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Study of cross-tail current carriers in magnetotail

Chen Lu
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Study of cross-tail current carriers in magnetotail

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University of New Hampshire, 1993
STUDY OF CROSS-TAIL CURRENT CARRIERS
IN
MAGNETOTAIL

BY

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DISSERTATION

Submitted to the University of New Hampshire
in Partial Fulfillment of
the Requirements for the Degree of

Doctor of Philosophy
in
Physics

December, 1993
This dissertation has been examined and approved.

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This dissertation is dedicated to

my parents Xian Lu and Rongfang Wang

my husband Guang Yang

my daughter Deedi
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ABSTRACT

STUDY OF CROSS-TAIL CURRENT CARRIERS
IN
MAGNETOTAIL

by
Chen Lu
University of New Hampshire, December, 1993

The purpose of this study is to gain physical insight into how charged particles, which violate the guiding center approximations, contribute to cross-tail current in a self-consistent plasma sheet. A technique to generate self-consistent one-dimensional (1-D) current sheets is described. Groups of monoenergetic protons are followed in a model magnetic field. The sample current sheets are characterized by resonant quasiadiabatic and stochastic orbits. The magnetic moment of a quasi-adiabatic ion which is injected from outside a current sheet changes substantially during an interaction with the current sheet, but returns to almost its initial value by the time the ion leaves. The resonant nature of the interaction is associated with a strong energy dependence. The magnetic moment of a stochastic ion changes substantially during an interaction with the current sheet. Several ion and electron groups are combined to produce a plasma sheet in which the charged particles carry the currents needed to generate the magnetic field in which the orbits are traced. An electric field also is required to maintain charge neutrality. Three distinct orbit types, one involv-
ing untrapped and two composed of trapped ions, are identified. Each class of ions carries a qualitatively different current distribution. Contributions from all three groups are needed when resonant ions are used to generate a typical quiet-time self-consistent current sheet. It was found that self-consistent current sheets can not be generated using only stochastic ions. A relatively small density of resonant protons must be added to a current sheet in which stochastic ions dominate. Distribution functions are evaluated so that the expected count rates and several fluid parameters could be plotted. Limitations associated with the use of a 1-D model also are investigated. It is found that the model can provide a good physical picture of an important component of the cross tail current. However, we conclude that 1-D models cannot adequately describe any region of the magnetotail in which the principal current sheet is separated from the plasma sheet boundary layer by a nearly isotropic outer portion of the central plasma sheet.
1.1 The Magnetosphere

The magnetic field of the Earth is very similar to a dipole. If the Earth were surrounded by a vacuum, the dipole field lines would extend out to vast distances. However, the Earth is immersed in the atmosphere of the Sun. The Sun is continuously emitting the solar wind, a highly ionized plasma that spreads out through the solar system. The pattern of the dipole field is thus significantly altered by the solar wind. The interaction of the solar wind flow and the Earth’s magnetic field creates a cavity called the magnetosphere. Many distinct plasma regions are established within it. Figure 1.1 shows the major regions of the magnetosphere in noon-midnight cross section. Figure 1.2 shows the three dimensional view. The unperturbed solar wind is shown on the left side of the picture. The bow shock is a shock front on the dayside of the Earth at the outer boundary of the magnetosphere. The boundary of the magnetosphere is called the magnetopause. The region between the bow shock and the magnetopause is the magnetosheath. The cylindrical part of the anti-sunward magnetosphere, which is dragged out by the solar wind, is called the magnetotail.

When the solar wind plasma connects the interplanetary magnetic field (IMF) to the Earth’s magnetic field, it produces open field lines with only one end on the Earth. These field lines extend far into the magnetotail and form the northern and southern tail lobes. These open field lines also cross the magnetopause, where they connect to the IMF in the magnetosheath. The field lines that reach down the magnetotail have opposite directions on the northern and southern sides of the equatorial plane. A current sheet therefore must exist to separate the northern and southern tail lobes. This is the so-called neutral sheet.
where the open field lines may reconnect to form closed field lines that are convected back to the Earth.

1.2 Geomagnetic Activity and the Magnetospheric Substorm

Variations of the Earth's magnetic field were first observed in the nineteenth century as a sudden increase followed by a slow decrease of the geomagnetic field, and were called geomagnetic storms. These geomagnetic storms were sometimes found to occur one or two days after a large solar flare. Further evidence for a link between the two phenomena came with the discovery of an 11-year periodicity in both solar and geomagnetic activities.

A typical magnetospheric storm consists of three phases, called storm sudden commencement (ssc), initial phase and main phase. Such storms begin when the interplanetary shock wave reaches the magnetosphere and compresses it. This sudden compression is responsible for the ssc. Its effect is most clearly recognized in geomagnetic field variations as a step-function-like increase on the ground and in the magnetosphere. After the ssc, there are a few hours of calmness, and then the geomagnetic field has a sudden decrease. The calm period from ssc to the sudden decrease is called initial phase, while the sharp decrease is referred to as the main phase. During initial phase, the magnetosphere appears to be located in the post-shock solar wind. The duration of this period varies considerably from one storm to another. It can be as short as ten minutes or it can last for more than six hours. The main phase of the magnetospheric storm begins at about the time when the shock-driven plasma reaches the magnetosphere, and is characterized by a succession of explosive processes, called magnetospheric substorms.

A magnetospheric substorm is often closely associated with a southward turning of $\mathbf{B}$ in the IMF. When the Earth is embedded in the region where $\mathbf{B}$ has a southward component, merging of the interplanetary and geomagnetic field lines takes place, and many substorms are expected to occur. A magnetospheric substorm may be considered to be the
process through which the magnetosphere tends to reduce the accumulated energy in the magnetotail. During the substorm, many phenomenon are occurring in the magnetosphere. They include an increase in auroral activity in the auroral oval region (auroral substorm), and a disturbance of the geomagnetic field at the Earth (magnetic substorm). The auroral substorm is the only visible manifestation of a magnetospheric substorm.

The magnetospheric substorm has three characteristic phases: the growth phase, the expansive phase, and the recovery phase. Different phenomena are happening in the auroral region, the polar region, and the magnetotail that correspond to these three phases. During the growth phase, the plasma sheet of the magnetotail becomes very thin and the magnetic field is stretched out more and more. At some point, a part of the magnetotail current is suddenly disrupted. This causes a field-aligned current to flow toward the morning part of the auroral oval from the magnetotail and back to the magnetotail from evening part of the oval, after flowing along the midnight part of the oval. The magnetotail current plays an important role in the auroral substorm growth phase.

### 1.3 The Cross-tail Current

As you have seen in Figures 1.1 and 1.2, the solar wind flow compresses the Earth's magnetic field on the dayside, and stretches it into the tail on the nightside of the Earth that forms a magnetotail. The structure of the magnetotail requires the presence of a current system. In the distant magnetotail, the magnetopause current flows from dusk to dawn, and the cross tail current which separates the field lines from the north pole and the south pole, flows from dawn to dusk. This cross tail current joins with the magnetopause current to form two closed circuits, one encircling each lobe of the magnetotail. If the \( B \) field of the magnetotail is known, then Ampere's law can be used to calculate the current \( j \) that must be carried by charged particles within the tail.

The cross tail current is important to a study of particle energization. As mentioned in
the last section, these currents also stretch the tail during substorm growth phase. Their
diversion forms field-aligned currents that flow to and from the ionosphere, and results in
tail collapse at substorm onset. The structure and behavior of the current sheet depends on
the charged particle population in the current sheet. In the present study, we will concen­
trate on how the charged particles carry the current that is self-consistent with the current
required by the local magnetic field.

1.4 The Motion of Charged Particles

The magnetosphere is full of low energy plasma. The solar wind can be viewed as one
of the particle sources, depositing charged particles into the magnetosphere where they are
energized, accelerated down into the ionosphere, and produce auroras during a magneto­
spheric substorm. The motion of plasma ions and electrons are determined by the local
electric and magnetic fields. Particle motion can be described by specifying the guiding
center drift motion as long as the gyro radius of the particle is much smaller than any gra­
dient scale length and the gyro period is far shorter than the time scale of any electric mag­
netic field variation. These approximations are valid within six Earth radii (R_E) of Earth
where the Earth’s magnetic field is still approximately dipolar and the magnetic moment
(first adiabatic invariant \( \mu = W / B \)) is nearly conserved. Guiding center drift motion
includes \( \mathbf{E} \times \mathbf{B} \) drift, polarization drift, curvature drift, gradient drift, and drift produced
by other external forces. The resulting electric current is not simply the sum of the above
drift currents; the magnetization current must be added.

When the particle gyroradius becomes large compared to the gradient or the radius of
curvature of the magnetic field (e.g. high energy particles in neutral sheet), then particle
motion becomes nonadiabatic. It is also true that the fluid description can break down in
the vicinity of current sheets where the gradient scale length can be on the order of or even
smaller than the particle gyroradius. The particle guiding center motion description is
invalid inside these regions, and an exact orbit calculation is needed to determine the trajectory of a particle. The motion of a single particle in the current sheet becomes important and needs to be examined in order to understand the structure and behavior of the current sheet.

1.5 Previous Studies

Since the magnetic field in the real tail is very complicated, the study of the motion of particles used to be performed in varied magnetic field models. In this thesis, concentration is on the motions of single particles in the model plasma sheet, and the calculation of current that the particles carry as they generate a self-consistent model plasma sheet. The structure of self-consistent plasma sheets has been studied extensively. Much of the early work was reviewed by Dungey [1972; 1975] and Schindler [1975; 1979]. Rogers and Whipple [1988] provide a good recent summary of the developments that are most relevant to our work.

Analytic solutions have been obtained in the outer part of the Central Plasma Sheet (CPS), where nearly all ions follow guiding center trajectories. A thin nonadiabatic region was included in early studies, in which complete analytic solutions could not be found. Both one dimensional and two dimensional magnetic field models were considered. One topic which has been studied extensively and is important in the sample case considered here is that one dimensional models require pressure anisotropy, which reaches the marginal fire-hose stability limit, in order to produce momentum balance. In a two or three dimensional model, some or all of the magnetic field tension can be balanced by pressure or flow gradients [Rich et al., 1972; Toichi, 1972; Kan, 1973; Cowley, 1973; 1978b; Cowley and Pellat, 1979; Schindler, 1979; Nötzel et al., 1985]. These macroscopic requirements also must be satisfied in a nonadiabatic study. The important consideration of plasma sheet stability has been addressed extensively in the above and other studies, but
will not be considered in this thesis. One feature that was recognized is the specific need for both trapped (particles that stay in the current sheet, see Chapter 2) and untrapped (particles that leave the current sheet, see Chapter 2) ions [see also Francfort and Pellat, 1967]. This division of particle groups is also used in our work.

Eastwood [1972; 1974; 1975] used individual orbit results to generate a self-consistent plasma sheet. We use essentially this same technique, although the specific cases studied and the orbit types used by Eastwood were quite different from those we consider. Swift and Allen [1987] and Burkhart et al. [1992] carried out self-consistent plasma sheet simulations which included a study of individual particle orbits.

A number of studies have followed single particle orbits to investigate the plasma sheet. The original studies by Speiser [1965] and those of Eastwood were primarily concerned with Speiser-type orbits. Wagner et al. [1979], Gray and Lee [1982], Lyons and Speiser [1982], Chen et al. [1990], Karimabadi et al. [1990], and Ashour-Abdalla et al. [1991] followed individual orbits in model plasma sheets to investigate the resulting distribution functions, but did not generate self-consistent models.

Rogers and Whipple [1988] considered cases in which \( J_z \) (\( J_z = \int v_z dz \)) is a good bounce invariant, and generated a self-consistent current sheet. The sample case used untrapped Speiser-type ions plus trapped and untrapped electrons, though both trapped and untrapped ions were included in the general derivation. Electrons and an electrostatic potential \( \phi(z) \) were used to maintain charge neutrality, and the results were iterated to produce the self-consistent field and particle distribution. Other than using the \( J_z \) invariant rather than tracing many particle orbits, the sequence of steps used by Rogers and Whipple is similar to the sequence used here. Brittnacher and Whipple [1991] and Savenkov et al. [1991] pointed out that \( J_z \) is not a good invariant for all orbits. Since quasi-adiabatic particles suffer little net change in magnetic moment, \( J_z \) appears to be conserved for these par-
ticles. However, our previous study [Kaufmann et al., 1993a] showed that there is a large net change in pitch angle and therefore in the magnetic moment of untrapped ions during a current sheet interaction for most ions in the mid magnetotail. Orbit tracing may be necessary in this region.

In this thesis, I shall start with a simple one dimension model that is commonly used for particle studies. In Chapter 2, a simple model of the tail magnetic field is discussed, and the motions of charged particles in this model field are investigated and classified. Chapter 3 will give a case study of a self-consistent current sheet formed by resonant particles. The features of stochastic particles are discussed in Chapter 4. The 3-keV particles, which are common in the magnetotail region, are used in the case studies. A method to generate self-consistent one-dimensional current sheets is also described in these two chapters. Discussions of the results from the case studies will be presented in Chapter 5. Summary and conclusions are in Chapter 6. Computational details are described in the Appendices.
Figure 1.1 Noon-midnight cross section of the magnetospheric plasma distribution.
Figure 1.2 Three dimensional view of the plasma mantle, the cleft (cusp) and the plasma sheet.
Chapter 2

Cross-tail Current Carriers in a One Dimensional Model:

An Overview

An understanding of the motion of individual charged particles in the magnetotail plasma sheet is an important first step in understanding the collective dynamics of the magnetotail plasma as a whole. In this chapter, a set of characteristic trajectories which are expected to be followed by thermal ions in the magnetotail will be defined. Our goals are to see how many distinct orbit types can be found, to gain a physical understanding of the cross-tail current distribution $j_y(z)$ carried by particles on each type of orbit, and to determine which particle groups are needed to model various portions of the magnetotail.

2.1 Tail Magnetic Field Model

The quiet time magnetotail field may be modeled, in its simplest form, by a neutral sheet magnetic profile $B_0(z)$ with a superimposed normal field $B_n \hat{z}$. We used solar magnetospheric coordinates where the x axis points from Earth to the Sun, the z axis points to the north, and the y axis points from dawn to dusk. In this thesis, a simple one dimensional modified Harris [1962] field

$$B_x(z) = B_{xo} \tanh \left( \frac{z}{L} \right)$$

$$B_y(z) = 0$$

$$B_z(z) = B_{zo}$$

is used to model various portions of the Earth magnetotail. The Tsyganenko [1989] magnetic field model (T89) was used to help select a representative value of the magnetic field constant parameters $B_{xo}$, $B_{zo}$, and the characteristic thickness of the plasma sheet $L$ in...
Equation (2.1) [Kaufmann et al., 1993a]. We include a one dimension electric field with
\( E_x = 0 \) and \( E_z (z) \) adjusted to maintain charge neutrality. The primary analysis is carried
out in a reference frame with \( E_y = 0 \). The use of such a simple model permits us to make
a thorough investigation of each orbit type and to survey a wide range of magnetotail
parameters.

The magnetic field (2.1) is not a self-consistent solution of the Vlasov-Maxwell equa­tions, but the field configuration described by Equation (2.1) does resemble the two-
dimensional analytic solution for the plasma sheet derived by Kan [1973] and the numeri­cal solution obtained by Toichi [1972]. Thus the field model in Equation (2.1) should be
adequate for examining the trajectories of test particles.

The magnetic field (2.1) was used to follow individual particle orbits when \( |z| \leq L \). The
Bulirsch-Stoer and Runge-Kutta algorithms were used to solve the equation of motion
[Press et al., 1986]

\[
\frac{dv}{dt} = \frac{q}{m} [E + v \times B]
\]

(2.2)

where \( E \) and \( B \) are the local electric and magnetic fields. The more efficient Bulirsch-Stoer
method was used whenever large groups of ions were traced. All single orbits shown in
the figures used the Runge-Kutta method, which took much smaller steps to obtain the
required accuracy.

When \( |z| > L \) we used

\[ B_x (z) = \frac{B_{x_0}}{\eta} \tanh \left( \frac{z}{L} \right) + B_{x_0} \left( 1 - \frac{1}{\eta} \right) \tanh (1) \]  

(2.3)

with \( \eta = 5 \) (Appendix B). The current required to produce the magnetic field given by
Equations (2.1) and (2.3) is
\[ j_y = \frac{1}{\mu_0} \frac{\partial B_x}{\partial z} = \begin{cases} \frac{B_{x0}}{\mu_0 L} \text{sech}^2 \left( \frac{z}{L} \right) & \text{for } |z| < L \\ \frac{B_{x0}}{\mu_0 L} \left( \frac{z}{L} \right)^{\eta-1} \text{sech}^2 \left[ \left( \frac{z}{L} \right)^{\eta} \right] & \text{for } |z| > L \end{cases} \] (2.4)

The modification (2.3) was selected because it does not involve any jump in \( B_x \) or in \( j_y \) at \( z = L \). The cross-tail current drops nearly to zero at \( z = 4L/3 \), so \( B \) is nearly constant and trajectory tracing can be stopped. See Appendix B for details. Since we are using a one dimensional magnetic field model (no \( x \) or \( y \) dependence), the orbits were arbitrarily started at \( x = y = 0 \).

### 2.2 Orbital Regimes

We found that the particle orbit types can be characterized by \( \kappa \), which was defined by Büchner and Zelenyi [1989]. Using the magnetic field (2.1), the \( \kappa \) parameter is

\[ \kappa = \left[ \frac{R_c}{\rho} \right]^{1/2} = B_{z0} \left[ \frac{qL}{B_{x0}mv} \right]^{1/2} \] (2.5)

where \( R_c \) is magnetic field line radius of curvature at \( z = 0 \), \( \rho \) is particle gyroradius at \( z = 0 \); \( q, m, \) and \( v \) are the ion charge, mass and velocity. The velocity is related to the ion thermal energy \( W \).

Four distinct regimes were found to exist in the magnetotail, each of which is associated with a specific set of ion orbits. In order of decreasing \( \kappa \), the four regimes are referred to as adiabatic, resonant, nonadiabatic, and nonresonant. The \( \kappa > 2 \) regime, where the guiding center approximations [Northrop, 1963] are adequate to describe \( j_y \), has been studied extensively. Guiding center particles are referred to as "adiabatic" because the magnetic moment \( \mu \) is nearly conserved. A guiding center orbit is analyzed by separating the rapid cyclotron or gyro motion from the slow drift of the cyclotron-averaged particle location or guiding center. Parker [1957] evaluated the resulting electric current density by counting the number of orbits which cross an arbitrary unit area each second. We use a
similar technique for the more complex trajectories studied here. The resulting electric current is commonly separated into guiding center drift and magnetization current terms. A sample orbit with $\kappa = 2$ is shown in Figure 2.1a. Such particles spiral around magnetic field lines, pass once through the equatorial plane, and leave through the opposite side of the plasma sheet with nearly the same pitch angle they had at the injection point.

The opposite extreme, or very small $\kappa$ limit, was originally studied by Speiser [1965]. Figure 2.1b shows a sample orbit for $\kappa = 0.02$. Here, particles bounce up and down many times in the solar magnetospheric $z$ direction as they drift around a semicircle in the $x$-$y$ or equatorial plane. Speiser orbits are well approximated by separating the rapid bounce motion in the $z$-direction from the slow gyro motion about $B_z$ of the bounce-averaged particle location. Ions in the distant tail, near a neutral line, and in the very thin plasma sheet that is present at the end of a substorm growth phase both satisfy these approximations. During the course of a plasma sheet interaction, the magnetic moment of a Speiser particle varies substantially from the initial value it had upon entering the current sheet. However, a different action integral, the bounce invariant $J_z$, is nearly conserved [Speiser, 1970; Sonnerup, 1971; Whipple et al., 1986]. A Speiser-type particle is referred to as quasi-adiabatic because its magnetic moment returns almost to its initial value by the time the ion leaves the current sheet. This is a consequence of the conservation of $J_z$. A particle on a Speiser-type orbit therefore suffers little net change in its magnetic moment. A characteristic feature of Speiser-type current sheets is that the north-south bounce motion of some ions reaches all the way to the edge of the sheet. This results in a current sheet that is insensitive to the value of $\kappa$, so that resonance effects are not observed. We refer to these as nonresonant quasi-adiabatic or simply "nonresonant" orbits.

Two intermediate $\kappa$ regimes, which will be the subject to this thesis, are dominant when magnetic field and the particle parameters characteristic of the mid-tail region are
used. These ion orbits are characterized by comparable gyro and bounce periods. We inject particles with known velocities at the edge of the sheet \( z = L \), and trace orbits until the particles reached a point 2 gyroradii beyond \( z = 4L/3 \), where \( B \) is essentially uniform. The variation of \( \langle v_y \rangle / v_0 \) as a function of \( z \) and \( \kappa \) for these particles is shown in Figure 2.2a, where \( \langle v_y \rangle \) is the average particle velocity in y-direction, \( V_0 \) is the total particle velocity, and \( z \) is the distance from the equatorial plane. Figure 2.2b is a contour plot of the same data.

The peaks in \( \langle v_y \rangle / v_0 \), at small \( |z| \) on Figure 2.2 occur at \( \kappa \) values of 0.53, 0.32, and 0.23. Most ions follow relatively simple trajectories when \( \kappa \) reaches those regimes. The resulting orbits are similar to Speiser trajectories that involve only \( N = \frac{0.40 - 0.25}{\kappa} = 1/2, 1, 3/2 \) or some other small half integral number of oscillations during each current sheet encounter. Figures 2.3a and 2.3b show examples of \( 1/2 \) and \( 1 \) bounce particle trajectories. Particles leave the plasma sheet at the opposite side from which they were injected when the number of oscillations is integer. Almost 100% of the \( 1/2 \) and \( 3/2 \) oscillation particles are reflected, so they leave the plasma sheet from the same side they entered. The orbits are roughly symmetric near these resonances [Chen and Palmadesso, 1986; Chen et al., 1990]. We use the term "resonant quasi-adiabatic" or simply "resonant" to describe these orbits. The orbit is quasi-adiabatic because the magnetic moment returns to nearly its initial value by the end of each current sheet interaction [Kaufmann et al. 1993a]. Unlike the "nonresonant" or Speiser ions, the \( j_y \) distribution carried by resonant ions depends sensitively upon \( \kappa \).

The valleys in \( \langle v_y \rangle / v_0 \) at small \( |z| \) in Figure 2.2 appear approximately at \( \kappa = 0.82, 0.4, \) and 0.27, which correspond to orbits with \( N = 1/4, 3/4, \) and \( 5/4 \). For these \( \kappa \), most ions follow stochastic orbits [Chen and Palmadesso, 1986; Büchner and Zelenyi, 1987]. About 50% of the particles are reflected at the current sheet so that they leave
through the same side of plasma sheet they entered. Figure 2.4a and 2.4b show examples of 1/4 ($\kappa = 0.82$) and 3/4 ($\kappa = 0.40$) bounce particle trajectories. We call such ions "nonadiabatic" because the magnetic moment makes a large net change as a result of each current sheet interaction.

2.3 Electric Current and Particle Number Density

There are three common types of orbits which carry different current distributions, $j_y(z)$, in the two intermediate $\kappa$ regimes. The goal of this section is to develop a physical understanding of the characteristic current distribution produced by each distinguishable class of resonant quasi-adiabatic orbit.

Figure-8 trapped particle orbits. Figures 2.5a-2.5c show 3 projections of a short segment of a 3/4-oscillation or $\kappa = 0.4$ proton trajectory ($B_{x_0} = 25$ nT, $B_{z_0} = 2.5$ nT, $L = 0.8$ RE, $W = 3$ keV in Equations (2.1) and (2.3)). This ion moves in the positive y direction, and therefore carries positive $j_y$ when it is the farthest from the neutral sheet ($0.35$ RE < $|z|$ < $0.58$ RE). The motion and $j_y$ are in the negative y direction whenever the ion is at $|z| < 0.35$ RE. When viewed from Earth (Figure 2.5a) the orbit looks like a "figure-8". When viewed from above (Figure 2.5c) the particle traces out an ellipse in each hemisphere for each traversal of the figure-8 pattern. The dashed curve in Figure 2.5b is a magnetic field line. Figure 2.5d is similar to Figure 2.5a except that the trajectory was traced longer to show the figure-8 pattern repeating. Particles would exactly retrace their orbits at a fixed point [Chen and Palmadesso, 1986].

The point at which $v_y = 0$ for fixed point figure-8 orbits is approximately $z_{o8} = 1.1$ $z_0$. The parameter $z_0 = m/v_0$ $[qB_x(z_0)]$ is the point at which a particle's local radius of curvature due to $B_x$ equals the distance from the center of the current sheet. The maximum $z$ reached before a figure-8 ion is turned back toward $z = 0$ is approximately $z_{\text{max}8} = 1.8$ $z_0$. Figure 2.5e shows the current carried by this one ion. We divided the
modeling region in to 40 slabs, each with a thickness $L/15$. The 40 slabs extend from $-4L/3$ to $+4L/3$. Since we assumed a symmetric source above and below $z = 0$, currents in boxes at the same $|z|$ were added to yield the current in each of 20 boxes between $z = 0$ and $|z| = 4L/3$. The location of a particle was retained each time it entered or left a box. The cross-tail current density is

$$j_y(z_k) = \frac{n_0 v_0 q \Delta y_k}{\Delta z} \quad (2.6)$$

where $\Delta z$ is the height of each box, $q$ is the ion charge, $n_0$ is a density normalization parameter, $v_0$ is the ion velocity at $z = L$, and $\Delta y_k$ is the distance an average ion traveled in the $y$ direction while located within the $k$'th box.

Figure 2.5f shows the particle number density, $n(z)$ corresponding to this single orbit. The density in the $k$'th box is

$$n(z_k) = \frac{n_0 v_0 \Delta t_k}{\Delta z} \quad (2.7)$$

where $\Delta t_k$ is the time an average ion spent in the $k$'th box. Figures 2.5a and 2.5d show that most time was spent near $z_{max} = 0.58 \text{ RE}$, where the particle was turning around.

The normalization parameter $n_0$ is arbitrary at this point, but the ratio of $j_y(z) / |l| n(z)$ gives the correct average cross-tail ion velocity $v_y(z)$. Since this ion nearly retraces the same trajectory, it is clear that an average over $z$ of $j_y(z)$ is zero (Figure 2.5e). Ions on figure-8 orbits carry equal positive and negative net currents at $|z| > z_{o8}$ and at $|z| < z_{o8}$, respectively. Stern and Palmadesso [1975] and Cowley [1978a] showed that trapped particles do not carry a net cross-tail current in a one-dimensional magnetic field. However, it will be evident in Chapter 3 that trapped particles and the magnetization current they carry are needed in a self-consistent current sheet.

Figure 2.6 shows trapped particle orbits in the $\kappa = 0.52$ field model ($B_{xo} = 32 \text{ nT}$,
$B_{z0} = 3.3 \text{ nT}, L = 1 \text{ R}_E, W = 3 \text{ keV}$). Away from fixed points, the figure-8 pattern is not retraced from orbit to orbit. Figure 2.6a is a short segment of the trajectory shown more fully in Figure 2.6b. Although the figure-8 pattern is more complex than that in Figure 2.5, the resulting current distributions are similar. The maximum $|z|$ reached by the $\kappa = 0.52$ proton is $z_{\text{max}}^8 = 0.35 \text{ R}_E$, so figure-8 ions cannot carry all the $j_y$ needed in Equation (2.4) to produce a $\kappa = 0.52$ model current sheet.

Figure 2.7a shows a short segment and Figure 2.7b a longer segment of a $\kappa = 0.23$ proton trajectory of the figure-8 type. The parameters $B_{x0} = 10 \text{ nT}, B_{z0} = 1 \text{ nT}, L = 1 \text{ R}_E$, and $W = 7 \text{ keV}$ were used. Since the orbit in Figure 2.7a and 2.7b executes $3/2$ an oscillation each time the neutral sheet is reached, there are alternately two loops at $z < 0$ with one at $z > 0$ (Figure 2.7a) and then one loop at $z < 0$ with two at $z > 0$. This $\kappa = 0.23$ figure-8 orbit carries current nearly out to the edge of the current sheet, because particles with $\kappa = 2\kappa_r = 2B_{x0}/B_{z0}$ have $z_{\text{max}}^8 = L$.

One conclusion is that few ions on figure-8 orbits with $\kappa < 2\kappa_r$ can be present in a self-consistent plasma sheet. If many were present, they would carry substantial positive $j_y$ outside the current sheet, and therefore be inconsistent with Equation (2.4). One possibility is that if a substantial number of figure-8 ions were injected into this $\kappa = 0.23$ model, then the current sheet would become thicker. Increasing $L$ would increase $\kappa$ for all ions.

Another possibility is that certain groups of orbits may be nearly unpopulated. This will produce empty regions of phase space in the self-consistent current sheet. The formation of phase space holes has been noted in previous simulation studies which use pre-selected source populations [Ashour-Abdalla et al., 1991].

The suggestion that phase space holes or other asymmetries develop may appear unphysical. One might ask: What happened to the particles that could have populated the nearly empty orbits, and why don't nearby particles scatter in to fill the holes? In regard to
the latter question, Holland and Chen [1991] showed that phase space boundaries persist in the presence of the pitch angle and energy scattering produced by typical magnetotail wave fields. It also should be noted that all current sheets characterized by a very small $\kappa$, including all nonresonant or Speiser-type current sheets, must have phase space depletions. The definition we use for nonresonant current sheets is $\kappa < \kappa_r$. This inequality implies that the average particle has enough energy to reach well beyond the edge of the current sheet during each north-south-north oscillation. However, if many particles were on trajectories that oscillated this far, they would carry substantial current outside the current sheet. Therefore few ions with energies large enough so that $\kappa < \kappa_r$ can follow such trajectories in a self-consistent current sheet.

Very thin current sheets are known to exist in the inner and midtail during substorm growth phase and may be common in the distant tail and near a neutral line. As an example, Mitchell et al. [1990] found brief intervals when ISEE 1 and 2 were on opposite sides of a current sheet with a total thickness of 400 km at a radial distance of only 11 $R_E$. The measured $B_x$ at the edge of the sheet was 45 nT, corresponding to $B_{xo} = 60$ nT and $L = 200$ km in Equations (2.1) and (2.3). The average measured $B_{zo}$ was 4 nT, though it fluctuated from slightly negative up to 8 nT during the period of interest. The average ion energy was 10 keV, with substantial fluxes to beyond 50 keV. The above figures give $\kappa = 0.07$ for an average proton and a smaller $\kappa$ for more energetic protons and other ions. The average $\kappa$ approximately equals $\kappa_r$ in this case. The corresponding $z_0$ is 275 km for a 10 keV proton, which implies that some of the associated figure-8 type orbits reach as far as $z_{max8} = 500$ km. Although some current is seen beyond the principal current sheet, the observed current density was very small beyond $|z| = 200$ km. Such observations, which show that a nonresonant current sheet sometimes is present in the midtail, require that for some reason there exist orbits which are almost unpopulated. Mitchell et al. spe-
cifically noted the presence of a non-field-aligned beam of ions, which indicates a strong phase space asymmetry.

**Mirror-type trapped particle orbits.** Figure 2.8 shows part of another common type of trapped particle trajectory. Beyond \(|z| = 0.5 \, \text{R}_E\) the magnetic moment of this sample ion is nearly conserved, and the orbit can be approximated by guiding center motion. The looping motion in Figure 2.8a involves guiding center gradient drift in the negative y direction while the particle is mirroring. This example is for a \(\kappa = 0.32\) or 1-oscillation orbit (\(B_{x_0} = 32 \, \text{nT}, \, B_{z_0} = 4 \, \text{nT}, \, L = 1 \, \text{R}_E, \, W = 50 \, \text{keV}\)). The particle makes one full oscillation (crosses \(z = 0\) three times) when it encounters the neutral sheet. If continued, the particle orbit would begin looping and gradient drifting in the negative y direction below the neutral sheet in a manner similar to that shown above. The cumulative distance traveled back and forth in the y direction is large while the particle is carrying out the looping motion. The resulting magnetization current dominates in the total \(j_y(z)\). The particle moves in the positive y direction at the top of each loop and in the negative y direction at the bottom of each loop, yielding the current shown in Figure 2.8c. The cross-tail current would be in the same direction when the particle is located at \(z < 0\) if the orbit was continued. Note that \(j_y\) is small near \(z = 0\) for this trapped particle because \(v_y\) is relatively small and can be either positive or negative when the particle crosses the \(z = 0\) plane. We will see that it is difficult to find groups of particles that carry a maximum positive \(j_y\) very near \(z = 0\) in this model field. Figure 2.8d shows the density distribution associated with the orbit.

Figure 2.6c is a longer segment of a 1/2-oscillation version of the mirror-type orbit. This figure illustrates two important properties of the nearly resonant \(\kappa = 0.52\) particles. First, the pitch angle is almost the same before and after each interaction with the neutral sheet (\(J_2\) is nearly conserved). This feature causes the ion to mirror at almost the same \(z\)
each time it moves into the northern half of the plasma sheet. Nonadiabatic particles have larger or smaller $\kappa$, and will mirror at different locations after each neutral sheet encounter.

The second important property is the tendency to make $1/2$ an oscillation ($2$ crossings of $z = 0$) each time the particle reaches the neutral sheet. The exact paths traced out at $z < 0$ in Figure 2.6c depend on the phase angle at which the particle happened to enter the neutral sheet. However, the tendency of this particle simply to “bounce off” the southern plasma sheet is evident. This tendency to bounce off the current sheet is found for all untrapped and mirror type $\kappa = 0.52$ ions.

The $\kappa = 0.32$ or approximately $1$-oscillation mirror-type orbit in Figure 2.7c alternately leaves the neutral sheet to mirror at $z > 0$ and at $z < 0$. This ion is farther from a resonance than the one shown in Figure 2.6c. The $|z|$ at which the ion in Figure 2.7c mirrors changes substantially during some interactions with the neutral sheet, implying a change in the magnetic moment. This type of orbit is not permanently trapped because eventually it will leave the neutral sheet with a small enough pitch angle to escape. We place it in the trapped category because it spends so much time following a mirror-type orbit that its $j_y$ is almost the same as for a permanently trapped ion.

To summarize, although there are a variety of trapped particle orbits, most carry similar $j_y(z)$. No net current is carried by permanently trapped particles in a purely one dimension field [Stern and Palmadesso, 1975; Cowley, 1978a; Rogers and Wipple, 1988]. Trapped particles are, however, essential to the formation of the model self-consistent one dimension current sheet. Figure-8 orbits involve positive $j_y$ at $z_{o8} < |z| < z_{max8}$, negative $j_y$ at $|z| < z_{o8}$, and no current beyond $z_{max8}$. Trapped mirror-type orbits also carry positive $j_y$ at the largest $|z|$ and negative $j_y$ at a smaller $|z|$. A second region of positive $j_y$ is possible near $z = 0$ for mirror orbits. The mirror-type orbits generally have a $j_y = 0$ point near $|z| = z_0$, which is the place at which $|B|$ becomes strong enough to reverse the $v_y$ of an
Untrapped particle orbits. It is convenient to separate groups of particles according to whether or not they ever leave the current sheet. An unrealistic feature of one dimensional models is that $B$ becomes constant at large $|z|$. Particles that reach large $|z|$ in the model field therefore do not drift and will never mirror or hit the ionosphere. In the actual magnetotail, $|B|$ continually increases along field lines, so that all ions eventually mirror or hit the ionosphere. However, those that mirror far outside the current sheet drift so far that they return to the equator in a different region of the tail. Trapped and untrapped particles carry substantially different current distributions in the resonant case. Several typical untrapped ion orbits are shown in Figures 2.1 and 2.3. Figure 2.9 shows projections, $j_y$, and $n$ for an untrapped $\kappa = 0.32$ (1-oscillation) trajectory. Most untrapped resonant ions tend to carry strong positive $j_y$ at $|z| < z_0$ and slightly negative $j_y$ at larger $|z|$. The untrapped stochastic particles show more chaotic features than the resonant particles. Except for the repeating figure-8 type trapped particles, almost all the stochastic particles eventually left the current sheet in a reasonable cutoff time. These untrapped trajectories can be changed a lot even if there is only a little change in phase space. Figure 2.10a and 2.10b show projections of a $\kappa = 0.4$ proton trajectory ($B_{xo} = 25$ nT, $B_{zo} = 2.5$ nT, $L = 0.8$ RE, $W = 3$ keV) which was injected at $z_i = L$ with pitch angle $\alpha = 170^\circ$ and phase angle $\psi = 165^\circ$. Note that the particle makes one tilted figure-8 orbit (viewed from Earth, Figure 2.10a) and crosses the neutral sheet several other times before it finally leaves the current sheet. When viewed from above (Figure 2.10b) the particle traces out one ellipse. Figure 2.10c shows the current carried by this one particle. The shape of $j_y(z)$ is similar to that for an untrapped resonant ion except that $j_y$ is negative near $z = 0$ which due to the figure-8 portion of the trajectory.

Figure 2.11a and 2.11b show a particle trajectory which uses the same field parame-
ters. This particle was injected at \( z_i = L \) with same pitch angle as in the above example, but the phase angle was \( \psi = 255^\circ \). This time the particle enters from above the current sheet, crosses the neutral sheet several times, and finally leaves above the current sheet (figure 2.11a). In the equatorial plane, the particle traces out one or more ellipses (figure 2.11b). Figure 2.11c shows that the current carried by the particle is very similar to the current carried by a mirror-type particle.

To summarize, although we call the particles that left the current sheet untrapped particles, the current carried by resonant particles and by stochastic particles show different features. The untrapped resonant particles carry strong positive \( j_y \) at \( |z| < z_0 \) and weak negative \( j_y \) at larger \( |z| \), while the untrapped stochastic particles carry a current that is most like that carried by the mirror-type particle which was described above. There are very small regions in phase space in which stochastic particles can follow pure figure-8 trajectories or the simple resonant untrapped type trajectories.

**Magnetization current.** The orbit in Figure 2.9 shows how magnetization current can dominate over guiding center drift current for an untrapped ion. This is a feature that is not always fully appreciated, even though it also is found in guiding center theory. Note that a group of protons following orbits identical to that in Figures 2.9a and 2.9b carry a net negative \( j_y \) within the current sheet (Figure 2.9c) but that the guiding centers move in the positive \( y \) direction. This nonintuitive result is a consequence of our selection of a sample particles that leaves the current sheet with a pitch angle near 90°.

Consider the ion as it leaves the current sheet in the upper right corner of Figure 2.9a. As an illustration, we will define \( z = 1.3 \, R_E \) as the edge of the current sheet. The particle spirals in the clockwise direction at this location in the projection shown. Three of the oval loops cross \( z = 1.3 \, R_E \), so that part of each loop is outside the region we define to be the current sheet and part is inside the current sheet. The proton moves primarily in the posi-
tive $y$ direction each time it is outside the current sheet, so this part of the cross-tail motion is not counted as a contribution to $j_y$ inside the current sheet. An equivalent viewpoint is to construct a vertical sheet that passes through the center of the spiral structure under discussion in Figure 2.9a. This vertical sheet starts at $z = 1.3 \, R_E$ and extends downward. The clockwise sense of the orbit causes the proton to cross this sheet several times while it is moving in the negative $y$ direction and to pass beyond the top end of the sheet while it is moving in the positive $y$ direction. Since current density is defined as the number of charges crossing a unit area per second, the net current density through the sheet that ends at $z = 1.3 \, R_E$ is in the negative $y$ direction.

It might be imagined that this result is an artifact of our definition of an edge of the current sheet at $z = 1.3 \, R_E$. However, note that the same conclusion is obtained if this edge is placed at $z = 1.4 \, R_E$, $1.5 \, R_E$, or any other point in the uniform field region, provided that orbits are traced until the ion is at least 2 gyroradii beyond the edge. Since $j_y$ is essentially zero at and beyond $z = 1.3 \, R_E$, the total sheet current is nearly independent of the exact placement of the edge. In fact, the net negative $j_y$ is small near $z = 1.3 \, R_E$ and is distributed over a large region (Figure 2.9c) as orbits change from north-south oscillations (at $|z| < 0.5 \, R_E$) to the looping motion.

One also might imagine that the negative $j_y$ or magnetization current is an artifact of the one dimensional model. For example, actual magnetotail field lines reach a maximum $|z|$ and then return closer to the equatorial plane before they reach the ionosphere. The spiral motion near the point of maximum $|z|$ causes all protons to move in the positive $y$ direction at the outermost 1/2 gyroradius of the plasma sheet or the Plasma Sheet Boundary Layer (PSBL). This positive $j_y$ magnetization current in the PSBL adds to the negative $j_y$ magnetization current in the current sheet, so that the total current integrated throughout the plasma sheet is given by the guiding center drift expression. However, the current
sheet in the inner tail and midtail usually is separated from the PSBL by a thick, nearly isotropic outer CPS in which \( j_y \) is very small. Magnetization currents are therefore important both in the current sheet and in the PSBL and are correctly described by \( j_y \) in Figure 2.9c. The resonant particle cross-tail current distribution \( j_y(z) \) cannot be understood by considering only the net motion of particle guiding centers.

Finally, as noted above, studies using guiding center equations or adiabatic particle orbits have arrived at similar conclusions [Bird and Beard, 1972]. A commonly used expression for the guiding center drift current is

\[
j = \frac{B}{B^2} \times \left[ \nabla p_{\perp} + \frac{P_{\parallel} - P_{\perp}}{B^2} (B \cdot \nabla) B \right]
\] (2.8)

The well known \( \nabla B \) guiding center drift current does not even appear in Equation (2.8), because guiding center \( \nabla B \) and magnetization \( \nabla B \) current cancel. The total field line curvature, or \( (B \cdot \nabla) B \) current, in Equation (2.8) is in the same direction as the guiding center curvature drift when \( P_{\parallel} > P_{\perp} \) but in the opposite direction when \( P_{\parallel} < P_{\perp} \), because magnetization currents dominate in the latter instance. These magnetization currents are real, produce magnetic fields that must be considered in a self-consistent model, and are not an artifact of the one-dimensional model or of our consideration of only the resonant class of orbits.

At this point, we have investigated the different trajectory types and the current distributions that they carried. In Chapters 3 and 4, we will see how these different types of particles can contribute to the model current sheet and to the formation of a self-consistent current sheet.
Figure 2.1 Sample orbits for $\kappa = 2$ and $\kappa = 0.02$, projected onto the $y$-$z$, $x$-$y$, and $x$-$z$ planes.
Figure 2.2a The variation of $\langle v_y \rangle / v_o$ as a function of $z$ and $\kappa$. 
Figure 2.2b. Contour plot of Figure 2.2a.
Figure 2.3 Similar to Figure 2.1 but for $\kappa = 0.52$ and $\kappa = 0.32$ resonant trajectories.
Figure 2.4 Similar to Figure 2.1 but for $\kappa = 0.82$ and $\kappa = 0.4$ stochastic trajectories.
Figure 2.5 (a-d) Projections of a single trapped ion orbit with a figure-8 type trajectory; (e) cross-tail current and (f) particle number density associated with ions distributed along this same trajectory.
Figure 2.6 Similar to Figure 2.1 but for $\kappa = 0.52$ ions on (a and b) figure-8 trajectories and (c) mirror-type trajectories.
Figure 2.7 Similar to Figure 2.1 but for (a and b) $\kappa = 0.23$ figure-8 trajectories; (c) $\kappa = 0.32$ mirror-type trajectories.
Figure 2.8 (a and b) similar to Figure 2.1 but for a $\kappa = 0.32$ ion on a mirror-type trajectory. (c) cross-tail current and (d) particle number density associated with ions distributed along this same trajectory.
Figure 2.9 Similar to Figure 2.8 but for an untrapped $\kappa = 0.32$ ion.
Figure 2.10 Similar to Figure 2.8 but for an $\kappa = 0.4$ stochastic untrapped figure-8 ion.
Figure 2.11 Similar to Figure 2.8 but for an $\kappa = 0.4$ stochastic untrapped mirror-type ion.
Chapter 3

Case 1: Resonant Particles

The calculation that will be described in detail in this chapter used 3 keV protons, \( B_{x0} = 32 \) nT, \( B_{z0} = 3.3 \) nT, and \( L = 1 \) R_E in Equation (2.1) and (2.3). These parameters are appropriate for \( x = -14 \) R_E in the \( K_p = 4 \) version of T89, and produce \( \kappa = 0.52 \), which is the resonance associated with orbits that undergo \( 1/2 \)-oscillation during each current sheet encounter.

3.1 Ion Current Distribution

A number of 1000-particle groups were injected at different z locations and with different angular distributions. The current carried by each such group was determined by tracing orbits. Figure 3.1a shows \( j_y(z) \) carried by all trapped ions from groups injected at \( z_i = 0 \). A large fraction of the ions randomly injected at \( z_i = 0 \) are trapped. The injection pitch angles were randomly picked so that the angular distribution would be the same as those of biMaxwellian plasmas with \( T_{||}/T_{\perp} = 0.1 \) (dotted line), 1 (solid line), and 10 (dashed line) if the particles were injected into a region with uniform B. All ions obey the guiding center approximations in such a uniform B region. The guiding centers move once through the injection box, producing the specified biMaxwellian pitch angle distribution and \( T_{||}/T_{\perp} \) ratio. The actual angular distribution at the injection point often bears little relationship to a biMaxwellian with the quoted \( T_{||}/T_{\perp} \). This is because some ions in the model magnetotail return many times to the injection point with completely different pitch and phase angles than they had initially. Figure 2.6a, 2.6c, 2.7a and 2.7c show examples of this effect for ions which were injected at \( z_i = 0 \). We will quote the "\( T_{||}/T_{\perp} \) ratio" of injected particle groups primarily as a way to label the groups, even though the final tem-
perature ratio may be quite different from the quoted value even in the injection region. All groups considered in this chapter had their injection phase angles randomly selected from a uniform distribution. We will show later that phase anisotropies develop near \( z = 0 \).

Our analysis in Chapter 2 provides physical insight into why \( \kappa = 0.52 \) trapped ions with \( z_i = 0 \) carry the current distribution shown in Figure 3.1a. Some orbits are of the mirror type (Figures 2.6c and 2.7c), but variations of the figure-8 pattern (Figures 2.5, 2.6a, 2.6b, 2.7a, and 2.7b) are dominant. The figure-8 orbits yield positive \( j_y \) beyond \( |z| = 0.2 \ R_E \) and negative \( j_y \) closer to the neutral sheet. Mirror type orbits produce positive \( j_y \) at the outer edge of the mirror region and negative \( j_y \) at the inner edge of this region. Mirror orbits can produce either positive or negative \( j_y \) near \( z = 0 \), with positive \( j_y \) usually dominating. It would be easier to visually estimate the total current if the curves in Figure 3.1a were drawn as histograms rather than line plots. We chose to connect \( j_y \) at the center of each box with lines so that several curves could be shown in each panel.

Figure 3.1b shows the current carried by untrapped \( \kappa = 0.52 \) protons that were injected at \( z_i = L \). Groups that are randomly injected at \( z_i = L \) contain a large fraction of untrapped particles. For example, it clearly is not possible for any trapped ions of the types that do not reach \( z = L \) (Figures 2.5, 2.6a, 2.6b, 2.7a, or 2.7b) to appear in this group.

The use of different injection points should be considered as merely another way to select groups of ions with different orbital characteristics. For untrapped particles, the orbit tracing routine starts at the injection point and follows the ion until it leaves the current sheet. Orbit tracing stops when the ion reaches a point 2 gyroradii beyond \( |z| = 4L/3 \). We then return to the injection point and trace the orbit backwards in time until the ion again is 2 gyroradii beyond \( |z| = 4L/3 \). The full orbit of an untrapped ion
therefore starts and ends outside the current sheet, even if the "injection point" is at
\( z = 0 \). The starting and ending points of trapped particles are more arbitrary. Such parti-
cles are traced to a predetermined cutoff time, which depends on the ion energy (usually
the cutoff time for a trapped particle is three times longer than a typical untrapped trajec-
tory). These times are long enough so the shapes of the \( j_y(z) \) and \( n(z) \) plots do not
change significantly if the cutoff time is doubled.

The absolute values of the density and current would become infinite if particles were
injected continuously and never escaped from the plasma sheet. The scales of all plots
were arbitrarily set to produce \( n = 1 \text{ cm}^{-3} \) if the ions had been injected into a uniform \( \mathbf{B} \)
region. The actual \( n \) and \( j_y \) of such plots depend both on how many times particles return
to a given box and, for trapped ions, on the cutoff time used. The \( j_y(z) \) and \( n(z) \) scales
of the single particle and single 1000-ion group plots therefore separately have little physi-
cal significance. However, their ratio always gives the correct drift velocity in \( y \)-direction
\(<v_y> = j_y(z) / [|q|n(z)] \). Each scale is physically meaningful for the final \( n(z) \) and
\( j_y(z) \) that will be discussed in section 3.4.

The current carried by a group of untrapped ions does not depend sensitively upon the
injection point or temperature ratio. Almost all untrapped groups carry positive \( j_y \) at
\(|z| < z_0 \) (\( z_0 \) is described in Chapter 2) and negative \( j_y \) at larger \( |z| \). In contrast, it is easy to
find groups of trapped 3 keV protons that carry substantially different current distributions
from those shown in Figure 3.1a. Injection away from \( z_i = 0 \), particularly with \( T_{\perp} > T_{\parallel} \),
tends to select ions which are mirroring near the injection point (Figures 2.6c, 2.7c, and
2.8). Figure 2.8 shows that such ions carry positive \( j_y \) at the outer extreme of the loops
produced as they mirror, and negative \( j_y \) at the inner edge of such loops. Figures 3.2a and
3.2b show the resulting \( j_y \) for the trapped components of 1000-ion groups with
\( T_{\parallel} / T_{\perp} = 0.1 \) and 1 which were injected at \( z_i = 0.25 \text{ R}_E \) and 0.5 \( \text{ R}_E \), respectively. Orbits
of the figure-8 type are not found in the group shown in Figure 3.2b because the injection point was beyond z_{max}. Elimination of this group and the dominance of mirror orbits that reach large |zl| permits j_y to be positive near z = 0. A comparison of Figures 3.1a and 3.2 shows that it is easy to move the principal region of positive j_y for trapped orbits to a desired z by appropriate selection of z_i.

3.2 Electron Current

Up to this point we have only considered the electric current produced by ion motion. Inner CPS electrons are observed to have 1/8 the kinetic energy of ions [Baumjohann et al., 1989; Christon et al., 1991]. We inject these electrons isotropically at z = 0. Equation (2.5) shows that an electron which has 1/8 the energy of a κ = 0.52 proton has κ = 5.7. The j_y carried by such electrons is well approximated by the ordinary guiding center drift expression (2.8).

Guiding center drift currents are proportional to particle energy (or plasma pressure). Therefore, electrons would not contribute a great deal to the total j_y if electrons and ions both followed guiding center motion. Since ion motion is not well approximated by the guiding center expression, it is necessary to compare ion and electron currents more carefully. Electron currents are estimated at this stage by assuming the electron density exactly equals the previously calculated ion density throughout the tail. This is not a consistent assumption at this point in the analysis because there is no E field with a component along B. Such an E field is needed in order to maintain charge neutrality all along a field line.

Figure 3.3a shows that with the above assumptions, electrons carry only a small fraction of the total cross-tail current in this particular trapped plasma group. The ions in Figure 3.3a are from the T_{\|}/T_\perp = 0.1 group in Figure 3.2a. Figure 3.3b shows similar results for an untrapped plasma group. The ion group in this case had T_{\|}/T_\perp = 0.1 and was injected at z_i = 0.25 \ RE. Note the similarity of this ion current to the corresponding
curve in Figure 3.1b, which was injected at $z_i = 1 \, R_E$. This similarity illustrates the relative insensitivity of untrapped $\kappa = 0.52$ ion current to the injection point. Electron currents are significant for some $\kappa$, but small compared to ion current for all resonant current sheets of interest here. A detailed description of our treatment of electrons is included in Appendix A.

### 3.3 Breakdown of Adiabatic Motion

Figure 3.9b compares $j_y(z)$ for a group of untrapped ions (solid line) to the current predicted using Equation (2.8), the guiding center approximation (dotted line). Section 3.6 will describe how the ion pressures needed in Equation (2.8) were calculated. In our model field, Equation (2.8) breaks down for 3keV protons ($\kappa = 0.52$) only in the few boxes closest to $z = 0$. Figure 3.9b used more energetic $\kappa = 0.32$ (1-oscillation) protons to extend the region in which Equation (2.8) is invalid. The guiding center approximation was found to produce a good estimate of $j_y(z)$ only beyond approximately $|z| = z_0$.

### 3.4 Combination of Plasma Groups

The solid curve in Figure 3.4a shows $j_y(z)$ from Equation (2.4) that is needed to produce the magnetic field in which orbits were traced. It is clear that no one of the plasma groups studied to this point carries a current distribution that even remotely resembles the solid curve in Figure 3.4a. Therefore, none of these plasma group can self consistently produce the model current sheet. However, the groups involving trapped ions carry current in the positive $y$ direction at relatively large $|z|$ and those containing untrapped ions carry current in the positive $y$ direction at small $|z|$. Thus it appears possible that a self-consistent current sheet could be generated by appropriately combining the various plasma groups.

The technique we use is to perform a least squares fit to the desired $j_y(z)$ [Daniels, 1966; Press et al., 1986]. We usually start with six to ten groups of ions and their accompa-
nying electrons. The numerical routine determines which groups contribute significantly to the fit. For example, if two of the original groups carry similar current distributions they usually will not independently produce significant improvements to the fit. We remove groups that do not meet the significance criterion and repeat the procedure. In most cases, reasonable fits are found with two to six remaining plasma groups. Each such group usually contributes to the fit at a 95% confidence level according to the Student t-test, though lower confidence limits sometimes are used. The dotted curve in Figure 3.4a shows a typical result. In this case, the $T_{\parallel}/T_{\perp} = 10$ untrapped source group in Figure 3.1b and the $T_{\parallel}/T_{\perp} = 0.1$ trapped source groups in Figures 3.1a, 3.2a, or 3.3a, and 3.2b were retained. A trapped source group with $z_i = 0.75$ $R_E$ (not shown) also contributed to the final fit in Figure 3.4a.

The locations of the peaks of dotted curve in Figure 3.4a were determined by the set of $z_i$ for the source groups we selected, as can be seen by comparing Figure 3.4a to Figures 3.1, 3.2, and 3.3. The specific location of any peak could be changed by selecting a different $z_i$, so the fit is not unique. However, the physical understanding gained previously of $j_y(z)$ carried by each orbit type leads to the conclusion that there is not much real choice in the selection of an appropriate set of injection points. Including more groups with different injection points can produce a smoother version of the dotted curve and therefore a "better" fit. However, doing so substantially reduces the confidence level that each separate plasma group is needed. The purpose of this study is to gain physical insight into the types of particle orbits that must be present in a plasma sheet with desired characteristics rather than to produce the smoothest possible simulation of a current sheet.

The dotted curve in Figure 3.4a shows that untrapped, trapped figure-8, and trapped mirror-type orbits all are needed to produce our model current sheet using 3 keV protons. For example, figure-8 orbits dominate the group in Figure 3.2a, which produces positive $j_y$.
near $z = 0.4 \ R_E$. Mirror-type orbits are needed to produce positive $j_y$ at $z = 0.5 \ R_E$ and beyond (Figure 3.2b). Untrapped ions are needed in the one dimension model both to produce a peak near $z = 0$ and also because only untrapped ions carry a net cross-tail current in this model. The fit in Figure 3.4a would be much poorer if any of these orbit types was omitted.

The result of our analysis to this point is a model for $n(z)$ and for $j_y(z)$ which is in rough agreement with the $j_y(z)$ needed to generate the one dimension magnetotail. The decrease in $j_y$ very close to $z = 0$ and $z = z_D$ were a persistent problem with $\kappa = 0.52$ ions and the particular magnetic field model given by Equation (2.1) and are discussed in Chapter 5.

3.5 Final 1/2-Oscillation Magnetotail Model

The density of ions throughout the current sheet was determined in the preceding section by following proton trajectories. If electron groups were injected with ions at the same $z_i$ and with the same $T_{\parallel}/T_{\perp}$ then the resulting electron density would not equal the ion density throughout the plasma sheet. Adiabatic electrons and quasi-adiabatic ions move differently along a field line. An electric field with a component along $B$ must naturally evolve in any self-consistent current sheet in order to produce charge neutrality. Although our steady-state analysis cannot follow this evolution, the process appears to be straightforward. If ions and electrons were injected in to a region with $E_{\parallel} = 0$, their markedly different orbits would result in a large charge separation. This would generate a very large $E_z(z)$ in the one-dimensional model. The $E_z(z)$ would evolve until the quasi-neutrality condition that generally is required in all plasmas was obtained. Our imposition of an $E_z(z)$, which has a component along $B$ at all $z$, is therefore required in order to maintain charge neutrality.

The dotted and solid lines in Figure 3.4b show the resulting electric potential $\phi(z)$.
and a Chebyshev polynomial fit respectively. In this example, the first approximation to \( \phi(z) \) is negative everywhere, so the simple Boltzmann relation \( n = n_0 e^{-q\phi/kT} \) is appropriate for electrons (Appendix A). Due to their lower energies, electron fluxes are much more strongly modified by the addition of \( E_z \) than are ion fluxes. The parallel electric field maintains charge neutrality by forcing more electrons into regions where their density otherwise would be too low and keeping some electrons out of regions in which their density otherwise would be too high.

The addition of \( E_z(z) \) requires us to retrace ion trajectories in order to be self-consistent. The entire procedure described to this point therefore is iterated to find a self-consistent magnetotail model. The iteration technique used for the present example imposes a final magnetic field configuration, and searches for the plasma and electric field distribution that will self-consistently produce the \( j \) needed to satisfy \( \nabla \times B = \nabla \times B \). Alternatively, it is possible to calculate the \( B \) produced by the present particle distributions and to retrace ion trajectories in the new \( E \) and \( B \) fields. We have used this technique, which is similar to a simulation analysis, in a few sample runs. However, we believe that the BGK-like method which imposes a fixed \( B \) is more realistic.

Several groups of 1000 ions each were once again randomly selected and traced with the added electric field \( E_z = -\partial \phi/\partial z \) associated with the polynomial fit in Figure 3.4b. Whenever an \( E_z(z) \) was present, protons at different \( z \) were started with whatever energy was required so they would have 3 keV at \( z = 0 \). The ion currents (dashed lines in Figure 3.5a and 3.5b) are very similar to the results obtained for the corresponding ion groups with \( E = 0 \) (the dotted line in Figure 3.2a and the dashed line in Figure 3.1b, respectively). This similarity was expected because we are dealing with 3 keV protons, and the maximum potential difference within the current sheet is only 100 volts (Figure 3.4b). The magnitude of \( \phi \) and the similarity of ion currents with and without the inclusion
of $E_z(z)$ are typical of resonant current sheet results when electrons have only 1/8 the energy of ions. Electron currents are shown by the dotted curves in Figure 3.5.

A small region of positive $\phi$ was present near the neutral sheet in this iteration (dashed line in Figure 3.4b), where an accelerated electron distribution function was used (Appendix A). Except for the 20 volts offset originating near $z = 0$, the two iterations produced similarly shaped $\phi(z)$ plots. Only $E_z(z)$ or $-\nabla\phi$ is significant for orbit tracing, so the 20 volts offset is not physically relevant. The polynomial fit to $\phi(z)$ (solid line in Figure 3.4b) was used to obtain a smoothly varying analytical expression which can be differentiated to get $E_z$. With $B$ exactly the same and $E_z$ (the slope of the smoothed $\phi(z)$ curve) nearly the same for the two iterations, the procedure has already converged. The dashed curve in Figure 3.4a shows the resulting $j_y$ carried by ions and electrons in the final model current sheet.

The solid curve in Figure 3.6a is the final number density, and illustrates an important difference between results from a one dimension modified Harris current sheet and the actual mid magnetotail. The model number density is several times higher than the observed CPS density of 0.3-0.4 cm$^{-3}$ [Frank et al., 1984; Baumjohann et al., 1989]. The problem is associated with the large number of trapped ions that was required to make the current sheet 1 $R_E$ thick. Since there is no $x$-dependence in a one dimension model, these trapped particles do not have a net drift velocity across the tail [Stern and Palmadesso, 1975; Cowley, 1978a; Rogers and Whipple, 1988]. The trapped particle orbits involve localized regions containing strong positive and negative $j_y$, but there is no net contribution to the total cross-tail current. We therefore are forcing the untrapped ions to carry all net cross-tail current.

Figure 3.6b shows the average (trapped and untrapped) ion cross tail drift velocity, $\langle v_y \rangle (z) = j_y(z) / [lq l n(z)]$. The untrapped ion component to the model plasma sheet
has a density that increases from 0.3 cm$^{-3}$ at $z = 0$ to 1.0 cm$^{-3}$ at $|z| > L$. The average untrapped ion drift velocity is 300 km/s at $|z| < 0.2 R_E$, and becomes very small at larger $|z|$. The average drift velocity required in the typical midtail current sheet is easy to evaluate. If the current sheet has a total thickness of 2 $R_E$ and a density of 0.4 ions and electrons per cm$^3$, an average relative drift between ions and electrons of 50 km/s would be required to carry enough current to reverse a 25-nT magnetic field (the value of $|B|$ at $|z| = 1 R_E$ in our model).

In summary, a straightforward procedure to create a self-consistent one dimensional Harris model current sheet was illustrated in this section. Individual field line shapes are approximately the same as the shapes of field lines in the T89 midtail current sheet. There may be times when this one dimensional model is adequate in the midtail, and a one dimensional model is more likely to be useful in the distant tail. Current sheet particle densities sometimes reach the required 1 cm$^{-3}$ in the regions surrounded by 25 nT lobe fields. However, we conclude that a one dimensional model is not adequate to describe a typical midtail current sheet, where the density is 0.4 cm$^{-3}$ or less and the current sheet full thickness is approximately 2 $R_E$. The few sample orbits that were traced in two and three dimensional models show that even trapped particles exhibit substantial cross-tail drift. However, the importance of magnetization currents that were found in the one dimensional model suggests that a full two or three dimensional analysis will be required before conclusions can be drawn.

3.6 Distribution Function and Fluid Parameters

Reduced Distribution Function In addition to $n(z)$ and $j(z)$, which have been used up to this point, information about the ion distribution function also is kept during the final iteration. The $v_\parallel$ and $v_\perp$ of an ion are stored each time a $z$ box is entered. The reduced two-dimensional distribution function
\[ f(\nu_{\|}, \nu_{\perp}) = \frac{n_0 \nu_{\|} \Delta t_k}{2\pi \nu \sin \alpha_k \Delta \nu_{\|} \Delta \nu_{\perp} \Delta z} \quad (3.1) \]

then is evaluated at the edge of each of the 20 pairs of \( z \) boxes. This is a reduced distribution function because only the magnitude of \( \nu_{\perp} \) is retained; i.e., phase angle information has been discarded. We also retain ion pitch and phase angles, \( \alpha \) and \( \psi \), which produces a second reduced or two-dimensional distribution function

\[ f(\alpha, \psi) = \frac{n_o}{\nu_{\|} \cos \theta_{\|} \Delta \Omega} \quad (3.2) \]

at the edge of each box, where phase space has been divided into bins of equal solid angle \( \Delta \Omega \), and \( \theta_{\nu} \) is the angle between \( \nu \) and the \( z \) axis. In this case the third variable \( \nu_{\|} \) has been set to a constant \( \nu_{\|0} \). Monoenergetic ions all have the same \( \nu_{\|} \) at each box edge, so no significant information actually was discarded for the special case considered here. The reduced distribution functions evaluated for each of the plasma groups are combined using the same weighting factors that were found in the least squares fit in order to produce reduced distributions for the self-consistent current sheet.

The dotted and dashed lines in Figure 3.6a show the density calculated by integrating \( f(\nu_{\|}, \nu_{\perp}) \) and \( f(\alpha, \psi) \), respectively. The solid line is the \( n(z) \) described previously. The solid line in Figure 3.6a is plotted using a point at the center of each box because \( n(z_k) \) is defined using the time in the \( z_k \) box. In contrast, the distribution functions refer to box edges, so the dotted and dashed lines in Figure 3.6a used points at each edge.

**Pressure and Force Balance** Figure 3.7a shows \( T_{\|} \) and \( T_{\perp} \), the average parallel (dotted) and perpendicular (dashed) “temperatures” in keV. These temperatures were obtained from integrations involving \( f(\nu_{\|}, \nu_{\perp}) \) at each box edge and were defined in term of average thermal energies by \( T_{\|} = 2W_{\|} \) and \( T_{\perp} = W_{\perp} \). Figure 3.8a compares the corresponding parallel \( (P_{\|} = nT_{\|}) \), perpendicular \( (P_{\perp} = nT_{\perp}) \), and the total
(P = 1/3P_\parallel + 2/3P_\perp) particle pressures.

To be self-consistent, particle and magnetic field forces should balance in the model plasma sheet. We have checked this only using the reduced distribution function information described above, i.e., by assuming the pressure tensor \( P \) is diagonal, with element \( P_\parallel \) and \( P_\perp \). We will see below that this is a good approximation everywhere except near \( z = 0 \). Writing the particle force \( \nabla \cdot P \) in Cartesian coordinates [e.g., Krall and Trivelpiece, 1973], equating this to the magnetic force \( (\nabla \times B) \times B/\mu_0 \), and integrating in the \( z \) direction gives

\[
\sin \theta_B \cos \theta_B \left[ P_\parallel - P_\perp - \frac{B^2}{\mu_0} \right] = 0
\]

if forces are balanced in the \( x \) direction and

\[
P_e + P_\parallel \cos^2 \theta_B + \left[ P_\perp + \frac{B^2}{2\mu_0} \right] \sin^2 \theta_B = P_e (0) + P_\parallel (0)
\]

if forces are balanced in the \( z \) direction. When viewed in the \( E_y = 0 \) reference frame, plasma in the model field (2.1) has no steady flow in the \( x-z \) plane. In the equations above, \( \theta_B \) is the angle between \( B \) and the \( z \) axis; \( P_e = n(z) T_e \) is the isotropic electron pressure, where we used \( T_e = (3/8) \text{keV} \); \( P_e (0) \) and \( P_\parallel (0) \) are the electron and parallel ion pressure at \( z = 0 \); and the solid line in figure 3.6a is used for \( n(z) \). The dotted and dashed lines in Figure 3.8b show the left sides of Equation (3.3) and (3.4), respectively. Each curve is nearly constant throughout the current sheet, so the model plasma sheet is close to being in force balance.

Figure 3.7b shows the ratio of particle to magnetic field energy densities, or plasma \( \beta \), in our model. The measurements of Baumjohann et. al. [1989] break the inner CPS, which is the region we are modeling, into two sections. The section closest to the neutral sheet, where \( B_{xy} = (B_x^2 + B_y^2)^{1/2} < 7.5 \text{nT} \), has an average \( \beta = 20 \), while the remainder of
the inner CPS has an average $\beta = 3$. The observed $\beta$ drops to 0.5 in most of the outer CPS and to 0.3 as the PSBL is approached. In our model, $B_{xy} = 7.5$ nT at $z = 0.24 R_E$. The model $\beta$ therefore agrees reasonably well observations in the inner CPS.

It has long been known [e.g., Rich et al., 1972; Eastwood, 1974; Hill, 1975] that force balance requires a one-dimensional plasma sheet to be near the firehose instability limit $\mu_0 (P_\parallel - P_\perp) / B^2 = 1$ at the edge of the current layer. The model result is plotted in Figure 3.9a. The large changes near $z = 0$ are associated with the small fluctuations in $P_\parallel$ and $P_\perp$ in Figure 3.8a.

**Detector Count Rates** One way to examine $f(\alpha, \psi)$ of a single ion group is to keep a list of the pitch and phase angles that each particle had whenever it crossed the edge of a box. As in Figure 3.5b, 1000 3-keV protons were injected at $z_i = 0.25 R_E$ with angular distributions characterized by $T_\parallel/T_\perp = 10$. Of these ions, 842 escaped from the current sheet and 158 were trapped. The pitch and phase of each crossing of a box edge near the center of plasma sheet, at $z = 0.067 R_E$, are plotted in Figure 3.10. Figure 3.11 is a similar plot near the edge of the current sheet, at $z = 0.93 R_E$. In these plots, protons with $\alpha < 90^\circ$ and $\alpha > 90^\circ$ are moving away from and toward $z = 0$, respectively.

Figure 3.10 and 3.11 are useful when studying fine structure and boundaries in the phase space distribution but do not provide a simple way to predict particle count rates. The untrapped particles forming clusters near $(\alpha, \psi) = (75^\circ, 230^\circ)$ and $(105^\circ, 130^\circ)$ in Figure 3.10b carry most of the current. At these points a 1/2-oscillation version of Figure 2.9a shows the ion being turned around after crossing the neutral sheet. The box edge at $z = 0.067 R_E$ falls midway between the first two points at small $z$ in Figure 3.5b. The second cluster of dots in each panel of Figure 3.10b, with $\alpha$ near $25^\circ$ or $155^\circ$, is produced by the particle as it first approaches the neutral sheet (before reflection) or when it leaves (after reflection). Figure 3.11a shows that trapped ions have pitch angles near $90^\circ$ at the
z = 0.93 RE box edge, as expected.

The asymmetry between $\psi = 90^\circ$ and $270^\circ$ of untrapped ions in Figure 14b is produced by the addition or subtraction of the $z$ components of $v_\parallel$ and $v_\perp$, and does not necessarily imply an asymmetry in particle detector count rates or in $f(\alpha, \psi)$. The density of dots must be weighted by the $|\cos \theta_v|$ factor (3.2). It is difficult to appropriately weight the size or density of points in a plot. Another problem is that Figures 3.10 and 3.11 show results from only a single group of 1000 ion trajectories. The technique described above involves combining several plasma groups with different injection points or temperature ratios in order to produce a self-consistent plasma sheet. The full self-consistent distribution function is a weighted combination of these groups. Once again, it is difficult to introduce weighting factors when plotting one marker for each box edge crossing.

It is for the above reasons that $f(\alpha, \psi)$ was evaluated for each group of 1000 ions. The $f(\alpha, \psi)$ for each group then was weighted according to the least squares fit results and combined to yield the total $f(\alpha, \psi)$ for the self-consistent plasma sheet. This function gives the predicted particle count rate.

Figure 3.12 shows a plot of $f(\alpha, \psi)$ at $z = 0.067$ RE for the single ion group in Figure 3.10. The asymmetry between $\psi = 90^\circ$ and $270^\circ$ is less in Figure 3.12a than in Figure 3.10a. Similar trapped and untrapped $f(\alpha, \psi)$ plots corresponding to the $z = 0.93$ box edge show almost no $\psi$ dependence.

Figure 3.13a and 3.13b show the final $f(\alpha, \psi)$ at $z = 0.067$ RE and $z = 0.93$ RE, respectively. All trapped and untrapped plasma groups have been combined after weighting according to the least squares fit. Note that fluxes at $z = 0.067$ RE are not gyrotropic. The clustering (Figure 3.10b and 3.12b), which is produced by the reflection of untrapped ions, remains. Our use of a diagonal pressure tensor to study force balance is therefore not accurate at $|z| < 0.15$ RE, where untrapped ions are most important (Figures 3.4a and
3.5b). The full three-dimensional $f(v)$ should be used here. There are no nearly empty regions of phase space because no $\kappa = 0.52$ ions can reach $z = L$ in a single $z$ oscillation.

Previous orbit-tracing studies have noted the tendency for field alignment at the edge of the current sheet. This effect is evident in Figure 3.13b. Note that predicted fluxes have a secondary maximum at $90^\circ$ and shallow minima near $\alpha = 60^\circ$ and $120^\circ$. These latter results are not consistent with observations in the midtail CPS, as will be discussed in the Chapter 5.
Figure 3.1 Each curve is the current carried by a group of 1000 3-keV proton. The ions were randomly selected from a source whose angular distribution is the same as a bi-Maxwellian with $T_{\parallel}/T_{\perp} = 0.1$ (dotted line), 1 (solid lines), or 10 (dashed lines). (a) Trapped ions injected at $z_i = 0$; (b) Untrapped ions with $z_i = 1 R_E$. 
Figure 3.2 Similar to Figure 3.1 except that these curves are for trapped ions with
(a) $z_i = 0.25 \, R_E$ and (b) $z_i = 0.50 \, R_E$. 
Figure 3.3 The small zero-order electron currents (dotted line) are added to the ion current shown in Figure 3.1 and Figure 3.2.
Figure 3.4 (a) The solid line is the $j_y$ needed to produce the model magnetic field. The dotted line shows currents carried in the zero-order approximation to the current sheet. The dashed line is the first-order result after iteration. (b) The dotted line is the potential needed to produce exact charge neutrality in the zero-order approximation, and the solid line is a polynomial fit. The dashed line is the final potential obtained after iteration.
Figure 3.5 Ion (dashed lines) and electron (dotted lines currents after the final iteration. (a) Trapped protons with \( z_i = 0.25 \, R_E \) and \( T_{\parallel}/T_{\perp} = 0.1 \); (b) untrapped protons with \( z_i = 0.25 \, R_E \) and \( T_{\parallel}/T_{\perp} = 10 \).
Figure 3.6 (a) Proton number densities evaluated by using Equation 2.7 (solid line), by integrating $f(v_\parallel, v_\perp)$ (dotted line), and by integrating $f(\alpha, \psi)$ (dashed line); (b) average ion cross-tail drift velocity.
Figure 3.7 (a) Parallel and perpendicular temperatures throughout the current sheet; (b) plasma $\beta$. 
Figure 3.8 (a) Parallel, perpendicular, and total pressures. (b) The dotted and dashed lines are plots of the left-hand sides of Equations 3.3 and 3.4, respectively.
Figure 3.9 (a) Firehose instability parameter. The plasma is unstable when this parameter exceeds 1. (b) One-oscillation or \( \kappa = 0.32 \) ions were used so the region in which the two calculations differ would extend over several boxes.
Figure 3.10 Pitch and phase angles of protons at the $z = 0.067 \, R_E$ box edge. A total of 1000 protons were followed. (a) Trapped ions; (b) untrapped ions. Ions with $\alpha < 90^\circ$ (left-hand panels) are moving away from the neutral sheet, and ions with $\alpha > 90^\circ$ (right-hand panels) are moving toward the neutral sheet.
Figure 3.11 Similar to Figure 3.10 but for a box edge at \( z = 0.93 \, R_E \).
Figure 3.12 Plots of $f(\alpha, \psi)$ for one group of 1000 protons as seen at the $z = 0.067 R_E$ box edge. (a) Trapped ions; (b) untrapped ions.
Figure 3.13 Similar to Figure 3.12 except that this plot is for the final combination of trapped and untrapped ions (a) at $z = 0.067 \, R_E$ and (b) at $z = 0.93 \, R_E$. 

$\log_{10} f(\alpha, \psi)$
Chapter 4

Case 2: Stochastic Particles

We used $B_{x_0} = 25 \, nT$, $B_{z_0} = 2.5 \, nT$ and $L = 0.8 \, R_E$ in Equations (2.1) and (2.3) to get $\kappa = 0.4$ for 3 keV protons. This $\kappa$, which is half way between the $\kappa = 0.32$ (1-oscillation) and $\kappa = 0.53$ (1/2-oscillation) resonances (the valley between the two peaks in Figure 2.2), corresponds to stochastic orbits. As shown in Chapter 2, the pitch angles of $\kappa = 0.4$ particles when they leave the current sheet appear to be almost unrelated to their entry pitch angles.

4.1 Ion Current Distributions

As in Chapter 3, we calculate the current by tracing ion orbits. Groups of 1000 particles were injected at different $z$ locations with different $T_{||}/T_{\perp}$ ratios. From our experience in Chapter 3, we picked one group with $T_{||}/T_{\perp} = 1$ injected at $z_i = 0$, and three groups with $T_{||}/T_{\perp} = 0.1$ injected at $z_i = 0.25 \, L$, $z_i = 0.5 \, L$ and $z_i = 0.75 \, L$. We also injected $T_{||}/T_{\perp} = 10$ groups at $z_i = 0.25 \, L$, $z_i = 0.5 \, L$, $z_i = 0.75 \, L$ and $z_i = L$.

Each group of $\kappa = 0.4$ particles had a larger average pitch angle change than the corresponding $\kappa = 0.52$ group. A typical orbit gets scattered each time it interacts with the current sheet. As mentioned in Chapter 2, we can not separate groups of $\kappa = 0.4$ particles by whether or not they ever leave the current sheet. Almost all $\kappa = 0.4$ particles left the current sheet within the cutoff time, regardless of their injection point and of their $T_{||}/T_{\perp}$ ratio. These particle trajectories can be very complicated, but we still can classify them as figure-8 type, mirror-type, and untrapped type. In this $\kappa = 0.4$ case, the figure-8 parameter $z_o \equiv 0.25 \, L$, and $z_{max8} = 1.8 \, z_o = 0.45 \, L$. Figure 4.1a shows $j_y(z)$ carried by ions from a group injected at $z_i = 0$ with $T_{||}/T_{\perp} = 1$. Although this $j_y$ is carried by the ions
which left current sheet (906 out of 1000 particles), the shape of the curve is very similar to those associated with figure-8 type trapped particles (Figure 3.1a). As we discussed in Chapter 2, figure-8 type orbits dominated at $z < z_{\text{max}8}$ for the group of particles which has $T_{\parallel}/T_\perp = 0.1$ or $T_{\parallel}/T_\perp = 1$ and an injection point $z_i < z_{\text{max}8}$.

Figure 4.1b shows a group of 1000 particles that was injected at $z_i = 0.5 \text{ L}$ or $0.4 \text{ R}_E$ with $T_{\parallel}/T_\perp = 10$. This $z_i$ is greater than $z_{\text{max}8}$, so that the figure-8 type feature is not seen here. The $j_y(z)$ curve in Figure 4.1b shows the basic feature of untrapped resonant particles (Figure 3.3b) in the region of $z < z_0$. The peaks near $z = 0.5 \text{ R}_E$ and $z = 0.9 \text{ R}_E$ show mirror effects.

Figures 4.2a and 4.2b give the $j_y$ carried by groups of $T_{\parallel}/T_\perp = 0.1$ particles with injection points at $z_i = 0.5 \text{ L}$ and $0.75\text{L}$, respectively. The two figures both have positive $j_y$ at $z > z_i$ and negative $j_y$ in the region between $z_0$ and $z_i$, which indicates a mirror effect. As is described in Chapter 2, there is also a possible region of positive $j_y$ for $z < z_0$.

Figure 4.3a shows the $j_y$ carried by a group of $T_{\parallel}/T_\perp = 10$ particles with injection point $z_i = 0.75 \text{ L}$. We still can observe the basic feature of the untrapped particles in the region of $z < z_0$, but the mirror effect plays an important role in the $z > z_0$ region. Figure 4.3b shows the $j_y$ carried by a group of ions that has $T_{\parallel}/T_\perp = 10$ and an injection point $z_i = \text{ L}$. In this Figure we can not see any similarity with Figure 3.3b. Particles just mirrored back and forth several times in the center of the plasma sheet before they left. For $\kappa = 0.4$, there is a very small region in phase space for which particles injected at $z = \text{ L}$ trace out a simple semicircle in x-y or equatorial plane like untrapped resonant particles do (Figure 2.3).

From the discussion above, we noted that all the $T_{\parallel}/T_\perp = 0.1$ groups have negative $j_y$ from $z = 0$ up to a point $z > z_0$, and that all the $T_{\parallel}/T_\perp = 10$ groups have negative $j_y$ at $z < z_0$. We can not remove the region of negative $j_y$ near $z = z_0$ by changing the injec-
4.2 Electron Current

So far, we only considered ion current. We also need to calculate the $j_y$ carried by electrons. The reason that electron current needs to be included in the total electric current was stated in Chapter 3. In the present case, an electron which has $1/8$ the energy of a $\kappa = 0.4$ proton has $\kappa = 4.4$. The $j_y$ carried by such an electron therefore is well approximated by the guiding center drift expression (2.8). We used the same method as described in Chapter 3, assuming the electron density exactly equals the previously calculated ion density throughout the tail, to determine the electron current.

Figures 4.4a and 4.4b show the electron (dotted curve), ion (dashed curve), and the total current for $\kappa = 0.4$. The ions in Figure 4.4a are injected at $z_i = 0.25 \text{L}$ or $0.2 \text{R_E}$ with $T_{\parallel}/T_{\perp} = 0.1$. The shape of the dashed curve in Figure 4.4a is very similar to Figure 4.1a. The ions in Figure 4.4b are the same as those used in Figure 4.1b. These figures show that the electrons do not contribute a great deal to the total $j_y$ for this stochastic case. This is the same result that was obtained with $\kappa = 0.53$ resonant particles.

4.3 Combination of Plasma Groups

3 keV monoenergetic particles The solid curve in Figure 4.5a shows the $j_y(z)$ (Equation 2.4) which is needed to produce a Harris magnetic field with the parameters $B_{x_0} = 25 \text{ nT}$, $B_{z_0} = 2.5 \text{ nT}$ and $L = 0.8 \text{ R_E}$. This is the field in which orbits were traced. None of the plasma groups studied up to this point carries a current distribution (Figure 4.1, Figure 4.2, Figure 4.3, and Figure 4.4a) that is similar to the solid curve in Figure 4.5a. We use the same technique that was used in Chapter 3 to perform a least squares fit to the desired $j_y(z)$. We start with eight groups of ions and their accompanying electrons. Although some of the groups carry current in the positive $y$ direction at relatively large $|z|$ and others carry current in the positive $y$ direction at small $|z|$, the fitting
failed. The best fit we could get to the desired current by combining the groups is the dashed curve shown in Figure 4.5a. This is not an adequate approximation to the solid curve. Note that there is a negative current region near $z = 0.25 L$ or $z = z_0$, and the current is very low at small $|z|$ compared to the desired current. As was mentioned in the previous section, we were not to be able to remove this dip by using the stochastic 3 keV protons alone.

**High energy ions** As shown in last section, the stochastic 3 keV protons do not carry a current distribution that is even close to the assumed Harris model requirements. There is always a big dip in the region near $z = 0.25 L$ or $z = 0.2 R_E$. In order to obtain a self-consistent current sheet, some resonant energetic protons are needed to provide a reasonable approximation to the assumed current in this region. We find that $3/2$-oscillation or $\kappa = 0.23$ particles, which have a positive peak in $j_y(z)$ near $z = 0.2 R_E$, can be used to fill in the dip. We therefore traced a group of 24 keV particles that was injected at $z = L$ with $T_\parallel/T_\perp = 10$ in the same magnetic field described in last section. The dashed curve on Figure 4.5b shows the current that this ion group carried. The electron current is the dotted curve shown on Figure 4.5b, which was calculated using the guiding center drift expression (2.8). An electron with 1/8 the energy of a $\kappa = 0.23$ proton has $\kappa = 2.5$ so that the guiding center description is still valid.

We again performed a least squares fit to the desired $j_y(z)$ using the one group of 24 keV ions and several groups of 3 keV ions. The fit program gave a reasonable fit this time, as shown by the dashed curve on the Figure 4.6a. The main contributions to this fit are given by the group of 24 keV ions shown in Figure 4.5b and the groups of 3 keV protons shown in Figures 4.1b, 4.4a, and 4.2a. The number density associated with this fit is the solid curve shown on Figure 4.6b. The dash curve on Figure 4.6b is the number density contributed by 3 keV protons, and the dotted curve is the 24 keV proton contribution. We
can see that the high energy particles make up less that 10% of the total number density, which is consistent with the observations [Lennartsson and Shelley, 1986; Daglis et al., 1991b].

4.4 Final Current Sheet

We got a reasonable fit for the combined current carried by 3 keV and 24 keV protons and also the number density throughout the current sheet in last section. An electric field with a component along B must evolve in the self-consistent current sheet in order to maintain charge neutrality. If ions and electrons were injected into a region with $E_n = 0$, their markedly different orbits would result in a large charge separation. This would generate a very large $E_z(z)$ in the one-dimensional model. The $E_z(z)$ would evolve until the quasi-neutrality condition that generally is required in all plasmas was obtained. Our imposition of an $E_z(z)$, which has a component along B at all z, is therefore required in order to maintain charge neutrality.

The resulting electric potential $\phi(z)$ and its polynomial fit are shown on Figure 4.7a as the solid and dotted curve respectively. In this case, the first approximation to $\phi(z)$ has both positive and negative regions, so the complicated form and the simple Boltzmann relation are both used for electrons (Appendix A). Groups of 1000 ions each were once again randomly selected and traced with the added electric field $E_z = -\partial \phi/\partial z$ in order to find a self-consistent magnetotail model. Whenever an $E_z(z)$ was present, protons at different $z_i$ were started with whatever energy was required so that they would have 3 keV (or 24 keV for high energy particles) at $z = 0$. It is expected that the ion currents (Figures 4.8a and 4.8b for 3 keV protons, Figure 4.7b for 24 keV high energy protons) are very similar to the results obtained for the corresponding ion groups with $E = 0$ (Figures 4.4, and 4.5b), since the maximum potential difference within the current sheet is only about 100 volts (Figure 4.7a) while we are dealing with 3 keV (or 24 keV) protons. The electron
currents are shown by the dotted curves in Figures 4.7b and 4.8.

The dashed curve in Figure 4.7a is the $\phi$ in this iteration. We can see that the two iterations produced very similarly shaped $\phi(z)$ plots. The polynomial fit to $\phi(z)$ (solid curve in Figure 4.7a) was used to obtain a smoothly varying expression which can be differentiated to get $E_z$ in the second iteration. With $B$ exactly the same and $E_z$ (the slope of the smoothed $\phi(z)$ curve) nearly the same for the two iterations, we say the procedure has already converged.

As we mentioned in the last section, the 3 keV monoenergetic stochastic protons cannot form the current that the model field required. We have to include 24 keV energetic resonant protons in order to get a reasonable current distribution. The dotted curve in Figure 4.9a shows the resulting $j_y$ carried by 3 keV and 24 keV ions and associated electrons in the final model current sheet. The dashed curve and the dot dashed curve shows the contributions from 24 keV particles and 3 keV particles respectively. The solid curve on Figure 4.9b is the final number density for this combined current sheet. It is also a little higher than observed densities. The dashed and the dotted curves present the number density for 3 keV and 24 keV particles respectively.

In summary, we conclude that a group of 24 keV protons must be added to the stochastic particles to get a reasonable fit to the desired current. The high energy particles contribute approximately 8% of the total number density.

4.5 Distribution Function and Fluid Parameters

Reduced Distribution Function Information about the ion distribution function is saved during the final iteration for each group. The $v_\parallel$ and $v_\perp$ of an ion are stored each time a $z$ box is entered. The reduced two-dimensional distribution function (3.1) then can be evaluated at the edge of each of the 20 pairs of $z$ boxes. Only the magnitude of $v_\perp$ is retained in this reduced distribution function; the phase angle information has been dis-
carded for this \( f(v_\|, v_\perp) \). We also retain ion pitch and phase angles, \( \alpha \) and \( \psi \), in a second reduced or two-dimensional distribution function (3.2) at the edge of each box. In this \( f(\alpha, \psi) \) the third variable \(|v|\) has been set to a constant \( v_0 \). Monoenergetic ions all have the same \(|v|\) at each box edge for each ion group, so no significant information actually was discarded for the special case considered here. The reduced distribution function \( f(\alpha, \psi) \) evaluated for each of the plasma groups are combined using the same weighting factors that were found in the least squares fit in order to produce a reduced distribution \( f(\alpha, \psi) \) for the self-consistent current sheet.

**Pressure and Force Balance** Figure 4.10a shows \( T_\| \) and \( T_\perp \), the average parallel (dotted) and perpendicular (dashed) “temperatures” in keV. These temperatures were obtained from integrations involving \( f(v_\|, v_\perp) \) at each box edge. They were defined in term of average thermal energies by \( T_\| = 2W_\| \) and \( T_\perp = W_\perp \) for each ion group, and then combined. Figure 4.10b compares the corresponding parallel (\( P_\| = nT_\| \)), perpendicular (\( P_\perp = nT_\perp \)), and the total (\( P = 1/3P_\| + 2/3P_\perp \)) particle pressures.

The dotted and dashed lines in Figure 4.11a show the left sides of Equation (3.3) and (3.4), respectively. Each curve is nearly constant throughout the current sheet, so the model plasma sheet is close to being in force balance.

Figure 4.11b shows the ratio of particle to magnetic field energy densities, or plasma \( \beta \), in our \( \kappa = 0.4 \) model. The observed \( \beta \) is shown in Chapter 3 section 6. In this case, \( B_{xy} = 0.75 \) nT at \( z = 0.248 \) R\(_E\). The model \( \beta \) therefore is higher than the observations in the inner CPS for this case.

It has long been known [e.g., Rich et al., 1972; Eastwood, 1974; Hill, 1975] that force balance requires a one-dimensional plasma sheet to be near the firehose instability limit \( \mu_0 (P_\| - P_\perp) / B^2 = 1 \) at the edge of the current layer. The \( \kappa = 0.4 \) model result is plotted in Figure 4.12. The large changes near \( z = 0 \) are similar to those seen in the
\( \kappa = 0.52 \) case, and are associated with the small fluctuations in \( P_\parallel \) and \( P_\perp \) in Figure 4.10b. These fluctuations appear to be so small that we independently generated several model plasma sheets to check the reproducibility of these results. Those models with \( B_y = 0 \) that we have investigated to date generally show that the firehose parameter first exceeds 1 as the neutral sheet is approached and then becomes negative \((P_\perp > P_\parallel)\) at \( z = 0 \).

**Detector Count Rates** As in Figure 4.8a, 1000 3-keV protons were injected at \( x_i = 0.2 \) \( R_E \) with angular distributions characterized by \( T_\parallel / T_\perp = 0.1 \). Of these ions, 906 escaped from the current sheet and 94 were trapped. The pitch and phase angle of each crossing of a box edge near the center of plasma sheet, at \( z = 0.054 \) \( R_E \), are plotted in Figure 4.13. Figure 4.14 is a similar plot near the edge of the current sheet, at \( z = 0.744 \) \( R_E \). In these plots, protons with \( \alpha < 90^\circ \) and \( \alpha > 90^\circ \) are moving away from and toward \( z = 0 \), respectively.

The \( \kappa = 0.4 \) stochastic particles change pitch angle each time when they interact with the current sheet. The trapped pure figure-8 particles form the triangular areas near \((\alpha, \psi) = (50^\circ, 320^\circ)\) and \((130^\circ, 40^\circ)\) in Figure 4.13. Figure 4.14b shows that stochastic particles can be mirrored back to the current sheet a couple of times before escaping. Figure 4.14a shows that trapped particles have pitch angles near \( 90^\circ \) at the \( z = 0.744 \) \( R_E \) box edge, as expected.

Since we included one group of \( 3/2 \)-oscillation or \( \kappa = 0.23 \) untrapped particles in our final \( \kappa = 0.4 \) model current sheet, the corresponding pitch and phase of each crossing of a box edge near the center of plasma sheet, at \( z = 0.054 \) \( R_E \), are plotted in Figure 4.15a. These untrapped particles forming clusters near \((\alpha, \psi) = (75^\circ, 210^\circ), (105^\circ, 150^\circ); (45^\circ, 180^\circ), (135^\circ, 180^\circ); (80^\circ, 105^\circ)\) and \((100^\circ, 255^\circ)\) carry most of the current. At these points a \( 3/2 \)-oscillation ion is being turned around after crossing the neutral sheet. The box
edge at $z = 0.054 \, R_E$ falls midway between the first two points at small $z$ in Figure 4.7b. The cluster of dots in each panel of Figure 4.15a, with $\alpha$ near 35° or 145°, is produced by the particle as it first approaches the neutral sheet (before reflection) or when it leaves (after reflection). Figures 4.13, 4.14 and 4.15a are useful when studying fine structure and boundaries in the phase space distribution but do not provide a simple way to predict particle count rates.

The distribution function $f(\alpha, \psi)$ was evaluated for each group of 1000 ions. Figure 4.15b shows a plot of $f(\alpha, \psi)$ at $z = 0.054 \, R_E$ for the high energy ion group. The density of dots in Figure 4.15a was weighted by the $|\cos \theta_v|$ factor (Equation 3.2). The $f(\alpha, \psi)$ for each group then was weighted according to the least squares fit results and combined to yield the total $f(\alpha, \psi)$ for the self-consistent plasma sheet. This function gives the predicted particle count rate.

Figures 4.16a and 4.16b show the final $f(\alpha, \psi)$ at $z = 0.054 \, R_E$ and $z = 0.744 \, R_E$, respectively. All 3 keV and high energy plasma groups have been combined after weighting according to the least squares fit. Note that fluxes at $z = 0.054 \, R_E$ are not gyrotropic. The clustering (Figure 4.15a), which is produced by the reflection of untrapped high energy ions, remains. Our use of a diagonal pressure tensor to study force balance is therefore not accurate at $|z| < 0.15 \, R_E$, where untrapped ions are most important. The full three-dimensional $f(v)$ should be used here.

Figure 4.16b shows the tendency for field alignment at the edge of the current sheet. The predicted fluxes have a secondary maximum at 90° and shallow minima near $\alpha = 60^\circ$ and 120°, similar to those shown in Figure 3.13.
Figure 4.1 Current carried by a group of 1000 3-keV proton for (a) $z_i = 0$ with $T_{\parallel}/T_\perp = 1$; (b) $z_i = 0.4 \, R_E$ with $T_{\parallel}/T_\perp = 10$. 
Figure 4.2 Similar to Figure 3.1 but for $T_\parallel/T_\perp = 0.1$ with (a) $z_i = 0.4 \, R_E$ and (b) $z_i = 0.6 \, R_E$. 
Figure 4.3 Similar to Figure 4.1 but for $T_\parallel/T_\perp = 10$ with (a) $z_i = 0.6 \, R_E$ and (b) $z_i = 0.8 \, R_E$. 
Figure 4.4 The small zero-order electron currents (dotted curve) are added to the ion current (a) $z_i = 0.2 \, R_E$ with $T_{/}/T_\perp = 0.1$ and (b) $z_i = 0.4 \, R_E$ with $T_{/}/T_\perp = 10$. 
Figure 4.5 (a) The solid line is the $j_y$ needed to produce the model magnetic field. The dotted line shows currents carried in the zero-order approximation to the current sheet. (b) Current carried by a group of 1000 24 keV protons that were injected at $z_i = 0.8 R_E$ with $T_{\parallel}/T_{\perp} = 10$ (dashed curve), the small zero-order electron currents (dotted curve) are added to get the total electric current (solid curve).
Figure 4.6 (a) The solid line is the \( j_y \) needed to produce the model magnetic field. The dotted line shows currents carried by one group of 24 keV protons and three groups of 3 keV protons in the zero-order approximation to the current sheet. The dashed line is the first-order result after iteration. (b) The solid line is the number density associated with (a), dotted line is the number density contributed by 24 keV protons, and the dashed line is the 3 keV proton contribution.
Figure 4.7 (a) The dotted line is the potential needed to produce exact charge neutrality in the zero-order approximation, and the solid line is a polynomial fit, the dashed line is the final potential obtained after iteration. (b) Ion (dashed lines) and electron (dotted lines) currents after the final iteration for 24 keV protons that were injected at $z_i = 0.8 \, R_E$ with $T_{\parallel}/T_{\perp} = 10$ (dashed curve).
Figure 4.8 Same as Figure 4.4 but for ion (dashed lines) and electron (dotted lines) currents after the final iteration.
Figure 4.9 (a) The solid line is the $j_y$ needed to produce the model magnetic field. The dotted line is the first-order result for 3 keV and 24 keV particles after iteration. The dashed line shows current contributed by 24 keV particles to the current sheet. The dot dashed line is the contribution from 3 keV particles. (b) Final number density (solid curve) for combined current sheet (a). The dashed and the dotted curve present the number density for 3 keV and 24 keV particles respectively.
Figure 4.10 (a) Parallel (dotted curve) and perpendicular (dashed curve) temperatures throughout the current sheet. (b) Parallel (dashed curve), perpendicular (dotted curve), and total (solid curve) pressures.
Figure 4.11 (a) The dotted and dashed lines are plots of the left-hand sides of Equations 3.3 and 3.4, respectively. (b) Plasma $\beta$. 
Figure 4.12 Firehose instability parameter. The plasma is unstable when this parameter exceeds 1.
Figure 4.13 Pitch and phase angles of protons at the $z = 0.054 \, R_E$ box edge. A total of 1000 protons were followed. (a) Trapped ions; (b) untrapped ions. Ions with $\alpha < 90^\circ$ (left-hand panels) are moving away from the neutral sheet, and ions with $\alpha > 90^\circ$ (right-hand panels) are moving toward the neutral sheet.
Figure 4.14 Similar to Figure 4.13 but for a box edge at \( z = 0.744 \, R_E \).
Figure 4.15 (a) Similar to Figure 4.13b but for a group of 24 keV particles. (b) \( f(\alpha, \psi) \) plot of (a).
Figure 4.16 Similar to Figure 4.15b except that this plot is for the final combination of trapped and untrapped ions (a) at $z = 0.054 \, R_E$ and (b) at $z = 0.744 \, R_E$. 
Chapter 5

Discussion

The motivation for this work was to improve our understanding of electric currents in the magnetotail. We traced monoenergetic ion trajectories on taillike field lines. The $E_y = 0$ reference frame was used so we could isolate the current carried by ions with each characteristic orbit type. However, the use of a one dimensional model resulted in magnetic field gradients that are not representative of those in the midmagnetotail. The model distribution functions are therefore substantially different from actual magnetotail distribution functions. The magnetic field pressures must be balanced by parallel particle streaming or by pitch angle anisotropies ($P_\parallel \neq P_\perp$) in the $E_y = 0$ reference frame. In the actual magnetotail and in two or three dimensional models, some of the magnetic force is balanced by pressure gradients and by the deflection of drifting plasma. The purpose of this section is to use physical arguments, which are guided by observations, in order to determine which one dimensional model results provide useful information about the magnetotail.

5.1 Observed Plasma Sheet Structure

Magnetometer data and models. Neutral sheets and current sheets are detected by magnetometers, which respond to the effects of electric currents. The observational definition of a mathematically thin "neutral sheet" is sometimes $B_x = 0$ and sometimes $|\mathbf{B}| = \text{minimum}$. The typical half thickness of the cross tail current sheet appears to be approximately $1 \text{ R}_E$ near midnight (see below). The term neutral sheet often is used to refer to the entire current sheet.

Magnetosphere models are derived from statistical averages of magnetic field mea-
surements, and therefore represent statistically averaged electric currents that flow in the magnetotail. Magnetosphere models do not necessarily provide a good approximation to a snapshot of the magnetotail. For example, a least squares fit to averaged magnetic field data tends to overestimate the instantaneous current sheet thickness if the sheet is in constant motion.

**Energetic particle data.** Particle detectors observe the dense CPS, which sometimes is discussed in terms of an inner and an outer region. A typical CPS thickness is 5-10 \( R_E \) [Frank, 1985; Fairfield, 1986]. The CPS therefore usually appears to be substantially thicker than the current sheet. We use the terms “current sheet” and “inner CPS” to refer to the same region. Beyond the outer CPS, particle and magnetic field detectors both observe the PSBL, with a thickness of about 1 \( R_E \).

**Current sheet thickness.** The only measurement we have seen of the instantaneous structure of a typical current sheet is that of McComas et al. [1986]. Other detailed measurements have been made in the extremely thin current sheets which form during substorm growth phase. McComas et al. used data from three sheet crossings which were made during a single day. The entire plasma sheet moved rapidly across the ISEE 1 and 2 satellites because a sharp discontinuity in the interplanetary plasma reached Earth. The region of strong currents on this day had a full thickness of 10,000 km during two crossings and 25,000 km during the other. For at least one crossing, the current sheet exhibited a two-layered structure: the central layer described above was imbedded within a several times thicker region of weak currents. No region of very small \( |B| \) was observed on this day. The field direction at the point of minimum \( |B| \) was substantially different during the 3 events. In each case \( B \) became uniform and the current density dropped to small values well before the outer edge of the CPS was reached. Figure 5.1 summarizes the above observations.
5.2 Analysis of One Dimensional Model Results

Two cases were presented in this thesis. In Chapter 3, the parameters $B_{xo} = 32$ nT, $B_{zo} = 3.3$ nT and $L = 1$ RE are used to represent a typical current sheet near $x = -14$ RE in the $K_p = 4$ version of the T89 model. These parameters yield $\kappa = 0.52$ for 3 keV protons, and therefore correspond to a $1/2$-oscillation quasi-adiabatic resonant current sheet. The other case studied was in Chapter 4. The parameters $B_{xo} = 25$ nT, $B_{zo} = 2.5$ nT and $L = 0.8$ RE were used to represent a current sheet near $x = -20$ RE in the $K_p = 4$ version of the T89 model. These parameters yield $\kappa = 0.4$ for 3 keV protons that correspond to a $3/4$-oscillation stochastic case. Huang et al. [1990; 1991] suggested that $B_{zo}$ is underestimated in T89. If so, then both the $\kappa = 0.52$ and $\kappa = 0.4$ regions modeled here would be found at larger $|x|$. The T89 current sheet half-thickness parameter $L$ is 1.5 RE at midnight and increases toward the flanks. We used thinner $L = 1$ RE and $L = 0.8$ RE current sheet because motion of the plasma sheet does not substantially change $B_{xo}$ or $B_{zo}$, but tends to inflate $L$, as noted above. The 1 RE value also is close to half the instantaneous current sheet thickness measured by McComas et al. [1986]. There are no net field-aligned currents in our symmetric one dimensional model; only the $B_y = 0$ case was considered in this thesis.

We found the procedure to generate a self-consistent resonant particle current sheet to be relatively straightforward. Although the model current sheet is not unique, we found that there really is not much choice in deciding which particle groups must be introduced. In particular, the $\kappa = 0.52$ current sheet required inclusion of untrapped ions and of trapped ions with both figure 8 and mirror-type orbits, and the $\kappa = 0.4$ current sheet required inclusion of high energy resonant particles. We have not carried out an extensive study, but have initiated the least squares fitting routine with several alternate sets of approximately 10 groups each. No significant difference in any macroscopic quantity was
noted in the final plasmas.

The ability to find a variety of distribution functions, each of which carries the same $j_y(z)$, illustrates the complexity of the concept of DC conductivity in a collisionless plasma [Speiser, 1970; Lyons and Speiser, 1985]. For example, the concept of conductivity generally is not considered useful when describing $j$ of the ring current. In this work we found it is relatively easy to generate a sample plasma which carries a rather arbitrary pre-selected $j_y(z)$, even though $E_y(z) = 0$.

Neutral sheet. A neutral sheet with distinct boundaries is included in many observationally-based sketches of the plasma sheet (Figure 5.1). It is not clear from magnetometer observations whether a neutral sheet current system, which is distinct from the principal cross-tail current sheet, exists. The limited data set and substantial fluctuations made it difficult to determine if distinct features exist in the region of smallest $|B|$.

Our sample calculation yielded unusually weak $j_y$ very near $z = 0$ (Figures 3.4a and 4.9a). This feature can be understood by studying individual orbits, so is not simply a peculiarity of one dimensional models. Current carried by untrapped (or the untrapped type orbit for $\kappa = 0.4$) protons injected at $z_i = L$ with a large $T_{||}/T_{\perp}$ ratio tended to peak slightly away from $z = 0$ (Figures 3.1b and 4.1b). Ion trajectories are nearly straight lines near $z = 0$ because $|B|$ is weak, and $v_y$ can be either positive or negative. Trapped protons on figure-8 orbits, which were most important in the $z_i = 0$ group (Figures 3.1a and 4.1a) tended to carry current in the negative $y$ direction at $z = 0$. The only positive current peaks that fell exactly at $z = 0$ for $\kappa = 0.52$ were broad ones, associated with groups of trapped protons which were injected at large enough $z_i$ so that figure-8 orbits were absent (Figure 3.2b). For $\kappa = 0.4$, there is hardly any group that has a positive current peak exactly at $z = 0$. Very low energy untrapped protons carry positive $j_y$ near $z = 0$, but the density required to fill in the hole at $z = 0$ in Figures 3.4a and 4.9a is
much larger than any which has been observed. Individual protons with $\kappa = 0.52$ on selected orbits can be found which carry maximum positive $j_y$ at $z = 0$. There is no physical reason to exclude a specially selected group of such ions. We simply found no such group using our procedure of randomly picking ions from distributions with different values of $T_{\parallel}/T_{\perp}$ and different $z_i$.

The addition of a uniform $B_y$ can substantially change the average $v_y$ near $z = 0$. Models therefore may have peaks in $j_y$ at $z = 0$ when a significant $B_y$ exists. At other times, the predicted weak neutral sheet current layer could be a real feature of resonant regions of the plasma sheet, which has not been detected due to the difficulty in making instantaneous measurements of the electric current.

Current sheet or inner CPS. Chapter 3 concentrated on a $\kappa = 0.52$ current sheet, in which most ions were on 1/2-oscillation resonant quasi-adiabatic orbits. This is the largest $\kappa$ for which a resonance exists, and the region in which resonant features extend throughout the largest energy range. For example, the peaking of untrapped ion current between $|z| = 0$ and $|z| = z_0$ (Figure 3.1b) is a characteristic of resonant orbits. This structure is not present in the adjacent nonresonant regions with larger and smaller $\kappa$. Three-keV protons have $\kappa = 0.52$ in the model plasma sheet. The characteristic peaking shown in Figure 3.1b was seen for protons with $1.5 \text{ keV} < W < 6 \text{ keV}$ in the model magnetic field. As a result, most protons in an actual magnetotail energy distribution can show resonant characteristics when $\kappa = 0.52$ for the average ion. The resonant energy band is narrower for protons near the higher resonances ($\kappa = 0.32, 0.23,$ etc. Figure 2.2), so a clear resonant $j_y$ structure is less likely to survive averaging over a distribution of energies.

Resonant untrapped ions were found to carry current primarily between $z = 0$ and $|z| = z_0 = 0.2\text{ R}_E$. It was necessary to add trapped proton populations in order to produce the self-consistent $1\text{ R}_E$ thick model current sheet. Figure-8 ions carried positive $j_y$. 
out to \(|z| = z_{m8} = 0.3 \, R_E\), and mirror-type ions carried positive \(j_y\) beyond this point. If

enough untrapped 3 keV \(O^+\) ions were present, they could carry positive \(j_y\) at larger \(|z|\).

However, protons are observed to be the major current carriers in a typical current sheet

[Lennartsson and Shelley, 1986; Daglis et al., 1991a].

In Chapter 4, we concentrated on a \(\kappa = 0.4\) current sheet, in which most ions were on

3/4-oscillation stochastic orbits. This is the \(\kappa\) half way between \(\kappa = 0.52\) and \(\kappa = 0.32\)

resonances, and shows the stochastic feature extending throughout a large energy range.

Most particles left the current sheet after several interactions regardless of whether the

orbit initially was of the figure-8 type (Figure 4.1a) or mirror type (Figure 4.1b). The cur­

tent is too low in the \(z < z_{o8}\) region (Figure 4.5a), so high energy untrapped resonant par­

ticles are needed in this region to produce a self-consistent current sheet.

It was noted that each particle group has a \(j_y = 0\) point near \(|z| = z_0\). This is a real

effect that was produced because \(|B|\) is not strong enough to reverse the \(v_y\) of most ions

when \(|z| < z_0\). The result is a dip in \(j_y\) at \(z = 0.2 \, R_E\) in Figure 3.4a. Since each orbit type

has \(j_y = 0\) at a slightly different point, \(j_y\) did not reach zero in the sample \(\kappa = 0.52\) cur­

tent sheet. The dip at \(|z| = z_0\) was more pronounced in resonant model current sheets

with smaller \(\kappa\). In such cases, we added a group of higher energy particles to fill in the dip

and provide a reasonable approximation to the \(j_y\) that is needed (Equation 2.4) in order to

generate a smooth self-consistent Harris model sheet. Figure 5.2 shows a example that

used 3 keV protons, \(B_{x0} = 25 \, nT\), \(B_{z0} = 2.5 \, nT\) and \(L = 0.5 \, R_E\) in Equations (2.1)

and (2.3). These parameters produce \(\kappa = 0.32\), which is the resonance associated with

orbits that undergo 1-oscillation during each current sheet encounter. In this figure, the

solid curve is the desired current that we need to fit, and the dashed curve represents the

current calculated from orbit tracing. We can see that the dip at \(z = z_0\) with \(z_0 = 0.16 \, R_E\). The alternative, which is suggested by the orbit calculations, is that a dip
in \( j_y(z) \) may actually be present near \( z = z_0 \) in resonant regions of the Earth current sheet. If such dips can be identified, they may occasionally provide a way to determine when a satellite is located a distance \( z_0 \) from the center of the current sheet. The dip in \( j_y \) at \( z = 0.2 R_E \) in Figure 4.5a, the \( \kappa = 0.4 \) case, extended into the negative region. We have not been able to generate a reasonable Harris-like current sheet using only stochastic protons. A relatively small density of more energetic resonant protons or heavier ions was needed in order to approximate the required current.

The requirement (for \( \kappa = 0.52 \)) that many ions must be trapped within the current sheet in order to self-consistently model a resonant particle region presents an apparent problem with regard to particle sources. This may largely be a problem with our use of a one dimensional model. When gradients are permitted in the \( x \) direction, regions in which resonant particles dominate alternate with regions in which most particles follow nonadiabatic orbits. Ions drift Earthward through such alternating regions in the presence of a dawn-to-dusk \( E_y \). Calculations similar to those described in this thesis, but using \( \kappa \) values corresponding to nonadiabatic orbits, show that most of velocity space can be populated using a source at \( z_i = L \). A study that reliably determines whether the resonant current sheet trapped particle regions (Figures 3.10a, 3.11a, and 3.12a) can be filled in by scattering from adjacent nonadiabatic regions will require detailed calculations in a two or three dimensional model. The trapped particle region of velocity space is largest and most clearly separated from the untrapped particle region in the \( 1/2 \)-oscillation resonant current sheet which was analyzed in this paper. Resonant current sheets characterized by more oscillations have more finely structured trapped and untrapped particle regions, so require less scattering to fill in the trapped orbits. Nonresonant current sheets, with \( \kappa < \kappa_r \) do not require trapped particles.

We found that velocity space holes are present in some self-consistent current sheets.
For example, it was seen that few figure-8 orbits can be populated if the orbits reach beyond the edge of the current sheet \((z_{m8} > L)\). Since the problem is clearly a feature of individual particle orbits, it is not related to the one dimensional nature of our model. The problem also is significant for the many-oscillation nonresonant or Speiser-type orbits. When \(k < k_r\), much of velocity space at \(z = 0\) corresponds to particles that would turn around, and therefore would carry current, well outside of the current sheet if they were present. Any process that creates the very thin nonresonant current sheets must produce or maintain a very low density of particles on these orbits. The resulting holes in velocity space could produce significant instabilities.

**Outer CPS and reference frames.** Since \(B\) is nearly uniform beyond \(|z| = 4L/3\) in our one dimensional model, untrapped ions maintain a fixed pitch angle and never mirror. The particle density remains high at the edge of the current sheet. Particles cannot undergo cross-tail drift or carry magnetization currents in the uniform \(B\) region beyond our model inner CPS. In the actual magnetotail, \(|B|\) continually increases as particles move Earthward along a field line, so almost all eventually mirror.

It was noted that one dimensional models must have a large \(T_{||}/T_{\perp}\) ratio or a large parallel streaming velocity at the edge of the current sheet (Figures 3.7a and 4.10a) in order to produce force balance. This anisotropy would be present throughout the outer CPS. Occasional strong anisotropies or streaming have been observed in the mid-tail CPS [Nakamura et al., 1991] and in the near-Earth plasma sheet during growth phase [Baker et al., 1978; Daglis et al., 1991a], but ions usually are nearly isotropic throughout most of the inner and outer CPS. Strong field alignment generally is not observed until the PSBL is approached. We conclude that a one dimensional model does not adequately represent any region in which the current sheet is surrounded by a nearly isotropic outer CPS. There may be no isotropic outer CPS in the distant tail, so the one dimensional model could be ade-
Another consequence of the inadequacy of a one dimensional model in the mid tail is that our calculated density is too large (Figures 3.6a and 4.9b). A smaller density of ions may carry the necessary $j_y$ if all trapped and untrapped ions have a cross-tail drift, as is the case in two and three dimensional models. Untrapped ions are forced to carry all the net current in one dimensional models for the $\kappa = 0.52$ case. High energy resonant protons carry most of the current in the one dimensional $\kappa = 0.4$ model. A satisfactory resolution of this question must await a full two dimensional study.

PSBL. Although we only modeled the current sheet, results from this analysis have implications regarding the PSBL. The guiding center approximations are valid whenever $|z| > z_0$, which includes all the outer CPS and beyond. After leaving the current sheet, ion guiding centers move away from $z = 0$ along the diverging magnetic field lines. In the Earth reference frame with $E_y \neq 0$, these ions also $E \times B$ drift toward the neutral sheet. The location of the PSBL is determined by a combination of drift, streaming, and neutral line effects in two and three dimensional models [Liu and Hill, 1986; Onsager et al., 1991; Kaufmann et al., 1993a]. When a neutral line is present, particle drifts assure that the neutral line separatrix is not the same as the edge of the PSBL. Figure 5.1 shows magnetic field lines crossing through the PSBL.

The field-alignment shown in Figures 3.13b and 4.16b is needed to provide force balance at the edge of a self-consistent one dimension current sheet. Some process must select appropriate groups of ions in order to avoid Earthward collapse of the one dimensional sheet due to force imbalance. It is evident (e.g. Figure 3.11) that the model field-aligned component is composed of untrapped ions, while trapped ions produce the small secondary peak near $\alpha = 90^\circ$. We again note that the magnitude of this field-aligned flux is strongly dependent upon the one dimensional nature of the model. Other forces could
provide major contributions to force balance in a typical two or three dimensional inner and mid tail current sheet.

When viewed from the \( E_y = 0 \) frame used for our calculations, field alignment is seen as either an orbit selection process or, more naturally, as a property of the source distribution. For example, a cold stationary plasma in the Earth frame appears as a beam in the \( E_y = 0 \) reference frame. Any process that generates cold stationary plasma at the edge of the current sheet therefore is selecting nearly field-aligned orbits as seen in the \( E_y = 0 \) frame. Particles are not accelerated in the \( E_y = 0 \) frame during a current sheet encounter. All particles in Figure 3.13b have approximately 3 keV when they approach and 3 keV when they leave the current sheet.

Field alignment, when viewed in the Earth reference frame, sometimes is considered a consequence of current sheet acceleration. An Earth or satellite observer would see the one dimensional model plasma sheet simply drifting Earthward at a speed \( u_x = E_y / B_z \) provided \( u_x \ll c \). Such plasma drifts frequently are observed, indicating that a mean \( E_y \) pointing from dawn to dusk is common. The Earthward drift causes trapped and untrapped ions moving tailward (Earthward) to have lower (higher) energies in the satellite frame than in the moving frame in which our calculations were performed.

It may be noted that the attribution of an energetic ion beam entirely to current sheet acceleration requires a relatively large \( u_x \) or Earthward drift speed, which should be observable throughout the CPS. For example, Lyons and Speiser [1982] used \( B_{zo} = 1 \) nT to obtain \( u_x = 250 \) km/s, or a net velocity change of \( 2u_x = 500 \) km/s. With these parameters, a zero energy (in the Earth frame) incident particle leaves the current sheet with an energy of 1.3 keV. Drift speeds as large as 250 km/s do not appear to be typical of the mid-tail CPS [Frank, 1985; Baumjohann et al., 1989], but may be common in the distant tail.
The presence of an isotropic outer CPS in the mid magnetotail suggests that any possible field alignment of ions at the edge of the current sheet or inner CPS in this same mid-tail region is not directly related to PSBL beams. It is hard to imagine how a given group of ions would become strongly field aligned at the edge of the current sheet, isotropic throughout the outer CPS, and then again become field aligned in the PSBL. It is possible that an isotropic outer CPS is absent in the distant tail. If so, then the distant tail current sheet could be well approximated by the 1-D model, and represent a direct source of PSBL beams.

The velocity filter effect [Liu and Hill, 1986; Onsager et al., 1991; Kaufmann et al., 1993a] rather naturally produces a low $v_{\|}$ cutoff in the PSBL. Distributions with such cutoffs often are referred to as beams. Both ion and electron beams are generated. Particles at the cutoff velocity are assumed to originate at a separatrix or neutral line. In a one dimensional model, which cannot contain a neutral line, particles are separated into two groups according to the signs of their guiding center $v_z$. In the Earth frame, a particle guiding center moves away from $z = 0$ because of its $v_{\|}$ along diverging field lines, and toward $z = 0$ due to $E \times B$ drift. Particles with higher $v_{\|}$ can be traced backwards to the current sheet. Ions with lower $v_{\|}$ trace directly to the lobes. These two groups are likely to have very different densities or $f(v)$. "Neutral line" type beams are formed when the flux of lobe particles with $v_{\|}$ slightly below the cutoff is much less than the flux of current sheet particles with $v_{\|}$ slightly above the cutoff. Zelenyi et al. [1990] presented evidence that PSBL ion beams originate in the $-100 \ R_E < x < -50 \ R_E$ region during typical conditions. Ashour-Abdalla et al. [1991] used orbit tracing in a two dimensional model to examine the source of these beams. The principal acceleration took place in a region of very weak $B_z$, within approximately $20 \ R_E$ of a distant (beyond $x = -100 \ R_E$) neutral line.

The location of the principal acceleration region is significant because most particle
energization takes place in the inner tail [Kaufmann et al., 1993b]. This result arises because the volume rate of charged particle energization is \( j \cdot E \). The integral, through the current sheet, of this quantity is much larger in the inner tail than in the distant tail or near a distant neutral line. If PSBL beams are associated with processes taking place beyond \( x = -50 \, R_E \) to \(-100 \, R_E\), then they are unlikely to be the major source of energy transport in the plasma sheet. The particles stream Earthward, are mirrored without significant energization, and move back to the tail in the velocity filter model. The beam-like property is simply a consequence of the particles drifting to different \( z \) locations for the inward and return portions of an orbit. Particles accelerated in the most intense energization region (the inner and mid tail current sheet) would not directly contribute to PSBL beams.
Figure 5.1. Sketch of the model plasma sheet structure.
Figure 5.2 The solid line is the $j_y$ needed to produce the model magnetic field. The dotted line shows currents carried by 3 keV $\kappa = 0.32$ particles.
Chapter 6

Conclusions

We studied particle motion in a one-dimension modified Harris field. We found that $\kappa$ is a good parameter to use when classifying particle orbits.

Nearly all the particles in the radiation belts follow guiding center motion. The magnetic moment of such particles is conserved. These particles are characterized by $\kappa > 2$, and are referred to as adiabatic. The very small $\kappa$ limit was referred to as nonresonant quasi-adiabatic because an ion's magnetic moment returns almost to its initial value by the time the ion leaves the current sheet. Such orbits exist in the distant tail, near a neutral line, and in the very thin plasma sheet that is present at the end of a substorm growth phase.

Two intermediate $\kappa$ regimes are common in mid-tail region. Particles follow relatively simple trajectories when $\kappa$ is 0.53, 0.32, or 0.23 and were referred to as resonant particles. Other particles, which have $\kappa$ values of approximately 0.82, 0.4, or 0.27 were referred to as stochastic. We called a particle that leaves the current sheet within a certain time period an untrapped particle, and a particle that stays in the current sheet longer than a certain time period a trapped particle.

We found that there are three common types of orbits which carry different current distributions $j_y$. Particles following figure-8 orbits carry positive $j_y$ at $z_{o8} < |z| < z_{max8}$, negative $j_y$ at $0 < |z| < z_{o8}$, and no current beyond $z_{max8}$. Mirror-type orbits also carry positive $j_y$ at the largest $|z|$ and negative $j_y$ at smaller $|z|$. A second region of positive $j_y$ is possible near $z = 0$ for mirror orbits. Most untrapped resonant particles carry strong positive $j_y$ at $|z| < z_o$ and slightly negative $j_y$ at larger $|z|$.

We studied the generation of a self-consistent one-dimensional Harris-like current
sheet. Figure-8 type, mirror-type, and untrapped particles are all needed to generate a self-consistent one-dimensional Harris-like current sheet since they carry positive \( j_y \) in different portions of the current sheet. In general, it is difficult to generate a smooth self-consistent Harris-like current sheet by using only 3 keV protons. None of the orbit types carries positive \( j_y \) in the region between \( z_0 \) and \( z_{08} \). The problem may not be serious for regions in which the protons follow resonant orbits. In fact, we have obtained good approximations by using only 3 keV \( \kappa = 0.52 \) resonant protons. The addition of a small density of higher energy protons or heavier ions to other resonant cases produces a smoother current sheet in the region between \( z_0 \) and \( z_{08} \). However, we have not been able to generate a reasonable Harris-like current sheet using only 3 keV stochastic protons. The stochastic particles can carry most of the required current at relatively large \( |z| \). It was necessary to add a relatively small density of more energetic resonant protons or of heavier ions in order to approximate the required currents between \( z = 0 \) and \( z = z_{08} \).

We generated a \( \kappa = 0.52 \) resonant current sheet and a \( \kappa = 0.4 \) stochastic current sheet with energetic protons. We also compared the results from the model current sheet with observations. In the neutral sheet, our sample calculation yielded unusually weak \( j_y \) very near \( z = 0 \). This predicted weak neutral sheet current layer could be a real feature of resonant regions of the plasma sheet, which has not been detected due to the difficulty in making instantaneous measurements of the electric current. In the inner CPS region, the model current sheet required us to include trapped proton populations in order to produce a thickness of \( 1 \, R_E \). The requirement that many ions must be trapped within the current sheet presents an apparent problem with regard to particle sources. It was noted that a large \( T_{||}/T_\perp \) ratio or a large parallel streaming velocity at the edge of the current sheet is also required in the one-dimensional models in order to produce force balance. This anisotropy would be present throughout the outer CPS, a feature which does not agree
with observations of the mid-tail CPS.

The field-alignment was needed at the edge of our self-consistent one dimension current sheet model in order to provide force balance. We again note that the magnitude of this field-aligned flux is strongly dependent upon the one-dimensional nature of the model. Other forces could provide major contributions to force balance in a typical two- or three-dimensional inner and mid tail current sheet.

Another inadequacy of a one-dimensional model of the mid-tail is that our calculated number density is too large. A smaller density of ions may carry the necessary $j_y$ if all trapped and untrapped ions have a cross-tail drift, as is the case in two- and three-dimensional models. Untrapped ions are forced to carry all the net current in one-dimensional models for the $\kappa = 0.52$ case. High energy resonant protons carry most of the current in the one-dimensional model when $\kappa = 0.4$.

The use of the one-dimensional model provides us with a clear physical picture of the cross-tail current carriers. Although the one-dimensional models cannot adequately describe any region of the magnetotail in which the principal current sheet is separated from the plasma sheet boundary layer by a nearly isotropic outer portion of the central plasma sheet, this work provides a meaningful step toward to the goal of understanding the structure of a current sheet. A satisfactory resolution of the general problem must await a full two or three dimensional study.
LIST OF REFERENCES


Appendix A: Treatment of Electrons

Electric current is defined by the difference between electron and ion drift. Although electron drift and the $\phi(z)$ needed to maintain charge neutrality are important in some regions of the tail, they turned out not to produce major effects in the particular examples studied here. The way we chose to introduce electron effects was guided by satellite observations. Frank et al. [1984] found that the thermal electron distribution function usually is nearly isotropic in the CPS. An elongation of contours of constant $f(v)$ along $B$ is seen at times [Hada et al., 1981]. These observed features place the strongest restrictions on our treatment of electron acceleration.

Isotropy is maintained everywhere in the guiding center approximation when $E_\parallel = 0$ if the distribution function is isotropic at the equator. We will refer to $z = 0$ as a "source" of isotropic electrons. If this source were placed elsewhere, e.g. at $z = L$, then the mirror effect would produce a strongly anisotropic or field-aligned source cone distribution at the equator. Such highly field-aligned distributions are seen both at synchronous altitude [McIlwain, 1975] and in the more distant plasma sheet [Hada et al., 1981]. However, as noted above, CPS electrons usually are nearly isotropic, and any observed field alignment usually involves a weak elongation of thermal electron distribution functions along $v_\parallel$. As with ions, the electron "injection point" parameter provides a convenient way to select an electron group with desired properties. An actual plasma source is not required at $z = 0$, though some electron interaction may be.

A.1 Deceleration: $\phi < 0$

If $E = 0$ and electrons were isotropic at the equator, then the electron density would be the same everywhere along a field line. However, $n(z)$ for the nonadiabatic ions is not
the same everywhere along a field line. Observations [Baumjohann et al., 1989] show that statistically there is not much variation in plasma density within the inner CPS. Nevertheless, purely isotropic electrons, a uniform $E$, and the ion groups studied previously do not produce exact charge neutrality. It is easy to create a region with electron number densities that monotonically decrease in an arbitrate manner as one moves away from the equator. To do this, electrons must be decelerated as their $|z|$ increases. We define the electric potential to be $\phi = 0$ at $z = 0$, so electrons have been decelerated in all regions with $\phi < 0$. The electrostatic force adds to the magnetic mirror force, causing electrons to mirror nearer to $z = 0$ than would be the case if $E_z = 0$. Figure A.1a is a sketch of constant $f(v)$ contours for an isotropic distribution function. This distribution function is largest at $v = 0$. Expressions in the present section will use a bi-Maxwellian with $R = T_\parallel / T_\perp$ specified at the equator. The distribution function and density can be written for any point along the field line where the field strength is $B$ and $\phi < 0$

$$f(v) = n_o R e^{-q\phi/T_s} \left[ \frac{m}{2\pi T_\parallel} \right]^{3/2} \exp \left[ -\frac{m (v^2 + rv^2)}{2T_\parallel} \right]$$

$$\frac{n}{n_o} = R e^{-q\phi/T_s}$$

$$r = 1 + (R - 1) (B/B_o)$$

where $n_o$ is the density, $B_o$ is the magnetic field, and $T_\parallel$ is the parallel electron thermal energy at the equator. It may be noted that the bi-Maxwellian parameter $T_\perp$ used to define $R$ equals the average equatorial perpendicular thermal energy only when $R = 1$. Both $q$, the electron charge, and $\phi$ are negative, so that an initially isotropic distribution remains isotropic (Figure A.1a). Only the magnitude of $f(v)$ must change because $n(z)$ is smaller than $n_o$. When $R = 1$, then $r = 1$ and Equation (A.1) is just the Boltzmann relation.

Evaluation of the guiding center approximation to electron current, based on (A.1), yields
where

\[ P_{\parallel} = \frac{n_e R T_{\parallel}}{r} e^{-q\phi/T}, \quad n T_{\parallel} \]  

\[ P_{\perp} = \frac{n_e R T_{\parallel}}{r^2} e^{-q\phi/T}, \quad \frac{n T_{\parallel}}{r} \]  

When \( r = R = 1 \), we define \( T_{\parallel} = T \) and (A.2) reduces to

\[ j_y = -\left[ \frac{B_x T}{B^2} \right] \frac{\partial n}{\partial z} \] (A.4)

in the one dimension model. This is the expression that was used in Chapter 3 to estimate electron currents. Although the form (A.2) is convenient to use when evaluating \( j_y \), it can be confusing in a discussion of orbit effects. Equation (A.2) [Parker, 1957] includes the pressure (or density and temperature) gradient, magnetic field gradient, and field line curvature \( [P_{\perp} (B \cdot V) B \text{ term}] \) magnetization currents. These currents all add up to be divergence free, involving no net cross-tail motion of particles. In addition, the magnetic field gradient and field line curvature \( [P_{\parallel} (B \cdot V) B \text{ term}] \) guiding center drift currents are included in Equation (A.2). The \( E \times B \) drift does not produce a current in a collisionless neutral plasma. The guiding center currents are not divergence free. These currents yield net cross-tail particle motion, and can couple to field aligned currents. The two magnetic field gradient terms are not seen in Equation (A.2) because their effects cancel for adiabatic particles (Appendix B).

Equation (A.4) was used with \( n(z) = n_0 \exp \left[ -q\phi(z) \right] / T_{\parallel} \) to evaluate both the electron current and the electric potential throughout most of the magnetotail, where \( n < n_0 \) and \( \phi < 0 \). Using \( n(z) \) that was determined from ion trajectories, the above equations were solved for \( \phi(z) \) which produces exact charge neutrality. However, even
though \( n(z) \) generally decreases monotonically from a maximum near \( z = 0 \) (Figures 3.6a and 4.6b), there sometimes are regions in which the electron \( n(z) \) must be larger than \( n_0 \) in order to maintain neutrality. This requires electrons to be accelerated away from \( z = 0 \). The observation that electron distribution functions sometimes are elongated in the \( v_\parallel \) direction suggests that electrostatic acceleration may be a real feature. There are, of course, a number of other possible explanations for elongated distribution functions, including broadened source cones and the effects of induced electric fields.

A.2 Acceleration: \( \phi > 0 \)

A treatment of accelerated electrons which is consistent with observations is difficult. First, if electrons were unmagnetized and sources were available everywhere surrounding the observer, the distribution function would look that in Figure A.1b. In this sketch, \( f(v) \) from Equation (A.1) still is valid for all \( v \) greater than the cutoff value, \( v_c = \left[ \frac{2|q|\phi}{m} \right]^{1/2} \). The density, obtained by integrating from \( v_c \) to \( \infty \), is slightly more complex than \( n(z) \) in Equation (A.1). We will not use this model; expressions for \( n(z) \) are not included. Such distributions sometimes are seen in ion measurements made on charged vehicles. However, electrons are magnetized in the regions we are considering, so Figure A.1b provides a poor description. Even so, this is the assumption that is made if one simply uses the Boltzmann relation for all \( \phi \).

Figure A.1c shows the magnetized version of Figure A.1b. Here electrons can freely accelerate only parallel to \( B \). It is assumed that the observer is located just beyond the acceleration region, so there is a cutoff at \( v_\parallel = v_c \) in Figure A.1c. If the observer is well away from the acceleration region, mirroring will modify the cutoff velocity to produce the results in Figure A.1d. In both cases, \( f(v) \) is given by Equation (A.1) beyond the heavy contour, where \( f(v) \neq 0 \), and the density is given by a more complex expression. One problem with the model used to prepare Figure A.1c is that so much of phase space is
devoid of electrons that integrating to evaluate \( n(z) \) gives numbers smaller than \( n_0 \) for all positive \( \phi \). The model which includes mirroring (Figure A.1d) can yield the desired \( n > n_0 \), for which we originally were searching, if the observer is located far enough from the equator. However, thermal electron distribution functions with sharp low energy cutoffs do not appear to be seen in the inner CPS. Electrons and ions with residual characteristics suggestive of such parallel acceleration are seen at relatively low altitudes in the auroral region. Observations in the PSBL show some similar properties.

Figure A.1e shows the \( f(v) \) that we finally selected for use when it is necessary to have \( n > n_0 \). The two caps, at \( |v_\parallel| > v_c \), are the same as in Figure A.1c. We have arbitrarily assumed \( f(v) \) in the region \( |v_\parallel| < v_c \) is independent of \( v_\parallel \) and that it smoothly matches \( f(v) \) given by (A.1) at \( |v_\parallel| = v_c \). Physically this structure could be generated by a Landau resonant instability such as the usual bump on tail instability. We assume some such process fills in the region of small \( |v_\parallel| \) in Figure A.1e until there is no region of positive \( \partial f/\partial v_\parallel \). We use an initial distribution with \( T_\parallel = T_\perp = T \) because scattering or diffusion is implied in this model. It is not possible to follow individual electron trajectories in phase space during the acceleration. The resulting \( f(v) \), with \( \phi > 0 \) and \( q < 0 \), is

\[
f(v) = n_0 e^{-q\phi/T} \left[ \frac{m}{2\pi T} \right]^{3/2} \exp \left[ -\frac{m(v_\parallel^2 + v_\perp^2)}{2T} \right]
\]

for \( |v_\parallel| > v_c \) \hspace{1cm} (A.5)

\[
f(v) = n_0 \left[ \frac{m}{2\pi T} \right]^{3/2} \exp \left[ -\frac{mv_\perp^2}{2T} \right]
\]

for \( |v_\parallel| < v_c \)

yielding a density

\[
\frac{n}{n_0} = 2 \left[ \frac{-q\phi}{\pi T} \right]^{1/2} + \left[ 1 - \text{erf} \left( \frac{-q\phi}{\sqrt{4qT}} \right) \right] e^{-q\phi/T}
\]

(A.6)

The resulting pressures are
\[ P_\| = nT \left[ 1 + \frac{4\lambda^3}{3\sqrt{\pi} \left[ 1 - \text{erf} (\lambda) \right] \exp (\lambda^2) + 6\lambda} \right] \]
\[ P_\perp = nT \]

where \( \lambda^2 = -q\phi/T \), yielding

\[ j_y = \left[ \frac{4n_0 T \lambda^3 B_z^2}{3\sqrt{\pi} B^4} \right] \frac{\partial B_x}{\partial z} - \left[ \frac{TB_x}{B^2} \right] \frac{\partial n}{\partial z} \]

To summarize, the observations that the electron distribution in the CPS usually is nearly isotropic and that our calculated ion density usually drops away from \( z = 0 \) requires the presence of an electric field in order to maintain charge neutrality. Wherever the ion orbit calculations yield \( n < n_0 \), Equation (A.1) is used to solve for the \( \phi(z) \) that is needed to maintain charge neutrality. Similarly, Equation (A.6) is solved for \( \phi(z) \) in the less common regions where ion analysis yields positive \( \partial n/\partial z \). Equation (A.4) or (A.8) then gives the electron contribution to \( j_y(z) \).
Figure A1. Sketches of possible electron distribution functions. The light lines are contours of constant $f(v)$. Fluxes suddenly drop to zero at the heavy contours in panels b, c, and d.
Appendix B: Computational Details

B.1 Magnetization Current

Orbit tracing studies can yield misleading results unless all particles are followed either until they mirror or until they reach a uniform B region. Figure 2.9 illustrates the potential problem. Note that the ion guiding center moves a net distance of approximately 1 $R_E$ in the positive y direction (Figures 2.9a and 2.9b). However, integration of Figure 2.9c shows that this positive ion carries a net current in the negative y direction within the plasma sheet. The area under the negative $j_y$ regions of Figure 2.9c is 18% larger than the area under the positive $j_y$ regions. If a plasma sheet was created by uniformly distributing many ions with exactly this same trajectory, the net plasma sheet current would be in the negative y direction even though all ion guiding centers move in the positive y direction.

This potential dominance of magnetization currents over guiding center drift currents is well known for adiabatic particles. Perhaps the most frequently mentioned guiding center example involves a region in which all magnetic field lines are straight and the only drifts are produced by a perpendicular magnetic field gradient. The guiding center drift produces a current

$$ j = \left[ \frac{B}{B^2} \right] \times \left[ \frac{p}{B} \nabla B \right] \quad (B.1) $$

The magnetization current is equal in magnitude but opposite in direction, so the total $\nabla B$ current is zero and no $\nabla B$ term appears in Equation (A.2). In the adiabatic case, the magnetization current can be understood physically as an orbit effect by shifting to a frame that is moving at the guiding center drift velocity. An observer in this frame sees only magnetization current. Ions on the side of the gradient with stronger $|B|$ move in smaller
circles at higher cyclotron frequencies than particles on the side with smaller $|\mathbf{B}|$. The areas of the circles and the gyrofrequency of both contribute to the magnetization current flowing at the observer's location. A larger gyroradius $\rho$ permits more particles to contribute, and a larger gyrofrequency yields more current per ion. Since the number of particles contributing is dependent on the area or $\rho^2$ and the gyrofrequency is proportional to $\rho^{-1}$, the area effect dominates. The net result in the adiabatic case is the exact cancellation of the guiding center drift current.

For the orbit in Figure 2.9, which does not follow guiding center motion near $z = 0$, the negative magnetization current is a consequence of the gyro-orbits that overlap in Figure 2.9a. At $z > 0$, the ion moves in the $+y$ direction at the top of each loop and in the $-y$ direction on the bottom side. If an imaginary plasma sheet boundary is drawn at $z = 4/3 \, L$, several loops will be cut by the line so that the top half is outside the plasma sheet, and therefore does not contribute to plasma sheet current. The net motion in the $-y$ direction on the bottom sides of these loops (and the top sides of the loops at $z > 0$) dominates over the net motion in the $+y$ direction near $z = 0$. This effect is strongly dependent on the pitch angle of the escaping ion. If the escape pitch angle was small, there would be little looping and the net current would be in the $+y$ direction. Figure 3.1b shows that when $T_{||}/T_{\perp} = 10$ with $z = L$ (dashed line), most particles escape with small pitch angles, so the positive current near $z = 0$ is much larger than the negative current at larger $z$. As the $T_{||}/T_{\perp}$ ratio decreases to 1 (solid line) and 0.1 (dotted line) more and more ions that escape with large pitch angles are present. The negative current at large $z$ becomes large and can dominate.

The above magnetization currents are real and not an artifact of the computational methods. The currents do not depend on the boundary selected, provided the boundary is a region with uniform $\mathbf{B}$. Particles have stopped drifting at $|z| = 4/3 \, L$ in Figure 2.9. Con-
continuing the orbit an extra Earth radius farther from \( z = L \) just adds more uniformly spaced, equal sized overlapping circles directly above (for \( z > 0 \)) the last circles shown. Moving the "edge of the plasma sheet" an extra \( 1 \, \text{RE} \) from \( z = L \) therefore just increases the number of circles that are inside the plasma sheet, but does not change the net current.

**B.2 The Cutoff of the Current**

It is because of the above considerations that we used (2.3) with \( \eta = 5 \) to cut the current off abruptly beyond \(|z| = L\) (solid line in Figure 3.4a) or, equivalently to quickly move into the uniform \( B \) region. All orbit tracing calculations therefore must give \( j_y = 0 \) at and beyond \( z = 4L/3 \), where \( B \) becomes uniform (e.g. see Figure 3.1b). The \( j_y \) boundary can be moved by varying \( \eta \) in (2.3). For example, the points at which \( j_y \) in (2.4) becomes 1% and 0.1% respectively of \( j_y \) at \( z = 0 \) are \( L=1.28 \) and \( 1.37 \, \text{RE} \) when \( \eta = 5 \), at \( L=1.81 \) and \( 2.13 \, \text{RE} \) when \( \eta = 2 \), and at \( L=2.99 \) and \( 4.15 \, \text{RE} \) when \( \eta = 1 \). Using \( \eta = 1 \) is the same as using (2.1) at all \( z \). We used \( \eta = 5 \) rather than \( \eta = 1 \) so we could be assured of reaching a uniform \( B \) region without tracing orbits out to such large \( z \) values.

We wanted to concentrate our efforts and the \( z \) boxes in the central part of the current sheet, where the guiding center approximation breaks down (Figure 3.9b). Several test cases were run with \( \eta = 2 \). The current in plots similar to Figure 3.1b approached zero more slowly, and at a larger \( z \), as expected. We did not find that the rapidity of the forced cutoff of current at \( z = 4L/3 \) produced any significant changes in the total integrated current at \(|z| \leq L\). It should be noted that no differential equations that explicitly required boundary condition at \(|z| = L\) were solved in the analysis. Currents were evaluated merely by tracing individual particle orbits in a fixed model field.

**B.3 The Procedure of the Computation**

In this section, we will list the computational steps that are used to generate a self-consistent current sheet.
First, groups of particles will be traced in a model magnetic field (iteration 0). The particles can be $e^-$, $H^+$, $He^+$, $He^{++}$, or $O^+$, and the model magnetic field can be the Modified Harris sheet (one dimensional B field), a uniform field (constant B field), a polynomial field (one dimensional B field), and the Tsyganenko 89 model (one, two, or three dimensional B field). In this thesis, only the modified Harris sheet model (Equations (2.1) and (2.3)) and protons ($H^+$) were used. The electric field was assumed to be $E = 0$ during this 0'th iteration. The program partrj.f traces particle orbits and generates files that contain all the information about 1) number density and current density (hereafter referred to as nj files); 2) the initial and final particle energy, pitch and phase angle (hereafter referred to as if files) for each group. This if file keeps all the information about the parameters of the model field and particles that allow us to study the individual particle trajectory.

The number density (Equation (2.7)) and current density (Equation (2.6)) are calculated in subroutine dencur.f. An auxiliary program called trsfdat.f can be used to read the nj file and output a two column data file that allows us to plot out $n(z)$ vs. $z$ or $j_y(z)$ vs. $z$.

A polynomial fit of the ion number density $n(z)$ for each group [fit.f] is then made. Then the electron current is calculated according to Equation (A.4) for each group [elecur.f].

Third, the electron current and ions current are summed to get the total electric current for each group [jeisum.f]. The desired current $j_y(z)$ is calculated according to Equation (2.4) [calubj.f]. All the groups of the total electric current then can be used to fit to the desired $j_y(z)$ [glsws.f]. After we get the best fit, the coefficients can be used to combine the correspond number density of each group to get the combined number density for the current sheet.

Fourth, based on the relation between $n$ and $\phi$ (Equations (A.1) and (A.6)), the electric
potential can be calculated by knowing the combined number density [phiton.f].

Fifth, the first two steps are repeated with \( E_z \neq 0 \) (iteration 1). The only difference between this and the previous iteration is that Equation (A.8) will be used to calculate the electron current for each group [elecur.f], since we may have both \( \phi < 0 \) and \( \phi > 0 \) in the current sheet.

Step three is repeated to get the total electric current of each group [jeisum.f]. Again, the groups of total electric current will be used to fit to the desired current \( j_y(z) \) as in step three [glsws.f]. The same set of coefficients which form the best fit to the desired current can be used to combine number densities of the corresponding groups. At this point we have the combined number density for the current sheet.

Step four then is repeated. The electric potential we had during the 0'th iteration is compared to the electric potential in step four of the present iteration. If there is a significant difference between the two, we need to go back to step five with this new \( \phi \) (or \( E_z \)) for another iteration. Otherwise, the iteration procedure has converged.

While we were doing step five, we also saved the reduced distribution functions \( f(\nu_{||}, \nu_{\perp}) \) (Equation 3.1), \( f(\alpha, \psi) \) (Equation 3.2), the points at which the particles crossed \( z = 0.067 \ L \) and \( z = 0.93 \ L \), and the nj file and if files. After the iteration has converged, we calculate the fluid parameters by using distribution function \( f(\nu) \) and the nj file [flupar.f] for each group that contributes to the final combined current. Then the total pressure can be determined by using the same set of coefficients [cbtpph.f]. The pressure balance parameter then is calculated [calupb.f].

Then using the previously described method, the combined distribution \( f(\alpha, \psi) \) can be calculated [cbtpph.f]. The black and white scale or the color scale plot of \( f(\alpha, \psi) \) (Figures 3.12 and 3.13) then is made [paphpt.f]. The plot of foot points (Figures 3.10 and 3.11) can be done by program circle.f.