I’m listening to exactly what they’re saying, and I’m focused on what they’re doing. And I can see the mistake, and I’m writing it. And I’ll just move forward with them. And then, if they don’t catch it, and another student doesn’t catch it, I’ll say, ‘I really don’t agree with everything that’s on the board right now.’ I want them to find it. (Interview #1)

Mrs. Kline felt strongly that students would benefit from finding their own mistakes. She also wanted them to be viewing written work with a critical eye, and not to assume that it was always correct.

One example of this technique occurred late in the semester, in an Algebra II class. The students were solving an application problem requiring the linear equation: 

\[ y = 40x + 68 \]

but one of the students had verbally transposed the values for slope and y-intercept. After writing the incorrect equation on the board, Mrs. Kline asked for a few more student responses until the mistake was discovered. She took the opportunity to address the wording of the problem to see if the students could avoid a similar mistake in the future.

Mrs. Kline: I’d like for someone to clarify for me, if we’re thinking about this linear style of an equation which everybody’s trying to write, what’s the slope in this problem? Someone raise their hand and tell me what the slope is in this problem. I saw a bunch of people. I’m still going to start on this side of the room. Student A?

Student A: Forty.

Mrs. Kline: Okay, and then Student B? You agree. Okay. If 40 is our slope, what in that process tells us that 40 is the slope? What do we know? Like this word problem is like dang, right there, there’s the slope. I know it.

Mr. Patrick: There’s key words that should be like popping out at you. Student C?

Student C: Because it says 40 per hour and the X would be less.

Mrs. Kline: What’s the X if you’re telling me forty per hour? X represents –.

Student C: Actually would be the amount of hours.
Mrs. Kline: Amount of hours. I love that you said that. Okay. We need to sort of isolate that idea. I love that we’re looking at per hour. That’s really helpful to pull that idea out. If we know that’s our slope and we know that we’ve got that per hour idea and that’s our variable, we can start looking at that piece.

In this vignette, Mrs. Kline demonstrates how a student mistake can be used to quickly address foundational learning concerns during instruction. She was able to focus on the underlying language contained in the application problem, resulting in a strategy that students could use to determine the value of the slope.

**Student Condition.** The second theme to emerge in the domain of KCS is *student condition*, which represents Mrs. Kline’s ability to incorporate knowledge of individual student psychosocial factors into classroom instruction. In her position as a special educator, Mrs. Kline has a comprehensive understanding of student learning disabilities. Additionally, she attends weekly meetings with a team of educators (including guidance counselors, student liaisons, and classroom teachers) to discuss factors influencing student performance. The meetings promote discussion concerning observed changes in student behavior or academic performance, so that interventions can be adjusted accordingly. This provides Mrs. Kline with a wealth of information concerning new and ongoing challenges for each student outside of the classroom. She described some of the questions that she typically brings to these meetings.

Have you been noticing these difficulties with test-taking anxiety? Have you been noticing lack of homework completion and follow-through? Have they stayed after school with you? What are the things that you have done? What can I do, to try to help this student? Do you know if they have a tutor? Do you know if they have anybody at home that can be helpful? Do you know if they’re here in the mornings or afternoons? What's their attendance like? We usually try to put those pieces together. (Interview #2)

This shows the wide range of concerns that Mrs. Kline incorporated into her knowledge of each student in her classroom.
This broad perspective on the psychological and social factors affecting students played an undeniable role in determining Mrs. Kline’s approach to mathematics instruction. She explained her strategy when working with an individual student in the classroom.

I don’t have a math background, so I’m not coming at it with a math-centered focus. So, I’m not instantaneously looking for the ins and outs and reasons, you know, the complete math steps. I’m not always looking for that. I’m looking more for what I know about the student, and how they’re going to approach a problem. (Interview #2)

In this statement, Mrs. Kline clearly indicates that she prioritizes her accumulated knowledge of each student before considering the mathematics. This allows her to individualize an instructional approach for each student, which can be augmented and adjusted over the course of a semester. She elaborated on this delicate balance.

I don’t think I necessarily baby students who are having a particularly tough time at home, but I don’t necessarily drill them, like I might a different student, or a student I think can handle it, in the moment. The thing that helps me the most is that I’m consistent with them. I’m not just going to get frustrated and give up on them. They know that I’m going to hold them to the same standards as everyone else. I might just approach them individually, because I know that they need an extra moment to deal with what’s going on. (Interview #2)

By continuously incorporating these factors into instruction, Mrs. Kline improved her ability to engage students in mathematical discussions. This, in turn, improved her ability to assess each student’s mathematical knowledge.

The primary methods used by Mrs. Kline to connect students’ mathematical knowledge with individual psychological and social factors included Instructional Assessment. This was especially true in cases where students appeared emotionally withdrawn from classroom activities. She commented that this did not come naturally, and had taken years to evolve.

I wasn’t originally calling on students, as a new teacher, because I didn’t want to make them feel uncomfortable. I also certainly wasn’t feeling as confident or direct in my
questioning, and I think I was asking very straightforward, basic, yes-or-no kinds of questions. (Interview #1)

Mrs. Kline paid close attention to physical or verbal cues that indicated that a certain student was not actively engaged. She described this approach as it pertained to her Geometry students.

I would say about half of the students are pretty confident with answering questions. Of the other half, probably about a quarter of those students can give us gestures, so we know that they’re cuing in, which is nice. The other quarter, sometimes, we have to directly engage them, and then prompt them, or preview a question for them before they’ll even respond, because they just don’t have the confidence level, or they don’t have the level of understanding. (Interview #1)

By not relying on traditional teaching methods, Mrs. Kline was able to reach every student in the classroom, whether they lacked confidence in their mathematical abilities or were dealing with the stress of outside factors.

Knowledge of Content and Teaching. Themes in the domain of KCT represent “mathematical knowledge that interacts with the design of instruction” (Hill, Ball, & Shilling, 2008, p. 375). The primary emphasis, therefore, is on the challenges of implementing teaching strategies, rather than assessing student understanding. When discussing the specific factors that contributed to Mrs. Kline's knowledge for teaching in this domain, one major theme emerged from the data: Differentiated Instruction. The main codes represented in this theme include Reflective Practice and Productive Disposition.

Differentiated Instruction. The only theme to emerge in the domain of KCT is differentiated instruction, which represents Mrs. Kline’s ability to meet individual student needs through flexible avenues of learning. During her time at Forest River, she had been exclusively tasked with teaching college preparatory (CP) courses (i.e., not honors (HN) or advanced placement (AP)). Students in CP courses represent a wide range of ability levels, with full
inclusion for students with learning disabilities. Mrs. Kline commented on the challenges this presented.

Nine times out of ten, with the students we work with, there is usually a very wide variety of students, in their level and ability on quizzes and tests. There’s a kid who is going to get 100, and there’s a kid who’s going to get a 50. And it’s not necessarily going to change my level of instruction for the entire class, but it’ll change how I approach that student who’s at a 50. (Interview #1)

Both co-teachers needed to be able to accommodate diverse learning styles while maintaining the overall pace of each course. However, in her role as special educator, Mrs. Kline was especially well-equipped to understand which students in each class required individualized attention. Through her training and experiences with students at all levels, she had established a responsive teaching style designed to address student needs directly and efficiently.

The unique co-teaching aspect of the classroom helped facilitate this process. During situations when Mr. Patrick was leading instruction for the entire class, Mrs. Kline could focus on working with individual students or small groups of students as necessary. This allowed the class to continue moving forward, while students were being simultaneously assisted. She emphasized that these interactions helped to determine her teaching methods.

I really do learn something new every day from these kids. If I don’t take the time to listen to them, then I’m not really informing my instruction at all. And it is about them. Because, if I don’t understand teenagers, and I don’t understand this group of kids in my room right now, I can’t really approach them at their level. (Interview #1)

Mrs. Kline stated that she attempts to be proactive with certain students who struggle to keep pace with the class, and checks in with them at least once every period. This helps her to monitor each student’s progress and isolate problem areas. She also said that it eases the tensions that can occur between teachers and students.

Because they’ve developed a level of understanding of me, and of us, in the classroom, and because I feel as though I have a non-threatening approach, then the majority of time,
they’re amenable and acceptable to me coming over and asking questions. They also know that, most of the time, I have a good idea of what they’re thinking, or where they get stuck, and where they make mistakes. (Interview #2)

This claim was substantiated during classroom observations. The students seemed generally comfortable when speaking with Mrs. Kline.

The use of differentiated instruction also provided a vehicle for Instructional Assessment. Focusing on the progress of individual students provided Mrs. Kline with regular opportunities to address areas of weakness. This, in turn, generated frequent feedback for students, and helped them to make necessary adjustments. One area where this process was especially useful was preparing students for the MCAS short-response questions. Students were required to explain their reasoning in addition to showing the solution steps. Because of this, both instructors had to make sure that the students understood their solutions and could also justify them in writing.

Mrs. Kline described a typical conversation.

I might say, ‘as the person grading the MCAS, they don’t know you in this classroom. So, they’re not going to guess what you’re thinking. You have to make sure they understand what you’re thinking. And, right now, I don’t understand what you’re thinking. Tell me what you’re thinking.’ (Interview #2)

This approach allowed students to communicate their understanding of a problem based on their level of understanding. The student remains the center of attention, and has control over how they choose to proceed.

Efforts were made to identify a second theme in the domain of KCT, but no such theme emerged from the interview data. One possible reason for this was Mrs. Kline’s tendency to phrase her statements in reference to the students, rather than to the ways in which she designed instruction. This occurred even when questions were specifically designed for her to reflect upon her teaching methods, and elicit the possible benefits to her knowledge for teaching. She
focused almost exclusively on the benefits to the students, and downplayed the advantages to herself. When pressed on this issue, she explained,

> It’s not about what I need, it’s about them. And I will center it about them. They need to talk about this material. They need to understand it. I’ve already graduated from high school and college. I don’t need to talk about this material. I want them to understand that I have a lot that I offer and bring to the table, and I want them to bring it to the table, too, so we can share what we got. It’s not all about me at the front of the room.  

(Interview #1)

This shows that Mrs. Kline held strong opinions about her role as a mathematics educator and advocate for student-centered learning. This identity is consistent with the themes established in this dissertation.

**Overarching Themes.** The themes presented thus far represent challenges experienced by Mrs. Kline in the domains of KCS and KCT. Two additional themes emerged from the data representing specific strategies employed by Mrs. Kline to navigate these challenges: Active Engagement and Scaffolding. The following sections provide evidence for these overarching themes, and illuminate some of the ways in which Mrs. Kline balanced instructional design with the assessment of student understanding. The main codes represented in these themes include Mathematical Thinking, Reflective Practice, and Probing Mathematics.

**Active Engagement.** The first overarching theme is *active engagement*, which represents Mrs. Kline's strategy of encouraging students to participate in classroom activities. While instructors often report wanting students to participate, research has shown that they frequently fall back on traditional teaching techniques (Shepard, 2000). However, in the case of this study, Mrs. Kline was regularly observed motivating her students to be involved with the instruction process. She explained that this helped her greatly to assess student understanding.

> When I’m working with the whole class, I feel mildly confident that I can assess understanding, because it’s a lot of understanding that needs to happen, for twenty kids.
If only two students are responding, I can’t really assess what the other eighteen are thinking, unless I see them writing and engaged, and I know what they’re doing. (Interview #1)

Here, Mrs. Kline illustrates the difficulty of monitoring student progress from the front of the room. Because of this, she placed a considerable amount of effort into working with students at their desks, either one-on-one, or in small groups. However, convincing the students to react accordingly was still an ongoing challenge.

Mrs. Kline talked at length about the struggle to maintain active engagement among her students. Determining why a certain student might not be participating was a complex process. In addition to dealing with the stress of outside factors, some students were quiet and reserved, while others could be talkative and distracting. In either case, she would attempt to redirect and focus on the instructional goals at hand. She spoke about how she incorporated many factors into determining the best style of intervention.

It depends on the student. If they don’t like how I’ve redirected them, and I think it’s because it doesn’t necessarily relate to what’s going on in the classroom, then I might just say, ‘well, this is what’s happening right now, and I’m asking you because everyone else is doing this.’ And I’ll walk away, and give them a minute, and then I’ll come back. (Interview #1)

Mrs. Kline also drew a connection between students’ comprehension of mathematics to the frequency of their participation. She spoke specifically about knowledge gaps developing in students who do not get involved.

I could name three students, right now, that are not doing well. But, that’s a lack of consistent effort on their part, not a lack of ability. They’ve developed Swiss cheese content for mathematics, in that they’re hit-or-miss with that you’re talking about, because they’re tuned out in class, and then when they tune in, they’re there, and sometimes they’re getting it, and sometimes they’re participating, but the retention is very weak. (Interview 2)
This shows that Mrs. Kline’s strategy of active engagement during classroom instruction was motivated by her intention to frequently assess student understanding.

During instructional discussions, one particular pattern of behavior displayed by Mrs. Kline was consistent praise directed at students who actively participated or answered questions posed to the class. Observation transcripts record her thanking students repeatedly for their contributions to class. She would also, sometimes, briefly explain why a particular action or response was helpful, especially in cases where the wording was not correct. This was very beneficial for acknowledging participation efforts and building trust with the students. The following example, taken from an Algebra II class, demonstrates a typical interaction of this type.

Mrs. Kline: So, for people that weren't here yesterday, can somebody describe how we identify the dimensions of a matrix

Student A: Rows and columns.

Mrs. Kline: Okay, and then, can you describe what it means to be in a row? If we put an example matrix up on the board, and then Student B, you can come down. Got the marker? Go to the board for me, while I'm talking, all right? If you had 0, 1, 2, and then, down, and then you had 5, -7, 10. And then down again, 3, 4, 92. And then make that look like a matrix with the brackets. Awesome, thank you. So, using that example matrix that's up on the board, that Student B just drew for you, can somebody else describe for me, Student A said they were talking about rows and columns. How do you describe where the rows are, in a matrix?

Student C: They’re on the side, top to bottom.

Mrs. Kline: Top to bottom. On the side.

Student D: Rows are horizontal.

Mrs. Kline: Okay, that’s a good word. Thank you for giving it. Student C was correct that rows are on the side, but I like your use of the word horizontal to describe which direction they’re going. Awesome.
In this vignette, Mrs. Kline prompts four students to engage in an introductory discussion on matrices. She invites one student to the board to provide a working example, and then asks for clarification on the meaning and location of matrix rows. In her responses, she thanks the students who have contributed, and speaks enthusiastically about the mathematics. This approach is designed to reward students for active engagement, and helps Mrs. Kline to quickly and efficiently assess student understanding.

*Scaffolding*. The second overarching theme is *scaffolding*, which represents Mrs. Kline's strategy of sequencing questions to develop understanding and promote independence. Given the stated concerns about gaps in students’ foundational knowledge, Mrs. Kline was often hesitant to ask higher-order questions until prerequisite concepts had been established. This process started at the classroom level, by reminding students of previous material prior to the start of instruction, and then continued during classroom activities. She explained how this style of questioning worked with individual students, and how it related to her ability to assess their understanding.

Working one-on-one with a student, I can say that I’m fairly confident that I can understand the level at which they’re working, depending on which questions I’m asking, and I sort of modify them, depending on their level of understanding. If I feel like they don’t understand that material, I’ll ask them a question that’s pretty basic. Hopefully, you know, I can get them to buy into it, and I can move forward, and ask them another question. (Interview #1)

This shows Mrs. Kline’s strategy of using an initial question to determine where a student is in the learning process. From this point, she can tailor follow-up questions based on physical and verbal cues, so that the path to full understanding is optimized.

Mrs. Kline also spoke about how her own knowledge of mathematical procedures impacted the scaffolding process. In a particularly noteworthy statement, she spoke of her
steadfast focus on the student’s mathematical thinking, rather than her own understanding of the problem at hand. She stated,

I go in with my blinders on, in how I approach a problem and think about a problem. I will say that I do that. It’s not a matter of, I want to do that, and I consciously do that. After so many years of teaching, I’ve learned not to say, ‘this is how I would do it.’ I need to understand where they’re coming from, and what they’re seeing, what they’re doing, and then that tells me how to scaffold the questions. (Interview #1)

She described this approach as “pretending not to know the answer,” even though she was usually aware of the correct solution method. This links Mrs. Kline’s instructional approach with her stated identity as a teacher without a math background. Furthermore, as the special education teacher in the room, she felt more comfortable focusing on student progress, rather than the mathematical details of the problem at hand. This also worked to provide students with an active role in determining the direction of instruction.

One example of Mrs. Kline’s question scaffolding technique was recorded during a Geometry class. The students were attempting to find the surface area of a triangular prism as part of a Think, Pair, Share activity. They worked individually for 15 minutes, and were then allowed to ask questions and work with other students. Mrs. Kline identified a student who was having difficulty, and spoke to him for almost eight minutes, asking over forty questions. The entire conversation is too long to contain within this document, but a summary is provided below.

[0:00 – 1:38] Mrs. Kline responds to student raising his hand. He says, “I’m not sure where to start, here.” She looks at his work, and begins asking questions about some of the measurements he has found. The student struggles to understand what she is asking, and after several questions, says, “This is hard.”

[1:39 – 2:03] Mrs. Kline agrees with the student that the problem is hard, and immediately changes her style of questioning to address the fundamental structure of the
problem. Here, she is trying to get the student to understand what the problem entails before returning to the calculations.

[2:04 – 2:55] Satisfied with the student’s responses, Mrs. Kline resumes asking questions about the shapes that make up the surface of the triangular prism, and their dimensions. The student is responding more confidently now, but still making mistakes about the measurements of unmarked edges.

[2:56 – 6:27] The student remarks, “I would have no idea how to do any of this, so, thanks for the help.” Mrs. Kline explains that some of this material had been covered during the previous class, when the student was absent. They return to solving the problem, where the student is now clearly doing most of the work, and Mrs. Kline is just asking short, clarification questions to check his progress.

[6:28 – 7:37] The student has successfully determined the dimensions of each shape comprising the surface of the prism. Mrs. Kline completes her questioning by asking, “So, then, how do you derive the total surface area?” The student motions to his paper, and Mrs. Kline agrees with his explanation. She finishes the conversation by asking the student to find the result based on their work. She then moves on to answer questions from other students.

In this teaching episode, Mrs. Kline demonstrates her ability to scaffold questions based on an immediate assessment of student understanding. This included an adjustment of her initial strategy once it became apparent that the student was not fully processing the concepts. This allowed her to re-establish a foothold for understanding, from which she could assist the student in discovering the individual pieces.

This concludes the presentation of the two case studies for this dissertation. In the following chapter, a concise summary of all findings derived from the data analysis will be presented, with specific attention to the role of Instructional Assessment.
CHAPTER V

CONCLUSIONS AND DISCUSSION

This dissertation study was designed to illuminate possible connections between Instructional Assessment practices and mathematics teachers’ professional self-efficacy, knowledge of mathematical content, and knowledge of students. Case studies were performed with two state-certified co-teachers in a Massachusetts secondary public school: a mathematics teacher and a special education teacher, both with eight years of experience. Each instructor was observed and audio-recorded teaching classes and interacting with students over the course of an academic semester. They were also interviewed and asked to complete surveys on professional self-efficacy, before and after the observation period. All data was analyzed through a process of categorizing, coding and the identification of themes outlining challenges and strategies experienced by each participant. The results provide important insight into the role of Instructional Assessment in the 21st Century mathematics classroom.

This chapter begins with a summary of the findings presented in Chapter IV. Themes related to professional self-efficacy and knowledge for teaching are revisited with particular attention to their relationship with Instructional Assessment. Each case study is presented separately, followed by a summary of cross-case findings. Next, the findings are interpreted into conclusions for the study, which are organized under each research sub-question, followed by the main research question. Finally, the implications of this study are considered for mathematics education researchers as well as secondary mathematics teachers and school districts.
Summary of Findings

This section provides a consolidated review of themes that emerged in the examination of each case study. It is important to note that while care was taken to avoid redundancy during data analysis, the themes presented herein are not mutually exclusive. Common threads appear across thematic boundaries, illustrating the complexity of classroom teaching. Furthermore, challenges and strategies related to each participant’s teaching practices are necessarily interconnected with their knowledge of mathematics, and their ability to accurately assess student understanding. Nevertheless, for the purposes of this summary, all themes are organized according to the original analytic matrix presented in Chapter III (see Table 8). Principal connections to Instructional Assessment are highlighted, and provide evidence for the eventual conclusions of this study. Following a review of each case study, a discussion of cross-case findings is presented.

Case Study 1 – Mr. Patrick

Mr. Patrick was designated as the first participant in this study due to his background and training in mathematics, and his position as a general mathematics educator. His experiences with the diverse student population at Forest River as a teacher and committee work as a curriculum developer made him a particularly suitable participant for this study. Moreover, his daily efforts to lead classroom discussions and design challenging activities with all of his students provided significant opportunities to observe episodes of Instructional Assessment. Following the analysis of interview and observation transcripts, a total of twelve themes were developed for Mr. Patrick in the areas of professional self-efficacy and knowledge for teaching. A summary of these themes is now presented.
Themes in Professional Self-Efficacy. The themes in this section represent Mr. Patrick’s perceived ability to perform the required daily activities of his position, taking into account the inevitable barriers and external constraints he encountered. These themes identify the challenges Mr. Patrick reported in his ability to sustain student engagement and curriculum management, as well as the overarching strategies he employed. Refer to pages 76 – 95 in Chapter IV for findings related to Mr. Patrick’s professional self-efficacy.

The main themes to emerge in the domain of student engagement were anticipation and student background. These represent ongoing challenges to Mr. Patrick’s ability to predict common student errors and areas of difficulty, coupled with their prior knowledge base, and willingness to engage in mathematical discussions. Mr. Patrick cited his long-term experience with Forest River students as being instrumental in his ability to effectively and efficiently address student needs during instruction. Each of the above characteristics was primarily informed and modified through daily student-teacher interactions. Mr. Patrick then used the real-time information provided by these interactions to influence the pace and direction of classroom learning. By regularly incorporating Instructional Assessment, he was better equipped to respond to student needs as they occurred, and customize instruction to reinforce student strengths and address potential weaknesses.

The main themes to emerge in the domain of curriculum management were sequencing and consistency. These represent ongoing challenges to Mr. Patrick’s ability to organize and pace mathematical topics, especially when teaching courses for the first time, or after a long hiatus. Mr. Patrick cited the frequent use of repetition and review as a primary teaching strategy for helping students to connect new concepts with previously established material. This process was enhanced through Instructional Assessment by helping Mr. Patrick to identify the concepts
that had become established, so he could focus on those requiring additional reinforcement.
Teaching the same courses from year to year further improved Mr. Patrick’s ability to manage curriculum requirements based on patterns of student misconceptions, and design appropriate instructional activities.

The main overarching themes to emerge under professional self-efficacy were balancing and negotiating. These represent reactive and proactive strategies employed by Mr. Patrick to better attend to student needs while adhering to curriculum standards and promoting student autonomy in the classroom. Mr. Patrick cited the use of active questioning during instruction as a technique for maintaining this equilibrium, and encouraged students to take pride in their contributions to class to help build responsibility and motivation. Activities such as Think, Pair, Share and group station work were regularly employed to elicit student participation, and provide further opportunities for Instructional Assessment. Over the course of an entire semester, Mr. Patrick was able to anticipate difficulties, implement effective feedback, and develop further opportunities for students to achieve success.

The themes presented in this section demonstrate challenges affecting Mr. Patrick’s professional self-efficacy, and the strategies he applied during classroom instruction to mitigate them. The findings of this study show that methods of Instructional Assessment improved his ability to anticipate problems and incorporate student experience into his teaching approach. Conversely, a comprehensive understanding of mathematics sequencing brought on by consistent teaching assignments improved his ability to implement methods of Instructional Assessment. The combined effect motivated Mr. Patrick to balance student needs and curriculum standards, while effectively negotiating control of classroom discussions.
**Themes in Knowledge for Teaching.** The themes in this section represent aspects of Mr. Patrick’s pedagogical content knowledge (PCK). These themes identify the challenges Mr. Patrick reported in his ability to assess student understanding (KCS) and implement teaching design (KCT), as well as the overarching strategies he employed. Refer to pages 95 – 115 in Chapter IV for findings related to Mr. Patrick’s knowledge for teaching.

The main themes to emerge in the domain of KCS were *learning trajectories* and *general proficiency*. These represent Mr. Patrick’s knowledge of the influence of mathematical concepts on future learning and broader topics, as well as the ability to distinguish between conceptual and procedural knowledge and misconceptions. Mr. Patrick cited the use of frequent informal classroom assessments as valuable resources for building his knowledge of how students respond to mathematical concepts. Methods of Instructional Assessment, including purposeful questioning and individual or group work serve to formalize the boundaries of student understanding and guide Mr. Patrick in the design and implementation of effective teaching strategies. Additionally, he is more informed on how certain topics will be received by the students, and is therefore better equipped to develop strategies for bridging the gap between procedural fluency and conceptual understanding.

The main themes to emerge in the domain of KCT were *mathematical disposition* and *curriculum standards*. These represent Mr. Patrick’s mathematical knowledge base, and his familiarity with students’ prerequisite knowledge across grades, courses, and assessments. Mr. Patrick described his educational background as being very formal and traditional, and recognized that this contrasted with the informal and non-technical perspectives of the students. His active role in curriculum development early in his career started the process of addressing this disparity. Eliciting student explanations in the classroom helped to further clarify student
learning styles and identify mastery of prerequisite knowledge. The combined effect was the development of flexible teaching methods that could be readily adjusted based on feedback from Instructional Assessment. Responses from Mr. Patrick showed that his knowledge of content and teaching was well defined for courses he had taught at Forest River, but lacked a complete connection with more advanced subject areas, such as Precalculus and Calculus.

The main overarching themes to emerge under knowledge for teaching were assessment diversity and instructional analogies. These represent Mr. Patrick’s strategies of using multiple assessment methods to illuminate student understanding, and his ability to connect new concepts to students’ foundational and real-world knowledge. Incorporating frequent informal assessments into class activities augmented traditional testing methods, and provided Mr. Patrick with multiple access points for diverse learning styles. This was particularly evident through his use of instructional analogies, designed to tap into students’ familiarity of applied contexts. Mr. Patrick presented daily opportunities for students to showcase their non-technical knowledge, and then linked it to formal mathematical concepts in the classroom. The use of Instructional Assessment was key in determining which analogies were appropriate, and when they should be incorporated into classroom discussions.

The themes presented in this section demonstrate challenges affecting Mr. Patrick’s knowledge for teaching, and the strategies he applied during classroom instruction to mitigate them. The findings of this study show that methods of Instructional Assessment improved his knowledge of students’ learning trajectories and their general proficiency, both procedural and conceptual. Conversely, a comprehensive understanding of traditional mathematics viewed from the perspective of student prerequisite knowledge improved his ability to implement methods of Instructional Assessment. The combined effect motivated Mr. Patrick to develop multiple
assessment opportunities for his students and consistently associate new concepts with students’
established knowledge base.

**Case Study 2 – Mrs. Kline**

Mrs. Kline was designated as the second participant in this study due to her limited
background and training in mathematics, and her position as a special educator. However, her
expertise with student learning disabilities, and her experiences teaching mathematics with the
diverse student population at Forest River made her a valuable participant for this study.
Moreover, her daily efforts to actively engage students and motivate classroom discussions
provided significant opportunities to observe episodes of Instructional Assessment. Following
the analysis of interview and observation transcripts, a total of eleven themes were developed for
Mrs. Kline in the areas of professional self-efficacy and knowledge for teaching. A summary of
these themes is now presented.

**Themes in Professional Self-Efficacy.** The themes in this section represent Mrs. Kline’s
perceived ability to perform the required daily activities of her position, taking into account the
inevitable barriers and external constraints she encountered. These themes identify the
challenges Mrs. Kline reported in her ability to sustain student engagement and curriculum
management, as well as the overarching strategies she employed. Refer to pages 116 – 134 in
Chapter IV for findings related to Mrs. Kline’s professional self-efficacy.

The main themes to emerge in the domain of student engagement were *student
disabilities* and *student beliefs and values*. These represent ongoing challenges to Mrs. Kline’s
ability to manage responsibilities for students with learning disabilities and address student
perceptions on learning mathematics. Mrs. Kline’s position as a special educator required her to
monitor the progress of multiple students on Individual Education Plans (IEPs). This was
partially facilitated through methods of Instructional Assessment, including individualized questioning and requiring several students to raise hands prior to selecting a respondent. Both techniques allowed Mrs. Kline to gather important information on the abilities and perceptions of her students, especially those with learning disabilities. This allowed her to be well-informed in effective methods for student engagement during instruction, which helped her to build trust throughout the classroom, improving her professional self-efficacy.

The main themes to emerge in the domain of curriculum management were *summative testing* and *time management*. These represent ongoing challenges to Mrs. Kline’s ability to operate classroom instruction efficiently, especially when preparing students for annual high-stakes testing. Mrs. Kline voiced strong negative opinions about the Mathematics MCAS, and was displeased about having to spend significant classroom time preparing students and proctoring the exams. However, she appreciated the fact that the MCAS materials provided supplemental resources for in-class problems. The inclusion of MCAS review into classroom instruction was largely seamless, and offered new opportunities for Instructional Assessment on topics that were often unrelated to the daily lesson. Despite these challenges, Mrs. Kline acknowledged that learning objectives were usually accomplished on time. Furthermore, the co-teaching approach she had developed with Mr. Patrick was flexible enough to accommodate schedule changes.

The main overarching themes to emerge under professional self-efficacy were *acknowledgment* and *exemplifying*. These represent reactive and proactive strategies employed by Mrs. Kline to communicate realistic expectations for student success, while serving as a role model in the classroom. In her time at Forest River, Mrs. Kline had developed a very hands-on teaching style. She preferred to be actively working with students in the classroom as often as
possible, despite the administrative responsibilities of her position as a special educator. This strategy allowed her to monitor student progress during instruction and provided frequent opportunities for Instructional Assessment. Additionally, she was confident in her ability to provide constructive feedback and encouragement based on real-time interactions and observations.

The themes presented in this section demonstrate challenges affecting Mrs. Kline’s professional self-efficacy, and the strategies she applied during classroom instruction to mitigate them. The findings of this study show that methods of Instructional Assessment improved her ability to attend to student learning disabilities and appeal to student beliefs and values. Conversely, a comprehensive understanding of challenges related to summative testing and time management improved her ability to implement methods of Instructional Assessment. The combined effect motivated Mrs. Kline to effectively communicate realistic expectations to the students while serving as a role model in the classroom.

**Themes in Knowledge for Teaching.** The themes in this section represent aspects of Mrs. Kline’s pedagogical content knowledge (PCK). These themes identify the challenges Mrs. Kline reported in her ability to assess student understanding (KCS) and implement teaching design (KCT), as well as the overarching strategies she employed. Refer to pages 135 – 152 in Chapter IV for findings related to Mrs. Kline’s knowledge for teaching.

The main themes to emerge in the domain of KCS were foundational learning and student condition. These represent Mrs. Kline’s ability to identify and address common gaps in students’ mathematical understanding, while incorporating knowledge of individual psychosocial factors into classroom instruction. In her time at Forest River, Mrs. Kline had become adept at recognizing students’ typical learning habits, especially concerning simple mistakes involving
prerequisite material. Additionally, she had become familiar with the ongoing challenges students faced outside of the classroom, and their impact on the teaching and learning of mathematics. This knowledge made her particularly attentive to student needs in the classroom, and helped her to fine-tune Instructional Assessment interventions based, initially, on physical and verbal cues. Once an approach was determined, she could focus on the structure of the problem at hand, and provide customized feedback.

The only theme to emerge in the domain of KCT was *differentiated instruction*. This represents Mrs. Kline’s ability to meet individual student needs through flexible avenues of learning. By working with a wide range of students in college-preparatory courses, Mrs. Kline was able to modify her instructional approach to respond to diverse learning styles. This primarily concerned her knowledge of students with learning disabilities, but also extended to the general population. She incorporated this knowledge into a model of individualized instruction, with frequent opportunities for Instructional Assessment. Employing this teaching strategy helped to build trust with the students, and permitted Mrs. Kline to maintain a student-centered approach. This was particularly effective for proactively monitoring student performance, and when assessing the mathematical reasoning of individual students during classroom discussions.

The main overarching themes to emerge under knowledge for teaching were *active engagement* and *scaffolding*. These represent Mrs. Kline’s overall strategies of encouraging students to participate in classroom activities, and sequencing questions to develop understanding and promote independence. Mrs. Kline felt more confident in her ability to assess student understanding when working with students one-on-one. She encouraged students to maintain active engagement during instruction, so that she could more easily identify when individual students were falling behind. This method of Instructional Assessment was helpful
for initiating student-teacher discussions in response. Mrs. Kline then used question scaffolding to pinpoint knowledge gaps and assist students in discovering the correct solution processes.

The themes presented in this section demonstrate challenges affecting Mrs. Kline’s knowledge for teaching, and the strategies she applied during classroom instruction to mitigate them. The findings of this study show that methods of Instructional Assessment improved her knowledge of students’ foundational learning skills as well as outside factors impacting their psychological and social behaviors. Conversely, a comprehensive understanding of the benefits of differentiated instruction improved her ability to implement methods of Instructional Assessment. The combined effect motivated Mrs. Kline to keep her students actively engaged during classroom activities, while scaffolding the learning process during student-teacher interactions.

**Cross-Case Findings**

From the outset of this study, the analysis of each case study was designed to be isolated prior to any cross-case examination. The unique co-teaching relationship between the two participants presented a formidable challenge to this goal, however, there are several reasons why it was possible to stay true to the original vision. First, there were multiple instances during observations where one of the participating teachers was not in the classroom for an extended period of time. This made it possible to collect data solitary to each case. Second, even when both teachers were present, they often took turns as lead instructor. Finally, it was readily apparent during the initial interview sessions that they did not share a common perspective on the challenges related to teaching high school math. This is not surprising, given the differences in their respective positions, and the disparity in their education backgrounds. Despite these
differences, after working together for four years, they had developed similar approaches to classroom instruction, and seemed comfortable working in a co-teaching environment.

In this section, similarities and differences between the two participating teachers will be presented for professional self-efficacy and knowledge for teaching. The development of cross-case themes was not considered necessary for addressing the research questions, and so this summary is presented as a brief discussion. Nevertheless, the descriptions contained herein remain essential to the narrative of individual and shared classroom experiences, leading to the conclusions of this study.

Both Mr. Patrick and Mrs. Kline presented themselves as capable and confident mathematics teachers, however, there were noticeable differences in how this was communicated in the classroom and during interviews. Mr. Patrick’s main focus on the challenges of his position was the mathematical content. In contrast, Mrs. Kline’s main focus was the students. This is evidenced by examining the themes in the domain of student engagement that emerged in this study. Mr. Patrick’s themes of *anticipation* and *student background* describe ways in which students perform during classroom activities, but derive primarily from his comfort level with the mathematics, rather than the students. Conversely, Mrs. Kline’s themes of *student disabilities* and *student beliefs and values* are firmly rooted in her comfort level with the students, rather than the mathematics. This is not to suggest that Mr. Patrick was not comfortable with the students or that Mrs. Kline was not comfortable with the mathematics, but it does illustrate a fundamental difference between their viewpoints and the challenges of maintaining professional self-efficacy.

Themes in the area of knowledge for teaching echo this difference. Mr. Patrick’s themes of *learning trajectories* and *general proficiency* describe ways in which students assimilate new
material, but derive primarily from his knowledge of mathematical content, rather than the students. Conversely, Mrs. Kline’s themes of foundational learning and student condition are centered on students’ understanding of mathematics, rather than her knowledge of mathematical content. This is not to suggest that Mr. Patrick was not concerned with student knowledge, just that he viewed it through the lens of his knowledge of mathematical content. Similarly, Mrs. Kline was concerned with mathematical content, but she consistently phrased her responses in ways that emphasized the student perspective. It is important to note that these contrasting viewpoints were not often noticeable during classroom observations, and may not have been detected without an analysis of the interview transcripts. This result demonstrates one benefit of including multiple interviews in the data collection process, due to their ability to capture each participant’s thoughts, feelings, and opinions.

Given the contrasting backgrounds of the participating teachers, these differences are not surprising, but they do provide important context for interpreting the findings of this study. It is reasonable to contend that case studies involving two mathematics teachers with similar credentials might result in conclusions from similar viewpoints. However, in the case of this study, the complementary nature of the participating teachers means that both content-centered and student-centered interpretations are represented in the findings. This is an unexpected benefit of examining co-teachers with complementary credentials and different backgrounds.

The next section presents the conclusions of this study.

**Conclusions**

The research sub-questions composed for this dissertation address fundamental aspects of teaching mathematics that occur whenever student understanding is assessed in ways that immediately impact the path of instruction. Central to this is an understanding of how teachers
perceive their ability to perform such assessments (professional self-efficacy) and a recognition of the diverse forms of expertise that are required (knowledge for teaching). Drawing on thematic analysis of the challenges and strategies related to these areas, the main research question then seeks to determine how teachers are empowered through methods of Instructional Assessment. The individual case study findings reveal important answers to each question, and the cross-case comparisons afford interpretations from multiple perspectives. This section reports the conclusions of this study, organized by research sub-question, followed by the main research question.

Research Sub-Question 1

The first research sub-question for this dissertation asks, “how does the integration of assessment into instruction relate to the professional self-efficacy of mathematics teachers?” Research in the area of formative assessment has shown that teachers are often uncomfortable with the prospect of informal assessments, despite evidence that they often occur as a natural consequence of instruction, and have been found to improve student performance on summative assessments (Black & Wiliam, 2006; OECD, 2005). Thus, teachers may rely on more traditional forms of instruction (such as lecture) and assessment (such as pencil-and-paper tests) under the pretenses of saving time and maintaining exclusive control over classroom activities. This inconsistency between research and practice is the focus of the first research sub-question. The themes developed to identify challenges and strategies for integrating student engagement and curriculum management revealed connections between the participants’ classroom practices and their professional self-efficacy. Conclusions will be presented for each case study participant, followed by a description of implications for teachers in general.
Mr. Patrick benefits from the frequent use of classroom discussions by gaining insight into his students’ personalities and their understanding of mathematics. This allows him to identify passive and active learners in the classroom more efficiently, so that he can modify his instructional approach accordingly. His ability to teach effectively in this manner is supported by a detailed understanding of mathematical content, brought on by teaching the same courses for multiple, consecutive years. The result is that Mr. Patrick feels more confident in his ability to balance the needs of students while meeting curriculum requirements in a timely manner. Furthermore, by incorporating activities such as Do Now, and Think, Pair, Share, Mr. Patrick consistently promotes autonomous learning situations, which lead to additional opportunities for Instructional Assessment.

Mrs. Kline benefits from the frequent use of classroom discussions by gaining student trust and maintaining a student-centered teaching approach. This is especially valuable for monitoring the progress of students with learning disabilities. Her ability to effectively teach in this manner is an ongoing challenge, frustrated by the requirements of preparing students for the MCAS exam within the allotted time constraints. However, she is able to mitigate these difficulties by consistently modeling and communicating positive attitudes toward mathematical exploration. Additionally, by frequently attending to her students on an individual level, she is more confident in her ability to address specific needs efficiently. Mrs. Kline is able to scale this approach when speaking with the entire class by asking for multiple volunteers before selecting a respondent. The observable effect is an increase in student confidence, which promotes further opportunities for Instructional Assessment.

In both cases, the data suggests a recursive process relating Instructional Assessment to professional self-efficacy. Teachers who make regular use of classroom discussions create
opportunities for Instructional Assessment, which leads to a variety of potential benefits for classroom teachers. From a pedagogical standpoint, information on individual student learning can be revealed, which informs appropriate interventions when difficulties arise. Over time, a culture of trust and active communication is promoted in the classroom, where students are more willing to participate, and teachers are more confident in deviating from standard instructional pathways. The final result is the motivation of further classroom discussions, which completes the recursion. This process is modeled in Figure 3.

Figure 3. Recursive Process of Instructional Assessment related to Professional Self-Efficacy

The connections between Instructional Assessment and professional self-efficacy can now be illustrated. Integrating assessment into instruction improves a teacher’s ability to effectively guide students toward achieving learning objectives. The result is an increase in control on the part of the teacher, both in identifying student roadblocks and suggesting appropriate detours. The appropriate analogy to this process is the use of GPS devices, commonly used in vehicle travel. Once a destination has been selected, the device uses real-time information to predict traffic problems and update travel times. If the current route is determined
to no longer be optimal, the device informs the driver as to the best options for avoiding trouble spots, and maintaining travel time. This improves the driver’s confidence to attempt alternate routes, especially in unfamiliar areas. Moreover, it saves the driver time by continuously responding to potential delays. Instructional Assessment serves as the teacher’s GPS, promoting and supporting classroom activities through repeated discussion and feedback.

**Research Sub-Questions 2 & 3**

The second research sub-question for this dissertation asks, “how do mathematics teachers reinforce or modify their understanding of mathematical content and processes through classroom interactions?” Analysis of the interview and observation data for each case study did not produce enough evidence to answer this question directly. Both participants felt that their knowledge of mathematics had already been largely established, and that any changes that occurred were centered around purely pedagogical matters. For this reason, it was decided to incorporate the third research question, which asks, “how do mathematics teachers make use of observation, listening, and questioning during instruction to construct models of student understanding?” The combination of these questions into one area of analysis helped to clarify the relationship between the teachers’ knowledge of content and teaching and their knowledge of content and students. The themes developed to identify challenges and strategies for integrating instruction design and the assessment of student understanding revealed connections between the participants’ classroom practices and their knowledge for teaching. Conclusions will be presented for each case study participant, followed by a description of implications for teachers in general.

Mr. Patrick possesses a detailed knowledge of secondary mathematics, combined with a traditional perspective toward teaching and learning that prioritizes the precision and rigor of
mathematical reasoning. However, he recognized early in his career that students didn’t always respond well to that sort of approach, requiring him to make adjustments to include individual student experiences. The result for him in making these adjustments is a broader understanding of where and why students typically have trouble, and leads to general knowledge of overall student learning trajectories. Mr. Patrick makes regular use of Instructional Assessment methods to inform this knowledge, constantly updating his archive of student perspectives. Over the years, he has incorporated the use of instructional analogies into classroom discussion to help students make connections between familiar activities and abstract mathematical concepts. He is also promoting students’ abilities to demonstrate their understanding of those concepts in diverse ways by not solely relying on traditional testing methods. Over the course of a semester, the information derived from these combined strategies helps Mr. Patrick to form individualized models of student understanding.

Mrs. Kline possesses a detailed knowledge of student disabilities, combined with a supportive approach toward teaching and learning. However, she recognized early in her career that it was difficult to monitor student progress from the front of the classroom, motivating her to engage with students individually as often as possible. The result is a teaching style that communicates mathematical concepts to students in ways that appeal to individual strengths, and avoids potential weaknesses. Mrs. Kline makes regular use of Instructional Assessment methods to make the most of this approach, by improving her knowledge of students’ mathematical abilities, while also attending to personal factors impacting student achievement. She models and encourages active engagement during instruction, and carefully scaffolds questions based on instant assessment of student understanding. Over the course of a semester, the information
derived from these combined strategies helps Mrs. Kline to form individualized models of student understanding.

In both cases, the data suggests a recursive process relating Instructional Assessment with knowledge for teaching. Teachers who use observation, listening, and questioning create opportunities for Instructional Assessment, which has the potential to reveal how students are thinking about mathematical concepts (KCS). This can improve teachers’ overall subject matter knowledge by illuminating multiple perspectives for approaching a certain problem or solution. Over time, teachers can learn to select appropriate problem-solving techniques, based on the moment-to-moment assessments of student understanding during instruction. This helps to fine-tune teaching methods, including the overall design of instruction to best meet student needs (KCT). The final result is the motivation of richer, more beneficial classroom interactions, which completes the recursion. This process is modeled in Figure 4.

Figure 4. *Recursive Process of Instructional Assessment related to Knowledge for Teaching*

The connections between Instructional Assessment and knowledge for teaching can now be illustrated. Integrating assessment into instruction improves a teacher’s ability to choose
interventions that address different levels of student proficiency. This occurs because knowledge of mathematical content is enhanced by an understanding of multiple perspectives, making the teacher more flexible in determining the best approach. Repetition of this process results in a fine-tuning of instructional design that is continually updated through trial and error. The appropriate analogy to this process is the use of adaptive software in customer marketing. Companies make use of customer data to determine what products and services are preferred. Over time, they can develop purchasing models specific to individual consumers. This information can then be used to suggest additional purchases, and offer discounts that will be well-received. Instructional Assessment serves as a teacher’s adaptive software, promoting the emergence of shared knowledge within the classroom.

**Main Research Question**

The main research question for this dissertation asks, “how are mathematics teachers empowered through methods of Instructional Assessment?” The conclusions presented thus far have been isolated to issues related to professional self-efficacy and knowledge for teaching. However, taken separately, the explanations do not provide a complete picture. This can be shown by contrasting the purposes of a GPS device with that of adaptive marketing software. A GPS is not concerned about the location of a driver’s destination, only the most efficient path to get there; the emphasis is on process over product. Conversely, adaptive marketing software is designed to promote repeat sales but is not concerned with how or when the consumer chooses to make a purchase; the emphasis is on product over process. These analogies provide methods for understanding the individual connections involving professional self-efficacy and knowledge for teaching, but do not fully answer the main research question.
In this section, generalized conclusions will be given that incorporate concepts from both areas of interest. The use of the word “empowered” is appropriate, since it combines notions of strength, control, and confidence. Translated to the context of teaching mathematics, empowerment represents the ability to accurately and consistently assess student understanding while effectively managing daily internal and external demands over the course of an academic year. Instructional Assessment represents the quantum interactions that occur on a moment-to-moment basis, helping to determine the most efficient path of instruction while simultaneously improving student knowledge. There is no particular emphasis on process or product in this case, because they are both required. For teachers, empowerment without attention to process can result in taking shortcuts and teaching to the test, because positive student results are the desired product. Conversely, empowerment without attention to product can result in a lack of underlying structure, with different teachers acting as freelance guides through the curriculum. For the purposes of answering this research question, both process and product must be included.

As previously described, both Mr. Patrick and Mrs. Kline seemed to welcome the challenge of classroom interactions. This study has shown how this related to their professional self-efficacy as well as their knowledge for teaching. Combining these concepts into one unified model of teaching finally begins to illustrate a source of empowerment that can be derived from Instructional Assessment: controlled creativity. By promoting a culture of active discussion in the classroom, both participants allow for observation, listening, and questioning to impact the direction of instruction. This provides them with frequent opportunities to adapt their feedback based on an immediate assessment of student understanding. Past experience has prepared them for typical student mistakes and general areas of difficulty, but moment-to-moment interactions require them to respond to specific and potentially unforeseen errors. A teacher with high
professional self-efficacy will view these moments as opportunities for creativity, where instruction can go in any number of directions. By incorporating subject-matter and pedagogical elements of knowledge for teaching mathematics, the skillful teacher can then determine which direction is best-suited for the student in question, and the topic at hand.

The model of teaching mathematics presented in the conclusions of this study successfully merges process with product. The teacher is focused on guiding students to achieve learning objectives while simultaneously monitoring the efficiency of the paths they are taking. Integrating assessment into instruction motivates creative techniques for maintaining this balance, and provides teachers with real-time information that builds individualized models of student learning. Of primary importance in this process is a recognition that the teacher remains in control at all times, despite sharing the stage with novice learners. Over time, regular inclusion improves professional self-efficacy by providing the teacher with frequent opportunities to regulate student learning in the face of external factors, including high-stakes testing. Additionally, the teacher gains important insight into the student perspective of mathematical content and processes, which helps to build knowledge for teaching. Thus, what has been described previously as a potential reversal of roles dictated by the classroom contract is not, in fact, the case. Instructional Assessment holds the potential to empower mathematics teachers by providing them with unrestricted access to student learning, and develops their ability to interact creatively in productive and meaningful ways.

**Discussion**

The final section examines the takeaways from this dissertation study and recommends avenues for additional research. First, the significance of this study is presented, including implications for mathematics education researchers as well as for schools and mathematics
teachers. Next, the limitations of this study are discussed. Lastly, suggestions for future research are outlined, followed by a few final thoughts.

**Study Significance**

This study was designed to provide valuable information on how secondary mathematics teachers formalize and incorporate formative assessment methods into planning and instruction. As discussed in earlier chapters, the current research base into formative assessment practices has shown multiple potential benefits for students (Black & Wiliam, 1998a, OECD, 2005), but has not fully considered the impact on teachers. Furthermore, a majority of these studies have been performed outside of the United States, and/or at the elementary level (Stiggins, 2002, Wiliam, 2007). This study prioritizes the role of the secondary mathematics teacher in the research literature, and highlights the types of assessments that have the most immediate impact on instruction. The conclusions are based on analyses of classroom observations and structured interviews designed to illuminate two main factors: teacher perceptions on the challenges of meeting curriculum standards (professional self-efficacy) and the associated characteristics of subject-matter expertise (knowledge for teaching).

Mathematics education researchers in the United States must incorporate Instructional Assessments into existing frameworks in order to fully understand their inclusion within the assessment continuum (see Figure 1). Despite continued pressure to focus on improving student performance on summative and evaluative assessments, attention should be shifted to incorporate the ways in which teachers interact with students on a daily basis. The reasons for this can be derived directly from the conclusions of this study. First, Instructional Assessment promotes individualized learning, trust, and active engagement within the classroom culture. Teachers are able to maintain control of the learning process, while correcting the course of instruction in
response to immediate student needs. Secondly, Instructional Assessment builds knowledge of content, students, and teaching by providing direct exposure to multiple perspectives and helping to build models of student understanding. These conclusions demonstrate the importance of informal assessments, despite their limited understanding outside of the classroom, and point toward the need for further research.

When considering the implications of this study for school districts, the focus should be on designing professional development opportunities for teachers to learn and implement Instructional Assessment. This includes understanding the advantages experienced by students, as well as their own empowerment. The mathematics classroom serves as an adaptable learning environment for teachers (Cobb, 2000), and they should be encouraged to develop methods for capturing information derived from student interactions to make informed instructional decisions. Teachers should also be supported by school administrators in the use of creative teaching techniques that promote active engagement among students, even when the benefits to achieving learning objectives may not be immediately evident. Consistent, long-term collaboration is required between teacher and students in order to build a thriving classroom culture, where emergent learning can take place without an overreliance on traditional teaching methods.

**Limitations**

One limitation in the data collection for this study was the lack of video evidence. This would have provided additional context for the audio transcripts and field notes, as well as depth to the narratives of the participating teachers (Davidson, 2003). It would have been potentially beneficial to be able to accurately account for detailed teacher movements during classroom instruction. Furthermore, the analysis of specific physical movements, such as hand signals and
facial expressions could have been incorporated into the study conclusions. A second limitation related to data collection was the limited number of interviews. Having two interviews was beneficial, but in retrospect, a third should have been scheduled following the administration of the MCAS. This would have given the participating teachers an opportunity to reflect upon the entire duration of the study period, which could have provided more insight into the long-term impacts related to Instructional Assessment.

Another limitation for this study was the use of two co-teaching participants. As described in Chapter III, this created difficulties for comprehensive data collection during classroom observations since both teachers were engaged in activities simultaneously. Additionally, both participants were close in age and their experiences as secondary mathematics teachers had always included preparing students for annual state-testing. It would have been preferable to have observed and interviewed at least one teacher with more experience, who could have provided some insight into the effects of accountability on classroom instruction. Nevertheless, the results of this study are not restricted to the individual cases, but rather are grounded in the case characteristics (Gobo, 2004). Since it is reasonable to infer that elements of Instructional Assessment are typically present in secondary mathematics classrooms, the conclusions herein represent potential effects experienced by teachers in relatively similar environments.

**Suggestions for Future Research**

Mathematics education research is sure to continue prioritizing studies across the assessment continuum (see Figure 1). However, increased attention needs to be given to the informal assessment methods that are performed frequently by the teacher during the natural flow of classroom instruction. A complete picture of student understanding cannot be limited to
performance on summative or evaluative tests. Teachers must be prepared to handle the high frequency of student interactions, and must be trained to respond in productive and meaningful ways. To this end, the development of a framework for capturing glimpses of student understanding as they occur during instruction is warranted. Further research into the use of observation, listening, and questioning can be especially beneficial for pre-service teachers as they learn to actively monitor their approach to instructional interventions in the classroom.

Mathematics teachers at all levels are increasingly relying on the use of educational software (Freeman, Becker, Cummins, Davis, & Giesinger, 2017). The use of these programs during classroom activities as well as providing students with online access can give much needed access to individual student learning trends. However, teachers need to be trained in the use of educational software so that they can make sense of the analytical tools that are included to monitor student progress and identify learning gaps. Programs that focus entirely on student performance are not being used to their full potential unless a qualified teacher is prepared to translate the results into effective classroom practices. Additionally, teachers certified as subject-matter experts can be trained to design learning modules that are based on course learning objectives, but that allow for question scaffolding, tutorial support, and multimedia resources. The teaching and learning of mathematics will continue to be a social activity, but there will be a growing need to investigate the nature of Instructional Assessment in technology-rich environments.

**Final Thoughts**

The conclusions presented in this dissertation provide useful insight into some of the demands placed on secondary mathematics teachers when assessing student understanding during instruction. The findings expand the current research literature by uncovering potential
benefits experienced by teachers as they attempt to balance student needs with curriculum requirements. Future studies can be designed to further refine these benefits for the purposes of pre-service education and ongoing professional development. As mathematics teachers continue to be held accountable for student performance, mathematics education researchers can provide support through a comprehensive understanding of assessment, along with practical methods for providing constructive feedback.
APPENDIX A

Interview Questions

Interview #1

1. Can you list your job experience as a mathematics teacher at all levels?
2. Can you list and describe all of the methods you use to assess student understanding in your classroom?
3. How confident are you in your ability to assess student understanding?
4. What types of assessments have the most impact on your instructional decision-making?
5. When interacting with students during class, do you find yourself questioning or challenging your knowledge of specific mathematical topics, and how do you react in such a situation?
6. How confident are you in your ability to answer student questions?
7. Are there any advantages or disadvantages we have not discussed that you have experienced by interacting with students during instruction?
8. How do you view your role as an assessor of student understanding within your classroom, and within the school?
9. What aspects of current educational policy impact the ways in which you determine your instructional methods, including assessment?
10. Do you feel that you are able to relate your instructional practices to external goals as effectively as you would like? If not, what are the main barriers?
Interview #2

1. How comfortable are the students with asking for guidance during instruction? Is the process more proactive or reactive for you?

2. How well do students respond to your guidance? What techniques do you use when a student doesn’t respond as predicted?

3. Do you find yourself negotiating with students about what constitutes an acceptable solution? How does this process serve to clarify your own mathematical knowledge?

4. Over the course of each semester, how accurately are you able to determine individual student strengths and weaknesses? What role does classroom instruction play in this process?

5. When a student asks for help, what do you do first: ask for clarification or immediately check student work?

6. How do you choose to respond when a student’s mistake is predictable? What happens when you can’t identify the problem immediately?

7. Do you differentiate your responses based on your knowledge of each student’s level of understanding?

8. What causes you to address the entire class based on evidence gathered from observations and/or interactions with individual students?

9. Are there ever times when you choose to do nothing and let a student find their own mistake? Is this due to time/accessibility concerns or some other reason?

10. Do you ever feel overwhelmed with student questions? How do you deal with the demand given the time constraints of the class/semester?
# APPENDIX B

Teachers’ Sense of Efficacy Scale (Long Form)

<table>
<thead>
<tr>
<th>Teacher Beliefs</th>
<th>How much can you do?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Directions:</strong> This questionnaire is designed to help us gain a better understanding of the kinds of things that create difficulties for teachers in their school activities. Please indicate your opinion about each of the statements below. Your answers are confidential.</td>
<td><strong>Nothing</strong></td>
</tr>
<tr>
<td>1. How much can you do to get through to the most difficult students?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>2. How much can you do to help your students think critically?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>3. How much can you do to control disruptive behavior in the classroom?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>4. How much can you do to motivate students who show low interest in school work?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>5. To what extent can you make your expectations clear about student behavior?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>6. How much can you do to get students to believe they can do well in school work?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>7. How well can you respond to difficult questions from your students?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>8. How well can you establish routines to keep activities running smoothly?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>9. How much can you do to help your students value learning?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>10. How much can you gauge student comprehension of what you have taught?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>11. To what extent can you craft good questions for your students?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>12. How much can you do to foster student creativity?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>13. How much can you do to get children to follow classroom rules?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>14. How much can you do to improve the understanding of a student who is failing?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>15. How much can you do to calm a student who is disruptive or noisy?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>16. How well can you establish a classroom management system with each group of students?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>17. How much can you do to adjust your lessons to the proper level for individual students?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>18. How much can you use a variety of assessment strategies?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>19. How well can you keep a few problem students from ruining an entire lesson?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>20. To what extent can you provide an alternative explanation or example when students are confused?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>21. How well can you respond to defiant students?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>22. How much can you assist families in helping their children do well in school?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>23. How well can you implement alternative strategies in your classroom?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
<tr>
<td>24. How well can you provide appropriate challenges for very capable students?</td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9)</td>
</tr>
</tbody>
</table>
APPENDIX C

IRB Approval and Informed Consent Form

University of New Hampshire
Research Integrity Services, Service Building
51 College Road, Durham, NH 03824-3585
Fax: 603-862-3564

04-Feb-2011

Wallinga, Willem
Mathematics, Kingsbury
19 Station Road
Salem, MA 01970

IRB #: 5050
Study: Formative Assessment in Mathematics Education: The Interplay Between Instructional Decision-Making and Student Understanding
Approval Date: 03-Feb-2011

The Institutional Review Board for the Protection of Human Subjects in Research (IRB) has reviewed and approved the protocol for your study as Exempt as described in Title 45, Code of Federal Regulations (CFR), Part 46, Subsection 101(b) with the following comment(s):

The researcher needs to submit to the IRB a copy of the release form for the file. Further, the researcher needs to review all the releases for the students in the class and if there are any, let the IRB know how he is going to handle students who have indicated that they do not want to be recorded.

Researchers who conduct studies involving human subjects have responsibilities as outlined in the attached document, Responsibilities of Directors of Research Studies Involving Human Subjects. (This document is also available at http://www.unh.edu/osr/compliance/irb.html.) Please read this document carefully before commencing your work involving human subjects.

Upon completion of your study, please complete the enclosed Exempt Study Final Report form and return it to this office along with a report of your findings.

If you have questions or concerns about your study or this approval, please feel free to contact me at 603-862-2003 or Julie.simpson@unh.edu. Please refer to the IRB # above in all correspondence related to this study. The IRB wishes you success with your research.

For the IRB,

Julie F. Simpson
Director

cc: File
    McCrone, Sharon
Informed Consent Form

The purpose of this study is to learn more about how secondary-level mathematics teachers determine student understanding during classroom activities and how this impacts instructional decision-making (a process called formative assessment). My primary focus during this study will be on observing interactions between teacher and student(s) that occur naturally during the course of instruction. Your participation in this study will enable me to interpret the relationships between formative assessment methods and several key aspects relevant to teaching mathematics.

This study will take place over the course of the spring 2011 semester and is expected to include no more than three participant teachers. All observations will occur in pre-determined classrooms (at your discretion) and all students will be briefly addressed as to the reason for my presence in the classroom. Additionally, I would like to hold monthly 1-hour interviews (at your convenience) in order to discuss the observations. The observations and interviews will be audio-recorded to assist with transcription. Finally, you will be asked to take a short survey on teacher beliefs before and after the study. No additional procedures will be required of you at this time.

Your participation in this study is voluntary. You may decide to withdraw at any time and you may refuse to answer any questions during interviews. Audio-files from observations and interviews will be transcribed and compiled personally. Your identity as well as your students’ will be kept confidential on all transcripts, and I will share all study data with only my dissertation committee members (Dr. Sharon McCrone, Dr. Timothy Fukawa-Connelly, & Dr. Ernst Linder). After transcription, all audio-files will be deleted. You will be given the opportunity to review all interview transcripts. Selected portions of these transcripts will be coded for use in the eventual dissertation, as well as at the dissertation defense.

There are rare instances when I may be required to share personally-identifiable information (e.g., according to policy, contract, or regulation). For example, in response to a complaint about the research, officials at the University of New Hampshire, designees of the sponsor(s), and/or regulatory and oversight government agencies may access research data.

There are no foreseeable risks to your involvement in this study. No compensation, monetary or otherwise, will be provided for your involvement. However, your participation will allow you a chance to reflect upon your current teaching practices and learn more about the processes of formative assessment.

If you have questions or concerns about the research or your participation, please contact Willem Wallinga ((978) 210-2363, wij3@unh.edu), Dr. Sharon McCrone ((603) 862-3587, smy72@unh.edu), or Dr. Julie Simpson at UNH Research Integrity Services ((603) 862-2003, Julie.Simpson@unh.edu).

Your signature below indicates that you have read and understood the above information and agree to be a participant in this study. You will receive a copy of the signed document for your records.
REFERENCES


