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Student interactions and mathematics discourse: A study of the development of discussions in a fifth-grade classroom

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Student interactions and mathematics discourse: A study of the development of discussions in a fifth-grade classroom

Abstract
Mathematics reform efforts are gaining attention and support in the years since the dissemination of the National Council of Teachers of Mathematics (NCTM) Standards documents. Encouraging student interactions in small and large group settings, and promoting discussion and argumentation of mathematical ideas among students are two possible implications of the vision presented in the Standards. The goal of having mathematics discussions, however, can present a variety of classroom challenges. Many factors influence classroom discourse and need to be addressed in ways that inform teachers as they work toward creating a more interactive, discussion-based mathematics classroom.

The study examines the development of mathematics discourse in a fifth grade classroom. Through extended observations, documentation and collaboration with the classroom teacher, various aspects of the classroom, the mathematics, and the participants’ interactions were investigated to determine those characteristics that play a part in the development of the interactions and the discourse.

It became evident that classroom interactions were regulated by the teacher, and in some cases by the teacher together with the students. In these cases, explicit discussion with students allowed the teacher and students to establish modes of interaction, to develop models for question posing and problem solving, and to negotiate expectations for participation within group discussions. Differences in the roles students assumed during whole class or small group discussions over the school year indicated that the development of discourse is linked to the development of the student as a learner and as a responsible participant in the mathematics community. A closer analysis of the setting and its changing characteristics revealed other factors influencing the development of the mathematics discourse including the choice of mathematical tasks, students’ social and mathematical roles, and the classroom environment.

The findings suggest that classroom teachers wishing to promote classroom interactions and discourse must be aware of the many aspects influencing or contributing to the discourse. Implications for mathematics pedagogy and suggestions for further research are given.

Keywords
Education, Mathematics, Education, Elementary, Education, Curriculum and Instruction

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STUDENT INTERACTIONS 

AND 

MATHEMATICS DISCOURSE: 

A STUDY OF THE DEVELOPMENT OF DISCUSSIONS 

IN A 

FIFTH GRADE CLASSROOM 

BY 

SHARON M. SOUCY MCCCRONE 

B.A. Dartmouth College, 1988 

M.S. University of New Hampshire, 1992 

DISSERTATION 

Submitted to the University of New Hampshire 

in Partial Fulfillment of 

the Requirements for the Degree of 

Doctor of Philosophy 

in 

Mathematics Education 

May, 1997
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ABSTRACT

STUDENT INTERACTIONS AND
MATHEMATICS DISCOURSE:
A STUDY OF THE DEVELOPMENT OF DISCUSSIONS IN A FIFTH GRADE CLASSROOM

by

Sharon M. Soucy McCrone
University of New Hampshire, May 1997

Mathematics reform efforts are gaining attention and support in the years since the dissemination of the National Council of Teachers of Mathematics [NCTM] Standards documents. Encouraging student interactions in small and large group settings, and promoting discussion and argumentation of mathematical ideas among students are two possible implications of the vision presented in the Standards. The goal of having mathematics discussions, however, can present a variety of classroom challenges. Many factors influence classroom discourse and need to be addressed in ways that inform teachers as they work toward creating a more interactive, discussion-based mathematics classroom.

The study examines the development of mathematics discourse in a fifth grade classroom. Through extended observations, documentation and collaboration with the classroom teacher, various aspects of the classroom, the mathematics, and the participants' interactions were investigated to determine those characteristics that play a part in the development of the interactions and the discourse.

It became evident that classroom interactions were regulated by the teacher, and in some cases by the teacher together with the students. In these cases, explicit discussion with students allowed the teacher and students to establish modes of interaction, to develop models for question posing and problem solving, and to negotiate expectations for
participation within group discussions. Differences in the roles students assumed during whole class or small group discussions over the school year indicated that the development of discourse is linked to the development of the student as a learner and as a responsible participant in the mathematics community. A closer analysis of the setting and its changing characteristics revealed other factors influencing the development of the mathematics discourse including the choice of mathematical tasks, students' social and mathematical roles, and the classroom environment.

The findings suggest that classroom teachers wishing to promote classroom interactions and discourse must be aware of the many aspects influencing or contributing to the discourse. Implications for mathematics pedagogy and suggestions for further research are given.
CHAPTER I

INTRODUCTION OF THE RESEARCH PROBLEM

Mathematics Reform and the Issue of Discourse

Mathematics reform efforts are gaining attention and support in the years since the dissemination of the National Council of Teachers of Mathematics Standards documents (National Council of Teachers of Mathematics [NCTM], 1989, 1991, 1995). Many new reform initiatives are being undertaken each year which aim to fulfill some of the vision of the Standards. These include teacher enhancement programs, development of new curricula, partnerships between schools and industries, and many more. Yet, there is still much to be done in considering the implications of the Standards on actual teacher practice and student learning.

The general aim of the NCTM Standards documents is to promote classroom mathematics that reflects the way mathematics is invented and used outside the classroom, "to shift toward classrooms as mathematics communities" (NCTM, 1991, p. 3). These documents furnish a vision of mathematics education that extends the traditional curriculum to provide skills needed to empower all students mathematically. This changing view of school mathematics carries with it many suggestions for changes in classroom teaching. Two possible implications of the vision presented in the Standards are to encourage student interactions in small and large group settings, and to stress discussion and argumentation of mathematical ideas among students.

The Professional Standards for Teaching Mathematics (NCTM, 1991) elaborate on these implications and make suggestions for the teacher's and students' roles in
establishing a mathematics classroom that promotes mathematics discourse. The authors of this document suggest that a view of mathematics as reasoning necessarily involves classroom discourse centered on mathematical evidence. In this view, both teachers and students play roles in shaping the discourse, as do the tools and tasks with which they engage. The teacher's role is seen to be central for initiating and orchestrating the discourse in ways that allow students to make sense of the mathematics. Discourse is also formed by the students as they use language to share ideas and mathematical evidence with others in making sense of the mathematics they encounter.

Mathematics education researchers and philosophers have theorized about the value of student interactions and discussions in mathematics learning (Bauersfeld, 1995; Cobb, Yackel, & Wood, 1992a; Hiebert & Wearne, 1993; Lo, Wheatley, & Smith, 1990; and Pimm, 1987). Many researchers and educators share the view put forth by the NCTM documents (1989, 1991) that mathematics classrooms where students express ideas, challenge those of each other and the teacher, and present convincing arguments are classrooms that facilitate students' development of mathematical concepts. Hiebert and Wearne (1993), for instance, offer the following view.

As students express their beliefs and opinions with their classmates, defend them in the face of questions, and question others' ideas, they are likely to recognize incongruities and elaborate, clarify, and reorganize their own thinking (p. 396).

Hiebert and Wearne have claimed that the theory lacks empirical support. But ongoing research in mathematics classrooms is beginning to build an empirical base for a theory of the role of discourse in mathematics learning. The research of Cobb, Yackel and their associates with classroom teachers and students, for example, focuses on social and cultural issues that are factors in developing a basis for communication in the mathematics classroom (Cobb et al., 1992a; Wood, Cobb, Yackel, & Dillon, 1993; Yackel & Cobb, 1996). The "Talking Mathematics" project includes school-based research and teacher support for promoting mathematical talk in the classroom (Corwin & Storeygard, 1995). The project focuses on pedagogical issues as well as mathematical issues that arise as
students generate conjectures, defend or challenge others' ideas, and actively participate in
discussions. Still other aspects of the pedagogy and the mathematics of classroom
discourse need to be explored, particularly the initiation and maintenance of discourse as a
regular element of learning mathematics.

The Problem

A traditional view of mathematics instruction portrays the teacher as the authority
who helps students come to know the rules and conventions established by mathematicians
through formal reasoning processes by mathematicians. Such a view calls up images of
what might be called a conventional classroom where the teacher provides the important
information for solving mathematical problems and then students are given similar
problems to solve. The recent suggestions from the mathematics education community for
changes in mathematics instruction carry with them new visions of the mathematics
classroom. Silver and Smith (1996), for example, advise that if teachers are to engage
students in active discussions around mathematical concepts, they must first help students
learn what is necessary to be full contributing participants. In addition, Lampert (1990)
suggests that students need to know what is meant by a mathematics discussion and what is
expected in a mathematics class where students work collaboratively in problem solving.

Knowing how to talk about mathematics is not immediate for many students,
particularly those who have participated in traditional mathematics classes. The
development of means of communicating in the mathematics classroom will most likely
take time. Discourse needs to be defined, and expectations of all participants must be made
explicit. Hiebert and his colleagues (Hiebert et al., 1997) also suggest establishing the
common goal of mathematical talk to discuss problem solving methods. They contend that
this goal envelops many subgoals familiar to traditional mathematics instruction such as
learning multiple strategies for problem solving and learning what counts as valuable
mathematics. To participate in such discussions, students must also know how to
appropriately verbally represent ideas and concepts, they must understand the relationship among many seemingly different ideas offered for consideration, and they must know what counts as a justification for a result. If classrooms are to become mathematics communities where discourse plays a central role, then teachers and students must work together to establish norms and patterns of discourse, develop skills for verbalizing mathematical ideas, and learn what is expected to become fully participating members (Lampert, Rittenhouse, & Crumbaugh, 1996; NCTM, 1991).

Although it seems easy enough to make a case for promoting mathematics discourse (Corwin & Storeygard, 1995; Hiebert et al., 1997; Lampert et al., 1996), making it happen in the elementary classroom is not as easy. Lampert et al. point out that "by using the word 'promote' to describe the pedagogical activity of the teacher who seeks to have such things happen in the classroom, NCTM sidesteps the question of what exactly teachers need to teach and students need to learn for this kind of talk to be seen as an appropriate mode of public interactions among school children and their teacher" (1996, p. 16). As can be inferred from the brief summary of literature above, developing mathematics practice consistent with the current reform requires paying attention to the role of discourse in the classroom, among other things. Many questions arise about appropriate content and tasks that lend themselves to mathematical discussions. The students' and the teacher's roles in the classroom must also be determined. In addition, the mathematics teacher will need to consider how the students will learn to work productively and cooperatively in this new model of teaching and learning. How do a teacher and a group of students develop ways of communicating mathematical ideas? What are the teacher's and the students' responsibilities in developing these discourse skills? What roles do the mathematical content area and the nature of the tasks play in establishing the discourse? These and other questions need to be addressed by teachers and researchers in order to provide a clearer vision for interpreting and implementing the recommendations of the NCTM Standards.
Research Issues

The purpose of this study is to look closely at the nature of discourse and the interactive situations that occur along with the discourse in the school mathematics classroom, in order to describe characteristics of the talk and of the classroom that contribute to the development of mathematics discourse. In particular, the focus will be on the development of discourse characterized by active participation of students as they share ideas about mathematical concepts and problem solutions, justify solutions, and interpret and challenge those of others. The emphasis will be almost exclusively on verbal mathematics discourse -- the formal (or not so formal) discussion, construction, and exchange of mathematical thoughts and information -- and its various structural components in the mathematics classroom.\(^1\) In one sense, the questions raised in the previous section about the development of discourse have to do with the creation of what Yackel and Cobb (1996) call classroom social and sociomathematical norms\(^2\). In another sense, the questions are best addressed by focusing on the mathematics, as well as the tasks and the tools being used to explore the mathematical ideas. It seems clear that investigating possible answers to the questions above will provide mathematics educators new perspectives for making shifts toward more interactive mathematics classrooms, for empowering students to contribute to the discussion of mathematical ideas, and for considering classroom structures that facilitate the development of discourse.

The Questions

The basic research question being explored in this study is: What aspects of the mathematics classroom contribute to the development of mathematics discourse? To

\(^1\)The term structural components refers to possible dialogue structures such as lecture, small group collaborative work, brainstorming sessions, and the imposed roles of the participants within these structures (e.g. facilitator of discussion, contributor of ideas, recorder of contributed information).

\(^2\)Social norms refer to normative aspects of behaviors, while sociomathematical norms are those which are specifically related to students' mathematical activities, such as a student's understanding of what is accepted as an appropriate mathematical definition.
answer this question it was necessary to focus on a single mathematics classroom, and a teacher whose intention was to establish mathematical inquiry and discussion as the basis for mathematics teaching and learning. An attempt was made to determine the various factors in the classroom that influenced the nature of the discourse, and how the teacher's and students' roles influenced the nature of the discourse. Several overarching hypotheses guided the research.

- The teacher's choice of tasks influences what the students say and how they talk about the mathematics involved in the tasks;
- The way the teacher structures interactions influences the kinds of interactions that occur in the classroom;
- The participants' roles in the discourse determine the nature of the discourse;
- The participants' expectations and beliefs about what mathematics is and how it is learned both shape the discourse and the classroom environment and are shaped by the discourse and the environment.

These hypotheses suggest several aspects of the mathematics classroom -- the mathematical tasks, the overall classroom structure, the students' and the teacher's role in the classroom, their beliefs, and the learning environment -- that were examined to determine how they contributed to the development of the mathematics discourse.

**Mathematics Tasks and Discourse Structures**

Research reported by Yackel and her colleagues (Yackel, Cobb, & Wood, 1991, 1993) indicates that choice of mathematical task and choice of discourse structure are major influences both on the kinds of interactions that take place and on the nature of the resulting discourse. Silver and Smith (1996) claim that such worthwhile tasks "often lend themselves to multiple solution methods, frequently involve multiple representations, and

---

3 Discourse structure refers to how the teacher organizes and structures interactions that involve discussion, such as small group work or whole class sharing.
usually require students to justify, conjecture and interpret. Thus, these tasks can furnish *rich opportunities* for mathematics discourse" (emphasis added, p. 24). It should be noted that Silver and Smith never claim that worthwhile tasks will require or produce rich mathematics discourse, but will "furnish rich opportunities" for discourse. So although the tasks themselves can be thought of as a prominent feature of discourse opportunities, the manner in which the teacher and students engage in the tasks is an important factor to consider when working to develop a discourse community. How do students learn to take advantage of these opportunities? How might teachers help students engage in the tasks in ways that develop their ability to draw conjectures, explain strategies, and interpret the work of others?

The authors of *The Curriculum and Evaluation Standards* (NCTM, 1989) clearly spell out how various discourse structures such as small group work and whole class sharing influence the discourse. Small group work, for instance, is described as an opportunity for students to exchange ideas, talk through differing strategies or representations, and develop the ability to reason and communicate with peers on a one-to-one basis. Participation in whole class discussions, on the other hand, "require[s] students to synthesize, critique, and summarize strategies, ideas or conjectures" (p. 67). It seems obvious that other discourse structures (e.g. lecture, student-as-teacher) will also contribute in varying ways to the development of the classroom mathematics discourse. Although there is much research that supports the use of small-group and whole-class discussions, there is often only a cursory mention of the ways in which these structures influence the classroom discourse. Which discourse structures afford students more opportunities for learning to communicate in mathematically appropriate ways?

**Participants’ Roles and Expectations**

The classroom teacher and the students create classroom discourse as they go through the motions of participating in mathematics discussions. They are the classroom
participants, and as such, each plays a role in forming the discourse. As mentioned above, discourse opportunities created by choice of task or by discourse structure depend on the roles the teacher and students take on as they engage in the tasks. Ball (1991) points out that although teachers and students together create the discourse of the mathematics classroom, teachers "play a crucial role in shaping the discourse of their classrooms through the signals they send about knowledge and ways of thinking and knowing that are valued" (p. 44). As this quote suggests, many students view the teacher as the holder of information about what does and does not count as appropriate mathematics. Furthermore, students often feel they must learn what it is they should be doing and saying. As such, the teacher is a model for what is expected and accepted in speech and action during mathematics class. Whether implicitly or explicitly, the teacher also frequently judges what is being said and how it is being said. If this is so, which paths should teachers follow to ensure they are leading students in the directions they intend? How do teachers establish expectations for their students?

The roles students take on in the mathematics classroom are also important to consider. As students engage in mathematical problem solving, as they interact with each other, and as they participate in whole class settings, they are creating the discourse. At the same time, the ways in which students' interact with each other and with the classroom teacher influences their conception of how to interact, and helps to establish the kinds of roles they play in mathematics discussions. The teacher's role in encouraging student participation and in modeling appropriate behaviors is important. Equally important in developing appropriate mathematics discussions is consideration of the students' roles as they take responsibility for working with others, sharing ideas, and providing explanations for solution methods. What roles do the students and the teacher play? What are the participants' responsibilities for contributing to the discourse?
Classroom Environment

There has been much research in recent years that focuses on the role of social interaction and the classroom environment on student learning (Cobb, Yackel, & Wood, 1992b; Lampert et al., 1996; Steffe, 1990). Cobb and his colleagues, for example, discuss the explicit negotiation of social norms between teacher and students as a way to establish expectations for participation in whole class and small group discussions. These negotiations also contribute to the students' understanding of what counts as meaningful mathematical activities. Although Cobb does not explicitly discuss establishing mutual respect in the classroom, he does imply that this is one goal of collaboratively negotiating the learning environment. In their classroom research, Silver and Smith (1996) found that when teachers were able to establish mutual respect and norms for mathematical interactions, students were more likely to engage in the discourse. Explicit discussions and cooperative setting of rules are two ways in which teachers and students can establish trust and respect for each other early in the school year. Participation in and facilitation of discussions with students also gives the teacher opportunities to show students that their talk is valued and valuable (Corwin & Storeygard, 1995).

A classroom in which students are allowed to explore activities with each other offers many opportunities for students to develop informal ways of communicating (Cazden, 1988). In a sense, Cazden reports, these occasions allow students to practice for academic discourse. As students learn to work with each other in these cooperative learning situations, they are given chances to discover alternate ideas and to establish models for problem solving. That is, students are given opportunities to develop working relationships with their peers, and to establish a learning environment. Thus, through discourse and interactions the students, along with the classroom teacher, shape the environment of the mathematics classroom. At the same time, the classroom environment influences the nature of students' interactions.
Summary

Communication and interaction in the mathematics classroom are critical elements for learning mathematics with understanding. When students are expected to contribute and support these contributions to the mathematics discussion, they are given an opportunity to build a clearer understanding or model of the concepts being discussed (Maher, Martino, & Alston, 1993). And through communication the teacher and students are encouraged to think deeply about their ideas in order to describe them clearly, explain them or justify them (Hiebert et al., 1997). What aspects of the mathematics classroom contribute to the ways students and the teacher interact and the ways the mathematics discourse develops? I hypothesize that the teacher’s choice of tasks and discourse structure, the participants’ roles as they interact with the mathematics and with each other, and the classroom environment influence the nature of the discourse. I have also briefly reviewed some of the current literature that supports the hypotheses. To extend the available research and to address the central research question of this study, it will be important to identify the underlying structure of a mathematics classroom and to understand the role of discourse in the mathematics activities (Erickson, 1982). With a close look at a mathematics classroom, I intend to highlight classroom features that contribute to the development of the discourse.

Overview of the Research

The development of discourse, the nature of student and teacher discourse and interactions, and the development of a learning environment are the major issues considered in the study. A fifth grade classroom was the object of observation and reflection to gain an understanding of these issues. The questions laid out in the sections above will be considered in the context of a particular classroom and its participants. The questions guide the investigation of what influences the discourse and how these characteristics determine the development of the discourse. The research questions are not intended to be a
comprehensive list to be followed exclusively. Rather, the questions provide avenues of investigation, that in turn suggest new avenues and factors for consideration.

Qualitative research methods, such as naturalistic inquiry described by Lincoln and Guba (1985), were used in this study since the questions raised above need to be addressed through close observation of a mathematics classroom, since observation of a mathematics classroom is most useful for describing the nature of the discourse and interactions over a period of time. The benefit of a qualitative approach is that it allows the researcher to construct a full picture of what is happening in the classroom as the discourse develops. Close watching and interacting with the participants also allows the researcher to take into account the perspectives of the participants. The overall view of the classroom from an outside observer's perspective as well as a closer look at the classroom from the participants' perspective frames the role of discourse within the classroom and provides insight into how the setting and the discourse are created through classroom interactions.

The observations took place in a fifth grade classroom with a teacher, who will be referred to as Brenda Miller, who had in the previous years invested time working with her fifth grade students to develop ways of exploring, talking about, and sharing mathematics. Brenda Miller's classroom was selected for several reasons. At the time of this study, Brenda was a participant in a teacher enhancement program in mathematics. As a participant in this program, Brenda was actively working with some colleagues in her school district and with teacher educators from a local educational resource center to investigate her practice in light of the current mathematics reform efforts. Mrs. Miller was an enthusiastic participant in this program and felt that the program validated her beliefs about mathematics teaching and challenged her to continue reflecting on how the recommendations for reform would play out in her classroom. Furthermore, preliminary observations during the 1994-95 school year suggested that students in the classroom had developed ways of working together, expressing ideas, and making conjectures, which assured me this would be an interesting classroom to watch. In talking with Mrs. Miller, it
became clear that she was interested in thinking about the research questions presented in my proposal, and she agreed to investigate the mathematics discourse of her classroom along with me.

The students in the mathematics class that was observed were a combination of students from Mrs. Miller's homeroom and from the homeroom of the school's other fifth grade teacher, henceforth known as Ms. Forest. Starting on the first day of school of the 1995-96 school year and continuing for the next several months, I observed and, with increasing frequency, participated in Mrs. Miller's mathematics class. My presence on the first day of school helped in establishing a relationship with the students. They were told that I would be observing the classroom on a daily basis, taking notes, and occasionally working with them. Mrs. Miller told them that I was interested in learning about how they talked with each other, and so I would be listening carefully to their conversations. The students were given an opportunity on that day and on a few other days to ask questions about the research and their participation in it. One student asked that she not be interviewed, video or audio taped. All other students agreed to participate fully in the study.

Observations continued less frequently after the winter break, providing me with extended periods of time for data analysis and possible reformulation of research questions and data collection methods. Daily participation and observation picked up again in the month of May and continued through the end of the school year. Thus, I had collected data from ten consecutive weeks in the Fall of 1995, five consecutive weeks during February and March, as well as many non-consecutive days, and 4 consecutive weeks in May and June of 1996.

Data was collected primarily by participant observation. That is, I took handwritten notes along with audio and video recordings of the happenings in the classroom, and also spent time with the students during small group and individual seat work. Student work, including individual homework papers and small group problem solving solutions, was
collected to supplement the audio tapes and field notes. In addition, six students were chosen by the teacher and myself for interviews based on our observations that these students had varying mathematical abilities and diverse methods for expressing mathematical ideas. A journal, which was kept jointly by the teacher and me, contained my daily reflections on observations and any questions raised by these reflections. The classroom teacher responded to the journal entries on a weekly basis.

The amount of data produced in the classroom through discussions and interactions was overwhelming. Although I could have audio recorded almost all of the talk in the classroom, and collected all student papers, this would have produced an unmanageable amount of information, much of which would most likely not be useful. Hand written notes necessarily cut down on the amount of data collected, and also required me to focus my attention more sharply on those aspects of the classroom activity that were pertinent to answering the research questions. There was the possibility, however, that I would miss data that were essential. As a compromise, almost all class periods were either audio or video taped, and my hand written notes consisted primarily of memos to myself about what to pay attention to when listening to or viewing the tapes. These notes were based on my research hypotheses which I organized into four areas of focus. The categories include the mathematics, the classroom culture or environment, the participants' roles during interactions, and the discourse. I predicted that there were interactions between categories as shown in Figure 1.

The hypotheses guiding the research, as described earlier in this chapter, determine the areas of focus for the data collection process. The mathematics content and format, for instance, are determined by the teacher’s choice of tasks and the way she structures interactions. I hypothesize that the choice of tasks influences how the students talk about the mathematics. I also hypothesize that it is important to pay attention to the teacher’s and students’ roles during discussions since the participants' roles and their expectations of the roles they and others take on determine who talks and what is being said. The teacher’s as
well as the students' beliefs about mathematics teaching and learning are also important to investigate. The participants' beliefs shape the classroom environment and contribute to their evolving sense of their role in the mathematics classroom. I hypothesize that the culture of the classroom influences the ways the discourse develops. In general, Figure 1 indicates that the mathematics, the participants' roles and the classroom culture all influence the nature of the discourse. More in depth analysis during the school year allowed me to begin to notice patterns in the data and characteristics that played a major part in determining the nature of the discourse. A closer look at the analysis process and how it evolved is provided in Chapter V.

![Figure 1. Areas of Focus to Guide Data Collection](image)

The research is reported in the following manner: Chapter II provides the theoretical perspective as well as a review of the relevant literature. Chapter III focuses on how the classroom was selected, how data was collected, and how analysis proceeded. In Chapter IV, I introduce the setting and its participants, including the larger community of the school and the fifth grade classrooms. Analyses, results, and a discussion of the results follow in Chapters V and VI.
CHAPTER II

THEORETICAL PERSPECTIVE AND LITERATURE REVIEW

Introduction

Chapter I clearly describes the mathematics education community's interest in promoting classroom interactions and mathematics discourse. I have also introduced my concern in understanding the nature of interactions and discourse in the elementary mathematics classroom. In educational research, one's theoretical perspective is guided by the situation being studied, but also guides the choice of situations to study and guides how one interprets the resulting events or phenomena. I claim that learning in school is a social endeavor, and thus the student's ability to take part in the society of the classroom determines, in part, her or his ability to construct useful or useable concepts. These claims are supported by a social constructivist perspective of learning, as well as theories related to classroom communication and interactions. Literature on the development of knowledge and understanding (Piaget, 1970; Vygotsky 1962, 1978), theories of constructivism and social interaction in mathematics (Bauersfeld, 1980, 1992, 1995; Cobb et al., 1992b; Lerman, 1989; Steffe, 1990; von Glasersfeld, 1987; Yackel & Cobb, 1996), and theories of classroom communication and discourse structures (Cazden, 1988; Erickson, 1982; Philips, 1983; Stubble, 1983; Wertsch, 1985) offer various complementary perspectives on discourse and mathematics learning. The purpose of this chapter is to develop the theoretical perspectives that frame the study and to show support for these theories through some of the corresponding empirical research. In particular, the review that follows highlights those aspects of the theories that support the research goals outlined in Chapter I.
This review also provides a theoretical grounding from which to interpret classroom events, to evaluate the importance of certain features in the development of the discourse, and to make sense of the results of the research to be reported in later chapters.

The discussion of perspective begins with theories of knowledge development that provide the foundations for a social constructivist perspective of learning. This is followed by a review of some research and expository by mathematics education researchers on the value of having a social constructivist perspective of learning. Thoughts on more socially based theories of mathematics learning, and the appropriate research, is also included. A discussion of theories of communication and the process of socialization into the classroom community (mathematics or otherwise) is also included. A review of the research that provides ways of looking at student interactions and understanding how students make sense of problematic situations is followed by a look at literature on establishing communities of learners.

Constructivism and Social Interactionism

A Constructivist Theory of Learning

In the late 1980's, with the release of documents such as NCTM's Standards (1989) and the National Research Council's Everybody Counts (1989), a constructivist perspective of mathematics learning grew quickly within the mathematics education community. Such a perspective -- a belief that all knowledge is necessarily a product of our own cognitive acts (Confrey, 1990, p. 108) -- is evident in many of the recent and current recommendations for changes in classroom mathematics teaching. The Curriculum and Evaluation Standards, for example, suggest that a “constructive, active view of the learning process” guide mathematics instruction, including opportunities for group work and discussions among students, to name a few specific suggestions (NCTM, 1989, p. 10). A constructivist perspective also frames much of the current research on the teaching and learning of mathematics, particularly at the elementary and middle school levels.
(Bauersfeld, 1990, 1992, 1995; Lo et al., 1990; Wood et al., 1993). Thus, I will explore constructivist theory and some of its derivatives, implications for classroom research, and my reflections on the value of taking such a theoretical stance.

Noddings (1990) and others (Lerman, 1989) concede that Piagetian theory is among the first to acknowledge that a person’s way of knowing comes through her or his actions within the world. Here, action can be defined as behavior by which we cause a change in the world around us or by which we cause a change in our relation to the world (Millroy, 1992). Such changes, Millroy contends, are the basis for the construction of new knowledge. Noddings (1990) further examines the relationship between the individual, actions, and the construction of knowledge. She points out that, by claiming Piaget’s theory is valid, one accepts that mathematical knowledge is constructed, at least in part, through a process of reflective abstraction — the mental process by which the individual (re)organizes or coordinates thoughts, actions and language. Reflection is also important in the constructive processes of knowledge development. According to Confrey (1990), reflection allows the learner to assess the relative worth of an individual mathematical “construct.” Furthermore, it is the individual’s cognitive structures that test and explain the results of constructions. And these cognitive structures are themselves under constant construction through a process of adaptation to new understandings (Noddings, 1990).

Von Glasersfeld’s theory of knowledge development, what he and others refer to as a radical constructivist perspective⁴, stems from the work of Piaget. Like Piaget, von Glasersfeld introduces the notion that all knowledge is actively constructed through experience, that the individual learner builds knowledge through adapting, interpreting, and organizing her or his experiential world (von Glasersfeld, 1990). Von Glasersfeld states that “making sense” of experience and of uses of language means “finding a way of fitting available conceptual elements into a pattern that is circumscribed by specific constraints”

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⁴ Radical constructivism differs from what may be called a general constructivist perspective in that a radical constructivist denies the existence of an external objective reality against which all constructed knowledge is tested.
(von Glasersfeld, 1987, p. 9). One meaning of constraint refers to an individual’s prior knowledge and her or his sense that the new concept “fits” with previous experiences.

Von Glasersfeld's work in explicating the constructivist paradigm in terms of the learning of mathematics, or any other subject matter, is often criticized as being subject to solipsism. Lerman (1989) addresses this issue, and more generally addresses questions about the constructivist hypothesis. He states that in coming to know about a mathematics concept, for example, one is not discovering a truth in a pre-existing (objective) world, nor is one creating a mathematical reality that cannot be shared within one's community. Lerman emphasizes that a constructivist outlook is concerned with the process by which the learner constructs a concept, and is less concerned with testing the validity of the concept.

To this point, Lerman (1989) elaborates:

>a concept is identified by its use, it gains its meaning from the shared social interpretation which is its use, and hence language, which itself is socially negotiated, and finds its meaning in its use, is integrally connected with the notion of a concept (p. 215)

Mathematical concepts and their meanings are thus determined by their use and an objective knowledge of mathematics is necessarily socially constructed, publicly negotiated, and “relative to a particular culture, in a time and a place” (Lerman, 1989, p. 219). It can be seen, then, that a constructivist view of mathematical knowledge development acknowledges the importance of understanding the learner’s experiences and interactions within the mathematics classroom, not as an individual apart from the social whole. I contend that a constructivist research perspective necessarily requires considering the nature of interactions and discussions that occur in the classroom and the part interactions play in mathematical knowledge development.

A Sociocultural Theory of Learning

Language and interactions play a role in knowledge development according to learning theories of cognitive psychology, information processing, behaviorism and constructivism. Taking these social interactions as the first step in knowledge development
is what sets a sociocultural theory apart from the others. The writings of Vygotsky and his students (Cole, 1985; Vygotsky, 1962, 1978) provide an overview of a sociocultural development of knowledge. Cole (1985) remarks that a central tenet of Vygotskian learning theory holds that the individual cannot be separated from the social and cultural environment. In fact, the development of knowledge is linked to the process of socialization or of "acquiring culture" (Cole, 1985, p. 148). Vygotsky’s theory (1978) emphasizes, for instance, that social relations among individuals constitute a first step in the construction of ideas (the inter-psychological level of development). The social reality of an interaction is then internalized or interpreted by the individuals involved and the idea reaches a higher cognitive or intrapsychological level.

With such an emphasis on social relations, it is not surprising that Vygotsky stressed the need to consider language as the framework for discussing the connections between social interactions and knowledge development within one’s cultural arena. That is, Vygotsky’s model (1978) supports the claim that the development of knowledge is parallel to the development of language, and it is through oral, inner and written speech that experiences are shaped. One might make sense of Vygotsky’s model by considering external communication as a starting point or stimulus for learning, as when a teacher works with a student, or when two students share problem solving strategies. This gives rise to internal speech as the individuals organize their thoughts, possibly comparing each other’s strategy. Internal speech and reflective thought then provide a source for the individuals’ development or construction of mental functions, such as a new strategy for solving similar problems that incorporate both strategies. Certainly, this is a very simplistic example, and I do not claim that a student’s construction stops with this untested new strategy. Even so, one can see that discussions with the teacher and with peers as well as time for reflection are important components of the student’s learning process when one considers a sociocultural theory of learning.
Cole also points to the work of Leontiev, one of Vygotsky's students, who emphasized the need to consider a cultural theory of cognition. Specifically, Leontiev asserted that human activity only exists within a system of social relations (Cole, 1985). And since psychology is chiefly concerned with human activity, the study of psychology cannot be removed from the study of social relationships within the given cultural system. Cole remarks that, hence, the study of child psychology naturally involves paying attention to the role of adults and the influence of cultural practices on children's development. In particular, a sociocultural perspective of learning accounts for learning opportunities occurring in all interactive classroom situations, those between a student and a teacher and those between students. Since these interactions occur within a specific cultural arena, a sociocultural theory takes into account the learning will occur within that given system of social relations. Furthermore, a sociocultural perspective acknowledges both internal processes and external interactions as important for the formation of concepts.

The theories of Piaget and Vygotsky support the constructivist claim that all learners are active organizers of their experiences. Piaget to some degree, and Vygotsky to a greater extent, have taken into account that learning occurs within a social and cultural arena, which in the case of this study can be thought of as the mathematics classroom and the larger school community. Piaget stresses that knowledge is acquired through one's actions within an environment and one's ability to make sense of and incorporate those experiences, while Vygotskian theories emphasize that interactions with other cognizing individuals are the basis for reflection. Thus, both perspectives lead to the conclusion that social aspects play an important role in the organization of the students' experiences and hence also in the development of concepts for individual learners. From this I conclude that a radical constructivist and a sociocultural theory of learning are not incompatible. In fact, these perspectives can be shown to be complementary. It is such a perspective, a blend of the two, that I take in this study.
Social Constructivism

Social constructivism is often referred to as a blend of a radical constructivist view of knowledge development and a sociocultural theory of learning (Bauersfeld, 1992; Cobb, Yackel, & Wood, 1993; Cobb & Yackel, 1995). Cobb, Yackel and Wood (1993), for instance, interpret von Glasersfeld’s radical constructivist perspective to view “mathematical learning as an active problem-solving process in which children reorganize their mathematical ways of knowing to resolve situations that, in their interpretation, give rise to obstacles or contradictions” (Cobb et al., 1993, p. 21). They also acknowledge the child’s place in the mathematics classroom and the influence of the classroom environment on learning, a position which evolves almost directly from the work of Vygotsky. Cobb and Yackel (1995) explain further that:

In general, analyses conducted from the psychological constructivist perspective bring out the heterogeneity in the activities of members of a classroom community. In contrast, social analyses of classroom mathematical practices conducted from interactionist perspective bring out what is jointly established as the teacher and students coordinate their individual activities. In drawing on these two analytic perspectives, the emergent approach focuses on both the individual and the community. This approach seeks to analyze both the development of individual minds and the evolution of the local social worlds in which those minds participate (p. 10).

The emergent perspective described by Cobb and Yackel (1995) corresponds to work of others who, for example, acknowledge the role of social interaction in cognitive development (Bauersfeld, 1980; Perret-Clermont, 1980). Perret-Clermont, for one, contends that students gain insights and understanding of new concepts through active participation, through theory building, and through reflection. Furthermore, students’ exposure to how “others” think about the mathematics they encounter in school helps them learn to verify, justify, and check their model against those of others in the mathematics community (Bauersfeld, 1992). Such a view of mathematics teaching and learning is congruous with a widely accepted view of the nature of mathematics and the process of constructing mathematics. The writings of Perret-Clermont (1980) and Bauersfeld (1980, 1992) support a social constructivist perspective of mathematics learning in that they claim

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social interaction is a valuable component of the learning process. In addition, their work endorses the claim that interactions potentially give rise to individual cognitive conflict as students' varying interpretations necessitate reflection on their activities and possibly the construction and (re)organization of mathematical knowledge.

The description of social constructivism found in this section fits well with the belief that mathematics learning is a social endeavor. The theory takes into account that student's classroom interactions involve formal and informal exchanges with the teacher and with peers, as well as argumentation and perhaps the elaboration of ideas. Although not explicitly stated in the description of a social constructivist theory of learning presented above, it seems clear that such a perspective signals the importance of paying attention to student interactions and developing ways of interacting, of talking, and of presenting oneself before others. A social constructivist perspective, then, legitimizes paying attention to the development of student interactions and the classroom discourse.

A Social Constructivist Perspective on Mathematics Learning

As discussed in the preceding section, there is a growing trend to acknowledge the child's place in the mathematics classroom, the child's role in constructing mathematical understanding, and the influence of the classroom environment on learning. This perspective evolves almost directly from the constructivist and sociocultural theories, and forms the basis of the social constructivist orientation shared by many mathematics education researchers, including those whose ideas are highlighted above.

The influence of the classroom environment and particularly the role of the teacher on student learning has been observed in the mathematics classroom and elsewhere (Bauersfeld, 1992; Carlsen, 1992; Cobb et al., 1992b; Yackel, Cobb, Wood, Wheatley, & Merkel, 1990). For instance, Cobb et al. (1992b) claim that the process of adopting mathematical modes of understanding begins with a student's earliest years in the mathematics classroom. That is, during the elementary school years a student develops a
sense of what counts as meaningful mathematics. This may be the student's interpretation of what is valued by the teacher -- whether it is demonstrating a solution method, finding a correct answer, or being able to justify a choice -- or possibly a deeper sense of what is an important mathematics concept. In turn, what is valued by the teacher is her or his interpretation of what is valued by the larger mathematics community (Carlsen, 1992). Carlsen's research shows that, thus, a person's understanding of what is valued shapes the construction of her or his mathematical knowledge. From these findings, it is not surprising that mathematics educators and researchers are increasingly suggesting that teachers acknowledge the connectedness of how they communicate mathematical ideas and the mathematical knowledge constructed by individual students (Confrey, 1990; Yackel et al., 1990). This view of mathematics learning also emphasizes the crucial role teachers play in appropriately representing mathematical concepts. A closer look at the research on teachers' roles, particularly with respect to classroom discourse, will be taken up in subsequent sections.

Bauersfeld, who characterizes learning as a process of interacting with one's environment, interpreting one's experiences, and organizing these experiences within the structures of previous constructions (1992), took this perspective to his research of elementary mathematics classrooms. In his analysis of mathematics classroom episodes, Bauersfeld (1980) defined mathematics learning (and classroom learning, in general) to be situations of "human interaction in an institutionalized setting" (p. 23), emphasizing both the importance of work with others and the importance of the setting. Analysis of an episode was used to introduce what Bauersfeld termed the "hidden dimensions" of the learning process. He noticed, from observations, that in the mathematics classroom the interactions between students were regulated in specific ways by the teacher and by the students together with the teacher. Further, the mathematical context of the setting influenced the formality of the discussions that occurred. As in all social situations, Bauersfeld comments, content and meanings are negotiated within the context of the
mathematics being discussed, hence creating hidden dimensions within which to work. He concludes that active participation in mathematics discussions and negotiation of expectations in interactions are crucial elements of social interaction for students' development of mathematical ideas (Bauersfeld, 1992).

The views presented by Bauersfeld (1980, 1992) are shared by Wood and her colleagues (Wood et al., 1993; Cobb et al., 1992a, 1992b). In their work with elementary school children, Cobb, Yackel and Wood (1992b, Wood et al., 1993) take a constructivist and social interactionist5 stance on the teaching and learning of mathematics, to explore students' construction of mathematical knowledge within the context of the classroom. Their research in a third grade classroom, for example, focused on work within small groups and pairs (Cobb et al., 1992b). They illustrate many instances of individual students working with ideas presented by others, or ideas of their own, with the goal of verbalizing their interpretation of the ideas in order to contribute to the work of the class. This individual work for the benefit of the group indicates a relationship between individual mathematical knowledge and the knowledge and practices of the mathematics community. Hence, interaction and communication were found to be useful in the individual's construction of mathematical knowledge and in the construction of the community's taken-as-shared mathematical knowledge.

The role of discussion and negotiation in the learning process, as highlighted in Bauersfeld's research reported above (Bauersfeld, 1992), naturally leads to the view that opportunities for interacting with others and for discussing mathematical solution methods are valuable for students. Lo and Wheatley (1994) investigated this hypothesis in their research on learning opportunities afforded through class discussion. Their research focused on whole class discussions, thought to be one setting that offers rich opportunities for mathematics learning. From a constructivist perspective, they argue that class

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5 A social interactionist perspective acknowledges that social interactions serve as a catalyst for cognitive development through the development of logical thinking, reflective thinking, and self-awareness (Wood et al., 1993).
discussions (1) provide students with chances to actively reflect on their own and other's problem solving activities, and (2) may develop into problematic situations for students (as they try to make sense of another's solution method, for example) which require further work to resolve, thus increasing learning opportunities.

Mathematics class discussions, then, can be rich opportunities for students to engage in mathematical activities such as hypothesizing, problem solving, explaining, and justifying. "In order to have a successful mathematics class discussion," Lo and Wheatley maintain, "social interactions (especially the verbal aspects) must be negotiated" (1994, p. 147). Through social interactions within the context of mathematics class, Lo and Wheatley contend, participants reorganize their beliefs and actions in the process of developing social norms, and these beliefs or norms reflexively determine the participants interactions. But participation is not immediate or easy for all members of a class (Forman & Cazden, 1985). Classroom discourse and an individual's ability to participate within the structure established or negotiated is discussed in more detail in the following section.

The research studies on mathematics learning discussed above all support, and are supported by a social constructivist outlook. In general, these examples highlight three major themes of the social constructivist perspective. First, the research of Cobb and his colleagues (Cobb et al., 1992a, 1992b; Yackel et al., 1990) shows that students' prior experiences, expectations, and interpretations of theirs and others' ideas influence the construction of mathematics knowledge. Second, the research of Wood et al. (1993), of Bauersfeld (1992), and of Lo and Wheatley (1994) acknowledges that students' constructions represent legitimate knowledge of mathematical objects and concepts. Although they might be tentative or partial, these constructions are judged against prior knowledge and are deemed viable if they fit with past and present understandings and help the student make sense of the present situation. Third, the researchers, particularly Bauersfeld (1980), take into account the students' place within the classroom. They recognize that students construct mathematical concepts within the classroom community,
and thus the environment and other social influences are important to account for when looking at student learning.

These three major themes of a social constructivist perspective of learning are useful for guiding mathematics education research that takes into account the complexities of the classroom, including the uniqueness of social interactions within a particular classroom, and an individual student's construction of mathematical concepts. In addition, the research discussed above supports the hypothesis that such a view of learning necessitates a close look at how interactions are developed and regulated within a specific classroom. The social and intellectual aspects of the mathematics classroom guide the nature of the discourse and hence the nature of the mathematical concepts developed.

**Discourse and Interactions in the Classroom**

As previously noted, Vygotsky's work emphasized both the role of social relations and the role of language in knowledge development. More generally, child psychology research within a sociocultural paradigm recognizes that an individual's development is guided by the social world in which she or he interacts. For the student, this social world is the classroom as well as the larger school community. Lave, for one, suggests that the study of cognition is most appropriate from within cultural contexts, more specifically, in everyday social settings such as the classroom, the store, or the workshop (Lave, 1988). Although Lave's major interests, and those of others studying culturally situated cognition (Millroy, 1992; Saxe, 1991a, 1991b) lie primarily in social settings outside the classroom, this type of research offers a framework for studying mathematics learning and the learners' roles in the mathematics classroom.

Until recently, there has been little research that considers the individual student's learning situation as one in a classroom of peers. Studies conducted by Vygotsky (1962, 1978), too, dealt primarily with one-to-one student and teacher situations rather than broadening the view to student interactions within the larger classroom setting. Even so,
his theory has been extended by researchers such as Forman and Cazden to address the issue of children's potential contributions to each other's intellectual learning through classroom social interactions (Cazden, 1988; Forman, 1981; Forman & Cazden, 1985).

Forman and Cazden (1985) name several earlier research studies that focus on the nature of peer interactions (e.g. Perret-Clermont, 1980) but remark that up to 1985, there had been relatively little research on the possible value of peer interactions for learning and cognition. In this section, I present the research of Cazden, Forman and others who pay particular attention to the role of peer and student-teacher interactions in the learning process. Research in situated cognition and inquiry-based or discussion-based mathematics classrooms are also briefly reviewed.

Interactions, Discourse, and Learning

When students interact with the teacher or when students interact with each other, they are given opportunities to test the viability of thoughts, ideas, and conjectures. Clearly, the classroom teacher, typically viewed by students as an authority in mathematics, is a valuable resource for students. Current research in peer tutoring and teaching experiments that use problem-centered instruction suggest that, in situations where students are expected to work with each other and help each other make sense of the concepts, students come to think of mathematical authority residing within the intellectual community of the classroom, not solely with the teacher (Cazden, 1988; Yackel et al., 1990). Particularly in peer tutoring situations, the student designated as the tutor is, in effect, given some of that mathematical authority. Research in peer tutoring, however, indicates that for students to work effectively as tutors the teacher must provide a model (of teacher) for the students that is learnable. Thus, students who take on the role of tutor will learn to speak to their peers in effective, teacher-like ways (Forman & Cazden, 1985). Forman and Cazden illustrate this with two examples. The first example is of a teacher and student, with the teacher first teaching the student an activity and then aiding her in learning to teach...
her peers. To provide an effective model for the student to follow, the teacher highlighted key words for the student to use when teaching others. Nonetheless, the student consistently shortened the process of instruction and left out many key words, making it difficult for other students to understand the activity. Since the student was unable to learn the model provided by the teacher, she was unable to use the model in teaching her peers.

In a second example from Forman’s and Cazden’s research on peer tutoring (1985), students were asked to take turns reading their journal assignments to a partner. It was noticed that as a student read her work to her partner, she was able to reflect on her work and to self-correct where needed. Her partner’s questions, similar to those the teacher would have asked, further prompted the reader to reflect on and make changes to her work. From these two examples, Forman and Cazden (1985) suggest that in situations where students are able to model teacher moves, or to work within the teacher’s expectations, student interactions create valuable opportunities for learning. This body of research indicates that teachers’ and students’ roles as they interact in the classroom become key issues in determining the value of learning situations. Although Forman and Cazden do not explicitly discuss issues of discourse, it is clear that as students learn to model teacher moves they are developing ways of communicating with their peers which reflect the teacher’s moves or goals.

In the research reported above, the researchers emphasized the teacher’s need to model expected behavior in ways that were learnable by students (Forman & Cazden, 1985). In a sense, this advocates that the teacher establish particular ways of creating learning opportunities. The teacher is thus establishing standards of conduct -- ways in which the students are expected to interact. Classroom standards of conduct, often implicitly established, influence the ways in which students interact and the ways in which they determine what the teacher expects of them, “which in turn influences both what mathematics the children learn and how they learn it” (Yackel et al., 1990, p. 12). It seems as though, with this statement, Yackel and her colleagues are making a strong case for
paying attention to the norms of conduct that are established, either implicitly or explicitly, within the classroom, or at least to consider the nature of and the role of classroom interactions in the development of mathematical ideas (Yackel et al., 1990). A more extensive discussion of the teacher’s and the students’ roles in the negotiation of classroom norms appears in the next section of this chapter.

**Student-Student Interactions**

Student-student interactions in which there is genuine collaboration provide rich opportunities for learning. Many education researchers and education curriculum specialists advocate collaborative or cooperative group settings within the classroom, and there is much recent research that supports such a structure (Cohen, 1994; Webb & Farivar, 1994). Prior to the early 1980’s, however, there was very little research conducted that focused on the effects of peer collaboration on logical reasoning skills, and even less that focused on the interactions themselves and the conditions responsible for influencing cognitive growth (Forman & Cazden, 1985). Although the research of Perret-Clermont and her colleagues from the mid-1970’s (Mugny, Perret-Clermont, & Doise, 1979; Perret-Clermont, 1980) indicates that small group collaboration improves students’ logical reasoning skills through reorganizations caused by cognitive conflict, these studies do not provide information on the interactions themselves. Forman and Cazden (Forman, 1981; Forman & Cazden, 1985) make the case that examining interactional patterns within groups would be useful for determining the role interactions play in cognitive growth.

In a study that focused on collaborative interactions in student pairs, Forman (1981) was able to identify three levels of procedural interactions -- parallel, associative, and cooperative -- that describe different approaches student pairs took in sharing ideas and performing tasks. Procedural interactions, defined as “all activities carried out by one or both children (in a pair) that focus on getting the task accomplished” (Forman & Cazden, 1985, p. 333) were thought to provide a basis from which the student pairs were able to
acquire strategies and processes for solving problems. Parallel interactions are those in which students share materials, exchange comments, but do not engage in direct sharing or monitoring of each other’s ideas. Students interacting associatively exchange information related to the task at hand, but do not necessarily coordinate and share roles. Working cooperatively implies that students monitor each other’s work, and coordinate roles in performing tasks.

Results from Forman’s research (1981) showed that most pairs progressed from parallel to associative to cooperative interactions. Furthermore, those pairs who used cooperative strategies most often also used more efficient problem solving strategies. Even so, when students were tested individually following weeks of working with partners, the collaborative effects on problem solving efficiency seemed to disappear. Forman and Cazden (1985) suggest that these results mirror Vygotsky’s zone of proximal development theory. In this case the partners assumed complementary roles, rather than one student being seen as the more capable peer. Taking on complementary social or academic roles, they hypothesized, may provide a type of “scaffolding” for the partners, thus allowing the pairs to work at higher levels than they could independently.

Extending this study to look at cognitive growth, Forman and Cazden (1985) noted that students’ problem-solving strategies that began as interactional were, at some later point, internalized and owned by the individuals. For example, they reported how one pair of students used a kind of sophisticated deductive method to generate all possible combinations of five items while working together, but that both students had difficulty when asked to complete a similar task on their own. Four months later, both the students had developed a deductive process for generating the combinations that was similar to the

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6 Vygotsky describes a child’s zone of proximal development as “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (Vygotsky, 1978, p.86).

7 Scaffolding is the process of providing guidance to a student so that she or he can perform a higher-order task than could necessarily be done on her or his own.
method they used when working together. Thus, collaborative problem solving was shown to offer some of the same experiences for children that peer tutoring provided — such as self-reflection encouraged by a visible audience, the need to verbalize ideas or instructions to peers, opportunities to respond to peer questions and challenges (Forman & Cazden, 1985).

The available research on student-student interactions supports the belief that true collaborative work provides opportunities for student learning. The particular studies reported above focus not only on how collaborative interactions influence student learning, but also emphasize the need to study the growth patterns of student-student interactions. Understanding the nature of interactions is shown to be just as valuable as understanding the role of student interactions.

**Teacher-Student Interactions**

When students participate in mathematics discussions with one or more other persons they do so for various reasons. For example, a student may wish to share an idea or a solution with others in the class or a student may verbally explain her or his strategy as a way of organizing or solidifying thoughts. Nonetheless, it has been found that when students talk in whole group situations, they do so primarily for the benefit of the teacher, to give the teacher access to their ways of thinking (Pimm, 1987). In contrast to small group work where students most often share the responsibility to monitor the group, Pimm discovered that whole class discussions are focused by and channeled through the teacher. From his observations, he concluded that the sharing of ideas was directed to the teacher in whole class situations, and students were less likely to listen to each others’ ideas. He characterized such discussions as a series of individual conversations between students and the teacher.

When the teacher is seen as the person controlling and focusing the talk, Pimm (1987) adds, the conversation often follows an evaluative format referred to as Initiation-
Response-Evaluation (IRE). That is to say, when the teacher controls the conversation, discussion revolves around teacher questions that elicit "expected" responses and are followed by the teacher's evaluation (e.g. "that's right," "you've made a good point," or "maybe someone else can help you out."). On the other hand, teacher questions that arise from genuine curiosity can lead to reflective thought on the students' part, Pimm contends. Unfortunately, students learn early on in their school experiences that even this kind of questioning may really be evaluative. Thus, even questions arising from the teacher's curiosity lead the students to try to give the expected answer. Here, I believe Pimm is cautioning that although teaching strategies are employed to enhance the discourse, the strategies may actually deter useful communication when student expectations do not match those of the teacher.

Every classroom has distinct characteristics, some of which determine the way participants (students) talk. Even within a particular classroom, the subject matter, whether science or reading, can determine how the discourse is organized (Stubbs, 1983). Stubbs' work in discourse analysis of classrooms confirms Pimm's claim that teachers can control the talk and the function of the talk, but that student expectations can also strongly influence the nature of the talk that occurs. Stubbs notes that students continuously work at picking up meaning from the teacher's actions and speech. That is to say, students try to interpret the teacher's speech to get themselves on the same wavelength as the teacher, to be able to anticipate how the lesson is organized and their roles within that organization. Thus, when a teacher asks for a definition or a problem result in mathematics class, the teacher also could be covertly calling for students' attention or to control the amount of speech. Stubbs suggests that paying attention to the teacher's and students' speech and the functions of their speech in conversations can reveal what is learned or reinforced through classroom conversations. In particular, listening to students' speech can exhibit what they have learned about how to participate in the classroom discourse.
Leder’s (1987) research on mathematics discourse and learning takes a different look at teacher-student interaction outcomes. Rather than investigating what students learn about participating in classroom talk, her research was intended to link teacher-student interactions to student achievement in mathematics. She looked specifically at teachers’ behaviors that occurred during teacher-student interactions by focusing on the following categories: (1) type of participation opportunity provided to students, (2) level of questions asked by teacher, (3) nature of students’ response, (4) type of feedback from teacher and other classmates, (5) length of time engaged in the interaction, and (6) frequency of interactions between the teacher and particular students. Leder hypothesized that differences in teacher-student behaviors led to differences in students’ levels of motivation and eventual differences in student achievement. To test the claims, a study was conducted in a grade six mathematics classroom. Over a period of three weeks, the mathematics class was videotaped and behaviors during teacher-student interactions were coded and analyzed according to the categories above. Further, two measures of student achievement, pre- and posttest, were obtained. The students were ranked as “best,” “average,” and “weakest” according to the pre-test, and these rankings were used as dimensions for analyzing teacher-student interactions. It was shown that those categorized as “best” were asked more questions by the teacher, and the questions were of a higher level than those asked of students rated “weak.” Weak students were given more wait time, in general, but the higher rated students were attended to by the teacher for longer sustained periods. Furthermore, greater differences in achievement level corresponded with greater differences in attention time and question type between the teacher and the students. The second measure of achievement revealed the same differences in the students, suggesting that although the “weakest” students were given the most wait time, that time was not necessarily constructive.

It should be noted that Leder did not consider variables such as previous knowledge, student expectations, ability to effectively communicate ideas, or inherent
mathematical ability when she examined learning differences in the students. The data gathered relates to a specific teacher and a specific group of students. However, by considering other variables, such as those suggested here, and by comparing the findings to those from other related reports (Forman & Cazden, 1985; Hiebert & Wearne, 1993), Leder's results could provide a broader picture and a better understanding of what does and can transpire in teacher-student interactions. For example, The work of Forman and Cazden highlighted that the teacher's speech is most often the model students use in learning to communicate effectively. Pimm's (1987) and Stubbs' (1983) findings, on the other hand, point to the influence of student expectations on their participation in group discussions.

Taken together, the research on teacher-student interactions indicates that such interactions play an important role as students learn to participate in mathematically rich discussions. Hence, the teacher together with the students must take responsibility for communicating in ways that enhance the development of mathematical ideas within the classroom community. I intend to extend this body of research in focusing on how the teacher's and the students' roles in the discourse influence the nature and the development of the discourse.

**Communicative Competence**

A third area of research that relates directly to the development of discourse in the classroom is communicative competence. Communicative competence refers to a person's ability to use language that is appropriate to a given social or academic context (Pimm, 1987). The use of language within learning situations serves to provide individuals with ways of communicating ideas with others and ways of gaining greater access to and control over their own thoughts. Thus, the study of communicative competence seems important to understand how mathematical discourse develops in the classroom.
Communicative competence is most often studied in relation to second language learning (Kleifgen, 1990; Kramsch, 1985), but it has been shown to be an important aspect for consideration when studying participation in instructional discourse in any subject area (Cazden, 1988; Kleifgen, 1990). The importance of communicative competence in mathematics is evidenced in the NCTM Standards (1989) which call for a vision of mathematics as communication. The NCTM 1996 Yearbook entitled Communication in Mathematics: K-12 and Beyond (Elliott, 1996) also reflects an emerging focus within the mathematics education community on students' ability to communicate mathematical ideas.

The mathematics education community's awareness of the value of communicating mathematical ideas (NCTM 1989, 1991; Elliott, 1996) emphasizes the need to understand how students learn to talk with each other and effectively communicate ideas. Curcio (1990) points out that, in a "traditional" mathematics classroom, symbolic representations and formal definitions are used to communicate mathematical concepts. Curcio sees these two forms of communication as too far from students' everyday experiences, and hence are often thought of as meaningless. Building mathematics discussions from students' informal talk may be the key to enhancing children's competence in communication, Curcio explains. She adds that important communication skills, such as listening, questioning, and articulating ideas, can be acquired through a language-experience approach to mathematics instruction. The language-experience approach emphasizes student interaction through verbal sharing of ideas and experiences, and through questioning of others ideas for the purpose of clarification.

Pimm would agree that language should develop from children's informal and socially oriented experiences (1987). Pimm argues, however, that communication in a classroom situation must move beyond the informal sharing of ideas. Active listening, Pimm contends, is an important component for a successful discussion of ideas.

In Cazden's (1988) seminal work on the development of communicative competence in the elementary classroom, she looked at social interactions of students and
teachers and their roles in establishing the classroom discourse. She suggests that communication in the classroom is governed equally by the cultural and instructional norms of the classroom. Cultural norms refer to those established as a result of implicit and explicit negotiations between all class participants (e.g., by "testing" the teacher, students develop an idea about what is appropriate and not appropriate to say in the classroom), while instructional norms refer to the teacher's expectations for the type and amount of talk that occurs (e.g., in a large group discussion, the teacher is seen as the facilitator while students share their understandings of a problem). Within a particular school subject, she adds, the basic structure of talk remains somewhat constant. Pimm (1987) suggests that this is the case since most instructors regulate conversations to ensure the use of "approved (mathematical) dialect."

Cazden's research on discourse in classroom settings led her to organize a framework of communication structures. The general framework includes categories such as (1) purpose of talk, which in part determines the role of the participants and the style of speech, and (2) the medium of interaction. The specific communication framework of a setting, then, is determined by the environmental and cultural surroundings and by its participants. Cazden has shown that these factors and others influence educational outcomes.

Kramsch's (1985) model of classroom discourse also looks at both the medium and the purpose of discourse. Her analysis of discourse in second language learning at the elementary school level placed "instructional discourse" and "natural discourse" along a continuum of classroom interactions. She categorized observations of discourse and established positions along a continuum to distinguish between the participants' roles in discourse, the types of tasks, and the focus of learning associated with different types of discourse. For instance, during instructional discourse, interactions focused primarily on

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8 Instructional discourse refers to the formal exchange of facts and concepts related to a particular subject area, while natural discourse refers to a more informal or stylized exchange of information (Kramsch, 1985).
exchanging content and on the accuracy of facts, while during natural discourse the focus was on the process of attainment of content and the ways in which interactions occurred. Kramsch also observed that through instruction-oriented discourse, information was delivered and received, while at the natural end of the continuum information was exchanged and meanings were negotiated between participants. There was more collaboration and more effort to make the discussion clear for all participants at the natural end of the discourse continuum.

Kramsch’s (1985) study also focused on the nature of the discourse in teacher-centered and participant-centered classrooms. She found that interactions and exchanges in a teacher-centered classroom revolved around the teacher as giver and the students as receivers of information. The participant- or student-centered classrooms focused on the interactional process (learning how to participate) as well as on the receipt of information. Kramsch also noted that student-student conversations on the lesson content tended to push students to produce more comprehensible, more articulate, statements.

Clearly, Kramsch (1985) presents a social-theoretical view of communicative competence. That is, she accounts for sociocultural factors in students learning to communicate and the relationship of these factors to interactions and discourse in the classroom. As such, the issues raised and points made with respect to instructional and natural discourse in the classroom are important for understanding communication issues in the mathematics classroom. The classroom discourse continuum provides many intermediate positions between “instructional” and “natural” discourse. Although Kramsch chose only to focus on the poles, most classroom situations warrant a consideration of the positions between the poles. It is probably at the points between the poles that students learn to take responsibility for active participation and collaboration in problem solving, and making sense of mathematics through discourse.

Baker’s (1992) qualitative and somewhat more theoretical study of discourse and interactions also took into account instructional talk and informal talk. Her research went
beyond Kramsch's work, however, and focused on finding links between the nature of the interactions and the students' construction of "classroom knowledge." She referred to "classroom knowledge" as the students' sense of appropriate relations with other classroom participants and their understanding of the classroom order -- how discussions are organized, for example. She claimed that by focusing on how the classroom participants describe and analyze interactions in their ongoing participation, researchers can get a better sense of the resources that produce classroom knowledge, classroom relationships, and participation structures.

To support her hypotheses, Baker (1992) provided text from various classroom conversations that illustrated how, for example, the teacher-student relationships governed the organization of lesson knowledge. In one example, she claimed that the teacher's deferrals of students' ideas helped to inform the students "of how school knowledge passes through the grid of (the teacher-student) relationship and needs to pass there in order to 'count'" (1992, p. 12). That is, students were expected to analyze the situation, and to take instructions or suggestions from the teacher as road signs, to use as guides through the lesson to the information or concepts that counted. Baker concluded that, "every classroom session is a site for describing and/or renegotiating the kind of talk-organization and knowledge-productions that can acceptably go on" (p. 12).

The research on communicative competence clearly makes a case for communication as a central component of classroom instruction. The research presented in this section also highlights many salient features of classrooms which contribute to or define the participants' roles within the classroom discourse. Curcio (1990), for one, discusses the need to understand how students develop ways of communicating with peers. Her work and the work of Pimm (1987) show that students perform better in discussion situations when they are allowed to begin with informal or natural ways of communicating. Secondly, the research of Cazden (1988) and Kramsch (1985) describe the roles participants' play, the subject matter, and cultural norms as important influences on the
nature of classroom discourse. Clearly, a case has also been made for communication as a central piece of classroom instruction. As Baker (1992) and others point out, the classroom setting affects the nature of interactions, the nature of the discourse, and hence the nature of learning opportunities. The mathematics classroom community and the nature of instruction, as it relates to the development of discourse, are discussed in this next section.

Establishing a Mathematics Community of Learners

The NCTM Standards present a case for the development of inquiry-based mathematics instruction as a means for encouraging student involvement in the active construction of mathematical meaning (Elliott, 1996; NCTM, 1989, 1991). Lo and Wheatley (1994) caution, however, that merely advocating this method of instruction does not guarantee that students will develop ways of constructing mathematical ideas that make sense to them. Rather, they underscore the importance of establishing a mathematics classroom community characterized by activities that include (1) student interpretation of the mathematics tasks and sharing of their strategies for solving tasks, (2) an emphasis on or support of student-student interactions, and (3) the teacher primarily as the facilitator of student conversations and not the giver of information or evaluator of student contributions (Lo & Wheatley, 1994; Lo, Wheatley, & Smith, 1994). Researchers have discussed issues around the complexities of creating such a community. Borrowing from the work of Cazden (1988), Lampert emphasizes the teacher's role in "creating and maintaining" a culture in which student mathematical activity is meaningful. Although not necessarily explicitly taught separate from content lessons, Lampert suggests that students learn how to participate and learn what kind of participation is legitimate through the mathematics tasks, through instructional activities, and through interactions with the teacher (Lampert, 1990).

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9 Inquiry-based mathematics instruction includes the use of active problem solving and discussion to introduce mathematical concepts. For a more detailed description of inquiry-based instruction, see Yackel and Cobb (1996).
What Lampert, Lo, and others do not consider is the students’ role in establishing a community of active mathematics learners. The research of Lo, Wheatley, and Smith (1994), however, does add to Lampert’s work in considering student participation. They propose that “a student’s participation in class discussion is influenced by his mathematics knowledge, beliefs, social competence, and how other students see him as a participant, both socially and mathematically” (p. 320). Thus, students do influence how the mathematics classroom conversations develop.

Silver and Smith (1996) recognize that an important component of inquiry-based instruction is an atmosphere of trust and respect, placing responsibility for developing trust equally on the students and the teacher. They claim that if students do not view the classroom as a safe environment, they will be reluctant to share ideas and participate in the community (for fear of being criticized, perhaps). In general, research on the development of inquiry-based mathematics instruction advocates joint negotiation of the learning process including establishing participants’ roles in the community and in establishing what counts as knowledge.

A Look at Inquiry-Based Instruction

Until as recently as 1990, there was little research on the role of the classroom community in the development of mathematical knowledge (for a general review of the research see Erickson, 1982; for other exceptions see Lampert, 1990, and Wood, Cobb, & Yackel, 1990). With growing recommendations for inquiry-based instruction as part of mathematics education reform, classroom research is turning toward this and other aspects of schooling that influence learning. Hiebert and Wearne (1993), for instance, compare traditional and inquiry-based approaches to teaching mathematics on the dimensions of learning environment and the nature of mathematics discourse to explain differences in student achievement. In their research, one set of classrooms used a traditional or “conventional” textbook approach for teaching place value and multidigit arithmetic that
focused on rote learning of prescribed algorithms. Another set of classroom teachers taught the same general content but provided activities for the students that emphasized constructing relationships between concepts and computation strategies in cooperative learning situations. The “alternative” classrooms’ activities also promoted the use of student discourse as a tool for learning. The intent of the study was to discover how differences in instructional approaches (and hence goals) were related to learning, and in particular, what features of discourse and of the overall classroom structure promoted learning. Discourse was analyzed separately from student understanding using qualitative techniques. The qualitative analysis of discussions occurring in the classrooms, however, did not attempt to make sense of how student interactions influenced the learning of concepts. Instead the analysis consisted of distinguishing types of teacher questions and resulting student responses, as well as how much the students talked as compared to the teacher. Overall student learning was measured by change in performance on written pre- and post-test assessment items.

Hiebert’s and Wearne’s (1993) research results indicate a positive correlation between the amount of improvement on assessment items (classrooms looked at as a whole) and type of instructional approach. That is to say, those classrooms based on inquiry learning showed greater achievement than did the classrooms classified as traditional. The authors caution, however, that many other classroom variables interacted with instruction, making it impossible to make a direct connection between instructional approach and student learning. They suggest that the nature of discourse and the choice of instructional tasks and procedures are salient features to consider in finding a relationship between instructional approach and learning in mathematics.

Another study of inquiry-based instruction explored student thinking and the ways in which the classroom teacher made the students’ thinking public (Putnam & Reineke, 1993). Putnam and Reineke collaborated with a classroom teacher to test the hypothesis that when student thinking is a focus of classroom activity students will learn to recognize
and develop flexible ways of working with mathematical concepts. Before entering into this teaching experiment, the classroom teacher's interactions with students followed a basic IRE interaction pattern. The model of teaching advocated by the researchers required the teacher to move away from the basic IRE pattern and invite students to give more than numerical answers. Students were expected to explain their solution methods and their choice of method. Through this collaborative research project, Putnam and Reineke found that students were not accustomed to paying attention to each other's work, and were also not skilled in assessing problem strategies. Thus, the teacher and students worked together early on and throughout the school year to establish patterns of interacting and participating in non-traditional (non-IRE) discussions. In a sense, the teacher and students worked to transfer some of the authority (deciding right or wrong, valid or invalid) to the students.

Putnam and Reineke concluded that this shift of authority and shift in traditional patterns of interaction involved changes in both the teacher's and the students' beliefs about mathematics teaching and learning. This conclusion is supported by the research of Lo and Wheatley (1994).

The research studies on inquiry-based instruction reviewed here emphasize that many factors contribute to student learning, and it is not feasible to separate out all possible variables. Both Hiebert and Wearne (1993) as well as Putnam and Reineke (1993) indicate the need to better understand the influence of the mathematics classroom environment or culture. Silver and Smith (1996) suggest that students are more likely to participate in class discussions if they feel the environment is "safe." Still other researchers (Lo & Wheatley, 1994; Millroy, 1992; Yackel & Cobb, 1993) advocate a focus on the teacher's and students' roles in mutually establishing this working and social environment. In general, research on inquiry-based instruction necessarily leads to a focus on the learning environment being created by the classroom participants.
Negotiating the Learning Environment

In Putnam's and Reineke's (1993) collaborative research with a classroom teacher, they highlighted that students were accustomed to particular ways of doing mathematics that included specific norms of interaction. When the teacher changed her expectations for how mathematics class would be conducted, it became necessary for the students to rethink their assumptions about mathematics learning and to learn new ways of interacting. To some extent, these new patterns of participation were socially negotiated within the classroom community. Mathematics tasks and the teacher’s changing role were other factors that determined new patterns of participation.

Similar to the work done by Putnam and Reineke (1993), the research of Lampert et al. (1996) is based on the theory that students’ beliefs, whether explicitly expressed or not, shape the interactions in which they engage, and hence shape how knowledge is acquired. Lampert et al. report that, in general, student beliefs about learning mathematics, referred to as their “folk learning theories,” conflict with how mathematicians invent and do mathematics, thus conflicting with a reform view of mathematics teaching and learning -- one that is more closely aligned with how mathematics is created. Hence, they argue that these folk theories must be addressed by the teacher and students explicitly through talk, choice of task, and expectations for student interaction and collaboration in problem solving. Lampert’s research explored how a teacher might address students’ folk theories through mathematics discourse and through discussions of mathematics instruction. This research is still in preliminary stages, but there is evidence that making a shift in students’ folk theories about mathematics learning requires explicit work by the teacher and the students to negotiate new cultural norms in the classroom.

Expressing one’s ideas, whether it be in a whole group or partner situation, carries with it responsibility for the speaker and for the listeners. When a student takes on the role of speaker -- sharing a solution method or a conjecture -- the student must also take responsibility for expressing ideas in ways that will make sense to the listeners. That is to
say, the speaker must agree to operate within the framework of the classroom community. The teacher must also maintain responsibility for trying to make sense of what students are saying (Yackel et al., 1990). Establishing conventions and rules for communicating ideas, for sharing solution methods, and for making conjectures requires negotiation. Yackel and her colleagues concur that the negotiation process often is implicit, and norms are established or developed as interactions necessitate. Even so, they share the belief that the teacher and students must take responsibility to resolve conflicts and to learn what counts as valuable mathematical knowledge. Yackel and Cobb (1996) refer to the learning of what counts as establishing sociomathematical norms — coming to learn what is viewed as a genuine idea, a different strategy, or a meaningful result.

Negotiation of social or cultural classroom norms (taken-as-shared beliefs) and the establishment of student expectations should be a goal of all classroom participants, and hence the responsibility of all participants. Lo's and Wheatley's (1994) look at this led them to list several social norms likely to encourage mathematics discussions. These include:

- taking the goal of mathematics discussions to be an opportunity for helping each other learn mathematics,
- realizing that doing mathematics involves getting stuck, making mistakes, and resolving apparent conflicts,
- trying to make sense of others methods or explanations and trying to explain one's own methods are difficult tasks that present opportunities to learn mathematics.

They also emphasize the need to keep in mind that "[t]he process of negotiation is dialectic; the negotiation of social norms makes possible the negotiation of mathematical meaning, while the meaning of social norms is formed and renegotiated in the social contexts of students attempting to communicate mathematical meaning" (Lo & Wheatley, 1994; p.48).

Mathematics discussions and inquiry-based instruction in mathematics are undeniably important components in students' development of mathematical relationships.
and concepts. A mathematics classroom focused on genuine inquiry moves instruction away from the lecture model and offers more of a balance between oral and written mathematical activities. There is a danger, however, that discussions come to be viewed as the goal in itself rather than as a means to an end. The base of empirical research on inquiry learning and classroom environment continues to build, and will provide valuable insights for the organization of mathematics instruction and for understanding the importance of developing mathematics discourse. For the purposes of the research to be reported here I focus on the various classroom features identified as salient for research on discourse, namely classroom environment, instructional approach, the negotiation of sociomathematical norms, and the nature of teacher and student interactions.

**Framing the Study Within Existing Research**

In order to summarize the research reviewed in this section, it is helpful to consider how the research addresses the questions pertinent to the present study and where the literature falls short of addressing the questions. Recall that the present study is concerned with the aspects of the mathematics classroom which contribute to the development of mathematics discourse. The corresponding research questions are presented below, along with a summary of the appropriate research.

- What aspects of the mathematics tasks, and discourse structures influence the kinds of interactions that occur and the nature of the students’ talk?

A social constructivist perspective of mathematics learning supports the view that students make sense of mathematics through interactive experiences with mathematics content. Hence, an individual’s participation in the social and academic world of the mathematics classroom is governed by the mathematics she or he experiences, and the ways in which she or he encounters the mathematics. Research on inquiry-based or problem-centered mathematics instruction, such as the research conducted by Hiebert and his colleagues (Hiebert & Wearne, 1993; Hiebert et al., 1997), suggests that instructional
tasks and the nature of the discourse are salient features to consider in creating an environment for learning mathematics with understanding. The research of Hiebert and others, nonetheless, does not discern the possible connections or mutual influences of these two classroom features.

In their research on inquiry-based instruction, Putnam and Reineke (1993) considered how a teacher's shift in instructional approach shifted the students' patterns of interacting. They suggest that such shifts involve changes in students' beliefs about mathematics learning. This research does not, however, specifically address the evolving nature of the discourse. It is my intent to extend the work of Putnam and Reineke by looking closer at how students' interactions evolve over the school year. I also investigate connections between instructional approaches and the developing nature of the classroom discourse, to add to the research of Hiebert and his colleagues (Hiebert et al., 1997)

- What roles do the participants' play in the discourse, and what are their expectations for theirs and others' roles? How do these roles influence the nature of the discourse?

Theories of communicative competence play a part in building the body of research on mathematics discourse and classroom interactions. This branch of research suggests that the classroom teacher plays a significant role in helping students' learn appropriate ways of communicating in the classroom (Forman & Cazden, 1985; Pimm, 1987; Stubbs, 1983). Forman and Cazden's (1985) study of how student interactions evolve followed a similar vein of research. Their research investigated the roles students' played as they learned to work cooperatively in problem solving situations.

Although much of the research reviewed supports the conclusion that both the teacher and the students' play essential roles in developing ways of interacting, very few studies investigate particular aspects of the teacher's role or the students' roles and the effect of their interactions on the nature of the classroom discourse. For the purposes of the present study, the changing nature of the participants' roles will be investigated. In
addition, the nature of the discourse will be analyzed in connection with the teacher's and students' changing roles.

- What is the relationship between the classroom environment, the participants' beliefs, and the nature of the mathematics discourse?

Much of the research discussed thus far includes at least a small focus on the classroom environment. Lampert and her colleagues (1996), for instance, discuss how students' beliefs about learning mathematics shape their interactions with others in the classroom. She and others (Lo et al., 1994) recommend explicit talk between teacher and learners in establishing ways of participating in classroom work. The intent, it seems, is to create an active classroom, and an environment for sharing ideas. The present study illustrates how a teacher and the students implicitly and explicitly work to establish such an environment.

Cobb and Yackel (1995) state that students' involvement in a changing classroom environment is typically not addressed in mathematics classroom research. The mathematics classroom community continually evolves as students participate in social and mathematical activities. This is due in part to the reflexive relationship between individual students' constructive activities and the community's practices within which the students participate. I claim that students' involvement in this community should continue to be explored, particularly how their participation influences the establishment of sociomathematical norms, and how their participation evolves over the school year.

Summary

Mathematics is commonly regarded as a socially constructed body of knowledge, and a specialized language for communicating about many aspects of our world. Hence, new knowledge (individual and community) is developed "through interactions and conversations between individuals and their community" (Corwin & Storeygard, 1995, p. 7). This view emphasizes the importance of paying attention to both the nature of
interactions and the nature of discourse in the mathematics classroom. Pimm (1987), for instance, notes that human language is the medium through which we communicate ideas, and so it is important for students to learn to use mathematical language to present, to share, and to generate new mathematical ideas. Hence, there is a need to build the base of research on discourse and interactions.

To address the research questions listed above, it is important to identify the elements of the classroom setting that contribute to the development of discourse. Yackel and her colleagues (1990) share their view of the complexities of the mathematics classroom.

When children learn mathematics in school, they do so in a classroom where certain standards of conduct are established either explicitly or implicitly. These standards, or norms, influence the way children interact with the teacher and with each other, which in turn influences both what mathematics the children learn and how they learn it (p. 12).

Research conducted in a single mathematics classroom promises to provide an overall picture of the nature of interactions and classroom discourse. This research is not intended to offer answers to the research questions, but rather to offer researchers and classroom teachers information that will help them to answer these questions in terms of their own specific needs and situations.
CHAPTER III

METHODOLOGICAL FRAMEWORK

Introduction

The present study was conducted over an entire 1995-96 school year in a fifth grade classroom. From the first to the last days of school, my presence in the classroom was primarily during mathematics class time, although I did often remain in the classroom for other classes. The classroom teacher and I scheduled weekly talk sessions so that I could better understand her perspective on student work during the mathematics class, and to come to know her as a mathematics teacher. It was equally important to come to know the students as mathematics learners, and so I spent time over the school year talking with the students, both during and outside of the regular mathematics class time.

In this chapter I describe and comment on the research approaches used and my role in the research process. I talk briefly about preliminary research and how that led to the choice of setting and participants. Several issues related to choosing the research setting, including entrée into the field and research ethics, are also discussed. Data collection and data analysis methods are reviewed as well as problems and changes in focus that resulted from these ongoing processes. Finally, I address the issue of the generalizability of a qualitative study.

A Qualitative Research Approach

Qualitative research methods were an appropriate choice for this study since the research questions of interest were best addressed through close observations of a single mathematics classroom over a period of time (see The Questions, page 5). I wanted to
understand not only how the students' ways of interacting and communicating had
developed, but also what about the learning environment had influenced this development.
Many educational researchers (Brown, 1985; Erickson, 1982; Jackson, 1968; Wilcox,
1988) agree that classroom life is complex, and is best understood if viewed from various
perspectives, including those of the participants. Brown (1985) argues further that
students and teachers bring to the classroom their own backgrounds and
personal histories. Thus, the ways in which teachers and students define
their situation in the classroom are constrained by their backgrounds and the
physical, temporal, and organizational context in which schools and
classrooms are embedded (p. 18).
Thus, although description of interactions and over-time comparisons of events are
important elements for looking at the development of mathematics discourse, it is equally
important to understand the classroom culture and the meaning the participants create
through their actions and interactions.
Erickson (1986) suggests the term interpretive research to refer to research methods
used to discover how members of a group or community make choices and to analyze how
their choices and actions constitute the learning environment. He specifies that such
methods necessarily involve taking meaning from the participants' point of view. One
might also refer to this form of research as taking an ethnographic research outlook -- one
that takes into account that human behavior and learning "are responsive to a context that is
interpreted by participants and that is dominated by social relationships" (Eisenhart, 1988,
p. 101). Whichever term is used, the qualitative research methods associated with an
interpretive or ethnographic approach allow the researcher to construct a full picture of what
is happening in the classroom and to answer the question Why is the mathematics discourse
developing in the way that it is? This "situation-based" process of inquiry (Erickson,
1982) provides a detailed look at classroom structure and environment, combining an
insider's and an outsider's view of the setting, for understanding the complexities of the
classroom and for understanding the factors involved in learning and negotiating
mathematics discussions.
The key to understanding the experiences of students in a particular social group (a classroom, for example), Eisenhart (1988) contends, is understanding the meanings they give to their beliefs and actions. These intersubjective meanings or social norms are generally implicitly held by the social group. To make sense of the meaning participants give to objects, actions and relations, the researcher must necessarily become an active member of the group. In my case, close watching and interacting with the mathematics students in the classroom allowed me to take into account the perspectives of the teacher and the students and how that influenced their actions. At the same time, I worked to maintain a broader view of the discourse within the classroom context, in order to describe the role of the discourse within the classroom and to provide insight into how the setting and the discourse were created and why they developed as they did.

An interpretivist research stance fits well with a constructivist view of learning. A constructivist view implies the need to consider an individual’s perspective, purposes, and reasons for actions in order to understand their behavior. And since one is best positioned to understand a person’s actions through understanding environmental and cultural effects on that person, a qualitative or interpretivist research approach seems like the best approach (Noddings, 1990).

Preliminary Studies and Selection of a Classroom

Classroom selection for my research reflected several particular features that I was interested in investigating. That is to say, since one of the primary goals of the research was to investigate the development of mathematics discussions in both small and large group classroom settings, it was necessary to locate a teacher who valued and promoted student collaboration and discussion of mathematics. Although a teacher may value mathematics discourse and claim to promote discourse, I quickly discovered that actively working to make it happen and to provide appropriate opportunities was not necessarily a regular occurrence in these teachers’ classrooms.
Thinking that middle school students might provide richer data than would elementary school students, I spent several consecutive days in a middle school mathematics classroom. The teacher was participating in a professional development program at that time and described herself as eager to make changes in her instructional style, wanting to incorporate more active learning situations for her students. I found that although students were encouraged to interact while working on mathematical activities, the interactions were more social than academic. The class instructional format did promote some investigatory work, but most of the work could be and was done as individual seat work. There seemed to be little impetus for the students to share ideas, strategies and conjectures except on occasions when the teacher asked for student answers to problems. For the most part, when students were asked to share, they seemed reluctant to do more than give a quick response. Sharing did not prompt discussion.

It was suggested by one of my colleagues, a teacher educator for a National Science Foundation-funded elementary level teacher enhancement project, that I visit a particular fifth grade teacher, Brenda Miller. Upon this recommendation, I spent two days in Mrs. Miller's classroom, late in the school year. The visit proved to be exciting. Prior to my visit, students had been making fractional pieces from large circles of colored paper (cutting wholes into fractional parts such as halves, thirds and fourths). On my first visit, the students were exploring one student’s question about why it was more difficult to create ninths than fourths or thirds. Many students presented conjectures about why this might be the case, and several offered more detailed explanations to support their conjectures. Students who spoke appeared confident, willing to "give it a try," and genuinely interested in finding an answer to the question. Mrs. Miller maintained a position at the front of the class and facilitated the discussion, giving all students an opportunity to participate. My curiosity of how the students had gotten to this point of being able to verbally explore a mathematical idea led me to approach Mrs. Miller with a more formal request for her participation in my research.
Classroom Selection Issues

Two major issues around my choice of teacher and classroom were quick to arise. The first was a fundamental question of ethics based on my professional relationship with the teacher as a participant in the teacher enhancement program. There was little to no concern about the fact that I had used this relationship to “find” a teacher interested in helping me with my research. The concerns raised by the staff of the teacher enhancement project were two-fold. It was mentioned that in forming this new relationship Mrs. Miller and I could compromise the confidentiality of the project. There was also the concern that in changing my professional relationship with this teacher I was apt to put undue pressure on her, and so was asked to consider what I might offer Mrs. Miller to relieve some of this pressure. In addressing these two concerns, Mrs. Miller and I agreed to set aside any conversations related to the teacher enhancement program. In addition, I agreed to help out in the classroom in ways that seemed reasonable (e.g. becoming an extra pair of eyes and ears, watching the classroom when errands needed to be run, running certain errands).

The second issue was one of obtaining permission to conduct my research in the classroom. The school district was hesitant to allow outside researchers into classrooms for fear of setting a precedent that would inundate their schools with researchers. Located near several well-known universities with graduate programs in education, the school district had learned from experience that researchers in the classroom could be distracting and even detrimental to student learning. It had become district policy to prohibit classroom research except when conducted by school or community members. My involvement in the mathematics teacher enhancement program that included many of the district’s elementary teachers proved valuable in gaining approval to conduct my research in Brenda Miller’s classroom.
Becoming a Community Member

To give students a sense of my place and permanence in the mathematics classroom, I felt it would be important to attend the first several mathematics classes of the new school year. The teacher introduced me to the students on the first day of school and gave them a rather brief explanation of my role in the classroom. It wasn’t until the sixth day of mathematics class that the teacher set aside time for me to talk with the students about my research, giving me a chance to demystify my presence and become more accepted as a member of the classroom. After introducing myself and describing my research, I talked to the students about how and what kind of data I would be collecting. When asked if they had any questions for me, several students eagerly raised their hands to be acknowledged. Mrs. Miller and I were not surprised that the students were more curious about my mathematics and education background (e.g. they wanted to know if I had been a “good” mathematics student in elementary school, and what I liked most about mathematics) than they were about the research. In a way, this “social” conversation began to establish a rapport between myself and the students.

Data Collection

The research questions of interest for this study are best answered through observation of a single classroom. The questions have to do with various aspects of the classroom and its participants, including the teacher’s and students’ roles during mathematics class, that are likely to influence the nature of the discourse and its development over the school year. I anticipated that the influencing factors included the teacher’s and students’ attitudes and beliefs about mathematics, their beliefs about the teaching and learning of mathematics, the mathematics content areas being studied, the culture of the classroom, and the types of interactions that take place among participants during a regular mathematics lesson.
To obtain a detailed picture of the mathematics discourse and to investigate some of the possible influencing factors listed above, methods common to the ethnographic tradition of data collection were employed. Eisenhart (1988) outlines four main methods -- participant observation, interviews, collection of artifacts, and researcher reflection -- used to obtain various perspectives of the classroom culture and the participants' roles within the culture. Audio and video recordings supplemented field notes gathered as a participant observer. The variety of data collection methods used led to a variety of perspectives which appear through the data. This allowed for cross-checking or triangulation so that my initial impressions could be clarified, validated, and corrected (Goetz & LeCompte, 1984). One data source in particular, a journal of the researcher's reflections shared regularly with the classroom teacher, was valuable for checking impressions and insights about classroom observations.

**Participant Observation**

The main source of data for this research were daily observations in the fifth grade classroom. Over a period of three months at the beginning of the school year, I collected observation notes both by hand and by audio recording on a daily basis. Additional observation notes were collected in February and March -- a total of fifteen days -- although not on as regular a schedule. Daily observations resumed in May and June, when I was able to collect data for two consecutive weeks. As a participant observer in the classroom, the taking of field notes was augmented by working directly with students, most often during small group work.

According to Goetz and LeCompte (1984), participant observation involves sharing in the activities of a community through face-to-face relationships with the community members, in order to elicit from people how they organize and make sense of the community in which they participate. By becoming an active member of a social group or school classroom, the observer is more likely to develop a framework for understanding
the particular behaviors and motivations of the students and the teacher. One the other hand, maintaining a more distant perspective allows the researcher to frame participants’ activities and beliefs within the broader picture of the classroom as a piece of the school community (Eisenhart, 1988).

Along a continuum of participant observation\textsuperscript{10}, as suggested by Glesne and Peshkin (1992), my position was one of observer as participant. Although I was not the mathematics teacher nor a student of mathematics I was able to find a place as a member of the classroom. My main focus was to gather data through observation. Frequently, however, I worked with groups of students, asking questions about their work and offering myself as a resource when the teacher was occupied with other students. On a few occasions, more frequently as the school year progressed, I was invited to facilitate whole class or group discussions related to problem solving strategies. I believe that these opportunities helped to legitimize my position as an active and interested member of the mathematics classroom community.

My primary role as observer and participant was to obtain a dynamic picture of the classroom activities both through my eyes and through the eyes of the teacher and students. This included learning the classroom social norms and expectations through interactions with the teacher and students and along with the students. Through my observations and through participation I collected written and audio records of:

1. Classroom activities -- whether students were working individually, with others, or as a whole class, as well as what mathematical concepts were being discussed;
2. Verbatim conversations through the use of audio and video recording devices;
3. Descriptions of student interactions and reactions -- how small groups were getting along in general and some specific cases;

\textsuperscript{10}Glesne and Peshkin (1992) suggest a continuum of participant observation ranging from mostly observation to mostly participation, with levels such as observer, observer as participant, participant as observer and full participant.
4. Explicit work by the teacher and students to negotiate “rules” for working together;
5. The teacher’s role during various classroom situations;
6. The range of students’ roles during various classroom situations;
7. My role – my participation and possible influence on the teacher’s and students’ choices;
8. The nature of the discourse – whether social or academic, and whether to convey specific or general messages.

Interviews

Interviews with selected focus students and the classroom teacher, both formal and informal, supplemented the field notes and provided a means to corroborate observations of classroom interactions. Formal interviews with students were conducted twice during the school year and utilized an interview protocol based on data gathered through participant observation.

Early in the school year, interviews were conducted individually with five students from Mrs. Miller’s homeroom class. The students were chosen with some recommendations from Mrs. Miller to represent a mix of mathematical ability and presentation style. Presentation style – referring to students’ willingness and capacity to express ideas or other contributions during whole class discussions – ranged from a student who was usually quiet during whole class discussions to students who almost always contributed ideas or solution strategies. Two students were chosen because they were particularly quiet during class discussion. One of these two was considered a strong mathematics student, while the other was considered average. Two other students were chosen to be interviewed because of their roles as question askers. These two were more apt to ask questions, such as asking for clarification or asking about alternate ways of representing a solution, then were their peers. Again, these two students were of differing mathematical ability. The fifth student chosen for a first interview appeared to be very well respected by his peers. His mathematical ability was about average, according to Mrs.
Miller, and there was nothing remarkable about his presentation style. Rather, I was interested in how his role as a class leader would play out in the mathematics class discourse.

These early interviews were conducted during the second month of school, and focused primarily on students’ beliefs about the nature of mathematics in and out of school. They were also asked to talk about recent mathematics class events and their views of experiences taken from these events (see Appendix A for October interview protocol). In this way, I was able to interpret the classroom happenings from the students’ perspectives.

Interviews conducted toward the end of the school year looked quite different from the first set of interviews. Students chosen for this second set of interviews had been selected as focus students earlier in the school year, but were not necessarily the same students interviewed in October. Most were selected based on quite specific characteristics displayed during mathematics class. For instance, one student, whom I will call Nathan, would occasionally speak up in whole class discussions to offer his view of the work of the group. On these occasions, he was pulling together the ideas or mathematical concepts that were being discussed over a period of time (one class period or several class periods). Nathan also found comfort in and often used a strategy I will refer to as finding “odd-even patterns.” Another student, Karen, was chosen for her skill in listening to other students’ contributions and seeing the other side of the coin -- making sense of varying perspectives. Only two of the original five students interviewed were re-interviewed near the end of the school year since the other three students were no longer in the mathematics classroom that became my focus for data collection.

The second set of interviews were conducted in groups of two to three students at a time. One of the students who had been interviewed individually earlier in the school year confided that being interviewed with other students made her feel more comfortable. My intent in organizing a group interview was to give the students a chance to talk with me and with each other. I revised my questions after the first round of interviews with the purpose

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of finding out more about how the students thought about and contributed to the work of a
group, whether large or small. Complete interview protocols are found in Appendix A.

Formal interviews with students are self-reports and often inaccurate. Although
subjective, qualitative researchers (Glesne & Peshkin, 1992; Goetz & LeCompte, 1984)
acknowledge that interviews are valuable for assessing "how individuals make judgments
about people and events" (Goetz & LeCompte, 1984, p. 122). They suggest that
descriptive questions can elicit students' depiction of some aspect of their community,
structural questions can generate or substantiate ways in which students structure their
interactions within a given environment, and contrast questions can get at students'
perceptions of relationships between various structures they use. The overall intent of
formal and informal interviews with students was to obtain student perspectives on what I
was seeing and to support the hypotheses I was forming about the mathematics classroom
environment and participants. Thus, student responses served to check my data and
hypotheses, and my observations served as a check on the accuracy of student responses.
Descriptive and structural questions played a large role in the student interviews, as did
questions that evolved out of immediate experiences during class time.

Goetz and LeCompte (1984) observe that questions should make sense to the
respondents in order to elicit necessary and accurate data through interviews with students.
The appropriate wording of questions proved more difficult than I had anticipated, and
required some revising with the help of the classroom teacher. Questions about specific
classroom events or a particular discussion elicited data that was useful for understanding
the students' actions and classroom contributions. Questions intended to elicit students'
beliefs about mathematics often were unsatisfactorily answered. Informal interviewing, or
impromptu question asking, occurred frequently throughout the school year both with
students and with the classroom teacher. As mentioned in the previous section, I often
questioned students as I observed them working together or independently. This form of
questioning provided immediate feedback for better understanding the students’ moves, ideas, contributions, and questions.

Prior to the start of the school year, I conducted an interview with Mrs. Miller to find out about her views of mathematics instruction and student learning. Some of her thoughts on teaching and learning mathematics were also shared with students during whole class discussions, and with me when we met for weekly talk sessions about the research. These occasions, although somewhat infrequent, gave me a beginning sense of Mrs. Miller and the culture she hoped to create in her classroom. Informal conversations with Mrs. Miller throughout the school year were another source of background and information about students, the school, the local community and her thoughts about mathematics education. Our talks most often centered around a particular student or group of students, about how a specific class discussion had played out or other things that might be on our minds. More formal discussions on similar topics occurred during the weekly talk session. A journal shared between Mrs. Miller and myself was a good place to record the events or ideas we talked about during the school day, and it gave us both a second chance to be reflective about what had occurred. Mrs. Miller and I did not sit down again for a more formal interview until the school year had ended (see Appendix A for interview questions).

Other Artifacts

As noted above, the journal I shared with Mrs. Miller gave us a place to continue conversations about daily events and other classroom curiosities. I wrote journal entries on a daily basis to record my thoughts about what happened during mathematics class, including any striking occurrences, and to record ideas related to the research questions. Mrs. Miller was given the journal to read, reflect on, and comment on over the weekend. The journal also served as a place to test hypotheses, contributing to the analysis process, and to obtain reactions to my observations from the Mrs. Miller’s perspective when there
was no time for a face-to-face conversation. Some of the journal entries merely documented changes in my thinking about the focus of the data collection and the research in general.

Written class work and homework was collected from the folders of the focus students. This included worksheets, small group problem solving and group as well as individual projects. Many written assignments solicited students’ explanations or understandings of a particular mathematical concept. These data samples gave me a better sense of a student’s ability to verbally express ideas about and perceptions of mathematics. In some cases, a student’s explanation reflected their understanding of the mathematics that had been shared by others during whole class discussions. Homework and class papers also allowed me to see some of the content of student activity that might otherwise have been missed, such as the work leading to the finding of a number pattern.

Methodological Issues and Data Analysis

Descriptive data collected through daily observations and recordings, data from interviews, student written work, and the shared journal provide multiple layers of meanings for the situations that arose. These various forms of raw data allow for “thick description,” as Wilcox (1988) refers to it, of the classroom events and ideally give the reader a better sense of the atmosphere in the classroom and the perspective of the participants. It is important, nonetheless, to keep in mind the influence of the researcher on the data that is gathered. Focusing on the lens through which data was collected is one component of the analysis process. Other components of this process include the ongoing comparison or analysis of data throughout the period of data collection, and pulling together and transforming the data to tell a story (Glesne, 1992; Wolcott, 1994).

Looking Through the Observer’s Lens

Clearly, the field notes taken as I observed and took part in classroom events are not without subjectivity. What I saw and how I recorded observations, the questions I
chose for interviews and the issues I chose to write about in the journal were all influenced by the lens I chose, developed by my beliefs and past experiences. How does the reader get a sense of why my attention rested on one matter and not another as I observed the overall happenings in the classroom? What might another observer have noticed that I didn’t? In my description of the data collection process, I have tried to lay out my choices and how these choices arose from the research questions and hypotheses. The observation lens I chose reflects my sense of what is important, my subjectivity. Thus, this study provides the reader with a view of Mrs. Miller’s classroom through the researcher’s eyes. Even so, the reader’s awareness of subjectivity, and the added perspective of the classroom teacher through my interaction with Mrs. Miller, allows the reader to make judgments about the validity of the research reported. Explicitly addressing my potential biases offers yet another means of assessing the value of the research.

Two sources of potential bias were my familiarity with how classroom relationships are built and my experiences as a mathematics educator. As a mathematics educator I hold ideas about how students learn to participate in the classroom, and about what I feel are best ways for teaching mathematics. These biases could potentially have led to my making unchecked assumptions about what was happening in the classroom. Wolcott (1994) advises the researcher to look at “nothing in particular” when entering a too-familiar setting, such as a school or classroom. Assuming classroom events and classroom structures are familiar, the researcher might take a new stance of curiosity, Wolcott suggests, and begin by focusing observations, for example, on student moves that keep the classroom running smoothly or that cause major interruptions. Wolcott and others refer to this strategy as “making the familiar strange” (Erickson, 1986; Spindler, 1982; Wolcott, 1994). Being aware of my biases and being explicit about how they might have influenced my views of the classroom supplies the reader with a guide for interpreting and making judgments about the consistency and validity of what is being reported (Wolcott, 1990, 1994). Thus, it was necessary to be able to distinguish between an imposed meaning of an event or situation.
based on my familiarity and meaning taken from the context through the examination of classroom relationships and interaction patterns.

The Role of Reflection

Ideally, the interpretivist research process involves the continuous testing of theories about the cultural and social organization of the group being studied (Erickson, 1986). In reflecting on the research questions asked and the research methods used to answer the questions, the researcher is positioned to make judgments about the explanatory power of the data or about the clarity of picture she or he is able to paint. When viewed as an ongoing process, analysis aids in the shaping and refining of the research methods. Glesne and Peshkin (1992) add that continuous reflection allows the researcher to take new perspectives, to reveal subjectivity and how it shapes data analysis, and to begin the process of organizing data. Others make a case for the early development of theory as a way to test possible explanations or meanings embedded in social activities.

Throughout the period of data collection, the regular sharing of my insights and hypotheses with the classroom teacher through the journal was an excellent check for misinterpretations, as well as a place to explore my biases and my initial coding schemes. For instance, one journal entry focused specifically on a few students and their means for entering whole class conversations (or for “getting the floor”). It was interesting to read Mrs. Miller’s reaction to how I grouped students according to various strategies they appeared to use. Her suggestion that I contrast these categories with roles students take on during class in general led to a new way of classifying student moves.

As noted in the previous section, it was important to learn about the nature of the judgments I made in recording and analyzing data. Descriptive data such as verbatim transcripts from class discussions provide further evidence for the reader to judge my interpretations. This focus allows the reader to make her or his own judgments about the development and the nature of mathematics discourse in this classroom. Erickson (1988)
adds that “thick description” together with a close look at the how (the ways interactions occur) helps the researcher uncover the why (why the discourse develops in the ways it does).

**Transforming Data**

Presenting “thick description” implies presenting a detailed story. Yet simply providing data -- the field notes, transcriptions, journal entries -- does not seem to fulfill the purpose of telling a story. Further, the story alone will not always fulfill the research purpose. Analysis, to provide a way of interpreting the data, seems to be an essential component of the research process. Analysis requires the researcher to pull together the data, to transform it in ways that tell a particular story. Wolcott (1994), for one, advises that analyses show “whatever it is we know we are getting right” (p. 175, emphasis in original), without necessarily answering the underlying question “So what?” Interpretation, Wolcott contends, where desirable and reasonable, takes on the “So what?” issue.

Another strategy commonly used in analysis of qualitative (and other forms) data is comparison. Comparison involves looking at situations at the beginning of the data collection period and at the end, in order to find and account for differences. In making before and after comparisons, the story of change is beginning to be told. To tell the whole story the researcher must consider what can be learned about the community being studied. It seems critical in developing the story of change to also tell the story of the community’s efforts to make changes and what can be learned from these efforts. The research presented in this dissertation is just such a case where telling the whole story necessarily includes describing the classroom participants’ efforts to make changes. Glesne and Peshkin (1992) also recommend more detailed analysis of events and community participants to uncover “systematic relationships,” not merely similarities, differences and
changes over time. A discussion of how these methodological recommendations were used in analysis of my data is found in Chapter V.

Other Issues Related to Methodology

Another issue related to the problem of subjectivity and bias in the recording of data is the influence of the researcher on the environment being studied. Mrs. Miller and her fifth grade mathematics students were aware of my presence, and were most likely also aware that I was recording just about all of their moves and what was being said. It was clear to me that although I had established a rapport with Mrs. Miller and most of the students within the first month of school, I needed to work to maintain that relationship, particularly with the students so they would remain at ease with my presence and carry on with their normal classroom activities. Even so, the presence of a researcher in any classroom most likely changes how the teacher prepares for instruction. In my case, my presence definitely imposed on the way in which the teacher reflected on her instruction and the students’ learning. Student behavior was also influenced by my presence. What child would not act differently with a researcher listening, watching and taking notes as she or he attends mathematics class? Thus, I worked to remain aware of the effect I had on daily events, and the influence of this on the research focus. Chapter VI contains more details of what I found.

Generalizing from Qualitative Research

One question related to the usefulness of this, or any, educational study is its generalizability to other classrooms in other schools. Can the results of this research inform other teachers and researchers who want to learn more about how discourse might develop in a particular classroom? How similar must a new classroom be to the one studied in order to extend research results to the new situation? Schofield (1990) remarks that a growing consensus appears to be emerging among qualitative researchers that generalizability of interpretive research is best thought of as "a matter of the 'fit' between
the situation studied and others to which one might be interested in applying the concepts and conclusions of that study" (p. 226). Thus, there is a need for attention to multiple complementary layers of description. With enough descriptive data and detailed accounts of the situation being studied, those interested in drawing generalizations from a qualitative work have information needed to make a judgment about the "fittingness", to use Schofield's word, of the study in question with the situation of another site. On the other hand, thick description and interpretation will highlight and distinguish the particulars of a situation, allowing the reader to verify the typicality, but also the uniqueness of the situation (Peshkin, 1993). Keeping an eye on the unique will help the reader determine when the study results cannot be "transferred" to a different setting.

Donmoyer (1990) subscribes to a different view of generalizability. His perspective of generalizability relies on the capacity of one's experiences to lead to meaning-making of a qualitative study. Thus, case studies, ethnographies, and other forms of qualitative research allow the individual to expand her or his experiences through those of others. Alternatively, a case study might help the reader form questions about their own social situation, and get ideas about how to investigate possible answers to the questions formed. Thus, the uniqueness of case studies becomes an asset to developing new views of reality.

In both Donmoyer's (1990) and Schofield's (1990) interpretations of the generalizability of qualitative research, there is an emphasis on detail. In one sense, greater detail highlights the peculiarities of the culture being studied which may narrow the possibilities for extending the research to other settings. However, the particulars of the study also allow the reader to more fully understand the conditions under which a hypothesis will hold, ensuring greater validity. Depending on the conditions or purpose of the study, Donmoyer's definition of generalizability as the ability to expand knowledge and Schofield's view of generalizability depending on "fit" take into consideration the
complexities of the study, and suggest the usefulness of findings, hypotheses, and theories that result.

**Observation Focus Areas**

What follows is a more detailed look at the questions guiding the research and the major categories for data collection, in order to provide more of a sense of the field work I engaged in over the school year. The overall research question -- What aspects of the mathematics classroom contribute to the development of mathematics discourse characterized by discussion and argumentation? -- was explored through research which focused on several subquestions:

- How do the mathematics tasks influence the kinds of interactions that occur and how students talk about the mathematics?
- What aspects of the mathematics classroom environment and of the community’s participants influence interactions and the nature of the discourse?
- What is the nature of participants’ roles and their expectations of others’ roles in mathematics class?
- What is the nature of the mathematics discourse and interactions that take place in the classroom throughout the school year?

One of the most important characteristics to be aware of in the mathematics classroom is the mathematics being taught, investigated, and talked about. More specifically, there is the mathematical content of a lesson, the format of the mathematical activities in which the students partake, and the vocabulary or the language of mathematics used by the students. To investigate the influence of the mathematics tasks and the mathematics content on classroom interactions and discussions, I explored the nature of tasks and the nature of interactions as students worked together to solve the tasks. Special attention was also paid to the manner in which the teacher interacted with the content and with the students in relation to the task and content.
Every mathematics classroom has its own way of functioning within the setting provided. The classroom participants develop and follow a daily or weekly routine for "math time." It is within this mathematics classroom "culture" that events take place and students learn. Thus it is important to understand some of the aspects of the culture, such as the teacher's and students' expectations for themselves and others during math lessons, their conceptions of mathematics, and their beliefs about doing and learning mathematics. An understanding of the culture of the classroom helps to make sense of observations within the classroom context. This category also encompasses the general classroom environment and individual differences which create the environment. To investigate the classroom environment and its potential influence on the mathematics discourse, I explored students' expectations and beliefs about their participation in mathematics class through formal and informal interviews. Analysis of the teacher's and students' moves also helped uncover how the participants established the learning environment, and hence ways of contributing to discussions.

The roles that the classroom participants take, both teacher and students, determine the ways in which participants will interact with one another. Certain roles, such as the teacher's overall authority, never change, while others change as the nature of the activity or the grouping of students changes. It is also true that over the school year, students come to see themselves or others in particular roles such as facilitator of discussion, recorder of ideas or answer verifier. Hence, the third data collection and analysis category addressed primarily the question of the nature of participants' roles and the interactions that took place in the classroom, how roles were established, and how various situations determined role differences.

The last category, the nature of discourse, addresses the general "flavor" of the discourse and the issues raised by the teacher and the students that involve talking specifically about talking about mathematics. Analysis of transcribed whole class and small group discussions as well as accompanying description provides a sense of the discourse at
various points during the school year. A look at the meta-discourse, occasions when Mrs. Miller and students talked about talking about mathematics, provides another access for analyzing how and why the discourse developed as it did over the school year. The four categories for data collection and analysis are not independent of one another. Rather, these categories provide many links for addressing each of the research questions.
CHAPTER IV

PORTRAIT OF THE CLASSROOM

The Community, the Classroom Setting, and the Participants

Mrs. Miller and the fifth graders of Mountainside Elementary School (MES) were residents of Hillsdale, a city of approximately 50,000 residents which borders on a large metropolitan area in the northeast. Unlike the surrounding metropolitan communities, Hillsdale’s population was not very ethnically diverse. Less than ten percent of the MES population included students of African-American, Asian or South American origin. In fact, the majority of students at MES belonged to the large Irish-American community of Hillsdale.

The student population at MES lived primarily in the neighborhoods bordering the school grounds. Most of the 300 students were close enough to walk to school, although many were driven to school by a parent. Only one bus served those students whose homes were designated too far for walking. Thus, the community of MES was close to the school in proximity, which most likely influenced parent involvement. Many parents of Mrs. Miller’s students did become involved in the functioning of the school in various ways. Some parents visited Mrs. Miller after school on a regular weekly basis. One helped Mrs. Miller by visiting one day a week to do photocopying and other similar tasks, while other parents were involved in the school Parent Teacher Organization [PTO]. Most parents visited with Mrs. Miller several times during the school year, during Parents’ Open House, Parent Visitation Day and for Parent-Student Conferences.
Three years prior to the time this research began, Mrs. Miller learned of and became involved in a National Science Foundation [NSF]-supported mathematics teacher enhancement project offered to elementary school teachers in the metropolitan area. After Mrs. Miller's participation in the preliminary year, she and five other teachers from MES signed up for the program (they were the only teachers from all Hillsdale elementary schools to sign up) and participated for its two year duration. The project, designed to support teachers in reflecting on and making changes in their mathematics teaching in directions suggested by the NCTM Standards documents (1989,1991), was of great interest to Mrs. Miller who had been somewhat dissatisfied with her mathematics teaching. She was invested in making changes, and was encouraged by the participation of her colleagues in the project and by the support of the project staff. Prior to her participation in the project, Mrs. Miller said she had worked hard to give students opportunities to show what they could do mathematically. She also wanted to enhance her students' confidence in doing mathematics.

In reflecting on her participation in the teacher enhancement project, Mrs. Miller felt that her participation helped answer some of her mathematics questions and validated her ideas for creating an environment in which students saw themselves as having mathematical ability. Mrs. Miller continued to participate in the project during the time this research was conducted (as a group facilitator), and continued to work on providing opportunities for students to become active participants in the mathematics classroom.

The Classroom

The layout of Mrs. Miller's classroom consisted of three distinct areas, the desk area, the work spaces, and material storage areas (see Figure 2). The students' desk area occupied most of the classroom floor space. Students' desks were moveable and were arranged in groups of 4, 5 or 6 desks. Students were assigned to a particular desk which changed at the beginning of each new month. Students sometimes were given a choice of
where to sit. Desk configuration also changed, although not as frequently as the students changed seats. The students' assigned seats did not always determine where they would work or with whom they would work during mathematics class. Mathematics work groups were most often chosen by the teacher at the start of a new assignment or project. These grouping decisions are discussed in more detail in the next chapter.

Figure 2. Brenda Miller's Classroom

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A second distinct area of the classroom was work space. Work spaces, from which students could choose when the opportunity arose, were plentiful and scattered around the room, although the greatest percentage of work spaces were located around the perimeter of the room. The front one-quarter of the room was mostly open work space, covered by an old pile rug. This area also contained two large work tables which were usually cluttered with various books and papers, and a computer work station separate from the large work tables. A large oval table to one side of the classroom was also open for student use. Students also frequently chose to use smaller corners of the room to work, where they would nestle with their papers and other materials.

Besides Mrs. Miller’s desk, the rest of the room was filled with classroom materials. Posters and student work covered most available wall spaces in the classroom. Books and various other classroom materials filled the bookshelves that extended around much of the room. Mrs. Miller’s desk sat to the back of the classroom, near the door. She rarely used her desk, other than to store papers, books, notices and student work. Behind the door that led to the hallway there was a sink and supply area which was used to store supplies such as paper clips, pens and markers, attendance lists, and any notices to be handed out to students.

In general, the classroom had a cozy atmosphere and the students in Mrs. Miller’s homeroom appeared quite comfortable in this environment. For example, during a vocabulary lesson, Mrs. Miller and the students would gather on the rug to share their understanding of the words on the weekly vocabulary list. All students were given opportunities to join this discussion and they all did. While reading, students chose comfortable spots to sit and read, sometimes at their desks, but most often on the rug or some other floor space in the room. While making state maps for a geography lesson, many students took advantage of the large work tables for spreading out their artwork and materials. They were familiar with the surroundings and used classroom tools and space as they felt necessary.
The Students of Mrs. Miller's Mathematics Classes

Mrs. Miller taught mathematics to all fifth grade students at MES. Since Mrs. Miller's homeroom consisted of only half the fifth graders, she would "trade-off" her students with Ms. Forest, the other fifth grade teacher, for just over an hour each day of the school week, except Fridays. At the start of the school year, the mathematics and social studies switches were whole-class switches. After mid-October, Mrs. Miller and Ms. Forest regrouped the students so each mathematics class (and social studies class) consisted of half Mrs. Miller's students and half Ms. Forest's students. It was at this time that I was able to narrow my focus to one mathematics class, and in fact, to a smaller subset of the chosen class. The five students chosen as focus students at this point in the school year were selected based on my observations and input from Mrs. Miller (as discussed in the Interview section of chapter III). More detailed portraits of these students are provided later in this chapter.

In her doctoral dissertation, Brown suggests that "students and teachers bring to the classroom their own background and personal histories" (1985, p. 18). Mrs. Miller, for one, was well aware that her mathematics background had contributed to how she now organized her mathematics class. She was also aware that the students' previous mathematics experiences would influence their responses to this new situation. Most of the fifth grade students had been students of MES since kindergarten or first grade, and all but one fifth grader had attended MES during their fourth grade year. Because of her familiarity with the students' fourth grade experiences, Mrs. Miller was aware that she would need to spend time at the start of the school year introducing the students to the kind of mathematics they would be interacting with for the remainder of their fifth grade year.

The following narrative of the first day of mathematics class illustrates the students' introduction to fifth grade mathematics with Mrs. Miller. The description and dialogue are based on field notes from the two mathematics classes and from a conversation with Mrs.
Miller. The narrative is intended to illustrate a beginning relationship between the students and Mrs. Miller.

**September 8.** During a phone conversation on the evening before the first day of school, Mrs. Miller shared that although she would be teaching two mathematics classes, she wanted to integrate the two homerooms to get a mix of student ability and interest in each mathematics class. She planned to talk with the students about this on the first day of school.

On that first day, I entered the classroom as students were completing a project for another subject. At eleven o’clock Mrs. Miller called the students to the rug, to sit and talk with her. Once everyone had settled onto the rug Mrs. Miller showed the students a list she had created to help her remember what she needed to do that day. She told the students that making a list is also a good way to organize thoughts, and that lists are often helpful in problem solving. She wondered aloud if anyone could think of other useful strategies for problem solving. No one responded. After a few seconds of silence Mrs. Miller shifted the conversation to mathematics, asking the students what they expected from mathematics class this year. Mrs. Miller asked “What do you think we do in math in fifth grade? What might I want to know about you in math class?” Several students responded.

“Our multiplication skills.”

“If we can do the kind of multiplication you do in fifth grade.”

“If we can solve problems, division word problems.”

Mrs. Miller then asked “Would it surprise you that in math class I need to find out who likes to act; who likes to draw; who likes to talk in front of others; who likes finding patterns? Are you surprised?” A few students nodded, but most students appeared to be waiting for something else to occur although it was clear they were not sure what would be asked of them. Mrs. Miller broke the short silence and explained that she wanted to know more about the students as mathematics learners, about what tools they use and about what works best for them. To do this, she explained, she wanted the students to answer some
questions. She then handed students a survey with questions that asked, for example, "whether making a graph is the way you like to learn math, or talking with others about the problem is the way you like to learn math."¹¹

Before sending the students off to fill in the survey, Mrs. Miller talked with them about her plan to integrate hers and Ms. Forest’s classes, and the possible benefits of having a heterogeneously mixed mathematics class. Two students indicated that they were concerned about such a mix, and Mrs. Miller suggested the three of them talk together about it later. Mrs. Miller had mentioned to me earlier that she expected a few students to be uncomfortable with this grouping, particularly those who had previously been labeled as above average and had been placed in a special mathematics grouping in previous years.

Other students began raising concerns about mathematics class, but these concerns were more about the nature of mathematics class than about who they would be working with. They wanted to know how long mathematics class would be, what kind of multiplication they would do, and how often they would be expected to answer surveys like the ones they were holding. While answering various questions of this sort, Mrs. Miller sent the class back to their desks to begin filling in the survey. This opening conversation with the students set a tone for future conversations. In this short discussion, Mrs. Miller indicated to the students that she would be open with them, that she was interested in what they could contribute to mathematics class, and that she was willing to listen to their concerns. Mrs. Miller had begun to lay a foundation for the mathematics community of fifth graders.

**Curriculum**

Many authors and researchers in education have written about the definition of and the nature of curriculum (Jackson, 1992; Schwab, 1960, 1970). Although no one seems to agree on a single definition, Schwab’s (1970) definition is widely used. He describes

¹¹ A copy of the survey can be found in Appendix E.
curriculum as being composed of four critical components or "four commonplaces of education" -- the teacher, the learner, the subject matter, and the milieu. These components afford one way of viewing the mathematics curriculum of Mrs. Miller's fifth grade classes, and will be used to fill in more of the portrait of the mathematics classroom.

The teacher is the first critical component of a mathematics curriculum, according to Schwab's (1970) definition. Thus, we focus first on Mrs. Miller and her role as mathematics teacher. Brenda Miller considered herself competent at doing mathematics and skilled as a mathematics teacher during her first few years teaching at MES\(^2\). She felt she was capable of solving equations and finding answers to story problems based on procedures she had memorized while a student. She realized, however, that there was more to mathematics than performing operations and following procedures. This realization was an increasing concern of Mrs. Miller's as many of her "lower track" fifth grade students spent much of their time learning procedures, while the "top track" students were given opportunities to investigate more interesting aspects of mathematics such as data gathering and analysis, graphing, and model construction. With her changing sense of what it means to be competent in mathematics spurred by her participation in the teacher enhancement program, Mrs. Miller had come to the conclusion that mathematical ability was not necessarily related to how quickly a person could find a correct answer. She believed that mathematical ability was related to being able to express ideas about mathematical concepts, being able to play with the mathematics, being able to recognize patterns, and being able to see the inherent beauty and the surprising connections within our mathematics system. When asked to describe how these thoughts translated to her mathematics classroom, Mrs. Miller offered the following:

My philosophy is ... to provide experiences that allow students to construct notions (such as prime or square numbers) previously told to them.... I want students to honestly construct and own the ideas, then to put the name

\(^2\) Brenda Miller was a veteran teacher at the elementary school level when she took several years off (working only part-time) to raise her children. Upon returning to teaching, she worked 9 years at another local school before joining the teaching staff of MES.
on (the ideas). I want to provide experiences that give them ways to make sense of how numbers work.

Mrs. Miller's beliefs about mathematics and mathematical ability guided the creation of a mathematics curriculum characterized by active learning. Rather than showing students step-by-step procedures or telling students what to think, Mrs. Miller expected her students to accept the responsibility to participate and to work with others to make sense of mathematical concepts encountered in problem solving. Mrs. Miller saw it as her responsibility "to keep up with what is going on for students." This meant asking questions that uncovered students' ideas or that pushed their thinking further or in new directions. "I also want to get them to ask these kinds of questions" of themselves and of each other, she confessed.

The second critical component of curriculum, as described by Schwab (1970), is the learner. The learner plays an important role in what happens in the mathematics classroom since the teacher's choices are often influenced by what she or he knows about the students. For instance, the students' background knowledge of multiplication facts most likely determines the amount of class time the teacher sets aside for the learning of multiplication. Students' beliefs about mathematics also dictate their actions and reactions to the teacher's expectations. Mrs. Miller was aware that sharing ideas and working with others during problem solving was somewhat new to most of her students. Thus she anticipated the need to work with the fifth graders, at least initially, to set guidelines for working with others, and to encourage students to use the ideas of others to further their own thinking.

Student interviews at the beginning of the school year revealed their beliefs about what it means to do mathematics and their thoughts on working and sharing ideas with others. Most of the students who were interviewed described mathematics as something people do with numbers to solve problems. When asked to comment on the mathematics they were doing in Mrs. Miller's class, the students talked about working on interesting and often challenging problems with friends, about helping each other figure out how to
find a solution. These students felt it was helpful to share one’s ideas and to hear what others had to say about the same problem. One student, for example, recalled having a difficult time figuring out a problem using “higher” numbers. After listening to what other students in her group were doing to solve the same problem, she was convinced that working with smaller or “lower” numbers would make the problem easier. She was sure that she then could use the solution strategy developed with smaller numbers to solve the problem with her larger numbers. This was quite different from past years, she added, when she was expected to work independently and could only rely on her own ideas. Other students agreed that working with a partner or group made problem solving easier since more ideas were generated with which to work. Thus, this student’s beliefs about mathematics instruction evolved as she learned to work with the ideas of other students and to share her ideas with students in her group. In a sense, her and other’s changing ideas about mathematics learning helped shape the mathematics curriculum. A more detailed analysis of students’ beliefs about mathematics learning is included in Chapter V.

Mathematics as an academic discipline, the third component of Schwab’s (1970) definition of curriculum, is the subject of much discussion in this era of educational reform. The NCTM Standards (1989) and other related documents on mathematics education reform (Mathematics Association of America, 1991; National Research Council, 1989) describe several important features of mathematics as it relates to student learning. For example, the NCTM documents (1989, 1991) suggest that to know mathematics one must do mathematics, and that school mathematics involves problem solving, communication, and reasoning (NCTM, 1989, pp. 7, 15). The NCTM documents also make it clear that content alone does not establish the discipline, but that the content (such as geometry and measurement, fractions and decimals, patterns and relationships) implies certain ways of thinking about and doing mathematics.

Goldenberg (1996) moves a step beyond this view and suggests that school mathematics focus on developing mathematical “habits of mind,” such as developing the
inclination to interpret diagrams, to visualize, to tinker, or to translate between visual and verbal information. After many informal interviews and after having observed her classroom for a school year, I believe that Mrs. Miller held a view of the discipline of mathematics similar to Goldenberg's. When discussing the subject matter to be taught in her mathematics classes, Mrs. Miller talked about uncovering some of the big picture of mathematics and getting at the important ideas of number operations. She wanted students to work together in exploring, interpreting, and visualizing what they were doing when they added, subtracted, multiplied or divided in problem situations. Although Mrs. Miller never explicitly expressed this to her students, she made clear her belief that "in order to learn you have to talk," since verbalizing one's ideas clarifies and adds structure to the ideas. Thus, one can conclude that Mrs. Miller's definition of school mathematics included specific content and actions as well as "habits of mind."

The Hillsdale school district provided each elementary school in the district with a standard textbook series and a curriculum outline establishing desired content knowledge for students at each grade level. The textbook series was traditional in that the content followed a general progression of topics, and topics were addressed through examples of standard computation problems followed by a series of similar problems for students to work. Deciding the textbook did not offer useful ways of exploring the various aspects of mathematics that she felt were important, Mrs. Miller chose to establish her own curriculum plan based on problem solving that involved inventing solution strategies, recognizing patterns, and building concepts from basic ideas. Mrs. Miller hoped that her problem solving curriculum allowed all students to enter into discussions, to share ideas, and become part of the community of learners.

The classroom milieu, the fourth component of curriculum, is described briefly in a previous section of this chapter, and can also be inferred through the narrative pieces that follow. The general atmosphere of the classroom was inviting. Students were respectful

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13 Personal communication with Mrs. Miller, October, 1996.
of adults and of each other. The arrangement of desks and work areas in the classroom afforded the students opportunities to work together in a space that was comfortable. The rug at the front of the room became an especially comfortable spot for sharing ideas and strategies. Problem solving with a partner or in a small group became the daily routine during mathematics class. Although students appreciated the time they spent working with others, a few reported during interviews that they were not always comfortable sharing their ideas with the entire mathematics class since they were afraid of giving a wrong answer in front of a large group. Even so, a large number of students did often seem eager to contribute to the whole class presentations.

In summary, Mrs. Miller's mathematics curriculum comprised a philosophy of active learning, a group of students who were willing to work with others in solving problems and sharing mathematical ideas, content that stressed problem solving and student construction of mathematical concepts, and an environment that encouraged and supported student participation. Although such a description seems to paint the portrait of an ideal mathematics classroom, that was not the case. Mrs. Miller's philosophy of active learning was not necessarily played out on a daily basis, and students were not always cooperative or motivated. The classroom environment did feel like a safe place to share and take chances, but these feelings were not always shared by the students. In addition, Mrs. Miller struggled to provide problem solving activities that aided the students in constructing deeper understandings of many mathematical concepts they had encountered in earlier grades. She worked at creating appropriate experiences without the guide of a published curriculum. At the same time, Mrs. Miller often worried about meeting the needs of the students in preparing them for sixth grade. In short, Mrs. Miller's was a "typical" fifth grade mathematics classroom.
Establishing a Daily Routine

This section takes more of a narrative stance to illustrate how the students and Mrs. Miller "negotiated" the daily routine of mathematics class. For the most part, the narrative includes transcriptions of occasions when the class, as a whole, explicitly discussed issues such as appropriate behavior for working with others, expectations such as having homework completed on time, providing justification for one's work, and contributing to the work of the group whether small or large. I have also included some discussion of my role in the classroom and how data collection became a part of the daily mathematics class routine.

On the second and third days of school Mrs. Miller’s mathematics classes collected data from their mathematics surveys and began to graph the data.14 The students worked in pairs chosen randomly by Mrs. Miller. Before getting back to work on their graphs on the fourth day, Mrs. Miller addressed the students and opened a discussion about daily expectations and student responsibilities when working with a partner. Mrs. Miller did just about all of the talking, but students occasionally nodded their heads to indicate they were listening. What follows is a transcript of the beginning of the fourth day of class. Ellipses in the transcription indicate omitted pieces of the dialogue.

Mrs. M: Let’s start by getting out the homework from last time. As we get going you’ll know what your responsibilities are and what you’ll need for math class each day.

Many students rustle through papers on their desks as they look for the data and graphs they had been working on the day before. When the rustling of papers and shuffling of books ceases Mrs. Miller picks up on a discussion that had begun on the previous day.

Mrs. M: Let’s go through what we talked about.... We spent time talking about our feelings, about working with partners. I’m a student, too,... sometimes I don’t like working with the person that I’m assigned to work with, and its hard, but we get our work done. When I put you with partners I may not know that

14 The surveys were filled out by students on the first day of mathematics class. Survey items included “Something about math that I really like is....” “Something about math that worries me is....” and “Some things I heard about math in grade 5 are....”

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two of you just had a big argument. I just know that we have math to do, and much of what we do, we do better when we work together.

Without elaborating much more, Mrs. Miller reminds the students that the graphs are to represent the work of both partners and that both are expected to be able to share the graph with the class.

A similar discussion occurred a few days later. On this day, Mrs. Miller and the students engaged in another discussion that focused on respecting one another’s ideas and maintaining a working relationship. Mrs. Miller called the entire class to sit on the rug and asked for students to share their thoughts.

Mrs. M: Now, what do you think would make for a good relationship with your partner?
Neil: Don’t fight. Just get the work done.
Mrs. M: (addressing Neil) What if you’re my partner and you don’t like my idea.
Neil: I would have to do it.
Mrs. M: Do you have to do it?
Neil: I could try to convince you to do it in a different way.

Mrs. Miller acknowledges Heather.

Heather: Maybe say that your way might be a good way, but let’s try another way.
Mrs. M: How can you work with a partner when you don’t like his or her idea?
Bethany: You could put the two ideas together, so you can both have your way.
Mrs. M: I like that, you can both have your way. OK, I think you get the idea. We’ll have chances later, too, to talk about what you say when you partner’s idea is way out in left field, or if your partner has a better idea than you do, but you want to hold onto yours and try it out.

Again, Mrs. Miller leads the students into and out of this discussion of working together. Many students took part in the discussion today, but there were a few who remained quiet.

These vignettes illustrate two of many conversations about behavior and expectations that Mrs. Miller conducted with the mathematics classes. Although it did not occur in the first of these discussions, Mrs. Miller almost always encouraged students to share their beliefs and expectations for working together. The students seemed to be a bit more comfortable about contributing to the discussion in the second vignette. More students had thoughts to offer, and Mrs. Miller encouraged them to share by asking them...
questions and setting up hypothetical situations. On one occasion, students were asked to comment in writing about working with partners in response to questions such as “What went well?” and “What difficulties did you and your partner encounter?” This explicit work in creating a working environment was a part of the daily routine early in the school year. By December Mrs. Miller was much less likely to engage students in conversations like the two presented above. By that time, students appeared to have established ways of working together.

The following excerpt from the end of the second week of school illustrates how quickly the students became accustomed to openly sharing their thoughts and entering into the whole class discussion. Ellipses in the transcript below indicate missing or omitted pieces of the dialogue.

The students and Mrs. Miller are sitting in a circle around the perimeter of the rug. Mrs. Miller has asked the students to talk about their experiences in collecting and graphing the data from their surveys. Then each pair of students is expected to show and explain their final graph of survey data. Before beginning the discussion, Mrs. Miller raises the following question:

Mrs. M: It just occurred to me that you are all in twos in this class. Why do you think I have one group of three in the other classroom?

Several students seem eager to offer an answer. They raise their hands quickly after the question is posed.

Mrs. M: Hands down. I want a little wait time.

Mrs. Miller explains that many people need time to find an answer, or to form a better and more complete answer. Wait time is for everyone, she says, but especially for those people who need extra time to think about the question. After about 30 seconds Mrs. Miller calls on Bethany.

Bethany: The other class has 19 (students) and we have 18.
Mrs. M: Say a little more about that.
Bethany: (hesitates, then asks the class) Can somebody help me out?
Mrs. M: Do you really need someone to help you or do you just need time to put your thoughts together?
Bethany: Um ... I know that 19 is odd, and 18 is even.

Mrs. Miller speaks to several students she believes are not listening.

Mrs. M: You can learn from the person who is speaking.
Jessica: I want to say something more about that. You could have done groups of 3 in our room.
Mrs. M: Why? I'm going to push you a bit on this.
Jessica: Because they are both equal to 18.
Mrs. M: What equals 18?
Jessica: 9 groups of 2 is 18 and 6 groups of 3 is 18.
David: (speaking quietly to people sitting nearby) cool... that's right.

Mrs. Miller picks up on Bethany’s hesitation and request for help. She addresses the entire class in response:

Mrs. M: It's OK to take a chance in this class. It's OK to say “I don’t have it all figured out yet, but...” Something really great can come out of it. ... Even if Mrs. M doesn’t get it, but you think you’re on to something, keep at it. Don’t give up. Now, what are we about today. What do we need to do during math class? (Ellipses represent missing or omitted pieces in the dialogue.)

Although there were several students who have not yet begun to contribute to these discussions, a few more students than usual could be heard on this day. Even though this vignette comes from just the second week of school, it does illustrate that the students were beginning to get a sense of when and how to contribute to the discussion. Bethany, in fact, took a bit of a risk in answering Mrs. Miller’s initial question, something that Mrs. Miller picked up on and encouraged from all students.

By this second week of working together in mathematics class, students seemed to be developing a sense that time spent “at the rug” indicated time for sharing thoughts, solution strategies, completed projects, stories, or for quietly listening. By midway through the school year, students almost automatically moved to sit on the rug when Mrs. Miller announced they were to share solution strategies with each other. The daily routine of working with partners, sharing work or strategies with the whole class was well established by the end of the first month of school. Although Mrs. Miller stressed other daily routines such as cooperating in problem solving, sharing ideas with group members, contributing to class discussions, students did not really develop these habits (or learn these routines) until much later in the school year. This issue is raised again in Chapter V.

My role as classroom researcher eventually became a part of the classroom participants’ daily routine. The students and Mrs. Miller became used to my presence, and essentially ignored me as I strolled from group to group, listening in on their conversations. By mid-October, however, my presence took on a different feel. Students
had discovered that I was useful for answering some of their questions, for pushing their ideas when needed, and for resolving minor conflicts. From that point on, I was a second “teacher” figure in the classroom. Even so, the students knew Mrs. Miller was the ultimate authority figure when important decisions needed to be made.

A second new aspect of my relationship with the students was in my role as researcher. After the first month or so of school, students became curious about the nature of my work. They asked more questions about what I was listening for, and what I was doing with my notes and audio tapes. A few students became involved in the data collection process as they “took charge” of the tape recorder during whole class discussions. In fact, just about all students seemed to have taken on the responsibility of being sure their contributions to the discussion were captured on tape. On occasion they would even pass the tape recorder from one speaker to the next as they shared solution methods.

In summary, Mrs. Miller and the students worked during the early weeks of the school year to explicitly establish a daily mathematics class routine. By the second month of school, much of what was discussed about expectations and appropriate behavior had become a part of their daily routine. Other daily routines, such as true cooperative problem solving and participation in discussions continued to develop as the year progressed.

**Portraits of the Focus Students**

Margaret, Kenny, Karen, Nathan and Lessa were chosen as focus students for this study. It was my intent to follow their contributions to small and whole class discussions in order to provide a more microscopic view of the development of the classroom discourse across a range of student types and abilities. The focus students’ unique personalities and contributions to the mathematics class are discussed below. Some direct quotes from the students are used to provide more of a sense of their personality and beliefs.
Margaret was chosen to be a focus student for this research for two main reasons. Early in the school year I noticed that Margaret was often a step ahead of her classmates. That is to say, she was quick to notice patterns, to find solutions, and to anticipate the next steps of the work being done. Other students regarded Margaret as a strong mathematics student, as did Mrs. Miller. Her mathematical ability was my first reason for choosing Margaret. A second reason for focusing on Margaret was her timidity to speak in front of a group of her peers. Over the course of the school year, students were expected to talk about their mathematics work, to share strategies and solution methods. Time and again Margaret would decline to share a new insight or a unique strategy with the rest of the class, leaving a partner or group member to explain Margaret's thoughts. Her unwillingness to speak in front of her peers may have been due, in part, to the fact that she was reluctant to "show off," but she was also rather shy in general.

Margaret was an interesting student to watch and talk with for many other reasons. She enjoyed mathematics class, especially when she discovered a pattern or formula that she could use to generate rows and rows of successive values. She enjoyed calculations as well as challenging investigative mathematics problems. Margaret usually picked up quickly on suggestions or a classmate's solution, but had a difficult time explaining or sharing her often "different" solution methods with others. Margaret preferred to work independently and was confident in her mathematics knowledge. She did not easily give up on a strategy, even when the strategy did not seem to be helpful. When asked to describe what mathematics meant to her, Margaret described two kinds of mathematics she had encountered in school. "There's the kind normal people do, and there's the kind we do in Mrs. M's class," she explained. Focusing on the development of Margaret's verbal contributions to the classroom discourse proved to be even more interesting than originally anticipated.

Kenny was primarily quiet during mathematics class and respectful of those working around him. He was productive when engaged in something of interest to him,
but easily distracted by friends when less interested in the task at hand. Kenny was well respected by his classmates, and in some sense this made him one of the class leaders. I was most interested in following Kenny because of his persistence in mathematics class and because of how he was perceived by his peers. He seemed to enjoy many of the more challenging problems, and was not easily discouraged when it took him more than a couple of days to find a problem solution.

Kenny was a frequent contributor to whole class discussions. He could often recognize and verbalize connections he saw between problems or mathematical concepts (e.g. recognized similar solution methods for seemingly different problems, described division as repeated subtraction), and seemed to hold onto ideas shared during mathematics class as if he was trying to organize them all. When asked to talk about group work, Kenny replied “I don’t really care who I work with, because sometimes when you find out.... sometimes you hear that they’re not real good workers. But then when you work with them... it depends what kind of math problem it is. If they really like it ... then you find out that [he or she is] a really good worker. So I don’t really mind.” Kenny did work well with all students in the class, whether in a small or large group setting. He would often take on the role of facilitator during small group work.

What impressed me most about Karen was her eagerness and lack of timidity for contributing to class discussions. Although she talked about being afraid to share an answer when she saw that her answer differed from those of other students, Karen did like to be heard by her classmates, and assured herself that “you might have the chance that all of them are wrong and you’re right. But... sometimes that doesn’t happen.” It appeared that she saw class discussions as a time to think through ideas, to listen to the ideas of others and to come away with new strategies. She did not participate only when she was confident of a final product, as was the case with many students.

Karen was a hard worker and truly enjoyed opportunities to explore mathematics with her classmates. She was not as quick as most students in recognizing patterns or in
using previously discussed concepts to build new ideas. She was, nonetheless, persistent in her attempts to make sense of the mathematics tasks. Karen was especially skilled at seeing and interpreting various perspectives shared by her classmates.

Nathan was a fair mathematics student in that he was not always a consistent worker. He was easily distracted by other things happening around him, and had a difficult time focusing on tasks, especially when he was asked to work independently. Nathan received assistance from a specialist for processing difficulties, so when he occasionally was able to clearly express his thought process Mrs. Miller and I were impressed. In fact, there were at least two instances during the school year when Nathan’s summary of the work of the whole class demonstrated his ability to reflect on his thought processes and to see the big picture. On one such occasion, the class had spent several days working through a basic combinatorics problem and extending it to greater values. In doing so, several students discovered a formula for calculating the number of possible combinations and had shared it with the class. Before leaving the problem and beginning a new one, Nathan shared his thoughts. He explained to the class that in working on the problem he had first needed to do out a chart showing all of the possible combinations in order to understand how the problem worked. He explained that when a first pattern had been found and shared by other students it hadn’t made sense to him. He had needed to work with the pattern and try it out with combinations that he had done in a chart form. When a second formula, involving factorials, was shared by a classmate, Nathan saw it as “another way of writing the (first) pattern.” Although the factorial method did not appeal to him, he demonstrated that he could use it to successfully calculate the number of combinations for larger numbers of sorting items.

Nathan was not ashamed to acknowledge when he did not understand, but he did admit that he was uncomfortable sharing his work with others because he often felt his ideas or answers were wrong. Even so, Nathan did share his work on occasion. In fact, Nathan often talked about basic strategies that he was confident about using, like finding an
odd-even pattern in a set of numbers, or using physical objects to represent a problem situation, and he applied them whenever the strategies seemed appropriate.

Lessa was chosen as a focus student because I felt she was a strong mathematics student, and I was interested in listening to how she expressed her ideas during mathematics class. As it turned out, Lessa was a very quiet student, especially when she was not working with her friends. Thus, I rarely had the opportunity to hear her talk. She explained that she preferred working with friends because it was easier to talk with them, and friends would not make her feel badly when she did not understand a problem. Lessa’s friends were her security, but she did also participate in group discussions when she felt confident of her work. Lessa worked best in small group arrangements where she had more opportunities to make herself heard. Even so, she was reluctant to take the lead and was not overly visible when assigned to work with more outgoing students. As the school year progressed, Lessa became more vocal during whole class discussions. She enjoyed some of the more challenging work done in class, and was happiest when, as she described it, “you’re the only one who got the problem right.”

In some respects, the five focus students represent many traits of the other fifth graders. Even so, they are all unique individuals and it is their individuality that made them interesting to follow throughout the school year. Although it was difficult at times to pick them out in the classroom of voices, an attempt was made to highlight their contributions to the daily mathematics discussions.

This chapter provides an overview of Mrs. Miller’s fifth grade mathematics classes. I have provided a portrait of the classroom that includes many perspectives. The classroom community, including the setting and the participants, is described, along with the mathematics curriculum. The daily routine is illustrated through vignettes and commentary, and portraits of the focus students round out this preliminary snap shot of the classroom.
CHAPTER V

RESULTS AND ANALYSIS

Introduction

The purpose of this chapter is to look closely at the data through description and analysis in order to investigate the changing nature of the mathematics discourse over the school year, and to investigate the research questions discussed in Chapter I. The research questions developed from my original hypotheses are:

• How does the teacher’s choice of tasks influence what students talk about during class discussions?
• How does class format or discourse structure in the classroom influence the students’ interactions?
• In what ways do the teacher’s and students’ roles in whole class discussions contribute to the development of the mathematics discourse?
• How do the participants’ expectations and beliefs about mathematics learning influence the development of the discourse?
• In what ways does the classroom learning environment shape the discourse?

These questions were investigated through qualitative analyses of the data (observation notes, audio recordings, interview transcripts, journal) in two basic forms. Evidence of the general nature of the mathematics discourse over the school year is provided in narratives or episodes from September, October, and February. The episodes were analyzed to provide a comprehensive look at (1) the nature of the classroom discourse and some of its changing features over the school year, and (2) the various and changing roles of the
participants within the discourse. Smaller data units (coded phrases and sentences) were
analyzed to investigate the development of the mathematics discourse and to determine
those aspects of the classroom that contributed to the development of the discourse.

Four episodes from early in the school year are presented in the second section of
this chapter to illustrate beginning small group and whole class discussions. Some
preliminary description provides the context of these episodes. The episodes are followed
by descriptive analyses, focusing on what the discussion participants were saying, and
what roles individuals played during discussions. Episodes from February are presented in
the third section of this chapter, following the same format as the previous section. The
general nature of the discourse at this point in the school year is examined with an eye
toward comparison with the earlier episodes. The February episodes serve to describe the
nature of the discourse at about midway through the students' fifth grade mathematics
experience. The fourth section looks more closely at the data from the first half of the
school year. The analysis in this section focuses on teacher and student “moves,” -- the
choices they make during whole class and small group discussions, the roles they take on,
and their contribution to the work of the entire group. In particular, these analyses focus
on how the students’ and teacher’s “moves” direct or establish sociomathematical norms.
Recall that sociomathematical norms refer to classroom norms for how to participate in the
mathematical work of the group. In this fourth section, the classroom norms are examined
and we take a look at how they may have influenced the nature of the discourse.

The fifth section documents the coding and analysis process and highlights factors
that potentially influenced the development of the mathematics discourse. In particular, five
major coding categories that arose from the research questions are used to examine the
discourse during early months as well as midway through the school year. The coding
categories include the nature of mathematics tasks, the students’ discourse or interaction
structures, the classroom environment, the teacher’s role in the discourse, and the students’
roles in the discourse. Teacher and student reflections from interviews and the shared
journal help to show the potential impact of these classroom factors on the mathematics discourse. Finally, the analysis and results are summarized.

**Beginning Conversations and Discussions**

Episodes are used as one unit of analysis since they offer rich, contextual illustrations of the discourse as it occurred. As suggested by both Miles and Huberman (1994) and Goetz and LeCompte (1984), this unit of analysis, the episode, was chosen or rather created through initial categorization of the data. Although episodes were not the only unit of analysis that emerged during the coding of the data, they proved to be helpful for getting a comprehensive view of the phenomena being studied. Spoken phrases, sentences, and groups of phrases were also used as the units of analysis. These smaller units of analysis proved helpful for analyzing the influence of particular classroom features on the mathematics discourse.

The term "episode" is suggested by Millroy (1992) to refer to a focused description of a collection of events occurring within a specific and usually short time frame. In addition, each episode is chosen to illustrate face-to-face interactions between participants that communicate key phenomena of the study, including those representative of the usual events and those which are distinct and noteworthy (Erickson, 1982). Each episode presented below follows a basic structure. A description of the context and the characters is followed by dialogue taken directly from observation notes or transcribed from audio taped conversations. This format allows the reader to get a sense of the events as they occurred and the impact on the class participants. The episodes are followed by comments and analysis.

The episodes found in this section were selected from observations notes and audio taped recordings of mathematics lessons during the months of September and October. They were chosen to represent a range of topics discussed by the class as a whole or in small groups. The episodes illustrate some of the general nature of class discussions.
during the early part of the school year. Although this section is primarily descriptive in nature, some elements of analysis are included. These preliminary analyses begin to examine the students' discourse skills, their mathematical knowledge revealed through discussion, the students' and teacher's roles in the discussion, and some classroom factors that potentially influenced what was being said, and how it was being said. The episodes are arranged chronologically to offer temporal perspective.

What is a Mathematics Discussion?

Discussions between the teacher and students occur frequently throughout the day in most any elementary classroom. Discussions are sometimes of a social nature, but most pertain to academic matters, whether the class is sharing what they saw under a microscope during a science lesson or learning how to set up a piece of paper for a spelling test. Mathematics discourse is the focus of this research, thus it is important to lay out what is meant by mathematics discourse, which I use interchangeably with the phrase mathematics discussion. What characterizes mathematics discourse? What makes a discussion mathematical?

Informal or formal talk between classroom participants that focuses on mathematical ideas is a basic definition that I will use for a mathematics discussion. Kramsch (1985) uses a similar definition to describe academic discussions, in general, and adds that academic discussions may be characterized by students and the teacher exchanging ideas about how to approach a task, why they made a particular choice, or how they interpreted the ideas of another student. Participants' talk may focus on observations (of mathematical phenomena), or they may simply repeat an idea to show agreement (Kramsch, 1985). Mathematics discourse, or a mathematics discussion, is characterized, then, by the sharing of thoughts, representations, observations, disagreements, and explanations; and is distinguished by the mathematical ideas being conveyed (Corwin & Storeygard, 1995). Taken as a definition, this description of mathematics discourse was used to choose several
episodes where the discourse was typical of discussions during the months of September and October.

The episodes illustrate three mathematical discussions and one discussion of a more social nature. As noted in Chapter IV, Mrs. Miller spent time during the early months of the school year to work with students on understanding how to participate in mathematics class. Episode 2 is included to provide another example of this type of discussion. It focuses on students' feelings when they encounter an answer different from their own. The four episodes, taken together, give the reader a general sense of the nature of the discourse early in the school year. Although not a major part of the analysis at this point in the study, the episodes do provide a first look at the nature of the tasks chosen by Mrs. Miller, how Mrs. Miller structured daily mathematics lessons, the roles the students took on during small group and whole class work, and Mrs. Miller's role during discussions, and the classroom environment.

The data for this first set of episodes comes from both Mrs. Miller's and Ms. Forest's homerooms. Both homerooms worked on the same assignments during the months of September and October, and since two of the five focus students were in Ms. Forest's homeroom, the episodes were chosen to include the voices of the five focus students.\footnote{As discussed in Chapter IV, the five focus students were chosen in consultation with Mrs. Miller to represent a mix of mathematical ability and ability for verbal expression of ideas. The focus students were Margaret, Nathan, Karen, Kenny, and Lessa (order does not indicate any hierarchy of ability).}

**Episode 1: The Bike Problem (September 27)**

During the last ten minutes of the previous day's class time (September 26) students were given "the bike problem" to work on with an assigned partner (a description of "the bike problem" is provided in Appendix E). For homework that same day they had been asked to write about the thinking they and their partner had done. On September 27 students were back with their partners, working on how to present their bike problem.
solutions to the class. Some pairs continued working on finding an acceptable solution while most others were planning creative ways to present their solutions.

Sarah and Lessa had not arrived at a solution the previous day, yet on her homework paper Sarah had written that the boy profited 10 dollars. Lessa had reworked the problem at home, and had come up with 20 dollars as the answer. When a student at a nearby table visited and shared his answer of a 10 dollar profit Sarah was quickly convinced that her solution was correct. Another pair of students shared their solution with Sarah and Lessa and tried to convince them the result should be a 20 dollar profit. It didn't take long for Sarah to change her answer to 20.

Lessa: (talking to David who is working at a nearby table) He ended up with twenty dollars. He didn't?

David shakes his head to indicate that twenty is not the answer he found.

David: Nope.
Sarah: He ended up with ten dollars?
David: Yup.
Sarah: Thank you! ... Maybe we did it right yesterday.
Lessa: I know why I had 70 there!
Sarah: Why?
Lessa: Because he sells it for 70. So now he has 70 dollars.
Sarah: Oh Geez. So he's going to end up with twenty dollars?
Lessa: Oh geez, Sarah, now it's 20. (Sounding more sure of herself.) Yah, yah, it's 20.
Sarah: (turning to another neighboring table of workers) Heather, I just changed it to ten dollars, now Lessa thinks....
Lessa: It's twenty dollars. That's what we think.
Heather: It has to be twenty dollars. We did it two different ways and got the same thing.
Sarah: (Sounding defeated) I guess it's twenty dollars.

Across the room Nathan and Lisa were having a difficult time coming to a decision. They were both holding to their different answers without listening to each other's explanation. I intervened to get them to listen and think through each different explanation.

Lisa had originally written that the answer should be 70 dollars since after selling the bike for 70 dollars the boy didn't lose or gain any more money. Nathan was convinced the boy's profit would be 10 dollars. After hearing Nathan talk about profit, loss, and gain Lisa came to realize she had not been thinking about the problem correctly. She did some
figuring on a piece of paper and changed her answer to a 20 dollar profit. Here is a piece of their conversation at this point. Ellipses in the transcript indicate a pause or hesitation in the speech.

Lisa: I think you're actually right, Nathan.
Nathan: What?
Lisa: I think you're actually kind of right.
Nathan: I know I am.
Lisa: (turns to the researcher) I think he's right... because ....
Researcher: Now you think he's right? Tell me why.
Lisa: Um, what?
Researcher: So you think Nathan is right, now. Can you tell me why?
Lisa: Because he sells it, because he sold it again for 50, so he's going to get 50 back. And then he's going to buy it back for 60, so there goes his 50 down the drain.
Researcher: Plus 10 more, right?
Lisa: Yah. So he's going to get 70 back, so... hold on.... (Silence as Lisa thinks through her work.)
Researcher: (Turning to see what Nathan is writing) Can I see what you wrote Nathan?
Nathan: Well he buys it for 40, then he sold it for 50. So then he buys it back for 60, then he sells it again for 70. And it equals 10.
Nathan: Because... see he buys it for 40, so he gives away 40 dollars. But then he sells it for 50, so now he has an extra 10 dollars.
Researcher: OK, ten more. OK.
Nathan: Ten more. Now he buys it again for 60. So now he has no more money. Now he has no money, now he has no money left. But then he sells it again for 70. So now he has 10 extra dollars again. That's how I think.
Lisa: Why don't we just put down both of our answers? (Sounding frustrated that they still have not come to agreement.)
Nathan: We can't (possibly in response to Mrs. Miller's request for pairs of students to share an agreed upon answer with the class).
Lisa: Hmm.

Episode 1 illustrates two conversations that show some of the students' early interactions and their discussions of a particular problem situation. The episode also provides a glimpse into the nature of tasks students worked on and how Mrs. Miller structured the students' interactions. Although students worked with a partner and were told to present a solution that represented the work of the pair, most students, including these two pairs, were not able to do so. Lessa's use of the word we in describing her solution of twenty dollars does not accurately represent the solution ownership by both Lessa and Sarah. With no discussion of how Lessa reached the result, Sarah accepted
Lessa's answer and then worked with her to find a creative way to present this answer to the class. Their presentation did not give much indication of how they arrived at the answer.

Although Nathan and Lisa appear to have put some thought into their answers, neither one was as willing as Sarah to accept the other person's result. Their banter seemed friendly enough, but the lack of real interaction with each other's ideas kept them and their results at some distance from one another. In both of these cases, the students in each pair shared some thoughts about the problem solution, but they were not successful in finding a solution strategy that represented the pairs' ideas.

**Episode 2: My Answer is Correct (September 28)**

The students were ready to present their “bike problem” solutions. Before gathering at the front of the room for presentations, Mrs. Miller told the students that she wanted to talk with them about the fact that not everyone would agree on the same answer. She asked the students “How do you think you are going to feel when you hear an answer that is different from yours?” Jeanne, who always appeared confident in her work, offered her opinion while the other students listened. Karen did not always appear to be as confident in herself as Jeanne was in herself, but Karen was eager to contribute her opinion, too. Ellipses in the transcript indicate pauses in the dialogue.

Jeanne: I'm going to depend on mine (my solution), and then I'm going to see... at the end... I'm going to look at theirs and say “well, that could be one... and mine could be one also.”

Mrs. M: OK, and yet there is only one answer to this.... So one of these answers has to be correct. And when there are different answers, we know they are not all correct.

Karen: Maybe if I hear somebody else's and mine is different, I can just have confidence in mine, maybe theirs is right, maybe ours is wrong, maybe theirs is wrong. All you have to do is have confidence in yours.

Mrs. M: Do you think it's possible to have confidence in an incorrect answer?

Jeanne: No.

Others: Yes.

Karen: Unless you don't know it's wrong.
Mrs. M: So is there a chance we may learn from somebody else’s way of thinking about it? That we might want to change our thinking about it?

Jeanne is shaking her head to indicate she would not change her mind.

Karen: Yes.
Mrs. M: Is there anyone here who is not willing to look at the other people’s answer and think about it? (Silence.)

One should notice that most students did not appear eager to participate in this discussion. In general, there were usually only two or three students who contributed to these early conversations intended to provoke discussion of feelings and beliefs. Jeanne and Karen were two of the most vocal students in the class, and often gave some sort of response to Mrs. Miller’s questions. In this case, their responses were the only ones. Karen and Jeanne were partners on the bike problem, and it is interesting that although Karen talks about needing to have confidence in her answer, she appeared much less confident than Jeanne. Was she talking the talk for the benefit of her partner? This episode also illustrates Mrs. Miller’s quite directive phrasing of questions. That is, she appears to be telling students of expected behavior, rather than asking them about how they might behave. There are examples, not presented here, where Mrs. Miller’s questioning was more inquisitive than directive.

This episode provides an example of Mrs. Miller’s explicit work with the students in creating an atmosphere for sharing thoughts about mathematics as well as feelings and beliefs about their work as a group. On this occasion Mrs. Miller invited students to respond to the issue of sharing different answers, and at the same time she took the opportunity to express some of what she expected from the students. The lack of response from most students may indicate that they were tentative about sharing their own expectations, or perhaps they were not yet sure what to expect in the whole class discussion of the bike problem. This type of discussion, initiated by Mrs. Miller was typical at the start of the school year. In the journal I shared with Mrs. Miller, she talked about setting a tone for the school year, and wanting to get students involved in that...
process. This discussion is one fairly typical example of how Mrs. Miller invited the students to share their thoughts on their participation in the mathematics discussions.

**Episode 3: I Found a Pattern (October 5)**

Students had been working on what one student called the “magic circle problem” (see Figure 3). On October 5 the class had gathered on the rug at the front of the classroom to share possible solutions to the five circle and seven circle problems. As solutions were being shared to the five circle problem, several students noticed patterns in the way the numbers were placed in the circles. For instance, Amir noticed that all three solutions shown on the board had an odd number placed in the middle circle. Students had then begun to share solutions for the seven circle problem when Kenny raised his hand to offer a “theory” to the class. Ellipses in the transcript indicate brief pauses, interruptions or hesitations in speech.

Kenny:  Um... you have the seven circles here... and you have to have a total of 12, and that’s even. So I think you would have to put an even number (in the middle circle). But if you wanted to make it eleven or something else, like thirteen, I think you would have to put an odd number in the middle.

Margaret:  That’s what I was thinking.

Mrs. M:  Just to be sure everybody is with us.... So you’re saying ... so when your sum was an even number ...

Kenny:  If the sum was even you’d have to put an even number in the middle circle, here. But if the sum was an odd number like thirteen, you would have to put an odd number right in the middle.

Mrs. M:  Anybody want to respond to that by looking at the five circles that are up there? Margaret?

Margaret:  What Kenny said is logical, but um... but that one, Lisa’s answer is eight, and her middle number is one.

Mrs. M:  All right. She’s testing Kenny’s theory.... She wants us to look at Lisa’s....

Lessa:  The sum of ten is like that, too. (Indicating that the five circle problem which resulted in a sum of ten also had an odd number in the middle.)

Kenny:  Well, I saw the sum of ten, and I thought, well ten can be also even and odd, because you can get there by fives or twos. But I didn’t look at Lisa’s and see that.

Mrs. M:  Could we back up to what you just said, that ten could be even or odd?

Kenny:  Yes, because you can get it by twos and you can get it by fives.
Place the numbers 1-7 in the circles so that when you add any straight line of three circles you get the same sum in every direction.

Figure 3. The Seven Circle Problem

Kenny continued to argue about the number ten being both even and odd. Most other students suggested to Kenny that since ten could be evenly divided into two parts it must be an even number. Although Kenny was persuaded by Margaret’s observation that his original theory about the magic circle problem did not hold, he did not seem to be persuaded about the evenness of the number ten. At this point the conversation quickly turned back to discussing more solutions to the magic circle problem.

Up to this point in the school year, whole class sharing consisted of student pairs presenting solutions or solution methods. Students rarely questioned each others’ work in front of the group, and they had only begun to look for and share possible patterns such as the one shared by Amir at the start of this episode. The extended Bike Problem episode found in Appendix B provides an illustration of a typical student presentation during September and early October. Episode 3 is the earliest example found in the data of students interacting in a fairly active way during whole class sharing. Kenny’s brainstorm was shared with the class, and Margaret almost immediately responded since she, too, had been thinking about the relationship between the choice of middle number and the resulting
sum. Margaret's observation spurred others to look more closely and test Kenny's theory with other solutions that had been presented. Although Mrs. Miller facilitated the discussion she was not an essential contributor to the ideas being discussed, tested, and rethought. Mrs. Miller did make sure everyone had heard and, ideally, understood Kenny's theory. As primary facilitator of the discussion, Mrs. Miller prompted Margaret and others to “take the floor,” and she repeated insights shared, perhaps to give emphasis to the “theory” and the students' methods for testing the theory.

Episode 3 also serves as an example of pattern identification, a practice the students were beginning to pick up on as a valued mathematical insight. As the year progressed, pattern identification and students' unique observations or theories became a major piece of the regular mathematics discourse. In this episode Kenny and a few other students, such as Amir, seemed eager to identify and share patterns. Margaret was also actively searching for patterns as evidenced by her remark in response to Kenny's suggestion: “That's what I was thinking.” She appeared less eager to share patterns and in this case she tested Kenny's theory of a pattern before offering her ideas to the class and before supporting the ideas of others. Mrs. Miller continued to encourage students to share such insights, at all stages of development in their minds, as a means of collaboratively investigating the wide variety of mathematical ideas found in the tasks students worked on.

**Episode 4: New Spin on an Old Problem (October 17)**

Jessica had been working independently for a day or so to come up with a new configuration for the magic circle problem that eliminated the need for a middle circle, and hence the need for an odd number of circles. She placed six circles in the shape of a triangle (see Figure 4) and filled in the numbers zero to five such that each side of the triangle produced the same sum. She showed her design to the class and explained that she had arranged the numbers so that the zero and five landed on the same side, the one and four were on the same side, and the three and
two were on the same side of the triangle, to balance the sums. Amir seemed fond of sharing observations with the class, and his question to Jessica sparked others' interest in similar observations. Ellipses in the transcript indicate brief pauses in speech or omitted dialogue.

![Figure 4. Jessica's Triangle Problem](image)

**Mrs. M:** Any questions for Jessica?

**Amir:** (Addressing Mrs. Miller, initially, then turning to Jessica) I have a question for Jessica. Don't you really have 3 middle numbers? 1, 3, and 5 are in the middle... and all are odd numbers.

**Mrs. M:** Ask everyone (i.e. invite others to think about the question) ... do you expect there is a reason for that? (All odds.) I'm asking myself, too.

**Amir:** I think odds have to be in the middle.

**Heather:** Jessica wanted the sum to be 7, she said. And you can't have 2 odds together... 2 odds and an even make an even number.

**Lessa:** What about 2 evens and an odd?

**Jessica:** I tried another method. I did it with odds at the corners and evens in the middle... it adds up to eight, it won't work.

**Mrs. M:** Say that again.

**Jessica:** It adds to 8.

Jessica draws this other diagram with even numbers at the center of each side and the odd numbers at the corners of the triangle.

**Amir:** I'm trying to say if you put a two there (in the middle of one side) and a 3 there (middle of another side) then it won't work.

**Mrs. M:** So the middles have to be all even or all odd, and the points have to be all one or the other, too?
As in Episode 3, this episode shows that students had been encouraged to explore new ideas within an old problem, and to share observations with the class. This episode also offers the reader a sense of some of the changes occurring in the discourse, the learning environment, and the teacher's and students' roles. For instance, the students were interacting more directly with each other than in past whole class discussions and Mrs. Miller played a more facilitative and less directive role in the discussion. The nature of the mathematics discourse had also taken a bit of a shift as students took on more responsibility for making observations about the work of others and for following through with their ideas. Amir's initial observation, for instance, that the three middle numbers were odd numbers, seemed as though he was merely doing as the teacher asked, questioning Jessica on her work. Amir did, however, return to his original question, this time stating a theory of his own about how the numbers must be placed in the circles. As facilitator of the discussion, Mrs. Miller pushed Amir's initial observation further and invited all students to think about it. She modeled curiosity by wondering aloud if there was a reason for all odd numbers in the middle circles. This question opened the discussion for further speculation. In this example, Mrs. Miller let the students know they were all responsible for thinking along with her to find out why all middle numbers were odd. The students who participated in this discussion did not necessarily defer to Mrs. Miller for authority or for the answer. Instead, they offered various answers and other questions. Heather came closest to giving evidence of why the only possible solutions were those presented by Jessica, but no one pursued her conjecture.

Analysis of the Episodes

Through their interactions in September and October, as evidenced in the episodes above, the students and Mrs. Miller had begun to partake in daily mathematics discussions and to establish their roles as participants in the mathematics classroom. The episodes provide snapshots of the students' interactions and the mathematics discourse, and also

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give the reader a sense of the mathematics tasks and classroom format, the participant’s roles in the discourse, and the classroom learning environment. Although four episodes provide only a limited view of the students’ interactions and the discourse, the episodes were chosen to be representative of the nature of the discourse during the early months of the school year. At the same time, this small set of examples highlights some unique characteristics of the classroom studied.

As Episodes 1 and 2 demonstrate, much of the discussion that occurred in September was prompted by and directed by Mrs. Miller. Nonetheless, as early as October, students were beginning to test the waters by offering conjectures, trying various strategies, and beginning to go beyond just finding an answer or just answering Mrs. Miller’s questions. Although there was not necessarily a consistent quality in the discourse that occurred between the students and Mrs. Miller at this time, September and October episodes show that the students were given many opportunities to learn from each other and from the teacher during whole class sharing. Mrs. Miller also shared some of her experiences as a learner and assured students she wanted to help them all get an idea of what was expected of them in small group and whole class situations. Changes in the nature of the students’ and Mrs. Miller’s contributions to the discourse are analyzed in more detail later in this chapter.

The mathematics tasks Mrs. Miller chose, and the class format she created for working on tasks played a role in forming the discussion and in establishing some early discourse skills such as listening and offering explanations. The Bike Problem (Episode 1), for instance, called for a single correct answer which spurred many interesting debates such as the conversation between Lisa and Nathan. By the time students began sharing solutions to the Bike Problem as a class, however, the discourse did not seem as significant since the majority of the student pairs had eventually arrived at the result of twenty dollars. The Magic Circle problem (Episode 3), on the other hand, prompted more discussion as students shared a variety of solutions and possible solution strategies. Episode 4 illustrates
how this problem was extended by a few students in the class. For all tasks worked during September and October, Mrs. Miller maintained the same format. She allowed student groups to work on finding problem solutions and then called for the groups to share or present their solutions to the entire class. This gave all student groups the opportunity to come to some closure on the problem solution, and to prepare a method for sharing the group's work. For example, Episode 1 illustrates a conversation between Lisa and Nathan as they worked on finding a solution to the Bike Problem. The Bike Problem Extended Episode found in Appendix B highlights an interesting shift in Lisa's and Nathan's mathematical ideas that had come about as they worked on preparing a presentation of their ideas. The expectation that they would share their solution led the pair to discover a new solution method and an agreeable solution. In this case, the class format was a catalyst for finding a solution and finding a way to share this solution with others.

The episodes presented here, those found in Appendix B, and other examples found in observation notes and audio recordings indicate that Mrs. Miller tended to direct and lead the conversations during the early months of the school year. Mrs. Miller spent quite a bit of time explaining her intentions and questioning students to bring them into the conversation. The convergent nature of the discourse illustrated in the Bike Problem Extended Episode (see Appendix B) and Episode 2 on page 98 did, however, slowly change as student interest in finding multiple solutions, new methods, and new patterns increased and became a regular part of whole class discussions (see Episode 4, for example). For the most part, the students' contributions to the discourse in September and early October consisted primarily of responses to Mrs. Miller's questions and solutions to the problem assignments. Only a few students took on leadership roles during small group work or when presenting solutions and new observations, such as Jeanne, Kenny, Amir, and Jessica (see Episodes 2, 3, and 4, respectively). Mrs. Miller maintained her role as discussion leader, always prompting students to respond to each other's explanations, to reason through new ideas, and to investigate possible patterns. The participants' roles in
the classroom discourse did change as the school year progressed, as will be shown in the February episodes.

Episode 2 is an example of many conversations Mrs. Miller and the students had in which the students were asked to share their beliefs about learning mathematics or their feelings on working with and learning from others. Mrs. Miller shared with me that these intentional discussions allowed her to set a tone or establish a climate for sharing.16 “Sharing at the rug” also began to take on significance at about this time. Mrs. Miller regularly called students to the front of the class to share each group’s work. For some, the rug seemed to signify a safe place to share solutions, new ideas, or interesting observations. This was true for even more students as the year progressed, and will be discussed again in more detail later in this chapter.

The beginning two months of the school year were a learning time for the students and Mrs. Miller. The students had begun to get a sense of what was expected of them as mathematics learners, namely working with others to solve mathematics problems and share solution methods through whole class discussions, and they had begun to offer their own unique ideas for others to consider. The mathematics discourse and the participants’ roles in the discourse had begun to take shape. The section that follows illustrates conversations from midway through the school, and is followed by a discussion of some of the significant changes in the nature of the discourse that had occurred by this time.

February Conversations: Sharing Relationships and Patterns

The episodes chosen from February observation notes and recordings were selected to illustrate the nature of the mathematics discourse at about midway through the school year. As with the September and October episodes, this section is primarily descriptive, and offers the reader a chance to get a sense of the nature of students’ interactions, the tasks they worked on, and the classroom environment. Analysis of the episodes focuses on the

discourse skills exhibited by the students and Mrs. Miller, and also on changes in the mathematics classroom as compared to earlier episodes. Hence, the general nature of the discourse is examined, as well as preliminary analyses of changes in the students’ and the teacher’s roles in the discourse.

Two episodes chosen from February observation notes represent a beginning and ending whole class discussion of the same mathematical problem. The students’ initial discovery mode for approaching the problem (Episode 5) carried over a week’s time as students discovered more and more to be curious about in relation to the problem solution (Episode 6). Sharing these curiosities with other members of the class became part of the daily routine. These February episodes illustrate occasions when (1) a student shared her understanding of the basic mathematical components of the problem, and (2) a student introduced an interesting pattern within the problem solution. As with earlier episodes, a description of the context and the participants precedes the actual classroom dialogue in the episodes found below. Although these two episodes cannot tell the entire story of the nature of conversations in February, the analysis that follows does take into account all observation notes from February, not only the data from these two episodes.

Episode 5: How Are These Related? (February 6)

On February 5, students had spent most of mathematics class time working with pattern blocks to get a feel for the “cake problem” in which students were asked to use the hexagon pattern piece as the basic cake shape, and to create progressively larger hexagonal cakes by adding rings to the basic cake. Students were also expected to record the price of each new ring and each new cake, given that the basic cake costs six dollars. Most of the students’ work on February 5 consisted of small group explorations with the pattern blocks in building the cakes and finding prices of the various pattern blocks. Class on February 6 began with students sharing what they discovered about the pattern blocks. Taylor had taken the initiative to record her work in her mathematics notebook and was eager to share
her group’s discoveries with the rest of the class. Although as a group Taylor, Margaret, David, and Heather had found the prices associated with each pattern block shape, Taylor chose to share only a small part of the group’s work. It may have been that Taylor had not yet made sense of how the others in her group had decided on prices.

Mrs. M: We’re going to start by considering what ... what you came to know from yesterday’s work. Taylor, would you start us please?
Taylor: OK, (reading from her notebook) I learned today, that’s yesterday, that we needed to know how many blocks is in the one ring, and how many rings is one whole cake. Also how much it costs. I know that a yellow block is six dollars. And the green triangle, you have to measure it to the yellow. And the blue diamond ... and...

As Taylor reads from her notebook, Mrs. Miller records the following information on the front chalk board:
- Number of blocks in a ring.
- Measuring shapes to each other and the yellow.

Taylor: I don’t know what the yellow is called. Octagon?
Eddie: No.
Nathan: Octagons have eight sides.
Taylor: It has six sides (referring to the yellow pattern block).
Mrs. M: It has six sides.
David: Hexagon.
Mrs. M: Hexagon. Good.
Taylor: The yellow hexagon.
Mrs. M: All right. I wrote down some of what I thought I heard from what you said. Read it and see if you would agree.
Taylor: (reading) Number of blocks in a ring. Measuring shapes to each other and the yellow hexagon. Yes.
Mrs. M: OK. You were using ... you were measuring things against the yellow. And you were comparing shapes to each other. And what did you find out in doing that, Taylor?
Taylor: Um, that the blue triangle was three... it took three for the yellow. And...
Mrs. M: Three blue ... it took three blue...
Taylor: To make the yellow. And six green to make the yellow. And we couldn’t do the beige because it wouldn’t....
Mrs. M: Did you come to know anything about the red?
Taylor: Oh, these were two (holds up a red trapezoidal piece). I forgot to put that in there.

This discussion continued, with other students explaining how they had come to know the monetary values of the various pattern blocks with relation to the yellow hexagon which had been given the price of six dollars. By the end of the conversation Mrs. Miller
had written several expressions on the board to represent what she heard students telling her, and she asked the students to be sure they agreed with what she had written:

- 2 Red = 1 Yellow
- 3 Blue = 1 Yellow
- 6 Green = 1 Yellow

1 Red = $3.00
1 Blue = $2.00
1 Green = $1.00

In general, the students all seemed to be in agreement with what Mrs. Miller had recorded.

This episode was chosen to demonstrate that although students continued to direct their contributions to Mrs. Miller, there is evidence that students were taking initiative in whole class discussions. In this case, Taylor had taken the initiative to record her understanding of the group’s work, and then shared this with the entire class. The conversation between Taylor and Mrs. Miller flowed smoothly as Mrs. Miller recorded Taylor’s suggestions, repeated them for emphasis, and occasionally asked questions to clarify the notions being shared. A few other students entered the discussion when Taylor faltered in response to Mrs. Miller’s questions. As the conversation continued, Taylor’s work was used by other students to explain their work in finding the prices corresponding to the various pattern blocks.

Although this episode does not reveal any immediately striking differences between October and February mathematics conversations, a closer look shows that the students’ roles in the discussion, as well as Mrs. Miller’s role had shifted. As Taylor presented her work to the class, for instance, we do not see others taking very active roles in the discussion. A few students did, however, call out answers to help Taylor, and to show what they understood from Taylor’s work. The episode also illustrates, through Taylor’s voice, the work of a student group who took responsibility for making sense of at least some of the mathematics they had encountered in the Cake Problem. Mrs. Miller then offered their work for use by all students. Notice that Mrs. Miller did not take up much time to share her expectations for student participation, to pull others into the conversation, or to interpret Taylor’s ideas. Instead, Mrs. Miller carefully facilitated the conversation by
recording and occasionally clarify Taylor's contributions. This marks an important change in Mrs. Miller's role. Episode 6 and other subsequent vignettes provide more examples of how Mrs. Miller's role in the discourse evolved, and many more instances of student-student interactions, which became a common occurrence in February.

**Episode 6: Counting By Twelve (February 13)**

After having worked on the Cake Problem for the previous five days, and having shared a few patterns or formulas for finding successively larger cakes and their prices, students were looking closer at the sequence of ring and whole cake prices they had listed on a large chart hanging at the front chalk board (see Figure 5). The students were gathered on the rug. Margaret and others shared number patterns they saw in the price of successively larger rings while Mrs. Miller kept the discussion focused.

Margaret: Um, in the tens column it goes one, three, four, five, and then the number six, and it keeps going, but it skips the two. And when it gets up...

Kenny: Oh yeah, I see that.

Margaret: And then when it gets up to another place, like... when it gets up to the thirteen in the tens column it skips the fourteen and then 15. First it skips the two, then it skips a four, well it's really fourteen. Um, and then it'll skip like a six,... twenty-six.

Mrs. M: Can you think about what's causing that to happen? (Turning to the entire class) I'm asking Margaret... she's talking about, she sees it skips decades... it's in the teens, then there are no numbers in the twenties... no digits... it's in the thirties, forties, fifties. And she's noticing it skips certain decades. Can you think what's causing it to do that?

Margaret: It's related to the twelve tables.

Mrs. M: It's related to the twelve tables?

Margaret: Because when ...

Mrs. M: Are those multiples of twelve? The 18, the 30, the 42, and the 54?

Margaret: It's still counting by twelve... but not starting at twelve.

Lessa: It's starting at six.

Mrs. M: It's starting at six, and what is that due to, Lessa? Why is it starting at six?

Lessa: Because the price of the first cake.

Mrs. M: Is it related to that first cake? Eddie?

Eddie: Well, I think that happens because, um, if it was on number 58, like say 58 was the next number. Um, if you add twelve to it, which it happens every time (i.e. "add twelve" was the pattern for finding the price of the next larger ring) ... it's just like
adding two... that would be 58 ... 60 ... but then you have to add another 10 and that brings it ... it goes to the next... it skips.

Mrs. M: It skips a decade? So 58, and you add twelve more, you're going to be in which decade?

Eddie: Sixties. I mean, no, seventies.

Mrs. M: The seventies, and adding twelve caused you to skip over all the decade of the sixties.

Kenny: Maybe it’s because twelve is a higher number than ten, and ten is the last number you can count... without skipping ... like fifties. And ....

Mrs. M: ... So if we count by tens we’ll still be in every decade. What if we go to eleven? Will that cause us to skip a decade sometimes?

Class: Yes.

Eddie: Anything over ten would.

At this point in the students’ work with the Cake Problem they were no longer using the pattern blocks, and most students could work with one or two distinct methods or formulas for finding the price of the cakes. However, even with what seemed to be common knowledge about how to find cake prices, students shared two different solutions to the price of the tenth cake, the basic hexagonal cake with ten rings (*not* part of the Episode 6 discussion). Mrs. Miller encouraged the students to find a way to use the pattern Margaret shared to check the two solutions offered for the price of the tenth cake.

<table>
<thead>
<tr>
<th>CAKE</th>
<th>RING</th>
<th>CAKE PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BASIC</td>
<td></td>
<td>$6.00</td>
</tr>
<tr>
<td>BIRTHDAY</td>
<td>1st</td>
<td>$18.00</td>
</tr>
<tr>
<td>GRADUATION</td>
<td>2nd</td>
<td>$30.00</td>
</tr>
<tr>
<td>WEDDING</td>
<td>3rd</td>
<td>$42.00</td>
</tr>
<tr>
<td>?</td>
<td>4th</td>
<td>$54.00</td>
</tr>
<tr>
<td>?</td>
<td>5th</td>
<td>$66.00</td>
</tr>
</tbody>
</table>

*Figure 5. The Cake Problem Chart*

The nature of the discussion described in Episode 6 is quite different from Episode 5, but not unlike the discussion that occurred in Episode 3 (page 100). As in Episode 3, Margaret and Kenny shared new ways of looking at an old problem, and in the process,
other students came to investigate the new idea. In Episode 3 it is Margaret who tested Kenny’s work and noticed a possible flaw, while in Episode 6 Kenny and others took ownership of the pattern identified by Margaret, and attempted to explain what they understood about the mathematics of the pattern. The general nature of the discourse shows active participation of many classroom members, with students in a discovery mode.

Here, as in the October episodes, the students took much more initiative or responsibility for contributing to the discussion of the Cake Problem, than they had during problem solving in September. Students who entered into the discussion laid out in Episode 6 were no longer interacting as pairs, nor sharing ideas from their small group work. Rather, they were interacting as individuals invested in making sense of ideas presented to the class. Thus, although in February students continued to work as a group during the initial problem solving phase of each task, they took individual responsibility for continuing to explore the mathematics found in the problem solutions. Although students had begun to take responsibility for such work by October, it did not become a regular part of their work on tasks until the middle of the school year.

**Analysis of February Episodes**

By midway through the school year, the students and Mrs. Miller had participated in many mathematical discussions, most arising from the students’ small group work on tasks provided by Mrs. Miller. The participants had established ways of interacting with each other, but these modes of interacting continued to evolve. The February episodes afford the reader snapshots of conversations occurring at this time of the year. They are also intended to offer illustrations of the nature of students’ interactions, the roles they took on during whole class discussions, Mrs. Miller’s changing role in the discourse, and the learning environment. As noted in the previous section of this chapter, the episodes were
chosen to be representative of February discussions, and to highlight some unique characteristics of the classroom.

Although Mrs. Miller continued to direct much of the discourse, as demonstrated in Episodes 5 and 6, the February discussions, in general, were qualitatively different from those in September and October. The February discussions illustrated in the episodes are characterized by an increase in the number of ideas exchanged by students during whole class sharing. The nature of the students interactions with one another had also shifted as students showed that they were eager to investigate and build on the ideas of others. The shift in their interactions most likely was influenced by the evolving classroom environment and the students' sense of the classroom sociomathematical norms (to be discussed in more detail in the next section).

The nature of the mathematics tasks and class format did not change much as the year progressed, but the students work with the tasks did change. The students continued to work in small groups and to share the work of the group with the entire class through presentations. By this point in the school year, the students seemed quite comfortable with the format, and could more easily anticipate what they would be asked to do, as evidenced by Taylor having taken initiative to record her thoughts from the previous days work (Episode 5).

In Episodes 5 and 6, as in examples from earlier in the school year, Mrs. Miller directed and focused much of the discussion. That is, most of the students' suggestions were directed at, channeled through, and commented on by Mrs. Miller. For example, as facilitator of the discourse in Episode 5, Mrs. Miller interpreted and summarized Taylor's explanations by writing short phrases and equations on the board. In Episode 6, and more frequently in general during February discussions, Mrs. Miller took a less active role in channeling the discussion and a more active role in extending the discussion, such as when she challenged Margaret and others to make sense of the mathematics behind the pattern. This episode also illustrates how several students accepted the challenge and took on the
responsibility of making sense of Margaret’s pattern as when Kenny, Eddie, and Lessa joined the discussion.

Another significant difference between February and earlier discussions is that, in February, students were more apt to build on the ideas of others. This is illustrated in Episode 6 when Eddie added on to Margaret’s explanation of “counting by twelves” by describing a particular situation in which adding twelve would result in a skipped decade. The analyses found in later sections of this chapter summarize further comparisons between February conversations and those from September and October such as changes in how students addressed the validity of a mathematical solution. To provide a few more illustrations of this, Examples 1 and 2 were pulled from October and February data, respectively. They highlight a change in the students’ focus when discussing a problem solution.

Example 1. Eddie and Neil were asked by Mrs. Miller to present their own version of the magic circle problem to the class and to explain their choices (see Figure 3 on page 101 for a description of the magic circle problem).

Eddie: What we did, ... we um, ... we didn’t ..., we tried to draw circles, but it was too hard to draw. Like, we couldn’t get the things even. So this is what we did.

Eddie holds up a six by three table of numbers. The middle column is completely covered by a large number seven.

Eddie: We thought this would be easy, so we thought of this idea. We tried to draw it, and we couldn’t figure out what we could do with the sum (of the numbers in the chart). So we did numbers one to thirteen. And then we went ... we counted backwards from thirteen down to ...

Neil: Down to one.

Eddie: So we counted thirteen down to eight, and then one down to six. And then um, ... the left over number that we couldn’t use was seven. And we put the seven in the middle.

This example illustrates how, early in the school year, students often shared their problem solutions by describing step by step procedures. In fact, Eddie went into quite a bit of detail as he shared with his classmates the choices he and Neil made in designing the array of numbers and filling in the correct number sequence.
Example 2. In this next vignette students offer explanations for and opinions of one student's solution for the Valentine Problem. Ellipses and the symbol /?/ indicate omitted or missing pieces of the dialogue.

Karen: I understand that what Amir did was that first he used Gregg. He said that Gregg will give valentines to Stephanie, Timothy, and Theresa. Then he moved up to Stephanie ... and he did the same thing with Timothy and Theresa.

Lessa: I think Amir /?/ ... I think Amir should have written the two times six out ... and he could have done it another way, too. And I show that on my paper.

Mrs. M: So you're uncomfortable that he doesn't indicate all the valentines? Heather, how do you address that?

Heather: I think Amir's method is OK. And what Sarah is doing ... she's doing it all out. She is doing it like Lessa said you could do.

This example is typical of many February conversations. It illustrates that by February the students were able to, and often did, comment on and explain the work of others. They had moved away from describing step by step procedures and instead often compared their work to that of others, or talked about what they understood was happening in the problem situation. These kinds of interactions indicate that, in presenting their work, students were explaining their reasoning in ways that allowed others to make sense of and use their ideas. Although not apparent in the examples provided thus far, students were also asking more questions of each other and offering alternative explanations when classmates expressed confusion.

The February episodes and the February data in general do not provide much evidence of how Mrs. Miller and the students worked to establish the classroom learning environment that one sees in through the episodes. These examples and the rest of the February data do, nonetheless, reveal changes in the learning environment. For instance, by February Mrs. Miller no longer initiated discussions about beliefs or expectations as illustrated in Episode 2, and these issues were hardly ever raised by students. The students seemed comfortable participating in mathematics discussions and appeared to have a sense

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17 The Valentine Problem asked students to find the total number of valentines distributed between four friends if each friend gave a valentine to the other three.
of their responsibility within that environment. The rug continued to be the place where students gathered to share their solutions and observations. The data from February and later in the school year indicate that Mrs. Miller rarely had to summon students to join her at the rug since they usually began moving in that direction when their work changed from finding solutions to sharing solutions.

In general, February observation notes reveal qualitative differences in the subject of discussions and in the nature of the teacher's and students' interactions during whole class sharing time. These changes are mentioned briefly here, but are taken up again in more detail later in the chapter. Before taking a closer look at changes in the classroom features and how these contributed to the development of the discourse, it will be helpful to better understand how Mrs. Miller and the students established a relationship with each other, and how they negotiated the classroom sociomathematical norms. This investigation looks closely at the participants' expectations and beliefs about what mathematics is and how it is learned, and provides the reader with more of a sense of the classroom and the participants' place within that setting.

**Teacher and Student "Moves" in Establishing Sociomathematical Norms**

A social constructivist perspective of mathematics learning implies that one believes mathematics is socially constructed, and that learning mathematics includes a "process of acculturation into the mathematical practices of a wider society" (Yackel & Cobb, 1996, p. 460). This acculturation process is continuously evolving and on some level must include interaction between the learner and knowledgeable individuals seen by the learner to have mathematical authority. In the elementary classroom, the mathematics teacher holds mathematical authority. Nonetheless, Yackel and Cobb (1996) give evidence that the teacher does not hold authority for creating and maintaining the sociomathematical norms of the classroom. Rather, the teacher and students together are responsible for creating and maintaining classroom norms. As discussed in Chapter II, sociomathematical norms are
the taken-as-shared beliefs of the classroom participants about what is involved in learning and doing mathematics.

Educational researchers such as Bauersfeld (1992) and Carlsen (1992) have written about the influence of the classroom environment and the role of the teacher on students’ mathematics learning and beliefs about mathematics. Yackel and Cobb (1996) claim that the students’ participation in the mathematics discourse also plays a large role in influencing the perception of what counts as valuable, meaningful mathematics. The analysis of data found in this section explores the teacher’s and students’ beliefs about mathematics, how the teacher and the students interactively negotiated what was valued mathematically, and hence what was discussed during whole class sharing. This section is included here to give the reader an understanding of the classroom perspective I held -- my sense of the classroom norms -- when analyzing the data and interpreting the analysis results found in later sections of this chapter.

Teacher Moves

Teacher “moves” play a large role in establishing mathematical and social classroom norms. The teacher is often seen as the authority over both the social and the mathematical domains, and so students tailor their participation based on their interpretation of these classroom norms (Cazden, 1988; Stubbs, 1983). That is, while either explicitly verbalizing or implicitly indicating her expectations, the teacher signals what counts as acceptable, valuable, and expected moves on the part of the students. For instance, a teacher’s reaction to a student’s solution is often interpreted by students as an indicator of how the response is valued by the teacher (Yackel & Cobb, 1996). Pushing students to give alternate responses signals that there are multiple solution methods, and when various solution methods are offered, the teacher’s response to each implicitly indicates what it means to have a mathematically different solution. Furthermore, the teacher’s reactions to
students' explanations give students a sense of what counts as verification, proof, or what counts as a valid attempt to find a solution.

The episodes above and observation notes from September and October observations of Mrs. Miller's mathematics classes provide an interesting view of Mrs. Miller's "moves" and her influence on developing classroom sociomathematical norms. The analysis of teacher moves focused on the coded data within three categories -- Nature of Discourse, Teacher Beliefs, and Teacher's Role -- and a total of six subcategories within these codes.18 Within the code Nature of Discourse, Mrs. Miller's repetition of student answers and her questioning of student responses highlighted indicators of what she accepted or expected as an answer. For instance, in September when students were presenting graphs of data collected from surveys, Mrs. Miller asked each pair of workers "How did you decide to do the graph this way?" and would then invite other students in the class to tell the story of the graph they were looking at. Such questions and phrases explicitly laid out for students that they were expected to explain their thought processes and they were also expected to make sense of the work of others.

The category Teacher Beliefs was used to code instances when Mrs. Miller explicitly or implicitly communicated her beliefs about the nature of mathematics and her expectations for the learning of mathematics. Mrs. Miller also often verbally shared her expectations for students' behavior and responsibilities for participating in discussions. On one occasion early in September, for example, Mrs. Miller interrupted a student who was presenting her group's work to remind this student and others that "[i]t's not I, it's we. We're working together in this class. That's something we'll have to get a hang of." Her influence on the developing sociomathematical norms was more subtly conveyed through her reactions to student responses and through her questioning techniques. I claim that her reaction to student responses or student presentations modeled the types of observations she wanted students to make. Similarly, her questioning of students' work often modeled

18 A full list of codes and subcodes used in analysis are found in Appendix C.
the kinds of questions she expected students to be asking of themselves and each other about the work being shared.

The following examples, identified by the codes discussed above, further illustrate how Mrs. Miller’s actions contributed to establishing classroom norms and establishing expectations for thinking through the work of others. These vignettes come from early October observation notes and audio recordings.

**Example 3.** Students were asked to use the digits 0 - 9 once each to fill in all blanks and make a true subtraction equation:

```
  4 , 5 , 6 , 7 , 8
- 3 , 0 , 5 , 1
```

A student from the first mathematics class offered the following as a solution:

```
  4 , 8 , 7 , 0 , 2 , 9
- 3 , 0 , 5 , 1
```

During a discussion of this result with the second mathematics class, many students offered advice and opinions in support of or in disagreement with accepting this student’s solution. After hearing from a few of the students, Mrs. Miller acknowledged the uncertainty she picked up in Jessica’s and others’ responses and admitted her own uncertainty.

Jessica: I don’t think the 6 should count ... (hesitates) ... when you use all 10 numbers ... (hesitates) ... when you borrow it ... the six doesn’t count ... I’m not sure.

Mrs. M: So you’re uncomfortable counting ... when he didn’t have enough tens he went into the hundreds and borrowed one of the hundreds... that left him with 600. You’re uncomfortable with this piece here? OK. But it’s giving you something to think about, isn’t it? You’re not feeling sure about it. What we’re not feeling sure about, ... I’m not feeling sure about it, either. He took me by surprise.

Not only did Mrs. Miller’s response echo the feelings of uncertainty expressed by many of the students, but she also took the opportunity to model how to communicate about the number values in this problem situation. For example, Mrs. Miller translated Jessica’s reference to the number 6 as the original student’s action of borrowing a hundred...
from 700 to give a value of 600. Notice that Mrs. Miller did not explicitly address this issue, but merely offered an alternate way of expressing the thought verbalized by Jessica. This example also illustrates how Mrs. Miller encouraged students to take chances and to share thoughts through her acknowledgment that Jessica would continue to think about the solution.

**Example 4.** Following the above conversation, Mrs. Miller asked the students to solidify their thoughts about the validity of the solution by writing their opinion about accepting or not accepting the solution. In so doing, she more explicitly informed the students that their opinions were valuable to her and that they were in fact responsible for making decisions about solution validity. Here is how she introduced the writing assignment.

> Mrs. M: Is your opinion on this [solution] important?
> Class: Yes.
> Mrs. M: Is it valid?
> Class: Yes.
> Mrs. M: Can your opinion differ from mine?

Some students respond “yes” but many others say “no.”

Mrs. M: All right. What I would like on this paper is your opinion. I want you to commit to your thinking on this.  
Eddie: So I could write, like, I don’t think this problem is acceptable because ....
Mrs. M: Oh, I like that ... with because. Right. I want you to tell me whether you think this fits. Does it follow the directions? Do you accept this as a solution? But I must have why or why not.
Eddie: I think the answer is right, but I don’t think it follows the directions.
Mrs. M: So the mathematics is correct.
Eddie: Yeah, but um....

Mrs. Miller interrupted Eddie here and asked him and all students to write why they would or would not accept the solution.

Through her questioning Mrs. Miller subtly told her students that she valued their opinions and expected the students to share opinions of each others work. In complimenting Eddie’s phrasing of his written response Mrs. Miller signaled what was acceptable, desirable and perhaps an expected way of expressing one’s opinion. Here
again, as with Jessica, Mrs. Miller also rephrased Eddie’s wording, suggesting that “the mathematics is correct” would be more appropriate than saying “the answer is right.” Eddie also played a role in the negotiation of what was expected. He made a suggestion and Mrs. Miller’s response provided him and the rest of the students with a better sense of her expectations. At this point in time, the notion of writing one’s opinion of a mathematical issue was fairly new but was understood by at least a few of the students, as shown in Jessica’s and Eddie’s contributions.

Examples 3 and 4 illustrate a few of the ways in which Mrs. Miller informed the students of the types of discussions she expected, such as expressing opinions and ideas, even when one’s thoughts are a bit shaky. She also demonstrated forms for communicating those ideas when she rephrased the contributions of both Jessica and Eddie. Although it was not always immediately apparent what influence Mrs. Miller’s moves had on the students, some of the students’ more immediate responses hinted that they had taken notice of and were working toward behavior that fit with those expectations. Lessa, for one, seemed to pick up on Mrs. Miller’s reference to appropriate place value (600 as opposed to 6) when she shared her thoughts on the validity of the problem solution (refer to Example 5, page 124). Lessa expressed her willingness to accept the digit 6 as valid, but she did not count the “extra” digit 1 since it represented ten tens, hence not the digit 1. The September and October data contain many more examples of students’ reactions to Mrs. Miller’s implicitly and explicitly expressed expectations. For instance, Mrs. Miller solicited and received students’ opinions in Episode 2 (page 98), she invited and received thoughts on Kenny’s conjecture in Episode 3 (page 100), and in Episode 4 (page 102) she redirected Amir’s question to the entire class, signaling that all students should consider it their responsibility, too.
Student Moves

Research on students' social and cultural beliefs about mathematics learning indicate that students' actions in the classroom are often shaped by these beliefs (Cole, 1985; Lampert et al., 1996). Cole claims students' beliefs will at least partially determine how they react to and are influenced by new classroom situations. Pimm (1987) and Stubbs (1983) also acknowledge that although the teacher is primarily responsible for directing and controlling the nature and function of talk in the classroom, the students' beliefs and expectations also exert an influence on the nature of the talk that occurs. Students continually work at interpreting the teacher's moves and anticipating how to participate. Thus, on the one hand, the teacher is responsible for constraining and developing the students' understanding of what is mathematically acceptable. On the other hand, the students' interpretations guide their reactions to and participation in the discourse, thus contributing to the development of norms for communicating during mathematics discussions (Stubbs, 1983).

Students develop a sense of what is expected, either socially or mathematically, through active participation in mathematics discussions as well as through explicit and implicit negotiation of expectations with the teacher. It is through participation, what they say and do, that students exhibit what they have learned about how to participate in the classroom discourse (Stubbs, 1983).

Here again, the September and October observation notes illustrate some of the students' "moves" that demonstrate their sense of what is expected and their attempts to establish ways of participating effectively in the mathematics discussions. The coding categories analyzed include Student Beliefs, Nature of Discourse, and Student Role (refer Appendix C for full description of these coding categories). Students' beliefs about school mathematics and the learning of mathematics were often revealed through students' contributions to the discussions. On occasion these beliefs were brought to light when Mrs. Miller or students addressed specific concerns such as whether it was OK to disagree...
with another person’s idea, especially when that other person was the teacher. Often, however, students’ beliefs about mathematics were revealed in more subtle ways. The subcategories of the code Nature of Discourse that were of interest for this analysis included students’ questions or curiosities that led to discussion of mathematical ideas, as well as times when a student was tentative about sharing her or his ideas. Questions were often raised by a student when, for example, she or he was testing whether or not a solution was acceptable, or when a student was curious about a particular concept or idea. A more tentative student may have shied away from questioning or sharing her or his own work, indicating that she or he was unsure that the idea was acceptable, possibly not feeling the environment was safe for taking risks. The roles students take on during class discussions also illustrate students’ “moves” in establishing classroom norms. Students questioning each other about something that arose in the discussion, students showing responsibility for being a contributing participant in the discourse, and students actively participating as evidenced through their insights shared about the mathematics being discussed all demonstrate that students are learning how to participate in the discussion and the activities occurring around them.

The two vignettes presented below exhibit some of the students’ actions and responses during whole class discussions. In particular, these examples expose some of the students’ beliefs about who has authority in the classroom, about what to contribute to the discussion, and about how to evaluate a contribution. Although these examples describe particular and somewhat unique situations and conversations, they are representative of discussions that revealed students beliefs about mathematics.

Example 5. The students from Mrs. Miller’s second mathematics class had been discussing the solution to the subtraction problem presented in Example 3, above. Mrs. Miller had just called on Bethany to share her thoughts on the validity of the solution.

Bethany: Um, he uses the number one twice ... to make the twelve and

Mrs. M: Do you see where the ones are, everyone?
Bethany: And when he crossed out the seven he got a six, so he used it once. So if we are supposed to count the six then we should count the one. (Turning to Mrs. Miller) I'm not sure if you're counting it though.

Lessa: Well, he used all the numbers but I don’t think that the one really counts. It’s not really a one. It’s like a ten.

Mrs. M: OK, ...

Bethany: In some ways it's up to the teacher ... whether you want to accept the six and the one, or just the six, or none at all.

Mrs. M: So the teacher would make that decision?

Bethany: Yes.

Mrs. M: Do I make that decision when I ... you put it up and I say “yes, I accept it,” or “no, I don’t accept it”? Or does that [decision] happen somewhere else?

Bethany: I think it happens somewhere else. In some ways, it’s the teacher’s decision whether they want to count the problem. If you’re going to count the problem, how are you going to count it? If he had the right answer? If he followed directions? I want to know what other people think.

Mrs. M: That’s interesting to you, to know what other people think.

At the start of this conversation, Bethany shared her thoughts about the validity of the problem solution being discussed. She then quickly added her belief that the teacher holds the authority to make decisions about the correctness of problem solutions such as this one. What is not clear is whether Bethany was deferring to Mrs. Miller because of the nature of this problem and the question of whether the solution was valid, or because she believed making any decision about another student's work was the teacher's domain. Nevertheless, Bethany indicated curiosity about what her classmates might say about the problem solution and about the issue of who has authority to make these kinds of decisions. Bethany's willingness to offer her own opinion on the validity of the solution shows that she was ready to participate in this kind of conversation. She was actually one of the first in the class to enter the discussion and was, in a sense, testing the waters for her classmates. She deferred to the teacher, the traditional classroom authority, for a final decision, yet invited her classmates to join in sharing their opinions. By this point in the school year, Mrs. Miller had initiated many discussions in which she encouraged students to express their opinions about topics such as taking chances in front of the group, and believing in one's work. Perhaps this experience was what led Bethany to open the discussion of who holds authority for making decisions about mathematical correctness.
Example 6. Students had presented three possible solutions to the five circle problem (Figure 3 on page 101 shows the analogous seven circle problem) when Amir and Kenny began sharing observations about the set of solutions on the board.

Amir: Do they all have to be odd numbers?
Mrs. M: There's something about this puzzle and odd numbers. Explain to the children what you mean by it has to be odd numbers.
Amir: In the middle. They are all odd numbers.
Mrs. M: OK, you see a pattern. Kenny, what do you want to add, dear?
Kenny: All of the numbers up there... were odd numbers... I think if you had 12, 12 circles, I think the middle numbers would be even, an even number. Because you gave us odd numbers to figure it out.

Mrs. M: (addressing the class) All right. There were two different ideas about odd numbers. Amir was talking about the digit that goes in the middle... is always odd. Now you're talking, using the word odd to describe the number of circles. This was five circles, then last night's was seven circles....

After a few more students had made conjectures about possible patterns and configurations for the five circle solutions, Kenny asked to be acknowledged again. It is obvious that his prior conjecture was still on his mind.

Kenny: I just did it on the back of my paper, and you can't make an even number of dots... if you match them up. If you have one in the middle, you can't make an even number of dots.
Mrs. M: OK, you're saying this can't be done with an even number... an even number of circles?
Kenny: No, you can't have an even number of dots and match up the circles with each other.
Micah: ... that was what I was going to... on the eleven one, if you did ten instead, there wouldn't be one in the middle.
Mrs. M: OK, there's something you've discovered about the number of circles.

Clearly, Amir initiated a form of interaction that was accepted by Mrs. Miller and hence was imitated by several other students. When Mrs. Miller responded to Amir's question with the statement "you see a pattern," she acknowledged this observation as an acceptable contribution to the discussion of possible solutions. To follow this example, Kenny presented a new observation and a conjecture about it. Although Mrs. Miller did not explicitly acknowledge Kenny's conjecture about what would happen if the number of circles was even, she did offer both observations to the class, as if to say that they were open for discussion. Micah and Kenny took the initiative to test Kenny's conjecture and

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report back to Mrs. Miller and the class about their results. This vignette is similar to Example 5 in that it illustrates how some students were learning and helping to establish what counted as an acceptable observation or conjecture. Kenny went a step further than Bethany, however, and took responsibility for verifying the conjecture. Micah’s confirmation of Kenny’s result from testing the conjecture indicates that other students were beginning to take responsibility for confirming the validity of a conjecture. Unlike Mrs. Miller’s role in Episodes 3 and 4 (pages 100 and 102, respectively), on this occasion Mrs. Miller did not need to prompt students to take on this responsibility.

Active participation by the students in Example 5 and Example 6 brought to the fore some of their beliefs and allowed them to test what was acceptable and expected by Mrs. Miller. Bethany’s initiation of discussion allowed her to test a very specific belief about who holds mathematical authority. Kenny, following Amir’s lead, helped to establish the activities of pattern finding and conjecture testing that became so familiar to the students as the school year progressed. Although Mrs. Miller and the students did not openly discuss these expectations, they became the norm through interactions such as those described in the examples above.

The vignettes and analysis above describe how Mrs. Miller and the students negotiated and established mathematics classroom norms. In summary, the data suggest that the following norms or expectations were established:

• problem solving in small groups should represent the understanding of all involved, and each group member should be able to explain the thinking of the group;
• students should take responsibility for making sense of the work of others and for checking the validity of solutions (theirs or others’ work);
• mathematics learning involves finding methods for completing tasks, exploring new ideas, noticing patterns, or making observations about the nature of problem solutions;
• opinions and beliefs about mathematics are to be respected and addressed;
• there are appropriate and accepted ways of expressing some mathematics concepts;
• the teacher has ultimate authority for decision making, although students are also
responsible for making some decisions.

The implicit and sometimes explicit negotiation of both social and mathematical
classroom expectations set a climate for Mrs. Miller’s fifth grade students to communicate
mathematical ideas. Up to this point in the school year, students primarily directed their
conversation to or through Mrs. Miller, and it was Mrs. Miller who took most of the
responsibility for focusing and directing the discourse. By October the students had begun
to take more responsibility, and, as Kenny had, they did at times direct the conversations
themselves. As the students learned more about what was valued and what was
meaningful in the mathematics discussions, their roles began to shift. This becomes
apparent in analyses of the discourse later in the school year.

Analyses of the Discourse

Preliminary analysis of factors influencing the mathematics discourse began during
the data collection period. The conceptual framework guiding the data collection as
presented in Figure 1, and reproduced here, was, in a sense, tested through these
preliminary analyses. During early stages, analysis consisted mainly of coding the data
along the four strands of the conceptual framework to test the usefulness and flexibility of
the framework. Codes were modified and new coding categories were added during the
coding process as themes emerged from the data. For instance, the general category of
Participants’ Roles was not specific enough and warranted separation into two distinct
categories to distinguish between Teacher Roles and Student Roles. The growing
significance of the rug area also led to the development of new codes. These changes are
reflected in Figure 7

In addition to coding the September, October and February data, analyses were
conducted to test hypotheses of the contribution of various classroom factors to the
mathematics discourse. More specifically, the coded data units were grouped in ways that
would potentially reveal relationships between characteristics of the classroom activities and the participants interactions. Common themes and recurring situations found in the data across time were examined to identify possible influences on the nature of the mathematics discourse.

Figure 6. Areas of Focus to Guide Data Collection

Figure 7. Areas of Focus to Guide Data Analysis
Tasks and the Mathematics Discourse

Mathematics tasks play an important part in the development of students’ mathematical understanding. I also hypothesize that mathematics tasks play an important part in the development of classroom mathematics discourse. To test this hypothesis, the mathematics tasks chosen by Mrs. Miller and the students’ discussions of the tasks were examined to determine the nature of the discourse related to the tasks, and the determine whether certain tasks elicited more or less discussion. One might guess that mathematical problems which require some thought and that have multiple solutions lend themselves to much discussion, while problems requiring few calculations or that have one possible correct solution require less overall discussion. The NCTM Standards documents (1989, 1991) support the notion that teachers should choose tasks that will promote and facilitate classroom discourse as students reason about possible strategies and solution methods.

Which tasks promote mathematics discourse? In what ways do the tasks influence the development of the discourse? Recent research on mathematics tasks (Stein, Grover, & Henningsen, 1996) suggests that when tasks encourage multiple solution strategies and require students to explain or justify their answers, then students are more likely to talk with others and share ideas about the mathematics they encounter.

Tasks that Mrs. Miller’s students worked on during the school year were categorized as either single-solution or multi-solution tasks. Within these categories, the tasks were identified as single-method or multi-method tasks. Single-method tasks are those tasks for which the students found or shared only one method for finding a solution or multiple solutions. If the students shared two or more strategies for finding a solution or multiple solutions, the tasks was labeled multi-method. In addition, several tasks originated from the work of the students, such as Jessica’s extension of the Magic Circle Problem (Episode 4, page 102), and were so noted.

Mrs. Miller primarily used unique and interesting problems with her students, through which they could investigate mathematical concepts and explore problem solving.
techniques. Table 1 provides a breakdown of the types of tasks students completed during their fifth grade year. The majority of the tasks students worked on allowed for a single solution, yet offered many avenues for the students to explore mathematics in arriving at the solution (see Appendix E for samples of various tasks). One example of such a task is the Cake Problem (refer to Episode 6, page 111, and Appendix E), which required students to find the price of increasingly larger cakes. Although each cake had a specific price, students found and discussed multiple ways of arriving at the price. Most students found initial cake prices by building successively larger cakes with the manipulatives provided with the problem. Once students had a few prices, they were encouraged to find patterns or formulas that would allow them to find cake prices without the manipulatives. As the extended Cake Problem episode illustrates (Appendix B), two very different formulas emerged, and were the topic of conversation for almost two class periods.

<table>
<thead>
<tr>
<th></th>
<th>Single-solution</th>
<th>Multi-solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-method</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>Multi-method</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Indistinguishable</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 1. Mathematics Tasks

Whole class discussions of the problems revolved around the solution strategies used by the students, and the students' understanding of the mathematical concepts involved. Hence, a closer look at the nature of discourse related to particular tasks provides a deeper understanding of how the tasks might have shaped the discourse. One discussion that grew out of the nature of the task occurred when students were asked to create a graph for visually representing survey data. The students shared their graphs,

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19 Only tasks from September, October, February, and March were available for categorization. One of the single-solution and two of the multi-solution tasks were generated by students.
explained how to read the graphs, and answered questions from fellow students and Mrs. Miller.

Example 7. (September 14) Students had been working with a second data set, and were asked by Mrs. Miller to try using a different type of graph to represent the data. Students who had used bar graphs for their first data set tried line graphs, pictographs and circle graphs. Margaret and Paul had experimented with circle graphs, and shared their experiences with their classmates.

Paul: (shows a pie chart) I kept on trying... kept trying so no lines were left over. First the circle was too big ... lines not in the right place. I had to keep starting over.
Mrs. M: How many (pieces) were you trying to make?
Paul: Ten.
Mrs. M: And is that harder to make than 8 pieces?
Paul: Yah, I guess so.

This question brings responses from many other students who have begun thinking about cutting pizzas in eight pieces and ten pieces. Someone mentions that it is probably easier to make an even number of pieces from a circle than an odd number of pieces.

After a few more students have shared graphs (Amir’s pictograph and Kenny’s bar graph), it’s Margaret’s turn. She has also created a circle graph.

Margaret: I made a pie graph and... (cut off)
Mrs. M: Was it hard?
Margaret: Yeah, I had made different size pieces and I didn’t notice ... I had to start all over.
Mrs. M: Was it harder or easier than the bar graph?
Margaret: Harder.
Mrs. M: Why?
Margaret: It’s harder to measure to get equal pieces.
Mrs. M: Any advice on circle graphs? Besides don’t do it.
Margaret: Before you make little pieces, make sure your addition is right so you won’t have to do it over.

Margaret talks about how she worked hard to make 19 equal pieces, but then realized she needed 21 equal pieces, so she had to start from scratch. A couple of students wonder aloud about how they might use eight equal pieces to get close to having 21 equal pieces. Before anyone can respond, Mrs. Miller turns to Karen to share her graph.

Although most of the discussion of the graphs on this day focused on naming the type of graph or chart used, the discussion of the pie charts brought up many other mathematical issues. Although Paul and Margaret were the principle speakers, a few other
students joined in the conversation and added their thoughts about dividing circles into equal pieces. These discussions were cut short by Mrs. Miller who seemed to want to “get through” the rest of the students’ presentations.

In another example, students had been discussing a particular pattern in relation to a problem that asked them to find the number of different ways to arrange trophies on a shelf, for successively larger numbers of trophies (the original problem asked students to find the number of ways three students could have placed first, second, and third in a spelling bee). When Kenny wondered out loud if the formula would work for figuring out all possible configurations of students’ desks in the classroom, many students got involved in the discussion and began asking about which desk configurations would allow them to use the pattern they had found. This discussion did not last long, however, since Mrs. Miller and the students agreed that many more factors would need to be considered in order to make sense of the problem. No one seemed willing to investigate it further.

The pie chart discussions, the students’ quick consideration of the problem Kenny proposed, and many other discussions arose from the students’ work with the mathematics tasks Mrs. Miller had chosen for them to investigate. In some cases, discussion of a task uncovered the students’ understanding of mathematical concepts (for example, Episode 3 revealed Kenny’s misunderstanding of even and odd numbers), and sometimes the discussion of a task exposed the students’ willingness to investigate related issues (Kenny’s question about desk configurations described above). There were also times when the whole class discussion of a task did not reveal much at all about the students’ or their mathematical understanding. The data on mathematics tasks indicate that mathematics discussions of multi-solution tasks lasted two or more class periods for all but two of these tasks. Similarly, mathematics discussions of single-solution tasks lasted two or more class periods for all but six of these tasks. Of these six tasks for which discussions lasted less than two days, all were categorized as single-method tasks. This would suggest that tasks which invite students to find multiple solutions or to use multiple solution strategies
encourage students to share their ideas and understandings, and may also encourage students to investigate related mathematical issues. Interestingly enough, both multi-solution tasks that elicited very little whole group conversation were categorized as multi-method.

Hiebert et al. (1997) suggest that tasks which engage students in solving genuine problems and that encourage reflection and discussion form the foundation of instruction. They add that when students are interested in and are challenged by the problem situation, they are more likely to participate in discussions of the problems. The two discussion situations described in this section provide examples of tasks chosen by Mrs. Miller that challenged students and led to discussions in which students pushed their understanding. Examples of discussions arising from student work on the bike problem, the magic circle problem, and the cake problem found in this chapter offer more proof that the mathematics tasks engaged students in thinking about and communicating about a variety of mathematical ideas. Many students also became engaged in reflecting on and contributing new ideas to these discussions. When asked about the tasks she chose, Mrs. Miller talked about creating or finding tasks for her students that would give them opportunities “to make sense of how numbers work.” She went on to say that class discussions were important follow-ups to doing the tasks since discussions “make available to everyone a variety of ideas.” Both reflections and sharing happened regularly in Mrs. Miller’s mathematics classes, in both large and small group settings.

Classroom Environment

The learning environment of the classroom can be defined as the “interplay of intellectual, social, and physical characteristics that shape the ways of knowing and working that are encouraged and expected in the classroom. It is the context in which the task and discourse are embedded.” (Connecticut State Department of Education, 1996). The subject matter, mathematics in this case, also determines in large part the learning
environment, for the mathematics content engenders how tasks are structured, the function
of talk, and students' knowledge — hence, students' willingness to take risks or to
participate in class discussions (Pimm, 1987; Stubbs, 1983). Thus, the mathematical
content, the task structure, and the classroom teacher's expectations for students' participation in the work of the mathematics class influence and are influenced by the learning environment of the mathematics classroom. The previous section of this chapter addressed how the teacher and the students interactively established classroom norms, and examined, to some degree, the nature of the evolving classroom environment. Here, the data are more broadly examined to provide an overall sense of the classroom environment and to take a first look at the interplay between environment and the nature of the mathematics discourse.

The observation notes and transcripts of audio recordings from September, October and February observations were systematically scanned and grouped along five coding categories that either describe the learning environment or describe factors that contribute to the learning environment. Units of data that described the task structure and the students' interactions were used to illustrate some of the physical characteristics of the environment. Data coded Student Beliefs were used to examine links between the students' interactions and their sense of the intellectual and social climate of the classroom. In addition, discussions initiated by the teacher that revealed her expectations for student participation, and discussions initiated by either the students or the teacher that revealed their beliefs about participation were coded to illustrate the underlying intellectual structure of the classroom and the participants' relationships. The data and codes that reflect the learning environment are analyzed below, and discussed in relation to the nature of the classroom discourse.

Throughout the school year students were asked to work with each other, most often in predetermined groups, in solving mathematics tasks that encouraged multiple solution methods. In these groupings, students were expected to share responsibility for
producing and understanding the group’s solutions. Students also worked as a whole class in sharing ideas, sharing solution methods, and in collectively making sense of mathematical concepts arising from the problems. Early in the school year, Mrs. Miller frequently stressed individual responsibility in the groups, and reminded the students that the work presented during whole class sharing was to be a “product of the pair” or group. The general task structure of working in problem solving groups and sharing work with the class remained relatively constant throughout the school year. The nature of the students’ interactions and Mrs. Miller’s attention to maintaining this routine, however, are two components of the task structure that did change.

The data from September and October suggest that Mrs. Miller and the students worked on mutually creating a social and an intellectual environment in the mathematics classroom. This is illustrated through conversations initiated by Mrs. Miller that engaged the students in thinking about appropriate social and working relationships during small group times, such as the pieces of conversations in Examples 8 and 9. Discussions of this type were coded Meta-discourse or Teacher Expectations and occurred as many as eleven times in September, and nine times in October. The February observation notes indicate only two such discussions.

Example 8. The students and Mrs. Miller engage in a conversation about maintaining a working relationship with a partner even when there is some disagreement.

Mrs. M: What would make for a good relationship with your partner?
Neil: Don’t fight, just get the work done.
Mrs. M: What if you’re my partner and you don’t like my idea?
Neil: I would have to do it.
Mrs. M: Would you have to do it?
Neil: I could try to convince you to do it another way.
Heather: May say “it might be a good way, but let’s try another way.”
Example 9. Mrs. Miller talks with the students about sharing solutions in front of the whole class.

Mrs. M: This is a little bit risky, because somebody may come up with an answer that you disagree with. And we've talked a little about how to disagree. Do you remember how we would do that?

In the fall of the school year, students were also learning what was expected during the whole class sharing time. As discussed in the section Teacher Moves, beginning on page 118, Mrs. Miller invited students to share their work, their solution strategies, their explanations, and also their beliefs about what to expect and to be responsible for in these situations. In response, students expressed tentativeness in sharing unfinished or incorrect work, and concerns that others might think they were “cheating” when the work being shown by another group was the same as their own. By February, students were exhibiting less tentativeness, and in some cases were eager to introduce new ideas and possibly unfinished thoughts. Nathan, for one, was more apt to volunteer or request to share work in February, whereas October notes show Nathan as quiet and almost invisible in large group settings, often nervous about sharing his work. Although Karen was outspoken from the start of the school year (see Episode 2, page 98), her role also grew from being primarily concerned with her own role in class discussions to the role of student advocate.

When the teacher initiates discussions of behavior and intellectual expectations and when the teacher initiates interactions that model these expected behaviors, she influences the students’ sense of the classroom environment (Baker, 1992). Baker, who reports on links between students' interactions and their sense of classroom order, claims that student-initiated interactions with the teacher or with classmates also contributes to everyone's sense of classroom climate. The data from September and October observation notes do not include much evidence of student-initiated interactions with the teacher, but a few examples such as the discussion Bethany initiated (see Example 5) do give the reader a
sense of the students' potential influence on the classroom climate. Evidence of student-initiated discussions are easier to find in data collected later in the school year, which would seem to attest to the students' growing collaboration in shaping the learning environment.

Teacher-initiated discussions of expectations remained consistent throughout the school year. In fact, as Table 2 reveals, the number of such occasions increased as the school year progressed. It is important and interesting to note, however, that the nature of the these interactions did evolve. For instance, data coded as Teacher Expectations in the September and October field notes suggest that Mrs. Miller was setting an agenda for the school year. She voiced her expectations that all students would contribute to discussions, that all students would be given opportunities to contribute to the group's mathematical understanding, and that all students should reflect on and share how their thinking changed or did not change as various problem solving strategies were being explored. The examples below, taken from mid-September observation notes, illustrate how Mrs. Miller explicitly shared her expectations.

<table>
<thead>
<tr>
<th>Teacher's Role (TROLE) Expectations</th>
<th>September</th>
<th>October</th>
<th>February</th>
</tr>
</thead>
</table>

Table 2. Frequency of Coded Data

Example 10. Mrs. M: Students, that way it is going to happen is... Sarah is going to say something and it's going to make us all think.... We need to be thinking about each child's answer.

Example 11. Mrs. M: I'm going to ask for a little wait time. Think it through and get your thoughts together. Then I'm going to ask you all to find a way to share that thinking with the class.

The February data which was also coded as Teacher Expectations exhibit what appears to be a shift in Mrs. Miller's expectations away from merely encouraging
participation toward explicitly encouraging the students to take greater intellectual risks.

Rather than informing students that they should offer something during class discussions, these units of data show that Mrs. Miller had begun to describe more specifically the nature of what the students should contribute. The three examples below, taken from February 12, 13 and 27 observation notes of whole class discussions, are representative of February observations.

Example 12.
Mrs. M: And why did you use three?
Karen: Because that's how much it was worth.
Mrs. M: What was worth? And how did you know it was three?
Karen: The red (pattern block) ... because it was half of the octa...
hexagon.
Mrs. M: All right. (Addressing the class) I'm going to be asking you things like that, just to remind you where you got that information.

Example 13.
Kenny: ... maybe it's because twelve is a higher number than ten, and ten is the last number you can count by without skipping ... /?/.
And ...
Mrs. M: Could somebody repeat what you think you understand Kenny to be saying?
Heather: Um, he thinks [the number pattern] skips because, um, twelve 
....

Example 14.
Mrs. M: (to Nathan) OK, the pattern you shared ... that is something, but .... people have dug deeper than that, now, Nathan. And that's where I would like you to be digging.

Each of the examples 10 through 14 give evidence of Mrs. Miller's changing expectations, and how she made these expectations known to the students. As will be shown in the sections to follow, Mrs. Miller's expectations did push the students in new directions when participating in class discussions. Example 14, in particular, signaled to all students that they would be challenged to push their ideas further or deeper to build on the ideas of those around them.

Another indicator of this growing sense of sharing was the physical movement of students from their desks to the rug area in the front of the classroom. Mrs. Miller
frequently called students to sit in a circle on the rug to share the work they had done with their partners. By the middle of February, the data reveals that students would begin moving toward the rug area of the classroom without Mrs. Miller's prompting. In fact, the students' movement toward the rug area would continue slowly as the discussion progressed from sharing thoughts to sharing mathematical strategies or solutions.

Teacher's Role in Discourse Development

Research on the development of discourse in the classroom setting, mathematics class or otherwise, supports the notion that the teacher plays a significant role in shaping the discourse (Ball, 1991; Hiebert & Wearne, 1993; Lampert et al., 1996; Tharp & Gallimore, 1988). Hiebert and Wearne (1993), for instance, acknowledge that the nature of the teacher's questions and the teacher's expectations for how students respond are salient features of the developing mathematics discourse. The research of Tharp and Gallimore as part of the Kamehameha Elementary Education Program (KEEP) also identifies questioning as an important strategy for promoting the learning of discourse (Tharp & Gallimore, 1988). Tharp and Gallimore also look at the significance of teacher feedback and verbal modeling as forms of assistance for continuing the discourse. Still others (Ball, 1991; Lampert et al., 1996) suggest other aspects to which one should pay attention in order to understand the influence of the teacher's role on the students' developing sense of mathematics discussions.

The Professional Standards for Teaching Mathematics of the NCTM (1991) suggest three particular aspects of the teacher's role in orchestrating mathematics discussions that fit well with the suggestions of Hiebert and Wearne (1993), Tharp and Gallimore (1988), Ball (1991), and Lampert (1996). These three aspects provide a useful framework for analyzing Mrs. Miller's participation in the discussions and her influence on the development of the mathematics discourse. These aspects are:

- how the teacher provokes student reasoning through the questions she asks;
• how the teacher changes her activities from doing most of the talking, modeling, and explaining to listening and encouraging student participation;
• how the teacher monitors and facilitates the students' participation in class discussions.

Table 3 illustrates the analysis of Mrs. Miller's role in the mathematics discourse using the general framework suggested above. The September and October data do not show much

<table>
<thead>
<tr>
<th>Nature of Questioning</th>
<th>September</th>
<th>October</th>
<th>December</th>
</tr>
</thead>
<tbody>
<tr>
<td>Questions mostly performance-oriented.</td>
<td>Again, majority of questions are performance oriented.</td>
<td>Shift to majority structure-oriented questions.</td>
<td></td>
</tr>
<tr>
<td>Several structure-oriented and a few theory-oriented questions.</td>
<td>More structure-oriented and a few theory-oriented questions.</td>
<td>Theory-oriented questions are more commonplace.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Talking versus Listening</th>
<th>September</th>
<th>October</th>
<th>December</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mrs. Miller spends a majority of discussion time explaining and modeling.</td>
<td>Data more evenly split between Mrs. Miller doing most of the talking and Mrs. Miller listening while students talked, explained, etc.</td>
<td>Mrs. Miller continues to spend time explaining and interpreting what she hears from students.</td>
<td></td>
</tr>
<tr>
<td>Only a few occasions noted as primarily listening, encouraging participation.</td>
<td></td>
<td>Spends equal time talking and encouraging participation.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Monitoring and Facilitating</th>
<th>September</th>
<th>October</th>
<th>December</th>
</tr>
</thead>
<tbody>
<tr>
<td>Much time spent making suggestions, modeling.</td>
<td>Beginning to encourage students to reflect and to think deeper about the ideas presented.</td>
<td>More focus on the mathematics being discussed.</td>
<td></td>
</tr>
<tr>
<td>Time spent emphasizing important and interesting ideas presented in solutions.</td>
<td>Few occasions when Mrs. Miller asked students to make decisions about how to structure the discussion.</td>
<td>Less focus on the mechanics of keeping the discussion flowing.</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Mrs. Miller's Role in Discourse

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change in her role, but it is interesting to note the differences between these data and the
data from February.

The first row of Table 3 summarizes the nature of questions asked by Mrs. Miller
during the months under investigation. The questions were first grouped into three major
categories and then identified as either questions that required a brief answer from one
student or questions that led to a discussion involving several students. The first two of the
three major categories of questions, performance-oriented and structure-oriented, are
defined by Renkl and Helmke (1992) in their research on the relationship between the types
of questions teachers ask and student learning. The third category, theory-oriented
questions, arose during the process of grouping the questions into general categories.

Many questions did not fit the two types defined by Renkl and Helmke, and so a third
category was created. Performance-oriented questions refer to those questions asked about
the details involved in applying a learned method, whereas structure-oriented questions are
those asked about properties of mathematical concepts or properties of solution results
(Renkl and Helmke, 1992). Theory-oriented questions refer to those asked about a
situation or concept not yet encountered by the student. For instance, Mrs. Miller might
have pushed a student’s thinking beyond what was required to solve a problem by asking
“What would you do if you could choose any number of objects instead of just 10?” The
performance-oriented questions were most likely to be closed or leading, requiring a
relatively brief response from a single student. The structure- and theory-oriented
questions, on the other hand, tended to be more open-ended, more of the Why or How
questions, and seemed to encourage participation from more students.

As Tables 3 and 4 clearly show, Mrs. Miller was much more likely to ask
performance-oriented questions than structure- or theory-oriented questions during
September and October. During the months of September and October, discussions often
revolved around the students sharing solution strategies in response to Mrs. Miller’s
question “What did you and your partner find?” “What operation did you use?” “What did
you do next?” or “How did you decide to ...?” Although less frequent, Mrs. Miller seemed to make a point of also asking structure-oriented questions that encouraged the students to justify or reflect on their work. Examples of structure-oriented questions include “Can you talk about what you were thinking when ...?” “How does this relate to ...?” “What makes your solution different from ....?” and “What can we learn from what you’ve shown us?”

In contrast to the earlier months, the questions Mrs. Miller asked in February were most often structure-oriented (E.g. “What’s happening to the value of the answer when you do that?” “How can you use division in what you’re saying is a subtraction problem?”). She was also twice as likely to ask theory-oriented questions in February than in September and October, such as “How do you know when you have a pattern?” or “What do you know that might inform us about any bigger value?”

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<tr>
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<tr>
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<td>5</td>
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<td>Modeling</td>
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<td>8</td>
<td>4</td>
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<tr>
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<td>Explaining Rationale</td>
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<td>20</td>
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<tr>
<td>Pushing by Suggestion</td>
<td>6</td>
<td>7</td>
<td>9</td>
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<tr>
<td>Pushing by Questioning</td>
<td>3</td>
<td>5</td>
<td>14</td>
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<tr>
<td>Encouraging Reflection</td>
<td>10</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>Repeating for Emphasis</td>
<td>10</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Redirecting</td>
<td>4</td>
<td>1</td>
<td>2</td>
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</tbody>
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*Table 4. Frequency of Events Initiated by Mrs. Miller*
Early in October, Mrs. Miller had shared with me that she wanted to pay attention to the questions she asked, with the hopes of improving her questioning techniques. As a participant in the teacher development program, Mrs. Miller had become aware of the value of asking questions that probed students' thinking, and caused the students to reflect on their work. Hence, she and I often had conversations about her questioning and about research related to questioning. In summary, Mrs. Miller's awareness of and interest in questioning was most likely the biggest factor in the changing nature of the questions asked.

The nature of Mrs. Miller's contributions to the mathematics discussion was examined by categorizing the types of things she said and by counting occurrences of these events. The category "Talking versus Listening," as found in Table 3, consists of four subcategories that describe Mrs. Miller's speech. The subcategories include (1) giving advice to one or all students; (2) modeling ways of verbalizing ideas or modeling what students should be discussing; (3) commenting on choices made by students or making suggestions for choices; and (4) explaining her intentions and rationale.

The data suggest that Mrs. Miller spent much time during September and October promoting the mathematics discourse through her own speech (refer to Table 3 and Table 4, Talking versus Listening). She gave many examples of what she expected, repeated students' responses to emphasize important concepts and methods, gave advice about when and how to ask questions of other students, and often took time to explain her rationale for choosing a particular task or for initiating a discussion around a particular topic. Mrs. Miller was also more likely to provide suggestions that would push students to clarify their explanations or their reasoning. Examples 15 and 16 are examples from the September data of Mrs. Miller giving advice and explaining her intentions, respectively.

Example 15. Mrs. Miller talks with the students about working in groups and the importance of resolving difficulties or differences of opinion.

Mrs. M: Now, next we'll talk about any difficulties you encountered. And with that, how you solved the difficulty. Sometimes there
is a lot to be learned when something is difficult and how you solved the difficulty. That's something we all want to learn.

**Example 16.** Mrs. Miller explains to the students that each pair will share their solutions to the Bike Problem.

**Mrs. M:** All right. Well, the reason I want to go through this is, some people may, after seeing the presentations think differently about their work. And is that OK with everybody? Let's begin.

During later months, Mrs. Miller was less likely to spend time explaining her rationale, but was more likely to push students to explain their work or to question the work of others. She did continue to repeat students' contributions to emphasize important concepts and to model appropriate responses. By February, Mrs. Miller was much less likely to share her experiences or to give examples to model her expectations (see Table 2 on page 138 for number of times Mrs. Miller verbally shared expectations).

Mrs. Miller’s verbal contributions to the mathematics discussion that could be categorized as monitoring and facilitating activities also consisted of four subcategories: (1) pushing suggestion or by questioning to explain, defend, or justify; (2) encouraging reflection and encouraging questioning; (3) emphasizing and modeling responses; and (4) redirecting the conversation. Early in the school year, Mrs. Miller was more apt to provide suggestions about how students could push their explanations to include justifications. By February she was more likely to directly question students about their ideas or about the ideas of others (primarily structure-oriented questions) by asking questions such as “can you tell me what you understand from this?” or “can you say, in another way, what you heard her saying?” Similarly, it has already been noted that Mrs. Miller spent much time during September and October repeating students' contributions to the discussion in order to emphasize important concepts or to model appropriate ways of verbalizing ideas. By February, Mrs. Miller appeared more focused on the mathematics being discussed and less on the modeling of appropriate behavior. She did, nonetheless, occasionally summarize or redirect the conversation to make a point (see Episode 6), to give closure, or to show the “big picture.”
As Table 3 and the closer analysis found in Table 4 indicate, Mrs. Miller's role in the mathematics discourse did evolve over time as she took on less of a modeling role and more of a facilitative role. This analysis is consistent with comments recorded and responded to in the journal I shared with Mrs. Miller. For instance, she agreed with my observations that in September and October she spent more time talking about her behavioral expectations for the students than her academic expectations, while February journal entries document our comments on how the focus of most discussions seemed to have shifted from non-mathematical issues to mathematics related issues (see Appendix F, journal entries from November 8, February 2, and February 20). I hypothesize that as the students learned to participate in ways acceptable to Mrs. Miller, she was able to change her focus to the discourse itself. I believe this gradual shift in Mrs. Miller's role influenced the student's developing ways of contributing to the mathematics discourse.

The Roles Students Take on During Whole Class Discussion

The roles that students play in the discourse of the mathematics classroom are also important to consider, for it is through the discourse that students engage in the mathematics. Both Piagetian and Vygotskian theory uphold that learning mathematics involves actively making sense of mathematical experiences (Cole, 1985; Confrey, 1990; Noddings, 1990). The work of Vygotsky and his students extends this theory of learning to claim that active sense making most often occurs in social environments such as the elementary school classroom. Learning experiences are thus shaped by social interactions and through formal and informal communication (Bauersfeld, 1980; Vygotsky, 1978). The ways students interact with each other and with the teacher influences the students' learning experiences. The ways students communicate ideas and the ideas they communicate influence their learning experiences. This portion of the analysis focuses on how the students interacted with each other, and how they communicated their ideas.
In examining the roles students took on during the mathematics discourse, the analysis centered on the following three aspects of the students interactions.

- in general, the ways in which students interacted with each other or with the teacher during whole class discussions (listening, explaining, responding, questioning, observing, etc.);
- when and how students took initiative (questioning another's work, offering alternative solutions, investigating a conjecture);
- the ways in which the students addressed mathematical understanding and validity.

Analyses also include a look at how the students' roles determined the discourse and how their roles evolved over time. Although no students were excluded from the data used in this section, the examples in this section concentrate primarily on the five focus students for convenience and for availability of data.

Tables 5 and 6 provide summaries of the data on the roles students played in whole class discussions. The nature of the student interactions were generalized from data along the three subcategories of sharing mathematics ideas, sharing non-mathematical ideas, and to whom interactions were directed.20 Tables 5 and 6 describe significant changes in the nature of students interactions as the school year progressed. These data displays clearly show that the students shifted from primarily directing their work or thoughts to the teacher to interacting directly with peers about ideas or solution methods. It is also interesting to note the increasing frequency with which students offered new ideas or insights on mathematical concepts, as reflected in the "Taking Initiative" section of Table 6. In September there were fewer than ten occasions when students presented interesting observations, while in February the number of such contributions rose to nineteen. In comparing these results with the data summary in Tables 3 and 4, it seems logical that the students interactions and contributions would have changed as they learned what kinds of

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20 Some categories described in Table 6 include two or more coding categories defined in Appendix C.
interactions were acceptable, which were expected, and which were encouraged by the teacher or by classmates.

The most striking difference between the students' interactions in September and their interactions in February was the students' increased willingness to take responsibility

<table>
<thead>
<tr>
<th>Nature of Interactions</th>
<th>Initiative</th>
<th>Addressing the Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Interactions occurred most often when invited by teacher to share.</td>
<td>• Explanation of work when invited by teacher.</td>
<td>• Greater amount of interactions between students.</td>
</tr>
<tr>
<td>• Likely to share math and non-math aspects of work.</td>
<td>• More likely to focus on the mathematical aspects of their work.</td>
<td>• Students question each other, add their understanding to the work of others.</td>
</tr>
<tr>
<td>• Interactions directed toward the teacher.</td>
<td>• Interactions directed toward the teacher.</td>
<td>• Eager to offer, defend, and justify ideas.</td>
</tr>
<tr>
<td>• Some take on leadership role.</td>
<td>• Beginning to acknowledge the work of others.</td>
<td>• Question the work of other students when unsure of their reasoning.</td>
</tr>
<tr>
<td>• Take responsibility to try new strategy.</td>
<td>• Offer or ask for alternative strategies, interpretation, ideas.</td>
<td>• Eager to investigate and share new ideas, patterns.</td>
</tr>
<tr>
<td>• Question own understanding and find help.</td>
<td>• Notice patterns and take responsibility for investigating.</td>
<td>• Beginning to ask each other for help.</td>
</tr>
<tr>
<td>• Share mathematics by reading solution procedures used.</td>
<td>• Offer explanations for solution strategies.</td>
<td>• Compare solution methods used in various situations.</td>
</tr>
<tr>
<td>• Not able to distinguish between solution procedure and justification for using the procedure.</td>
<td>• Able to talk about changes in one's understanding, and to apply another student's strategy.</td>
<td>• Able to follow reasoning of others and make suggestions, ask questions.</td>
</tr>
<tr>
<td></td>
<td>• Rely on learned math facts or problem criteria to verify solutions.</td>
<td>• Give answers and justification.</td>
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</table>

Table 5. Roles Students Play in the Discourse

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for digging into their work and making sense of the work of others. The “Taking Initiative” and “Addressing the Mathematics” rows of Table 5, taken together, summarize this difference and highlight that by February the students were not only beginning to investigate number patterns and mathematical concepts, but they were taking responsibility for making sense of the patterns they or others shared. It should be noted, however, that even in February there were many missed opportunities, occasions when students did not dig deeper into understanding why a particular solution method worked, and occasions when no one asked questions about differing results to the same problem. The nature of the tasks students worked might explain some of the changes in the students interactions, or provide reasons for why students occasionally passed on investigating alternate solutions. Further research in this area is needed.

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<tr>
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<tbody>
<tr>
<td>Sharing Mathematical Ideas</td>
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<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Sharing Non-mathematics</td>
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<td>1</td>
</tr>
<tr>
<td>Student-student Interactions</td>
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<td>11</td>
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<tr>
<td>Taking Initiative</td>
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<td>Leadership Roles</td>
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<td>Making Observations</td>
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<tr>
<td>Addressing the Mathematics</td>
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<tr>
<td>Sharing Mathematical Ideas</td>
<td>4</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Explaining, Justifying Ideas</td>
<td>6</td>
<td>8</td>
<td>21</td>
</tr>
<tr>
<td>Understands Work of Others</td>
<td>4</td>
<td>4</td>
<td>23</td>
</tr>
</tbody>
</table>

Table 6. Frequency of Events Initiated by Students

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21 Table 6 indicates which coding categories were used to summarize the results and how often such event occurred. Table 5 describes these results in more detail, such as describing the nature of the mathematical ideas shared in September versus those shared in February (“Addressing the Mathematics”).

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The students' changing roles, particularly how they interacted with each other and what they shared during these exchanges, are best described through examples from the classroom. Although choosing a single vignette does not provide a general view of the nature of students' exchanges over a month's time, the following three illustrations were chosen to represent what might be considered typical along general characteristics such as subject of conversation or amount of details given when describing one's work.

**Example 17.** (September 26) David and Kenny shared their answers to a story problem, in preparation for going before the class to explain their solution method. It appeared they had done their work independently before sharing ideas with each other.

David: I got the right answer but I didn't do it right.
Kenny: What did you do?
David: I just did minus sixty plus seventy and I got ten. I didn't do the forty and fifty.
Kenny: Ten is the right answer?
David: Yeah.

From his tone, Kenny seemed to be unsure of his own result which differed from David's answer of ten. What is also striking is that Kenny comments on what David claims is the answer, and not on his method for finding the answer. David appeared sure that the answer was ten, but didn't know how to arrive at it. He was also not sure how to explain his thinking except for replaying the procedure he used to arrive at ten ("I just did minus sixty plus seventy ...").

**Example 18.** (October 16) Nathan and David had been working together to produce a new version of the magic circle problem, one that used subtraction rather than addition (see Episode 3 on page 100 for an explanation of the magic circle problem). At this point they were sharing their work with a group of other students, facilitated by Mrs. Miller.

David: We started out real small, with like five circles.
Nathan: Yeah, we started out with five.
David: And we kept getting bigger and bigger and bigger.
Mrs. M: Oh, you did? I think that's really important to hear.
David: And we're going to try a bigger one ... we're going to try and do twenty circles.
Mrs. M: What did you find in doing it small? How did it help you?
David: It was easier. But after we figured out the ten circles one, we figured out the trick ....
Nathan: That you had the number one in the middle.

Here, David took the lead in sharing with others how he and Nathan found a pattern or "trick" for completing larger and larger magic circles. Mrs. Miller helped emphasize their method when she commented that this was important information and then asked what about their method they found useful.

Example 19. (February 13) On the previous day various students had shared patterns or formula they used to solve the "cake problem." In this problem, larger and larger cakes are obtained by adding a ring of a set width to the previous cake (see Appendix E for a full description of the problem). Students are revisiting one of these patterns.

Mrs. M: (summarizing results from Lessa's group) You did this pattern.

Mrs. Miller writes on the board "price of cake = nxnx6."

Mrs. M: And this one (pointing to a different pattern) you didn't work with. You went to this first one.

Kenny: (addressing Lessa) How did you get the next ring number? You need the ring number to get the next price of the cake. Did you just um, ... you know, you needed to add by twelve to get the next ring. And then if you went to get the next answer... in order to do that you have to get the next ring. And you guys, did you find [the pattern] after building [cakes with manipulatives]?

Lessa: I don't understand.

Heather: (A member of Lessa's group) We didn't really use the ring answer.

Kenny: So, how did you get the thirty if you didn't know to add twelve?

Lessa: I don't think we did that.

Heather: We didn't use the rings to find this ....

It seems as though Kenny wanted to know more about how Lessa and her group members found the pattern they shared. Although it was not clear to Lessa what Kenny was asking, Heather understood that Kenny wanted to know how or if they had used the "add twelve" result that most other groups used. Kenny had tried to follow Lessa's reasoning but was unable to do so. This scenario shows Kenny trying to relate Lessa's method to one he understood. Heather's response started to uncover the new method described by the formula nxnx6..
The three examples above show an evolution in the nature of verbal exchanges between the students in Mrs. Miller's mathematics class. Certainly, the number and nature of student-to-student interactions cannot be categorized by a single heading or sentence. Nonetheless, the observation data from the month of September do suggest that students' interactions during problem solving or solution sharing primarily consisted of talking through the actions each performed to obtain an answer. As the students became more sophisticated in thinking through their problem solutions, the exchanges between students became dialogues in which they were able to share their thoughts, to compare solution methods, and make sense of each others' ideas.

Summary

The analysis of small group and whole class discussions over two periods of time during the school year highlight three main factors that contributed to the development of the mathematics discourse. First, the analysis focused on the participants' beliefs and expectations in order to get a sense of how classroom sociomathematical norms were established that would determine or contribute to how the participants interacted and what they communicated to each other. As Yackel and Cobb (1996) suggest, the notion of sociomathematical norms provides one way of describing the mathematical aspects of interactions in the classroom. As shown in this study, sociomathematical norms were interactively established by Mrs. Miller and her students as they worked together in the classroom. These norms included:

- problem solving in small groups should represent the understanding of all involved, and each group member should be able to explain the thinking of the group;
- students should take responsibility for making sense of the work of others and for checking the validity of solutions (their work or others' work);
- mathematics learning involves various methods for completing tasks, exploring new ideas, noticing patterns, or making observations about the nature of problem solutions;
• opinions and beliefs about mathematics are to be respected and addressed;
• there are appropriate and accepted ways of expressing some mathematics concepts;
• the teacher has ultimate authority for decision making, although students are also responsible for making some decisions.

The teacher's and students' expectations and the negotiated classroom norms this here accounted for some of the ways in which the students and the Mrs. Miller worked together, and most likely influenced some of the changes in the participants interactions as the school year progressed. For instance, as students learned about or accepted their responsibility for checking the validity of a solution, they were more likely to offer justifications for their solutions as they were shared, as opposed to waiting for the teacher to ask for an explanation. These norms were determined by analysis of data from the months of September and October, and thus it is possible the norms evolved over the school year as did the students interactions.

The second main classroom feature analyzed was the nature of the mathematics tasks and their possible influence on the development of the classroom discourse. The analysis of tasks focused on the nature of the tasks and the nature of amount of discussion that occurred around and about the task. It was noted that although the majority of tasks students worked were single-solution tasks, most tasks were multi-method tasks. Recent research (Stein, Grover, & Henningsen, 1996) supports the belief that multi-method tasks encourage students to investigate different strategies to share ideas about the mathematics they encounter. In fact, analysis and examples of the discourse related to multi-method and single-method tasks revealed that discussions were twice as likely to carry over for two or more days when the tasks were multi-method tasks. In addition, the discussions were more likely to lead to investigations of related tasks or new aspects of the tasks when the tasks were multi-method (Example 7, page 132, and Episode 6, page 111). Hence, Mrs. Miller's choice of tasks contributed to the discussion of mathematical concepts. Episodes
from September, October, and February offer more examples of the nature of the discourse related to specific tasks.

Thirdly, I focused on the classroom environment and how that influenced the nature of the discourse. Early in the school year, Mrs. Miller initiated conversations to raise issues related to the learning environment in the classroom. For instance, Mrs. Miller asked students to voice their opinions about appropriate small group working relationships, and on what to do when members of the group disagree about problem solutions. These conversations also brought to the fore some of Mrs. Miller’s expectations for the students as mathematics learners, as well as some of the students expectations for their work and their relationship with Mrs. Miller (Examples 3, 4, and 5, pages 120, 121, 124, respectively).

Mrs. Miller continued to discuss her expectations throughout the school year, and challenged the students to push their thinking further, to share their work, and to build on the work shared by others. The September data indicate that not many students entered discussions of their beliefs and expectations, and they were hesitant about sharing their work when it was not quite finished or when they thought it might be incorrect. By October and even more so by February, students appeared to be working toward Mrs. Miller’s expectations as they began sharing unfinished work and tentative ideas. Students more frequently introduced new ideas and seemed more eager to participate in some discussions by February. In general, it was found that as the students began to feel more comfortable or safe in the classroom environment they began participating in discussions more often, and their contributions to the discussion were closer to Mrs. Miller expectations.

The fourth major area of analysis was the participant’s roles in the discourse and how their changing roles contributed to the development of mathematics discussions. I provided a somewhat detailed look at the teacher’s role and the students’ roles in the discourse. This revealed an almost natural growth in the students’ confidence and sense of
responsibility as they moved from interacting primarily with the teacher about their own ideas, to taking responsibility for investigating and extending the ideas of others. Although Mrs. Miller's role in questioning, monitoring and facilitating the discourse remained consistent, her emphasis and expectations did not. As the nature of Mrs. Miller's questions to students changed, so did her expectations of students' contributions to the discussion. At the same time, Mrs. Miller's emphasis in monitoring discussions shifted from a focus on the mechanics of discourse to the substance of the discourse.

The use of episodes and smaller units of data allowed for both a comprehensive and a focused look at the mathematics discourse within the context of Mrs. Miller's classroom. The results of analyzing both data units suggest that the teacher's role is the most important factor to consider in thinking about how the discourse develops.
CHAPTER VI

DISCUSSION

Introduction

The purpose of this final chapter is to discuss, in a more general sense, the findings of the research presented in this dissertation, the significance of the findings, the implications for mathematics teaching, and the possibility of further research in this area. The first section summarizes the findings presented in Chapter V, and provides a discussion of some of the significant implications for mathematics teaching and classroom discourse. The second section addresses methodological issues related to the study, such as my biases as a researcher, and my influence on classroom activity. This second section also includes some discussion of the benefits and limitations of the qualitative methodology used in conducting the research. The third section of this chapter focuses on other limitations of the present study and possible areas of future research that build on the research reported in this dissertation.

Summary of Findings and Implications

Chapter V offers analyses of various aspects of the classroom interactions and the discourse that occurred over the school year. In this section I summarize the significant findings of the analysis, and discuss these findings in relation to the original research question: What aspects of the mathematics classroom contribute to the development of mathematics discourse? More specifically, the summary focuses on (1) changes in the nature of the discourse and interactions; (2) changes in various aspects of the classroom, such as environment, the mathematics tasks, and the focus of whole class discussions, that
are directly related to the participants’ interactions and the nature of the discourse; (3) the teacher’s role in the development of the mathematics discourse; and (4) the importance of the students’ contributions to discussions in developing the mathematics discourse.

Changes in the Nature of the Discourse and Interactions

One major purpose of the study was to investigate the changing nature of the mathematics discourse in a classroom where the teacher worked to promote and develop the discourse. The portrait of Mrs. Miller, the classroom setting, and the students presented in Chapter IV makes the case that this teacher did actively promote the development of mathematics discourse. How did the nature of the discourse change over the school year?

Episodes one through six, as well as the extended episodes of Appendix B, offer snapshots of Mrs. Miller’s mathematics classes and provide the reader with a perspective from which to get a sense of the classroom activity and the discourse from two reference points during the school year. The episodes from September and October reveal several characteristics of how the students’ interacted with one another early in the school year. Episode 1, for instance, illustrates how two different pairs of students worked together to arrive at a problem solution. In both situations the students shared ideas about the solution, but the answer that was presented to the whole class reflected the work or thinking of only one student in each pair. It is interesting that early in the school year the students often talked about fairness and sharing responsibility yet they did not always share a common understanding of the solution method and the answer they presented to others (refer to Appendix B, the Bike Problem).

Examples of the mathematics discourse from October indicate that students were beginning to show curiosity about some simple patterns and were beginning to ask themselves why such patterns occurred (refer to Episodes 3 and 4 on pages 100 and 102, respectively). These episodes, as well as other examples from Chapter V, reveal that the students observations of patterns and their questions about these patterns were primarily
directed toward the teacher, and redirected by her to all students. In general, mathematics class discussions from September and October focused on solutions to mathematics problems (as in Episode 1, page 95), some non-mathematical issues (as in Episode 2, page 98), and discussions of simple patterns found in problem solutions (as in Episodes 3 and 4, pages 100 and 102). Small group interactions were characterized by the sharing of work but not necessarily shared understandings. Furthermore, whole class discussions were dominated by teacher-student interactions, with very few peer interactions taking place.

A look at the classroom in February reveals interesting differences in the participants' interactions and in the nature of the discourse. First of all, whole class discussions in February primarily revolved around the mathematics of problem situations and related mathematical issues. Students were less likely to share non-mathematical aspects of the problem solving process. A second difference arises in the nature of the ideas shared during whole class discussions in February. By February, students were more likely to take intellectual risks by sharing unfinished thoughts and unchecked ideas. Furthermore, as the February episodes illustrate, students were building on the ideas and observations of others and were thus creating shared understandings (shared at least by those contributing to the discourse, such as Margaret, Kenny, and Eddie in Episode 6, page 111).

Student-student interactions were more common during whole class discussions in February than in the months of September and October, such as the exchange between Kenny and Lessa in Example 19 (page 151). Although Mrs. Miller continued throughout the school year to channel the students' contributions to the discussion, by February she was less likely to control the conversation and more likely to act as facilitator of the conversation. That is to say, Mrs. Miller continued to play a significant role in whole class discussions, but her role had changed. She listened to responses, reworded or emphasized student contributions, asked questions to push ideas further, and redirected many questions back to the entire class as opposed to doing much more talking, directing, and explaining as
she had done early on. Episode 6 and the February extended episode found in Appendix B provide examples of this behavior.

The episodes and examples of discourse and interactions from the beginning and middle of the school year illustrate the general nature of the mathematics discourse in a fifth grade classroom. At the same time, these examples afford a view of some distinct characteristics of the setting and activities of the participants. The February vignettes show significant changes in the mathematics discourse, and also in the nature of the teacher’s and the students’ interactions. I claim that the episodes and vignettes offer classroom teachers examples of what mathematics discourse can be. More importantly, this study illustrates beginning mathematics conversations which gives mathematics teachers a realistic view of initiating and promoting student interactions and the resulting discourse. Although these examples come from one specific classroom, they add to past empirical studies of mathematics discourse to provide a more comprehensive view or a range of views on the nature of the classroom discourse.

Changes in the Classroom

The more detailed analysis of data reported in Chapter V reveals many interesting changes in the focus of the mathematics discussions, the learning environment of the classroom, and the nature of the discourse structures. These changes appear to be, in some sense, a natural growth over the course of the school year. The changes noted also played a role in the development of the mathematics discourse. As such it is important to acknowledge and understand how these features contributed to the discourse.

I claim that one important change in the classroom that affected the development of the discourse was in the focus of mathematics discussions as influenced by the nature of the questions Mrs. Miller asked. Analysis of Mrs. Miller’s role in class discussions found in Chapter V (see Table 3 on page 141) indicated that her format for questioning students changed from primarily performance-oriented questions at the start of the school year to
many more structure-oriented and some theory-oriented questions by mid-year. Table 3 and Table 5 (page 148) indicate a relationship between the nature of Mrs. Miller's questions and the nature of the mathematics addressed by the students. As Mrs. Miller's questions shifted, the students' responses shifted from describing their problem solving activities to explaining and justifying their problem solving activities. Hence, we see the effect of the questioning on the focus of the students' mathematical conversations.

The focus of mathematics discussions not only changed from principally description of procedures to explanation of procedures, but the discourse also began to include verbal explorations of the mathematics being shared. Although this change in the focus of discussions began in October (see Episode 3 on page 100 for one example), many more students took responsibility for finding patterns or extending problems in new directions as the year progressed. In general, students appeared to take more initiative and more responsibility for participation in mathematics discussions later in the school year. This supports research on changes in instructional focus (Hiebert et al, 1997; Hiebert & Wearne, 1993; Putnam & Reineke, 1993) which suggests that teachers' shifts in instructional approaches influence students' patterns of interacting. The research presented here shows that, in particular, the shift in the nature of Mrs. Miller's questions is one type of change that influenced both the nature of the students' interactions and the nature of their contributions to the discussion of mathematical concepts.

As mentioned above, the changes noted thus far appear to be due in part to a natural learning process over the course of the school year for Mrs. Miller and the students. By participating in discussions and responding to Mrs. Miller's questions, students came to learn what was expected of them, and what behaviors were acceptable. In addition, Mrs. Miller's journal entries from February (see Appendix F) report her reflections on her own growth and learning about how her questioning and facilitating influenced the discourse. Mrs. Miller and the students also explicitly worked together to establish shared ideas of expectations, as I describe in the section on establishing sociomathematical norms of
Chapter V (page 117). I claim that the teacher and the students together continued to negotiate these expectations throughout the school year, which then determined the nature of their interactions and their contributions to the discourse. Some of these changes in interactions are described above. Another difference between October and February interactions is that by February students were more likely to acknowledge the work of others and to work cooperatively in building new understandings from each other's contributions. This change can be accounted for, in part, by considering the students' growing familiarity with the class format and their growing sense of safety in sharing ideas. Changes in the classroom learning environment are described in more detail in Chapter V (see Classroom Environment, page 134).

The analysis of changes in various aspects of the classroom highlights two important implications for mathematics teachers. First, changes in the focus of the mathematics discussions towards investigating and justifying mathematical ideas highlights the importance of a teacher's choices of mathematics tasks, and her or his questioning techniques. As previously discussed, Mrs. Miller chose mathematics tasks that allowed all students to use their prior knowledge to enter into and engage in mathematical problem solving. Mrs. Miller also encouraged her students to share their ideas, and to take part in mathematics discussions whether in a small group or whole class setting. Through questioning and establishing expectations, Mrs. Miller positively encouraged students to take part in the mathematics discourse. This is supported by the observation data which show that changes in the nature of the teacher's questions were followed by changes in the nature of the students' contributions to the discourse. In fact, the students' contributions in mathematics discussions changed from primarily descriptions of actions to addressing issues and providing justifications for their solutions.

Second, the changes in the classroom toward an environment that supported all students' contributions to the discussion highlights the importance of the teacher's and students' working together to establish sociomathematical norms. As was discussed in
Chapter V, Mrs. Miller and her students together were responsible for developing ways of working with and communicating about mathematical concepts and ideas. The learning environment does effect the mathematics discourse, and thus the teacher’s and students’ awareness of the environment can lead to useful discussions and mathematics learning.

In general the changes in the classroom and the changes in the mathematics discourse reported here support the current reform trends in mathematics education (Cobb & Yackel, 1995; Hiebert et al., 1997; Lampert et al., 1996) and are consistent with suggestions found in the NCTM Standards documents (1989, 1991, 1995). The focus of mathematics discussions, the learning environment, and discourse structures (the ways in which students are encouraged and expected to participate) were found to affect the nature of the mathematics discourse. Thus, it is important that teachers wishing to promote mathematics discourse be aware of and actively work toward creating conditions that support and encourage students to engage in mathematics discourse. That is, teachers should maintain a discussion focus that allows all students to contribute. Teachers should be aware of the learning environment being established and work with students to create a safe place for sharing mathematical ideas. Also, teachers should encourage all students to participate in mathematics discussions by helping to establish a sense that mathematics involves investigating and constructing ideas with others in the mathematics classroom community.

Teacher as Most Valuable Player

I point out in Chapter V and in the sections above that the mathematics teacher plays a very important role in the development of the mathematics discourse. From the start of the school year Mrs. Miller spent time talking with her mathematics students about issues related to working as a community of learners. She shared her own experiences as an adult learner and as a mathematics student. Mrs. Miller also invited students to express their beliefs about working relationships with peers and about doing mathematics in school.
Early in the school year Mrs. Miller also established a mathematics class format -- small group problem solving followed by whole class discussion of methods, solutions, and new observations. Through her intentional moves to structure both the working environment and daily activities, Mrs. Miller influenced the nature of the students’ mathematical experiences and the nature of their interactions.

Results of the analysis presented in Chapter V show that at least three aspects of Mrs. Miller’s role changed as the school year progressed and had an impact on the nature of the mathematics discourse. First, as mentioned in the previous section, the nature of Mrs. Miller’s questioning evolved and influenced the nature of the students’ responses. Secondly, Mrs. Miller’s feedback to students -- making suggestions, rewording their responses, repeating for emphasis, modeling presentations -- influenced the nature and the level of the mathematics discourse. The type of feedback that Mrs. Miller gave to her students reflected active listening throughout the school year, yet her feedback did change from September to February. Table 3 on page 141, for instance, indicates that as the school year progressed, Mrs. Miller spent even more time listening carefully to students ideas, asking students to clarify their contributions to the discussion, and interpreting or emphasizing their contributions for the whole class. As the research suggests (Cazden, 1988; Yackel & Cobb, 1996) feedback from the teacher that shows students their ideas are valued will encourage continued contributions to the discourse of the group. Yackel and Cobb (1996) take it a step further and suggest that the teacher’s role as a model for what to say and for what counts as an acceptable and valuable solution strategy influences how subsequent solutions are presented, and influences “the mathematical aspects of the knowledge students construct” (Yackel & Cobb, 1996, p. 474). Thus, as Mrs. Miller modeled expected behaviors and made suggestions to students she positively affected the students’ participation in the mathematics discourse.

The third way in which Mrs. Miller influenced the mathematics discourse was in her role as facilitator of the discussion. I have already mentioned, and repeat here for
emphasis, that Mrs. Miller was committed to promoting mathematics discussions. As such, she actively worked to develop her role in monitoring and facilitating the discourse. Table 3 on page 141 and Mrs. Miller’s responses to my journal entries (Appendix F) give the reader a sense of the changes Mrs. Miller made in developing her role as facilitator of the discourse, as opposed to lecturer or other possible more directive approaches to mathematics teaching.

One other important aspect of Mrs. Miller’s role was her choice of tasks. In conversations with Mrs. Miller she talked about choosing tasks that allowed all students a way into thinking about the problem situation. The tasks were also chosen to encourage student creativeness, to allow for multiple solution methods, and to give students something to talk about and to be curious about. I note in my analysis of the influence of the tasks (see Tasks and the Mathematics Discourse page 130) that many interesting discussions of mathematical concepts arose from student curiosities about the mathematics involved in the problems or from related mathematical issues, such as the discussion of why it is difficult to divide circles into eighteen equal portions. Mrs. Miller’s choice of tasks greatly influenced the substance and the level of many whole class discussions.

The discussion of the importance of the teacher’s role in the development of the mathematics discourse suggests that the mathematics classroom teacher cannot easily step out of the discussion. The teacher is responsible for setting up the working environment and the classroom format, for choosing appropriate tasks that will give all students entry into problem solving and hence into discussions, and for facilitating the discussion. Furthermore, mathematics teachers at all level who wish to promote mathematics discourse need to evaluate the usefulness of their questions for probing student understanding of the mathematics and for clarifying the students’ contributions. Teachers should also take time to actively and reflectively listen to students’ contributions in order to (a) provide opportunities for students to work with one another in developing ideas, and (b) make

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22 Personal communication with Mrs. Miller, October 1996.
decisions about the need to clarify an issue or about the need to encourage other students to participate.

As mentioned in Chapter II, researchers of communicative competence and of classroom interactions (Forman & Cazden, 1985; Pimm, 1987; Stubbs, 1983) suggest that the teacher plays a significant role in aiding students as they learn appropriate and useful ways of communicating in the mathematics classroom. Thus, mathematics teachers wishing to promote discourse also need to be aware of ways to model or teach such behaviors. As was true for Mrs. Miller's, teachers should also be aware that they will continue to learn about their role in the classroom discourse, through their own participation in it.

**Importance of Students' Contributions**

The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) make a case for students communicating mathematical ideas. Recent educational research supports the NCTM recommendations through studies that indicate increased learning opportunities for students who partake in collaborative inquiry-based instruction (Forman & Cazden, 1985; Lo & Wheatley, 1994; Webb & Farivar, 1994; Yackel et al., 1990). The *Professional Standards for Teaching Mathematics* (NCTM, 1991) complement the *Curriculum Standards* and make recommendations for the students' roles in the mathematics discourse. I note in Chapter II, however, that research on the students' potential contributions to developing and maintaining mathematical communication is scarce. In the research presented here, I have shown that students do play an important role in developing the classroom learning environment, in developing all students' sense of the mathematics discourse in general, and in developing mathematical autonomy. In this context, mathematical autonomy refers to the students' ability to make problem solving decisions based on their understanding of mathematical concepts.
In a previous section of this chapter, I note that Mrs. Miller and her students all were responsible for establishing ways of working together in mathematics class. Clearly, when Mrs. Miller invited students to express their beliefs and ideas about how to work together, she signaled to the students that their contributions were valued (see Episode 2, page 98, for one example). Throughout the school year, Mrs. Miller continued to encourage student participation in mathematical and non-mathematical discussions. It is interesting to note that the students just as often were the ones to initiate conversations that raised important issues related to the learning environment or related to a mathematical concept being discussed. These opportunities contributed to establishing a learning environment in which most students felt safe, as was witnessed by their increasingly frequent participation in investigating new mathematics and sharing new ideas (refer to Table 6, page 149).

Cazden (1988) and Yackel et al. (1990) make a case for developing students' mathematical autonomy so that students may help each other and help themselves make sense of mathematical concepts encountered in problem situations. In Mrs. Miller's classroom, the students' contributions to mathematics discussions pushed all students toward intellectual autonomy. Bethany, for one, introduced the issue of who holds mathematical authority in the classroom when she conceded the decision about solution correctness to Mrs. Miller, as illustrated in Example 5 of Chapter V (page 124). Mrs. Miller challenged Bethany's belief about who makes these kinds of decisions, which Bethany then made an issue for discussion for the entire class. This issue arose several more times during the school year when hesitant students turned to Mrs. Miller for help, and she advised them to seek help from their peers. In this way, students' questions and concerns became important contributions that pushed students' beliefs and shaped their intellectual autonomy. The students' beliefs in turn shaped the classroom learning environment and, hence, shaped the nature of the discourse.
Individual students and groups of students also contributed to the development of the mathematics discourse through their questions of, observations about, and concerns with the mathematics content or problem situation. For instance, Margaret often hunted for patterns, as did Amir and Kenny, and usually stimulated other students to do the same. Episode 6 (page 111) is one example of a student's pattern seeking and questions from Mrs. Miller that spurred Margaret and others to investigate the mathematics of the pattern. Chapter V and Appendix B provide more examples of how students initiated or otherwise influenced the mathematics discussions.

I claim that students play an important part in creating the mathematics discourse. As such, it is important for the classroom teacher to recognize the value of students' contributions, and to allow for flexibility in solution methods, and for the expansion of problem situations. In promoting classroom discourse, the mathematics teacher should encourage students to contribute to the dialogue. Furthermore, both teachers and students should be aware of how their contributions will influence the work of all in coming to understand mathematics.

**Implications for Teacher Education**

Thus far, I have presented some general findings from the research. In particular, I highlight the potential influence of the teacher's role, the students' roles, the mathematics tasks, and the classroom environment on the development of the mathematics discourse. The analyses of Chapter V provide illustrations as well as details of the features of the mathematics classroom that were important contributors to the nature of the discourse. These results suggest that the mathematics classroom teacher and the educational researcher need to be aware of possible classroom structures, including the nature of tasks, discourse structures, and other physical characteristics, that contribute to mathematics discussions.

For the most part, mathematics classroom teaching reflects the instructor's experiences as a learner. If an instructor has not taken part in an inquiry-based learning
environment where discourse is encouraged, the instructor may not have a sense of the value of mathematics discourse. She or he also may not have a sense of how to initiate and maintain discourse as a regular part of mathematics class. This study suggests important implications for how one might introduce and promote discourse in mathematics teacher education.

First, preservice and inservice mathematics methods and content courses should model possibilities for discourse in the mathematics classroom. This can be accomplished through inquiry-based instruction or other instructional formats that encourage the participants to interact with one another and collaboratively build understandings of mathematical concepts. The instructor of such a course should model active listening and questioning that promotes discussion of the mathematics students encounter. In this way, students (whether inservice or preservice) can discover for themselves how communicating about mathematics offers new insights, and allows them to make important mathematical connections (Lo & Wheatley, 1994).

This research also suggests that mathematics teaching methods courses focus on the classroom features that contribute to the development of mathematics discourse. If teachers value mathematics discourse and wish to promote it in their classrooms, it is important that they examine and reflect on possible tasks. Tasks that provide opportunities for a variety of learners to enter into the problem solving process will most likely allow the students to enter into the resulting conversations. The tasks alone will not, however, determine the success of the mathematics discussion. Classroom teachers also need to examine questioning techniques and learn to formulate structure-oriented and theory-oriented questions that will elicit student understanding of mathematical concepts. The literature review and the research presented here also suggest that teachers be given opportunities to consider and discuss how to establish a learning environment in which students feel comfortable sharing new ideas and taking risks (Bauersfeld, 1992; Lo & Wheatley, 1994; Yackel, Cobb, & Wood, 1993). The Professional Standards for Teaching Mathematics.
(NCTM, 1991) elaborate on the benefits of such a learning environment, but the results of this study make specific suggestions for how teachers might go about creating such an environment. For instance, this study shows that when students are invited to share their beliefs about mathematics teaching and learning they are likely to reflect on their underlying assumptions and possibly consider new ideas, such as students holding some authority for making mathematical decisions. The mathematics teacher must also account for her or his own expectations and beliefs, and have a sense of how these beliefs will affect the learning environment.

As previously mentioned, it is important for both preservice and inservice teacher recognize that establishing mathematics discourse as a regular part of mathematics class is a learning process for teachers and students alike. Teachers and students must together accept the responsibility of their roles in the ongoing mathematics conversation. A next step in this line of research is to focus more carefully on the particular effects of those factors that influenced the mathematics discourse. The nature of classrooms makes this a difficult but important task. Since there are so many factors mentioned above that influence each other as well as the discourse, teasing out the direct effects of any one factor will prove challenging. If successful, such an effort will greatly inform mathematics teachers choices and behaviors as they work to initiate and promote mathematics discourse.

The Nature of Classroom Research

The qualitative research approach used in this study allowed me to characterize the mathematics classroom, the mathematical context, as well as specific phenomena within the context such as the participants' activities and the mathematics discourse. This approach provided an overall view of the classroom, the participants, and the evolving nature of the discourse and interactions. At the same time, a more interpretive stance was used to make sense of the participants' interactions by involving their point of view and by understanding the meaning they gave to theirs and others' interactions. Even so, a qualitative
methodology is not always easy to work with and has its limitations. Qualitative research methods generate large volumes of data that require plenty of time for sorting, reducing, and focusing the analysis. The complex nature of classrooms adds to the complex nature of qualitative research.

**Complexities and Other Constraints on Data Collection**

Eisenhart (1988) and others (Brown, 1985; Erickson, 1988; and Wolcott, 1994) attest to the complex nature of the classroom and the difficulties involved when attempting to account for all possible interacting variables (students' prior mathematical knowledge, students' beliefs about mathematics class, social relationships, and mathematics content to name a few variables to consider). As a participant observer, I hoped to talk with students to get a better understanding of the variables involved in their decision making and in their resulting contributions to the discussion. In actuality, the variables were difficult to identify and separate out, but my conversations with students did give me more information with which I could make inferences about their behavior. For instance, during an interview early in the school year, Margaret shared with me that she was quite confident in her mathematical ability. This allowed me to make judgments about some of her interactions with fellow classmates, such as when she challenged Kenny’s hypothesis for the circle problem (Episode 3, page 100). I also had many conversations with Karen during small group work that allowed me to see much of her “behind the scenes” thinking. Although she often expressed a lack of confidence in her work, it was interesting to note that this did not translate into shyness since she was often eager to share her work, whether it was correct or not. It would have been interesting to delve further into Karen’s thoughts about what was acceptable to share during whole class discussions and how these beliefs influenced her participation. Many other instances of students sharing ideas and concerns were observable and recordable, but it was difficult to account for all variables in order to interpret or make judgments about what influenced the situations at hand.
The busy classroom environment was another factor that complicated the collection and analysis of my data. The data collection process involved daily visits to the fifth grade classroom, note taking, tape recording, talking with students as they worked in small groups, and many conversations with Mrs. Miller during and after the school day. Since there was so much happening in the classroom at any one time, it was difficult to capture many of the details. Instead, I usually focused on one particular small group, or let the tape recorder pick up most of the whole class discussions while I recorded notes on non-verbal interactions. Video recordings offered a different view of classroom happenings, but even these did not capture all of the students' interactions. Hence, on any given day I was only able to pay some attention to some of the classroom features and some of the participants' activities. Audio and video recordings of daily classroom events greatly enhances the data collected, and will allow me to continue investigating other aspects of the classroom. Possible extensions and other areas of further research are presented in more detail later in this chapter.

Discussion of Potential Biases

In a sense, the complexities of the classroom and situation constraints influenced my analysis of the data. That is, I was only able to analyze and draw conclusions from the data that I collected, thus leaving possible holes in the analysis. I addressed this potential bias by choosing what I felt were representative episodes based on my daily experiences in the classroom, not just representative of the observation notes.

A second potential influence on the data collected was the effect of my presence on the teacher's and the students' daily activities. Although Mrs. Miller did not turn to me to help her plan activities or to find appropriate problem solving situations, our talks during and after mathematics classes did pertain to specifics of the students' work and the flow of the discussions. Mrs. Miller seldom asked for advice about what to do next, but we did
talk about the role of discourse in the classroom, which most likely influenced some of the choices she made in preparing for and orchestrating mathematics lessons.

My influence on the students' participation was sometimes obvious, but not always. There were several students, for example, who "hammed up" their participation in small group work when the audio recorder was nearby. One student, in sharp contrast to the majority, often requested that I place the audio recorder with his group since it helped him to stay on task, knowing that someone (or something) was listening. It is less obvious how my physical presence as researcher and occasional classroom assistant influenced the students' work, their mathematical understanding, and their participation in whole class discussions, although Margaret once shared with me that she spoke less when the audio recorder or I was nearby.

My own perspective of mathematics teaching and learning was a third potential bias on the data collection and analysis processes. As discussed in Chapter III, my biases as an educator of mathematics could have led to my making assumptions about what was happening in the classroom and why these events were occurring. To counter this possible bias, I tried to enter the classroom each day with a sense of curiosity for what might happen. I tried to listen to the students and let their actions and their talk help me understand their beliefs and their conceptions of what was occurring. As with any qualitative or interpretive research, the researcher must be sure to support interpretation with description and with the actors own voices. Through the episodes and examples of Chapter V, I have allowed the classroom participants to speak, and I have supported my analyses with their actions and words.

Limitations of the Methodology

In the previous section of this chapter, I summarize the study findings and discuss how the findings answer or address the research questions. In addition, I suggest implications of this research for the classroom teacher and for education researchers.

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Analyzing the data from a single mathematics classroom allowed me to reduce some of the complexities of the study and to focus on providing a more descriptive and detailed perspective of the developing classroom discourse. Even so, the choice to study a single classroom eliminates the possibility of drawing stronger, more generalized conclusions. In the preface of his book *Life in Classrooms*, Jackson (1968) states that "classroom life ... is too complex an affair to be viewed or talked about from one single perspective.... We must not hesitate to use all the means of knowing at our disposal" (p. vii). It was my intention, in designing this research study, to provide classroom mathematics teachers, teacher educators, and education researchers a new perspective on the development of and the role of classroom discourse within the discipline of mathematics. I agree with Jackson that we need to account for various perspectives of classroom life in order to come to an understanding of what is possible and what is necessary to pay attention for initiating and maintaining classroom discourse. Hopefully the study I present offers both teachers and researchers answers to some of their questions related to interactions and discourse in the mathematics classroom.

Educational researchers caution the use of direct translation of qualitative research results. Qualitative research results, particularly case studies, often are scrutinized by researchers and practitioners who wish to draw generalizations from particular situations to inform their practice, as I addressed in Chapter III. In Chapter III I highlight the opinions of two qualitative or ethnographic researchers (Donmoyer, 1990; Schofield, 1990). In summary, I make the case that "thick description" (Wilcox, 1988) allows the reader to make a judgment about how the classroom studied fits with other situations and other classroom sites. Thus, even though this research details particular aspects of a particular classroom situation, aspects such as Mrs. Miller's intentions, expectations, and curricular choices may match with those of another teacher who is thinking about ways of promoting classroom discourse. The particulars of the study allow the reader or mathematics teacher to more fully understand the conditions under which a hypothesis will hold, ensuring
greater validity. I claim that this research study will provide answers to other classroom teachers who wonder what features of the classroom and of the participants' interactions are important to pay attention to for developing mathematics discourse. These findings are detailed above.

**Implications for Future Research**

The results of this research have raised as many questions as they have answered. Having studied the growth of the mathematics discourse in just one classroom, it would be extremely informative to replicate this study in a different classroom to compare the potential impact of the various factors identified here. It might also be interesting to locate teachers with varying pedagogical views and make comparisons of the nature of the mathematics discourse in these classrooms.

Research that looks more closely at any of the individual influencing factors on the mathematics discourse would also be in order. It would be useful, for instance, to investigate the influence of students' beliefs or "folk learning theories" by conducting frequent interviews with students while observing their actions and talk during the school year. A closer examination of how the mathematical content and context influences the nature of the discourse would also be important and informative for educators. Bauersfeld (1980) suggested that specific mathematical context might account for the formalness of some discussions. One might also be interested in how students' understanding of mathematical concepts affects the formalness or the general nature of discussions. These results could provide insights about assessing student understanding based on the discussion and about the changing nature of the mathematics discourse based on the classes mathematical knowledge.

Another avenue of research would be to investigate other forms of discourse (written, idiosyncratic communication, etc.) and the influence on student learning. These recommendations for further research are not necessarily new ideas, but all seem important.
for making sense of the value of mathematics discourse for student learning, and for providing teachers with ways of answering their own questions about initiating and maintaining mathematics discussions in their classrooms.
APPENDIX A

INTERVIEW PROTOCOL

October 1995 Student Interview Protocol

Read to students prior to interview: There are no correct or incorrect answers. I do not want to test you, I only want to hear what you are thinking about. Answer to the best of your ability. If you don't understand a question or if you don't want to answer one, let me know.

General/Structural Topics

1. How do you feel math class is going this year? What do you like or dislike about it? Why?
2. What's your favorite part of math class? What activities did you like working on? Why?
3. Tell me more about math class this year.
   - What kind of math are you doing?
   - What do math lessons look like?
4. Tell me about math discussions.
   - What part do you usually play in the discussions?
   - What part does Mrs. Miller play?
   - What makes a discussion useful or helpful to you?
   - What did you learn from today’s (yesterday’s) discussion about ________?
5. What about work with partners:
   - What do you like or dislike about working with a partner or group?
When is it helpful to you to be working with others? When isn't it helpful?
When you are assigned a new partner, how do you decide who does what?
What might you say to your partner to get started?
What if there's a disagreement, how would you settle it?

6. What do you think of class presentations?
   Do you like showing others your work?
   Do you have an easy time explaining your work so others can understand?
   Why or why not?

Mathematical Topics

7. How would you describe what mathematics is?
   How would you describe mathematics that's not just in schools... what does it look like outside of school?

8. How do you think the bike problem was different from the t-shirt problem or the M, F, and A prize problem?

9. What new pieces of mathematics did you learn from doing these investigations?
   What mathematics do you think you understand better after doing these problems?
   Explain why you think so.

10. When do you use a calculator?
    Do you use it to get an answer? to check an answer? to think about a strategy for finding an answer?
    Do you "trust" the answer the calculator gives you? Why or why not?

12. Suppose you and two of your friends are planning to organize a student raffle. You have collected all the prizes and now have to print raffle tickets to be sold. If the machine that prints raffle tickets uses only the numbers 1, 2 and 3, how many different tickets will you be able to print? (Hint: No two tickets can have the same number, but tickets can have 1, 2, or 3 digit numbers).
May 1996 Student Interview Protocol

Read to students prior to interview: There are no correct or incorrect answers. I do not want to test you, I only want to hear what you are thinking about. Answer to the best of your ability. If you don't understand a question or if you don't want to answer one, let me know.

1. Complete the following two phrases:
   Math is ...
   In math class we ...

2. What kind of math student would you say you are? Why?
   (like math, dislike math, like some math, very good student, etc.)

3. How do you feel when Mrs. Miller hands out a problem you've never seen before and she asks you to work on it alone?
   Would you feel differently if she said you could work with others? Why?

4. Describe how a small group works ... (Prompt student with the following) you are given an investigation, and Mrs. M assigns you to work with three other students.
   What happens next?

5. Do you think it's important to be able to work with others? Why?
   When is it most helpful to work with others? When isn't it helpful? Explain.

6. What does a math discussion look like?
   What makes it useful or helpful to you? Why? When isn't it helpful? Why?
   Who talks during a big class discussion about a math problem?
   When do you participate? What do you contribute?

7. How is the mathematical talk different when you're working with just one or three other(s) than when it's the whole class? Why?

8. How do you feel when others disagree with what you have said (either in a small group or large group)?
   What do you do when you disagree with what someone else has said?
9. What did you learn in class today? (OR What can you tell me about divisibility?) Did you discover this on your own? Or did you come to understand it based on what someone else said? Do you think others learned that too? 

10. Who do you listen to the most during class discussions or small group work? Why?
Interview Protocol for Mrs. Miller

1. Talk about one activity/investigation the students worked with this year that is typical of the kind you give.
   
   What are some of the things you think about when creating or preparing such an investigation for the students?
   
   Talk about what you anticipated to occur when the students were given the task.
   
   What did occur? And what did you learn from that about the students? About the activity?
   
   What mathematics learning occurred?
   
   What factors other than mathematical ones need to be considered?

2. Describe your model of mathematics teaching.
   
   What do you do in the classroom during mathematics time? Why?
   
   What do students do?
   
   How do your beliefs about learning fit with your model of mathematics teaching?

3. Nature of mathematics

4. Describe your beliefs about mathematics learning
   
   How do individuals come to learn about mathematical concepts?
   
   How do they come to know about the nature of mathematics?
APPENDIX B

EXTENDED EPISODES

The Bike Problem (September 27)

During the last ten minutes of the previous class time students were given "the bike problem" to work on with an assigned partner. For homework they had been asked to write about the thinking they and their partner had done. On this day students were back with their partners, working on how to present their bike problem solutions to the class. This was the first occasion they had to present a mathematical solution to the class. Some pairs continued working on finding an acceptable solution while most others were planning creative ways to present their solutions.

Nathan and Lisa were having a difficult time coming to a decision. They were both holding to their different answers without listening to each other's explanation. I intervened to get them to listen and think through each different explanation.

Lisa: He has 70 dollars. He didn't do anything else with it after selling it for 70.
Nathan: He got 10.
Lisa: 70.
Nathan: How could he get 70 dollars?
Lisa: OK... because when he sold it the last time he got 70 dollars. And it says he didn't do anything with the 70 dollars.
Nathan: Yah, because he bought it back again for 70, so he ends up with no money... only ends up with 10.
Lisa: No, he's going to sell it again for 70.
Nathan: No he buys it for 70.... (reads) But I do ... I'm going to sell it again... Eeii yeii yeii.
Lisa: He's going to sell it again for 70, so when you're selling it you're going to get more money. 70!
Nathan: Uhhh... you can't end up with 70 dollars.
Lisa: Why not?
Nathan: 'Cause everybody is getting 10 and 20 and then you come up with 70. (Lisa laughs.) How can he get 70?
Researcher: Now, Nathan, I heard just a minute ago, you were saying 10. Why do you think 10 is right? You were just saying 20 a couple of minutes ago.

Nathan: Well, he buys it 40 and he sells it for 50. That's 10 dollars he gets. Then um... he plans on buying it back for 60. So he loses 10 dollars. But then when he sells it again for 70 he gains 10 dollars back.

Researcher: So he gains 10 dollars, loses 10 dollars, then gains 10 dollars back? (Nathan "yah. So it's 10.") So, gain, lose, gain you end up with...

Nathan: 10.
Lisa: 70.
Nathan: How did you end up with 70?
Lisa: Look, he doesn't have 40 dollars because he just spent 40 dollars... "I sold it for 50" so he got 50 dollars back. Then ....
Nathan: He plans on buying it back for 60. But then he sells it again for 70, which equals 10... (Lisa: Which equals 70!)
Lisa: 70!
Nathan: ARG. How can he get so much?
Researcher: Lisa, maybe you should think of it as, he started with a certain amount of money and he ends with a different amount of money, so what's the difference between the starting amount and the ending amount? You know that he ends up with 70 dollars. But how much more is that than what he starts with?

Nathan: 10.
Researcher: Does he really start with 40?
Nathan: Yah.
Researcher: He gives away 40.
Nathan: No, he starts with 50. He gets 50... (Researcher: OK.)
Lisa: So 20.
Nathan: What? 20?
Lisa: Yah. Think about it.
Researcher: If he starts with 50 and ends with 70, is that what you're saying? If he starts with 50 and ends with 70 then the difference is 20?
Lisa: Yah. 70 - 50 dollars (punching into the calculator as she speaks)... look, it's 20.
Nathan: It's 10.
Lisa: 20.
Nathan: Nah ah.
Researcher: But what about... I was going to suggest that you write down how you found 20. Go through all the work that you did in your head mathematically to figure that out. And you (Nathan) do the same for 10. (Nathan: OK.) You can use words or you can just use numbers. And maybe if you explain to each other... Sometimes if you have it written down it's easier to explain.

After a several seconds of silence, Lisa turns to Nathan....

Lisa: I think you're actually right, Nathan.
Nathan: What?
Lisa: I think you're actually kind of right.
Nathan: I know I am.
Lisa: (turns to the researcher) I think he's right... because ....
Researcher: Now you think he's right? Tell me why.
Lisa: Um, what?
Researcher: So you think Nathan is right, now. Can you tell me why? Can you tell me what you’re thinking?
Lisa: Because he sells it, because he sold it again for 50, so he's going to get 50 back. And then he's going to buy it back for 60, so there goes his 50 down the drain.
Researcher: Plus 10 more, right?
Lisa: Yah. So he’s going to get 70 back, so... hold on.... (Silence as Lisa thinks through her work.)
Researcher: (Turning to see what Nathan is writing) Can I see what you wrote Nathan?
Nathan: Well he buys it for 40, then he sold it for 50. So then buys it back for 60, then he sells it again for 70. And it equals 10.
Nathan: Because... see he buys it for 40, so he gives away 40 dollars. But then he sells it for 50, so now he has an extra 10 dollars.
Researcher: OK, ten more. OK.
Nathan: Ten more. Now he buys it again for 60. So now he has no more money. Now he has no money, now he has no money left. But then he sells it again for 70. So now he has 10 extra dollars again. That's how I think.
Lisa: Why don’t we just put down both of our answers? (Sounding frustrated that they still have not come to agreement.)
Nathan: We can’t.
Lisa: Hmm.

On the following day of class Mrs. Miller asked students to present their solutions to the class. I was quite surprised that Lisa and Nathan had finally agreed on a solution and had found a clear way of presenting the solution to the class.

Mrs. M: All right. Well, the reason I want to go through this is, some people may, after seeing the presentations think differently about their work. And is that OK with everybody? (Class: Yes.) You know, "I thought one way, and now Rose showed me this, and now I think maybe I want to change my answer because of what she told me or I thought about her work.”
Let’s begin. Let’s start over here with Lisa and Nathan.

Nathan: This is a bike, but it’s a car (holds up a toy car), so we’re just going to .... (the play begins, and Nathan turns to Lisa) I’d like to buy that bike, how much do you want?
Lisa: 40 dollars. (Nathan hands Lisa 4 pieces of green paper.)

Lisa: I’d like to buy that bike back from you.
Nathan: OK. That’ll be 50 dollars. (This time Lisa hands Nathan some "cash.") OK.

Nathan: I’d like to buy that bike back. How much do you want for it?
Lisa: 60 dollars. (Nathan hands Lisa some "cash.")
Lisa: I’d like to buy that bike back again.
Nathan: That'll be 70 dollars. (They exchange “cash” again.)

Nathan: Now she has the bike at the end. But you know how I ... I started with 100 dollars... and I’ll count my money. 1, 2, 3, ..., 12. Now I have 120 dollars. If I started with 100, that means I have 20 dollars extra. That’s how we figured out our answer.

Mrs. M: (to the class) Any questions? Remember the day we sat here and I asked “are there students who like to act?” And acting was a way to help you with the thinking of that. Or did you do the thinking first and then thought of the play as a way to present it?

Lisa: We did the thinking first.
Heather's Formula (February 13)

On the previous day various students had shared patterns or formula they used to solve the "cake problem." In this problem, larger and larger cakes are obtained by adding a ring of a set width to the previous cake. Students are revisiting one of these patterns.

Lessa and Heather hold up a poster made by their group (along with Jessica and Jeanne) to show their thinking, and a possible formula.

Heather: If you look here... We did this on that (poster). Um, if you had called it... instead of calling it the basic cake, call it the first ring, because it only has one. And you could go 1 times 1 times 6 and you get 6. And you call the second 2...

Mrs. M: All right... if I were to call this...

Heather: The basic cake is one.

Mrs. M: (Writes a 1 next to the basic cake on the chart at the board) I'm going to call that one.

Heather: And you do 1 times 1 times 6.

Mrs. M: All right. Where do I get that extra one?

Lessa: You multiply it by the same number it is.

Jessica: You do number times number times 6.

Mrs. M writes 1x1x6 = 6 on the board. Heather continues to read her group's poster:

Heather's group's poster:
1st row 1x1x6 = 6 (price of basic cake)
2nd row 2x2x6 = 24 (price of cake with 1 ring)
3rd row 3x3x6 = 54

Heather: And you get 6. And this is two (points to second cake -- with one ring-- on her poster). And you do 2x2x6 and you get 24.

Lessa: And that's how we found out 100s.

Mrs. M: Before we go any further let's be sure that we have, um...

Nathan: I have a pattern.

Mrs. M: Nathan, there's something here. OK. One thing at a time. I want to be sure that before we go any further with this that everybody is engaged with it. And I need to be sure that I understand, too, what Heather was saying. Heather, if I understand what you and Jessica and Lessa and Jeanne were working on .... (silence)... let's see what we have... So, if I understand the work of the 4 girls, if we call each of these cakes by a number, they're saying the number of the cake times the number of the cake times 6 is going to give the new price.

Mrs. M writes the following formula on the board:

\[ n \times n \times 6 = \text{price} \]
Mrs. M: Now, I don’t know about you, but I want to see what they’re doing with this.

Mrs. M: (summarizing results from Lessa’s group) You did this pattern (she writes price of cake = nxnx6). And this one (price of rings increases by 12) you didn’t work with, you went to this first one.

Kenny: (addressing Lessa) How did you get the next ring number? You need the ring number to get the next price of the cake. Did you just um, ... you know, you needed to add by twelve to get the next ring. And then if you went to get the next answer... in order to do that you have to get the next ring. And you guys, did you find [the pattern] after building [cakes with manipulatives]?

Lessa: I don’t understand.

Heather: (member of Lessa’s group) We didn’t really use the ring answer.

Kenny: So, how did you get the thirty if you didn’t know to add twelve?

Lessa: I don’t think we did that.

Heather: We didn’t use the rings to find this ....

Lessa: We can’t find out the rings with that (formula).

Mrs. M: Um, Eddie and Kenny, you seem particularly interested in that. How did they get those numbers?

Kenny: I don’t... !?!

Mrs. M: Just from what they told you, how did they get the next one?

Kenny: Um, by um... just figuring it out.

Mrs. M: They built it, right.

Kenny: Yeah.....!?

Mrs. M: They kept building... I think you asked an excellent question. They got these numbers by building each cake. They built the first ten cakes, if I remember. And at that point they didn’t know... they weren’t looking at it as increasing by 12. Do I understand that’s what you were saying? (To Lessa, and Heather) Are you OK with that (Kenny)? Eddie, are you OK with that answer?

It seems as though Kenny wanted to know more about how Lessa and her group members found the pattern they shared. Although it was not clear to Lessa what Kenny was asking, Heather understood that Kenny wanted to know how or if they had used the “add twelve” result that most other groups used. Kenny had tried to follow Lessa’s reasoning but was unable. This scenario shows Kenny trying to relate Lessa’s method to one he understood. Heather’s response is the start of an uncovering of this new method.
APPENDIX C

CODING CATEGORIES USED IN ANALYSIS

Description of Codes

TASK
This code is used to highlight descriptions of tasks or mathematical content being discussed or worked on, as well as places where the nature of the task obviously influences the discourse. The data in this category helps answer the question How does mathematical content/task play a role in the development of discourse?

Subcategories of TASK include:
description
influence

ENVR
To code notes on the classroom environment or aspects of the classroom culture. This code is primarily speculative of inferential.

One subcategory was used to distinguish between those notes that give some sense of the environment and those that indicate a disruption in the activity (e.g. loudspeaker announcement changing a discussion from mathematical to social):
disruption

NATR
To code notes that illustrate the nature of the discourse (what is being said? How are participants communicating? What are they communicating about?) This is primarily descriptive data.

The SG suffix indicates that the discourse occurred in a Small Group (2 to 5 students, and possibly an instructor).

Subcategories include:
meta (talk about talk)
minimal (not sure what to say) SQ to discuss (std Q leads to disc.)
SG disagree std-std (share thoughts with peer)
SG idio (idiosyncratic talk) tchr repeats (rephrase, highlight)
SG share ideas tentative (timid about sharing)
SG std work (sharing work on task) TQ and A (primarily tchr Q/prompt)
share M (share mathematical work) TQ to discuss (Q prompts talk)
share NM (share non-math work) student missed
RROLE
To code any units of data that highlight the researcher’s role as a participant in the classroom and/or the discourse. Also highlights researcher’s potential influence on the direction of the discourse.

RUG
Codes all occasions when students or the teacher refer to the rug or move toward the rug. Data coded RUG helps to give a sense of the significance of the rug for the teacher and students.

SBLF
To code units of data that indicate a student’s belief about mathematics, either implicit or explicit. To help answer the question How do beliefs about mathematics and learning mathematics shape or get shaped by the discourse?

SROLE
This code is used for all data units that describe the students’ roles or responsibilities in the classroom, particularly roles played during mathematical discussions. To help answer the question What are the participants’ roles and responsibilities during mathematics class?

Subcategories include:
defends (supports work of another) Q understd (std Qs understanding)
disagree S resp (stds talk about/show resp)
explains (std explains work) TQ role (tchr questions stds roles)
leader (std takes lead in grp work) understnd (std shares, interprets)
obs (std makes observation) verbal thinking

STR
This code highlights the discourse structures such as whole class discussion, small group discussion, discussion led heavily by teacher prompting, discussion a result of some event, etc. To help answer the question Does discourse structure influence interactions and the nature of the discourse that occurs?

TBLF
To code units of data that indicate the teacher’s beliefs about mathematics, either implicit or explicit. To help answer the question How do beliefs about mathematics and learning mathematics shape or get shaped by the discourse?

TROLE
This is used to code data that describes the teacher’s role or responsibility in the classroom, whether this role is during a discussion or some other event/time. To help answer the question What are the participants’ roles and responsibilities during mathematics class?

Subcategories include:
advise (advises students) model Q (models questioning)
agenda (sets agenda for work) model obs (models observation)
comment (on student work) push (push stds to explain, defend)
direct (focus or lead conversation) Q sharing (questions to share ideas)
expect (reminds stds of resp) Q to push ideas (questions push idea)
interp (interprets std ideas, clarifies) shares (curiosity, personal interest)
Codes Used to Analyze Data

Classroom Environment

The following coding categories were used to describe the classroom learning environment:

- TASK with subcategories description and influence
- ENVR
- NATR with subcategories meta, tentative
- SBLF
- TBLF
- TROLE with subcategories expect, shares

Teacher's Role in the Discourse

Nature of Questions: The following codes were used to analyze the nature of the questions asked by Mrs. Miller.

- NATR with subcategories TQ and A, TQ to discuss
- TROLE with subcategories agenda, model Q, Q sharing, Q to push ideas

Talking versus Listening: The following codes were used to analyze the changes in amount of talking and listening done by Mrs. Miller.

- TROLE with subcategories advice, comment, interp, shares

Monitoring and Facilitating: The following codes were used to analyze how Mrs. Miller facilitated discussions.

- NATR with subcategory tchr repeats
- TROLE with subcategories interp and push

Roles Students Play in the Discourse

Nature of Interactions: The following codes were used to analyze the nature of students interactions during whole class (and some small group) discussions.

- NATR with subcategories share NM, share M, SQ to discuss, TQ to discuss
- SROLE with subcategories disagree, defends, explains, Q understnd, obs, understand, verbal thinking

Taking Initiative: The following codes were used to analyze when and how students' took initiative during discussions.

- NATR with subcategories std to std and SQ to discuss
- SROLE with subcategories disagree, leader, obs, Q understnd

Addressing the Mathematics: The following codes were used to analyze how students addressed their mathematics understanding during whole class discussions.

- NATR with subcategory share M
- SROLE with subcategory defends, explains, obs, understnd, verbal thinking
APPENDIX D

FREQUENCY OF CODES

Frequency of Codes in September Data

**TASK, 13**
- ENVR, 9
- ENVR disruption, 3

**NATR:**
- disagree, 1
- minimal, 1
- SG idio, 3
- SG std work, 4
- share NM, 3
- SQ to discuss, 3
- tchr repeats, 7
- TQ and A, 7

**meta, 7**
- SG disagree, 5
- SG share ideas, 10
- share M, 4
- SQ to discuss, 1
- std to std, 1
- tentative, 1
- TQ to discuss, 6

**RROLE, 1**

**RUG, 5**

**SBLF, 6**

**SROLE:**
- disagree, 1
- leader, 4
- Q understnd, 1
- T support role, 5
- understnd, 4

**explains, 6**
- obs, 5
- S resp, 7
- TQ role, 4

**STR, 8**

**TBLF, 7**

**TROLE:**
- advice, 7
- comment, 2
- interp, 3
- model Q, 3
- Q sharing, 7
- shares, 3

**agenda, 4**
- expect, 6
- model obs, 5
- push, 6
- Q to push ideas, 3

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### Frequency of Codes in October Data

**TASK, 6**

**ENVR, 9**

**ENVR disruption, 1**

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**RROLE, 12**

**RUG, 6**

**SBLF, 4**

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**STR, 3**

**TBLF, 2**

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## Frequency of Codes in February Data

**TASK, 8**

**ENVR, 7**

**ENVR disruption, 5**

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**RROLE, 6**

**RUG, 8**

**SBLF, 11**

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**STR, 1**

**TBLF, 4**

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EXAMPLES OF STUDENT HANDOUTS

**Student Survey (September 8)**

1. Something about math that I really like is ...
2. Something about math that worries me is ...
3. Some things I heard about math in grade 5 are ...
4. When I solve problems in math I like to use some or all of the following:
   - A. Draw a picture or diagram
   - B. Make a chart of the information
   - C. Talk it over with a friend
   - D. Use objects to figure it out
   - E. Guess and then check it
   - F. Try a simple but easier one first
   - G. Act out the information
   - H. Work backward
   - I. Find a pattern
   - J. Other? ___________________
The Bike Problem (September 26)

(Adapted from *Math for Smarty Pants* by Marilyn Burns.)

Directions

1. Read the comic strip.
2. Discuss it with your partner.
3. For homework, write out a plan you think you could use to solve it with.
4. Both partners share plans and work together to solve it.

Comic Strip Dialogue

Kid: Hey, Billy, Speedo. I just bought a bike for $40.
Billy: Where is it?
Kid: I sold it for $50.
Speedo: That was fast.

Kid: Well, I plan to buy it back for $60.
Billy: Some people never know what they want.
Kid: But I do. I'm going to sell it again for $70.
Speedo: Make up your mind.

Kid: Well, what I want to know is, will I make money on the deal or lose money -- and how much? Or will I come out even?
Speedo: That's cinchy. You'll be ahead by $10.

Kid: That's what I thought at first, but now I don't think that's right. I think I'll make $20.
Billy: You're both wrong. You'll break even, wheeler dealer.

Speedo: No way! It's a clear $10 profit.
Kid: But I'll make $10 on each sale.

Speedo: But it'll cost you too.
Billy: I'd rather think about magic. Actually, that's not a bad way to make the bicycle disappear.
The Magic Circle Problem (October 5)

Looking for a Strategy!
Is there more here than meets the eye?

Place the numbers from 1 to 7 in the above circles. Use each one only once. Place them in any circle so that when you add any straight line of three circles you get a sum of 12 in any direction.

Thinking and Writing about Math (October 13)

In the circle project that we have just completed we looked at the problem solving strategy of taking a large problem and trying to understand it by looking at a smaller but similar one. I'd like you to write for me about how this worked for you. Explain what you saw in the smaller puzzle that helped you to understand the puzzles as they grew larger. Be sure to include how you found a strategy and if it was useful to you for all the larger puzzles.
The Price of Cake (February 6)

The price of each cake is determined by its size. A cake that is twice as large costs twice as much. All of the cakes discussed below are hexagonally shaped.

The yellow pattern block represents a cake worth $6.

1. To make a birthday cake, the baker adds a ring of red blocks to the basic cake.
   - What is the price of the added ring?
   - What is the price of the birthday cake?

2. To make a graduation cake, the baker adds a ring to the birthday cake.
   - What is the price of the added ring?
   - What is the price of the graduation cake?

3. To make a wedding cake, another ring is added. Again, what is the price of the added ring and the entire cake?

4. If the process of adding rings to the cakes is continued,
   - What is the price of the tenth ring?
   - What is the price of the tenth cake?
   - What is the price of the nth ring?
   - What is the price of the nth cake?
The Cake Problem (February 12)

You’ve been working on the cake problem now a few days. It is now time to see if we can put together the information we have learned and the cake prices.

<table>
<thead>
<tr>
<th>CAKE</th>
<th>RING</th>
<th>CAKE PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BASIC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BIRTHDAY</td>
<td>1st</td>
<td></td>
</tr>
<tr>
<td>GRADUATION</td>
<td>2nd</td>
<td></td>
</tr>
<tr>
<td>WEDDING</td>
<td>3rd</td>
<td></td>
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<tr>
<td>?</td>
<td>4th</td>
<td></td>
</tr>
<tr>
<td>?</td>
<td>10th</td>
<td></td>
</tr>
</tbody>
</table>

To think about:

Do we have enough information to indicate to a customer what the price of any size cake would be without actually building it first? Write about any patterns you have found and how you could determine larger cake prices.
September 15, 1995

(Mrs. Miller's responses are in italics.)

I felt the purpose or focus of the first day of fifth grade mathematics was to set the tone for a year of mathematics that would most likely feel different to this group of students. I would guess that students got the point that this math class was going to be different, but I wonder if they have any ideas about what mathematics can be?

A setting of tone has begun, a beginning look at math delights and fears has begun. Can the students who have not experienced an avenue to learn math in a way that is meaningful to them trust that there is a place for them and their way of knowing here? I'm left wondering if I know all of what math can be. Did all students feel "invited" into this math class?
September 21, 1995

I want to be sure I spend a moment thinking about the students working together and the various ways in which they negotiated their roles.

Ashanti, Karen, and Taylor: My two visits to their work area today informed me that they were trying to equally share the making of at least the final draft of their graph. Karen told me she and Ashanti were each drawing and coloring four categories and Taylor was doing only three but that her three had more items, so they were doing equal work. My most recent visit witnessed them deciding how to divide up the writing of the title. How did their “equal share” system come about? Do you have any insights? Did you witness or encourage it’s creation?

No insights here. I neither witnessed nor encouraged the creation. I remember asking if symbol sizes in the pictograph needed to be the same size. I feared, on a different note, that Ashanti would be overshadowed by Karen and Taylor.

Brad and David: After their final draft was complete, Brad made a suggestion for changing the ordering of categories. David didn’t agree with it, but Brad was ready to make the changes anyway. It was nice to see that Brad did eventually “hear” David’s suggestion to first test the proposed changes on scrap paper. David then came up with a modification of Brad’s idea that was less likely to damage the appearance of the final draft they had just finished. They both seemed pleased with the result.

Are there social skills they have acquired that are useful and just part of what they do to get along that they don’t “see” them as clearly as we do?

It will be interesting to continue to watch these two students, and others who seem to have interesting ways of negotiating or getting along.

Once in each mathematics class this week you talked to the students about bringing new ideas, possibly incomplete ideas, to the group ... about feeling safe to share. I’d like to keep looking at this, but I wonder what my evidence will be? How will we know that students feel safe (and excited) about sharing?
- when those who hold back come forth?
- when "eye darters" do this less?
- when a language becomes natural -- such as "I see and understand the solution, and here's one I came up with, too."
- when students hold confidence in their solutions in face of oppositions, continue to test it and still hold on.
- when some students start tripping over each other to present their ideas.
November 8, 1995

Although many students are still working on making their explanations clear, at your urging, I am noticing that in general the students are doing more talking during whole class discussions. Your talking is still frequent, but you have shifted from doing lots of telling to clarifying, probing, monitoring. Here are some examples:

- paraphrasing or repeating for emphasis: E.g. “Multiply by 4 to get 24. Right?”
- asking probing questions to get at details of students’ work or understanding: E.g. “Where did you get the idea to try that?”
- monitoring and facilitating the discussion: E.g. “OK, now let’s turn to what Kelly has said.”
- commenting on ideas, making suggestions: E.g. “It might help if you line these up....” “What do we need to ask Jeanne to understand this better?”

Is there a balance between my clarifying and my leading the students? How much of this questioning will shift to the students as the year progresses?
February 2, 1996

A difference I've noticed in your contributions to the discourse since the beginning of the year is the kind of non-mathematical issues you discuss with the class. Earlier in the school year you and the class spend time talking about appropriate behavior for working with partners and for participating in group discussions. There was also talk encouraging student participation, sharing thoughts and thinking about the ideas of others. Now you talk with students about taking responsibility for their work and for making sense of the work of others. You talk about needing them to explain clearly, and to justify answers verbally and in writing. You continue to “push” students to ask questions of each other and to share their thoughts. I think their contributions to the discourse are changing, too.

It will certainly be of interest to me to see how my discussions with students changed over the year. What can I learn from it? How can I modify it?

I like the fact that you often share your new mathematical discoveries with students (such as when they come up with a way of thinking about a problem that you hadn’t considered). Highlighting your own learning is one piece I’ve been keeping track of since September. I often find examples of you expressing how you learn best, too.

I enjoy sharing my discovery -- honest discovery with students. I want them to know that I am a learner too, and that I am on this journey with them. I think that a true belief on the part of the students that I am a learner gives them an authentic place to take ownership of ideas, presenting their conjectures, and gives them confidence in making me see their thinking when I don’t see it at first.
February 20, 1996

The discussion of students' work on the cake problem was really fascinating today. I didn't expect to hear much since the two "patterns" were shown yesterday. Obviously, there was much more to be learned today.

As Lisa was explaining her work to the first math class she mentioned need to find the price of the next larger ring, but didn't explain why she needed to know this. She did, however, add that she had found a way to get the price (adding $12 to the price of the previous ring). Justin's question of why she knew to add $12 and not some other number was great and really gave Lisa something to think about. How fitting that Justin ask such a great question since he was the person who talked yesterday about not knowing how to ask good questions! Lisa got some help from others and was then able to answer Justin's questions in a satisfactory way (in my opinion, but I don't know what Justin thought about it).

Yesterday when I asked questions of Tim when he was presenting his information about the cakes, I was very aware that again I was the only one asking questions. I turned to you when it occurred to me that the students of course have no questions to ask if they are "with" Tim in that they too determined the price the same way that he had. This struck me as important. They are not as interested here in Tim's thinking as I am. I want him to explain his thinking but I also want his explanation to aid students who aren't clear where the numbers come from. When students got to the part today of sharing discovered patterns it involved different kinds of questions. Hooray for Justin!
REFERENCE LIST


