Some observations of magnetic clouds and simulations of a normal fast shock interaction with an idealized magnetic cloud

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SOME OBSERVATIONS OF MAGNETIC CLOUDS
AND SIMULATIONS OF A NORMAL FAST SHOCK
INTERACTION WITH AN IDEALIZED MAGNETIC
CLOUD

BY

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DISSERTATION

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ABSTRACT

SOME OBSERVATIONS OF MAGNETIC CLOUDS AND SIMULATIONS OF A NORMAL FAST SHOCK INTERACTION WITH AN IDEALIZED MAGNETIC CLOUD

by

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University of New Hampshire, December, 1998

We looked at three magnetic clouds observed by the WIND satellite and find that though they are temporally the same, their effects on the Earth were different. The magnetopause is slightly expanded for all the three clouds from its average position during the $B_z < 0$ phase of the clouds and in the $B_z > 0$ phase there was a compression of the magnetopause taking place. During the $B_z < 0$ phase of the October 1995 cloud the bow shock expanded from its average position much more then the January 1997 but the May 1996 cloud hardly affected the Earth’s bow shock position during this phase. For the later $B_z > 0$ phase, we find that all three clouds compressed the bow shock closer to the Earth from its average position.

We studied a number of discontinuities in the field and plasma observations for the October 18-19, 1995 magnetic cloud. Except for the front cloud boundary, which was a tangential discontinuity, all other discontinuities were rotational. We also
could identify 3 different coherent structures within the October 18-19, 1995 cloud. We also found regions for which no coherent structure existed.

From our simulations of a shock with a static force-free Lundquist flux tube, we find that the width of the tube will decrease but the boundaries of the tube were still clearly defined. The magnetic field components retain their original orientation but with an increased amplitude. Depending on the density ratio's between the tube and surrounding plasma, the possible waves generated by the shock interaction at the tube boundary are: (1) transmitted and reflected shock; (2) reflected expansion wave and transmitted shock; and (3) only a transmitted shock. We also find that the shock speed in the tube was increased, decreased or remain unchanged. Some numerical results are supported by GEOTAIL observation's of the October 18-19, 1995 magnetic cloud crossing the bow shock of the Earth.
Chapter 1

Introduction

The Earth is an obstacle in the path of the solar wind, a stream of magnetized plasma which is ejected from the Sun. The solar wind at 1 AU, which is the distance from the Sun to the Earth, is supersonic and superalfvenic and has to flow around the Earth. This is done with the help of the Earth’s bow shock which slows and heats the solar wind plasma so that it may flow past the Earth. Both I. A. Axford and P. J. Kellog independently had predicted that a shock wave would be present in front of the Earth’s magnetosphere to slow down the wind and deflect it around the magnetosphere and their predictions proved correct from spacecraft observations in 1963 (Sonnet et al. [1963]). Shocks in space are produced by collisionless processes and they may be both steady state and transient. An example of a steady state shock is the bow shock of the Earth, while examples of transient shocks would be those generated by solar flares and supernova explosions.

There is a boundary that separates the Earth’s magnetosphere from the shocked solar wind, and this boundary is referred to as the magnetopause. The region between the bow shock and the magnetopause is referred to as the magnetosheath. It is at the magnetopause boundary that reconnection of magnetic field lines can take place that would allow solar wind plasma to flow into the magnetosphere, and,
vice versa, energetic magnetospheric particles to escape into the solar wind. The most favorable magnetic field configuration for reconnection to occur is a southward pointing Interplanetary Magnetic Field (IMF). The reconnection process allows for energy to be transferred into the magnetosphere by way of magnetic field energy to the plasma in the magnetotail. Increase in auroral particle precipitation, energization of plasma sheet particles, ring current and joule heating is the end result of a solar wind-magnetosphere energy transfer. The bulk of the energy from the solar wind goes to the buildup of the ring current located in a radius of \( \sim 4-6 \, R_E \) from the Earth. An increase in the ring current would decrease the horizontal magnetic field component at mid to low latitudes of the Earth (Sugiura and Chapman [1960]). This disturbance of the horizontal magnetic field component is measured by the \( Dst \) index.

Burlaga et al. [1981] first observed an interplanetary ejecta and coined the term "Magnetic Cloud" to describe it. Three conditions need to be satisfied for an interplanetary ejecta to be considered a magnetic cloud: (a) a large and smooth rotation in the magnetic field; (b) a larger than average magnetic field, and (c) a lower than average proton temperature. These magnetic clouds are ejections from the sun with a well defined magnetic field topology which can be approximated by force-free magnetic fields (i.e., \( \vec{j} \times \vec{B} = 0 \)). The cylindrically symmetric Lundquist solutions (see Figure's 5.1 - 5.3) has been used to describe the magnetic fields of the clouds (Lundquist [1950]). These clouds have been approximated to be 0.25 AU.
in width near the Earth after being ejected from the Sun (Burlaga et al. [1981]). It has also been noticed that these clouds may be connected to the Sun (Burlaga et al. [1990], Farrugia et al. [1993]) and their large scale structure is best described as a bent flux rope whose "feet" are anchored at the Sun. Magnetic clouds are a natural source of southward IMF since their magnetic field configuration is such that they have a large southward IMF which rotates to a northward IMF or vice versa and the rotation would last at least for a day. With this magnetic field configuration there would be an energy transfer primarily via reconnection from the magnetic cloud to the Earth’s magnetosphere as it passes by the Earth. Shocks may also be driven by magnetic clouds, and magnetic clouds would also have to interact with the Earth’s bow shock as they propagate away from the Sun. An example where both these shocks (dmi...shock and bow shock interaction) are seen is in the October 18-20, 1995 magnetic cloud observed by the WIND and GEOTAIL satellites. WIND and GEOTAIL satellites provide observations for the International Solar-Terrestrial Physics Science Initiative (ISTP) whose objective is to carry out multipoint studies of the solar-terrestrial relationship. The WIND satellite is generally located upstream of the Earth, thereby providing observations of the solar wind prior to its interaction with the Earth. GEOTAIL on the other hand is in an orbit which extends from the near-earth regions (as close as 8 \(R_E\)) to the distant tail regions (200 \(R_E\)) providing observations of the magnetosphere and magnetic cloud interaction.
The purpose of this research is to look at some observations made on magnetic clouds and their interaction with the Earth. To this end three examples of magnetic clouds are considered. October 18-20, 1995, May 27-29, 1996 and January 9-11, 1997. Though these clouds temporally look the same, we find their effects on the Earth are different. Some of their characteristics are described, noting both similarities and differences between them in terms of their interplanetary and derived parameters such as the dynamic pressure, total pressure which is the sum of the thermal pressure and the magnetic pressure, the $\beta$ parameter which is the ratio of the thermal pressure to the magnetic pressure, the Alfvén Mach number $M_A$ and the Akasofu $\epsilon$ parameter (Perreault et al. [1978]; Akasofu [1981]) which gives a measure of the Poynting flux into the Earth’s magnetosphere. The dynamic pressure and the $M_A$ are used to estimate the Earth’s subsolar bow shock and magnetopause positions during cloud passage. Also the $Dst$ index is looked at for these three clouds to get a measure of the intensity of the storms they generate due to the energy they transfer to the Earth’s magnetosphere.

Larson et al. [1996] in their investigation of dropouts of heat flux electrons in the energy range $\sim$0.1-1 keV and high energy electrons for the October 18-20, 1995 magnetic suggested some of the magnetic field lines for this cloud are severed from the Sun and the cause for this is the reconnection of adjacent field lines in the magnetic cloud. This would imply a more complicated magnetic field topology for the October 18-20, 1995 magnetic cloud than the simple bent flux rope attached at
both “ends” to the Sun, mentioned above. We look for other signatures in the October 1995 magnetic cloud to investigate Larson et al’s. argument for reconnection taking place in this magnetic cloud. One such signature is to look for rotational discontinuities in the magnetic field which has never been done before. Finding a rotational discontinuity may suggest a reconnection of the magnetic field lines taking place whereas a tangential discontinuity would imply two separate plasma objects are present. Finding discontinuities would also imply that the rotation of the magnetic field in the cloud is not smooth. Selected discontinuities in the magnetic field for the October 18-20, 1995 magnetic cloud are examined to determine their nature such as rotational, tangential or others. The selection of these discontinuities are made by requiring the rotation in the fields for a time period be larger than an expected smooth rotation in the magnetic field for the same time period. An attempt is also made to determine the large scale structure of the October 18-20, 1995 magnetic cloud. From Burlaga et al’s. [1981] work we know that magnetic clouds are bent on a large scale and using minimum variance analysis on the magnetic field components of the magnetic cloud the axis of cloud may be determined locally. The minimum variance technique was applied to hour long segments on the October cloud to determine the direction of the axis and from which a global topology for this cloud was pieced together.

Having looked at some aspects of the October cloud, the question asked now is, what would happen to a magnetic cloud when it interacted with a strong fast shock
such as the Earth's bow shock. To get past the Earth, a magnetic cloud would have to interact with the bow shock first. Signatures of such an interaction would be useful when observing and understanding shock-magnetic cloud interactions. We turn here to numerical simulations to help provide us with some guidelines as to what is likely to happen. The method chosen to simulate the MHD equations is a Gudonov method referred in the literature as PPMMHD (Dai et al. [1994]). A Gudonov method is one in which the fluxes used to update the solutions in time are determined by solving a Riemann shock tube problem. The PPM in PPMMHD means Piecewise Parabolic Method and in this method a parabolic distribution function is constructed to describe a variable between the two interfaces of a computational zone. These distribution functions are then used to determine the initial conditions for the Riemann problem. This numerical scheme is applied to a problem of a fast shock interacting with an idealized magnetic cloud. The static Lundquist force free cylindrically symmetric solutions are used to describe the magnetic field topology of this idealized magnetic cloud. A normal fast shock comparable in strength to the Earth's bow shock is used to simulate a shock interaction with the cloud and in this way the complication of an obstacle is avoided. Magnetic clouds have been observed to have a density that varies through a range of values from the front cloud boundary to the rear cloud boundary. The simulations are simplified to consider three cases: (1) the density of this idealized magnetic cloud is one-half that of the surrounding plasma; (2) the density of this idealized magnetic cloud is equal
to that of the surrounding plasma; and (3) the density of this idealized magnetic cloud is twice that of the surrounding plasma. Another simplification from the more general case is to take the surrounding solar wind plasma of this idealized cloud to have only a component along the axis of the tube. The approach taken here for the three cases is to consider first a 1-D simulation along an axis that cuts the idealized flux tube into two equal \( Y \) portions of the tube. Having familiarized oneself with the solutions along this axis for the three cases, 2 1/2-D simulations are carried out. The numerical results from these cases are then applied to an observation made on the October 18-20, 1995 magnetic when it crossed the bow shock of the Earth. This observation was made by the GEOTAIL satellite. Finally the results of this research are summarized along with recommendations for future work.
Chapter 2
Numerical Code

2.1 Introduction

Magnetohydrodynamics involves the interaction of a conducting fluid with the electromagnetic field on a macroscopic scale. The basic equations for Magnetohydrodynamics (MHD) are the hydrodynamic equations coupled with Maxwell’s equations that describes electrodynamics through the external body force equation given by the Lorentz force equation. In this chapter, we present the MHD equations used for the simulations and describe the numerical method to integrate them.

2.2 Basic Equations

The ideal MHD equations (Landau and Lifshitz [1960]) used neglects the effects of displacement currents, viscosity, resistivity, and heat transfer. The fluid is also considered to be infinitely conducting. The governing equations are as follows:

Continuity:

\[ \frac{\partial}{\partial t} \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (2.1) \]

Momentum:

\[ \frac{\partial}{\partial t} \rho \mathbf{u} + \nabla \cdot (\rho \mathbf{u} \mathbf{u} + \mathbf{P}) = 0 \quad (2.2) \]
Energy:
\[ \frac{\partial}{\partial t} \rho E + \nabla \cdot (\rho u E + \mathbf{P} \cdot u) = 0 \] (2.3)

Maxwell's Equations:
\[ \frac{\partial}{\partial t} \mathbf{B} + \nabla \cdot \mathbf{T} = 0 \] (2.4)

\[ \nabla \cdot \mathbf{B} = 0 \] (2.5)

Equation of State:
\[ p = (\gamma - 1) \rho \epsilon \] (2.6)

Stress Tensor:
(Pressure and Maxwell's Stress Tensor)
\[ P_{ij} = p \delta_{ij} + \left( \frac{1}{8\pi} \mathbf{B}^2 \delta_{ij} - \frac{1}{4\pi} B_i B_j \right) \]

Total Specific Energy:
\[ E = (\epsilon + \frac{1}{2} u^2) + \frac{1}{8\pi \rho} \mathbf{B}^2 \]
\[ T_{ij} = u_i B_j - u_j B_i \]

Here \( \rho \) is the mass density, \( u \) is the flow velocity, \( \mathbf{B} \) is the magnetic field, \( p \) is the thermal pressure, \( \epsilon \) is the specific internal energy and \( \gamma \) is the ratio of specific heats.

The equation of state used is that which describes an ideal gas.

Thermal conduction though important are not include in the simulations. The
reason being that, in the cloud, the temperature of the electrons \( T_e \) is much greater than the temperature of the protons \( T_p \) and the polytropic index for the electrons \( (\gamma_e) < 1 \) while that of protons \( (\gamma_p) \approx 1 \) but still less than 5/3 (Osherovich et al. [1997]). In the solar wind however, \( T_e \approx T_p \) with \( \gamma = 5/3 \) for both electrons and protons. It is for this reason, we feel that an MHD simulation is not the best suited method to model the temperature dependent variations found in the cloud. But we use this method instead, to study the shock interaction with the magnetic field topology of a static force-free Lundquist flux tube with plasma density variations (Chapter 5).

### 2.3 Numerical Method

The numerical scheme makes use of the operator splitting method (Strang, G. [1968]) in which a multidimensional problem is reduced to solving a series of one dimensional (1-D) problems. The above set of equations can be written in the following form:

\[
\frac{\partial}{\partial t} U + \nabla \cdot F(U) = 0 \quad (2.7)
\]

The discretization scheme applied to the 1-D case of the above equation takes the following form
Figure 2.1: Flowchart for the Numerical Computation
where $D^-$ is the backward difference operator acting on $F_{x_i}$. We define the operator $L^*_{ij}$ which advances $U_i$ in a time step $\Delta t$ as follows

$$L^*_{ij} = L_{ij} = L_{i+1,j} - D^- F_{x_i}$$  \hspace{1cm} (2.9)

A computational cycle for a 2-D problem consists of four one dimensional sweeps and in each sweep the spatial gradients perpendicular to the direction of sweep are set to zero. In the 2-D case the sweep is first made in the X-direction followed by a sweep in the Y-direction, then a sweep in the Y-direction and finally a sweep in the X-direction. This procedure can be represented as follows

$$U^{n+1}_i = (L^*_{x} L^*_{y} L^*_{y} L^*_{x}) U^n_i$$  \hspace{1cm} (2.10)

For this splitting method to remain symmetric in the two respective directions, an alternate direction splitting is used. For odd numbered time steps the sweep in the X-direction is made first and for even numbered time steps the y-sweep is made first. This directional sweep method is second order accurate in time as long as the 1-D sweeps are second order accurate in space. Based on the numerical simulation results (see Chapter 5), we find that the use of the alternate direction method did not change circular structures into square shaped structures. This is evident by the shape of the shock and expansion wave (curved in structure) propagating upstream.
of the front tube boundary (Chapter 5) for the life of the simulation.

The set of 1-D MHD equations are solved in a Lagrangian frame. If we divide up a 1-D region into N cells of width \( dx \), than the mass in each cell is given by \( dm(\equiv \rho dx) \). In a Lagrangian frame the mass \( dm \) will be conserved in each time step even though the grid may get distorted. The method for solving the 1-D MHD equations is as follows; (1) solve the 1-D MHD equations on a Lagrangian grid, and (2) map the updated solutions back onto a fixed Eulerian grid. With this in mind, the set of 1-D MHD equations are written in Lagrangian mass coordinate \((dm)\) form and are as follows (Dai and Woodward [1994]):

\[
\frac{\partial}{\partial t} U + \frac{\partial}{\partial m} F(U) = 0 \tag{2.11}
\]

and

\[
\frac{\partial}{\partial t} V B_x - \frac{\partial}{\partial m} B_x u_x = 0 \tag{2.12}
\]

with

\[
U = \begin{bmatrix}
    V \\
    u_x \\
    u_y \\
    u_z \\
    V B_y \\
    V B_z \\
    E
\end{bmatrix}
\]

\[
F(U) = \begin{bmatrix}
    -u_x \\
    P \\
    \Lambda_y \\
    \Lambda_z \\
    -B_x u_y \\
    -B_x u_z \\
    P u_x + \Lambda_y u_y + \Lambda_z u_z
\end{bmatrix}
\]

\[
P = p + \frac{1}{8\pi}(B_y^2 + B_z^2 - B_x^2)
\]

\[
\Lambda_y = -\frac{1}{4\pi}B_x B_y
\]

\[
\Lambda_z = -\frac{1}{4\pi}B_x B_z
\]
Here $V = 1/\rho$, $P$, $\Lambda_y$ and $\Lambda_z$ are the diagonal and off-diagonal parts of the total pressure tensor. Equation 2.7 is in conservation form (LeVeque [1992]). Integrating Equation 2.7 w.r.t. $m$ and noting that the fluxes at $\infty$ are zero, means that $\int_{-\infty}^{\infty} U dm$ is a constant with respect to time. $F(U)$, is the flux function and it determines the rate of change of each variable $U$ at $(m,t)$. The above system of equations is hyperbolic by which we mean that the Jacobian matrix $\frac{\partial F(U)}{\partial U}$ has 7 real eigenvalues and corresponding linearly independent eigenvectors (Jeffrey and Taniuti [1964]). The numerical scheme used to solve the 1-D equations is a high order Godunov method, PPMMHD (Dai and Woodward [1994]) which is an extension of the Piecewise Parabolic Method (PPM) (Collela and Woodward [1984]) for fluid flows to Magnetohydrodynamic problems. The discretization scheme applied to equation (2.7) gives:

$$< U(\Delta t) >_i = < U(0) >_i + \frac{\Delta t}{\Delta m_i} (\bar{F}_i - \bar{F}_{i+1})$$  \hspace{1cm} (2.13)

Where $\Delta t$ is the time step and $\Delta m_i$ is the mass in the zone, $< U >_i$ is the average value of $U$ at time $t$ over the zone and $\bar{F}_i$ is the time averaged flux at the interface $x_i$ (Fig. 2.2), i.e.

$$< U(t) >_i = \frac{1}{\Delta x_i} \int_{x_i}^{x_{i+1}} U(t, x) dx$$  \hspace{1cm} (2.14)

$$\bar{F}_i = \frac{1}{\Delta t} \int_0^{\Delta t} F(U(t, x_i)) dt$$  \hspace{1cm} (2.15)
PPMMHD makes use of Godunov's method (Gudunov [1959]) in which the solution to the magnetohydrodynamic equations are pieced together from discontinuous solutions. These discontinuous solutions approximate closely the solutions in the smooth regions where applicable but also approximate the true solution when the flow is not smooth. Godunov used the solutions from the Riemann shock tube problem to describe the nonlinear flow that develops from the discontinuous jumps separating two constant states. A Riemann problem is an initial value problem for Equation 2.7 with the following initial conditions (Jeffrey and Taniuti [1964]);

\[
U(x, 0) = \begin{cases} 
U_R, & \text{if } x > 0; \\
U_L, & \text{if } x < 0
\end{cases}
\]

where \( U_R \) and \( U_L \) are two constant states with a discontinuity at \( x = 0 \). To better understand the Riemann problem, one can imagine a partition at the interface that separates the 2 computational zones which, when removed, will generate waves through the 2 zones for the system to reach an equilibrium state. This forms the basis of Godunov’s method which is to approximate the flow by a large number of constant states, compute the interactions at the interfaces exactly and calculate
the fluxes at these interfaces. This procedure gives an accurate and well behaved representation of shock discontinuities. A Riemann solver is an important element for this method and it allows the scheme to have narrow discontinuities without introducing unphysical oscillations in the solution. The Eulerian calculation used in PPMMHD is made up of two steps. The first step is to advance the solutions using a Lagrangian method which involves a reconstruction step to determine the interface values which separate 2 zones in the computational domain. The next step is to determine the domain of dependence for each interface, i.e., the region of a computational zone from which information can reach or propagate to the interface in a time step $\Delta t$. This averaged value of the variables $U$ will then serve as initial conditions for the Riemann shock tube problem. The solution to the Riemann problem at the interface will be used to determine the time averaged fluxes to advance the solutions in a time step $\Delta t$. Since the solutions are advanced in the Lagrangian frame, the grids are free to move. Thus there is the need to remap the solutions onto a fixed Eulerian grid. Each sweep, either $x$, $y$ or $z$ has the following set of steps best represented by the figure shown below.
Figure 2.3: Steps involved for each 1-D sweep

1. Reconstruction Step: Construct Interface Values for Each Cell
2. Domain of Dependence: Construct Initial Conditions for Riemann Problem
3. Solve for Fluxes at Cell Interfaces Using a Riemann Solver
4. Advance the Discretized Cell Variables, $\rho$, $\mathbf{V}$, $E$, and $\mathbf{B}$
5. Remap the Conserved Variables back onto a Fixed Eulerian Grid
2.3.1 Reconstruction Step

First, an interpolation is done to find the interface values using zoned averaged values \(< \mathbf{U} >_{i-1}, < \mathbf{U} >_i, < \mathbf{U} >_{i+1} \) and \(< \mathbf{U} >_{i+2} \) (Figure 2.4). For a uniformly spaced grid, the interpolated interface value is given by Collela and Woodward [1984]

\[
\mathbf{U}_{L,i} = \frac{7}{12}(< \mathbf{U} >_i + < \mathbf{U} >_{i+1}) - \frac{1}{12}(< \mathbf{U} >_{i+2} + < \mathbf{U} >_{i-1})
\] (2.16)

with

\[
\mathbf{U}_{R,i-1} = \mathbf{U}_{L,i} \]

\[
\begin{array}{cccc}
< \mathbf{U}_{i-1} > & < \mathbf{U}_i > & < \mathbf{U}_{i+1} > & < \mathbf{U}_{i+2} > \\
\mathbf{U}_{L,i+1} & \mathbf{U}_{R,i+1} \\
\end{array}
\]

Figure 2.4: Zones used to determine interface values

For a nonuniform grid, \( \mathbf{U}_{L,i} \), the left interface of the \( i \) th zone is given by

\[
\mathbf{U}_{L,i} = < \mathbf{U} >_i + f_a(< \mathbf{U} >_{i+1} - < \mathbf{U} >_i) + f_{da} \Delta \mathbf{U}_{i+1} + f_{dal} \Delta \mathbf{U}_i
\] (2.17)

where

\[
\Delta \mathbf{U}_i = g_{dal}(< \mathbf{U} >_i - < \mathbf{U} >_{i-1}) + g_{dar}(< \mathbf{U} >_{i+1} - < \mathbf{U} >_i)
\]
and \( f_a, f_{da}, f_{dal}, g_{dal}, \) and \( g_{dar} \) are geometric factors which depend only on the grid size and are of the form

\[
g_{dal} = \Delta x_i (2\Delta x_{i+1} + \Delta x_i) / [(\Delta x_{i-1} + \Delta x_i)(\Delta x_{i+1} + \Delta x_i + \Delta x_{i-1})]
\]

\[
g_{dar} = \Delta x_i (2\Delta x_{i-1} + \Delta x_i) / [(\Delta x_i + \Delta x_{i+1})(\Delta x_{i+1} + \Delta x_i + \Delta x_{i-1})]
\]

\[
f_{da} = -\Delta x_i (\Delta x_{i-1} + \Delta x_i) / [(2\Delta x_i + \Delta x_i + 1)(\Delta x_{i-1} + \Delta x_i + \Delta x_{i+1} + \Delta x_{i+2})]
\]

\[
f_{dal} = \Delta x_{i+1} (\Delta x_{i+1} + \Delta x_{i+2}) / [(2\Delta x_{i+1} + \Delta x_i)(\Delta x_{i-1} + \Delta x_i + \Delta x_{i+1} + \Delta x_{i+2})]
\]

\[
f_a = [\Delta x_i - 2(\Delta x_{i+1} f_{da} + \Delta x_i f_{dal})] / (\Delta x_i + \Delta x_{i+1})
\]

With the interface values and the zone averaged value, a parabola can be defined for each zone. The parabola defined within each zone using \(< \vec{U} \>_i, \vec{U}_{L,i}\) and \(\vec{U}_{R,i}\) is of the form

\[
\vec{U}_i^{(p)}(\xi) = \vec{U}_{L,i}(1 - \xi) + \vec{U}_{R,i}\xi + \vec{U}_{i,6}(1 - \xi)\xi \tag{2.18}
\]

\[
\xi = \frac{x - x_{i-1/2}}{\Delta x_i}, \quad x_{i-1/2} \leq x \leq x_{i+1/2}
\]

\[
\vec{U}_{i,6} = 6 < \vec{U} >_i - 3(\vec{U}_{L,i} + \vec{U}_{R,i})
\]

The PPM scheme has been found to create shocks which are too narrow because of the small amount of dissipation used in the method. When this happens, unphysical oscillations will develop in the flow. In order to remove these oscillations, additional dissipation is added to the scheme by flattening the parabolic profile of those zones.
which are near shocks that are found to be too steep. The effect of flattening
(Collela and Woodward [1984]) is to reduce the order of the method in the region
where it is applied. For the case when maximum flattening is applied, the scheme
locally reduces to Gudunov's first order method. The flattening is implemented as
follows:

\[
U_{L,i}^{flat} = \langle U \rangle_i^n f_j + U_{L,i}(1 - f_i) \quad (2.19)
\]

\[
U_{R,i}^{flat} = \langle U \rangle_i^n f_i + U_{R,i}(1 - f_i) \quad (2.20)
\]

Where \(0 \leq f_i \leq 1\). The coefficient \(f_i\) is equal to zero away from strong shocks and
equal to 1 if the shock profile is sufficiently steep. The flattening algorithm used
here is described by Collela and Woodward [1984] and is implemented as follows:

\[
y = \frac{|p_{i+1} - p_{i-1}|}{\min(p_{i+1}, p_{i-1})}
\]

where \(y\) determines if a shock is present in the computational cell and \(p\) is the
pressure.

\[
w_i = \begin{cases} 
1 & \text{if } (u_{i-1} - u_{i+1}) > 0, \quad y > \epsilon; \\
0 & \text{otherwise}
\end{cases}
\]

\[
s_i = \begin{cases} 
1 & \text{if } (p_{i+1} - p_{i-1}) < 0, \\
-1 & \text{if } (p_{i+1} - p_{i-1}) > 0,
\end{cases}
\]

\[
f_i = \max(\tilde{f}_i, \tilde{f}_{i+1}) \quad (2.21)
\]

where

\[
\tilde{f}_i = 1 - w_i \max(0, \frac{p_{i+1} - p_i - 1}{p_{i+2} - p_i - 2} - \omega^1) \omega^2
\]
The constants are set at $\omega_1 = 0.75$, $\omega^2 = 10$ and $\epsilon = 0.33$ $u_i = 1$ gives an indication that a shock is present in the $i$th zone and 0 elsewhere. Index $i+s_j$ gives the index for the zone just upstream of zone $j$ where the shock is present.

The interpolated values are not always monotonic even if they are interpolated from monotonic data. They give rise to over and undershoots in the zone averaged data due primarily to the interpolation scheme used and not related to the Gibb's phenomenon which arises from the representation of a piecewise smooth function with a series of partial sums. *Van Leer* [1977], preserved the monotonicity of the initial data by flattening any non-monotone interpolated values so that they remain monotonic. *Van Leer*’s monotonicity constraint states that no interpolated values within a zone can lie outside the range defined by the zone averages of this zone and its two neighboring zones. There are two cases that need to be considered (*Collela and Woodward* [1984]) when applying the monotonic constraint. If $U_i$ is a local maximum or minimum, the interpolated values are set to a constant. The second case is when $U_i$ lies between $U_{L,i}$ and $U_{R,i}$ but is sufficiently close to one of the interface values such that the parabola gives values that do not lie between $U_{L,i}$ and $U_{R,i}$. The conditions when the coefficients of the parabola do not give rise to overshoots is $|\Delta U_i| \geq |U_{i,6}|$. When this condition fails, either $U_{L,i}$ or $U_{R,i}$ is reset such that the parabola is monotonic and the derivative at the opposite edge of the reset zone is set to zero. The monotonicity constraint is applied as follows.
\[ U_{i,L} \rightarrow U_i \quad U_{i,R} \rightarrow U_i \quad \text{if} \quad (U_{i,R} - U_i)(U_i - U_{i,L}) \leq 0 \]

\[ U_{i,L} \rightarrow 3U_i - 2U_{i,R} \quad \text{if} \quad (U_{i,R} - U_{i,L})(U_i - \frac{1}{2}(U_{i,L} + U_{i,R})) > \frac{(U_{i,R} - U_{i,L})^2}{6} \]

\[ U_{i,R} \rightarrow 3U_i - 2U_{i,L} \quad \text{if} \quad \frac{(U_{i,R} - U_{i,L})^2}{6} > (U_{i,R} - U_{i,L})(U_i - \frac{1}{2}(U_{i,L} + U_{i,R})) \]

(2.22)

### 2.3.2 Domain of Dependence

The parabola representing \(U(x,t)\) is continuous inside each zone but it may be discontinuous across the interface between two neighboring zones. During the time step \((t^n, t^{n+1})\), information for an MHD problem is propagated to the interface at three wave speeds. These are the Fast, Alfven and the Slow wave. We integrate the parabolic profile over the spatial domain which influences the zone interface values to get the initial conditions for the Riemann problem (Fig. 2.5). This domain averaged value for the \(k\)th wave is defined as \((Dai and Woodward [1995])\)

\[
< U >_{d,k} = \frac{1}{x_i - x_{dk}} \int_{x_{dk}}^{x_i} U(0,x)dx \quad \text{for either} \quad c_k > 0 \quad \text{or} \quad c_k < 0 \quad (2.23)
\]

where

\[ x_{dk} = x_i - c_k \Delta t \]

for both cases of \(c_k\). Using the parabolic interpolation one obtains the following

\[
< U >_{d,k} = U_{i-1,R} - \frac{1}{2}\sigma_{i-1,k}[U_{i-1,R} - U_{i-1,L} - U_{i-1,6}(1 - \frac{2}{3}\sigma_{i-1,k})] \quad \text{for} \quad c_k > 0 \quad (2.24)
\]
\[<U>_d,k = U_{i,L} + \frac{1}{2} \sigma_{i,k}[U_{i,R} - U_{i,L} + U_{i,0}(1 - \frac{2}{3} \sigma_{i,k})] \quad \text{for} \quad c_k < 0 \quad (2.25)\]

with \(\sigma_{i,k}\) is the courant number, i.e. \(|c_k|\Delta t/\Delta x\). The expressions for the fast \(C_f\), Alfven \(C_a\) and slow \(C_s\) waves are given as follows:

\[
C_{f,s}^2 = \frac{1}{2}[(C_0^2 + C_a^2 + C_i^2) \pm \sqrt{(C_0^2 + C_a^2 + C_i^2)^2 - 4C_0^2C_a^2}]
\]

\[
C_a^2 = (B_z^2\rho/4\pi)
\]

\[
C_i^2 = [(B_y^2 + B_z^2)\rho/4\pi]
\]

\[
C_0^2 = (\gamma\rho\rho)
\]

With these initial conditions for the Riemann problem, the next step is to obtain the solution to the Riemann MHD problem.

Figure 2.5: Two domains of dependence for a zone interface for a time step determined by tracing the paths of waves arriving at the interface at the end of the time step.
2.3.3 Riemann Solver

The numerical scheme makes use of a nonlinear and a linear Riemann solver. This is because the nonlinear solver uses Newton’s method to iteratively solve for the unknowns, as will be described shortly. If an improper initial guess is made, Newton’s method will fail to converge to the solution, and if this happens the scheme switches over to the linear solver to obtain the fluxes at the boundary.

Nonlinear Riemann Solver

In general, there are four kinds of discontinuities possible in ideal MHD (see Jeffrey and Taniuti [1964], Landau and Lifshitz [1960] and Parks [1991]). They are contact discontinuities, fast shocks, slow shocks and rotational discontinuities. For the case when $B_x = 0$, tangential discontinuities and magnetoacoustic shocks are present. For contact discontinuities, only jumps in density and energy are allowed. It can be viewed as a surface that separates two parts of the fluid and with no flow through.

![Figure 2.6a: Fast Shock](image)

![Figure 2.6b: Slow Shock](image)
this surface. A tangential discontinuity occurs when the longitudinal component of the magnetic field vanishes but there are discontinuous jumps in the density, transverse components of the magnetic fields and the velocity flows across the surface. The total pressure and the longitudinal velocity field remain unchanged for a tangential discontinuity. For a shock discontinuity, jumps in the density, pressure, magnetic field and flow velocity take place. The pre-shock state is the state of the fluid that enters the shock discontinuity and the state after the discontinuity is called the post-shock state. For a fast shock, there is an increase in the magnitude of the transverse magnetic field across the shock front, i.e., there is an increase from the pre-shock state to the post-shock state. This increase is due to two factors, the compression and shearing of the fluid. The compression increases the magnitude of the transverse components. The fluid is also sheared against the direction of the transverse components of the magnetic field and it has the effect of increasing the magnitude of the transverse components of the magnetic field. In the case of a slow shock, there is a decrease in the magnitude of each transverse component of the magnetic field across the discontinuity. The compression as in the fast shock case increases the magnitude of the transverse components of the magnetic field but the shearing on the fluid in the direction of the transverse components of the magnetic field results in a reduction in the magnitude of the transverse components of the magnetic field. In a rotational discontinuity, the transverse magnetic fields are rotated about the normal to the surface of the rotational discontinuity. There
are jumps in the transverse flow velocities but no jumps in the density, thermal pressure and the longitudinal velocity. The magnitude of the the magnetic field remains unchanged across this surface and the speed with which the rotational discontinuity travels is at the Alfven speed $C_A^2 = \rho B_z^2$. In the limit as $B_x \to 0$, the contact discontinuity, slow shock and the Alfven discontinuity collapse to a tangential discontinuity. The solution of the Riemann problem in general contains six waves (Fast, Alfven and Slow), which may be discontinuous, traveling leftward and rightward of the contact discontinuity or, in this case, the interface (Figure 2.7). There are 8 possible regions in $t vs m$ space, and following Dai and Woodward [1994] they are labeled R1, R2, R3, R4, R5, R6, R7 and R8. With the initial conditions for the Riemann problem given by R1 and R8, the purpose of the solver is to determine the types of waves, strengths, speeds and the flow properties in the other six regions. The algorithm that determines the solution to the six different regions is called the Riemann solver in MHD. This nonlinear Riemann solver due to Dai and Woodward [1994] is based on conservation laws across any discontinuity. The jump conditions across any discontinuity is given by:

$$W[V] = -[u_x]$$  \hspace{1cm} (2.26)

$$W[u_x] = [P]$$  \hspace{1cm} (2.27)

$$W[u_y] = [A_y]$$  \hspace{1cm} (2.28)

$$W[u_z] = [A_z]$$  \hspace{1cm} (2.29)
\[
W[V B_y] = -B_x[u_y]
\]
\[
W[V B_z] = -B_x[u_z]
\]
\[
W[E] = [u_z P] + [u_y \Lambda_y] + [u_z \Lambda_z]
\]

W is the speed of the discontinuity in the mass coordinate and \([..]\) denotes the difference of the values between the two sides of the discontinuity, for example, \([V]\). To make use of the nonlinear Riemann Solver, a formula is needed for W and it is given in terms of the pre-shocked state and one transverse component of the magnetic field in the shocked state Dai and Woodward [1994];

![Figure 2.7: The waves generated and the various regions for the MHD Riemann problem](image)

\[
W_{f,s} = \frac{1}{2(1 + A_0)} \left[ (C_s^2 + C_f^2 + A_1) \pm \sqrt{(C_s^2 + C_f^2 + A_1)^2 - 4(1 + A_0)(C_s^2C_f^2 - A_2)} \right]
\]

(2.33)
where

\[ A_0 = -\frac{1}{2}(\gamma - 1)[\Lambda_y]/\Lambda_y \]

\[ A_1 = \frac{1}{2}[(\gamma - 2)\lambda \rho B_y[\Lambda_y]/B_x + 2C_s^2 - (\gamma - 4)C_i^2 - 2\gamma C_o^2][\Lambda_y]/\Lambda_y \]

\[ A_2 = \frac{1}{2}[\lambda \rho^2[\Lambda_y]^2 + (\gamma + 2)\lambda \rho^2\Lambda_y[\Lambda_y] + (\gamma + 1)C_s^2C_i^2 + (\gamma + 1)C_o^4 - 2C_o^2C_s^2][\Lambda_y]/\Lambda_y \]

\[ \lambda = 1 + (B_y/B_z)^2 \]

The variables \( A_{0,1,2} \) are evaluated in the pre-shocked state, \([\Lambda_y]\) is the difference across the discontinuity. The orientation of the transverse magnetic fields are unchanged across a fast and slow shock. The orientations in regions \( R3 \) and \( R6 \) are

\[ \frac{B_{z3}}{B_{y3}} = \tan \psi_3 \]
\[ \frac{B_{z6}}{B_{y6}} = \tan \psi_6 \]

But from the conditions at the contact discontinuity, the two orientations must be equal since only discontinuities in \( \rho \) and energy are permitted. Thus

\[ \tan \psi_3 = \tan \psi_6 \]

and this common field orientation angle will be referred as \( \psi \). For a given left and right state, the solution to the Riemann problem can be summed up as follows.
1) Guess one transverse component of the magnetic field \( B_y \) in regions R2, R4 and R7 and the orientation \( \psi \), where \( \tan \psi = \frac{B_{z3}}{B_{y3}} \) in region R3.

2) Taking the initial conditions as pre-shocked states, calculate the two fast shock speeds using Equation 2.33. With these shock speeds along with the initially guessed \( B_y \) components, the states R2 and R7 can be determined using Equations 2.26-2.31.

3) Rotate the fields using \( \psi \), calculate the Alfvén speed in R3 and R6, using this for \( W \) in Equations 2.26-2.31, and the states R3 and R6 are determined.

4) Using R3 and R6 as pre-shocked states, one repeats step 2 to find the states R4 and R5 but using the slow shock values for \( W \) in Equations 2.26-2.31.

5) At the contact discontinuity, \( p \), the thermal pressure and the velocities must be equal. One makes use of these conditions to make an improved guess for \( B_y \) in regions R2, R4 and R7 and \( \psi \) in region R3.

Once the desired accuracy is reached, the fluxes are evaluated and the solution is advanced.

Explicitly, from equations 2.26-2.31, relationships for region R7 and R8 which has a fast shock discontinuity are as follows

\[
U_{y7} = U_{y8} - \frac{B_x}{4\pi W_f} (B_{y7} - B_{y8})
\]

\[
U_{z7} = U_{z8} - \frac{B_x}{4\pi W_f} (B_{z7} - B_{z8})
\]

\[
V_7 = V_8 - \frac{1}{B_{y7}} [\frac{B_x}{W_f} (U_{y7} - U_{y8}) + V_8 (B_{y7} - B_{y8})]
\]
\[ U_{x7} = U_{x8} - W_s(V_{x7} - V_{x8}) \]

\[ p_7 = p_8 - W_f^2(V_{x7} - V_{x8}) + \frac{1}{4\pi}(B_{y7}^2 - B_{y8}^2) + \frac{1}{4\pi}(B_{z7}^2 - B_{z8}^2) \]

where \( W_f \) is the shock speed calculated using Equation 2.33. Between R7 and R6 there is a rotational discontinuity present. The relationship between these two states, with \( C_{x7} \) the Alfvén speed in R7, are

\[ B_{y6} = \cos \psi \sqrt{B_{y7}^2 + B_{z7}^2} \]

\[ B_{x6} = \sin \psi \sqrt{B_{y7}^2 + B_{z7}^2} \]

\[ U_{y6} = U_{y7} - \frac{B_x}{4\pi C_{x7}}(B_{y6} - B_{y7}) \]

\[ U_{x6} = U_{x7} - \frac{B_x}{4\pi C_{x7}}(B_{x6} - B_{x7}) \]

\[ V_6 = V_7 \]

\[ U_{x6} = U_{x7} \]

\[ p_6 = p_7 \]

Finally the states between R6 and R5 which are separated by a slow shock (\( W_s \)) have the following relationship between them

\[ U_{y5} = U_{y6} - \frac{B_x}{4\pi W_s}(B_{y5} - B_{y6}) \]

\[ U_{z5} = U_{x6} - \frac{B_x}{4\pi W_s}(B_{z5} - B_{x6}) \]

\[ V_5 = V_6 - \frac{1}{B_{y5}}[\frac{B_x}{W_s}(U_{y5} - U_{y6}) + V_6(B_{y5} - B_{y6})] \]

\[ U_{x5} = U_{x6} - W_s(V_{x5} - V_{x6}) \]

\[ p_5 = p_6 - W_s^2(V_{x5} - V_{x6}) + \frac{1}{4\pi}(B_{y5}^2 - B_{y6}^2) + \frac{1}{4\pi}(B_{z5}^2 - B_{z6}^2) \]
There are similar expressions that relate regions R1 and R2, R2 and R3 and R3 and R4. The differences from the above expressions are replacing \( W_{f/s} \) by \(-W_{f/s}\) and the indices for the different states above, i.e. 8→1, 7→2, 6→3 and 5→4. The contact discontinuity that exist between regions R4 and R5 allows us to relate the velocities and pressures between the two states.

\[
U_{x4}(B_{y2}, B_{y4}, \psi) = U_{x5}(B_{y7}, B_{y4}, \psi)
\]

\[
U_{y4}(B_{y2}, B_{y4}, \psi) = U_{y5}(B_{y7}, B_{y4}, \psi)
\]

\[
U_{z4}(B_{y2}, B_{y4}, \psi) = U_{z5}(B_{y7}, B_{y4}, \psi)
\]

\[
p_4(B_{y2}, B_{y4}, \psi) = p_5(B_{y7}, B_{y4}, \psi)
\]

From the above set of equations \( B_{y2}, B_{y4}, B_{y7} \) and \( \psi \) are solved by iteration using Newton’s method. The Numerical Recipes [15] algorithm for Newton’s method was used to obtain the solutions. For Newton’s method, we need equations that will determine how to modify the initial guesses for \( B_{y2}, B_{y4}, B_{y7} \) and \( \psi \). These are given by taking variations with respect to the unknowns from the above equations, i.e.

\[
\frac{\partial U_{x4}}{\partial B_{y2}} \delta B_{y2} + (\frac{\partial U_{x4}}{\partial B_{y4}} - \frac{\partial U_{x5}}{\partial B_{y4}}) \delta B_{y4} - \frac{\partial U_{x5}}{\partial B_{y7}} \delta B_{y7} + (\frac{\partial U_{x4}}{\partial \psi} - \frac{\partial U_{x5}}{\partial \psi}) \delta \psi = U_{x5} - U_{x4}
\]

\[
\frac{\partial U_{y4}}{\partial B_{y2}} \delta B_{y2} + (\frac{\partial U_{y4}}{\partial B_{y4}} - \frac{\partial U_{y5}}{\partial B_{y4}}) \delta B_{y4} - \frac{\partial U_{y5}}{\partial B_{y7}} \delta B_{y7} + (\frac{\partial U_{y4}}{\partial \psi} - \frac{\partial U_{y5}}{\partial \psi}) \delta \psi = U_{y5} - U_{y4}
\]

\[
\frac{\partial U_{z4}}{\partial B_{y2}} \delta B_{y2} + (\frac{\partial U_{z4}}{\partial B_{y4}} - \frac{\partial U_{z5}}{\partial B_{y4}}) \delta B_{y4} - \frac{\partial U_{z5}}{\partial B_{y7}} \delta B_{y7} + (\frac{\partial U_{z4}}{\partial \psi} - \frac{\partial U_{z5}}{\partial \psi}) \delta \psi = U_{z5} - U_{z4}
\]

\[
\frac{\partial p_4}{\partial B_{y2}} \delta B_{y2} + (\frac{\partial p_4}{\partial B_{y4}} - \frac{\partial p_5}{\partial B_{y4}}) \delta B_{y4} - \frac{\partial p_5}{\partial B_{y7}} \delta B_{y7} + (\frac{\partial p_4}{\partial \psi} - \frac{\partial p_5}{\partial \psi}) \delta \psi = p_5 - p_4
\]
The dependence of $U_{x5}$ on $B_{y7}$ involves a number of nested functions. For example, $W_f$ has a dependence on $B_{y7}$, the transverse magnetic field components $B_{y6}$ and $B_{z6}$ also have a dependence on $B_{y7}$, and the $W_z$ has a dependence on $B_{y7}$ through $B_{y6}$. Rather than develop an expression for the partial derivative, these are evaluated numerically. As was pointed out earlier, Newton’s method requires a reasonable initial guess for convergence. The initial guess for $B_{y2}, B_{y4}, B_{y7}$ and $\psi$ are set equal to those variables in regions R1 or R8 and if it fails the scheme switches over to the linear solver.

**Linear Riemann Solver**

A higher-order linear MHD Riemann solver developed by *Dai and Woodward* [1995] is used to determine the fluxes at the interfaces for this scheme. For the MHD problem there are seven conservation laws for which correspond 7 characteristic waves. The Riemann invariant for the $k$th characteristic wave along a characteristic curve $dx = c_k dt$ where $L(U)_k$ is the left eigenvectors of the matrix $A(U)(\equiv \frac{\partial F(U)}{\partial U})$ can be written as (*Dai and Woodward* [1995]):

$$dR_k = L_k^T(U)dU$$

The MHD differential Riemann invariants are

$$R_0 = \rho V^\gamma \quad \text{along} \quad \frac{dm}{dt} = 0 \quad (2.34)$$
\[ dR_{f \pm} = (C_f^2 - C_a^2)(dP \pm C_f du_x) + \rho \Lambda_y (d \Lambda_y \pm C_f du_y) + \rho \Lambda_z (d \Lambda_z \pm C_f du_z) \]

along \[ \frac{dm}{dt} = C_f \] (2.35)

\[ dR_{s \pm} = (C_s^2 - C_a^2)(dP \pm C_s du_x) + \rho \Lambda_y (d \Lambda_y \pm C_s du_y) + \rho \Lambda_z (d \Lambda_z \pm C_s du_z) \]

along \[ \frac{dm}{dt} = C_s \] (2.36)

\[ dR_{a \pm} = \pm C_a (B_z du_y - B_y du_z) + (B_z d \Lambda_y - B_y d \Lambda_z) \] along \[ \frac{dm}{dt} = C_a \] (2.37)

Following \textit{Dai and Woodward [1995]}, for given initial conditions, \( U_L \) and \( U_R \), \( \bar{U} \) is obtained from the linear Riemann equations \( k=1,2,7 \) from which the time averaged fluxes are obtained.

\[ \bar{L}_k^T (\bar{U} - U_R) = 0 \text{ for all } k \text{ with } c_k < 0 \] (2.38)

\[ \bar{L}_k^T (\bar{U} - U_L) = 0 \text{ for all } k \text{ with } c_k > 0 \] (2.39)

\[ \bar{L}_k^T (\bar{U} - U_0) = 0 \text{ for } c_0 = 0 \] (2.40)

where

\[ U_0 \equiv \frac{1}{2} (U_L + U_R) \]

\[ \bar{L}_k \equiv \frac{1}{2} \text{sign}[D_k(U_R)] [abs(L_k(U_L)) + abs(L_k(U_R))] \text{ for } c_k < 0 \]

\[ \bar{L}_k \equiv \frac{1}{2} \text{sign}[D_k(U_L)] [abs(L_k(U_L)) + abs(L_k(U_R))] \text{ for } c_k > 0 \]
$D(U)_k$ is a diagonal matrix whose $j$th element $d_{jj}$ is equal to the $j$th element of the vector $L(U)_k$ When applying the above equations, the $j$th component of $\tilde{L}_k$ is the absolute sum of the $j$th components of $L_k(U_L)$ and $L_k(U_R)$ and adjust the sign of the $j$th component of the left or right vector based on the characteristics propagation direction. For example,

\[
C_f = \frac{1}{2}(C_{fast,left} + C_{fast,right})
\]
\[
C_s = \frac{1}{2}(C_{slow,left} + C_{slow,right})
\]
\[
C_a = \frac{1}{2}(C_{alfven,left} + C_{a,right})
\]

The Riemann Invariant equations 2.35-2.37 are next normalized as suggested by Brio and Wu [1988] so that they remain defined in the limit of $B_x \to 0$ and $B_\perp \to 0$. The singularities can be removed by using the following identities

\[
C_s = \frac{C_o|C_a|}{2} C_f
\]
\[
C_s^2 - C_a^2 = -\frac{C_a^2(C_y^2 + C_z^2)}{C_f^2 - C_a^2}
\]

Multiplying Equation 2.35 by $\alpha_f$ (defined below), Equation 2.36 by $\alpha_s$ (defined below) and Equation 2.37 by $\frac{1}{\sqrt{C_y^2 + C_z^2}}$ along with the identities above, the Riemann Invariants are rewritten as

\[
dR_{f,\pm} = \alpha_f(dP \pm C_f du_z) - \alpha_s\beta_y \text{sgn}(B_z)(d\Lambda_y \pm C_f du_y) - \alpha_s\beta_z \text{sgn}(B_z)(d\Lambda_z \pm C_f du_z)
\]

(2.41)
\[ dR_{s\pm} = \alpha_s(dP \pm C_s du_x) + \alpha_f \beta_y \text{sgn}(B_x) (d\Lambda_y \pm C_s du_y) + \alpha_f \beta_z \text{sgn}(B_z) (d\Lambda_z \pm C_s du_z) \]  
(2.42)

\[ dR_{a\pm} = \pm C_a \text{sgn}(B_x)(\beta_z du_y - \beta_y du_z) + (\beta_z d\Lambda_y - \beta_y d\Lambda_z) \]  
(2.43)

where

\[
\begin{align*}
\alpha_f &= \sqrt{\frac{C_f^2 - C_a^2}{C_j^2 - C_s^2}} \\
\alpha_s &= \sqrt{\frac{C_s^2 - C_j^2}{C_f^2 - C_s^2}} = \frac{|C_a|}{C_f} \sqrt{\frac{C_j^2 - C_o^2}{C_j^2 - C_s^2}} \\
\beta_y &= \frac{|C_y|}{\sqrt{C_s^2 + C_j^2}} \\
\beta_z &= \frac{|C_z|}{\sqrt{C_s^2 + C_j^2}}
\end{align*}
\]

From these set of equations, the fluxes are obtained for updating the difference equations. The numerical solution of an initial value problem for hyperbolic system of conservation laws is a weak solution if and only if

\[ w_t + f(w)_x = 0, \quad w(x, 0) = w_0(x), \quad -\infty < x < \infty \]  
(2.44)

the following conditions are met, i.e.,

(1) \( w \) satisfies \( w_t + f(w)_x = 0 \) pointwise in the smooth regions
(2) Across each discontinuity the Rankine-Hugoniot condition

\[ f(w_r) - f(w_l) = S(w_r - w_l) \]

have to be satisfied. \( S \) is the speed of propagation of the discontinuity, and \( w_r \) and \( w_l \) are the states to the right and left of a discontinuity (entropy conditions).

The weak solutions to Equation 2.44 are not uniquely determined by the initial data and, as such, an additional condition is required to admit the relevant solution. A method for determining the appropriate weak solution is to examine the equivalent viscous equations

\[ u_t + f(u)_x = \epsilon u_{xx}, \quad \epsilon > 0 \]  

(2.45)

and consider only those weak solutions that can be constructed from the viscous equation in the limit \( \epsilon \to 0 \). The question of whether the Riemann solver used satisfies the entropy condition is not answered with a formal proof. Rather, the method chosen here is to compare the solutions of the Riemann solver to that of a first order approximate Riemann solver known to satisfy the entropy inequalities.

The first order solver used for comparison purposes is the Harten, Lax, van Leer and Einfeldt (HLLE) solver (Harten et al. [1983], Rider [1994])

\[ F_{LR} = \frac{C_{LR}F(U_L) + C_{LR}F(U_R)}{C_{LR} + C_{LR}} - \frac{C_{LR}C_{LR}}{C_{LR} + C_{LR}}(U_R - U_L) \]  

(2.46)

where \( C_{LR} \) is the largest signal speed at the interface.
2.3.4 Solution Step

Having solved the Riemann problem and the fluxes calculated, the difference equations are updated as follows:

\[
x_{i+1/2}^{n+1} = x_{i+1/2}^{n} + \Delta t \bar{u}_{x}
\]

\[
V_{i}^{n+1} = V_{i}^{n} + \frac{x_{i+1/2}^{n+1} - x_{i-1/2}^{n+1}}{\Delta m_{i}}
\]

\[
u_{x_{i}}^{n+1} = u_{x_{i}}^{n} + \frac{\Delta t}{\Delta m_{i}} (\bar{P}_{i-1/2} - \bar{P}_{i+1/2})
\]

\[
u_{y_{i}}^{n+1} = u_{y_{i}}^{n} - \frac{\Delta t}{\Delta m_{i}} (\bar{\Lambda}_{y_{i-1/2}} - \bar{\Lambda}_{y_{i+1/2}})
\]

\[
u_{z_{i}}^{n+1} = u_{z_{i}}^{n} - \frac{\Delta t}{\Delta m_{i}} (\bar{\Lambda}_{z_{i-1/2}} - \bar{\Lambda}_{z_{i+1/2}})
\]

\[
V B_{y_{i}}^{n+1} = V B_{y_{i}}^{n} + \frac{\Delta t}{\Delta m_{i}} (B_{y_{i-1/2}}^{n} - B_{y_{i+1/2}}^{n})
\]

\[
V B_{z_{i}}^{n+1} = V B_{z_{i}}^{n} + \frac{\Delta t}{\Delta m_{i}} (B_{z_{i-1/2}}^{n} - B_{z_{i+1/2}}^{n})
\]

\[
E_{i}^{n+1} = E_{i}^{n} - \frac{\Delta t}{\Delta m_{i}} ((P u_{x})_{i-1/2}^{n} - (P u_{x})_{i+1/2}^{n} + (\Lambda_{y} u_{y})_{i-1/2}^{n} - (\Lambda_{y} u_{y})_{i+1/2}^{n} +
\]

\[
(\Lambda_{z} u_{z})_{i-1/2}^{n} - (\Lambda_{z} u_{z})_{i+1/2}^{n}
\]

\[\text{(2.47)}\]

2.3.5 Remap Step

After updating the solutions on the lagrangian grid, a mapping is made of the conserved quantities, \(\rho, u_{x}, u_{y}, u_{z}, B_{y}, B_{z},\) and \(E\) onto an Eulerian grid. First, cubic
polynomials are used to interpolate $\rho$, $u_x$, $u_y$, $u_z$, $B_y$, $B_z$, and $p$ to find the interface values at each computational zone, after which the monotonicity constraint is applied to these values. In addition to the above, interface values for the internal, kinetic and magnetic energy are also calculated, and they will be used to update the energy.

\[
(E_{\text{int}})_{i,L} = (\rho e)_{i,L} = \frac{1}{\gamma - 1} (p)_{i,L}
\]

\[
(E_{\text{ke}})_{i,L} = \frac{1}{2} ((u_x)^2)_{i,L} + (u_y)^2_{i,L} + (u_z)^2_{i,L})
\]

\[
(E_{\text{mag}})_{i,L} = \frac{1}{8\pi} ((B_x)^2)_{i,L} + (B_y)^2_{i,L} + (B_z)^2_{i,L})
\]

The monotonicity constraint need not be applied to these values since they are already monotonic. A parabola is next defined using the zone average value and the interface values to describe the above variables inside each zone. Two cases need to be considered for the remapping, see Figure 2.8. The amount of mass flux
due to the change in the Lagrangian grid is given by

\[ \delta m_{i+1/2} = \int_{x_{i+1/2}^n}^{x_{i+1/2}^{n+1}} \rho(x) dx \]  

(2.48)

The integral is evaluated using the parabola describing \( \rho \) in the zones and are as follows If \( \delta x_{i+1/2} \equiv x_{i+1/2}^{n+1} - x_{i+1/2}^n > 0 \) then

\[ \delta m_{i+1/2} = \delta x_{i+1/2}(\rho_{Ri}^{n+1} - \frac{\delta x_{i+1/2}}{\Delta x_{i+1}^{n+1}}(\frac{1}{2}\Delta \rho_{i+1}^{n+1} - \rho_{6i}^{n+1}(\frac{1}{2} - \frac{\delta x_{i+1/2}}{3\Delta x_{i+1}^{n+1}}))) \]

If \( \delta x_{i+1/2} < 0 \) then

\[ \delta m_{i+1/2} = \delta x_{i+1/2}(\rho_{Li}^{n+1} + \frac{\delta x_{i+1/2}}{\Delta x_{i+1}^{n+1}}(\frac{1}{2}\Delta \rho_{i+1}^{n+1} + \rho_{6i+1}^{n+1}(\frac{1}{2} - \frac{\delta x_{i+1/2}}{3\Delta x_{i+1}^{n+1}}))) \]

The momentum flux for each component \( \delta P_{x,y,z} \) is calculated here and is given by

\[ \delta(P_{x,y,z})_{i+1/2} = \int_{x_{i+1/2}^n}^{x_{i+1/2}^{n+1}} (U_{x,y,z}) dm \]

for the case when \( \delta x_{i+1/2} > 0 \)

\[ \delta(P_{x,y,z})_{i+1/2} = \delta m_{i+1/2}((U_{x,y,z})_{Ri}^{n+1} - \frac{\delta m_{i+1/2}}{\Delta m_{i}^{n+1}}(\frac{1}{2}(\Delta U_{x,y,z})_{i}^{n+1} - (U_{x,y,z})_{6i}^{n+1}(\frac{1}{2} - \frac{\delta m_{i+1/2}}{3\Delta m_{i}^{n+1}}))) \]

and if \( \delta x_{i+1/2} < 0 \) then

\[ \delta(P_{x,y,z})_{i+1/2} = \delta m_{i+1/2}((U_{x,y,z})_{Li}^{n+1} + \frac{\delta m_{i+1/2}}{\Delta m_{i}^{n+1}}(\frac{1}{2}(\Delta U_{x,y,z})_{i+1}^{n+1} + (U_{x,y,z})_{6i+1}^{n+1}(\frac{1}{2} - \frac{\delta m_{i+1/2}}{3\Delta m_{i+1}^{n}}))) \]

Only the transverse components of the magnetic flux are calculated, for remap, keeping in mind that \( B_x \) is treated as a constant in 1-D sweeps;

\[ \delta(\Phi_{y,z})_{i+1/2} = \int_{x_{i+1/2}^n}^{x_{i+1/2}^{n+1}} B_{y,z}(x) dx \]
for the case $\delta x_{i+1/2} > 0$

$$\delta(\Phi_{y,z})_{i+1/2} = \delta x_{i+1/2}((B_{y,z})_{Ri}^{n+1} - \frac{\delta x_{i+1/2}}{\Delta x_i^{n+1}}(\frac{1}{2}(\Delta B_{y,z})_{i}^{n+1} - (B_{y,z})_{6i}^{n+1}(\frac{1}{2} - \frac{\delta x_{i+1/2}}{3\Delta x_i^n}))$$

and if $\delta x_{i+1/2} < 0$ then

$$\delta(\Phi_{y,z})_{i+1/2} = \delta x_{i+1/2}((B_{y,z})_{Li}^{n+1} + \frac{\delta x_{i+1/2}}{\Delta x_i^{n+1}}(\frac{1}{2}(\Delta B_{y,z})_{i+1}^{n+1} + (B_{y,z})_{6i+1}^{n+1}(\frac{1}{2} - \frac{\delta x_{i+1/2}}{3\Delta x_i^{n+1}})))$$

Next the kinetic, internal and magnetic energies to be re-mapped are calculated in the following manner; Kinetic Energy,

$$\delta(E_{ke})_{i+1/2} = \int_{x_i^{n+1/2}}^{x_{i+1/2}^{n+1}} E_{ke} dm$$

for the case $\delta x_{i+1/2} > 0$

$$\delta(E_{ke})_{i+1/2} = \delta m_{i+1/2}((E_{ke})_{Ri}^{n+1} - \frac{\delta m_{i+1/2}}{\Delta m_i^{n+1}}(\frac{1}{2}(\Delta E_{ke})_i^{n+1} - (E_{ke})_{6i}^{n+1}(\frac{1}{2} - \frac{\delta m_{i+1/2}}{3\Delta m_i^n}))$$

If $\delta x_{i+1/2} < 0$ then

$$\delta(E_{ke})_{i+1/2} = \delta m_{i+1/2}((E_{ke})_{Li}^{n+1} + \frac{\delta m_{i+1/2}}{\Delta m_i^{n+1}}(\frac{1}{2}(\Delta E_{ke})_{i+1}^{n+1} + (E_{ke})_{6i+1}^{n+1}(\frac{1}{2} - \frac{\delta m_{i+1/2}}{3\Delta m_i^{n+1}})))$$

Internal Energy,

$$\delta(E_{int})_{i+1/2} = \int_{x_i^{n+1/2}}^{x_{i+1/2}^{n+1}} E_{int}(x) dx$$

for the case $\delta x_{i+1/2} > 0$

$$\delta(E_{int})_{i+1/2} = \delta x_{i+1/2}((E_{int})_{Ri}^{n+1} - \frac{\delta x_{i+1/2}}{\Delta x_i^{n+1}}(\frac{1}{2}(\Delta(E_{int})_i^{n+1} - (E_{int})_{6i}^{n+1}(\frac{1}{2} - \frac{\delta x_{i+1/2}}{3\Delta x_i^n}))$$

and if $\delta x_{i+1/2} < 0$ then

$$\delta(E_{int})_{i+1/2} = \delta x_{i+1/2}((E_{int})_{Li}^{n+1} + \frac{\delta x_{i+1/2}}{\Delta x_i^{n+1}}(\frac{1}{2}(\Delta(E_{int})_{i+1}^{n+1} + (E_{int})_{6i+1}^{n+1}(\frac{1}{2} - \frac{\delta x_{i+1/2}}{3\Delta x_i^{n+1}})))$$
Magnetic Energy

$$\delta(E_{mag})_{i+1/2} = \int_{x_i^{n+1}}^{x_{i+1}^{n+1}} E_{mag}(x) dx$$

for the case $\delta x_{i+1/2} > 0$

$$\delta(E_{mag})_{i+1/2} = \delta x_{i+1/2} ((E_{mag})_{Ri}^{n+1} - \frac{\delta x_{i+1/2}}{\Delta x_{i+1}^{n+1}} \left( \frac{1}{2} \Delta (E_{mag})_{i+1/2}^{n+1} - (E_{mag})_{6i}^{n+1} \left( \frac{1}{2} - \frac{\delta x_{i+1/2}}{3\Delta x_{i}^{n}} \right) \right))$$

and if $\delta x_{i+1/2} < 0$ then

$$\delta(E_{mag})_{i+1/2} = \delta x_{i+1/2} ((E_{mag})_{Li}^{n+1} + \frac{\delta x_{i+1/2}}{\Delta x_{i+1}^{n+1}} \left( \frac{1}{2} \Delta (E_{mag})_{i+1/2}^{n+1} + (E_{mag})_{6i+1}^{n+1} \left( \frac{1}{2} - \frac{\delta x_{i+1/2}}{3\Delta x_{i+1}^{n}} \right) \right))$$

Next update $\Delta m$ for the Eulerian Grid and the conserved quantities, i.e.

Density:

$$\Delta m_{i}^{E.G.} = \Delta m_{i}^{n} + \delta m_{i-1/2} - \delta m_{i+1/2}$$

$$<\rho>_{i}^{E.G.} = \frac{\Delta m_{i}^{E.G.}}{\Delta x_{i}^{n}}$$

Velocity components:

$$<(U_{x,y,z})>_{i}^{E.G.} = (<U_{x,y,z})_{i}^{n+1} \Delta m_{i}^{n} + \delta(P_{x,y,z})_{i-1/2} - \delta(P_{x,y,z})_{i+1/2})/\Delta m_{i}^{E.G.}$$

Magnetic components y,z:

$$\Delta (B_{y,z})_{i}^{E.G.} = (B_{y,z})_{i}^{n} \Delta x_{i}^{n} + \delta(\Phi_{y,z})_{i-1/2} - \delta(\Phi_{y,z})_{i+1/2}$$

$$<B_{y,z}>_{i}^{E.G.} = \frac{\Delta (B_{y,z})_{i}^{E.G.}}{\Delta x_{i}^{n}}$$

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Energy:

The energy flux contribution from the kinetic, internal and magnetic terms are

\[ \delta E = \delta(E_{\text{int}})_{i-1/2} - \delta(E_{\text{int}})_{i+1/2} + \delta(E_{\text{ke}})_{i-1/2} - \delta(E_{\text{ke}})_{i+1/2} + \]

\[ \delta(\Emag)_{i-1/2} - \delta(\Emag)_{i+1/2} \]

\[ < E >_{i}^{E,G.} = (< E >_{i}^{n+1} \Delta m_{i}^{n} + \delta E) / \Delta m_{i}^{E,G.} \]  

(2.49)

The thermal pressure for each Eulerian Grid is given by

\[ < p >_{i}^{E,G.} = (\gamma - 1) < \rho >_{i}^{E,G.} (< E >_{i}^{E,G.} - \frac{1}{2} (< U >_{i}^{E,G.})^{2} - \frac{1}{8\pi} < \rho >_{i}^{E,G.} (< B >_{i}^{E,G.})^{2} \]

The stability condition for this scheme is given by

\[ \Delta t_{cfl} = \min_{i}(\frac{\Delta x_{i}^{n}}{C_{f_{i}} + | < U >_{i}^{n} |}) \]

2.3.6 Imposing the Divergence-Free Condition for the Magnetic Field

The divergence-free condition for the magnetic field is a direct result of the absence of magnetic charges that would give rise to a magnetic field. Therefore the divergence free condition is a constraint that is imposed on the system even as it evolves. For the present numerical scheme, this condition is imposed by following the method introduced by Brackbill and Barnes [1980]. The method is as follows:
\[ B' = B + \nabla \Phi \]

where \( B \) is the solution from the numerical scheme for each time step. One corrects for this field by adding to it the gradient of a potential \( \Phi \) such that

\[ \nabla \cdot B' = 0 \]

and

\[ \nabla \cdot B + \nabla^2 \Phi = 0 \quad (2.50) \]

The difference equation for the source term \( \nabla \cdot B \) at the node \((i,j)\) with a truncation error of \( O(h^2) \) is

\[ \nabla \cdot B \approx \frac{B_{i+1,j} - B_{i-1,j}}{2\Delta X} + \frac{B_{i,j+1} - B_{i,j-1}}{2\Delta Y} \]

Figure 2.9: Grid representation for the Poisson Solver

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and the five point finite difference representation for Laplacian operator acting on $\Phi$ is

$$\nabla^2 \Phi \approx \frac{\Phi_{i+1,j} + \Phi_{i-1,j} - 2\Phi_{i,j}}{\Delta X^2} + \frac{\Phi_{i,j+1} + \Phi_{i,j-1} - 2\Phi_{i,j}}{\Delta Y^2}$$

Figure 2.9 gives the grid geometry for $B$ and $\Phi$ from which difference equations are formed for the Poisson equation, and $A_{i,j}$ is a dummy variable. To complete the set of equations to determine $\Phi$ we specify the boundary conditions for $\Phi$. For the problems solved, the Dirichlet boundary conditions are applied, i.e., $\Phi = 0$. The assumption here is that the domain for the simulation is large enough that the field $B$ is constant at the boundaries. If we define a vector

$$\Psi^T = [\Phi_{1,1}; \Phi_{1,2}; \ldots; \Phi_{1,M-1}; \ldots; \Phi_{M-1,1}; \ldots; \Phi_{M-1,M-1}]^T$$

where the components of the vector are the scalar potential $\Phi$ at the nodes or cell centers arranged row by row. The Poisson equation describing $\Phi$ can be written in matrix form as

$$\overline{A} \overline{\Psi} = \overline{b}$$

with $\overline{A}$ containing the coefficients of $\Phi$ and $\overline{b} = -\nabla \cdot B$ are the source terms at each nodes. The solution $\Psi$ are obtained by an iterative method called the Conjugate Residual method [21]. We define the residual

$$\overline{\mathcal{R}}(\overline{\Psi}) = \overline{A} \overline{\Psi} - \overline{b} = \overline{0} \quad (2.51)$$
where \( \overline{p} \) is the solution we wish to find. The steps for obtaining the solution are as follows. Step 1: Calculate the residual \( \overline{r}_v \) for a guessed solution \( \overline{p}_v \)

\[
\overline{r}_v = \overline{b} - \overline{A} \overline{p}_v
\]

Step 2: Calculate \( \overline{P} \overline{r}_v = \overline{q}_v \), which is a guess of the solution to \( \overline{A} (\overline{p}_v + \overline{q}_v) = \overline{b} \). The matrix \( \overline{P} \) referred to as the preconditioning matrix is as close an approximation to the inverse of the matrix \( \overline{A} \). The preconditioning matrix \( \overline{P} \) used in this simulation is the Jacobi preconditioning matrix given by

\[
P_{ij} = \begin{cases} 
1/a_{ij} & \text{if } i = j \\
0 & \text{if } i \neq j 
\end{cases}
\]

where \( a_{ij} \) are the components of the matrix \( \overline{A} \). Step 3: Orthogonalize \( \overline{q}_v \) with respect to the previous solution vector \( \overline{q}_{v-1} \). This method for orthogonalizing the vector is known as the Gram-Schmidt orthogonalization process.

\[
\overline{q}_v = \overline{q}_v - \lambda_v \overline{q}_{v-1}
\]

where

\[
\lambda_v = \frac{[\overline{q}_v, \overline{q}_{v-1}]}{[\overline{q}_{v-1}, \overline{q}_{v-1}]}
\]

with \([X_1, X_2]\) is the inner product for the two vectors \( X_1 \) and \( X_2 \). The vector \( \overline{q}_v \) is the unit vector for the new, guessed solutions is to be constructed in. Step 4: To find the components \( \delta p_{v+1} \) of \( p - \overline{p}_v \) in the direction \( \overline{q}_v \). Note that \( p \) is the exact solution.
The new guess for the exact solution is $\bar{p}_{v+1} = \bar{p}_v + \delta \bar{p}_{v+1}$. To begin the iterative process for $v = 0$, we take $\bar{p}_0 = \bar{0}$. For the first time through this process Step 3 is bypassed and set $\bar{q}_0 = \bar{q}_0$. To evaluate $\lambda_v$ and $\alpha_v$ the inner product needs to be evaluated. By definition, the inner product is

$$[X_1, X_2] \equiv (\bar{A} \bar{P} \bar{A} X_1, X_2)$$

where $(\cdot)$ is the usual inner product for two vectors: that is $(a_1, b_2) = \sum_{i=1}^{N}(\bar{a}_i)(\bar{b}_i)$.

With the above definition for the inner product, $\lambda_v$ and $\alpha_v$ are

$$\lambda_v = \frac{(\bar{A} \bar{q}_v, \bar{P} \bar{A} \bar{q}_{v-1})}{(\bar{A} \bar{q}_{v-1}, \bar{P} \bar{A} \bar{q}_{v-1})}$$

and

$$\alpha_v = \frac{(\bar{A} (\bar{p} - \bar{p}_v), \bar{P} \bar{A} \bar{q}_v)}{(\bar{A} \bar{q}_v, \bar{P} \bar{A} \bar{q}_v)}$$

From the above expressions, expressions for $\bar{A} \bar{q}_v$, $\bar{A} \bar{q}_v$ and $\bar{A} (\bar{p}_v - \bar{p}_v)$. First we note that

$$\bar{r}(\bar{p}_v) = \bar{b} - \bar{A} \bar{p}_v$$

$$\bar{r}(\bar{p}_v + \bar{q}_v) = \bar{b} - \bar{A} (\bar{p}_v + \bar{q}_v)$$
Subtracting the two equations gives an expression for evaluating $\vec{A} \vec{q}_v$, i.e., evaluating residuals:

$$\vec{A} \vec{q}_v = \vec{r}(\vec{p}_v) - \vec{r}(\vec{p}_v + \vec{q}_v)$$

From Step 3 and the linearity of the matrix $\vec{A}$ we can evaluate, an obtain an expression for $\vec{A} \vec{q}_v$:

$$\vec{A} \vec{q}_v = \vec{A} \vec{q}_v - \lambda_v \vec{A} \vec{q}_{v-1}$$

Finally for $\vec{A} (\vec{p}_v - \vec{p}_u)$ we first note that $\vec{p}_v = \vec{p}_{v-1} + \delta \vec{p}_v$ such that

$$\vec{r}(\vec{p}_v) = \vec{A} (\vec{p} - \vec{p}_u)$$

$$\vec{r}(\vec{p}_v) = \vec{A} (\vec{p} - \vec{p}_{v-1} - \delta \vec{p}_v)$$

$$\vec{r}(\vec{p}_v) = \vec{r}(\vec{p}_{v-1}) - \alpha_{v-1} \vec{A} \vec{q}_{v-1}$$

From the above expressions, data is stored from the previous iteration steps for $\vec{A} \vec{q}_{v-1}$, $\alpha_{v-1}$ and $\vec{r}(\vec{p}_v)$ and only the vector $\vec{r}(\vec{p}_v + \vec{q}_v)$ needs to be evaluated for each iteration. The convergence criteria used to judge the solution vector $\vec{p}_v$ is for each node or cell of the residual vector $\vec{r}_v$ is $\leq 10^{-8}$.

### 2.4 Test of the Numerical Code

The numerical code has been applied to a number of problems in MHD involving shock interactions. Three problems are discussed in the 1-D case and 2 simulations for the two dimensional problems. For the 1-D case, the numerical solutions are
compared against the solutions from the nonlinear Riemann solver (see Section 2.2.3). Since \( \int_{-\infty}^{\infty} U dm \) is a constant with respect to time, one could use this to determine if the numerical solutions are correct. This however was not done for the simulations, but instead is compared against published results. The simulations results presented below used uniform Cartesian grids for the computational mesh.

### 2.4.1 1-D Problem

The first problem *Dai and Woodward* [1994] in the 1-D case, the code has been applied to uses 200 computational zones and generates two fast shocks with Mach number 25.5. The initial conditions for this problem is \((\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (1, 1, 36.87, -0.155, -0.0386, 4, 4, 1)\) for \(x < 0.5\) and \((\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (1, 1, -36.87, 0, 0, 4, 4, 1)\) for \(x > 0.5\), with \(\gamma = 5/3\). The structure at \(x = 0.5\) is due to the discontinuity of the initial conditions used. The solutions from the nonlinear Riemann solver are; Figure 2.10 shows the results of the simulation for \(t = 0.03\). The profile at \(x = 0.5\) is due to the pure discontinuity in the initial conditions of the problem. Figure 2.11 shows this same simulation but using the HLLE first order Riemann solver.

<table>
<thead>
<tr>
<th>Region</th>
<th>(\rho)</th>
<th>(p)</th>
<th>(U_x)</th>
<th>(U_y)</th>
<th>(U_z)</th>
<th>(B_y)</th>
<th>(B_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>1</td>
<td>1</td>
<td>36.87</td>
<td>-0.155</td>
<td>-0.0386</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>R2</td>
<td>3.98</td>
<td>1800</td>
<td>0</td>
<td>-0.08</td>
<td>-0.02</td>
<td>16.0</td>
<td>4.0</td>
</tr>
<tr>
<td>R8</td>
<td>1</td>
<td>1</td>
<td>-36.87</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>
The second one-dimensional problem chosen, the nonlinear Riemann solver fails to find a solution for the given initial conditions. The left state \((x < 0.5)\) for the 1-D problem is \((1, 20, 10, 0, 0, 5, 5, 0)\) and \((\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (1, 2, -10, 0, 0, 5, 5, 0)\) for \(x > 0.5\), with \(\gamma = 5/3\). The solutions for the eight regions from the nonlinear Riemann solver with appropriate initial guess, are as follows.

<table>
<thead>
<tr>
<th>Region</th>
<th>(\rho)</th>
<th>(p)</th>
<th>(U_x)</th>
<th>(U_y)</th>
<th>(U_z)</th>
<th>(B_x)</th>
<th>(B_y)</th>
<th>(B_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>20</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>2.7</td>
<td>151</td>
<td>0.7</td>
<td>0.2</td>
<td>0</td>
<td>13.6</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>R3</td>
<td>2.7</td>
<td>151</td>
<td>0.7</td>
<td>0.2</td>
<td>0</td>
<td>13.6</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>R4</td>
<td>2.7</td>
<td>150</td>
<td>0.7</td>
<td>0.4</td>
<td>0</td>
<td>14.3</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>R5</td>
<td>3.9</td>
<td>150</td>
<td>0.7</td>
<td>0.4</td>
<td>0</td>
<td>14.3</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>R6</td>
<td>3.7</td>
<td>143</td>
<td>0.7</td>
<td>-0.4</td>
<td>0</td>
<td>19.2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>R7</td>
<td>3.7</td>
<td>143</td>
<td>0.7</td>
<td>-0.4</td>
<td>0</td>
<td>19.2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>R8</td>
<td>1</td>
<td>2</td>
<td>-10</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Numerical zones were used in this simulation and the results in Figure 2.12 are for \(t = 0.08\). The figure shows two fast shocks moving to the left and right, a contact discontinuity and a weak slow shock. The numerical solution using the first order HLLE solver is shown in Figure 2.13.

The final 1-D problem is also a Riemann shock tube problem which generates seven discontinuities. The initial conditions are \((\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (0.18, 0.36, 3.89, 0.54, 2.48, 4, 2.39, 1.19)\) for \(x < 0.5\) and \((0.1, 0.1, -5.5, 0, 0, 4, 2, 1)\) for \(x > 0.5\). The profiles in Figure 2.14 are at \(t = 0.15\), and this simulation used 400 zones with \(\gamma = 5/3\). The solutions for the eight regions from the nonlinear Riemann
solver are as follows. There are two fast and two slow shocks traveling leftward and rightward of the initial discontinuity. There is also a rotational discontinuity and two slow shocks. The location of the rotational discontinuity is best seen using both the transverse components and the magnitude of the field. For a rotational discontinuity, the field magnitude remains unchanged.

### 2.4.2 2-D Problem

For the 2-D case, the solutions are compared against published numerical results. The first two dimensional problem examined is a spherical explosion \textit{Zachary et al.} [1994] type problem. The explosion is driven by large overpressure. The initial conditions for the problem is a uniform density $\rho = 1$, velocities $U_x = U_y = U_z = 0$ throughout the region and the fluid has a pressure $P_e = 1$ surrounding a region ($r = 0.1$) having a large overpressure of $P_e = 100$. The simulation considered two different values for the uniform magnetic field, $B_x = B_z = 0$, $B_y = 10$ and $B_y = 100$ and are shown in Figures 2.15 and 2.16. The computational mesh uses $120 \times 120$
Figure 2.10: Shows two fast shocks with Mach Number 25.5. Initial conditions: \((\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (1, 1, 36.87, -0.155, -0.0386, 4, 4, 1)\) for \(x < 0.5\) and \((\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (1, 1, -36.87, 0, 0, 4, 4, 1)\) for \(x > 0.5\), with \(\gamma = 5/3\). The line plots are for \(t = 0.03\).
Figure 2.11: Shows two fast shocks with Mach Number 25.5. Initial conditions: \((\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (1, 1, 36.87, -0.155, -0.0386, 4, 4, 1)\) for \(x < 0.5\) and \((\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (1, 1, -36.87, 0, 0, 4, 4, 1)\) for \(x > 0.5\), with \(\gamma=5/3\). The line plots are for \(t = 0.03\).
Figure 2.12: This simulation uses 200 zones and the profile is for $t = 0.08$ Initial conditions: $(\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (1, 20, 10, 0, 0, 5, 5, 0)$ for $x < 0.5$ and $(\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (1, 2, -10, 0, 0, 5, 5, 0)$ for $x > 0.5$, with $\gamma=5/3$. 

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Figure 2.13: This simulation uses 200 zones and the profile is for $t = 0.08$ Initial conditions: \((\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (1, 20, 10, 0, 0, 5, 5, 0)\) for $x < 0.5$ and \((\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (1, 2, -10, 0, 0, 5, 5, 0)\) for $x > 0.5$, with $\gamma = 5/3$. 

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Figure 2.14: This simulation uses 200 zones and the profile is for $t = 0.08$ Initial conditions: $(\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (0.18, 0.36, 3.89, 0.54, 2.48, 4, 2.39, 1.19)$ for $x < 0.5$ and $(\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (0.1, 0.1, -5.5, 0, 0, 4, 2, 1)$ for $x > 0.5$, with $\gamma = 5/3$. 

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zones to cover a radius of $r = 1.0$.

Second Problem. The numerical code has also been applied to a 2-D problem ([Dai and Woodward [1994]]) using 200 X 200 zones and carried out on a square domain whose dimensionless length is 1 X 1. This is an interaction of a Fast MHD shock with a denser cloud. The initial conditions are for an initial shock of Mach 10, traveling towards a cloud that is five times denser than its surrounding. The initial conditions in the shocked state are $(\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (3.88, 14.26, 0, -0.05, 1, 0, 3.93)$, the surrounding unshocked plasma has initial conditions $(1, 0.04, -3.31, 1, 0, 1)$ and the denser cloud $(5, 0.04, -3, 31, 1, 0, 1)$. The radius of the cloud is 0.18. The simulation results show the presence of two fast shocks, one transmitted into the cloud and the other propagates upstream into the shocked gas. Contour plots for $t = 0.16$ are shown in the figures 2.17 - 2.24. The results show that the initial shock has wrapped around the cloud. Figure 2.25 is a plot of the field in the X - Y plane is also shown with the length of the arrow is proportional to the strength of the field.
Figure 2.15: This spherical explosion simulation used 120 X 120 zones and the profile is for 48 time steps. Initial conditions: $(\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (1,100,0,0,0,0,10,0)$ for overpressure region of radius $r = 0.1$ and $(\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (1.0, 1.0, 0, 0, 0, 0, 10, 0)$ for the surrounding plasma, with $\gamma = 5/3$. 

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Figure 2.16: This spherical explosion simulation used 120 X 120 zones and the profile is for 48 time steps. Initial conditions: \((\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (1, 100, 0, 0, 0, 0, 100, 0)\) for overpressure region of radius \(r = 0.1\) and \((\rho, p, u_x, u_y, u_z, B_x, B_y, B_z) = (1.0, 1.0, 0, 0, 0, 0, 100, 0)\) for the surrounding plasma, with \(\gamma = 5/3\).
Figure 2.17: The density contour plot for the shock cloud simulation used 200 X 200 zones and the profile shown is for $t = 0.16$. The initial conditions and the geometry are described in the text.
Figure 2.18: The thermal pressure contour plot for the shock cloud simulation used 200 X 200 zones and the profile shown is for $t = 0.16$. The initial conditions and the geometry are described in the text.
Figure 2.19: The contour plot of the x-component of the velocity for the shock cloud simulation used 200 X 200 zones and the profile shown is for $t = 0.16$. The initial conditions and the geometry are described in the text.
Figure 2.20: The contour plot of the y-component of the velocity for the shock cloud simulation used 200 X 200 zones and the profile shown is for t = 0.16. The initial conditions and the geometry are described in the text.
Figure 2.21: The contour plot of the $z$-component of the velocity for the shock cloud simulation used 200 X 200 zones and the profile shown is for $t = 0.16$. The initial conditions and the geometry are described in the text.
Figure 2.22: The contour plot of the x-component of the magnetic field for the shock cloud simulation used 200 X 200 zones and the profile shown is for $t = 0.16$. The initial conditions and the geometry are described in the text.
Figure 2.23: The contour plot of the y-component of the magnetic field for the shock cloud simulation used 200 X 200 zones and the profile shown is for $t = 0.16$. The initial conditions and the geometry are described in the text.
Figure 2.24: The contour plot of the z-component of the magnetic field for the shock cloud simulation used 200 X 200 zones and the profile shown is for t = 0.16. The initial conditions and the geometry are described in the text.
Figure 2.25: This is a plot of the magnetic field in the X-Y plane for the shock cloud simulation at $t = 0.16$ and used 200 X 200 zones. The initial conditions and the geometry are described in the text.
Chapter 3
Magnetic Clouds

3.1 Introduction

In this chapter, we describe some features of magnetic clouds and discuss both interplanetary and derived parameters useful in understanding magnetic clouds and their interaction with the Earth. Some of the interactions of a magnetic cloud with the Earth considered here deals with the clouds effect on the subsolar bow shock and magnetopause boundaries. We also look at the result of the transfer of energy from the solar wind (magnetic cloud) into the magnetosphere and the disturbances (or geomagnetic storms) generated as a result of this. Though magnetic clouds may look similar temporally, their effects on the Earth can be different.

3.2 What are Magnetic Clouds

For an interplanetary ejecta to be considered a magnetic cloud it must satisfy the following three properties: (1) there is a large and smooth rotation in space in the magnetic field lasting for about a day; (2) an enhanced magnetic field when compared to the average interplanetary values; and (3) lower proton temperatures then the average interplanetary or solar wind values. At 1 AU (distance from the Sun to the Earth) the width of these magnetic clouds are approximately 0.25 AU.
Burlaga et al. [1981]. Locally, the magnetic field configurations can be considered to be approximately force-free (see Goldstein [1983], Marubashi [1986]), that is, the cloud has a magnetic field configuration for which the Lorentz force vanishes.

\[ J \times B = 0 \]  
(3.1)

This implies that the current \( J \) is parallel to \( B \).

\[ J = \alpha B \]  
(3.2)

where \( \alpha \) can in general be a function of position. A subset of this more general case is the constant \( \alpha \) case studied by Burlaga et al. [1988]. For the case when \( \alpha \) is a constant, taking the cross product of Equation 3.2 gives

\[ \nabla \times J = \alpha \nabla \times B \]  
(3.3)

Since \( J = \nabla \times B \)

\[ \nabla \times (\nabla \times B) = \alpha J \]  
(3.4)

\[ -\nabla^2 B = \alpha^2 B \]  
(3.5)

The solutions for the case of a constant \( \alpha \) force-free cylindrically symmetric configuration are referred to as the Lundquist solutions Lundquist [1950]. The solutions to Equation 3.5 are written as

\[ B_\theta = B_f H J_1(\alpha \frac{r}{a_0}) \]

\[ B_z = B_f J_0(\alpha \frac{r}{a_0}) \]

\[ B_r = 0 \]
where $B_f$ is a constant to be determined (the field strength on the axis), $\alpha$ is the first zero of the zeroth order Bessel function $J_0$, and $a_0$ is the radius of the magnetic cloud. $J_1$ is the Bessel function of order 1. $r$ is the distance from the axis of the tube and $H = \pm 1$ is the helicity of the magnetic field. $B_z$ is component of the magnetic field along the axis of the magnetic cloud, $B_\theta$ is the component of the magnetic field in the azimuthal direction, and $B_r$ is the radial component. Plots of these field configurations are shown in Chapter 5. Locally, a reasonably good approximation to the magnetic field of a magnetic cloud can be made using the force-free constant alpha cylindrically symmetric Lundquist solutions. By fitting the magnetic field observations of a magnetic cloud to these solutions one could determine locally the axis of the cloud. By making use of multiple spacecraft observations of a magnetic cloud observed in January 1978, Burlaga et al. [1990] were able to determine the radius of curvature of the axis of the magnetic cloud by using the above idea, and obtained the radius of curvature of the axis for the January 1978 magnetic cloud as $1/3$ AU. From this, Burlaga et al. concluded that not only are they huge but they were also bent flux tubes (see Figure 3.1 reproduced from Burlaga et al. [1990]). The authors could not, however, determine if the cloud magnetic field lines were connected to the surface of the sun using this method. A discussion on the large scale structure of magnetic clouds can be found in Burlaga [1995]. If the magnetic field lines for a magnetic cloud were connected to the sun, Kahler and Reames [1991] argued that solar energetic particles from a specific solar event.
Figure 3.1: Shows a sketch reproduced from Burlaga et al. [1990] of a magnetic cloud at 1 AU.
would be detected by a satellite in a magnetic cloud that had been created earlier then the solar event itself. This would indicate that these solar particles would have traveled along the clouds magnetic field lines from their origin on the sun to where they were detected in the magnetic cloud by a satellite. This would prove that at least one end of the magnetic field lines of the cloud was connected to the sun. A satellite passing through the magnetic cloud would detect a unidirectional flow of energetic particles for the above case. If the other end of the cloud magnetic field lines were also connected to the sun, then bidirectional streaming of particles would be detected. The bidirectional streaming would be the result of the solar energetic particles being reflected from mirror points located near the sun. Such observations have been made by Kahler and Reames [1991], Farrugia et al. [1993b], Richardson and Cane [1995] and others from which it has been concluded that the ends of a magnetic cloud are connected to the sun. Sometimes shock waves are observed being driven by magnetic clouds. If the speed of the front shock boundary relative to the ambient plasma is greater then the magnetosonic speed for the ambient plasma, then a shock will be driven by the magnetic cloud (Burlaga [1995]). Due to the high magnetic field found inside a magnetic cloud the magnetic pressure \(B^2/8\pi\) is higher then the ambient plasma pressure surrounding the cloud which will cause the cloud to expand as it propagates. Models for expanding clouds will not be considered for this dissertation, but material on this subject can be found in Burlaga [1995], Farrugia et al. [1992, 1993]; Osherovich et al. [1993].
[1993a,b. 1995]; Vandas et al. [1995, 1996]; Kumar and Rust [1996]; and Cargill et al. [1996]. Combining all of the observed features, a model for the magnetic cloud would be a bent flux tube with its end connected to the sun with the magnetic field configuration satisfying the static constant $\alpha$ force free Lundquist solutions. Sources for magnetic clouds are a solar event referred to as coronal mass ejections which are due to disappearing filaments, and Burlaga et al. [1982] was able to show evidence linking magnetic clouds with coronal mass ejections. However, it is not possible to determine which coronal mass ejection would result in a magnetic cloud in the solar wind. It has been approximated that 1/3 of solar ejections end up as magnetic clouds in the solar wind Gosling [1990].

3.3 Solar Wind Magnetosphere Coupling

Only a qualitative discussion is presented here on the energy transfer from the solar wind into the magnetosphere. Figure 3.2 shows a sketch reproduced from Cowley [1980] of an open magnetosphere in the noon-midnight meridian plane for a southward interplanetary magnetic field (IMF). In the figure the magnetic field are shown as solid lines, the bow shock and magnetopause boundaries are represented as long dashed lines. Also shown is the direction of the Poynting vector ($\mathbf{E} \times \mathbf{B}$ drift) which are represented as short dashed lines. The currents that are directed out of the plane are represented as circled dots and for these current flows there is a transfer of energy ($\mathbf{J} \cdot \mathbf{E} > 0$) from the magnetic field to the plasma. The
circle crosses indicate the current is flowing into the plane and the energy transfer 
\((\mathbf{J} \cdot \mathbf{E} < 0)\) will be from the plasma to the magnetic field. For a southward IMF 
\((B_z < 0)\), the magnetic field lines of the solar wind will connect with the magnetic 
field lines of the Earth at the magnetopause boundary. This connection allows 
for the flow of plasma from the solar wind and the addition of magnetic field lines 
to the magnetotail. For a continuous period when the solar wind \(B_z < 0\) there 
will be an increase in the magnetic energy and also a flow of solar wind plasma 
into the magnetotail \((\mathbf{J} \cdot \mathbf{E} < 0)\). The magnetic field lines in the magnetotail will 
be pushed together by forces from the tail lobes towards the tail center while they 
are being stretched. The stretching of these magnetic field lines is the result of 
the frozen-in flux condition, that is, the field lines of the solar wind are frozen into 
the plasma flow. As the solar wind flows downstream of the Earth the fields lines 
that are connected to the Earth will get stretched. This results in the reduction 
of the plasmasheet thickness. As the plasmasheet thickness decreases, an X-type 
neutral line is formed in the tail current sheet and a merging of the field lines will 
take place. One portion of the merged field lines will be ejected anti-sunward into 
the solar wind, while the other will be accelerated sunward or towards the Earth 
\((\mathbf{J} \cdot \mathbf{E} > 0)\). This is also referred to as a geomagnetic storm onset. An analogy of 
this process would be the release of a stretched rubber band. A parameter used to 
measure the transfer of solar wind energy into the magnetosphere is the Akasofu 
\(\epsilon\) parameter defined as the integrated Poynting flux and is given Perreault et al.
Figure 3.2: Shows a sketch reproduced from Cowley [1980] of an open magnetosphere in the noon-midnight meridian plane for a southward interplanetary magnetic field (IMF). The magnetic field are shown as solid lines. The bow shock and magnetopause boundaries are represented as long dashed lines. Also shown is the direction of the Poynting vector ($\mathbf{E} \times \mathbf{B}$ drift) shown as short dashed lines. The currents that are directed out of the plane are represented as circled dots and for these current flows the energy ($\mathbf{J} \cdot \mathbf{E} > 0$) is transfered from the electromagnetic field to the plasma. The circled crosses indicate the current is flowing into the plane and the energy ($\mathbf{J} \cdot \mathbf{E} < 0$) is transfered from the plasma to the electromagnetic field.
Figure 3.3: A sketch reproduced from Baker et al. [1979] showing the sequence of events associated with a substorm onset and recovery.
where $V_p$ is the bulk flow of the plasma, $B$ is the total magnetic field, $\psi$ is the clock angle defined below, and $l_0$ is an empirical constant equal to $7$ $R_E$.

The energy that was stored in the magnetic field in the magnetotail before the X-type merging will be transferred to the plasma particles ($\mathbf{J} \cdot \mathbf{E} > 0$) in the tail region after the merging process and will appear as the buildup of the plasma sheet, auroral particles, ring current and Joule heating of the ionosphere. Figure 3.3 is a sketch reproduced from Baker et al. [1979] showing a sequence of events illustrating a substorm onset and recovery. The bulk of this energy goes into the buildup of storm time ring current which are trapped magnetospheric particles located between 4-6 $R_E$ that drift as a result of magnetic field gradients, curvatures in the magnetic field and gyration orbit effects. The ions will drift in the direction from midnight to dusk while the electrons will drift from midnight to dawn. A measure of the daily mean values of the horizontal magnetic field at middle and low geomagnetic latitudes gives an indication if there is an enhancement of particles in the ring current due to geomagnetic storms Sugiura and Chapman [1960]. This measure is referred to as the $Dst$ and a decrease in this index would indicate that particles have been injected into the ring current. Corrections have to be made to the $Dst$ for contributions from the magnetopause currents and is written Burton et
al. [1975] as

\[ Dst^* = Dst - bP_{dyn}^{1/2} + c \]  

(3.7)

where \( b \) is a constant with a value of 0.2 nT, \( c \) is the quiet time solar wind dynamic pressure contribution and \( P_{dyn} \) is the solar wind dynamic pressure.

The classification of geomagnetic storms based on the terminology of Sugiura and Chapman [1960] are given in Table 3.1. For quiet days the \( Dst^* \) will vary from -20 nT to -10 nT. For a review on the correlation of magnetic clouds and geomagnetic storm activity see Farrugia et al.’s [1997a] work on this.

### Table 3.1: Geomagnetic storm terminology

<table>
<thead>
<tr>
<th>( Dst^* )</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>\leq -250 nT</td>
<td>great</td>
</tr>
<tr>
<td>-250 \leq Dst^* \leq -100 nT</td>
<td>major</td>
</tr>
<tr>
<td>-1000 \leq Dst^* \leq -50 nT</td>
<td>moderate</td>
</tr>
<tr>
<td>-50 \leq Dst^* \leq -30 nT</td>
<td>weak</td>
</tr>
</tbody>
</table>

3.4 **WIND Observations of Magnetic Clouds**

We present here three examples of magnetic clouds observed in the solar wind and some observations one can draw from the interplanetary data. The interplanetary data presented in this chapter come from the SWE (Ogilvie et al., [1995]) and MFI (Lepping et al., [1995]) instruments on the Global Geospace Mission spacecraft WIND. The SWE (Solar Wind Experiment) instrument provides the solar wind proton plasma data and the MFI (Magnetic Field Investigation) instrument
gives the magnetic field data of the solar wind. Figures 3.4-3.6 are plots of three interplanetary data sets at ~1.5 minute averages for October 18-20, 1995, for May 27-29, 1996 and for January 9-11, 1997. All these plots have the same format, and starting from top to bottom the panels show the proton number density \((cm^{-3})\), magnitude of the velocity for the protons \((kms^{-1})\), proton temperature \((K)\), the magnetic field components \(B_x, B_y, B_z\) in Geocentric Solar Magnetospheric (GSM) coordinates, total field \(B\) \((nT)\) and the clock angle. The clock angle defined as \(\tan^{-1}(By/Bz)\) is the polar angle measured from the GSM \(Z\) axis of the magnetic field projected onto the \(Y-Z\) GSM plane. In the GSM coordinate system the \(X\)-axis is pointing from the Earth to the Sun and the \(Y\)-axis is perpendicular to the Earth's dipole with the \(Z\)-axis is in the direction of the northern magnetic pole. The horizontal axis shows the Universal Time (UT) in hours starting at 00 UT for the first day of each plot. Figures 3.7-3.9 are plots of parameters derived from the interplanetary observations for the above three magnetic clouds. All these figures have the same format with the horizontal axis representing the time starting at 00 UT for the first day for the days plotted. The derived parameters plotted in the panels from top to bottom are: (1) dynamic pressure \(P_{dyn}\) (nPa)

\[
P_{dyn} = m_pN V_p^2
\]  
(3.8)
with $m_p$ being the proton mass, $N$ is the proton number density and $V_p$ is the bulk flow of the plasma; (2) the total pressure $P_t$ (nPa)

$$P_t = P_b + P_p$$  \hfill (3.9)

where $P_p$ (nPa) is the plasma thermal pressure and $P_b$ (nPa) is the magnetic pressure; (3) thermal pressure $P_p$

$$P_p = N k T$$  \hfill (3.10)

where $k$ is the Boltzmann constant and $T$ is the temperature; (4) the magnetic pressure $P_b$

$$P_b = B^2/(8\pi)$$  \hfill (3.11)

where $B$ is the total magnetic field; (5) the proton $\beta$ parameter

$$\beta = P_p/P_b$$  \hfill (3.12)

which is the ratio of the thermal pressure to the magnetic pressure; (6) Alfven Mach number $M_A$

$$M_A = V_p/V_A$$  \hfill (3.13)

where $V_A = B/(4\pi m_p N)^{1/2}$ is the Afven speed; and (7) the Akasofu $\epsilon$-parameter ($mW/m^2$) normalized with respect to square of $l_o = 7R_E$ which is a constant scale area factor.

When a fast stream overtakes a slower stream, Alfven waves are known to be generated in regions where they interact Belcher and Davis [1971], Gonzalez and
If we let $B_0$ and $V_0$ be the average magnetic field and bulk flow of the plasma and let the fluctuations due to a wave in the magnetic field and velocity be defined as $\Delta B = B - B_0$ and $\Delta V = V - V_0$ then the fluctuations due to an Alfven wave can be written \textit{Belcher and Davis} \,[1971]\ as

$$\Delta B = \pm (4\pi m_p(N + N_\alpha))^{1/2} \Theta \Delta V \quad (3.14)$$

where $N_\alpha$ is the \(\alpha\) particle number density and $\Theta$ is the pressure anisotropy factor defined as

$$\Theta = [1 - \frac{4\pi (P_\parallel - P_\perp)}{B_0^2}]^{-1/2} \quad (3.15)$$

where $P_\parallel$ and $P_\perp$ is the pressure parallel and perpendicular to $B_0$. And if there is a strong correlation between the components of the fluctuations for the magnetic field and the solar wind velocity, one could argue that Alfven waves are present for the time period considered. The negative sign in Equation 3.14 would indicate that the Alfven waves are propagating along the field away from the Sun if $B_x < 0$ and vice versa for a positive sign.

We also look at the effect that these magnetic clouds have on the Earth's magnetopause and bow shock boundaries. The magnetopause subsolar point $R_{mp}$ is known to vary as a function of the solar wind dynamic pressure \textit{Choe et al.} \,[1973].

$$R_{mp} = \left[ \frac{(f B_{eq})^2}{8\pi P_{st}} \right]^{1/6} \quad (3.16)$$

where $B_{eq} = 3200nT$ is the Earth's magnetic field strength at the Equator, $f=2.45$ is a measure of the dipole field compression and $P_{st}$ is the stagnation pressure.
at the subsolar magnetopause position. A correction is made to the the solar wind dynamic pressure to account for an average 4% $\alpha$-to-proton relative particle concentration in the solar wind in the calculation for the subsolar magnetopause position. Also taken into account is that the stagnation pressure at the subsolar point is 88% of the solar wind dynamic pressure.

$$P_{st} = 1.02P_{dyn} \quad (3.17)$$

Based on gasdynamic simulations, Spreiter et al. [1966] showed that an empirical linear relationship existed between the magnetosheath thickness $\Delta_{ms}$ and the density jump $X = \rho_{sw}/\rho_d$ across the bow shock where $\rho_{sw}$ is the solar wind density and $\rho_d$ is the density downstream of the shock:

$$\frac{\Delta_{ms}}{R_{mp}} = kX \quad (3.18)$$

where $R_{mp}$ is the magnetopause standoff distance and $k = 1.1$. Fairfield [1971] in his observations of the Earth's bow shock had noticed six bow shock positions that were further away from the average position of 14.6 $R_E$ which are attributed to low $M_A$. For example, IMP 4 on July 30 at 2320 UT observed the bow shock 17 $R_E$ beyond the Earth's average position and the solar wind plasma had an Alfven Mach number $M_A$ of 1.4 Fairfield [1971]. Cairns et al. [1995] observations of Earth's bow shock on 24-25 September 1987 for $M_A \sim 1 - 3$ also required an increase in the subsolar bow shock position to account for the observation. Using MHD simulations, Cairns et al. [1995] found that the above empirical relation
needed some adjustments for MHD flows and the relationship they found was

\[ \frac{R_{sh}}{R_{mp}} = 3.4X + 0.4 \quad (3.19) \]

where \( R_{sh} \) is the bow shock standoff distance. Using the above equation and the MHD jump conditions for \( X \), Cairns et al. [1994]

\[ X = \frac{1}{2(\gamma + 1)} \left( A + \sqrt{A^2 - 4(\gamma - 2)(\gamma + 1)M_A^2} \right) \quad (3.20) \]

where \( A = (\gamma - 1) + \gamma/M_A^2 + 2/M_s^2 \), \( M_s = v_p/c_s \) is the sonic Mach number, the Earth's bow shock standoff distance can be estimated during the cloud passage.

### 3.4.1 October 18-20, 1995

The WIND satellite during this period of observation was \( \sim 175 \) \( R_E \) upstream of the Earth. Looking at Figure 3.4 the magnetic cloud is located between the dashed lines at \( \sim 1900 \) UT on October 18, 1995 (front boundary) and \( \sim 4800 \) UT on October 19, 1995 (back boundary), cloud passage thus lasting \( \sim 29 \) hours. The dot-dashed line in the figure at \( \sim 3400 \) UT indicates the time the \( B_z \) component of the magnetic field turns truly positive (northward). We can see that the magnetic cloud identified above satisfies the 3 conditions required; (1) a large rotation in the magnetic field \( (B_z \text{ component}) \sim 180^\circ \) from \( \sim 19 \) nT at the front boundary of the cloud to \( \sim 21 \) nT at the back boundary of the cloud. The rotation is relatively smooth (although we noted the presence of RD's, see Chapter 4); (2) an enhanced total B field which is of order \( \sim 20 \) nT and having a flat profile; and (3) lower proton temperatures.
Figure 3.4: Proton plasma and magnetic field observations for the October 18-20, 1995 magnetic cloud. The vertical dashed lines from the left to right indicate the driven shock, front cloud boundary, $B_z$ transition from negative to positive and the back cloud boundary. The panels from top to bottom show the proton density, bulk flow speed, temperature, magnetic field components $B_x$, $B_y$, $B_z$, the total field $B$ and the clock angle.
compared to the surrounding ambient solar wind plasma. The $B_y$ magnetic field component for the magnetic cloud has a larger-than-average negative magnitude and the $B_x$ component of the magnetic field is slightly higher than the surrounding solar wind field data. The dashed line at 1042 UT in the figure indicates a shock seen propagating which is $\sim 8$ hours ahead of the front magnetic cloud boundary. The cloud as a whole is traveling at an average speed of $\sim 410 \, kms^{-1}$. The magnetic cloud is seen overtaking the ambient solar wind plasma upstream of the front cloud boundary while the cloud itself is being overtaken by a faster stream at its rear. The proton density in the cloud is rising as one travels from the front to the back boundary of the cloud. The proton density reaches its maximum value near hour 4600 UT close to the back boundary of the cloud. The front cloud boundary is a clear tangential discontinuity (Lepping et al. [1997]; Janoo et al. [1998]), while the back boundary is not as clearly defined but this boundary was chosen from the proton temperature profile, that is, when the temperature begins to rise from its low profile to the solar wind values. Typically, identification of magnetic cloud boundaries is not easy although, in this case, identification of the front boundary is unproblematic. This cloud is being overtaken by a faster stream and there is an interaction taking place between the cloud and the faster stream ($\sim 4700 \, UT$ to $\sim 5400 \, UT$). From the magnetic field data for this region there are a number of directional discontinuities observed and Lepping et al. [1997] have argued that this cloud-stream interaction cannot be understood by data from one satellite but
have put forward a number of “possibilities” to explain this interaction. From the $B_z$, $B$ and $V_p$ data, (Lepping et al. [1997]) inferred a shock-like structure seen in the magnetic cloud at \( \sim 4200 \) UT. Lepping et al. [1997] found that this feature is similar to a shock structure except for two serious problems: (1) The temperature was higher downstream of this “shock”, which contradicts the results from the MHD equations; (2) the shock speed determined from the data would imply that the solar wind speed would speed up as it passed through the “shock”, violating the entropy condition, that is, the entropy would decrease across the “shock”.

**Derived Parameters**

The derived parameters for the October 18-20, 1995 time period are shown in Figure 3.5. Since the velocity of this magnetic cloud is relatively steady, so any large scale variations in the dynamic pressure would correspond to large scale changes in the proton density. The rapid changes in the dynamic pressure at the driven shock boundary and at the front cloud boundary are related to sharp changes in the proton density. One other factor to note here is that coinciding with the rapid decrease in $P_{dyn}$ at the front cloud boundary there is the large and rapid change in the $B_z$ magnetic field component of the magnetic cloud (turning southward). $P_{dyn}$ in the cloud from the front cloud boundary to hour 3600 UT is lower then the average solar wind values (\( \sim 2-3 \) nPa) but then rises to values above average (\( \sim 10 \) nPa) with a maximum near hour 4400 UT, and thereafter drops to average.
Figure 3.5: Derived parameters from the solar wind proton plasma and magnetic field observations for the October 18-20, 1995 magnetic cloud. The vertical dashed lines from the left to right indicate the driven shock, front cloud boundary, $B_z$ transition from negative to positive and the back cloud boundary. The panels from top to bottom show the dynamic pressure, total pressure, magnetic pressure, thermal pressure, the proton $\beta$, the Alfvén Mach number and the $\epsilon$ parameter.
values at the cloud back boundary. From the plot of the total pressure $P_t$, the magnetic cloud has a much higher $P_t$ than the surrounding solar wind plasma and the dominant component to the total pressure is the magnetic pressure $P_b$. This difference in $P_t$ between the cloud and the surrounding plasma would generate a force that would cause the magnetic cloud to expand unless opposed by other forces, such as magnetic curvature forces. Typically at 1 AU the thermal pressure $P_p$ and the magnetic pressure $P_b$ for the solar wind are approximately equal, which would correspond to a $\beta \sim 1$. For this cloud one finds that it is much lower ($\sim 0.03$) than this average value. The Alfven Mach number $M_A$ ($\sim 3$) is also lower than in the surrounding plasma which at times is greater than 10, considered high for typical solar wind values ($\sim 10$). The epsilon parameter increased by a factor of 15 to $\sim 0.15 \ mW/m^2$ at the front cloud boundary and then decreased to values corresponding to the surrounding plasma when the $B_z > 0$ transition begins for this cloud. This large rate of energy input into the magnetosphere produced a major geomagnetic storm and considerable auroral activity (Lepping et al. [1997]).

Effect of Ring Current

Figure 3.6 is a plot of the raw $Dst$ and the corrected $Dst$ ($Dst^*$) as a function of time. The starred symbols are $Dst^*$ and the diamond symbols represent the raw $Dst$ values. The time for Figure 3.11 starts at 00 UT on October 18, 1995. The dashed and dot-dashed lines indicate the driven shock, magnetic cloud boundaries.

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and the $B_z$ transition time, the same as in the previous figures. The data from WIND were delayed by 44 minutes so as to correlate the drop in the dynamic pressure at the front cloud boundary with the large drop in the uncorrected $Dst$ values. The solar wind plasma between the driven shock and the front cloud boundary only produced a weak storm which picks up when the front cloud boundary and the succeeding $B_z < 0$ period interact with the magnetosphere. The $Dst^*$ drops in $\sim 3$ hours to an average value of -120 nT which lasts for 6 hours. From the classification of Tsurutani et al. [1988] this storm would be considered major. There was a further energization of the ring current at the $B_z$ transition phase, but then the magnetosphere had recovered when the clouds back boundary passed the Earth. The fast stream interacting with the cloud produced moderate storms coinciding with the $B_z < 0$ phases found in this region. The fluctuations in the $B_z$ are the result of Alfven waves riding on the faster stream.

Alfven Waves in the Faster Stream

When inspecting the data for October 20-21, 1995 visually, an anticorrelation between the large magnetic field fluctuations and the solar wind velocity fluctuations could be seen. The number of data points used for the Alfven wave study for these days was 1069. The $\alpha$ particle number density for these days was taken to be zero (though in fact it was highly variable, A. J. Lazarus, Private Communication, 1996) and the pressure was considered to be isotropic ($P_\parallel=P_\perp$). Figure 3.7 is a plot...
Figure 3.6: $Dst$ (diamond) and the corrected $Dst^*$ (starred) index for quiet time and magnetopause currents for October 18-20, 1995 magnetic cloud. The vertical dashed lines from the left to right indicate the driven shock, front cloud boundary, $B_z$ transition from negative to positive and the back cloud boundary.
Figure 3.7: Plot of the fluctuations of solar wind magnetic field ($\Delta B$) and the normalized solar wind velocity ($\Delta A = \Delta V/(4\pi N m_p)^{1/2}$) components in GSE coordinates. The number of points used are 1609 and $R_{x,y,z}$ are the correlation coefficients for the straight line fits. The fitted regression lines passes through 0 and have slope $\approx -1$. 

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of the components of the magnetic field fluctuations and the solar wind velocity components normalized with respect to $(4\pi m_p N)^{1/2}$ (defined as vector $A$). The correlation coefficients are $R_x = -0.79$, $R_y = -0.54$ and $R_z = -0.67$ with the fitted regression lines passing through 0 and have slope $\approx -1$ which would indicate the presence of large amplitude Alfvén waves for October 20-21, 1995 (Lepping et al. [1997]).

Magnetopause and Bow Shock Subsolar Position

Figure 3.8 is a plot of the estimated Earth's subsolar magnetopause $R_{mp}$ and bow shock position $R_{sh}$ for October 18-20, 1995 after subtracting the statistical average Earth's magnetopause subsolar position of 11 $R_E$ and bow shock position of 14.6 $R_E$ Fairfield [1971]. The presence of a high dynamic pressure found upstream of the front cloud boundary and the cloud driven shock compressed the magnetopause and the bow shock of the Earth. At the front cloud boundary the magnetopause expanded back to its average position while the bow shock moved by as much as 9 $R_E$ beyond its average position. With the increase in the dynamic pressure in the cloud going from the front to the rear cloud boundary there is a corresponding compression of the Earth's magnetopause. This can be attributed to the cloud-fast stream interaction (Farrugia et al. [1998]). The bow shock remained about $+10$ $R_E$ beyond its average position during the $B_z < 0$ phase and 2 hours beyond the $B_z$ transition time. The only exceptions during this period occur when the solar
Figure 3.8: Magnetopause and bow shock subsolar position for Oct 18-20, 1995 after subtracting the average locations for the magnetopause (11 $R_E$) and the bow shock (14.6 $R_E$) Fairfield [1971]. The vertical dashed lines from the left to right indicate the driven shock, front cloud boundary, $B_z$ transition from negative to positive and the back cloud boundary.
wind dynamic pressure increases to 2 nPa from values that are lower than average. With the rise in the solar wind dynamic pressure, both the bow shock and the magnetopause are compressed. The bow shock returns to its average position at the back cloud boundary while the magnetopause remains compressed by 2 $R_E$ from its average distance. In the cloud fast stream interaction region where there is a $P_{dyn}$ peak a compression takes place for both the shock and the magnetopause. There is compression of the magnetopause and bow shock due to the cloud passage that is 0.8 - 1.5 $R_E$, however, the bow shock returned to its original position after cloud passage.

3.4.2 May 27-29, 1996

During this period of observation WIND was ~150 $R_E$ upstream of the earth. The magnetic cloud is located between the dashed lines (Figure 3.9) at ~1445 UT on May 27, 1996 (front boundary) and ~5315 UT on May 29, 1996 (back boundary) lasting ~39 hours. The dot dashed line at ~2445 UT is the time when the $B_z$ component of the field turns truly positive. There is no shock being driven by this magnetic cloud but instead there is a region ~2 hours ahead of the front cloud boundary where the magnetic field is low and the dynamic pressure is ~1 nPa. The three required signatures for an event to be a magnetic cloud are present in this figure. There is the large rotation in the $B_z$ magnetic field component lasting ~39 hours from ~9 nT to ~15 nT. The total magnetic field is higher than the average ambient magnitude and has a flat profile of ~10 nT except for a hump near the
back boundary of the cloud. There is a net positive $B_y$ magnetic component for the cloud and this is mostly constant ( $\sim 9$ nT ). There is a negative $B_z$ component ( $\sim 4$ nT ) for this cloud. The proton temperature in the magnetic cloud on the average is lower than the surrounding solar wind plasma temperature. The front boundary for this magnetic cloud is again clearly defined but the back boundary is not. Apart from using the three conditions to determine the cloud and its boundaries, the clock angle can also be used to give a good indication when the magnetic field has finished rotating, thereby helping to establish the back boundary of the cloud. From the proton bulk velocity profile the magnetic cloud on average is travelling slower than the solar wind plasma surrounding the cloud.

**Derived Parameters**

Figure 3.10 is a plot of the derived parameters for the May 27-29, 1996 magnetic cloud. From 1200 UT to the front cloud boundary the dynamic pressure for the plasma that is propagating ahead of the cloud is $\sim 5$ (nPa), which than decreases sharply at the front boundary of the cloud coinciding with the $B_z$ component turning southward. From the front boundary of the cloud to the $B_z$ transition time (2445 UT ) $P_{dyn}$ is approximately 1.5 nPa which is slightly lower then the average solar wind values (2.5 nPa). From 2445 UT onwards, $P_{dyn}$ rises to $\sim 8$ at the back cloud boundary. As was the case with the October 18-20, 1995 cloud, the profile matches closely the proton density for this cloud. The total pressure $P_t$ for
Figure 3.9: Proton plasma and magnetic field observations for the May 27-29, 1996 magnetic cloud. The vertical dashed lines from the left to right indicate the driven shock, front cloud boundary, $B_z$ transition from negative to positive and the back cloud boundary. The panels from top to bottom show the proton density, bulk flow speed, temperature, magnetic field components $B_x$, $B_y$, $B_z$, the total field $B$ and the clock angle.
Figure 3.10: Derived parameters from the solar wind proton plasma and magnetic field observations for the May 27-29, 1996 magnetic cloud. The vertical dashed lines from the left to right indicate the driven shock, front cloud boundary, $B_z$ transition from negative to positive and the back cloud boundary. The panels from top to bottom show the dynamic pressure, total pressure, magnetic pressure, thermal pressure, the proton $\beta$, the Alfven Mach number and the $\epsilon$ parameter.

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most of the cloud ranges from 0.02 - 0.04 nPa except near the back boundary where it rises to \( \sim 0.1 \) nPa which is a result of the faster stream overtaking the magnetic cloud (Farrugia et al. 1998). The dominant contribution to the \( P_t \) comes from the magnetic pressure \( P_b \). The \( \beta \) in the cloud when \( B_z \) is negative is on average \( \sim 0.03 \) and in the region where \( B_z > 0 \) there is a steady rise to \( \sim 0.3 \) at the back boundary of the cloud. The Alfvén Mach number for this cloud is \( \sim 5 \) and \( \sim 10 \) on average, i.e. higher than typical values for this parameter inside magnetic clouds \( (\sim 3) \), for the plasma surrounding this cloud with some regions in the surrounding plasma reaching values \( \sim 40 \). The \( \epsilon \) parameter jumps at the front cloud boundary (when \( B_z \) is large and turns negative), remains fairly constant at \( \sim 0.02 \, mW/m^2 \) for 5 hours, and thereafter starts to decrease to values comparable to the plasma surrounding the cloud.

**Effect of Ring Current**

A time delay of \( \sim 40 \) minutes was used to correlate the interplanetary parameters with the raw \( Dst \) values (Figure 3.11). As before, the starred symbols are \( Dst^* \) and the diamond symbols represent the raw \( Dst \) values. The passage of this magnetic cloud only produced a weak storm starting from the front cloud boundary to the \( B_z \) transition time after which the magnetosphere began its recovery phase.
Figure 3.11: $Dst$ (diamond) and the corrected $Dst^*$ (starred) index for quiet time and magnetopause currents for May 27-29, 1996 magnetic cloud. The vertical dashed lines from the left to right indicate the driven shock, front cloud boundary, $B_z$ transition from negative to positive and the back cloud boundary.
Figure 3.12: Magnetopause and bow shock subsolar position for May 27-29, 1996 after subtracting the average locations for the magnetopause (11 \( R_E \)) and the bow shock (14.6 \( R_E \)) Fairfield [1971]. The vertical dashed lines from the left to right indicate the front cloud boundary, \( B_z \) transition from negative to positive and the back cloud boundary.
Magnetopause and Bow Shock Subsolar Position

Figure 3.12 is a plot of the estimated Earth's subsolar magnetopause $R_{mp}$ and bow shock position $R_{sh}$ for May 27-29, 1996 after subtracting the average Earth's magnetopause and bow shock subsolar positions. The presence of high dynamic pressure upstream of the front cloud boundary and the cloud driven shock compressed the magnetopause by $2R_E$, but at the cloud front boundary the magnetopause expanded back to its average position. The bow shock during this period was also compressed by as much as $3R_E$ and recovered to its average position at the front cloud boundary. For the period when $B_z < 0$ in the cloud, both the magnetopause and bow shock are to be found at their average positions, corresponding to average solar wind $P_{dyn}$ values. With the increase in $P_{dyn}$ for $B_z > 0$, both the bow shock and the magnetopause are compressed. After the cloud passage, both the bow shock ($4R_E$) and the magnetopause ($2.5R_E$) remain compressed.

3.4.3 January 9-11, 1997

The WIND satellite during this period was $\sim 110R_E$ upstream of the Earth when it recorded the passage of a magnetic cloud. The magnetic cloud during these days is located between the dashed lines at 2900 UT (front boundary) and 5050 UT (back boundary) lasting $\sim 22$ hours in Figure 3.13. The dot dashed line at 4140 UT in the figure indicates the time when the $B_z$ component of the magnetic field turns truly positive. The other dashed line in the figure at $\sim 2450$ UT indicates a shock

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Figure 3.13: Proton plasma and magnetic field observations for the January 9-11, 1997 magnetic cloud. The vertical dashed lines from the left to right indicate the driven shock, front cloud boundary, $B_z$ transition from negative to positive and the back cloud boundary. The panels from top to bottom show the proton density, bulk flow speed, temperature, magnetic field components $B_x, B_y, B_z$, the total field $B$ and the clock angle.
that is being driven by this cloud. The $B_z$ component of the magnetic field for the cloud rotates relatively smoothly from $\sim-15$ nT to $\sim16$ nT. There is a large $B_y$ component with a maximum of $\sim-14$ nT and a slightly larger $B_x$ component for the cloud compared to the surrounding solar wind plasma. The temperature for this cloud is lower than the surrounding plasma. From the velocity profile the magnetic cloud is overtaking the plasma upstream of the driven shock while it itself is being overtaken by a faster flow, as was the case for the other two magnetic clouds. Again, there is a noticeable rise in the proton density as one travels from the front to the back boundary of the cloud. Near the back boundary of this magnetic cloud the total magnetic field drops from 20 nT to 2 nT in $\sim5$ minutes and rises just as rapidly again.

**Derived Parameters**

Figure 3.14 is a plot of the derived parameters for the January 9-11, 1997 magnetic cloud. The plasma between the cloud driven shock and the front boundary of the cloud has a relatively steady dynamic pressure ($P_{\text{dyn}}$) of 4 nPa with a large drop to 0.5 nPa at the front cloud boundary. It then rises to $\sim2$ nPa in one hour and from this time to the $B_z$ negative-to-positive transition time (4140 UT) the dynamic pressure averages out to $\sim2$ nPa comparable to average solar wind values. With the start of $B_z > 0$ phase $P_{\text{dyn}}$ rises and reaches a maximum of $\sim11$ nPa at the back boundary of the cloud. Looking at the data plotted for the $P_t$, $P_b$, $P_p$ and $\beta$ within
Figure 3.14: Derived parameters from the solar wind proton plasma and magnetic field observations for the January 9-11, 1996 magnetic cloud. The vertical dashed lines from the left to right indicate the driven shock, front cloud boundary, $B_z$ transition from negative to positive and the back cloud boundary. The panels from top to bottom show the dynamic pressure, total pressure, magnetic pressure, thermal pressure, the proton $\beta$, the Alfven Mach number and the $\epsilon$ parameter.
the cloud the magnetic pressure dominates, resulting in a $\beta$ value of $\sim0.03$ in the region from the front cloud boundary to the $B_z$ transition phase and then rises to $\sim0.1$ at the cloud back boundary. At the region near the back cloud boundary where $B$ drops to $\sim2$ nT the plasma pressure $P_p$ dominates. The plasma between the cloud driven shock and the front cloud boundary has a slightly higher magnetic pressure and corresponds to an average $\beta$ of $\sim0.7$ for this region. The Alfvén Mach number for this cloud is $\sim4$ but the plasma upstream of the cloud driven shock have values that exceed 10. Corresponding with the large southward change for the $B_z$ component at the front cloud boundary there is a rise in the $\epsilon$ parameter which reaches a maximum about 3 hours into the cloud, and subsequently decreases as the $B_z$ component rotates to positive values. The energy input from this magnetic cloud produced a moderate ($-100$ nT $\leq Dst \leq -50$ nT ) geomagnetic storm ( Figure 3.15 ).

**Effect of Ring Current**

The corrections to the raw $Dst$ were made by sampling the delay time of the dynamic pressure at the shock driven magnetic cloud and the rise in the raw $Dst$ and the delay in response to the rise in the dynamic pressure at the rear of the cloud ( 4600 - 5600 UT ) to the raw $Dst$. The time delays were 18 and 26 minutes, respectively, and an average of 22 minutes was used to correct the raw $Dst$ except from hours 4600 to 5600 UT where the delay was taken as 26 minutes. For this
Figure 3.15: $Dst$ (diamond) and the corrected $Dst^*$ (starred) index for quiet time and magnetopause currents for January 9-11, 1997 magnetic cloud. The vertical dashed lines from the left to right indicate the driven shock, front cloud boundary, $B_z$ transition from negative to positive and the back cloud boundary.
cloud, its passage past the Earth produced a moderate storm ( ~85 nT ) which started with the interaction of the front cloud boundary with the magnetosphere. It took ~3 hours for the storm to grow ( storm main phase ) and lasted for ~7 hours, after which the recovery phase began near the $B_z > 0$ transition time but the magnetosphere did not recover to its state before the cloud passage.

**Magnetopause and Bow Shock Subsolar Position**

Figure 3.16 is a plot of the estimated Earth's subsolar magnetopause $R_{mp}$ and bow shock position $R_{sh}$ for January 9-11, 1997 after subtracting the average Earth's magnetopause and bow shock. There is a compression of the Earth's bow shock for the region downstream of the cloud driven shock and upstream of the front cloud boundary. This compression corresponds to the higher than average solar wind dynamic pressure. The magnetopause and the bow shock expand beyond its average positions at the front cloud boundary the magnetopause, expanding by as much as $3 R_E$ and the bow shock by as much as $18 R_E$ more than double its statistical average position according to Farfield [1971]. For the $B_z < 0$ period in the magnetic cloud, the magnetopause returns to its position prior to the cloud passage but the bow shock remains slightly expanded. From the $B_z$ negative-to-positive transition time to the back cloud boundary both the magnetopause and the bow shock are compressed by as much as $5 R_E$. After the cloud passage both

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Figure 3.16: Magnetopause and bow shock subsolar position for Jan 9-11, 1997 after subtracting the average locations for the magnetopause (11 \( R_E \)) and the bow shock (14.6 \( R_E \)) Fairfield [1971]. The vertical dashed lines from the left to right indicate the driven shock, front cloud boundary, \( B_z \) transition from negative to positive and the back cloud boundary.
the magnetopause and bow shock return to their average positions by the end of January 11, 1997.

### 3.5 Conclusion

Three magnetic clouds which were and are subject of deep study by the ISTP community, were observed and their effect on the Earth's magnetosphere were noticeably different. The storms generated by these clouds ranged from major for October 18-20, 1995, moderate for January 9-11 and weak for May 25-27, 1996. Through their low Alfven Mach number and variable $P_{\text{dyn}}$, these clouds also affected both the magnetopause and bow shock of the Earth. A comparison of the effectiveness in the coupling of these 3 magnetic clouds to the Earth was made by Farrugia et al. [1998]. In this work Farrugia et al. concluded that despite the similarities in the interplanetary data for the three clouds, their effects on the Earth were significantly different. Thus the rate of energy input into the magnetosphere was greatest in the October 18-20 1995 cloud, the January 9-11, 1997 cloud inputted only half as much as the October 1995 cloud and ~5-6 times less than the October cloud for the May 27-29 cloud. This power also correlated well with the peak values generated for the $Dst$ index. It is however interesting to note the relationship between the expansion of the Earth's bow shock (due to low $P_{\text{dyn}}$ and $M_A$) from its average position and the transfer of solar wind energy to the magnetosphere. During the $B_z < 0$ phase the estimated subsolar bow shock
position for the 3 magnetic clouds considered were sufficiently different and their expansion from average bow shock locations correlated with the energy transfer and eventually the strength of the geomagnetic storms observed.
Chapter 4

Field and flow perturbations in the October 18-19, 1995, magnetic cloud

4.1 Introduction

A global topology for magnetic clouds proposed by Burlaga et al. [1990] is a large and bent flux tube where magnetic field lines may be anchored to the Sun. On examining 3 sec. resolution magnetic field and plasma data from the WIND (Lepping et al. [1995], Lin et al. [1995]) satellite instruments there were noticeable discontinuities in the field and plasma data. In some instances there were simultaneous discontinuities in both field and plasma data, but there were also cases when such correspondence between field and plasma data were not present. This would imply that the rotation of the field for a magnetic cloud is not smooth. In this chapter we examine and classify the discontinuities that occurred in both the field and plasma data which has never been done before. Discontinuities in the magnetic field are selected by requiring that the rotation in the magnetic field lines be larger than the expected rotation for a smoothly rotating field for the same time period. If we consider a smooth rotation of $\sim 180^\circ$ in the field lasting $\sim 29$ hours, than in $\sim 1$ minute, the field would rotate by as little as $0.1^\circ$. In addition to discussing
discontinuities, we examine the large scale structure of the October 1995 magnetic cloud. We make the assumption here that the magnetic cloud is not expanding since the flat $V$ and "boxcar" $B$ profiles for the October cloud may indicate that no further expansion is taking place for this cloud Farrugia et al. [1992, 1993]. The enhancement in the fields and proton density which Burlaga et al. [1998] and Lepping et al. [1997] reported is the result of a faster corotating stream overtaking the October cloud at its rear. With the simplifying assumption that a static field configuration is valid for the October 1995 cloud, we apply minimum variance analysis to hour-long segments in the October 1995 magnetic cloud. In this way the axis of the cloud at each hour long segment is determined and using these directional vectors we attempt to piece together the large scale structure for the October cloud.

4.2 Wind Observations

4.2.1 Overview: Magnetic Field and Plasma

The Wind spacecraft, launched in November 1, 1994, is one of NASA’s spacecraft in the Global Geospace Science Initiative, forming part of the International Solar Terrestrial Physics Program (ISTP). The ISTP, which includes many other spacecraft and ground-based facilities, is meant to investigate solar and interplanetary phenomena and their effects on the Earth’s magnetosphere. Among the scientific objectives of the Wind mission are to (1) provide interplanetary plasma and
energetic particle and magnetic field measurements in support of magnetospheric studies and (2) investigate basic plasma processes in the solar Wind close to Earth [see Acuna et al., 1995]. In this chapter data from three instruments on Wind were used: the Magnetic Field Investigation (MFI Lepping et al., 1995); the Solar Wind Experiment (SWE Ogilvie et al., 1995); and the Three-Dimensional Plasma and Energetic Particle Investigation Lin et al., 1995. Figure 4.2 shows magnetic field data at 3-s resolution from the MFI. The panels from top to bottom show the total field $B$ (nT) and the magnetic field components $B_x$, $B_y$ and $B_z$ (nT) in GSE coordinates. The horizontal axis shows the universal time (UT) in hours starting from 1800 UT on October 18 to 4800 UT or 2400 UT on October 19, 1995. Following Lepping et al. [1997], the October magnetic cloud is located between the dotted-dashed lines from $\sim 1900$ UT on October 18, 1995 (front boundary) to $\sim 4700$ UT (rear boundary) in Figure 4.2. As discussed in Chapter 3, the October cloud satisfied the 3 requirements of: (1) large rotation in the magnetic field; (2) larger than average magnetic field strength; and (3) lower than average proton temperatures. The front boundary for this cloud is clearly defined, while the back boundary is somewhat arbitrary. Both the rise in the temperature and the clock angle (see Figure 3.4) rotation are useful in determining the October clouds rear boundary, but the determination of cloud boundaries is in general uncertain (see Burlaga, 1990), as this example shows. At the cloud front boundary the magnetic fields is due south, as seen by the large negative $B_z$ component, with small contributions
from \( B_x \) and \( B_y \). At the rear cloud boundary, the field is pointing northwest. The total field is smooth only to a first approximation since fluctuations are clearly seen in all components of the field. The perturbations are all perpendicular to the local field direction since \(|\vec{B}|\) does not change.

\[ \delta B \\
\overrightarrow{B} \rightarrow \overrightarrow{B_\delta} \]

Figure 4.1: Perturbation perpendicular to the local field direction.

\[ \overrightarrow{B_\delta} = \overrightarrow{B} + \delta \]

where \( \overrightarrow{B} \) be the local total field and \( \delta \) the perturbation. Then

\[ \frac{B_\delta^2}{B^2} = 1 + \frac{\delta^2}{B^2} \]

As was noted earlier, a smooth rotation of \( \sim 180^\circ \) in the field lasting \( \sim 29 \) hours, it follows that in \( \sim 1 \) minute the field would rotate by as little as \( 0.1^\circ \). Using this as a guide, we selected to examine those directional discontinuities in the fields that rotated more then \( 15^\circ \) in \( \sim 1 \) minute. 16 discontinuities were selected in this manner and are numbered in Figure 4.2. Figure 4.3 shows proton parameters for the same interval as in Figure 4.2. The panels from top to bottom show the number density \((cm^{-3})\) from the SWE instrument, velocity magnitude \((km \text{ s}^{-1})\),
Figure 4.2: Wind magnetic field data at 3-s resolution for the period 1800 UT, October 18 to 2400 UT, October 19, 1995. The data are plotted in a GSE coordinate system. The magnetic cloud interval is between the dotted-dashed lines. The field directional discontinuities studied are numbered in the figure.
Figure 4.3: Plasma (proton) observations for the same period as in Figure 1. The vertical lines are drawn at the times of the directional discontinuities shown in Figure 4.2.
temperature (K), and the velocity components $V_x$, $V_y$ and $V_z$ in GSE coordinates from the Three-Dimensional Plasma and Energetic Particle Experiment for the protons. The data from the Three-Dimensional Plasma and Energetic Particle Experiment instrument are at 3-s temporal resolution whereas data from the SWE instrument had a resolution of 1.5 minute. Ahead of the cloud, the density is higher than average ($\sim 60$ cm$^{-3}$) and Burlaga et al. [1998] had argued in connection with another magnetic cloud (that on January 1997), this might be due to the ejecta having been formed within the streamer belt, where densities are typically high. At the front boundary of the magnetic cloud, the density drops from $\sim 60$ to $\sim 4$ cm$^{-3}$, a tenfold decrease. It remains steady around 4 cm$^{-3}$ until hour 31, after which it rises, at first gradually (until hour 36) and then more steeply until hour 45, reaching a peak of 60 cm$^{-3}$ at the rear of the cloud, followed by a decrease to 20 cm$^{-3}$ around hour 47. The temperature drops at the front boundary by a factor of $\sim 2.5$ to a value of $4 \times 10^4$ K, and begins a quasi-linear rise to $6 \times 10^4$ K up to hour 22.5, where a sudden rise to $10^5$ K takes place. From hour 22.5 to hour 36 the temperature profile then decreases linearly to around $4 \times 10^4$ K. Subsequently, it rises to $6 \times 10^4$ K at the cloud boundary. Qualitatively, the $V_y$ component is positive till around hour 36; remains around 0 km s$^{-1}$ until hour 41; and then goes negative for the rest of interval in Figure 2. On the other hand, the $V_z$ component starts out with a small negative value till about hour 22.5, and then remains positive for the remainder of the interval. The major flow component,
$V_x$, can be approximated by $-425 \text{ km s}^{-1}$ for the whole cloud. It can be seen that the large field rotations are often accompanied by impulsive changes in the plasma parameters. The directional discontinuity (DD) marked A is a shock-like feature advancing in the cloud from the rear and has been discussed by Lepping et al. [1997]. Among the points made by Lepping et al. in favor of this interpretation are (1) that it is a thin transition; (2) that consistent estimates of the normal direction to this front are reached by various methods; (3) satisfaction of the MHD jump conditions (except for the temperature); and (4) the observed sense of change of field and plasma parameters, which is consistent with a fast forward shock. But we also note that Lepping et al. could not definitively interpret the DD marked A as a shock (see Chapter 3).

### 4.2.2 Analysis Method

Many people have examined DDs in the solar Wind [see, e.g., Burlaga, 1969]. Table 4.1 lists the number of directional discontinuities whose magnetic field rotates $\geq 15^\circ$ and the average time the rotation takes for the discontinuities. We apply

<table>
<thead>
<tr>
<th>Number of DDs</th>
<th>Angle Field Rotates</th>
<th>Average Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>$15 \leq \phi \leq 20$</td>
<td>15 s</td>
</tr>
<tr>
<td>2</td>
<td>$20 &lt; \phi \leq 30$</td>
<td>19 s</td>
</tr>
<tr>
<td>2</td>
<td>$30 &lt; \phi \leq 40$</td>
<td>45 s</td>
</tr>
<tr>
<td>1</td>
<td>$\phi &gt; 40$</td>
<td>3 min</td>
</tr>
</tbody>
</table>

the minimum variance analysis technique of Sonnerup and Cahill [1967] to the
directional discontinuities selected. In the minimum variance method a matrix $M$ is generated whose elements are

$$M_{ij} = \langle B_i B_j \rangle - \langle B_i \rangle \langle B_j \rangle$$

$$\langle B_i \rangle = \frac{1}{N} \sum_{k=1}^{N} B_i$$

where $N$ is the number of field observations and $\langle B \rangle$ is the average value. The eigenvalues and the eigenvectors of matrix $M$ are next determined and only those DDs are retained for which the ratio of intermediate to minimum eigenvalues $(\lambda_2/\lambda_3) > 2.0$ are considered reliable [see Lepping and Behannon, 1980]. The vector along the axis of the cloud will be given by the eigenvector corresponding to the minimum eigenvalue and all fluctuations of the field will be in the plane defined by eigenvectors corresponding to the large and intermediate eigenvalues. We form a matrix $A$ whose column entries are the eigenvectors of the matrix $M$ which are then used to transform each field observation into the minimum variance coordinate frame.

$$\mathbf{B}_{\text{minvar}} = A \mathbf{B}_{\text{obs}}$$

where $\mathbf{B}_{\text{obs}}^T = (B_x, B_y, B_z)$ and $\mathbf{B}_{\text{minvar}}^T = (B_i, B_j, B_k)$. $B_i$ is the component along the eigenvector corresponding to the largest eigenvalue, $B_j$ is the component along the eigenvector corresponding to the intermediate eigenvalue and $B_k$ is the component along the eigenvector with the lowest eigenvalue. The magnitude $B_n$ used in the field analysis below is the average over the number of observations for the
discontinuity.

\[ B_n = \frac{1}{N} \sum_{k=1}^{N} B_k \]

where \( N \) is the number of field observations for the discontinuity. We apply Neugebauer et al.'s [1984] criteria for classifying the DDs, which are as follows: For a rotational discontinuity (RD)

\[ B_n/|\mathbf{B}| \geq 0.4 \quad ||\mathbf{B}||/|\mathbf{B}| < 0.2 \]

For a tangential discontinuity (TD)

\[ B_n/|\mathbf{B}| < 0.4 \quad ||\mathbf{B}||/|\mathbf{B}| \geq 0.2 \]

(4.1)

where \( |\mathbf{B}| \) is the larger of the field magnitudes on either side of the discontinuity, \( B_n \) is the absolute value of the component of \( \mathbf{B} \) normal to the plane determined by minimum variance, and \( ||\mathbf{B}|| = |\mathbf{B}_2| - |\mathbf{B}_1| \) is the difference in absolute values of the fields across the DD. Neugebauer et al. had also classifications termed "Either" and "Neither", but they will not be of importance here. We also apply tests based on the plasma data [Neugebauer et al., 1984; Parks, 1991]. Hudson et al. [1970] shows that in MHD theory rotational discontinuities satisfy the following relationship:

\[ [\mathbf{V}] = \pm \sqrt{\rho A/\mu_0} [\mathbf{B}/\rho]. \]

(4.2)

\( A \) is the pressure anisotropy of the plasma, defined as

\[ A = 1 - \frac{(P|| - P_{\perp})\mu_0}{B^2}, \]
where $P_\parallel$ and $P_\perp$ are, respectively, the thermal plasma pressures parallel and perpendicular to the magnetic field; $\mu_0$ is the permeability of free space; and [...] denotes the difference between the quantities before and after the discontinuity. For a DD propagating antisunward, the plus sign in Equation 4.2 is used when the average interplanetary magnetic field points towards, and the minus sign when it points away from, the Sun. In the absence of more exact knowledge, we assume the plasma to be isotropic. In the normal solar wind, parameter A is estimated as $\sim 0.9$ [Burlaga, 1971]. In coronal mass ejections (of which magnetic clouds form a subset) observed by ISEE 3, $T_\parallel$ is generally greater than $T_\perp$, with a typical ratio $T_\parallel/T_\perp \sim 2$ [Gosling et al. 1987]. Using $T = 1/3(2T_\perp + T_\parallel)$, we obtain $A = 1 - 0.75\beta$, where $\beta$ is the average proton beta. For the October 1995 magnetic cloud, $\beta < 0.05$ [Lepping et al., 1997] which gives $A \sim 0.96$. Thus the assumption of isotropy seems to be fulfilled in our case. We define $\theta$ to be the angle between $[V]$ and $[B/\rho]$. For an RD, $\theta$ should ideally be 0 or 180°, while it can take any value for a TD. Another parameter considered is the angle $\alpha$, introduced by Belcher and Solodnya [1975] and defined by the following relation:

$$\tan(\alpha) = \frac{\sqrt{\mu/\rho}[V]}{|[B/\rho]|} = \frac{\sqrt{\mu/\rho}|V_2 - V_1|}{|B_2/\rho_2 - B_1/\rho_1|}$$

(4.3)

Ideally (i.e., not taking account of observational and other errors), for an RD, the angle $\alpha$ should be close to 45°. For a TD the angle $\alpha$ can take on any value between 0° and 90°. Besides these field and flow relations, we also consider jump conditions on the density and the temperature across the discontinuity. For a TD and assum-
ing isotropy, there can be arbitrary jumps for the density and temperature with
the requirement that there should be no changes in the total pressure (magnetic
+ thermal pressure) across the discontinuity. For an RD and assuming isotropy,
there should be no jumps in either the density or the temperature [Parks, 1991].
We now give three examples of discontinuities that were examined.

D8

Figure 4.4 shows plasma and field data in GSE coordinates for the discontinuity D8
(Figure 4.2). The horizontal axis indicates the time interval (~3 mins.) in hours
for the discontinuity D8 centered at hour 38.005. The panels from top to bottom
show the density, bulk flow speed, temperature and the three, paired components
of the velocity and magnetic fields. The horizontal lines indicate the average values
for each variable before and after the discontinuity. The magnetic field rotated by
as much as 15° and the minimum variance analysis gave a reliable normal with an
intermediate-to-minimum eigenvalue ratio of $(\lambda_2/\lambda_3) = 71.1$. Applying Neugebauer
et al.'s [1984] test to this discontinuity which has $B_n = 21 nT$, $B_n/|B|$ of 0.99, and
$[|B|]/|B| = 0.01$ satisfied the condition for a rotational discontinuity (RD). The
normal vector for this discontinuity is (-0.26,0.91,-0.33) mainly in the $y$
direction. This result is also supported by the plasma tests where it was found that angles $\theta$
(the angle between $[V]$ and $[B/\rho]$) and $\alpha$ are $\sim 9^\circ$ and $33^\circ$, which are reasonably
close to the theoretical values of $0^\circ$ or $180^\circ$ for $\theta$ and $45^\circ$ for $\alpha$ for an RD. The
Figure 4.4: Wind magnetic field and plasma data at 3-s resolution for the discontinuity around 38.005 hours (~14.005 UT, October 19, 1995). The solid lines are the average values before and after the discontinuity. There is a flow speed enhancement and a proton temperature rise at the discontinuity, but the density stays constant.
low resolution proton density shows no change but there is a $\sim 20\%$ rise in the
temperature and the plasma is speeded up by $\sim 35 \text{ km/s}^{-1}$ across the discontinuity.

**D10**

This second example is numbered D10 in Figure 4.2. Figure 4.5 is a plot of the
same variables as Figure 4.4 but for a time interval of 2.4 minutes showing the
discontinuity D10 which is centered at $\sim 42.58$ hours. Minimum variance analysis
for D10 gave an intermediate-to-minimum eigenvalue ratio of 5.5 and a normal field
component $B_n$ of 28.4 nT with $B_n/|B|$ of 0.95, and $||B||/|B| = 0.015$ thus satisfying
the conditions for an RD in Equation 4.1. The normal vector for this discontinuity
was found to be (0.50, -0.59, 0.63). From the plasma tests the angles $\theta$ and $\alpha$ are $\sim$
3° and 37° which is again reasonably close to the expected theoretical values of an
RD for these angles. Again no density or bulk flow changes are observed for this
discontinuity but there is a temperature rise.

**D6**

This third example is numbered D6 in Figure 4.2. Figure 4.6 is a plot of the
same variables as Figure 4.4 but for a time interval of 1.2 minutes showing the
discontinuity D6 centered at $\sim 30.97$ hours. Minimum variance analysis of this
discontinuity resulted in a $(\lambda_2/\lambda_3) = 64.2$, with $B_n = 18.0$ nT. The field test for
this discontinuity classified it as an RD ($B_n/|B| = 0.95$ and $||B||/|B| = 0.03$). The
Figure 4.5: Wind magnetic field and plasma data at 3-s resolution for the discontinuity centered at 42.58 hours (18.58 UT, October 19, 1995). The solid lines are the average values before and after each discontinuity. Across the discontinuity there is no flow enhancement, the density is constant, but the temperature increases.
Figure 4.6: Wind magnetic field and plasma data at 3-s resolution for the discontinuity centered at 30.97 UT, hours (6.97 UT, October 19, 1995). In this case, the flow speed decreases, the proton temperature is depressed, and the density stays constant.
Figure 4.7: Comparison of experimental results with theoretical expectations for the discontinuity shown in Figure 5a. For further details, see text.
normal for this discontinuity is (-0.38, 0.64, 0.67). An alternative to the angles $\theta$ and $\alpha$ for the plasma and field tests is a graphical vector representation of the field and flow tests as used by Sonnerup et al., [1981] for their studies of the dayside magnetopause. For this vector representation a reference value is first determined by averaging the values for the field and flow variables before the discontinuity. As one progresses through the discontinuity the observed values are subtracted from the average values before the discontinuity to generate the following quantities.

$$\Delta V_{\text{obs}} = V_{\text{ref}} - V_{\text{obs}}$$

(4.4)

$$\Delta V_{\text{th}} = \sqrt{(\rho_{\text{ref}}/\mu_0)[B_{\text{ref}}/\rho_{\text{ref}}]} - \sqrt{(\rho_{\text{obs}}/\mu_0)[B_{\text{obs}}/\rho_{\text{obs}}]}$$

(4.5)

The observed differences $\Delta V_{\text{obs}}$ are normalized with respect to the theoretical values $\Delta V_{\text{th}}$ and the resulting vector plotted in Figure 4.7. For perfect agreement, the vectors in Figure 4.7 should all be pointing along the horizontal line and be of unit length. We find here that these vectors all lie within 20° of the horizontal line and their normalized magnitudes are greater than 50%. These results are at least comparable to Phan et al., 1996 work on the magnetopause when it is an RD. Figure 4.6 shows the bulk flow for the plasma decreased by as much as $\sim 60 \text{ km s}^{-1}$ as one progresses through the RD but recovers to average values before the discontinuity. Bulk flow speed decreases in RDs have been discussed by Scudder [1984], who pointed out that plasma flow enhancements ("jetting") are not a necessary condition for an RD since Equation 4.2 is a vector equation. The
temperature as one progresses through the discontinuity in Figure 4.6 first decreases and then increases by \(~12\%\) after the discontinuity suggesting the discontinuity may not be just a pure RD.

### 3.2.3 Summary of Results

Table 4.2 summarizes the results of the field analysis for the discontinuities numbered in Figure 4.2. The column entries in Table 4.2 from left to right are the

| DDs | Time   | \(B_n\) | | \(|B|\) | \(|\Delta B|\) | Normal Vector | Class |
|-----|--------|---------|---|---------|----------------|---------------|--------|
| D1  | 18.97 - 19.02 | -0.76 | 19.9 | 13.3 | 16.3 | (0.99,0.08,0.02) | TD |
| D2  | 22.96 - 22.97 | -19.6 | 20.0 | 0.43 | 10.1 | (-0.12,0.21,0.97) | RD |
| D3  | 23.22 - 23.223 | -19.2 | 20.2 | 0.22 | 9.34 | (-0.24,0.08,0.97) | RD |
| D4  | 23.96 - 24.967 | -18.3 | 19.0 | 0.17 | 30.4 | (-0.43,0.35,0.83) | RD |
| D5  | 28.33 - 28.375 | -18.4 | 19.0 | 0.30 | 8.11 | (-0.38,0.34,0.86) | RD |
| D6  | 30.96 - 30.973 | 18.0 | 18.9 | 0.51 | 64.3 | (0.38,-0.64,-0.67) | RD |
| D7  | 32.627 - 32.628 | -18.5 | 18.9 | 0.32 | 7.01 | (-0.49,0.56,0.67) | RD |
| D8  | 37.995 - 38.015 | -21.4 | 21.6 | 0.09 | 71.1 | (-0.26,0.91,-0.33) | RD |
| D9  | 40.328 - 40.333 | -19.7 | 21.2 | 0.28 | 5.13 | (-0.18,0.94,-0.28) | RD |
| D10 | 42.57 - 42.59 | 28.4 | 30.0 | 0.45 | 5.45 | (0.50,-0.59,0.63) | RD |
| D11 | 45.567 - 45.585 | 16.3 | 21.3 | 0.15 | 3.10 | (0.71,-0.57,0.42) | RD |
| D12 | 46.455 - 46.458 | 22.5 | 24.0 | 0.51 | 7.42 | (0.49,-0.44,0.75) | RD |
| D13 | 46.464 - 46.466 | 23.0 | 24.4 | 0.32 | 7.17 | (0.02,0.26,0.97) | RD |
| D14 | 46.474 - 46.475 | 24.0 | 24.4 | 0.16 | 13.0 | (0.29,-0.60,0.75) | RD |
| D15 | 46.881 - 46.882 | 24.4 | 24.8 | 0.14 | 88.4 | (0.28,-0.43,0.86) | RD |
| D16 | 46.896 - 46.897 | 24.3 | 24.7 | 0.14 | 13.7 | (0.15,-0.41,0.89) | RD |

The index number of discontinuities studied found in Figure 4.2, the time interval of discontinuity from 0000 UT October 18, the average \(B_n\) for the interval, \(|B|\) the larger of the field magnitudes on either side of the discontinuity, \(|\Delta B|\) the difference
Table 4.3: Results of Field and Plasma Discontinuity Test

<table>
<thead>
<tr>
<th>DDs</th>
<th>Time</th>
<th>(\alpha)</th>
<th>(\theta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>18.97 - 19.02</td>
<td>1.29</td>
<td>96.4</td>
</tr>
<tr>
<td>D2</td>
<td>22.96 - 22.97</td>
<td>2.66</td>
<td>117.1</td>
</tr>
<tr>
<td>D3</td>
<td>23.22 - 23.223</td>
<td>29.13</td>
<td>154.9</td>
</tr>
<tr>
<td>D4</td>
<td>23.96 - 24.967</td>
<td>11.21</td>
<td>85.93</td>
</tr>
<tr>
<td>D5</td>
<td>28.33 - 28.375</td>
<td>14.01</td>
<td>160.8</td>
</tr>
<tr>
<td>D6</td>
<td>30.96 - 30.973</td>
<td>31.15</td>
<td>4.31</td>
</tr>
<tr>
<td>D7</td>
<td>32.627 - 32.628</td>
<td>35.16</td>
<td>5.63</td>
</tr>
<tr>
<td>D8</td>
<td>37.995 - 38.015</td>
<td>33.40</td>
<td>9.44</td>
</tr>
<tr>
<td>D9</td>
<td>40.328 - 40.333</td>
<td>9.64</td>
<td>160.5</td>
</tr>
<tr>
<td>D10</td>
<td>42.57 - 42.59</td>
<td>37.3</td>
<td>3.14</td>
</tr>
<tr>
<td>D11</td>
<td>45.567 - 45.585</td>
<td>32.0</td>
<td>40.5</td>
</tr>
<tr>
<td>D12</td>
<td>46.455 - 46.458</td>
<td>12.3</td>
<td>38.9</td>
</tr>
<tr>
<td>D13</td>
<td>46.464 - 46.466</td>
<td>27.3</td>
<td>44.3</td>
</tr>
<tr>
<td>D14</td>
<td>46.474 - 46.475</td>
<td>54.2</td>
<td>164.4</td>
</tr>
<tr>
<td>D15</td>
<td>46.881 - 46.882</td>
<td>41.9</td>
<td>166.5</td>
</tr>
<tr>
<td>D16</td>
<td>46.896 - 46.897</td>
<td>35.9</td>
<td>139.0</td>
</tr>
</tbody>
</table>

in the total field across a discontinuity, \(\lambda_2/\lambda_3\) the ratio of intermediate to minimum eigenvalues, the normal vector for the discontinuity, and the classification of the discontinuity: RD is rotational, and TD is tangential. Table 4.3 summarizes the results for the field and plasma-based tests. The column entries in Table 4.3 from left to right are the index number of discontinuities studied found in Figure 4.2, the time interval of discontinuity from 0000 UT October 18, \(\alpha\) defined by Equation 4.3, and \(\theta\) the angle between \([V]\) and \([B/\rho]\). We found the front boundary to be a clear TD, which agrees with Lepping et al. [1997] results. The normals for the discontinuities studied were well defined normals, \(B_n\) values are large, and from the field tests of Equation 4.1 the discontinuities were all RD's. The plasma based tests

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Equations 4.2 and 4.3 were found at times to agree with the field based results but there have been cases when they did not (Table 4.3). We do find changes in the temperature across the discontinuities but no noticeable changes for the density. We do note that the density is at a lower resolution (1.5 minute). The changes in the temperature suggest a more elaborate structure than just an RD.

3.3 Large-Scale-Perturbations

In this section we examine the coherence of the October 1995 magnetic cloud: Is it a single or are there multiple structures to be found in the October 1995 magnetic cloud? We do this by taking hour long segments of cloud data and apply the minimum variance analysis to these data sets. As we form these contiguous reliable normals we check to see if the normal deviates from the previous hour’s normals. If it deviates by a prescribed “tolerance” angle, we say that the structure in this hour long segment is no longer coherent with the previous hour long segments. The tolerance angle is arbitrary and for this work was taken as 22°. Table 4.4 summarizes the results found. The columns from left to right indicate the universal time of the hour stretches, δ the angle between successive normals, the normal vector to a given hour segment in GSE coordinates, the ratio $\lambda_2/\lambda_3$ (a measure of how reliable the normal is), and a parameter $\psi$ giving the angle between the normal to a given hour data stretch and the average normal to the coherent structure to which the hour-long data stretch belongs. From the above analysis we find three
Table 3.4: Minimum Variance Analysis on Successive 1-Hour Segment Intervals

<table>
<thead>
<tr>
<th>Hour Intervals</th>
<th>$\delta$</th>
<th>Normal Vector</th>
<th>$\frac{2\sigma}{\lambda}$</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Segment 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19.1- 20</td>
<td>11.4</td>
<td>(0.14, 0.02, 0.99)</td>
<td>6.8</td>
<td>21.4</td>
</tr>
<tr>
<td>20-21</td>
<td>10.2</td>
<td>(0.06, -0.14, 0.99)</td>
<td>40.3</td>
<td>23.2</td>
</tr>
<tr>
<td>21-22</td>
<td>21.3</td>
<td>(-0.28, 0.001, 0.96)</td>
<td>3.0</td>
<td>10.6</td>
</tr>
<tr>
<td>22-23</td>
<td>6.7</td>
<td>(-0.16, 0.038, 0.91)</td>
<td>11.7</td>
<td>7.5</td>
</tr>
<tr>
<td>23-24</td>
<td>21.3</td>
<td>(-0.36, 0.34, 0.87)</td>
<td>69.5</td>
<td>14.6</td>
</tr>
<tr>
<td>24-25</td>
<td>8.9</td>
<td>(-0.21, 0.37, 0.90)</td>
<td>46.2</td>
<td>12.2</td>
</tr>
<tr>
<td>25-26</td>
<td>5.4</td>
<td>(-0.29, 0.32, 0.90)</td>
<td>37.3</td>
<td>10.8</td>
</tr>
<tr>
<td>26-27</td>
<td>8.6</td>
<td>(-0.38, 0.42, 0.83)</td>
<td>118.</td>
<td>19.4</td>
</tr>
<tr>
<td>27-28</td>
<td>6.3</td>
<td>(-0.42, 0.32, 0.85)</td>
<td>10.2</td>
<td>16.8</td>
</tr>
<tr>
<td><strong>Segment 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29-30</td>
<td>8.8</td>
<td>(-0.29, 0.62, 0.73)</td>
<td>21.7</td>
<td>5.04</td>
</tr>
<tr>
<td>30-31</td>
<td>4.4</td>
<td>(-0.22, 0.61, 0.76)</td>
<td>5.3</td>
<td>8.47</td>
</tr>
<tr>
<td>31-32</td>
<td>11.4</td>
<td>(-0.37, 0.66, 0.64)</td>
<td>3.9</td>
<td>3.70</td>
</tr>
<tr>
<td>32-33</td>
<td>12.1</td>
<td>(-0.27, 0.81, 0.53)</td>
<td>4.2</td>
<td>11.2</td>
</tr>
<tr>
<td><strong>Segment 3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35-36</td>
<td>4.1</td>
<td>(-0.25, 0.97, -0.02)</td>
<td>40.0</td>
<td>13.2</td>
</tr>
<tr>
<td>36-37</td>
<td>7.2</td>
<td>(-0.21, 0.97, -0.14)</td>
<td>16.4</td>
<td>7.68</td>
</tr>
<tr>
<td>37-38</td>
<td>7.3</td>
<td>(-0.29, 0.93, -0.23)</td>
<td>10.4</td>
<td>0.89</td>
</tr>
<tr>
<td>38-39</td>
<td>17.7</td>
<td>(-0.48, 0.77, -0.41)</td>
<td>7.6</td>
<td>17.4</td>
</tr>
<tr>
<td>39-40</td>
<td>18.2</td>
<td>(-0.22, 0.78, -0.59)</td>
<td>3.2</td>
<td>22.0</td>
</tr>
</tbody>
</table>
coherent structures and a 6-hour-long stretch of data at the trailing edge of the cloud where no coherency could be determined because the normals from minimum variance analysis were not reliable. The first coherent structure (Segment 1) was found to be between hour 19.1 (just downstream of the front cloud boundary) and hour 2900 UT. The normals in this segment were all well determined. The angle between contiguous normals did not exceed 21.3° and all the normals for this region lie in a cone of half angle ~23°. We find that a second coherent structure (Segment 2) occurred between the hours 2900 and 3400 UT. Again the normals are well defined and the normals in this segment lie in a cone half angle of ~11°. For hour 3400 to 3500 UT we found the normal deviated considerably from both preceding and following hour segments. After this interruption, a further coherent structure (Segment 3) was found between hours 3500 and 4100 UT. The normals for Segment 3 are well defined and they all lie in a cone half angle of ~22°. The average normals to each of the three coherent segments labeled $n_1$, $n_2$, and $n_3$ are listed in Table 4.5. The columns from left to right indicate the time intervals for the three coherent structures, the average normal vector for each segment, the designation of this vector in Figure 4.9, and $\gamma$ the angle between the normals of the

<table>
<thead>
<tr>
<th>Time, UT</th>
<th>Normal</th>
<th>Designation</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.1 - 29</td>
<td>(-0.19, 0.16, 0.92)</td>
<td>$n_1$</td>
<td>$\gamma_{12} = 35^\circ$</td>
</tr>
<tr>
<td>29 - 34</td>
<td>(-0.31, 0.67, 0.66)</td>
<td>$n_2$</td>
<td>$\gamma_{23} = 58^\circ$</td>
</tr>
<tr>
<td>34 - 41</td>
<td>(-0.28, 0.90, -0.24)</td>
<td>$n_3$</td>
<td>$\gamma_{13} = 91^\circ$</td>
</tr>
</tbody>
</table>
Figure 4.8: Magnetic field in principal axes coordinates for each of the three several-hour-long segments discussed in the text.
Figure 4.9: Orientation of the normals for the three coherent segments in a GSE coordinate system.
three segments where the angle $\gamma_{ij}$ between the normals to segment $i$ with segment $j$ ($i, j = 1, 2, 3$). These angles are $\gamma_{12} = 35^\circ$, $\gamma_{23} = 58^\circ$, and $\gamma_{13} = 91^\circ$. The field data for each of these segments are shown plotted in minimum variance coordinates in Figure 4.8. For each segment the total field, and the field components in the minimum variance coordinate system are plotted from top to bottom in Figure 4.8. Despite the three very different normals, the data plotted in the respective principal axes coordinates are in all three cases very satisfactory. The orientation of these normals in a GSE coordinate system is shown in Figure 4.9. If we were to carry out a minimum variance analysis on the data for the entire $\sim 29$ hours of the cloud interval in Figure 4.2, we would get a normal of $(0.96, 0.29, 0.00)$ with $\lambda_2/\lambda_3 = 5.6$, i.e., very different from the the normals to each of the three coherent segments. In summary, we may say that there is clear distortion in the October 1995 cloud. Forming the angle between the DD normals and the average normal to the segment where a given DD occurs, we find that all DD normals are closely aligned with the segment normal, the largest deviation being $\sim 20^\circ$ and occurring in Segment 1.

### 4.4 Discussion and Conclusions

From the high resolution magnetic field and plasma data we noticed a number of directional discontinuities for the October 18-19, 1995 magnetic cloud. Selecting discontinuities for which the field rotates larger than $15^\circ$ in $\sim 1$ min we applied field
and plasma tests to these selected discontinuities. The findings from the field tests showed the front cloud boundary to be a tangential discontinuity while all other discontinuities were found to be rotational. Tests which included plasma data confirmed some of the field results but not all. We did also find temperature jumps across the discontinuities indicative of further structure to these discontinuities.

Another magnetic cloud we examined in which there were RD’s is that on the Dec 24-25, 1996 (see Farrugia et al. Graz [1998]). We also looked for large-scale perturbations and suggested a method based on minimum variance analysis to search for coherent structures. We found evidence of three several-hour-long segments each having a well defined normal and the normals between segments were significantly different. If a minimum variance analysis is carried out for the whole cloud, the normal obtained is different from the normals found for the 3 segments. We found no evidence for a coherent structure in the last 6 hours of the October 1995 cloud and also between hours 3400 to 3500 UT between Segments 2 and 3. The normals of each segment do not appear to be aligned, and each segment normal is found to be different, with the normal obtained for the whole cloud which is found to be x direction.

Larson et al. [1996] using data from the Three-Dimensional Plasma and Energetic Particle Experiment on Wind detected particles produced from solar flares. They found that prior to 3100 UT electron heat flux energies of 118 and 290 eV were found streaming bidirectionally along magnetic field lines of the October 1995
magnetic cloud. At 3100 UT Larson et al. found that the bidirectional streaming was replaced by unidirectional streaming, and at 3400 UT there was a dropout of electrons in both these energy channels. From their observations, Larson et al. [1996] proposed that some lines of the bent flux rope model for a magnetic cloud are connected at both ends while others were connected at one end, and some disconnected from the Sun entirely. Larson et al. also noted that for the last 6 hours of the October magnetic cloud there were repeated connections and disconnections of the magnetic field lines. This is the same time period where we could not find any coherent structure. As noted earlier, the cloud is running into dense material ahead, and is being overtaken by faster material from behind.

We can conclude that there is evidence for large scale distortions of the static force-free Lundquist flux tube model for the October 18-19, 1995 magnetic cloud which may be the result of interactions with the surrounding plasma.
Chapter 5
Simulation and Results

5.1 Introduction

There has been an interest in understanding magnetic clouds, their motion in the solar wind, and their interaction with the Earth as obtained through measurements. To this end, simulations involving magnetic clouds play a part in adding to our understanding of magnetic clouds. Simulations involving propagation of magnetic clouds or flux tubes have been studied by various authors (e.g. Vandas et al. [1995], Vandas et al. [1996], Cargill et al. [1996]). Studies involving shock interactions with a magnetic cloud (Vandas et al. [1997]) have been carried out, in which the authors simulated an interaction of a shock wave overtaking a magnetic cloud that was propagating away from the Sun. They concluded that the magnetic field increased inside the cloud, the magnetic cloud was compressed in the radial direction, and the driven shock and the faster interplanetary shock merged together. Shock wave interactions with denser clouds, both gas dynamic clouds and magnetized clouds (not magnetic flux tube topology), have also been studied in the past (Bedogni et al. [1990], Dai et al. [1994]). The purpose for the simulations carried out here is to develop an understanding of strong shock interaction, like the Earth’s bow shock with a magnetic cloud. For the simulation, we represent a magnetic cloud as
a static force-free Lundquist flux tube. In the simulation, three constant density profiles were considered for the tube plasma. The strong fast shock chosen has a strength comparable to the Earth's bow shock. The simulations carried out here makes use of the ideas of Cargill et al. ([1996], [1995]) for the representation of the Lundquist flux tube and from Bedogni et al. [1990] and Dai et al. [1994] for the setup of a shock interacting with a magnetic cloud. The simulations are carried out in one and two and one half dimensions (2 1/2-D) and in a Cartesian coordinate system. In 2 1/2-D simulations, the simulation variables have no Z dependence but components like $U_z$ and $B_z$ are functions of $X$ and $Y$.

The outline of this chapter is as follows: (1) we discuss the normalization of the system of equations used in the simulation; (2) present the flux tube model used in the simulation; (3) establish the shock relations used to determine the initial conditions of the simulation; (4) present 1-D simulations results; and (5) 2 1/2-D simulation results; and (6) draw conclusions from the results.

### 5.2 Normalization

We begin first by renormalizing the system of equations which are to be solved numerically using the method discussed in Chapter 2. The reason for doing this is to avoid dealing with numbers that are small in the simulations, as solar wind parameters tend to be. For convenience, we repeat here the 1-D MHD equations
which will be solved written in the Lagrangian mass coordinate \((dm = \rho dx)\) frame:

\[
\frac{\partial}{\partial t} U + \frac{\partial}{\partial m} F(U) = 0
\]  

(5.1)

with

\[
U = \begin{bmatrix} V \\ u_x \\ u_y \\ u_z \\ V B_y \\ V B_z \\ E \end{bmatrix} \quad F(U) = \begin{bmatrix} -u_x \\ P \\ \Lambda_y \\ \Lambda_z \\ -B_x u_y \\ -B_x u_z \\ P u_x + \Lambda_y u_y + \Lambda_z u_z \end{bmatrix}
\]  

(5.2)

\(E = p/(\gamma - 1)\rho + \frac{1}{2}u^2 + \frac{1}{8\pi\rho}B^2\)

\[P = p + \frac{1}{8\pi}(B_y^2 + B_z^2 - B_x^2)\]

\[\Lambda_y = -\frac{1}{4\pi}B_x B_y\]

\[\Lambda_z = -\frac{1}{4\pi}B_x B_z\]

With \(V = 1/\rho\). The parameters are normalized as follows \(\tilde{t} = t/t_0, \tilde{x} = x/L_0, \rho_0, \tilde{U}_x = U_x/U_0, \tilde{U}_y = U_y/U_0, \tilde{U}_z = U_z/U_0, \tilde{B}_x = B_x/B_0, \tilde{B}_y = B_y/B_0, \tilde{B}_z = B_z/B_0, \tilde{\rho} = p/(B_0)^2\). The parameters with the subscripts "0" will be given explicit expressions below and these are constants. With the use of the above definitions, the partial derivatives with respect to time and the Lagrangian mass coordinate become

\[
\frac{\partial}{\partial \tilde{t}} = \frac{1}{t_0} \frac{\partial}{\partial \tilde{t}}
\]  

(5.3)
Using the above expression the continuity equation becomes

\[
\frac{1}{\rho_0 t_0} \frac{\partial 1/\tilde{V}}{\partial t} + \frac{1}{\rho_0 L_0} \frac{\partial}{\partial \tilde{m}}(-\tilde{U}_x U_0) = 0 .
\]  

(5.5)

By defining

\[ t_0 = \frac{L_0}{U_0} , \]

(5.6)

the non-dimensionalized continuity equation is obtained.

\[
\frac{\partial 1/\tilde{V}}{\partial t} + \frac{\partial}{\partial \tilde{m}}(-\tilde{U}_x) = 0
\]

(5.7)

Similarly, the X momentum equation becomes

\[
\frac{U_0}{t_0} \frac{\partial \tilde{U}_x}{\partial t} + \frac{B_0^2}{\rho_0 L_0} \frac{\partial}{\partial \tilde{m}}(\tilde{P}) = 0 .
\]

(5.8)

Using the previous definition for \( t_0 \), and defining

\[ U_0^2 = \frac{B_0^2}{\rho_0} , \]

(5.9)

the X momentum equation is written in dimensionless form as

\[
\frac{\partial \tilde{U}_x}{\partial t} + \frac{\partial}{\partial \tilde{m}}(\tilde{P}) = 0 .
\]

(5.10)

The Y momentum equation can be expressed as

\[
\frac{U_0}{t_0} \frac{\partial \tilde{U}_y}{\partial t} + \frac{B_0^2}{\rho_0 L_0} \frac{\partial \tilde{\lambda}_y}{\partial \tilde{m}} = 0 ,
\]

(5.11)
and for the Z momentum we have

\[ \frac{U_0}{t_0} \frac{\partial U_z}{\partial t} + \frac{B_0^2}{\rho_0 L_0} \frac{\partial \lambda_z}{\partial \tilde{m}} = 0. \]  

(5.12)

Making use of the definitions for \( U_0 \) and \( t_0 \), the \( Y \) and the \( Z \) momentum equations become

\[ \frac{\partial \tilde{U}_y}{\partial t} + \frac{\partial \hat{\lambda}_y}{\partial \tilde{m}} = 0 \]  

(5.13)

and

\[ \frac{\partial \tilde{U}_z}{\partial t} + \frac{\partial \hat{\lambda}_z}{\partial \tilde{m}} = 0. \]  

(5.14)

The \( Y \) magnetic flux equation becomes

\[ \frac{B_0}{\rho_0 t_0} \frac{\partial \tilde{V} \tilde{B}_y}{\partial t} - \frac{B_0 U_0}{\rho_0 L_0} \frac{\partial \tilde{B}_z \tilde{U}_y}{\partial \tilde{m}} = 0 \]  

(5.15)

and the \( Z \) magnetic flux equation is

\[ \frac{B_0}{\rho_0 t_0} \frac{\partial \tilde{V} \tilde{B}_z}{\partial t} - \frac{B_0 U_0}{\rho_0 L_0} \frac{\partial \tilde{B}_z \tilde{U}_z}{\partial \tilde{m}} = 0. \]  

(5.16)

Again, with the use of the definitions for \( U_0 \) and \( t_0 \), the equations result in

\[ \frac{\partial \tilde{V} \tilde{B}_y}{\partial t} - \frac{\partial \tilde{B}_z \tilde{U}_y}{\partial \tilde{m}} = 0 \]  

(5.17)

and

\[ \frac{\partial \tilde{V} \tilde{B}_z}{\partial t} - \frac{\partial \tilde{B}_z \tilde{U}_z}{\partial \tilde{m}} = 0 \]  

(5.18)

The total specific energy can be expressed in the normalized parameters as

\[ E = p/(\gamma - 1)\rho + \frac{1}{2} u^2 + \frac{1}{8\pi \rho} B^2 = \frac{B_0^2}{\rho_0} \tilde{E}. \]  

(5.19)
Using this, the energy equation becomes

\[ \frac{B_0^2}{\rho_0 \epsilon_0} \frac{\partial E}{\partial t} + \frac{B_0^2 U_0}{\rho_0 L_0} \frac{\partial (\hat{P} \hat{U}_x + \hat{\lambda}_y \hat{U}_y + \hat{\lambda}_z \hat{U}_z)}{\partial \hat{m}} = 0 \]  \hspace{1cm} (5.20)

and with the use of the expression for \( t_0 \), we can divide out the constants, and the energy equation becomes

\[ \frac{\partial E}{\partial t} + \frac{\partial (\hat{P} \hat{U}_x + \hat{\lambda}_y \hat{U}_y + \hat{\lambda}_z \hat{U}_z)}{\partial \hat{m}} = 0 \]  \hspace{1cm} (5.21)

The form of the equations in normalized parameters have the same structure as the original equations. All results presented in the numerical calculations are simulated using normalized initial conditions. The normalization parameters (which are representative of solar wind conditions) used in this numerical study are:

\[ B_0 = 5.0 \times 10^{-8} \text{ gauss} = 5 \text{nT} \] the ambient magnetic field

\[ \rho_0 = 8.363 \times 10^{-24} \text{ g cm}^{-3} \] \( \rho_0 \) corresponds to 5 particles \( \text{cm}^{-3} \)

\[ U_0 = \frac{B_0^2}{\rho_0^{1/2}} = 172.89 \text{ km s}^{-1} \]

\[ p_0 = B_0^2 = 0.25 \text{ nPa} \]

\[ R = 0.18 \text{ AU} \] is the radius of the magnetic flux tube

\[ L_0 = 100R/18 = 1 \] is the normalized width of the simulation zone

\[ t_0 = L_0/U_0 = 10.04 \text{ days} \]

### 5.3 Flux Tube

A static constant alpha force-free field is used to model the flux tube (Burlaga [1988], Cargill et al. [1996]) and in cylindrical coordinates it is the Lundquist
Figure 5.1: Magnetic field component $B_\theta$ of the Lundquist Flux Tube Model with helicity of -1.

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Figure 5.2: Magnetic field component $B_z$ of the Lundquist Flux Tube Model

$x_{\text{center}} = 0.8, \ y_{\text{center}} = 0.5$
$\beta_0 = 0.75, \delta = 0.5, \ B_\phi = 1.0$
$\alpha = 2.4, \ a_0 = 0.18, \ h = -1$

$B_x = \sqrt{(B_{x0}^2 + (B_{\phi0} a_0 a_0))^2} / \sqrt{(B_{\phi0} (1-h))}$
Figure 5.3: Total magnetic field component $B$ of the cylindrically symmetric Lundquist Flux Tube Model

$x_{center} = 0.8, y_{center} = 0.5$
$b_0 = 0.75, \delta = 0.5, B_0 = 1.0$
$a = 2.4, a_0 = 0.18, h = -1$

\[ B_z = h B_{0z} (\alpha_{0z}/a_0) \]
\[ B_z = \sqrt{B_{0z}^2 + (B_{0z} (\alpha_{0z}/a_0))^2} \]
\[ B_z = (B_{0z}/A_1(\alpha)) \sqrt{(b_0^2 (1-h))} \]
flux tube model. The ambient plasma that the flux tube is moving in is considered to have only a $B_z$ component for the magnetic field, direction parallel to the axis of symmetry of the tube. The assumptions made here for modeling the shock interaction with a magnetic cloud, though simplified, will give us an understanding of the interaction of a shock wave with a magnetic cloud. The mathematical representation for the fields in the magnetic cloud are:

$$B_\theta = B_f J_1(\frac{r}{a_0})$$

$$B_z = \sqrt{B_{z0}^2 + (B_f J_0(\frac{r}{a_0}))^2}$$

With $B_f$ a constant to be determined, $\alpha$ is obtained from the first zero of the zeroth order Bessel function $J_0$, and $a_0$ is the radius of the flux tube. With this representation for the flux tube, the field configuration is

$$B_{tube} = (0, B_\theta(r), B_z(r))$$

and the ambient field is

$$B_0 = (0, 0, B_{z0})$$

The constant $B_f$ (the axial field strength) is determined by requiring the tube be a total pressure balanced structure with respect to the ambient plasma. With these representation for the fields, the total pressure balance outside the tube is

$$p_\omega = p_0 + \frac{B_{z0}^2}{8\pi}$$
while inside the tube it is

\[ p_{tf} = p_f + \frac{1}{8\pi} \left[ (B_f J_1(x))^2 + B_{20}^2 + (B_f J_0(x))^2 \right] \]

with \( x = \alpha r / a_0 \). Equating \( p_{tf} = p_{i0} \) at the tube surface ( \( x = \alpha \) the constant \( B_f \) can be determined in terms of known initial conditions as follows

\[ B_f^2 = \frac{\left( p_0 - p_f \right)}{J_1(\alpha)^2} \]
\[ B_f = \frac{B_{20}}{J_1(\alpha)} \sqrt{\beta_0 (1 - \delta)} \]
\[ \delta = \frac{p_f}{p_0} = \frac{p_{\text{inside tube}}}{p_{\text{outside tube}}} \]
\[ \beta_0 = \frac{8\pi p_0}{B_{20}^2} \]

Figure 5.1 gives a vector plot of \( B_\theta \) in the X-Y plane for a field of helicity of -1. The physical domain of this plot is 1x1 and the number of cells used for this figure is 75x75. The ratio of the thermal pressure to the magnetic pressure, \( \beta_0 \), is equal to 1 with \( \delta = 0.5 \) and \( B_{20} = 1 \). One can see an anti-clockwise rotation of the field in the X-Y plane. The \( B_z \) magnetic field component shown in Figure 5.2 looks like an inverted bowl. The tube is immersed in an ambient field having only \( B_z = 1 \). The total magnetic field ( Figure 5.3 ) for the tube which is cylindrically symmetric, shows a jump in the magnetic field at the cloud boundaries and reaching a maximum at the center of the tube.
5.4 Shock Relations

Consider the general case of a shock propagating in a coordinate frame with speed $S$. The MHD shock relations (Kulikovskiy et al. [1965]) in a frame in which the shock is stationary are:

\[ B_{z1} = B_{z2} = B_n \]

\[ \rho_1(U_{x1} - S) = \rho_2(U_{x2} - S) \]

\[ B_n(U_{y1} - U_{y2}) = B_{y1}(U_{x1} - S) - B_{y2}(U_{x2} - S) \]

\[ B_n(U_{z1} - U_{z2}) = B_{z1}(U_{x1} - S) - B_{z2}(U_{x2} - S) \]

\[ \rho_1U_{y1}(U_{x1} - S) - \rho_2U_{y2}(U_{x2} - S) = \frac{B_n}{4\pi}(B_{y1} - B_{y2}) \]

\[ \rho_1U_{z1}(U_{x1} - S) - \rho_2U_{z2}(U_{x2} - S) = \frac{B_n}{4\pi}(B_{z1} - B_{z2}) \]

\[ p_1 + \rho_1(U_{x1} - S)^2 + \frac{B_{y1}^2 + B_{z1}^2}{8\pi} = p_2 + \rho_2(U_{x2} - S)^2 + \frac{B_{y2}^2 + B_{z2}^2}{8\pi} \]

\[ (U_{x1} - S)\rho_1[\varepsilon_1 - \varepsilon_2 + \frac{1}{2}(U_{y1}^2 - U_{y2}^2) + \frac{1}{2}(U_{z1}^2 - U_{z2}^2) + \frac{1}{2}(U_{x1} - S)^2 - \frac{1}{2}(U_{x2} - S)^2 \]

\[ + \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} + \frac{1}{4\pi}\left(\frac{B_{y1}^2 + B_{z1}^2}{4\pi} - \frac{B_{y2}^2 + B_{z2}^2}{4\pi}\right) \]

\[ = \frac{B_n}{4\pi}[(B_{y1}U_{y1} - B_{y2}U_{y2}) + (B_{z1}U_{z1} - B_{z2}U_{z2})] \]

The strength of the shock is given by the following relation.

\[ M_{\text{shock}} = \frac{|U_{x1} - S|}{C_f} \]

\[ C_f = \frac{1}{2}[(C_o^2 + C_a^2 + C_i^2) + \sqrt{(C_o^2 + C_a^2 + C_i^2)^2 - 4C_o^2C_i^2}] \]

\[ C_a^2 = B_{x1}^2/4\pi\rho_1 \]
\[ C_i^2 = \frac{(B_{y1}^2 + B_{z1}^2)}{4\pi\rho_1} \]
\[ C_o^2 = \frac{\gamma p}{\rho_1} \]

where 1 denotes downstream of shock and 2 upstream of shock. Consider a shock propagating through ambient plasma. A transformation is made in which the shock remains stationary and the ambient plasma is traveling towards the shock. After transformation into the rest frame of the shock, the shock speed in terms of a fast...
shock Mach number will be:

\[ S = U_{x1} + C_{fast} M_{shock} \]

5.4.1 Check on Relations

Consider a plasma with a field orientation of \( \mathbf{B} = (0, 0, B_1) \). The check is made on the solutions to the shock relations by those given in Ferraro et al. [1966] in Table 1 page 103. Their relation \( M^* \) in equation 4.60 can be rewritten as follows:

\[
M^* = \frac{N}{\sqrt{2Q + \gamma}}
\]
\[
N = \frac{\sqrt{\gamma} U_{z1}}{C_{o1}}
\]
\[
Q = \frac{B_1^2}{8\pi p_1} = \frac{1}{\beta_1}
\]
\[
C_{o1} = \frac{\gamma p_1}{\rho_1}
\]
\[
M^* = \frac{U_{z1}}{C_f}
\]

For infinitely strong shocks, the density jump across the shock

\[ \rho_2/\rho_1 = (\gamma + 1)/(\gamma - 1) \]

which for

\[ \gamma = 5/3 \]

is 4. Taking \( \gamma = 5/3 \), \( \rho_1 = 1 \), \( B_{x1} = 10^{-6} \), \( B_{z1} = 1 \) and \( p_1 = B_{z1}^2/8\pi Q \) we compare some results as a check on the algorithm for solutions downstream of the shock with those from Ferraro et al. [1966]: A non zero value is used for \( B_x \) so as to avoid

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any singularities in the numerical calculations of the MHD equations. The Mach number for the shock was chosen such that the components of the velocity behind the shock are small or nearly stationary. This was done primarily so as to avoid the use of a larger simulation region such that the flux tube still remains in the computational zone. Table 5.2 shows the initial conditions used in the simulations for the ambient and shocked plasma.

Table 5.2: Initial conditions of plasma downstream of shock and ambient plasma

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Downstream of $M_f = 11.215$ shock</th>
<th>Ambient Plasma</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>3.8831052</td>
<td>1</td>
</tr>
<tr>
<td>$p$</td>
<td>13.1038365</td>
<td>$1/8\pi$</td>
</tr>
<tr>
<td>$U_x$</td>
<td>-5.892092×10^{-4}</td>
<td>-3.18109</td>
</tr>
<tr>
<td>$U_y$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$U_z$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$B_x$</td>
<td>1.0×10^{-6}</td>
<td>1.0×10^{-6}</td>
</tr>
<tr>
<td>$B_y$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$B_z$</td>
<td>3.8831</td>
<td>1</td>
</tr>
</tbody>
</table>
5.5 Results of 1-D Simulations

The 1-D simulations using 800 computational zones were first carried out to obtain some understanding of the interaction of a shock with an idealized magnetic cloud before looking at the 2 1/2 D simulation results. The mesh size for the 1-D simulations were chosen so as to resolve any discontinuities (e.g. shock) present in the simulation. The axis chosen for the simulation is along the line running through the center of the tube in the $X$ direction, which will be referred to as the centerline $X$-axis of the tube. This line cuts the flux tube in half. Figure 5.6 gives the representation of the magnetic field along this line. The panels from the top to the bottom show the $B_x$, $B_y$, $B_z$ and the total field $B$. The $B_x$ component is taken to be zero inside and outside of the tube for the simulation. The $B_y$ component has a jump at the tube boundary and a rotation of $180^\circ$ in the field inside the tube. The $B_y$ component of the ambient plasma is zero. For the $B_z$ component the ambient value is 1 and in the flux tube it has a maximum at the center of the tube which decreases to 1 at the tube boundaries. The total field $B$ has a value of 1 in the ambient plasma with a jump in the field at the tube boundaries and rises to a maximum at the center of the flux tube. The tube has its center at 0.8. The positive $X$ direction in the plots will be from the viewers left to right. Three cases were considered: (1) Simulation 1 for $p_{\text{tube}}/p_{\text{ambient}} = 0.5$; (2) Simulation 2 for $p_{\text{tube}}/p_{\text{ambient}} = 1.0$; and (3) Simulation 3 for $p_{\text{tube}}/p_{\text{ambient}} = 2.0$. 
1-D Simulation Parameters

Shock Strength: $M_{fast} = 11.215$

$\gamma = 5/3, B_x = 1.0 \times 10^{-6}, B_{x0} = 1, \alpha = 2.4048$ and $\psi = 0 \text{ or } 180^\circ$.

$B_f = \frac{B_{x0}}{J_1(\alpha)} \sqrt{\beta_0(1 - \delta)}$

$\beta_0 = 1$

### 5.5.1 Simulation 1

The results of this simulation are shown in Figures 5.7-5.10. Figure 5.7 is a representation of the solutions at different times and are plotted on the same panel to give a time dependent image of the solutions as a function of time (see Forbes [1982]). The panels from the top to bottom show the density ($\rho$), thermal pressure ($P_{thermal}$), the $X$-component of the velocity ($U_x$) and the magnetic field components in the y and z direction ($B_y, B_z$). Each line in each panel for each variable is a numerical solution of the MHD equations at a different time. Each plot in the figure has been shifted upwards from the previous plot to create the time depen-

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Shocked State</th>
<th>Ambient State</th>
<th>Magnetic Tube</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>3.8831052</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>$p$</td>
<td>13.1038365</td>
<td>$1/8\pi$</td>
<td>$0.5/8\pi$</td>
</tr>
<tr>
<td>$U_x$</td>
<td>$-5.892092 \times 10^{-4}$</td>
<td>$-3.1810964$</td>
<td>$-3.1810964$</td>
</tr>
<tr>
<td>$U_y$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$U_z$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$B_y$</td>
<td>0</td>
<td>0</td>
<td>$B_fJ_1(\alpha_{x0})\sin(\psi)$</td>
</tr>
<tr>
<td>$B_z$</td>
<td>3.8831052</td>
<td>1</td>
<td>$\sqrt{B_{z0}^2 + (B_fJ_0(\alpha_{x0}))^2}$</td>
</tr>
</tbody>
</table>
Flux Tube along the axis

Figure 5.6: Magnetic field components and the total field along the centerline x-axis of a Lundquist flux tube.

- \( X_{\text{center}} = 0.8, \beta_0 = 1.0, \delta = 0.5, B_{z0} = 1.0 \)
- \( \alpha = 2.4, \alpha_0 = 0.18, \eta = 0 \) or \( 180 \)

\[
\begin{align*}
B_x &= B_{0x} J_1(\alpha r/r_0), \\
B_y &= \sqrt{B_{z0}^2 + (B_{0y}(\alpha r/r_0))^2}, \\
B_z &= (B_{z0}/J_1(\alpha)) \sqrt{\beta_0^2(1-\delta)}, \\
B &= B_0 \sin(\eta), \\
B_y &= B_0 \cos(\eta)
\end{align*}
\]
Figure 5.7: Stacked plots of the 1-D numerical simulation of Simulation 1 at different times (from top to bottom) for the density, thermal pressure, $U_x$ component of the velocity, $B_y$ and $B_z$ components of the magnetic field. Each plot of the solution have been shifted upwards from the previous solution result for the stacked effect. For example, for the density, each solution has been shifted by $40q$ from its previous plot, where $q=0.01$. The dashed line indicates the initial condition used for the problem.
Figure 5.8: Results of the 1-D numerical simulation of Simulation 1 at various time steps (from top to bottom and color coded) for the density, thermal pressure, total pressure and $\beta$.
Figure 5.9: Results of the 1-D numerical simulation of Simulation 1 at various time steps (from top to bottom and color coded) for the components of the velocity and the total velocity.
Figure 5.10: Results of the 1-D numerical simulation of Simulation 1 at various time steps (from top to bottom and color coded) for the components of the magnetic field and the total magnetic field.
dent image, for example, each density plot is shifted by 40q, where q=0.01. The $X$ coordinates for the panels is the non-dimensional computational domain of the simulation region, which is equal to 1. The positive $X$-axis in the plots is in the direction from 0 to 1.0. The dotted lines in Figure 5.7 indicate the initial conditions of the simulation. The magnetic flux tube is initially located in the region $0.62 \leq x \leq 0.98$ with the shock located at 0.6. The ratio of the density of the tube over the ambient plasma for this simulation is 0.5. The tube and the surrounding ambient plasma is propagating towards the shock from the viewers right to left with a speed of $\sim -3.18$. Figures 5.8-5.10 also plots of variables which are color coded to indicate the solutions at different times in the simulation. The panels in Figure 5.8 from top to bottom show the density ($\rho$), thermal pressure ($P_{\text{thermal}}$), total pressure ($P_{\text{total}}=P_{\text{thermal}}+P_{\text{magnetic}}$) and $\beta$ the ratio of thermal to magnetic pressure. In Figure 5.9 the panels from top to bottom show the components of the velocity ($U_x, U_y, U_z$) and the total velocity $U$. The magnetic field components ($B_x, B_y, B_z$) and the total field $B$ are shown in Figure 5.10.

When the initial fast shock interacts with the front tube boundary, an expansion wave propagates into the previously shocked plasma and a shock wave is transmitted through the flux tube. As a result of this interaction, the front tube boundary propagates in the positive $X$ direction. The whole region containing the expanded plasma and the shocked tube plasma is propagating towards the back boundary of the tube, which is unaware of any changes and still propagating at its original speed.
in the negative $X$ direction. The $B_y$ component of the magnetic field downstream of the transmitted shock increases in magnitude but still retains its orientation. One can clearly see an expansion wave reflected from the front tube boundary into the initially shocked ambient plasma as well as a transmitted shock in the tube for the $B_z$ component. The front boundary of the tube is still clearly defined after the shock interaction. When the transmitted shock in the tube meets the back boundary of the tube, a shock is reflected back into the tube and another shock transmitted into the ambient plasma trailing the tube from this boundary. The effect of this interaction is an increase in magnitude of the $B_y$ and $B_z$ component of the magnetic field in the tube. There is no noticeable change in the ambient $B_y$ component, but there is an increase in magnitude of the $B_z$ component of the magnetic field.

As a result of the shock interaction, the velocity of the tube back boundary has a small negative $U_x$ component after this interaction. At this time, the front boundary of the tube is propagating in the positive $X$ direction with an internal shock propagating towards it. When this internal shock meets the front boundary a shock can be seen to be transmitted into the previously expanded plasma and a weaker shock reflected back into the tube. There is no noticeable change in the $B_y$ magnetic field component of the tube, but there is a change for the $B_z$ component in the previously expanded ambient plasma. The back boundary, on meeting this reflected shock in the flux tube will again produce a reflected and transmitted

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shock in the tube and the previously shocked ambient plasma trailing the tube. An observation we can make is that the width of the tube is found to have decreased (an observation also seen by Vandals et al. [1997]) and the magnetic field still retains the rotation of $\sim 180^\circ$ for the $B_y$ magnetic field component. Also observed in the simulation is an increase in the magnitude for $B_z$ magnetic field component thus the total field an observation made by Vandals et al. [1997]. The magnetic field configuration is still similar in shape to the initial magnetic field components. The density profile of the resulting shock flux tube interaction in Figure 5.8 shows a bowl-like structure surrounded by shocked plasma, the boundaries of the tube are still clearly defined. This is also true for the thermal pressure and the $\beta$ parameter but they are not as deep a bowl-like structure as the density. The ratio of the density between the final and the initial tube is found to be greater than 4 (2.2/0.5) and this result can be explained if we consider multiple shock reflections off the tube boundaries which increase the density in the tube though with diminishing shock strengths. If one looks at the solution for a time $t \geq 0.2$, it appears the shock has propagated through the tube into the ambient plasma behind the tube, with an expansion and shock wave propagating ahead of the flux tube. The ambient plasma surrounding the tube approaches values downstream of the initial shock results.
5.5.2 Simulation 2

The ratio of the density of the tube over the ambient plasma for this simulation is 1.0. The results of the numerical simulation are shown in Figures 5.11-5.14.

Table 5.4: Initial conditions for simulation 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Shocked State</th>
<th>Ambient State</th>
<th>Magnetic Cloud</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>3.8831052</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( p )</td>
<td>13.1038365</td>
<td>( 1/8\pi )</td>
<td>( 0.5/8\pi )</td>
</tr>
<tr>
<td>( U_x )</td>
<td>(-5.892092 \times 10^{-4})</td>
<td>(-3.1810964)</td>
<td>(-3.1810964)</td>
</tr>
<tr>
<td>( U_y )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( U_z )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( B_y )</td>
<td>0</td>
<td>0</td>
<td>( B_f J_1(\alpha \omega) \sin(\psi) )</td>
</tr>
<tr>
<td>( B_z )</td>
<td>3.8831052</td>
<td>1</td>
<td>( \sqrt{B^2_{20} + (B_f J_0(\alpha \omega))^2} )</td>
</tr>
</tbody>
</table>

Figure 5.11 is plotted in the same format as Figure 5.7. The density, thermal pressure, \( U_x \) component of the velocity, \( B_y \) and \( B_z \) components of the magnetic field plotted at various times in the same panel. Figures 5.12-5.14 are also plotted as in Figures 5.8-5.10 and the variables are color coded to indicate the solutions at various times in the solutions. From Figure 5.11 we can observe a kink in the density and the \( B_z \) component of the magnetic field. This is due to the discontinuity in the initial conditions, where the shock and tube boundaries are spread over only 1 computational cell. This behavior in the solution can also be seen in the earlier figures describing the results of Simulation 1. Ignoring this, the effect of the shock on the Lundquist flux tube is to pass through the tube without generating any reflected waves (a shock or expansion wave) on interacting with the tube.
Figure 5.11: Stacked plots of the 1-D numerical simulation of Simulation 2 at different times (from top to bottom) for the density, thermal pressure, $U_x$ component of the velocity, $B_y$ and $B_z$ components of the magnetic field. Each plot of the solution have been shifted upwards from the previous solution result for the stacked effect. For example, for the density, each solution has been shifted by 40q from its previous plot, where q=0.01. The dashed line indicates the initial condition used for the problem.
Figure 5.12: Results of the 1-D numerical simulation of Simulation 2 at various time steps (from top to bottom and color coded) for the density, thermal pressure, total pressure and $\beta$.
Figure 5.13: Results of the 1-D numerical simulation of Simulation 2 at various time steps (from top to bottom and color coded) for the components of the velocity and the total velocity.
Figure 5.14: Results of the 1-D numerical simulation of Simulation 2 at various time steps (from top to bottom and color coded) for the components of the magnetic field and the total magnetic field.
boundaries. But with the passage of the shock through the front boundary, the velocity of the shocked ambient and tube plasma is \( \sim 0 \). But the back boundary of the tube is still propagating to the left at \(-3.181\). When this back boundary reaches the transmitted shock, the velocity of this boundary is \( \sim 0 \). Before this happens, however, the width of the tube has shrunk from its original size. The shock in this simulation basically treats the tube as if it were just ambient plasma with a different magnetic field configuration. The components of the magnetic field still retain the same shape as the initial field but with an increase in magnitude. The boundaries of the tube are still clearly defined after the shock passage. The other plasma variables such as density and thermal pressure are found to be slightly lower then the surrounding shocked ambient plasma. The ratio of the density between the final and the initial tube is found to be \( < 4 \). The beta in the tube region has increased but it is still lower then the surrounding shocked ambient plasma.

5.5.3 Simulation 3

The ratio of the density of the tube over the ambient plasma for this simulation is 2.0. The results of the numerical simulation for this set of initial conditions are shown in Figures 5.15-5.18 plotted in the same format as in the previous two simulations. Figure 5.15 also shows that when the shock meets the front tube boundary there is a shock reflected back into the previously shocked plasma and a shock transmitted into the tube. The resulting velocity of the front boundary is still in the negative \( X \) direction but with a much reduced magnitude. The back
Figure 5.15: Stacked plots of the 1-D numerical simulation of Simulation 2 at different times (from top to bottom) for the density, thermal pressure, \( U_x \) component of the velocity, \( B_y \) and \( B_z \) components of the magnetic field. Each plot of the solution have been shifted upwards from the previous solution result for the stacked effect. For example, for the density, each solution has been shifted by 40q from its previous plot, where q=0.01. The dashed line indicates the initial condition used for the problem.
Figure 5.16: Results of the 1-D numerical simulation of Simulation 3 at various time steps (from top to bottom and color coded) for the density, thermal pressure, total pressure and $\beta$.
Figure 5.17: Results of the 1-D numerical simulation of Simulation 3 at various time steps (from top to bottom and color coded) for the components of the velocity and the total velocity.
Figure 5.18: Results of the 1-D numerical simulation of Simulation 3 at various time steps (from top to bottom and color coded) for the components of the magnetic field and the total magnetic field.
Table 5.5: Initial conditions for simulation 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Shocked State</th>
<th>Ambient State</th>
<th>Magnetic Cloud</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>3.8831052</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>p</td>
<td>13.1038365</td>
<td>$1/8\pi$</td>
<td>$0.5/8\pi$</td>
</tr>
<tr>
<td>$U_x$</td>
<td>$-5.892092 \times 10^{-4}$</td>
<td>$-3.1810964$</td>
<td>$-3.1810964$</td>
</tr>
<tr>
<td>$U_y$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$U_z$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$B_y$</td>
<td>0</td>
<td>0</td>
<td>$B_f J_1(\alpha_{\alpha_0}^x) \sin(\psi)$</td>
</tr>
<tr>
<td>$B_z$</td>
<td>3.8831052</td>
<td>1</td>
<td>$\sqrt{B_{\alpha_0}^2 + (B_f J_0(\alpha_{\alpha_0}^z))^2}$</td>
</tr>
</tbody>
</table>

 flux tube boundary is still moving at its original speed and direction. When the transmitted shock in the tube meets the back boundary a shock is transmitted into the ambient plasma but an expansion wave is reflected from this boundary back into the tube. The result of this expansion wave is to reduce the density, thermal pressure, total pressure and the $B_y$ and $B_z$ magnetic field components in tube. The effect of the expansion wave on the velocity is to reduce its magnitude to $\sim 0$. The back flux tube boundary will be stationary from this time onwards in the simulation but the front flux tube boundary will still be moving to the left until the expansion wave reaches it. During this time, the width of the tube is expanding but it will not expand back to its original width. When this internal expansion wave hits the front tube boundary one can see an expansion wave transmitted into the previously shocked plasma ahead of the front tube boundary. There is no observable reflected expansion or shock wave in the tube. The ratio of the density between the final and the initial tube for this simulation is $\approx 3$. The magnetic field components in the

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tube increased in magnitude but still retained the original tube profile, that is, 180° rotation in the $B_y$ component and the hump like structure in the $B_z$ component after the shock passage.

For the three 1-D simulations we can draw the following conclusions. First we note that the only differences in the initial conditions of the 1-D simulations is in the density of the flux tube. In Simulation 1 the density is 0.5 while in Simulation 2 it is 1 and in Simulation 3 it is 2. This affects the value of the fast wave in the tube, that is,

$$C_f^2 = \frac{1}{2}[(C_\alpha^2 + C_a^2 + C_t^2) + \sqrt{(C_\alpha^2 + C_a^2 + C_t^2)^2 - 4C_\alpha^2C_a^2}]$$

$$C_\alpha^2 = B_{z1}^2/4\pi\rho_1$$

$$C_t^2 = (B_{y1}^2 + B_{z1}^2)/4\pi\rho_1$$

$$C_a^2 = \gamma p/\rho_1$$

In Simulation 1, the fast wave in the ambient plasma is lower then in the tube. For Simulation 2, they are approximately equal, while in Simulation 3, the fast wave is higher for the ambient plasma than in the tube. Consider a boundary which separates two different plasmas designated by 1 and 2. A shock will be considered to be traveling from 1 to 2 at an imaginary flux tube boundary. If the unshocked fast wave has $C_{fast,2} > C_{fast,1}$, an expansion wave is reflected from the boundary and a shock transmitted through. This is seen in Simulation 1 when the shock interacts with the front boundary and in Simulation 3 when the transmitted shock
interacts with the back boundary of the tube. For the case when $C_{fast,1} \approx C_{fast,2}$, a shock passes through the boundary without any waves reflected back from the boundary, as seen by the shock interaction at the front and back boundaries of the tube in Simulation 2. In the case when $C_{fast,2} < C_{fast,1}$, a shock is transmitted and reflected from the boundary. We can see this in Simulation 1, when the transmitted shock meets the back boundary of the tube, and in Simulation 3, when the shock interacts with the front tube boundary. Another observation we can make is that for all three cases the width of the tube is reduced when a shock has completely passed through the tube. The ratio of the density of the tube after all the interactions is found to be greater than 4, the maximum possible for an infinitely strong, single shock passing through a plasma for the case when the ratio of the initial flux tube density to the surrounding ambient plasma is 0.5. For the other two cases considered, the density for the flux tube after a fast shock interaction is $\leq 4$. The magnetic field topology for the flux tube is still retained after the shock interaction but with an increase in magnitude of the components. The boundaries of the flux tube have not eroded as a result of the fast shock interaction.

5.6 Results of 2 1/2-D Simulations

We extend the numerical simulations carried out above to 2 1/2-D. The simulations were all done using 200×200 computational zones, and the domain of the simulation is 1×1. The differences for the 3 simulations are in the initial conditions for the
Figure 5.19: Shaded and contour plots of the initial densities used in the 2 1/2-D simulations.
Figure 5.20: Shaded and contour plots of the initial velocity ($U_x$) and pressure used in the 2 1/2-D simulations.
Figure 5.21: Shaded and contour plots of the initial Lundquist flux tube magnetic field components $B_x$, $B_y$ and $B_z$ used in the 2 1/2-D simulations.
density of the Lundquist flux tube. The figures which show the initial conditions used in the 21/2-D simulations have a shaded and a contour plot. For an observer viewing the shaded plot, the $X-Y$ plane is at an angle into the figure. The $Z$ axis will be perpendicular to this plane in the direction from the bottom to top indicating the magnitude of the variable plotted. The positive $X$ axis for the shaded plots will be from the viewer's left to right. The $X-Y$ plane indicates the computational cells used in the simulation. For the contour plots the positive $X$ axis is also in the direction from an observer's left to right when viewing the plots and the positive $Y$ axis will be along a line from the bottom to the top of the viewed figure. Figure 5.19 shows the initial density for the three cases considered, and the shock is represented by the sharp drop in the shaded plots, while in the contour plots it is located on the right half of the figure as a band of straight lines. This representation of the shock as straight lines will also be true for the other variables such as pressure, velocity and the magnetic field components. The top plot in Figure 5.19 represents the case when the density in the Lundquist flux tube is one half that of the surrounding ambient plasma which has been normalized to be 1. The middle two plots represent the case when the densities in the tube and surrounding ambient plasma are equal. The last two plots at the bottom of the figure refer to the case when the tube density is twice that of the surrounding ambient plasma. Downstream of the shock the plasma has a density of $\sim$3.88. The tube and surrounding ambient plasma are traveling in the negative $X$ direction with a normalized speed of $\sim$3.18 (see Figure
5.20 top panel). This will be from right to left if one is looking down at the figure. There is no $U_y$ and $U_z$ components to the velocity field. Downstream of the shock the velocity $U_x$ is of the order of $10^{-4}$. Also plotted in the bottom panels in Figure 5.20 are the pressure profile at $t=0$. The pressure in the ambient plasma is taken as $1/8\pi$ and in the tube it is $0.5/8\pi$. The contour plots do not show this clearly, but if one looks closely at the shaded plot for the pressure one can see a circle surrounding a slight depression in the figure which is boundary of the tube. The normalized pressure downstream of the shock is $\sim 13.1$. The initial magnetic field components are shown in Figure 5.21. The top plot in the figure is the $B_x$ component, the middle plot the $B_y$ component and the bottom plot is the $B_z$ component of the magnetic field. Looking at the $B_x$ and $B_y$ magnetic field plots one can see that the $B_y$ component has the same magnitude and rotation as the $B_x$ component except that the $B_x$ components have been rotated counterclockwise by $90^\circ$ about the $Z$ axis. Along the centerline $X$-axis of the tube (which is the line drawn parallel to the $X$ axis from the $100^{th}$ $Y$ computational cell) the $B_y$ component of the magnetic field rotates $180^\circ$. For the $B_x$ component of the magnetic field the rotation of $180^\circ$ occurs in the $Y$ direction. There are no components for the magnetic field in the $X$ and $Y$ direction for the ambient plasma and also downstream of the shock. For the $B_z$ component plotted at the bottom of Figure 5.21 the Lundquist tube can be distinguished from the ambient plasma by the hill or mound-like structure in the figure. The $B_z$ component ambient plasma surrounding the tube is set at 1
and downstream of the shock $B_z$ is $\sim 3.88$. Again the only difference with the three initial conditions for the numerical simulations are the density ratio of the Lundquist flux tube with respect to the surrounding ambient plasma.

For the discussion that follows Case 1 will refer to the set of initial conditions as in Simulation 1 for the 1-D case, that is, with $\rho_{tube}/\rho_{ambient} = 0.5$; Case 2 as in Simulation 2 will be for initial conditions of $\rho_{tube}/\rho_{ambient} = 1$; and Case 3 will be for $\rho_{tube}/\rho_{ambient} = 2$ as was the case for the 1-D results of Simulation 3. The plots of the simulation results in each figure are shown from different views for each variable. Beginning with the top left plot, this shaded plot is in the same view used to show the initial conditions of the simulation. The top right plot in the figure is a view in the direction an observer has way downstream of the initial shock and flux tube configuration. The middle left plot is a contour plot of the simulation corresponding to the top left shaded plot. The middle right plot is a view an observer has looking at the results in the opposite direction (from inside the page looking out) of the top left plot. The bottom left plot is from a direction upstream of the shock and tube, that is, as seen from a position in the ambient plasma.

### 5.6.1 Case 1

Figure 5.22 - 5.29 are plots of the density, pressure, components of the velocity $(U_x, U_y, U_z)$, components of the magnetic field $(B_x, B_y, B_z)$ at $t=0.17$. The results of this simulation will be discussed in the following order: (1) the reflected expansion
Figure 5.22: Density plots for Case 1 at $t=0.17$. See text for description of the various views.
Figure 5.23: Pressure plots for Case 1 at $t=0.17$. See text for description of the various views.
Figure 5.24: $U_x$ velocity component plots for Case 1 at $t=0.17$. See text for description of the various views.
Figure 5.25: $U_y$ velocity component plots for Case 1 at $t=0.17$. See text for description of the various views.
Figure 5.26: $U_z$ velocity component plots for Case 1 at $t=0.17$. See text for description of the various views.
Figure 5.27: $B_x$ magnetic field component plots for Case 1 at $t=0.17$. See text for description of the various views.
Figure 5.28: $B_y$ magnetic field component plots for Case 1 at $t=0.17$. See text for description of the various views.
Figure 5.29: $B_z$ magnetic field component plots for Case 1 at $t=0.17$. See text for description of the various views.
wave; (2) the plasma downstream of the reflected expansion wave and the front boundary of the flux tube (Region 1); (3) the flux tube itself covering $[115-145] \times [60-140]$ computational cells in the contour plots (Region 2); (4) The plasma from the back boundary of the flux tube to the transmitted shock; and (5) the transmitted shock.

**Reflected Expansion Wave**

Looking at Figure 5.22 for the density the top right plot from the viewers left to right one comes across a V shaped structure. This structure is due to the discontinuity of the initial conditions used in the simulation where the shock and the boundaries of the tube are spread over 1 computational cell. In the contour plot this V shaped structure is seen as two straight lines running parallel to the y-axis located near the $30^{th}$ computational cell in the $X$ direction. Right after this structure one sees the cylindrical expansion wave. The interaction of the initial shock with the front tube boundary produces a reflected cylindrical expansion wave propagating in the negative $X$ direction (to the viewers left) downstream of the initial shock and ahead of the front tube boundary. This expansion wave is seen in the contour plot as the further most curved line. The expansion wave is also seen in the figures for the pressure (Figure 5.23), velocity components $U_x$ and $U_y$ (Figures 5.24 - 5.25) and the $B_z$ magnetic field component (Figure 5.29).
Region 1

This region covers the plasma downstream of the expansion wave and the front boundary of the flux tube which in the contour plot is located between 30 – 100th X computational cells (see Figure 5.22). The density is lower than values upstream of the expansion wave and the plasma in this region has a positive velocity component in the X direction and a symmetric structure along the centerline X-axis of the tube (which is the line drawn parallel to the x-axis from the 100th Y computational cell) as seen in Figure 5.24. The plasma here has a small $U_z$ velocity component (maximum magnitude of ~0.04, see Figure 5.26) and has an antisymmetric structure about the centerline X-axis of the tube. From this structure for $U_z$ we can infer that the tube plasma below the tube centerline X-axis is moving in the negative Z direction while the plasma above this axis are moving in the positive Z direction. In the $U_y$ velocity component (Figure 5.25) one can see in this region an antisymmetric structure about the centerline X-axis of the tube. The plasma below the centerline X-axis of the tube the plasma has a positive $U_y$ velocity component and for the plasma above this axis it has a negative $U_y$ velocity component. From the velocity profiles for $U_x$ and $U_y$ the plasma is moving to fill the void left by the flux tube whose front boundary has been pushed inwards but with a slight shear in the Z direction. As expected, there are no corresponding $B_x$ and $B_y$ magnetic field components observed in this region (see Figures 5.27-5.28).

For the $B_z$ component (Figure 5.29) of the magnetic field, there is a reduction in
magnitude due to the expansion wave. Near the 90th X computational cell a shock can be seen propagating away from the front tube boundary in the density, pressure and the $B_z$ component of the magnetic field. From the results of Simulation 1 this would be the transmitted shock due to an internal shock (that was reflected earlier from the back tube boundary from the initial transmitted shock) interaction with the front tube boundary.

**Region 2 (Flux Tube)**

The flux tube can now be seen as a bag-like structure covering [115-145] x [60-140] computational cells in the Figure 5.22 of the density contour plot. The tube is still symmetrical along the centerline $X$-axis of the tube but not along its initial centerline $Y$-axis. The 'cup' like structure seen in the pressure contour plot (Figure 5.23) without the 'handle' is the tube. Ahead of the 'cup', to the viewer's left, is the shock propagating away from the front tube boundary. From the $U_x$ velocity plot (Figure 5.24) the outer edges of the tube (furthest edges perpendicular to the centerline $X$-axis of the tube) has a negative component (moving to the viewer's left) while the middle of the tube has a positive $U_x$ component. The tube plasma has a $U_y$ component that is moving away from the centerline $X$-axis of the tube. With the two motions combined, we can say that there is a flow of plasma away from the center $X$-axis of the tube towards the edges of the tube in the $Y$ direction followed by motion in the positive $X$ direction at these outer edges. It appears that
the tube plasma has to travel around the expanded plasma upstream of the front boundary. The $B_x$ and $B_y$ component’s of the magnetic field (Figures 5.27-5.28) for the tube still rotate from negative to positive after the interaction with the shock. For the $B_x$ component (Figure 5.27) the rotation is no longer smooth, as seen by the presence of an additional structure near the front boundary of the tube. In the contour plot for the $B_x$ component this additional structure is seen as the islands just before the first series of closely drawn contour lines. These islands are antisymmetric about the centerline $X$-axis of the tube. The $B_x$ field component in the tube appears to be more dense for regions away from centerline $X$-axis of the tube, that is, pushed away from the middle. The $B_y$ magnetic field component (see Figure 5.28) is found to be compressed in the $X$ direction. Looking at the $B_y$ field component along the centerline $X$-axis of the tube starting from the viewers left to right, the field starts at zero at the front boundary drops to a minimum negative value and then rises rapidly to a positive maximum value only to drop to zero at the back boundary of the tube. At the outer edges away from the centerline $X$-axis of the tube and at the front boundary we can see additional structures in which the field $B_y$ has a negative component. The $B_y$ component of the field still rotates but through a much smaller region and thus more rapidly when compared to the initial $B_y$ field configuration. In the tube, the $B_z$ component has a positive magnitude (Figure 5.29) and compressed in the $X$ direction and curved at the back boundary of the tube.
Region 4 (Back tube boundary to transmitted shock)

The view of the bottom left plots in each figure are from a direction a viewer has if the person were standing at a point in the ambient plasma that lies on a line that passes through the centerline X-axis of the tube. Shocks can be seen propagating away from the back boundaries (top and bottom) of the tube but with speeds no faster than the transmitted shock (for example see Figure 5.29). These shocks appear to be connected to the transmitted shock (see below). In the region between the shocks which includes the centerline X-axis of the tube the plasma has a positive $U_x$ component near the back tube boundary, which approaches zero as one travels towards the transmitted shock with $U_y$ components that are moving away from the centerline X-axis of the tube. Beyond the shocks, the plasma is moving away from the centerline X-axis of the tube but with a negative $U_x$ component. This indicates that the plasma downstream of the back boundary of the flux tube is traveling around the tube and not penetrating it. The density in the region between the shocks are lower than those beyond the shocks. This observation is also true for the thermal pressure and the $B_z$ magnetic field component.

Transmitted Shock

The shock that has been transmitted through the tube can be seen in the contour plots as the dark band of closely spaced lines near the 150th X computational cell. For the shaded plots like the density this transmitted shock is seen as a sharp drop.
in magnitude to the viewer’s right. The portion of the shock that passed through
the tube is found to have traveled further hence faster than for the portion of the
shock that had passed the ambient plasma. This accounts for the curved structure
seen for the transmitted shock.

5.6.2 Case 2

This is the case where the density in the tube is equal to the surrounding ambient
plasma. The magnetic field components are still described by the Lundquist flux
tube model. Figures 5.30-5.37 are plots of the simulation for the density, pressure,
velocity components \((U_x, U_y, U_z)\) and the magnetic field components \((B_x, B_y, B_z)\)
at \(t=0.13\).

Looking at the density plot (Figure 5.30) the shock is seen to have passed through
the tube with the magnitude of the fluid and magnetic variables in the tube and
the surrounding plasma downstream of the shock approximately equal. One can
see two v-shaped structures which are due to discontinuities of the initial shock
and front tube boundary. The transmitted shock can be seen around the 137\(^{th}\) \(X\)
computational cell and it is straight, that is, no curvature is seen to the portion
of the shock that had passed through the tube. This would indicate that there is
no difference in speed for the portion of the shock that traveled through the tube
from the portion of the same shock that traveled through the surrounding ambient

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Figure 5.30: Density plots for Case 2 at $t=0.13$. See text for description of the various views.
Figure 5.31: Pressure plots for Case 2 at t=0.13. See text for description of the various views.
Figure 5.32: $U_x$ velocity component plots for Case 2 at $t=0.13$. See text for description of the various views.
Figure 5.33: $U_y$ velocity component plots for Case 2 at $t=0.13$. See text for description of the various views.
Figure 5.34: $U_z$ velocity component plots for Case 2 at $t=0.13$. See text for description of the various views.
Figure 5.35: $B_z$ magnetic field component plots for Case 2 at $t=0.13$. See text for description of the various views.
Figure 5.36: $B_y$ magnetic field component plots for Case 2 at $t=0.13$. See text for description of the various views.
Figure 5.37: $B_z$ magnetic field component plots for Case 2 at $t=0.13$. See text for description of the various views.
plasma. A similar observation can be made for the pressure (Figure 5.31). The pressure in the tube is slightly lower than the surrounding plasma after the shock passage. The $U_x$ component in the tube is found to have the same magnitude as in the surrounding plasma, which is close to zero. There are small non-zero components for $U_y$ and $U_z$, which implies there is motion in the $Y$ and $Z$ direction for the tube but its magnitude is small. There is an antisymmetric structure seen about the centerline $X$-axis of the tube for $U_y$ and $U_z$. From Figure 5.34 for $U_z$, the bottom half of the tube centerline $X$-axis has a motion in the negative $Z$ direction while above this axis the motion of the plasma is in the positive $Z$ direction. As with Case 1 the tube is distorted as a result of the shock passage but again the boundaries are still clearly defined. From the magnetic field components ($B_x$, $B_y$, $B_z$) plotted we find that the tube has been squeezed in the $X$ direction and the rotation in the $B_x$ and $B_y$ components of the magnetic fields are still present after the shock has passed through the tube. There are no added structures to be seen as in Case 1 for the $B_z$ and $B_y$ magnetic field components. The maximum values and rapid changes for $B_x$ are to be found near tube boundaries away from the centerline $X$-axis of the tube. The magnitude of the $B_y$ component of the magnetic field is enhanced, with a sharper descent to zero found at the back boundary of the tube. From the plot of the $B_z$ component of the magnetic field (Figure 5.37) we can see that the magnitude has been increased by the shock passage but the overall shape (hill structure) is still the same though compressed in the $X$ direction. For
this case, the results of the simulation would indicate that the initial shock passed through the tube without generating any expansion or shock waves as it crossed the boundaries of the tube. The end result was for the tube to be squeezed in the $X$ direction with an increase in magnitude for the density, pressure, magnetic field components ($B_x, B_y, B_z$) and a small velocity field in the $Y$ and $Z$ direction.

5.6.3 Case 3

This simulation is for the flux tube whose initial density is twice that of the surrounding ambient plasma. Figures 5.38 - 5.45 are the results of the simulation of the density, thermal pressure, velocity components ($U_x, U_y, U_z$) and magnetic field components ($B_x, B_y, B_z$) at $t=0.17$. The results of this simulation will be discussed in the following order: (1) the reflected shock wave from the front tube boundary; (2) the plasma downstream of the reflected shock wave and the front boundary of the flux tube (Region 1); (3) the flux tube itself covering $[85-120] \times [60-140]$ computational cells in the contour plots (Region 2); (4) The plasma from the back boundary of the flux tube to the transmitted shock; and (5) the transmitted shock.
Figure 5.38: Density plots for Case 3 at $t=0.17$. See text for description of the various views.
Figure 5.39: Pressure plots for Case 3 at $t=0.17$. See text for description of the various views.
Figure 5.40: $U_x$ velocity component plots for Case 3 at $t=0.17$. See text for description of the various views.
Figure 5.41: $U_y$ velocity component plots for Case 3 at $t=0.17$. See text for description of the various views.
Figure 5.42: $U_z$ velocity component plots for Case 3 at $t=0.17$. See text for description of the various views.
Figure 5.43: $B_x$ magnetic field component plots for Case 3 at $t=0.17$. See text for description of the various views.
Figure 5.44: $B_y$ magnetic field component plots for Case 3 at $t=0.17$. See text for description of the various views.
Figure 5.45: $B_z$ magnetic field component plots for Case 3 at $t=0.17$. See text for description of the various views.
Reflected Shock Wave

In the density plot (Figure 5.38) is a cylindrical reflected shock propagating away from the front tube boundary towards the viewer's left. The leading edge of this shock is seen as the first curved line in the contour plot for the density.

Region 1

This is the region between the cylindrical reflected shock wave and the tube front boundary and ranges from the 25\textsuperscript{th} to the 80\textsuperscript{th} X computational cells. There is an increase in magnitude of the density and pressure (Figure 5.39) due to the shock passage, while the $U_x$ component of the velocity (Figure 5.40) indicates the plasma has a negative component, i.e., the plasma in this region is traveling to the viewer's left. For the $U_y$ component of the velocity field (Figure 5.41), the plasma in this region is moving away from the centerline X-axis of the tube. There is no $U_z$ component for the plasma in this region. The net effect of the velocity field in this region is for the plasma to be propagating outwards (away) from the front tube boundary. In this region the $B_x$ and $B_y$ magnetic field are zero (see Figures 5.43-5.44). However, looking at Figure 5.45 for the $B_z$ component of the magnetic field, we can see an increase for this region downstream of the reflected shock and the front boundary of the tube.
Region 2 (Flux Tube)

The flux tube located between [85-120]×[60-140] computational cells is seen clearly in the density contour plot but it has been distorted and its width in the X direction reduced from its original shape by the shock passage but still having clearly defined boundaries. A mushroom structure comes to mind when viewing the tube contour profile. From the $U_x$ velocity field plot (Figure 5.40) the plasma in the tube that is furthest away from the centerline X-axis of the tube has a positive $X$ component while in between the plasma in the tube has a negative $X$ component. The plots for $U_y$ (Figure 5.41) has a complicated structure but overall one can see that the tube plasma is moving away from the centerline X-axis of the tube. There is a small $U_z$ (Figure 5.42). Below the centerline X-axis of the tube the plasma in the tube has a negative $U_z$ component while above this axis it has a positive $U_z$ component, indicating that the tube plasma is being pulled in opposite directions about this axis. The magnitude of the density (Figure 5.38) in the tube is higher than the surrounding plasma, and the opposite is true for the pressure (Figure 5.39), i.e., it is lower in the tube than the surrounding plasma. We also find that the tube plasma pressure is lowest in regions at the front boundary and in the regions of the tube furthest away from the tubes centerline X-axis in the Y direction. This band of low pressure indicates the passage of the expansion wave propagating away from the front tube boundary. The magnetic field component $B_x$ (Figure 5.43) is still bipolar, going from negative to positive in the tube, but there is some
complicated structure seen at the back boundary of the tube. In the contour plot these complicated structures are the closely spaced lines indicating there are rapid changes in the $B_x$ component of the magnetic field taking place in these regions. The $B_x$ structure in the tube is still antisymmetric after the shock passage. For the $B_y$ component (Figure 5.44), the original field is found to be compressed in the $X$ direction, more so in the region of the tube with the positive magnitude. The rotation from negative to positive is rapid as seen in the $B_y$ closely lines at the back boundaries of the tube. The $B_z$ magnetic field component has lower positive values (the surrounding plasma) around the front boundaries of the 'mushroom' tube and rises to values higher than the surrounding plasma. As was noted earlier, this region of lower $B_z$ values indicates an expansion wave propagating away from the front tube boundary.

**Region 4 (Back tube boundary to transmitted shock)**

The plasma in the region between the transmitted shock (where the shock bulges in the positive $X$ direction) and the back boundary of the tube has positive $U_x$ (Figure 5.40). This region will be referred to as Region A in further discussions. The plasma outside Region A but still in Region 4, referred to as Region B for further discussion, has a negative $U_x$ velocity component. In both these regions the plasma has a $U_y$ component (see Figure 5.41) directed towards the centerline $X$-axis of the tube. $U_z = 0$ in both these regions (Figure 5.42). The density of
the plasma in Region A is lower when compared to the density in Region B (see bottom left plot in Figure 5.38). As was noted in Simulation 1, the shock travels faster in the region with the lower density which would account for the bulge in the positive direction for the transmitted shock, discussed below. The thermal pressure (Figure 5.39) in Region A is lower than in Region B. There are no magnetic field components for $B_x$ and $B_y$ (see Figures 5.43 - 5.44). For the $B_z$ component of the magnetic field (Figure 5.45) one finds that Region B has a higher magnitude than Region A which when added with the thermal pressure would imply that the plasma in Region A is being squeezed towards the centerline $X$-axis of the tube and as a result of this the plasma is jetting out in the positive $X$ direction.

**Transmitted Shock**

The initial shock was transmitted through the flux tube can be seen in the contour plots of the figures as a series of closely spaced lines located near the 150th $X$ computational cells. An interesting feature for this transmitted shock which differs from the two previous simulations is that the portion of the shock that traveled in the tube moved slower then the portion of the same shock that had traveled in the ambient plasma. This would account for the curvature in the shock profile in the contour plots. The density of the plasma in Region A is lower when compared to the density in Region B. As was noted in Simulation 1, the shock travels faster in the region with the lower density, and this would account for the bulge in the
positive direction for the shock that transmitted through the tube.

5.7 Conclusion

From the 1-D and the 2 1/2-D simulation results we can conclude the following for interactions of shock with a Lundquist flux tube boundary:

1. $\rho_{\text{tube}} < \rho_{\text{ambient}}$

An expansion wave is reflected and a shock transmitted into the tube when the initial shock interacted with the front tube boundary. A shock can be seen propagating away from the front tube boundary and this is the result of the initial transmitted shock interacting with the back tube boundary. This interaction will generate a reflected shock in the tube which, on meeting the front tube boundary, is transmitted into the previously expanded plasma. The transmitted shock travels faster in the tube than in the surrounding ambient plasma. The width of the tube is also reduced in the direction of shock propagation. The edges of the tube (regions perpendicular to the centerline $X$-axis of the tube) and curved in the direction opposite of the shock propagation direction. The rotations in the magnetic fields are still preserved but with added structures at the front boundaries of the tube. There is a much flatter profile to the $B_z$ component as if one had taken the initial $B_z$ profile, squeezed it from both sides in the $X$ direction and curved the back boundary to give it triangular structure.
\( (2) \ \rho_{\text{tube}} = \rho_{\text{ambient}} \)

No reflected waves are seen. The width of the tube is also reduced in the direction of shock propagation. The edges of the tube (regions perpendicular to the centerline x-axis of the tube) are found not to be curved. The rotations in the magnetic fields are still preserved and no added structures are to be found at the front or back boundaries of the tube. There is a much flatter profile to the \( B_z \) component as if one had taken the initial \( B_z \) profile, squeezed it from both sides in the \( X \) direction.

\( (3) \ \rho_{\text{tube}} > \rho_{\text{ambient}} \)

A shock is reflected and transmitted from the boundary when the initial shock interacted with the front tube boundary. The transmitted shock travels slower in the tube than in the surrounding ambient plasma. Also noticed at \( t=0.17 \) there is an expansion wave propagating away from the front tube boundary. This expansion wave is the result of the transmitted shock interacting with the back tube boundary producing a reflected expansion wave in the tube. On interacting with the front tube boundary this reflected expansion wave, produces a transmitted expansion wave into the previously shocked plasma. The width of the tube is reduced in the direction of shock propagation. The edges of the tube (regions perpendicular to the centerline \( X \)-axis of the tube) are curved in the direction of shock propagation. The plasma inside and outside the tube have velocity components in the \( X-Y \) plane that
‘conform’ to the geometry of the tube and the surrounding plasma. The rotations in the magnetic fields are still preserved but with added structures at the back boundaries of the tube. There is a much flatter profile to the $B_z$ component as if one had taken the initial $B_z$ profile, squeezed it from both sides in the $X$ direction and flattened the top of the resulting structure.
Chapter 6

Observation of A Magnetic Cloud-Shock Interaction

6.1 Introduction

The purpose of this chapter is to correlate aspects of the numerical results of Chapter 5 with an observation involving a shock with a magnetic cloud. A number of magnetic clouds have been observed by WIND (see examples given in Chapter 2) upstream of the Earth. Since the Earth is an obstacle to the flow of the supersonic and superalfvenic solar wind a bow shock will be present upstream of the Earth to slow down the solar wind plasma so it may flow past the Earth (see Spreiter et. al. [1966]). Typically the subsolar point of the Earth’s bow shock is $\sim 14 \, R_E$ upstream of the Earth Fairfield [1971]. A magnetic cloud was observed by WIND from October 18-20, 1995 approximately $176 \, R_E$ upstream of Earth and GEOTAIL, another member of the Global Geospace Mission spacecraft family. During this time, GEOTAIL had a dawn to dusk orbit upstream of the Earth and managed to observe portions of the October magnetic cloud interacting with the bow shock of the Earth since GEOTAIL was alternatively in the magnetosheath and in the solar wind. It is not our intent to establish a 1-1 correspondence of the results from the numerical simulations with observations but rather to make use the results of the
simulations from Chapter 5 to: (1) discuss points where they agree: (2) discuss any disagreements or problems with matching observation with numerical results: and (3) recommend future studies of shock-magnetic cloud interactions.

6.2 Observations

Taken from the Web page “http://www-istp.gsfc.nasa.gov/istp/geotail/geotail.html” the mission statement for GEOTAIL reads as follows “The GEOTAIL mission is a collaborative project undertaken by the Institute of Space and Astronautical Science (ISAS) and the National Aeronautics and Space Administration (NASA). Its primary objective is to study the dynamics of the Earth’s magnetotail over a wide range of distance, extending from the near-Earth region (8 Earth radii (Re) from the Earth) to the distant tail (about 200 Re). The GEOTAIL spacecraft was designed and built by ISAS and was launched on July 24, 1992.” The GEOTAIL data plotted in this chapter comes from two instruments, the Comprehensive Plasma Instrument (CPI) instrument for the plasma data and the Magnetic Fields (MGF) instrument for the magnetic field data plotted in GSE coordinates. Information on these instruments can be found from the ISTP Science Planning and Operations Facility Web page at “http://www-istp.gsfc.nasa.gov/istp/geotail/geotail_inst.html”. In the GSE coordinate system the X-axis is pointing from the Earth towards the Sun and the Y-axis is in the ecliptic plane pointing towards dusk (opposite planetary motion) with the Z-axis parallel to the ecliptic pole. Figure 6.1 is a plot
Figure 6.1: WIND and GEOTAIL data with WIND data lagged by 0.525 hours of the October 18-19, 1995 magnetic cloud. The magnetic cloud front boundary is at 1940 UT. The blue and red trace represent data from GEOTAIL and WIND respectively.
showing both GEOTAIL and WIND spacecraft proton plasma and magnetic field data in GSE coordinates. The data from the WIND spacecraft was lagged by 0.525 hours so as to align the ‘shock’ like feature Lepping et. al. [1997] observed in the magnetic cloud data and seen by both spacecraft when they were in the solar wind. By doing this, the interplanetary data for the October 18-19, 1995 magnetic cloud from the WIND spacecraft is overlapped with the same cloud data observed by the GEOTAIL satellite. The panels from top to bottom in the figure show the total magnetic field $B$ (nT), the components of the magnetic field $B_x$, $B_y$, $B_z$ (nT), proton density $n$ (cm$^{-3}$) and the proton plasma bulk flow $V_p$ (km s$^{-1}$). The blue trace in the figure represents data from GEOTAIL and the red trace is data from WIND. At times when the red and blue trace agree, GEOTAIL is in the solar wind, and where they disagree GEOTAIL is downstream of the Earth’s bow shock. The horizontal axis shows the time measured from 00 UT from October 18, 1995. During October 18-19, 1995 the position of the spacecraft GEOTAIL in GSE coordinates is as listed in Table 6.1 below. Table 6.2 gives the estimated region occupancy for GEOTAIL during October 18-19, 1995. Information on the position and region occupancy was obtained from ISTP Science Planning and Operations Facility Web page “ftp://www-spof.gsfc.nasa.gov/pub/ltsp/”. The orbit of GEOTAIL for these days is from dawn to dusk and from north to south with its orbit such that it would traverse plasma downstream of the bow shock that is near perpendicular. The front boundary of the magnetic cloud is located at 1940 UT in
Table 6.1: GEOTAIL spacecraft position for October 18-19, 1995

<table>
<thead>
<tr>
<th>October</th>
<th>Time (hours)</th>
<th>X (R_E)</th>
<th>Y (R_E)</th>
<th>Z (R_E)</th>
<th>Radius (R_E)</th>
</tr>
</thead>
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<td>-29.18</td>
<td>-3.21</td>
<td>29.42</td>
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<td>6.00</td>
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<td>-2.81</td>
<td>28.44</td>
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<tr>
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<td>-26.47</td>
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<td>27.06</td>
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<tr>
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<td>18.00</td>
<td>8.38</td>
<td>-23.76</td>
<td>-1.75</td>
<td>25.26</td>
</tr>
<tr>
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<td>-1.11</td>
<td>23.03</td>
</tr>
<tr>
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<td>-15.15</td>
<td>-0.40</td>
<td>20.36</td>
</tr>
<tr>
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<td>-9.01</td>
<td>0.34</td>
<td>17.29</td>
</tr>
<tr>
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<td>13.86</td>
<td>-1.68</td>
<td>1.02</td>
<td>14.00</td>
</tr>
<tr>
<td>19</td>
<td>24.00</td>
<td>9.41</td>
<td>5.89</td>
<td>1.41</td>
<td>11.19</td>
</tr>
</tbody>
</table>

Table 6.2: GEOTAIL region occupancy for October 18-20, 1995

<table>
<thead>
<tr>
<th>Start Time (hours)</th>
<th>Stop Time (hours)</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct. 18, 01:00</td>
<td>Oct. 19, 16:24</td>
<td>Interplanetary Medium</td>
</tr>
<tr>
<td>Oct. 19, 16:36</td>
<td>Oct. 20, 00:36</td>
<td>Dayside Magnetosheath</td>
</tr>
<tr>
<td>Oct. 19, 00:48</td>
<td>Oct. 20, 06:48</td>
<td>Dayside Magnetosphere</td>
</tr>
</tbody>
</table>

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Figure 6.1 where the total magnetic field $B$ rises sharply to $\sim 20$ nT in the solar wind and for GEOTAIL data it rises to $\sim 40$ nT. The $B_z$ component at the front cloud boundary has a larger magnitude ($\sim 40$ nT) and still negative (southward). No information is available for the passage of the back boundary of the October 1995 magnetic cloud through the bow shock of the Earth. From the position of GEOTAIL the satellite had entered the magnetosphere near hour 47.

6.3 Comparisons between observations and numerical results

6.3.1 Front cloud boundary

The numerical simulations predict that if the density ratio $\rho_{\text{cloud}}/\rho_{\text{upstream plasma}} < 1$ then an expansion wave would be reflected upstream of the front cloud boundary as would be the case for the October 18-19, 1995 magnetic cloud. The observations that were made by GEOTAIL showed that the plasma upstream of the front cloud boundary was still in the solar wind. However, both the front cloud boundary and the plasma downstream of this boundary are to be found downstream of the Earth’s bow shock. The front boundary of the cloud is preserved, which supports the results of the numerical simulations. This result of finding the plasma upstream of the front cloud boundary in the solar wind and the magnetic cloud downstream of the bow shock is the result of the Earth’s bow shock expanding. The bow shock responded impulsively by moving outwards and away from the Earth at the front magnetic cloud boundary due to the drop in the dynamic pressure and the Alfven
Mach number (see Figure 3.5).

6.3.2 Width of the cloud

The average bulk flow velocity for the plasma downstream of the front cloud boundary as measured by GEOTAIL (and downstream of the bow shock) is \( \sim 250 \text{ kms}^{-1} \). In the solar wind, the October, 1995 cloud was traveling at a speed of \( \sim 410 \text{ kms}^{-1} \). From this we can get an estimate on the reduction in the width of the tube if there were no obstacles (Earth) to the plasma flow and the shock is normal to the flow. From the data the front cloud boundary has slowed down after passing through the bow shock but the back cloud boundary should still be traveling at its original speed. For the analysis that follow, the expansion of the cloud due to total pressure imbalances is ignored and we take the diameter in the Earth-Sun direction of the initial tube to be a constant \( L_0 \). During its passage through a fast shock the diameter of the cloud \( (L_f) \) is given by

\[
L_f = L_0 - (V_{bcb} - V_{fcb}) \delta t
\]

(6.1)

where \( V_{bcb} \) is the speed of the back cloud boundary, \( V_{fcb} \) is the speed of the front cloud boundary and \( \delta t \) is the time it takes for the whole cloud to pass through the shock. From GEOTAIL data \( V_{fcb} = 250 \text{ kms}^{-1} \), from WIND data \( V_{fcb} = 410 \text{ kms}^{-1} \) and \( \delta t = 29 \text{ hours} \). Substituting this into Equation 6.1 and dividing by \( 1.5 \times 10^{11} \) to get the distance in AU one finds that

\[
L_f = L_0 - 0.111
\]

(6.2)
The width of the tube would decrease as suggested by the numerical simulations. The October 1995 magnetic cloud had a width that was estimated at \( \sim 0.27 \text{ AU} \) by Lepping et al. [1997]. This simple analysis assumes that the cloud is not expanding (just as in the simulations of Chapter 5) and the shock is normal and not responding to changes in the dynamic pressure and Alfvén Mach number. Further work would be required to consider the case involving the effects of a fast shock and the resulting flow past an obstacle like the Earth.

### 6.3.3 Magnetic Field

The numerical simulations have indicated that the rotation in the magnetic field will be preserved by the fast shock passage through a cloud. But there should be an increase in the magnitude of the field. Downstream of the front cloud boundary, the \( B_z \) component of the magnetic field has increased in magnitude and pointing in the same direction (southward) and this observation seems to hold for all \( B_z < 0 \) except when GEOTAIL was in the solar wind. The transition to \( B_z > 0 \) is also preserved and in the region where \( B_z > 0 \) its magnitude has increased and is still pointing in the positive \( z \) direction. Even though the observations of this magnetic cloud was made at different times and at different locations in the cloud, this seems to locally support the numerical results that the rotation of the magnetic field in the cloud will be preserved after an interaction with a fast shock. The \( B_x \) and \( B_y \) components of the magnetic field have increased magnitudes for regions where the cloud is downstream of the bow shock.
6.3.4 Density Effects on the Bow shock

The following conclusions were drawn from the simulation results (Chapter 5) for a shock traveling from medium 1 into medium 2. If the density upstream of the cloud (medium 1) is greater than the cloud itself, the fast shock speed would be increased in the cloud. If the densities were reversed, for the two mediums, the fast shock will travel slower in the cloud. No observations could be made to support or refute the simulation results.

6.4 Recommendations and Conclusions

There are some agreements between observation and numerical results. The agreements are in the prediction that the width of the cloud would decrease as it passes through the Earth’s bow shock, the orientation of the components of the magnetic field are preserved with its magnitude enhanced as a result of the passage of the shock. The front boundary of a magnetic cloud will be preserved during an interaction with a fast shock.

More numerical studies will be required to fully understand the interaction of the front cloud boundary with the bow shock. One complication is the presence of a shock being driven by the magnetic cloud. The first interaction that will take place will be a shock-shock interaction for which no simulations have been carried out to predict the results of such an outcome. Another complication is the presence of a finite region of high density plasma propagating upstream of the front cloud.
boundary. From the results of Chapter 5 (Simulation 3), the shock interaction with the front boundary of a high density plasma would result in a shock being reflected and transmitted at this boundary. The transmitted shock on reaching the front cloud boundary would result in an expansion wave reflected back and a shock transmitted through the front cloud boundary. What would happen to this picture if one were to combine a preceding shock followed by a high density plasma region interacting with a strong fast shock is not clear.

From an observational standpoint more observations would have to be made downstream of the Earth for a magnetic cloud that has just passed through the bow shock of the Earth. This would give us an indication if any expansion or shock waves have resulted due to the interaction of the magnetic cloud with the bow shock. It will be somewhat difficult to determine the shape of the overall magnetic cloud after an interaction with the bow shock due to its size, but it may be possible to infer its shape by looking at the plasma flow surrounding a magnetic cloud. The results of the numerical simulations of Chapter 5 indicate that the plasma flow surrounding a Lundquist flux tube after a shock interaction was different for the three cases considered. The surrounding plasma treated the flux tube as a boundary that it had to flow around. By mapping the plasma flow surrounding a magnetic cloud it may be possible to infer the shape of the cloud after its interaction with the bow shock of the Earth.

Numerical studies will also be needed for a Lundquist flux tube topology with an
increasing density profile from the front to the back cloud boundary. All three clouds observed by WIND seem to have such a profile and what the final shape for an initial cylindrical magnetic cloud will be after a fast shock interaction will be interesting to study.

The numerical simulations predict a flow in the $Z$ direction (small in magnitude) for a force free Lundquist flux tube model and, as such, it would be useful to do a 3-D simulation with symmetric boundary conditions for the $Z$ direction.
Chapter 7

Conclusion

In this work we first looked at the interplanetary structure of magnetic clouds and their interaction with the Earth. Three clouds observed by the WIND satellite were examined. They were the October 18-20, 1995, May 25-27, 1996, and January 9-11, 1997 magnetic clouds. Although they had similar interplanetary features (with respect to the time series data), such as a large rotation in the magnetic field, larger than average total magnetic field and lower than average temperatures their effect on the Earth was significantly different. The October 1995 magnetic cloud produced a major geomagnetic storm while the January 1997 cloud produced a moderate storm and the May 1996 cloud produced only a weak storm. All these clouds share another common feature in that they were all been overtaken by a faster stream resulting in a peak in the density near the rear of the cloud. In the region downstream of the October cloud (October 20-21, 1995) there were Alfvén waves generated on the faster stream due to this fast-stream cloud interaction. We also find that the magnetopause is slightly expanded for all the three clouds from its average position during the $B_z < 0$ phase of the clouds and in the $B_z > 0$ phase there was a compression of the magnetopause taking place (but we did not consider erosion when $B_z < 0$). The bow shock, however, behaved differently for the
three clouds. During the $B_z < 0$ phase of the October 1995 cloud the bow shock expanded from its average position much more then the January 1997 but the May 1996 cloud hardly affected the Earth's bow shock position during this phase. For the later $B_z > 0$ phase, we find that all three clouds compressed the bow shock closer to the Earth from its average position.

High resolution magnetic field and plasma data for the October 18-19, 1995 magnetic cloud showed a number of discontinuities in the field and plasma observations. We studied these and determined that, except for the front cloud boundary, all other discontinuities were rotational. The front cloud boundary was found to be, by contrast, a tangential discontinuity. There were jumps in the temperature across these discontinuities which would indicate further structure associated with these discontinuities. We further examine this October cloud for large scale structures using minimum variance analysis and found that we could identify 3 different coherent structures within this cloud. We also found that no coherent structure existed for a 6 hour period at the rear of the cloud and also between the hours 3400 and 3500 UT. The lack of coherency in the cloud structure could be the result of distortions of the flux tube due to the cloud overtaking the plasma upstream of the cloud and itself being overtaken by a faster stream from the rear of the cloud.

All these magnetic clouds will interact with the bow shock of the Earth. To better understand this process, a numerical simulation of the MHD equations was carried out on a idealized Lundquist flux tube. Static conditions were assumed for
the flux tube. The method chosen to simulate the MHD equations is a Gudonov method referred in the literature as PPMMHD, in which the fluxes used to update the solutions in time are determined by solving a Riemann problem. The ambient plasma surrounding the tube had a simple field configuration. We simulated three cases for which the density of the flux tube was; (1) less than; (2) greater than; and (3) equal to the plasma surrounding the tube.

From the 1-D simulation results we conclude the following. The numerical results showed that the width of the tube decreased for all cases. The boundaries of the tube were still clearly defined after the shock interaction. The magnetic field components retain their original orientation but with an increased amplitude. If the shock is traveling to a tube boundary with a higher density then its interaction with the boundary will generate a transmitted and reflected shock. On the other hand, if the shock is traveling towards the tube boundary which has a lower density, then its interaction with the boundary will generate a reflected expansion wave and a transmitted shock. For the case when the densities of the tube and surrounding plasma are equal, then there is only a shock transmitted through the tube boundaries.

The 2 1/2-D simulations resulted in the following conclusions. In all cases the width of the tube decreased. In the case for a tube with a higher density then the surrounding plasma the portion of the shock that traveled through the tube was slowed down when compared to the portion of the shock that had traveled through
the surrounding plasma. For the case when the tube had a lower density the shock traveled faster in the tube then in the surrounding plasma. When the densities of the tube and surrounding plasma were equal, both portions of the shock traveled at the same speed. The overall shape was found to be distorted differently for all three cases. The plasma surrounding the tube flowed around the distorted tube without penetrating it. In the case when the density of the tube is lower then the surrounding plasma, the resulting tube structure has a parabolic profile with its focus located upstream of the tube. The plasma upstream of the tube flows towards and around the front tube boundary towards the edges. The plasma downstream flows around the back boundary towards the edges of the tube where it meets the plasma from the front. For the case when the density of the tube is larger then the surrounding plasma the front tube boundary has a parabolic curve with its focus located downstream of the back cloud boundary. The back tube has a structure that is somewhat difficult to describe but its overall shape resembles a mushroom. The plasma upstream of the tube flows away from the front tube boundary and around the tube. Downstream of the back boundary of the tube, the plasma flows towards the centerline \(X\)-axis of the tube and jets away along this axis in the positive \(z\) direction. For the case when the densities are equal the tube appears to be squeezed in from the front and back boundary.

We applied these numerical findings to an observation of a bow shock magnetic cloud interaction. GEOTAIL was in an orbit which allowed it to make on observa-
tion of the October 18-19, 1995 magnetic cloud crossing the bow shock of the Earth. The data supported the numerical result that the width of the cloud would decrease as a result of the bow shock interaction. This was done by observing the reduction in the speed of the cloud when it passed through the bow shock of the Earth while other portions of the cloud still traveled at previously observed solar wind speeds. There would be a difference in the distance traveled by the shocked and unshocked cloud plasma and the end result would be a reduction in the width of the cloud.

The data also confirmed that the front cloud boundary still remains clearly defined after the bow shock interaction. The orientation in the field components are still retained but their magnitudes have increased as the cloud passed through the bow shock. More information could be gathered from the bow shock cloud interaction if the observations are made downstream of the Earth. This would help determine if there are any waves reflected from the front cloud boundary as a result of the bow shock interaction. It may also be possible to infer the resulting shape of the cloud by making observations of the plasma flow surrounding the cloud.

More simulations would also be needed to better understand the shock magnetic cloud interaction. One observation common to the three clouds examined is the presence of a density step upstream of the front cloud boundary and also shocks being driven upstream of the front boundary for two of these clouds. Another feature noticed in all three clouds is a faster stream overtaking the cloud from the rear resulting in a density rise at the rear of the cloud. It would be worthwhile
to simulate flux tubes with the density configurations as described above along with driven shocks upstream of the tube. Also it would be worthwhile to consider simulating a 3-D case with a general ambient magnetic field configuration.
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