Fiber-reinforced plastic grids for the structural reinforcement of concrete beams

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University of New Hampshire, Durham

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Fiber-reinforced plastic grids for the structural reinforcement of concrete beams

Yost, Joseph Robert, Ph.D.

University of New Hampshire, 1993
FIBER REINFORCED PLASTIC GRIDS FOR THE STRUCTURAL REINFORCEMENT OF CONCRETE BEAMS

by

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B.S., State University of New York College of Environmental Science and Forestry, 1983
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DISSERTATION

Submitted to the University of New Hampshire
in Partial Fulfillment of the
Requirements for the Degree of

Doctor of Philosophy
in
Engineering

December, 1993
This dissertation has been examined and approved.

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07 September 1993
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Diese Seite wurde, zum offensichtlichen Ursachen, an Marc und Helmut Girardelli, Albert Schweitzer und Madonna gewidmet.
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ABSTRACT

FIBER REINFORCED PLASTIC GRIDS FOR THE STRUCTURAL REINFORCEMENT OF CONCRETE BEAMS

by

Joseph Robert Yost
University of New Hampshire, December 1993

The research presented in this dissertation evaluates the performance of simply supported concrete beams reinforced with a 2 dimensional Fiber Reinforced Plastic (FRP) grid. The non-corrosive, high strength properties characteristic of FRP materials make them a desirable structural reinforcement for concrete in environments where high concentrations of chloride ions are present.

The FRP grid under investigation is called NEFMAC and it is manufactured by Turay Industries of Tokyo Japan. NEFMAC is dimensionally fabricated as orthogonally intersecting longitudinal and transverse bars. The bars are continuous at the intersection points and, as such, there exists no preferred or strong direction within the grid. Tensile strengths of the material used to reinforce test beams range from 99 ksi to 178 ksi and modulus values range from 6000 ksi to 12300 ksi. These properties suggest that substituting NEFMAC for steel on an equal area basis will result in significantly higher deflections and correspondingly greater flexural capacity. As a consequence,
deflection limitations will be an important component in design considerations.

Test data from 31 beams reinforced with NEFMAC is presented in detail. Comparison of test results and current ACI strength predictions conclude that flexural strength is accurately quantified but shear strength is being significantly overestimated. A modification to the code shear strength prediction is proposed for design with NEFMAC. Deflection compatibility between test results and theoretical predictions employing the Branson equation for calculating the cracked-section effective moment of inertia was dependent upon the percentage of reinforcement provided. For sections reinforced greater than 2 times a balanced design, deflection prediction was good for the duration of the test. For section below this level deflection was significantly underestimated.

Unlike steel, FRP material behave nearly linearly elastically to ultimate, at which point a brittle failure occurs. As such, application of an ACI flexural design criterion that is founded upon the yield capabilities in the reinforcement is not appropriate for FRP. Knowing that FRP reinforced beams can only experience brittle failure, a design criterion that considers energy reserve as a measure of safety is proposed. The result is low working stress levels in the reinforcement providing a high degree of reserve strength and acceptable compliance with deflection criteria.
CHAPTER I

INTRODUCTION

1.1 History of Reinforced Concrete

The use of reinforced concrete as a structural building material is an established part of modern engineering design. Concrete often represents an attractive alternative to structural steel and other building materials because of its high strength, economy, durability and moldability.

The first structural application of reinforced concrete took place in Europe during the 1860s. The Frenchman, Xavier Dujon, is generally credited with making the first practical use of reinforced concrete in 1867. Dujon ultimately used reinforced concrete in constructing pipes, tanks, flat plates, railroad ties, bridges and irrigation channels.

Since the beginning of the twentieth century, advancements in the understanding of reinforced concrete behavior and materials technology have developed rapidly. Of particular interest in this regard has been the development of high strength fiber-reinforced plastic (FRP) materials and their subsequent application as a reinforcing material for concrete structures. Since the 1940's, when FRP materials were first being developed, engineers recognized that the high strength, low weight and corrosion-resistant
properties characteristic of FRP make them an attractive material for reinforcing concrete. [1] [57] [58]

Present day understanding of FRP reinforced concrete behavior is by no means complete. Researchers are continually providing design engineers with more comprehensive information describing more accurately the behavior of FRP reinforced concrete structural systems. From the knowledge provided by research, building codes and specifications that will dictate design and construction procedures are currently being developed.

1.2 Concrete Reinforcing Materials

Concrete is a brittle material, strong in compression and comparatively weak in tension. As a consequence, concrete is rarely used in a structural application without the use of tensile reinforcing. Several different materials have been employed for the purpose of reinforcing concrete, including; wood, steel, bamboo and a number of different synthetically manufactured composite materials. Steel, because of its competitive cost and favorable mechanical properties, is presently the most extensively used material for reinforcing concrete.

1.3 Steel Reinforcement

Some properties that make steel an attractive reinforcement for concrete include its high tensile and compressive strengths, its ductility and its coefficient of thermal expansion is very near to that of concrete. Steel reinforcement is available as deformed bars, welded wire fabric, or wires and
tendons. Generally, steel bars are used in many reinforced concrete structures.

1.3.1 Corrosion of Reinforcing Steel

Unfortunately, the anticipated design life of many steel reinforced concrete structures is shortened due to a deterioration in the ability of steel to effectively transfer force with the surrounding concrete. [2] This condition is namely the result of corrosion taking place on the steel surface. The corrosion process is instigated when chloride ions present in deicing salts and marine environments migrate through the concrete, ultimately coming in contact with the reinforcing steel causing corrosion of the steel surface. The by-products of the corrosion process (rust) cause an expansion of material on the bar's surface. The expansion pressure against the surrounding concrete causes cracking and spalling to occur, ultimately exposing the reinforcing steel and eliminating bond as a mechanism of force transfer. [3]

Under optimal conditions, concrete inherently provides good protection against the corrosion of the reinforcing steel. This condition is developed due to the high pH of the concrete pore water. As a result of the high pH, a thin film of iron oxide coats the steel bars. This film offers protection in the form of a passive layer that electrochemically neutralizes the steel surface. [4] However, even small amounts of chloride ions are capable of breaking down the iron oxide film coating the steel bars. The disintegration of this protective passive layer instigates the corrosion process and, under favorable environmental conditions, the rate of corrosion can be dramatically accelerated. [5]
In an attempt to control this condition, science and engineering have developed the following methods for controlling the processes responsible for corrosion of reinforcing steel [6]: (1) decreasing concrete porosity, (2) providing steel bars with an epoxy protective coating, and (3) cathodic protection methods. These methods are, however, more successful in suppressing the corrosion process than they are in eliminating it.

1.4 Fiber Reinforced Plastics as a Steel Substitute

The inherent incompatibility existing between concrete and steel resulting from the corrosion process has initiated the development of alternative concrete reinforcing materials. Fiber Reinforced Plastics (FRP) represent a corrosion-resistant, high strength potential substitute for steel in reinforcing concrete structures. FRP materials possess three physical properties of major interest in their application as structural reinforcement for concrete: (1) high tensile strength, (2) low modulus of elasticity, and (3) they are corrosion-resistant in saline environments. The tensile strength and modulus values are relative to steel in magnitude.

By definition, FRP is a composite of two material groups: (1) a plastic matrix, and (2) reinforcing fibers. Reinforcing fibers are generally made from glass, carbon, aramid and other high strength fibrous materials. [7] The plastic matrix has form and mechanical properties which are customized by the techniques of applying heat and pressure during manufacturing. [7] Plastics are generally one of two types; thermosetting plastics and thermosoftening plastics. Thermosoftening plastics do not experience a chemical change when
going from a heated state to a cooled state. They can therefore be molded like wax or metal by heating and formed upon cooling. Thermosetting plastics, however, undergo an irreversible chemical change between heated and cooled states. They are generally molded under heat and very high pressure and, upon cooling, are permanent in their application. Both thermosetting and thermosoftening plastics are used in the manufacturing of FRP materials.

In order that FRP materials function effectively within the context of reinforcing concrete, they must be capable of developing their strength potential through an effective force transfer mechanism with the surrounding concrete. The challenge for the materials and structural engineers is in providing the necessary components of strength and force transfer capability within a single FRP reinforcing product.

1.5 NEFMAC Fiber Reinforced Plastic

NEFMAC (New Fiber Material for Reinforcing Concrete) is an FRP material manufactured for the purpose of reinforcing concrete and soil structures. As an FRP material, NEFMAC is made from high performance continuous fibers impregnated within a vinyl ester resin. [8] Several different types of reinforcing fibers are used including, carbon, glass and aramid.

Geometrically, NEFMAC is manufactured in continuous two and three dimensional grid shapes. A typical two dimensional grid is shown if Figure 1.1.
As can be seen from the figure, a typical 2 dimensional NEFMAC grid consists of intersecting longitudinal and transverse bars. The bars are crudely rectangular in shape, possessing smooth top and bottom surfaces and rough fibrous side surfaces. Typically, the lateral-bar-spacing and transverse-bar-spacing are equal and both bars are equivalent in material and strength properties.

The research presented in this thesis considers the performance of simply supported NEFMAC reinforced concrete beams subjected to 4 point loading applied both monotonically and cyclically. A detailed discussion of the physical and mechanical properties of NEFMAC is developed in Chapter 3.
1.6 Research Objectives

Prerequisite to the widespread implementation of NEFMAC as a structural reinforcement for concrete beams and slabs is a comprehensive understanding of how its structural performance might be reliably predicted. The ability to accurately quantify strength and deflection characteristics of structural components reinforced with NEFMAC must be developed from research data collected from laboratory testing. In this regard, test results provide a reference against which the suitability of analytical procedures can be determined.

From the load, deflection and reinforcement strain data presented in this thesis, a preliminary database documenting how NEFMAC and concrete perform together as a structural system is provided. Using this information as a reference, the accuracy of analytical strength and deflection procedures as relevant to NEFMAC reinforced concrete beams can be determined. The following items represent the specific objective of this thesis:

1. Determine the effectiveness of traditional analytical steel reinforced concrete procedures in predicting the flexural strength, shear strength and deflection behavior of NEFMAC reinforced concrete beams. Basic flexural mechanics are employed, assuming that plane sections remain during bending. Stress distributions in the concrete are assumed to be either rectangular or bilinear.

2. Discussion of how an energy criterion could be employed for specifying service loads of concrete beams reinforced with NEFMAC. The uniaxial tensile stress-strain behavior of FRP materials is linearly elastic up to ultimate, at which point a brittle failure occurs. This is significantly different from the desirable elastic-plastic behavior characteristic of reinforcing grade
steels. As such, application of flexural design criteria founded upon the yield capabilities in the reinforcement are not relevant for FRP. Knowing that FRP reinforced beams can only fail in a brittle manner, a methodology for calculating service loads relative to energy levels is described.

1.7 Research and Thesis Organization

Chapter 2 highlights the testing methods and conclusions published by research institutions relative to the performance concrete beams reinforced with deformed FRP bars. A detailed discussion of NEFMAC is presented in Chapter 3. The discussion includes a comprehensive description of the relevant geometric, physical and mechanical properties of the material as well as the manufacturing process used in its fabrication. Also included in Chapter 3 is a description of the relevant material properties of concrete and steel as required for analytical calculations. Analytical techniques as applied in the study of flexural, shear and deflection behavior are covered in Chapter 4. Test sample specifications are provided in Chapter 5 and monotonic and cyclic test results are given in Chapters 6 and 7, respectively. A criterion for calculating service loads of NEFMAC reinforced concrete beams is discussed in Chapter 8. Conclusions and Recommendations are presented in Chapter 9. Appendix A documents the data acquisition and instrumentation techniques used in measuring load, deflection and strain data. The analytical methods used for deflection prediction are detailed in Appendix B and a Finite Element modeling discussion is presented in Appendix C. Finally, each chapter is concluded with a section identifying the results and findings which are relevant to the research objectives stated in Section 1.6.
CHAPTER II

BACKGROUND AND SIGNIFICANCE

2.1 Introduction

Chopped glass and steel fibers have been used as admixtures in concrete for many years. FRP as a substitute for steel reinforcing bars is, however, relatively new, and the amount of technical information available on its performance within this capacity is currently limited. [9]

The limited attention given FRP in this application is mainly the result of difficulty in providing effective force transfer with the surrounding concrete [10] and, also, the low modulus values characteristic of the material. Modulus values for FRP generally range between 6 and 10 million psi, roughly 1/5 to 1/3 that of Grade 60 steel. [11]

The successful performance of FRP materials as a main reinforcement for concrete structures depends on exploiting their high tensile strength potential while simultaneously compensating for their modulus deficiencies and providing an effective mechanism of force transfer. Building codes must be sensitive to the advantages and disadvantages of FRP materials when defining analytical procedures on which engineers will rely for design. This may well require a modification of current design philosophy so as to be more compatible with the specific performance limitations of FRP.
This chapter highlights some of the more significant research findings concerning performance and design considerations of FRP reinforced concrete beams.

2.2 FRP Deformed Bars

To date the majority of FRP manufactured for reinforcing concrete is fabricated as deformed bars or cables for prestressing applications. The deformed bars are circular in cross-section and similar in appearance to that of rolled steel bars.

Manufactures of deformed FRP bars generally use a pultrusion method during fabrication, as is shown in Figure 2.1. The first step during manufacturing process is the bundling of fibers into a single, continuous ribbon like strand known as a roving. The roving then travels in an over-under fashion through steel rollers submerged in a resin bath. As the roving exits the resin bath it is pulled through a die, giving the rod its final cross-sectional shape and simultaneously stripping off excess resin. The resulting smooth composite bar is typically 30% to 40% resin and 60% to 70% reinforcing fibers. [11] Additional bundles of fibers are often wound around the circumference of the smooth bar in a spiral manner. This provides the deformed shape necessary for improving bond. [11]
FRP bars can be made in standard ASTM geometric sizes from #2 to #8. As a consequence of their manufactured shape, FRP deformed bars employ a bonding mechanism for transferring force to the surrounding concrete. The mechanics of bond transfer in deformed FRP bars are similar to that of deformed steel bars and is shown in Figure 2.2.

![Figure 2.1 Pultrusion Method](image)

**Figure 2.1 Pultrusion Method**

![Figure 2.2 Force Transfer in Deformed FRP and Steel Bars](image)

**Figure 2.2 Force Transfer in Deformed FRP and Steel Bars**
As can be seen, the transfer of force is accomplished through the bearing and shearing action of the concrete against the deformations in both of the deformed reinforcing bars.

2.3 Flexural Performance of Beams Reinforced With FRP Deformed Bars

Of particular interest in the study of FRP reinforced concrete beams is the analysis of their load-deflection behavior and load carrying capacity, and determining how these quantities might be reliably predicted.

2.3.1 Deflection Prediction

The load-deflection analysis required by the ACI-9.5.2.3 uses an effective moment of inertia to provide a smooth transition between the gross uncracked moment of inertia, $I_g$, and the transformed cracked-section moment of inertia, $I_{cr}$. [12] The effective moment of inertia was developed by Branson [54] and is calculated as:

$$I_e = \left[ \frac{M_{cr}}{M_a} \right]^3 I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \quad [\text{ACI 9.5.2.3}]$$

where:

- $I_e$ = effective moment of inertia
- $I_g$ = gross moment of inertia
- $I_{cr}$ = cracked transformed moment of inertia
- $M_a$ = moment at the deflection level sought

The compatibility between laboratory measured and ACI predicted strength and deflection behavior for FRP reinforced concrete beams has been
investigated at several research institutions with varying results. A study published by the University of West Virginia concluded that theoretical deflections calculated for FRP R/C (reinforced concrete) beams employing a cracked-section moment of inertia are accurate to within 5% of the experimental results. [13]

A much different conclusion regarding the predictability of FRP reinforced concrete flexural behavior was reached from a study conducted at Drexel University in 1988. [14] For concrete beams reinforced with glass-fiber FRP deformed bars the researchers learned that at load levels up to about 30% of ultimate, the experimental deflection values are fairly well predicted using the transformed section assumption. As the load level increases past 30% of ultimate, however, the deflection values become increasingly inaccurate.

A similar finding was reported by Rutgers University where measured deflections were also found to be underestimated using an effective moment of inertia. [15] However, the accuracy of deflection predictions for glass-fiber FRP reinforced concrete beams was found to vary with the amount of reinforcement provided. [15] Researchers found that $I_e$, as calculated according to Eq. 2.1, became increasingly overestimated with decreasing amounts of fiberglass reinforcement. At 35% of the samples' laboratory tested ultimate strength, results showed measured deflections were underestimated by almost 60% for beams with a reinforcement ratio of $\rho_{frp}=0.696\%$ (where $\rho_{frp} = \{A_{frp}/bd\} \times 100\%$). However, for samples reinforced at $\rho_{frp}=2.187\%$ predicted deflections were found to underestimate measured values by as low as 8%. Thus, the study concluded, that for all beams tested, predicted deflections underestimated measured values, but that the amount
of underestimation was inversely proportional to the amount of reinforcement ($\rho_{frp}$) provided.

2.3.2 Strength Prediction

The prediction of an FRP reinforced beam's ultimate strength seems to provide results more consistent with laboratory test data than does deflection analysis. At the University of Arizona good correlation was found to exist between the experimentally determined flexural capacity of glass-fiber FRP reinforced concrete beams and that theoretically predicted using analytical procedures developed for concrete beams reinforced with steel bars, but using the mechanical properties of FRP. [16] Beams were tested in which the full strength of the FRP bars was developed resulting in a brittle tensile failures. Calculated ultimate loads based on tensile failure of the FRP reinforcement and flexural compression failure of the concrete were in very close agreement with measured laboratory values.

A similar conclusion regarding the theoretical flexural capacity of glass-fiber FRP reinforced beams was reached at Rutgers University in 1971. [17] The results from a series of flexural tests conclusively demonstrated that it was possible to accurately predict the flexural capacity of FRP reinforced concrete beams. Beams failed in flexural tension and compression at loads close to predicted levels. A second study conducted at Rutgers University in 1977 also concluded that the ultimate load of glass-fiber FRP reinforced concrete beams could be predicted with the same accuracy as for steel R/C beams. [15]
Generally, beams for which the ultimate flexural capacity was inaccurately predicted failed in shear. Researchers at Drexel University identify shear as possibly limiting the achievable flexural strength of beams based upon full development of the FRP tensile capacity. [14] This being the case, a method for predicting shear strength of FRP reinforced concrete beams without shear reinforcement was considered necessary.

The results from these research programs suggest that flexural strength is a quantity more accurately predicted using traditional concrete analysis procedures than is deflection or shear strength.

2.3.3 Cracking and Bond Properties

When a reinforced concrete beam is subjected to a flexural loading condition an internal moment is created between the tension force in the reinforcing and an equal compression force in the concrete. As the applied load increases the tensile force in the reinforcing increases and flexural cracks develop along the tension side of the beam. When effective force transfer exists between the concrete and reinforcing, flexural cracks develop at fairly uniform intervals, starting in the region of highest moment and propagating outwardly toward the support points with increasing load.

The pattern of flexural cracking in concrete beams reinforced with deformed FRP bars has been observed to be similar to that of beams reinforced with deformed steel bars. [14] [15] [16] [17] The studies conducted at the University of West Virginia, University of Arizona, Drexel University and Rutgers University all reported cracking patterns that reflected good mechanical bond
was being developed between the deformed FRP bars and surrounding concrete. In some cases, the bond was strong enough to develop the full strength of the FRP reinforcing bars, resulting in a brittle tensile failure. [16]

The cracks that developed in the FRP reinforced beams were generally reported to be wider than those of steel reinforced beams. This condition reflects the large deflections that result as a consequence of the material's low modulus. It has been suggested that using FRP as a reinforcement should permit higher tolerable crack widths because corrosion of the reinforcement is no longer a major concern. [12] Certainly, deflection criteria must be considered as well.

2.4 NEFMAC Research Significance

The analytical procedures ultimately developed for designing concrete structures reinforced with deformed FRP bars are not necessarily applicable to NEFMAC. The geometric shape, force transfer properties and grid arrangement of NEFMAC constitutes an FRP reinforcing material whose behavior is likely to be different than that of deformed FRP bars. Thus, the performance of NEFMAC as a concrete reinforcement should be investigated independently.

The generation of test data documenting the performance of NEFMAC as a reinforcement for concrete is prerequisite to understanding how its behavior might be reliably predicted. Research, like that performed at UNH [18] [19] [42], represents only a preliminary contribution to this effort. Much
additional research is currently necessary so that a more comprehensive documentation of NEFMAC performance can be assembled.

2.5 Conclusions and Findings

The tests referenced here support the following conclusions regarding the performance of concrete beams reinforced with deformed FRP reinforcing bars:

(1) Good bond development is possible with the winding of spiral rovings around the bar circumference.

(2) Flexural strength can be predicted with an acceptable degree of accuracy using traditional reinforced concrete analytical procedures.

(3) Shear strength is of major concern and may be a limiting factor in design.

(4) A methodology for shear strength prediction is needed.

(5) Deflection prediction using a Branson effective moment of inertia consistently underestimates laboratory results. The amount by which predicted deflections underestimated lab measured deflections was found to increase as the percentage of FRP reinforcement provided decreased.
CHAPTER III

MATERIAL PROPERTIES

This chapter presents the material properties of NEFMAC, concrete and reinforcing grade steel as required for analytical calculations. The NEFMAC discussion includes a detailed description of all relevant physical, mechanical and geometric properties characteristic of the grid.

3.1 NEFMAC

NEFMAC is composed of continuous reinforcing fibers impregnated within a vinyl ester resin. Using a layering process, the FRP is formed into rigid two and three dimensional grid shapes. Examples of NEFMAC grid shapes available from the manufacturer are shown in Figure 3.1.
NEFMAC has been used extensively in many different construction projects throughout Japan. Between May of 1986 and March of 1991, 1,048,000 m² of NEFMAC sheets have been used to reinforce concrete structures. [20] Some of these applications include reinforcing for tunnel linings, curtain walls, storage tanks, silos and pontoons.

Because of the grid's effective reinforcing capabilities, NEFMAC has also been used as a geotextile reinforcement in soil structures.

3.1.1 Manufacturing Process

The manufacturing process used in forming the grid is known as the "pin-winding" process. [8] The pin-winding process is similar to the filament winding process wherein the final bar cross-section is developed through a "building up" action of individual resin-fiber layers.

The reinforcing fibers and matrix resin used in forming the individual layers are mixed together during the bathing process. The amount of fiber material present by volume is referred to as the "Volume Fraction of Fibers" or $V_f$ and is typically around 40%. [8] It is the fiber component of the FRP that is responsible for its high tensile strength. The resulting fiber-resin composite is then guided either vertically or horizontally by spools over a grid, building-up individual longitudinal and transverse laminations. This process is referred to as "grid-forming" and is shown in Figure 3.2. It is during the grid-forming that the dimensions of bar width, bar thickness, longitudinal-bar-spacing and transverse-bar-spacing are determined.
The layering process is repeated in alternating longitudinal and transverse directions until the desired cross-sectional thickness is achieved.

### 3.1.2 Bar Geometric and Cross-sectional Properties

As a result of the layering process, the bar's sides are irregular in shape and a series of smooth surfaces, seen as interlaminar bond lines on the bar cross-section, have been introduced, as is shown in Figure 3.3. These lines represent boundaries between adjacent layers of FRP and are characterized by weak interlaminar shear strength. This condition is, however, of secondary importance, in that the interlaminar shear weakness of the bar does not affect its tensile strength.
Smooth top surface

Irregular side profile

Void

Individual fiber-resin layers

Interlaminar lines

Smooth bottom surface

**Figure 3.3 Typical Bar Cross-Section**

The irregularity of the side profile is due to the fibrous nature of the unpressed FRP material upon drying, and the unevenness is a consequence of the different fiber-resin layers not aligning themselves exactly during the grid-forming process. The out of alignment arrangement existing between neighboring laminations results in the random propagation of voids into the bar cross-section (Figure 3.3). Although the cross-section is somewhat irregular in shape, the fiber content is very accurately controlled insuring constant axial strength throughout the bar length.

Within a grid, the longitudinal and transverse bars are continuous and orthogonal at their intersection points (Figure 3.2). Because the bars are continuous at the intersections, there exists no "preferred" or strong direction within the grid. Thus, a grid is two-dimensionally symmetric with respect to its mechanical and geometric properties in the longitudinal and transverse directions.
There is a subtle flaring of the bar width in the vicinity of the orthogonal intersections (Figure 3.4). This is a consequence of the grid layering process which forces twice as much material to occupy a unit length at the intersection points.

Figure 3.4 Flaring of Bar Width Near Intersections

The minor flaring of the bar width shown in Figure 3.4 varies slightly from one grid intersection to another and can affect the force transfer properties of the grid. [21]

3.1.3 Longitudinal and Transverse Bar Identification

The identification of a grid bar as being "longitudinal" or "transverse" is a function of the bar orientation with respect to the axis about which bending is
occurring. In this context, a bar is identified as "longitudinal" if it is oriented perpendicular to the axis of bending and "transverse" if it is oriented parallel to the axis of bending, as is shown in Figure 3.5.

The designation of bars as being either longitudinal or transverse also describes how the bars function physically as flexural reinforcement. This definition identifies longitudinal bars as those supporting axial force and transverse bars as those supporting bearing force, as is shown in Figure 3.5. The development of a bearing force against the transverse bar can be interpreted as a force resisting axial pullout. In effect, the transverse bar is transferring the axial force from the longitudinal bar to the concrete. Thus, transverse bars are responsible for force transfer and longitudinal bars are responsible for tensile reinforcement. There is also the potential for a limited amount of force transfer along the longitudinal bar. This could develop as a
consequence of the bar's course and irregular side profile (Figures 3.3 and 3.4) providing a bonding surface for the surrounding concrete.

3.1.4 Fibers Used in Reinforcing NEFMAC

Three different categories of fibers are used in manufacturing NEFMAC: glass fibers (GF), carbon fibers (CF) and aramid fibers (AF). The different fiber-types available within each fiber-category are; E-glass and T-glass for GF, High Modulus (HM) and High strength (HS) for CF, and Kevlar 49 and Technora for AF. Thus, a total of six different types of fibers are available for reinforcing NEFMAC. Importantly, more than one type of fiber may be used in the manufacturing of a given NEFMAC grid. Table 3.1 shows the mechanical properties of the six fibers available for reinforcing NEFMAC. [8]

Table 3.1 shows carbon fibers have the highest modulus of elasticity and the glass fibers the lowest modulus values. As compared to Grade 60 steel ($E_s=29000$ ksi, $F_y=60$ ksi) the modulus of all carbon fibers (HM and HS), is however, superior and in the case of the high modulus carbon fiber (HMCF) as much as 98 % higher. Also, the tensile strength of all fibers is significantly greater than steel and it is this property of FRP that makes them attractive as tensile reinforcement of concrete.

NEFMAC may be manufactured using one or more than one type of fiber. The uniaxial tensile strength and elastic modulus of the bar are then proportional to the percentage by volume of reinforcing fibers and resin from which the composite material is formed. [23] Because the tensile strength and modulus of the resin is sufficiently low compared with that of the fibers, it is
often neglected in calculating the theoretical strength and stiffness properties of the bar. [24]

**Table 3.1 Properties of NEFMAC Reinforcing Fibers**

<table>
<thead>
<tr>
<th>Type of Fiber</th>
<th>Density (lb/ft$^3$)</th>
<th>Texture (lb/1000ft)</th>
<th>Fiber Diam. (10$^{-6}$m)</th>
<th>Tensile Strength (ksi)</th>
<th>Young's Modulus (ksi)</th>
<th>Ultimate Strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>E-Glass</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>monofilament</td>
<td>157.9</td>
<td></td>
<td></td>
<td>497.7</td>
<td>10523</td>
<td>4.8</td>
</tr>
<tr>
<td>roving*</td>
<td>157.9</td>
<td>1.49</td>
<td>22</td>
<td>238.9</td>
<td>10523</td>
<td>2.27</td>
</tr>
<tr>
<td><strong>T-Glass</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>monofilament</td>
<td>154.8</td>
<td></td>
<td></td>
<td>675.5</td>
<td>12230</td>
<td>5.5</td>
</tr>
<tr>
<td>roving*</td>
<td></td>
<td>1.51</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>carbon</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>high strength (HS)</td>
<td>110</td>
<td>0.54</td>
<td>7</td>
<td>479.7</td>
<td>34128</td>
<td>1.4</td>
</tr>
<tr>
<td>high modulus (HM)</td>
<td>112.5</td>
<td>0.244</td>
<td>7</td>
<td>383.9</td>
<td>56880</td>
<td>0.6</td>
</tr>
<tr>
<td><strong>aramid</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kevler 49</td>
<td>90.2</td>
<td>0.85</td>
<td>11.9</td>
<td>398.2</td>
<td>18486</td>
<td>2.4</td>
</tr>
<tr>
<td>Technora</td>
<td>86.4</td>
<td>0.112</td>
<td>12</td>
<td>440.8</td>
<td>10096</td>
<td>4.4</td>
</tr>
</tbody>
</table>

Note: Roving - a number of glass fiber strands collected into a parallel bundle with little or no twist. [22]

### 3.1.5 Resin and Thermal Coefficient of Expansion

While fibers provide strength, the general purpose of the resin is in providing protection for the fibers from mechanical abuse and chemical attack. Also, it is the resin that provides resistance to alkali, acid, salt and
chemical attack. The elastic modulus of the resin is \(4.9 \times 10^5\) psi and the tensile strength of the resin is 11,800 psi. [25]

The coefficient of linear Expansion for NEFMAC reinforced with glass-fiber is reported by the manufacturer as \(9 \pm 10 \times 10^{-6}/^\circ\text{C}\). [8]

3.1.6 Grid Identification System

A NEFMAC grid is identified using a letter-number combination (ex. H10). The letter is either A, C, G or H and has the following meaning:

- **A** - indicates that aramid reinforcing fibers are used
- **C** - indicates that high strength carbon reinforcing fibers are used
- **G** - indicates that E-glass roving reinforcing fibers are used
- **H** - indicates that a combination of both E-glass and high strength carbon fibers are used

The second component of the grid identification is the bar number. The number corresponds to the diameter, in millimeters, of a Grade 70 steel reinforcing bar having the same ultimate strength as the NEFMAC bar. Thus, a bar designated as **G10** would identify an E-glass fiber reinforced bar having the same tensile strength as a 10 mm diameter Grade 70 steel bar.

3.1.7 Experimental Determination of Mechanical Properties

The following NEFMAC bars were used as reinforcing for test samples in this thesis: H10, H19, H22, C19 and C22. The fiber composition, average cross-
sectional area and thickness for these bars are given in Table 3.2. [26] The cross-sectional areas listed in Table 3.2 were determined by averaging the results of repeated volumetric measurements.

Table 3.2 Material Composition and Geometric Properties of NEFMAC [26]

<table>
<thead>
<tr>
<th>Bar Type (ID #)</th>
<th>% Carbon Fiber (% volume)</th>
<th>% Glass Fiber (% volume)</th>
<th>% Resin (% volume)</th>
<th>Thickness (in)</th>
<th>Area (in²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H10</td>
<td>5</td>
<td>41</td>
<td>54</td>
<td>.39</td>
<td>.124</td>
</tr>
<tr>
<td>H19</td>
<td>5</td>
<td>41</td>
<td>54</td>
<td>.79</td>
<td>.465</td>
</tr>
<tr>
<td>H22</td>
<td>5</td>
<td>41</td>
<td>54</td>
<td>1.02</td>
<td>.620</td>
</tr>
<tr>
<td>C19</td>
<td>43</td>
<td>0</td>
<td>57</td>
<td>.49</td>
<td>.248</td>
</tr>
<tr>
<td>C22</td>
<td>43</td>
<td>0</td>
<td>57</td>
<td>.83</td>
<td>.326</td>
</tr>
</tbody>
</table>

Table 3.3 summarizes the mechanical properties describing the bars' stress-strain behavior which were determined from uniaxial tensile tests conducted on full cross-section bar samples. [26]

Table 3.3 Mechanical Properties of NEFMAC [26]

<table>
<thead>
<tr>
<th>Bar Type (#)</th>
<th>Pₚ (kips)</th>
<th>Fₚ (ksi)</th>
<th>εᵤ (in/in)</th>
<th>Eᵥ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H10</td>
<td>14.9</td>
<td>120</td>
<td>.0200</td>
<td>6000</td>
</tr>
<tr>
<td>H19</td>
<td>46.0</td>
<td>99</td>
<td>.0160</td>
<td>6200</td>
</tr>
<tr>
<td>H22</td>
<td>57.0</td>
<td>92</td>
<td>.0150</td>
<td>6100</td>
</tr>
<tr>
<td>C19</td>
<td>46.0</td>
<td>185</td>
<td>.0149</td>
<td>12400</td>
</tr>
<tr>
<td>C22</td>
<td>58.0</td>
<td>178</td>
<td>.0145</td>
<td>12300</td>
</tr>
</tbody>
</table>

where:  Pₚ, Fₚ, and εᵤ = load, stress and strain at ultimate (tensile failure), respectively.

The information given in Tables 3.2 and 3.3 provides a comprehensive listing of all material and dimensional properties required in analysis and design procedures.
Stress and strain in NEFMAC are nearly linearly elastic up to ultimate at which point a brittle tensile failure occurs. The materials stress-strain behavior is mathematically modeled as follows:

\[
\begin{align*}
    f_{frp} &= E_{frp} \varepsilon_{frp} \quad \text{for } \varepsilon_{frp} < \varepsilon_u \\
    f_{frp} &= 0 \quad \text{for } \varepsilon_{frp} \geq \varepsilon_u
\end{align*}
\]  

(3.1)

where:

- \( f_{frp} \) = nominal tensile stress (ksi)
- \( E_{frp} \) = tensile modulus of elasticity (ksi)
- \( \varepsilon_{frp} \) = nominal tensile strain (in/in)
- \( \varepsilon_u \) = ultimate tensile strain (in/in)

### 3.2 Concrete

Typical stress-strain curves for 28 day concrete cylinders loaded in uniaxial compression are shown in Figure 3.6. [40] The maximum uniaxial compressive strength of the concrete is designated as \( f'_c \).

![Figure 3.6 Uniaxial Compression Stress-Strain Curves for Concrete [40]](image-url)
The first part of each curve is nearly a straight line. This behavior continues to an $f_c$ value of about $0.5f'_c$, after which the curve becomes nonlinear and continues so until $f_c$ is reached. This nonlinearity results from the formation and subsequent propagation of microscopic internal cracks that lower the concrete's stiffness. The post $f_c$ curvature for low strength concrete is relatively flat. For high strength concrete, however, the post $f_c$ curvature is very sharp.

At $f_c$, the concrete's strain, $\varepsilon_c$, is between $0.002$ in/in and $0.0025$ in/in for all strengths. [44] In beams of normal strength concrete ($f'_c \leq 6000$ psi), compression failure occurs by crushing of the concrete at strain values between $0.003$ in/in and $0.0045$ in/in. [43] ACI-10.2.3 specifies an ultimate compressive strain, $\varepsilon_{cu}$, value of $0.003$ in/in be used in all analysis and design equations. [12]

The slope of the initial straight portion of a stress-strain curve represents the concrete modulus of elasticity, $E_c$. For $f'_c \leq 6000$ psi, ACI-8.5.1 specifies the calculation of $E_c$ from the empirical equation: [12]

$$E_c = 33w_c \sqrt{f'_c} \tag{3.2}$$

where:
- $E_c$ = concrete modulus of elasticity (psi)
- $w_c$ = the unit weight of the hardened concrete (pcf)
- $f'_c$ = uniaxial compressive strength of concrete (psi)

For normal concrete with $w_c = 145$ pcf, $E_c$ may be calculated as:

$$E_c = 57,000 \sqrt{f'_c} \tag{3.3}$$

where:
- $f'_c$ = compressive strength (psi)
The tensile strength of concrete in flexure is usually measured in terms of the modulus of rupture, $f_t$, which represents the computed flexural tensile stress ($f_t = Mc/I$) at which point a test beam of plain concrete fractures. ACI-9.5.2.3 recommends that $f_t$ be calculated as: [12]

$$f_t = 7.5 \sqrt{f'_c}$$  \hspace{1cm} (3.4)

where: $f'_c =$ concrete compressive strength (psi)

### 3.3 Steel

Grade 60 steel was used in the design and construction of all test beams for this thesis. For Grade 60 steel, the yield strength, $F_y$, is taken to be 60 ksi. The steel's modulus of elasticity, $E_s$, is 29,000 ksi and the corresponding strain at yield, $\varepsilon_y$, is calculated as .0021 in/in. For analysis computations stress-strain behavior is either perfectly elastic or perfectly plastic and calculated as:

$$f_s = E_s \varepsilon_s \quad \text{for} \quad \varepsilon_s < \varepsilon_y$$

$$f_s = F_y \quad \text{for} \quad \varepsilon_s \geq \varepsilon_y$$  \hspace{1cm} (3.5)

where:

- $f_s =$ nominal uniaxial tensile stress in steel
- $\varepsilon_s =$ nominal strain (in/in)
- $F_y =$ yield stress = 60 ksi
- $\varepsilon_y =$ yield strain (in/in)
- $E_s =$ steel modulus of elasticity = 29000 ksi
3.4 Summary of Material Properties

The material properties of NEFMAC are significantly different from those of Grade 60 steel. In particular the following three items are identified:

(1) NEFMAC behaves linearly-elastic to ultimate at which point a brittle failure occurs.

(2) The modulus of NEFMAC is as low as 1/5th that of Grade 60 steel.

(3) The tensile strength of NEFMAC is as much as 3 times the yield strength of Grade 60 steel.

These properties suggest that concrete beams reinforced with NEFMAC will experience much larger deflections than steel reinforced beams with the same reinforcing ratio. Also, for beams with equal reinforcement stiffnesses (EA), the NEFMAC reinforced section will have a much greater flexural strength than the steel reinforced section.

Additional information describing NEFMAC is available from the manufacturer. [27] [28] [29] [30] [31] [32]
CHAPTER IV

ANALYTICAL TECHNIQUES

This chapter presents the analytical techniques used to predict stress-strain relations, flexural strength, shear strength and deflection behavior of reinforced concrete beams.

4.1 Analytical Assumptions

The fundamental assumptions relating to the analysis of bending stresses in analyzing reinforced concrete beams are as follows:

1. A cross section that was plane before loading remains plane after loading. This means that the strain distribution in a beam above and below the neutral axis are linearly related. (ACI-10.2.2)

2. Vectors normal to originally plane surfaces remain normal after bending.

3. The cross-section does not change shape during bending.

4. The bending stress at any point depends on the strain at that point in a manner given by the stress-strain diagram of the material.

5. The maximum usable concrete strain at the extreme compression fiber, $\varepsilon_{cu}$, is limited to .003 in/in. (ACI-10.2.3)
4.2 Stress and Strain Relations

Compression stress-strain relations in the concrete are analyzed using the bilinear material model shown in Figure 4.1. [63]

The bilinear model assumes concrete behaves either perfectly elastic or perfectly plastic. For $\varepsilon_c \leq .001$ (elastic case), flexural stress and strain distributions on a cracked section of a R/C beam are as shown in Figure 4.2.
Figure 4.2 Flexural Stress and Strain Distributions for $\varepsilon_c \leq .001$

where:

- $A_r, f_r, \varepsilon_r = $ reinforcement area, stress and strain, respectively
- $\rho = \frac{A_r}{bd} = $ reinforcement ratio
- $\varepsilon_c = $ variable $\leq .001 \text{ in/in}$
- $E_c = \frac{.85 f'_c}{.001} = 850 f'_c$ (4.1)
- $f_c = E_c \varepsilon_c$ (4.2)
- $c = \frac{\sqrt{1 + \left\{ \frac{(2 E_c b d)/(A_r f_r)}{(E_c b)/(A_r f_r)} \right\}} - 1}{(E_c b)/(A_r f_r)} = $ constant (4.3)
- $f_r = E_r \varepsilon_r = E_r \varepsilon_c \frac{d - c}{c}$ (4.4)
- $T = $ Resultant internal tensile force
- $M_a = $ internal resisting moment
- $M_a = T \left( d - \frac{c}{3} \right) = A_r E_r \varepsilon_c \left( \frac{d - c}{c} \right) \left( d - \frac{c}{3} \right)$ (4.5)

When the concrete strain exceeds .001 in/in, stress-strain distributions on a cracked-section of a R/C beam are assumed as shown in Figure 4.3.
Cracked section | Strain distribution | Stress Distribution and Resultant Forces

Figure 4.3 Flexural Stress and Strain Distributions for $\varepsilon_c > .001$ in/in

where:

$\varepsilon_c = \text{variable} > .001$

$c_1 = c \left( \frac{\varepsilon_c - .001}{\varepsilon_c} \right)$ (a)

$c_2 = c \left( \frac{.001}{\varepsilon_c} \right)$ (b)

$C_1 = .85 f_c b c_1 = .85 f_c b c \left( \frac{\varepsilon_c - .001}{\varepsilon_c} \right)$ (c)

$C_2 = \frac{1}{2} .85 f_c b c_2 = \frac{1}{2} .85 f_c b c \left( \frac{.001}{\varepsilon_c} \right)$ (d)

$T = A_r \varepsilon_r = A_r E_r \varepsilon_r = A_r E_r \varepsilon_c \left( \frac{d - c}{c} \right)$

From $T = C_1 + C_2$, the compression block depth "c" is calculated as:

$$c = \frac{-A_r E_r \varepsilon_c + \sqrt{(A_r E_r \varepsilon_c)^2 + [4 \times .85 f_c b d (1 - (.001/2\varepsilon_c)) A_r E_r \varepsilon_c]}}{2 \times .85 f_c b (1 - (.001/2\varepsilon_c))}$$ (4.6)
Designating $C = C_1 + C_2$ as the resultant compressive force acting in the concrete of Figure 4.3, the centroid of the bilinear compressive stress distribution located a distance $Y$ above the neutral axis, is calculated from:

$$C(Y) = C_1(c_2 + \frac{C_1}{2}) + C_2(\frac{2}{3}c_2)$$

Substituting Eqs. (a) through (d) into the above equation, $Y$ is calculated as:

$$Y = \frac{\frac{C}{2}(\varepsilon_c^2 - \frac{1}{3}.001^2)}{(\varepsilon_c^2 - \frac{.001 \varepsilon_c}{2})}$$

(4.7)

Taking moments about the resultant compressive force, the internal resisting moment is computed as:

$$M_a = T(d - c + Y) = A_f E_f \varepsilon_c \left(\frac{d - c}{c}\right) (d - c + Y)$$

(4.8)

4.3 Flexural Strength - Whitney Rectangular Stress Distribution

For flexural strength prediction, the shape of the bilinear stress distribution shown in Figure 4.3 is replaced by a simplified rectangular stress block. [43] This stress block is called the Whitney Rectangular Stress Distribution and is shown in Figure 4.4.
Figure 4.4 Whitney Stress Distribution

where:

\[ a = \text{depth of the Whitney stress block} \]

\[ a = \beta_1 c \quad (4.9) \]

\[ \beta_1 = 0.85 \quad \text{for } f_c \leq 4000 \text{ psi} \quad (4.10) \]

\[ \beta_1 = 0.85 - 0.05 \left( \frac{f_c - 4000}{1000} \right) > 0.65 \quad \text{for } f_c > 4000 \text{ psi} \]

The Whitney stress block shown in Figure 4.4 simplifies the calculations necessary in computing the resultant compressive force acting in the concrete. Both ACI [12] and PCA [45] sanction the use of the Whitney stress block and, as such, it is widely employed in design.

According to ACI-10.3.2 the balanced strain condition for a steel reinforced beam is defined as the state where the reinforcement reaches its yield strain, \( \varepsilon_y \), just as the strain on the extreme compression fiber of the concrete is simultaneously equal to its assumed ultimate value of 0.03 in/in. From internal equilibrium of the tensile and compressive forces, the corresponding balanced reinforcing ratio, \( \rho_b \), assuming a Whitney stress block in the concrete is calculated as:
Thus, failure of a concrete beam reinforced with steel at $\rho = \rho_b$ would occur as simultaneous yielding of the reinforcement and crushing of the concrete. This is referred to as a balanced failure. Correspondingly, when $\rho < \rho_b$, a flexural tensile failure is predicted and when $\rho > \rho_b$ a flexural compression failure is expected.

It must be appreciated that a balanced failure as defined above is not possible for an FRP reinforced beam, as it does not possess any yield capabilities. Thus, for concrete beams reinforced with FRP the definition of a balanced design, as given in Eq. 4.11 (but with the substitution of the FRP’s $\varepsilon_u$ for $\varepsilon_y$ of the steel), provides only a reference against which flexural failure can be expected as either compression or tension.

**Strength Prediction - Overreinforced Design**

When $\rho > \rho_b$ the concrete fails in compression before the reinforcement fails in tension. From internal equilibrium, the section's moment capacity is calculated as:

$$M_n = .85 f_c' b a \{ d - \frac{a}{2} \}$$  \hspace{1cm} (4.12a)

where:

$$a = \frac{(-A_r E_r \varepsilon_{cu}) + \sqrt{(A_r E_r \varepsilon_{cu})^2 + 4 \{ .85 \beta_1 f_c' b d E_r A_r \varepsilon_{cu} \}^2}}{2 \times .85 f_c' b}$$  \hspace{1cm} (4.12b)
Strength Prediction - Underreinforced Design

When \( \rho < \rho_b \) the reinforcement fails in tension before the concrete fails in compression. From internal equilibrium, the section's moment capacity is calculated as:

\[
M_n = .85 f'_c b a \left\{ d - \frac{a}{2} \right\}
\]

(4.13a)

where:

\[
a = \frac{A_s F_y}{.85 f'_c b} \quad \text{for steel}
\]

(4.13b)

\[
a = \frac{A_{frp} F_u}{.85 f'_c b} \quad \text{for NEFMAC}
\]

(4.13c)

and:

\[A_s, F_y = \text{steel area and yield stress, respectively}\]

\[A_{frp}, F_u = \text{FRP area and tensile strength, respectively}\]

NEFMAC Considerations

For NEFMAC reinforced beams where \( \rho < \rho_b \) a tensile failure is also predicted. However, unlike steel, the NEFMAC has no yield capacity and when the reinforcing stress \( f_{frp} \) reaches \( F_u \) the failure is sudden and brittle. As a consequence of this condition, FRP reinforced beams can only experience a brittle mode of failure in flexure.
4.4 Deflection Prediction

The load point deflection for a simply supported beam subjected to two equal concentrated loads of magnitude $P/2$ symmetrically placed a distance "a" from the supports is given as:

$$y_a = \frac{3La - 4a^2}{6E_c} \left( \frac{M_a}{I_e} \right)$$  \hspace{1cm} (4.14)

where:

- $y_a =$ deflection at load point
- $L =$ span length
- $E_c =$ concrete elastic modulus
- $I_e =$ effective moment of inertia
- $a =$ shear span
- $M_a =$ moment at load point $= (P/2) a$

Two methods are presented for calculating deflection according to the above equation: (1) $I_e$ is calculated according to the Branson equation and (2) according to a moment-curvature analysis, where $(M_a/E_cI_e)$ in Eq. 4.14 is replaced with $(\varepsilon_c/c)$. Both of these procedures are detailed in Appendix B.

4.5 Shear Analysis

The analysis of shear in reinforced concrete beams is not directly concerned with the action of vertical shearing forces as such. The direct shear stresses in most beams are far below the shear strength of the concrete. When shear problems do occur they are invariably the consequence of principal stresses
resulting from the combined action of direct shearing and flexural stresses. The principal stresses represent the largest and smallest normal stresses acting on an element and can be either tension or compression. When the maximum principal stress is in tension the name "diagonal-tension stress" is used.

4.5.1 Shear Strength of Concrete Beams

ACI 11.3.1.1 employs the following simplified equation in calculating the nominal shear strength of beams subjected to shear and flexure only with no shear reinforcement: [12]

\[
V_c = 2 \sqrt{f_c} \cdot b \cdot d
\] (4.15)

The value calculated by Eq. 4.15 often overestimates \( V_c \) for steel reinforced beams of low reinforcing ratios. [50] [51] For values of \( \rho \) lower then about 1.2%, the following expanded form of Eq. 4.15 is recommended: [52] [53]

\[
V_c = (0.8 + 100 \frac{A_s}{b \cdot d}) \sqrt{f_c} \cdot b \cdot d
\] (4.16)

4.5.2 Shear Failure of Concrete Beams

The process of shear failure in reinforced concrete beams usually begins at the crack located farthest from the beam centerline. Initially this crack is oriented vertical or slightly slanted. Under increasing load the crack propagates upward becoming more and more inclined as it penetrates through the beam. The crack encounters resistance as it moves upward into the compression
zone eventually coming to a halt. With further loading, the crack will suddenly and without warning traverse the entire depth of the beam splitting it in two causing brittle failure.

In general, the amount of reserve strength available after the formation of inclined cracking as well as the amount of time spent in the growth stage is strongly influenced by the shear-span to depth ratio (a/d). In the case where all design, load and support conditions are kept constant, with only a variation in a/d, beams can be classified relative to their shear strength and failure characteristics as follows: [43]

**Short Beams:** (1 < a/d < 2.5) Short beams are able to sustain loads greater than that at which inclined cracking occurs. Thus, short beams have a shear strength that exceeds the inclined cracking strength.

**Intermediate Beams:** (2.5 < a/d < 6) The reserve strength available after the formation of inclined cracking is small for intermediate length beams. When the inclined crack forms, the beam is not able to redistribute the load and establish equilibrium. As a result, the crack propagates rapidly through the beam, in the process becoming progressively more inclined. At failure the crack is almost horizontal at which point the beam splits in two. This type of failure is referred to as "diagonal-tension failure".

**Long Beams:** (a/d > 6) Failure in long beams usually occurs as the result of flexural deficiencies. The cracking pattern at failure is characterized by nearly vertical flexural cracks located at the section of maximum bending. Near the point of failure cracks located between the load and support points may become slightly inclined from the vertical. They do not, however, propagate through the beam and, as such, the strength of the beam is entirely dependent of its flexural weaknesses.
4.5.3 Shear Strength Considerations for FRP Reinforced Beams

The transfer of shear force in reinforced concrete beams occurs by a combination of the following mechanisms: (1) aggregate interlock, (2) dowel action of the reinforcement, and (3) shear resistance of the uncracked concrete. [43] Shear transfer through aggregate interlock occurs when the surfaces on each side of a crack are close enough to cause interlocking of the aggregates. However, when a crack opens to the extent that the aggregates do not interlock, this component of shear transfer is eliminated. The relationship given in Eq. 4.16 recognizes that where deflections are large, there is a deterioration in shear strength. Thus, the large deflections and wide crack widths characteristic of FRP reinforced concrete beams are expected to lower the section's shear capacity relative to that predicted according to ACI-11.3.1.1, Eq. 4.15.

4.6 Summary of Analytical Procedures

The code-specified procedures for calculating flexural strength, shear strength and deflection have essentially been developed for the analysis of steel reinforced concrete beams. Test results will determine their effectiveness in application to NEFMAC reinforced beams.

Tensile failure of NEFMAC reinforced beams will require very low reinforced sections resulting in very large deflections. Thus, in practical design, the strength of NEFMAC reinforced beams will likely be controlled by compression failure in the concrete or a shear rupture.
CHAPTER V

TEST SETUP AND SAMPLE SPECIFICATIONS

5.1 Introduction

All physical characteristics describing each beam tested are presented in this chapter. The information provided includes specifications identifying: (1) type and amount of reinforcement used, (2) dimensions of beam cross-section and reinforcement location, and (3) material properties of concrete and reinforcement.

5.2 Test Setup and Variables

Figure 5.1 details the components used to load and support the samples during testing.
The load and support points consisted of solid 2 inch diameter rolled steel bars. Sample beams were subjected to a four point load system providing an area of constant moment and zero shear at a center portion of the beam. Figure 5.2 identifies the nomenclature used for all geometric variables required for design and analysis calculations.
Figure 5.2 Test Setup and Variables

where:

\[ \begin{align*}
P & = \text{total load} \\
P/2 & = \text{shear force} \\
a & = \text{shear span} \\
A_r & = \text{area of reinforcement} \\
b,d,h & = \text{cross-sectional dimensions} \\
cv & = \text{clear cover} \\
j & = \frac{1}{2} \text{length of constant moment region} \\
L & = \text{length of beam}
\end{align*} \]

5.3 Test Sample Specifications

Sample beams are grouped together in Test Groups (TG) according to the date they were poured. Thus, all beams within a given Test Group were poured at the same time and possess the same concrete material properties. Each beam tested is identified using an identification number having the format; #B#. The first number refers to the Test Group and the second number refers to the beam number within that Test Group. Thus, beam 3B4 represents the fourth
A total of 32 beams from 6 different Test Groups are detailed. Test samples were subjected to both monotonically and cyclically applied loads.

Concrete compression strength was determined on the day of testing using 6"x12" cylinders loaded according to the provisions of ASTM C39-84. All samples were cast using NHDOT standard bridge mix design. Form work was removed after 7 days and curing duration's were all in excess of 28 days.

The reader should understand that the Test Groups listed in the following sections are not related to each other in any specific way. The grouping were done for convenience of identification. Also, the amount of reinforcement provided each test sample is specified relative to a balanced design as \( \%p_b \) where \( \%p_b = \left( \frac{\rho}{\rho_b} \right) \times 100\% \).

### 5.3.1 Test Group 1

Test Group 1 consisted of 3 beams, 2 of which were reinforced with a single C22 NEFMAC bar and 1 reinforced with 3-#4 steel bars. The geometric, reinforcing, load and support conditions for the 3 test beams in Test Group 1 are given in Table 5.1. Figure 5.3 details the cross-sectional geometry showing how the reinforcing was placed.

The test was designed to study cyclic performance and the distribution of force that occurs in the FRP along the length of a longitudinal bar. This was accomplished by instrumenting the longitudinal NEFMAC bar of sample
Beam 1B1 with a total of 15 strain gages. The locations of the individual strain gages are presented with the test results in Chapter 7.

Table 5.1 Test Group 1 Sample Parameters

<table>
<thead>
<tr>
<th>Beam ID</th>
<th>b</th>
<th>h</th>
<th>d</th>
<th>Rein.</th>
<th>(A_r)</th>
<th>Load</th>
<th>%(\rho_b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#</td>
<td>(in)</td>
<td>(in)</td>
<td>(in)</td>
<td>(type)</td>
<td>(in^2)</td>
<td>(m/c)</td>
<td>(%)</td>
</tr>
<tr>
<td>1B1</td>
<td>12</td>
<td>6</td>
<td>4.8</td>
<td>1xC22 FRP</td>
<td>.326</td>
<td>cyclic</td>
<td>186</td>
</tr>
<tr>
<td>1B2</td>
<td>12</td>
<td>6</td>
<td>4.8</td>
<td>1xC22 FRP</td>
<td>.326</td>
<td>mono</td>
<td>186</td>
</tr>
<tr>
<td>1B3</td>
<td>12</td>
<td>6</td>
<td>4.8</td>
<td>3x#4 steel</td>
<td>.600</td>
<td>mono</td>
<td>33</td>
</tr>
</tbody>
</table>

Test Group Constants
- \(f'_c = 4.5\) ksi
- \(F_{u}^{C22} = 178\) ksi
- \(a = 28\) in
- \(E_{C22} = 12300\) ksi
- \(\epsilon_{u}^{C22} = .0145\) in/in
- \(F_y = 60\) ksi

Figure 5.3 Test Group 1 Reinforcing Detail

5.3.2 Test Group 2

A total of 6 beams were tested in this group. The design and testing characteristics for the test beams of this group are given in Table 5.2. The cross-sectional geometry and reinforcement location are shown in Figure 5.4.
Table 5.2 Test Group 2 Sample Parameters

<table>
<thead>
<tr>
<th>Beam ID</th>
<th>b (in)</th>
<th>h (in)</th>
<th>d (in)</th>
<th>Rein.</th>
<th>$A_r$ (in$^2$)</th>
<th>Test</th>
<th>%$p_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2B1 - 2B3</td>
<td>12</td>
<td>8.5</td>
<td>7.56</td>
<td>2xH10 FRP</td>
<td>.248</td>
<td>mono</td>
<td>69</td>
</tr>
<tr>
<td>2B4 - 2B6</td>
<td>8.5</td>
<td>12.5</td>
<td>11.56</td>
<td>2xH10 FRP</td>
<td>.248</td>
<td>mono</td>
<td>64</td>
</tr>
</tbody>
</table>

Test Group Constants

- $f'_c = 5.5$ ksi
- $F_u^{H10} = 120$ ksi
- $a = 31$ in
- $E^{H10} = 6000$ ksi
- $j = 4$ in
- $\varepsilon_u^{H10} = .02$ in/in

All beams in Test Group 2 were tested monotonically without strain gage instrumentation. Two different designs were tested, with three beams per design. Both designs were provided with equal amounts of NEFMAC reinforcing, however, $p$ was slightly different (Table 5.2). The test was designed to determine how accurately analytical equations predict the flexural capacity of FRP reinforced beams assuming a tensile failure condition. However, shear failure occurred during testing in all samples.

**Figure 5.4 Test Group 2 Reinforcing Detail**
5.3.3 Test Group 3

The parameters of the 12 beams making up Test Group 3 are given in Table 5.3. The cross-sectional details are shown in Figure 5.5.

<table>
<thead>
<tr>
<th>Beam ID</th>
<th>b (in)</th>
<th>h (in)</th>
<th>d (in)</th>
<th>Rein.</th>
<th>$A_r$ (in$^2$)</th>
<th>Test</th>
<th>Test %pb</th>
</tr>
</thead>
<tbody>
<tr>
<td>3B1 - 3B3</td>
<td>12.5</td>
<td>8.5</td>
<td>7.56</td>
<td>1xH10 FRP</td>
<td>.124 m/c mono</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>3B4 - 3B6</td>
<td>15</td>
<td>8</td>
<td>7.06</td>
<td>1xH10 FRP</td>
<td>.124 m/c mono</td>
<td>37</td>
<td></td>
</tr>
<tr>
<td>3B7 - 3B9</td>
<td>8</td>
<td>6</td>
<td>4.90</td>
<td>4xH10 FRP</td>
<td>.496 m/c mono</td>
<td>403</td>
<td></td>
</tr>
<tr>
<td>3B10 - 3B12</td>
<td>7.5</td>
<td>6</td>
<td>4.90</td>
<td>4xH10 FRP</td>
<td>.496 m/c mono</td>
<td>430</td>
<td></td>
</tr>
</tbody>
</table>

Test Group Constants

- $f'_c = 4.0$ ksi
- $F_u^{H10} = 120$ ksi
- $E_{H10} = 6000$ ksi
- $\varepsilon_{uH10} = .02$ in/in

This test group contained 4 different designs with 3 beams per design. The first 2 designs, Beams 3B1 - 3B3 and 3B4 - 3B6, were both provided with reinforcing ratios well below that calculated for a balanced condition. The last 2 designs, Beams 3B7 - 3B9 and 3B10 - 3B12 were provided with reinforcing ratios well above that calculated for a balanced condition. As such, the test was designed to conclusively fail FRP reinforced beams in both flexural tension and flexural compression while avoiding a diagonal shear failure. This effort was successful and all Test Group 3 samples experienced flexure failure.
In anticipation of a tensile failure, Samples 3B1 and 3B3 were instrumented with one strain gage located on the bottom surface of the longitudinal NEFMAC bar at centerspan. Sample 3B4 was instrumented with 2 strain gages, one located at centerspan and a second located one transverse bar spacing away (4"). Both gages were located on the bottom surface of the longitudinal NEFMAC bar.

5.3.4 Test Group 4

The dimension and reinforcing properties of Test Group 4 beams are given in Table 5.4. Figure 5.6 identifies the reinforcing detail for these beams. The test
was designed to study the cyclic performance of concrete beams reinforced with H10 and H19 NEFMAC grids. For this reason, two identical samples reinforced with each grid type were cast, with one sample being tested cyclically and the second sample acting as a control and tested monotonically.

Table 5.4 Test Group 4 Sample Parameters

<table>
<thead>
<tr>
<th>Beam ID</th>
<th>b (in)</th>
<th>h (in)</th>
<th>d (in)</th>
<th>Rein.</th>
<th>A_r (in²)</th>
<th>Test</th>
<th>%ρ_b</th>
</tr>
</thead>
<tbody>
<tr>
<td>4B1</td>
<td>10</td>
<td>7.0</td>
<td>5.80</td>
<td>1xH19 FRP</td>
<td>.465</td>
<td>mono</td>
<td>160</td>
</tr>
<tr>
<td>4B2</td>
<td>10</td>
<td>7.0</td>
<td>5.80</td>
<td>1xH19 FRP</td>
<td>.465</td>
<td>cycle</td>
<td>160</td>
</tr>
<tr>
<td>4B3</td>
<td>10</td>
<td>7.0</td>
<td>6.06</td>
<td>2xH10 FRP</td>
<td>.248</td>
<td>cycle</td>
<td>120</td>
</tr>
<tr>
<td>4B4</td>
<td>10</td>
<td>7.0</td>
<td>6.06</td>
<td>2xH10 FRP</td>
<td>.248</td>
<td>mono</td>
<td>120</td>
</tr>
</tbody>
</table>

Test Group Constants

- $f'_c = 4.5$ ksi
- $F_u^{H10} = 120$ ksi
- $F_u^{H19} = 99$ ksi
- $E^{H10} = 6000$ ksi
- $E^{H19} = 6200$ ksi
- $ε_u^{H10} = .02$ in/in
- $ε_u^{H19} = .016$ in/in

Figure 5.6 Test Group 4 Reinforcing Detail

Sample 4B1 was instrumented with 2 strain gages located on the top and bottom surface of the NEFMAC bar at centerspan. The test was designed to
evaluate the cyclic load response of intermediate length beams reinforced with H10 and H19 type grids.

5.3.5 Test Group 5

The dimensions and reinforcing properties of the two test samples in Test Group 5 are given in Table 5.5 and the cross-sectional reinforcing details are shown in Figure 5.7.

Table 5.5 Test Group 5 Sample Parameters

<table>
<thead>
<tr>
<th>Beam ID (ID#)</th>
<th>b (in)</th>
<th>h (in)</th>
<th>d (in)</th>
<th>Reinforcing</th>
<th>$A_r$ (in$^2$)</th>
<th>Test</th>
<th>%$\rho_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5B2 &amp; 5B3</td>
<td>9</td>
<td>15</td>
<td>13</td>
<td>2xC19 FRP</td>
<td>.496</td>
<td>mono</td>
<td>148</td>
</tr>
</tbody>
</table>

Test Group Constants

- $f'_c$ = 4.5 ksi
- $F_u^{C19}$ = 185 ksi
- $a$ = 31 in
- $E^{C19}$ = 12400 ksi
- $j$ = 4 in
- $\epsilon_u^{C19}$ = .0149 in/in

Figure 5.7 Test Group 5 Reinforcing Detail
Both test beams in this test group were provided with the same amount of NEFMAC C19 longitudinal reinforcement. The test was designed to evaluate the shear performance of beams classified as 'short' according to their shear span to depth ratio. As can be seen, a/d for these samples is 2.38, the smallest of all test samples.

5.3.6 Test Group 6

The dimensions and reinforcing details for the 5 beams of this test group are given in Table 5.6 and Figure 5.8, respectively. Five NEFMAC reinforced samples were tested in Test Group 6 with two different reinforcing ratios. The test was designed to study the failure mode of 'long' beams reinforced well over a balanced design.

<table>
<thead>
<tr>
<th>Beam ID</th>
<th>b</th>
<th>h</th>
<th>d</th>
<th>Rein.</th>
<th>Ar</th>
<th>Test</th>
<th>%ρb</th>
</tr>
</thead>
<tbody>
<tr>
<td>6B1 - 6B3</td>
<td>8</td>
<td>4</td>
<td>3.06</td>
<td>2xH10 FRP</td>
<td>.248</td>
<td>mono</td>
<td>290</td>
</tr>
<tr>
<td>6B4 - 6B5</td>
<td>8</td>
<td>4</td>
<td>3.06</td>
<td>3xH10 FRP</td>
<td>.372</td>
<td>mono</td>
<td>436</td>
</tr>
</tbody>
</table>

Test Group Constants

- $f'_c = 4.6$ ksi
- $F_{uH10} = 120$ ksi
- $a = 24$ in
- $E_{H10} = 6000$ ksi
- $j = 6$ in
- $\varepsilon_{uH10} = .02$ in/in
Note: All samples have dimensions of b=8", h=4" and d=3.06"

Figure 5.8 Test Group 6 Reinforcing Detail

5.4 Test Sample Summary

In summary, a total of 32 samples were designed, poured and tested. The information detailing each individual beam is provided in the Tables 5.1 through 5.6 and Figures 5.3 through 5.8. Table 5.7 summarizes the relevant analytical, material and dimensional properties of all beams tested.

Table 5.7 Test Sample Parameter Summary

<table>
<thead>
<tr>
<th>Beam</th>
<th>b (in)</th>
<th>d (in)</th>
<th>a (in)</th>
<th>j (in)</th>
<th>$f_{c}$ (ksi)</th>
<th>Reinforcement</th>
<th>type</th>
<th>$A_r$ (in$^2$)</th>
<th>$F_u$ (ksi)</th>
<th>$e_u$ (%)</th>
<th>E (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1B1 - 1B2</td>
<td>12</td>
<td>4.8</td>
<td>28</td>
<td>4</td>
<td>4.5</td>
<td>steel</td>
<td>C22</td>
<td>.326</td>
<td>178</td>
<td>.0145</td>
<td>12300</td>
</tr>
<tr>
<td>1B3*</td>
<td>12</td>
<td>4.8</td>
<td>28</td>
<td>4</td>
<td>4.5</td>
<td>steel</td>
<td>.600</td>
<td>60**</td>
<td>120</td>
<td>.0021</td>
<td>29000</td>
</tr>
<tr>
<td>2B1 - 2B3</td>
<td>12</td>
<td>7.56</td>
<td>31</td>
<td>4</td>
<td>5.5</td>
<td>steel</td>
<td>H10</td>
<td>.248</td>
<td>120</td>
<td>.020</td>
<td>6000</td>
</tr>
<tr>
<td>2B4 - 2B6</td>
<td>8.5</td>
<td>11.56</td>
<td>31</td>
<td>4</td>
<td>5.5</td>
<td>steel</td>
<td>H10</td>
<td>.248</td>
<td>120</td>
<td>.020</td>
<td>6000</td>
</tr>
<tr>
<td>3B1 - 3B3</td>
<td>12.5</td>
<td>7.56</td>
<td>31</td>
<td>4</td>
<td>4.0</td>
<td>H10</td>
<td>.124</td>
<td>120</td>
<td>6000</td>
<td>.020</td>
<td>6000</td>
</tr>
<tr>
<td>3B4 - 3B6</td>
<td>15.0</td>
<td>7.06</td>
<td>31</td>
<td>4</td>
<td>4.0</td>
<td>H10</td>
<td>.124</td>
<td>120</td>
<td>6000</td>
<td>.020</td>
<td>6000</td>
</tr>
<tr>
<td>3B7 - 3B9</td>
<td>8.0</td>
<td>4.9</td>
<td>31</td>
<td>4</td>
<td>4.0</td>
<td>H10</td>
<td>.496</td>
<td>120</td>
<td>.020</td>
<td>6000</td>
<td></td>
</tr>
<tr>
<td>3B10 - 3B12</td>
<td>7.5</td>
<td>4.9</td>
<td>31</td>
<td>4</td>
<td>4.0</td>
<td>H10</td>
<td>.496</td>
<td>120</td>
<td>.020</td>
<td>6000</td>
<td></td>
</tr>
<tr>
<td>4B1 - 4B2</td>
<td>10.0</td>
<td>5.8</td>
<td>31</td>
<td>4</td>
<td>4.5</td>
<td>H19</td>
<td>.465</td>
<td>99</td>
<td>6200</td>
<td>.016</td>
<td>6200</td>
</tr>
<tr>
<td>4B3 - 4B4</td>
<td>10.0</td>
<td>6.06</td>
<td>31</td>
<td>4</td>
<td>4.5</td>
<td>H10</td>
<td>.248</td>
<td>120</td>
<td>.020</td>
<td>6000</td>
<td></td>
</tr>
<tr>
<td>5B2 - 5B3</td>
<td>9.0</td>
<td>13.0</td>
<td>31</td>
<td>4</td>
<td>4.5</td>
<td>C19</td>
<td>.496</td>
<td>185</td>
<td>.0149</td>
<td>12400</td>
<td></td>
</tr>
<tr>
<td>6B1 - 6B3</td>
<td>8.0</td>
<td>3.06</td>
<td>24</td>
<td>6</td>
<td>4.6</td>
<td>H10</td>
<td>.248</td>
<td>120</td>
<td>.020</td>
<td>6000</td>
<td></td>
</tr>
<tr>
<td>6B4 - 6B5</td>
<td>8.0</td>
<td>3.06</td>
<td>24</td>
<td>6</td>
<td>4.6</td>
<td>H10</td>
<td>.372</td>
<td>120</td>
<td>.020</td>
<td>6000</td>
<td></td>
</tr>
</tbody>
</table>

* steel reinforced sample  ** $F_Y$
CHAPTER VI

MONOTONIC TEST RESULTS

6.1 Introduction

Discussion of the monotonic test results begins with a detailing of load-deflection and cracking behavior for each Test Group. A summary table is provided at the end of each section from which general conclusions regarding sample strength, failure mode, and flexural and shear behavior are discussed. This is followed by a presentation of strain data recorded on the longitudinal NEFMAC bar during testing of Samples 3B1, 3B3, 3B4 and 4B1. Finally, a summary is provided wherein all test results are tabulated and the accuracy of strength (flexure and shear) and deflection predictions are discussed.

6.2 General Test Results

Unless otherwise specified, all tests were executed in load control using a programmed loading rate of 1000 lb/min. At a given point in time, load, deflection and strain data were all recorded simultaneously using the data acquisition techniques detailed in Appendix A. A sampling frequency of 1 Hz was used for all data acquisition unless otherwise stated.

As mentioned above, a summary table is provided for each Test Group from which results are discussed and compared with theoretical predictions. In these summary tables, failure of NEFMAC reinforced beams is considered as
the complete loss of load carrying capacity and the corresponding "failure type" is recorded as the mechanism responsible for this condition, i.e. flexural compression, flexural tension or shear (diagonal-tension).

6.2.1 Test Group 1 Results

The crack pattern and load-deflection results for test Samples 1B2 and 1B3 are provided in Figures 6.1 through 6.3. Sample 1B2 was reinforced with 1xC22 NEFMAC bar at 186% $\rho_b$ and Sample 1B3 was reinforced with 3x#4 steel bars at 33% $\rho_b$. The shear span to depth ratio ($a/d$) for both samples was 5.8, classifying them as "intermediate" length beams. Sample 1B1 is of identical design to Sample 1B2 but was loaded cyclically. A discussion of it's results are presented in Chapter 7.

Figure 6.1 Crack Pattern and Moment Diagram - Samples 1B2, 1B3

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Sample 1B2

(2xC22 NEFMAC bars, b=12", d=4.8", a=28", L=64")

Figure 6.2 Load-Deflection Results - Sample 1B2

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Sample 1B3

(3x#4 Steel bars, b=12", d=4.8", a=28", L=64")

Figure 6.3 Load-Deflection Results - Sample 1B3

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The pre-crack response of both beams was identical and linear up to a load and deflection of 3 kips and .05 inches, respectively. Under an applied load of 3 kips the tensile strength of the concrete on the extreme fiber was reached and the first crack formed. For both Samples, this first crack occurred within the region of constant moment.

As expected, a ductile flexural failure occurred in Beam 1B3 with yielding of the reinforcing steel at a load of 10.16 kips. From this point, the stress in the steel remained constant and the reinforcing deformed plastically. The stretching of the steel caused the neutral axis to rise slightly, increasing the internal moment arm between the resultant tensile and compressive forces. This action slightly increased the moment capacity of the beam as can be seen from the post-yield test results. The rising of the neutral axis also increased the unit stress in the compression block until secondary failure eventually occurred by concrete crushing. The advantage of steel as reinforcing is demonstrated in the ability of the beam to absorb considerable amounts of energy through plastic deformation in the reinforcement. Although the beam had mathematically failed at the point of yielding in the steel, the sections capacity to sustain the applied load remained intact absorbing large amounts energy.

The cracked-section load-deflection performance of NEFMAC reinforced Beam 1B2 was closely linear up to ultimate. Brittle failure then occurred as a result of diagonal-tension shear. As can be see in Figure 6.1, it was the flexural-shear crack located farthest from the beam centerline that propagated upwardly through the beam's depth eventually causing brittle failure. No compression failure in the concrete was observed prior to collapse.
Although the load and deflection at ultimate were roughly the same for both samples, there was more and wider cracking in the NEFMAC reinforced sample than in the steel reinforced sample. The NEFMAC sample experienced a well developed cracked-section over 44 inches, about 70% of its length. The extent of visible cracking in the steel sample was mostly confined to the middle 16 inches of the beam, about 25% of its length and the cracks were mostly vertical, characteristic of flexural cracking. It is possible that micro-cracks, undetectable to the naked eye, existed outside of the visibly cracked section length. Unlike the steel beam, the cracks in the NEFMAC beam were mostly inclined, the result of diagonal-tension.

Test Summary

Test results are summarized in Table 6.1.

<table>
<thead>
<tr>
<th>Beam</th>
<th>%Pb</th>
<th>Failure</th>
<th>Pfail</th>
<th>Predicted</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(type)</td>
<td>(k)</td>
<td>P_fail</td>
<td>P_fail/P_pred</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(ksi)</td>
<td>P_u-flex.</td>
<td>ffrp</td>
</tr>
<tr>
<td>1B2</td>
<td>186</td>
<td>shear</td>
<td>10.90</td>
<td>106</td>
<td>12.79</td>
</tr>
<tr>
<td>1B3</td>
<td>33</td>
<td>flex.-ten.</td>
<td>10.16*</td>
<td>60**</td>
<td>11.33</td>
</tr>
</tbody>
</table>

Premature shear failure occurred in Sample 1B2 at a load 29% lower then that predicted according to ACI-11.3.1. At failure the stress in the NEFMAC (f_{frp})
was only 60% of the bars ultimate strength ($F_u$). The crack pattern reflected well developed diagonal-tension stresses in excess of the concrete's tensile capacity. The location and width of all cracks were plainly visible to the naked eye. Catastrophic failure occurred by complete separation on the flexural-shear crack located farthest from the beam centerline. Deflection prediction using the Branson I$_E$ was poor. Moment-curvature deflection prediction was good up to about 50% of ultimate (=6k).

As expected, steel reinforced Sample 1B3 experienced a ductile failure through yielding of the reinforcing steel. This was followed by a large amount of plastic deformation until ultimately brittle failure by concrete crushing occurred. The cracked pattern reflected only the development of flexural cracking suggesting that diagonal-tension stresses remained within the concrete's tensile capacity. Theoretical strength and deflection predictions compared very well with the test results.

### 6.2.2 Test Group 2 Results

Design properties and testing conditions were constant among Beams 2B1, 2B2 and 2B3, and again among Beams 2B4, 2B5 and 2B6. For this reason the results from each of these two groups are discussed separately.

**Samples 2B1, 2B2 and 2B3**

The cracking pattern and load-deflection results for Test Group 2 Samples 2B1, 2B2 and 2B3 are shown in Figure 6.4 and Figure 6.5, respectively. These samples were reinforced with 2xH10 NEFMAC bars at 69%$\rho_b$ and designed to
fail in flexural tension (tensile rupture of the NEFMAC bars). The shear span to depth ratio \((a/d)\) was 4.1, classifying them as "intermediate" length beams.

Figure 6.4 Crack Pattern and Moment Diagram - Samples 2B1, 2B2, 2B3
Samples 2B1, 2B2 and 2B3
(2xH10 NEFMAC bars, b=12", d=7.56", a=31", L=70")

Figure 6.5 Load-Deflection Results - Samples 2B1, 2B2, 2B3
A high degree of consistency characterized the load-deflection response of these three samples. Failure occurred in all three beams at about the same load and deflection magnitudes of 12 kips and 1.2 in. respectively, and the slopes of the pre- and post-crack load-deflection curves were very close.

The evolution of cracking generally started within the region of constant moment and propagated outwardly to a distance roughly halfway between the applied load and the roller support. The cracked load-deflection history for all 3 samples was characterized by a total of 7 individual cracks. The corresponding length of the cracked-section ranged between approximately 32" to 40" or 46% to 60% of the sample length. The individual crack locations consistently started at transverse bars. However, cracking was not observed to occur at each transverse bar within the cracked-section region.

Although reinforced at only 69% of \( \rho_b \), the test was unsuccessful in producing a flexural tensile failure. Rather, a shear failure occurred in all three beams. For each sample, it was the flexural-shear crack located farthest away from the beam centerline that ultimately caused brittle failure to occur. The shape this crack assumed was initially inclined about 45° with respect to a horizontal datum. The crack followed this inclined direction from its point of inception at the transverse bar to a point located roughly 12" outwardly away from the load point and 1.5" down from the top of the beam. From here the crack propagated directly towards the load point causing brittle failure. This failure mode is typical of beams of intermediate length without shear reinforcement. [44]
Test Results Summary

Test results are summarized in Table 6.2.

Table 6.2 Summary of Test Results - Samples 2B1, 2B2, 2B3

<table>
<thead>
<tr>
<th>Beam</th>
<th>%P_b</th>
<th>Failure</th>
<th>P_fail</th>
<th>Predicted</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>(%)</td>
<td></td>
<td>(type)</td>
<td>(k)</td>
<td>f_{frp}^1</td>
<td>P_{u-f}^2</td>
</tr>
<tr>
<td>2B1</td>
<td>69</td>
<td>shear</td>
<td>11.50</td>
<td>98</td>
<td>14.00</td>
</tr>
<tr>
<td>2B2</td>
<td>69</td>
<td>shear</td>
<td>12.00</td>
<td>103</td>
<td>14.00</td>
</tr>
<tr>
<td>2B3</td>
<td>69</td>
<td>shear</td>
<td>12.00</td>
<td>103</td>
<td>14.00</td>
</tr>
</tbody>
</table>

Although these samples were theoretically designed to fail in flexural tension, it was ultimately a shear failure in the form of diagonal-tension that terminated the test. For all samples failure was brittle with complete separation of the beam along the diagonal-tension crack located farthest from the beam centerline. All cracks initiated at transverse bar locations. Load at shear failure was only 45% of that predicted by ACI-11.3.1. Although reinforcement stress levels were close to ultimate, the NEFMAC did not fail and load-deflection behavior remained linear for the duration of the test.

Deflection prediction using the Branson Eq. was better at ultimate then at 50% of ultimate. This suggests the Branson equation overestimated the cracked-section moment of inertia at low post-crack loads. This likely reflects the low amount of reinforced provided (69%P_b). Moment-curvature deflection prediction was very good for the duration of the test.
Samples 2B4, 2B5 and 2B6

Due to errors in programming the test machine, Beam 2B4 was unexpectedly preloaded to 10 kips and Beam 2B5 was accidentally tested under deflection control with a programmed displacement rate of 4 in/min. The 10 kip preload applied to Sample 2B4 was removed and the test executed using the desired 1000 lb/min loading rate. Sample 2B6 was tested without incident. A typical crack pattern is given in Figure 6.6 and the load-deflection results for these samples are shown in Figure 6.7 but are questionable.

Shear span to depth ratio for these samples was 2.7, classifying them as "intermediate" in length. Reinforcement was provided at 64% $\rho_b$ (2xH10 NEFMAC bars) and a flexural tension failure was expected from this test group. However, all three samples failed prematurely in shear.

![Figure 6.6 Crack Pattern and Moment Diagram - Samples 2B4](image-url)
Samples 2B4, 2B5 and 2B6

(2xH10 NEFMAC bars, b=8.5", d=11.56", a=31", L=70")

Figure 6.7 Load-Deflection Results - Samples 2B4, 2B5, 2B6
Despite the loading problems, the cracked pattern observed is arguably legitimate test data. The locations of all cracks were consistently aligned with that of the transverse bars. The extent of cracking was approximately 50% of the length for Sample 2B4, 70% for Sample 2B5 and 60% for Sample 2B6. For all samples brittle failure occurred when the shear capacity of the beams was reached. No compression failure in the concrete was observed at ultimate. This mode of failure again characterized the shear performance of intermediate length beams without shear reinforcement. [44]

Test Summary

Test results are summarized in Table 6.3.

Table 6.3 Summary of Test Results - Samples 2B4, 2B5, 2B6

<table>
<thead>
<tr>
<th>Beam</th>
<th>%Pb</th>
<th>Failure (type)</th>
<th>Pfail (k)</th>
<th>f_{frp} (ksi)</th>
<th>P_{u-flex} (in/in)</th>
<th>P_{u-shear} (k)</th>
<th>P_{fail}/P_{pred}</th>
<th>f_{frp}/F_{u}</th>
</tr>
</thead>
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<tr>
<td>2B4</td>
<td>64</td>
<td>shear</td>
<td>14.97</td>
<td>83</td>
<td>21.50</td>
<td>29.15</td>
<td>(.51)</td>
<td>(.69)</td>
</tr>
<tr>
<td>2B5</td>
<td>64</td>
<td>shear</td>
<td>18.30</td>
<td>102</td>
<td>21.50</td>
<td>29.15</td>
<td>(.63)</td>
<td>(.85)</td>
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<td>64</td>
<td>shear</td>
<td>13.20</td>
<td>73</td>
<td>21.50</td>
<td>29.15</td>
<td>(.45)</td>
<td>(.61)</td>
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</tbody>
</table>

1 at P_{fail}, 2 (Equation 4.15)*2, 3 comparison made with bold faced predicted P_{u}

All three beams failed in a brittle manner as a result of diagonal-tension shear. Load at failure was much lower than that predicted by ACI-11.3.1. At failure NEFMAC stress levels ranged from 61% to 85% of ultimate. All cracks started at transverse bars. This test group failed to develop a flexural tensile failure because of the unexpected low shear strength of the design. As can be
seen from Table 6.3, the ratio of measured shear strength to predicted shear strength ranged from a low of .45 to a high of .63. These results demonstrate the inadequacy of Equation 4.15 (ACI-11.3.1.1) in estimating the shear strength of intermediate length concrete beams reinforced with low amounts of FRP. The large deflections that resulted from the low amount of reinforcement likely lowered the contribution of aggregate interlock as a mechanism for shear transfer, thus reducing the sample shear strength.

6.2.3 Test Group 3 Results

Test Group 3 consisted of 4 different designs with 3 beams per design for a total of 12 samples. Using the results from Test Group 2, the test was designed to avoid a shear failure and conclusively fail NEFMAC reinforced beams in flexural tension and flexural compression. Beams 3B1, 3B2, 3B3 were reinforced at 42%ρb. Beams 3B4, 3B5, 3B6 were reinforced at 37%ρb. These 6 samples had 1xH10 NEFMAC bar and were designed to fail in flexural tension. Beams 3B7, 3B8, 3B9 were reinforced at 403%ρb. Beams 3B10, 3B11, 3B12, were reinforced were reinforced at 430%ρb. These 6 samples had 4xH10 NEFMAC bars and were designed to fail in flexural compression. The cross-section of all beams was designed to provide adequate shear strength and thus ensure a flexural failure.

In anticipation of a tensile failure, beams 3B1, 3B3 and 3B4 were instrumented with a strain gage located on the longitudinal NEFMAC bar at centerspan. Using measured strain results, a comparison of theoretical and experimental reinforcement stress values is presented in Section 6.3, Strain Data Results, of this chapter.
All properties of Samples 3B1, 3B2 and 3B3 were identical and nearly the same as Samples 3B4, 3B5 and 3B6. Likewise, Samples 3B7, 3B8 and 3B9 were constant and nearly the same as Samples 3B10, 3B11 and 3B12. For this reason, the results of the first 6 beams (3B1 - 3B6) are presented together as the "Tensile Failure Group" and the results from the last 6 beams (3B7 - 3B12) are presented together as "Compression Failure Group".

Samples 3B1, 3B2 3B3, 3B4, 3B5 and 3B6 - Tensile Failure Group

The load-deflection results and typical crack patterns for Samples 3B1 through 3B6 are provided in Figures 6.8 through 6.10. All six beams were reinforced with 1xH10 NEFMAC bar and designed to fail in flexural tension (rupture of the NEFMAC reinforcement). This failure mode was accomplished.

The test successfully developed the ultimate strength of a single H10 NEFMAC bar. Tensile rupture of the H10 NEFMAC bar occurred in all six beams. The strain recorded at failure from four different strain gages concluded an ultimate strain value in the FRP of 2%. The results demonstrate clearly the brittle nature of FRP materials. No evidence of yielding in the reinforcing was observed as the ultimate strength was approached. Both deflection and strain remained linear with respect to applied load at ultimate.
The observed cracking pattern for these samples was radically different from that reported for Test Groups 1 and 2. In comparison, the amount of cracking that characterized the failed sections was very small. At failure a minimum of two cracks over a length of 8" was observed in Sample 3B6. This represents only 11% of the beam length. A maximum of 4 cracks over a length of 28" was observed at failure in Sample 3B1. Failure of the H10 NEFMAC bar in beams 3B1, 3B5 and 3B6 occurred at the transverse bar within the region of constant moment. For Samples 3B2, 3B3 and 3B4 failure occurred at the location of the first transverse bar outside of the constant moment region.
Samples 3B1, 3B2 and 3B3

(1xH10 NEFMAC bar, b=12.5", d=7.56", a=31", L=70")

Figure 6.9 Load-Deflection Results - Samples 3B1, 3B2, 3B3
Samples 3B4, 3B5 and 3B6

(1xH10 NEFMAC bar, b=15", d=7.06", a=31", L=70")

Figure 6.10 Load-Deflection Results - Samples 3B4, 3B5, 3B6

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Test Summary

Test results are summarized in Table 6.4.

Table 6.4 Summary of Test Results - Samples 3B1 - 3B6

<table>
<thead>
<tr>
<th>Beam</th>
<th>%ρ_b</th>
<th>Failure</th>
<th>P fail</th>
<th>Predicted</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(#)</td>
<td>(type)</td>
<td>(k)</td>
<td>f_{frp}</td>
<td>P_u-flex.</td>
</tr>
<tr>
<td>3B1</td>
<td>42</td>
<td>flex. - ten.</td>
<td>7.78</td>
<td>7.09</td>
<td>23.91</td>
</tr>
<tr>
<td>3B2</td>
<td>42</td>
<td>flex. - ten.</td>
<td>7.57</td>
<td>7.09</td>
<td>23.91</td>
</tr>
<tr>
<td>3B3</td>
<td>42</td>
<td>flex. - ten.</td>
<td>7.46</td>
<td>7.09</td>
<td>23.91</td>
</tr>
<tr>
<td>3B4</td>
<td>37</td>
<td>flex. - ten.</td>
<td>6.57</td>
<td>6.64</td>
<td>26.79</td>
</tr>
<tr>
<td>3B5</td>
<td>37</td>
<td>flex. - ten.</td>
<td>7.24</td>
<td>6.64</td>
<td>26.79</td>
</tr>
<tr>
<td>3B6</td>
<td>37</td>
<td>flex. - ten.</td>
<td>6.57</td>
<td>6.64</td>
<td>26.79</td>
</tr>
</tbody>
</table>

1 at P_{fail}, 2 (Equation 4.15)*2, 3 comparison made with bold faced predicted P_u

All samples failed in flexural tension as was expected. This was accomplished by reducing the amount of NEFMAC reinforcement to only 42% and 37% of a balanced design. Predicted flexural strength using the Whitney stress block together with the material properties of the NEFMAC was very close to measured values. Tensile rupture of the H10 NEFMAC bar occurred with no apparent deterioration in force transfer at the transverse bar locations. Failure was sudden and brittle with no yielding in the NEFMAC as ultimate was approached. Cracking was very limited, with a minimum and maximum of 2 and 4 individual cracks, respectively. All cracks started at transverse bars. Deflection prediction calculated according to the Branson effective moment of inertia was very poor, while moment-curvature deflection prediction was very good. This demonstrates that the Branson
equation was vastly overestimating the effective moment of inertia, $I_e$, for these NEFMAC reinforced beams.

Samples 3B7, 3B8 3B9, 3B10, 3B11 and 3B12 - "Compression Failure Group"

The 6 samples in this group were designed to fail in flexural compression. Each sample was reinforced with 4xH10 NEFMAC bars. The cross-sections varied slightly, with $b=8''$, $d=4.90''$ and $h=6''$ for Samples 3B7, 3B8 and 3B9 and $b=7.5''$, $d=4.90''$ and $h=6''$ for Samples 3B10, 3B11 and 3B12. The shear span to depth ratio for these six samples was 6.3, classifying them as "long beams". Failure of long beams usually occurs as the result of flexural deficiencies before shear rupture. [44] Typical crack patterns are shown in Figure 6.11. The load-deflection results are shown in Figures 6.12 and 6.13.

Flexural performance of the samples in this group was is seen to be very consistent. All samples cracked at a load of 1.5 kips and load-deflection of the cracked-section was very similar.

The amount of cracking was extensive, starting within the region of constant moment and propagating outward towards the support points. All cracks were observed to originate at the location of transverse bars. Beams 3B7 through 3B9 developed 14 individual cracks for a total cracked-section length of 52'', about 75% of the beams' length. For all three beams, cracking occurred at every transverse bar within the 52'' length. For Beams 3B10 through 3B12 the cracked length distance was 44'', approximately 63% of the length. There occurred in this group several instances where cracking did not develop at the location of transverse bars within the 44'' cracked length. Cracks within and
near the constant moment region were observed to grow in a vertical direction. This orientation became progressively more inclined as the distance from the beam centerline increased.

Figure 6.11 Crack Pattern and Moment Diagram - Samples 3B8, 3B12
Samples 3B7, 3B8 and 3B9

(4xH10 NEFMAC bars, b=8", d=4.9", a=31", L=70")

Figure 6.12 Load-Deflection Results - Samples 3B7, 3B8, 3B9
Samples 3B10, 3B11 and 3B12

(4xH10 NEFMAC bars, b=7.5", d=4.9", a=31", L=70")

Figure 6.13 Load-Deflection Results - Samples 3B10, 3B11, 3B12
All six beams developed crushing in the concrete. The incidence of compression failure was, in the case of Beams 3B7 through 3B9, followed soon thereafter by a brittle shear failure. Post compression failure of beams 3B10 through 3B12 was characterized by a continual crushing of the compression block. These samples eventually failed in a brittle manner as a result of shear rupture.

Upon close examination of the load-deflection curves, the incidence of flexural compression failure can be identified by a change in slope. Based upon concrete crushing at a strain of .003 in/in, the predicted flexural capacity for Samples 3B7 - 3B9 is approximately 7.8 kips. Figure 6.14 shows that load and deflection for Samples 3B7,8,9 are linear between first crack and 7.8 kips. However, above 7.8 kips the slope of the load-deflection curves become gradually smaller until failure at an average load of 9.07 kips. This non-linear activity is likely the result of concrete crushing.

![Figure 6.14 Compression Failure - Samples 3B7, 3B8, 3B9](image)
This same flexural compression failure behavior was observed in Samples 3B10, 3B11 and 3B12. The predicted flexural capacity for these samples is 7.5 kips. Figure 6.15 shows that load and deflection are linear between first crack and 7.5 kips. However, above 7.5 kips the load-deflection curve slopes become gradually smaller until failure at an average load of 9.30 kips. This non-linear activity is again likely the result of continual crushing of the concrete.

Test Sample 3B10,3B11,3B12

(b=7.5 in d=4.9 in reinf=4xH10 a/d=6.3 Pcr=1.5 k)

![Figure 6.15 Compression Failure - Samples 3B10, 3B11, 3B12](image)

Test Summary

Test results are summarized in Table 6.5. Note, because brittle shear failure terminated all tests, strength comparisons are made based on the sections predicted shear strength.
Table 6.5 Summary Test Results - Samples 3B7 - 3B12

<table>
<thead>
<tr>
<th>Beam</th>
<th>%ρ&lt;sub&gt;b&lt;/sub&gt;</th>
<th>Test Results</th>
<th>Predicted</th>
<th>Comparison&lt;sup&gt;3&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(##)</td>
<td>(类型)</td>
<td>P&lt;sub&gt;fail&lt;/sub&gt;</td>
<td>f&lt;sub&gt;frp&lt;/sub&gt;&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td>3B7</td>
<td>403</td>
<td>shear</td>
<td>9.02</td>
<td>66</td>
</tr>
<tr>
<td>3B8</td>
<td>403</td>
<td>shear</td>
<td>8.90</td>
<td>65</td>
</tr>
<tr>
<td>3B9</td>
<td>403</td>
<td>shear</td>
<td>9.30</td>
<td>68</td>
</tr>
<tr>
<td>3B10</td>
<td>430</td>
<td>shear</td>
<td>9.36</td>
<td>69</td>
</tr>
<tr>
<td>3B11</td>
<td>430</td>
<td>shear</td>
<td>9.52</td>
<td>71</td>
</tr>
<tr>
<td>3B12</td>
<td>430</td>
<td>shear</td>
<td>9.02</td>
<td>66</td>
</tr>
</tbody>
</table>

1 at P<sub>fail</sub>, 2 (Equation 4.15)<sup>2</sup>, 3 comparison made with bold faced predicted P<sub>u</sub>

All samples failed initially in flexural compression followed soon thereafter by brittle shear failure. Significant compression crushing of the concrete was visible at failure. Load and deflection were linear up to about 80% of ultimate after which a gradual decrease in slope occurred. The point where load and deflection become nonlinear corresponds to a concrete strain of roughly .003 in/in and represents the onset of concrete crushing. Brittle failure eventually terminated all tests and occurred as a result of combined compression crushing and shear rupture. Failure loads were very close to ACI-11.3.1 predicted shear strengths. NEFMAC strain at failure ranged between 54% and 59% of ultimate. Individual cracks grew in a vertical direction reflecting well developed flexural cracking.

Deflection prediction up to the onset of concrete crushing was very good for both Branson and moment-curvature techniques. Thus, the Branson equation more accurately estimated the cracked-section effective moment of inertia for these samples than was the result for Samples 3B1 - 3B6.
6.2.4 Test Group 4 Results

Test Group 4 was designed to evaluate the cyclic performance of intermediate length beams overreinforced with H10 and H19 type NEFMAC. The first two beams of this group, Samples 4B1 and 4B2, were reinforced with 1xH19 NEFMAC bar at 160%\(\rho_b\). The second 2 beams, Samples 4B3 and 4B4 were reinforced with 2xH10 NEFMAC bars at 120%\(\rho_b\). All 4 beams were classified as "intermediate" according to their respective a/d ratios. Samples 4B1 and 4B4 were loaded monotonically and Samples 4B2 and 4B3 were loaded cyclically. The results of the monotonically loaded samples are presented in this section. Figures 6.16 through 6.18 show the cracking and load-deflection results for Beams 4B1 and 4B4.

![Diagram of Crack Pattern and Moment Diagram - Samples 4B1, 4B4](image)

**Figure 6.16 Crack Pattern and Moment Diagram - Samples 4B1, 4B4**
Sample 4B1
(1xH19 NEFMAC bar, b=10\" , d=5.8\", a=31\", L=70\")

Figure 6.17 Load-Deflection Results - Sample 4B1
**Sample 4B4**

(2xH10 NEFMAC bars, b=10", d=6.06", a=31", L=70"

![Graph showing load-deflection results for Sample 4B4.](image)

**Figure 6.18 Load-Deflection Results - Sample 4B4**

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As can be seen from Figures 6.17 and 6.18, the initial load-deflection response for both beams was linear up to a load of 2 kips at which point the first crack formed. For both samples, this first crack was located within the constant moment region.

Sample 4B1 failed in a brittle manner as a result of diagonal-tension at a load of 9.9 kips. A total of 7 individual cracks were observed at failure covering a longitudinal distance of 49" or 70% of the beam's length. All cracks originated at the location of transverse bars, however only 1 of the 2 transverse bars located within the region of constant moment developed a crack. Shear failure occurred when the flexural-shear crack located farthest from the beam centerline traveled through the beam depth penetrating at the load point causing complete rupture of the section. There was no compression failure observed at failure.

The failure mode of Sample 4B4 was identical to that of Sample 4B1. At a load of 10.90 kips the sample failed in a brittle manner from diagonal-tension shear. The cracking pattern at failure was characterized by 8 individual cracks covering a longitudinal distance of 40" or 57% of the beam's length between the supports. All cracks were observed to originate at transverse bars, however 3 of the 11 transverse bars located within the cracked section did not develop cracks.

Test Summary

Test results are summarized in Table 6.6.
Table 6.6 Summary Test Results - Samples 4B1, 4B4

<table>
<thead>
<tr>
<th>Beam</th>
<th>%ρ_b (%)</th>
<th>Failure (type)</th>
<th>P_{fail} (k)</th>
<th>f_{frp} (ksi)</th>
<th>P_{u-flex} (k)</th>
<th>P_{u-shear} (k)</th>
<th>P_{fail}^{1} P_{pred.}</th>
<th>f_{frp}^{2} P_{u}</th>
</tr>
</thead>
<tbody>
<tr>
<td>4B1</td>
<td>160</td>
<td>shear 9.90</td>
<td>61</td>
<td>12.3</td>
<td>15.56</td>
<td>.64</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 at P_{fail}, 2 (Equation 4.15)*2 3 comparison made with bold faced predicted P_u

Both beams failed in a brittle manner as a result of diagonal-tension shear. The failure loads were well below those predicted according to ACI-11.3.1. Failure occurred when the flexural-shear crack located farthest from the beam centerline penetrated through the beam. Many well developed diagonal-tension cracks were evident at failure. Although Sample 4B4 reached its theoretical flexural strength, no compression failure was evident at collapse. Also, referring to Figure 6.18, load and deflection for this sample remained linear up to ultimate.

Deflection prediction for both samples was acceptable as calculated according to the Branson equation and moment-curvature techniques. However, moment-curvature deflection prediction was more accurate than the Branson equation at low post-crack loads.

6.2.5 Test Group 5 Results

The two beams tested in Test Group 5 were reinforced with 2xC19 NEFMAC bars. The shear span to depth ratio (a/d) for these samples was 2.38,
classifying them as "short beams", the smallest of all samples tested. The test was designed to evaluate the shear strength of short beams reinforced at 148%ρb with a 2xC19 NEFMAC bars. A shear failure was expected from these two samples. Crack pattern and load-deflection results are given in Figures 6.19 and 6.20, respectively.

![Diagram showing crack pattern and moment diagram for samples 5B2 and 5B3.](image)

**Figure 6.19 Crack Pattern and Moment Diagram - Samples 5B2, 5B3**

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Samples 5B2 and 5B3

(2xH19 NEFMAC bars, b=9", d=13", a=31", L=70")

Figure 6.20 Load-Deflection Results - Samples 5B2, 5B3
The pre-cracked load-deflection response for both beams was linear to a load of 10 kips at which point the first crack formed. For Sample 5B2 the first crack occurred at the first transverse bar outside of the constant moment region while the first crack in Sample 5B3 developed at the transverse bar within the constant moment region. The change in slope of the load-deflection curve that develops as a result of cracking was very small for both samples. The slope of the cracked-section load-deflection curve was only about 1/2 that of the uncracked-section. This behavior is likely the result of support settlement that was occurring at pre-crack loads. During testing the sample was possibly not squarely resting on the support points until after cracking occurred.

Shear failure with no observed crushing in the compression zone occurred in both beams. The crack located farthest from the beam centerline eventually grew into the terminal crack along which shear failure occurred. The shape of this crack resembled that of an arch. The crack initially grew vertically, typical of flexural cracking, to about half way through the beams depth. From here the crack followed an approximately circular arc to the load point. Failure then occurred by complete separation of the beam. Both beams had a total of 5 individual cracks at ultimate. Cracking occurred over a distance of 36” or 52% of the beams length. Of the 7 transverse bars located within the cracked distance, 2 did not develop cracking. These transverse bars were located one inside the constant moment region and the second was the first transverse bar outside the constant moment region. The development of this cracking pattern and subsequent failure mode is very consistent with that expected for short beams without shear reinforcement. [44]
Test Summary

Test results are summarized in Table 6.7.

Table 6.7 Summary of Test Results - Samples 5B2, 5B3

<table>
<thead>
<tr>
<th>Beam</th>
<th>%ρb</th>
<th>Failure</th>
<th>Pfail</th>
<th>f_{frp}</th>
<th>P_{u-flex}</th>
<th>P_{u-shear}</th>
<th>( \frac{P_{fail}}{P_{pred}} )</th>
<th>( \frac{f_{frp}}{F_u} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5B2</td>
<td>148</td>
<td>shear</td>
<td>28.9</td>
<td>73</td>
<td>56.8</td>
<td>31.4</td>
<td>.92</td>
<td>.40</td>
</tr>
<tr>
<td>5B3</td>
<td>148</td>
<td>shear</td>
<td>34.6</td>
<td>88</td>
<td>56.8</td>
<td>31.4</td>
<td>1.10</td>
<td>48</td>
</tr>
</tbody>
</table>

1 at \( P_{fail} \), 2 (Equation 4.15)*2 3 comparison made with bold faced predicted \( P_u \)

Both beams failed in shear at loads very close to that predicted according to ACI-11.3.1. The shear-span to depth ratio for these samples was 2.38, classifying them as short beams. Short beams have a shear strength that exceeds the inclined cracking strength. This statement is consistent with test results where failure occurred well after inclined cracks were first observed. Because shear failure occurred well before the design flexural strength, stress in the NEFMAC was only 40% and 48% of ultimate at failure.

6.2.6 Test Group 6 Results

Samples 6B1, 6B2 and 6B3 were reinforced with 2xH10 NEFMAC bars at 290%\( \rho_b \). Samples 6B4 and 6B5 were reinforced with 3xH10 NEFMAC bars at 436%\( \rho_b \). All 5 samples had an a/d ratio of 7.8 classifying them as "long beams". The test was designed to the measure failure mode of long beams.

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reinforced well above a balanced design. The load-deflection results for these 5 beams are given in Figure 6.21 and 6.22.

**Samples 6B1, 6B2 and 6B3**

(2xH10 NEFMAC bars, b=8", d=3.06", a=24", L=60")

![Graph of Load-Deflection Results](image)

Figure 6.21 Load-Deflection Results - Samples 6B1, 6B2, 6B3
Samples 6B4 and 6B5

(3xH10 NEFMAC bars, b=8", d=3.06", a=24", L=60")

flexural compression/shear failure(s)

Figure 6.22 Load-Deflection Results - Samples 6B4, 6B5
Failure in Samples 6B1 through 6B3 was initially caused by flexural compression failure in the concrete. This did not, however, occur suddenly as can be seen from the load-deflection results. These samples ultimately did fail in a brittle manner, the consequence of significant compression failure in the concrete and shear rupture. This did not, however, occurred until after the beams absorbed a limited amount of energy during crushing of the concrete. As can be seen from Figure 6.21, each sample experienced a small region where the load was maintained (or increased slightly) while experiencing an accelerated growth in deflection. All beams failed initially in flexural compression at about the same load and deflection values of 4 kips and 1.50 inches, respectively. Brittle failure ultimately occurred as a combination of significant concrete crushing and diagonal-tension shear at an average load and deflection of 4.48 kips and 2.00 inches, respectively.

Beams 6B4 and 6B5 failed more suddenly than did samples 6B1,2,3. Their failure mode was characterized by simultaneous flexural compression failure of the concrete and shear rupture which is the result of a diagonal-tension cracking condition. The slope of the load-deflection curves for these beams was about 70% greater than that for Samples 6B1 through 6B3. This is slightly higher than the 50% increase in the amount of reinforcement provided. It is likely that "d" was slightly greater for Samples 6B4,5 thus increasing the section's transformed moment of inertia.

Test Summary

Test results are summarized in Table 6.8. Note, strength comparisons for Samples 6B1 - 6B3 are based on the predicted flexural capacity. This is because
significant compression failure was evident at failure. For Samples 6B4 and 6B5 strength comparison is based on the predicted shear strength. For these samples, failure was more the result of shear rupture.

Table 6.8 Summary Test Results - Samples 6B1 - 6B5

<table>
<thead>
<tr>
<th>Beam</th>
<th>%(\rho_b)</th>
<th>Test Results</th>
<th>Predicted</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Failure (P_{\text{fail}})</td>
<td>(f_{\text{fail}})</td>
<td>(P_{\text{u-flex.}})</td>
</tr>
<tr>
<td>6B1</td>
<td>290</td>
<td>flex.-comp. 4.24</td>
<td>74</td>
<td>3.87</td>
</tr>
<tr>
<td>6B2</td>
<td>290</td>
<td>flex.-comp. 4.42</td>
<td>78</td>
<td>3.87</td>
</tr>
<tr>
<td>6B3</td>
<td>290</td>
<td>flex.-comp. 4.78</td>
<td>85</td>
<td>3.87</td>
</tr>
<tr>
<td>6B4</td>
<td>436</td>
<td>shear 6.12</td>
<td>76</td>
<td>4.53</td>
</tr>
<tr>
<td>6B5</td>
<td>436</td>
<td>shear 5.28</td>
<td>64</td>
<td>4.53</td>
</tr>
</tbody>
</table>

1 at \(P_{\text{fail}}\), 2 (Equation 4.15)\(^*\), 3 comparison made with bold faced predicted \(P_u\)

Initial failure of Samples 6B1, 6B2, and 6B3 was characterized by compression crushing in the concrete. This was followed by a region of limited ductility in the load-deflection behavior where continual crushing of the concrete was occurring while maintaining the applied load. Ultimately these sections failed in a brittle manner from a combination of significant compression failure in the concrete and shear rupture. The predicted flexural strength was very close to lab results. Samples 6B4 and 6B5 failed more suddenly as a result of combined flexural compression and diagonal shear. The predicted shear strength for these samples was very close to the failure load.

6.3 Strain Data Results

Strain data recorded on the longitudinal NEFMAC bar during testing is graphically presented in the form of reinforcement strain vs. applied load.
plots. Experimental results are shown together with the theoretical predictions calculated using the techniques of Section 4.2 and assuming an initially cracked section.

6.3.1 Samples 3B1, 3B3 and 3B4

These three beams failed in flexural tension, the result of tensile rupture of a single NEFMAC H10 bar. For all 3 samples strain was recorded at centerspan. Sample 3B4 was instrumented with two strain gages, one located at centerspan and a second located one transverse bar away from centerspan (4"). All gages were bonded to the bottom surface of the NEFMAC bar. In Figures 6.23 through 6.25 lab measured strain results are compared with predicted values calculated using the procedures of Section 4.2 and assuming an initially cracked section.

Figure 6.23 Centerspan Strain vs. Load - Sample 3B1

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Figure 6.24 Centerspan Strain vs. Load - Sample 3B3

Figure 6.25 Strain vs. Load - Sample 3B4
The incidence of cracking can be clearly identified in all of the above 3 figures. As can be seen, simultaneous with the opening of a new crack there occurred a sudden and proportional decrease in applied load and strain. Because the magnitude of reinforcement strain is proportional to the magnitude of the applied load this activity should be expected. This behavior is perfectly consistent with the load-deflection behavior for these samples shown back in Figures 6.9 and 6.10. As was shown in these figures, simultaneous with the opening of a new crack there was also a sudden decrease in applied load.

Strain at failure was measured to be 2% by all gages. The cracked-section results show strain remains fairly linear up to failure with no apparent yielding in the NEFMAC. From Figure 6.25, the amount of force transferred at the transverse bar can be calculated from the horizontal strain offset (Δε in Figure 6.25) between the centerspan gage and the gage located one transverse bar away. For loads between the first and second crack this is equal to approximately .5%, and the corresponding force transferred at the transverse bar, ΔT, is calculated as follows:

\[
ΔT = Δε \times E_{frp} \times A_{frp} = (.5%/100\%) \times (6000 \text{ ksi}) \times (.124 \text{ in}^2) = 3.72 \text{ k}
\]

Unfortunately, the strain gage on Sample 3B1 was damaged as a result of cracking at a strain of about 1.6%. This value is, however, close to the ultimate strain of the material (2%) and the data recorded provides information to load levels near ultimate.

The predicted reinforcement strain level at the beam centerspan was very close to that recorded during testing. As can be seen from Figures 6.23
through 6.25, the post-crack predicted and measured centerspan strains are very close to each other. This compatibility suggests a flexural analysis employing a bilinear concrete model and assuming a linear strain distribution through the beam cross-section represents well the internal flexural stresses and forces acting in the concrete and reinforcement.

6.3.2 Samples 4B1

Sample 4B1 was instrumented with 2 strain gages located one on the top and one on the bottom surface of the longitudinal bar at centerspan. The purpose of this arrangement was for measuring any strain gradient that might exist through the thickness of the NEFMAC H19 bar. The bar thickness of .80 inches was thought to be relatively large compared with the beam depth of 5.8 inches. As such, the assumption of plane sections remaining plane before and after bending would suggest a measurable strain gradient existing over the bar depth.

A measurement of about .005% strain was made from the centerline gage when cracking first occurred. The entire centerspan-strain vs. load history is shown in Figure 6.26, where it is noted that strain as measured both on the top surface and bottom surface of the NEFMAC bar at the beam centerspan is given. Brittle failure occurred as a result of shear rupture. The predicted strain was calculated using the analytical techniques of Section 4.2 assuming an initially cracked section and represents the strain at the centroid of the FRP bar cross-section.
The strain on the bottom surface of the NEFMAC longitudinal bar is seen to compare favorably well with that predicted using analytical techniques. The compatibility of the calculated strain is within about 10% of that measured in the lab. The predicted strain could be lower than the measured strain because of an error in the assumed beam dimensions. It is likely that the value of "d" used for analytical calculations was slightly in error of the actual measurement. A strain value of 0.9% was measured from the bottom gage at failure. This corresponds to about 56% of the material's ultimate strain, $\varepsilon_u$, of 1.6% (from Table 3.3). Thus, 44% of the tensile strength of the bar was left in reserve at failure.

The anticipated strain gradient is quite evident from the test results shown in Figure 6.26. As can be seen from the figure, the strain level measured on the
top surface of the bar was consistently lower than that recorded on the bottom surface. Assuming the strain gradient through the bar thickness is linear, these results can be used to check the assumption that plane sections are remaining plane after bending. Referring to Figure 6.27, the distance from the bottom surface of the NEFMAC bar to the neutral axis, designated as "x", can be calculated using similar strain triangles as:

\[
x = d_b \left( \frac{\varepsilon_b}{\varepsilon_b - \varepsilon_t} \right) \text{ calculated from experimental strain data} \quad (6.1)
\]

where, at a given load:

- \( \varepsilon_b \) = strain recorded on the bottom surface of the NEFMAC bar
- \( \varepsilon_t \) = strain recorded on the top surface of the NEFMAC bar
- \( d_b \) = thickness of bar = .80 inches

From Figure 6.27, the distance "x" is also equal to \( d - c + \frac{d_b}{2} \), where the value for 'c' for a cracked-section is calculated according to Section 4.2.

Figure 6.27 Strain Distribution Assuming Plane Sections
The depth "x" calculated substituting $\varepsilon_b$ and $\varepsilon_t$ from the test results into Equation 6.1 is shown together with the predicted depth in Figure 6.28 as a function of applied load.

![Graph showing distance to neutral axis as a function of load](image)

**Figure 6.28 Distance to Neutral Axis Assuming Plane Sections**

The results show the assumption of plane sections remaining plane before and after bending is a valid approximation for analytical calculations. This conclusion is founded upon the results shown in Figure 6.28 where it is recognized that the predicted value for "x" compares well with the value of "x" as calculated from test strain data assuming similar strain triangles. The degree of compatibility between these two results is surprisingly high considering the sensitivity of the analysis to variability in cross-sectional geometry, reinforcement bar depth ($d_p$) and concrete dimensions.
6.4 Summary of Test Results

Test results presented in the preceding sections are summarized in the following section according to flexural strength, shear strength and deflection criteria. Accuracy of analytical techniques in predicting flexural and shear strengths and deflection behavior is considered based upon comparison with laboratory results.

6.4.1 Flexural Strength

Flexural failure occurs as either crushing of the concrete or tensile failure in the reinforcement. Analytically, compression failure occurs when the concrete strain reaches .003 in/in and tensile failure occurs when the reinforcement strain reaches its yield limit, for steel, or ultimate, for FRP. For those samples which experienced flexural failure, Table 6.9 compares their ultimate load as measured during testing to that predicted using the Whitney rectangular stress block. Note that because brittle failure of Samples 3B7 through 3B12 and Samples 6B1 through 6B5 occurred as a combination of compression failure in the concrete (flexure) and shear rupture, their failure loads are compared with both the predicted flexural strength and predicted shear strength.
Table 6.9 Flexural Strength Summary

<table>
<thead>
<tr>
<th>Beam (ID#)</th>
<th>%( \rho_b )</th>
<th>Failure (type)</th>
<th>Reinf. (type)</th>
<th>( P_{u-lab} ) (k)</th>
<th>( P_{u-pred.} ) (k)</th>
<th>Lab/Pred.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1B3*</td>
<td>33</td>
<td>ten. steel</td>
<td>10.16**</td>
<td>11.33</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>3B1</td>
<td>42</td>
<td>ten. H10</td>
<td>7.78</td>
<td>7.09</td>
<td>1.10</td>
<td></td>
</tr>
<tr>
<td>3B2</td>
<td>42</td>
<td>ten. H10</td>
<td>7.57</td>
<td>7.09</td>
<td>1.07</td>
<td></td>
</tr>
<tr>
<td>3B3</td>
<td>42</td>
<td>ten. H10</td>
<td>7.46</td>
<td>7.09</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>3B4</td>
<td>37</td>
<td>ten. H10</td>
<td>6.57</td>
<td>6.64</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>3B5</td>
<td>37</td>
<td>ten. H10</td>
<td>7.24</td>
<td>6.64</td>
<td>1.09</td>
<td></td>
</tr>
<tr>
<td>3B6</td>
<td>37</td>
<td>ten. H10</td>
<td>6.57</td>
<td>6.64</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>3B7</td>
<td>403</td>
<td>comp. H10</td>
<td>9.02</td>
<td>7.83</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td>3B8</td>
<td>403</td>
<td>comp. H10</td>
<td>8.90</td>
<td>7.83</td>
<td>1.14</td>
<td></td>
</tr>
<tr>
<td>3B9</td>
<td>403</td>
<td>comp. H10</td>
<td>9.30</td>
<td>7.83</td>
<td>1.19</td>
<td></td>
</tr>
<tr>
<td>3B10</td>
<td>430</td>
<td>comp. H10</td>
<td>9.36</td>
<td>7.53</td>
<td>1.24</td>
<td></td>
</tr>
<tr>
<td>3B11</td>
<td>430</td>
<td>comp. H10</td>
<td>9.52</td>
<td>7.53</td>
<td>1.26</td>
<td></td>
</tr>
<tr>
<td>3B12</td>
<td>430</td>
<td>comp. H10</td>
<td>9.02</td>
<td>7.53</td>
<td>1.20</td>
<td></td>
</tr>
<tr>
<td>6B1</td>
<td>290</td>
<td>comp. H10</td>
<td>4.24</td>
<td>3.87</td>
<td>1.10</td>
<td></td>
</tr>
<tr>
<td>6B2</td>
<td>290</td>
<td>comp. H10</td>
<td>4.42</td>
<td>3.87</td>
<td>1.14</td>
<td></td>
</tr>
<tr>
<td>6B3</td>
<td>290</td>
<td>comp. H10</td>
<td>4.78</td>
<td>3.87</td>
<td>1.24</td>
<td></td>
</tr>
<tr>
<td>6B4</td>
<td>436</td>
<td>comp. H10</td>
<td>6.12</td>
<td>4.53</td>
<td>1.35</td>
<td></td>
</tr>
<tr>
<td>6B5</td>
<td>436</td>
<td>comp. H10</td>
<td>5.28</td>
<td>4.53</td>
<td>1.16</td>
<td></td>
</tr>
</tbody>
</table>

* steel reinforced ** steel yield

The results in Table 6.9 demonstrate that the flexural strength of NEFMAC beams can be quite accurately predicted using the analytical procedures developed for steel reinforced concrete beams but with the material properties of FRP. It would, thus, seem appropriate to calculate the theoretical flexural strength of NEFMAC reinforced concrete beams using the same analytical procedures as are employed in predicting the flexural strength of steel reinforced beams.

In most cases, the predicted theoretical strength was less than that measured, implying design predictions tend to be more conservative in estimating flexural strength. This is especially true for the compression failures and possibly reflects concrete failure at strain levels in excess of the code assumed \( \varepsilon_{cu} = 0.03 \text{ in/in} \).
6.4.2 Shear Strength

Shear failure tends to be a sudden and brittle condition where the beam collapses, becoming unable to support any appreciable load. It is therefore of utmost importance that this type of failure be avoided at all costs. For all beams classified as having failed in shear, brittle collapse resulted from the propagation of a diagonal shear-flexure crack completely through the beam cross-section. This condition is commonly called "diagonal-tension failure". In Samples 3B7 through 3B12 and 6B1 through 6B5 compression crushing in the concrete between the load points was also observed when shear failure occurred.

Unlike flexural strength, shear strength prediction tends to be more empirical in nature, employing design constants in calculating strength values. Using Eq. 4.15 shear strength of test samples is calculated according ACI-11.3.1 as:

\[
\frac{P_v}{2} = V_c = K \sqrt{f_c'} b d
\]

or

\[
P_v = K \sqrt{f_c'} b d (2)
\]

where:

- \( P_v \) = sample shear strength
- \( K = 2 \) according to ACI-11.3.1
- \( b, d \) = beam cross sectional dimensions
- \( f_c' \) = concrete compressive strength (psi)

Presented in Table 6.10 are the measured shear strengths together with those predicted according to ACI-11.3.1 (Eq. 6.2b).
Table 6.10 Shear Strength Summary

<table>
<thead>
<tr>
<th>Beam (ID#)</th>
<th>( \rho )</th>
<th>a/d</th>
<th>( P_{V,\text{lab}} )</th>
<th>( P_{V,\text{pred.}} )</th>
<th>( \frac{P_{V,\text{lab}}}{P_{V,\text{pred.}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1B2</td>
<td>.566</td>
<td>5.8</td>
<td>10.90</td>
<td>15.46</td>
<td>0.71</td>
</tr>
<tr>
<td>2B1</td>
<td>.273</td>
<td>4.1</td>
<td>11.50</td>
<td>26.91</td>
<td>0.43</td>
</tr>
<tr>
<td>2B2</td>
<td>.273</td>
<td>4.1</td>
<td>12.00</td>
<td>26.91</td>
<td>0.45</td>
</tr>
<tr>
<td>2B3</td>
<td>.273</td>
<td>4.1</td>
<td>12.00</td>
<td>26.91</td>
<td>0.45</td>
</tr>
<tr>
<td>2B4</td>
<td>.252</td>
<td>2.7</td>
<td>14.97</td>
<td>29.15</td>
<td>0.51</td>
</tr>
<tr>
<td>2B5</td>
<td>.252</td>
<td>2.7</td>
<td>18.30</td>
<td>29.15</td>
<td>0.63</td>
</tr>
<tr>
<td>2B6</td>
<td>.252</td>
<td>2.7</td>
<td>13.20</td>
<td>29.15</td>
<td>0.45</td>
</tr>
<tr>
<td>3B7</td>
<td>1.265</td>
<td>6.3</td>
<td>9.02</td>
<td>9.92</td>
<td>0.91</td>
</tr>
<tr>
<td>3B8</td>
<td>1.265</td>
<td>6.3</td>
<td>8.90</td>
<td>9.92</td>
<td>0.90</td>
</tr>
<tr>
<td>3B9</td>
<td>1.265</td>
<td>6.3</td>
<td>9.30</td>
<td>9.92</td>
<td>0.94</td>
</tr>
<tr>
<td>3B10</td>
<td>1.350</td>
<td>6.3</td>
<td>9.36</td>
<td>9.30</td>
<td>1.01</td>
</tr>
<tr>
<td>3B11</td>
<td>1.350</td>
<td>6.3</td>
<td>9.52</td>
<td>9.30</td>
<td>1.02</td>
</tr>
<tr>
<td>3B12</td>
<td>1.350</td>
<td>6.3</td>
<td>9.02</td>
<td>9.30</td>
<td>0.97</td>
</tr>
<tr>
<td>4B1</td>
<td>.802</td>
<td>5.3</td>
<td>9.90</td>
<td>15.56</td>
<td>0.64</td>
</tr>
<tr>
<td>4B4</td>
<td>.409</td>
<td>5.1</td>
<td>10.90</td>
<td>16.26</td>
<td>0.67</td>
</tr>
<tr>
<td>5B2</td>
<td>.424</td>
<td>2.4</td>
<td>28.9</td>
<td>31.4</td>
<td>0.92</td>
</tr>
<tr>
<td>5B3</td>
<td>.424</td>
<td>2.4</td>
<td>34.6</td>
<td>31.4</td>
<td>1.10</td>
</tr>
<tr>
<td>6B1</td>
<td>1.013</td>
<td>7.8</td>
<td>4.24</td>
<td>6.64</td>
<td>0.64</td>
</tr>
<tr>
<td>6B2</td>
<td>1.013</td>
<td>7.8</td>
<td>4.42</td>
<td>6.64</td>
<td>0.67</td>
</tr>
<tr>
<td>6B3</td>
<td>1.013</td>
<td>7.8</td>
<td>4.78</td>
<td>6.64</td>
<td>0.72</td>
</tr>
<tr>
<td>6B4</td>
<td>1.520</td>
<td>7.8</td>
<td>6.12</td>
<td>6.64</td>
<td>0.92</td>
</tr>
<tr>
<td>6B5</td>
<td>1.520</td>
<td>7.8</td>
<td>5.28</td>
<td>6.64</td>
<td>0.79</td>
</tr>
</tbody>
</table>

The results of Table 6.10 show that shear strength is significantly overestimated by ACI-11.3.1, especially where low amounts of reinforcement are provided. This result is assumed to reflect a deterioration in shear force transfer through aggregate interlock that occurs with large deflections and wide crack widths. This same result is recognized for steel reinforced beams, for which the following reduced form of \( K \) in Eq. 6.2b is recommended when \( \rho < 1.2\% \): [43]

\[
K = 0.8 + 100 \frac{A_s}{bd} \quad \text{Steel reduced form} \quad (6.3)
\]

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The empirical relationship given in Eq. 6.3 assumes steel reinforcing is provided. Where FRP reinforcing is used, the value of FRP area, $A_{frp}$, required in Eq. 6.3 should be converted into an equivalent area of steel. This conversion is accomplished by equating the two material stiffness values and solving for $A_s$ as follows:

$$A_{frp} E_{frp} = A_s E_s \quad \text{or} \quad A_s = A_{frp} \frac{E_{frp}}{E_s}$$

Substituting Eq. 6.4 into Eq. 6.3, the reduced shear strength constant calculated for FRP reinforced beams, $K_{frp}$, is given as:

$$K_{frp} = \{.8 + 100 \frac{A_{frp}}{bd} \frac{E_{frp}}{E_s} \} = \{.8 + 100 \rho_{norm.}\}$$

where:

$$\rho_{norm.} = \text{normalized reinforcement ratio} = \frac{A_{frp} E_{frp}}{bd E_s}$$

The format of Eq. 6.5 normalizes the material properties of FRP relative to those of steel.

Using the test results given in Table 6.10 a measured value of $K$ can be calculated from Eq. 6.2b as:

$$K_{meas.} = \frac{P_v}{\sqrt{f_c} \ bd \ (2)}$$

Plotted in Figure 6.29 are the predicted reduced shear strength constant, $K_{frp}$, calculated according to Eq. 6.5, and measured shear strength constant, $K_{meas.}$, calculated according to Eq. 6.6 vs. a normalized reinforcement ratio, $\rho_{norm.}$.
The results presented in Figure 6.29 suggest that there is a direct correlation between the nominal shear strength constant, $K$, and a normalized $\rho$. As can be seen, the nominal value of $K$ as measured from test results decreases slightly with decreasing amounts of reinforcing. This relationship is, however, underestimated by Eq. 6.5. The measured value of $K$ is seen to become increasingly underestimated by Eq. 6.5 with increasing $\rho_{\text{norm}}$.

The form of Eq. 6.5 seems appropriate, but in need of slight modification. The following is proposed:

$$K_{frp} = .8 + 200 \frac{A_{frp}}{bd} \frac{E_{frp}}{E_s} = .8 + 200 \rho_{\text{norm}} \quad (6.7)$$

Figure 6.30 shows the results of Eq. 6.7.
It seems that the empirical form of Eq. 6.7 is more effective at approximating the shear strength constant for NEFMAC reinforced beams than is Eq. 6.5. The approximation is conservative in that it bounds the data from below and captures more closely the increases in shear strength that comes with increasing $p$. The form of Eq. 6.7 is observed to be different from that of Eq. 6.5 only in that $\rho_{\text{norm.}}$ is multiplied by 200 rather than 100. Although using 205 or 208 in Eq. 6.7 could perhaps yield a slightly better match, the relatively simple empirical format of the analysis and limited number of data points do not warrant such a refinement.

The shear strength constant calculated according to Eq. 6.7 must not exceed the limit imposed by ACI-11.3.1 of $K=2$ (Eq. 6.2b). Eq. 6.7 reaches this limit at a normalized reinforcement ratio of .6%. However, the data plotted in Figure
6.30 supports Eq. 6.7 only within the range of \(0.05\% < \rho_{norm} < 0.32\%\). Thus, the relevancy of Eq. 6.7 is in question when \(0.32\% < \rho_{norm} < 0.6\%\). Until further testing is done to validate the suitability of Eq. 6.7 within this range, a limit should be placed on Eq. 6.7 according to the results shown in Figure 6.30. Thus, referring to the test data plotted in Figure 6.30, \(K_{frp}\) should be less than or equal to 1.4, from which Eq. 6.7 is more accurately written as:

\[
K_{frp} = 0.8 + 200 \frac{A_{frp} E_{frp}}{bd E_s} = \{0.8 + 200 \rho_{norm,}\} \leq 1.4
\] (6.7)

6.4.3 Deflection

Comparisons between laboratory and theoretical deflections are given in Table 6.11 at 35% and 50% of ultimate. Theoretical calculations are made using the Branson Equation and a moment-curvature analysis using a bilinear concrete model.

<table>
<thead>
<tr>
<th>Beam</th>
<th>%\rho_b</th>
<th>at 35% (M_u)</th>
<th>at 35% (M_u)</th>
<th>at 50% (M_u)</th>
<th>at 50% (M_u)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lab Branson</td>
<td>Lab Curvature</td>
<td>Lab Branson</td>
<td>Lab Curvature</td>
</tr>
<tr>
<td>1B2</td>
<td>186</td>
<td>2.00</td>
<td>0.91</td>
<td>1.72</td>
<td>1.15</td>
</tr>
<tr>
<td>1B3</td>
<td>33</td>
<td>1.02</td>
<td>1.05</td>
<td>1.02</td>
<td>1.08</td>
</tr>
<tr>
<td>2B1 - 2B3*</td>
<td>69</td>
<td>1.89</td>
<td>0.83</td>
<td>1.75</td>
<td>0.93</td>
</tr>
<tr>
<td>3B7 - 3B9*</td>
<td>403</td>
<td>1.11</td>
<td>0.91</td>
<td>1.02</td>
<td>0.98</td>
</tr>
<tr>
<td>3B10 - 3B12*</td>
<td>430</td>
<td>1.22</td>
<td>0.91</td>
<td>1.14</td>
<td>1.04</td>
</tr>
<tr>
<td>4B1</td>
<td>160</td>
<td>1.41</td>
<td>0.83</td>
<td>1.12</td>
<td>0.95</td>
</tr>
<tr>
<td>4B4</td>
<td>120</td>
<td>1.35</td>
<td>0.94</td>
<td>1.25</td>
<td>0.86</td>
</tr>
<tr>
<td>6B1 - 6B3*</td>
<td>290</td>
<td>0.88</td>
<td>0.89</td>
<td>0.93</td>
<td>0.91</td>
</tr>
<tr>
<td>6B4 - 6B5*</td>
<td>436</td>
<td>0.80</td>
<td>0.81</td>
<td>0.81</td>
<td>0.83</td>
</tr>
</tbody>
</table>

* for these individual Sample Groups design parameters are constant and the laboratory measured deflection values represents an average for the Group.

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The deflection predictions for steel reinforced Sample 1B3 are seen to be very good at 35% and 50% of ultimate for both methods. For NEFMAC reinforced beams from Test Groups 3, deflection prediction at 35% and 50% is quite good using both techniques. These samples were reinforced at 403% and 430% of a balanced design and failed in flexural compression. Also, deflection predictions for Test Group 6 samples are acceptable. These samples are also reinforced well above a balanced design and failed in flexural compression.

The Branson equation is seen to significantly underestimate deflections for Samples 1B2 and 2B1 - 2B3. All of these beams failed in a brittle manner from diagonal-tension shear. Samples 2B1 - 2B3 are reinforced well below a balanced design and Sample 1B2 about 90% above. The moment-curvature deflection predictions are seen to be good for all beams. The comparison suggests the Branson Equation suffers when reinforcement below about 200% of a balanced design is provided. This is directly related to the reinforcing ratio \( \rho \) and suggests that the Branson equation overestimates the cracked section moment of inertia where deflections are large and the depth of the compression block is small. Considering that \( I_e \) is a function of the square of the depth to the neutral axis (crack depth) and the cube of the compression block thickness, a small error in calculation of these dimensions will significantly effect stiffness and deflection prediction. Because \( I_e \) is being overestimated, so is the depth of the compression block.

Deflection predictions employing a moment-curvature analysis are good for all samples at 35% and 50% of ultimate. This technique does not suffer the limitations of the Branson equation when low amounts of reinforcement are provided. This result suggests that substitution of \( M/EcI_e \) with \( \epsilon_c/c \) into
the elastic curve equation provides a more accurate representation of internal mechanics for lightly-reinforced sections after cracking has occurred. Both $E_c$ and $I_e$ are calculated using empirical relationships, whereas $e_c$ and $c$ are derived from elastic theory together with a bilinear concrete model.

6.5 Conclusions and Findings

The results from monotonic testing have shown that relative to flexural strength predictions, NEFMAC reinforced beams behave as expected. However, the shear strength of NEFMAC reinforced beams is significantly lower than that predicted according to code equations. The large deflections, and hence wide crack widths, characteristic of FRP reinforced beams are hypothesized to dilute or eliminate aggregate interlock as a mechanism for shear force transfer, thus reducing the section's shear capacity.

The grid shape of NEFMAC forces cracking to occur at discrete points coincident with the location of transverse bars. This reflects the transfer of force that occurs as a result of a bearing force that develops between the transverse bar and concrete. Force transfer in the grid was effective and allowed development of an H10 bar's full tensile strength.
CHAPTER VII

CYCLIC TEST RESULTS

7.1 Introduction

Three beams were cyclically tested: Sample 1B1, reinforced at 186%\(\rho_b\) with 1xC22 NEFMAC bar, Sample 4B2, reinforced at 160%\(\rho_b\) with 1xH19 NEFMAC bar and Sample 4B3, reinforced at 120%\(\rho_b\) with 2xH10 NEFMAC bars. The longitudinal NEFMAC bar of Sample 1B1 was instrumented with 15 strain gages. For each beam cyclically loaded, a control sample of identical NEFMAC reinforcement and geometric design was tested monotonically. Results for each sample are presented separately in the following sections.

7.2 Sample 1B1

The reinforcement and geometry of cyclically loaded Sample 1B1 were identical to monotonically loaded Sample 1B2. Both samples were reinforced at 186%\(\rho_b\) with 1xC22 NEFMAC bar and the shear-span to depth ratio was 5.8. As was the result for Sample 1B2, Sample 1B1 experienced a shear failure.

7.2.1 Instrumentation and Load Schedule

Sample 1B1 was instrumented with two strain gages located on each longitudinal section of the C22 NEFMAC reinforcement bar. Strain gages
were installed on a milled key-way and oriented in a vertical plane. Location and identification of each gage is shown in Figure 7.1. The instrumentation was designed to measure force transfer on longitudinal bar sections and transverse bars sections of the grid.

![Diagram of instrumentation](image)

**Figure 7.1 Instrumentation of Sample 1B1**

Sample 1B1 was subjected to 200 load cycles as shown in Table 7.1. The load cycling is composed of 20 "Groups", where each Group represents 10 equal load cycles. Table 7.1 shows that the cyclic maximum and minimum loads for all odd numbered Groups was held constant. The maximum load increased approximately linearly for the even numbered Groups. Using this arrangement, the even and odd numbered Groups are referred to as "overload" and "reference-load" Groups, respectively.
Table 7.1 Load Cycling Schedule - Sample 1B1

<table>
<thead>
<tr>
<th>Group (#)</th>
<th>Type (-)</th>
<th># Cycles (#)</th>
<th>$P_{\text{max}}$ (k)</th>
<th>$P_{\text{min}}$ (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>reference</td>
<td>10</td>
<td>3.004*</td>
<td>1.334</td>
</tr>
<tr>
<td>2</td>
<td>overload</td>
<td>10</td>
<td>3.784</td>
<td>1.564</td>
</tr>
<tr>
<td>3</td>
<td>reference</td>
<td>10</td>
<td>3.004</td>
<td>1.334</td>
</tr>
<tr>
<td>4</td>
<td>overload</td>
<td>10</td>
<td>4.564</td>
<td>1.894</td>
</tr>
<tr>
<td>5</td>
<td>reference</td>
<td>10</td>
<td>3.004</td>
<td>1.334</td>
</tr>
<tr>
<td>6</td>
<td>overload</td>
<td>10</td>
<td>5.234</td>
<td>2.224</td>
</tr>
<tr>
<td>7</td>
<td>reference</td>
<td>10</td>
<td>3.004</td>
<td>1.334</td>
</tr>
<tr>
<td>8</td>
<td>overload</td>
<td>10</td>
<td>6.014</td>
<td>2.564</td>
</tr>
<tr>
<td>9</td>
<td>reference</td>
<td>10</td>
<td>3.004</td>
<td>1.334</td>
</tr>
<tr>
<td>10</td>
<td>overload</td>
<td>10</td>
<td>6.464</td>
<td>3.004</td>
</tr>
<tr>
<td>11</td>
<td>reference</td>
<td>10</td>
<td>3.004</td>
<td>1.334</td>
</tr>
<tr>
<td>12</td>
<td>overload</td>
<td>10</td>
<td>7.354</td>
<td>3.454</td>
</tr>
<tr>
<td>13</td>
<td>reference</td>
<td>10</td>
<td>3.004</td>
<td>1.334</td>
</tr>
<tr>
<td>14</td>
<td>overload</td>
<td>10</td>
<td>7.794</td>
<td>3.784</td>
</tr>
<tr>
<td>15</td>
<td>reference</td>
<td>10</td>
<td>3.004</td>
<td>1.334</td>
</tr>
<tr>
<td>16</td>
<td>overload</td>
<td>10</td>
<td>8.684</td>
<td>4.234</td>
</tr>
<tr>
<td>17</td>
<td>reference</td>
<td>10</td>
<td>3.004</td>
<td>1.334</td>
</tr>
<tr>
<td>18</td>
<td>overload</td>
<td>10</td>
<td>9.354</td>
<td>4.564</td>
</tr>
<tr>
<td>19</td>
<td>reference</td>
<td>10</td>
<td>3.004</td>
<td>1.334</td>
</tr>
<tr>
<td>20</td>
<td>overload</td>
<td>10</td>
<td>10.35</td>
<td>4.792</td>
</tr>
</tbody>
</table>

* constant for all reference-load Groups

7.2.2 Load-Deflection and Failure Mode

Cyclically loaded Sample 1B1 and monotonically loaded Sample 1B2 are of identical design. Load-deflection and crack results for both beams are shown in Figures 7.2 and 7.3, respectively.

Sample 1B1 failed at a load of 10.35 kips, about 5% lower than the 10.90 kips recorded for the monotonically loaded Sample 1B2. However, at failure Sample 1B1 had a deflection of .97 in, about 8% less than the deflection of Sample 1B2 at a load of 10.35 kips. The slope of the load-deflection curve for the cracked-section of Sample 1B2 appears to be slightly less than that measured by connecting the load peaks for Sample 1B1. This indicates firstly
that there was no deterioration or softening in the flexural stiffness of Sample 1B1 as a result of the limited load cycling. Also, this behavior could possibly reflect a slight stiffening phenomena that has been observed to occur in carbon fibers subjected to cyclic loading. [62] Much further cyclic testing is required to substantiate this phenomena in FRP reinforced concrete beams.

Figure 7.2 Load-Deflection Results - Samples 1B1, 1B2
The failure mode for both samples was identical, the result of diagonal-tension shear with no compression failure observed in the concrete. Referring to Figure 7.3, the pattern of cracking is nearly the same for both beams. Sample 1B1 developed 7 individual cracks over a length of 36 inches. Sample 1B2 had 9 individual cracks covering a slightly longer length of 44 in. In both beams, it was the flexural-shear crack located farthest from the beam center line that eventually caused brittle failure. At failure, this crack was identical in shape for both samples.

At failure, the strain in gage #1 located at centerspan was recorded to be .786%. Using a modulus value of 12300 ksi for a C22 NEFMAC bar, the corresponding stress level is (.786%/100%)*12300 ksi = 97 ksi or 55% of the
bar's ultimate strength of 178 ksi. This value compares very well with a theoretical reinforcement stress at failure of 98 ksi.

In conclusion, the cracking pattern, load-deflection response, ultimate load and failure mode for these two beams were very similar. This behavioral compatibility suggests that the limited cyclic loading did not have any measurable effect on the load-deflection or shear performance of Sample 1B1.

7.2.3 Damage Analysis

Damage is being defined as permanent growth in deflection that occurs between cycles of equal maximum load and can occur in either of two ways: (1) each time the system is cycled from a reference load to a new maximum load and then returns to the reference load, this is referred to as "overload" damage and (2) during load cycling at a constant load, this is referred to as "constant-load" damage. These two damage conditions are hypothetically shown in Figure 7.4.

Figure 7.4 Damage in a Cyclically Loaded Beam
Referring to Table 7.1, the first cycle of all even Groups represents a new maximum. Thus, overload damage can be measured as the growth in deflection between the 10th cycle of a previous odd Group (or reference load) and the 1st cycle the current odd Group (return to reference load). Constant-load damage is measured as the growth in deflection over the 10 cycles of an odd Group (or reference load). Figure 7.5 shows the results of this analysis for Sample 1B1.

![Figure 7.5 Damage Results - Sample 1B1](image)

With the exception of Group 1, Figure 7.5 shows that, within the precision of the test equipment, no constant-load damage was detected. Maximum deflections are unchanged between the 1st and 10th cycles for odd Groups indicating that no permanent growth in deflection occurred. There is, however, overload damage. Referring to Figure 7.5, there is a permanent change in deflection each time cycling returns to a reference load Group from an overload Group. For example, the deflection of the 1st cycle of Group 9 is offset from the deflection of the 10th cycle of Group 7. Between these two
Groups the beam was cycled to an overload of 6.014 kips. Thus, overload damage occurred at 6.014 k and is observed as the deflection offset indicated in the figure. With the exception of Group 1, zero constant-load damage was observed for odd Groups up to failure and remained unchanged regardless of the overload level applied to the sample.

7.2.4 Longitudinal Strain Distribution and Force Transfer

Strain distribution along the longitudinal NEFMAC bar was recorded at 15 locations using the instrumentation shown in Figure 7.1. Strain readings were used to calculate force transfer as is hypothetically shown in Figure 7.6.

Referring to Figure 7.6, the force transfer model assumes the total force transferred along a given longitudinal bar section, $F_{LS}$, is the result of friction and calculated according to Eq. 7.1. The strain gradient on the longitudinal bar section is assumed linear (between points a and b) and represented with a solid line. The total force transferred along a given transverse bar section, $F_{TS}$, is calculated according to Eq. 7.2 and assumed the combined result of friction, wedge action and bearing at the transverse bar. The shape of the strain gradient on these sections is unknown due to the complicated force transfer mechanics. Thus, the dotted line used on the transverse bar section (between points b and c) of Figure 7.6 does not indicate the strain gradient.
recorded strain readings assumed linear shape unknown

Figure 7.6 Force Transfer Model

where:

\[
F_{LS} = \text{total force transferred along 2" longitudinal bar section}
\]

\[
F_{LS} = F_a - F_b = (\epsilon_a - \epsilon_b)A_{frp}E_{frp}
\]  \hspace{1cm} (7.1)

\[
F_{TS} = \text{total force transferred along 2" transverse bar section}
\]

\[
F_{TS} = F_b - F_c = (\epsilon_b - \epsilon_c)A_{frp}E_{frp}
\]  \hspace{1cm} (7.2)

\[
\epsilon_a, \epsilon_b, \epsilon_c = \text{measured strains at locations a, b, and c, respectively}
\]

\[
F_a, F_b, F_c = \text{axial tensile forces in NEFMAC at a, b, and c, respectively}
\]
Nomenclature for identification of the individual transverse bar sections and longitudinal bar sections, relative to load geometry and strain gage numbering for a 22" section of Sample 1B1 is shown in Figure 7.7. Note that the figure identifies the location of the first crack as being at centerspan.

![Figure 7.7 Transverse Bar and Longitudinal Bar Section Identification](image)

Calculation of force transfer on LS1 and TS1 requires knowing the strain at point "p" located 1 inch to the right of sg1 in Figure 7.7. For this purpose, the strain gradient on LS1 was assumed equal to that on LS2 (between sg2 and sg3). The gradient on LS2 was selected because of its proximity to LS1 and, also, 1/2 of it's length is within the constant moment span. From this assumption, the strain at point "p" in Figure 7.7 is calculated as:

\[ \varepsilon_p = \varepsilon_{sg1} - \left( \frac{\varepsilon_{sg2} - \varepsilon_{sg3}}{2"} \right) \times 1" \]  

(7.3)

where:

- \( \varepsilon_p \) = calculated strain at point p
- \( \varepsilon_{sg1}, \varepsilon_{sg2}, \varepsilon_{sg3} \) = measured strains in sg1, sg2, and sg3, respectively

The strain distribution along the longitudinal bar recorded at 3.004 kips on the first cycle of Group 1 is considered. This load and cycle combination has
been selected to represent the strain distribution with no previous cycling history (i.e. monotonic conditions). At this point in the test, there was one crack located at the beam centerline as shown in Figure 7.7. The crack opened at a load of 2.784 kips. The post-crack strain distribution in the longitudinal bar is shown in Figure 7.8. It should be noted that in Figure 7.8 the solid lines connecting the strain points on longitudinal bar sections represent the assumed linear gradient and the dotted lines connecting strain points on transverse bar sections indicate that the gradient shape is unknown.

Figure 7.8 Strain Distribution For P=3.004 k, Cycle 1

As expected, the centerline crack activated the longitudinal bar section on which strain gage 1 was located. From this point outwards, the strain
distribution is seen to decrease becoming zero at gage 10, located 19 inches away. The strain is seen to decrease across each of these 5 transverse bar sections indicating that force is being transferred at these locations. In addition, there is a measurable strain gradient along the longitudinal bar sections. These gradients indicate that the tensile force in the NEFMAC reinforcement is being transferred at transverse bar locations as well as along the length of the longitudinal bar. The percentage of force transferred at each longitudinal and transverse bar section is calculated relative to the centerline gage reading of .25\% strain. Table 7.2 summarizes the amount of force transferred at each location.

<table>
<thead>
<tr>
<th>Force Transfer Section (Type and #)</th>
<th>Measured Change in Force</th>
<th>Force Transferred (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Between Gages (#'s)</td>
<td>strain, $\Delta \varepsilon$ (%)</td>
</tr>
<tr>
<td>LS1</td>
<td>1 - p*</td>
<td>.018</td>
</tr>
<tr>
<td>TS1</td>
<td>p* - 2</td>
<td>.090</td>
</tr>
<tr>
<td>LS2</td>
<td>2 - 3</td>
<td>.036</td>
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<td>TS2</td>
<td>3 - 4</td>
<td>.064</td>
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<tr>
<td>LS3</td>
<td>4 - 5</td>
<td>.022</td>
</tr>
<tr>
<td>TS3</td>
<td>5 - 6</td>
<td>.007</td>
</tr>
<tr>
<td>LS4</td>
<td>6 - 7</td>
<td>.004</td>
</tr>
<tr>
<td>TS4</td>
<td>7 - 8</td>
<td>.000</td>
</tr>
<tr>
<td>LS5</td>
<td>8 - 9</td>
<td>.005</td>
</tr>
<tr>
<td>TS5</td>
<td>9 - 10</td>
<td>.003</td>
</tr>
</tbody>
</table>

Sum Transverse Bar Sections (TS) 66\%
Sum Longitudinal Bar Sections (LS) 34\%
Sum all Components (TS + LS) 100\%

* p is identified in Figure 7.7 as the location where a strain value is calculated according to Eq. 7.3.

Force transfer along the length of the longitudinal bar sections is likely the result of concrete adhering to the course, fibrous and irregular texture of the
NEFMAC bar sides. Comparing total amounts, 66% of the centerline force was transferred along transverse bar sections and 34% along longitudinal bar sections.

Table 7.3 summarizes the amounts of force transferred at individual transverse bar and longitudinal bar sections during load cycles for which there existed only a single crack located at the beam centerline. Note the strain results given in Table 7.3 are for a constant load of 3.004 kips recorded during cycles 1, 10, 11, 20, 22, 32 and 33 and, that for these cycles, only one crack existed as was shown in Figure 7.8. A second crack did not open until cycle #34.

Table 7.3 Force Transfer for $P = 3.004 \text{k}, \text{Cycles 1, 10, 11, 20, 22, 32, 33}$

<table>
<thead>
<tr>
<th>Section (type and #)</th>
<th>Gages (#s)</th>
<th>% Force Transferred During Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1 (%) 10 (%) 11 (%) 20 (%) 22 (%) 32 (%) 33 (%)</td>
</tr>
<tr>
<td>LS1 1 - p*</td>
<td>7 (%)</td>
<td>4 (%) 4 (%) 3 (%) 3 (%) 3 (%)</td>
</tr>
<tr>
<td>TS1 p* - 2</td>
<td>36 (%)</td>
<td>34 (%) 33 (%) 24 (%) 25 (%) 24 (%) 18 (%)</td>
</tr>
<tr>
<td>LS2 2 - 3</td>
<td>14 (%)</td>
<td>7 (%) 8 (%) 6 (%) 6 (%) 6 (%)</td>
</tr>
<tr>
<td>TS2 3 - 4</td>
<td>26 (%)</td>
<td>28 (%) 28 (%) 31 (%) 32 (%) 32 (%) 33 (%)</td>
</tr>
<tr>
<td>LS3 4 - 5</td>
<td>9 (%)</td>
<td>13 (%) 11 (%) 12 (%) 12 (%) 10 (%) 11 (%)</td>
</tr>
<tr>
<td>TS3 5 - 6</td>
<td>3 (%)</td>
<td>8 (%) 9 (%) 13 (%) 13 (%) 14 (%) 16 (%)</td>
</tr>
<tr>
<td>LS4 6 - 7</td>
<td>2 (%)</td>
<td>2 (%) 3 (%) 4 (%) 4 (%) 5 (%) 6 (%)</td>
</tr>
<tr>
<td>TS4 7 - 8</td>
<td>0 (%)</td>
<td>2 (%) 1 (%) 3 (%) 3 (%) 2 (%) 4 (%)</td>
</tr>
<tr>
<td>LS5 8 - 9</td>
<td>2 (%)</td>
<td>1 (%) 2 (%) 1 (%) 0 (%) 3 (%) 2 (%)</td>
</tr>
<tr>
<td>TB5 9 - 10</td>
<td>1 (%)</td>
<td>1 (%) 1 (%) 1 (%) 2 (%) 1 (%) 1 (%)</td>
</tr>
<tr>
<td>Sum TS</td>
<td>66 (%)</td>
<td>73 (%) 72 (%) 72 (%) 75 (%) 73 (%) 72 (%)</td>
</tr>
<tr>
<td>Sum LS</td>
<td>34 (%)</td>
<td>27 (%) 28 (%) 28 (%) 25 (%) 27 (%) 28 (%)</td>
</tr>
</tbody>
</table>

* p is identified in Figure 7.7 as the location where a strain value is calculated according to Eq. 7.3.

Table 7.3 shows a steady redistribution of force transfer taking place between cycles 1 and 33. As can be seen, there is a steady decrease in the amount of
force transferred through TS1 (from 36% during cycle 1 to 18% during cycle 33). Also, a gradual increase in the force transferred through TS2 (from 26% to 33%) and a pronounced increase in that of TS3 (from 3% to 16%) takes place. Longitudinal bar sections 3 and 4 experience a slight increase as does transverse bar section 4. In general, there seems to be a migration of force transfer in a direction away from the beam centerline, and the first transverse bar and longitudinal bar sections in particular. Referring to Table 7.3, during the first cycle, a total of 57% of the centerline force was transferred through LS1, TS1 and LS2 combined (7% + 36% + 14%). By the 33rd cycle this total amount is reduced to only 27% (3% + 18% + 6%). However, the summations remain very constant at about 72% for transverse bar sections and 28% for longitudinal bar sections.

As the load cycling continued past the 33rd cycle a second crack opened at a load of 4.564 kips during the 34th cycle, as is shown in Figure 7.9.

![Figure 7.9 Opening of 2nd Crack on Cycle 34](image-url)
The 2nd crack was located at the first transverse bar outside of the constant moment region as is indicated in Figure 7.10. Referring to Figure 7.9, the load identified as 4.454 kips represents pre-2nd crack conditions and the loads identified as 3.454 kips and 3.004 kips represent post-2nd crack conditions. The pre- and post-2nd crack strain distributions for these three loads are shown in Figure 7.10.

Figure 7.10 Pre- and Post-2nd Crack Strain Gradients for Cycle 34
The post-2nd crack strain distributions between gages 2 and 3 (on LS2) are seen from Figure 7.10 to experience an unexpected reversal in gradient. As can be seen, for both post-2nd crack loads shown, the strain in gage 3 is greater than that in gage 2. This condition indicates that the resultant force acting between the concrete and NEFMAC along LS2 has reversed direction relative to its pre-2nd crack orientation.

Moving in a longitudinal direction outwardly away from the beam centerline, the 2nd crack causes a sharp increase in the strain level of gages 4, 5, 6 and 7. For gages 4 and 5 this sharp increase is the result of the second crack activating longitudinal bar section 3 (LS3). The opening of the second crack exposes this region of the reinforcement to the full magnitude of the moment in that section. For both post-crack loads (3.454 k and 3.004 k), there is a strong strain gradient observed along longitudinal bar section 3 and, to a lesser degree, along longitudinal bars section 4 and 5.

As the sample was continually cycled to alternating overload and reference-load levels the strain distribution in the longitudinal NEFMAC bar at 3.004 k was observed to remain relatively consistent with that shown in Figures 7.10. A reversal in strain gradient was also recorded along longitudinal bar section 4 (LS4) during a later cycle. This condition is shown in Figure 7.11 where it is noted that strain distributions are given for cycles 1, 50, 70 and 90 and for a constant load of P=3.004 k. These load and cycle combinations were selected to evaluate how the longitudinal strain distribution at reference-load is effected by load cycling. Also, no additional cracks opened during cycles 50, 70 and 90 and, as such, recorded strains for these cycles reflect the existence of only two cracks.
The figure clearly shows that for cycles 50, 70, and 90 the strain gradient along the second longitudinal bar section between strain gages 2 and 3 (LS2) has reversed relative to the 1st cycle. As can be seen, the strain in gage 2 is greater than that in gage 3 for the first cycle. However, for cycles 50, 70 and 90 the strain in gage 2 is now less than that in gage 3, indicating a force reversal has taken place. This same trend is observed to occur between gages 6 and 7. The results show that for the 50th cycle the strain in gage 6 is only slightly greater than that in gage 7, indicating the gradient along this longitudinal bar segment is almost zero. Then, for the 70th cycle the strain in gage 6 is less than that in gage 7. The strain gradient reversal is then observed to increase for the 90th cycle.

The strain gradient reversal indicates that the friction force acting along the longitudinal bar section still exists, but has reversed direction. The cause of
this gradient reversal is currently unknown. It could reflect the intensity of the bearing force that develops at the transverse bar location. The bearing force develops at the interface between the concrete and NEFMAC and acts to resist the change in axial force that develops as a result of the moment gradient. It is thus assumed that the bearing force develops on the centerline side of transverse bars causing cracks to open on the support side, as is shown in Figure 7.12. Much further testing with more refined instrumentation on the NEFMAC and concrete is required to validate these assumptions.

![Figure 7.12 Crack Location Relative to Transverse Bar](image)

7.3 Sample 4B2

The reinforcement and geometric properties of cyclically loaded Sample 4B2 were identical to those of monotonically loaded Sample 4B1. Both beams were reinforced with a single H19 NEFMAC bar at 160%ρb. Sample 4B2 failed in shear. This result was expected based on the test results from Sample 4B1.
Due to an error in programming the test machine, Sample 4B2 was accidentally preloaded monotonically to 6 kips before data acquisition commenced. Thus, the test results will not reflect any cracking activity for loads lower than 6 kips.

7.3.1 Load Schedule

The cyclic loading schedule used during testing is shown in Figure 7.13 where it is noted that the sample was subjected to a total of 62 load cycles. The intended purpose of the test was to determine how steadily increasing the cyclic load amplitude would effect the load-deflection and strength properties of the sample. However, due to difficulties with the test equipment, the load amplitude did not increased as uniformly as was desired.

![Figure 7.13 Load Cycling Schedule - Sample 4B2](image-url)
7.3.2 Load-Deflection and Failure Mode

The load-deflection results for Samples 4B1 and 4B2 are shown in Figure 7.14 and the crack pattern as observed at failure is given in Figure 7.15.

![Load-Deflection Results - Samples 4B1, 4B2](image)

Figure 7.14 Load-Deflection Results - Samples 4B1, 4B2
The flexural performance of Samples 4B1 and 4B2 are seen from the load-deflection curves to be very similar. Both beams failed at approximately the same deflection of 1.1 inches, and the corresponding ultimate loads were very close, 9.9 kips for 4B1 and 10.7 kips for 4B2, about a 7% difference. Also, the slope of the load-deflection curve for Sample 4B1 is very close to the load-deflection slope measured by connecting the load peaks of Sample 4B2, although they are offset by about 0.6 kips.

The cracking pattern of Sample 4B2 was characterized by 6 individual cracks, all initiating at the location of transverse bars. As with Sample 4B1, only one crack opened in the region of constant moment. The pattern of cracking was
characteristic of flexural-shear cracking in beams of intermediate length. The crack located within the constant moment region grew in a nearly vertical direction, characteristic of flexural cracking. However, the path of the individual cracks became progressively more inclined as the distance from the beam centerline increased. The incidence of inclined cracking represents a lack of tensile capacity in the concrete to resist the diagonal-tension stresses that develop as a result of combined shear and bending action. As is predictable in beams of this shear-span to depth ratio (a/d=5.3), it was the flexural-shear crack located farthest from the beam centerline that ultimately propagates through the beam depth causing brittle failure. At failure this crack was nearly horizontal. (Figure 7.15).

In conclusion, the similarities in crack pattern, load-deflection behavior and failure mode between Samples 4B1 and 4B2 suggests that the limited cyclic loading used on Sample 4B2 had little or no effect on the beam's load-deflection or shear performance.

7.4 Sample 4B3

The reinforcement and geometry of cyclically loaded Sample 4B3 were identical to monotonically loaded Sample 4B4. Both samples were reinforced at 120% $p_b$ with 2xH10 NEFMAC bars and the shear-span to depth ratio was 5.1. The maximum cyclic load applied to Sample 4B3 was 8.5 k. This is 22% less than the sample's expected ultimate strength of 10.9 k (based upon the strength of Sample 4B4). The test was stopped after a region of significant concrete spalling develop just outside the constant moment span.
7.4.1 Load Schedule

Sample 4B3 was subjected to a total of 212 load cycles. The minimum and maximum loads for each cyclic are shown in Figure 7.16. The purpose of the test was to determine how varying the maximum cyclic load would effect the sample’s load-deflection and stiffness properties. It was intended to successively hold the maximum cyclic load constant at 2.5 kips for 15 cycles followed by 15 cycles where the maximum cyclic load was increased so as to be 1 kip larger than all previous loadings. However, due to difficulties with the test equipment, this effort was only partially successful.

![Load Cycling Schedule - Sample 4B3](image)

Figure 7.16 Load Cycling Schedule - Sample 4B3

7.4.2 Load-Deflection and Failure Mode

All design parameters for cyclically loaded Sample 4B3 were identical to those of monotonically loaded Sample 4B4. The load-deflection and crack pattern results for these two samples are shown in Figures 7.17 and 7.18, respectively.
Both samples are observed to crack at about the same load and deflection. The flexural performance of Sample 4B4 was linear up to an ultimate load of 10.90 k. Load-deflection measured by connecting the load peaks for Sample 4B3 is also linear up to a load of about 8 kips. The slope of the cracked-section load-deflection curve for Sample 4B4 appears to be slightly greater than that of Sample 4B3 as measured by connecting the load peaks up to cycle 169. The
opposite result was observed for Sample 1B1, where a slight increase in the slope of the load-deflection curve was observed as a result of load cycling. This could potentially reflect the response of different NEFMAC fiber types to load cycling. Sample 4B3 was reinforced with an H10 grid (glass/carbon fiber hybrid) and Sample 1B1 was reinforced with C22 grid (100% carbon fiber). Further cyclic testing is required to determine whether these different grids do in fact respond differently to load cycling.

Figure 7.18 Crack Pattern and Moment Diagram - Samples 4B3, 4B4

The failure of Sample 4B4 was brittle, occurring as a result of diagonal-tension shear. Although the cracked pattern of Sample 4B3 suggested diagonal-tension deficiencies, the beam did not fail in a brittle manner. As can be seen from Figure 7.16, the maximum load applied to the sample was only 8.5 kips.
Thus, the beams ultimate capacity (10.90 kips as derived from the results of Sample 4B4) was never reached. However, as the sample was cycled between 8.5 kips and 0.5 kips, significant concrete spalling developed just outside of the constant moment region (Figure 7.18). Near the end of the test a "fragment" of concrete (roughly 1" deep by 6" long) had broken loose and was physically removed. The loss of this concrete volume did not, however, result in brittle failure. Although significant deflection growth was sustained with each load cycle, the section remained physically intact and able to support the applied load. The test was terminated after a significant growth in deflection had occurred.

The steady growth in deflection observed between cycles 169 and 212 (Figure 7.17) reflects the response of concrete to load cycling at high stress levels. During these cycles, the maximum cyclic load is 8.5 kips and the corresponding stress in the concrete is above .85*f_c. At this stress level, the unloading-reloading curve for concrete exhibits strong nonlinearities causing the permanent growth in deflection observed between cycles 169 and 212. [59]

7.5 Conclusions and Findings

The limited load cycling did not effect the failure mode of the samples reinforced with C-22 NEFMAC and H-19 NEFMAC grids. For both monotonically loaded and cyclically loaded samples reinforced with these bar types, the failure mode was identical and the result of diagonal-tension shear. Also, ultimate loads were very close (within 10%) indicating that there was no deterioration in shear strength. At failure the cracked pattern of the cyclically loaded and monotonically load beams were nearly identical.
In general, the slope of the load-deflection curve measured by connecting the load peaks of the cyclically loaded samples followed very closely that of the monotonically loaded control samples. This result was observed for the entire duration of the test for samples reinforced with C22 and H19 NEFMAC grids. The similarity between the monotonic and cyclic load-deflection curves suggests that there was no measurable deterioration or softening in the flexural stiffness of these samples as a result of the limited load cycling. This same result was observed for the first 169 load cycles of the sample reinforced with the H10 type grid. For this sample, the slope of the load-deflection curve measured by connecting the load peaks up to cycle 169 followed closely that of the monotonically loaded control sample.

Strain gradients measured on the longitudinal NEFMAC bar indicate that about 72% of the tensile force is transferred on transverse bar sections and about 28% through friction on the longitudinal bar sections. The force transferred at the transverse bar sections is a combination of direct bearing at the transverse bar, wedge action at the grid intersection points and longitudinal friction.
CHAPTER VIII

DESIGN CONSIDERATIONS

8.1 Introduction

The widespread use of NEFMAC as a reinforcement for concrete beams requires a design guide that provides adequate safety from brittle failure. For steel reinforced beams, brittle failure is avoided by specifying underreinforced sections (i.e. $\rho \leq 0.75\rho_b$). This ensures the steel will yield before the concrete crushes. The result is a ductile failure followed by the absorption of large amounts of energy through plastic straining in the reinforcing steel. Ultimately, a steel reinforced beam will fail in flexural compression, but only after large deflections have taken place forcing the strain in the concrete to reach its ultimate value. This secondary failure of concrete crushing is considered brittle. \[43\]

Unfortunately, FRP materials respond linearly elastic up to failure and, as such, are not capable of any yielding or ductility. As a result, failure of FRP R/C beams, whether the result of shear, flexural compression or flexural tension, is unavoidably brittle.
8.2 Energy Criterion as a Factor of Safety

At the limit state it can be said that both steel and NEFMAC reinforced beams are equivalent in that their failure modes are ultimately brittle. For properly reinforced steel beams this behavior is, however, of secondary consideration whereas for NEFMAC reinforced beams this failure mode is primary.

Considering a working strength design (WSD) where it is specified that service level stresses are not to exceed \(0.40 \times F_y\) for Grade 60 steel or \(0.45 \times f_c\) for concrete, [12] the theoretical service load calculated for steel reinforced Sample 1B3 would be 4.5 k, as is shown in Figure 8.1.

![Sample 1B3](image)

**Figure 8.1 Service Level Load - Sample 1B3**

Employing the Trapezoidal Rule for numerical integration, the areas under the load-deflection curve in Figure 8.1 at service and ultimate (concrete crushing) loads are 0.266 k-in and 13.8 k-in, respectively. Or, the area at service is about 1/50th that under the curve at compression failure in the concrete.

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The factor of 50 represents a measure of service level performance relative to brittle failure by concrete crushing. Thus, had the service load for Sample 1B3 been founded upon an energy absorption criterion with a safety factor of 50, the resulting design would have yielded service load levels for which the steel stress is about 40% of yield. For steel reinforced Beam 1B3 these two service level criteria (energy and stress level) are equivalent.

In general, the energy factor of safety (EFS), or ratio of area under the load-deflection curve at ultimate load to service load, for steel reinforced beams can be expected to vary inversely with the amount of reinforcement provided relative to a balanced design. That is to say, the smaller the amount of reinforcement provided the higher the ratio of area under the load-deflection curve between ultimate and service levels (as calculated according to working stress criteria). Thus, according to current ACI reinforcement limits, the EFS for steel reinforced beams can be expected to take on a minimum value at .75 $\rho_b$ (the maximum amount of reinforcement allowed by the code) and a maximum value at $200/F_y$ (the minimum amount of reinforcement allowed the code).

Theoretical bilinear load-deflection curves for Sample 1B3 reinforced at $\rho_{\text{max}}$ and $\rho_{\text{min}}$ are shown in Figures 8.2. In the figure, service loads were calculated according to a working stress criteria (i.e. $f_s \leq .40F_y$ for steel and $f'_c \leq .45f'_c$ for concrete). Yield and ultimate loads were calculated according to Section 4.2 and the corresponding deflections calculated according to a moment-curvature analysis as detailed in Appendix Section B.3.
where:

\[ A_{\text{ult.}} = \text{area under load-deflection curve at ultimate (concrete crushing)} \]
\[ = .5 (P_y)(Y_u) + (P_y)(Y_u - Y_y) + .5 (P_u - P_y)(Y_u - Y_y) \]
\[ A_{\text{sv.}} = \text{area under load-deflection curve at service load} \]
\[ = .5 (P_{sv})(Y_{sv}) \]
\[ P_u, P_y = \text{load at ultimate (concrete crushing) and yield, respectively} \]
\[ Y_u, Y_y = \text{deflection at ultimate (concrete crushing) and yield, respectively} \]
\[ s = \text{pre-yield load-deflection curve slope} = P_y / Y_y \]
\[ P_{sv} = \text{service load calculated according to WSD} \]
\[ Y_{sv} = \text{service deflection} = P_{sv}/s \]

The hypothetical limits of the EFS for Sample 1B3 are seen from Figure 8.2 to range between 20 (at \( \rho_{\text{max}} \)) and 240 (at \( \rho_{\text{min}} \)). Between these limits, Figure 8.3 shows how the EFS would theoretically vary as a function of \( \rho \) for Sample 1B3.
Figure 8.3 shows the EFS to decrease exponentially with increasing \( \rho \), becoming asymptotic at about 2%. Considering the distribution shown in Figure 8.3, an EFS=50 is an acceptable value, if not conservative, relative to limits placed on \( \rho \).

The above discussion has shown that when employing an energy criterion for calculating service loads, safety is measured relative to brittle failure. Because FRP reinforced beams can only experience brittle failure, it is logical to establish this condition as a reference from which service loads can be calculated. The following sections consider an energy factor of safety for calculating the service loads of NEFMAC reinforced beams.
8.3 Service Load Calculations

For discussion purposes only, the following section considers service load calculations of NEFMAC reinforced beams tested in this thesis according to the proposed energy level criterion using an EFS of 50. The implementation of such a design criterion requires the calculation of areas under load-deflection curves. For this purpose, the flexural response of NEFMAC reinforced beams can be closely approximated by assuming a linear pre-crack load-deflection response followed by a linear cracked-section load-deflection response up to ultimate. Shown in Figure 8.4 are comparisons of actual flexural behavior recorded in the lab and bilinear approximations for 2 NEFMAC reinforced samples.

![Figure 8.4 Bilinear Flexural Approximation for NEFMAC Reinforced Beams](image)

For cases where several beams of identical design were tested, load and deflection coordinates were chosen so as to produce the smallest area under the load-deflection curve at ultimate, as is shown in Figure 8.5.

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Figure 8.5 Bilinear Approximation for NEFMAC Reinforced Beam Groups

Shown in Figure 8.6 is a generic representation of the load-deflection curve for a NEFMAC reinforced beam. The figure identifies all load and deflection coordinates required for calculation of service level loads using an energy level criterion.

Figure 8.6 Idealized Load-deflection Coordinates
From the coordinates describing the load-deflection curve shown in Figure 8.6, the areas under the load-deflection curve at ultimate and service are calculated as:

\[
A_{\text{ult.}} = \frac{1}{2} (P_{\text{cr}})(Y_{\text{cr}}) + P_{\text{cr}} (Y_u - Y_{\text{cr}}) + \frac{1}{2} (Y_u - Y_{\text{cr}})(P_u - P_{\text{cr}}) \quad (8.1)
\]

\[
A_{\text{sv}} = \frac{1}{2} (P_{\text{cr}})(Y_{\text{cr}}) + P_{\text{cr}} (Y_{sv} - Y_{\text{cr}}) + \frac{1}{2} (Y_{sv} - Y_{\text{cr}})(P_{sv} - P_{\text{cr}}) \quad (8.2)
\]

For post crack conditions, load and deflection are related through the slope as:

\[
s = \frac{P_u - P_{\text{cr}}}{Y_u - Y_{\text{cr}}} = \frac{P_{sv} - P_{\text{cr}}}{Y_{sv} - Y_{\text{cr}}} \quad (8.3)
\]

or, solving the above equation for the service level displacement:

\[
Y_{sv} = Y_{\text{cr}} + \frac{P_{sv} - P_{\text{cr}}}{s} \quad (8.4)
\]

Substituting into (8.4) into Eq. (8.2) and setting the result equal to \(A_{\text{ult.}}/50\), yields:

\[
\frac{A_{\text{ult.}}}{50} = \frac{1}{2} P_{\text{cr}} Y_{\text{cr}} + P_{\text{cr}} (Y_{\text{cr}} + \frac{P_{sv} - P_{\text{cr}}}{s} - Y_{\text{cr}}) + \frac{1}{2} (P_{sv} - P_{\text{cr}})(Y_{\text{cr}} + \frac{P_{sv} - P_{\text{cr}}}{s} - Y_{\text{cr}})
\]

Solving the above equation for \(P_{sv}\) yields:

\[
P_{sv} = \sqrt{s \left( \frac{P_{\text{cr}}^2}{s} + \frac{A_{\text{ult.}}}{25} - P_{\text{cr}} Y_{\text{cr}} \right)} \quad (8.5)
\]

where:

\[P_{sv} = \text{service level load}\]
\[A_{\text{ult.}} = \text{area under the load-deflection curve at ultimate} = \text{Eq. 8.1}\]
\[s = \text{slope of the post crack load deflection curve} = \text{Eq. 8.3}\]
Equation 8.5 represents the service load for which an energy level criterion provides a factor of safety of 50. The load at service is seen to be a function of load and deflection at cracking and ultimate only.

8.4 Test Sample Service Load Performance

Considering the energy absorption criterion for calculating service loads developed above, safety is measured relative to the limit state of brittle failure. For NEFMAC reinforced beams brittle failure occurs as a result of shear, flexural tension or flexural compression. From the test results, shear failure and flexural tension failure were seen to occur suddenly with an immediate and terminal loss in load carrying capacity. However, flexural compression failure was, in some cases, more gradual. For example, Samples 3B7 - 3B12 and 6B1 - 6B3 were able to maintain the applied load after concrete crushing had began. Considering this failure performance together with the low modulus and high tensile strength of FRP materials, practical design will likely dictate overreinforced sections, where compression failure limits strength. For this reason, service loads calculated according to an EFS in the following tables are limited to those NEFMAC reinforced samples that experienced flexural compression failure.

Using the load and deflection results from laboratory testing, Table 8.1 summarizes the service level loads calculated for NEFMAC reinforced Samples 3B7-3B12 and 6B1-6B5 based upon the proposed energy criterion and using an energy factor of safety equal of 50.
Table 8.1 Proposed Service Level Conditions for EFS=50

<table>
<thead>
<tr>
<th>Beam (ID#) (type)</th>
<th>Cracked &amp; Ultimate</th>
<th>Service</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>YCr</td>
<td>Pcr</td>
</tr>
<tr>
<td>3B7-9 comp.</td>
<td>.050</td>
<td>1.5</td>
</tr>
<tr>
<td>3B10-12 comp.</td>
<td>.050</td>
<td>1.5</td>
</tr>
<tr>
<td>6B1-3 comp.</td>
<td>.091</td>
<td>.67</td>
</tr>
<tr>
<td>6B4,5 comp.</td>
<td>.090</td>
<td>.78</td>
</tr>
</tbody>
</table>

where:

\[
Psv = \text{service level load}
\]

\[
Ysv = \text{service level deflection}
\]

\[
Pcr = \text{load at first crack}
\]

\[
Pu = \text{load at failure}
\]

\[
Ycr = \text{deflection at first crack}
\]

\[
Yu = \text{load at ultimate}
\]

\[
fsv = \text{service level stress}
\]

\[
fu = \text{stress at ultimate}
\]

\[
Fu = \text{tensile strength}
\]

The results shown in Table 8.1 reveal some interesting conclusions regarding service level conditions of stress levels in the reinforcement and deflection properties. First, at service load the deflection for NEFMAC reinforced beams ranges from a maximum of L/420 to a minimum of L/600. From Figure 8.1 deflection at service load for steel reinforced Sample 1B3 is 0.10 inches or L/640. These levels are all well within the maximum allowable deflection imposed by ACI-9.5.2.6 and some meet the minimum allowable deflection. Based upon ACI-Table 8.5(b) [12] the acceptable immediate live load deflection ranges from a maximum of L/360 for floor beams supporting elements not likely to be damaged by large deflections, to L/480 for floor beams supporting elements likely to be damaged by large deflections (i.e. plaster ceilings, glass windows ...etc.).
NEFMAC service load stress levels in Table 8.1 range from 10% to 13% of the material's ultimate strength. These are considerably lower than the 40% observed for the steel reinforced Sample 1B3 (i.e. .40*F_y). The NEFMAC stress levels at ultimate in Table 8.1 range from 53% to 62% of the material's maximum tensile strength. Also, NEFMAC ultimate loads are 4.9 to 5.3 times greater than service loads, which average about 1.25 times greater than cracking loads. Thus, a high degree of reserve strength is available when specifying service loads using an energy criterion with a safety factor of 50. In general, however, the amount of reserve strength will vary depending on the choice of an EFS.

8.5 Flexural Compression Group - Limit State Considerations

NEFMAC reinforced Samples 3B7 through 3B12 were observed clearly to fail in flexural compression. The onset of compression failure in the concrete began at a strain level of .003 in/in, at which point the load-deflection curve became slightly nonlinear. The structural integrity of these samples, however, remained intact and the test was continued until brittle failure ultimately occurred. The concrete strain at failure was calculated to be about .005 in/in. A basic assumption on which the ACI code is founded states that the maximum usable concrete strain on the extreme compression fiber is limited to .003 in/in, and that at this strain level concrete fails in a brittle manner. The test results are partially in agreement with this assumption in that compression failure did occur at a strain level of .003 in/in. However, failure at this level was not brittle as is assumed by the code. The test results showed the code assumption of brittle failure occurring at a strain of .003
in/in to be a conservative estimate in that the concrete was able to develop higher strain levels before brittle failure occurred by shear rupture.

In order that an energy design philosophy be consistent with imposed code strain limitations, ultimate load and deflection levels should correspond to a concrete strain level of .003, and, as such, all service level calculations made relative to this state. For this purpose, the failure load, $P_u$, is taken to be the theoretical flexural strength, i.e. that calculated for $\varepsilon_{cu}=0.003$. Table 8.2 gives the results of this calculation ($P_u$) and the corresponding service load, stress and deflection conditions.

### Table 8.2 Limit State Service Level Conditions

<table>
<thead>
<tr>
<th>Beam (ID#) (type)</th>
<th>Cracked &amp; Ultimate</th>
<th>Service</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y_{cr}$ $P_{cr}$ $Y_u$ $P_u$ $f_u$</td>
<td>$P_{sv}$ $Y_{sv}$ $f_{sv}$</td>
</tr>
<tr>
<td>3B7,8,9 comp.</td>
<td>.050 1.5 1.11 7.83 55</td>
<td>1.7 .088 12</td>
</tr>
<tr>
<td>3B10-12 comp.</td>
<td>.050 1.5 1.15 7.53 54</td>
<td>1.7 .089 12</td>
</tr>
<tr>
<td>6B1 - 3 comp.</td>
<td>.091 .67 1.40 3.87 67</td>
<td>.77 .132 13</td>
</tr>
<tr>
<td>6B4 - 5 comp.</td>
<td>.090 .78 .96 4.53 54</td>
<td>.84 .105 11</td>
</tr>
</tbody>
</table>

The results shown in Table 8.2 are little different from those shown in Table 8.1. The service level loads calculated according to a limit state defined by an ACI maximum concrete strain are slightly smaller than those determined based upon laboratory measured ultimate load results. This reflects the reduced amount of available energy at ultimate (brittle failure) that results from the using the theoretical values for $P_u$, which are about 15% less than laboratory values.
Service loads average about 20% of ultimate ($P_u$) and 110% of the cracking load ($P_{cr}$). For steel reinforced Sample 1B3, the service load was 40% of the yield load (i.e. $f_s = 0.40F_y$). Thus, service loads for NEFMAC R/C beams are significantly smaller relative to ultimate than are the service loads for steel R/C beams. This result is perfectly consistent with the area-ratio criterion on which an EFS design is based. Because NEFMAC R/C beams lack the section ductility characteristic of concrete beams underreinforced with steel, service loads must be drastically reduced relative to ultimate levels in order that the required area-ratio (or EFS) be maintained.

Deflections at service loads are seen to be lower than those listed in Table 8.1. This reflects both the smaller service loads and also a slight increase in the slope of the theoretical bilinear load-deflection curve that results for the cracked-section. This occurs because the nonlinear region where concrete was crushing has been neglected. The result is a decrease in service load deflection. As can be seen, service load deflections for Samples 3B7-3B12 and 6B4-6B5 are all well within the most conservative code requirement of $L/480$, while that for Samples 6B1-6B3 is very close to this limit.

Reinforcement stress levels for NEFMAC at service and ultimate loads are seen from Table 8.2 to average about 10% and 48%, respectively, of the materials ultimate tensile strength. Thus, at ultimate less than half of the bar's tensile strength was developed. It must be appreciated, that this result is a consequence of the large amount of NEFMAC reinforcement provided (between $290\%\rho_b$ and $436\%\rho_b$) for these test samples. Assuming that in practical application adequate shear reinforcement is used, NEFMAC reinforced beams can be designed with lower amounts of reinforcement and
still satisfy the proposed requirement of a flexural compression failure at ultimate. This will result in increased stress levels at both service and ultimate loads.

8.6 Conclusions and Findings

The use of an energy absorption criterion for the design of FRP reinforced concrete beams is a reasonable approach to ensuring adequate safety is provided from brittle failure. Because FRP reinforced beams can only experience brittle failure, it is logical to establish this condition as a reference from which service loads can be calculated. An acceptable EFS should be determined from the load-deflection results of concrete beams tested to ultimate with varying NEFMAC reinforcement ratios ($\rho_{frp}$). This is necessary to determine that a section's ultimate strength and failure mode, upon which an energy criterion is founded, can be predicted with an acceptable degree of accuracy independent of the amount of reinforcement provided.

In practice, adequate shear reinforcement will be assumed provided, so that strength limitations are determined according to flexural deficiencies. Considering the high tensile strength and low modulus of NEFMAC, practical design will require that flexural compression failure limit strength. The specified energy factor of safety will likely be provided according to limiting stress levels in the reinforcement (Working Stress Design Method) and/or modifications to load and reduction factors (Ultimate Strength Design Method). The consequence will be low reinforcement working stress levels resulting in a high degree of reserve strength and acceptable compliance with deflection criteria.
CHAPTER IX

CONCLUSIONS AND RECOMMENDATIONS

9.1 Research Conclusions

From the research presented in this thesis the following conclusions are derived regarding the performance of NEFMAC as a reinforcement for concrete beams.

9.1.1 Flexural Strength

Flexural strength of NEFMAC reinforced beams can be predicted with an acceptable degree of accuracy using traditional reinforced concrete analysis methods but with the material properties of NEFMAC. Ultimate strength predictions calculated using a Whitney stress distribution in the concrete and assuming a linear strain distribution on a cracked-section were consistently in close agreement with test results. This conclusion was found for both flexural compression failure and flexural tension failure.

It must be appreciated that in practical design application, the high tension strength and low modulus of NEFMAC make flexural tension failures unlikely. The tensile strength and modulus of a C-type and H-type NEFMAC bars are 178 ksi and 12300 ksi, and 120 ksi and 6000 ksi, respectively. As reinforcement for normal strength concrete where adequate shear
reinforcement is provided, it is most likely that deflection criteria will force a design in which concrete compression failure occurs well before a tensile rupture of the NEFMAC is possible.

### 9.1.2 Shear Strength

The nominal shear strength of concrete beams without shear reinforcement is specified by ACI-11.3.1 as $V_c = 2 \sqrt{f_{cd}} bd$. The form of ACI-11.3.1 is seen to be more an empirical derivation from laboratory test data than a mathematical derivation from elastic theory. As such, the shear capacity calculated using ACI-11.3.1 provides a statistical prediction, derived from data points characterizing the shear strength of steel reinforced concrete beams.

It has been determined from the research presented herein that shear strength of NEFMAC reinforced beams of intermediate length ($2.5 < a/d < 6$) without shear reinforcement is substantially lower than that predicted according to ACI-11.3.1. The consequence of this inequality requires a modified form of ACI-11.3.1 be derived, one that is sensitive to the specific shear behavior characteristic of NEFMAC reinforced beams.

The test results have shown that a correlation exists between the amount of longitudinal NEFMAC reinforcement provided and the shear strength of the section. This result is consistent with studies that have shown ACI-11.3.1 to overestimate shear strength for beams reinforced with steel at $\rho \leq 1.2\%$. In both cases, the decrease in shear strength is a result of large deflections and wide cracks that act to reduce or eliminate aggregate interlock as a mechanism of shear force transfer. Using the measured shear strength of 22 beams, the
The following modified form of ACI-11.3.1 is recommended for calculating the nominal shear strength of NEFMAC reinforced beams:

\[
V_c = K_{frp} \sqrt{f_y} bd
\]  

(9.1)

where:

\[
K_{frp} = (0.8 + 200 \frac{A_{frp}}{bd} \frac{E_{frp}}{E_s}) = (0.8 + 200 \rho_{norm}) \leq 1.4
\]  

(9.2)

\[
\rho_{norm} = \frac{A_{frp} E_{frp}}{bd E_s}
\]

The above equation considers the amount of longitudinal reinforcement provided as a variable in calculating shear strength, while maintaining the basic empirical form of ACI-11.3.1. The relationship conservatively predicts the shear strength as a function of the amount of NEFMAC reinforcement provided.

The shear strength constant \(K_{frp}\) calculated according to Eq. 9.2 must not exceed the limit imposed by ACI-11.3.1 of \(K=2\). From Eq. 9.2 this limit is reached at a normalized reinforcement ratio \(\rho_{norm}\) of 0.6%. However, Eq. 9.2 was derived from data in the range \(0.05\% < \rho_{norm} < 0.32\%\). Thus, the relevancy of Eq. 9.2 is in question when \(0.32\% < \rho_{norm} < 0.6\%\). Until further testing is done to validate the suitability of Eq. 9.2 within this range, the limit of \(K_{frp} \leq 1.4\) is recommended.

**9.1.3 Cracking and Force Transfer**

Force transfer in the NEFMAC grid was measured using bonded strain gages located on the longitudinal bar at 2" intervals. From this instrumentation
force transfer was modeled relative to longitudinal bar sections and transverse bar sections as is shown in Figure 9.1.

**Figure 9.1 Force Transfer Components**

Force transfer on the 2" longitudinal bar sections is assumed the result of friction that develops between the course and fibrous texture characteristic of the NEFMAC bar side profile and the surrounding concrete. On the 2" transverse bars sections the total force transferred is assumed a combination of friction, wedge action and direct bearing at the transverse bar locations. Longitudinal strain gradients on the grid suggest that approximately 72% of the axial force is transferred at the transverse bar sections and 28% along the longitudinal bar sections.

The grid shape of NEFMAC insures adequate force transfer is provided to develop a flexural tensile failure in beams reinforced with a single H10 longitudinal bar. Beams were tested where a tensile rupture of the H10-type NEFMAC bar occurred with no apparent deterioration in force transfer mechanics. The grid intersection of the transverse and longitudinal bars remained rigid and no bearing or shear failure between the concrete and transverse bar was detected. Also, beams were tested in which 60% of the C22
type NEFMAC bar's tensile strength was developed with no deterioration in force transfer observed.

The crack pattern of NEFMAC reinforced beams reflects the transfer of force that is occurring at the transverse bar locations. Individual cracks consistently initiated at transverse bar locations. However, cracking was not always observed to occur at all transverse bars located within the cracked length of the beam.

9.1.4 Deflection Prediction

The Branson equation employs an effective moment of inertia, \( I_e \), based upon the cracked transformed-section for calculating post-crack stiffness and deflection. Test results have shown the effectiveness of the Branson equation is influenced by the amount of longitudinal reinforcement provided. For test samples reinforced with NEFMAC between 300% and 430% of a balanced design the Branson equation predicted deflections with an acceptable tolerance. Deflections calculated at 35% and 50% of the tested ultimate strength were within ±20% of lab measured results. However, for test samples reinforced with NEFMAC between 33% and 186% of a balanced design predicted deflections underestimated lab results by as much as 100% at 35% of ultimate strength and 70% at 50% of ultimate strength. These results suggest that the Branson equation overestimates \( I_e \) where NEFMAC reinforcement was provided below approximately 200%\( \rho_b \) and that the amount of overestimation was increasing as \( \rho \) decreased below 200%\( \rho_b \).
Deflection prediction employing a moment-curvature analysis was very good for all samples at 35% and 50% of ultimate. Theoretical deflections calculated using this technique were generally within ±10% of lab results for all samples at both 35% and 50% of ultimate. This technique did not suffer the limitations of the Branson equation for those samples reinforced with NEFMAC below 186% of a balanced design. Although a moment-curvature deflection analysis requires more detailed calculations as compared to the Branson equation, its use is recommended for deflection prediction of concrete beams reinforced with FRP.

9.1.5 Cyclic Loading

Load cycling did not visibly effect the load-deflection or shear performance of samples reinforced with C22 and H19 type NEFMAC grids. Comparing test results of monotonically and cyclically loaded samples reinforced with these grid types, showed the cracked pattern, load-deflection response, ultimate load and failure mode to be very similar. This behavioral compatibility suggests that the limited cyclic loading did not have any measurable or visible effect on the flexural stiffness, force transfer properties or ultimate strength of the beams reinforced with these grid types.

For the sample reinforced with an H10 type grid, the slope of the load-deflection curve measured by connecting the load peaks for the first 169 load cycles followed closely that of the monotonically loaded control sample. This sample was not loaded to it's ultimate strength and, as such, no conclusion regarding the effect of load cycling on section strength is stated.
9.2 Design Recommendations

Design of steel reinforced beams requires sections to be underreinforced relative to a balanced strain condition. This ensures a ductile failure followed by plastic deformation. Ultimately, flexural compression failure will occur, but only after large deflections have taken place forcing the strain in the concrete to reach its ultimate value. This secondary failure mode is considered brittle.

Unlike steel sections, NEFMAC reinforced concrete beams can only experience a brittle failure mode. This will be the result of shear, tensile rupture of the NEFMAC or compression failure in the concrete.

Recognizing that NEFMAC R/C beams do not have the ductility characteristic of beams undereinforced with steel, a design criterion is discussed in which service loads for NEFMAC reinforced beams are specified relative to energy levels existing at brittle failure. For calculation purposes, energy levels are considered relative to areas under the load-deflection curve. Thus, service is defined as the load for which the energy in the beam is some fraction of that at ultimate and the corresponding margin of safety, or Energy Factor of Safety (EFS), is equal to the energy at ultimate divided by the energy at service. For discussion purposes only, an energy factor of safety (EFS) was taken to be 50. This value of 50 was derived from the load-deflection results of a beam underreinforced with steel at 33%ρ_b and is equal to the area under the load-deflection curve at the point where the stress in the reinforcement was .40F_y divided by the area under the load-deflection curve at secondary failure of concrete crushing (ε_cu=.003 in/in). Thus, the procedure emulates ACI
working stress design (WSD) criteria, but considers energy levels as a measure of safety and not reinforcement stress levels.

Theoretical EFS limits were calculated for the steel reinforced sample and found to range between 20 (at $\rho_{\max} = 75\% \rho_b$) and 240 (at $\rho_{\min} = 200 / F_y = 11\% \rho_b$). Between these limits, the EFS was found to decrease exponentially with increasing $\rho$, asymptotically approaching EFS=20 at about $\rho = 64\% \rho_b$. Thus, the distribution of EFS vs. $\rho$, shows EFS=50 to be an acceptable value, if not conservative, relative to the allowable limits placed on $\rho$.

In practice, adequate shear reinforcement will be assumed provided, so that ultimate strength is determined according to flexural limitations. From test results, flexural tension failure was seen to occur suddenly with an immediate and terminal loss in load carrying capacity. However, flexural compression failure occurred, in some cases, more gradually. Considering this failure performance together with the low modulus and high tensile strength of FRP materials, the design recommends overreinforced sections, where flexural compression failure limits strength. In design application, an acceptable energy factor of safety will likely be provided by restricting reinforcement stress levels and/or modifications to load and capacity reduction ($\phi$) factors. The consequence will be low reinforcement working stress levels resulting in a high degree of reserve strength and acceptable compliance with deflection criterion.

Hypothetical service load calculations are presented for NEFMAC reinforced test samples that experienced flexural compression failure. Using the proposed energy criterion with an EFS of 50, the results show working stress
levels in the NEFMAC to be only 10% of the material's tensile strength. At this level, service loads were 20% of ultimate and deflections were well within the most conservative required by the ACI code. These service load conditions directly reflect the selection of an EFS=50. It has yet to be determined what an acceptable energy factor of safety for FRP reinforced beams should be.

A detailed discussion of the proposed methodology is presented in Chapter 8.

9.3 Future Research

The information presented in this thesis provides only a preliminary understanding of how concrete beams reinforced with NEFMAC can be expected to behave. Much further research is required before engineers are able to confidently design concrete structures reinforced with NEFMAC grids. The following items represent some of the more significant areas where further research is warranted based on results presented in this thesis: (1) Monotonic testing of beams with shear reinforcement; (2) Validation of the proposed shear strength equation for beams reinforced with NEFMAC at \(0.32%<\rho_{\text{norm}}<0.6\%\); (3) Investigation of an acceptable energy factor of safety; (4) Amount and distribution of force transfer on longitudinal and transverse bar sections; (5) Strength and stiffness performance under cyclic loading; (6) Finite Element Modeling of NEFMAC R/C beams. These research issues are explained in the following sections.
9.3.1 Monotonic Testing of Beams with Shear Reinforcement

In many of the samples tested for this thesis premature shear failure occurred before flexural strength could be developed. Where flexural failure occurred, it was necessary to either severely overreinforce (flexural compression failure) or severely underreinforce (flexural tensile failure) test samples relative to a balanced strain condition. Thus, the relevance of a balanced design for predicting the type of flexural failure and corresponding ultimate load is only substantiated for extreme amounts of reinforcement. It is necessary to determine if balanced failure does occur at a theoretical $\rho_b$, and correspondingly, that flexural compression and tensile failures do occur when $\rho > \rho_b$ and $\rho < \rho_b$, respectively. In providing shear reinforcement, shear failure is eliminated and a finer understanding of how accurately flexural strength and failure mode are predicted relative to the amount of reinforcement becomes possible.

9.3.2 Shear Strength

An empirical equation for predicting the shear strength of NEFMAC R/C beams was presented. The equation was derived from a limited number of data points describing the shear strength of samples reinforced between $.05\% < \rho_{\text{norm}} < .32\%$. Extensive testing is required to validate the accuracy and limits of the proposed equation with respect to reinforcement ratio and shear-span to depth ratio ($a/d$).
9.3.3 Energy Factor of Safety

Presented in this thesis was a design criterion where service loads for NEFMAC reinforced beams are calculated relative energy levels existing at ultimate. An energy factor of safety of 50 was discussed based on the results of a steel reinforced beam and the corresponding theoretical limits and distributions for this parameter presented. These theoretical limits and distributions need to be validated from laboratory testing of steel reinforced beams before an acceptable EFS for NEFMAC R/C beams can be determined.

9.3.4 Force Transfer

Preliminary results suggest that longitudinal bar sections account for 28% of force transfer and transverse bar sections for the remaining 72%. These results were derived from strain gage instrumentation that defined longitudinal bar sections as only 2" in length. In reality, the longitudinal bar length along which force transfer through friction was occurring was 3.5". Thus, the value of 28% does not reflect the total potential contribution of friction as a mechanism for force transfer. In order to more accurately define the percentages and distributions of force transfer relative to longitudinal and transverse bar dimensions more refined (closer spacing) strain gage instrumentation is recommended.

9.3.5 Load Cycling

The cyclic results presented in this research are very preliminary in nature. Conclusions regarding the flexural and shear performance of NEFMAC R/C
beams subjected to load cycling require much more extensive testing. It is recommended that multiple samples of identical design be tested under more severe cyclic loading conditions regarding to the total number of load cycles and load amplitude. Also, monotonically loaded control samples of identical design should always be tested so as to provide a reference from which the effects of load cycling can be measured.

9.3.6 Finite Element Analysis

When designing NEFMAC reinforced concrete structures, engineers will need to satisfy requirements of strength, deflection, crack width and stress level to ensure adequate performance and safety at service load. The finite element (FE) technique has the potential to contribute much valuable information regarding these aspects of NEFMAC R/C behavior. The FE method has already contributed much valuable information in the research, design and analysis of steel R/C structures. Thus, the analysis of NEFMAC R/C structures using the FE technique is a logical extension of this steel R/C work. A preliminary FE model is presented in Appendix C.
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APPENDIX A

EXPERIMENTAL INSTRUMENTATION AND DATA ACQUISITION

A.1 Introduction

The data acquisition techniques used to collect load, deflection and strain information during the testing of steel and NEFMAC reinforced concrete beams are presented in this appendix. The system described herein was designed and built by the author so as to provide the facility for measuring multiple channels of the above mentioned test data.

A.2 System Description

The data acquisition system used for this research project can be identified by four fundamental components: (1) signal generator, (2) analog/digital converter, (3) micro computer and (4) data acquisition system software. The test information flows sequentially from the signal source through the A/D converter and into the micro computer. It is then read by the system software, stored and displayed to the user.

The signal generator component of the system represents the experimental testing apparatus and includes the test machine and strain gages, together with any other sensing equipment. These devices output analog signals that are functionally representative of the load, deflection and strain information.
generated during the test. Using a micro computer, this information can be quickly and efficiently recorded. However, before the test information can be read by a micro computer it must first be converted from its continuous analog form into a discrete digital form.

The analog/digital board or A/D board is responsible for converting the analog signals generated by the test equipment into digital signals that the micro computer can record. The digital signals that exit the A/D board are then read by the computer and ultimately stored as a data file. As with the analog signals exiting the test equipment, the digital signals exported from the A/D board are not randomly generated, but functionally related to the load, deflection and strain data generated during testing. The transformation functions used to convert load, deflection and strain DC voltage signals into quantities of kips, inches and % strain respectively, were determined during the calibration process.

The data acquisition software provides the interface between the user and the data acquisition system. Specifications detailing how the signal generator, A/D board and micro computer interact with each other are prescribed by the software. For example, the software tells the A/D board on what channels the strain, load and displacement signals are coming in and what maximum and minimum values are expected.

The complete data acquisition system includes the four basic components mentioned above together with a bridge completion board and multiplexing board. The bridge completion board is necessary for use with strain gages and
the multiplexing board increases the number of channels that the A/D board is capable of handling.

A detailed description of the individual system components is given in the following sections.

A.3 Component Description

The following is a description of the signal generator, A/D board, microcomputer and system software. Not included in this section is a detailing of the strain gage instrumentation. This will be presented in Section A.4.

Signal Generator

The signal generator component of the data acquisition system represents the test machine and strain gages. The test machine used in the lab was an Instron Model 1335. [33] From this device the load and deflection signals were directly input to the A/D board. Strain measurement was accomplished through the use of 350 ohm bonded resistance strain gauges. These devices are not able to export an analog signal directly into the A/D board. The strain signal first passed through a signal conditioning board which provided the electronics necessary to produce an analog strain signal that can be input directly into the A/D board.
Testing machine

An Instron Model-1335 testing machine was used for testing all samples. The machine outputs a full scale load and full scale deflection signal of +/- 10 volts, respectively. Two different load cells are available providing full scale load capability as either +/- 5 kips or +/- 200 kips. The load cell measures the force applied to the sample using resistive Wheatstone bridges, from which the output signal is generated. Full scale deflection capability was +/- 2.5 inches. A linear Variable Displacement Transducer (LVDT) is mounted inside the actuator to provide the displacement signal. The load and displacement voltage signals are proportional to the applied force and actuator travel, respectively.

The driving functions applied to the test specimen are controlled and monitored through an electronic console. The +/- 10 volt load and displacement signals are all output from the electronic console via three separate coaxial cables. From the electronic console panel, a test can be executed monotonically or dynamically using programmed load, displacement or strain rates.

Analog/Digital Board

The Analog-to-Digital board converts the original analog signals generated by the test machine and strain gages into computer-readable digital form. The A/D board is mounted directly into an expansion slot inside the micro computer. All connections are made through a 50 pin connector that is attached to the board and extends out the rear of the computer. Wiring
connections to the A/D board are made through a separate external screw terminal board.

A MetraByte DAS-20 A/D board was used. [34] The DAS-20 can receive analog input voltages as either single ended or differential. Differential inputs use a separate positive and negative terminal for each channel. Single ended inputs have a single ground return common for all channels. All connections made to the DAS-20 board were made as differential inputs. The DAS-20 can accept 8 separate differential inputs. However, capacity can be expanded with the use of a multiplexing board. A multiplexing board is simply a switch arrangement that allows many input channels to be serviced by one output channel. Using an OMEGA Engineering Inc. EXP-20 multiplexing board [35], an additional 16 channels of differential input were added to the capacity of the DAS-20, increasing it's total to 23. The EXP-20 board is connected directly to the system through the screw terminal board. The channel input connections for the DAS-20 are shown in Figure A.1.

![Diagram](image)

**Figure A.1** A/D Board Channel Connections
All analog load and deflection signals are input directly to the A/D board through the + and - terminals shown in Figure A.1.

The DAS-20 provides 12 bit resolution. A 12 bit system provides 1 part in 4096 ($2^{12}$) or approximately .025% of full scale. The DAS-20 accepts incoming analog voltage on any one of 6 selectable full scale voltage input ranges (bipolar ranges: +/- 10 volts, +/- 5 volts, +/- .05 volts, unipolar ranges: 0-10 volts, 0-1 volt, and 0-.1 volts). Thus, the board voltage resolution ranges from a maximum of:

$$\frac{20 \text{ volts}}{4096 \text{ parts}} = .00488 \text{ volts} = 4.88 \text{ millivolts} \quad (A.1)$$

...to a minimum of:

$$\frac{.1 \text{ volts}}{4096 \text{ parts}} = .0000244 \text{ volts} = 24.4 \text{ microvolts} \quad (A.2)$$

When an input signal change is smaller then the system's set voltage resolution, that event goes undetected.

The full-scale input range for each of the DAS-20's input channels and those of the EXP-20 multiplexing board can all be set independently.

**Computer**

A DEC Station 316sx PC computer was used to support the A/D board and data acquisition software. [36] The station is IBM PC/XT /AT-compatible and features a 16MHz Intel 80386SX microprocessor. Three 16-bit standard
expansion slots are available, one of which was occupied by the DAS-20 A/D board.

System Software

Integration of the signal generators, A/D board and micro computer into a single smoothly functioning data acquisition unit is performed by the data acquisition software. The software used for this purpose was Lab Tech Notebook. [37] Lab Tech Notebook is completely menu driven and requires no programming interface from the user.

A.4 Strain Gage Instrumentation

Strain measurement was accomplished with the use of 350 ohm bonded electrical resistance strain gages. [38] In an unstained state the resistance of the gage is constant. As the material to which the gage in applied strains, the gage stretches and its resistance value changes. The amount of strain causing the gage to stretch is directly proportional to the change in resistance of the gage. Employing this basic relationship, a Wheatstone bridge circuit was used to measure the change in gage resistance and thus provide strain measurement capability.

Each strain gage used during testing was installed as the single active arm of a quarter bridge Unbalanced Wheatstone Bridge circuit. The output voltage obtained from the quarter bridge circuit is a function of the change in resistance of the active gage only, and is therefore directly related to the strain applied to the gage. The dimensionless relationship between these two
quantities is called the gage factor and is mathematically expressed in the following relationships:

\[ F = \frac{(dR_g/R_g)}{(dL/L)} = \frac{dR_g/R_g}{\epsilon} \]  
\[ \epsilon = \frac{4 (E_0/E)}{F \left(1 - (2E/E_0)\right)} \]

where:

- \( F \) = gage factor = constant = 2.0
- \( R_g \) = initial gage resistance (ohms)
- \( dR_g \) = change in gage resistance (ohms)
- \( L \) = original gage length (in)
- \( dL \) = change in gage length (in)
- \( \epsilon \) = strain (in/in)
- \( E \) = excitation voltage
- \( E_0 \) = bridge output voltage

When a single active gage is connected to the Wheatstone bridge with only two wires both wires of the gage are in series in the same leg of the bridge circuit. The result of this arrangement, is that temperature induced resistance changes in the lead wires contribute to \( dR_g \) in Eq. A.3 and are, thus, manifested as apparent strains.

Lead wire resistance changes, \( R_L \), can be eliminated in single active gage Wheatstone bridge circuits by using a three wire arrangement in connecting the gage to the circuit. In this case a third lead wire is brought out from one of the gage terminals and installed as shown in Figure A.2. Assuming all lead
wire resistance values $R_L$ are nominally equal, the same amount of leadwire resistance is connected in series with both the active gage and resistor $R_2$. As such, the effects of $R_L$ are algebraically nulled and the bridge balance is maintained for any value of $R_L$.  

\[
R_l = R_g + dR_g
\]

![Figure A.2 Three Wire Active Gage Circuit]

The three-wire active gage Wheatstone bridge circuit shown in Figure A.2 was used in all strain gage instrumentation. The output voltage from the circuit, $E_0$, is in analog form and can now be directly input to the A/D board through the + and - terminals as shown in Figure A.1.

**Shunt Calibration**

The change in strain experienced by the active gage as it stretches can be artificially introduced using shunt calibration resistors. Using shunt resistors of predefined resistance values, specific quantities of strain can be artificially simulated, providing a corresponding output voltage $E_o$ from the system.
For different shunt resistor values, different voltage-strain points are generated and the system voltage-strain calibration determined.

Figure A.3 shows the shunt calibration circuit for a single active gage (quarter bridge) Wheatstone bridge circuit.

![Figure A.3 Shunt Calibration for Single Active Gage Circuit](image)

Note: $R_g = R_2 = R_3 = R_4$

When the calibration resistor, $R_c$, is shunted (switch is closed) across $R_1$, the net resistance value of the bridge arm, $R_n$, becomes:

$$R_n = \frac{R_g R_c}{R_g + R_c}$$  \hspace{1cm} (A.5)

The corresponding change in the bridge arm resistance before and after shunting is thus:

$$dR = R_n - R_g = \frac{R_g R_c}{R_g + R_c} - R_g$$  \hspace{1cm} (A.6)
or:

\[
\frac{\text{d}R}{R_g} = \frac{R_g}{R_g + R_c} \quad (A.7)
\]

Recalling from Eq. (A.3) the relationship between change in resistance and strain is expressed through the gage factor \( F \) and can be written as:

\[
\frac{\text{d}R}{R_g} = F \varepsilon \quad (A.8)
\]

Substituting Eq. A.8 into Eq. A.7:

\[
F \varepsilon = \frac{R_g}{R_g + R_c} \quad (A.9)
\]

Solving Eq. A.9 for the simulated strain, \( \varepsilon \), and the corresponding shunt calibration resistance, \( R_c \), gives the following relationships:

\[
\varepsilon = \frac{R_g}{F (R_g + R_c)} \quad (A.10a)
\]

\[
R_c = \frac{R_g}{F \varepsilon} - R_g \quad (A.10b)
\]

The substitution of a value for \( R_c \) into Eq. A.10a provides a corresponding simulated strain value, \( \varepsilon \), and also a circuit output voltage \( E_0 \) is generated from the system (Figure A.3). Thus, for each value \( R_c \) substituted into Eq. A.10a, a unique point with coordinates (voltage, strain) is defined.
A shunt calibration circuit providing 8 different values for $R_C$ was built for strain gage calibration. The values of the 8 shunt resistors and their corresponding simulated strains (according to Eq. A.10a) are given in Table A.1.

Table A.1 Shunt Calibration Resistors and Strains

<table>
<thead>
<tr>
<th>Gage (#)</th>
<th>$R_C$ (ohms)</th>
<th>$\varepsilon$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>349650</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>174560</td>
<td>0.10</td>
</tr>
<tr>
<td>3</td>
<td>87150</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>43400</td>
<td>0.40</td>
</tr>
<tr>
<td>5</td>
<td>34650</td>
<td>0.50</td>
</tr>
<tr>
<td>6</td>
<td>17150</td>
<td>1.00</td>
</tr>
<tr>
<td>7</td>
<td>14880</td>
<td>1.15</td>
</tr>
<tr>
<td>8</td>
<td>11880</td>
<td>1.43</td>
</tr>
</tbody>
</table>

A rotary switch was used to select individual values of $R_C$. Upon selection of a shunt resistor $R_C$, the circuit shown in Figure A.3 output a corresponding voltage of magnitude $E_0$. Thus, the shunt calibration circuit provided a total of 8 unique strain, voltage points for each strain gage. Using these 8 points, a linear regression was performed to determine the gage's calibrated strain/voltage slope and offset.

Bridge Completion Board

A bridge-completion/calibration board (BCB) was built providing the three wire bridge completion circuit shown in Figure A.2 and the shunt calibration circuit shown in Figure A.3. The BCB was designed to provide bridge completion and calibration for a total of 16 strain gages. This section details the BCB.
The analog voltage output from the Wheatstone bridge circuit, $E_0$, is directly related to strain, $\varepsilon$, as defined by Eq. A.4 or through a shunt calibration process. For data acquisition, this signal is input directly into the A/D board through the + and - terminals of either the EXP-20 multiplexing board, or the STA-U screw terminal board, as was shown in Figure A.1. Thus, strain measurement using electrical bonded strain gages requires a Wheatstone bridge circuit for each individual gage.

Strain gage bridge completion was provided using commercially available Bridge Completion Module (BC). [39] The modules provide the Wheatstone bridge circuitry shown in Figure A.2 in a compact miniaturized form. An individual module is capable of providing bridge completion for a single active gage in either a two or three wire arrangement. A sample module and its wiring arrangement are shown in Figure A.4.

![Bridge Completion Module](image)

**Figure A.4 Bridge Completion Module**
A total of 16 individual Bridge Completion Modules were wired together and installed on the BCB providing 16 strain gage channel capacity for the data acquisition system. The wiring of each BCM is identical to that shown in Figure A.4.

As mentioned previously, a rotary switch was used to select one of the 8 shunt calibration resistors shown in Table A.1. A second rotary switch was used to select the strain gage channel on which a calibration was to be performed. Both of these rotary switches together with the 8 shunt resistors were installed on the BCB. The bridge completion board in its entirety is shown in Figure A.5.
Figure A.5 Bridge Completion Board (BCB)
Strain Gage Calibration and Accuracy (noise and linearity)

The voltage ($E_o$) vs. strain ($\varepsilon$) calibration curve for each strain gage used in testing was derived from a linear regression performed on 9 points of known voltage/strain coordinates. The 9 known strain values were selected from the 8 shunt calibration strain resistors given in Table A.1 together with a zero strain condition. The corresponding output voltage ($E_o$) at a given constant strain was read by the data acquisition system. For each constant strain point, the output voltage ($E_o$) was taken to be the average of 200 voltage readings. This was accomplished by running a data acquisition for 10 seconds at a sampling rate of 20 Hz. From the resulting 9 strain/voltage points a linear regression determine the calibrated slope ($m=\varepsilon/volt$) and offset ($b=\varepsilon$). The measured strain values recorded during testing were then calculated as:

$$\text{strain} = m\ (E_o) + b \quad (A.11)$$

where:

\begin{align*}
    m &= \text{strain/voltage slope from linear regression} \\
    b &= \text{offset strain from linear regression} \\
    E_o &= \text{voltage measured during test}
\end{align*}

The existence of low voltage electronic noise generated from machinery such as lights, computers, power supply etc. is impossible to eliminate. Because the DAS-20 is very sensitive to low voltages, the electronic noise signals, if
strong enough, can be recorded together with the strain gage output signal as $E_0$. The resulting voltage reading ($E_0$) is thus:

$$E_0 = (E_{0\text{strain}}) + (E_{0\text{noise}}) \tag{A.12}$$

where:

- $(E_{0\text{strain}}) = \text{voltage output from BCB for strain gage}$
- $(E_{0\text{noise}}) = \text{voltage from random electromagnetic noise signals}$
- $(E_0) = \text{voltage read by DAS-20 A/D board}$

The relative amount of electronic noise in the system was measured during calibration as the difference between the maximum and minimum voltages taken over the 200 voltage points recorded at a constant shunt calibration strain point "i" divided by the full scale voltage range. Full scale was taken to be the difference in average voltage readings between 1.43% strain and 0% strain. This relationship can be written as:

$$\% \text{ noise} = \frac{(E_{0i})_{\text{max}} - (E_{0i})_{\text{min}}}{(E_0)_{\varepsilon=1.43\%} - (E_0)_{\varepsilon=0\%}} \times 100\% \tag{A.13}$$

where:

- $i = \text{strain from Table A.1}$
- $(E_{0i})_{\text{max}} = \text{max voltage over 200 points recorded at } \varepsilon = i \text{ strain}$
- $(E_{0i})_{\text{min}} = \text{min voltage over 200 points recorded at } \varepsilon = i \text{ strain}$
- $(E_0)_{\varepsilon=1.43\%} = \text{avg. voltage over 200 points recorded at } \varepsilon = 1.43\% \text{ strain}$
- $(E_0)_{\varepsilon=0\%} = \text{avg. voltage over 200 points recorded at } \varepsilon = 0.00 \text{ strain}$
The results showed a voltage drift in the system that was very low, generally on the order of .5 to .8 % of full scale (quite good). A sample calibration sheet showing the linear regression calibration process and % noise readings is given in Table A.2.

Table A.2 Sample Strain Gage Calibration Sheet

<table>
<thead>
<tr>
<th>Shunt Strain (10^{-2})</th>
<th>V-avg. (volts)</th>
<th>dV (volts)</th>
<th>Noise (% full scale)</th>
<th>Strain = mx+b (10^{-2})</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.00067</td>
<td>.00112</td>
<td>.1677</td>
<td>.00115</td>
<td>-</td>
</tr>
<tr>
<td>.05</td>
<td>.00316</td>
<td>.00017</td>
<td>.2376</td>
<td>.05075</td>
<td>1.5</td>
</tr>
<tr>
<td>.1</td>
<td>.00563</td>
<td>.00014</td>
<td>.1957</td>
<td>.10016</td>
<td>.17</td>
</tr>
<tr>
<td>.2</td>
<td>.01062</td>
<td>.00014</td>
<td>.1957</td>
<td>.19950</td>
<td>-.25</td>
</tr>
<tr>
<td>.4</td>
<td>.02060</td>
<td>.00014</td>
<td>.1957</td>
<td>.39864</td>
<td>-.34</td>
</tr>
<tr>
<td>.5</td>
<td>.02561</td>
<td>.00017</td>
<td>.2376</td>
<td>.49839</td>
<td>-.32</td>
</tr>
<tr>
<td>1.0</td>
<td>.05084</td>
<td>.00014</td>
<td>.1957</td>
<td>1.0013</td>
<td>.14</td>
</tr>
<tr>
<td>1.43</td>
<td>.07221</td>
<td>.00014</td>
<td>.1957</td>
<td>1.4275</td>
<td>.18</td>
</tr>
</tbody>
</table>

Regression Output
Slope = 19.9375 %/volt
offset = -.0122 %
R squared = .9998

The results from the calibration process demonstrated that the gages performed almost perfectly linear between 0 strain and 1.43x10^{-2} strain. An additional shunt calibration resistor with a shunted strain value of 2.93x10^{-2} was installed on the BCB to check the gage linearity up to a strain value of 2.93x10^{-2}. The results showed that although not perfectly linear at 2.93x10^{-2} strain, the strain reading recorded using Eq. A.10a was only 2.8% in error as compared to the shunted value.
A.5  System Configuration and Wiring

The complete data acquisition system componentry and inter component wiring is shown in Figure A.6.
Figure A.6 Data Acquisition Componentry and Wiring
APPENDIX B

DEFLECTION ANALYSIS

B.1 Introduction

The load point deflection for a simply supported beam subjected to two equal concentrated loads of magnitude P/2 symmetrically placed a distance "a" from the supports is given as:

\[ y_a = \frac{3La - 4a^2}{6Ec} \left( \frac{M_a}{I_e} \right) \]  
(B.1)

where:

\[ y_a = \text{deflection at load point} \]
\[ L = \text{span length} \]
\[ Ec = \text{concrete elastic modulus} \]
\[ I_e = \text{effective moment of inertia} \]
\[ a = \text{shear span} \]
\[ M_a = \text{moment at load point} = (P/2)a \]

Two methods are presented for calculating deflection according to the above equation: (1) \( I_e \) is calculated according to the Branson equation, (2) according to a moment-curvature analysis, where \( (M_a/EcI_e) \) in Eq. B.1 is replaced with \( \epsilon_c/c \).
B.2 Branson Equation

The incidence of cracking creates a situation where member stiffness ($E_c I_e$) is changing continuously. Recognizing that $E_c$ remains constant, $I_e$ in Eq. B.1 is calculated either as a function of the gross uncracked section dimensions or the transformed cracked-section properties. The Branson Equation employs an exponential curve fitting of the cracked and uncracked section modulus for calculating $I_e$ of the cracked transformed section. [12] [54] Referring to Figure B.1, the effective moment of inertia, $I_e$, is calculated according to the Branson Equation as follows: [54]

\[ I_e = I_g \] for $M < M_{cr}$  \hspace{1cm} (B.2a)

\[ I_e = \left[ \frac{M_{cr}}{M_a} \right]^m I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^m \right] I_{cr} \] for $M > M_{cr}$  \hspace{1cm} (B.2b)

where:

Figure B.1 Uncracked and Cracked Section Properties

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\[ I_g = \text{moment of inertia of the gross uncracked cross section about the horizontal centroidal axis, neglecting reinforcement.} \]

\[ M_{cr} = \text{cracking moment} = \frac{f_t I_g}{y_t} \] \hspace{1cm} (B.3)

- \( f_t = \text{concrete modulus of rupture} = \text{Eq. 3.4} \)
- \( y_t = \text{distance from neutral axis to extreme fiber in tension} = \frac{h}{2} \)

\[ M_a = \text{moment existing at load point} = \left( \frac{P}{2} \right) a \]

\[ n = \text{modular ratio} = \frac{E_r}{E_c} \]

\[ A_t = \text{transformed area of reinforcement} = n A_r \]

\[ c = \text{depth of compression block} = \frac{\sqrt{A_t^2 + 2b d A_t - A_t}}{b} \] \hspace{1cm} (B.4)

\[ I_{cr} = \text{moment of inertia of the cracked transformed section} \]
\[ = \frac{1}{3} b c^3 + A_t (d - c)^2 \] \hspace{1cm} (B.5)

\[ m = \text{transition constant} \]

Although the ACI code considers \( m=3 \) as constant, [12] Branson recommends a value of \( m=4 \) for use in the analysis of simply supported beams. [54] Consistent with Branson’s recommendation, a value of \( m=4 \) was used in Eq. B.2 for deflection calculations in this thesis.
The relationship between the bending moment at the load point, $M_a$, and the corresponding concrete stress on the extreme fiber, $(f_c)_a$, is expressed as:

$$(f_c)_a = \frac{M_a}{I_e} = E_c (\epsilon_c)_a$$

or

$$\frac{M_a}{E_c I_e} = \left( \frac{\epsilon_c}{c} \right)_a = \frac{1}{R_a}$$ \hspace{1cm} (B.6)

where:

- $R_a$ = radius of curvature at the load point
- $(\epsilon_c)_a$ = concrete strain on the extreme compression fiber at the load point
- $(c)_a$ = depth of the compression block at the load point

Substituting Eq. (B.6) into Eq. (B.1), deflection at the load point is calculated as a function of $\epsilon_c$ and "c" as:

$$y_a = \frac{1}{6}(3L_a - 4a^2) \left( \frac{\epsilon_c}{c} \right)_a$$ \hspace{1cm} (B.7)

Where "c" is calculated as a function $\epsilon_c$ as described in Section 4.2. The application of Eq. (B.7) implies that the beam is cracked along its entire length and that, like $(M/E_c I_e)$, $(\epsilon_c/c)$ decreases linearly to zero at the support point. In reality only part of the beam length is uncracked at a given load and, thus, the $(\epsilon_c/c)$ diagram is discontinuous at a distance corresponding to the cracking moment. The uncracked beam length can be calculated as the distance where the
applied moment is less than the moment at which cracking of the concrete occurs, $M_{cr}$. This relationship is shown in Figure B.2.

![Figure B.2 Cracked and Uncracked Beam](image)

Figure B.2 Cracked and Uncracked Beam

Referring to Figure B.2, for a given load $P > P_{cr}$ the uncracked length of the beam is calculated using similar triangles as:

$$x_{ucr} = \frac{2M_{cr}}{P} \text{ for } P > P_{cr}$$  \hspace{1cm} (B.8)

The deflection at any point along the beam can now be determined using the Moment Area method and the $\frac{M}{EI}$, or $\frac{\varepsilon_{c}}{c}$ diagram shown in Figure B.2. First, the following coordinates on the vertical axis are calculated:
Referring to the diagram in Figure B.2, the deflection at the load point is calculated as:

\[ y_a = M^1_A + M^2_A + M^3_A + M^4_A - M^1_B \]  

(B.11)

where:

- \[ M^1_A = \text{moment of area 1 about point } A = \frac{1}{8} \frac{\varepsilon_c}{c} (L^2 - 4a^2) \]
- \[ M^2_A = \text{moment of area 2 about point } A = \frac{1}{2} \frac{\varepsilon_c}{c} (x_{ucr} a - \frac{x_{ucr}^3}{a}) \]
- \[ M^3_A = \text{moment of area 3 about point } A = \frac{1}{6} \frac{\varepsilon_c}{c} (a - x_{ucr})^2 \left( \frac{x_{ucr}}{a} + 2 \right) \]
- \[ M^4_A = \text{moment of area 4 about point } A = \frac{1}{3} \left( \frac{\varepsilon_c}{c} \right)_{ucr} (x_{ucr})^2 \]
- \[ M^1_B = \text{moment of area 1 about point } B = \frac{1}{2} \frac{\varepsilon_c}{c} \left( \frac{L}{2} - a \right)^2 \]

The application of Eq. B.11 is made in conjunction with a Bilinear concrete model. The analysis procedure begins by assuming a value for concrete strain, \( \varepsilon_c \). Then using the techniques outlined in Section 4.2, a corresponding internal resisting moment is calculated. This moment is balanced from an external load \( P \). Finally using Eq. (B.11) the deflection for that magnitude of load is calculated.
APPENDIX C

FINITE ELEMENT MODELING

C.1 Introduction

The implementation of the finite element method in the study of reinforced concrete has been an accepted method of analysis for 20 years. Owing to this history, a significant quantity of reference data has been generated. Although this volume of information is extensive, it is by no means comprehensive or complete. This is especially true for reinforced concrete, where modeling the composite behavior is complicated owing to concrete's nonhomogeneity, the low strength of concrete in tension, the sudden nonlinear properties of concrete (cracking) and the difficulty in modeling force transfer and bond properties. The existence of these conditions constitute a nonhomogenous, anisotropic body and, thus, render the direct application of classical continuum mechanics in its study difficult.

As a result, a more empirical approach is necessary in modeling the material and boundary conditions that define the composite behavior existing within reinforced concrete. Based upon the results of extensive laboratory testing, material models have been developed that predict the behavior of concrete as it strains both elastically and plastically. This information is then used in developing failure envelopes from which the limits of the yield and post yield states are predicted.
The finite element analysis of reinforced concrete must inevitably address the issue of scale versus accuracy, and how one affects the other. This condition has forced the divergence of the FE techniques in two different directions: 1) the discrete crack modeling technique, and 2) the smeared crack modeling technique. The first method accounts for the components of concrete and reinforcement separately, with each material being represented by different elements. In this discrete-element approach, the reinforcing elements are connected to the concrete at nodal points using special link or bond elements. The resulting model attempts to incorporate mechanics that can simulate cracking and bond capability so that the stresses in the vicinity of a crack can be computed.

The second method is known as the composite element approach or smeared approach and assumes perfect bond exists between the concrete and reinforcement, thus, eliminating the need for bond elements at the material interfaces. Using this approach, the thickness of the member is generally divided into a number of layers. The reinforcing is now modeled as an isotropic solid layer(s) bound to the concrete above and below by compatible nodal point geometry. This is analogous to smearing the reinforcing through the plane in which it is lying. The resulting model approaches the analysis on a more global scale, attempting to model zones of cracking and determine how crack development effects overall structural behavior.

C.2 3 Dimensional Model

Preliminary work has been initiated on constructing a three dimensional FE model for NEFMAC reinforced concrete beams. Using the program ADINA
[60] a discrete element approach was used in constructing a FE mesh representing NEFMAC grid reinforced concrete beam. The model makes use of symmetry in the longitudinal and transverse directions so that only 1/4 of the sample need be modeled for analysis. Figure C.1 identifies the proposed FE model relative to the full scale section.

![Beam Section Modeled For Finite Element Analysis](image)

**Figure C.1** Beam Section Modeled For Finite Element Analysis

In modeling the individual components of the beam, the vertical direction (z) was divided into three different layers in the xy plane. The first layer represents the thickness of the concrete cover and is composed of all concrete elements. The second layer represents the thickness of the NEFMAC and is composed of both NEFMAC and concrete elements. The third layer
represents the concrete above the plane in which the reinforcement is located. Figure C.2 identifies the elements in each of the three vertical layers.

![Diagram of FE Model Element Groups](image)

**Figure C.2 FE Model Element Groups**

All NEFMAC and concrete elements are 20 noded and 3 dimensional. In describing the nodal point topology, concrete elements in layers 1 and 2 are joined together by common nodes at the interface. This is also the case for concrete elements in layers 2 and 3. Using a discrete element approach, nodal
points are not common between NEFMAC and concrete elements. Referring to Figure C.3, the physical interface of concrete and NEFMAC is modeled using different nodal points for each of these two material groups.

![Figure C.3 Nodal Point Topology at Material Interface](image)

Concrete elements are separated from NEFMAC elements using a contact surface. The contact surface is a 3 dimensional element that allows the transfer of compression and shear forces. In doing so it is able to model bearing and bonding or friction as mechanisms for force transfer. The resulting FE model is restrained by rollers on both planes of symmetry (xy and yz) as well as at the support nodes.

The concrete material model supported by ADINA identifies the material's behavior as existing within one of the following five physical states: 1) linear tensile, 2) tensile failure from cracking, 3) linear compression, 4) nonlinear...
compression, and 5) compression failure by crushing. Concrete failure criteria are governed by tensile cracking and compression crushing failure envelopes. The general triaxial constitutive relationships required for describing the material's behavior during each of the five mentioned physical states are derived from basic uniaxial stress-strain relations together with a triaxial failure envelope that is input in terms of 24 discrete stress ratios. A typical triaxial failure surface for normal weight concrete was used for inputting the stress ratios. [61] A linear elastic isotropic material model was used in representing NEFMAC elements.

Execution of the analysis was met with limited success. The model was very unstable relative to an energy convergence criteria, and as a result, only very low stress levels were achieved in the NEFMAC reinforcement. This result was reached invariably the consequence of out-of-balance loads being greater then incremental loads after the iteration limit was reached. Increasing the iteration limit was not a solution. Numerically, the model was not able to reestablish a state of equilibrium between the internal and external forces. This equilibrium condition is expressed as follows:

\[ t^{+dt}R - t^{+dt}F = 0 \]  

(C.1)

where: 
- \( t^{+dt}R \) = the vector of externally applied loads at time step \( t^{+dt} \)
- \( t^{+dt}F \) = the nodal point force vector at time step \( t^{+dt} \)

Because the solution of nonlinear problems employs an incremental strategy (over \( t \)) based upon iterative methods (over \( i \)), the equality of Eq. (C.1) is replaced with an inequality that measures equilibrium relative to an
acceptable convergence criterion or "energy tolerance". The energy convergence criteria then requires equilibrium between the internal nodal point force vector and the externally applied load vector to be within the designated "energy tolerance" before continuing on to the next time step. Thus, the program uses a relative measure of force equality in determining when equilibrium is satisfied.

The results indicated the model became numerically very unstable soon after cracking. The incidence of cracking creates sudden nonlinearities and significant stress redistributions take place in the concrete for each increment in applied load. This is especially true for NEFMAC reinforced beams, where the low modulus of the material results in a radical change in flexural stiffness during cracking. As a result cracks propagate very quickly deep into the beam resulting in a very small thickness of the compression zone. Thus, the amount of reinforcement provided in the model will likely effect the ability of the solution to satisfy convergence criterion. The analysis run modeled NEFMAC sample 1B2 which was reinforced at 186%ρ_b. The cracked moment of inertia for this sample is only 1/11-th that of the gross section. For steel reinforced sample 1B3 the cracked moment of inertia was 1/3.4 that of the gross section, significantly greater. Perhaps increasing to amount of NEFMAC reinforcement will allow for better energy convergence at higher loads. Also, the mesh was refined by subdividing layer 3 into additional elements through its thickness. This was, however, unsuccessful in rectifying the convergence problem.
C.3 2 Dimensional Model

A 2D analysis might prove more successful and perhaps serve as a barometer for mesh refinement. Eliminating the transverse direction would provide the basis for using a smeared approach, and therein, significantly reducing the computational effort required for calculating a solution. Figure C.4 shows a hypothetical 2D FE model.

![2D Finite Element Model](image)

**Figure C.4 2D Finite Element Model**

For a 2D analysis concrete elements would be modeled using 8 noded, plane stress, isoparametric elements. The use of 8 noded elements allows for a second order displacement in describing the elements strained geometric shape. The smeared approach requires identifying the reinforcement with the same nodes as the concrete. The NEFMAC bars are modeled using 1D truss elements with 3 nodes. This arrangement provides displacement compatibility with the concrete elements above and below them. The truss element assumes that the normal stress is constant over the entire cross sectional area, and that during deformation the area itself remains constant. It is likely that such an analysis could prove valuable at predicting the global flexural performance of a NEFMAC reinforced beam's relative to load-deflection behavior. The 2D analysis is, however, limited in its ability to
model force transfer in the transverse direction. The bearing force against the transverse bar must be physically represented in three dimensional space.

Future work in 3D should absolutely maintain a discrete element approach in modeling the NEFMAC grid. Perhaps a different mesh arrangement would allow for a smoother redistribution of strain energy that is released with cracking and therein render the model numerically more stable.