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Extending the National Council of Teachers of Mathematics' "Recognizing and Recording Reform in Mathematics Education" documentation project through cross-case analyses

Loren Phaffle Johnson
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Extending the National Council of Teachers of Mathematics' "Recognizing and Recording Reform in Mathematics Education" documentation project through cross-case analyses

Abstract
The primary emphasis of this study was to broaden the understanding of data collected in the National Council of Teachers of Mathematics' (NCTM) Recognizing and Recording Reform in Mathematics Education (R³M) project in order to more fully clarify the processes by which reform in mathematics education was occurring across five high school sites. A secondary emphasis was to develop a model of doing cross-case analyses and identifying those methodological elements and linkages that could be applied generally in large-scale studies of this sort.

R³M documenters obtained data that resulted from interviews of mathematics teachers, administrators, and students; classroom observations; and documents collected at the sites. That data and summary reports of documenters constituted the data pool for this study. There were six guiding issues from the R³M project which determined the clusters used in this study: the mathematical vision held by people at each high school; the pedagogical vision held, relative to mathematics, by the people in the site; the contextual features which influenced, both positively and negatively, curriculum reform in mathematics at each site; the influences on students of the mathematical and pedagogical practices; the evolution of the reform efforts at each high school; and the impact of the NCTM Standards on these efforts of reform.

The study consisted of immersion in the data through four levels of analysis and reduction: coding of data, sorting and summarizing of data by codes for each school, summarizing the data by clusters for each school, and analyzing each of the six clusters across the five cases of the study. The coding and sorting of data were facilitated by the computer program HyperResearch (Version 1.55).

Some of the findings center on how pedagogical shifts by teachers influenced the way students do and understand mathematics; how teachers learned and adapted when confronted by the complexities involved in mathematics reform efforts; the different ways in which the Standards influenced the efforts of reform; the possible value of the cross-case methodology to similar studies; and issues raised in study and the implications of those issues for the field of mathematics education.

Keywords
Education, Mathematics

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EXTENDING THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS' "RECOGNIZING AND RECORDING REFORM IN MATHEMATICS EDUCATION" DOCUMENTATION PROJECT THROUGH CROSS-CASE ANALYSES

BY

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DISSERTATION

Submitted to the University of New Hampshire in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy

in

Education

May, 1995
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Date: March 15, 1975
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Thomas Schram, my other major advisor, has provided a balance to my educational development and to my research. As a cultural anthropologist, his insights and experience have permitted me to gain a more integrated sense of what I want to do as a researcher. I progressed from being one of Professor Schram's students to teaming up with him as a documenter for one of the schools in this study. That field experience allowed me to see research through a different perspective and has been especially helpful in completing this study. For the many opportunities to sit and talk with Professor Schram, I will always be grateful.

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on-going input throughout my study. Charles Ashley has been a constant source of inspiration and encouragement from the very beginning of my program. His ability at group skills allowed for a close-knit camaraderie to develop in his Proseminar—a setting which provided colleagues and friends throughout this journey. He also removed "fear" from the vocabulary of his doctoral students and substituted "can do." Sharon Nodie Oja has taught me to look around all the corners. She has made me very aware of the "others" who are involved in our research collaborations and to lean more heavily upon reflection. Donovan Van Os dol has provided a mathematician's perspective to the research. His guidance, encouragement, and sense of humor are a valuable piece of this scenario.

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ABSTRACT

EXTENDING THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS' "RECOGNIZING AND RECORDING REFORM IN MATHEMATICS EDUCATION" DOCUMENTATION PROJECT THROUGH CROSS-CASE ANALYSES

by

Loren P. Johnson
University of New Hampshire, May, 1995

The primary emphasis of this study was to broaden the understanding of data collected in the National Council of Teachers of Mathematics' (NCTM) Recognizing and Recording Reform in Mathematics Education (R³M) project in order to more fully clarify the processes by which reform in mathematics education was occurring across five high school sites. A secondary emphasis was to develop a model of doing cross-case analyses and identifying those methodological elements and linkages that could be applied generally in large-scale studies of this sort.

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across the five cases of the study. The coding and sorting of data were facilitated by the computer program HyperResearch (Version 1.55).

Some of the findings center on how pedagogical shifts by teachers influenced the way students do and understand mathematics; how teachers learned and adapted when confronted by the complexities involved in mathematics reform efforts; the different ways in which the Standards influenced the efforts of reform; the possible value of the cross-case methodology to similar studies; and issues raised in study and the implications of those issues for the field of mathematics education.
SECTION I

CONCEPTUAL AND METHODOLOGICAL FOUNDATION
CHAPTER 1

INTRODUCTION

Background

An area of particular interest and concern to researchers and educators in mathematics education is in trying to understand the complexity of the current reform efforts in mathematics learning and teaching. Teachers and school administrators are looking for ways to effectively bring about changes in mathematics content and pedagogy that will better meet the needs of the students in their classes and schools. Often their search is for a quick solution to a host of very complex problems. Pedagogical shifts impact on the belief systems of those who would bring about change; those shifts also impact upon the entire school community. There is a need to promote a deeper discussion of the complex issues associated with changing mathematics programs in schools, a discussion which must include how other teachers, administrators, and communities are wrestling with change. This study is motivated by my desire to extend this deeper discussion and to provide necessary research in the field of mathematics education.

Mathematics Reform

A look at the history of mathematics education in this country provides an ongoing panorama of change which is reflected in the growth of the field of mathematics (and mathematics education) and the changing needs of the society in which we live (National Council of Teachers of Mathematics, 1970). The rapid growth of mathematics and technology point to a need for a changed content in K-12 mathematics, and research on how students learn mathematics suggests changes in pedagogy which will facilitate the learning process (Driscoll & Lord, 1990; Ferrini-Mundy, 1992a; National Council of Teachers of Mathematics, 1989, 1991; Silver, 1990). These advances in technology and the rapid development of new mathematics and new uses for mathematics have tended to
obsolesce much of what we call "current" educational practice in mathematics education (National Council of Teachers of Mathematics, 1989). Do these current issues and forces imply that change in the way mathematics is taught is inevitable? Many seem to think so.

Recognizing these needs for changes in the K-12 mathematics curriculum and in teaching practice, the National Council of Teachers of Mathematics (NCTM) saw the need to provide some direction for the reform effort. Its two documents, Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) and Professional Standards for Teaching Mathematics (NCTM, 1991), have become not only the standards for change in mathematics for grades K-12, but they have also provided a model for other disciplines to follow in their attempts to bring about reform (Brunner & Fisher, 1994). The impact of the Standards documents on the community of mathematics practitioners, researchers, and policy makers at the local, state, and national levels has been phenomenal and continues to grow, according to Blank and Gruebel (1993).

As testimony to the impact of these Standards documents, forty-three states have adopted Standards-like mathematics frameworks (verbal communication with NCTM headquarters, December 1994), and membership in NCTM has increased by thirty-eight percent during the past five years (Gates, 1994). As a professional organization presently representing approximately 106,000 members, NCTM is struggling with its future role in this reform process.

Though there is widespread circulation of the two Standards documents, the number of teachers actually attempting to develop their mathematics curriculum to follow Standards' recommendations may be somewhat overstated (Weiss, 1992; 1994). Weiss' studies indicate that there is greater understanding of the Standards at the secondary level, but implementation may be slower there than at the elementary level. However, there is evidence emerging from the field of mathematics education research (e.g., the R$^3$M project, QUASAR$^2$, CGI$^3$) that teachers of mathematics are seeking ways to bring about change in the way they teach mathematics. But there remain many questions that are being
raised as schools, districts, and states attempt to redesign their mathematics content and pedagogy. A national survey of mathematics and science education was funded by the National Science Foundation (NSF) which provided indicators of the level of recognition of the Standards documents (Weiss, 1994). Survey instruments are limited, however, in the depth of information they provide. Lacking from such surveys will be the successes and problems that teachers and schools face as they try to conceptualize what a Standards-like curriculum looks like.

Recognizing and Recording Reform in Mathematics Education. A concern "that the mathematics education community be able to describe the changes that occur in conjunction with these two documents and related activities, and as best we can, to explain how the changes occurred" (Ferrini-Mundy, 1992a) was the motivation behind an NCTM task force's recommendation to monitor the changes taking place as individual schools and school districts began to implement their interpretations of what the Standards were saying (Schoen, Porter & Gawronksi, 1989). As a result, a two-year monitoring effort was funded by Exxon Education Foundation in order to find out whether the Standards were being implemented and what effects they were having on student learning (Ferrini-Mundy, 1992a). NCTM recognized the importance that this documentation study would have on a wide variety of audiences which would include practitioners, policy makers, and funders.

The R³M project was seen by NCTM as one of the studies of a "multidimensional endeavor to understand and facilitate mathematics reform" (Ferrini-Mundy, 1992a, p. 6). The project is attempting to accomplish three basic tasks: determining the influence and knowledge of the Standards in various communities; developing useful descriptions where change in mathematics education is occurring for the benefit of several audiences; and to disseminate these descriptions in a variety of ways to several audiences.

To accomplish these tasks, the R³M project staff developed, with input from documenters, the guiding issues for the project which dictated the creation of interview guides, classroom observation guides, and guidelines which would guarantee the integrity
of the individual site (e.g., issues of anonymity, non-evaluative nature of site visits). That orientation (Appendix A) served as a focus for the collection of data at individual sites and included:

1. The mathematical vision held by the people in the site.
2. The pedagogical vision held, relative to mathematics, by the people in the site.
3. The contextual features which influence, both positively and negatively, curriculum reform in mathematics at each site.
4. The ways in which the mathematical and pedagogical practices are affecting students.
5. The evolution of reform efforts at each site.
6. The place the Standards play in the reform efforts of each site.

A focus on these six components provides direction for the main things studied—"the key factors, constructs or variables--and the presumed relationships between them" (Miles & Huberman, 1994, p. 18).

Twenty-two documenters from various parts of the country have gathered data for the project, Recognizing and Recording Reform in Mathematics Education: Surveying and Documenting the Effects of the National Council of Teachers of Mathematics Curriculum and Evaluation Standards and Professional Standards for Teaching Mathematics (R³M), during a two-year period. These field researchers are comprised of mathematics education researchers, ethnographers, mathematics educators, teacher educators, and mathematics practitioners.

Documenters traveled in pairs to seventeen broadly representative sites in the U.S. and Canada gathering data in the form of interviews, classroom observations, and written sources of data from the sites. The selected sites were not considered to be "model" sites; rather, they were schools or districts that were thought by project leaders to typify a variety of attempts to bring about change in their mathematics programs. Some sites had just started the reform process; others were seasoned veterans at change. The period in which
some form of identifiable, planned change had been going on in the observed sites ranged from one to eight years duration.

Those of us who participated as documenters in the R³M project were interested not only in what schools were changing in their mathematics programs but how they were going about the process of change. We found that local interpretations of the direction of change varied considerably, and there seemed to be no real consistency in how the Standards documents were guiding the local implementation process. This created a dilemma for the R³M researchers because at some sites very traditional mathematics programs (highly teacher directed and textbook driven) were touted as being avant-garde. At another site, the mathematics teachers were curious about why their school had been selected: "We haven't done anything that special to be selected for your study." Yet, this site had been trying out many of the suggestions in the Standards documents.

It was not the intent of the documentation study to assess or evaluate the mathematics programs at the sites. These documenters were never given a set of criteria by which to assess the quality or stage of implementation of a site's reform efforts since that was never the purpose of the study. Judgment calls by documenters were strictly taboo in the study, and documenters avoided phrases like "a very traditional program" or "only 60 percent implemented" in their final written portrayals of the sites (Ferrini-Mundy, 1992a). Documenters were interested in developing accurate descriptions of local perceptions of change and letting readers draw their own interpretations from those descriptions.

One of the dilemmas faced by the documenters was how to make the data more manageable and still portray the complexity of change since the data generated from these visits were so abundant. In an effort to reduce the data into a more manageable form, documenters were asked to identify two or three things that stood out about each site's efforts towards bringing about reform, and each of these characteristics became the basis of a separate story or "scenario." The scenario became the medium by which documenters began to reduce data to begin the development of final products for dissemination. In
some instances, several scenarios were combined to form case studies; in other instances, scenarios from several sites were grouped to form "composites" which followed a common theme. Some of the scenarios stood alone in their portrayal of a site.

While the use of scenarios satisfied our need to communicate the stories of reform efforts, there were much data that went unused through this process of writing scenarios to capture the stories of the various sites. There was also little attempt to draw comparisons across sites. This study extends the original analysis of data for the secondary school sites, which were a part of the K-12 effort of the R^3M project, to include cross-case analyses.

**Purpose of the Study**

There is a two-fold emphasis for this study. The primary emphasis is to broaden the understanding of data collected in the Recognizing and Recording Reform in Mathematics Education project in order to more fully clarify the processes by which reform in mathematics education is occurring across multiple cases for grades 9 through 12. This will entail efforts to attain an understanding of how reform processes for different cases are shaped by "specific local contextual variations" (Miles & Huberman, 1984, p. 151) and whether those local variations permit commonalities in the change process to exist across the multiple cases. A secondary emphasis will be to develop a model of doing cross-case analyses and identifying those methodological elements and linkages that can be applied generally in large-scale studies of this sort. This calls for demonstrating the viability and applicability of cross-case analyses to the R^3M project and the possible applicability to other research studies in mathematics education.

**Questions Guiding the Research**

The questions which directed the research for the area of primary emphasis were:

- What is the range of interpretations of the Standards held by site curriculum developers across the multiple cases of secondary schools in the R^3M project? Specifically, to what extent have the Standards impacted practice at those sites?
• What factors influencing reform in mathematics education emerge from the cases of secondary schools in the R³M project?
• What factors contribute to the persistence of change in mathematics programs across the multiple cases of secondary schools?

Questions which guide the methodological emphasis of this research are:
• How can cross-case analyses inform the R³M project in ways that are qualitatively different from everything else that has been done with the study?
• How is the feasibility and viability of cross-case analyses in this research influenced by the data collection processes of the R³M project?
• What implications does the model of cross-case analysis used in this study hold for similar types of research in mathematics education?

Rationale for the Study

Broadening the Understanding of Data Collected in the R³M Project for Grades 9-12

It is the intent of this study to broaden the base of understanding of data of the R³M project by extending the analysis and interpretation of data across the five cases which were conducted at the high-school level. The rationale for this extension is driven by the following four reasons:

Why Select Grades 9-12? Most of the current research in mathematics education focuses on elementary level (Ball, 1990; Ball, 1992; Cohen, 1990; Lampert, 1990; Lappan & Theule-Lubienski, 1992; Silver, 1994; Wood, Cobb & Yackel, 1993). The kind of unsupported wisdom in the field of mathematics education is much different at the secondary level than at the elementary level. There is a lot of speculation about how to change mathematics education at the high-school level, but there is little available research about the secondary effort. The small amount of literature at the secondary level is concerned with specific materials and texts being developed for use at that level (e.g., the University of Chicago School Mathematics Project [UCSMP] and the Interactive

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Understanding the change process in mathematics education. The most significant reason for extending the analysis of the data in the R3M project is to contribute to understanding of the change process in mathematics education. Silver (1990) cautions that our current understanding may lack definitive answers to questions about change, and that the value of research is its cumulative effect over a long period of time.

A number of multiple-case projects to promote and understand reform of K-12 mathematics are currently in progress across this country, each with its particular perspective on how changes in mathematics programs are to be accomplished. For example, there are development and research projects such as Cognitively Guided Instruction (CGI), QUASAR, and the Interactive Mathematics Project9 (IMP); there are teacher enhancement projects such as the Atlanta Mathematics Project (AMP) and Mathematics for Tomorrow (MFT); there are documentive studies such as Implementing the Standards of the NCTM in Geometry (Wiske, Levinson, Schlichtman & Stroup, 1992) and the R3M project. Variations in the goals of these various projects, including those of the R3M project, allow for an accumulation of greater understanding of the change process itself, including that of the R3M project.

Limitations of scenarios and case studies. The scenarios and case studies provide single case stories that will help teachers and other educational planners see the complexities of the change process in particular settings. This will have value for those contemplating change and those currently struggling with mathematics reform.

A vast amount of data at each site is overlooked through the scenario process. For example, themes of "leadership", "university partnership", and "mathematics for all students" characterized the three scenarios constructed for one of the sites at which I was a documenter. On the other hand, the data that documenters were asked to collect included: the mathematical and pedagogical visions of the site; the context in which change was
taking place; the causal factors for reform at the site; examples of classroom activities which teachers thought were particularly good; examples of the not-so-good attempts at change; the level of investment of teachers in the mathematics reform effort; observed classroom activities; changes in teaching practice by individual teachers during the past five years; what is needed in the school setting to optimize the change process; a description (by teachers) of "the student" who functions well in the mathematics program and what exit skills they would expect from such a student; finding out what things are most difficult for teachers to change content and pedagogy; and the list goes on and on. (Interview and observation forms are available upon request.) Though it may be argued that the type of reporting afforded by the scenarios essentially addresses the goals listed for the documentation study (see Appendix B), nonetheless it captures only a few of the foci of data from each site visit. As a result, much of the data from the site visits are excluded from any final reporting.

Neither does the present analysis of data allow for comparisons across cases. To illustrate: "leadership" became the focus of a scenario at each of two of the visited sites. Though not mentioned as one of the prevalent characteristics for the remaining fifteen sites, one might ask: What role does leadership play in the reform process of mathematics education? Is leadership a necessary ingredient for curricular changes in mathematics to occur? Administrative support, external funding, collaboration among teachers were important characteristics of some of the sites. What role did these factors play in the other sites? It is the intent of this extended research to capture this unassimilated data in order to delimit the analysis afforded by the scenarios.

**Generalizability.** Increasing generalizability becomes an important reason for doing this additional analysis of R³M data. Studies which use qualitative methodology rely on comparability and translatability in order that generalizations can be drawn from the research (Goetz & LeCompte, 1984). Or, as Lincoln and Guba (1985) suggest "the degree of transferability is a direct function of the similarity between the two contexts, what we
shall call "fittingness" (p. 124). Miles and Huberman (1994) believe that doing cross-case analyses increases the likelihood of revealing any comparability that may exist between cases. Certainly common characteristics emerging from otherwise divergent settings would strengthen the case for generalizability.

**Organization of the Study**

In this section, I will briefly describe the organization of my dissertation study. The text is organized in three sections with a total of seven chapters. Section I lays the conceptual and methodological foundation for the study. The background, the purpose, and the rationale for the study were described in Chapter 1. The second chapter is devoted to the development of the conceptual framework and will include review of selected literature on the learning of mathematics, mathematics pedagogy, combined research in both the learning and teaching of mathematics, the complexity of educational change, and how mathematics education may conform or differ to those patterns of complexity. Chapter 3 explains the methodology of the study and is prefaced by a brief description of some of the research studies of reform in mathematics education which have relevance to this study. The review of literature will also include the cross-case methodology as applied to the general field of education.

The Second Section will deal with the first two levels of data reduction and analysis. In Chapter 4, the first level of data reduction is discussed in the development of the meta-matrix for the study. A further reduction of data into clusters is discussed in the fifth chapter and leads to the cluster matrix, which becomes the basis for the analyses of clusters across cases.

Section III is devoted to analysis and interpretation of clusters and implications of the data. The analysis of each of the six clusters across the five cases constitutes the third level of data analysis and will be presented in Chapter 6, along with an interpretation of each of these six analyses. In Chapter 7, some additional interpretation of data between clusters is
included. Additionally, the conclusions drawn from the study are discussed in this final chapter as well as implications, and directions for further research.
Chapter Footnotes

1 NCTM is currently compiling a "scan" of projects, documents, and news related to the two NCTM Standards documents (Brunner & Fisher, 1994).

2 Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR), "is a development and research project aimed at improving mathematics instruction for students attending middle schools in economically disadvantage communities" (from a brochure explaining the project).

3 CGI follows a "process-product" research paradigm in which teachers' knowledge, beliefs, and decisions interact with students' cognition, learning, and behaviors to produce classroom instruction (Carpenter & Fennema, 1988).

4 "Curriculum" is used here to mean "an operational plan for instruction that details what mathematics students need to know, how students are to achieve the identified curricular goals, what teachers are to do to help students develop their mathematical knowledge, and the context in which learning and teaching occur" (NCTM, 1989, p. 1). Curricular as used here refers to mathematical content and pedagogy.

5 "Traditional" mathematics programs are characterized by a pedagogy that typically includes a given sequence of events: correction of homework; lecture by the teacher of the new material; demonstrating solutions to several of the new problems; and assigning homework. The role of the teacher is essential that of a dispenser of information, and the student is expected to absorb the information by listening to the lecture and doing the homework.

6 A "scenario" is a "story" that captures the most compelling feature of a site through a compilation of field notes and interviews with relatively minimal interpretation by the documenters. A compelling feature is not necessarily a strength of the site.

7 Miles and Huberman (1984, 1994) use the words "case" and "site" interchangeably, shifting to preference for the latter in their more recent publication. Throughout this research, I shall employ the term "cross-case analysis" to signify any analysis of data that exists across the multiple sites of the R3M study. A "site" in the R3M project was an individual school or a school district.

8 A "cross-case analysis" is the analysis of data across multiple cases where a case can be "an individual, a family, a tribe, a formal organization, a community, or even a culture as a whole" (Miles & Huberman, 1994, p. 172). In this study, a "case" refers to a single school or a school district.

9 IMP is a problem-based mathematics curriculum for high school students consisting of four to eight-week units that are organized around a central problem or theme.
CHAPTER 2

THEORETICAL AND CONCEPTUAL PERSPECTIVES

Levels of Theory

Goetz and LeCompte (1984) identify three levels of theory which are "commonly distinguished in the social sciences: grand theory and related theoretical models, middle-range and formal theory, and substantive theory" (p. 36). Because of the complexity of human behavior, few social scientists believe that the human sciences will ever be reducible to some universal grand theory. Lacking a grand theory, "social scientists substitute for grand theory their disciplinary orientations" (Goetz & LeCompte, 1984, p. 37). In mathematics education, "disciplinary orientations" are those theories that provide the basis of further theorizing in the field. For example, the developmental learning theories of Piaget and Vygotsky serve in that capacity and have become the middle-range or formal theory for the field of mathematics education. Formal theories are more limited in scope than a grand theory and "explain some abstract class of human behavior" (Goetz & LeCompte, 1984, p. 37).

From the formal theory or disciplinary orientations, a number of substantive theories may emerge, and it is at this substantive level that the theorists in the field of mathematics education operate. There are a group of theorists in the field of mathematics education who are identified as radical constructivists and claim that their theory for learning mathematics derives from Piagetian theory (Confrey, 1990; Steffe, 1991; von Glasersfeld, 1991). Another camp of theorists claim their origin in the theories of both Piaget and Vygotsky and are referred to as social constructivists (Bauersfeld, 1992; Cobb, Wood, & Yackel, 1990). Genetic epistemologists such as Dubinsky (1989) and Sfard (1989), whose research focuses on undergraduate mathematics, also build their theory on the works of Piaget and consider themselves as constructivists. (Discussion of each theory will be
addressed later in this chapter.) Each of these groups of constructivists has its own substantive theory by which the group carries on its research in mathematics learning and teaching.

These operating theories in mathematics education represent, according to Goetz and LeCompte (1984), the level of substantive theory which refers to "interrelated propositions or concepts lodged in particular aspects of populations, settings, or times" (p. 38). That is, the field of mathematics education is the "particular setting" for the purposes of this theoretical development. Educational change theories are also substantive in nature.

Miles and Huberman (1994) suggest that the map which guides researchers in their collection of data is made up of the orientations of the researchers. What the researchers want to know gives direction to the way data is gathered, but it is theory that drives or determines that orientation. It is from this third, or substantive, level that I developed a conceptual framework, and it is from that framework that I built a set of orienting assumptions that guided this study.

For example, in the R^3M project, the map which guided the collection of data at each school site consisted of six areas of particular interest to the documenters: (1) the mathematical vision; (2) the pedagogical vision; (3) the evolution of the reform efforts; (4) the impact of contextual features on change; (5) the influence of change on students; and (6) the impact of the Standards documents on the changes taking place at individual sites (see Chapter 1).

There are any number of contexts which influence the way we shape our views about teaching and learning mathematics, and those views or beliefs rarely remain static, nor are they the same from individual to individual. In the R^3M project, an attempt was made to standardize the data collection instruments, yet the orienting assumptions of the different documenters may have differed widely. Those differing orientations affected the way documenters asked interview questions, the way they recorded field notes from classroom observations, and the final writings for the project. Each of the documenters came into the
study with different theoretical backgrounds and with different orienting assumptions, even though the different documenters worked collaboratively towards common goals.

Through a focused review of research and literature at the substantive level, I developed a conceptual framework for this study, and, from that framework, I derived a set of orienting assumptions which influenced how I personally engaged in the collecting, analyzing, and interpreting of data.

A Conceptual Framework

How do students learn mathematics? What are the most effective pedagogical paths to follow in order to optimize student learning in mathematics? What factors assist or hinder reform efforts in mathematics education? How important are teacher beliefs in the reform process? Are there problems or dilemmas that slow down or prevent mathematics reform from taking place? The answers to the questions seem always to be changing. Nor does the field of mathematics education operate in isolation in addressing these questions. Contributions and insights from the fields of psychology, cultural anthropology and sociology, and the broader field of education with its various disciplines help to clarify and address these important issues in mathematics education.

A framework of concepts to guide my responses to the above questions began to emerge from the exploration of writings and research in these related fields. The exploration involved making choices of those ideas which help in building a logic for those responses--choices which are tempered by my experience and my perceived needs as an investigator. Some ideas are accepted or rejected based upon the evidence of research; others pass or fail the test of my personal logic. This framework is a dynamic entity always capable of some adjustment to make itself more viable to my research needs.

Once satisfied with the logic of this framework, I begin to develop assumptions based on this conceptual framework. These assumptions guide me in any number of research endeavors and become the tools with which the I manage data of research. These orienting assumptions assist in making decisions about how and what data is to be collected; how it
is to be analyzed; and finally, how it is to be interpreted. Through this process, the assumptions themselves are viable and subject to realignment or adjustment, continually shaped by new insights and new experience.

To give form to this conceptual framework, I have gathered focused research and literature which will lend support to the framework. This focused review will address current research and thinking on each of the following: how students learn mathematics; what research says about various pedagogical practices in mathematics classrooms; research which focuses on both cognition and pedagogy in mathematics; and finally readings and research that deal with the complexities of bringing about educational change—particularly as it applies to mathematics teaching and learning. I have embedded various orienting assumptions for my research in appropriate places throughout the development of the conceptual framework, and I give a complete statement of these assumptions at the conclusion of that framework.

**Learning Mathematics: Constructivism**

The radical constructivists (Confrey, 1990; Steffe, 1991; von Glasersfeld, 1990; 1991), the social constructivists (Bauersfeld, 1992; Cobb, 1994), and the genetic epistemologists (Dubinsky, 1989; Sfard, 1989) take epistemological positions which support the view that the learner actively participates in the construction of his/her own knowledge, and this construction builds upon the previous experiences of the individual.

All constructivists generally agree that knowledge is an **adaptive function** (von Glasersfeld, 1991). According to von Glasersfeld:

[Constructivism] has taken seriously the revolutionary attitude pioneered in the 1930s by Jean Piaget, the Swiss founder of cognitive psychology. This attitude is characterized by the deliberate redefinition of the concept of knowledge as an **adaptive function**. In simple words, this means that the results of our cognitive efforts have the purpose of helping us cope in the world of our experience, rather than the traditional goal of furnishing an "objective" representation of a world as it might "exist" apart from us and our experience. (pp. xiv, iv)
It is because of the radical constructivists' refusal to be forced by others into making claims that their "theory of knowing is true" that von Glasersfeld (1990) has coined the phrase "radical" for this particular group of constructivists. According to von Glaserfeld:

To claim that one's theory of knowing is true, in the traditional sense of representing a state or feature of an experiencer-independent world, would be perjury for a radical constructivist. One of the central points of the theory is precisely that this kind of "truth" can never be claimed for the knowledge (or any piece of it) that human reason produces. (p. 19)

Confrey (1995) interprets Piaget as a radical constructivist. She summarizes the program of the radical constructivist as having four planks: genetic epistemology, radical epistemology, scheme theory, and model building and the construction of others.

**Genetic epistemology.** Genetic epistemology is the belief that the "construction of knowledge occurs over time; to understand an idea, one needs to examine its construction, ontogenetically and phylogenetically" (p. 195). Dubinsky (1989), a genetic epistemologist, defines mathematical knowledge as "an interconnected collection of schemas corresponding to individual mathematical concepts" (p. 286). According to Dubinsky, acquiring mathematical knowledge is done through a process of "reflective abstraction," which he defines as consisting of **interiorization** (the formation of an internal process corresponding to some mathematical transformation), **generalization** (assimilation of a new phenomenon to an existing schema), **coordination** (linking together two or more schemas), and **encapsulation** (making a mathematical object out of a cognitive process). Dubinsky believes that "inducing learning" by understanding this process of reflective abstraction becomes a main function of mathematics education.

**Radical epistemology.** The radical constructivist rejects the "picture theory of knowledge (that we are progressing toward an increasingly accurate view of the way things really are)" and believes that "a requirement to know something is to act on it" (Confrey, 1995, p. 195). Acting on it is done through building schemas.

**Scheme theory.** Developing a schema becomes important in an individual's knowing of reality. Piaget referred to these schemas as the "systems of transformations" which a
learner constructs in order to "act on it"; that is, in building a reality or knowing for the individual (Piaget in Confrey, 1995, p. 196). According to Piaget:

These [transformations] are more or less isomorphic to transformations of reality. The transformational structures of which knowledge consists are not copies of the transformations in reality; they are simply possible isomorphic models among which experience can enable us to choose. Knowledge, then, is a system of transformations that become progressively adequate. (p. 196)

Model building and the construction of others. Confrey (1995) identifies model building as an essential part of the radical constructivist's program in which it is believed that "no privileged access is accorded to our knowledge of others" (p. 199). To understand others is to build "viable models of others in our environment through experience" (p. 199).

This brief summary of the four planks of radical constructivism allows us now to look at another branch of constructivism, that of social constructivism. The social constructivists, according to Cobb et al. (1990) find that Piaget's theory "considers only broad areas of intellectual development. His theory therefore constitutes a general orienting framework but leaves much unsaid about the nature of cognitive development in specific conceptual domains" (pp. 125, 126).

Social constructivists (Cobb et al., 1990) believe that Piaget's framework is limited in its interpretation of intellectual development. It is the lack of attention to "specific conceptual domains" in which learning takes place (particularly social interaction) that has prompted social constructivists to be influenced by Vygotsky's "analysis of the crucial role that social interaction plays in learning" (Cobb, 1990, p. 126). There are areas of incompatibility between the theories of Piaget and Vygotsky which this group recognizes--especially around the notion of "internalization" which plays a major role in the theory of Piaget and has no place in Vygotskian theory, but social constructivists have managed to work around this problem of internalization by specifying that social interaction is not a source of processes to be internalized. Instead it is the process by which individuals create interpretations of situations that fit with those of others for the purposes at hand. In doing so, they negotiate and
institutionalize meanings, resolve conflicts, mutually take others' perspectives and, more generally construct consensual domains for coordinated activity. These compatible meanings are continually modified by means of active interpretive processes as individuals attempt to make sense of situations while interacting with others. (Cobb, 1990, p. 127)

Social constructivists believe that social interaction provides a rich source of opportunities to learn mathematics since the "process of constructing mathematical knowledge involves cognitive conflict, reflection, and active cognitive reorganization" (p. 127). Mathematics learning, to the social constructivist, embraces both social interaction and constructive activity.

Both the radical constructivists and the social constructivists have developed research in mathematics learning that supports their respective positions (Cobb et al., 1990; Steffe, 1991; Steffe & Cobb, 1988).

**Learning Mathematics: Cognitive Science**

Resnick (1991), who takes a cognitive science view of learning, agrees with von Glasersfeld (1987) that knowledge is not transmitted passively and that the learner must be actively engaged in his/her own construction process. Resnick believes that individuals construct their knowledge but that most of this construction occurs in a social context. That is, people build their "knowledge structures" by what others tell them through oral, written, and visual processes. As individuals question each other or make points in conversation, those individuals shape this cognitive process. "The idea that individuals' social categories might predict what they know is rooted in the assumption that members of social groups share common knowledge and conceptions about the world" (p. 11). Resnick argues that every cognitive act "must be viewed as a specific response to a specific set of circumstances" (p. 4).

Resnick (1992) believes that learning in mathematics consists of successive layers of mathematics which consist of protoquantities, quantities, numbers, and operators. Thus, at any given moment, a learner can be functioning at several different layers of mathematical thought. Resnick believes that constructivism must consider two hypotheses, which at
times, may seem at odds with each other: the first, the syntax-semantic hypothesis, often ignores or discourages the learner from bringing "developed intuitions" into the learning process when the focus of learning is on formal symbol manipulation; the second, the abstract entities hypothesis, identifies a need for formal mathematics participation that "cannot be directly experienced through the physical world." That is, mathematical competence "on the streets" may do little to prepare the learner for this formal form of mathematics. She believes her theory of learning to be epistemologically grounded and that it "accounts for the differences between everyday and formal mathematics knowledge" and it describes "a set of processes by which informal knowledge is transformed into formal mathematics" (Resnick, 1992, p.374). The position of Resnick and other researchers of cognitive science toward the process of mathematics learning would indicate that a pedagogy which embraces this theory would also be compatible with that of constructivism.

This selection of literature has purposely excluded theories of learning which have come and gone in mathematics education or those which I believe hold little promise for general application. Few of the information processors (Simon, 1978), for example, remain true to their original ideology that the human mind processes information like a computer in a serial fashion and moves practiced and memorized chunks from short term to long term memory. Toulmin's (1972) perspective on conceptual change may explain the often painful process by which researchers abandon their "commitment to one set of conceptual understandings and the adoption of another irreconcilable set" (Geuther, 1986, p. 5).

How students learn mathematics may be tied to their feelings and attitudes about mathematics. Researchers and teachers recognize that students enjoy mathematics during the first two or three years of schooling and then begin to develop a negative shift in their attitudes toward mathematics (McLeod, 1989). McLeod believes that a linking of cognitive
and affective factors to learning is necessary. He feels that research to determine the role of the emotions in learning mathematics is a badly neglected area of research.

There is currently general support for a constructivist theory of learning in the mathematics education field, regardless of whether it is radical or social. A goal of the two Standards documents (NCTM, 1989, 1991) is a sound and significant mathematics program for all students. The writers of these documents believe that learning mathematics occurs through the learner's active participation in his/her own construction of mathematics knowledge. Constructivism provides the theoretical basis for the goal of mathematics for all to be achieved.

Orienting Assumption 1: Learning mathematics is a process by which the learner actively participates in the construction of his/her own knowledge, and this construction builds upon the previous experiences of the individual.

Pedagogy

Glaser and Bassok (1989) indicate that cognitive research during the past quarter century has been focused primarily on the analysis of competence, but two additional areas have been receiving attention during the past few years: (1) analysis of the learner's initial state of knowledge and skills, and (2) the process of instruction by which the learner makes the transition from this initial state to a desired state. A major shift in emphasis during recent years is away from "content" to "process" in describing "what mathematics is and what one hopes students will learn from studying it" (Schoenfeld, 1992, p. 343).

The presence of an underlying learning theory will greatly influence what is selected as content and pedagogy for the curriculum. In spite of the recent research in how students learn mathematics, the prevalent pedagogy in this nation's schools is based in behaviorist principles which encourage the passing of information along to students as if they were intellectual sponges capable of processing this new information at precisely the same rates and in the same ways as that intended by the teacher (Confrey, 1990; Good, Mulryan, & McCaslin, 1992). A "good" behaviorist teacher is one who can break down information
into palatable (and sometimes interesting) tidbits for the sponges to absorb. A study by Palardy (1992) suggests that the three common practices of emphasizing student retention of mathematics facts, discipline, and basic skills instruction may contribute to the overall perceived problem in public education. The practices appear positive in the short term but may become negative throughout a student's academic career.

Howson et al. (1981) identifies five theoretical approaches that impacted on pedagogy which became popularized in the U.S. from about the 1950s onwards: the behaviorist approach, the new-math approach, the structuralist approach, the formative approach, and the integrated-teaching approach.

The behaviorist approach prompted a pedagogy that developed suitable "stimulus-response" programs in which learning outcomes could be "objectivized." Achieving a complex learning objective required locating properly sequenced elementary objectives. Programmed learning and computer-assisted instruction were applications of the behaviorist approach.

The new-math approach dealt with the selection of mathematical content which would emphasize the structure and precision of mathematics. Making mathematical proofs from an axiomatic base became more important than acquiring "trivial computing skills" (p. 101). Because of the emphasis on content, pedagogy was geared to the transmission of knowledge through carefully developed lectures.

The structuralist approach advocates that cognitive structures are combinations of acquired concepts and thinking abilities. There is a progression from simple structures, which are made up of a few concepts, to more elaborate ones "through the addition of new concepts" (p. 108). The highest stage of development consists of all "insights, concepts and procedures of the sciences" (p. 108). The "process character" of learning is more important to the structuralist than the content, and, as a result, "discovery learning" motivates teaching practice. Discovery learning allows the student to behave like a scientist and acquire structures as needed. "The major task of the curriculum developer is to devise
appealing and meaningful teaching models for these processes of discovery as embodiments of the underlying structures" (p. 109).

The **formative** approach is similar to the structuralist's approach, except the emphasis is on devising suitable teaching situations that are based on the concreteness of reality instead of models. This approach derives from Piaget's theory that the growth of mathematical concepts in children are "formed through the internalization of schemata of activity in the manipulation of concrete objects and their abstraction" (in Howson et al., 1981, p. 116). The emphasis of the **Standards** (NCTM, 1989) on making mathematical connections certainly seems to fit this pedagogical format.

The **integrated-teaching** approach employs the same cognitive-theoretical basis as the formative one, but it eliminates the division of learning into subjects of instruction and considers content which is geared to the interests and needs of students. Thus, the "real context of a mathematical idea becomes the subject of the teaching and learning process" (p. 122).

While a structuralist approach may encourage the exploration of mathematics in a scientific way, typical whole-class instruction provides little opportunity for such explorations to occur. Instead of providing students with opportunities to conceptualize mathematical ideas through exploratory problem solving, in typical whole-class instruction, teachers follow a pedagogy that emphasizes "exposure rather than understanding" (Good, et al., 1992). Good et al. believe that valuable time is wasted in most mathematics classes with needless drill, review, and individualized seatwork, and that small-group instruction may be a possible solution to this problem. The emphasis here is on "may," for they realize that small group activity may also result in less coherence of mathematical ideas which may, in turn, create a greater need for review and increased drill. Good and his colleagues conducted a review of hundreds of studies on the relationship between group processes and mathematics instruction. Their conclusion is that the "form of grouping (whole class, two groups, three groups) is less important than the quality of instruction"
They do feel that "increased use of appropriate small-group instruction can make mathematics learning more meaningful" (p. 193).

**Additional research needed.** There is an indication that many of the research needs in mathematics teaching practices of just five or six years ago are being addressed by current research, particularly at the elementary and middle-level grades (Koehler & Grouws, 1992). Koehler and Grouws (1992) believe it is imperative that we look at the secondary level to see whether the theories of learning and teaching currently being discussed "would be viable in a more complex mathematical setting, with older students, and with teachers who have (usually) a more thorough mathematics background" (p. 125). There is a need for research on student assessment, particularly in finding ways of assessing cognitive levels of learning. Teaching practice that develops growth in understanding of mathematics is needed (Hiebert & Carpenter, 1992). For example, a dynamic, multidimensional, multidirectional model to develop the growth of understanding of mathematics, suggested by Kieren and Pirie (1991), might be the basis for further research.

**Orienting Assumption 2: Pedagogy can be changed to improve the mathematical learning opportunities of students.**

**Combining Teaching and Learning in Research**

Koehler and Grouws (1992) reviewed different types of research programs that combined research on learning and research on teaching. The three types of research which are of interest here include the constructivist approach, cognitively guided instruction, and the epistemological view.

**Constructivist approach.** The Purdue Problem-Centered Mathematics Project (Wood, Cobb, & Yackel, 1993) has a research focus using project-developed student instructional activities which are designed to facilitate a problem-centered approach to teaching and learning. Based on social constructivism, the teacher's role is that of a facilitator for the students' construction of knowledge. Social interactions are important in the process.
where "teaching is viewed on a continuum between negotiation and imposition" (Koehler and Grouws, 1992, p. 123).

Cognitively Guided Instruction (CGI). The underlying philosophy of CGI is that pedagogy should encourage teachers "to make instructional decisions based on knowledge from cognitive science about how students learn particular content" (Koehler and Grouws, 1992, p. 119). This research includes a close monitoring of teachers' behavior in the classroom after teachers have been given information about how students learn a particular mathematics topic. The CGI paradigm "assigns a central role to teachers' and students' thinking" (Carpenter & Fennema, 1988, p. 7).

The Cognitive Instructional Approach is a similar type of research to CGI in which "studying learning to inform teaching" becomes central to the research (Koehler and Grouws, 1992, p. 122). Koehler and Grouws (1992) reported that a study by Hiebert and Wearne revealed that students in grades 4-6 showed significant gains in understanding the addition and subtraction of decimal fractions using that methodology.

Epistemological view. Learning mathematics the way it was developed by mathematicians becomes the basis for the underlying theory of the epistemological view of teaching and learning mathematics. The teacher's role in this research is one of "helping students construct knowledge in the discipline through problem posing and engaging students in mathematical discourse so that they might examine their own assumptions about mathematics" (Koehler and Grouws, 1992, p. 123). This methodology involves the researcher as "teacher-scholar."

Regardless of which cognitive theory drives instruction, proponents of each theory share a common goal of wanting to bring about the most effective student learning. The process by which each plans for instruction may vary however. Confrey (1990) deals first with an analysis of direct instruction, (i.e., what has come to be known as the traditional approach to teaching: "an introductory review, a development portion, a controlled
transition to seatwork and a period of individual seatwork" (p. 107). In contrast, constructivism provides for a model of instruction which consists of six components:

- the promotion of student autonomy, the development of reflective processes, the construction of case histories, the identification and negotiation of tentative solution paths, the retracing and group discussion of the paths, and the adherence to the intent of the materials. (p. 107)

Confrey believes that in order for a teacher to apply constructivism to teaching, it is necessary to "reject the assumption that one can simply pass on information to a set of learners and expect that understanding will result" (p. 109).

Steffe (1991) discusses six requirements for the constructivist mathematics educator:

1. using the mathematics of children as the basis on which to teach mathematics;
2. determining the mathematics for children through interactive communication;
3. interpreting children's mathematical activity in learning environments through interactive communication;
4. taking assimilation as the fundamental relation involved in learning and learning as consisting in the modifications of schemas;
5. treating operative mathematical concepts as constructed by children as a result of goal-directed activity in learning environments; and
6. taking the responsibility for learning children's mathematical knowledge.

It is Steffe's contention that until teachers "learn the mathematical knowledge of the involved children and how they construct it" (p.178), teachers will be ill equipped to provide meaningful learning experiences in mathematics.

Cognitive science has researched ways in which students learn particular strands of mathematics. This research is then used to influence pedagogy with the teacher as decision maker for the instructional practice. Cognitively Guided Instruction and the Cognitive Instructional Approach are examples of this type of research. Constructivism, on the other hand, looks at how individual children are constructing their own mathematical knowledge in an attempt to influence pedagogy.
Orienting Assumption 3: Mathematics teaching and learning should be interwoven with each other in order that learning experiences provided for students fit what is known about how students learn mathematics.

Complexity of Change in Mathematics Teaching and Learning

Sarason (1991) cautioned that "to confuse change with progress is to confuse means with ends" (p. 8). Effecting change is a complex process which those committed to change may not fully be aware of (Fullan, 1991). In fact, Fullan said there is often an inverse relationship between "commitment to what should be changed" and the "knowledge about how to work through the process of change" (p. 95). In a later book, Fullan (1993) offers eight import dictums or lessons that he calls the "new paradigm of change." There does seem to be some agreement between Sarason and Fullan that for significant change to occur in schools, it is important that those who would effect such change first develop an understanding of the school community or, as Lieberman (1992) indicates, a "conceptual knowledge about teachers, school cultures, and the process of change" (p. 11).

Fullan (1993) makes the distinction between "detailed complexity" and "dynamic complexity." "The former involves identifying all the variables that could influence a problem," according to Fullan, but he recognizes that "detailed complexity is not reality" (p. 20). It is by considering dynamic complexity that we come to realize that "obvious interventions" do not produce the "expected outcomes" and unexpected, "unplanned factors dynamically interfere" (p. 20). Fullan believes that change is dynamically complex and that change does not follow a linear progression. Thus, the process of change cannot be predicted with any certainty.

Several studies which have sought to document change includes those of Driscoll (1987); Stake and Easley (1978); Wiske, Levinson, Schlichtman, and Stroup (1992); and, currently, the R³M project. The literature on change in mathematics teaching and learning suggests a number of places in which change must occur. The curriculum (pedagogy as well as content), teacher beliefs, and the attitudes of the community are three
important places in which change should occur. That is, it becomes important to consider the impact of change on the school community, including the dilemmas and constraints to reform efforts.

**Curriculum.** Will social constructivists be able to bring about the "radical" change in the school mathematics curriculum that seems necessary? Romberg (1992) suggests this change will be impossible unless there is a curriculum theory "to interpret the vast quantities of information now being gathered" as a result of the shift occurring from absolutist to social constructivist positions. According to Romberg (1992):

Theory in scientific epistemology refers to statements about causal relationships between variables. Such statements are produced by persons about complex phenomena in the world to explain their puzzling aspects. However theories differ from most causal statements (speculations, hypotheses, etc.) in two ways. First, there is a group of persons who finds the causal statements useful; and second, they have been put to test several times and have survived. It is the "testing" portion that is missing at the present time. ... [T]here is no scholarly group with an agreed-upon ideology, syntax, and vocabulary ready to build, model, and test these phenomena in light of the philosophic shift so that a theory might emerge. (p. 781)

Romberg indicates that curriculum theories have been developed during the past half century but they are "static rather than dynamic" and are inadequate to deal with the changes suggested by the social constructivists. He cites five ingredients which he believes necessary for an adequate theory: (1) there must be a scholarly group with common interests; (2) the common "interest of the group is in relationship to some real world phenomena"; (3) a set of statements must exist which give explanations of the theory (e.g., conjectures about relations or about causality) so that "predictions can be made about the phenomena"; (4) "the statements use an agreed-upon vocabulary and syntax; and (5) the statements are tested by gathering evidence using agreed-upon procedures to verify, falsify, or, with respect to predictions, either confirm or reject the propositions" (p. 781).

In a pluralistic society such as the U.S., schools are always in the middle of competing ideologies over what should be taught in schools and for what purpose. Eisner
(1992) believes that ideologies do not translate to school practice without going through a political process first.

There is a political process that inevitably must be employed to move from ideological commitment to practical action. When a society is characterized by value plurality and when the political strength of groups is comparable, the process almost always leads to certain compromises. As a result, the public school curriculum seldom reflects a pure form of any single ideological position. (p. 304)

Eisner sees the necessity for an ideology being capable of modification in order to survive, "just as a tightrope walker must correct for movement in the wire to remain on it" (p. 305).

While I have indicated that the Standards documents of NCTM (1989; 1991) are characterized by a constructivist perspective, there is more to the Standards than just a learning theory. The evidence of an ideology or belief system is apparent from such phrases as: "transmit aspects of the culture to the young," "provide [students] with an opportunity for self-fulfillment," "mathematically literate workers," "lifelong learning," "opportunity for all," "informed electorate," and others.

The importance of using more than one ideology "may be among the important changes that can be made in the field of education" (Eisner, 1992, p. 319). Recognizing that several ideologies may need to be woven into changes in mathematics education can influence and give direction to research activities in the field.

Changing Teacher Beliefs. The importance of teacher beliefs has been the focus of an abundance of research literature. Underhill (1988) did a review of the literature on four categories of teachers' beliefs: teachers' beliefs about mathematics, mathematics learning, mathematics teaching, and some other important teacher beliefs. Thompson (1992) is among a group of researchers who believes that classroom practices are dictated by teachers' beliefs in what is most essential. Romberg (1988a) argues that "The most important barriers to reform are the beliefs, attitudes, and expectations strongly held by all persons involved in education in relation to specific aspects of reform" (p. 35). He believes that reform efforts in mathematics teaching and learning directly challenge long
held perceptions of many people involved in mathematics instruction. Romberg (1988b) reasons that one of the biggest obstacles to change is the belief system of those "insiders" who would effect such change. A change must first occur in the beliefs of would-be changers before actual reform can occur (Anderson, Anderson, & Romagnano, 1993; Romberg, 1988b).

Anderson, Anderson, and Romagnano (1993) indicate that the beliefs of teachers, administrators, members of the community, and even the reformers are among the barriers to reform. The belief system of teachers is an often neglected component of this complex process of educational reform. How teachers perceive mathematics reform in their schools is influenced by what teachers believe to be good instructional practice; those beliefs will, in turn, determine their involvement in the change process. Fullan (1991) said that, "Situations vary, and we never fully know what implementation is or should look like until people in particular situations attempt to spell it out through use. Implementation makes further policy; it does not simply put predefined policy into practice" (p.92).

This review of the literature revealed little quantitative research on teachers' beliefs in mathematics education. A recent study was done for NCTM (Weiss, 1992) in order to provide baseline data for the R3M project. Teachers of mathematics in grades K-12 from 121 schools in eleven states across the country were surveyed about their attitudes toward teaching, their instructional practices, and their knowledge of the Standards. In addition, there were telephone interviews with elementary and secondary teachers, college and university faculty, consultants, district curriculum specialists, and directors of mathematics-education projects. Weiss' study indicated that teachers generally were not very aware of the Standards, especially at the elementary level. The study also revealed that a very traditional pedagogy still persisted in most mathematics classes. A National Science Foundation survey was recently completed by Weiss (1994) which supported her earlier findings. A survey by Peterson (1991) had similar findings. Peterson found that most elementary teachers employed a softened version of drill-and-practice in their
classrooms. While these results are discouraging to those who are promoting reform of mathematics education, there are teachers from both surveys who have indicated that their teaching practice has changed.

Other research studies on teacher beliefs exist outside of mathematics. One such study by Fuller and Izu (1986) cuts across all subject-matter disciplines. The survey examined teachers' beliefs regarding teaching philosophy, categorical aid programs, feelings about job satisfaction and efficacy, and perceptions of the principal's leadership skills. Questionnaire responses showed that teachers' beliefs were sensitive to environmental and internal school factors, including heterogeneity of students, size of administrative staff, and the community's social class.

Shavelson and Stern (1981) did an extensive review of research literature on teachers' beliefs. Teachers' beliefs encompass subject matter, the students in their classes, and how they plan for instruction. These beliefs influence how teachers make quick decisions many times during classroom practice.

Prawat (1992) discussed teachers' beliefs that prevent or impede their adoption of a constructivist view of teaching and learning. These impediments included: a view of the learner and the curriculum as separated, instead of in interaction; the notion that student interest and involvement is requisite for worthwhile learning to occur; the belief in a distinction between comprehension and application, including the notions that learning is hierarchical and that generalization leads to transfer; and the view that the curriculum is a fixed agenda. Because of the social context of constructivist teaching and learning, change is particularly difficult since the focus is on ideas and not skills or strategies. Negotiation is a difficult change in a teacher's belief system but a requirement in the constructivist's classroom (Tinto, 1990; Wiske et al., 1992).

Brown and Gray (1992) reported on a correlational study to determine whether teachers' mathematics beliefs (anxiety) might inhibit the introduction of more problem solving and abstraction in elementary schools, thus preventing more ninth graders to enroll
in algebra. The study indicated that anxiety decreased with increased mathematics content studied by teachers and the increased age of teachers.

Newmann (1991) indicated one of the most prominent themes in restructuring the power base of the schools is through decentralization of the decision-making process and empowering parents, teachers, principals, and students. Benson and Malone (1987) studied the relationships between teachers' perceptions of their status in making decisions in their work and the deprivation they felt in their work when not allowed to make decisions. Sarason (1991) and Fullan (1993) both discussed the importance of teachers and others in the school community sharing in the decision-making process.

The importance of teacher beliefs cannot be over-emphasized. Those interested in the reform of mathematics education are interested in finding answers to important questions about teacher beliefs. What are the relationships between practitioners' beliefs and practice? Are there differences in belief systems of practitioners who are involved in mathematics education reform and those who are not? Can significant reform in mathematics education occur without this prior change in beliefs? A different view on teacher beliefs and teaching practice is that beliefs and practice are interactive (Ferrini-Mundy, 1994). That is, changed practice can influence beliefs, and vice versa—each feeding off the other. The National Science Foundation and other funding agencies are currently showing great interest in projects for teacher enhancement. An expected outgrowth of such projects is changing teachers' belief systems about teaching mathematics through this interactive process.

**Impact of Change on the School Community.** The organizational structure of schools pose additional obstacles to change (Fullan, 1991; Fullan, 1993; Romberg, 1988a; Sarason, 1991). Sarason (1991) indicated the dangers to reform that may be imposed by those who are not informed about the complexity of the school system, and he cautioned that being part of the system is no guarantee that one understands the system in any comprehensive way. Both Sarason and Fullan argue that effective reform must not be
piecemeal but should include changing the complex system of the school. The changer must know the context in which intervention is to take place.

There is widespread dissatisfaction with public education in this country. Lay "experts" seem to be coming in all directions with their solutions for solving the vast array of problems facing schools, and educators can ill afford to ignore these public outcries of dissatisfaction. The recent rejection of educational reform efforts by the voters in Connecticut (Immerwahr, 1994) shows the power of organized opposition.

**Dilemmas and constraints to reform efforts.** Efforts to reform mathematics education are usually accompanied by constraining influences and dilemmas which may slow down or even curtail the change process. Interpretation of reform will vary according to the interpreter (Ball, 1990). This becomes problematic if the teacher believes she/he is changing teaching practice when others see instead a very traditional practice (Ball, 1990; Cohen, 1990; Cohen & Ball, 1990; Johnson & Ferrini-Mundy, in press).

The Standards recommend that teachers develop learning experiences for students by using concrete models, calculators, and computers in their classrooms, but the presence of the items in a classroom does not guarantee that they will benefit learning. Ball (1992) found that in her class of third graders her students do not draw the same conclusions that teachers do when using concrete objects. Cohen and Ball (1990) did case studies of five teachers which revealed that teachers are both the problem (superficial and mechanical instruction) and the solution (those with an understanding of new mathematics) for such a massive reform effort. The very nature of the Standards documents allows for multiple interpretations and raises issues of what it means to align local practice with the intent of the Standards (Schoen & Ferrini-Mundy, 1991).

The lack of authentic assessment is seen by Boyer (1990) as a major constraint for mathematics reform to take place. Dossey (1990) indicates that an important role of assessment is to the improvement of teaching, and this explains the growth of assessment activities now occurring in mathematics education. Problematic for teachers is the impact
of mandated testing. Some of the resistance to change, particularly in schools with large minority populations, results from the importance placed on the mandated test results (Madaus, 1992). NCTM recognizes the importance of providing direction to teachers on this important component of change. NCTM's third Standards document, Assessment Standards, is currently in the final stages of draft revision (NCTM, 1993).

External funding is no guarantee that reform efforts will succeed. Teachers may be looking for a quick fix to curricular problems in mathematics. The lack of understanding among mathematics teachers about the complexity of the change process and their focus on superficial change was the cause for the failure of the Cleveland Math Collaborative according to Bruckerhoff (1990). Other factors which contributed to the demise of the collaborative were the traditional systematic programming of courses, traditional teaching and learning conceptions, and poor attendance at meetings planned for the would-be change agents (Bruckerhoff, 1991).

The literature and research on change indicate that there are many variables involved in the process of change. One of the more important prerequisites for changing mathematics learning and instruction is an understanding of the complexity of change by all of the stakeholders in the process of change.

Orienting Assumption 4: The processes for bringing about change in mathematics learning and instruction are complex due to the many (oftentimes unpredictable) factors and people involved. An understanding of this complexity by all of the key players is an important step in the change process.

Orienting Assumptions

Based on my review of the research, there are four ideas that have emerged to organize and direct my thinking in this study--the orienting assumptions which influence how I personally engaged in the collecting, analyzing, and interpreting of data:
(1) Learning mathematics is a process by which the learner actively participates in the construction of his/her own knowledge, and this construction builds upon the previous mathematical experiences of the individual.

(2) Pedagogy can be changed to improve the mathematical learning opportunities of students.

(3) Mathematics teaching and learning should be interwoven with each other in order that learning experiences provided for students fit what is known about how students learn mathematics.

(4) The processes for bringing about change in mathematics learning and instruction are complex due to the many factors and people involved. An understanding of this complexity by all of the key players is an important step in the change process. Later, I will discuss how these assumptions affected decisions with regard to the methodology and my interpretation of the analyses across the five cases of this study.
Chapter Footnotes

10 "Curriculum" is used here to mean "an operational plan for instruction that details what mathematicians need to know, how students are to achieve the identified curricular goals, what teachers are to do to help students develop their mathematical knowledge, and the context in which learning and teaching occur" (NCTM, 1989, p. 1). Curricular as used here refers to mathematical content and pedagogy.

11 Technically speaking the "new-math" approach is not an approach; it is an emphasis on a particular content. The pedagogical approach during the new-math era was still based on behaviorism, though there were some exceptions such as that of the Madison Project (Davis, 1964) which was a very constructive curriculum.

12 Again, Howson et al. (1981) categorize different emphases on content as different approaches. The structuralist approach encompasses what the authors refer to as the formative approach and the integrated-teaching approach. The differentiated core curriculum which has been modeled in NCTM's Addenda Series: A Core Curriculum (Hirsch, 1992) illustrates how the integrated-teaching model might be accomplished. An example of a currently operating program is the Interactive Mathematics Project (The Interactive Mathematics Project, 1993) which has developed a high school mathematics program in which students work cooperatively in groups on five or six problem-solving units each year.
CHAPTER 3

METHODOLOGY

Related Research

Since one of the purposes for this study is to develop a model of doing cross-case analyses, it seems appropriate that the review of the literature and related research connected with the methodology should be included in the chapter. It is the purpose of this review to look at two areas of related research: (1) research on the reform of mathematics education, and (2) research which utilizes cross-case analyses to reduce and interpret data.

Research on the Reform of Mathematics Education

Lane and Silver (1994) suggest there is no shortage of reform efforts in mathematics education, but there is a shortage of data regarding "their efficacy and impact" (p. 2). In any attempt at analysis of data that may give us an understanding of how change is occurring in various school sites, there is the reality that change is not some fixed goal at which the change agents finally arrive; it is rather something that they continue to do. As Prestine and Bowen (1993) suggest, "[A]ny assessment of an ongoing change process can inherently represent only a partial understanding of the whole" (p. 317). No single piece of qualitative research (or quantitative for that matter) will provide the substantive understanding of the change process which would generalize to all sites seeking change. Each research contributes to the "cumulative" effect to which Silver (1990) alludes.

I have found that research in mathematics education falls into several broad categories as suggested below, and this list does not attempt to include all research activities taking place in the reform of mathematics education.

Developmental/research studies. Developmental/research projects have as their main purpose the development of instructional strategies and materials in an attempt to improve student understanding and learning of mathematics. Since research is also an important
part of such projects, these projects often develop their own assessment instruments as part
of that research. For many of these studies, data for the research component is still being
collected or is in the early stages of analysis. Few of the projects have produced completed
analyses of research data.

One such program is the Purdue Problem-Centered Mathematics (PPCM) project, a
problem-centered approach to learning mathematics (Cobb, Wood, Yackel, & Nicholls,
1991). Research-based cognitive models guided the development of the activities, and
there is assessment data which shows significant growth among second graders in
conceptual understanding and the ability to apply understanding to mathematical
relationships (Wood, Cobb, & Yackel, 1993).

Some cross-case studies are beginning to emerge from the Quantitative Understanding:
Amplifying Student Achievement and Reasoning (QUASAR) project (Brown & Smith,
1994; Lane & Silver, 1994; Silver, 1994; Stein, Grover, & Henningsen, 1994). Extensive
data collection instruments are used in this multi-year demonstration and research study.
This project began in 1989 to improve the mathematical thinking and reasoning of students
attending middle schools in six economically disadvantaged communities scattered around
the country, and, greatly influenced by the two Standards documents, emphasizes
thinking, reasoning, and problem solving in mathematics learning and pedagogy.

QUASAR employs a variety of alternative assessments. One assessment instrument is
a prototype which has been developed to reflect the evaluation criteria of the Standards
(Lane & Silver, 1994). This instrument, the QUASAR Cognitive Assessment Instrument
(QCAI), is used to measure students' mathematical thinking and reasoning. Results from
student assessment using the QCAI during the first three years of QUASAR indicate that
students involved in the project are showing gains in mathematical thinking and reasoning
(Lane & Silver, 1994).

Another middle-grades project is the Connected Mathematics Project (CMP) which is a
five-year project to develop a complete mathematics curriculum for grades 6-8 that follows
the Standards (Connected Mathematics Project, 1993). This NSF funded project seeks to develop student mathematical knowledge that is rich in connections. Mathematical content is organized into three to eight-week units that "investigate important mathematical ideas" (p. 1). Each unit consists of six to nine investigations. Student writing and group activity are a big part of the program.

The Interactive Mathematics Program (IMP) is a four-year, problem-based mathematics program for high school students (Interactive Mathematics Program, 1993). Students are presented with a central problem or unit which require four to eight weeks to complete. The program allows for heterogeneity in the classroom and enables all students, both college bound and non-college bound students access to the core of mathematics at the high-school level. Originally funded by the California Postsecondary Education Commission (CPEC), it has been awarded an additional five-years of funding by NSF for the purpose of national dissemination. Formal, long-range evaluation of IMP will be conducted by the Center for Educational Research, University of Wisconsin.

Teacher enhancement projects. The intent of the Exxon K-3 Specialist project is to determine whether changing the way mathematics is taught at the early grades would affect children's understanding of and attitude toward mathematics beyond third grade. Each site participating in the project is required to develop K-3 mathematics specialists who are encouraged to follow the guidelines of the two Standards documents. Another feature of the project is to develop a parent component. Additionally, each site is expected to show evidence of continued local support.

Several research projects promote teacher change. The Atlanta Mathematics Project (AMP) (Hart, Schultz, Najee-Ullah, & Nash, 1991) and the Mathematics for Tomorrow Project (Nelson, 1993) are examples of teacher enhancement and research projects designed to strengthen teachers' mathematical understanding and change pedagogy through reflective practice.
Hart (1993) explains that the research component of the AMP deals with trying to understand how teachers change their practice. The researcher's role is not a passive one; rather, researchers engage in participatory action research and actively try to influence the participants. The research of the project assumes a "naturalistic" (constructivistic) paradigm and adheres to an inductive analysis of data which provides for the development of theories grounded from a particular setting.

Documentive research. There are other documentive studies in mathematics education besides the R3M project. For example, Wiske et al. (1992) conducted a documentive study entitled, Implementing the Standards in High School Geometry, which sought to clarify how the shifts in the way students learn geometry and the changes in pedagogical practices can be "aided through appropriate policies and supports" (p.1). Researchers conducted fifty telephone interviews with geometry teachers in a variety of high schools who had been pioneers in changing their teaching practice to more closely align with the NCTM Standards. From the interviews, three composite case studies were prepared and epitomized through three "representative teachers." These thematic case studies include commentary which relates the themes to those raised by the entire sample.

The researchers found that changes in pedagogy required a renegotiation of intellectual authority in the classroom. Many teachers had received "hands on" development and follow-up support for their changed teaching practice. They also found that changing teacher beliefs was easier than changing classroom practice. Factors which were needed to effectively change practice included: common planning time to develop new lessons and adapt materials was important; students need time to acclimate to this changed pedagogical practice; restructuring of the organization and management of the school was often required to bring about effective reform in mathematics education, which in some instances resulted in additional demands being placed on teachers' time; and standardized assessment practices often served to constrain changes in curriculum content and practice.
Use of Cross-Case Analyses

Firestone (1993) believes that case-to-case transfer can occur only when there is "thick description" provided in the individual case studies. This view is also held by Yin (1989) as he describes "explanation-building" in multiple-case studies. An initial theoretical statement, according to Yin, undergoes a change when examined against the findings of a particular case study. The revised theoretical statement is further subjected to other details of the case, and further revision of the theoretical statement occurs. Yin explains that this process may go on as many times as needed in shaping the explanation (theoretical statement) for a particular case.

Generalizability may be an important aim in the decision to do cross-case analyses, but there is a risk in simply identifying those steps which may be required in the change process. Researchers and would-be change agents may see particular schools as being the "exception" to such criteria. It is also important to understand how the events and processes are "qualified by local conditions," allowing the researcher "to develop more sophisticated descriptions and more powerful explanations" (Miles & Huberman, 1994, p. 172). A goal for the cross-case researcher is to put the factor of believability into any generalizable results.

Explanation building in multiple-case studies follows this iterative process "to build a general explanation that fits each of the individual cases, even though the cases will vary in their details" (Yin, 1989, p. 114). Original hypotheses can be strengthened, modified, or yield to new ones during the fieldwork (Glaser & Strauss, 1970), and field researchers code or categorize data--implicitly or explicitly--to verify these hypotheses or explanations of cases. Miles and Huberman (1994) support an explicit coding of data which is loosely formulated in the early stages (unordered meta-matrices) and which is later clustered by common explanations (ordered meta-matrices) as the case studies begin to emerge. Goetz and LeCompte (1984) also support a grounded approach which avoids a precise pre-planning strategy and allows for searching of data to explore hypotheses of the researcher.
A review of literature shows a lack of cross-case studies in the reform of mathematics education, though some are starting to emerge. The Journal for Research in Mathematics Education compiles a somewhat exhaustive list of journal and dissertation research in mathematics education in its July issue each year. There have been no studies cited in that journal for the past four years (1990-1993) that use "cross-case" or "cross-site" in their titles. However, the geometry study by Wiske (1992) is an example of a cross-case study. Wiske and her colleagues developed a meta-matrix by which they analyzed the relationships between variables.

The QUASAR study will provide a large-scale cross-case study when its data analysis has been completed. In fact, multiple-case analyses will be available from most of the research projects in mathematics education already cited. The Exxon K-3 Specialists Evaluation Project, for example, is just beginning to share its cross-case analyses through preliminary reports.

There are an increasing number of educational cross-case studies which relate to the educational change process and to policy analysis. Pristine and Bowen (1993) did a cross-case study on the restructuring efforts taking place in four schools which belong to the Coalition of Essential Schools. The intent of this study was evaluative in the sense that the primary focus of the study was to determine progress toward restructuring using the Coalition's benchmarks of change. The researchers found that these benchmarks of change provided only a "partial understanding" of the change process, and it was necessary to modify the conceptual framework for the study.

Pristine and Bowen (1993) conducted site visits and collected data through intensive open-ended interviews and follow-up interviews. Their study extended over a three-year period and included 48 to 57 individuals who were interviewed at each site. To compensate for the partial understanding obtained using the Coalition's benchmarks, the researchers found it necessary to do a second cross-case analysis in which emerging themes from the data were used for comparative purposes. The authors identified six such
themes which emerged from their data which they appended to the benchmarks criteria to enable a more comprehensive analysis of data.

Mills (1992) used the cross-case analysis of data to do an in-depth study of the role of school personnel in participative decision making. He developed case studies of seven schools (five elementary schools, one middle school, one high school) and used a variety of data collection techniques, including non-participant observation, interviews, written sources of data (including a questionnaire), and non-written sources of data which extended over a six-month period. He used a site-ordered descriptive matrix (Miles & Huberman, 1984) to determine what was going on across sites. For example, Mills (1992) was able to order sites in the matrix according to such variables as "authority and influence over site-based decisions in terms of; principal's expectations, teachers' expectations, forms of participation, and authority for decision making" (p. 9).

Advantages of Using Cross-Case Analyses to Extend the R³M Research

There is a need for documentive research data on how changes are taking place in mathematics programs at the high school level. As a methodology, the use of cross-case analyses, as adapted to the R³M project, holds promise for providing a depth of understanding not always available in individual case studies and comparative reporting of similar studies.

The rationale for extending the analysis of data in the R³M project was explained in Chapter 1. The methodology of cross-case analyses as explained by Miles and Huberman (1994) has a number of advantages which include: generalizability of qualitative research, gaining a more in-depth understanding of the data, providing a means of handling extended text, allowing additional theories to emerge, providing opportunity for modifying and verifying conclusions, providing other opportunities to apply the model, providing supportive evidence for change, and providing for a greater triangulation of methodology. Each of these advantages is discussed is discussed below.
Generalizability

Increasing generalizability is one aim in studying multiple cases, and cross-case analyses can provide assurance "that the events and processes in one well-described setting are not wholly idiosyncratic" (Miles & Huberman, 1994, p. 172).

Understanding the Data

An even more important reason for developing a model for cross-case analyses is the process of discovering relations between characteristics or variables that might otherwise go unnoticed. This, in turn, permits a broader and deeper "understanding and explanation" of the data (Miles & Huberman, 1994).

A Way of Handling "Extended" Text

The analysis of data often requires that much important data is eliminated simply because humans have limited information-processing capabilities. Miles and Huberman (1994) suggest that the human tendency is "to reduce complex information . . . into easily understood configurations" (p. 11) which may result in inaccurate portrayals of data. They warn, too, of researchers' tendencies to focus on and give too much emphasis to "vivid" or "exciting" events as they reduce data prior to their reporting of results. Miles and Huberman explain what is often quite typical in data reduction:

We drastically overweight vivid information, such as the exciting event that jumps out of page 124 of the field notes after a long, "boring" passage. Pages 109 through 123 may suddenly have been collapsed, and the criteria for weighting and selecting may never be questioned. (p. 11)

Extended text, while not as exciting, can be reported in ways other than narrative form (resulting in countless pages of field notes) in cross-case analyses using matrices, graphs, charts, and networks.

Emergence of Additional Theories.

The cross-case methodology allows for a "grounding" of theory in which the data becomes the basis for additional theory building (Glaser & Strauss, 1970). Miles and
Huberman (1994) describe theory building through ethnographic methods which have been extended by work in grounded theory:

[The analytic sequence] moves from one inductive inference to another by selectively collecting data, comparing and contrasting this material in the quest for patterns or regularities, seeking out more data to support or qualify these emerging clusters, and then gradually drawing inferences from the links between other new data segments and the cumulative set of conceptualizations. (p. 14)

Listening carefully to what the data says allows the researcher to inductively build relational patterns from the ground up. This process sometimes provides justification for modification of existing theoretical constructs or the development of additional theories.

Modifying and Verifying conclusions

Researchers organize data collection techniques based on preconceived conclusions which they hold. Experienced researchers, according to Miles and Huberman (1984), hold these conclusions lightly, and wait for "final" conclusions until the data analysis is over. Use of cross-case analysis permits a more thorough verification of conclusions than is afforded by a single-case study.

Other Applications of the Model

Subjecting data to the further scrutiny of cross-case analysis provides another avenue for the triangulation of data (Denzin, 1978). While the focus of the research will be on reform efforts in mathematics, the local variations on the change processes for mathematics programs may also have implications for other subject areas as well (Eisenhart, 1988; Prestine & Bowen, 1993).

Potential for Providing Supportive Evidence for Change

Sarason (1991) and Fullan (1991, 1993) address the complexities involved in the change process. Sarason, in particular, is not optimistic that educational change is possible in the present educational climate of our schools. A change that is produced in one place (e.g., mathematics education) will impact on other areas of the school which have a natural

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inclination to resist the change. Reform efforts usually become "ameliorative" in the sense that they placate the change agents in some temporary fashion before returning, more strongly entrenched, to the prior status quo (Goodman, 1992). Silver (1990) also cautions those who would use research to impact teaching practice not to expect too much from the research. He advises educators that the "widespread belief" that "ultimate" answers to our questions about educational practice is unrealistic. Answering the question, "How is significant change effected in a school community?", becomes as important as the change itself.

Trying to understand those cases where change is persistent and viable may provide input to a deeper discussion of the tensions involved in the change process in mathematics education. The investigation of data across multiple cases has potential for providing some supportive evidence for effecting significant change.

**Triangulation of Methodology**

Are there ways in which the analysis of data across cases will allow the research community to benefit from a deeper, more expansive, investigation of data? Qualitative researchers (Denzin, 1978; Fetterman, 1989; Wolcott, 1990) indicate the need for triangulation in the research process. One of the basic types of triangulation is methodological triangulation (Denzin, 1978) by which multiple methods are utilized to verify conclusions that are drawn from the data. In an effort to increase the interpretive validity (Mills & Schram, 1994) of the R³M study, the cross-case study proposed in this research is another form of documentive analysis.

**Description of the Methodology**

**Data Gathering**

As mentioned earlier, data collection instruments used by the documenters in the R³M project included: interview guides for teachers, administrators, supervisors, and students; classroom observation forms; and the collection of pertinent data from the sites (e.g., curriculum guide, samples of mathematical tasks, demographic data, etc.). After each site
visit, the pair of visiting documenters met to develop a preliminary summary of their data. The format for that summary was governed by the orienting information mentioned in Chapter 1. Each of the six types of information is described below:

**Mathematical Vision.** A number of questions guided interviews and classroom observations to determine the mathematical vision of each site. We were interested in whether each school's mathematics department had goals for its program and if those goals played an active role in actual classroom practice. We wanted to know the kinds of mathematics learning teachers at the site hoped their students would experience. We wanted to know what the people in the site saw as worthwhile mathematical tasks, or important mathematical ideas. We wanted to find out if alignment among teachers and administrators existed around this vision. We were interested in whether the mathematics program emphasized problem solving, communication, reasoning, conjecturing, and mathematical connections. Can the classrooms be described as mathematical communities? Why? We were looking for specific evidence that their mathematical vision was being brought to life in the classroom.

**Pedagogical Vision.** Documenters were asked to describe the pedagogical vision held, relative to mathematics, by the people in the site. That required documenters to look at the pedagogical philosophy being articulated in the site and describe any observations which illustrated that the pedagogical vision was being brought to life in the classroom. They were looking for the approaches, strategies, and ways of teaching mathematics that those at the site valued. Documenters were asked to determine and describe the level of alignment that existed among teachers and administrators around that vision.

**Contextual Features.** Fullan (1993) discusses the need to examine the connection which change will have to the "wider environment" and the importance of realizing that "the best organizations learn externally as well as internally" (p. 21). Documenters were curious about the contextual features that influenced the changes taking place. They wanted to know what was happening with inservice, with outside consultants, with outside
funding, with the school and district administration, with the community, among the teachers; what was the role of materials, scheduling and structural matters, and policies about evaluation and testing. Documenters were attempting to describe how contextual features were influencing, both positively and negatively, the teachers' efforts to change their mathematics practice (Ferrini-Mundy, 1992a).

**Influence on Students.** We wanted to find out if the mathematics program was impacting on students in a positive way. We wanted to know the kind of mathematics which students were learning and if teachers perceived their classroom practice to be effective. We were anxious to discover examples of mathematical discourse between teacher and student and between student and student. We were interested in teachers' perceptions of students and whether the program had been effective in developing students' ability to reason and solve mathematics problems.

**Evolution of the Mathematics Reform.** We were interested in how the mathematics program had gotten to be the way it is and in what direction the mathematics teachers and administration want it to go with their program.

**Influence of the Standards.** We felt it important to determine what features of the Standards emerged as teachers described their mathematical and pedagogical visions and whether the Standards were serving as a catalyst for reform or as validation of that reform. We were interested in the local site peoples' familiarity and interpretation of the Standards.

**The Cases Comprising This Study**

In the R^3^M project, sites for the documentation study "might be classrooms, school buildings, or even school districts, where significant change in mathematics teaching and learning is occurring" (Ferrini-Mundy, 1992a). The R^3^M project contacted practitioners, researchers, and policy makers in mathematics education for school sites. From the 350 letters which were mailed, we received 190 recommendations for school sites where the reform efforts toward implementing the NCTM Standards were perceived to be in progress. Each nominated site was contacted by letter, and we received 76 completed
Preliminary Information Questionnaires. Five documenters reviewed those questionnaires and narrowed that number to twenty-six. Three of the documenters conducted telephone interviews with the 26 sites. From those interviews and the Preliminary Information Questionnaires, Advisory Board members made the final selection of the twelve sites that comprised the first round of our documentation effort.

The second round of five sites were solicited sites. We felt we needed additional urban sites in the study. We were also trying to identify one or more sites that were addressing the issue of "algebra for all" in their reform efforts. We wanted our study to have a balanced geographic distribution, so we attempted to combine these three needs in the selection of the additional sites.

Five of the seventeen sites in the R^3M documentation project were high-school schools, and these five schools become the cases for this study. As explained before, the lack of descriptive research at the secondary level prompted this study. Those five schools, pseudonymed for this study, are briefly described in terms of their demographics:

East Collins High School. Once a farming community, East Collins is an easy commute to a large metropolitan area. It's student population comes from diverse socio-economic backgrounds. Professionals live here, as do those who live in subsidized housing projects. The population of the school has a racial mix of Caucasian (about 70%) and African-American (about 30%). There is rapid growth in new housing, and there is need for additional schools.

Desert View High School. This school is located near the border to Mexico and is faced with the constant movement of students back and forth across the border. The school's population, mostly Hispanic has been declining since the opening of another high school in the city and some recession in the local economy. A state university is located nearby. Absenteeism is a prevalent problem in this school of declining enrollment.

Green Hills High School. Situated in a rapidly growing suburban area in the midwest, this high school is one of the two high schools serving a district with approximately
11,000 students. The student population is nearly all Caucasian. Recently, this school district spent nearly two-million dollars to revise its K-12 curriculum, including mathematics.

**Pinewood High School.** This magnet school has an African-American population which is approximately 70% of the total student body. This urban high school is located in one of the more affluent areas of a large city and is part of a large county school district with many layers of administration. It is in the process of providing Algebra and Geometry for all of its students.

**Scottsville High School.** This school is located in an affluent suburb of large midwestern city. Students traditionally score very high on standardized tests such as the SAT, ACT, and Advanced Placement. It's proximity to a large university makes it a natural place to pilot programs, and university professors sometimes take their sabbaticals and serve as part of the school's teaching staff to further their research. The school provides an abundance of technology for its students.

**Extended Analysis of R³M Data**

A number of questions are raised after two years of gathering and analyzing data from the R³M project which now motivate the final reporting of the project: Do commonalities exist between the various sites in the R³M study? If so, what are they? Are there insights to be gained from the processes of reform observed at the various sites that would allow for any generalizability in the reform process itself (Kennedy, 1979; Yin, 1989)? If so, to what extent are these processes tempered by local conditions? Are local variations between sites so great as to preclude such generalizations? If so, should we be seeking instead the discovery of "relevant explanations" (Patton, 1990)? According to Miles and Huberman (1994), "Each case must be understood in its own terms, yet we hunger for the understanding that comparative analysis can bring" (p. 172).

The more extensive analysis and interpretation of data as envisioned through this research would be accomplished through various cross-case analyses (Miles & Huberman,
1994) of the site data already collected in the R³M project. These cross-case analyses would provide an orientation for a more comprehensive level of data interpretation by recapturing large amounts of data which were not used in the scenarios and case studies.

This researcher's role. My position as the project assistant for the R³M study provided me the opportunity to participate in the development of field instruments and interview questions which served as criteria for the selection of data from the various sites. I also served as a documenter on two of the five sites which were selected for this extended study.

My role in this extended research will be a different one. Instead of collecting additional data through interviews and observation, I see the "fieldwork" for the cross-case analyses as consisting of immersing myself in the analysis of data collected by documenters on the five sites. I will not be looking for just two or three characteristic features of each site which informed the writings of scenarios; rather, I will be searching through the field notes of observations and the transcriptions of interviews for many variables which may contribute to common threads across cases.

Data from the R³M project exist in the form of taped interviews, field notes from classroom observations, written materials furnished by the sites, the written summaries of documenters, notes from documenters' meetings, scenarios, case studies, and publications by documenters. An attempt will be made to impose no additional a priori outcomes on the data (Lincoln & Guba, 1985), letting the data speak for itself (Glaser & Strauss, 1970). Because of possible interpretive bias by documenters, I will not plan to use scenarios, case studies, or research papers of documenters as a source of data for this study. The other sources of listed data will constitute the base from which I will develop the cross-case analyses for this research.

This study drew heavily on the methodological frameworks of Miles and Huberman (1994). Their suggestions for the various steps and strategies for doing cross-case analyses indicated a degree of creative latitude on the part of the researcher to fit the methodology to
the particulars of the cases being analyzed. Finding the most appropriate methods of reporting the data occupied a central position in this research. The methodology proposed for this research borrowed from several of those strategies. The stages of analysis consisted of the following:

- **Coding Individual Cases (Chapter 4):**

  Coding data is one of the ways by which the researcher reduces data. Initially, I developed some coding categories during spring 1993 based on an inductive analysis of transcripts from the two sites I visited in the R³M project. That is, any data that seemed relevant to the documentation study was coded to reflect the category (or categories) in which the data might fall. The data itself dictated the coding categories that were selected in those earlier analyses. This list was expanded by including codes suggested by cross-case matrices displaying areas of commonality between the sites developed in documenters' meetings. Such a structured\(^{15}\) inductive approach has support from Strauss (1987 cited in Miles & Huberman, 1994, p. 58). Miles and Huberman also indicate a preference for a type of structured coding system--especially for doing cross-case data analysis. I developed a "starting list," or initial set of codes, by using the guiding issues (Chapter 1, page 4), some matrix development in documenters' meeting of important themes that were emerging from the sites, and the coding categories which I developed earlier through critical examination of transcripts of two sites. The "critical examination" was influenced by my personal experiences as a mathematics educator and by the conceptual framework presented in Chapter 2. The final set of codes, grouped by guiding issues are displayed in Chapter 4, Figure 4.1.

  For each site I coded data from transcribed interviews, classroom observations, written documents from the school sites, and reports from documenters (especially Form F from the R³M Documenters' Handbook, Appendix A). It was anticipated that coding data in this way would allow for a greater understanding of each site. This process was flexible enough to allow "leeway for uniqueness" (Miles & Huberman,
1994) to emerge as each site was analyzed in depth. Variables (coding categories) were added or deleted during this preliminary stage of analysis.

- Building the Single Case-Summarizing Matrix:

  Miles and Huberman (1994) recommended that some condensing of data occur for each case before bringing all the data from the various cases together for comparative purposes, but they cautioned against moving too quickly to establish too much internal order. While some patterns emerged from an individual site's data, data was kept loosely organized by six clusters for ease in coding and handling in order not to predetermine the clustering patterns that might emerge in the cross-case matrix (unordered meta-matrix).
Figure 3.1

ORGANIZATIONAL FLOW OF DATA ANALYSIS AND INTERPRETATION

SINGLE-CASE SUMMARIZING MATRIX (Code, Single Case)

META-MATRIX (Code, Case)

One School (case)

CLUSTER MATRIX (Cluster, Case)

10 codes summarized for each of 5 cases

10 codes above summarized to single cell
Figure 3.2
ORGANIZATIONAL FLOW OF DATA ANALYSIS AND INTERPRETATION

CLUSTER MATRIX
(Cluster, Case)

CROSS-CASE CLUSTER MATRIX
(Cluster, All Cases)

R^3 M INTERPRETATION VECTOR
(Cluster, Interpretation)

10 codes summarized for each of 5 cases

10 codes above summarized to single cell

Page 56
I developed Figures 3.1 and 3.2 as a convenience to assist the reader in following the various stages of the methodology. There were, in fact, no actual matrices hanging on the walls of my workspace. Cell entries were stored in computer files and consisted of data that ranged from one or two paragraphs to several pages.

The single-case summarizing matrix is a simple matrix containing the pseudonym of a particular site and data entries for the initial codes. For example, the East Collins High School (ECHS) summarizing matrix can be visualized as a single column with sixty cells, each representing a separate code. There were five such matrices—one for each of the sites in this extended study (columns 1-5 in Figure 3.1). The sixty cells were grouped into six general categories or clusters\textsuperscript{16} which included: evolution of program, mathematical vision, pedagogical vision, context of change, influence of program on students, and impact of Standards (see Figure 4.1). I avoided moving too quickly at this stage to solidify internal order in the data since such a move had the potential of prejudicing the results by suppressing some data patterns from emerging across sites\textsuperscript{17}.

- Unordered Meta-Matrix (Chapter 4):

  When completed, the in-depth coding of data for each of the sites was brought together into one large matrix (unordered\textsuperscript{18} meta-matrix) and stacked by sites according to those codes (Miles and Huberman, 1994). This juxtapositioning of single-case summarizing charts provided an opportunity for an unhurried look at data across cases by common variables. For example, moving across row 5 would represent case summaries for the code "basic skills emphasis" (one of 10 such codes listed under the cluster heading "mathematical vision"). Mills (1993) characterizes this careful study of the unordered meta-matrix as one of "making gradual sense" of the data, a process that "cannot be hurried" (p. 1).

  According to Miles and Huberman (1984), the cell entries in the unordered meta-matrix will have "done little violence to the original data" (p. 154) and these early
displays may have only limited internal order where "[t]he first cut at cross-case analysis is often exploratory, to see what the general territory looks like" (Miles & Huberman, 1994, p. 177).

It may be argued that this process is cumbersome and the single-case summarizing matrices should first be condensed to fewer codes through a clustering of categories (codes). There is the risk that eliminating codes in one site may turn out to be important variables in another site. This process will be slow and tedious, and it will require multiple approaches at reviewing the data for patterns.

I chose to prefix the codes to correspond to the orienting information sought in the R^3M project (Chapter 1, p. 4). For example, the code "M curriculum changes" was one of several codes loosely grouped in the "mathematical vision" cluster (Figure 4.1). There were six such clusters, each of which corresponded to one of the six components of the orienting information.

"Researchers often look for themes that cut across cases" (Miles & Huberman, 1994, p. 175). Inductive coding of data (both descriptive and interpretive) leads to these common themes. The themes which I chose again correspond to the six clusters of the orienting information: evolution of program, mathematical vision, pedagogical vision, contextual features, influence on students, and Standards' influence. Cross-case analysis may clarify variables that were not clear in a single-case study, and I was careful not to move too quickly at this stage to summarize data across cases.

- Cluster Matrix (Chapter 5):

  This was the first attempt at looking at the case-partitioned data in the unordered meta-matrix in order to summarize the data according to the six clusters of the orienting information from the R^3M study. The 300 individual cells from the meta-matrix were reduced to 30 cells during this process. For example, there were ten separate codes within the "mathematical vision" cluster. Figure 3.1 illustrates this second level of data analysis. In column 5 of the "Meta Matrix," the solid shading
represents the 10 codes of "mathematical vision." All 10 cells (of approximately one or two paragraphs each) were summarized into a single cell in the "Cluster Matrix" (single shaded cell in row "M" and column 5). Each of these cells were summaries of a single school (case) for a single cluster of codes and were from two to four pages in length.

- Cross-Case Analyses and Interpretation by Cluster (Chapter 6):

  The thirty cell entries were further summarized from the 6 by 5 Cluster Matrix (Chapter 5) to the 6 by 1 Cross-Case Cluster Matrix. The single column heading of this matrix was titled "Across All Cases," and there were now just six entries in the column, one for each cluster. In Figure 3.2, the shaded cell represented the "all cases" summary for the cluster, "Mathematical Vision." That is, every one of the cluster summaries for each school (case) in the second row of the Cluster Matrix was now summarized together into a single (across all cases) summary for "Mathematical Vision." It was at this point of analysis that real comparisons across all five cases of the study were able to be made. There were six such separate analyses across all cases, one for each cluster. The interpretive piece for each cluster was placed at the conclusion of each cluster summary.

  Drawing conclusions based on the analyses of clusters across cases provided a means by which to gain a deeper understanding of the data from the R^3M project. However, findings of data across all cases did not always fall conveniently into a cluster designation. Additional findings "across clusters" were presented in the final chapter.

  The next chapter will be devoted to explaining in detail the coding of data and the first cut at data analysis--the Meta-Matrix.
Chapter Footnotes

13 It is important to recognize that such techniques of data collection are not necessarily limited to evaluative studies. For example, documentive studies also employ open-ended interviews.

14 "A site-ordered descriptive matrix contains first-level descriptive data from all sites, but the sites are ordered according to the main variable being examined, so that one can see the difference among high, medium, and low sites. It puts in one place the basic data for a major variable across all sites" (Miles & Huberman, 1984, p. 160).

15 The use of the word "structured" refers to the derivation of some of the codes from the orienting information (Form F) of the R3M study (Ferrini-Mundy, 1992b).

16 The convenience of the clusters does not mean that data has been compacted in any way to fewer codes. Later, in the analysis of data in Chapter 5, clustering did result in summarizing data from separate codes to a single cluster summary.

17 Development of a conceptual framework for this study admits to a certain bias which I had as I made decisions about which codes I used, how I reduced and organized the data, and, finally, how I interpreted data.

18 There are several ways that data displays some internal order. If there is no intention to group or organize data in some way, then it is categorized as "unordered." Miles and Huberman (1994) identify several types of ordered displays of data: time-ordered, role-ordered, conceptually-ordered.
SECTION II

SINGLE-CASE ANALYSIS OF DATA
CHAPTER 4

THE R³M META-MATRIX FOR SECONDARY SCHOOLS

It is the purpose of this chapter to explain how data from the five high school sites in the R³M project were coded; how the coded data was reduced through a summarizing process to form the cells of the Meta-Matrix for the study; and how the cells within a given cluster of the Meta-Matrix were collapsed to a single summary in preparation for the Cluster-Matrix which will be explained in the next chapter.

Coding of Data

Data from transcribed interviews, classroom observations, written documents from the school sites, and reports from documenters (especially Form F in the R³M Documenters' Handbook) were coded for each case (site). In an effort to ground the study with the data, no attempt at data reduction was made until the data had been coded. I wanted to minimize my influence on the data as much as possible and let the data speak for itself, a process supported by other researchers (Glaser & Strauss, 1970; Miles & Huberman, 1994). I assigned some multiple codes to much of the data, and found that to preserve the integrity of the data, I had created an enormous mass of coded data. I found later that that sacrifice of valuable research time could have been partially avoided by limiting any data string (usually a sentence or paragraph) to two codes. Coding the data required a larger block of data analysis time than any of the other analyses that followed.

Organization of Codes by Clusters

The origination of codes was explained in Chapter 3. For convenience in coding and later analysis, the codes were grouped by clusters as shown in Figure 4.1. This grouping made it much faster to do the coding since I was able to first identify to which cluster a string of data belonged; then I could assign a variation (a particular code) within that cluster.
**Figure 4.1**

**Coding by Clusters**

**Evolution of Program:**

<table>
<thead>
<tr>
<th>E advice to others</th>
<th>E evolution of math program</th>
</tr>
</thead>
<tbody>
<tr>
<td>E partners or collaborators</td>
<td></td>
</tr>
</tbody>
</table>

**Mathematical Vision:**

<table>
<thead>
<tr>
<th>M basic skills emphasis</th>
<th>M curriculum changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>M math as communications</td>
<td>M math as connections</td>
</tr>
<tr>
<td>M math as problem solving</td>
<td>M mathematical content</td>
</tr>
<tr>
<td>M mathematical vision</td>
<td>M role of technology</td>
</tr>
<tr>
<td>M staff perception of program</td>
<td>M textbooks and materials</td>
</tr>
</tbody>
</table>

**Pedagogical Vision:**

<table>
<thead>
<tr>
<th>P evidence of math discourse</th>
<th>P examples of pedagogy</th>
</tr>
</thead>
<tbody>
<tr>
<td>P mathematical tasks</td>
<td>P pedagogical vision</td>
</tr>
<tr>
<td>P site view of pedagogical changes</td>
<td>P teachers as risk takers</td>
</tr>
</tbody>
</table>

**Influence of Program on Students:**

<table>
<thead>
<tr>
<th>I all students or select groups</th>
<th>I assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>I effect of program on students</td>
<td>I student activity</td>
</tr>
<tr>
<td>I teacher perception of students</td>
<td>I teacher student interaction</td>
</tr>
</tbody>
</table>

**Context of Change:**

<table>
<thead>
<tr>
<th>C additional needs</th>
<th>C administrative support of program</th>
</tr>
</thead>
<tbody>
<tr>
<td>C change is slow process</td>
<td>C classroom characteristics</td>
</tr>
<tr>
<td>C collaboration with others</td>
<td>C common planning time</td>
</tr>
<tr>
<td>C community outreach and input</td>
<td>C community support of program</td>
</tr>
<tr>
<td>C cultural diversity</td>
<td>C difficulties in implementation</td>
</tr>
<tr>
<td>C duration of reform</td>
<td>C expectation on teachers</td>
</tr>
<tr>
<td>C feelings of doubt</td>
<td>C funding as an impetus</td>
</tr>
<tr>
<td>C helpful things in developing program</td>
<td>C impact of testing</td>
</tr>
<tr>
<td>C impact on rest of school</td>
<td>C inservice development</td>
</tr>
<tr>
<td>C leadership</td>
<td>C local and state policy</td>
</tr>
<tr>
<td>C mathematical community</td>
<td>C pride in program</td>
</tr>
<tr>
<td>C problems with site</td>
<td>C scheduling changes</td>
</tr>
<tr>
<td>C teacher difficulty in changing</td>
<td>C teacher openness to change</td>
</tr>
<tr>
<td>C teacher ownership of program</td>
<td>C teacher preparation</td>
</tr>
<tr>
<td>C teacher role in developing curriculum</td>
<td>C teacher support of program</td>
</tr>
<tr>
<td>C teaching experience</td>
<td></td>
</tr>
</tbody>
</table>

**Influence of Standards:**

<table>
<thead>
<tr>
<th>S impact of standards</th>
<th>S site familiarity of standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>S standards as catalyst</td>
<td>S standards as validation</td>
</tr>
</tbody>
</table>
The Cells of the Meta-Matrix: Preparing Code Summaries

The coding of data was done using HyperResearch (Version 1.55 for MacIntosh). This software allowed for multiple assignment of codes to a single string of data. For example, suppose I wanted to assign the two codes, "I: all students or select groups" and "I: effect of program on students" to a string of data from an interview transcript of a teacher named "Jones" from East Collins High School\(^\text{19}\). There are a number of formats that can be selected when sorting data, but I chose the following:

**EC Jones Interview, I: all students or select groups.**
Source Material: I've said this before and I really mean it. Kids who would have never been in an advanced algebra are there. They are there! 91\% of kids go from algebra I to algebra II, that's just unbelievable.

**EC Jones Interview, I: effect of program on students.**
Source Material: I've said this before and I really mean it. Kids who would have never been in an advanced algebra are there, are there. 91\% of kids go from algebra I to algebra II, that's just unbelievable.

In this particular school, most students took Algebra I, so this data seemed significant in both categories. As data reduction progressed through different stages, I had the option of keeping this string of data in both categories, only one category, or none at all. A decision to discard one of the categories was not made if I thought it meant the loss of valuable supporting data. In this case, I decided to keep the double coding. If I was in doubt about whether to keep the second code for a string of data, I usually kept it.

For this particular school, there were 34 interview transcripts, 40 classroom observations, and artifacts from the school that comprised the data pool that had to be coded. The computer page count was in excess of 600 pages of coded data after the HyperResearch sort by codes; so there might be on the average about 10 pages of data for each of the 60 codes. I summarized the data from those 10 pages to one or two paragraphs. It was the summary for each code that became a cell entry in the Meta-Matrix.

The end product of this first level of data sorting and reduction resulted in a Meta-Matrix with dimension of 60 by 5, or 300 separate cells arranged in five columns (school names) with 60 entries in each column. The dimension of the matrix would be difficult to
display on a single sheet, even if the entries were single words instead of one or more paragraphs. Figure 4.2 shows how a portion of the Meta-Matrix would look for a single case (East Collins High School) and a single cluster of codes.

<table>
<thead>
<tr>
<th>Code</th>
<th>East Collins High School</th>
<th>4 Other Schools</th>
</tr>
</thead>
<tbody>
<tr>
<td>all students or select groups</td>
<td>Cell (1, 1)</td>
<td>Cell (1, 2)</td>
</tr>
<tr>
<td>assessment</td>
<td>Cell (2, 1)</td>
<td>Cell (2, 2)</td>
</tr>
<tr>
<td>effect of prog on students</td>
<td>Cell (3, 1)</td>
<td>Cell (3, 2)</td>
</tr>
<tr>
<td>student activity</td>
<td>Cell (4, 1)</td>
<td>Cell (4, 2)</td>
</tr>
<tr>
<td>tchr percept of students</td>
<td>Cell (5, 1)</td>
<td>Cell (5, 2)</td>
</tr>
<tr>
<td>tchr student interaction</td>
<td>Cell (6, 1)</td>
<td>Cell (6, 2)</td>
</tr>
<tr>
<td>54 other codes</td>
<td>Cell (7, 1)</td>
<td>Cell (7, 2)</td>
</tr>
</tbody>
</table>

In Figure 4.2, Cell (1, 1) represents the summarization of multiple pages of coded data for East Collins High School. This particular cell is the summary for "all students or select groups". The contents of Cell (1, 1) are displayed in Figure 4.3.

Again, there has been an attempt to reduce data without interpreting it, but a documenter through his/her choices cannot keep the data completely free of bias. It was my goal to minimize the influence of my bias, not to conceal it. To say I was bias free would be comparable to saying I had no personal theoretical positioning in this study.

There were only three pages of coded data for the code "all students or select groups", but keep in mind that the average number of pages of back data is approximately 10 pages per cell entry on the Meta-Matrix. A sampling of raw data from the coding sheets (sorted by using HyperResearch) for Cell (1, 1) is shown in Figure 4.3.1.
Most ninth graders are in algebra. The sequence of mathematics courses is Algebra I, Algebra II, then Geometry. About 91% of students in Algebra I go on to take Algebra II. The state requires three years of mathematics for graduation, but most students at ECHS will take four. There is an emphasis to include minority students (mostly African-American which range from 10% to 30% of the classroom totals) and female students. The goal of the mathematics department (and of the state) is that all students completing high school must be successful in algebra. The school goal is that all students will also be successful in a Geometry course. The staff recognizes that a modified three or four-semester algebra class may be needed for some students to succeed in algebra. The content of such a course would include much more than a one-year course. There is an on-going collaboration and sharing of content and pedagogy between teachers in the voc-tech program and the mathematics teachers to provide a mathematics program that is interesting and meaningful to every student. Mathematics teachers counsel students to determine what student goals are and to encourage students to take as much mathematics as possible to help meet those goals.

Multiple coding became a problem during this early phase of data analysis. I realized that in my zealous attempt not to lose any data, I had created massive amounts of sorted data. I had coded some data strings with as many as five or six separate codes. (In Figure 4.3.1, each of the entries is what I refer to as a data string.) This process produced approximately 600 pages of sorted data for East Collins High School (ECHS). It became apparent that no loss of data occurred by limiting coding choices to two. Through a long and tedious process I managed to cull out data strings that only slightly affiliated with a particular code and reduce the number of pages of sorted data for ECHS to 277 pages.
### Figure 4.3.1

**Sample Data Sheet Sorted by Code**
(From HyperResearch Printouts)

The actual report follows:

**File, Code,**

**EC Hopkins Interview, I: all students or select groups,**
Source Material: I'd hate to say what percentage of ninth graders are in algebra, but most of them are.

**EC Hopkins Interview, I: all students or select groups,**
Source Material: most of us try in our math classes to counsel them too, so we do a lot of encouraging and really trying to convince them if they don't want to go into a math class next year, we talk about their needs and try to see what they want to do after high school and we try to channel them in the direction that would be best for them.

**EC Hopkins Interview, I: all students or select groups,**
Source Material: I guess starting with next year's ninth grade, the graduation requirements in [our state] will include either algebra I or two years of applied math and we're going to be making some changes along those lines to try to prepare those students.

**EC Hopkins Interview, I: all students or select groups,**
Source Material: I don't think they are going to specify that algebra I has to be a one year course. We're looking at maybe making it a three semester or a two year course and rather than just spending more time doing the same stuff, adding lots of extras that we do in [our new] program. Making it almost like a beefed up [part of the] program.

**EC Hopkins Interview, I: all students or select groups,**
Source Material: they'll be ready to do the two years of algebra or whatever they need. So [the mathematics supervisor's] really going to try to address that situation starting maybe in sixth or seventh grade.

**EC Jones Interview, I: all students or select groups,**
Source Material: I've said this before and I really mean it. Kids who would have never been in an advanced algebra are there. They are there! 91% of kids go from algebra I to algebra II, that's just unbelievable.

...
Figure 4.3.1 represents a sample of a sorted data sheet that is produced using HyperResearch software. I selected two-letter codes to remind me of the case to which the data belonged. In this sample, "EC" represents East Collins High School. Note that the code for this sample belongs to the cluster "T" (influence of program on students), and the specific code within that cluster is "all students or select groups."

The Meta-Matrix becomes the basis for further analysis and reduction of data. It allows the researcher to see if certain codes are consistently important across the cases. For example, if everyone at a particular school is saying the same thing about the school's mathematical vision, it is more significant than if just one person is saying it. If that same level of frequency persists across several of the sites, then it may emerge as one of the common findings of the study. The Meta-Matrix also provides some early perceptions about how different codes within a cluster may complement or overlap each other and possibly revealing strengths and weakness of the data-gathering instruments used in the R3M study. (In the concluding chapter, I address some of the issues relating to how data was gathered.)

The remaining cells for East Collins High School for the cluster, "Influence of Program on Students", are shown in Figure 4.4 through Figure 4.8. This amount of detail is being displayed only for illustrative purposes, for it is from these six cells of the Meta-Matrix that I have further summarized the data to form a Cluster Summary for "Influence on Students" for ECHS (see Figure 3.1). It is the Cluster Summary that becomes the basis for making the next and final stage of analysis--the Cross-Case Analyses (Chapter 6). The Cluster-Matrix is the subject of Chapter 5.
Alternative forms of assessment are just beginning to take place within the department. Some teachers are beginning to use portfolios, and inservice development at the district level is geared towards familiarizing teachers with the use of portfolios. Self-assessment, parent assessment, group quizzes, homework quizzes, tests which accompany the UCSMP books, and teacher-made tests are used in varying degrees by the teachers to assess student understanding. Teachers have expressed concern that evaluating students in "the old way" (looking for a single correct answer to problems) contradicts the purposes of the newly evolving mathematics program.

There is also an interest in program assessment. Besides the large percentages of students taking more advanced mathematics classes, the mathematics teachers seek student input to improve their mathematics program and a "before" and "after" attitude check towards mathematics and their mathematics teaching. Observation of changes in student attitudes towards mathematics is one of the ways in which mathematics teachers feel their mathematics program is working.

Teachers and administrators see a number of signs which indicate that the new mathematics program is impacting in a positive way on students. Among these signs are the reduction of students' mathematics anxiety by focusing on doing relevant, connected mathematics instead of memorizing mathematics skills; large numbers of students moving on to the next course; improving relations between teachers and students as a result of changed content and practice; greater responsibility on students to learn through an emphasis on reading mathematics and on group learning activity; increased levels of confidence by students in doing mathematics; greater success among students who would normally be considered at risk; increased ability of how to get started and continue through the problem-solving activity; less fear and anxiety over "word problems" since nearly all problems are word problems; student recognition that there may be multiple ways of reaching solutions to mathematical tasks; greater interest through doing hands-on activity and using calculators and computers; greater access to help when it is needed, either through group members or the teacher; and positive reaction of students toward the UCSMP texts and materials.
Students are engaged in varying degrees of group activities with 1 to 3 other students. In some classes, students immediately get into groups when they enter the classroom. In others, teachers will go over homework problems with the whole class before students move into groups after an initial correction and review of homework. Student activities consist of working on assigned problems that accompany the text and lab activities. Students are encouraged to be responsible for their mathematics learning by reading their mathematics text and discussing problems in the groups before they ask the teacher for assistance. Varying degrees of mathematics discourse take place in the group settings and can become very intense as students try to persuade each other about the soundness of their ideas. The level of responsibility of individual group members differs from teacher to teacher. Lab activities usually take place once a week and can occur within the class or the computer lab. In these labs, students engage in numerous activities which require students to analyze data and make conjectures about outcomes. Examples include rolling marbles down ramps to discover linear relationships, discovering the Fundamental Counting Principle in probability through explorations which connect to the students’ personal world, or developing an understanding of variables by using algebra tiles. Calculators are becoming an important component of student learning in mathematics. Some of the mathematics teachers encourage student writing by requiring careful write-ups of labs including, in some classes, a story-line. All students are expected to do much of their mathematics activity through regular homework assignments outside the class.

Teachers see a number of changes taking place with students in the [new mathematics] program, and these changes may take time as students adjust to a different mathematics learning environment. Students are eager and more comfortable with and have less anxiety in learning mathematics. This is evidenced by the larger numbers of students who take more advanced mathematics. Teachers characterize students as being more cooperative, though some teachers report behavior problems still exist. Students are more confident and they are finding avenues for dealing with their frustrations when solving mathematics problems. Students are becoming better and more independent readers of mathematics and are adjusting to the prevalence of word problems. Students are enjoying mathematics more and are taking more risks in striking out in new directions and expressing themselves in supportive classrooms. Students who have weaknesses in reading skills usually have to work much harder and require additional assistance and support from teachers and group members.
Figure 4.8

Cell (6, 1): (Teacher Student Interaction, ECHS)

Teachers see their role as that of facilitator and not a lecturer, though this will vary by teacher. Typically directions by teacher are followed by group activity. Class activities are predominately teacher directed on some occasions (even when students are in smaller groups), especially when reviewing for tests, and there is little opportunity for interaction between student and teacher. The interaction is mostly respectful, but not always, and it is often interspersed with humorous comments. There are varying degrees of mathematical discourse in these interchanges; some promote student conjectures, others illicit short responses.

Cluster Summary

As indicated in Diagram 4.1, the coding summaries for a given cluster (e.g., "Influence on Students") are considered as a group and further summarized to form a Cluster Summary. There are six clusters (Figure 4.1) for each of the five cases in this study, yielding a total of thirty such cluster summaries. These thirty summaries will become the cells for the Cluster-Matrix which is displayed in Chapter 5. The Cluster Summary for "Influence on Students" is given below (Figure 4.9):

Figure 4.9

Cluster Summary (Influence on Students, ECHS)

Teachers are attempting to provide for the mathematical needs of all students at ECHS, including minority students, female students, and at-risk students. Successful completion by students of a course in Algebra and Geometry is a district-wide goal. Algebra for the at-risk students will have a different content than the regular algebra course and will require three or four semesters for completion. Collaboration with other departments such as vocational education is providing mathematics experiences with connections to real life. Student input is valued in the continuous upgrading of the mathematics program.

There has been a shift away from general mathematics courses, and nearly all ninth graders are taking Algebra I or Algebra II. Approximately 91% of all students completing Algebra I go on to take Algebra II. Pedagogical shifts to a more facilitative role varies among teachers, and these shifts are sometimes difficult adjustments for some teachers and for some students. Teachers feel their choice of the UCSMP texts and materials provide students with mathematics that is more relevant to the students' world, though teachers are
constantly looking for additional activities that connect to the real world. There is much emphasis on student reading of the texts and on pairwise and small-group activity. Varying degrees of mathematics discourse take place in the group settings and sometimes become very intense as students try to persuade each other about the soundness of their ideas. The most prevalent activity consists of working on assigned problems that accompany the UCSMP text. Lab activities, which are encouraged by special state funding, usually take place once a week and can occur within the class or the computer lab. In these labs, students engage in numerous activities which sometimes require students to analyze data and make conjectures about outcomes.

Calculators are becoming an important component of student learning in mathematics. Student writing is encouraged through careful write-ups of labs including, in some classes, a story-line. All students are expected to do much of their mathematics activity through regular homework assignments outside the class. Teachers admit they have a long way to go in developing the program to its fullest potential.

The interaction between teacher and students is mostly respectful, but not always. Most teachers enjoy an excellent rapport with students. Teachers are finding that some of the first year students in the program require a longer adjustment period. Teachers indicate that students who have made the adjustment more readily assume greater responsibilities for their learning. There are varying degrees of mathematical discourse between teachers and students. Some promote conjectures and deeper understandings of mathematics; others illicit short responses.

There are a number of indications that the mathematics program is impacting in a positive way on students. A teacher describes the change: "It's amazing how they enjoy. They think they're having fun and not learning, and it's real subtle you know." As a result of a program that emphasizes exploration of relevant connected mathematics and a de-emphasis on memorizing mathematics skills, teachers see improved relations with students, a decrease in students' mathematics anxiety as students gain confidence in doing word problems, and students taking greater responsibility for their own learning. Students enjoy doing hands-on activities and using calculators and computers. They are becoming better and more independent readers of mathematics and are adjusting to the prevalence of word problems. Students are enjoying mathematics more and are taking more risks in striking out in new directions and expressing themselves in increasingly supportive classrooms.

Quizzes and tests prepared commercially to accompany the UCSMP materials are often used for student assessment, but some teachers see this as contradicting the purposes of the newly evolving mathematics program. Alternative assessments of student learning are being explored by individual teachers. These include self-assessment, parent assessment, group quizzes, homework quizzes, and portfolios.

The Cluster-Matrix will be completed in Chapter 5 with thirty entries similar to the Cluster Summary displayed above.
Chapter Footnotes

19 The prefix "I" for these two codes was selected in order to keep codes by clusters together while using the HyperResearch software. "I" is a reminder that this code is in the "Influence on Students" cluster.
CHAPTER 5

THE R3M CLUSTER MATRIX

The Cluster-Matrix

Data Reduction

Approximately eighty pages of coded data were reduced through two levels of summary to the resulting one-page Cluster Summary at the end of the previous chapter. Thirty cluster summaries are presented in this chapter, each of which will constitute a cell entry in the Cluster-Matrix in Figure 5.1. The Cluster-Matrix is crucial to the cross-case analyses and interpretations which follow in Chapters 6 and 7. Data in the Cluster-Matrix consist of "subsuming particulars into the general" (Miles & Huberman, 1994, p. 255), a process which involves clumping (or clustering) things together that seem to fit. At this stage of analysis, the Cluster-Matrix provides data in a more manageable size and form to facilitate comparisons and interpretations of data.

<table>
<thead>
<tr>
<th>Cluster-Matrix (Cluster, Case)</th>
<th>ECHS</th>
<th>DVHS</th>
<th>GHHS</th>
<th>PHS</th>
<th>SHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evolution of Prog</td>
<td>Cell (E, E)</td>
<td>Cell (E, D)</td>
<td>Cell (E, G)</td>
<td>Cell (E, P)</td>
<td>Cell (E, S)</td>
</tr>
<tr>
<td>Standards</td>
<td>Cell (S, E)</td>
<td>Cell (S, D)</td>
<td>Cell (S, G)</td>
<td>Cell (S, P)</td>
<td>Cell (S, S)</td>
</tr>
</tbody>
</table>
The Cells of the Cluster-Matrix

The primary purpose of this chapter is to describe the results of the third and final level of data reduction. (Recall that the first two were the coding of data strings and the development of coding summaries for the Meta-Matrix.) There will be thirty such summaries as indicated by the Cluster-Matrix (Figure 5.1) in this chapter. The six clusters are indicated in the first column of Figure 5.1 and include: evolution of program, mathematical vision, pedagogical vision, context in which change occurs, influence of program on students, and impact of the Standards.

Cluster 1: Evolution of Program

R³M researchers were interested in how the mathematics program in each site had evolved and in what direction the mathematics teachers and administration wanted their program to proceed. Were others outside the school helpful in some collaborative or partnership capacity? We also wanted to know what advice those involved in changing their mathematics program had for others who might take similar steps. The codes which were summarized for Chapter 4 and used as components of this cluster summary include (from Figure 4.1):

- evolution of mathematics program
- advice to others
- partners or collaborators

East Collins High School. Prior to an offer by local industry of a fully equipped computer lab and software to teach Algebra, the mathematics teachers at ECHS were already talking of change. Some were familiar with the Curriculum and Evaluation Standards. There was also some knowledge of Usiskin's work with the University of Chicago School Mathematics Project (UCSMP). However, teachers felt that the computer lab offered by local industry hindered teacher contact with students, and that the software which would come with the package lacked the pedagogical direction that the Standards recommend. When the teachers indicated that the offer was not quite what they were looking for, the industry spokesperson asked them what they would prefer instead.

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A committee consisting of the county mathematics coordinator, the associate principal of ECHS, a company spokesperson, the state mathematics coordinator, two university mathematics professors, and an ECHS teacher representing the business education partnerships began to investigate how a different plan might be developed. The associate principal appointed two high school mathematics teachers to assist with the planning. Those two would also launch the first year's implementation. Planning occurred from December 1990 until school opened the following September (1991). Teachers were given discretionary power to spend the $55,000 contributed by local industry. UCSMP Algebra texts and calculator purchases were the principal expenditures for the first year in which all students taking Algebra I became participants.

During the second year, the use of the UCSMP texts and materials expanded downward to one of the middle schools where Transition Math was offered, and upward to include Algebra II. Five additional teachers participated. By the third year all eleven teachers in the department were participating in the program.

There were a number of factors that the original implementers and the associate principal found were helpful in getting the program of reform underway:

- It became evident that one teacher should never try this alone. The two teachers who implemented the program the first year found that they constantly needed the other to share ideas and concerns and to offer support during periods of discouragement.

- It was helpful not to give students options during the program's startup. All students in Algebra I during that first year were enrolled in the new program. One teacher commented: "I would hate to think what would happen if we'd have half of our Algebra I classes [doing it the new way] and the other half the traditional way."

- It was critical to get the information out to parents and the community before introducing major change.

- Students had so many hang-ups about mathematics that it was important to catch students before ninth grade with these efforts to make mathematics more interesting.
• Changing attitudes of other teachers and parents toward mathematics was essential in order to change student attitudes. Getting these people to agree not to say negative things about the difficulty they have experienced with mathematics was important.
• It was important to keep up the momentum.
• Teachers must be dedicated and believe in what they were doing.

The county mathematics coordinator believed it was important to "immerse" teachers in new ideas, instead of telling them that their way of teaching was wrong and they needed to change. This immersion was accomplished by going to conferences, workshops, visiting other schools, etc. which really helped in the evolutionary process of the new program.

Desert View High School  The partnership which developed between mathematics professors at the local university and the mathematics teachers at Desert View High School had a serendipitous beginning. The university professors had received a grant two years previously (1988) to develop projects--multilayered problems that students had to solve and then explain in technical reports--for their calculus classes at the university. They thought the concept should be expanded to include a local high school. Several mathematics professors started developing a funding proposal and were just getting ready to contact a local high school with their ideas. Mathematics teachers at the high school, in an attempt to establish a better line of communication with the local university, went to the Mathematics Department at the university since no mathematics education department existed at the university. The high school mathematics teachers were interested in determining university expectations of high school students who might enter the university's mathematics program. Teachers at the high school had some familiarity with the projects since students often returned to ask their high school teachers for assistance in developing them. When the two groups of teachers met quite by chance, a collaborative effort was hatched to expand the projects concept to the high school. An NSF grant was applied for by the university professors and granted for the development of mathematics projects at the high school.
In the summer of 1990, several Desert View High School math teachers collaborated with the mathematics faculty at the university in a program designed to train teachers to incorporate student projects in the high school mathematics curriculum, with an initial focus on Algebra II, Geometry, and Trigonometry. Over the next three years, more DVHS math teachers became involved in the writing, editing, and implementation of projects. Currently, all members of the department use projects--to varying degrees--in most of their classes. In January of 1993, approximately one hundred projects were on file in the department. Interviews with faculty confirmed continued widespread commitment to the development of projects and the implementation of cooperative group instructional strategies, even though the funding for the grant phased out in the spring of 1994.

Although projects have been the central focus of reform efforts at this high school, the process of incorporating projects into the curriculum has prompted corresponding changes in regular classwork including: (1) math-related writing exercises on a daily or weekly basis; (2) cooperative learning as an instructional strategy; (3) hands-on laboratory activities, particularly in Technical Math and Math Applications (remedial level) classes; and (4) the discovery method of learning, at all levels. The process also spurred such initiatives as a pilot project on Michael Serra's (1991) Discovering Geometry and wider use of graphing calculators in joint efforts to work through math problems.

The partnership that existed between the university mathematics professors and the mathematics teachers at Desert View High School developed on different levels. The external funding placed the professors in charge of the funding and the program. The professors chose not to exercise the "power" of their official capacity as the grant holders. Instead there developed a close collaboration that resulted in frequent visits by the professors to the high school to visit mathematics classes and to meet regularly with teachers for the purpose of sharing ideas. The professors saw their role more as a sounding board against which the high school teachers would bounce their ideas. The reflection which developed around mathematics and pedagogy was a two-way-street for the
separate faculties: the high school teachers felt their ideas were being mathematically validated by the mathematics professors; the professors were learning a great deal about practical pedagogical ideas which they, in turn, used to modify components of their university projects program. Teachers were greatly impressed that the university mathematicians would even want to sit in on their classes.

Teachers offered the following advice to others who might start a projects program:

- Look to the university mathematics professors as possible partners in developing the projects for mathematics classes.
- Schools may benefit from collaborating with each other in order to get enough projects written and also to share outside funding.
- Have at least several teachers doing it. It's important to share ideas and have group support.
- If another school has developed a successful program, ask them to share ideas and possibly lead workshops.
- The attitude of teachers is important. If most teachers are against it, don't bother.
- There should be flexibility in developing the projects to allow for the different teaching styles of teachers.
- Plan to spend a lot of extra time developing the projects, even if there is release time built into the budget.

Advice offered by the mathematics professors:

- Realize that using projects might not work with a different group of people.
- Teachers who are not sure how to effectively grade the projects should have an experienced colleague with whom to talk and work with.

Green Hills High School. An innovative mathematics teacher was given the position of district mathematics coordinator and organized the process by which a large representative group of K-12 mathematics teachers developed a new curriculum. A research committee and a writing committee were formed and during a two-year period

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developed a mathematics curriculum that was ratified by all of the mathematics teachers in the district. A stipulation imposed by the mathematics coordinator prevented teachers from bringing in any packaged or preconceived mathematics programs. This stipulation prevented teachers from being influenced by a particular series of texts and allowed them to discuss, in an unprejudiced manner, what they wanted in their mathematics program in terms of content and pedagogy.

The district was in its first year of implementation during the documenters' visit, and there was evidence that some teachers were beginning to facilitate cooperative learning groups in their classes. For example, conjecturing was becoming a more important part of problem solving, and a Mathematics Learning Lab was available for students who wanted to explore mathematics or just get help when they needed it. The evolution included community cooperation and support, particularly parents. Technology was becoming increasingly important, and there was an emphasis to include calculator usage in mathematics classes. Writing in mathematics was also beginning to find its way into the mathematics curriculum.

Pinewood High School. The county mathematics supervisor had been in that position for a number of years and had endeavored to eliminate an elitist point of view toward mathematics and squarely face issues of equity. It had been her goal to improve the quality of mathematics education for all the students in the school district. She felt that low test scores in mathematics were too easily accepted and that teachers and others in the district conveyed a feeling that the students of the county just weren't able to do well in mathematics. Her recommendations for change in content and pedagogy predated the Curriculum and Evaluation Standards but ran parallel to suggestions for change found in the Standards.

In spite of the fact that the predominantly African-American student population was from one of the more affluent black communities in the country, students in this school system did poorly on mathematics related tests. Data supported the fact that the school
system was "not getting the job done." The school board and the superintendent decided to set up a task force made up of business people and representatives from higher education to look into the problem regarding mathematics education and to make recommendations for improving the program. The task force recommended that the curriculum honor the Standards and accept the challenge of Algebra and Geometry for all graduating seniors (which was precisely the position taken by the mathematics supervisor earlier). The board accepted the recommendations of the task force and mandated a change in the mathematics program to the full implementation of the Standards by the end of the 1990-91 school year.

There were many skeptics among administrators and among teachers. Many teachers refused to get involved, and there remained much resistance. A new superintendent was appointed to start with the 1991-92 school year, and that is when the mathematics supervisor no longer felt an outsider to the process. (The old superintendent had hoped the task force would discredit the supervisor's mathematical and pedagogical vision for the system, but it served instead to reinforce it.)

Formal restructuring began in the summer of 1991 with a Pre-Algebra Institute and was followed in successive summers with Algebra I, Geometry, Algebra II/Trigonometry (1994). Equity 2000 became part of the restructuring and the new Algebra I and Geometry courses were instituted in successive years. Teachers were paid stipends to attend summer inservices in preparation for the new classes. Pacesetter, a capstone experience for seniors, began its second year of implementation in 1993. Two middle grades teachers from each school attended a summer institute during the summer of 1994 as the program began to expand beyond the confines of the high school. The College Board provided funding for the Equity 2000 and the Pacesetter programs and collaborated on inservice institutes and on inservice release days.

Scottsville High School A number of factors played into the evolution of the mathematics program and continued change at Scottsville High School (SHS). About ten years ago, the mathematics supervisor began to plan for a mathematics program that would
be rich in technology and problem solving. The influence of Usiskin and the development of the University of Chicago School Mathematics Project (UCSMP) provided nearby resources for the high school. Usiskin frequently worked with groups of students in the school, and SHS became a pilot site for his materials. Professors on sabbatical leave were frequently hired on a part-time basis to teach and do research in the school.

Much of the change and creativity, especially for the computer and calculator activities, had taken place with teachers working individually or in collaboration with each other. Teachers often made reference to the hours of preparation required to develop just a one-hour computer lab. Teachers wanted classroom practice to be assisted by technology, and they often indicated that they were not using calculators and computers for the sake of technology. The evolution of the mathematics program witnessed more and more mathematics being compressed into the K-8 programs. The norm was that students in grade 9 took geometry. The supervisor's vision of calculus for everyone by the year 2000 was seen as within the realm of possibility by teachers at the site.

Teachers agreed that the success of their program depended on strong leadership (mathematics supervisor). To avoid the danger of teacher burnout, teachers suggested time should be factored into the school day for adequate planning and collaborating with peers. There should also be more time and opportunity for teachers in such a high-tech program to keep up with the rapidly changing technology, particularly software advances and changing calculator capabilities. Time was emphasized over and over as being in short supply for their needs.

Cluster 2: Mathematical Vision

R³M documenters were interested in whether each school's mathematics department had goals for its program and if those goals played an active role in actual classroom practice. They wanted to know the kinds of mathematics learning teachers at the site hoped their students would experience. Documenters wanted to know what the people in the site saw as worthwhile mathematical tasks, or important mathematical ideas. They
wanted to find out if alignment among teachers and administrators existed around this vision. Documenters were interested in whether the mathematics program emphasized problem solving, communication, reasoning, conjecturing, and mathematical connections. Can the classrooms be described as mathematical communities? Why? Documenters were looking for specific evidence that the teachers' mathematical vision was being brought to life in the classroom. The specific codes comprising this cluster include:

- mathematical vision
- basic skills emphasis
- curriculum changes
- mathematics as communications
- mathematics as connections
- mathematics as problem solving
- mathematical content
- role of technology
- textbooks and materials
- staff perception of program

**East Collins High School.** Mathematical competence in the workforce became the main component of the mathematical vision of the ECHS mathematics program. Eliminating mathematics anxiety, particularly among girls and minority students, was an important part of that vision. Another part of this vision was that all students would take algebra at some level before they graduated. Teachers made a distinction between the UCSMP texts and the vision which was the ECHS mathematics program. This vision required "teacher excitement, student interest, parent involvement, and administration and community support." Teachers felt the UCSMP texts were the best available to help meet the goals of the ECHS mathematics program, but their vision was not limited to any book or series of books. "Mathematics is everywhere" was a recurring theme that guides the mathematical vision.

Mathematics teachers developed the curriculum that they wanted the students to experience and then looked for the supporting materials and books that would aid in that process. Teachers were moving away from traditional courses and pedagogy and moving towards courses whose content and pedagogy followed the Standards. Pre-algebra and
general mathematics classes had been eliminated. Curriculum changes emphasized making mathematical connections to real life problems, problem solving, mathematical reasoning, and developing communication skills in mathematics. These changes in content were facilitated by including computers, calculators, and manipulatives in students' mathematics learning activities. Teachers were also in the process of changing their role to a more facilitative one.

Important mathematical ideas were not skills or skill development, according to teachers at ECHS. Content that focused on important mathematical tasks was a goal for the teachers to find or develop. Geometric proofs was a topic of much discussion among the teachers, whether to de-emphasize them or not. They finally decided to de-emphasize proofs and adopted the UCSMP Geometry text. One teacher observed:

That's one of the things I like about the Chicago series, it tends to eliminate a lot of the obsolete stuff. We sometimes fuss in the car about this. I believe a lot of what we teach is just obsolete. I have a math book from 1905, I could use it today until we started using this.

Communication in mathematics occurred through the verbal discussion between pairs or small groups of students. Instead of giving students answers to questions, teachers were trying to develop skills in promoting mathematics discourse. This was one of the most difficult processes for teachers and few teachers were observed who possessed this skill. Students were also encouraged to develop writing skills in the way they presented their finished mathematics tasks. Sometimes students were expected to create story lines that went with mathematics problems. Teachers and students indicated that understanding of mathematics was heightened through these various modes of explaining what one was thinking mathematically. Teachers found that students got so excited about mathematics that they often argued over each others' ideas and solutions.

Making mathematics more relevant to the students' needs through connections to real life situations was a main focus of the mathematics program. Connections between mathematical concepts were also made between mathematics and other subject areas.

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People from the community often explained the relevance of mathematics to their occupations. Students researched magazines and newspapers for applications of mathematics. Personal experiences related to mathematics were drawn into class discussions. Teachers felt that the UCSMP texts did a good job of making meaningful connections in mathematics.

It was difficult for first year students to get comfortable with the emphasis on problem solving. Prior mathematics texts had very few problem solving experiences. Now, practically all that students did was problem solving. The teacher of an Algebra class engaged her students with some elementary concepts of the fundamental counting principle by demonstrating unions, and intersections in terms of student clothing. In another class, students were led to conjecture about the relationship of perimeter and area if one or the other were held constant. This was done by cutting from grid paper as many objects as possible with an area of four square units. Students were rolling marbles down an inclined plane to discover relationships about the height of the incline and its length in determining the distance marbles rolled. Students were asked to make conjectures (e.g., what happens when the length of the incline is increased) and test them based on the accumulating data. Teachers believed that developing skills in the process of problem solving allowed students to be responsible for their own learning and gain interest and confidence in doing mathematics. Students "may not know how to conquer that task, but they're going to know how to start. They're going to know where to go, they're going to know how to research, how to look it up."

Becoming good at using mathematical discourse in classes was a difficult skill for teachers to develop, but a few well-placed questions during discussions with students allowed the students an opportunity to reason through their solutions. For example, one boy questioned why the answer to "$y < 2$ and $y > 5$" was the null set. His solution had been the solid graph between 3 and 4, inclusive. Through a series of questions, the teacher was able to allow the student to reason through his own mistake. Students working in
pairs and small groups often find opportunities to challenge or question the reasoning of others.

A teacher observes: "I see the others changing like I did. They're enjoying the creativity. They're enjoying the technology." Incorporating technology in a mathematics class was not always easy. Teachers found that it took a great deal of time to search for software that would meet their needs. Learning how to use the software often demanded a great deal of teacher time, which was always in short supply. Since the ECHS reform efforts began, there have been a number of technological acquisitions: every classroom had a computer; the computer lab was more fully equipped than it was before; there was a set of calculators in every mathematics classroom; students in Algebra 2 were issued a TI-81 graphing calculator with their books; and there was a growing number of manipulatives available--some obtained commercially, others made by the teachers and students. Some teachers wanted a computer on every student's desk. Student interviews also indicated that students were enjoying the use of technology in their mathematics classes. Another program component that encouraged the use of technology was the special funding available from the state for lab classes. To qualify as a lab, students had to be in lab setting 25% of the time. This special state provision also required reduced class size for labs.

After an extensive search for texts, teachers decided that the UCSMP texts would best meet the students' mathematical needs. Many hands-on experiences in mathematics were being provided through the use of materials such as manipulatives. The first year after the curriculum was developed, all Algebra 1 classes used the UCSMP Algebra 1 text. Algebra 2 and Transition Mathematics were then added the next year, followed by Geometry. Teachers felt authors of the UCSMP books had done such a good job of organizing the flow of the mathematics. Teachers were encouraged by the improvement they noticed in students' study habits, especially the reading of mathematics books. Students said this was because "they put it in real English that you can understand." Teachers supplemented
the text with other published materials and teacher-created activities. Manipulatives were increasing in abundance and many were teacher made.

Teachers' perceptions of their mathematics program was one of optimism. They felt they were on the right track. Students were enthusiastic about continuing on with more advanced courses (91% of Algebra 1 students go on to Algebra 2), and teachers shared that enthusiasm. "I feel like it makes a difference in their attitude towards me, which is 90% of the battle I think." Teachers felt the program was more student oriented, and teachers were developing confidence as they gained more experience working with students in the program. "I think one of the things that makes our math department so successful, I think we've got a great math department because, the new mathematics program isn't something that began last year here. It's been in the minds of these people, all of us, in one way or another, for years. ... The program is the focus of that." Several teachers had remarked that they were able to release some of the creativity that had been stifled under the old program.

Teachers saw the process of developing the mathematics program as on-going. Typical comments include: "It's not something that's going to blow in and out; it's going to stay. It's here to stay, it's working, let's keep going." "We are expanding in whatever direction seems to be the next logical thing to do." "I don't see anything with the program that has been difficult or that I really would change--except add more to it." "I hope that they don't hold back, and that's letting this trickle on down to the elementary level." "You need to stay open-minded about things."

Desert View High School. One of the co-chairs explained the mathematical vision behind including projects in their mathematics classes: "Get students used to long-term projects and themes and writing up the results; working cooperatively in groups also relates to the real world of work; trying to get some students away from just a regurgitation of mathematical facts and algorithms." One of the assistant principals (an ex-mathematics teacher) did not believe the mathematics teachers had a common vision for their
mathematics program. He added, "I think it's better than it's been in the past." Interviews with teachers indicates that an unofficial vision may be the Standards. There is to be a district adoption of mathematics texts next year, and one teacher explained that for the past six months committee members have been studying the Standards to help in this process of textbook selection. Documenters observed that the mathematics teachers were making a real effort at providing a meaningful mathematics program for all students attending DVHS.

A curriculum was viewed by people at the site as being different from teaching practice. For example, teachers didn't feel that the inclusion of the projects into all their mathematics classes were changing the curriculum. This view of curriculum was also shared by the curriculum coordinator: "Any curriculum reform we do is going to be district wide, it's not going to be limited to Desert View. Any textbooks we choose that we think will support us in meeting the Standards, that's going to be a district wide kind of thing." Teachers explained that they had a "very in-depth curriculum guide" in the district, but it "doesn't say you will see this on page 1, page 2 and it doesn't say when, but it does say you will cover these things and how we cover them is up to us."

Content changes in mathematics were not the primary function of including projects and themes into the school's instructional format. The projects were pointing to a need for more relevant mathematics, something that connected with the experiences of students. They were seen as providing students with meaningful problem-solving opportunities. With the input and collaboration of the university mathematics professors, teachers were developing a new vehicle to present much of the mathematics content. Certainly, if one were to look at the mathematical content students in the school were being exposed to, there was significant change. According to an assistant principal a "pull-up" of mathematics has resulted:

The trend to fewer remedial classes during the past four years from 33 to 12 is a positive one. ... I think part of this story is that by trying something new and attempting some things that were kind of avant-garde, and getting support and
doing it, we were able to, I think, get more kids involved in the math mainstream than before; and that it was done without kicking and screaming and crying."

Greater numbers of students taking mathematics, in spite of a declining school population was another strong indicator that mathematics content for students is changing.

"I'm seeing a bigger and bigger push across the whole nation that people have got to have better communication skills," said one teacher, "and I'm all for that." Teachers pointed to the changes in the way students communicated in mathematics that had resulted from working with the projects. Students' verbal and written communication skills in mathematics had shown tremendous growth "especially if you look at the first time they do [a project] and the last time they do one." Another teacher remarked, "The long term, large scale transformation of their writing abilities is remarkable. [They are] learning through this process of writing and explaining to each other within their groups. They are learning how to communicate mathematics to each other or to someone else." Writing in mathematics classes was extending beyond the projects and themes. Teachers often asked students to write explanations of their mathematical thinking on their regular mathematics activities. One teacher explained that she gave a set of discussion questions at the beginning of each unit of work. For each question, she expected students to prepare one paragraph responses. And teachers were insisting that students write correctly. These activities helped students crystallize their mathematics thinking and carried over into better verbal discussions of mathematics. Not all teachers were doing the same thing with student writing in mathematics. A teacher explained: "I think some of the other teachers are implementing [writing] in other ways, not necessarily just in the projects and the themes."

Documenters' observations of classroom activity indicated the interest and fun generated by writing a story line for each project.

By the time of our return visit to Desert View High School in 1993, teachers had collaborated with each other and the university mathematics professors to develop in excess of one hundred projects. Every project was a mathematical task (NCTM, 1991) which
made a connection to students' experiences. Not every task was connected to a real-life situation, but they were certainly relevant to the students' mathematical experience.

Projects offered students opportunities to assume different roles in their story lines (e.g., a flooring contractor, a designer of shields, an irrigation engineer, a mathematician looking for patterns in a sequence).

In most classes, group activity allowed students opportunities to share ideas and seek out the best possible way to solve these mathematical tasks. Teachers encouraged students to consider multiple-options in solving problems. Many of the projects allowed students of varying mathematics ability to be engaged at a challenging level. The projects were not designed to produce a single correct answer, and there were many places in which students had to make sense of the mathematics that they would need to reach a solution. Even in those classes which consisted primarily of at-risk students, there were opportunities for students to engage in meaningful problem solving activity.

One of the mathematics professors made a distinction about the level of verbal and written communication that were taking place in the projects: "Understanding why the solution works is much more interesting than solving the problem. ... We're actually asking them about understanding. We never asked that question before. We said can you do this problem, and we didn't require that they do it neatly or anything." Students discussed (sometimes argued) the logic of possible solutions. There were many opportunities for students to make conjectures and then test them. Defending a position required students to reason mathematically.

Teachers were seeing the computer as something other than a way to keep track of student grades. Some of the projects were developed around student use of computers and calculators, and this had caused many of the mathematics teachers to venture into the unfamiliar waters of technology. One teacher confessed that "most of the rest of us would not go near the computer room ... the projects opened [technology] to us." As more and more teachers used the computer lab and graphing calculators, others were getting the
courage to learn more. For example, one of the co-chairs of the department had developed a graphing calculator training packet for her colleagues. Usage of technology was increasing to the point where teachers were wanting more and better equipped computer labs and graphing calculators for all students in Algebra 2 classes or higher.

A spin off from the mathematical tasks which were developed as projects was the search for mathematics resource (or text) books which made better mathematical connections. One geometry teacher was using Serra’s (1991) geometry text, and other teachers were starting to move away from an emphasis on two-column proofs. Teachers complained about the need to use old textbooks (1982 copyright in one case), a problem created because of the district’s low per capita funding. One teacher said, "My textbook is difficult for the students to read, so I do a lot more with it. I really don’t like it, but, you know, you just adjust." Teachers were looking for better books than the ones that had the traditional "three word problems at the end of the exercises."

Mathematics teachers perceived that using projects in their mathematics classes was changing student attitudes. Student reactions to the projects and themes were positive. Teachers believed students remembered mathematics better when they learned it through doing projects. "You have a better chance of passing," a student remarked. Another who worried at first about all the writing requirements in the projects admitted, "It turned out to be really fun though, I learned a lot." Another offered: "It helps me to see what it has to do with my life." A perception which some teachers shared was that they may not cover quite as much material by using the projects, but the quality of instruction was much better.

Teachers expected to continue developing the projects even after the funding ends. They attributed many improvements in their pedagogy to the projects, improvements which they wanted to develop further (e.g., working as a facilitator among group activity, improving their skills and the skills of students in developing mathematical discourse, getting better with the use of technology). Several teachers echoed the sentiments of one teacher who wanted to expand the use of the projects: "I think my next goal is bringing it
down so my remedial classes have some success because they have not been part of the projects program."

Green Hills High School. According to teachers, mathematics was now much more student oriented. "You've got to find a level where you can take them where they can find some success." Technology provided a means of extending mathematical concepts which before would have not been possible using paper and pencil techniques. Understanding why a particular solution worked became more important than the solution itself. One teacher observed: "Our curriculum is saying to students why this is important, why you need to know this, what is the reasoning behind it, and I think that is important."

The three different general mathematics classes had been eliminated, and the lower-ability students were now taking the three-year sequence, Integrated Mathematics 1, 2, 3 which emphasized algebra, informal geometry, and probability and statistics. Another change was the Algebra III/Trigonometry course, a course designed "for students who really aren't quite ready for Pre-Calculus but they're still hungry for more math." The new K-12 curriculum was closely aligned to both the State Mathematics Guidelines and the NCTM Standards but also included Standards-recommended changes in pedagogy. The mathematics curriculum did not depend upon a particular textbook (or series) but represented a two-year effort by teacher research and writing teams. Content and sequencing of courses had also changed dramatically (e.g., discrete mathematics was a strand that was now in all of the mathematics courses). Though tracking still occurred, the objectives for a particular course (e.g., geometry), was the same across the levels.

With a greater emphasis on problem solving, less learning time was being used for drill and practice and rote memorization. Teachers observed that the UCSMP books have eliminated, in their judgement, the unnecessary content and followed closely the Standards recommendations.

Communication in mathematics took place in different ways. Students explained what they find displayed on graphing calculators. Students were expected to do more writing in
communicating their ideas and understandings of mathematics. Mathematical discourse was a skill that some teachers were beginning to develop through practice and experience.

Students were making connections between mathematical concepts (e.g., equations and graphic displays, connections between plane and solid geometry, the relevance of ratios to many applications of mathematics) to determine mathematical patterns. Many of the problems they solved dealt with real life situations. There were connections with other content areas—especially in science.

Teachers were finding that mathematical problem solving was everywhere. A teacher observed: "I find now that everything is a story problem." Students were spending less time on drill and practice. Instead, they were being presented with a variety of problem-solving situations. Problem-solving activities were in evidence during the site visit such as folding of origami paper to discuss such concepts as axes of symmetry, geometric figure identification, central angles, interior angles, and how to construct certain sized angles or designing a six-pack designer wrapper. One teacher avoided giving students formulas: "I just talk about the process and try to get them to think, because if they could think through the process they'll remember what they need to know."

Allowing students opportunities to reason mathematically was observed by documenters as one of the most difficult areas for teachers. Teachers, who for years had been ready with answers to student questions, were finding it difficult to allow students time to reflect sufficiently on mathematics problems. Making conjectures and testing those conjectures was still teacher driven. Varying degrees of discourse which would foster mathematics reasoning were observed by documenters.

"There's so many things there's no point in spending class time doing them because you have a tool that you can use that does that for you. You can put your energies towards the problem solving aspects of it." This statement by a teacher was indicative of the direction the teachers were following. The use of calculators and computers allowed for extension and more in-depth probes of mathematical concepts that were impossible a few
years ago. For example, the calculator allowed students to work with larger dimensioned matrices and to explore exponential and polynomial equations that would have been too complex and time consuming using paper and pencil methods. And these explorations were occurring in earlier courses (e.g., in Algebra 1 instead of Algebra 2) in a student's mathematical career.

The mathematics coordinator indicated that there was no single textbook that covered all the objectives of any course, and that teachers needed to "go out and find something that will support that vision." Teachers were realizing that it was more important to teach objectives and not blindly follow a book. A wide range of other materials were being used in mathematics classes: tissue paper, newsprint, scissors, soda cans, rulers, calipers, wrapping paper, foil, tape, CSL Kit (lights, mallet, hole punch, batteries, desk protector pads), paper patterns. Other mathematics books were used as supplements to the texts. Many of these additional materials were used in conjunction with the calculators or computers.

Teachers were aware that change would be continuous. Through the process of continuous program evaluation, shifts would be made as teachers' experiences with the new curriculum influenced future teaching decisions. No one at the site indicated they felt the program was complete. Teachers saw the need to expand their knowledge of mathematics, particularly in the area of technology. Some teachers recognized a need to more fully integrate student learning and teaching with technology.

**Pinewood High School.** The administration (primarily the mathematics supervisor) had a clear vision for the mathematics program in the district. This vision included providing a non-elitist mathematics program for all students. This vision meant that a "constructivist" pedagogy would extend beyond the reach of just college bound students to include all students. As part of that vision all students would take both Algebra and Geometry courses in high school. The mathematics supervisor believed it was important to "empower teachers to be instructional decision makers and to be accountable for the
decisions they make." She strongly believed that using a single textbook for a course did not empower teachers, that they should use multiple resources (which are provided) to support the mathematics program which follows the Standards and had school board approval. Her vision was "whole math programs, not separate courses."

Teachers, on the other hand, did not seem to have a big picture of where they wanted their reform efforts to go. They placed great importance on the day to day problems they faced and had difficulty seeing beyond them. They professed the need for making mathematics relevant and based on real life applications, but documenters observed that that rarely showed up in actual classroom practice. There was a preoccupation by teachers to stress simple mathematics skills and back away from a mathematics program that would allow students to experience mathematics as something to explore, to conjecture or speculate about with peers and the teacher. Teachers seemed to be satisfied if students could get the correct answers and score well on tests. It did not seem important to the teachers that students demonstrate an understanding of what they had done to come up with answers. Though some teachers were discovering that working with students in groups had positive implications for learning, teachers felt that mathematical tasks (e.g., those provided by the Pacesetter program) were too long and complex and should be broken down into smaller pieces--ones which emphasized learning basic skills.

There were major changes taking place in the Algebra 1 and Geometry courses as a result of the Equity 2000 program. Teachers talked of plans to provide classes with flexible time arrangements for students having difficulty getting through the algebra material, schedules which would allow students to receive additional help in larger blocks of time. In that way, students could still finish an algebra course in one year. Much of the content of Algebra 2 courses had been shifted down to Algebra 1. The Pacesetter course was a capstone experience in mathematics with an intended level about the same as a pre-calculus class. The course was offered to those seniors who were not taking calculus or the regular pre-calculus classes. A Pacesetter teacher commented: "This is an excellent
capstone class. It's a real good class for seniors who have all kinds of misconceptions of math, and who have had problems in math. It's a good place for them to find themselves."

There was no curriculum guide which detailed the mathematics content to be covered in each class. There was, instead, a four-page document that identified the student outcomes which were expected in the program. According to the mathematics supervisor, "they're broad, big idea outcomes." The Criterion Referenced Tests, recently developed at the county level, were designed to drive the instruction identified by the objectives.

One of the Pacesetter teachers believed an important part of the program was the emphasis on getting students to discuss mathematics. It allowed students to get over their fear of discussing what they were thinking and asking questions about things they didn't understand. Reading mathematics was also becoming more important as students worked together in groups. Students were developing writing skills too. In some mathematics classes they were writing in notebooks and keeping portfolios, and relating mathematics to other subjects.

The mathematics supervisor believed that unless students saw the relevance of mathematics, they got turned off with mathematics. She said,

That was one thing we really wanted to impress upon our teachers, that unless you can have the students buy in early, and they see that this is connecting with the lives they lead right now, non-traditional algebra takers are going to have problems. We've got to find a way of making the mathematics accessible to them.

Observations by documenters indicated that, except for the Pacesetter classes, the depth of these connections was very shallow and required that students go through very few steps to solve problems. A team of teachers from the mathematics department and the vocational education department met several times to integrate practical applications into the mathematics program, but many of these connections were for warm-up activity in mathematics classes.

Interviews with teachers revealed that the notion of "problem solving" had a wide range of interpretation. One teacher saw it as simply the ability to solve an equation.
Pacesetter teachers saw the need to reduce the length and complexity of problems that are available through the program. "In the Pacesetter you don't cover as much material. I think it can work, the tasks are long and laborious, they go on forever, and that's the short one. But they go on for so long that the kids kind of get lost and lose focus of what they are really trying to do." Other teachers felt that incoming ninth graders had no experience doing word problems, which slowed down the process of problem solving. "It's appalling that kids come to ninth grade, and they don't know how to think," said one teacher.

Some teachers were breaking away from traditional models of teaching and encouraging students to explore multiple ways of coming up with solutions. The mathematics coordinator at the high school believed it was important to "follow all the information" in reaching their solutions. Creating this environment for multiple reasoning was developing thinking skills in the students.

There was resistance by some teachers when it came to learning how to use calculators and computers, even when they were given the opportunity. The mathematics supervisor indicated that some teachers lacked confidence in their mathematics abilities and it became an issue that they needed to know more than their students knew. Other teachers were enthusiastic about the possibilities of using graphing calculators in their mathematics classes and learning alongside their students. Classroom sets of graphing calculators had been made available to students in the Equity 2000 program, and some teachers were requiring that students purchase their own. The calculators allowed students to work with graphing patterns without getting lost in the detail of graphing all the points. Teachers were encouraged to use the computer lab in their mathematics classes, but teachers were frustrated because the lab was seldom available. A Pacesetter student reported: "We make charts and take regression of charts and find out the various formulas, make graphs, we do everything with the calculators."

Textbooks had become a real issue at the school. Teachers cited the difficulties in trying to teach Algebra 1 without textbooks. At the time of the documenters' visit, there
had been no agreement on a text that would be suitable to meet the objectives of the mathematics program. The mathematics supervisor and other administrators felt that mathematics teachers would need to draw upon several resources, and, as professionals, the teachers should be able to do that. Three principal text resources had been provided along with multiple supplementary materials. The mathematics supervisor used the Professional Standards for Teaching Mathematics to support her position that teachers should be informed decision-makers and not depend upon a single text.

Teachers were beginning to see that "algebra for all" was manageable. As one teacher observed:

There are still some who feel, "Boy this is just not a task we can deal with," but with what I've seen this year, I think that more of them will come over to this side and say, "Okay, this is okay." Teachers just felt they would not be able to teach all students algebra. But they are finding that they can, almost as easy as division if they go about it in a different manner.

Some teachers were finding that using manipulatives made it easier to teach and easier for some students to learn mathematics. "A lot of kids won't get it any other way."

Teachers felt they needed a text in each course, while the administration indicated a need for teachers to use multiple resources to match the objectives outlined in the Standards and in the district's mathematics program. Teachers who had never taught without a text or learned without a text were finding this transition difficult. According to one teacher, "I think the fact is that we really don't have any input. We can't say this is not working. Can we do it this way? The county has already bought us into this, everybody doing everything the same way."

The administration perceived the challenges in getting teachers to change their beliefs about learning and teaching as being nearly insurmountable. Inservice opportunities were being made available to teachers through the Equity 2000 and Pacesetter programs, but change would be slow for teachers who had experience only with traditional, teacher-directed mathematics. The mathematics supervisor was exploring a number of ways to give teachers greater responsibility in developing the district's mathematics program. One
way was to have teachers conduct their own inservice workshops. One teacher expressed the confusion that teachers faced as they struggled with change: "But I feel kind of bad in the fact that we were supposed to... we were doing this Pacesetter program and we're really not doing what they asked us to do. I feel bad about it. But a whole lot of good stuff has come out of this. The kids have learned a whole lot in math, they really have."

**Scottsville High School.** Mathematics for everyone was the vision that many mathematics teachers had, and they wanted to see that every student performed "up to their capabilities." They saw this as being possible through the use of technology in many lab opportunities in mathematics. Teachers saw the importance of "making mathematics live" for the students by constantly striving to make mathematical connections to real life. Pulling in an opposite direction was the need expressed by some teachers to address skills development in the basic concepts of mathematics. A longer range vision of the mathematics supervisor was calculus for everybody by the year 2000. He says, "We can't keep adding. We have to do some subtracting," and he believed it was important that the mathematics teachers "invent a new curriculum, new ways of teaching."

Most curriculum changes had been influenced by the mathematics supervisor's push for technology and by the utilization of the UCSMP materials. All Algebra 2 classes would be using the UCSMP materials. There had been some structural changes in some mathematics classes. There was an Algebra 2 class which takes two years to complete. It was not a slower class, but it offered additional topics in trigonometry and geometry, and a lot of problem solving. There was a computer-intensive algebra course that permits ninth-grade students who may be weak in algebraic skills to explore algebra using computer software such as Mathematica.

Much of the change in content was influenced by the presence in the school of one or more authors of the UCSMP books and materials. There were two tracks for students to pursue, the UCSMP and a more traditional skills oriented program. The content of the two tracks differed appreciably with much more time spent on skill development in the
traditional track. Because of this difference, students who moved from the UCSMP program to the traditional one faced difficulties. Teachers indicated that there was a need "to let go of some content" if you were going to use technology. Pushing students through mathematics faster was an alternative to exploring mathematics in greater depth and motivated teachers to cover the content. There was also an attempt to integrate more probability and statistics into the different mathematics courses.

The superintendent had mandated that all subject areas emphasize writing skills. Some teachers were finding that writing in mathematics was helpful in gaining an understanding of what students were thinking. The development of some verbal skills in explaining mathematics ideas and understanding in small group settings was starting to occur, but teachers recognized that this was an area that needed a lot more attention.

Making the connection of mathematics to technology was the primary focus of the program. Connections to real-life applications and connections of mathematics to other areas such as science, social sciences (consumerism), and vocational education were also aims of the program, though this aspect of the program was very new. Teachers believed the UCSMP books had an abundance of problem solving for students, but teachers were constantly looking for applications to tie in technology to further extend student exposure to problem solving opportunities.

Teachers gave credit to the mathematics supervisor for pushing hard for state of the art technology in the school, and he supported the teachers in providing opportunities for teachers to learn about the computers and the calculators. There were some who saw all of this technology as producing a constant pressure to just keep up with others in the department. The in-house study groups provided teachers with opportunities to learn and discuss the technology. The UCSMP materials were seen as particularly well suited for the application of technology. Teachers felt a responsibility to prepare students to take their places in a world of technology. They were finding that as they gained more experience at working technology into their mathematics that they got better at it, and they found
additional ways to incorporate technology the next year. One teacher summed it up: "I think we are trying to be careful to use technology to aid instruction and not to use it for technology's sake." Student reaction to the technology used in their mathematics classes seemed very positive. Some students had indicated that they didn't use it as often as they would like. The school board was very supportive of requests to provide technology and adequate technological training for teachers.

Teachers who taught a particular course usually recommended certain textbooks for selection. The mathematics supervisor generally had final say on textbook selection. More and more of the courses were using the UCSMP texts. The Forester books have application problems which teachers had found helpful, but the UCSMP books had a consistency and continuity the teachers appreciated. According to the teachers, students were learning to read mathematics and seemed to enjoy the mix of activities in the UCSMP materials.

Teachers perceived of themselves and their mathematics program as being ahead of other schools. "I really don't have a good pulse, but I have a feeling we are way ahead of everybody." Mathematics teachers saw their program as very demanding, a program which required teachers to keep up with rapidly advancing technology. Much of what a teacher understood mathematically was developed while they were teaching. Sharing among teachers helped, and the development of new mathematics application was often done collaboratively. Some teachers found it easier just to create their own application problems than to find ones which later needed to be modified. There are teachers who feel that all the various parts of the program needed to be organized into some coherent whole to avoid replication of effort. Teachers saw themselves as "people who are willing to try things if they think it's going to help the students to learn." Their perceptions were that students were sharing and discussing mathematics together, especially through the use of technology. "Technology teaches them to discover and to be able to conjecture."

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Teachers felt that in order "to do it right--it takes a long time." They realized they must personally develop skills in the use of technology and in facilitating cooperative learning groups. There was concern that some students had been pushed "everywhere so hard that there is a core of kids ... that gets blown away a little bit, particularly in terms of appreciating mathematics for what it is." Teachers were concerned that these students would come to think of mathematics as "more drudgery." Sometimes it was difficult for teachers to see where they wanted students to go with their mathematics, "because we are changing all the time."

Cluster 3: Pedagogical Vision

Documenters were asked to describe the pedagogical vision held, relative to mathematics, by the people in the site. That required documenters to look at the pedagogical philosophy being articulated in the site and describe any observations which illustrated that the pedagogical vision was being brought to life in the classroom. They were looking for the approaches, strategies, and ways of teaching mathematics that those at the site valued. Documenters were asked to determine and describe the level of alignment that existed among teachers and administrators around that vision. The codes comprising this cluster include:

- pedagogical vision
- evidence of mathematical discourse
- examples of pedagogy
- mathematical tasks
- teachers as risk takers
- site view of pedagogical changes

East Collins High School. There were several things teachers at ECHS included in their pedagogical vision: develop a program to get students more involved in more problem solving; reduce students' mathematics anxiety by providing more comfortable surroundings for learning to take place; make mathematics interesting so students will become more confident in using math and excited enough about mathematics to want to pursue higher levels of mathematics; and help students develop critical thinking skills.
Teachers tried to achieve that vision in a variety of ways. Group activity took place in just about every class, except on review days. Since students were working in the UCSMP books, they spent some time in their groups reading the current material before they began discussing problems with their partner or other members of the group. In most classes, students first had an opportunity to go over the previous night's homework as a whole class. Generally students found group activity helpful and more relaxing. Group activity in some classes was highly organized, with each student assuming a different responsibility. Students engaged in regular lab activities (at least one per week) in which they worked with hands-on materials, calculators, or computers. For example, students could be found sitting on carpeted floors rolling marbles down inclines; working with blocks; gathering data from the color of clothing; and generating a rule based on their conjectures. Some teachers expected students to keep notebooks or portfolios. Many discussions were interactive ones in which students engaged each other as well as the teacher. Students often turned mathematics into stories so that students were also developing writing skills in mathematics. The use of graphic calculators allowed students to work with more difficult kinds of problems.

Teachers frequently used the UCSMP activities books, but they often brought mathematical tasks from other sources. They felt that such mathematical tasks were part of their program even if they were not part of the UCSMP program. Teachers made it clear that their mathematics program was not limited to a textbook series.

Students indicated they liked the relaxed atmosphere that existed in the groups, a place where they could "chat" about mathematics together. In one class, students had developed a radio commentary for a turtle race (which was described mathematically by a quadratic equation). Students engaged interactively during the commentary using their graphing calculators. Another class also used teacher-led discourse in discussing the relationships (or lack of them) between areas and perimeters of various rectangles. Another teacher was
observed asking her students a lot of questions to "bring the students in" as active participants in mathematical discourse.

Since teachers were spending less time in front of the class lecturing, they felt they were getting to know their students better. The facilitative role of the teacher allowed more time for the one-on-one communication between teacher and student. Teachers noted big changes in the way they taught when compared to a year or two ago. They were empowering students to take on responsibility for their own learning, and were seeing students who hated math in the past starting to like it. Those who had special training in working with cooperative groups found it helpful in bringing about major changes in their roles as classroom teachers. Teachers saw themselves as frequently moving between the roles of an information dispenser (especially on review days) and of a facilitator. One teacher said that "seeing things from the kids' side of it" helped make changing easier. Another said: "I saw a quote somewhere that I really like. It said something like the best teaching is done when we have more questions than answers." Another: "In my regular classes (those not yet included in the reform effort), I feel that I'm teaching students to shoe horses when they are going to be driving cars."

Administrators saw the self-esteem of teachers growing since the start of the program, which, in turn, allowed teachers the confidence to try new things. The cooperation and collaboration among teachers had made it safe for teachers to take risks. The department chair described the mathematics teachers as being open to new ideas, and teachers shared their failures as well as successes. "I'm learning constantly," seemed to be the attitude of the teachers.

Desert View High School. The chair of the mathematics department saw the student projects as a means of accomplishing several pedagogical goals: students learned to work cooperatively on long-term projects and shared in writing up results; working in groups also related to reality in the world of work; the project activity moved students beyond thinking of mathematics as "just a regurgitation of mathematical facts and algorithms," and
students began to see the usefulness of mathematics. "It means converting math to something real," says one teacher who helped get the program started. Part of the pedagogical vision was to let students know that the teachers cared about them and letting students take more responsibility for their learning. "Reaching all of the kids mathematically," was repeated time and again by teachers as one of their more important goals.

Pedagogy was essentially broken into two pieces: the pedagogy employed when students worked with projects and the pedagogy teachers used for "regular" classwork. Projects required that students collaborate in groups, and teachers provided three or four days of class time for students to work in class. Students were expected to get together outside of regular classtime to finalize preparation of the projects. Peer editing was commonplace, and students had several opportunities to improve their finished product before turning it in as a group project. The teacher's role during these classes was a facilitative one. Interestingly, much of the pedagogy thought to be appropriate for the projects also spilled over into the "regular" mathematics class activity. Except for days when teachers reviewed for tests, students were given a large portion of each class period to work together in groups on daily assignments. Teacher lectures became much shorter, and the facilitative role was becoming a regular part of all classes. Writing and verbalizing became important components of the projects, and teachers praised the improvement they had seen in students' written and verbal skills. Teachers saw vast improvement in students' writing and verbal skills in expressing mathematical ideas—especially when comparing early efforts of students who are now in their third year of working with the projects. Many teachers required student journals or portfolios.

With over 100 projects developed by the documenters' second visit, the repertoire of mathematical tasks was growing to include all mathematics classes being taught in the high school. There was a wide range of tasks which were observed. A sampling included: determining the cost of painting a king's palace; developing flooring patterns using
tessellations; making discoveries about polyhedra using paper models; performing experiments in which students had to measure and collect data and then analyze and report results; using biorhythm to optimize sporting and academic events, and the list went on and on.

The mathematics projects allowed many opportunities for rich mathematical discourse. Teachers often posed problems that would require a long-term (2 to 4 weeks) effort from student groups. Since the projects never had single-answer results, students had to engage in meaningful discourse with each other in order to develop solution strategies—which differed from group to group. Students often introduced and summarized their mathematics solution and understanding through the use of a story-line. Teachers were observed leading students with questions rather than giving answers. "We're actually asking students about understanding," said one of the mathematics professors who collaborated in the project program.

Students, at first cautious about the projects, had come to enjoy them, and students who were not successful before were finding real success. They liked working together, and mathematics learning was more enjoyable. "The kids are freer to communicate something that they are not understanding." Many cited mathematics as their favorite subject. As one of the university mathematics professors saw it: "I see the projects as a learning environment." Students saw mathematics learning as something besides doing thousands of homework problems. A teacher observed that the students did not throw their "priceless" projects in the trash after they had been assessed. Teachers noted that they began to shift their pedagogical positions three years ago, when the projects started. Teachers were finding that "students will give 105%" if they thought the teachers really cared about them. Teachers were finding that they understood their students better than ever before.

Teachers in this school worked closely as a team and shared successes and failures. When the program started its first year of implementation, only three teachers were
involved in working with the projects. Three years later all of the teachers in the mathematics department were active participants and supporters of the project concept. The level at which teachers took risks varied. As an assistant principal observed: "I think it was contagious. I think people saw that it's not going to bite you. Teachers are gaining a level of expertise that you can only get by trying it, attempting it." Teachers felt that the administrative support to experiment with new ideas had developed in them the habit where "it's almost expected" that they were going to be trying new things. "The mathematics teachers in this school are always first to jump up and say, 'I'll try it.'"

Green Hills High School. The state and local curriculum guides embodied the Standards and have a strong focus on the learner, equity in opportunities to learn, hands-on active learning, and the use of technology. A total integration of all subject areas through technology was an ultimate goal of the district. Some teachers felt the new curriculum was unique because instructional practice had also been revised. Group activity was a part of pedagogical vision, but much of the observed group activity was primarily limited to students checking each other's work. Additional opportunities for students to learn mathematics were provided through evening tutoring sessions and the availability of a Math Resource Room which was staffed throughout the day and after school. Teachers saw more problem solving and less drill and practice as part of the vision, and some teachers indicated that they needed to be willing to take more risks in their teaching to accomplish this vision. "Mathematics should be enjoyable, and it is not going to be enjoyable if they don't like it," said one teacher.

Teachers attempted to move to new pedagogical experiences for students. Some indicated the need to be flexible and provide for those spur-of-the-moment experiences or those activities where students have hands-on experiences that add interest to mathematics: measuring the depth of snow on a snowy day; designing and making six-pack beverage wrappers; developing circuit boards to check answers to homework; or working with folded paper models of geometric solids. Much of the pedagogy centered around work in
textbooks. Textbooks have improved a great deal, according to teachers. Teachers described the new texts as going beyond simple numerical answers and leading students to delve into multiple solutions and mathematical processes such as making conjectures and problem solving. Some classes were highly structured, and this was observed even in classes where students were working with manipulatives. Teachers demonstrated how to use the manipulatives first, with students following in precise order what the teacher demonstrated. However, there were indications that teachers generally were moving away from a pedagogy where they gave students all the information they would need to solve a particular problem. They also indicated that they were moving away from drill and practice to more meaningful problem-solving activities.

Observations revealed that mathematical discourse was limited only by teacher expectations. If a teacher asked questions that elicited only short answers, that was what the students gave. Deeper levels of discourse existed within student groups where group members had to negotiate solutions with each other and reach group consensus. Again, this activity varied among classes. Some students reported that group activity was very frequent "nine out of ten days" while others indicated they had group activity only on rare occasions. Some teachers saw students as being the instigators of teacher-student discourse. For example, in describing the successful student, one teacher responded that it was a student who was "successful at something, they talk about it, they seek out other people to help them out." Much group activity centered around students checking each other's work, which was assigned to be done individually. There was little mathematical discourse in such groups.

There were various levels of mathematics tasks which were observed. Some of the activities in which students engaged required that students make conjectures from their explorations. Most tasks are modeled by teachers first, then followed by the students. Teachers were looking for a variety of mathematical tasks to include in their mathematics classes, and some teachers were starting to allow students to assume more responsibility in

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exploring these mathematical tasks. Some teachers indicated a reluctance in providing
mathematics projects for students because it means giving up time which could be used to
cover important topics. There was a sense that if only more time were available, more
mathematical tasks would be provided. Some tasks which teachers thought students would
think were "silly" turned out to be well received by students. Writing was also becoming
an important feature in some of these tasks. Documenters observed that most mathematical
tasks were not designed for independent exploration by individual students or groups of
students.

How did teachers perceive the pedagogical changes taking place? Teachers saw
themselves as being willing to go outside the texts to find resources, as becoming more
tolerant of multiple ways of getting answers, as looking at process rather than just
answers, as relinquishing more control to students, and as offering more support and
encouragement to students than they used to. Teachers felt they needed to take more risks,
especially in letting students assume more responsibility for their own learning. According
to one teacher, "I say, 'This is your classroom and there's a question on the floor, don't
look at me for the answer. I want you as a class to bring this discussion to a solution,
and it's your class and how much you get out of it depends on what you put into it.' Not
as, 'What you should do is sit back and wait for me to give the answer to you.' So that
control needs to be kind of let out." Some teachers identified their peers as "being afraid to
take a risk" because of concerns they had that students will discover there are things they
didn't know about mathematics.

Pinewood High School. The administration (county mathematics supervisor, school
mathematics coordinator, principal) saw constructivism as outlined in the Standards as the
basis for a pedagogical vision in which students would find mathematics interesting and
even exciting. This would be accomplished through the use of technology, small-group
instruction, manipulatives, and connections to real-life problems. There was evidence that
teachers were starting to move in a similar direction, but teachers were struggling with their
own vision which included years of experience dominated by drill and practice on
fundamental mathematics skills. Sometimes, group work was perceived by teachers as a
convenience for taking care of classroom routine such as checking homework rather than
the construction of learning by students. In the Pacesetter classes, students focused on
problem solving activities that involved making conjectures and doing mathematical
explorations. Even in these classes, there was a strong pull to insert the basic skills
practice on top of the planned curriculum.

Most teachers used small groups all of the time or at least part of the time. Teachers
found that an effective use of their time was to explain something to one person in a group,
who, in turn, explained it to the other group members. Some teachers felt the need to be
flexible and present mathematics in different ways so all students could grasp it: a lot of
drawing on the board and use manipulatives for the visual learners; verbalization for those
who learn auditorially; and for some students, "they just need that one on one." One
teacher played music in the background. If the students got too noisy, off went the music.
"It works pretty well," she explained. Notebooks and portfolios were required in some
classes. In several classes, teacher presentations were very structured, and teachers
expected students to follow their instructions precisely.

Typically a class consisted of a warm-up, discussion and collection of homework,
students doing work on new material in groups, and students coming back together as a
single group at the end of the class period to discuss current classwork. In the Pacesetter
classes, a task set took several weeks to complete. The Pacesetter program also
emphasized problem-solving activity almost exclusively. There was an attempt being made
to make mathematics more relevant through a collaborative effort with the vocational
education department.

There were examples of tasks that teachers were starting to use, especially in the
Equity 2000 Algebra I and Geometry classes and the task sets in the Pacesetter program.
Documenters observed that "the communication working in groups was quite active.
Students were engaged throughout the period on their mathematics. Students used math vocabulary, showed patience in helping each other, treated each other with respect, and did not seem to be afraid to ask peers for help." Students asked a variety of questions both of the teacher and of their peers. This active discourse was not present in all classes, however. Some teachers observed it was much slower to develop student discussion in the Algebra II classes than the Pacesetter classes (where students "argue back and forth" in a constructive manner about mathematics). One teacher who loved to lecture, observed that "the better they get at this, the less they need us [to lecture]." Some teachers avoided giving the answers to student questions. Instead they asked questions that would encourage students to answer their own questions. A student indicated, "We're not just solving problems; we learn how to explain problems and the steps that we go through to solve them so we understand it. I like it. I understand it more."

Teachers saw several advantages to the changes in teaching practice: group work was better for building student confidence and socialization skills; it also provided a means for students to check homework and get questions answered by others in the group, thus freeing the teacher from having to explain problems. Pacesetter teachers saw many positive aspects of the program, but felt the task sets were too long.

Teachers were starting to use manipulatives (algebra tiles to teach multiplication of binomials and factoring; spaghetti pieces to determine the possibility of a triangle; project dealing with pi; statistical projects where students gathered and analyzed data such as M&Ms). Portfolios and writing assignments were finding their way into mathematics classes. Over 180 activities to do at the beginning of the period had been developed by the teachers (some through a collaborative effort with the vocational education department). Some teachers equated the new Equity 2000 program to the elimination of the algebra textbooks. As a result they saw this as a great inconvenience (e.g., difficulty with the copy machine, lack of paper supplies, etc.) rather than as an opportunity to be resourceful (as the supervisor would like to see).
Except in the Pacesetter classes, teachers were slow to take risks. Even the teacher who was held up as a model by the administration had some reservations—especially in relinquishing some parts of his instructional program such as basic skills development. However, this teacher was impatient with colleagues for not getting more actively involved in the change process (e.g., not going to conferences and workshops which teach skills in using calculators and group techniques). Several teachers were unwilling to provide anything other than a highly-structured instruction to their students—whom they expected to model step by step.

Scottsville High School. The pedagogical vision seemed to be an imposed one, and did not appear to originate from the teachers. Writing across the subject areas was prompted by the superintendent; the abundant use of technology in mathematics classes originated with the mathematics supervisor; the emphasis on mathematical applications through student reading and discussion came from those who authored the UCSMP program. While teachers did create tasks for students, there was no clear cut pedagogical vision found in their remarks. Teacher interviews revealed that many of them believed they were offering the best program to their students. The mathematics supervisor said of his teaching staff that they had talent and they "should be inventing new curriculum, new ways of teaching."

Typical pedagogical practice included an explanation by the teacher followed with several examples, and then students working individually or in groups on worksheets. While there was an emphasis on writing across all subjects, students were expected to do a lot of independent reading from the UCSMP texts. Teachers indicated a need to know more about cooperative groups. Technology was used in a variety of ways. For example, one teacher used a computerized animation to illustrate the notion of limits for finding the area under a curve. Calculators were used frequently in all classes for graphing and other computations. Computer activity consisted of students working in pairs or individually, primarily on worksheets which were teacher made or which came from UCSMP materials.
Students generally were very independent and sought little assistance from teachers during the computer lab activities. In other classes, a typical routine consisted of teachers reviewing homework, followed by explanation of new material; students given worksheets; and students presenting worksheet solutions on the board near end of class period. There was evidence that manipulatives were being used in some classes.

Teachers spent a lot of time creating new projects and tasks. Examples included an ESP experiment, science connected projects that required measurement, statistical tasks which required students to analyze data and make conclusions (e.g., find the formula from data for the state's growth rate and then making population projections), using mirrors and lenses to do variation problems, and using a seesaw for proportions and variation.

The level of discourse in several classes was the "prompting" of students by the teacher to offer the next step in a solution. The documenters noted that many students were not engaged in these solutions. Some teachers indicated that it was important for students to read the book and listen carefully in class.

Perceptions of those in the school site seemed very positive. Students indicated that there is abundant use of technology and felt fortunate to be in this particular school. Students indicated that to do well in a mathematics class, a student must listen and read carefully and keep up. Teachers saw themselves as creative, but they recognized that creativity (especially project and task development) required a lot of time. So, teachers saw their creativity being hampered by the time constraints, and many felt overworked. Teachers felt successful and considered themselves to be among the best mathematics teachers in the country. They felt that the presence of prominent mathematics educators on campus gave them positive feedback that they are on the right track and bolstered their self-confidence.

The mathematics supervisor bragged about his teachers and saw them as being very talented. With many available resources, he felt that they were and will continue to be on the "cutting edge" of reform in mathematics education. The principal perceived the
mathematics teachers as "not being afraid to try something new. If it doesn't work, that's okay." Teachers agreed. They felt that they had been empowered as professionals and were "willing to try things if they think it's going to help the student learn. "If this is going to help the kid learn, we're going to do it or we're going to try it." Teachers indicated collaboration with their colleagues was a big help and encouragement in taking these new steps.

Cluster 4: The Context in Which Change Occurs

Documenters were curious about the contextual features that influenced the changes taking place. They wanted to know what was happening with inservice, with outside consultants, with outside funding, with the school and district administration, with the community, among the teachers; what was the role of materials, scheduling and structural matters, and policies about evaluation and testing. Documenters were attempting to describe how contextual features were influencing, both positively and negatively, the teachers' efforts to change their mathematics practice (Ferrini-Mundy, 1992a).

This was by far the most difficult cluster to analyze since there were so many variables to consider when one discussed the context in which change occurs (Fullan, 1993; Sarason, 1991). The mathematics program was a part of the larger school community, and a change in one place has a rippling effect on all parts of that larger community (Fullan, 1993). Of the sixty codes used in the analysis of data, nearly half of them fell into this single cluster. The twenty-nine codes were reorganized for practical reasons into four subcategories for ease of analysis. The codes were sorted according to these subcategories as follows:

1. factors depending upon mathematics teachers and students: change is a slow process, collaboration with others, common planning time, feelings of doubt, leadership, mathematical community, pride in program, teacher difficulty in changing, teacher openness to change, teacher ownership of program, teacher preparation (academic

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preparation in mathematics), teacher role in developing curriculum, teacher support of
program, teaching experience

(2) factors depending upon support of others in the school or district that work actively in
the mathematics program: administrative support, expectation on teachers, helpful
things in developing program, impact on rest of school, scheduling changes

(3) factors depending upon available funding (i.e., materials, equipment, staff
development): additional needs, difficulties in implementation, duration of reform,
funding as an impetus, inservice development, and

(4) external factors (i.e., factors outside of the school building which influence change):
community outreach and input, community support of program, cultural diversity,
impact of testing (standardized or district testing), local and state policy

The above sort is not without fault. It can be argued that some of the codes could be
placed in one or more of the subcategories, but the choices above are based on the
frequency of actual coding of data. The choice of which category to list first is based on
level of importance, and, again, the choices are not without counter-examples. The intent
here was to manage the various code summaries in this cluster so that a coherent summary
of the cluster could be developed. These subcategories do not appear in the cluster
summaries.

**East Collins High School.** Teachers recognized that the students would need time to
adjust to the changes. Teachers also recognized that they, themselves, would require time
to make the transition to this new pedagogical vision. Sometimes feelings of doubt
prevented or slowed the change process. These doubts came in a variety of forms: older
teachers who were unwilling to take the risk included in reform; the pressures produced
when teachers are expected to develop a model that could be replicated; change may take
longer than expected, and some teachers become discouraged because it's hard "to hold the
momentum": giving up power to students was difficult too; and some students are not
always willing to accept responsibility for their own learning. "The mathematics chair

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observed this about himself: "It is interesting to note that even this teacher who is one of the prime movers for mathematics reform still makes compromises in pedagogy--often moving between roles as an information dispenser and as a facilitator." "Teachers as a group are not good changers," said the associate principal. "I would venture to say that most of us teach the way we were taught and that's one reason change is so slow. Change is frightening. You don't always have a model for that." It was very risky for teachers to take these changes lightly or make them too quickly.

Collaboration had been instrumental in assisting with change. There was the collegial collaboration between mathematics teachers in the school; the collaboration between teachers in other subject areas at the high school; and there was a collaboration that existed between schools. Collaboration was viewed by the mathematics teachers as the strongest force in bringing about change. "The best resources are just other teachers," a high school mathematics teacher remarked about her peers. During the first year of implementation, just two teachers piloted the program. They found that sharing successes and failures with each other saw them through that first year. Having been validated by others outside the department was helpful, especially that which was received from the university mathematics educators. Collaboration was nurtured when there was a common time for teachers to get together. The mathematics teachers have seen the value of having common planning time through the special funding provided by the project through inservice meetings, summer workshops, and release time. A common planning time built into the school schedule was an important goal for these teachers.

The focus of leadership at ECHS centered on two mathematics teachers, one of whom was the department chair. The county mathematics supervisor and the associate principal have also played major leadership roles, especially during the early stages of the project. They saw the wisdom in relinquishing that leadership to the two teachers who they felt would be energetic and adaptable and good spokespeople for the program. Even though the primary leadership role resided in two teachers, the other teachers felt important
members in the mathematical community. "They didn't make a move that they didn't consult with all of us." The stability of the mathematics department was also cited by the chairperson as a real advantage. The mathematical community that resulted from the reformed program expanded to include the other high school and the three middle schools.

Self-esteem resulted from the recognition that mathematics department members have received from their accomplishments with the new program. Teachers were allowed to voluntarily participate in the program, and by the third year of implementation, all teachers were participating in the new program in some way. An openness to change prevailed among the mathematics teachers, and that openness played out in sharing ideas with other mathematics teachers outside their school. The fact that administrators turned over total control of the program, including the way industry's contribution was spent, greatly empowered the teachers in this school. They saw the program as "their" program, as coming up from them and not down to them. The mathematics teachers at ECHS cared very much about their students and their program, and teachers perceived of themselves as being the ultimate authorities in knowing what was best for students in their mathematics classes.

Teachers acknowledged that their relationship with district and school administrators was unusually good. A very strong sense of trust and respect was evident in interviews with the school and district administrators. "You see, we trusted them to know what to do, to try, and to let us know if it was wanted." In the early stages of implementation, parents had strong reservations about what their children were being subjected to in the mathematics program. Teachers found support in a protective layer afforded by both levels of administration during that first year of implementation. The school's administration and counseling staff reasoned with parents to give the teachers enough time to get the program underway. The purchase of new geometry and trigonometry texts outside the normal acquisition cycle was another example of administrative support. District funding was also provided for teachers to attend workshops in Chicago on how to effectively use the new

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textbooks. A teacher remarked, "That makes us feel good, that they think we're worth it."
The county curriculum coordinator pushed hard for extra moneys for the mathematics
department to attend regional and national professional meetings of mathematics teachers.
At the school site, the associate principal was an ardent supporter of what the mathematics
department was attempting to accomplish, and her efforts paved the way for adequate
planning time, inservice development, and release time for several important work
meetings during the year.

Teachers were treated as professionals, and the expectations placed on teachers by the
school community were highly professional. These expectations included the following:
teachers should be student centered; teachers must be "responsive to parents, responsive to
criticism, responsive to sudden changes, responsive to long waits" and be flexible, patient,
and adaptable; teachers must be in charge of their classrooms and be willing to change; they
are trusted by administrators to have good judgement and be a source of good ideas and
have the confidence to express those ideas; they should plan for their classes and know
where they are going with those plans; and teachers should seek ways to maintain and
improve their professionalism. Administrators encouraged teachers to gain in
professionalism by providing funding and opportunities for them to attend regional and
national NCTM meetings and to study the Standards as they created their mathematics
program.

There was evidence that the philosophy underlying the mathematics program was
spilling over into other subject areas, and administrators were encouraging it. This was
especially true in some of the science classes. An important goal for the mathematics
teachers was to change the negative attitudes that other teachers in the school had toward
mathematics (e.g., "I was never good at math.").

Time was the biggest need of teachers at ECHS. They needed more time at school to
prepare for classes, to collaborate with each other, to investigate the availability of
materials--especially different mathematics labs and computer software, to explore and
become familiar with computer software, to pursue self-improvement in areas such as cooperative learning and student discipline, to do long-term planning individually and in groups, and to attend mathematics conferences. Some teachers also felt the class period was too short (50 minutes) and that the school year should be arranged differently with more frequent breaks for teachers and students.

There were several concerns or needs noted by teachers. Shortage of space and equipment was a problem. Either larger classrooms or smaller classes were needed. There just wasn’t room for 35 students in a classroom. Teachers in the mathematics department had to share the computer lab with other departments. They needed more lab space and more computers. They also needed rooms with tables where students could work together doing mathematics. There was a need for more graphing calculators, enough for every student to be able to check them out. There was a need to organize lab activities in a manual so that the mathematics teachers knew exactly what labs are available, how to set them up, etc. More hands-on activities were needed to add to the labs—which should be held more frequently. There was also a need for workshops during the summer to last longer than two weeks. One teacher indicated a need to get rid of irrelevant content (e.g., rationalizing denominators).

Though teachers were very satisfied with progress so far, they still felt there were a number of problems which they had to deal with. Parents saying "I can’t do mathematics" or "I was never good at math" was a big problem according to teachers. The Associate Principal told parents "that what you say influences the student more than what we say." Reading was the mainstay of the UCSMP texts, and those students who had weak reading skills posed a special challenge to teachers. According to the Associate Principal, some teachers simply did not relate well to students, and those teachers particularly found it difficult to adapt to the new pedagogy. Teachers felt the colleges and universities do not adequately develop teachers for teaching in programs like theirs. Handling assessment adequately was always a big problem.
One day each school year was provided for inservice development. Teachers complained that that was not enough. Some teachers attended summer workshops or took university classes, but the mathematics teachers felt there was additional time needed for teachers to simply get together and share ideas. The mathematics supervisor provided opportunities for many teachers to go to mathematics conferences, but teachers felt there was need for much more in the way of inservice development (e.g., a hands-on workshop on how to develop cooperative learning groups and some training on the use of different software). Some peer observations of other teachers' classrooms did occur, but those were limited to a single prep period and teachers needed to go beyond that one-period limitation.

Teachers and funders expected that it would take about three years to really get the program well established, but teachers planned to continue improving the program even after the funding runs out. The $55,000 from private industry was not a large sum of money, but teachers had complete control over how it was spent, and it allowed mathematics teachers an opportunity to "experiment" with new ideas. The funding allowed for a fully furnished computer lab and a computer in every mathematics classroom. Teachers were able to attend workshops at the University of Chicago which provided valuable information on using the UCSMP materials. The funding also served to motivate a commitment for additional funding from the school system.

Initially the outreach and input to the community was not entirely positive. Parents misconstrued the intent of the mathematics program and felt that teachers were not teaching anymore—that their children were learning mathematics on their own. Parents' early concerns were replaced with satisfaction in the program. A teacher commented: "We've also had parents tell us that they think this is the best program that they've ever heard of, and they are very excited about it." Locally, within the school, the mathematics program was impacting on curriculum reform in other departments. There were outreach activities with the middle schools and the other district high school. Industry officials, as the
principal funder of the project, and the mathematics coordinator would like to see the
program expand beyond the local level—even see it replicated nationally. The mathematics
teachers had an attitude of "can do" that allowed them to get just about anything they could
justify for the mathematics program.

The mathematics teachers were confident that their students would do well on
standardized testing. One teacher said: "I think we're going to see our scores go up, and I
think it's going to make a lot of people very happy. I wish it was not an issue." Another
teacher said: "My personal opinion is that [our program] will make their scores go up."
But there was some concern over statewide testing being mandated before students were
allowed to graduate from high school. Teachers indicated that they would teach to the test
in preparing students. Teachers were concerned that politicians made important educational
decisions—especially about testing—and they knew very little about what they were doing.
The state mandated that each student who graduates must have three years of mathematics
and Algebra or Applied Math must be included. Applied Math requires a pre-algebra
(regular algebra preferred) preparation. The school district was required to adopt texts that
were on the state list and which met the district's objectives. Most of the mathematics
classes were designated as "lab" classes, which required that 25% of the time in class was
spent doing lab work. Besides the extra money that was available, the maximum number
of students in a lab was 28.

The area around ECHS was becoming more culturally diverse. Once a farming area,
people who worked in the nearby metropolitan area were moving to the community:
factory workers, airline employees, lawyers. School administrators indicated that most of
the African-American students (32%) lived in government subsidized housing.

Desert View High School. There was an appreciation expressed by all teachers for
being a part of "such a good staff." Documenters observed a very friendly, close-knit
group of teachers who expressed wit and humor towards each other in a very open and
non-threatening way. There was, however, a "wait and see" attitude among most of the

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mathematics teachers when three of the teachers began to pilot the program, and it took three years of collaborative effort before all the teachers were using projects. Some teachers were afraid to take risks with a pedagogy that was so foreign to them. Getting past "being afraid to criticize or afraid about damaging egos" was a big but important step for these teachers. This friendly collaboration extended to the mathematics professors at the nearby university. They met one day every other month with the professors to share and discuss ideas for improving their high school mathematics program. One of the mathematics professors commented: "We were offering the pedagogy and the mathematical strength, and they were offering teaching experience and understanding of the society involved." The willingness of the professors to actually observe high schools mathematics classes helped to foster the collaborative mood.

The adjustment to mathematics projects took time for the students, too. Teachers who had experienced working with projects in their classes noted that students who had been in projects classes previously required little time to adapt to a different mathematics class that also used projects. There was a longer adjustment for students during their first year of exposure to the projects, especially in developing their writing skills. The group activity, which became necessary in working with projects, was spilling over into the regular mathematics class work, and the role of the teacher was gradually shifting to a more facilitative one.

"Shared leadership" was the term that documenters used to describe the leadership at Desert View High School. Initially three teachers took the lead in working with the mathematics professors at the university. Later, as most of the teachers began participating in using and developing the projects, the co-chairs took an administrative leadership role, but the leadership for the reform effort was an equally shared event among those who participated. Teachers at Desert View felt that the funding provided by the NSF teacher enhancement grant has allowed them opportunities for common planning times. It was because of the value they saw in these sharing times that they did not want to lose those
opportunities. They are looking for other outside funding to continue to support that part of the project in a regularly scheduled way.

When the partnership with the university mathematics professors began, the mathematical community within the high school strengthened at a professional level through a sharing of each other's ideas. The community of mathematics teachers began to expand and include the other two high schools and middle schools in the district. Showing their projects to teachers outside Desert View High provided a vehicle for enhancing self-esteem among the Desert View mathematics teachers. They were invited to speak at mathematics conferences in the state, and the two co-chairs made a presentation at a regional MAA meeting held in Cincinnati. The mathematics professors were putting all the projects together for publication as a book. Every teacher with whom the documenters talked displayed a sense of pride in what was being accomplished.

An openness to change evolved with the program. An important part of this openness to change was the humanizing effect it has had on students. Through the constant sharing among teachers of successes and failures, teachers have developed a support mechanism that has made mistakes or failure less threatening. They found that some things that worked for one teacher (or class of students) would not work for another. Teachers found that collaboration consists of two pieces, "creating" the projects and "using" the projects. A couple of teachers almost ready to retire were also caught up in the department's enthusiasm. Recognition helped too, particularly the validation they received from the mathematics professors at the university.

District and school administrators prided themselves on treating teachers as professionals. They believed the mathematics teachers were the best qualified to make decisions regarding content and pedagogy. Teachers also viewed themselves as professionals with expertise in mathematics. The mathematics professors from the local university reinforced the notion of professionalism. According to the program's director, "I may have been the first person who ever called them professional mathematicians. And

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they were all taken aback. I told them they get paid for doing mathematics. I don’t know what else a professional mathematician is.” Even though teaching experience ranged from 4 to 28 years, teachers recognized that their pre-service training had not prepared them for the new pedagogy demanded by the projects, and preparing to teach in new ways became a big part of their inservice preparation. Much of that preparation came as a result of stipends from the teacher-enhancement grant that allowed teachers to participate in a month-long summer workshop at the local university. As new mathematics teachers were hired, they undertook staff development to prepare them for working in a “projects” environment.

The mathematics teachers at Desert View have had a great influence in developing mathematics curriculum at the school and district levels. They were well represented on the district curriculum committee and influenced all decisions made by that group. DVHS teachers described themselves as “always on the alert for something new.” The curriculum director saw the impact of what the mathematics department did in mathematics reform as being the forerunner of school restructuring at Desert View.

Support from administration was shown in the confidence they felt toward the mathematics teachers to make their own decisions about the mathematics program. Though the per student financial support is only about $2500, the curriculum director found money to support trips to NCTM conferences. This director did believe that, through state Eisenhower money, she would be able to buy some substitute time for additional release days—though not at the level afforded by the NSF grant. The expectations on teachers were for the most part self-imposed as a result of the confidence placed in them and their freedom to try out new ideas. These self-imposed expectations appeared to documenters to result from the collegiality that existed among the mathematics teachers. Some of these expectations included developing writing skills and developing verbal discourse in mathematics through group activities.

There were a number of helpful factors that contributed to development of the mathematics program and of the mathematics teachers themselves. Much of the
cohesiveness that was apparent among the mathematics teachers could be attributable to the equity in the way classroom assignments were made. Teachers were allowed to select the classes they wanted to teach. During a two-year cycle, they were guaranteed to have those choices honored. Those classes which no one chose were evenly distributed among the staff. The way the university mathematics professors gave feedback to the teachers proved a real asset. According to one of the professors: "All the ideas came out of their mouths. So when they did something good, we told everybody. And two meetings later, everybody was doing it."

The mathematics teachers at Desert View felt there were additional needs that must be addressed in order to further strengthen their program. Time was an important factor, not only for long-term planning, but also for the "day-to-day, week-to-week kind of things." They needed time to get together as teachers and share ideas, and they needed longer class periods—perhaps some sort of block scheduling. There was a need for more inservice opportunities. They needed a computer lab just for the mathematics department and appropriate software. To facilitate group activities they needed to have furniture which is conducive to group activity (e.g., tables that fit together). Teachers also felt class sizes of 29-30 students were too large for the size of the rooms. There was also a need to connect with resources outside the school, especially in terms of the uses of technology. One assistant principal saw the Carnegie units as being an impediment in changing the six-period day. He felt that some day that restriction will be removed.

There are problems which the teachers faced which makes their jobs more challenging. Lack of maturity among ninth grade students caused teachers to take a more traditional stance with the projects in Algebra I. Teachers limited the group activity and expected more to be done on an individual basis. Students' dependence on parents for transportation also made it difficult for 9th and 10th graders to meet with other group members outside regular class times. Absenteeism was a problem in classes where there was a layering of content such as algebra. It was particularly a problem among Hispanic
high school students who often stayed at home to care for younger brothers and sisters while the parents worked. It was difficult to move some teachers away from a traditional curriculum—especially in the "sacred" course of geometry. Teachers felt there was a big gap in what the university mathematics educators said are the best pedagogical practices and the actual preparation of preservice teachers.

Teachers saw the need for more and more projects; so they perceived their reform efforts as never ending. New projects were always needed to replace those that became familiar to most students. Some teachers were starting to include "themes" along with the projects. Because of the shorter time required to complete a theme (usually a week or two), some teachers were starting to include more themes as a substitute for content coverage from the textbooks. A different pedagogy was starting to emerge that is less dependent on a particular text. Teachers had not decided on a "common vision" according to the assistant principals, and that such a common vision would occur as teachers gain more expertise with the projects and themes. Teachers described their program as "always ongoing."

All teachers seemed to agree that the extra funding through the teacher enhancement grant was essential to get the projects developed. Teachers said they will continue developing new projects and extending their use even if future funding was unavailable because they discovered that "once they had the success, there were more reasons than just the money to be involved" in the projects.

The program was spreading to the middle schools, and the other two high schools in the district were showing an interest in working with projects. The program was being shared with schools in the district and throughout the state. Teachers were trying to reach parents with the message that it's not okay to keep making negative comments about mathematics to their sons and daughters. One teacher said to a group of parents, "It's not okay for you to keep saying, 'It's okay honey, you are just like me, and I didn't do well in math either.'" The outreach effects of the mathematics professors' involvement in this
project had implications that may not be easy to address. The mathematics professors recognized the importance of their roles in helping the high school teachers develop their projects program, but they were uncertain if that partnership role was necessary to the transportability of a similar program to other school settings. One of the assistant principals commented: "Well one thing is that you can take a fairly traditional high school in a very traditional setting, and you can break some ground. You can do some non-traditional things that have not been imposed by edict or force."

There was a feeling among teachers that the number of Mexican children crossing the border to attend school in the U.S. would have an increasing effect on their school and the school district. This would mean a need for greater numbers of ESL classes in mathematics. Teachers indicated that there was little impact of standardized testing on the mathematics program. They do provide a mini-course for SAT preparation. "Other than that, I don't think we teach to tests at all." The state has a three-year mathematics requirement for graduation from high school, but there is no specification as to the required content of those courses.

Green Hills High School. Most of the high school mathematics teachers in both high schools served on the research and writing committees for the new district-wide K-12 mathematics curriculum. This curriculum development required a collaboration with university professors and business people. It provided an opportunity for elementary and secondary mathematics teachers to work closely together and resulted in a mutual respect for each other's level and a common focus on what is important in a mathematics curriculum. The coordinator believed that this common focus would not have been possible without the NCTM Standards: "After these Standards came out, people started changing immediately, so it's been kind of fun to see that process." Teachers have had a great deal of input into much of the change that was taking place in the mathematics program. Much of the leadership for change came from the Mathematics Coordinator.
Change for some teachers was difficult. One difficulty, according to teachers at the school, was the reluctance of "traditionalists" to change to anything different. Teachers felt the changes would take time. Student resistance to problem solving activities was troublesome for some teachers. In spite of these difficulties, most of the mathematics teachers openly supported the change. The majority of mathematics teachers in both high schools served on the curriculum revision committee. Most of the meetings were run by teachers, and preconceived ideas for programs were not allowed. Even though that restriction was imposed by the Mathematics Coordinator, teachers, working with parents and business representatives, developed a program that was based on the Standards. Teachers did not feel that the changes were being imposed from the top. Once teachers believed in something they were willing to "go the extra mile." Teachers talk about being "sparked" by the ideas that were shared with them by a colleague.

Teachers felt that experience in implementing the newly developed program would be helpful in making adjustments later: "Why did I spend all that time doing that? I want to do some things differently next year. It just takes experience getting familiar enough to identify the things we want to change about the courses."

Teachers indicated that administrators' lack of support for inservice development was a lack of support for the changes taking place, even though the administrators verbally supported the changes. Frustration by building principals over the same people always going off to conferences led to a limit of one professional day per year. As a result, state money was lost for inservice development. According to teachers: "Of course last year when we envisioned this, we had more sub days in our contract. They took all of our sub-days away from us except one sub-day for the entire year that we can use to leave the building to do staff development or visit or anything." The principal saw his role as being "pretty much problem solver, encourager--listening, seeing what is going on." He spoke of the benefits of sending teachers to conferences and to teacher development sessions.
The documenters reported that the interviews with administrators seemed to contradict the perception of non-support at the administrative level.

Teachers spoke of the creation of the role of math coordinator as being key to the support they needed to make changes. Outside consultants were used only in the case of the state university. Six teachers attended an NSF workshop. Teachers from the high school were on the Curriculum Committee. A Teacher Center was established by the District and has a lending library of materials, video tapes for professional development and equipment to create lesson materials. Teachers felt they needed to expand their knowledge and abilities in mathematics—especially in technology. Teachers also felt that the best inservice development would be time to share ideas with peers in their school. At the present time most of the money being spent in mathematics education was for textbooks instead of on inservice development.

There was a feeling of openness that existed between teachers and the administration—especially the mathematics coordinator. Until this year, the policy of the district allowed teachers to get away from the school and see what others were doing in mathematics. Peer coaching was helpful, and teachers shared in providing coverage for their classes. Other helpful things included the excitement of most students towards mathematics; the K-12 perspective of working together. The principal felt the dedication of the mathematics teachers was the most important and helpful part of developing the program (e.g., giving up their personal time to staff the math room).

The teachers said that they felt no pressure from the administration to perform in a particular way. Expectations were self-imposed, but teachers did feel that the tradition in the school to put out good mathematics students carried a responsibility. Limitations of time and space made for some hard to manage expectations of setting up and taking down equipment and having rich mathematics experiences for students—all in an interval of 40 minutes. The principal expected teachers to be willing to take risks and try new ways of
teaching. "You've got to be a risk taker. You've got to be willing to put your neck out on the chopping block and be willing to get burned."

Some teachers felt a need for a restructuring of the class times to allow for a more flexible teaching arrangement. One teacher offered: "Look at things a little differently and realize that it's not the old ground rules we had before. The whole spectrum probably has to change, and people might need to have two planning periods and team teach with another teacher all day long in order to get this thing going the way we want it." Two new courses have been added to the curriculum: Math 1,2,3 is an integrated mathematics class for students who don't follow the college-prep sequence, and Algebra III/Trig is designed for students not quite ready for a Pre-Calculus class.

Time was the most important need mentioned by teachers. Except for those mathematics teachers who served on the district curriculum revision committees, there has been little time for teachers to do any common planning together, and this was a problem among the teachers. They cited that there was not enough time to plan the activities. Including projects, some teachers claimed, takes away the time for content coverage. They worried that testing would cover things that weren't stressed in class. Also some concern about students getting noisy and off task in group settings. Better equipped classrooms were seen as particular needs. There needed to be more time to share with each other. The mathematics coordinator's biggest wish: "I wish we could be the best math department in the nation, and I wish we had all the money we could ever have to do that." She went on to say she would spend the money on "whatever my teachers needed."

Teachers didn't feel that the new curriculum was perfect; there may need to be adjustments to the curriculum guide that has just recently been developed, but that they first needed to gain experience with the new program. No one expected the changes to be quick. According to the coordinator: "It's not something that you can jump into and you can do overnight, and I think that it's going to take a lot of people. I think it's going to take us all a few years to get to where we really want to be with the Standards. A lot of us
have talked for a long time, and it takes forever." There was a sense that change would be continuous. The impact of the present mathematics reform would take another three years to be felt in all the K-12 grades.

Community support was strong. Two million dollars was spent developing a community wide K-12 mathematics curriculum, but there were real indications of budget cutbacks. Meetings were held with parents to explain the changes taking place in the mathematics program. The coordinator for mathematics felt that they did a good job of selling the program. The principal felt that if you educate parents properly, "they will listen to change and growth." The coordinator tried to get the three A's needed for success in mathematics:

Attendance, attitude, and attention. One of them is the attitude. I think it's the very last one is that math is not hereditary, and I always read that to the parents and they laugh and I said, "I'm being quite honest here I don't know how many times I've sat through parent conferences and the first thing out of mom's and dad's mouth is, 'I wasn't very good at math.' You just lowered their expectations dramatically, just dramatically, don't tell them that."

A K-12 mathematics community began to evolve when teachers from the various grade levels served to shape a Standards-like curriculum for the district. As a result, the development of the K-12 mathematics curriculum produced a sense of community pride. People at the site reported that there was no resistance to the changes in courses and course structures developed because there had been an effort to communicate with parents with a handbook describing the mathematics program and through regular information meetings (e.g., Parent Club, Meet A Teacher Night). There was strong support in the community to keep accelerated mathematics courses, even though the teachers and the coordinator wanted to eliminate them. While the community was informed about changes, the community, including parents, residents, and business people, were actually interviewed in focus groups before the restructuring of the curriculum. The committee that was put in place to rewrite the mathematics curriculum, held a series of focus groups to listen to the community. They noted that it was the input from the business and the military.
surrounding this area that made them very aware that they must make changes and the changes suggested were consistent with the Standards. 

The State Math Test, a criterion referenced test, was started in 1993. It was Standards-based and was designed to drive instruction since the state has adopted a framework based on the Standards. Students in this school tended to score well on the Test of Academic Proficiency, but there was a move to eliminate testing of this type because of its lack of relevance to the mathematics program. The mathematics coordinator saw this kind of testing as being very political. "Tell our people something that in comparison with other people we're doing a pretty good job in these areas—that's political—that's the reality parents like to see." The principal saw the current mathematics program as being beneficial in preparing students for tests such as PSAT, SAT, and ACT.

Principal: "But if we do find objectives that we have put in the wrong places or mistakenly included, we do have the audit. The yearly audit built in that will start next fall when everybody gets together by grade level." Teachers indicated that building administrators throughout the district establish much of the local policy decisions. Their emphasis on textbooks over teacher development in pedagogical areas was viewed as short-sighted by the mathematics coordinator.

Very few minority students were in this school.

Pinewood High School. There was not overwhelming support for the new program during the first year, but teachers did ask for "another year to work out" what they thought might be improvements in the program. Some teachers resisted changing the way they taught, and some refused to use calculators in their classes. Pinewood High was one of the more progressive schools in the district. Some teachers wanted to pull back on the change process, but the mathematics supervisor felt that no progress would ever be made if she allowed teachers to pull back. One teacher explained: "They are reluctant to change right now, but they will slowly grow into it I think. They don't have much choice." Another teacher remarked: "I think if they see what else is going on, and it's working

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successfully with other people, and other people have good things to say about it, I think you can bring them around."

Each course had its own "division" meetings where teachers met every other week and shared ideas. Teachers indicated that they need more opportunities to get together and collaborate. All of the teachers that taught a particular course became a member of a division. It was through those division meetings which occurred biweekly that teachers had a common planning time. There were occasions where the administrators "baby sat" students while teachers worked together during the regular school day. There was some pairing of teachers with the electronics teacher, the electricity teacher, a drafting teacher, and a carpentry teacher to try to develop mathematics with connections. Not having a textbook in the Algebra I classes forced teachers to share ideas and materials with each other.

Teachers were required to give departmental exams in their mathematics classes, and there was a lot of pressure on teachers not to have more than one-third of their students fail or get D's. The perception of some teachers, who gave too many failing grades, was that they were punished by assigning them all lower level classes the following year, though the principal denies that this practice still existed. Sometimes, teachers felt that they don't know enough mathematics to turn students loose to explore. "I need to know more than my students know." Many doubted that just because people like Michael Serra could teach students to discover geometry, there was no reason to believe that they could do it. Fear of grouping and losing control of students bothered some teachers. Several doubted the program would work without modifying it to emphasize skill development. Teachers doubted they could adequately cover the material they were expected to cover and do all the preparation that was expected of them. There were concerns about administrators giving poor evaluations if teachers couldn't make the new methods work.

The real leader for reform was the county mathematics supervisor. She was pushing very hard to get past mathematics being so elitist. Teachers have resisted her efforts to
have a Standards-like mathematics program—especially to her insistence that a single text was not necessary.

The high school mathematics coordinator was helpful in carrying out and assisting in this mission, though teachers at times felt that they had no choice but to conform. Some teachers were beginning to see that a different pedagogy could make a difference in student learning, and several teachers were beginning to assume leadership roles within the mathematics department.

A mathematical community began to develop near the end of the first year of implementing the Equity 2000 algebra. Teachers asked for "another year to work out" what they thought might be improvements to the program, which the mathematics leaders took as a positive sign. More and more teachers were beginning to see advantages in using groups in their classes. There seems to be opportunity for teachers to have more involvement in the direction the reform effort should take, but there was some reluctance, even among new teachers, to become involved.

The mathematics supervisor was absolutely convinced that the program that had been developed was correct. She seemed to take pride in certain individuals (teachers and coordinators) who were inspiring others to become involved. Some teachers admitted there were benefits in making the changes and to make it work well required a good deal of preparation and planning by the teacher. There were no strong indications of pride in the total program among teachers.

The mathematics supervisor saw many teachers as being unwilling to change, citing a variety of reasons: no book, no formal curriculum, no training, etc. The teachers saw themselves as being forced to change, though some agreed that the changes were for the better. There were some teachers who saw the benefits of the new program, and they were, in turn, trying to convince their peers to change too. The division meetings were helping teachers to change what and how they taught mathematics. The mathematics supervisor spoke about trying to develop teachers into decision makers so that they could
use multiple resources in planning their instruction. Teachers felt very controlled and believed they had very little input into the process of change. A number of mathematics teachers at Pinewood have had experience teaching in other areas. A problem seemed to be that teaching experience in mathematics for many teachers was weak, and teachers (approximately half) have been re-certified to teach mathematics.

With a new superintendent, there was more support for the efforts of the mathematics supervisor and the Equity 2000 and Pacesetter programs. There was not that support before. There were many layers of administrative authority, and each layer seemed not to overlap onto the next. The mathematics supervisor felt that she was "finding that the principals don't want to know about the mathematics program. The less they know, the more comfortable they are." She had prepared a notebook for principals which explained the why and how of the mathematics program and even conducted a three-day workshop for principals. The other problem that teachers faced was that they "change principals every year." She felt that support from the director of instruction (her boss) and the associate superintendent of instruction was good. One teacher admitted that she had very little contact with the principal, that she didn't know what level of support the principal offered. The principal felt that the inservice programs provided for mathematics teachers provide the support the teachers needed to implement the new program. The school's mathematics coordinator offered a "how to" level of support for her teachers. The principal has allowed some of the mathematics teachers to be freed of their daily 15-minute duty so that they could meet after school for 75 minutes each week.

There was a real push by the mathematics supervisor and the mathematics coordinator for teachers to develop skills in working with groups. Teachers were expected to be effective so that they didn't have large numbers getting D's or failing in mathematics. They were also expected to be creative in developing some of their own manipulatives and materials. Some teachers placed additional goals on themselves (e.g., working on
correcting grammatical usage of students, attending workshops and math conferences to get new ideas).

Trying to provide teachers with the administrative support that will empower them as decision makers was a long-range goal which the math supervisor felt will help teachers. The enthusiasm of some teachers was infectious. The sharing of ideas, especially in the division groups was seen as helpful. Some of the teachers also conducted mini-workshops after school (e.g., algebra tiles, work with the calculator, etc.). Parent contact was also helpful to some teachers. A geometry teacher admitted that it really helped when he finally switched to groups.

Several teachers indicated that the 45-minute class period was too short. Student scheduling was a problem in the school, and many students were apparently placed in the wrong classes. Schedules for using the computer lab allow only one period per day for the mathematics department. There was a need for a more proportionate number of African-American teachers to teach in a school with a 2/3 African-American student population. Teachers needed a better understanding of mathematics themselves. They needed to be able to look up and down the grade levels and understand what was going on to properly facilitate learning.

Additional needs of teachers included:

- Proper placement of students, especially in the more advanced classes.
- The tasks in Pacesetter were too long and should be shortened. First, there needed to be skill building.
- Time to really collaborate.
- Greater availability of computer lab (only available for the math dept. one period per day).
- They needed texts for their students.
- They needed a better copy machine with adequate paper supply (one machine served 160 teachers).
• Teachers needed class periods that were longer than 45 minutes.
• There should be fewer school activities that pull students out of class (field trips, etc.).
• The county needed an attendance policy (too lax).
• There was need for more parent contact.
• The department needed more calculators.
• Mathematics teachers should also expect students to use proper grammar.
• Teachers wanted a written curriculum. Instead the district had mathematics outcomes based on the Standards, but teachers wanted the curriculum to tell them exactly what to do instead of relying on teacher resourcefulness.
• A criticism of the supervisor and the coordinator was that the teachers' expectations of students were too low.

Outside funding from the College Board to pilot the Equity 2000 and Pacesetter programs had been available. Additional funding from the school system, Eisenhower program, and community college grants allowed for much of the inservice development and payment of stipends for teachers to attend. Some teachers were concerned that when the pilot period was over that many of the benefits presently available would disappear.

Levels of inservice were planned by the county mathematics supervisor. The mathematics supervisor indicated that since the introduction of the Pacesetter and Equity 2000 programs, the summer workshops have progressed from being a "farce" ("here's some stuff") to a forum where they looked at important issues (e.g., "Something's wrong, and we can't blame the kids.") that prepared teachers to be "instructional decision makers." Team time was provided at these workshops ("institutes") where a team could "come up with strategies for how it's going to happen in [their] building." The progression was pre-algebra the first summer; then Algebra I; and last summer it was Geometry, with Michael Serra discussing the pedagogy he used with his book. Next summer the focus would be Algebra II/Trigonometry.
The mathematics coordinator at the high school also worked with teachers at the elementary and middle levels. Often the teachers didn't want her there unless they asked for her. "Teachers will come over here if they need help."

There were a number of teachers who were concerned about mandated testing (local CRT and the Statewide Functional Math Test [SFMT]). Though the SFMT had been moved out of the high school to the middle school, a number of teachers felt that the Pacesetter and Equity 2000 programs do not adequately prepare students in the basic skills which they felt were important for success. Teachers admit that they coached students on thirty objectives before students took the SFMT. Teachers worried about the CRT, even though the first year of testing showed favorable results. The local Criterion Referenced Test was developed in collaboration with the local community college. The county mathematics supervisor and the mathematics coordinator at the high school both saw the local CRT as a means of driving instruction. Departmental exams were given for all mathematics classes as a way to insure higher expectations of students. In prior years, teachers who failed more that a third of their students were given poor evaluations. The principal said that that practice no longer exists.

There was a very top-down structure in the district and in the school. Teachers felt that they must conform or receive low evaluations. However, mathematics teachers were beginning to admit that the reform in mathematics at Pinewood High was beginning to work. Grading policy, which had been very restrictive, was starting to open up as teachers began to employ alternative assessments such as journal, notebooks, etc.

Because Pinewood was a magnet school, it catered to three groups of students: visual and performing arts; the university school; and the comprehensive high school. About 87% of the student population was African-American and the county was considered the most affluent African-American community in the country.

**Scottsville High School.** "Let the teachers choose to change" seemed to be the motto of the mathematics department and the administration. Some teachers were biased toward a
traditional way of teaching. Teachers were not pushed to change, but they were often teamed with another teacher who was enthusiastic to introduce technology into the classroom and use the UCSMP materials. Another belief was that change took a long time if it's going to be done correctly. An outside factor that slowed the process was the preservice teacher development that took place in universities and colleges. People at Scottsville didn't believe that that training was relevant to the Standards or what they were trying to do at their school. Adequate funding allowed the program to move forward. If the district were to get "financially bashed," then the program would slow down. Collaboration among teachers existed in study groups and in sharing ideas, particularly with new teachers.

There was also collaboration with other departments (e.g., technical education dept.). Writing became a focus of the district, and writing skills in mathematics was becoming a big part of the program as a collaborative effort with the English teachers. A veteran teacher indicated that a significant shift from individual teachers working alone to a team effort had occurred. The supervisor believed that a "wonderful collaboration" existed among the mathematics teachers. Teachers' desks were in a common area which allowed for a natural collaboration to occur. Some collaborative activity was being done with the science department, particularly measurement, drawing conclusions, analyzing data, doing statistical work. Professors on sabbatical were hired part-time to teach and do research. Teachers were willing to get together and work on mathematics projects.

The need for common planning time was mentioned by teachers as their greatest need. Teachers who had the same preparation period did have some opportunities for this, but teachers felt they needed to mix with each other. There was also a need for long blocks of time to work on projects together. Some teachers worried that skill development was being de-emphasized. One teacher felt that he couldn't keep up with his colleagues on the technological front and questioned his own ability to contribute fully to his students. Much
of the doubt centered around just being able to meet the demands of the mathematics program.

Everyone attested to the leadership of the mathematics supervisor. His style was to share ideas and constantly push for change. He believed it was important to empower teachers and did not force his views. He believed that dissenting opinion was important to clearing and shaping the change process. He went after the district support for mathematics and convinced the school board and parents of the need for change and the necessary funding to bring it about. He has pushed for a technology-enriched program and seemed unsatisfied with the notion of completeness of the mathematics program.

The mathematics supervisor was very active in local and state mathematics groups and was the recognized leader of mathematics reform in the school. Strong leadership (mathematics supervisor) and empowered teachers existed side by side. The mathematics community of the district expanded to include close ties with the university. A particularly difficult problem facing the mathematics community was the necessity to quit adding courses and start subtracting. It's a community that encourages those who "are well educated, articulate, involved, willing to experiment." There was little room for the complacent teacher. This community had confidence and pride in its accomplishments, had a strong technological orientation, and was willing to collaborate and share with each other. A teacher says, "I have a feeling we are way ahead of everybody."

There were infrequent top-down decisions from the superintendent with regards to curriculum. He did, however, mandate a writing component to all subject areas, and he expected all teachers to conform to it. The administration expected the subject-area supervisors to be "leaders in their area" and that included keeping abreast of current developments in the field, being leaders in professional organizations, and spreading that leadership to other schools.

Teachers expressed pride in more than just the program. There was a pride in the community, the students who attended the school, and the leadership. "They taught
harder" after receiving the Excellence in Education Awards. Teachers felt empowered and recognized their uniqueness. Teachers saw themselves as always trying to do a better job. Teachers felt there were just so many things to do to properly implement the program that they couldn't get to them all. Most teachers were very open to change because it was expected. They saw a particular area that they needed to strengthen, and they went about fixing it. Some took mathematics classes or attended special workshops (e.g., cooperative learning, use of technology). Careful selection of teachers who were hired to experiment with new ideas in mathematics was an important factor for this openness to change to exist. Inservice opportunities for teachers to share ideas helped. A teacher comment that seemed to fit: "If this is going to help the kid learn, we're going to do it or we're going to try it."

Several teachers talked about the on-going preparation needed to effectively be a teacher. This was especially the case in regard to using technology in the classroom. Teachers organized study groups where they taught each other how to use software. Teachers in the school were expected to continue updating their preparation through coursework and conferences. Special teaching skills required to implement the UCSMP program had been gained at special workshops. While only a couple of the teachers had formal training in working with cooperative groups, they were helping other teachers. Others used the techniques as "gifted amateurs." Many of the teachers had been in the department for several years and enjoyed being in the school. One teacher with four years in the school saw himself as being "here the least amount of time."

The superintendent sought information and advice about the mathematics program from the mathematics supervisor. He was very supportive, particularly in recommending funding for the program. The associate principal of instruction worked closely with the supervisor and mathematics teachers. The principal viewed the administrators' jobs as serving the teachers; "not to encumber them, to support them, and we are not there to get in their way." He explained that the teachers had a lot of release time.
The supervisor expected teachers to have a high energy level; to be willing to experiment; to not sit in their classrooms in isolation; to enter into dialogue with others about their ideas; and to act as professionals. As the principal stated: "They need to know what's going on in the world around them. They need to have an idea of where they want to take the program."

The Mathematics Supervisor had created a risk capital fund from which anybody can apply for moneys to try out new kinds of things." Teachers viewed the community as demanding, wanting the best for its kids, wanting its teachers to be great teachers, and expecting a stability in the district which allowed for consistency. Visiting mathematics educators/researchers from the university greatly influenced the mathematical and pedagogical vision of the school. Administrators indicated that the forward looking, experimental nature of the mathematics department was something that existed in other departments as well. There was no indication that the mathematics department sparked that mindset in other departments.

The school had a split schedule with a seven period and a nine-period day. This allowed for a flow of people through the math office, a place for collaboration and idea sharing. Teachers suggested alternative scheduling plans might allow more time for teacher preparation and collaborating with colleagues.

Teachers felt the school's facilities were outstanding, but that their number one problem without any question was the aspect of time. This need was mentioned by every teacher interviewed. Keeping up with technology required time for teachers to understand it. Restructuring the school day into modules could help lessen the pressure of time. There was also a need to develop a coherence between the various applications being developed by teachers. Teachers were concerned they would "burn out" if they didn't get more time to do things. During the past two years, teachers began working with students in groups. For some teachers this has been difficult, and they felt a need to develop more expertise in group activity. Large classes of 26-30 students created additional difficulties.
A big problem was the pressure brought by parents for students to do well on the ACT and SAT tests. Some teachers found it difficult to strike a balance between the skill development they think students would need on those tests and the problem solving which the mathematics program tried to promote. Movement between tracks was often difficult for students. The UCSMP emphasized problem solving while their regular program emphasized development of skills. As a result they covered only about 60% of each book’s content. Some teachers felt it was impossible to meet all the expected commitments of the program and were starting to set priorities on their time to avoid burn out.

People involved in the mathematics program at Scottville viewed the reform as something that was continuous. "We're changing all the time." An administrator described the change that was going on as incremental, as something that showed major change over even a short period of time. Teachers were motivated to get better and better. A number of inservice opportunities were provided for the mathematics teachers at Scottville. There was lots of release time, and teachers had been encouraged to go to professional conferences and meetings. Visiting professors spent half-day sessions talking with teachers about mathematics from the teachers' perspective. There had been many half-day inservices targeted at technology with a hands-on approach. In fact, a budget of $500,000 per school year was appropriated for the district to bring in technology consultants to infuse new ideas. Some additional needs of teachers were inservices on cooperative learning groups. Teachers earned credits for attending the study groups. The mathematics supervisor provided a "steady stream of articles" geared to the needs of teachers. In spite of all this attention to inservice opportunities, teachers felt they needed release time to do more collaboration with each other.

The mathematics supervisor perceived of Scottville as being a school where there was great consistency, and as a result of the nurturing environment for change, the mathematics program should be on the cutting edge as a model for others. There was a mutual respect and participation among the district administration, the board members, and the people in
the schools. The community was demanding but supportive of the mathematics program.
The people in the community were very united. The support came from being able to
"operate the politics of the situation." Teachers were empowered in the school and in the
community, and there was a close relationship between teachers, board members,
administration, and community. Parents were very involved with their children.

The people in the community had a keen interest in SAT and ACT scores. Fortunately
the mathematics program had proven a positive impact on those scores, which were
traditionally among the highest in the state. The AP Calculus results were also a very
political issue in the district, but student numbers passing that test had quadrupled over a
ten-year period. The principal claimed that "the entry level ability of our students, although
good, was not near what it was at these other schools around the area. But when they
leave they're achievement scores far exceed all the others." An associate principal stated:
"I would tell you that almost everything we do here was couched in evaluative terms."

Says one teacher:

It was clear to me that the board of education was very interested in
maintaining those high scores. If it meant skills at the sacrifice of innovation
and experimentation, I think they would go for skills. If that meant it would
maintain those high SAT scores and ACT scores.

Teachers felt that this emphasis on standardized testing was inconsistent with the goals of
their program, but they felt they were caught in the middle on this issue.

A zero-based budget was being started. The mathematics supervisor felt that the
mathematics department was ready for this, but cautions that "nothing is immune from
being questioned." The district had a high degree of latitude in staffing part-time personnel
(e.g., professors on sabbatical) since teachers did not belong to any bargaining group.

Some Hispanics come into the school with no literacy in their native tongue, so an
initial challenge for the school was to get them to be able to read and write in their own
language. After that, they had the same opportunities in ESL classes to get the mathematics
courses that they needed.
Cluster 5: Influence of Program on Students

We wanted to find out if the mathematics program was impacting on students in a positive way. We wanted to know the kind of mathematics that students were learning and whether pedagogical practice seemed effective. We were anxious to discover examples of mathematical discourse between teacher and student and between student and student. We were interested in teachers' perceptions of students and whether the program had been effective in developing students' ability to reason and solve mathematics problems. There were six coding categories included in this cluster:

- all students or select groups
- assessment
- effect of program on students
- student activity
- teacher perception of students
- teacher-student interaction

East Collins High School. Teachers were attempting to provide for the mathematical needs of all students at ECHS, including minority students, female students, and at-risk students. Successful completion by students of a course in Algebra and Geometry was a district-wide goal. Algebra for the at-risk students would have a different content than the regular algebra course and would require three or four semesters for completion.

Collaboration with other departments such as vocational education was providing mathematics experiences with connections to real life. Student input was valued in the continuous upgrading of the mathematics program.

There was a shift away from general mathematics courses, and nearly all ninth graders were taking Algebra I or Algebra II. Approximately 91% of all students completing Algebra I went on to take Algebra II. Pedagogical shifts to a more facilitative role varied among teachers, and these shifts were sometimes difficult adjustments for some teachers and for some students. Teachers felt their choice of the UCSMP texts and materials provided students with mathematics that was more relevant to the students' world, though
teachers were constantly looking for additional activities that connected to the real world. There was much emphasis on student reading of the texts and on pairwise and small-group activity. Varying degrees of mathematics discourse took place in the group settings and sometimes became very intense as students try to persuade each other about the soundness of their ideas. The most prevalent activity consisted of working on assigned problems that accompany the UCSMP text. Lab activities, which were encouraged by special state funding, usually took place once a week and could occur within the class or the computer lab. In these labs, students engaged in numerous activities which sometimes required students to analyze data and make conjectures about outcomes.

Calculators were becoming an important component of student learning in mathematics. Student writing was encouraged through careful write-ups of labs including, in some classes, a story-line. All students were expected to do much of their mathematics activity through regular homework assignments outside the class. Teachers admitted they had a long way to go in developing the program to its fullest potential.

The interaction between teacher and students was mostly respectful, but not always. Most teachers enjoyed an excellent rapport with students. Teachers were finding that some of the first year students in the program required a longer adjustment period. Teachers indicated that students who have made the adjustment more readily assumed greater responsibilities for their learning. There were varying degrees of mathematical discourse between teachers and students. Some promoted conjectures and deeper understandings of mathematics; others elicited short responses.

There were a number of indications that the mathematics program was impacting in a positive way on students. A teacher described the change: "It's amazing how they enjoy. They think they're having fun and not learning, and it's real subtle you know." As a result of a program that emphasized exploration of relevant connected mathematics and a de-emphasis on memorizing mathematics skills, teachers saw improved relations with students, a decrease in students' mathematics anxiety as students gained confidence in
doing word problems, and students taking greater responsibility for their own learning. Students enjoyed doing hands-on activities and using calculators and computers. They were becoming better and more independent readers of mathematics and were adjusting to the prevalence of word problems. Students were enjoying mathematics more and were taking more risks in striking out in new directions and expressing themselves in increasingly supportive classrooms.

Quizzes and tests prepared commercially to accompany the UCSMP materials were often used for student assessment, but some teachers saw this as contradicting the purposes of the newly evolving mathematics program. Alternative assessments of student learning were being explored by individual teachers. These included self-assessment, parent assessment, group quizzes, homework quizzes, and portfolios.

**Desert View High School.** Teachers made an effort to provide a significant mathematics program that met the diverse needs of the school's student population. There was a spin-off effect from the projects which was changing pedagogy and content for all the mathematics courses, particularly in the types of mathematical tasks teachers were assigning students. Teachers were including more and more group activity in their classes, and they had much latitude to experiment with new ideas. A number of efforts were being made for the ESL and at-risk students, especially finding connections of mathematics to the students' lives.

An interesting feature of student assessment on the projects was the opportunity students had for several edits of their work before turning in the final product. Assessment emphasized what students know instead of what they don't know and included portfolios, group quizzes, peer assessment, group grades on projects, take home exams, a project sometimes serving as a test, homework quizzes, and assessment of written and verbal discourse.

Teachers perceived of the projects as allowing students to see the stimulating side of mathematics which humanized it, breaking down students' resistance and mathematics
anxiety and increasing students' confidence in doing mathematics. The projects, which offer long-term opportunities for students to work together, provided numerous student activities which included writing introductions and conclusions for the project, peer editing, peer evaluations, self evaluations, and using technology (calculators and computers). Decreasing enrollments in remedial courses were being replaced by increasing enrollments in higher level courses (in spite of an overall decrease in school enrollment). Students took pride in their work on the projects, and teachers found students hanging on to their finished products. Mathematics had become something more than a regurgitation of facts and algorithms and students characterized the changes as being "more fun." Students were observed to be engaged in a number of long-term mathematical tasks (projects).

Teachers indicated that students had shown "surprising" improvement during the three years of using projects in their classes. This was particularly true in the at-risk classes. Excellent rapport between teachers and students was obvious in most of the observations, and in several instances the level of discourse between teacher and students motivated students to explore conjectures and seek alternative solutions.

**Green Hills High School.** The mathematics department felt that providing opportunities for success for all students was important and that it was meeting those needs through its three mathematics tracks. Teachers indicated that the new mathematics program was focused on the needs of students and that teachers were now more accessible to students. General and remedial mathematics courses were being replaced by an Integrative Math 1, 2, 3 sequence which allowed lower ability students to take algebra and geometry before completing high school. There were two additional tracks, the "normal" and "honors" track for the college-intending students. Teachers indicated that they were always looking for different learning activities, ones that had less orientation towards drill and practice.
The state had an open-ended assessment test to go along with its Standards-like framework which was intended to drive local curriculum changes. The sophomores were given the Test of Academic Proficiency which was the only national gauge of how students compare with other schools and states. Teachers were just beginning to experiment with alternative assessments in their classrooms. Rubrics were being used by some teachers in the assessment of mathematical tasks, and one teacher talked about having students do an oral test explaining how they used the calculator on problems. Teachers also sought program assessment through interviews with students.

All students were expected to complete algebra and geometry before they graduated, and many students took more than the required three years of mathematics. Students were encouraged to seek multiple strategies in solving problems. They were expected to develop verbal and writing skills through mathematical explorations which often occurred in group settings. Students were expected to use calculators and computers in some of their mathematics classes. One teacher used HyperCard to provide students with assignments. Student activity included working with a variety of mathematical tasks which teachers hoped were better connected to reality and the lives of the students. The adjustment to more problem solving activity was difficult for some students.

Teacher perceptions of students were positive, but they viewed students at Green Hills as being overly concerned about grades, which made alternative assessments more difficult to implement. Teachers indicated that students were eager, more involved with, and seemed to be enjoying their mathematics. Students were honest in their appraisal of different mathematics activities which gave teachers valuable feedback in order for teachers to adjust their instructional programs. "Kids love to work on the calculator and computer." Teachers saw students as being interested in doing mathematics and reported that students came in after school, went to the mathematics learning center for additional help, and were always asking parents for assistance. There were some students that didn't fit this pattern.
Some had difficulty staying on task—usually those students with poor mathematics
motivation and/or poor mathematics skills.

The interaction between teachers and students varied according to the classroom
setting. Some teacher questions sought out only a very limited response from students;
others were more probing and promoted a deeper discourse. Teachers described student-
teacher interactions variously as being enjoyable, challenging, and sometimes frustrating.

**Pinewood High School.** The vision for providing a mathematics program that insured
that all students must complete algebra and geometry before graduating from high school
came from the county mathematics supervisor and was receiving increasing support from
teachers—though in varying degrees. The supervisor indicated the need for a "smarter"
methodology to accomplish this goal.

The county established a locally developed criterion referenced test (CRT) to drive
instruction to accomplish a mathematics curriculum which supported the supervisor's
vision. Teachers were concerned that the new approaches and content coverage would not
allow students to master basic skills, which they felt were very important and were
included on a statewide functional mathematics test (for which students were coached).
The Equity 2000 and Pacesetter programs were getting underway, and assessment
provided by those programs was also used. Performance on the CRT had been noted by
teachers as okay, but the students scored poorly on the Pacesetter tests.

Students were in the process of adjusting to the changes, but teachers observed that
most students "enjoy math." This was particularly true in the Equity 2000 and Pacesetter
classes. Group work provided opportunities for students to understand mathematics, and
one teacher observed that he didn't have to repeat explanations "nearly as often."

Documenters observed that students were engaged throughout the classes on their
mathematics, that students used math vocabulary, showed patience in helping each other,
treated each other with respect, and did not seem to be afraid to ask peers for help.

Students often worked together in small groups and there was some use of calculators and

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manipulatives in classes. Students were accepting more responsibility for their learning. Activities varied between teachers, and many teachers were "rule oriented" and emphasized teaching basic skills and just one way of finding answers. Others were encouraging students to seek alternate paths to solve problems.

Teacher perceptions of students vary. Some saw students getting lost on the lengthy Pacesetter tasks. Most teachers perceived the students as weak in basic skills. They also saw some students as benefiting from group activity--especially the "brighter" students. They felt that students were understanding mathematics better because they were now approaching problems from multiple perspectives: numerical, graphical, and algebraic.

Documenters observed that there was good communication between teacher and students in both whole-class and small-group settings, though deeper mathematical discussions were not observed. Students generally felt that their interaction with teachers was good.

Scottsville High School. Teachers indicated their desire to provide for the most appropriate mathematics course for all students in a program that has four tracks. Teachers were encouraging students in the UCSMP texts to be independent readers and learners ("cutting the infant strings"), while nurturing special education students with connected and relevant mathematics activities. Teachers admitted it was difficult to move upward through the tracks, and that they needed to pay more attention to the kids in the lower tracks because "this whole community is college bound."

There were no indications that alternative assessments were being used, but there was a heavy emphasis on test results--especially standardized tests such as SAT, ACT, and AP exams. Test scores were very political in this community where students traditional achieve high scores, especially on the AP Calculus exams. Departmental exams predominated. Students in the honors classes had a "scaled-down" grading system that encouraged students to remain in that track. Some teachers were concerned about the
school's preoccupation with results on national tests. Careful monitoring existed at the administrative level, including outside evaluation.

Students' reaction to the UCSMP materials seemed to be positive ("just kind of fun"). One student described his mathematics learning activity as consisting of doing the homework, reading the book, and maybe listening to the teacher in class. Students thought the availability of calculators and computers were a real benefit and as a way of eliminating a lot of not very useful mathematics ("older mathematics"). Using technology was encouraging students to explore mathematics and look for patterns, make conjectures, and make conclusions based on their analysis of data. Some teachers provided students with mathematics that had connections to real life; others, admittedly, taught in a very "traditional" way. Students were being encouraged to do more and more writing in their mathematics classes.

Teachers recognized students as "high caliber" and highly motivated to achieve well in mathematics. They perceived of students as being content with the program ("very few grumpy, angry kids who are walking around the hallways"). Teacher perception of "good" students varied between one who was well disciplined in class ("listens carefully") to one who achieved well in mathematics. Interaction between teachers and students centered on the teacher giving information to students (i.e., answering student questions), even when students were in group settings. Little evidence existed of mathematical discourse.

Cluster 6: Impact of the Standards

We felt it important to determine what features of the Standards emerged as teachers described their mathematical and pedagogical visions and whether the Standards were serving as a catalyst for reform or as validation of that reform. We were interested in the local site peoples' familiarity and interpretation of the Standards. The following four codes were included in these summaries:

impact of Standards

Page 152
sit familiar of Standards
Standards as catalyst
Standards as validation

East Collins High School. During the developmental stages of their new mathematics program, the working committee (consisting of teachers, administrators, and business representatives) looked to the Standards in conjunction with the community needs. The Standards acted to confirm the thinking of this diverse group. With the program still evolving, the chair of the mathematics department admitted that they were looking for those things to include in their program that "are the Standards."

"Our teachers have the spirit of the Standards. They may not be able to quote them word for word, but they understand them." Teachers viewed the Standards as a "kind of mind set" that helped them make decisions about what content they covered in mathematics and how they taught mathematics. One teacher saw the Standards as making common sense: "A lot of it is basically what we teach anyway."

The understanding which the two teachers and the county mathematics supervisor had of the Standards helped to shape the program that resulted from the collaborative efforts with the business community, the local universities, and a representative from the state department of education. The Standards were certainly an influence in the selection of the UCSMP materials and other supplementary materials for the program according to teachers at East Collins. The teachers' perception of the UCSMP texts as the closest embodiment of the Standards made these texts a natural choice. One of the administrators involved on the planning committee noted: "The Standards just happened to be a form, and you could use it as a guide, these are the things, we believe. [The Standards] and our planning were both happening at the same time." The program which had been offered by industry was rejected because, as the chair put it, "We didn't see that program fitting the Standards well at all." Another teacher said, "Nothing has been initiated here without the Standards."

According to the mathematics supervisor:
We read the Standards, became immersed in them and probably, hopefully, the words that came out of our months to create [our mathematics program], were words that we had put in our minds from the Standards and from other good stuff too besides the Standards.

"I see our program and the Standards are almost one and the same. I mean that the Standards are what we are employing," said one teacher who had for years followed the work of Usiskin and his work at the University of Chicago. She saw their mathematics program as being the philosophy and the UCSMP books as the "hard copy" of that philosophy.

Desert View High School. There were indications that the Standards would have more of an impact in the future than they had during the early stages of projects development. One of the teachers who served on the district-wide mathematics curriculum committee indicated that the old curriculum was being weighed against the Standards and was being changed where needed. The Standards would also serve as criteria in the selection of textbooks. There was currently a planned process by which mathematics teachers were studying different parts of the two Standards documents.

Staff familiarity of the Standards was evidenced in many of the statements made by teachers about how they felt the projects were being validated by the Standards. As the district curriculum coordinator expressed it, "We've done a fairly serious look at the Standards, and we need to do a nitty gritty one, planning kind of thing."

The Standards did not serve as a catalyst in the early stages of developing mathematics projects. The projects were an off-shoot of a similar program at the local university. Currently projects being developed were being done with the Standards as a criteria. Also, most of what was being done at the district level was now being guided by the Standards documents.

The Standards did not appear to be the reason for the mathematical vision of this school's mathematics program, though there had been workshops where teachers discussed the Standards. Interviews with teachers revealed differing perceptions of the
impact the Standards had on their mathematical vision. The mathematics department chair said it this way:

After we really had a successful program with writing and mathematics and we went back to look at the Standards and realized that it was aiming our students a lot more towards many of the Standards, and it was kind of a neat. It was an affirmation that that's another reason why we must be doing the right thing. But it wasn't originally an attempt to address the Standards.

Most teachers see the Standards, as validating their mathematical vision. Much of what the mathematics teachers were doing in their classes aligned with the Professional Teaching Standards.

Green Hills High School. The school district spent an entire year developing curriculum objectives for K-12 which used the Curriculum and Evaluation Standards as their basis. The community input from various sectors first identified mathematical needs for the community. People at the school felt that these fit perfectly what the Standards were saying, and the mathematics curriculum was developed by a broad base of representation and ratified by the teachers. District assessment was developed which would drive the instruction intended by the newly adopted curriculum objectives. Without the NCTM document, the mathematics supervisor felt that the finished product would not have turned out as complete as it did. The Standards gave their efforts "coherence."

Those teachers serving on the curriculum research committee (45 members representing all K-12 schools) were each given copies of the Curriculum and Evaluation Standards with the admonition to read them during the year of their work on the committee. Almost immediately these teachers began to change in the way they perceived mathematics instruction. This core of teachers reported regularly to teachers at the various schools, so other teachers were starting to get the message of the Standards. The ratification process by the teachers prompted teachers to better understand the Standards.

Without the Curriculum Standards to guide the efforts of the curriculum research committee, those serving on the committee admitted that it would have been impossible to develop such a coherent document that all teachers later ratified. The supervisor said that
the Standards was the first thing that members of the committee looked at in preparation for their work. Teaching suggestions were later developed by elementary teachers for their level. The Standards also prompted the development of an assessment program that would drive the newly developed curriculum objectives of the district. Though a community needs assessment preceded the study of the Standards, the Standards were used as criteria to check the value of those different needs.

Pinewood High School. The present district-wide curriculum for mathematics resulted in 1991 from a task-force recommendation to develop a curriculum based on the Curriculum and Evaluation Standards. The School Board authorized that the mathematics program be Standards-based. Some teachers recognized the impact of the Standards through changes in pedagogy: the involvement of students in hands-on mathematical experiences, greater use of manipulatives and technology (especially calculators), and small group activity. However, most teachers had little familiarity of the Standards. Though the district curriculum was Standards based, there were only a few teachers who had input into that process. There was evidence that the school's mathematics coordinator was attempting to make teachers aware of the changes the Standards addressed.

From the perspective of administrators (the mathematics supervisor and the school mathematics coordinator), the Standards prompted changes taking place in the mathematics curriculum—including teaching practice. The teachers, on the other hand, found their catalyst for change to be the mandates being made at the district level—though some teachers were seeing the wisdom of some of the Standards-driven vision. The mathematics supervisor often supported her position by referencing the Standards. Particularly was this evident in the dispute over a textbook for Algebra 1 classes. The supervisor backed up her position that until the right text came along, teachers should be resourceful in using several sources.

Scottsville High School. There was no clear indication that the Standards had an impact on the program that was in place. There appears to be a sense among mathematics
staff that they didn't need to go outside the building for ideas, that they were capable of creating their own—though there was an almost carte blanche acceptance of the UCSMP materials. Usiskin was perceived to be one of them (the mathematics teachers), and there was a built-in trust for the UCSMP materials (which have largely been piloted at the school). Any real link to the Standards appeared to come through Usiskin. Even the mathematics supervisor makes little reference to the Standards, even though he was credited for being influential in their development. Integrating technology into classrooms seemed more of an impact on what was happening than the Standards, though it seemed certain that the Standard's references to the use of technology made a natural connection to those at the site. One teacher sort of typified the faculty's sense of the Standards: "

Personally, I don't think--I read it--but I don't go through my lesson plans or a section or chapter of the book and say, "Oh, there was one of those goals," or "Oh, I'll get that copy right now." Its kind of like, our curriculum that we have, if we do the curriculum, it was going to cover those things we are looking for.

And a comment by the principal seemed to reflect the taken-for-granted impact of the Standards:

Well I don't think there is any question that the NCTM Standards have been implemented, to what degree I'm not certain, but I would be surprised if the NCTM Standards weren't almost completely implemented into our program, because I know the Mathematics Supervisor not only philosophically is in tune with them, I understand that he contributed in many ways into their development.

There were a number of references by teachers of the exact match between their mathematics program and the Standards (e.g., "Our courses were built with the Standards, that is how they were designed. The curriculum that we have here is all within the Standards." "We exceed all of those Standards." "If you look at any one of our courses, or came in and observed our teaching you could just go down the list of your Standards and be assured that we are right there." )

There doesn't seem to be any evidence that the mathematics teachers did any development toward the vision of mathematics or pedagogy as portrayed in the Standards.
It was not even clear that the mathematics supervisor was motivated by the Standards. From statements made by teachers and key personnel involved with the mathematics program, it seems that the Curriculum and Evaluation Standards were only being used to lend credibility to changes that had occurred. The use of the UCSMP materials, which were probably a nuance of Usiskin's proximity to the school, may be what teachers were actually referring to when they said they were meeting all the Standards. The advanced state of technology in the school may be what teachers perceived as "exceeding the Standards." Pedagogy, in spite of grouping arrangements in some classes, was perceived by the documenters as being very traditional and not necessarily following suggestions in the Professional Teaching Standards.

Analyses of Cluster Summaries: The Next Step

In the next chapter, the cluster summaries will be analyzed across the five cases of this study. The five cases, though different in many ways, do have similarities worth noting. The setting for each case shows great variation; the cases consist of one rural high school (East Collins), a rural-suburban high school (Desert View), an urban high school (Pinewood), and two suburban high schools (Green Hills and Scottsville)—though there were vast differences between these two suburban cases. The size of the school districts varies considerably, as does the administrative structure and the socio-economic makeup of each case. Each cell in the Cluster Analyses Across All Cases Matrix (Figure 6.1) will represent a separate analysis of a particular cluster.
SECTION III

ANALYSES ACROSS CASES, INTERPRETATION, AND IMPLICATIONS
Prefacing Remarks for Section III

The format of Section III needs some explanation. The placement of interpretations, particularly, was troublesome because there are two different classes of interpretations which the analyses seemed to prompt. First, there was a cross-case interpretation by cluster. Secondly, there were interpretations of results which crossed clusters. This second class of interpretations was more integrative and seemed to require a different positioning from the clusters in the interpretative portion of this study.

The interpretation of each of the six cluster analyses will be placed directly after each analysis. For example, the interpretation of "Evolution of Reform Efforts" will immediately follow the analysis of that cluster. Likewise, for each of the other five clusters, the respective interpretations will follow each analysis. These six analyses and the six interpretations will comprise Chapter 6 of this study.

The "integrative" interpretations do not seem to belong with the six cluster interpretations in this chapter and will be included in the final chapter of the study with the conclusions, implications, and future directions.
CHAPTER 6

CROSS-CASE ANALYSES AND INTERPRETATION

The Next Stages

Data have been reduced through two levels up to this point in the study. In Chapter 4, the first level of coded data reduction resulted in a 60 by 5 matrix (300 cells) in which each cell represented one of the sixty codes (Figure 4.1) and one of the five schools (cases). Each cell in that Meta-Matrix represented a reduction of approximately ten pages of data down to one or two paragraphs. Chapter 5 documented a further reduction of data to a manageable format that would enable the cross-case analyses, the reduction to the 6 by 5 Cluster-Matrix (Figure 5.1) which consisted of 30 cells. Each cell in the Cluster-Matrix was approximately two to four pages in length and resulted from condensing and further summarizing data according to the six clusters (Figure 4.1).

I deliberately avoided ranking the importance of data and doing any interpretation or evaluation of data during these first two levels of data reduction. Any evaluative statements which have been reported have been those of site participants or the original site documenters. I did not want to lose extended data that might not qualify as "vivid" or "exciting" (Miles & Huberman, 1994). Redundancy of data (e.g., several people saying essentially the same thing at a given school) was the only data removed at this third level of analysis in an attempt to preserve all of the information included in the extended text.

The cell entries in the Cluster Analysis Across All Cases Matrix (Figure 6.1) consist of the analysis of data across the five cases for each of the six clusters. (Referring to Figure 5.1, this represents single-row summaries.) By analysis, I mean my examination of the five cells of a particular cluster in the Cluster Matrix (e.g., Evolution of Program across the five schools) and the way I organized and arranged that information so that patterns or themes emerged. In some instances of analysis, the frequency of particular strands of data
were useful; in others, a composite pattern was possible across the sites. There were times when patterns and similarities did not emerge. Figuring a way of presenting diversity of data was also a very important part of the analysis.

Several questions guided the analysis. Is there an obvious pattern in the data? If not, are there more subtle arrangements? What similarities and differences exist between cases? Are data so different that contrasts become important in the analysis? Are there plausible explanations for arranging the data in certain ways?

Any interpretation of data was being avoided until I had developed the six cells of the Cluster Analyses Across All Cases Matrix in Figure 6.1. By interpretation, I refer to a deeper level of thinking about the data, a level in which meaning was assigned to the data. This meaning was derived from a synthesis which combined the analyses of data across cases with the conceptual framework for this study. I was looking for that "mesh with the initial framework" in this process which Goetz and LeCompte (1984, p. 198) describe. Were there ways in which the data challenged that framework, or lent credence to it? I was trying to determine what the data of this study meant in terms of the questions I asked as I began the study (Chapter 1). Were the orienting assumptions (Chapter 2), which influenced the way I conducted this study, supporting and enabling this probing for meaning?

Part of the interpretive process included further checks against first impressions that had been made as data was sifted, organized, summarized, and reorganized. Miles and Huberman (1994) cautioned against leaping to conclusions based on plausible explanations or first impressions: "So the moral is: Trust your 'plausibility' intuitions, but don't fall in love with them. Subject the preliminary conclusions to other tactics of conclusion drawing and verification" (p. 247). By moving up the "abstraction ladder" to interpretation (Miles & Huberman, 1994), the emerging patterns were giving meaning to the data.

Wolcott (1994) indicates that when a researcher interprets qualitative or descriptive data "it is our colleagues' presence we feel over our shoulders; our interpretive 'rightness'
is judged within traditions, not in the correspondence between our accounts and Truth or a
strict adherence to procedures" (p. 258). He also refutes the common tradition that
interpretation is at a loftier position than analysis and argues that one does not have to
follow the other. He believes that there is "potential for interaction between analysis and
interpretation," and interpretation becomes the answer to the question, "So What?" (pp.
256, 257).

I have chosen to follow the more traditional path of following analysis with
interpretation for this study.

Figure 6.1
Cluster Analyses Across All Cases Matrix

<table>
<thead>
<tr>
<th>Cluster</th>
<th>All Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evolution of Program</td>
<td>Cell (E, A)</td>
</tr>
<tr>
<td>Mathematical Vision</td>
<td>Cell (M, A)</td>
</tr>
<tr>
<td>Pedagogical Vision</td>
<td>Cell (P, A)</td>
</tr>
<tr>
<td>Context of Change</td>
<td>Cell (C, A)</td>
</tr>
<tr>
<td>Influence on Students</td>
<td>Cell (I, A)</td>
</tr>
<tr>
<td>Standards Influence</td>
<td>Cell (S, A)</td>
</tr>
</tbody>
</table>

The figure above serves as a visual convenience in identifying the six analyses that
follow. For example, (E, A) is an abbreviation for (Evolution of Program, All Cases).
Similarly, (M, A) represents the analysis of the cluster, Mathematical Vision, across all five
cases of the study.

I experimented with different orderings of the six clusters in Figure 6.1 before
deciding that the one above was the most appropriate to allow a "natural" flow to the
analyses. By natural, I mean that I tried to arrange the clusters in an order that closely
approximated the chronology\textsuperscript{21} of events at a particular school. Following this line of reasoning, the evolution of reform efforts seemed a logical place in which to start.

\textbf{Evolution of Reform Efforts Across All Cases}

\textbf{Different Beginnings}

Each of the five cases had a different beginning for its reform efforts in mathematics education. A partnership with industry was the impetus for the changes taking place in the mathematics program at East Collins High School. At Desert View, a partnership with professors from the Mathematics Department of the nearby State University initiated change efforts. The changes in the mathematics program at Green Hills High School resulted from a revision of the K-12 mathematics curriculum for the district. Participation in the College Board's Equity 2000 and Pacesetter programs assisted Pinewood High School's efforts to implement the county mathematics framework in its mathematics program, and a visionary leader at Scottsville High School was the prime moving force in that school's technology rich mathematics program.

\textbf{Disposition as a Factor in Change}

A disposition for change was present in all of the five cases. Disposition for change refers to both the recognition that some things needed to be changed in the existing mathematics program and a willingness by leaders to act on those needs. In two of the five cases, policy-makers believed a change in the mathematics content and teaching practice was necessary (Pinewood and Scottsville). In the case of Pinewood, the mathematics supervisor had a vision of change which paralleled, and was modified later in response to, the two \textit{Standards} documents. This vision for a changed mathematics curriculum was supported by a district task force's report. The supervisor's desire to eliminate the elitism that existed in the mathematics program at Pinewood (and other high schools) coincided with the aims of the College Board's two programs.

Both of the mathematics supervisors in Pinewood and Scottsville had persisted over time in trying to convince others of their vision. Scottsville High was in a district where
there was ongoing support at the district level for the supervisor's vision. There was evidence that the emphasis on technology was working for the students (e.g., high scores on standardized testing), so it became easy for the supervisor to get the support he needed to carry out his plan for change.

Desert View had a handful of teachers who were poised to modify their instructional practice. Through a serendipitous beginning, three teachers were put in contact with several of the mathematics professors at the nearby university. The disposition of these teachers to alter existing practice allowed them to jump at the chance to collaborate in a funded project with the university professors and to develop mathematics projects which were used at Desert View.

In the other two cases, the proclivity for change was evident across a broad spectrum of the key players in the mathematics program. East Collin's teachers were supportive and participated in the planning stage where a new mathematics program took shape. The county mathematics coordinator and the associate principal in charge of curriculum were also ready for this change. One of the teachers observed: "The mathematics program isn't something that began last year here. It's been in the minds of these people, all of us, in one way or another, for years and it's just that the program is the focus of that."

According to the District Mathematics Coordinator, all of the K-12 mathematics teachers in the Green Hills District had to ratify the new district mathematics curriculum. Fifteen of 22 teachers in the two high schools had served on the committee to draft the changes, so there was a mindset for change among nearly all of the high school mathematics teachers and they had helped to define the process (and the K-8 teachers as well). Administrators and community representatives also served on the research and writing committees. The state had already adopted a Standards-like mathematics framework which, along with the Curriculum and Evaluation Standards, helped to shape the collective propensity for mathematics reform among this broad base of participants.
Conditions that Facilitate Change

According to Fullan (1993), the change processes are "uncontrollably complex" and "unpredictable." Data from the five schools of this study confirmed Fullan's (1993) observations about change. Changing a mathematics program was a complex process and entailed different components at each of the sites. Sites varied in their emphasis to change mathematical content and/or pedagogy. It seemed important in this study to identify those conditions that facilitated or precipitated the sought after changes.

External funding seemed critical in three of the cases to get the changes started and to support the evolution of the reformed mathematics programs. Interviews with many of the key players at East Collins indicated that there was a desire to improve their mathematics program, but it wasn't until local industry made its no-strings-attached contribution that changes really began to take place. A teacher enhancement grant allowed Desert View teachers the opportunity to develop projects and participate in inservice teacher development opportunities. Pinewood's participation as a pilot site for the College Board provided the needed funding to carry its reform efforts forward.

The decision by the Green Hills School District's administration and school board to open up the new position of mathematics coordinator allowed the new appointee to coordinate efforts for the development of the K-12 mathematics curriculum. The input from a lot of people over a 29-month period produced the revision for their K-12 mathematics curriculum, but without this position of leadership, the impetus for reform might have been delayed. The district mathematics coordinator explained:

Never before has such an impressive and extensive process been employed to revise a math curriculum in our district. Never before has a math curriculum revision involved such camaraderie, teamwork, and agreement from such a large number of teachers, patrons, and administrators. Never before has the professionalism, dedication and effort poured into a new math curriculum been more appreciated. Every [Green Hills] teacher contributed to this new curriculum.
The constancy of strong leadership at Scottsville coupled with the receptivity of the central office administration provided the setting for continuous improvement in technology to occur at that site.

**Levels and Forms of Planning**

Planning for change took on different forms in the five cases, and there was no clear evidence what the effects of the planning would be. There was a range of planning involved across the sites.

The curriculum planning and development at Green Hills was the most extensive among the sites. Planning in Green Hills engaged many people over a very long period and addressed mathematical content and pedagogy across grades K-12. Planning at East Collins extended over a nine-month period and was intended only for grades 9-12 in a single school. What developed there was a framework and a philosophy which would guide mathematics content and instruction.

Only a few of the Pinewood teachers participated in curriculum development at the district (county) level. The four-page framework for mathematics which resulted from the report of a task force validated the mathematical and pedagogical visions of the county mathematics supervisor and represented a set of objectives which students should achieve in the program.

The perceptions of participants involved in planning at East Collins, Green Hills, and Pinewood had several similar features. Participants at each site perceived their plans for reform to include mathematical and pedagogical visions that paralleled those of the Standards. The planning in each of these three cases resulted in a complete restructuring of the school’s (or district’s) mathematics curriculum framework.

Teachers at Desert View felt that their efforts at developing mathematics projects had nothing to do with reshaping their mathematics curriculum. The projects became an "addon" to the existing curriculum. In fact, the county curriculum coordinator indicated that curriculum revision could only occur at the district-wide level. Even the adoption of
particular texts for mathematics classes was a district decision. Of course teachers from Desert View served on the district's mathematics curriculum committee and had input into the process. Through their collaborative partnership with university mathematics professors, teachers at Desert View were following the lead of those professors without any thought of changing their curriculum. The planning by teachers at Desert View followed a different path—how best to develop long-term mathematical projects for their students.

At Scottsville, there was an emphasis on including more technology in the mathematics program, but this decision was not precipitated by curriculum changes. Neither was the use of the UCSMP texts and materials a planned part of implementing a particular curriculum. Teachers at this site didn't necessarily have a clear mathematical or pedagogical vision of their own, but they were following the strong leadership of their mathematics supervisor.

Focus for Reform

Each case in the study had a different focus for its reform efforts; that is, a rallying point around which the energy of participants at each of the sites was directed. The foci of reform included mathematics projects, technology, a changed philosophy, revision of a K-12 curriculum, and equity issues. Desert View teachers were looking for long-term mathematical tasks that would provide students with opportunities to explore relevant mathematics in a cooperative setting. The projects which were being used in undergraduate mathematics classes at the nearby university served as a model to inspire teachers to develop their own projects for the different mathematics courses being taught at the high school.

The inclusion of technology—particularly computers—into the mathematics program at Scottsville had started as a goal of the mathematics supervisor ten years prior to the site visit by documenters. Data from teacher interviews indicated that progress to meet those goals would not have been achieved without the strong leadership of the supervisor. The
compatibility of the UCSMP texts to the application of technology provided a further focus on technology and allowed nearby university researchers an opportunity to pilot their materials at Scottsville High School.

The mathematics department at East Collins had as its focus a philosophy which would guide future as well as present decisions about their mathematics program. It became important to the teachers at East Collins High School to distinguish between texts and materials used in the program and the philosophy of the program itself. According to one of the teachers:

We don't want our mathematics program to be known as a particular textbook. I think if you look at pedagogy, that sometimes the textbook may not spell out exactly what we've spelled out in our program. So that's why we're trying to capture both [content and pedagogy].

Equity was the focus at Pinewood High School. According to the County Mathematics Supervisor, elitism prevailed in the way teachers conducted their mathematics classes. She indicated that only the more capable mathematics students were being afforded instruction which allowed for rich mathematical experiences and explorations. The students who performed at lower ability levels were often denied access to teaching practice that prevailed in the higher mathematics tracks. The supervisor expressed the view that until lower performing students were allowed access to the same kinds of instruction, an inequity was allowed to persist. The new mathematics framework sought to address those inequities and provide for greater opportunity for all students in all mathematics classes.

The complete overhaul of the K-12 curriculum in mathematics was the focus of change occurring at Green Hills. This restructuring of the mathematics program addressed pedagogy as well as content. It was a careful process requiring over two years to complete. At the time of the visit by documenters, the implementation phase of the program was just beginning.
Stages of Teacher Support

Desert View and East Collins both had modest implementation phases at the inception of their reform efforts. Not all teachers at Desert View were in favor of developing and using mathematics projects during the first year, and most of the teachers assumed a wait-and-see attitude. In fact, just three of the thirteen mathematics teachers at Desert View participated in the development and utilization of projects during the first year. All but three teachers became active participants during the second year. At the beginning of the third year, all of the mathematics teachers were actively involved in the changes that were taking place.

The reform effort at East Collins had the support of the entire mathematics department throughout the planning stage. The department agreed that the best way to launch the new program was to start it with just one course and add additional courses each year. During the first year, all students who took Algebra 1 were automatically placed in the new program. The second year, the offerings moved in two directions: the Algebra 2 classes at the high school and Transition Math at one of the middle schools. All teachers were teaching at least some of their classes in the new program by the start of the third year.

The ratification of the new mathematics curriculum was nearly unanimous (96% or higher on each of the curriculum components) among teachers in the Green Hills District. The data from documenters were gathered only a few months after the ratification, and it was too soon to determine the various ways in which teachers might become engaged in mathematics reform.

A curriculum framework and the supporting pedagogy was mandated at Pinewood from the district's administration. Support for the program was mixed. Some teachers recognized the need to adjust pedagogy and content accordingly. However, active support was not commonplace. Data indicated that a real barrier to general teacher acceptance of the framework and a different pedagogy was the reluctance of many mathematics teachers to move away from a strong emphasis on mathematics skills acquisition.

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Useful Suggestions for Those Contemplating Change

Documenters asked those at the sites what suggestions they might have for others contemplating change in their mathematics programs. Some suggestions were strikingly similar; others were quite different.

The most frequently cited bit of advice was to allow teachers adequate time to plan and develop materials for contemplated classroom practice. Teachers felt there was a need, particularly in the early implementation phase, to have someone to collaborate with, someone to lean on and offer support.

Letting parents know of the changes taking place seemed important in four of the sites. Parent input was planned for in the development of the new mathematics curriculum at Green Hills, and there were a number of information nights planned to explain the changes that would be taking place. Staff at East Collins decided not to give students any options when they began in Algebra 1. Every student taking Algebra 1 would be placed in one of the "new" classes. Teachers and administrators felt it was important to get the message across to parents that students could not be transferred into some other type of Algebra 1 class. Scottsville parents were actively involved with the school, and keeping parents informed about change was more or less taken for granted.

There were close ties with parents at Desert View too. The inclusion of projects into the school's mathematics program was viewed by parents and teachers as add-ons to regular courses. Thus, parents perceived no dramatic changes taking place with respect to the sequencing or elimination of content, even though pedagogy was altered appreciably.

One of the mathematics professors who collaborated with the mathematics teachers at Desert View made the observation about the replicability of the project program: "Realize that with a different group of people, using projects might not work."

Interpretation: Evolution of Reform Efforts

Understanding some of the factors which have influenced the ways in which different reform efforts in mathematics have evolved in the five schools of this study provides
important insight for researchers, teachers, administrators, NCTM, and parents who may be trying to understand the complexity of modifying mathematics content and pedagogy. Such data may also be useful to researchers or mathematics educators who are collaborating with schools that may be contemplating reform of their mathematics programs. The school sites represented in the five cases were very different from each other. In spite of these differences, there are inferences that can be drawn from the analysis of this data across the five cases that may be useful to researchers in the field of mathematics education.

The data indicated the existence of a central focus for reform at each site, and the recorded perceptions of those at each site supported the need for such a focus. This study sought to go beyond describing those perceptions and look more deeply at the possible implications of this phenomena. A narrow focus has the possibility of limiting the evolution of reform. For example, if technology is thought of as an end point in the reform effort and not "part of the journey" as Fullan (1993) suggests.

A focus may play a more expansive role in which other paths of change may evolve. The projects at Desert View, for example, evolved into pedagogical shifts that were not evident at the inception of the school's reform efforts. Those pedagogical shifts were spilling over into many parts of mathematics instruction, and there were early attempts at the district level to begin aligning the mathematics curriculum to the Standards. The philosophic focus at East Collins offered expansive opportunities for teachers to develop their mathematics program.

When might a particular focus be problematic? There are no easy answers to this question. The focus on equity issues at Pinewood lacked a general support from teachers, but that school was just beginning its mathematics reform efforts. It seems difficult to imagine the absence of a focus for reform efforts. The movement away from a present condition to something different implies a focus.
There may be a temptation to generalize that change follows some sort of linear progression. That is, if certain conditions are satisfied, then significant change will surely result. Fullan (1993) believed that the processes of change are dynamically complex and warned against a too simplistic interpretation of the processes.

Data from this study suggested that a disposition to modify existing mathematics programs was a necessary condition for reform to take place. Once "disposition" had been dispensed with, there was an inclination to look for the "next" steps in the development of more viable mathematics programs. A number of conditions which facilitated change were identified, as was the focus for reform at each of the five schools. Data also indicated a progression in teacher support for reform efforts to persist, a build up of involvement in the reform efforts until all teachers were active participants. Thus, there may be an inclination to apply a "check list" logic to initial efforts to align local mathematics teaching practice to some "ideal" model.

Potential risks exist in generalizing too quickly from the above analysis of data. The brief period in which four of the schools had been engaged in reform efforts makes generalization premature. There are those unpredictable factors which enter into the dynamic complexity of change which should be expected as part of the process—the events which support a non-linear process (Fullan 1993).

For example, what if one of the mathematics supervisors (Pinewood or Scottsville) decided to take a similar position in another district? It is impossible to predict the consequences of such a change in leadership. Do all mathematics teachers experience change in the same ways? If the assumption that learning takes place as a result of the learner actively participating in the construction of his/her own knowledge and prior experiences, then believing that all mathematics teachers must follow similar stages of development runs counter to that assumption. Personalities and needs of individual teachers enter into this complex equation which, in turn, interact with others in the school community in often unsystematic fashion. Understanding that these unpredictable, but
expected, forces exist to help or hinder the efforts of the key players in mathematics reform seems just as important as the specific indicators which the data suggest and provides a more realistic view of the evolutionary process.

**Mathematical Vision Across All Cases**

A mathematical vision identifies areas of mathematics that teachers and specialists feel are important for students to learn. Such a vision also identifies how that mathematics content is organized and what mathematical processes it embodies.

**What Were the Mathematical Visions?**

The interview guidelines for the $R^3M$ documenters included the question, "What is the mathematical vision in your mathematics department?" The answers given by teachers and other key players in the mathematics reform effort varied considerably across the sites and between participants at each site. The data from Desert View and Scottsville indicated that the mathematical vision was often personalized, but teachers generally articulated a commitment to providing meaningful mathematics for their students. Teachers at Desert View perceived that mathematical projects would best serve their vision, while teachers at Scottsville felt technology would become a part of the vision as well as assisting students in the attainment of that vision.

Green Hills had a vision that was clearly defined and articulated by mathematics teachers. The content and pedagogy presented in their district-wide mathematics curriculum was well understood by the teachers, since so many had played an active part in its development. Once the curriculum committee agreed that it was important for all students at Green Hills to complete Algebra 1 and Geometry before students were permitted to graduate from high school, members began to investigate the Standards more thoroughly. The district mathematics coordinator explained: "Once they made that decision, and then looked at the Standards and said, 'Yes let's pattern our new curriculum after the NCTM Standards,' then we started to write." It was common to hear different teachers voicing the same Standards-based vision.
The interesting thing about the mathematical vision at Pinewood was that there were in fact two visions—the one held by the mathematics supervisor and the one held by most of the mathematics teachers. The common response among teachers to the question of a mathematical vision was their determination to continue their emphasis on developing mathematics skills. They felt that the low test scores of students on standardized tests would go even lower without the customary drill and practice on basic skills. This prevalent vision among teachers was very different from that of the mathematics supervisor and others who worked closely with her. Her vision focused on equity issues and included the abolition of an elitist mathematics program that had denied access to quality mathematics instruction to all except those who excelled in mathematics. Her vision exemplified conceptual understand as more important than basic skills development.

The role of mathematics frameworks. Mathematics frameworks were mentioned in interviews at East Collins, Green Hills, and Pinewood. Pinewood, for example, had no detailed curriculum guide, but there was a four-page framework document that gave direction to the county-wide mathematics program. Data was inconclusive regarding the impact which these documents played in reform efforts and how these frameworks may have influenced actual classroom practice.

Some Common Threads

The data analysis done in Chapter 5 suggested a number of places in which the mathematical vision of one case appeared in similar or comparable form to other cases. The following components of a mathematical vision were frequently found in all or nearly all of the cases.

Algebra and Geometry for all. The vision in four of the cases included Algebra 1 and Geometry as required subjects before students could graduate from high school. Integrated Mathematics replaced general mathematics courses at Green Hills. This three-semester course emphasized algebra, informal geometry, probability, and statistics. The Equity 2000 Algebra and Geometry courses at Pinewood were designed to meet the needs of those
students who normally don't take either course at the high school level. The Computer-Intensive Algebra course at Scottsville allowed students with weak algebraic skills to explore algebra using computer software. Teachers at the fifth site (Desert View) reported that remedial mathematics classes had declined in section numbers from 33 to 12 during the past four years, and projects are being introduced into the remedial mathematics classes.

**Connections/relevance in problem solving.** The need to include more problem-solving opportunities for students was frequently cited as an important part of a mathematical vision across the cases of this study. Teachers believed that developing skills in the process of problem solving depended upon making mathematics more meaningful and connected to real life. Teachers at Desert View perceived of the projects as making those connections. Pinewood teachers participating in the Pacesetter program often found that the tasks in that pilot project were noted for their mathematical relevance.

**Encouraging multiple approaches.** The importance of allowing students multiple paths by which to approach mathematical tasks was voiced by the mathematics supervisor who oversaw the mathematics program at Pinewood. Teachers at Desert View encouraged students to seek multiple approaches in completing mathematics projects. Teachers and students at East Collins indicated that understanding of mathematics was heightened through the use of various strategies for reaching solutions.

**Increased uses of technology.** Technology was becoming a more important part of mathematics learning and instruction across the five cases. The integration of technology with mathematics was most prevalent at Scottsville, particularly the use of computers and graphing calculators. Scottsville teachers found that software such as Mathematica was especially helpful for students who were weak in algebraic skills. The Pacesetter program at Pinewood emphasized the use of technology, especially mathematics which required the use of graphing calculators. Teachers found that technology allowed students to visualize mathematical relationships and recognize patterns which were not feasible using only paper and pencil techniques.
East Collins turned down industry’s offer of a fully equipped computer lab, but they managed to acquire a computer and printer for every mathematics classroom. Their mathematics philosophy which they believed was based on the Standards encouraged the use of technology, and the textbooks they were using depended upon the availability of technology. Classroom sets of calculators were made available for regular student use.

Teachers at Desert View utilized technology on a smaller scale because of their limited district funding. Mathematics teachers sold pencils to students and held other fund raisers during the year to raise money for classroom sets of graphing calculators. Teachers realized the potential of the calculators and shared the one classroom set that they did own.

Green Hills teachers believed that mathematical concepts could be extended through the use of technology, and there was support for this belief at the district’s administrative level. Every K-12 classroom had a set of calculators for student use, and there were three computer labs shared among the different departments in the school.

**Concepts versus basic skills.** There was a strong emphasis on basic skills development at two of the schools. Teachers at Pinewood found it difficult to de-emphasize the time spent on learning basic skills because of low test scores in the school. Scottsville, a school where students traditionally scored high on standardized tests, also had a mathematics department committed to insuring that students master important mathematics skills. This was particularly true in Scottsville’s traditional, upper track of their mathematics program. Teachers at both sites felt that time spent on conceptual development borrowed valuable time from skills development and prevented teachers from covering needed material.

Teachers in the other three sites were finding that they could make the transition towards a focus on mathematical concepts and processes. In the case of Green Hills, alternative statewide assessment in mathematics was encouraging or driving instructional practice that encouraged conceptual development. Teachers at East Collins and Desert View indicated that they felt little pressure from standardized testing, and they were readily
adapting to a process-oriented curriculum. Teachers in those two schools felt that skills acquisition followed naturally if mathematical concepts were emphasized.

**Reduction and elimination of general and remedial mathematics classes.** Every school indicated a reduction in the number of students taking general or remedial mathematics classes. At East Collins, the general mathematics classes were being phased out because such high percentages of students were taking more advanced mathematics (Algebra 2 or higher). This was also true at Desert View. Even though there was a declining enrollment in the school, more students were taking higher level courses.

With the emphasis on all students taking Algebra and Geometry, the mathematics department at Green Hills had eliminated general mathematics classes. A similar emphasis of Algebra and Geometry for all students at Pinewood was expected to have a similar impact on general mathematics courses. Pushing Geometry to grade 9 for most of its students, the Scottsville mathematics program by its very nature de-emphasized general mathematics.

**Communications skills in mathematics.** Writing in mathematics classes was becoming a common practice in all of the cases. The mathematics teachers at Scottsville had to follow a mandate from the superintendent to provide writing opportunities for students in their classes. The weekly lab reports at East Collins required that students explain the process by which they arrived at their solutions. Some of the teachers at East Collins expected students to embed their mathematical reasoning in a fictitious storyline. The projects at Desert View required extensive write-ups. Students were required to write an introduction (often a fictitious storyline), explain the mathematical processes used in solving the problem, and write a concluding or summarizing statement. Extensive peer editing of the projects became an important part of the process for Desert View students. Developing communication skills, including writing, was listed as part of Green Hills' mathematical vision. The Pacesetter task sets at Pinewood also required the students to develop writing skills in mathematics.
Early concerns about students’ ability to write correctly later gave way to a sense of pride in students’ accomplishments at Desert View. A teacher observed:

The quality of the writing is incredible compared to what students were doing. And it’s the same age kids, so I can assume doing the projects has brought up their skill level so much that they’re actually having a lasting effect. ... Their projects are priceless treasures.

Teachers across cases observed that group activity provided a safe place in which to discuss mathematics with peers. Explaining or defending a position helped to reinforce learning and allowed students to develop a sense of logic in the way they reasoned.

Students became more comfortable with admitting that they had made a mistake, especially when they discovered that their logic was faulty. Teachers observed that developing good group skills also takes time. A Green Hills teacher explained the transition which occurred when she expected her students to develop good communication skills in mathematics:

When I first started it in my classroom, my kids said, "Wait, this isn’t English." Now they’re getting used to it, little by little. I have my kids do a lot of writing and explaining things—not only in their writing but also in conversation with each other.

**Types of mathematical tasks.** When asked about mathematical tasks, a range of interpretations was given. Some teachers felt that alternate or multiple solutions should be possible in mathematical tasks. Others perceived any mathematics problem or exercise to be a mathematical task. Teachers at Pinewood found it difficult to work with the length of the tasks provided in the Pacesetter program. They felt the tasks should be broken down into shorter segments in order that students could manage them better.

Teachers at East Collins found the tasks in their text series to be useful, but these teachers were always looking for additional tasks that would extend student exploration with more challenging material. The approximately 100 projects at Desert View were identified by those at the site as long-term mathematical tasks.²²
Interpretation: Mathematical Vision

Hart et al. (1991) believe that teacher beliefs determine what teachers find important in their mathematics practice. She and her co-workers have found in the Atlanta Mathematics Project (AMP) that teachers learn new belief structures through construction or reconstruction of knowledge that is in conflict with the teachers’ existing knowledge. According to Hart:

The coordination of new knowledge ... with already existing knowledge structures is facilitated through reflection. When learners are conscious of and are in control of their beliefs, understandings and procedures, they are more likely to change or alter their knowledge.

Lieberman (1995) also supports the need for teachers to construct their own knowledge much as teachers expect their students to do. According to Lieberman, "The ways teachers learn may be more like the ways students learn than we have previously recognized" (p. 592).

The mathematical vision held by teachers in the five sites of this extended study reflected what those teachers believed important for students to learn. The wide variations in these different mathematical visions is supported by the constructivist perspective that teachers construct or reconstruct knowledge based upon perceived needs or through conflict resolution.

The active participation in the mathematics curriculum revision by 15 of 22 high school teachers in the Green Hills School District placed those teachers in a position where resolution to a shared vision was the result. The many meetings over a period of 29 months allowed mathematics teachers numerous opportunities to share ideas with colleagues and representatives from the community. A shared vision also resulted during the planning stages of East Collins' mathematics curriculum. It can be argued that a shared vision also existed at Desert View since the importance of the projects at Desert View enabled teachers to do what they wanted, which was to bring mathematical meaning to their students.
Teachers at Pinewood valued the development of basic mathematical skills, and that became their vision. Some teachers at that school found themselves caught in the middle between their vision and that of the mathematics supervisor. What is the basis for a dichotomy between basic skills development and an emphasis on conceptual processes? Those who developed the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) emphasized process over skills development, but there was never any statement by the writers that basic skills development should be eliminated. In fact, the authors of the Standards document felt that a process or conceptual orientation would motivate the acquisition of needed mathematical skills. A resolution of this conflict may lead teachers at Pinewood to reconstruct their belief structure to provide for the development of both concepts and skills.

The data support the importance of a mathematical vision in bringing about and perpetuating reform of mathematics, but the data also suggest that as teachers construct and reconstruct their beliefs of what mathematical meanings are important for their students those visions become more dynamic and viable.

The Curriculum and Evaluation Standards (NCTM, 1989) supports a mathematical vision that provides for relevant and meaningful mathematics for all students. Each of the grade level clusters (i.e., K-4, 5-8, and 9-12) in the Standards recommends a greater emphasis on algebraic and geometric concepts. The data from this study indicate that mathematics courses with more substantive content are replacing courses which have traditionally lacked relevance for high school students. The difficult part of this process seems to be in providing tasks which make strong mathematical connections to students' experiences. Teachers must be willing to reflect on how they translate their mathematical vision into classroom practice. Hart et al. (1991) indicate that this process is slow and must involve continuous learning for the teacher as well as the student.

The data was not as definitive regarding the need for revision of curriculum frameworks as a prerequisite for or as a causal factor of changing mathematics practice in a
school. This included choices of materials and pedagogy. Data indicated that the county-wide framework had few consequences on classroom practice at Pinewood High School. When it comes to translating policy into practice, the limited data from Pinewood at least suggests the possibility of convergent findings with that which other researchers found at the elementary level (Ball, 1990; Cohen, 1990; Cohen et al., 1990). Green Hills had a framework in place, but data was not yet available to determine whether the framework played out in actual classroom practice. Further research is indicated.

Fullan (1993) warns that change is complex and unpredictable and believes that "change is a journey and not a blueprint." (p. 21). He further suggests that "vision and strategic planning should come later" (p. 21), that a too careful planning can actually "blind" attempts at change. The fact that reform efforts were occurring at the schools in this study without reliance on formalized mathematics frameworks suggest that teachers may see those frameworks as blueprints they would prefer not to follow. I speak from experience in saying that many teachers really do prefer to do classroom practice their own way.

A number of factors seemed to be contributing to the reluctance which some teachers had for embracing a Standards-like curriculum. One of the administrators of a high school made the observation:

Teachers as a group are not good changers. I would venture to say that most of us teach the way we were taught and that's one reason change is so slow. Change is frightening. You don't always have a model for that.

There was a real concern among some high school teachers that the "hands-on," exploratory activities designed for learning and understanding mathematics would circumvent the computational and algorithmic procedures which have and continue to dominate the mathematics curriculum in grades K-12 (Koehler & Grouws, 1992; Weiss, 1992).

Data across cases indicated that spending time on exploratory activities was a major concern in two of the schools. Teachers felt that if they allowed time to explore
mathematics, they would deprive students of time needed to master fundamental skills (i.e., memorizing and manipulating formulas). The influence of teacher beliefs in determining classroom practice has been the focus of much research (Anderson, Anderson, & Romagnano, 1993; Romberg, 1988; Thompson, 1992; Underhill, 1988). Allowing teachers opportunities to collaborate and reflect on teaching practice is one way in which teachers can engage in constructing their own knowledge about how they can best serve their students (Hart et al., 1991; Lieberman, 1995).

There were a number of factors that distinguished the urban school in this study from the schools in rural or suburban surroundings. The layers of administrative policy-making were very different, and, as a result, teachers at the urban site were often far removed from the policy-making process. Teachers in the urban site were wary of change, and there was suspicion that they might get caught once again in the middle of frequently changing administrations and changing policy. As a consequence, change appeared to be much more difficult for teachers in the urban school. Consistency at the administrative level seems an important need for teachers at Pinewood.

Providing more problem solving opportunities for students was identified in the data as one of the more frequent mathematical visions. The data across cases revealed that there was no consistent meaning of what problem solving actually was, but that everyone was trying to do more of it. "Connections" to real life and "relevance" to the students mathematical background were commonly used descriptors for the process referred to as problem solving.

Problem solving took on various forms from routine exercises in a problem set to full-scale, long-term mathematical tasks as defined in the Professional Standards for Teaching Mathematics (NCTM, 1991). The references to the UCSMP materials as being rich in problem-solving activities was probably a reference to the abundance of "verbal" types of problems. These differences in meanings would explain why documenters witnessed a very traditional pedagogy in some schools, while those at the sites teachers perceived their
classroom practice as being avant-garde. Schoenfeld (1992) addressed the multiple meanings of problem solving and how just about any use of the term could be the correct one. Problem solving has been defined in various ways ranging from "anything required to be done" to "a question ... that is perplexing or difficult" (p. 337). According to Schoenfeld, "'Problems' and 'problem solving' have had multiple and often contradictory meanings through the years--a fact that makes interpretation of literature difficult" (p. 337).

The data indicate that growing numbers of teachers were looking for meaningful mathematical experiences which provide multiple approaches or strategies as part of their mathematical vision. The need for different approaches to solving mathematics tasks is indicated in the literature. Multiple approaches is identified in the Profession Standards for Teaching Mathematics (NCTM, 1991) as being one of the criteria for a "worthwhile" mathematical task. An earlier publication, the 1985 Mathematics Framework (California State Department of Education, 1985) stressed the need for classroom environments in which students are free to try out several ideas or strategies in solving problems that "may require original approaches" (p. 14). Lieberman (1995) sees the importance of multiple approaches to problem solving in areas other than mathematics.

Tied closely to the indicated needs for mathematical experiences rich in exploration opportunities and multiple approach possibilities is the vision which fosters communication of mathematical ideas to others. "Mathematics as communications" is cited as one of the process standards of NCTM (1989), and the means by which this vision is carried forward properly belong in the realm of pedagogy. Interpretation of this component of the mathematical vision will be addressed along with other pedagogical interpretations.

The importance of technology as part of the mathematical vision is indicated throughout the Standards (NCTM, 1989). Technology is important as mathematical content and as a means of facilitating mathematical understanding. The extended use of data from the R³M project has revealed increased use of technology occurring across the five cases of this study, but that data also revealed the reluctance of many teachers to the

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use of technology in their mathematics classes. Kaput (1992) downplays the technological
limitations of the school and identifies the "old habits and social structures" of teachers as
limiting technological applications to mathematics education. Kaput believes that no
serious use of computers is part of the present school culture.

Fey (1989) identifies the potential of technology in extending the possibilities for
mathematical understanding. Technology allows teachers and students to skip over menial
tasks and more productively utilize time to extend the mathematics curriculum to
mathematical ideas and "applications of greater complexity" than those presently available.
The expanded use of technology to facilitate mathematics learning for students will also
require that teachers be provided a structure which will enable teachers' construction of
technological knowledge.

Pedagogical Vision Across All Cases

A pedagogical vision represents the instructional practice which the mathematics
teachers interpret as being the most ideal in providing students with an environment in
which to learn mathematics. It includes teacher and student activity in the classroom as
well as the materials that will be needed for instruction. The Cluster Matrix (Chapter 5)
provides a summary of the pedagogical vision for each of the five cases. Those summaries
indicate whether or not there was a shared vision among the key players at a particular site.
If so, what was that vision, and was it explicit or implicit? In other words, were there visions which were articulated by those at the site, or were they implied through practice?

Pedagogical Visions Held By Those at the Sites

The pedagogical vision was explicitly stated by several of the teachers and
administrators in three of the sites (East Collins, Desert View, and Green Hills). The
articulation by Desert View teachers was less structured and reflected two pedagogical
positions. The first seemed to apply to teaching practice when students were engaged in
doing mathematics when working with a project. This pedagogical vision was very similar
to visions held by East Collins and Green Hills. Desert View teachers articulated a second vision for their "regular" classwork (i.e., daily work using a single textbook).

The County Mathematics Supervisor and Pinewood's Mathematics Coordinator both expressed a clear pedagogical vision, and teachers had little input into developing that vision—though teachers were beginning to try out some of the components of that vision. The pedagogical vision at Scottsville was an imposed one, too, and had a narrower focus on technology.

There were portions of the pedagogical vision which were common to all five cases: Equity should be an important consideration in developing a pedagogical vision, and all students in the school should be provided opportunities for a meaningful and sound mathematics program; technology, including graphing calculators, computers, and quality software, should be available to help students learn mathematics; hands-on learning experiences, which utilize manipulatives, should be part of a student's mathematics development; and it is important that students develop verbal communication skills in mathematics.

Those pedagogical components found common to four cases included: Teaching practice should focus on the learner; there should be more problem solving and less drill and practice; mathematics should be enjoyable, interesting, and fun; there should be regular experiences which promote written communication in mathematics; and types of group activity should be encouraged which relinquish more responsibility for learning to students.

In three cases, the vision stated that it was important to build students' confidence and success in mathematics. Textbooks became part of the vision at two of the sites. That is, teachers at those two sites identified with a particular text as their vision because they felt that the texts provided the kind of mathematics they wanted to teach. The pedagogical visions at East Collins and Pinewood identified a need for multiple resources instead of relying too heavily on a single textbook in each course. Teachers at Green Hills and Desert
View emphasized the use of multiple strategies for exploring and solving long-term mathematical tasks.

Several features of a pedagogical vision were frequently mentioned in only one of the cases: Teaching practice should promote a constructivist pedagogy (Pinewood). Teachers should find ways to relieve mathematics anxiety (East Collins). It is important for teachers to become pedagogical decision makers in their classrooms (Pinewood supervisor). Drill and practice of mathematics fundamentals is important (Pinewood teachers only). All subject areas should be integrated through technology (Green Hills)

Pedagogical Practice

Group activity. The most prevalent practice across cases was the increased use of small groups of students in mathematics classes. Group activities were observed at all of the sites, and group activity varied from class to class and from day to day. Most teachers admitted that they didn’t use groups all of the time. In one school teachers had found that through group activity, they could explain something once to one student in a group and that the need for repetition was greatly reduced. Those teachers admitted that group activities became a "convenience" for them. Another convenience of the group setting, according to the same teachers, was that groups provided a place for students to go and do their work.

Group activity prevailed at East Collins and Desert View on days when lab activities or projects were planned. On other days when students were following regular assignments in their texts, there was a mix of group activity utilized by the different teachers. However, teachers at both schools reported an increased use of group activities was beginning to take place on those days of regular mathematics activity.

Activities within groups varied greatly too. In some classes, student exploration prevailed. The type of group activity was greatly influenced by the mathematical tasks provided by teachers. In another school, where computer worksheets guided student activity, there was little exchange of information between group members. The projects at
Desert View were long-range tasks that necessitated a good deal of collaborative effort of all group members.

**Mathematical discourse.** The Professional Standards for Teaching Mathematics (NCTM, 1991) identifies the teacher's role in discourse as consisting of several pedagogical practices:

- posing questions and tasks that elicit, engage, and challenge each student's thinking
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during discussion;
- deciding when and how to teach mathematical notation and language to students' ideas;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate. (p. 35)

Mathematical discourse was observed at different levels. Teachers at East Collins were making efforts to allow for more mathematical discussion to occur at different levels between teachers and students and between students and students. Teachers at East Collins were always looking for mathematical tasks that would provide for more student explorations and would furnish settings for discourse to develop and expand. Mathematical discourse was particularly prevalent in classes at Desert View; especially when students were working on projects. There was a deeper level of discourse taking place where the mathematical tasks provided options for multiple solution strategies. In those mathematical situations, students were expected to explore various ways of solving problems by making conjectures and then testing them.

Many teachers were struggling to change teaching practice to a more facilitative one. The data may suggest that mathematical discourse follows a progression of slow steps to move away from a traditional teaching practice in which teachers assume sole responsibility for dispensing mathematics information. This struggle was most apparent in the sites of
the study that were just beginning reform efforts. Teachers at Pinewood, for example, were just starting to experiment in working with small groups of students.

**Technology.** Some form of technology was in use at each school. Graphic calculators were used at all sites, particularly in Algebra 2 and more advanced mathematics courses. Usage of computers varied among the five schools. Students at Scottsville and Green Hills had more computer labs available and more opportunities to integrate technology and mathematics. In each of the other three sites, computers were used less frequently because it was difficult to schedule computer usage when they were needed. Usage of technology was limited to classes where teachers felt comfortable with the technology.

**Teachers as risk takers.** Many of the teachers were trying new ways of teaching. Working with students in groups was a big change for many of these teachers. Teachers indicated that they were doing less lecturing as a result of their increasing facilitative roles. A few teachers felt they had inadequate mathematics and technical preparation to cope with the these new pedagogical roles. It was difficult for some teachers to admit to students that they might not know the answer to a question about mathematics, and that possibility became a real risk as teachers yielded more responsibility for learning to the students.

**Interpretation:** **Pedagogical Vision**

I was interested in finding the answer to two questions in the analysis of this cluster on pedagogical vision: Was there common features in the pedagogical visions across the five cases? If so, how did that vision play out in actual classroom practice?

The results of the analysis revealed that there several common features among the different pedagogical visions. These included:

- teaching practice which focuses on the learner;
- allowing all students to receive quality mathematical experiences;
- using more technology and hands-on experiences;
• furnishing experiences which promote written and verbal communication in mathematics;
• providing for more "problem solving" and less drill and practice;
• making mathematics enjoyable, interesting, and fun;
• promoting group activity and relinquishing more responsibility for learning to students; and
• building students' confidence and success in mathematics.

In terms of the actual pedagogical practice, the cross-case analysis indicated that changes were occurring in classroom practice. The more common observations included the following:

• Small group activity was common at all of the sites and was seen by most teachers as providing a less threatening and more enjoyable environment in which students could learn mathematics. This was a significant pedagogical shift for most teachers, and most of the teachers recognized that they needed to learn more effective ways in facilitating group activity.

• Teachers felt that the level of problem-solving activity was improving. This was being accomplished through changes in texts and materials. Teachers in several of the sites were developing their own problem-solving activities.

• The role of the teacher was changing in varying degrees from a dispenser of information to a facilitator in the learning process. For many teachers, this was unfamiliar terrain, and change would be slow.

• Technology was being used by some teachers. The use of graphing calculators was a more common practice than computers. The availability of computers for student use was still a problem in most schools.

• The impact of these changes on students' attitudes towards mathematics was, for the most part, positive.
Interviews with teachers across the sites indicated a pattern towards a more uniform pedagogical vision and practice was taking place at East Collins and Desert View, the two schools which had been engaged in reform efforts for several years. Small-scale strategies appeared to provide a better vehicle for change (i.e., just a few teachers trying out the new pedagogy during the first year, etc.).

Giving up a familiar practice was not easy for teachers who often found themselves moving between the roles of an information dispenser and a group facilitator. The incentive to change frequently came from a desire to follow the lead of colleagues. In some schools, the initial desire was not always that of the teachers. Desire for change originated with mathematics supervisors in two of the schools (Pinewood and Scottsville). Fullan (1993) indicated that change may depend on both top-down and bottom-up strategies. What was initially a top-down strategy in Pinewood was beginning to transform to a combination of both strategies as teachers began to see some benefits of changed practice.

An important question seems to be: What factors influenced the pedagogical visions across sites to be similar? This question will be addressed in Chapter 7 since it requires looking at several of the cluster analyses to reach a conclusion.

**Context of Change Across All Cases**

There were a number of contexts in each site which interacted with the change process in mathematics education. The more significant included: collaboration among staff and others; leadership; administrative support; funding; and the role of the teacher in developing change. Each of these factors was addressed across the five cases.

**Collaboration Among Staff and Others**

A close bond among teachers was observed in two of the cases (East Collins and Desert View). This collegiality went beyond the professionalism and cohesiveness observed at Green Hills and Scottsville. Characteristic of this collegiality was a collaboration based on friendship, humor, and trust. This friendly esprit de corps provided a non-competitive, non-threatening environment in which teachers were able to share ideas,
concerns, and failures. It also provided a safe mathematical community in which to take risks.

We're comfortable with one another as people and as teachers. But there again, you know, I think we have just such a wonderful opportunity here because we've been together for so long, we know one another. (Teacher, East Collins)

So it was the chemistry at the time. The department was very collegial--got along very well and had for quite some time. And I don't think there were any real principalities there that you had to knock down in order to try something new. ... I think, more than ever, right now that the math faculty works together toward mathematical goals. ... I think working on the projects really made them a lot closer as a faculty. (Assistant Principal, Desert View)

The mathematics teachers in each of these two schools let two or three of their peers pilot new instructional practices while the other teachers took a supportive wait-and-see approach. It took three years in each case for the total mathematics staff to embrace the changes that were piloted. Yet, teachers kept their autonomy as classroom teachers, and there was never any coercion for teachers to submit to these new ideas.

Collaboration existed at all of the other sites but in varying degrees, and collaboration was considered by teachers to be an important part of the change process. Most of the Green Hills secondary mathematics teachers (15 of 22) served on the K-12 mathematics curriculum committees for a two-year period and, as a result, collaborated on a regular basis. After the revised curriculum had been drafted, teachers in the district ratified it almost unanimously. Since the new curriculum carried a number of pedagogical implications, teachers were careful not to simply think of the committee's work as a set of curriculum guides for each of the K-12 grades.

There were a number of ways in which teachers collaborated at Scottsville. They helped each other develop assignments and computer lab worksheets. Teachers taught each other through their study groups program. Much of the collaboration at Scottsville was focused on sharing knowledge about the technology which was constantly being integrated into the mathematics classes. As one teacher suggested, it was not easy keeping up with all the new technology, especially all of the new software:
I feel a constant source of pressure that I have to try and keep up with a very bright department, with a lot of people that know a lot of technology. So that is something that I want to work on and improve.

Pinewood teachers met regularly by "divisions" (e.g., all Algebra 1 teachers belong to a division). The division meetings provided opportunities for teachers to share ideas and discuss concerns which arose in their classes and were helping teachers to change what and how they taught mathematics. While the overall unity of teachers, evident in other schools, was not as prevalent at Pinewood, there were indications that teachers were moving in that direction. The fact that teachers wanted to give the Equity 2000 classes and the Pacesetter classes another year of effort was seen by the mathematics supervisor and the school mathematics coordinator as a sign of real progress. A growing group of teachers were trying out group activity in their classes, and some were collaborating on the use of calculators and manipulatives.

Leadership

Strong leadership was emerging as an important ingredient for change to occur, but leadership across the cases came in various forms. The mathematics supervisor or the mathematics coordinator was the instructional leader in three of the cases (Green Hills, Pinewood, and Scottsville). Two teachers (one of whom was the chair) and the associate principal at East Collins were the prime movers of change. The entire mathematics department became the leaders for Desert Hills.

The chair of the mathematics department at East Collins accepted a leadership role in the planning and implementation phases of their mathematics program because he was afraid that, left in the hands of others, the effort would not succeed.

It seems like a lot of times a good idea comes up and everybody gets excited, and for two or three years it is there. Then it just kind of fades away. And I really feel strongly about this. I don't want this [program] to do that. And kind of selfishly, I was afraid that if I didn't accept being head of the math department, that somebody else who wasn't interested in our changes would let it fizzle, and I don't want that to happen. So I'm hoping that I can continue to push it.
The other teachers at East Collins felt that they were an important part of the decision-making process as their mathematics program unfolded. According to one teacher: "They didn't make a move that they didn't consult with all of us."

After the one-year start-up period in which three teachers implemented projects in their mathematics classes at Desert View, most of the remaining teachers of the mathematics department became participants. Interestingly, the three who started the program did not continue on in any strong leadership capacity. All participants seemed to share equally in developing new projects and moving the program forward. According to one of the teachers:

When we first got together as a project group...there were definitely some barriers between us as far as being afraid to criticize or damage egos. Through the process of editing projects for each other, we have taken each other's worth. And it's helped level everybody out to an even playing field where they feel more comfortable communicating back and forth....Getting over the barrier of having done something dramatically different in the classroom, and being successful at it, has made everyone believe that it's not that tough to try new things. Now it's almost expected that you are going to try new things, see what happens, and not be afraid to report back to your peers about what worked or what didn't work.

In each of the other cases, the leadership for reform efforts came from mathematics educators at the administrative level. Green Hills' teachers were contributed to the development of the K-12 mathematics curriculum, but it was the newly appointed District Mathematics Coordinator that organized and led that effort.

For Pinewood High School, it was the County Mathematics Supervisor who had the vision for mathematics reform that would impact on the school. According to one of the mathematics teachers:

We really do have a dynamic supervisor in math who is very energetic and really pushing for change and just really active, ... She's well respected, and I think everybody respects her and appreciates what she's trying to do.

The Mathematics Supervisor was also the leader in change at Scottsville High. Teachers and administrators recognized his ability to get what he felt was needed for the
mathematics department and his skills at motivating teachers to try new things in the
classroom. According to the school principal:

I also think the reason the math department is so willing to try new ideas and
do the kinds of research that they themselves were, is primarily the
supervisor's leadership. I mean, he will not allow them to sit back on their
laurels. I mean, if they do something really good and really unique, all of a
sudden he wants them to do something else. They don't sit back and enjoy the
fruits of their labor that long. I mean they're just moving on to something
else.

Administrative Support

There was strong administrative support for the mathematics teachers and their reform
efforts in all five cases. Interviews with school and district administrators were very
positive about the changes taking place. As indicated above, some of the district-level
mathematics administrators (supervisors, coordinators) were and continue to be
instrumental as leaders in change.

The associate principal and the county mathematics supervisor for East Collins early
relinquished their leadership roles to two of the mathematics teachers at the school.
Besides encouraging teachers through the change process, administrators at the school site
acted as buffers against early criticisms of the program from parents. Their admonitions to
parents to "give the teachers a little more time" really helped pave the way for public
acceptance and support of the program.

School administrators generally viewed their mathematics staff as professionals.
Principals showed a confidence in the decisions which the mathematics teachers made, and
there was rarely any interference at the school level. The district curriculum director for
Desert View indicated that the changes being brought about by the mathematics staff could
well serve as a model for whole school restructuring at that school. School officials had
often bent over backwards finding necessary funds to help the emerging mathematics
program along.

A move by principals in the Green Hills District to curtail the frequent absences (of
some teachers), due to attendance at professional meetings, resulted in eliminating all but

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one release day per year for teachers to attend outside conferences. The move was perceived by mathematics teachers (and others) as an indication of lack of administrative support, even though the principals indicated a strong verbal support for the mathematics program. The district mathematics coordinator was also concerned about the shortsightedness of school principals who make decisions to spend most of the mathematics budget on textbooks and allow very little for teacher development.

The mathematics supervisor for Pinewood indicated that she would like to see the building principals more aware of the changes going on in mathematics. She even prepared a notebook which would keep the principals abreast of the program and gave them a three-day workshop. She admitted that several of the principals hadn't read the contents and don't really know what's going on in the program. A notable exception is the principal at Pinewood who seemed quite articulate about the mathematical vision of the new curriculum. Administrators at Pinewood were supportive of efforts being made by the mathematics teachers. They freed mathematics teachers of other duties to enable them to meet regularly with each other. According to the school principal, administrators "baby sat" with all the mathematics classes while the mathematics department had inservice staff development meetings.

**Funding**

As mentioned previously, external funding was the catalyst in three of the cases (East Collins, Desert View, and Pinewood). This is not to say that funding was not important at Green Hills and Scottsville, but the additional funds in those two schools came from a local commitment to fund the reform efforts.

The mathematics teachers at East Collins saw the benefits that the funding from industry provided: release time allowed teachers to participate in inservice meetings to plan their program; summer workshops gave teachers opportunities to develop skills in using the new texts and materials; a computer and printer were purchased for each mathematics
classroom; classroom sets of calculators (including graphing calculators) were purchased; and the cost of new texts was partially provided by industry's contribution.

The teacher enhancement grant, which university mathematicians directed, gave Desert View teachers stipends to attend workshop to prepare projects, and it provided release time for periodic work meetings during the year.

The College Board's pilot program at Pinewood was helping to fund the Equity 2000 Algebra and Geometry classes and the Pacesetter program. Teachers who had been or would be teaching one of these classes received staff development during summer institutes. Many of the teaching materials were also provided, particularly for the Pacesetter program. According to the mathematics supervisor, the goals of the College Board were compatible with the county's mathematics framework, and the external funding was pushing the implementation of the new framework.

Teacher Role in Developing Change

Teacher input in the developmental stages of mathematics reform was clearly visible in four of the five cases. Even in the fifth case (Pinewood), there was some participation by a few teachers on a district committee who were active participants in the development of the new curriculum. Teachers at Pinewood made no reference to that participation and apparently felt that they had little or no input into the whole process.

Potentially the most helpful thing in developing the East Collins mathematics program was the recognition afforded teachers to develop their mathematics program on their own. "I felt that it started with us," said one of the teachers. The involvement of the whole mathematics department in the planning of the reform efforts at East Collins made it easy for all of the staff to become active participants in the program within three years.

A similar situation developed at Desert View. Teachers at the school had enjoyed a close camaraderie for years, but when the partnership with the university mathematics began, that mathematical community strengthened at a professional level. The collaboration and trust continued throughout the testing or pilot period of the projects.
More teachers decided to become involved in the development of projects the second year. The entire program has developed as a result of teacher input. One of the university mathematics professors explained the process: "All the ideas came out of their mouths. So when they did something good, we told everybody. And two meetings later, everybody was doing it."

There was tremendous involvement of the high school mathematics teachers from Green Hills in the planning and development of the district's K-12 mathematics curriculum. The nearly unanimous ratification by teachers of the curriculum indicated a strong teacher commitment and ownership.

The agreement of teachers at Scottsville with the mathematics supervisors' vision has promoted teacher collaboration and development of activities that utilize the technology of computers and calculators. Some teachers were having a difficult time changing the way they had been used to teaching. Said one teacher, "I'm biased for traditional mathematics. That is what I believe mathematics is. It has been my whole career." Teachers were not pushed to change, but they were often teamed with another teacher who was enthusiastic to introduce technology into the classroom and to use the developmental materials being piloted by university professors.

Some teachers at Pinewood were beginning to change their beliefs about how to teach. For example, facilitating group activity was a real change for these teachers, but one of the reluctant teachers decided to give it a try at the bidding of the school's mathematics coordinator. His comments indicated that long established patterns of teaching can change:

But it works better in groups. They really do help each other in everything. I have to split them up sometimes because you always change the groups and stuff. It really has changed my opinion on that. Plus, it's been hard getting used to. I was telling before about getting used to the fact that the better they get at this, the less they need us and I love [lecturing]. But I'm getting more comfortable with that and I'm more patient with them.

"Teachers are reluctant to change right now, but they will slowly grow into it I think. They don't have much choice," voiced a different teacher. Another teacher remarked: "I
think if they see what else is going on, and it's working successfully with other people, and other people have good things to say about it, I think you can bring them around."

There are other contextual features that are influencing the way change is occurring at the five schools of this study, and though they may not be significant across all cases, they should be identified. The impact of standardized testing influences decisions which teachers are making in their individual classes. Teachers at Scottsville indicate the difficulty in striking a balance between the skill development they think students will need on standardized tests and the problem solving which the mathematics program tries to promote. Says one teacher:

It was clear to me that the board of education was very interested in maintaining those high scores. If it meant skills at the sacrifice of innovation and experimentation, I think they would go for skills. If that meant it would maintain those high SAT scores and ACT scores.

County-developed criterion referenced testing, intended to drive instruction, was replacing standardized testing at Pinewood, but teachers were still hesitant about abandoning a strong emphasis of skills development. At Green Hills, tests which compared students achievements between schools and between other parts of the country were very important to many people in the community. The reliance on such standardized testing was a "political reality" which made the shift to more authentic assessment difficult, according to the district's mathematics coordinator.

Changing existing mathematics programs is not always easy. For example, Green Hills' parents wanted to keep accelerated mathematics courses, and it became important to get parents involved in the early planning stages of the new K-12 mathematics curriculum before such courses could be eliminated. Scottsville was faced with similar problems. The supervisor felt that they must balance "adding on new classes" by deleting others, but parents resisted such moves.
Interpretation: Context of Change

The importance of the individual classroom teacher cannot be underestimated if reform efforts are proceed (Ball, 1990; Ball, 1992; Cohen et al., 1990). But how important is teacher input in the initial stages of the reform effort? If teachers do not participate in the development of mathematics reform, what steps can mathematics supervisors, for example, take to insure that teachers will adopt and implement the changes?

In all of the cases of this study, the role of mathematics teachers in mathematics reform efforts seemed critical. Several contextual requirements were indicated from the analysis of this cluster:

- The role of the individual teacher cannot be underestimated. Broad-based collaboration among teachers during the ongoing reform efforts is indicated at the sites of this study and should probably be encouraged. This may require a period of building trust between administrators and teachers. Teachers must feel that they are not being controlled by others, and teacher input must be valued. The failure of previous attempts at reform in mathematics education have failed, according to Lappan and Theule-Lubienski (1992), "because researchers and curriculum developers failed to take into account the existing knowledge, beliefs, values, and purposes of teachers ... and the cultures and contexts in which teachers work" (p.19).

- Data indicates that teacher participation in the implementation phase should be voluntary. The data from East Collins and Desert View support this interpretation. Teacher support of the reform efforts progressed more smoothly when teachers were allowed to voluntarily participate than at those sites where participation was "expected." Fullan (1993) indicates that many instructional reform efforts fail because they fail to address "collaborative cultures among educators" (p. 46).
- If there is not sufficient support among at least a few mathematics teachers to implement change, then change should be delayed until that support is in place. Mandating reform may work, but not until some teachers are willing to support the reform. Reluctance among teachers at Pinewood later turned to mild support, but it was not until a few teachers had become mildly enthusiastic about changing pedagogy that their mathematics reform efforts started to move forward.

Peterson et al. (1992) have found among elementary teachers in three states (California, Florida, and Michigan) that changing teaching practice is gradual. Fullan (1993) indicates that administrators should not try to mandate things that really matter, such things as "skills, creative thinking, and committed action" (p. 22) in trying to achieve important educational goals. Fullan does indicate that in achieving complex educational goals that it may be necessary to have both top-down and bottom-up strategies.

- Leadership is necessary, but it must be the kind that teachers want to follow. This was illustrated in the cases of East Collins, Desert View, Green Hills, and Scottsville.

- Various forms of administrative support are useful, but most important to teachers is the support which comes from being treated as professionals. This was the case in four of the schools in this study. Noddings (1992) believes it is important that teachers be given opportunities to earn the trust of administrators and others who control their work. It is through this process of administrators working with teachers that promotes "faculty cultures that exemplify professional relations and attributes" (p. 206).

- Teachers need opportunities to develop different instructional strategies. Lappan and Theule-Lubienski (1992) believe these opportunities must extend beyond a one-day or a one-week workshop and suggest "an intensive summer experience
with follow-up support during the year" (p. 19). The model suggested by Lappan and Theule-Lubienski was witnessed at East Collins and Desert View.

Collegial collaboration was found in this study as one of the more important ways of developing different instructional strategies. Fullan (1993) supports this view and believes that "teachers must work in highly interactive and collaborative ways, avoiding the pitfalls of wasted collegiality..." (p. 81). Just getting together with colleagues is no guarantee that beneficial instructional strategies will emerge. Gripe sessions too often occupy the valuable time in which teachers meet together.

- Funding serves as a catalyst for change efforts, but funding also serves to validate what teachers believe to be important. The importance of teacher self esteem was indicated in several of the cases of this study, and the recognition teachers received as a result of outside funding helped foster that self esteem.

Influence of Mathematics Program on Students Across All Cases

There were a number of factors which I wanted to determine in terms of the influence that the five mathematics reform efforts were having on students. Were these reform efforts impacting on students in a positive way? Were provisions being made for the mathematical needs of all students? What kinds of mathematics were students learning, and was pedagogical practice effective? I was anxious to learn about the nature of mathematical discourse between teacher and student and between student and student and about teachers' perceptions of students and whether the program had been effective in developing students' ability to reason and to solve mathematics problems.

Provision for All Students

Teachers and administrators at all of the five schools indicated that one of their major objectives was to provide a meaningful mathematics program for all of the students. Two of the cases (Green Hills and Scottsville) had multiple tracks, and it was believed that all students would find their needs being met in one of those tracks. There were various
provisions being made for students with special needs at the schools: a modified algebra class was being developed at East Collins; the Integrated Math 1, 2, 3 at Green Hills provided a three-year sequence of topics that included algebra and geometry; a computer-intensive algebra class at Scottsville; the development of projects for use in high risk classes and the ESL classes at Desert View; and provisions for Algebra and Geometry for all students at Pinewood and East Collins.

A pull-up effect, in which students were taking higher level mathematics courses, was taking place in at least four of the cases, resulting in fewer and fewer students taking general mathematics courses. East Collins indicated that nearly all of its ninth graders were either taking Algebra 1 or Algebra 2 (Geometry follows Algebra 2), and approximately 91 percent of students taking Algebra 1 went on to take Algebra 2. Green Hills was eliminating its general mathematics courses in favor of a focus on algebra and geometry through its Integrated Math 1, 2, 3. Desert View indicated dramatic decreases in its general mathematics offerings (from 12 to 3 in the past three years), and more students are taking Algebra and higher level classes. Scottsville expected that Calculus for all its students would be a reality by the year 2000. Pinewood's participation in the Equity 2000 program was an attempt to pull its students to higher levels of mathematics and possibly meet its goal of Algebra and Geometry for all of its students.

Activities of Students

Examples of pedagogy have already been discussed, and my intention here is to summarize the various mathematics activities high school mathematics students have been engaged in across the five cases. Reading in mathematics was receiving increased emphasis in two of the schools (East Collins, Scottsville). Teachers in four of the schools reported that writing in mathematics classes was becoming increasingly important. Daily homework was assigned in all five sites. Increased use of technology was taking place across all cases, especially the use of the graphing calculator. Two schools have sufficient labs to encourage regular use of computers, and two other schools were trying to move in
that direction. Long-term mathematics activities (projects or tasks) were important at three of the sites, as were multiple solution strategies.

Assessment

Program assessment and student assessment were both observed. Student input for program assessment was actively sought in two of the cases. East Collins encouraged its students to anonymously make written comments about strengths and weakness of its mathematics program. At Green Hills, interviews of students were conducted to yield "honest appraisals" of its mathematics program.

A number of alternative assessment strategies of student progress were observed in classes at the various schools of this study. Teachers at East Collins were experimenting with a variety of assessment options which include self-assessment, parent assessment, group quizzes, homework quizzes\textsuperscript{25}, and portfolios. Desert View teachers employed some or all of the following assessment strategies: peer editing\textsuperscript{26}, group quizzes, portfolios, peer assessment by others in a student's group, group grades on projects, take home exams, project grades sometimes serving as a test grade, homework quizzes, and assessment of verbal and written discourse. Rubrics were used at Pinewood to assess mathematical tasks.

Alternative assessments were also being given at state and district levels. An open-ended assessment intended to drive a Standards-like state framework was given at Green Hills. A locally developed criterion referenced test to push a Standards-like framework at the county level was used at Pinewood.

Departmental exams were used at Pinewood and Scottsville, and program assessments were provided for the Equity 2000 and Pacesetter programs at Pinewood and the UCSMP materials at East Collins and Scottsville. Student scores on tests such as the SAT, ACT, and AP exams were "politically" very important to the communities of Scottsville and Green Hills where student scores were traditionally high.

Teacher-Student Interaction

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We were interested in determining the levels of teacher-student discourse which students were experiencing. Were students being led to explore and make conjectures in mathematics as a result of this discourse? Generally the level of communication between teachers and students was perceived by both teachers and students as being good. A teacher at Desert View explained some changes he had observed:

We used to get the students in our algebra classes geared toward making sure that they hand the teacher a paper at the end of the period; and it's out of their face at that point, and they don't want to see it again. Here we're trying to tell them, "Hang on to this, bring this back, we'll do more work with this, we'll make improvements on it and we'll keep improving the quality of this even though you might have enough material worked out and things like that, we're just going to make it better." So it's a quality over quantity type of thing that you're trying to get them focused to and adapted to.

Observations of documenters revealed that the effort to allow students time to discuss and think through mathematics may be one of the more difficult transitions for teachers to make as they shift to new teaching styles that emphasize facilitative roles.

Waiting for student responses was reported by teachers as a real important part of this process. Teachers admitted they were more skillful at giving information, and it was difficult for many to receive information—especially from students. Listening to students' mathematical ideas was witnessed by documenters in some of the classes. According to documenters, the nature and design of the projects at Desert View promoted such discourse, and classroom observations revealed that a few of the teachers were actively encouraging mathematical discussions. Mathematical discourse between students was also observed as students collaborated on what strategies would best meet the criteria of a particular project.

Teacher Perceptions of Students

Teachers perceived increased levels of students' enjoyment of mathematics in all cases of this study. Other superlatives offered by teachers were that students were finding mathematics "exciting" and "stimulating." All five of the schools were employing group activity to varying degrees, and teachers in three of the cases were finding this to be a more
supportive and comforting environment for students to learn mathematics. As a result of these pedagogical changes, teachers in two of the schools indicated that there was less student anxiety associated with mathematics; this, in turn, resulted in greater confidence in doing mathematics—especially word problems. Teachers felt students were generally understanding mathematics better, but some teachers were concerned that students were weak in basic skills. Some of these positive changes in student attitudes towards mathematics became increasingly apparent to teachers of at-risk students, particularly as teachers de-emphasized drill and practice (three cases).

Teachers who promoted group activity also found that they had to repeat explanations less frequently, and they found they were getting to know their students better. Teachers recognized that they were changing too and were becoming more accessible to their students. In two of the schools, teachers felt students were too preoccupied with grading.

Student interviews were conducted in only three of the schools of this study. Student perceptions of the reform efforts were almost always positive. When asked what was their favorite subject in school, most students responded that it was mathematics. Again, most students indicated that they liked working in groups and that they learned better that way. A student at Desert View commented:

Well I think the math program is really extraordinary here. We not only have regular problem sets to do, but we get to do group work, projects, themes and I think the group work has really helped me to bring up my grade to learn, to experience, because some things that I can't get by myself, someone around the table usually can get.

Several students thought group activity was okay, but they preferred to do mathematics by themselves. Students who completed projects at Desert View, for example, felt that they retained mathematics longer than they did in their "regular" class activity. A couple of students at East Collins admitted that they "used to hate math," but the "way they were doing it now was better."

**Interpretation: Influence of Mathematics Program on Students**

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Equity issues were indicated as important in all of the schools of this study, but there is some question of whether providing "more substantial" mathematics courses in lieu of general mathematics and remedial mathematics courses will properly address issues of equity. Secada (1992) views this nation's present system of mathematics instruction as operating to perpetuate and broaden the inequities that exist for the "poor and ethnic minorities." What is needed, according to Secada, is that reform efforts "should first become effective with these students, and then applied to other populations" (p. 654).

If Secada is correct in his analysis, then the efforts to provide for equity in mathematics at the four schools of this study may be based on faulty premises. The pattern across four of the schools was to first get the reform effort established for those students who were considered to be not-at-risk. Then, if successful with the "mainstream" students, the reform efforts would be extended to include those students considered at-risk (i.e., the poor and ethnic minorities). The Mathematics Supervisor for Pinewood saw elitism in mathematics instruction as being the single most important obstacle to mathematics reform, and the data indicated that reform efforts in that school were addressing the concerns of Secada (1992) (i.e., the emphasis on reform by piloting the Equity 2000 Program to insure that all students would be complete mathematics courses that would include Algebra and Geometry).

There was no attempt to gather standardized test results on students in this study. Instead, documenters were gathering descriptive data with which to portray change efforts in mathematics. However, even if such test results were available, they would offer little in terms of assessment of students' understanding of mathematics since such tests are designed for different purposes. Webb (1992) refers to mathematical assessment as "the comprehensive accounting of an individual's or group's functioning within mathematics or in the application of mathematics" (pp. 662, 663) and believes that both quantitative and qualitative approaches are needed to gain this comprehensive accounting. The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) stress the importance of
fitting the assessment methods to the type of information being sought and caution that "standardized norm-referenced have difficulty measuring the generation of ideas, the formulation of problems, and the flexibility to deal with mathematical problems that are not well structured" (pp. 201, 202).

A constructivist approach to student assessment begins with the individual student and recognizes that students differ in their attempts to make meanings from mathematics. The Standards (NCTM, 1989) identifies the cyclic nature of student assessment and the need for assessment to be continuous and most often informal in nature. It is this "informal" approach to student assessment that data from this study offer the most information on how well these various reform efforts are influencing student learning in mathematics.

Comments by teachers indicated on a number of occasions that students' understanding of mathematics was improving as a result of the reform efforts:

A lot of students that are generally interested in learning have a good attitude about the learning, but never saw math as offering anything stimulating to think about. Many of those students find that they are successful in the program that we have now because, partly because of the group interaction. (DVHS teacher)

The quality of the writing is incredible compared to what students were doing. And it's the same age kids, so I can assume doing the projects has brought up their skill level that much that they're actually having a lasting effect. And we know that a lot of times if we ask them, "Tell us how to complete this query" for something they did several years ago that they will be able to do it. (DVHS teacher)

I think for the most part with the group activities and relying on the group to help them, that most students do better on the tests. (PHS teacher)

Data indicated that teacher expectations of students' abilities to do mathematics was increasing as a result of the reform efforts.

The "qualitative" assessments referred to by Webb (1992) and the "informal" assessments mentioned in the Standards (NCTM, 1989) offer alternatives to student assessments that traditionally occur at the end of a chapter or unit in a textbook. Much of what teachers do on a day-to-day basis as they make decisions about what strategies are most appropriate for each student is the one of the major components of student assessment.
(NCTM, 1989). Authentic assessments instruments\textsuperscript{27} were non-existent in some of the sites and received only limited use in others. The fact that teachers generally see the limitations of traditional testing may be an indication that as pedagogy changes so will assessment practice.

The general acceptance by the lay public of norm-referenced achievement tests may prove to be problematic as teachers struggle with issues of reform. In three of the districts (Pinewood, Green Hills, and Scottsville) parents and others in the communities put great emphasis on students' performance compared to other schools, other communities, and other states. Green Hills' model of curriculum reform involved a cross-section of representation from the entire community. Such a model may be indicated as other schools approach the complex task of aligning assessment practices to match changing pedagogy.

Teachers were finding that facilitating learning was complex and required skills that were often difficult to develop. Mathematical discourse, for example, is an area of classroom practice that some teachers found difficult to manage. For some teachers, just being aware of what the Standards (NCTM, 1991) recommend regarding mathematical discourse may be helpful. The data suggest that teachers feel a need for frequent opportunities to collaborate with each other. These findings parallel what Hart et al. (1991) found in Atlanta Math Project. Teachers will vary in their needs for providing more discourse in their mathematics classes, but it should be recognized that there are probably no specific steps by which a teacher learns the techniques of discourse. Reflection through collaboration can become an important link in the constructive process (Hart et al., 1991; Lieberman, 1995) and allows teachers opportunities "to look back on their teaching strategies, to reflect on the outcome of their behaviors, and to learn from experience" (Hart et al., 1991, p. 6).

Findings from this cross case analysis indicate that programs which were student centered (as opposed to teacher centered classrooms) provided greater opportunities for students to do mathematics in non-threatening ways. As teachers relinquished more
responsibility for learning to students, students and teachers both reported improvements in students' attitudes toward mathematics. These findings coincide with those of Wiske et al. (1992) and Tinto (1990) in which negotiation of classroom authority became a requirement in the constructivist's classroom.

**Impact of the Standards Across All Cases**

The mission of the R^3M project was to monitor the changes taking place as individual schools and school districts began to implement their interpretations of what the Standards were saying. What were the perceived influences of the Standards in bringing about the changes in mathematics learning and teaching already discussed in this study? Was there general familiarity of the Standards among all the key players in these reform efforts? Were the Standards a catalyst to change, or a validation of change that had already occurred?

The more detailed analysis of the data in this study suggested that there was a third function of the Standards. As the different mathematics programs continued to evolve, the Standards documents were being used to guide program adjustments and additions. This certainly seemed to be the role of those documents in all cases but one. Desert View teachers indicated a role shift of the Standards from a "wonderful validation of what we've been doing" to a more proactive function as a recent curriculum revision took place in their school district and as more projects were developed. Teachers reported that they had regularly studied and discussed different portions of the two Standards documents. One teacher explained that as a result of becoming familiar with the Standards, she and others who served on the district mathematics curriculum committee were evaluating the district's mathematics curriculum against the Standards and were making recommendations to align the curriculum to the Standards. The teacher also explained that the Standards would be used as criteria in the selection and adoption of new mathematics textbooks.

The department chairperson at East Collins indicated that their ongoing reform efforts were judged against the Standards:
We read the Standards, became immersed in them and probably, hopefully, the words that came out of our mouths to create [our mathematics program], were words that we had put in our minds from the Standards...

The mathematics supervisor for Pinewood indicated that as program changes were underway in her county during the summer of 1989, the Curriculum and Evaluation Standards helped to make changes credible to school board members and to a task force which had been appointed by the school board to make recommendations regarding the county's mathematics program:

The Standards gave us a vehicle by which we could de-personalize the change. Even now as I do inservices, I'm learning that whatever I say, black or white, handed out, this is from the Standards. Then people can't personalize it as my idea.

The Standards were a major influence on the reform of the mathematics program for three of the cases in this study (East Collins, Green Hills, Pinewood). Desert View saw the Standards as a validation of what they had done with projects, though the teachers admit that there was little direct influence from the Standards during the early development of their program.

Familiarity of the Standards was mixed among those connected with the mathematics programs in the five cases. Teachers at East Collins frequently cited the Standards as the basis for their reform efforts. That was also the case at Green Hills--especially those who spent two years working on the curriculum revision. The county mathematics supervisor and the school mathematics coordinator at Pinewood made frequent references to the Standards during their interviews and that reform efforts were based on the Standards. Teacher interviews at Pinewood did not indicate that the Standards were influential in the program changes they were experiencing. Scottsville teachers and administrators tended to refer to their progress in technology as evidence that they were implementing the Standards, but the data there suggest that the changes going on were not motivated by teachers' knowledge of the Standards. Familiarity with the Standards at Desert View took
place after the projects program was well underway, though teachers at that school recently seemed well versed in the content of both Standards documents.

Interpretation: Impact of Standards

The primary mission of the R\textsuperscript{3}M project was to describe the changes taking place as individual schools and school districts began to implement their interpretations of what the Standards were saying. It is important to recognize that the sites were not randomly selected for the R\textsuperscript{3}M documentation study. The five sites for this study were selected because there were changes taking place at each site which were believed to exemplify the changes recommended in the Standards documents. It is important that any interpretation of the impact of the Standards be seen from that "selective" perspective. That is, interpretation of the data for the five sites in this study might not have the same level of generalizability for sites who are not engaged in Standards-like reform.

The Standards "invite and encourage multiple interpretations" which added to the difficulty of discussing "alignment" of local practice with the intent of the Standards (Schoen & Ferrini-Mundy, 1991). Requiring that all high school students complete a traditional algebra course might be one school's interpretation of implementing the Standards. For another, it might be the restructuring of the classroom seating arrangement into groups. The word "implement" meant different things to different people and was subject to local interpretation, and it was this local interpretation that the R\textsuperscript{3}M project sought to document.

Prior to this analysis of data, documenters had envisioned that there were two uses being made of the Standards documents in these reform efforts--either as a catalyst or motivator to change or as a validation of change. This cross-case analysis revealed that the Standards also directed further changes being taken in reform efforts. This is an important distinction which became especially apparent in two of the cases in this study.

The notion of "being done" with reform efforts runs counter to the assumption of how students and teachers construct new knowledge and admits to a sameness or uniformity in
vision and in practice. Such a notion would also contradict the second orienting assumption (see Chapter 2) that there are always ways of improving teaching practice. Teachers and administrators at all of the five sites had indicated that change was ongoing; it was something that they had come to expect. Many felt that the reform efforts were only partially developed, and they would always be seeking ways to make their programs better. The cross-case analysis indicated that these additional changes were being made in an effort to more fully align with the Standards.

Fullan (1993) indicates that "dealing with change is endemic to post-modern society" which requires a shift of mind that prevents "a continuous conservative system" from replacing "a continuous change theme" (p. 3). Thus, there is inherent danger in a viable, dynamic entity moving towards stagnation the moment it is conceptualized as "complete."

**Concluding Remarks About the Cross-Case Analyses**

I have attempted to separate analysis from interpretation and vice versa, but that became increasingly difficult during the analysis of each of the six clusters across the data from the five high schools of this study. There were many references to the Standards throughout the analyses, and it was difficult to save everything that dealt with the Standards until the analysis of that final cluster.

I had indicated that the six clusters were selected for their convenience in tying the analysis of data in this extended study to the R3M project. Teaching practice and the impact of the reform efforts on students were also hard to separate, and there were other instances where the six clusters seemed to overlap. It is for those reasons that additional interpretation seemed justified. In Chapter 7, I will present several "integrative" interpretations that do not seem to fit a single cluster.

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20 I have made number of procedural decisions which are not entirely free of making interpretations of the data. (e.g., changing the order of clusters in a table, grouping certain codes within a cluster, etc.). Miles and Huberman (1994) address these issues of drawing conclusions from the data in making these kinds of decisions.

21 Miles and Huberman (1994) refer to this type of ordering "according to some variable of interest" (p. 187) as a case-ordered descriptive meta-matrix.

22 Most teachers expected students to do at least two projects per year. Typically, students did about four per year.

23 There were co-chairs for the mathematics department, and one was a real expert at taking care of administrative detail and following through with the department's wishes. Other teachers may have perceived of her as the leader, but one sensed that she thought of herself as just one of the teachers in the department. All teachers' ideas seemed to carry equal value in meetings which the documenters observed.

24 "Authentic" assessment refers to the selection of assessment methods and instruments based on the type of information being sought (NCTM, 1989, p. 199). For example, if a teacher wants to assess a student's ability to reason mathematically, then the student should be given opportunities to demonstrate mathematical reasoning. Asking the student to explain why he/she decided on certain steps may be one of several ways to do this.

25 The student is allowed to use his/her homework solutions for this quiz which consists of one or more of the homework problems. If the student has completed his/her assignment correctly, he/she is assured of a perfect score on the quiz.

26 Groups or individuals exchange their projects when it is still in draft form for one or more peer edits before turning in the project for a final assessment.

27 Research which may be needed for new assessment instruments will require a long-term commitment and funding.
CHAPTER 7

INTEGRATIVE INTERPRETATION, CONCLUSIONS, IMPLICATIONS, AND FUTURE DIRECTIONS

Integrative Interpretation

I restricted my interpretation of the cross-case analyses in Chapter 6 to the individual clusters. Just as data crosses over boundaries between the clusters, so it becomes essential to integrate analyses from more than one cluster or from other research into some of the interpretation. What follows is such an integration of analyses.

Fostering A Disposition for Change

The data had indicated that disposition for change was observed at some level in each of the five cases of this study. If disposition is necessary before reform efforts can take place, how can it be fostered in a school or in a district? Or, better yet, how can it be cultivated among a whole mathematics department?

The disposition for change depends upon the beliefs of individuals, so the task becomes one of trying to replace the "long held perceptions" of mathematics teachers (Romberg, 1988b). The belief system of teachers is an often neglected component of this complex process of educational reform (Anderson, Anderson, & Romagnano, 1993; Carpenter & Fennema, 1991; Prawat, 1992). The data from several sites indicated that inservice opportunities to attend regional and national mathematics conferences was one of the most effective ways of changing beliefs in teachers who are contemplating change. Summer workshops geared to teachers' specific needs were also useful mechanisms for changing beliefs.

Collaboration among peers was identified as the most useful way of sharing and developing new ideas and beliefs. The data identify "time to collaborate with colleagues" as the most prevalent request among mathematics teachers. A significant change in teacher
beliefs was witnessed among three teachers at one of the schools in a period of just eight months (the time between first and second visits by documenters). The changes in these teachers' beliefs resulted from collaborating with other teachers who were enthusiastic about the reform efforts.

Oja and Smulyan (1989) found in their research that collaborative activity is more effective if it draws "on theory in the fields of group dynamics and adult development to explain how individuals and groups develop through involvement in action research" (p. viii). The notion that every teacher is at a different stage of adult development is important for researchers to recognize. It makes it easier to accept that learning is a constructive activity for teachers as well as students and to recognize that beliefs which foster a disposition for change will come in varying degrees for different teachers.

Prawat (1992) suggests several impediments that teachers have in adopting a constructivist view of teaching and learning. One of these impediments is a dichotomous view of the learner and the curriculum instead of viewing them interactively. This may explain why some teachers change seating arrangements of students into groups but still dispense mathematics information as though all students learned at the same rate and in the same way.

Teachers varied in the degree which they felt motivated to engage in the mathematics reform effort. In this study, some teachers' disposition towards change allowed them to alter pedagogy in a number of new ways. For other teachers, an incremental and slower approach was needed.

Renegotiating Authority at Various Levels

Wiske (1992) had indicated in her study of reform efforts in geometry how changes in pedagogy required a renegotiation of intellectual authority in the classroom. She was referring to the need for teachers to be willing to relinquish more of the responsibility for learning to students. This change in authority was also observed in many of the high school mathematics classes which were visited during the R3M project. As one teacher
observed in referring to the shift in authority and responsibility for learning: "They don’t need us any more."

Prawat (1992) believes that social context is key to constructivist teaching and learning, particularly when the focus is on ideas and not skills or strategies. According to Prawat, in this type of classroom, negotiation is required to set up rules of interaction as well as arrive at "disciplinary truths."

The renegotiation of authority was also witnessed at a different level in the extended R³M study. Teacher authority within the classroom has customarily been recognized as fact in education, but what about at the policy-making level? Through the reform efforts in mathematics, changes in pedagogy required more than just a rearrangement of students in a classroom. There were implications for different structures of authority in high schools to allow mathematics teachers (and other teachers) more latitude in making important decisions about what mathematics will be taught, how it will be taught, and how money is allocated to support these changes.

Data analyses revealed that self esteem had a lot to do with teacher beliefs. If administrators (or, in the case of Desert View, university mathematicians) made teachers feel that they were the mathematics professionals of a school, teachers were found to adopt a professional stance which fostered a desire among teachers to look for additional ways to improve instructional practice. Carried to a different level, the renegotiating of authority that took place in the classroom between teacher and students could serve as a model for the entire school community by allowing teachers a greater voice in their learning and in their contributions to decision-making processes.

For example, the teachers in one of the high schools of this study felt less need for textbook purchases and a greater need for professional staff development. The policies of the school district, however, mandated that a fixed percentage of money had to be spent on mathematics textbooks. The real needs of the mathematics teachers were being denied in order to follow guidelines established by those who are far removed from the mathematics
classroom. These kinds of policies may have the effect of severely limited the scope of change. The data did indicate that teachers in some schools were being given more authority to make important policy decisions that would affect their classroom practice. 

How Does Time Interact with Other Contextual Features?

The analyses of data across cases indicated that at two of the sites reform efforts were well underway by the end of the third year. Teachers and other key players in those two sites perceived of their first year of change efforts as being very successful. Those early successes encouraged all of the mathematics teachers to participate in change during the following two years. Is a similar time span what other schools should expect in their efforts at change? What about schools where mathematics teachers are struggling over many difficult issues and every new step taken seems monumental? Will the change process take longer in such sites?

Teachers at Pinewood were facing many difficulties at the end of their first year of reform efforts, and it was considered a victory by the mathematics supervisor that some teachers were willing to continue their change efforts into the second year. Was change less significant because it was proceeding at a slower pace?

It is difficult to draw conclusions about the time required to effect change, or about the quality of change taking place. Change does not have to occur in a linear progression until some "ideal" pedagogy is reached. Efforts of reform may be filled with stops and starts and reversals to prior experience. What happens to change efforts when the funding dries up and the persistence of leaders becomes less persistent? These are difficult judgment calls. Further investigation may reveal that bringing about reform in mathematics programs need not necessarily be characterized as efficient transformations.

The decisions by local teachers about which students get served first in efforts of reform may also impact upon the time it takes a mathematics staff to bring about the reform they envision. Pinewood teachers' primary reform efforts were seeking to address the problem of elitism in mathematics courses by first providing quality learning opportunities
in mathematics to those students characterized as being at-risk. Would it be logical to assume that such an effort might take longer to establish than reform directed at other levels of students? Probably.

Staff development was just one of many of the contextual factors which this study found to impact upon these efforts of reform and may have something to do with the time it takes reform to occur. The data indicated that certain kinds of staff development activities were perceived by teachers as particularly useful in promoting their reform efforts (e.g., attendance at mathematics conferences and release time for peer collaboration).

These findings are corroborated in research done by the Center for Research on the Context of Secondary School Teaching (CRC) at Stanford University (McLaughlin, 1991). McLaughlin reported that staff development activities which are "decontextualized" or "disembodied" from the daily realities of teaching (e.g., bringing in outside speakers unfamiliar with the school setting) have only limited value for teachers. She reported that staff development was of greater value to reforming classroom practice when the "nowness" of teachers' classrooms were the focus of staff development activities. The length of time required for reform efforts to occur will certainly be influenced by the staff development activities in a school and whether those activities are useful to teachers.

There are of course many other contextual factors which would influence how much time may be required to bring about the vision of reform (Fullan, 1993; Sarason, 1991).

**Why a Common Pedagogical Vision?**

When examining the pedagogical data across cases, the analysis revealed strikingly similar pedagogical visions. What factors influenced the pedagogical visions across sites to be so similar? Analyses across several clusters are required to address this question. The analyses of "context for change" and the "influence of the Standards" are two clusters which provide some insight into why this "common" vision exists.

The study revealed that different uses of the Standards were impacting upon change in the five schools. There was an awareness of the Standards by those who developed each
school's pedagogical visions, and that awareness came from reading the Standards and from attending mathematics meetings, conferences, or workshops where the Standards or Standards-like pedagogy were being addressed. For example, the prevalence of group activity in classes observed at all of the sites may have originated from seeds planted at workshops or meetings which featured speakers who were demonstrating or explaining pedagogy that would foster student learning along the lines of pedagogy suggested in the Standards. (This was certainly the case at East Collins and Desert View.)

Teachers who attend mathematics conferences, for example, will not find sessions which promote rote memorization or drill and practice. Teachers in this study indicated the value of such conferences during their interviews. Mathematics supervisors indicated a need to provide such inservice experiences for their mathematics teachers. What is suggested here is that the influence of the Standards permeates the mathematics conferences and meetings which the teachers, supervisors, and coordinators attend. Staff development opportunities at each of the schools enables teachers to attend those meetings and conferences, which, in turn, provide the context for "common" pedagogical visions to emerge.

Conclusions

The primary purpose of the study was to broaden the understanding of the data collected in the R^3M project in order to more fully clarify the processes by which reform in mathematics education was occurring across the five cases for grades 9 through 12. As a secondary reason for conducting the study, I sought to develop a model of doing cross-case analyses and identifying those methodological elements and linkages that can be applied generally in large-scale studies of this sort. There were six areas of interest which were addressed in the extended study (see pp. 7, 8): the range and interpretation of the Standards; the factors influencing reform; the factors contributing to the persistence of change; the additional qualitative analyses provided by the methodology; the feasibility of methodology with data gathering sources; and the implications for the methodology to
other research studies in mathematics education. These six foci of the study will form the framework for a brief summary of findings.

**Range of Interpretations of the Standards**

The two Standards documents (NCTM, 1989; 1991) were found to have influenced reform efforts in the five high schools in three ways: as a catalyst to those efforts; as a validation of the reform which took place; and as providing direction to future or contemplated reform in mathematics. Teachers and others responsible for the reform efforts in mathematics at these five sites admit that any additional changes to those reform efforts are subjected to a criteria test based on the Standards.

**Factors Influencing Reform**

The factors which influenced reform efforts in this study were numerous, but there are undoubtedly many other factors which impacted on those efforts that went unnoticed or unreported by those at the site. Fullan (1993) suggests that in order to detect the complexity of the underlying forces which impact on reform, it is necessary to take any educational policy or problem and start listing all the forces that could figure in the solution and that would need to be influenced to make for productive change. Then, take the idea that unplanned factors are inevitable--government policy changes or gets redefined, key leaders leave, important contact people are shifted to another role, new technology is invented, immigration increases, recession reduces available resources, a bitter conflict erupts, and so on. Finally, realize that every new variable that enters the equation--those unpredictable but inevitable noise factors--produce ten other ramifications, which in turn produces tens of other reactions and on and on. (p. 19)

Viewed from a constructivist perspective, reform is further complicated by the levels at which individual teachers decide is most appropriate for them to contribute to those reform efforts (Hart et al., 1991; Lieberman, 1995). Additionally, stages of adult development enters into this whole process of individual motivation to participate in those efforts (Oja & Smulyan, 1989).

Collaboration among colleagues was found to be the most valuable (and most often mentioned) influence for teachers as they engaged in new pedagogical practice. Interviews of teachers indicated that new ideas about teaching mathematics were often...
obtained at local and regional meetings of mathematics teachers and through opportunities afforded by summer workshops.

The importance of teachers is paramount. Teachers should be engaged at all levels of decision making that will impact on classroom practice. Teachers in the study were found to gain in self esteem and professionalism when they were recognized by administrators and other policy makers as the experts in the field of mathematics education.

Leadership was an important ingredient in efforts of reform in mathematics, but the form of that leadership varied from school to school. In three of the schools, individual leadership prevailed. A shared leadership prevailed at the other two sites. Administrative support and external funding were also found to be important contributing influences in efforts of reform.

Factors Contributing to the Persistence of Change

Those factors that influenced reform also contributed to the persistence of those reform efforts. When teachers were asked by documenters how long they perceived the task of reform would take, teacher responses indicated that there would be no end to the process. According to the teachers, collaboration among colleagues and participation in the decision-making process were most important. External funding was seen as helpful but not critical to the persistence of change—though teachers often cited the need for release time to collaborate with each other.

The Standards were serving to give direction to these ongoing changes, and many teachers at the schools in the study felt that their mathematics programs would go through adjustments to bring mathematics instruction into alignment with the Standards documents.

Additional Qualitative Analyses Provided by the Methodology

Broadening the understanding of data. In order to support my original claim that the methodology of this study would provide a greater depth and breadth of understanding of the data from the R^3M project, I decided to take a closer look at the emerging scenarios
and case studies of the R₃M project. I subjected the scenarios and case studies to the same coding categories which were used in the extended study. The codes which emerged are delineated below:

East Collins:
- partners or collaborators
- mathematical vision
- pedagogical vision
- administrative support of program
- effect of program on students
- teacher role in developing curriculum
- student activity
- teacher ownership of program

Desert View:
- partners or collaborators
- collaboration with others
- leadership
- administrative support of program
- all students or select groups
- mathematical tasks

Green Hills:
- teacher role in developing curriculum
- role of technology
- community outreach and input

Pinewood:
- all students or select groups
- teacher support of program
- expectations on teachers

Scottsville:
- role of technology
- leadership
- administrative support of program

Data presented in this form would have made cross-case analyses impractical because there was no consistency of categories reported across all cases. The coverage of data from the five sites provided only a small sampling of the sixty available codes and certainly did not approach the depth and breadth of analysis provided by this extended study.

Patterns emerged as a result of subjecting data from the five schools to cross-case analyses which would not have been possible using the scenarios from the R₃M project. For example, the common pedagogical vision which emerged across all cases would not
have been detected. Also, the focus of reform on the mainstream mathematics students in four of the sites and the focus on students characterized as at-risk at Pinewood offers a contrast which becomes important in the discussion of what students should first be targeted for reform efforts. In fact, all the interpretations made in Chapter 6 were made possible through the extended analyses of the R^3M data.

Feasibility of Methodology with Data Gathering Sources

Generally speaking, the data-gathering instruments used in the R^3M project proved to be effective in providing data for this study. I found that there were only two areas of difficulty in the way in which data was gathered in the original design of the R^3M project: consistent use of data gathering instruments, and inadequate provision for gaining information about mathematical content.

Consistency versus multiple perspectives. With twenty-two documenters representing several fields of educational endeavor, multiple perspectives abounded. Even if the documenters had consisted of only mathematics educators, there still would have been wide differences in research perspectives. A very consistent collection of data would have necessitated multiple site visits by just a few documenters. Such a choice would stand the risk of putting a sameness into the way descriptions emerged from the data. For example, we found that pairs of documenters differed greatly in the way they chose to collect and summarize data and in the way they wrote their scenarios and case studies.

Schram and Mills (1995) address the methodological insights that arose as a result of the "cross disciplinary nature of the research team" (p. 4). The choice to have a mix of documenters from several fields of study made the reporting and written stories more diverse and interesting, but it posed a bit of a challenge when it came to the analysis of data. According to Schram and Mills, there were questions which persisted throughout the R^3M project: "How do we balance breadth of analysis with the need for in-depth understanding of specific settings? How do we co-construct meaningful analytical
categories? To what extent do we go 'beyond' our data in developing an interpretive framework?" (p. 5).

There were several instances that the types of data gathered in interviews were not always consistent. In those instances, documenters either didn't have time to complete the interview questions or they carried their own agendas into the interviews and classroom observations and tended to focus on those items that were important to their special interests and perspectives. As Schram and Mills (1995) suggest, documenters were "influenced by their own conceptual frameworks of mathematics reform" (p. 11). Time limitations during the interviews may have been a factor, but often it was the particular perspective of the documenter that allowed digressions from the interview guides to occur. This was especially obvious in some interviews where documenters stayed on one or two items of the interview guide throughout the session.

In several instances, it was necessary to fill in the gaps of information by referring to internal reporting forms that were used in the R²M project. The Write-Up Questions form (Appendix A) was particularly useful and represented summary responses jointly prepared by the visiting pair of documenters.

**Data on mathematics content.** The most difficult data to obtain was the mathematics content at each of the five sites of the study. We had hoped to gain some understanding of how a school's reform efforts played out in terms of the choices of content which students were expected to learn. This did not prove to be the case, and there were several reasons why documenters did not collect much information about content. First, that kind of data is difficult to obtain during short visits of two to six days, since classroom observations would probably be the best means of obtaining such data. Second, many of our documenters lacked the mathematical and research orientation to know specifically what to look for. Third, our instructions to documenters about obtaining this kind of data were not very thorough. We included among the instructions for classroom observations the following: "What mathematical topics were addressed (try to group these into major
themes); how much time was spent on each theme?" This limited requirement for content coverage gave us just that--limited information about content. Finally, obtaining useful data in terms of mathematical content would likely constitute a separate study, and that was not the intent of the R³M project.

**Implications for the Methodology to Other Research Studies in Mathematics Education**

**Research in the field of mathematics education.** The findings of this extended study of the R³M project will be an important addition to the small body of research dealing with the current Standards-influenced reform efforts taking place in secondary mathematics education. This is particularly true at the secondary level where little research has been done. The cross-case study by Wiske (1992), which focused on the influence of the Standards on the teaching of geometry, sought to clarify how the shifts in the way students learn geometry and pedagogical practices can be "aided through appropriate policies and supports" (p. 1). This study represented a beginning of a base for this type of investigation at the secondary level. There will be an additional demand to document the influence of NCTM's Assessment Standards soon to be released.

**Research on change.** The study will also contribute to a broader base of research and theory relating to the complexity of change in education. Most of the educational change literature and research comes from outside the field of mathematics education. For example, do the findings of this study agree with the complexities involved in educational change? How much of what Fullan (1993) and Sarason (1991) say about educational change actually applies to reform efforts in mathematics education?

Fullan (1993) identifies eight basic lessons of his "New Paradigm of Change." These eight lessons generalize to much of what was found in this study. For example, mandated change at Pinewood ran into a number of stumbling blocks. The mathematics supervisor initially started the change efforts with a top-down management style, then she began to encourage teachers to take a more active role in the process of instructional decision-making. Efforts at change were slow, and many teachers felt that little had been
accomplished during a very difficult first year. A number of teachers resisted change and were finding it difficult to leave the comfort of a pedagogy that had been familiar for so long. Other teachers were finding that trying out new classroom strategies (e.g., working with students in groups) really did allow students a better environment in which to learn mathematics. Finally, at the end of the first year, teachers decided that they really wanted to give another year to the mathematics change effort.

Desert View teachers were enthusiastic about developing long-term mathematical tasks (projects) with which to supplement their regular mathematics program. Teachers found that pedagogical shifts which they employed during "project" days were carrying over into the days of "regular" instruction. All of the mathematics teachers at Desert View shared in the leadership of the change efforts. The support and professionalism afforded by administrators developed a high level of self-esteem in the mathematics teachers, and teachers felt no pressures to change their teaching practice. Teachers in both of these schools found benefits in attending regional mathematics meetings and workshops.

Even though the content of mathematics may be different, much of the pedagogy that has been prevalent in the social sciences (e.g., group work and group discussions) are now finding a place in mathematics pedagogy. The factors which influence change are also indicated in this extended study of the R^3M projects.

**Research using cross-case analysis.** The methodology employed in this study holds promise for conducting documentive or descriptive research of a similar nature in the field of mathematics education. Researchers who employ the cross-case methodology used in this study will undoubtedly need to adapt the methodology to their particular study. This study focused on five schools, but the use of cross-case analysis would have application to larger studies. Miles and Huberman (1994) believe it is "easier to move from a limited but well-delineated small sample to a larger one and to make some educated guesses about what we are likely to find, than to go directly from a single case to a larger set" (p. 31). In fact, such a methodology might be the only way to manage descriptive data in larger
studies. However, this methodology is very labor intensive, particularly if the researcher works inductively from the data.

The methodology also has the potential of being mixed with other forms of data collection such as questionnaires or survey forms. For example, Prestine and Bowen (1993) found that the descriptive data for their study of the Coalition of Essential Schools in the Chicago area was missing important data to answer questions they felt important to their study. They used telephone interviews and questionnaires as add-ons to the original data to find the missing information they were seeking. Their cross-case methodology involved coding and analyzing original data with the newly collected data to complete their study.

Miles and Huberman (1994) also illustrated the flexibility of the cross-case methodology to a number of different settings and to a number of different fields of research.

**Retrospective Thoughts on the Cross-Case Methodology**

In retrospect, the model of cross-case analyses which evolved through this extended study of data from the R3M project has accomplished what I had envisioned at the start, that is, a broader interpretation and understanding of the data. There were times during the study that minor adjustments in the methodology were necessary to improve the model. A log was kept throughout the study as suggested by several members of the research committee, and, among the entries, periodic shifts in my methodological strategies and techniques were recorded.

Among the changes were the following:

- Three coding categories were removed from the list: "features of the program," "classroom characteristics," and "problems with site." The first was removed because it was redundant and was covered by other coding categories. The second was removed because the physical characteristics of an observed classroom offered very
little in the way of useful information for the study. "Problems with site" covered the same ground as "difficulties in implementation."

- "Basic skills" was added as a category when its importance surfaced in two of the cases.
- Originally, an alphabetical order of clusters of codes had been used. Later, it made more sense to follow a chronological orientation (e.g., "Evolution of program" seemed a logical starting place).
- On future research studies employing these cross-case techniques, no more than two coding categories would be assigned for each string of data. During this study, as many as five or six codes were assigned to the same piece of data. This created volumes of coded files and did nothing extra to insure that all the data was being preserved through the coding process.
- Computer software (HyperResearch) was utilized only during the first two levels of data analysis--data coding and data sorting. I experimented on several occasions in looking for relationships between variables using the software, but this process was highly subjective and very time consuming. It was decided, after consultation with my two major advisors, that such relationships would add little to the findings of the study, and that area of analysis was dropped. (HyperResearch does allow for Boolean logic and hypothesis testing.)

Two other aspects of the cross-case methodology deserve comment: the potential for generalizability using the methodology and the influence of the conceptual framework on the methodology.

**Generalizability**

Tinto (1990) believed that generalizability in qualitative research must be limited to the research being conducted, but she indicated that similar situations arising from research "might not be irrelevant to those from other situations" (p. 211). That is, she was suggesting the possibility that similar research in a similar setting might produce like
results. Tinto believed that validity and generalizability were increased when the research
was conducted objectively, and an attempt was made to minimize bias. She accomplished
this objectivity by analyzing data inductively and utilizing computer software to facilitate
the process. The methodology of cross-case analyses was seen by Miles and Huberman
(1994) to increase the likelihood of revealing any comparability that may exist across cases.

The findings which emerged from this cross-case study allowed for the interpretation
of results which were presented in Chapter 6. Certainly, if the same features found across
cases in this study are translatable to other studies of a similar nature, the argument for
generalizability is strengthened (Goetz & LeCompte, 1984; Tinto, 1990). Miles and
Huberman (1994) argue that the factors in single case studies that allow for replicability to
similar settings preserves generalizability in multiple-case studies. The difference between
single-case and multiple-case studies, according to Miles and Huberman, is that "We are
generalizing from one case to the next on the basis of a match to the underlying theory, not
to a larger universe. The choice of cases usually is made on conceptual grounds, not on
representative grounds" (p. 29).

Influence of the conceptual framework. The orienting assumptions for this study were
derived from the conceptual framework developed in Chapter 2. Those orienting
assumptions influenced the decisions I made as I gathered, analyzed, and interpreted data
in this study. In spite of my attempt at objectivity and to remain non-evaluative, the
organization of the study itself is tied to two conceptual frameworks--that which was
developed in Chapter 2 and the underlying framework for the R3M project (Ferrini-
Mundy, Graham, & Johnson, 1993). To suggest that this study is free of the influences of
those conceptual frameworks would be in error.

The interpretations which were drawn in Chapter 6 and at the beginning of this chapter
were based heavily on the orienting assumptions for this study. Learning as a constructive
experience was used to interpret the pedagogy employed and the ways in which teachers
constructed their own knowledge in preparation for changed pedagogy. The importance of
considering teaching practice as something that can always be improved upon permeates much of the interpretive sections—especially the ongoing and persistent nature of reform found in the cases of this study. The need to match pedagogy with what is known about learning was a persistent theme. The complexity of reform was important in interpreting many of the findings.

**Implications**

**Implications for Instructional Practice**

The extended analyses of the data from the five high schools of the R³M project have contributed important findings to the field of mathematics education. The study focused on a particular area of mathematics reform that has received little attention in the research, namely, the reform efforts in mathematics at the secondary school level (Koehler & Grouws, 1992; Silver, 1990).

When documenters asked students what they would change about the way their mathematics classes were taught, documenters found that most students felt satisfied with what their mathematics teachers were doing. Of course, we were asking students whose classroom practice had undergone significant change in the last two or three years. These were students who were accustomed to working in pairs or small groups during much of their class period. Students were also accepting more responsibility for their own learning.

What are the implications for high school mathematics teachers who find themselves confronting different mathematics content and different pedagogical practice? Do the findings of this study provide direction for these teachers?

Some teachers will find that the way in which they change their beliefs about mathematics learning will be a highly individual process. This study indicates that there are many complex factors at both the individual and institutional levels that may impact on teachers' needs and desires to change their knowledge of how students learn mathematics and what pedagogy best suits that knowledge. Changes in an understanding of how students learn mathematics may then motivate teachers to take the other steps towards
ensuring that teaching practice matches their different understanding of learning. These steps are not always easy. This study portrays some of the struggles teachers faced in bringing about change—in the way they believed that students learn and in the way they taught.

Collaborating with peers, attending professional meetings of mathematics teachers, actively participating in curriculum development at the school and district levels, participating in partnerships with others outside the school, and being willing to take risks in changing the way they teach were ways in which teachers in this study were bringing about reform in mathematics practice. Sometimes the struggle became intense as cherished beliefs were relinquished in favor of different models. Most often, change took longer than expected, and teachers must had to exercise patience to endure drawn-out reform efforts.

Implications for Administrators and Policy Makers

This study indicates that policy makers may need to change their thinking about what is needed to operationalize reform efforts. For example, diverting funding away from textbook purchases to include more inservice teacher development may be an important step for some administrators and policy makers. Recognizing that mathematics teachers in the schools are the experts with respect to mathematics education will encourage more teacher input into curriculum development and instructional strategies. With this recognition of professionalism comes an obligation for policy makers to value the input from these professionals in policy decisions.

Future Directions

Is change occurring at the secondary level of mathematics instruction in different ways than it might be expected to occur at either the K-4 or 5-8 levels? If so, why are there differences? There may be important connections between the cross-case analyses provided by this study and a similar cross-case study for the remaining K-8 data from the
R³M project. A fuller understanding of differences between grade levels could provide valuable information to the mathematics education research community.

Research similar to that done at the elementary level by Ball (1990; 1992b), Cohen (1990), Cohen and Ball (1990), and Peterson (1990) could be expanded by other researchers to include grades 9-12. That research which focused on how policy gets translated into classroom practice is particularly relevant since forty-three states have adopted Standards-like curriculum frameworks for mathematics.

More research needs to be done in terms of student assessment. Data were inconclusive on whether students were learning and understanding more mathematics as a result of their participation in mathematics programs that had undergone recent reform efforts. Authentic assessment instruments were non-existent in some of the sites and received only limited use in others. All but one site had been in the implementation stage of their reform efforts for three years or less, which further complicated the issue of assessment—which will require a longer look. In view of the soon to be published Assessment Standards for School Mathematics by NCTM, more documentive research is certainly indicated in terms of different assessments to gauge students' understanding of mathematics.

Lieberman (1995) and Hart et al. (1991) have looked at how teachers learn new knowledge which, in turn, influences teaching practice. Additional transformative research is needed at the secondary level. A valuable contribution to existing research would be an in-depth study of how teachers' mathematical visions change through the process of reflective collaboration with colleagues. Settings such as the K-12 curriculum revision at Green Hills certainly warrant a more thorough investigation by researchers.

Providing support mechanisms for teachers to engage in reflective practice is indicated in this study. Schoenfeld (1992) identifies the need for research at the systemic level to address the question: "What changes in school and district structures are likely to provide teachers with the support they need to make the desired changes in the classroom?" (p.
365). Such research is needed to enable reform efforts in mathematics education, but it also has implications across disciplines.

Determining the role of professional organizations such as NCTM in fostering change seems an important next step. Is there any obligation for a professional group representing approximately 106,000 members (Gates, 1994) beyond the development of the Standards documents? In what ways does this study inform future actions of NCTM? This study has found the influence of the Standards to have been considerable, at least at these "selective" sites. Ferrini-Mundy (1993) suggested that through the R³M project the role of NCTM would be expanded to describing "the implementation of these Standards."

Hart et al. raised two questions which have possible implications for further research which could be important for NCTM:

How do teacher beliefs about learning mathematics, about teaching mathematics, and about the classroom environment change over time?

How do teacher beliefs about mathematical tasks and content change over time? (Hart et al., 1991, p. 7)

NCTM's role might be to encourage additional research which addresses these two questions, particularly if NCTM is to take a more proactive stance in terms of helping teachers to make changes in teaching practice as recommended in the Professional Standards for Teaching Mathematics (NCTM, 1991).

The six analyses of data across the five cases of this study have produced results that will contribute to a body of research literature that has been lacking at the secondary level of mathematics education. Some of the findings from the analysis of data from these five high schools parallel findings found in studies done at the elementary level (Ball, 1990; Ball, 1992; Cohen, 1990; Lampert, 1990; Wood, Cobb & Yackel, 1993). There are a number of factors that make teaching at the high school level different from grades K-4 and 5-8. Such factors include: the specialization of teachers in one area of study; the success which high school teachers had in learning mathematics; the long-established tradition of a pedagogy dominated by directed teaching in mathematics at the secondary level; and the
different structure of the high school as compared to the elementary school (e.g.,
departmentalization and movement of students to different classes during the school day).
Certainly, an extension of the present study to include cross-case analyses for grades K-8
(and ultimately for K-12) data from the R³M project would provide insights into the
differences in which reform efforts occur at the different grade levels.

Different types of research may be required to determine and describe some aspects of
the reform efforts in mathematics education. Collaborative action research (Oja &
Smulian, 1989; Oja, 1991; Elliott, 1991) and teacher enhancement studies (Ferrini-
Mundy, 1994; Hart, 1991) have the potential of going into greater depth with certain areas
of reform. For example, the limited time available for site visits in the R³M project
prevented a deeper look at mathematical content across the cases of this study.
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APPENDIX A

WRITE-UP QUESTIONS (Form F)

INSTRUCTIONS: Feel free to create your own form, but please respond to these questions. We suggest that you formulate an answer to the question, and then that you provide as much supporting evidence as possible. Supporting evidence can come from anywhere; documents, interviews, observations, etc. Be very specific as you do so; for example, say "The principal said . . ."; the mathematics department's mission statement says " . . ."; in her classroom, teacher X did . . . and the students did . . . ."; in the teachers' room two teachers were talking and they said " . . . .". If you draw a conclusion based on your own impressions but cannot find clear documentation, please include but make it clear that you have only an impression, without clear support.

Note: these write-ups will be used only by R3M staff at this point. PLEASE DO NOT SHARE THIS WITH THE SITE AT THIS TIME; FOR SITES THAT ARE TO BE VISITED A SECOND TIME, WE MAY LATER DECIDE ABOUT SHARING THIS IN SOME WAY. FOR NOW, THE ONLY COPIES SHOULD BE IN YOUR PERSONAL FILES, AND IN THE PROJECT FILES.

Notice that each question calls for description and then interpretation.

1. DESCRIBE THE "MATHEMATICAL VISION" HELD BY THE PEOPLE IN THE SITE. (What are their goals for their mathematics program? What kinds of mathematics learning do they hope their students will experience? What features of the Standards emerge as they describe their mathematical vision? What do the people in the site see as worthwhile mathematical tasks, or important mathematical ideas? Is there alignment among teachers and administrators concerning this vision? Does the mathematics program emphasize: problem solving, communication, reasoning, conjecturing, and mathematical connections? Can the classrooms be described as mathematical communities? Why?

IS THE MATHEMATICAL VISION THAT THEY HOLD BEING BROUGHT TO LIFE IN THE CLASSROOM? WHAT IS HAPPENING, MATHEMATICALLY, IN THIS SCHOOL? PROVIDE AS MUCH SPECIFIC EVIDENCE AS POSSIBLE.

2. DESCRIBE THE "PEDAGOGICAL VISION" HELD, RELATIVE TO MATHEMATICS, BY THE PEOPLE IN THE SITE. (What pedagogical philosophy is articulated in the site? How can you tell? What approaches, strategies, and ways of teaching mathematics are important here? What features of the Standards emerge as they describe their pedagogical vision? What do the people in the site believe to be effective pedagogical practices? Is there alignment among teachers and administrators concerning this vision?)

IS THE PEDAGOGICAL VISION THAT THEY HOLD IN THIS SITE BEING BROUGHT TO LIFE IN THE CLASSROOM? WHAT IS HAPPENING, PEDAGOGICALLY, IN THIS SCHOOL? PROVIDE AS MUCH EVIDENCE AS POSSIBLE.
3. **DESCRIBE HOW CONTEXTUAL FEATURES ARE INFLUENCING, BOTH POSITIVELY AND NEGATIVELY, THE TEACHERS' EFFORTS TO CHANGE THEIR MATHEMATICS PRACTICE.** (What has happened with inservice, with outside consultants, with outside funding, with the school and district administration, with the community, among the teacher, role of materials, scheduling and structural matters, policies about evaluation and testing.)

4. **DESCRIBE THE WAY THAT THE MATHEMATICAL AND PEDAGOGICAL PRACTICES IN THIS SCHOOL ARE AFFECTING STUDENTS.** (Are students engaged? What kind of mathematics are they learning? Do the pedagogical approaches seem effective? What is the nature of the discourse and mathematical communication? Are students learning to reason and to solve problems? Are they experiencing mathematical connections?)

5. **DESCRIBE THE EVOLUTION OF THE MATHEMATICS PROGRAM IN THIS SCHOOL.** (How has the program gotten to be the way it is? What factors are important? Where do people in the site feel there is a need for continued growth, support, and development?)

6. **OTHER.** (Comment on other important observations that will help people understand the way in which this site is interpreting and adapting the ideas of the NCTM Standards.)
APPENDIX B

R³M PROJECT GOALS

Project Objectives

The proposed project is a multidimensional endeavor to understand and facilitate mathematics reform, and in particular to assess movement within the mathematics education community toward the visions offered by NCTM's *Curriculum and Evaluation Standards* and the *Professional Standards for Teaching Mathematics* (hereafter called the *Standards* documents). NCTM sees a critical need for undertaking a documentation and demonstration project at this time, in order to further the implementation of the reform spirit, by providing guidance to practitioners and by helping the public understand the richness and complexity of the process.

This is a comprehensive plan for a project to: assess the influence and depth of knowledge about the NCTM *Standards* documents in various communities; develop useful descriptions of sites where change in mathematics teaching and learning is occurring; communicate and share what is learned with a variety of audiences, in a form leading to model development.

The project goals are:

1. To understand the status quo in terms of the level of penetration of the *Standards* documents, and to measure the breadth and depth of knowledge about the *Standards* documents in various communities.

2. To develop useful descriptions of teachers, classrooms, and children in settings where significant attempts at change in mathematics education, consistent with the *Standards* documents, are underway.

3. Within the context of these rich descriptions, to describe the effects of this enactment on classrooms and on children's learning of mathematics, casting these descriptions in ways acceptable as evidence by teachers, parents, the public, business, and industry.
4. To increase our understanding of the circumstances, forces, and situations in which change in the teaching and learning of mathematics occurs.

5. To synthesize and disseminate insights and findings about contextual features that promote and hinder change in mathematics teaching and learning as envisioned in the Standards.

6. To assist classroom teachers with the process of change in mathematics education by communicating descriptions of implementation activity.