STABILITY OF UPGOING AURORAL ION BEAMS AT 33 R(E) (PLASMA, MAGNETOSPHERE, WAVE-PARTICLE INTERACTIONS)

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University of New Hampshire, Durham

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STABILITY OF UPGOING AURORAL ION BEAMS AT 3.3 $R_e$

BY

G. RICHARD LUDLOW
BA University of New Hampshire

DISSERTATION

Submitted to the University of New Hampshire in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy in Physics

May, 1986
This dissertation has been examined and approved.

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ABSTRACT

STABILITY OF UPGOING AURORAL ION BEAMS AT 3.3 $R_e$

by

G. Richard Ludlow III
University of New Hampshire, May, 1986

Simultaneous upflowing hydrogen and oxygen beams of ionospheric origin are seen at altitudes near 3.3 $R_e$ in the auroral zone by the DE-1 satellite. The beam observations indicate that, in addition to bulk acceleration, substantial heating in both hydrogen and oxygen has taken place either near or above the cold ionospheric source region. Velocity space distributions of the two ion species show hydrogen having a higher streaming velocity than oxygen, but also a smaller streaming energy. The region of velocity space between the two ion beam peaks has been filled in, with ions in the oxygen beam having been accelerated and hydrogen ions having been decelerated. This suggests wave-particle interactions at lower altitudes enabled the transfer of energy from the hydrogen to the oxygen. Maxwellian fits to the beam distributions are used to study the stability of the beams with respect to various low frequency electromagnetic waves. By identifying the growing wave modes generated by the observed distribution functions, one can obtain information about any heating that is taking place in the vicinity of the satellite. This also gives one information on the nature of the heating mechanism at lower altitudes. Using the homogeneous plasma approximation, the Maxwellian fits produce a weak instability of the electrostatic slow hydrogen ion
acoustic wave. The wave resonates both with hydrogen and oxygen and can heat both species by quasilinear diffusion. This process seems to be a viable one at satellite altitudes but is not responsible for the bulk of the heating which took place at lower altitudes.
INTRODUCTION

Ionosphere-Magnetosphere Interaction

This study is concerned with processes in the near earth space environment, which consists of the ionosphere and the magnetosphere. The ionosphere is a layer at the top of our atmosphere made up of neutral atoms and free ions and electrons. It has three layers, called the D, E, and F layers. The D layer is located between 60 and 85 km with a temperature that varies from 130-250 °K. The composition is similar to air at ground level with the electron density varying from 10 cm^-3 at the lower boundary to \( \sim 10^3 \) cm^-3 at the higher boundary. The second layer (E layer) extends from an altitude of 85-130 km and has an electron density of \( 10^4-10^5 \) cm^-3 and a temperature of \( \sim 10^3 \) °K. The F layer is located between 250-500 km and has an electron density that varies from \( 3 \times 10^5 - 10^6 \) cm^-3. Temperatures can reach \( 10^4 \) °K in the F layer. This is less than 1 eV, which corresponds to a temperature of \( 1.161 \times 10^4 \) °K. The ion composition varies as a function of height, as different ionization processes become important. The ion composition of the ionosphere based on rocket measurements is shown in figure 1.

The magnetosphere is the region of space occupied by the earth's magnetic field above the ionosphere. The magnetosphere is compressed on the dayside and stretched out on the nightside. This is due to the solar wind-magnetosphere interaction. The solar wind plasma has a velocity of \( \sim 400 \) km/sec and a temperature of \( \sim 4 \times 10^4 \) °K for singly ionized hydrogen. Because of the extremely high conductivities of all space plasmas, as well as the solar wind, the earth's magnetic field wants to
convect with the solar wind. This so-called frozen-in effect of the earth's magnetic field produces its configuration. The earth's magnetic field connects to the interplanetary magnetic field on open field lines. Some of the major regions of the magnetosphere are shown in figure 2.

Not shown is the bow shock, which is a shock front on the dayside at the outer boundary of the magnetosphere where the solar wind changes from super-Alfvénic to sub-Alfvénic flow. One can view the solar wind as being a source, depositing particles into the magnetosphere where they get energized and are accelerated down into the ionosphere, producing aurora. In this respect, the ionosphere acts as a sink, experiencing joule heating as a result of this process. The ionosphere can also act as a source of plasma, which has been verified over the past decade with satellite measurements (Ghielmetti et al., 1986; Collin et al., 1981; Sharp et al., 1977). Ionospheric ions were seen flowing upward along the geomagnetic field lines at altitudes \( > 1 \, R_E \) in the auroral zone. These ions, which are the subject of this thesis, are an important contribution to the ion composition of the magnetosphere.

Early in this century, during the 1902-1903 auroral expedition, Birkeland discovered polar magnetic substorms (Birkeland, 1908, 1913). He argued that the observed magnetic perturbations were caused by horizontal currents at an altitude of 200 km, and that these currents must be maintained by an external source. Thus, he deduced from the observations the existence of field aligned currents. These currents are also known today as Birkeland currents. This was quite a remarkable result when one considers that the conducting ionosphere would not be discovered until a quarter century later (Dressler, 1983).

Field aligned currents enter the ionosphere at a point in the polar
region, then travel for some distance horizontally along a region called the auroral oval, and finally are fed back into the magnetosphere at some point different from the point of entry. The field aligned currents are actually current sheets which can be seen from photographs of the aurora. Figure 3 illustrates these processes in the northern auroral oval and polar cap. Field aligned currents can be characterized by their position in latitude at the ionosphere. Region 0, 1, and 2 currents refer to the polar cap, auroral oval, auroral oval and mid-latitude regions, respectively (Heikkila, 1983). Figure 4 shows the distribution of field aligned currents in the northern hemisphere observed by the Triad spacecraft during weakly disturbed conditions. On the dawnside Region 1 currents flow into the ionosphere, while on the duskside Region 2 currents flow into the ionosphere. Also in figure 4 are shown field aligned currents believed to flow in the cusp region on open field lines.

Ionospheric currents flow in directions perpendicular to the earth's magnetic field. For ions, an ionospheric electric field \( \mathbf{E} \) perpendicular to \( \mathbf{B} \) will generate a drift in two directions. One direction is perpendicular to both \( \mathbf{E} \) and \( \mathbf{B} \) and is called the Hall drift due to the \( \mathbf{E} \times \mathbf{B} \) drift of charged particles in a magnetic field, \( \mathbf{v} = \frac{c\mathbf{E} \times \mathbf{B}}{\mathbf{B}^2} \). The other direction is parallel to \( \mathbf{E} \) and is called the Pedersen drift. The Pedersen drift results from collisions. The electron-neutral collision frequency remains much smaller than the electron gyrofrequency above an altitude of 80 km, causing the electrons to have only the Hall drift. In these regions the ion-neutral collision frequency is comparable to the ion gyrofrequency. Thus, in the direction perpendicular to \( \mathbf{E} \), which in the absence of collisions ions and electrons have the same velocity,
a current is set up because the ions are prevented from obtaining the \( \vec{E} \times \vec{B} \) velocity due to collisions. In the direction parallel to \( \vec{E} \), where ions and electrons respond in opposite directions, the current depends on the ratio of the ion-neutral collision frequency to the ion gyrofrequency. At any given height in the ionosphere the Pedersen and Hall currents have strengths which depend on the Pedersen and Hall conductivities at that altitude. There are also currents parallel to \( \vec{B} \), due to the presence of an \( E_\parallel \). The parallel conductivity is much higher than the perpendicular components, and consequently \( E_\parallel \) is much smaller than \( E_\perp \). These currents and their associated electric fields have been verified with rocket measurements (Kintner et al., 1974).

There are many mechanisms which generate large scale electric fields in the ionosphere and magnetosphere (Stern, 1977). These include electric fields generated by the rotation of the earth and its magnetic field, electric fields resulting from the flow of hot plasma within the magnetosphere, and electric fields associated with differential motions of the neutral atmosphere and ionospheric plasma. Also, there is the convection electric field generated by the solar wind drifting through the interplanetary magnetic field \( \vec{E} = -\vec{v} \times \vec{B}/c \). Because the conductivity parallel to the earth's magnetic field lines is essentially infinite, the convection electric field is mapped down to the area of the ionosphere intercepted by these field lines, which form equipotentials.

**Parallel Electric Fields and Ion Acceleration Mechanisms**

Parallel electric fields make up a special class of electric fields in the magnetosphere. The magnetospheric plasma has a very high conductivity parallel to the magnetic field, making the field lines equipoten-
tials. Any voltage drops perpendicular to $\mathbf{B}$ that occur in the ionosphere due to finite Pedersen resistivities will be communicated outward to the magnetosphere at the Alfvén speed by means of polarization currents flowing parallel to $\mathbf{B}$. In certain regions of the magnetosphere this condition does not hold, and one finds a field aligned potential difference. This is the case for auroral field lines. Rocket and satellite measurements of precipitating auroral electrons show marked characteristics.

When a satellite traverses the auroral region, the field aligned flux of electrons increases in energy from $\sim 100$ eV to several keV and then decreases again, forming a time spectrogram resembling an inverted V (Arnoldy et al., 1974; Lin et al., 1979; Hoffman et al., 1981). Potential structures which would have to exist on auroral field lines in order to produce the inverted-V are shown in figure 5 (Kaufmann and Kintner, 1984). These are the so-called U and S-shaped potential structures. Although the existence of U and S-shaped potential structures is based on experimental results (Temerin et al., 1981; Mizera et al., 1982), Lyons (1980) has shown theoretically that U-shaped potential structures result from discontinuities in the magnetospheric convection electric field, $\mathbf{E} = -\mathbf{v} \times \mathbf{B}/c$, where $\mathbf{B}$ is the earth's magnetic field and $\mathbf{v}$ is the velocity of the convecting plasma. The average magnetospheric convection is antisunward over the poles and sunward at lower latitudes. The boundary between the two flows produces a finite divergence of the convection electric field, generating U-shaped potential structures which cause upward field aligned currents at the discontinuity on the evening side and downward field aligned currents on the morning side.

Figure 5 shows the ion and electron distributions and the frequen-
cies of ion cyclotron waves that would be seen by a satellite traversing the U and S-shaped potential structures. For the U-shaped structure, ionospheric ions would be accelerated upward to form a beam. The electron distribution will show a widened loss cone due to downgoing electrons escaping at small pitch angles and upgoing electrons which are decelerated in parallel velocity by the potential. The S-shaped structure can produce conic distributions in the ions where the electric field has components both perpendicular and parallel to the magnetic field. Cold ionospheric electrons can be reflected by the potential to form weak downgoing field aligned beams. These beams are unstable and can produce growing upper hybrid waves (Kaufmann and Ludlow, 1981).

Conic distributions in velocity space are so named because when they are rotated around the \( v_\parallel \) -axis they produce the shape of a cone. Conic distributions produced by perpendicular electric fields at low altitude will fold into ion beams at higher altitude as they adiabatically evolve up the field lines. Upgoing ion beams formed this way or by direct acceleration have been detected at altitudes \( \geq 1 \, R_E \) by Chielmetti et al., (1978), Collin et al., (1981), and Yau et al., (1984) with data from the S3-3 and DE-1 satellites. Potential structures like the S-shaped one containing an electric field oblique to the magnetic field are called oblique double layers and can be responsible for the energization of ion beams as well as the production of conics (Greenspan, 1984; Yang and Kan, 1983; Borovsky and Joyce, 1983).

There are also other proposed ion acceleration mechanisms responsible for the production of conics and the heating, in addition to bulk acceleration of ion beams. These ion beams show thermal energies at high altitude on the order of 100 times the thermal energy in the cold
ionospheric source plasma, $\lesssim 1$ eV (Kaufmann and Kintner, 1982; Kaufmann and Ludlow, 1986). Dusenbery and Lyons (1981) proposed that ion conics at low altitudes could be generated by upgoing ionospheric electrons in regions of diffuse auroral precipitation. The upgoing ionospheric electrons excite electrostatic ion cyclotron waves which cyclotron resonate with upgoing thermal ionospheric ions. The waves then heat the ions by quasilinear diffusion to perpendicular energies on the order of 100 times their initial thermal energy. Another process involving lower hybrid waves has been developed by Chang and Coppi (1981), Retterer et al., (1983). Precipitating electron beams generate broad band lower hybrid waves at low altitude which can resonate with both the electrons and the ions, thus transferring energy from the electrons to the ions. The ions are energized transverse to the magnetic field, producing conic distributions. Using quasilinear heating rates in the unmagnetized approximation, Chang and Coppi (1981) heat 1 eV ions to energies of 100 eV or more.

Lysak, Hudson and Temerin (1980) have proposed a nonlinear mechanism where coherent electrostatic hydrogen cyclotron waves, as observed on S3-3 (Kintner et al., 1978, 1979), heat 1 eV source ions to perpendicular energies of 100 eV. Bergmann and Lotko (1986) have provided a theoretical scenario where upgoing $H^+$ and $O^+$ beams evolve first under the influence of the potential drop, $\Delta \phi$, and the earth's diverging magnetic field. The relative drift of the $H^+$ and $O^+$ beams generates an explosive $H^+-O^+$ two-stream instability at low altitudes with higher growth rates than the current driven ion acoustic instability with one ion species. This instability could provide a mechanism where energy could be transferred from the $H^+$ to the $O^+$, thus heating the oxygen.
This thesis focuses on the nature of these H\(^+\) and O\(^+\) ion beams at 3.3 \(R_e\) geocentric and how they interact with one another through plasma waves. The Dynamics Explorer I satellite crosses the auroral zone at 3.3 \(R_e\) geocentric, accumulating ion composition measurements of the auroral plasma. At this altitude, the H\(^+\) and O\(^+\) ion beams have different streaming velocities. This is because a single ion species \(\propto\), having experienced the same potential drop \(\Delta \phi\), will acquire a parallel velocity \(V_{\parallel \alpha} = \left(2q_\alpha \Delta \phi/m_\alpha\right)^{1/2}\). The difference in the hydrogen and oxygen masses results in a different streaming velocity. The relative drift between two ion species can produce unstable plasma waves. These waves can interact with particles in the beams and can cause the beams to diffuse in velocity space. This process, known as quasilinear diffusion, heats the ion beams and gives them a larger thermal velocity.

The ionospheric source of plasma is cold, \(\leq 1\) eV. The beam observations at 3.3 \(R_e\) on DE-1, presented in Chapter II, reveal that significant heating has taken place, yielding beam temperatures of 100-200 eV. By identifying unstable waves, one can get a clue as to what process is responsible for heating the ion beams. Direct measurements of plasma densities, composition, and ion fluxes by the DE-1 satellite are used in the theoretical calculations, thus providing an accurate description of the observed plasma. First, a search for unstable low frequency electromagnetic waves is carried out. The dispersion relation for electromagnetic waves as derived in Chapter I is used. After following in k-space the dispersion of various right and left polarized electromagnetic waves generated in a plasma with two ion components, it is found that no low frequency electromagnetic waves are unstable. This is because the free energy needed to drive electromagnetic waves is not pro-
vided by the ion beams. This problem is explored in Chapter III.

It is found, finally, that the plasma is unstable to a hydrogen ion acoustic wave. This wave is electrostatic, thereby enabling the electrostatic dispersion relation to be used. Quasilinear diffusion is explored as a possible mechanism for heating the ion beams. The results of quasilinear theory as derived in Chapter I are explored with the unstable ion acoustic wave and are presented in Chapter III. Conclusions and future work are described in Chapter IV.
Ion composition of the ionosphere. (From Banks and Kockarts, 1973)

Figure 1
Regions of the magnetosphere. (From Heikkila, 1972)

Figure 2
Northern auroral oval with field aligned and ionospheric currents.  (From Heikkila, 1974)

Figure 3
Currents into Ionosphere

(From Iijima and Potemra, 1976)

Figure 4
U and S-shaped potential structures. (From Kaufmann and Kintner, 1984)
CHAPTER I

LINEAR AND QUASILINEAR THEORY

Formulation of the Problem

In order to derive the equations that will provide a complete description of the plasma, one must define the regime in which one is working. This includes defining length and time scales and how these parameters affect particle interactions. One fundamental quantity in a plasma is the Debye length. Because the electrons are much lighter than the ions, the electrons will respond to positive charges by forming a cloud which shields the positive charge from the surrounding volume. The Debye length is a measure of the distance at which the potential field around a positive charge gets screened out by the induced electron field. By considering the Poisson equation for the effective field and assuming the electrons obey Maxwell-Boltzmann statistics, one finds the potential field for the positive point charge $q_0$ to be

$$\phi(r) = \frac{q_0}{r} e^{-r/\lambda_{De}}$$

$$\lambda_{De} = \left( \frac{K T_e}{4 \pi e^2 n_e} \right)^{\frac{1}{2}} \text{ (cm) Debye length} \quad (1.1)$$

$K =$ Boltzmann's constant $= 1.38 \times 10^{-16}$ erg/°K

$T_e =$ temperature °K

$n_e =$ density

For distances shorter than the Debye length, the effective potential is equivalent to the bare Coulomb potential. Outside the Debye sphere, the
potential decreases exponentially, and one sees the collective behavior of the plasma. Another collective phenomena in a plasma is the plasma frequency.

\[ \omega_{pe} = \left( \frac{4\pi n_e^2}{m_e} \right)^{1/2}, \quad (s^{-1}) \]  (1.2)

The plasma frequency is the result of suddenly introducing a static electric field into the plasma. The electrons respond and move so as to quench the field. Because of their finite mass their inertia causes them to overshoot, producing another nonequilibrium situation. The Coulomb field provides the restoring force, and an oscillation results.

The important result here is whether the particles are responding in a collective sense, described by the parameters \( \lambda_{De} \) and \( \omega_{pe} \), or whether the particles behave in a discrete sense, with Coulomb collisions being the strongest response. If one calculates the Coulomb collision frequency for ion-electron scattering, assuming a Maxwellian distribution for the electrons and letting the impact parameter assume a maximum value of \( \lambda_{De} \), the result is

\[ \nu = \left( \frac{\pi}{2} \right)^{3/2} \frac{n_e^2}{m_e^{3/2} T_e^{1/2}} \ln \Lambda, \quad (s^{-1}) \]  (1.3)

where \( \ln \Lambda \) is the Coulomb logarithm, \( \ln \Lambda = \ln \left( \frac{2 \pi n \lambda_{De}^3}{1} \right) \).

The ratio of the Coulomb collision frequency to the plasma frequency is given by the following expression.

\[ \frac{\nu}{\omega_{pe}} = (32 \left( \frac{2 \pi}{n_e} \right)^{1/2} \lambda_{De}^{3})^{-1} \ln \Lambda \]  (1.4)
Thus, one sees that as long as \( \eta e^{3} \gg \omega_{pe} \gg \nu \). The quantity \( \eta e^{3} \) is the average number of particles in a Debye sphere, which must be large for collective behavior to dominate. For the plasma in the auroral zone at 3 \( R_e \), \( \eta e^{3} \sim 10^{13} \), thus making Coulomb collisions insignificant.

With these insights in mind, one can use the quantity \( (\eta e^{3})^{-1} \) as the small parameter describing a plasma. This parameter is known as the plasma parameter \( g \), and is used as an expansion parameter in the theory of an ordinary plasma.

The equations which describe a system of charged particles are the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy equations (Bogoliubov, 1962).

\[
\left[ \frac{2}{\tau} + \sum_{i=1}^{s} L(i) - \frac{1}{n} \sum_{i \neq j}^{s} \nabla (i,j) \right] f_{s}(1,\ldots,s) = \sum_{i=1}^{s} \int \nabla (\hat{x}_{i},s+1) f_{s+1}(1,\ldots,s+1) d(s+1)
\]

(1.5)

\( i = (\vec{x}_{i},\vec{v}_{i}) \), position and velocity of the \( i^{th} \) particle

\( f_{s}(1,\ldots,s) \), \( s \)-particle distribution function
\( n = \frac{N}{V} \) = average number density

\( d(s) = d^3x_d^3v_s \)

\( L(i), \) single particle operator

\[
L(i) = \mathbf{v}_i \cdot \frac{\partial}{\partial \mathbf{r}_i} + \frac{q}{m} \left[ \mathbf{E}_{\text{ext}}(\mathbf{r}_i) \right] + \frac{\mathbf{V}_i}{c} \times \mathbf{B}_{\text{ext}}(\mathbf{r}_i) \cdot \frac{\partial}{\partial \mathbf{v}_i}
\]

\( V(i,j), \) two particle operator

\[
V(i,j) = \frac{q^2\mu}{m} \left( \frac{\partial}{\partial \mathbf{x}_i} \frac{1}{|\mathbf{x}_i - \mathbf{x}_j|} \right) \cdot \frac{\partial}{\partial \mathbf{v}_i}
\]

The distribution functions have normalization

\[
\frac{1}{\sqrt{V}} \int f_i(i) \, dl = 1
\]

\[
\frac{1}{\sqrt{V}} \int f_{s+1}(1, \ldots, s+1) \, d(s+1) = f_{s}(1, \ldots, s)
\]

It can be seen that the BBGKY hierarchy is an infinite series of equations. The equation for the single particle distribution function
depends on the two particle distribution function. The equation for the two particle distribution function depends on the three particle distribution function, and so on. The BBGKY hierarchy needs to be truncated in order to be applicable. Because one is interested in solutions for an ordinary plasma, one can try to solve the BBGKY hierarchy by expanding \( f_s \) in a power series in \( g \), the plasma parameter. A convenient way to express the higher order distribution function \( f_s \) is to introduce \( s \)-particle correlation functions. The \( s \)-particle distribution functions are then written in terms of these correlation functions, a scheme similar to the Mayer cluster expansion (Mayer, 1940).

\[
S=1 \quad f_1(1) = F(1) \quad (1.6a)
\]

\[
S=2 \quad f_2(1,2) = F(1)F(2) + G(1,2) \quad (1.6b)
\]

\[
S=3 \quad f_3(1,2,3) = F(1)F(2)F(3) + F(1)G(2,3)
+ F(2)G(3,1) + F(3)G(1,2)
+ H(1,2,3) \quad (1.6c)
\]

\( F(1), F(2), \ldots \) are introduced to represent the single particle distribution functions \( f_1(1), f_1(2), f_1(3), \ldots \). Thus, \( f_2(1,2) \)
is represented as a product of the two single particle distribution functions \( F(1) \) and \( F(2) \) plus the pair correlation function \( g(1,2) \).

One now assumes that
\[
F(1) \approx O(1)
\]
\[
\frac{g(1,2)}{F(1)F(2)} \approx O(q)
\]  \hspace{1cm} (1.7)

The reason for doing so is that the joint distribution of two particles in a small volume \( V \), such that \( n^{-1} << V << \lambda_0^3 \) will be determined by the surrounding plasma volume, and not by the separation between the two particles. This means that
\[
\int_2 (1,2) \approx F(1)F(2)
\] as long as \( q << 1 \). Following this argument to higher order, one has
\[
\frac{H(1,2,3)}{F(1)F(2)F(3)} \approx O(q^2)
\]  \hspace{1cm} (1.8)

and so on. If one substitutes (1.6) into (1.5), one obtains for the first equation
\[
S = \int \left[ \frac{2}{\partial x} + L(1) - \int v(1,2) F(2;x) \, dx \right] F(1;x) \nonumber \\
\hspace{2cm} = \int v(1,2) G(1,2;x) \, dx . \hspace{1cm} (1.9)
\]

The right hand side of (1.9), which depends on the pair correlation function \( g(1,2) \), describes the collisional effects and is known as the collision term. In the limit \( q \to 0 \), the right hand side goes to zero. In this limit, equation (1.9) is called the Vlasov equation. By putting in the explicit forms of all the symbols in equation (1.9), and by letting the single particle distribution function for particles of species
\( \alpha \) be \( f_\alpha (\vec{x}, \vec{v}, t) \), the Vlasov equation assumes the form
\[
\left[ \frac{\partial}{\partial t} + \vec{V} \cdot \frac{\partial}{\partial \vec{x}} \right] \psi_f (\vec{x}, \vec{v}, t) = \mathcal{L} \psi_f (\vec{x}, \vec{v}, t) \tag{1.10a} \]
\[+ \frac{q_\alpha}{m_\alpha} \left( \psi_f (\vec{x}, \vec{v}, t) + \frac{\vec{V}}{c} \times \mathcal{B} (\vec{x}, t) \right) \cdot \frac{\partial}{\partial \vec{v}} \right] f_\alpha (\vec{x}, \vec{v}, t) = 0. \]

where
\[
\mathcal{E} (\vec{x}, t) = \mathcal{E}_{ex} (\vec{x}, t) - \sum_{\alpha} q_\alpha n_\alpha \frac{\partial}{\partial \vec{x}} \int d\vec{x}' \int d\vec{v}' \frac{f_\alpha (\vec{x}', \vec{v}', t)}{|\vec{x} - \vec{x}'|} \tag{1.10b} \]
\[\mathcal{B} (\vec{x}, t) = \mathcal{B}_{ex} (\vec{x}, t) \tag{1.10c} \]

The second term in (1.10b), involving the two-particle operator, represents the electrostatic field produced by the average distribution of particles and is called the Hartree field. Collisional effects are due to fluctuations in the Hartree field, which, as shown above, are of order \( g \) and can be neglected. This study concerns itself with homogeneous plasmas which are charge neutral, in which case the Hartree field vanishes. Thus, the plasma does not produce its own macroscopic field.

As for the time scales involved, one can construct three characteristic time scales \( \tau_0 \), \( \tau_1 \), and \( \tau_2 \), known as Bogoliubov's hierarchy of characteristic time scales. The hydrodynamic time scale \( \tau_0 \) is the characteristic time for hydrodynamic fluctuations to relax and is given by
\[
\tau_0 \sim \frac{L}{c_s} \tag{1.11} \]
where \( L \) is the length of the system and \( c_s \) is the sound speed. The
second characteristic time scale is \( \tau_1 \), and is the time for the single particle distribution function to relax to its local equilibrium value. The time \( \tau_1 \) is given by

\[
\tau_1 \sim \frac{\lambda}{\langle \mathbf{v} \rangle},
\]

(1.12)

where \( \lambda \) is the mean free path and \( \langle \mathbf{v} \rangle \) is the mean velocity of the particle, \( \langle \mathbf{v} \rangle \sim \left( \frac{kT}{m} \right)^{1/2} \). Since \( \zeta_s \sim \langle \mathbf{v} \rangle \), and \( \zeta \gg \lambda \), one has \( \tau_0 \gg \tau_1 \). The third time scale \( \tau_2 \) is the characteristic time associated with pair correlations and is given by the average time for a particle to travel a correlation distance, which is the order of a Debye length.

\[
\tau_2 \sim \frac{\lambda_D}{\langle \mathbf{v} \rangle}
\]

(1.13)

Because \( \tau_1 \sim \nu^{-1} \) and \( \tau_2 \sim \omega_p^{-1} \), we have \( \tau_1 \gg \tau_2 \) as long as the plasma parameter, \( q = (n \lambda_d)^{-1} \), is very small. This means that pair correlation functions relax much faster than single particle distribution functions and can be neglected in this description.
Linear Theory

The Vlasov equation

\[
\left[ \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{e}{m_\alpha} \mathbf{v} \cdot \nabla_x \right] f_\alpha (\mathbf{x}, \mathbf{v}, t) = 0
\]  

(1.14)

describes the time evolution of \( f_\alpha (\mathbf{x}, \mathbf{v}, t) \), the single particle distribution function in phase space \( \mathbf{x}, \mathbf{v} \). The dimensions of \( f_\alpha \) are \((\# \text{ particles of species } \alpha)/\text{cm}^3/(\text{cm/s})^3\). The Vlasov equation is also known as the collisionless Boltzmann equation. It can be used in the magnetosphere where the collision frequency of particles is much less than the plasma frequency, as was seen in the previous section. Thus particle distributions change as a result of wave interactions rather than collisions. Waves are generated in a plasma by small perturbations. In a plasma immersed in a uniform magnetic field \( \mathbf{B}_0 \), density perturbations will cause ions and electrons to deviate from their helical paths. This gives rise to a current density perturbation \( \mathbf{j}_i (\mathbf{x}, t) \), and induces microfields \( \mathbf{E}_i (\mathbf{x}, t) \) and \( \mathbf{B}_i (\mathbf{x}, t) \). The ions and electrons respond to these fields so as to achieve equilibrium. Their inertia causes them to overshoot the equilibrium state, and the induced fields oscillate. A variety of electromagnetic and electrostatic or compressional wave modes can be set up. In the following, the dispersion relation for waves in a homogeneous plasma in a uniform magnetic field is obtained.

The theoretical impetus is to allow a transition to take place so that the initial state described by \( f_\alpha \) evolves to a final state in which waves are present, \( f_\alpha = f_\alpha_0 + f_\alpha_1 \). \( f_\alpha_0 \) describes the plasma at \( \tau' = -\infty \). As one lets \( \tau' \) increase, the plasma is perturbed, which generates \( f_\alpha = f_\alpha_0 + f_\alpha_1 \) at time \( \tau = \tau' \). The
perturbing distribution function $f_{\alpha_1}(x_i, v, t)$ is related to the induced current density by $\mathbf{j}_I(x_i, t) = \sum_{\alpha} n_{\alpha} \mathbf{q}_{\alpha} \int \mathbf{v} f_{\alpha}(x_i, v, t) \, dv$.

If one can obtain an analytical expression for $f_{\alpha_1}$ in terms of known quantities $f_{\alpha_0}$, $\mathbf{E}_1$, and $\mathbf{B}_1$; where $\mathbf{E}_1$ and $\mathbf{B}_1$ vary as $\exp[i (k \cdot x - \omega t)]$, then the induced current $\mathbf{j}_I(x_i, t)$ can be found. The wave equation, obtained from Maxwell's equations with $\mathbf{j}_I$, can then be written for electromagnetic waves in a plasma.

To find $f_{\alpha_1}(x_i, v, t' = t)$, note that in both initial and final states the Vlasov equation must be valid. Thus,

$$\left[ \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{\mathbf{F}_{\alpha_0}}{m_{\alpha}} \cdot \nabla_{\mathbf{v}} \right] f_{\alpha_0} = 0$$

and

$$\left[ \frac{\partial}{\partial t'} + \mathbf{v'} \cdot \nabla + \frac{\mathbf{F}_{\alpha}}{m_{\alpha}} \cdot \nabla_{\mathbf{v'}} \right] f_{\alpha} = 0.$$  \hspace{1cm} (1.15)

$\mathbf{F}_{\alpha_0}$ is solely the magnetic force at time $t' = t$, since one is assuming that there is no external electric field present.

$$\mathbf{F}_{\alpha_0} = \frac{q_{\alpha}}{c} \mathbf{v} \times \mathbf{B}_o$$  \hspace{1cm} (1.17)

and

$$\mathbf{F}_{\alpha} = \frac{q_{\alpha}}{c} \left[ \frac{\mathbf{v} \times (\mathbf{B}_o + \mathbf{B}_1)}{c} + \mathbf{E}_1 \right]$$  \hspace{1cm} (1.18)

If only terms linear in perturbing quantities are kept in (1.16) and (1.17) is used, the equation for $f_{\alpha_1}$ is

$$\left[ \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{\mathbf{q}_{\alpha}}{m_{\alpha}} \frac{\mathbf{v} \times \mathbf{B}_0}{c} \cdot \nabla_{\mathbf{v}} \right] f_{\alpha_1} =$$

$$= - \frac{q_{\alpha}}{m_{\alpha}} \left( \frac{\mathbf{v} \times \mathbf{B}_1}{c} + \mathbf{E}_1 \right) \cdot \nabla_{\mathbf{v}} f_{\alpha_0}.$$  \hspace{1cm} (1.19)
One can obtain an explicit form of $f_{\alpha l}(\mathbf{x}, \mathbf{v}, t)$ by integrating the equation above. As one integrates from $x' = -\infty$ to $x' = x$, the particle's path in phase space will correspond to the orbit of a charged particle in equilibrium field $B_0$, since the perturbation is assumed to be small. Thus,

$$\frac{d\mathbf{v}'}{dt'} = \frac{q_\alpha}{m_\alpha} \left( \frac{\mathbf{v}' \times B_0}{c} \right)$$

(1.20)

and

$$\frac{d\mathbf{x}'}{dt'} = \mathbf{v}'(t')$$

(1.21)

With boundary conditions

$$\mathbf{x}'(x' = x) = \mathbf{x}(x)$$
$$\mathbf{v}'(x' = x) = \mathbf{v}(x)$$

(1.19) becomes

$$\frac{df_{\alpha l}(\mathbf{x}', \mathbf{v}', t')}{dt'} = \left[ \frac{\partial}{\partial t'} + \mathbf{v}' \cdot \nabla + \frac{d\mathbf{v}'}{dt'} \cdot \nabla_{\mathbf{v}'} \right] f_{\alpha l}(\mathbf{x}', \mathbf{v}', t')$$

$$= -\frac{q_\alpha}{m_\alpha} \left[ E_1(\mathbf{x}', t') + \frac{\mathbf{v}' \times B_1(\mathbf{x}', t')}{c} \right] \cdot \nabla_{\mathbf{v}'} f_{\alpha l}(\mathbf{x}', \mathbf{v}', t').$$

(1.22)

integrating

$$\int_{-\infty}^{t} dt' \frac{df_{\alpha l}}{dt'} = f_{\alpha l}(\mathbf{x}, \mathbf{v}, t) - f_{\alpha l}(\mathbf{x}(\infty), \mathbf{v}(\infty), t(-\infty))$$

$$\downarrow$$

0
so that

\[
\tilde{f}_{\alpha\lambda}(x',v',t) = -\int_{-\infty}^{t} \frac{q_{\alpha}}{m_{\alpha}} \left[ \mathcal{E}_{i}(x',t') + \frac{v' \times B_{i}(x',t')}{c} \right] \hat{\nabla}_{v} \tilde{f}_{\alpha0}(x',v',t') dt'.
\]

(1.23)

At this point one assumes an oscillatory solution for

\[
f_{\alpha\lambda}(x',v',t) = f_{\alpha\lambda} \hat{k} e^{i(\hat{k} \cdot x' - \omega t')}
\]

where \( \hat{k} = \frac{2\pi}{\lambda} \hat{n} \), \( \hat{n} \) = wave normal unit vector

and \( f_{\alpha\lambda} \hat{k} \) = amplitude associated with mode \( \hat{k} \).

\( \mathcal{E}_{i} \) and \( B_{i} \) are also written in a similar fashion with an explicit oscillatory dependence.

\[
\mathcal{E}_{i}(x',t') = \mathcal{E}_{i} \hat{k} e^{i(\hat{k} \cdot x' - \omega t')}
\]

\[
B_{i}(x',t') = B_{i} \hat{k} e^{i(\hat{k} \cdot x' - \omega t')}
\]

Thus (1.23) becomes

\[
\tilde{f}_{\alpha\lambda} = -\frac{q_{\alpha}}{m_{\alpha}} \int_{-\infty}^{t} \left[ \mathcal{E}_{i} \hat{k} + \frac{v' \times B_{i} \hat{k}}{c} \right] \hat{\nabla}_{v} \tilde{f}_{\alpha0}(x',v',t') \cdot i \hat{k} \cdot (x - x') \ e^{i\omega(t - t')} \ e^{i\omega(t' - t)} \cdot dt'.
\]

(1.24)
The following definitions are introduced.
\[ \vec{X} = \vec{X} - \vec{X} \]
\[ \vec{\Gamma} = \vec{X} \]
\[ \vec{B}_0 = B_0 \frac{\hat{z}}{\hat{z}} \]
\[ \omega_{c\alpha} = \frac{2\alpha B_0}{m_\alpha c} \]

One has
\[ \vec{v}' (\vec{r} = 0) = \vec{v} \]
\[ \vec{x}' (\vec{r} = 0) = \vec{x} \]

as boundary conditions. The equations of motion are
\[ \frac{d\vec{v}'}{d\tau} = \omega_{c\alpha} (\vec{v}' \times \hat{z}) \]
\[ \frac{d\vec{x}'}{d\tau} = \vec{v}' . \]
With the coordinate system below, cylindrical geometry is used.

Figure 6
The equations of motion yield

\[ v'_x = v_\perp \cos(\phi - \omega_c \tau) \]

\[ v'_y = v_\perp \sin(\phi - \omega_c \tau) \]

\[ v'_z = v_\parallel \]

and

\[ \Delta x = x'_1 - x = \frac{v_\perp}{\omega_c} \sin \phi - \frac{v_\perp}{\omega_c} \sin(\phi - \omega_c \tau) \]

\[ \Delta y = y'_1 - y = -\frac{v_\perp}{\omega_c} \cos \phi + \frac{v_\perp}{\omega_c} \cos(\phi - \omega_c \tau) \]

\[ \Delta z = z'_1 - z = v_\parallel \tau. \]

Therefore,

\[ \vec{x} = \hat{x} \Delta x + \hat{y} \Delta y + \hat{z} \Delta z \]

\[ i \left( \vec{k} \cdot \vec{x} - \omega \tau \right) e^{-i(\vec{k} \cdot \vec{x} - \omega \tau)} = \frac{k v_\perp}{\omega_c} \sin \phi - \frac{k v_\perp}{\omega_c} \sin(\phi - \omega_c \tau) \]

\[ f_{\alpha k} = -\frac{2 \pi}{m_k} \int_{-\infty}^{0} d\tau \left[ \vec{E}_k + \frac{\vec{v} \times \vec{B}_k}{c} \right] \cdot \nabla_x f_{\alpha 0} \left( \vec{x}'_1, \vec{v}'_1, \tau' \right) \cdot e^{-i(\vec{k} \cdot \vec{x} - \omega \tau)}. \]

(1.25)

By Faraday's Law,

\[ \nabla \times \vec{E}_1(\vec{x}, \tau) = -\frac{1}{c} \frac{\partial}{\partial \tau} \vec{B}_1(\vec{x}, \tau) \]
or
\[ \mathbf{B}_k^* = \frac{\mathbf{c} \times \mathbf{E}_k^*}{\omega}, \]

thus,
\[ \frac{\nabla \times \mathbf{B}_k^*}{c} = \frac{\nabla \times (\mathbf{c} \times \mathbf{E}_k^*)}{\omega} = \frac{1}{\omega} \left[ (\nabla \cdot \mathbf{E}_k^*) \mathbf{k} - (\nabla \cdot \mathbf{k}) \mathbf{E}_k^* \right]. \]

Because the plasma considered here is homogeneous,
\[ f_{\alpha_0}(\mathbf{x}', \mathbf{v}', t') = f_{\alpha_0}(\mathbf{v}_1', \mathbf{v}_2', t'). \]

There is no \( \phi \)-dependence in \( f_{\alpha_0} \) and also
\[ f_{\alpha_0}(t' = -\infty) = f_{\alpha_0}(t' = \infty). \]

Thus (1.25) becomes
\[ f_{\mathbf{k}} = -\frac{2\alpha}{m_x} \int_{-\infty}^{0} d\gamma \left[ (1 - \frac{\mathbf{k} \cdot \mathbf{v}'}{\omega}) \mathbf{E}_k^* + (\mathbf{v} \cdot \mathbf{k}) \mathbf{E}_k^* \right] \left[ (\mathbf{k} \cdot \mathbf{v}') \frac{2}{\mathbf{v}_1'} + \lambda \frac{2}{\mathbf{v}_2'} \right] f_{\alpha_0}(\mathbf{v}', t') e^{i(\mathbf{k} \cdot \mathbf{x}' - \omega t')}. \]

(1.26)
The terms in brackets are written explicitly as

\[
\begin{aligned}
&\frac{\partial f_{do}}{\partial V_L} \left[ (1 - \frac{k_z V_x}{\omega}) (E_x \frac{V_x'}{V_L} + E_y \frac{V_y'}{V_L}) + V_z E_z \frac{k_x V_x'}{\omega V_L} \right] \\
+ &\frac{\partial f_{do}}{\partial V_{II}} \left[ (1 - \frac{k_x V_x}{\omega}) E_z + (V_x' E_x + V_y' E_y) \frac{k_z}{\omega} \right] \\
= &\left[ \frac{\partial f_{do}}{\partial V_L} \left(1 - \frac{k_z V_z'}{\omega}\right) + \frac{\partial f_{do}}{\partial V_{II}} \frac{k_z V_z'}{\omega} \right] E_x \cos(\phi - \omega_{ca} \tau) \\
+ &E_y \sin(\phi - \omega_{ca} \tau) \right]
+ \left[ \frac{\partial f_{do}}{\partial V_L} \frac{k_x V_x'}{\omega} - \frac{\partial f_{do}}{\partial V_{II}} \frac{k_x V_x'}{\omega} \right] E_z \cos(\phi - \omega_{ca} \tau)
+ \frac{\partial f_{do}}{\partial V_{II}} E_z
\end{aligned}
\]

where \( E_x, E_y, E_z \) are the components of \( \vec{E} \).

The following Bessel function identities are used to simplify the above.

\[
e^{-i a \sin(\phi - \omega_{ca} \tau)} = \sum_{\ell = -\infty}^{\infty} J_\ell(a \omega) e^{-i \ell \phi - i \omega_{ca} \tau}
\]

\[
\cos(\phi - \omega_{ca} \tau) e^{-i a \sin(\phi - \omega_{ca} \tau)} =
= \frac{1}{-i a} \frac{d}{d\phi} \left\{ \sum_{\ell = -\infty}^{\infty} J_\ell(a \omega) e^{-i \ell (\phi - \omega_{ca} \tau)} \right\}
\]
Thus, (1.26) becomes

\[
\tilde{f}_{\alpha k} = - \frac{\alpha}{m} \int_0^\infty \sum_{l=-\infty}^\infty \left[ \rho \frac{l}{a_\alpha} J_l + i J'_l \right] e^{i(k_{n\parallel}v_{n\parallel} - \omega t + l\omega t_{\alpha})} J_n(a_\alpha) e^{i(n-l)\phi} \]  

(1.27)

where

\[
\rho = \frac{\partial f_{x0}}{\partial v_{\parallel}} \left( 1 - \frac{k_{n\parallel}v_{n\parallel}}{\omega} \right) + \frac{\partial f_{x0}}{\partial v_{n\parallel}} \frac{k_{n\parallel}v_{n\parallel}}{\omega}
\]

\[
Q = \frac{\partial f_{x0}}{\partial v_{\parallel}} \frac{k_{x\parallel}v_{n\parallel}}{\omega} - \frac{\partial f_{x0}}{\partial v_{n\parallel}} \frac{k_{x\parallel}v_{n\parallel}}{\omega}
\]

\[
a_\alpha = \frac{k_{x\parallel}v_{n\parallel}}{\omega_{\alpha}}
\]
With the use of Maxwell's equations, the dielectric tensor can be evaluated, which then is used in the wave equation. One has

\[ \nabla \times \mathbf{B}_i = \frac{1}{c} \frac{\partial \mathbf{E}_i}{\partial t} + \frac{4\pi}{c} \mathbf{J}_i, \quad \text{Ampere's Law} \]

\[ = \frac{1}{c} \frac{\partial \mathbf{E}_i}{\partial t} + \frac{4\pi}{c} \sum_{\alpha} n_a g_{\alpha} \int \mathbf{V} f_{\alpha k} (x, y, z, t) \, d\mathbf{V} \]  \hspace{1cm} (1.28)

The quantities \( \mathbf{B}_i, \mathbf{E}_i \), and \( \mathbf{J}_i \) can be written in terms of their Fourier amplitudes, i.e. \( \mathbf{J}_i = \frac{2}{\pi} \exp[\pm i(k \cdot \mathbf{r} - \omega t)] \) enabling the substitution of (1.27) into (1.28).

That is,

\[ \nabla \times \mathbf{B}_i = \nabla \times \mathbf{B}_k \exp[i(k \cdot \mathbf{r} - \omega t)] = i k \times \mathbf{B}_k \exp[i(k \cdot \mathbf{r} - \omega t)] \]

\[ = \left( -\frac{i\omega}{c} \mathbf{E}_k + \frac{4\pi}{c} \sum_{\alpha} n_a g_{\alpha} \int \mathbf{V} f_{\alpha k} \, d\mathbf{V} \right) \exp[i(k \cdot \mathbf{r} - \omega t)] \]

\[ i k \times \mathbf{B}_k = -\frac{i\omega}{c} \left[ \mathbf{E}_k \exp[i(k \cdot \mathbf{r} - \omega t)] + \frac{4\pi}{\omega} \sum_{\alpha} n_a g_{\alpha} \int \mathbf{V} f_{\alpha k} \, d\mathbf{V} \right] \]  \hspace{1cm} (1.29)

\[ \mathbf{E} = \mathbf{E}_k \cdot \mathbf{E}_k, \quad \mathbf{E} = \text{dielectric tensor} \]

and

\[ \int \mathbf{V} f_{\alpha k} \, d\mathbf{V} = \int \mathbf{V} f_{\alpha k} \exp[i(k \cdot \mathbf{r} - \omega t)] \, d\mathbf{V} \]
where $\mathbf{V} = (V_x, V_y, V_z) = (V_{\perp} \cos \phi, V_{\perp} \sin \phi, v_u)$. Averaging over $\phi$, the following relations are used.

$$\frac{1}{2\pi} \int_0^{2\pi} e^{i(n-\lambda)} d\phi = \delta_{n, \lambda}$$

$$\frac{1}{2\pi} \int_0^{2\pi} e^{i(n-\lambda)} \frac{1}{2\lambda} (e^{i\phi} + e^{-i\phi}) d\phi = \frac{1}{2\lambda} (\delta_{n, \lambda-1} + \delta_{n, \lambda+1})$$

$$\frac{1}{2\pi} \int_0^{2\pi} e^{i(n-\lambda)} \frac{1}{2\lambda} (e^{i\phi} - e^{-i\phi}) d\phi = \frac{1}{2\lambda} (\delta_{n, \lambda-1} - \delta_{n, \lambda+1})$$

$$\frac{J_{\lambda-1}(\alpha) + J_{\lambda+1}(\alpha)}{2} = \frac{\lambda}{\alpha} J_\lambda(\alpha)$$

$$\frac{J_{\lambda-1}(\alpha) - J_{\lambda+1}(\alpha)}{2} = J'_\lambda(\alpha)$$

The $\tau$ integration yields

$$\int_{-\infty}^{0} d\tau e^{i(k_n v_u + n\omega_{c}\alpha - \omega)} E_{x,y,z} = \frac{E_{x,y,z}}{\lambda (k_n v_u + n\omega_{c}\alpha - \omega)}$$

where $E_{x,y,z} (\lambda' = -\infty) = 0$. From (1.27) and (1.29), changing the final indice to $n$, one obtains

$$\Xi = \frac{1}{\Xi} - \sum_{\alpha} \frac{\omega_{c}^2}{\omega} \sum_{n=-\infty}^{\infty} \int d\mathbf{v} \frac{5}{(k_n v_u + n\omega_{c}\alpha - \omega)}.$$
\[
\mathcal{S} = \begin{bmatrix}
\nu_\perp \frac{n^2}{a_\alpha} J_n^2 p & \nu_\perp J_n J_n' \frac{n}{a_\alpha} p & \nu_\perp J_n^2 \left(\frac{nQ}{a_\alpha} + \frac{\partial f_{\delta_0}}{\partial V_{\parallel}}\right) \\
-\nu_\perp \frac{n}{a_\alpha} J_n J_n' p & \nu_\perp (J_n')^2 p & -\nu_\perp J_n J_n' \left(\frac{nQ}{a_\alpha} + \frac{\partial f_{\delta_0}}{\partial V_{\parallel}}\right) \\
\nu_\parallel \frac{n}{a_\alpha} J_n^2 p & \nu_\parallel J_n J_n' p & \nu_\parallel J_n^2 \left(\frac{nQ}{a_\alpha} + \frac{\partial f_{\delta_0}}{\partial V_{\parallel}}\right)
\end{bmatrix}
\]

where \( \omega_{p_\alpha}^2 = \frac{4\pi n g_{\omega}^2}{m_\alpha} \)

With the use of the relations

\[
\frac{\nu_\parallel p}{k_{\parallel} + n\omega_{\alpha}} + \frac{\nu_\parallel \frac{\partial f_{\delta_0}}{\partial V_{\parallel}} - \nu_\perp \frac{\partial f_{\delta_0}}{\partial V_{\parallel}}}{\omega} = \frac{\nu_\perp}{k_{\parallel} + n\omega_{\alpha}} \left[ \frac{n}{a_\alpha} Q + \frac{\partial f_{\delta_0}}{\partial V_{\parallel}} \right]
\]
and
\[ \sum_{n=-\infty}^{\infty} J_n^2 = 1 \quad \sum_{n=-\infty}^{\infty} J_n J_n' = 0 \quad \sum_{n=-\infty}^{\infty} n J_n^2 = 0, \]

\[ S_{x\xi,y\xi,\zeta} \] can be rewritten in terms of \( P \). Thus,
\[ \mathcal{E} = \mathcal{I} - \sum_{\alpha} \frac{\omega^2_{p\alpha}}{\omega} \sum_{n=-\infty}^{\infty} \int \frac{P_{\alpha} d\vec{v} \cdot v_{\perp}^{-1}}{R_{\alpha} n_{\|} + n \omega_{c\alpha} - \omega} \]

\[ = \sum_{\alpha} \frac{\omega^2_{p\alpha}}{\omega^2} \left( 1 + \int \frac{v_{\perp}^2}{V_{\perp}} \frac{\partial f_{\alpha 0}}{\partial V_{\perp}} d\vec{v} \right). \quad (1.31) \]

\( L = 0 \) except for \( L_{z\zeta} = \mathcal{I} \).

\[ \mathcal{I} = \begin{bmatrix}
  v_{\perp}^2 \frac{n^2}{a^2} J_n^2 & i v_{\perp}^2 \frac{n}{a^2} J_n J_n' & v_{\perp} V_{\perp} \frac{n}{a^2} J_n^2 \\
  -i v_{\perp}^2 \frac{n}{a^2} J_n J_n' & v_{\perp} (J_n')^2 & -i v_{\perp} V_{\perp} J_n J_n' \\
  v_{\perp} V_{\perp} \frac{n}{a^2} J_n^2 & i v_{\perp} V_{\perp} J_n J_n' & v_{\perp}^2 J_n^2
\end{bmatrix} \]

By looking at the form of (1.31), one can see that \( \mathcal{E} \) has been separated into a part that couples \( E_x, E_y \) and a part that separates out \( E_z \) from \( E_x \) and \( E_y \). \( L_{z\zeta} \) is independent of \( \omega_{c\alpha} \) and describes perturbations parallel to the magnetic field which are not due to cyclotron motion. A more compact form for \( \mathcal{E} \) can be obtained
by the use of the following relations.

We have

\[
\frac{P}{k_n v_n + n \omega_c - \omega} = -\frac{1}{\omega} \frac{\partial f_{d0}}{\partial v_\perp} + \frac{1}{\omega (k_n v_n + n \omega_c - \omega)} \left( n \omega_c \frac{\partial f_{d0}}{\partial v_\perp} + k_n v_\perp \frac{\partial f_{d0}}{\partial v_\parallel} \right)
\]

and

\[
\sum_{n=-\infty}^{\infty} (J_n')^2 = \sum_{n=-\infty}^{\infty} \frac{n^2 J_n^2 (a_n)}{a_n^2} = \frac{1}{2}.
\]

The final form becomes

\[
\varepsilon = (1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega^2}) I
\]

\[
- \sum_{\alpha} \sum_{n=-\infty}^{\infty} \frac{\omega_{p\alpha}^2}{\omega^2} \int_0^I \frac{1}{k_n v_n + n \omega_c - \omega} \left( n \omega_c \frac{\partial f_{d0}}{\partial v_\perp} + k_n v_\perp \frac{\partial f_{d0}}{\partial v_\parallel} \right)
\]

\[\text{(1.32)}\]

If Faraday's Law is combined with Ampere's Law, (1.29), the wave equation results.

\[
\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad \text{Faraday's Law}
\]

or

\[
\mathbf{1} \times \mathbf{E} = \frac{\omega}{c} \mathbf{B}
\]
When written in the form
\[ \nabla \cdot \mathbf{E} = 0, \quad \nabla = \hat{n} \hat{n} - n^2 \mathbf{T} + \mathbf{E}, \] (1.33)
a nontrivial solution exists for \( \mathbf{E} \) when \( \det \nabla = 0 \). The roots of \( \det \nabla \) are the electromagnetic modes in the plasma. In this study \( f_{\alpha \phi} \) is taken to be a Maxwellian with streaming effects and perpendicular and parallel temperatures. The streaming and the temperature anisotropy are the sources of free energy in the plasma, which is available for the growth of plasma waves. The Maxwellian distribution is given by
\[
f_{\alpha \phi} = n_{\alpha} \left( \frac{m_{\alpha}}{2\pi K_{T_{\parallel}}} \right)^{\frac{3}{2}} \left( \frac{m_{\alpha}}{2\pi K_{T_{\perp}}} \right)^{\frac{1}{2}} \exp \left[ -\frac{m_{\alpha}}{2K_{T_{\parallel}}} (V_{\parallel} - U_{0})^2 - \frac{m_{\alpha}}{2K_{T_{\perp}}} V_{\perp}^2 \right], \]
(1.34)

where
- \( m_{\alpha} \) = mass of species
- \( T_{\parallel} \) = parallel temperature
- \( T_{\perp} \) = perpendicular temperature
- \( K \) = Boltzmann's constant
- \( U_{0} \) = streaming velocity

The streaming velocity \( U_{0} \) is put in the \( V_{\parallel} \) direction because the cyclotron motion dominates the \( V_{\perp} \) direction.
When (1.34) is substituted into (1.32) and the integrations over velocity are performed, the following expression for \( \varepsilon \) results.

\[
\varepsilon_{xx} = 1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} \left[ 1 - \frac{1}{\sigma_{\alpha}} + \sum_{n=-\infty}^{\infty} \frac{1}{\sigma_{\alpha}} \frac{n^2 \Lambda_n}{\lambda_{\alpha}} \right]
\]

\[
\varepsilon_{xy} = -\varepsilon_{yx} = -i \sum_{n,\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} \frac{n \Lambda_n}{\sigma_{\alpha}} \Lambda_{\alpha} \lambda \nu \nu \gamma
\]

\[
\varepsilon_{xz} = \varepsilon_{zx} = -\sum_{n,\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} \left[ \frac{n \Lambda_n m_{\alpha}}{k_{\perp} k_{\parallel} k_{T_{\parallel}}} (\nu - n \omega_{c\alpha}) \omega_{c\alpha} \Lambda \gamma \nu \alpha \right. 
\]

\[
\left. - \frac{k_{\perp}}{k_{\parallel}} \frac{\sigma_{\alpha} - 1}{\sigma_{\alpha}} \right]
\]

\[
\varepsilon_{yy} = 1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} \left[ 1 - \frac{1}{\sigma_{\alpha}} + \sum_{n} \frac{1}{\sigma_{\alpha}} \left( n^2 \Lambda_n - 2 \Lambda_{\alpha} \Lambda_n \right) \Lambda \gamma \nu \alpha \nu \gamma \right]
\]

\[
\varepsilon_{yz} = \varepsilon_{zy} = i \sum_{n,\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} \frac{1}{\sigma_{\alpha}} \frac{k_{\perp}}{k_{\parallel}} \left( \nu - n \omega_{c\alpha} \right) \Lambda_n \Lambda \gamma \nu \alpha \nu \gamma
\]

\[
\varepsilon_{zz} = 1 - \sum_{n,\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} \left[ \frac{m_{\alpha}}{k_{T_{\parallel}} k_{\parallel}} \left( \nu - n \omega_{c\alpha} \right)^2 \Lambda_n \Lambda \gamma \nu \alpha \nu \gamma - \frac{m_{\alpha} \omega^2}{k_{T_{\parallel}} k_{\parallel}^2} \right.
\]

\[
\left. - \frac{k_{\perp}^2}{k_{\parallel}^2} \frac{1 - \sigma_{\alpha}}{\sigma_{\alpha}} \right]
\]

\[
\chi = 1 + \frac{\sigma_{\alpha} n \omega_{c\alpha}}{\nu - k_{\parallel} \nu_0 - n \omega_{c\alpha}}, \quad \gamma = \left| -W(F) \right|
\]

(1.35)
\[ W(f) = 1 + \frac{f}{\sqrt{2}} \mathcal{Z}(\frac{f}{\sqrt{2}}), \quad \mathcal{Z}(f) = \left( \frac{m_{\alpha}^2}{K_{T_{\perp}}} \right)^{\frac{1}{2}} \frac{(\omega - k_{\parallel} u_{0} - n\omega_{c\alpha})}{k_{\parallel}} \]

\[ \mathcal{Z}(f) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-\beta^2)}{\beta - \chi} \, d\beta \]

\[ \sigma_{\alpha} = \frac{K_{\parallel}}{T_{\perp} \chi}, \quad \lambda_{\alpha} = \frac{K_{\parallel} k_{\parallel}^2}{m_{\alpha} \omega_{c\alpha}^2}, \quad \Lambda_{\alpha}(\lambda_{\alpha}) = \Gamma_{\alpha}(\lambda_{\alpha}) e^{-\lambda_{\alpha}} \]

If one assumes that \( \nabla \times \vec{E}_1 = -\frac{1}{c} \frac{\partial \vec{B}_1}{\partial t} = 0 \) or \( \vec{k} \times \vec{E}_1 = 0 \) in the previous derivation, the dispersion relation for electrostatic waves is obtained.

\[ \mathcal{E}(\omega, \vec{k}) = 1 + \sum_{\alpha, \gamma} \frac{\omega_{p\alpha}^2}{\omega^2} \int d\vec{r} \frac{J_{\gamma}^{2}(a_{\gamma})}{(\omega - n\omega_{c\alpha} - k_{\parallel} v_{\parallel})} \left[ \frac{n\omega_{c\alpha} x_{\gamma}}{v_{\perp}} \frac{\partial f_{\alpha 0}}{\partial v_{\perp}} + k_{\parallel} \frac{\partial f_{\alpha 0}}{\partial v_{\parallel}} \right] \]

\[ = 0 \]

(1.36)

If (1.34) is substituted into (1.36), the following form results.

\[ \mathcal{E}(\omega, \vec{k}) = 1 + \sum_{\alpha} \frac{K_{D,\perp}^2}{k_{\parallel}^2} \left[ 1 - \sum_{\gamma} \chi \gamma \right], \quad K_{D,\perp}^2 = \frac{4\pi n e^2}{K_{T_{\perp}}} \]

(1.37)

This dispersion relation has the interesting property that it involves a sum over species. Thus one can see the individual contributions to \( \mathcal{E}(\omega, \vec{k}) \). This property is hidden in (1.33), where the determinant involves the multiplication of three terms. It should be noted that (1.33) contains the solution for both electromagnetic and electrostatic waves, while (1.37) is just for electrostatic waves.
Both $\mathbf{E}(\omega, \mathbf{k})$ and $\omega$ are complex quantities. For growing waves, $\text{Im} \omega > 0$, as can be seen from the expression for a plane wave.

$$\mathbf{E}(\mathbf{x}, t) = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{x} - \omega_R t)} e^{i\omega_I t}$$

At a root one has $\mathbf{E}(\omega) = \mathbf{E}_R(\omega) + i \mathbf{E}_I(\omega) = 0$. In linear theory one imposes the following conditions at the root frequency.

$$\omega_I \ll \omega_R, \quad \mathbf{E}_I(\omega) \ll \mathbf{E}_R(\omega)$$

This enables one to expand $\mathbf{E}(\omega)$ in a Taylor series about $\omega_R$ in powers of the small quantity $\omega_I$, and also ensures the waves are growing weakly. Thus at the root,

$$\mathbf{E}(\omega + i \omega_I) = 0 = \mathbf{E}(\omega) + i \omega_I \frac{\partial \mathbf{E}(\omega)}{\partial (i \omega_I)} + \frac{(i \omega_I)^2}{2!} \frac{\partial^2 \mathbf{E}(\omega)}{\partial (i \omega_I)^2} + \ldots$$

$$\omega = \omega_R$$

Both the real and imaginary parts of the Taylor series above must equal zero. Thus,

$$0 = \mathbf{E}_R(\omega_R) + \text{higher order terms} \quad (1.38)$$

$$0 = \mathbf{E}_I(\omega_R) + \omega_I \frac{\partial \mathbf{E}_I(\omega_R)}{\partial \omega_I} + \text{higher order terms} \quad (1.39)$$

Equations (1.38) and (1.39) are solved for the real and imaginary frequencies. Equation (1.39) can be rewritten in the following form.

$$\omega_I = - \frac{\mathbf{E}_I(\omega_R)}{\left( \frac{\partial \mathbf{E}_R}{\partial \omega_R} \right)} \omega_R = \text{root frequency} \quad (1.40)$$
where the Cauchy-Riemann conditions have been used for an analytic function, i.e.

\[
\frac{\partial \varepsilon_I (\omega_R + i\omega_I)}{\partial \omega_I} = \frac{\partial \varepsilon_R (\omega_R + i\omega_I)}{\partial \omega_R}
\]

One sees from (1.40) that \( \frac{\partial \varepsilon_R}{\partial \omega_R} > 0 \), \( \varepsilon_I < 0 \) produces a positive \( \omega_I \). One can also have \( \frac{\partial \varepsilon_R}{\partial \omega_R} < 0 \) and \( \varepsilon_I > 0 \) to produce a positive \( \omega_I \). The analysis above will give similar results for \( \det \Delta = 0 \), the dispersion relation for electromagnetic waves.
Quasilinear Theory

Quasilinear theory is so named as it is not a completely non-linear theory but uses the results of linear theory as derived previously. In linear theory an unstable distribution will produce growing waves, the growth rates of which are predicted by the theory. As waves begin to grow they may interact with the particles in the distribution causing them to acquire different energies and pitch angles. Linear theory does not take into account the possibility of changes in \( f(\mathbf{v}) \), but quasilinear theory does. To summarize, we may distinguish between two different kinds of wave-particle interactions. One is the wave-particle interaction of linear theory, where an unstable distribution of particles produces growing waves, and the other is that of quasilinear theory, where \( f(\mathbf{v}) \) changes as a result of the growing waves. The details of the latter interaction are the subject here.

In quasilinear theory the quantities \( f_{\alpha 0}, f_{\alpha 1}, \mathbf{E}_1, \mathbf{B}_1 \), and \( \mathbf{B}_0 \) will have the same meaning as in the linear theory. In the following derivation the space average is used.

\[
\langle \mathbf{f} \rangle = \frac{1}{V} \int \mathbf{f} d^2 \mathbf{x}.
\]

The space average of \( f_{\alpha} = f_{\alpha 0} + f_{\alpha 1} \) is

\[
\frac{1}{V} \int f_{\alpha} (\mathbf{x}, \mathbf{v}, t) d^2 \mathbf{x} = \frac{1}{V} \int [f_{\alpha 0} + f_{\alpha 1}] d^2 \mathbf{x} = \frac{1}{V} \int f_{\alpha 0} d^2 \mathbf{x} = f_{\alpha 0}
\]

since \( f_{\alpha 0} \) is the equilibrium distribution function, homogeneous throughout infinite space. We have the additional definitions

\[
\langle f_{\alpha 1} \rangle = \langle \mathbf{E}_1 \rangle = \mathbf{B}_1 = 0.
\]
One will consider only electrostatic waves. Upon taking the space average of the Vlasov equation for $f_\alpha$, one obtains

$$
\frac{\partial}{\partial t} \langle f_\alpha \rangle + \langle \nabla \cdot \mathbf{V} f_\alpha \rangle + \frac{q_\alpha}{m_\alpha} \langle (\mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c}), \nabla \cdot \mathbf{V} f_\alpha \rangle = 0.
$$

Looking at each term separately, one has

**first term**

$$
\frac{\partial}{\partial t} \langle f_\alpha \rangle = \frac{\partial}{\partial t} f_{\alpha 0}
$$

**second term**

$$
\langle \nabla \cdot \mathbf{V} f_\alpha \rangle = \mathbf{V} \cdot \langle \nabla f_\alpha \rangle = \mathbf{V} \cdot \langle \nabla (f_{\alpha 0} + f_{\alpha 1}) \rangle
$$

$$
\nabla f_{\alpha 0} = 0
$$

$$
\langle \nabla f_{\alpha 1} \rangle = \lim_{V \to \infty} \frac{1}{V} \int_{\mathbf{V}} \nabla f_{\alpha 1} \, d^2x = \lim_{s \to \infty} \int_s \hat{n} f_{\alpha 1} \, ds = 0
$$

since $f_{\alpha 1} \to 0$, $|\mathbf{x}| \to \infty$.

**Third term**

$$
\langle \mathbf{E}_1 \cdot \nabla \cdot (f_{\alpha 0} + f_{\alpha 1}) \rangle = \langle \mathbf{E}_1 \rangle \cdot \nabla \cdot f_{\alpha 0} + \langle \mathbf{E}_1 \cdot \nabla \cdot f_{\alpha 1} \rangle
$$

$$
= \langle \mathbf{E}_1 \cdot \nabla \cdot f_{\alpha 1} \rangle = \nabla \cdot \langle \mathbf{E}_1 f_{\alpha 1} \rangle \quad \text{since} \quad \nabla \cdot \mathbf{E}_1 = 0
$$

$$
\langle \frac{\mathbf{V} \times \mathbf{B}_0}{c}, \nabla \cdot (f_{\alpha 0} + f_{\alpha 1}) \rangle = \frac{\mathbf{V} \times \mathbf{B}_0}{c} \cdot \nabla \cdot f_{\alpha 0} + \frac{\mathbf{V} \times \mathbf{B}_0}{c} \cdot \nabla \cdot f_{\alpha 1} \rangle
$$

$$
= \frac{\mathbf{V} \times \mathbf{B}_0}{c} \cdot \nabla \cdot f_{\alpha 1} \rangle
$$
Thus, one is left with

\[
\frac{\partial f_{\alpha \omega}}{\partial t} + \frac{q\omega}{m\omega} \frac{\nabla \times \mathbf{B}_e}{c} \cdot \nabla f_{\alpha \omega} =
\]

\[
= - \frac{q\omega}{m\omega} \nabla \cdot \left\langle \frac{\mathbf{E}_1}{c} f_{\omega \omega} \right\rangle.
\]

(1.41)

This is the quasilinear equation for electrostatic waves in a plasma immersed in a uniform external magnetic field. The equation above can be written in terms of Fourier transforms for \( \mathbf{E}_1(x_1, t) \) and \( f_{\omega \omega}(x_1, v_1, t) \).

\[
\mathbf{E}_1(x_1, t) = \int \frac{d^3k}{(2\pi)^3} E_{\omega \omega}(k) e^{i\mathbf{k} \cdot \mathbf{x}}
\]

\[
f_{\omega \omega}(x_1, v_1, t) = \int \frac{d^3k}{(2\pi)^3} f_{\omega \omega}(k, v_1) e^{i\mathbf{k} \cdot \mathbf{x}}
\]

Thus,

\[
- \frac{q\omega}{m\omega} \nabla \cdot \left\langle \frac{\mathbf{E}_1}{c} f_{\omega \omega} \right\rangle = - \frac{q\omega}{m\omega} \nabla \cdot \frac{1}{V} \int d\mathbf{x} \int d^3\mathbf{k} \int \frac{d^3k}{(2\pi)^3} \mathbf{E}_{\omega \omega} f_{\omega \omega}.
\]

\[
\cdot e^{i\mathbf{k} \cdot (\mathbf{q} + \mathbf{k})} = - \frac{q\omega}{m\omega} \nabla \cdot \frac{1}{V} \int d\mathbf{x} \int d^3\mathbf{k} \int \frac{d^3k}{(2\pi)^3} \mathbf{E}_{\omega \omega} f_{\omega \omega} \delta(\mathbf{q} + \mathbf{k})
\]

and (1.41) becomes

\[
\frac{\partial f_{\omega \omega}}{\partial t} + \frac{q\omega}{m\omega} \frac{\nabla \times \mathbf{B}_e}{c} \cdot \nabla f_{\omega \omega} = - \frac{q\omega}{m\omega} \nabla \cdot \frac{1}{V} \int d^3k \mathbf{E}_{\omega \omega} f_{\omega \omega}.
\]

(1.42)
\( f_{\alpha k} \) is the perturbation on \( f_{\alpha 0} \) obtained from linear theory, equation (1.25).

\[
\dot{f}_{\alpha k} = - \frac{q_{\alpha}}{m_{\alpha}} \int_{-\infty}^{0} d\tau \mathbf{E}_{k} \cdot \mathbf{\nabla} f_{\alpha 0}(\mathbf{x}, \tau, V_{\alpha}, t) e^{i(k \cdot \mathbf{x} - \omega \tau)}
\]

(1.43)

where \( \mathbf{B}_{k} = 0 \) since this treatment is for electrostatic waves only.

After substituting (1.43) into (1.42), the same mathematical manipulations will be used on (1.42) as were used in the derivation of the linear dispersion relation (1.33).

The result is

\[
\frac{\partial f_{\alpha 0}}{\partial t} + \frac{q_{\alpha}}{m_{\alpha}} \frac{\mathbf{v} \times \mathbf{B}_{0}}{c} \cdot \mathbf{\nabla} f_{\alpha 0} =
\]

\[
= \frac{q_{\alpha}^{2}}{m_{\alpha}^{2}} \sum_{n, k} \frac{1}{\nu_{L}} \int \frac{dk}{(2\pi)^{3}} \partial_{\mathbf{op}} \frac{\mathbf{E}_{k} \cdot \mathbf{\Phi}_{k}}{\nu_{R}} \frac{\mathbf{J}_{n} \cdot \mathbf{J}_{k}}{\mathbf{\omega}_{\mathbf{cd}} - \omega} \partial_{\mathbf{op}} f_{\alpha 0}(\mathbf{V}, t)
\]

(1.44)

where

\[
\partial_{\mathbf{op}} = \frac{n\nu_{c, \alpha}}{\nu_{L}} \frac{\partial}{\partial \nu_{L}} + k_{n} \frac{\partial}{\partial \nu_{n}} \quad \mathbf{J}_{n} = \mathbf{J}_{n}(\nu_{L} \frac{k_{n} \nu_{c, \alpha}}{\mathbf{\omega}_{\mathbf{cd}}})
\]

\[
\mathbf{E}(\mathbf{x}, t) = -\mathbf{\nabla} \mathbf{\Phi}(\mathbf{x}, t) = -\mathbf{\nabla} \int \frac{dV}{(2\pi)^{3}} \mathbf{\Phi}_{k}(t) e^{i(k \cdot \mathbf{x})} \Rightarrow \mathbf{E}_{k} = -i k \mathbf{\Phi}_{k}
\]

Because \( f_{\alpha 0} \) is independent of \( \phi \), equation (1.44) is averaged over and the identity

\[
\frac{1}{2\pi} \int_{0}^{2\pi} d\phi e^{i(n-\ell)\phi} = \delta_{n, \ell}
\]

is used. The second term on the LHS of (1.44) vanishes.
Thus,

\[ \frac{\partial f_{ao}}{\partial t} = \frac{2a}{m_{\alpha}} \sum_n \frac{1}{V} \int \frac{d^3 k}{(2\pi)^3} \rho \left( \frac{1}{J_n^2(k_\|, n\omega_{\alpha} - \omega)} \right) \delta_{op} f_{ao} \]

Because \( \omega = \omega_R + i\omega_I \) where \( \omega_I \ll \omega_R \), one can use the result from complex analysis,

\[ \frac{1}{k_\| n\omega_{\alpha} - \omega} = P \frac{1}{k_\| n\omega_{\alpha} - \omega} + i\pi \delta\left(k_\| n\omega_{\alpha} - \omega\right) \]

where \( P \) = Principal Value.

Also, the energy density in the electrostatic field is defined as

\[ \langle \mathcal{W}_E \rangle = \left\langle \frac{E^2}{8\pi} \right\rangle = \frac{1}{V} \int \frac{E_k^* \cdot E_{-k}}{8\pi (2\pi)^3} d^3 k \]

\[ = \int E_k^* d^3 k \]

\( \langle \mathcal{W}_E \rangle \) (ergs/cm\(^3\) in mode \( k \))

Note that \( k^2 \vec{\Phi}_k \cdot \vec{\Phi}_{-k} = \vec{E}_k^* \cdot \vec{E}_{-k} \).

Thus,

\[ \frac{\partial f_{ao}}{\partial t} = \frac{8\pi q^2_a}{m_{\alpha}} \sum_{n=-\infty}^{\infty} \int d^3 k \delta_{op} \left( \frac{1}{J_n^2(k_\|, n\omega_{\alpha} - \omega)} \right) \]

\[ + \pi \delta\left(k_\| n\omega_{\alpha} - \omega\right) \]

\[ \frac{\partial f_{ao}}{\partial t} = \inf \frac{E_k^2}{k^2} \delta_{op} f_{ao} \]

(1.45)
Because the electric field \( \vec{E}(\vec{x},t) \) is a real quantity, the following relations are obtained.

\[
\vec{E}(\vec{x},t) = \frac{1}{(2\pi)^3} \int \vec{E}_k \, e^{i(\vec{k} \cdot \vec{x} - \omega_k t)} \, dk
\]

\[
\vec{E}^*(\vec{x},t) = \frac{1}{(2\pi)^3} \int \vec{E}_k^* \, e^{-i(\vec{k} \cdot \vec{x} - \omega_k^* t)} \, dk
\]

Since \( \vec{E} = \vec{E}^* \), it follows that

\[
\vec{E}_{-\vec{k}} = \vec{E}_\vec{k}^*
\] (1.46a)

\[
(\omega_k^*) = -\omega_k \Rightarrow \omega_R(-\vec{k}) = -\omega_R(\vec{k}), \omega_I(-\vec{k}) = \omega_I(\vec{k})
\]

One also sees that

\[
\frac{\partial}{\partial t} \, \vec{E}_k(x) = 2 \omega_I \, \vec{E}_k(x)
\] (1.46b)

With the use of (1.46a),(1.45) can be put in the final form as

\[
\frac{\partial f_{\alpha\rho}}{\partial x^\alpha} = \vec{\nabla}_\rho \cdot \vec{D} \cdot \vec{\nabla}_\rho \, f_{\alpha\rho} = \left[ \frac{1}{v_L} \frac{2}{\partial v_L} \, v_L \, D_{\perp \perp} \frac{2}{\partial v_L} + \frac{1}{v_L} \frac{2}{\partial v_L} \, v_L \, D_{\parallel \parallel} \frac{2}{\partial v_L} 
\right. \\
\left. + \frac{2}{\partial v_L} \, D_{\perp \parallel} \frac{2}{\partial v_L} + \frac{2}{\partial v_L} \, D_{\parallel \perp} \frac{2}{\partial v_L} \right] \, f_{\alpha\rho}
\]

\[
= \left( \frac{1}{v_L} \frac{2}{\partial v_L} \, v_L \right) \left( \begin{array}{cc} D_{\perp \perp} & D_{\perp \parallel} \\ D_{\parallel \perp} & D_{\parallel \parallel} \end{array} \right) \left( \frac{2}{\partial v_L} \right) \, f_{\alpha\rho}
\]

\[
\vec{D} = \vec{D} \quad \text{(resonant)} \quad \text{and} \quad \vec{D} \quad \text{(nonresonant)}
\]
Equation (1.47) was obtained by assuming $\omega_\perp \ll \omega_R$. This allowed the integrand to be split into two parts. One involves a principal value integral, the nonresonant part, and the other involves the pole where $k_\parallel v_\parallel + n\omega_{c\alpha} - \omega = 0$, the resonant part. The nonresonant part involves particles that are not in resonance with the wave, and thus $k_\parallel v_\parallel + n\omega_{c\alpha} - \omega \neq 0$. Since $\omega = \omega_R + i\omega_\perp$, there is a finite $\omega_\perp > 0$ for nonresonant particles that ensures that there is a growing wave in the plasma which will exist for a time period long enough to allow nonresonant particles to diffuse. On the other hand, the resonant term is valid in the limit $\omega_\perp \to 0$. Thus $\omega$ inside the delta function is set equal to $\omega_R$. In practice, $\omega_\perp > 1$ and the separation into resonant and nonresonant parts is valid as long as $\omega_\perp \ll \omega_R$. 

$$D = \frac{8\pi q^2}{m^2_{\alpha}} \int d^3k \sum_{n=\infty}^{\infty} J_n^2 \left( \frac{k_\parallel v_\parallel}{\omega_R} \right) \cdot \left[ \pi \delta \left( k_\parallel v_\parallel + n\omega_{c\alpha} - \omega \right) + P \frac{\omega_\perp}{(k_\parallel v_\parallel + n\omega_{c\alpha} - \omega_R)^2 + \omega_\perp^2} \right].$$

$$\begin{pmatrix} n\omega_{c\alpha}^2 & k_\parallel n\omega_{c\alpha} \\ k_\parallel n\omega_{c\alpha} & k_\parallel^2 \end{pmatrix} \begin{pmatrix} \left( \frac{n\omega_{c\alpha}}{v_\perp} \right)^2 \\ k_\parallel^2 \frac{n\omega_{c\alpha}}{v_\perp} \end{pmatrix}$$

(1.47)
CHAPTER II

OBSERVATIONS

Macroscopic Quantities

The measurements made by instruments on the satellite give one information on various parameters that characterize the plasma around the satellite. In this study, one makes use of particle measurements obtained by ion and electron detectors. With these measurements, it is easy to obtain distribution functions, \( f_\alpha(\vec{v}) \), for a species \( \alpha \). Once \( f_\alpha(\vec{v}) \) is known, then macroscopic quantities can be found by taking moments of the distribution. The various moments of the distribution which are of interest in this study are given below.

density: \( n_\alpha = \int f_\alpha(\vec{v}) \, d\vec{v} \), (cm\(^{-3}\)) \hspace{1em} (2.1)

drift velocity: \( u_{\alpha d} = \frac{1}{n_\alpha} \int v_{\parallel} f_\alpha(\vec{v}) \, d\vec{v} \), (cm s\(^{-1}\)) \hspace{1em} (2.2)

parallel temperature:
\[
K T_{\parallel,\alpha} = \frac{m_\alpha}{2} \int (v_{\parallel} - u_\alpha)^2 f_\alpha(\vec{v}) \, d\vec{v} \hspace{1em} (eV) \hspace{1em} (2.3)
\]

perpendicular temperature:
\[
K T_{\perp,\alpha} = \frac{m_\alpha}{2} \int v_{\perp}^2 f_\alpha(\vec{v}) \, d\vec{v} \hspace{1em} (eV) \hspace{1em} (2.4)
\]

Other macroscopic quantities which are of interest but which are not calculated here are the following:

field aligned current density: \( j_{\parallel,\alpha} = n_\alpha q_\alpha u_{\alpha d} \), (amp m\(^{-2}\)) \hspace{1em} (2.5)
parallel momentum flux:
\[ \lambda_{\parallel\alpha} = \int |m_{\alpha} v_{\parallel}| v_{\parallel} f_{\alpha}(\vec{v}) \, d\vec{v} \quad \text{(g cm}^{-1} \text{s}^{-1}) \] (2.6)

parallel energy flux:
\[ \Theta_{\parallel\alpha} = \int \left( \frac{1}{2} m_{\alpha} \vec{v}^2 \right) v_{\parallel} f_{\alpha}(\vec{v}) \, d\vec{v} \quad \text{(erg cm}^{-2} \text{s}^{-1}) \] (2.7)

parallel heat flux:
\[ q_{\parallel\alpha} = \frac{m_{\alpha}}{2} \int (v_{\parallel} - u_{\parallel\alpha}) |\vec{v} - \vec{u}_{\parallel\alpha}| f_{\alpha}(\vec{v}) \, d\vec{v} \quad \text{(erg cm}^{-2} \text{s}^{-1}) \] (2.8)

In the following analysis, an \( f_{\alpha}(\vec{v}) \) is obtained by fitting the measured distribution with Maxwellians. This is done in order to perform the stability analysis, which uses Maxwellian distributions. With the fitted distributions, the density, drift velocity, and temperature are explicit in \( f_{\alpha}(\vec{v}) \), and these moments do not have to be taken. Also, one is interested primarily in the macroscopic properties of the beams, which are part of the total distribution. When taking the moment of the total distribution, the beam properties cannot be separated out. If a number of Maxwellians are fitted to the distribution, one of which is the beam Maxwellian, then the beam Maxwellian contains all of the beam properties.

There are discrepancies in fitting the data with Maxwellians. As it turns out, the electron distribution fits extremely well with a Maxwellian. The electrons have attained an equilibrium at the satellite, except for the non-Maxwellian features of a loss cone and a small hole in the downgoing particles. The loss cone and the small hole are not characteristic of the entire distribution. Thus, one would expect a species in thermal equilibrium to be a Maxwellian distribution.

The ions are a different case. They have been accelerated through a potential drop to DE-1 altitudes to form beams. As one looks at the distribution functions in the next section, one sees a distribution that
resembles a Maxwellian, especially in hydrogen, but has non-Maxwellian features. Such features could be a steepening on one side of the peak and a flattening on the other side. Such an asymmetric distribution could be fit by two Maxwellsians. This would provide a better fit, but would complicate the stability analysis by giving two streaming components when one beam is seen in either H$^+$ or O$^+$. When these fits are put into the stability analysis, one would get relative streaming interactions between the two components describing a single beam. One fits a single Maxwellian to the beam distribution in order to get a single streaming velocity and a characteristic perpendicular and parallel temperature.

**Particle Measurements**

This study is based on wave and particle observations of the space environment at 3.3 $R_e$, the altitude at which the Dynamics Explorer I satellite passed through the auroral zone. DE-1 contained an ion mass spectrometer, enabling one to obtain measurements of both the energy and composition of the auroral plasma. The Energetic Ion Composition Spectrometer (EICS) (Shelley et al., 1981) made measurements of H$^+$ and O$^+$ at 24 pitch angles and 15 energies in the range 10 eV to 17 keV. The instrumental full widths at half maximum were 160 to 200 eV, enabling one to resolve beams peaked at 1 keV or higher.

In the sampling mode, the detector would sample H$^+$ and O$^+$ on different six second spin periods, sampling the lower eight energies first, and finally, the higher seven energies. Thus, completed distributions of H$^+$ and O$^+$ were obtained during a 24 second time period, or four spin periods. The center energies of the 15 energy channels are, in eV, 62
In Figure 7, data from the EICS is shown for October 4, 1981 during a satellite crossing of the auroral zone in the midnight sector with magnetic local time 21 hours 57 minutes and invariant latitude 68°. Universal time is on the horizontal axis, and energy in keV is on the vertical axis. The count rate is approximately ten counts per sample. Data for hydrogen is in the top panel. The middle panel is oxygen. The lower panel does not discriminate ion species, and the pitch angle of the instrument is on the bottom.

The satellite passes through the auroral acceleration region at 1404 UT where one can see a transition from a field aligned isotropic flux to field aligned beams with an energy of approximately 4 keV. It is interesting to note that there is no background present in the oxygen, and that any background present in hydrogen outside the auroral zone is reduced considerably when beams are seen. It is possible that a cold H\(^+\) background exists with an energy in the range of 10-30 eV. The detector is unable to make a reliable measurement of a cold component with just its lowest energy channel.

With the energy and pitch angle information, one can make distribution function plots of each ion species in velocity space. The coordinate \(V_H = V \cos \theta\) is the velocity of the particle along the magnetic field, and \(V_L = V \sin \theta\) is the velocity of the particle perpendicular to the magnetic field. The detector measures the particle energy flux, \(j (\# \text{ particles} \cdot \text{cm}^{-2} \cdot \text{sec}^{-1} \cdot \text{sr}^{-1} \cdot \text{eV}^{-1})\). This is converted to \(f\) (# particles \cdot \text{sec}^{-3} \cdot \text{km}^{-6}) through the relation \(j = 2Ef/m^2\), where \(E\) is the particle's energy, and \(m\) is its mass. Distribution functions were extracted in this way for the time period during which the beams were
present in Figure 7. These are shown in Figures 8-11. Figures 8-11a are hydrogen distributions. Figures 8-11b are oxygen distributions.

In Figures 8-11c, the combined distribution function is shown where \( f_c = f_{H^+} + \frac{m_H}{m_O} f_{O^+} \). The mass ratio factor is inserted because in the stability analysis we evaluate \( \varepsilon(\omega_i k^2) \), which involves a multiplication by \( \omega_p^2 \) as each species is summed. Thus, the combined distribution function is an effective \( H^+ \) distribution, where the \( O^+ \) counts are interpreted as if they were \( H^+ \). In the combined distribution, one can see the \( H^+-O^+ \) interaction as the filling in of the valley between the two peaks in parallel velocity. The species with the higher streaming velocity, \( H^+ \), has transferred energy to the species with the lower streaming velocity, \( O^+ \).

In Figure 12, the 1/2 count distribution is shown. All plots contain a 1/2 count level at points where the detector measured \( O^+ \) counts.

The ion distribution functions reveal some interesting features over this time period. All distributions have evolved from what they would look like in the source region of the ionosphere, where \( T \leq 1 \text{eV} \). All \( H^+ \) distribution functions exhibit a steepening on the high velocity side of the beam, which is a feature of the cold source region along with any adiabatic cooling that may occur as the ions accelerate through a diverging magnetic field. The \( H^+ \) also reveals perpendicular heating as well as parallel heating on the low energy side of the peak. The \( O^+ \) possesses a high energy ion tail which is more pronounced than the \( H^+ \). This tail extends in the perpendicular as well as in the parallel direction, as can be seen in Figures 8-11b. This suggests that the \( H^+ \) has transferred energy to the \( O^+ \) by wave-particle interactions. In this energy transfer, \( O^+ \) ions would scatter to higher velocities, and \( H^+ \)
ions would scatter to lower velocities. The separate H\(^+\) and O\(^+\) distribution functions show the features that support this conclusion.

A Maxwellian fit to the separate ion distributions is used to perform a linear stability analysis to see which waves are made unstable by the observed distribution. Three Maxwellians fitted the data, including one beam, and two background. The two background Maxwellians fitted only the 1/2 count level in the case for oxygen. In the case for hydrogen, the two background Maxwellians fitted the 1/2 count plus hot background. Only the beam Maxwellians were used in the stability analysis. This worked well with oxygen, since no background counts were detected. The hot hydrogen background is present, but its temperature and density are uncertain. Many channels detected 0 counts, making a subtraction of the beam plus 1/2 count from the total ion density unreliable, since the result would have counts in every channel. It is concluded that the hot hydrogen background density is small, less than the beam density, and is characterized by a temperature on the order of 1-5 keV. Thus, it would contribute little to the damping of any growing plasma waves.

The fitted parameters of the beam Maxwellians are given in Table 1. The hydrogen density is constant at .05 cm\(^{-3}\), while the oxygen fluctuates during the beam observations. The hydrogen streaming velocity is always higher than the oxygen streaming velocity as a result of the difference in mass between the two species. This is to be contrasted with the streaming energy of oxygen, which is approximately twice that of hydrogen. The non-Maxwellian properties of the beams will introduce some errors in these results. This is explored below. The perpendicular to parallel temperature ratio of hydrogen is consis-
tently \( < 1 \), while \( \frac{\tau_n}{\tau_L} \) is \( \geq 1 \) for oxygen except for the fourth fit. Thus, one sees that there is more perpendicular heating of oxygen than hydrogen.

Because the beams are steep on one side in velocity space and stretched out on the other, the single Maxwellian fit will peak at a different parallel velocity than in the data. For hydrogen, which is stretched out on the low velocity side, the fitted peak will be less than the peak in the data. For oxygen, which is stretched out on the high velocity side, the fitted peak will be greater than the peak in the data. Thus, the streaming energies obtained from the fit in Table 1 are a little misleading.

A better determination of the streaming energy can be made by examining the contour plots in figures 8-11. \( \text{H}^+ \) and \( \text{O}^+ \) are plotted in velocity space, but on the same energy scale. Thus, a comparison of the peak energy of the \( \text{H}^+ \) and \( \text{O}^+ \) beams for a given time period can be obtained by visually examining the position of the peak on the \( v \)-parallel axis. For the first and fourth time periods, figures 8 and 11 show the \( \text{H}^+ \) and \( \text{O}^+ \) beams peaking at the same energy. For the second and third time periods, it is very apparent that oxygen peaks at a higher energy than hydrogen. One suggestion as to why this occurs could be that wave-particle interactions at lower altitudes enabled the transfer of streaming energy from hydrogen to oxygen.

Another possibility is that oxygen was preferentially heated perpendicular to the magnetic field either at or above the source region. As the oxygen falls through the potential drop to form a beam, conservation of \( \mathcal{U} = 1/2(mv_{\perp})^2/B \) would decrease the perpendicular energy of the heated particles enough to make them appear almost field aligned.
Conservation of energy requires $v_{\parallel}$ to increase in this process. As $B$ decreases from .5 gauss to .014 gauss at DE-1 altitudes, this corresponds to a decrease in perpendicular energy by a factor of 36. Simultaneous measurements of the $O^+$ and $H^+$ distributions at low altitude would confirm whether this is a correct conclusion.

Two parameters that will effect the stability of plasma waves are the density and temperature of the electrons. The electron data was obtained from the Southwest Research Institute's High Altitude Plasma Instrument (HAPI) detector (Burch et al., 1981). A complete distribution is obtained during one six second spin period. The detector contained 28 energy bins. Each energy bin accepted ions in the energy range $\Delta E$, with $\Delta E/E = 32\%$, $E$ being the center energy of the bin. The center energies were, in eV, 6, 8, 10, 13, 18, 23, 31, 42, 56, 74, 99, 132, 176, 235, 313, 417, 554, 739, 984, 1319, 1753, 2339, 3121, 4187, 5569, 7425, 9900, and 13206.

The distribution functions corresponding to the ion distribution in figure 9, $T = 14:04:35 - 14:04:59$, are shown in figures 13-17. Positive $v_{\parallel}$ points down into the ionosphere. An artificial 1/2 count is inserted where the detector measured 0 counts. A hole is seen in the downgoing electrons at $v_{\parallel} = 10 \times 10^8$ cm/sec. This is most likely due to potential structures that exist above the satellite, and suggests that a potential parallel to the magnetic field exists at the satellite. The electron precipitation associated with the ion beams is not seen at the satellite because the satellite is above the potential drop which accelerated the ion beams out of the ionosphere. At lower altitudes electrons pass through the potential drop to produce aurora.

In the upgoing electrons there is a widened loss cone which appears
at small pitch angles. The most distinctive feature is the transition from a warm electron component to a very steep, cold component at small $|\mathbf{v}|$. This cold component is produced by photoelectrons when sunlight impinges on the satellite and is not characteristic of the surrounding plasma.

The warm distribution is characterized very well by an isotropic Maxwellian with $T_e = 500$ eV and $n = 0.2 \text{ cm}^{-3}$. This can be seen in Table 2, where the Maxwellian fit parameters are given for five consecutive time periods roughly corresponding to the ion distribution $T = 14:04:35 - 14:04:59$. The fit also produced a low density $4.4 \text{ keV}$ component. The average temperature of the five fits was used in the stability analysis. Both the warm and hot components were used. The electron density was set equal to the total ion beam density in order to preserve charge neutrality. This value came to approximately $0.1 \text{ cm}^{-1}$, which is within the factor of 2 uncertainty in the measurement of very low densities (Burch, 1985). Any cold hydrogen plasma that might be present was not included. A reliable estimate of this value cannot be extracted from the measurements. One cannot measure a cold background with this detector since all counts would be in the lowest energy channel. Using a technique that involves the upper cutoff of whistler mode auroral hiss (Persoon, et al., 1983), the satellite measured a background density that dropped from $4 \text{ cm}^{-3}$ to $0.2 \text{ cm}^{-3}$ as the satellite entered the auroral zone.

Because of the uncertainties involved with this measurement at very low densities, one concludes that it is possible that a cold component with a density equal to the beam density actually exists. Also, it is equally possible that no cold hydrogen is present, since if it were present in the lower acceleration region during time periods of upgoing ion
beams, it would be accelerated upward to become part of the beam.

**Wave Observations**

Wave observations during this time period were made by the University of Iowa Plasma Wave Instrument (Shawhan et al., 1981). Unfortunately, the measurements are not of high quality in the frequency range of interest. In figure 18, the differential electric field intensity is plotted as a function of frequency. In the low frequency range, which is of interest here, the measurements are sparse with few data points. Typical electric field amplitudes at frequencies near 5 Hz are .5 mV/m. This is a low fluctuation level and is to be expected, as the beams have achieved a state of marginal stability.
Figure 7

Energy (keV)

All ions

$O_+ \quad H_+$
Figure 8
Figure 9
Figure 12
Figure 13
Figure 14
Figure 15
Figure 16
Table 1

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Table 2

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</table>

Maxwellian Fits: DE-1 October 4, 1981

Definitions: N = density (cm^{-3}), u = streaming velocity (10^8 cm/s), W = streaming energy (keV), T_n = parallel temperature (eV), T_l = perpendicular temperature (eV)
Figure 18
CHAPTER III

STABILITY OF THE OBSERVED PLASMA

Electromagnetic Waves

Originally this study was designed to investigate the stability of electromagnetic waves in the observed plasma at 3.3 R_e geocentric. If an electromagnetic wave with a phase velocity between the H^+ and O^+ beam velocities is found to be unstable, it could be a mechanism which mediates the energy transfer from the H^+ to the O^+, as is revealed in the observed distribution functions. Low frequency electromagnetic waves are observed on S3-3 and a stability analysis shows that downward streaming electrons are responsible for wave growth (Temerin and Lysak, 1984). The plasma in question here is stable to low frequency electromagnetic waves simply because the source of free energy needed to drive the waves is not available. To compare with a study of hot ion beam instabilities (Smith, 1985), beam velocities on the order of 10 V_A were used, where V_A is the Alfvén speed or the velocity of harmonic vibrations of resonating magnetic field lines.

\[ V_A = c \left[ \frac{\omega_{PH}^2}{\omega_{CH}^2} + \frac{\omega_{PD}^2}{\omega_{CO}^2} \right]^{1/2} \text{ (cm/sec)} \]  

(3.1)

The beam velocities observed on DE-1 are .01-.07 V_A, which are too small to drive Alfvén waves, much less electromagnetic waves at higher frequencies.

With these considerations it was still informative to look at the dispersion of low frequency electromagnetic modes in a two-ion component plasma and to test program DISP3, which numerically solves the
electromagnetic dispersion relation as derived in Chapter I. A description of program DISP3 is given in Appendix B. Also, electromagnetic waves undergo polarization changes in a two-ion component plasma, and it was worthwhile to investigate possible wave growth due to this effect.

Low frequency electromagnetic waves in a two-ion component plasma have been studied by Smith and Brice, 1964; Young et al., 1981; and Andre, 1985. The plasma that was input to DISP3 contained the two fitted Maxwellians to the separate H\(^+\) and O\(^+\) beams in the distribution \(T = 14:05:23 - 14:05:59\) plus an equal number of electrons with a temperature of 100 eV. In figure 20 an \(\omega\) vs. \(k\) plot of the various right and left polarized electromagnetic waves that exist in this plasma is shown. The electric field of right polarized waves rotates in the same sense as electrons, which is counterclockwise when the magnetic field is out of the page. Left polarized waves rotate in the same sense as ions, i.e. counterclockwise. The situation is illustrated below.

![Figure 19](image)
In figure 20 the wave normal angle $\Theta$ is $0^\circ$ so that the waves depicted propagate parallel to $B_0$. For finite $k_\perp$, and $k_\perp \ll k_h$, the dispersion does not change much and figure 20 can be used to identify the modes. At frequencies below the oxygen cyclotron frequency there are two modes, an R and an L-mode. For $\omega \ll \omega_{ch}, \omega_{co}$, the dispersion relations of the two modes are

$$\omega^2 = \frac{k^2 c^2}{1 + \delta^2} \quad R, \quad \omega \ll \omega_{ch}, \omega_{co} \quad (3.2a)$$

$$\omega^2 = \frac{k_\parallel^2 c^2 \cos^2 \Theta}{1 + \delta} \quad L, \quad \omega \ll \omega_{ch}, \omega_{co} \quad (3.2b)$$

$$\delta = 4\pi (n_h m_h + n_o m_o) c^2 / \beta_0^2$$

The right polarized mode is called the fast compressional mode or magnetosonic mode. The left polarized mode is called the slow Alfvén mode. Since the fast mode is right polarized, it is unaffected by the resonance at $\omega_{co}$, and passes right through it. The Alfvén mode is left polarized and is affected by the resonance at $\omega_{co}$. For $\omega$ near $\omega_{co}$ this mode is called the electromagnetic ion cyclotron mode.

At higher frequencies the R-mode meets another branch of the L-mode near the hydrogen cyclotron frequency. This second L-mode is a feature of a two-ion component plasma. At $\Theta = 0^\circ$ the R and L-modes appear to cross each other, when actually what happens is a polarization reversal at the crossover frequency, $\omega_{cr}$. Thus, the R-mode becomes the L-mode and the L becomes the R. The L-mode has a resonance at $\omega_{ch}$ and a cutoff at $\omega_{cr}$. One has a resonance or cutoff when the index of refraction, $n = kc / \omega$, goes to infinity or zero,
respectively. The cutoff frequency, $\omega_c$, and the crossover frequency, $\omega_{cr}$, are given by the following cold plasma relations (Andre, 1985).

$$\omega_c = \eta_H \omega_{co} + \eta_0 \omega_{CH} = 91.7 \text{ s}^{-1} \quad (3.3)$$

$$\omega_{cr} = \left[ \eta_H \omega_{co}^2 + \eta_0 \omega_{CH}^2 \right]^{1/2} = 117.4 \text{ s}^{-1} \quad (3.4)$$

$\eta_H, \eta_0$ - relative abundances of hydrogen and oxygen

These frequencies will be slightly shifted because of the hot ion and electron components used here.

Above $\omega_{CH}$ in figure 20 the R-mode continues to propagate and is called the whistler mode. For $\omega_{ce} < \omega_{pe}$, the whistler mode has a resonance at $\omega_{ce}$. For $\omega_{pe} < \omega_{ce}$, the whistler mode has a resonance at $\omega_{pe}$. Another branch of the L-mode can be seen in this frequency range and is the left polarized light wave. This wave has a lower cutoff given by

$$\omega_L = \frac{1}{2} \left( -\omega_{ce} + \left( \omega_{ce}^2 + 4 \omega_{pe}^2 \right)^{1/2} \right) \quad (3.5)$$

Because of its left polarization this wave is unaffected by the resonance at the electron cyclotron frequency. Dispersion relations for the L and R-modes at high frequencies are given by the following.

$$n^2 = \left[ - \frac{\omega_{pe}^2}{(\omega + \omega_{ce} \cos \theta) \omega} \right] \quad L, \omega \gg \omega_{CH}, \omega_{ce} \ll \omega_{pe} \quad (3.6)$$

$$n^2 = \left[ - \frac{\omega_{pe}^2}{(\omega - \omega_{ce} \cos \theta) \omega} \right] \quad R, \omega \gg \omega_{CH}, \omega_{ce} \ll \omega_{pe} \quad (3.7)$$
In figure 21 one has the same dispersion plot for $\theta = 60^\circ$, except for the $0^\circ$ L-mode, which is plotted for $\theta = 40^\circ$. At all $\theta > 0^\circ$, the R-L and L-R branches do not touch each other at the crossover frequency. The gap in the L-R mode near $\omega_{cH}$ where the linear dispersion relation loses the mode is due to cyclotron damping.

The waves in figure 21 are damped, and continue to be damped as the wave normal angle is increased toward $90^\circ$ or decreased toward $0^\circ$. This can be understood by focusing on the band of parallel phase velocities for $10^7 < \omega/k \parallel < 10^8$ cm/s shown in figure 21. One is looking for a growing wave with a parallel phase velocity in this range, which covers the range of hydrogen and oxygen beam velocities. Waves in this region will interact strongly with the beam ions. This is done by means of the Landau resonance, where particles with parallel velocity $v_\parallel$ are in resonance with a wave with parallel phase velocity $\omega/k \parallel$. In this way beam ions can transfer energy to waves and waves can transfer energy to the ions. As can be seen in figure 21, the parallel phase velocities of the waves are much greater than $10^8$ cm/s, since $\omega/k < \omega/k \parallel$. Thus, $\omega/k \parallel > U_H$, the beam velocity of hydrogen, which is always higher than oxygen. The two L-modes in figure 21 seem to extend toward the dotted lines, but stop before they reach this region. This is because these waves undergo strong cyclotron damping when $\omega \rightarrow \omega_{cH}$.

For completeness the search is extended to higher wave normal angles. The perpendicular waves have very high parallel phase velocities, making a Landau resonance with beam ions unlikely. This is because the beam velocities are much less than $\omega/k \parallel$. There might be a possibility of a cyclotron resonance with these waves.
At Θ near 90° the perpendicular electromagnetic wave at low frequencies is the two-ion counterpart to the X-mode in a single ion species plasma. This mode is called the e-mode and has two branches which are shown in figure 22. The upper branch connects to the upper R-mode at smaller Θ and the lower branch connects to the lower R-mode at smaller Θ. The upper e-mode has a resonance at the lower hybrid frequency, ω_{LH}, and a cutoff at ω_{cf}, where ω_{LH} is given by the following.

$$\omega_{LH} = \left[ \frac{\rho}{\alpha} \left( 1 + \left( 1 - \frac{4Q}{3} \right)^{\frac{1}{2}} \right) \right]^{\frac{1}{2}}$$

(3.8)

where

$$P = \omega_{CH}^2 + \omega_{CO}^2 + \frac{\omega_{ce}^2 (\omega_{PH}^2 + \omega_{PD}^2)}{\omega_{ce}^2 + \omega_{pe}^2}$$

$$Q = \omega_{CH}^2 \omega_{CO}^2 + \frac{\omega_{ce}^2 (\omega_{PH}^2 \omega_{CO}^2 + \omega_{PD}^2 \omega_{CH}^2)}{\omega_{ce}^2 + \omega_{pe}^2}$$

The lower branch has a resonance at a new frequency called the bi-hybrid frequency, ω_{bA} (Andre, 1985).

$$\omega_{bA} = \left[ \omega_{CH} \omega_{CO} \frac{\eta_{H} m_{H} + \eta_{O} m_{O}}{\eta_{O} m_{H} + \eta_{H} m_{O}} \right]^{\frac{1}{2}} \left( \omega_{PH}^2 + \omega_{PD}^2 \right) \frac{\omega_{CH}^2}{\omega_{CH}}$$

(3.9)

As can be seen in figure 22, the dispersion of the e-mode is a sensitive function of k when k \(>\) 1 x 10^{-5} cm^{-1}. The e-mode goes to a resonant frequency which is slightly above the cold plasma expression given above, ω_{bA} = 43.5 s^{-1}. This is because of the presence of oxygen ion cyclotron modes below the e-mode that exist in a hot plasma. This structure can be seen in more detail by looking at plots of the
dispersion relation, $|\Delta|$ . In figures 23-25, the dispersion relation, $|\Delta|$ , is plotted as a function of frequency.

In figure 23 the two branches of the e-mode are shown. At $k = 1 \times 10^{-5} \text{cm}^{-1}$ the upper branch is just above the lower hybrid resonance and the lower branch is between the oxygen cyclotron waves and $\omega \omega_{CH}$. The 6th harmonic of the oxygen cyclotron wave exists at these parameters and has a frequency of $\omega_R = 61 \text{s}^{-1}$. There are 6 $O^+$ ion cyclotron waves because 6 harmonic terms were part of the input to program DISP3. The number of harmonic terms for $H^+$ was two. The number of ion cyclotron waves increases as the number of harmonic terms increase because of $\omega \omega_{CH}$ in the dispersion relation.

The large positive structure in $|\Delta|_I$ in figure 23 is the Landau damping from the three species. A measure of the amount of damping is given by the thermal velocity of the particular species. The peak of the electron contribution to Landau damping occurs at $k \nu_{TE}$, where $\nu_{TE}$ is the thermal velocity of the electrons, $\nu_{TE} = (kT_e/m_e)^{1/2}$ . The peaks in the two ion contributions occur at lower frequencies, and are off scale in figure 23. All of the waves in figure 23 have positive slopes and are damped. The harmonics of the $O^+$ and $H^+$ ion cyclotron frequencies are marked and are doppler shifted by their respective beam velocities. The ion cyclotron waves occur between multiples of the ion cyclotron frequencies. Note that the first $O^+$ ion cyclotron wave does not appear at this value of $k$. Finally, there is cyclotron damping, which occurs at the cyclotron frequencies, and at each harmonic. This can be seen as the narrow positive structure in $|\Delta|_I$ at $\omega_{CH} + k \nu_{HH}$ and at $2 \omega_{CH} + k \nu_{HH}$. In oxygen the damping is barely visible as small
ripples on top of the Landau damping.

Figure 24 illustrates the structure of $|\Delta|_R$ at the crossover frequency, $\omega_{cR}$. In figure 24a, $k = 1 \times 10^{-8} \text{ cm}^{-1}$, the L-mode has the higher frequency and the R-mode has the lower frequency. As $k$ is increased the two modes move toward each other as can be seen in figure 24b where $k = 1.17 \times 10^{-8} \text{ cm}^{-1}$. This value of $k$ almost corresponds to the crossover frequency. At $\Theta = 0^\circ$, $|\Delta|_R$ becomes a spike because the two modes practically touch each other, as can be seen in figure 24b. At higher $\Theta$ the L-R and R-L modes are separated in $k$-space. In figure 24c $k$ is increased to $1.2 \times 10^{-8} \text{ cm}^{-1}$. Here the two modes have higher frequencies above $\omega_{cR}$ and have reversed their polarization. Note that the slopes of the modes stay the same before and after the polarization reversal. Determination of the polarization of the waves was done by observing how the modes behave at the ion cyclotron resonances. Left polarized modes have an upper cutoff at an ion cyclotron resonance.

Figure 25 illustrates how the e-mode behaves at high $k$ values. In figure 25a $k = 2 \times 10^{-5} \text{ cm}^{-1}$ and the 5th and 6th harmonics of the $0^+$ ion cyclotron waves are present. The e-mode and the 6th $0^+$ ion cyclotron wave have become the same wave. Ordinarily in a cold plasma the e-mode goes to the bi-hybrid resonance at $\omega_{bi}$. When the plasma has a finite temperature and ion cyclotron waves are present, the e-mode resonance is shifted to one of the ion cyclotron resonances. In figure 25b where $k$ has increased to $2.5 \times 10^{-5} \text{ cm}^{-1}$ the ion cyclotron modes do not exist, as well as the e-mode.
Figure 20
Figure 21
Figure 22

\[ \theta = 89.6^\circ \]

e-mode
Figure 23
Figure 24
\[ \Theta = 89.6^\circ \]
\[ k = 2.5 \times 10^{-5} \text{ cm}^{-1} \]

\[ |\Delta|_R \quad \text{and} \quad |\Delta|_I \]

\[ k \parallel \omega_0 + 6 \omega_{CO} \]

\[ \omega_R, \text{S}^{-1} \]

Figure 25
**Electrostatic Waves**

It was found in the previous section that the observed plasma was stable to electromagnetic waves. This is not the case for electrostatic waves. An electrostatic wave is defined by the condition \( \mathbf{k} \times \mathbf{E} = 0 \). Some electromagnetic waves propagating parallel to the magnetic field become partially or completely electrostatic when the wave normal angle is increased. This can be depicted by drawing wave normal diagrams for electromagnetic waves. Wave normal diagrams are constructed by plotting the phase velocity vector \( \omega \frac{\mathbf{k}}{k} \). For the whistler mode at frequencies below the electron plasma frequency and above the lower hybrid resonance (\( \omega_{pe} < \omega_{ce} \)), the wave normal diagram looks like the following.

![Wave normal diagram for whistler mode](image)

**Figure 26**

Thus, the whistler mode will propagate until some maximum angle with respect to the magnetic field where it becomes electrostatic. From figure 26 it is seen that the phase velocity of the wave goes to zero at \( \Theta_{\text{max}} \) and the wave is at a resonance. At this point the component of the electric field perpendicular to \( \mathbf{k} \) vanishes. At the lower hybrid resonance \( \Theta_{\text{max}} = 90^\circ \).
Other waves never experience the resonance at $\Theta_{\text{max}}$ and propagate at all angles. At high $\omega$ the left polarized light wave exists parallel to the magnetic field. It propagates at all angles and becomes the ordinary mode at large $\Theta$ and $\omega > \omega_{pe}$.

The extraordinary mode is a high frequency electromagnetic wave that propagates perpendicular to the magnetic field. This wave is a hybrid wave with the $\hat{E}$ vector rotating in an ellipse in the plane perpendicular to $\hat{B}_0$. In the frequency range $\omega_{pe}, \omega_{ce} < \omega < \omega_{uh}$, the extraordinary or X-mode experiences a resonance as $\Theta$ is decreased. Thus, the wave normal diagram looks like figure 26 only rotated by 90°. At the upper hybrid resonance, $\omega_{uh} = \left(\omega_{pe}^2 + \omega_{ce}^2\right)^{1/2}$ this wave becomes purely electrostatic with $\Theta = 90°$.

The waves that exist between the harmonics of the oxygen and hydrogen cyclotron frequencies in figure 23 are electrostatic ion cyclotron waves. These waves exist only at large $\Theta$. As $\Theta$ is decreased from 90° the waves become damped.

Another ion wave, which will be the topic for the rest of this chapter, is the ion acoustic wave. This wave propagates with phase velocity

$$C_s = \left(\frac{kT_e}{m_i}\right)^{1/2}, \quad T_e \gg T_i.$$  \hspace{1cm} (3.10)

In order to see the characteristics of the ion acoustic wave, let us look at the final plasma parameters chosen for study.

The final distribution which was chosen was $T = 14:04:35 - 14:04:59$ for ions. The electron temperature was set equal to the average temperature obtained from the fitted Maxwellians to the 500 eV electron component during this time period. The electron density was set equal to the total ion density in order to preserve charge neutrality. The low
density, 5 keV electron component which came out of the fit was also included for completeness. This component has a small effect on the stability of waves compared to the others, and will be left out of the following discussion. Thus, there are essentially three components, two ion beams drifting through a warm electron background. The physical picture can be seen in figure 27a. Figure 27a shows four waves associated with the ion beams. Each beam has two waves, a fast wave and a slow wave. The fast acoustic waves are doppler shifted forward in the direction of the beams, with parallel phase velocities \( v_{ph\parallel} = U_0 + C_0, U_H + C_H \), and the slow acoustic waves are doppler shifted backward in the opposite direction of the beams, with parallel phase velocities \( v_{ph\parallel} = U_0 - C_0, U_H - C_H \). The sound speed for species \( \alpha \) in a two ion component plasma is \( C_\alpha = \left( \frac{kT_\alpha n_\alpha}{m_\alpha n_\perp} \right)^{1/2} \), where \( n_\alpha \) and \( n_\perp \) are the densities of the species \( \alpha \) and the total ion density, respectively. The two fast waves are positive energy waves, and grow when their phase velocities lie in regions where there exist positive slopes in the distribution function. A positive energy wave grows as resonant particles give energy to the wave. Thus, resonant particles will diffuse to lower energies. The fast hydrogen acoustic wave is damped since all resonant particles exist in regions of negative slope. The fast oxygen acoustic wave is growing if the resonant growth due to the positive slope in hydrogen overcomes Landau damping from the electrons. The slow ion acoustic waves are negative energy waves which grow from a resonant interaction with regions of negative slope. Negative energy waves grow as resonant particles take energy from the wave. Thus, resonant particles diffuse to higher energies in this case. When a negative energy wave is present, the total kinetic energy of the ions is less than when the wave is absent. The total energy of
the waves plus particles is still a positive quantity. If one transforms to the hydrogen rest frame as in figure 27b, it is clear that the two slow waves are growing. Both are positive energy waves as a result of the transformation and are resonantly driven by positive slope regions in velocity space.

The two modes of interest are the slow $H^+$ wave and the fast $O^+$ wave, since their phase velocities lie in between the two beam velocities. When the streaming velocities of the two ion species are well separated, the modes also separate and assume their fluid characteristics. If the distributions overlap, as is the case in this study, then resonant effects can take place with the other species. Resonant effects will also be introduced when a magnetic field is present, even if the distributions are well separated. This is because particles with velocities outside the Landau resonant region can resonate with the wave through a harmonic of the cyclotron frequency according to the resonance condition $k_n v_{H} + n\omega_c - \omega = 0$.

Finally, if the phase velocities of the two waves are such that $U_H - C_H \sim U_O + C_O$, then an explosive instability which is fluidlike will be generated. This instability has been studied in detail by Bergmann and Lotko, (1986).

With the fitted Maxwellians to $T = 14:04:35 - 14:04:59$ it is found that the slow hydrogen acoustic wave is the growing mode. Other waves which might be important in this plasma are the oxygen electrostatic ion cyclotron waves, but these waves were seen to be damped.

In figure 28a is a plot of the dielectric response function, (1.23). The growing mode is marked with a dot and is the slow hydrogen acoustic mode. The imaginary part of $\epsilon(\omega, \vec{k})$ is positive at the root, signifying growth. The angle of propagation is $72.6^\circ$ which means this is the oblique
acoustic wave. The wavenumber $k$ is $1.7 \times 10^{-6} \text{cm}^{-1}$ which is much smaller than the Debye wavenumber, $K_\text{De} = \left(\frac{4\pi ne^2}{kT_e}\right)^{1/2} = 2 \times 10^{-5} \text{cm}^{-1}$.

This meets the criterion for ion acoustic waves, i.e. the wavelength must be larger than the Debye length. The frequencies $k_H U_0$ and $k_H U_H$ are marked in figure 28a and occur where $\xi_i$ passes through zero with positive slope. This is a characteristic of these frequencies and is not a general statement about the zeros of $\xi_i$. Because $\xi_i$ changes sign at these frequencies they determine boundaries for wave growth, as was seen in figures 20 and 21 for the case of electromagnetic waves. Thus the parallel phase velocity of the wave is between the two beam velocities and can be a mechanism for the transfer of energy from hydrogen to oxygen. Figures 28b-28d show the separate contributions to $\xi = \sum_\alpha \xi_\alpha$ from each species.

The hydrogen contribution, figure 28d, is negative at the root frequency in both $\xi_R$ and $\xi_i$. The large bumps in $\xi_i$ are the amount of Landau damping present from hydrogen and the location of their peaks are determined by the hydrogen thermal velocity $k_H v_{TH}$. The oxygen contribution, figure 28c, has a large positive imaginary part at the root frequency and is responsible, together with the electrons, for the resonant growth of the wave. The first bump in $\xi_i$ is the Landau growth contribution with a peak at $k_H (U_0 + v_{TO})$. The additional bumps in $\xi_i$ occur at the doppler shifted harmonics of the oxygen cyclotron frequency and represent cyclotron growth, as opposed to cyclotron damping in the case of a positive energy wave. The small bump in $\xi_R$ near $\omega_R = 27 s^{-1}$ is the first oxygen ion cyclotron wave which has not formed a root at this value of $k$.

At higher $\theta$ it will form a root and will couple strongly with the oblique acoustic wave. In figure 28b the electron contribution is shown. Both $\xi_R$ and $\xi_i$ are positive over the frequency range of interest. The
positive $\xi_\Sigma$ adds to the oxygen $\xi_\Sigma$ contribution to produce growth. The positive $\xi_\Sigma$ adds to the oxygen and hydrogen $\xi_\Sigma$'s to make the wave. In figure 29 contours of the growth rate $\omega_\Sigma$, the parallel growth length $D_\parallel = \frac{v_{\|}}{\omega_\Sigma}$, and the perpendicular growth length $D_\perp = \frac{v_{\perp}}{\omega_\Sigma}$, are shown in the growing region of $k$-space. The solid line boundary marks the region where the wave is damped before the root is lost. The dashed line marks the region where the root is lost while it is still growing. The growth rate is $1-3 \text{ s}^{-1}$ and exists over a small region of $k$-space. It has a frequency range of $18-27 \text{ s}^{-1} = \omega_R$. Thus, the beams are in a state of marginal stability. This supports the observations, which indicate that waves must have been strongly unstable at lower altitudes in order to produce the significant heating observed at satellite altitudes. The growth lengths are larger in the parallel than the perpendicular direction with group velocities $v_{\|} = 1 \times 10^7 \text{ cm/s}$ and $v_{\perp} = 1 \times 10^5 \text{ cm/s}$ being typical values.

Near the lower border in figure 29 the wave has a parallel phase velocity near $v_{\Phi\parallel} = U_H - C_H$. At this $k_\parallel$, the wave is in cyclotron resonance with $O^+$ beam ions so that the resonance condition $k_\parallel U_O = \omega_R - \omega_{co}$ is satisfied. Here, the beam ions see the wave doppler shifted to $\omega_{co}$. Inserting this $v_{\Phi\parallel}$ into the resonance condition and solving for $k_\parallel$ gives a lower cutoff of $k_\parallel = (\omega_{co}/(U_H - C_H - U_O))$, which is marked in figure 29. At the higher boundary the parallel phase velocity approaches the difference in the beam velocities, $\omega_R = k_\parallel (U_H - U_O)$. Note that the parallel phase velocity of the wave does not correspond exactly to the fluid value of $v_{\Phi\parallel} = U_H - C_H$. This is because the wave is being driven by only resonant oxygen ions and electrons and is not fluidlike. Also, at the upper boundary the wave frequency is given approximately as $\omega_R =$
\( k_U 0^+ \frac{3}{2} \omega_c \), which is the doppler shifted frequency of the first oxygen electrostatic ion cyclotron wave. The growth rate peaks at this boundary, suggesting wave coupling between the ion acoustic and the oxygen cyclotron wave. This effect is seen more dramatically in a case when the hydrogen beam temperature is dropped to 50 eV, thus increasing the growing region in k-space. The inverse oxygen cyclotron radius is marked on figure 29, indicating a rough boundary in \( k_L \) where the wavelength is approximately \( \frac{\rho}{\omega} \).

Because of the oblique propagation of the wave, waves generated near the edges of the beam will escape and become damped as they encounter the surrounding dense plasma outside the auroral zone. The satellite traversed a beam region of approximately 300 km so that waves generated inside the beams having perpendicular growth lengths of 1 km will not be able to escape. The beams extend over a distance of several \( R_e \), and consequently all parallel transport of wave energy is confined to the beams.

The existence of a cold \( H^+ \) background component would considerably affect the stability of plasma waves. Cold \( H^+ \) background with a temperature of 10 eV was added to the stability analysis above to see the effect of damping. When the cold \( H^+ \) density was increased to .035 cm\(^{-1}\) the root was lost by the linear dispersion relation and the growing region in k-space vanished. This is a density roughly equal to the beam density. If the beams are streaming through a cold \( H^+ \) background at this altitude, the cold \( H^+ \) must have entered the auroral zone from the sides, because any cold \( H^+ \) background of magnetospheric or ionospheric origin already in the acceleration region would experience the potential drop and become part of the beams. One concludes that a definite
measurement is needed here.

If the electron temperature is increased to 690 eV or higher, the parallel H+ acoustic instability is excited. The dielectric response function and its components are shown in figures 30a-d. At \( \Theta = 0^\circ \) none of the ion cyclotron harmonics appear in \( \varepsilon_R \) and \( \varepsilon_T \) for oxygen. One sees in figure 30c the single structure at \( \omega = \omega_0 \). Consequently, the parallel phase velocity of the wave lies closer to the peak of the oxygen distribution than the oblique wave. The phase velocity has the value

\[
\nu_p = U_o + C_o = 2.9 \times 10^7 \text{cm/sec.}
\]

The effect of the oxygen can clearly be seen here, since the ordinary H+ electron instability has \( \nu_p = U_H - C_H \). Here, the electron temperature is too low to excite the wave if the oxygen were absent, and consequently doesn’t determine the phase velocity of the wave. The parallel mode is probably going to be unstable in ion beams if the electron temperature fluctuations are on the order of 200 eV, which is very reasonable.

During the early stages of this study the exact value of the electron temperature was not known at satellite altitudes. With the Maxwellian fits to \( T = 14:05:23 - 14:05:41 \) the electron temperature was artificially increased to excite both the parallel and oblique ion acoustic instabilities. The growth rates and growth lengths are shown in figures 31 and 32 for the parallel and oblique waves, respectively. For the parallel mode the electron temperature was set equal to 2 keV, which is unrealistic for DE-1 altitudes in the auroral acceleration region. The mode is growing in a huge region of k-space with the growth rate increasing to very high values near the top border. \( D_\perp \) and \( D_\parallel \) have values similar to figure 29, covering a slightly higher range. The dotted line indicates that the mode is lost by the linear dispersion relation before it gets damped. At
Electrons - Stationary Frame

H+ - Stationary Frame
Figure 28
Figure 29
$k_\parallel = 4.0 \times 10^{-7} \text{ cm}^{-1}$

$k_\perp = 1.0 \times 10^{-8} \text{ cm}^{-1}$

$T_e = 724 \text{ eV}$

Figure 30
Figure 31
Figure 32
Table 3

<table>
<thead>
<tr>
<th></th>
<th>$V_T$</th>
<th>$\varrho$</th>
<th>$C_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>e$^-$</td>
<td>$9.6 \times 10^8$ cm/s</td>
<td>$3.9 \times 10^3$ cm</td>
<td></td>
</tr>
<tr>
<td>H$^+$</td>
<td>$1.0 \times 10^7$ cm/s</td>
<td>$7.5 \times 10^4$ cm</td>
<td>$1.5 \times 10^7$ cm/s</td>
</tr>
<tr>
<td>O$^+$</td>
<td>$3.6 \times 10^6$ cm/s</td>
<td>$3.5 \times 10^5$ cm</td>
<td>$4.2 \times 10^6$ cm/s</td>
</tr>
</tbody>
</table>

Definitions: $V_T$ = thermal velocity, $\varrho$ = cyclotron radius, $C_s$ = sound speed, $B_0 = .014$ gauss

Plasma parameters for the fitted Maxwellians to $T = 14:04:35 - 14:04:59$
the extreme upper left the Taylor Series test was performed to ensure
the series expansion of $\varepsilon (\omega, k) \rightarrow \varepsilon_R (\omega_R, k)$ in the limit $\omega_L \rightarrow 0$.
The results were positive here, although throughout most of the region
in $k$-space higher order terms in the expansion had finite values at the
root frequency. The $k$-space boundary at $\Theta = 45^\circ$ and $k_z \sim 7 \times 10^{-7}$ cm$^{-1}$
becomes jagged and irregular. Here, the parallel dispersion surface does
not connect smoothly to the oblique surface and is subject to cyclotron
effects. At small $k$ the mode is continuous to oblique angles. When $T_e$
is reduced to 1 keV the parallel mode ceases to exist, but the oblique
wave is still growing. The oblique mode is shown in figure 32, which is
similar to figure 31 except for $T_e = 1$ keV. Because the electron temper­
ature is higher than in figure 29 the mode is growing in a much larger
region of $k$-space. The Taylor Series converges to $\varepsilon_R (\omega_R, k)$ along the
border where the mode is damped, making this wave more acceptable to the
linear approximation used here. The parallel phase velocity of the wave
has a higher range than in figure 29, due to the expanded region in $k$-
space. The waves in figures 31 and 32 provide one with information on
the magnitude of the instability as the parameter $T_e$ is varied.

The Quasilinear Approach
The validity of using quasilinear theory to describe plasma process­
es depends on the nature of the interaction. For the problem at hand, the
waves are growing weakly, with $\omega_L \ll \omega_R$, enabling quasilinear theory
to be used. Also, collisions are negligible at satellite altitudes, so
particles will be scattered by waves rather than other particles. Finally,
these waves are electrostatic, enabling the quasilinear diffusion coef­
ficients for electrostatic waves to be used.

There are ion heating mechanisms other than quasilinear theory, some
of which were mentioned in the Introduction. It would be useful to remark on the applicability of other approaches to the problem at hand. For example, the nonlinear theory developed by Lysak et al., 1980 involving strong turbulence and the coherency of the waves would not be applicable here, where weak turbulence is observed. By weak turbulence one means that the wave amplitude increases slowly over many wave periods. Strong turbulence involves large amplitude waves which are near saturation. The heating rates predicted by the nonlinear theory are on the order of four times the quasilinear heating rates for ions being heated by ion cyclotron waves, for example. The beams at DE-1 altitudes are marginally stable. This means that if the beams were to suddenly become colder as a result of adiabatic flow the waves would grow more rapidly and heat the beams. The heated beams would then reduce growth rates which would produce the original state of marginal stability. In this way, the beams never become strongly unstable in a nonlinear sense.

Another approach to this problem is to do a numerical simulation. There have been reports of simulations of ion acoustic double layers (Barnes et al., 1985; Sato and Okuda, 1981). The double layers, which exist at lower altitudes, are believed to cause transverse heating as well as parallel acceleration of ion beams. If the O\(^+\) and H\(^+\) beams pass through the double layer simultaneously, an H\(^+\)-O\(^+\) interaction could result.

Ashour-Abdalla and Okuda (1985) have done simulations of counter-streaming ion beams in the plasma sheet. The ion beams generate the ion acoustic instability which heats plasma sheet ions and electrons only slightly, as revealed by the simulations. Instead, an ion-ion
two stream instability is the stronger process for heating the plasma sheet and beam ions. Dusenbery and Lyons (1985) have also looked at this problem in the linear regime. Their model includes cold ionospheric $H^+$ streaming in the plasma sheet with temperatures less than 50 eV.

When performing a quasilinear calculation one can solve for a number of different quantities. Many people want to know what $f(v,t)$ looks like at various stages of evolution (Dusenbery and Lyons, 1981). Others will take moments of the distribution to obtain $T_{||}(t)$ and $T_{\perp}(t)$, providing parallel and perpendicular heating rates (Chang and Coppi, 1981; Rogers et al., 1985).

A considerable amount of information can be obtained by mapping the resonant and nonresonant regions in velocity space, and evaluating the diffusion coefficient for a given k-spectrum. Diffusion time scales for resonant and nonresonant particles can be obtained in this way (Ashour-Abdalla and Kennel, 1978; Ashour-Abdalla and Thorne, 1978). In the following sections this approach is taken.

In the next section we are going to evaluate the velocity space dependent part of the diffusion coefficient $D$ for a given k-spectrum and also plot the resonant and nonresonant regions in velocity space for $H^+$ and $O^+$. This analysis will enable us to identify the resonant ions and to see how these ions diffuse with the k-spectrum of the growing mode at the satellite, figure 29. The nature of the diffusion in velocity space, that is, perpendicular, parallel, or both, is independent of the electric field energy density and depends only on the k-spectrum. This is because the electric field energy density depends only on $\mathbf{k}$ and $\omega_\mathbf{k}(\mathbf{k})$. The electric field energy density determines the
rate of diffusion, and this will be explored in the last section of Chapter III.

One quantity of interest is the resonant width. The nonresonant diffusion coefficient is proportional to \( \frac{1}{(k_{\parallel} v_{\parallel} + n \omega_{c\alpha} - \omega_k)^2 + \omega_\Sigma^2} \).

The splitting into resonant and nonresonant terms came about because \( \omega_\Sigma < < \omega_R \), producing a contribution from the pole at \( k_{\parallel} = (\omega_R - n \omega_{c\alpha})/v_{\parallel} \). The finite \( \omega_\Sigma \) gives a width such that nonresonant diffusion is dominant when \( k_{\parallel} v_{\parallel} + n \omega_{c\alpha} - \omega_R > \omega_\Sigma \), and resonant diffusion is dominant when \( k_{\parallel} v_{\parallel} + n \omega_{c\alpha} - \omega_R < \omega_\Sigma \). This gives a resonant width in velocity space of magnitude \( v_{\parallel} \sim 2 \omega_\Sigma / k_{\parallel} \).
Characteristic Time

The characteristic time, $\tau^\|^R$, is obtained from the diffusion coefficient, $D = \frac{\partial^2}{\partial x^2}$.

$$D = \begin{pmatrix} D_{\perp\perp} & D_{\perp\|} \\ D_{\|\perp} & D_{\|\|} \end{pmatrix} = \frac{8\pi n^2}{m^2} \sum_k \xi^R_k(t) \tau^\|^R(x, v)$$

(3.11)

The characteristic time is a different quantity from the diffusion time, which gives one an order of magnitude estimate of the time it takes particles to diffuse to the final state. The characteristic time multiplies the electric field energy density, $\xi^R_k$, at a given $k$. $\tau^\|^R$ contains all the velocity space dependence of $D = \frac{\partial^2}{\partial x^2}$, as well as the tensor components. Thus, it tells one both where the diffusion is taking place in velocity space, and the nature of the diffusion, i.e. perpendicular or parallel. By looking at the explicit form of $\tau^\|^R$, one can see more of its properties.

$$\tau^\|^R = \tau^R + \tau^{NR} = \frac{1}{k^2} \sum_{n=-\infty}^{\infty} \int_{\mathbb{R}} \left( \frac{\omega - k_n v_n - n\omega_{ci}}{v_L} \right)^2 \left( \frac{\omega - k_n v_n - n\omega_{ci}}{v_L} \right)^2 \left( \frac{k_n v_n}{v_L} \right)^2 \left( \frac{k_n v_n}{v_L} \right)^2$$

(3.12)
Let us first consider the oxygen distribution, figures 8-11b. In figure 33a, $\gamma^R_k$ is evaluated for the oblique mode at $k_\perp = 1.6 \times 10^{-6}$ cm$^{-1}$ and $k_\parallel = 5.0 \times 10^{-7}$ cm$^{-1}$. To evaluate $\gamma^R_k$ for a given $k$, a corresponding value of $\omega^R_k$ and $\gamma_0$ are given by the dispersion relation. This specifies all parameters, and $\gamma^R_k(\gamma)$ can be plotted as a function of $v_\perp$ and $v_\parallel$. Because of the delta function in the resonant coefficients, $\gamma^R_k(\gamma)$ will be zero except at values of $v_\parallel$ where the resonance condition $\omega^R_k - k_\parallel v_\parallel - n \omega_{\text{ci}} = 0$ is satisfied. The delta function is removed when the k-integration is performed. For two different values of $k$, figures 33a and 33b show $\gamma^R_{k_\perp} > \gamma^R_{k_\parallel}$ in the region $v_\perp < 0.1 \times 10^8$ cm/s. The two examples show that perpendicular diffusion is dominant in $O^+$ and $\gamma^R_{k_\perp}$ is weakly dependent on $k_\perp$. For higher values of $k_\perp$, $\gamma^R_{k_\perp}$ has a zero at lower perpendicular velocities than when $\gamma^R_{k_\parallel}$ is evaluated at low $k_\perp$ values. Because $\gamma_0$ is obtained by summing over $k$, particles with small perpendicular velocities, $v_\perp < 0.05 \times 10^8$ cm/s are equally weighted by all modes and particles with perpendicular velocities $> 0.1 \times 10^8$ cm/s will be affected by modes with small $k_\perp$ values.

By looking at the contours of phase space density in figures 8-11b, one can see the effects of perpendicular diffusion. The contours are stretched out in the perpendicular direction on the high velocity side of the $O^+$ beams. The n = 1 resonant region is responsible for this effect. This can be seen in figure 34, which shows the resonant and nonresonant regions of the oxygen distribution $T = 14:04:35 - 14:04:59$. In figures 35-37, the nonresonant characteristic time, $\tau^\text{NR}_k$, is plotted as a function of $v_\parallel$ with the same value of $k$ as figure 33a. One can see the resonant width $\Delta v_\parallel = 2 \gamma/k_\parallel = 2 \times 1.4/(5. \times 10^{-7}) = 5.6 \times$
$10^6$ cm/s. The resonant width defines a rough boundary for resonant and nonresonant diffusion. For example, $\gamma_{\perp\perp}^{NR}$ peaks at the $n = +1, -1$ resonant parallel velocities. Outside the region defined by $\Delta v_{\parallel} = 2\gamma/k_{\parallel}$ centered on the peak, the diffusion is nonresonant, and $k_{\parallel} v_{\parallel} + n\omega_{\perp\perp} - \omega_{k}\gamma$. Because there is a range in $k_{\parallel}$, rather than one $k$ value, as shown here, the resonant width applies only to the boundary of the resonant region. In figures 36 and 37 one sees that $\gamma_{\parallel\parallel}^{NR}$ and $\gamma_{\perp\parallel}^{NR}$ are smaller than $\gamma_{\perp\perp}^{NR}$, in accordance with the resonant characteristic time.

In figure 38 the resonant and nonresonant regions for the oblique mode are shown for hydrogen. Most of the distribution lies in the nonresonant region except for the area of Landau resonance at $-5 \times 10^7$ cm/s $< v_{\parallel} < -4.1 \times 10^7$ cm/s. The characteristic time for this resonance is shown in figure 39. This Landau resonance region can be important in the parallel heating of the $H^+$ beam. In the hydrogen rest frame, as the resonant oxygen ions and electrons give energy to the wave, the wave grows and the electrostatic energy in the wave increases. This process competes with hydrogen Landau damping. The electrostatic energy in the wave will increase as oxygen ions and electrons diffuse and will peak at the point where the growth mechanism is balanced by damping. As the wave damps, its electrostatic energy is absorbed by Landau resonant hydrogen ions. In the hydrogen rest frame, the wave has positive energy, which, when absorbed by Landau resonant hydrogen ions causes them to diffuse to higher parallel velocities. When transforming to the electron rest frame, Landau resonant $H^+$ ions are decelerated and fill in the valley between the two beams. In the electron rest frame Landau resonant $O^+$ ions are accelerated because they take energy from a
negative energy wave, and also fills in the valley. To see how both species fill in the valley one can view the process in both the hydrogen and electron rest frames, where the wave has negative and positive energy, respectively.

In figure 40 $\Gamma_{NR}^{H}$ is plotted for hydrogen. Once again one sees that nonresonant diffusion is a weaker process than resonant diffusion. In light of this, nonresonant processes must be included in our treatment in order to define the boundary of resonant diffusion.

For the oblique mode discussed here, the Landau resonant region lies at the edge of the $O^+$ beam distribution. Thus, not many $O^+$ beams will be affected by it. Rather, the $n = 1$ cyclotron resonant region occupies the $O^+$ distribution. Thus, the $O^+$ will undergo pitch angle diffusion, producing perpendicular heating as revealed by the velocity space plots of $f(\Psi)$.

Resonant and nonresonant regions for the parallel mode are shown in figures 41 and 42. The Landau resonance region is reduced to a band of width $\Delta v_H = 2 k_T v_H \approx 1.6 \times 10^7 \text{cm/s}$ centered on $-3.1 \times 10^7 \text{cm/s}$. This is because the parallel phase velocity is equal to a constant for the parallel mode and the only spread in $v_{res}^H$ is due to the resonant width. In figures 41-42 the electron temperature is set equal to 724 eV, slightly above the threshold value of 695 eV. The value of the electron temperature determines the position of the Landau resonance region in velocity space, $v_{ph}^H = U_H - C_H$. As the beams flow into different regions of the magnetosphere this position will change as the electron temperature changes. It will also change if $U_H$ changes. At lower altitudes the $H^+$ and $O^+$ beam velocities will have smaller values and the separation in streaming velocities will be smaller. Thus, the Landau
resonance region could be moved into the oxygen distribution in this way, provided the oxygen temperature has increased enough above its ionospheric value to allow this to happen. This would depend on how much heating was caused by the ion-ion two-stream instability during the initial stages of acceleration. The electron temperature would also be lower at low altitudes which counteracts the streaming velocity dependence of $v_{ph_{H^+}}$. It is clear that after the fluid two-stream instability shuts off and the beams have a heated thermal distribution, resonant processes will take over. This is accompanied by the excitation of oblique modes. It is not clear whether the slow parallel $H^+$ acoustic mode is growing significantly at lower altitudes and if it provides any additional heating, because of the lack of information on the plasma temperatures and streaming velocities where resonant processes become important.

Returning to figures 41 and 42, one can see how the parallel mode might affect the beams at satellite altitudes. In figure 41 the parallel phase velocity of the wave lies outside the region of the $H^+$ beam, $v_{ph_{H^+}} < U_{H^+} - v_{TH^+}$. One sees that the $n = 1$ resonance region cuts the $H^+$ beam in half on the high velocity side and can be responsible for perpendicular diffusion of the beam at these velocities.

The $O^+$ beam in figure 42 is dominated by $n \geq 1$ resonant diffusion for the parallel mode. This could be a strong process although two considerations must be taken into account. At the lower boundary of these regions where $n = 1$, the growth rate diverges and linear theory is invalid. In the region close to this, where there are large growth rates, diffusion times may be comparable to the oxygen cyclotron period, $= 1.2$ sec. This violates the requirement of the theory that ion
gyroperiods be small compared to diffusion times, in order that there is no $\phi$-dependent diffusion. The unmagnetized diffusion coefficients could be used for oxygen in this region, where frequencies are much higher than the oxygen cyclotron frequency.
Figure 33
Figure 34

Oblique mode $0^+$
Resonant and Nonresonant Regions

$V_{\parallel}, 10^8$ cm/sec

$V_{\perp}, 10^8$ cm/sec

$R$, $N = 2$
$NR$, $N = 1$
$NR$, $N = 0$
Characteristic Time

$O^+ \text{ Nonresonant}$

$V_{\perp} = 0.2 \times 10^7 \text{ cm/sec}$

$k_{\parallel} = 5.0 \times 10^{-7} \text{ cm}^{-1}$

$k_{\perp} = 1.6 \times 10^{-6} \text{ cm}^{-1}$

$\omega_k = 21.8 \text{ s}^{-1}$

$\gamma = 1.3 \text{ s}^{-1}$

Figure 35
Figure 36

Characteristic Time

$\tau_{NR}$

$\omega_{co}$

$k_h = 5.0 \times 10^{-7} \text{ cm}^{-1}$

$k_L = 1.6 \times 10^{-6} \text{ cm}^{-1}$

$\omega_R = 21.8 \text{ s}^{-1}$

$\gamma = 1.3 \text{ s}^{-1}$

$V_{||} \times 1 \times 10^8 \text{ cm/sec}$
Characteristic Time

$O^+ \text{ Nonresonant}$
$V_{\text{perp}} = 0.2E7 \text{ cm/sec}$

$\gamma = 1.3 \text{ s}^{-1}$
$\omega_R = 21.8 \text{ s}^{-1}$
$k_H = 5.0 \text{ E-7 cm}^{-1}$
$k_L = 1.6 \text{ E-6 cm}^{-1}$
$\bar{v}_{NL} \frac{1}{c_0}$

Figure 37
Resonant and Nonresonant Regions

Oblique mode

Figure 38
Characteristic Time

\[ \frac{\tau_{||}}{\omega_{\text{ch}} (\omega - k_{||} v_{||})} \]

H+ Resonant

\[ k_{||} = 5.8 \times 10^{-7} \text{ cm}^{-1} \]
\[ k_{\perp} = 2.4 \times 10^{-6} \text{ cm}^{-1} \]
\[ \omega_{\text{R}} = 24.6 \text{ s}^{-1} \]
\[ v_{||} = 4.25 \times 10^{7} \text{ cm/sec} \]
\[ n = 0 \]
\[ \gamma = 3.0 \text{ s}^{-1} \]

Figure 39
H^+ Nonresonant
U_{perp} = 1.0 E^7 cm/sec

k_{II} = 5.8 E^7 cm^{-1}
k_{II} = 2.4 E^6 cm^{-1}
\omega_{R} = 24.6 s^{-1}
\gamma = 3.0 s^{-1}

Figure 40
Parallel mode
Resonant and
Nonresonant
Regions

\( V_{\parallel}, 10^8 \text{ cm/sec} \)

\( V_{\perp}, 10^8 \text{ cm/sec} \)

Figure 41

\( T_e = 724 \text{ eV} \)
Resonant Diffusion Time Scales

Since $D$ has units of $v^2/t$, one can construct a resonant diffusion time scale $v^2/D_R$, where $v$ is the velocity of the resonant particle. The diffusion time scale gives one an estimate of how long it will take the resonant region of the distribution to evolve to the later stages of quasilinear diffusion. Resonant diffusion time scales can be calculated for the hydrogen and oxygen resonant regions defined by the oblique a-coustic wave. One has

$$T_{\perp\perp}^R (0^+) = \frac{v_L^2}{D_{\perp\perp}^R}$$

$$T_{\parallel\parallel}^R (H^+) = \frac{v_H^2}{D_{\parallel\parallel}^R}$$

where $D_{\perp\perp}^R \gg D_{\parallel\parallel}^R$, $D_{\parallel\parallel}^R$ is used to construct $T_{\perp\perp}^R (0^+)$. The resonant region for $H^+$ is the Landau resonant region. To calculate $T^R$ one needs to know how the energy density $E_k^R$ is distributed in k-space. When calculating the total electrostatic energy density $W_e = \sum_k E_k^R$, one considers modes occupying a cylindrical volume of length $L = 2\pi/\Delta k_\perp$ and radius $R = 2\pi/\Delta k_\perp$, where $\Delta k_\parallel$ and $\Delta k_\perp$ are the grid spacing in k-space. In integral form,

$$W_e = \lim_{v \to \infty} \frac{v}{(2\pi)^3} \int 2\pi k_\perp dk_\perp \int \int \int dk_\parallel k_\parallel E_k^R$$

$$\lim_{\Delta k_\parallel \to 0} \lim_{\Delta k_\parallel \to 0} \frac{\pi}{(\Delta k_\parallel)^3} \sum_k \frac{2\pi^2 k_\parallel}{\Delta k_\parallel} \Delta k_\parallel \Delta k_\parallel E_k^R = \sum_k \frac{2\pi^2 k_\parallel}{\Delta k_\parallel} E_k^R$$
The expression above is dimensionally correct, keeping $E^r_k$ in units of ergs/cm$^3$. When calculating the diffusion coefficient one has the same criterion.

In the case for oxygen,

$$D_{\perp}^R = \frac{8\pi e^2}{m_0^2} \int d^2k E^r_k \sum_n J_n^2 \left( \frac{k_L v_L}{\omega_n} \right) \frac{\pi \delta \left( \omega_0^2 - \omega_n^2 \right)}{k^2} \left( \frac{\omega_0}{\omega_n} \right)^2 \left( \frac{n\omega_0}{v_L} \right)^2$$

which, for the $n = 1$ resonant region, becomes

$$D_{\perp}^R = \frac{8\pi e^2}{m_0^2} \int 2\pi k_L dk_L J_1^2 \left( \frac{k_L v_L}{\omega_0} \right) \left( \frac{\omega_0}{v_L} \right)^2 \frac{\pi E^r_k}{k^2} \left( \frac{\omega_0}{v_L} \right)^2 \left( \frac{\omega_0}{v_L} \right)^2 \left( \frac{n\omega_0}{v_L} \right)^2$$

When transforming the integral to a sum, and keeping $\Delta k_L$, $\Delta k_n$ finite, one obtains,

$$D_{\perp}^R = \frac{8\pi e^2}{m_0^2} \frac{\pi}{(\Delta k_L)^2\Delta k_n} \sum_{k_L} 2\pi k_L \Delta k_L E^r_k J_1^2 \left( \frac{k_L v_L}{\omega_0} \right) \left( \frac{\omega_0}{v_L} \right)^2 \frac{\pi}{k^2} \left( \frac{\omega_n}{v_n} - \frac{\partial}{\partial k_n} \right) \left| \frac{k_n = \omega - \omega_0}{v_n} \right|$$

Again, one introduces the unit volume in $k$-space $\frac{\pi}{(\Delta k_L)^2\Delta k_n}$ in order to keep the integration dimensionally correct. One can introduce the quantity

$$E \left( k_L, k_n \right) = \frac{2\pi^2 k_L}{\Delta k_L} E^r_k \left( \text{ergs/cm}^3 \right)$$

without any loss of generality. One obtains finally,

$$D_{\perp}^R = \frac{8\pi e^2}{m_0^2} \sum_{k_L} E \left( k_L, k_n \right) J_1^2 \left( \frac{k_L v_L}{\omega_0} \right) \left( \frac{\omega_0}{v_L} \right)^2 \frac{\pi}{k^2} \left( \frac{\omega_n}{v_n} - \frac{\partial}{\partial k_n} \right) \left| \frac{k_n = \omega - \omega_0}{v_n} \right|$$

(3.15)
The factor $\Delta k_{\|}$ appears because the $k_{\|}$ integration was performed to remove the $s$-function.

In the same manner, one can calculate the diffusion coefficient for $H^+$.  

$$
D_{\|\|}^R = \frac{8\pi e^2}{m_e^2} \sum_{k_L} \frac{\epsilon(k_L, k_{\|})}{\Delta k_{\|}} \int_0^2 \left( \frac{k_{\|} v_L}{\omega_C H} \right)^2 r_{\|}^2 \left| \frac{\pi}{k_{\|}^2 |v_{\|} - v_{g_{\|}}|} \right|_{k_{\|} = \frac{\omega}{v_{g_{\|}}}}
$$

Normally, when one is performing a simulation on the computer, $\omega_e$ is spread uniformly throughout the growing region in $k$-space at $t = 0$. $\epsilon_R^p(t)$ evolves according to  

$$
\epsilon_R^p(t) = \epsilon_R^p(0) e^{2 \omega_x t}.
$$

When calculating the diffusion time scales for the oblique mode the electric field measurements made during the beam observations on DE-1 are used. From figure 18 it can be seen that measurements at low frequencies are of poor quality. There is a large spacing between the data points. A best straight line through the data in the low frequency range is used to get $E^2 (V^2 m^{-2} Hz^{-1})$ as a function of frequency. The range in $\omega$ for the oblique mode is $18.5 - 26 s^{-1}$, corresponding to a $\Delta f$ of $1 s^{-1}$ with $f_o = 3.5 s^{-1}$. The electric field intensity in this range is $\sim 4 \times 10^{-7} V^2/m^2$. This corresponds to an energy density in MKS of $\omega_e = \epsilon_o E^2/2 = 1.77 \times 10^{-18} J/m^3$ or $1.77 \times 10^{-17} \text{erg/cm}^3$. Thus, the range in $\omega(k)$ corresponding to the growing wave contains $\omega_e$ given above.

The resonant diffusion coefficients are evaluated at particular resonant parallel velocities $v_{\| res}$ corresponding to a particular $k_{\| res}$. For the oblique mode, $\omega$ is primarily a function of $k_{\|}$, but at $k_{\|}$ near the upper boundary $\omega$ varies with $k_{\perp}$. Thus, a particular value of $k_{\| res}$
corresponds to a particular $\omega(k_{\parallel res})$. In equation (3.15) and (3.16) $\Delta k$ is set equal to the value corresponding to $\Delta \omega = 1$. This corresponds to a fraction $\sum_{k_L} \mathcal{E}(k_{\parallel}, k_L) = (1/2\pi)W_e$. For $k_{\parallel res}$ near the lower boundary it does not matter what $\Delta k_{\parallel}$ is set equal to since the ratio $\sum_{k_L} \mathcal{E}(k_{\parallel}, k_L)/\Delta k_{\parallel}$ will be a constant as long as $\mathcal{E}(k_{\parallel}, k_L)$ does not vary with $k_L$. At $k_{\parallel res}$ near the higher boundary, $\sum_{k_L} \mathcal{E}(k_{\parallel}, k_L)$ is set equal to $(\Delta \omega/2\pi)W_e$, where $\Delta \omega$ spans $\Delta k_{\parallel}\Delta k_L$. For the oblique mode, three values of $k_{\parallel res}$ were chosen. These were $4 \times 10^{-7}$, $5 \times 10^{-7}$, and $6 \times 10^{-7}$ cm$^{-1}$, roughly corresponding to three $v_{\parallel res}$ which span the $n = 1$ and $n = 0$ resonant regions in the $O^+$ and $H^+$ distributions, respectively.

Diffusion time scales for the oblique mode are shown in figures 43-46. In figures 43 and 44 the measured electric field fluctuation level was used. Realistic diffusion time scales would correspond to the time it takes the beams to travel a distance of approximately $1 R_e$, which is about 10 seconds for hydrogen and about 40 seconds for oxygen. As one can see in figures 43 and 44, the ions diffuse in time scales longer than these times except for oxygen ions with small perpendicular velocities, less than $0.4 \times 10^7$ cm/s. This result is consistent with the observations, which show that considerable heating of the beams has taken place at lower altitudes. Any additional heating at or above the satellite would depend on how much wave turbulence is present. Near the satellite this additional heating is small, as is shown by the results presented here. At higher altitudes significant heating could occur if the beams entered a region with a hotter electron background. This would increase the growth region in k-space and would also shift the Landau resonance region into the oxygen distribution.
In figures 45 and 46 the electric field fluctuation level is increased to 2 mV^2/m^2 with all other parameters the same as figures 43 and 44. The shortest diffusion time in figure 45 corresponding to resonant hydrogen ions at \( v_{\parallel \text{res}} = -4.2 \times 10^7 \text{cm/s} \) is approximately 18 seconds, which is comparable to the time it takes a hydrogen ion to cover a distance of an earth radius. In figure 46 the oxygen diffusion times are small for the bulk of the resonant beam ions. Diffusion times for oxygen ions at \( v_\perp > 0.75 \times 10^7 \text{cm/s} \) are greater than the transit time for 1 R_e. These ions will diffuse little in velocity space if all other parameters are kept constant. As already mentioned, if the electron temperature suddenly increased or the beams got colder, the growth region in k-space will expand, giving faster diffusion times. It should also be mentioned that any heating of the beams will contribute to damping due to increased thermal velocities. This process will compete with any growth mechanism, such as an increase in electron temperature.

The divergent behavior of \( D_\perp \) at both small and large \( v_\perp \) in figures 44 and 46 result because \( D_\perp \propto \left( \frac{\omega_0 v_\perp}{v_\perp} \right)^2 \). When performing a computer simulation, the velocity space grid spacing is chosen so as to avoid numerical divergence.

Thus, figures 45, 46 show that with a slightly higher electric field fluctuation level than the extrapolated result from data, the beams should be heated in times comparable to their time of flight through the high altitude auroral zone. This still does not explain the parallel heating in both oxygen and hydrogen, which is the major feature in the observed distribution function.

At satellite altitudes the oblique wave does not heat oxygen para-
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The Landau resonance region lies outside the distribution, and occupies the low velocity side of the hydrogen beam. Its width is small, involving a fraction of the hydrogen ions. It is clear that the oblique mode can do little of the observed heating, but the Landau resonance region can be moved into the oxygen distribution by reducing the $H^+$ beam velocity to $5 \times 10^7 \text{ cm/s}$, keeping the $O^+$ parameters fixed. The lower boundary of the Landau resonance region in this case is $3.0 \times 10^7 \text{ cm/s} = -v_{\text{H}}$. As the $H^+$ beam velocity was lowered, it was also cooled to 50 eV. The cooling increased the growth region in k-space, but did not affect the location of the Landau resonance region, $\sim U_H - C_H$. The increased growth region also did not affect the width of the Landau resonance region. This is because as $k_{\text{H res}}$ increases, $\omega_K$ also increases, causing $v_{\text{H res}}$ to stay constant.

Also, in an attempt to model the beams at lower altitude, one must include the cooling of oxygen, which would put the oxygen ions further from the Landau resonance region. A good candidate for parallel heating, as mentioned earlier, is the slow parallel ion acoustic mode, the resonant regions of which are shown in figures 38 and 39. The Landau resonance region for this mode is extremely narrow, because the parallel phase velocity of the wave is a constant. This velocity will change as the beam velocities change or if the electron temperature changes. Thus, if these two parameters change over a distance small compared to an earth radius, say 100 - 200 km, different $H^+$ and $O^+$ ions will be in resonance with the wave at different altitudes. This could produce parallel heating. But, as mentioned earlier, the $H^+$ beam velocity and the electron temperature increasing together will change the Landau resonant velocity for the parallel mode slightly, since $v_{\text{phH}} = U_H - C_H$. 

The fluid two-stream instability most likely produced the bulk of the parallel heating. The instability occurs when $U_O + C_O = U_H - C_H$. This instability, which involves all particles and is explosive, would be most effective in filling in the valley between the $O^+$ and $H^+$ beams.
Diffusion Time (seconds)

$T_{HH}$ (sec)

$\nu_{||} = 4.4 \times 10^7 \text{ cm/s}$

$E^2 = 0.4 \text{(mV/m)}^2$

$\nu_{||} = 4.2 \times 10^7 \text{ cm/s}$

$V_\perp \times 1 \text{E8 cm/sec}$

Figure 43
Diffusion Time seconds $v_I = 2.6 \times 10^7 - 2.8 \times 10^7$ cm/sec

$0^\circ$ Resonant

$E^2 = 0.4 (mV/m)^2$

Figure 44
Figure 45

Diffusion Time (sec) $v_{ll} = 4.8 \times 10^7 \text{ cm/s}$

$H^+$ Resonant

$E^2 = 2 (mV/m)^2$

$T_{llll}$ (sec)

$V_\perp \times 10^8 \text{ cm/sec}$

$v_{ll} = 4.4 \times 10^7 \text{ cm/s}$

$v_{ll} = 4.2 \times 10^7 \text{ cm/s}$
Diffusion Time seconds \( v_{\parallel} = 2.6 \times 10^{7} - 2.8 \times 10^{7} \text{ cm/sec} \)

\[ E^2 = 2 \left( \frac{m}{e} \right)^2 \]

Figure 46
CHAPTER IV

CONCLUSION

This study has looked at aspects of plasma processes occurring in the high altitude auroral acceleration region. The major feature in the \( H^+ \) and \( O^+ \) distribution functions was the extended high energy tail in \( O^+ \) and the corresponding heating on the low parallel velocity side of \( H^+ \) which produced a filled in valley between the two ion beam peaks.

In order to identify the mechanism responsible for the parallel energy transfer from \( H^+ \) to \( O^+ \), we investigated low frequency electromagnetic instabilities generated by the relative streaming of the \( H^+ \) and \( O^+ \) beams. Although the beam velocities were much less than the Alfvén velocity, it was worthwhile to look for instabilities near the crossover frequency, \( \omega_c \). These efforts showed that the electromagnetic waves were damped, but the study found an instability of the oblique slow \( H^+ \) ion acoustic mode. This mode produced heating in both \( H^+ \) and \( O^+ \) through quasilinear diffusion, but it was concluded that it was not responsible for most of the parallel energy transfer from \( H^+ \) to \( O^+ \) which occurred at lower altitudes.

A good possibility to explain the parallel heating is the parallel ion acoustic wave. The parallel \( H^+ \) ion acoustic mode was found to be unstable if the electron temperature was increased to values of \( \sim 200 \) eV above the measured value at the satellite. The mode also gets excited when the ion beams are cooled. For example, if all plasma parameters at the satellite are fixed and the \( H^+ \) and \( O^+ \) beam temperatures are dropped
to 10 eV, the slow parallel H\(^+\) ion acoustic mode is excited. Continuing the work in the linear regime by Bergmann and Lotko (1986), one could investigate the ion heating produced by this mode and the H\(^+\)-O\(^+\) two-stream instability, which lies on the same branch.

The theoretical framework used to describe the plasma turbulence at satellite altitudes is adequate and gives accurate results, but it is by no means a unique solution. In the present theoretical scenario the dielectric tensor \(\epsilon_0(\omega, k)\) is evaluated with the real frequency, \(\omega_R\). This came about from the condition \(\omega_\| < \omega_R\). At certain regions in \(k\)-space the growth rate diverges and the mode cannot be followed with the present theory. This is a consequence of the approximation made on \(\omega_\|\).

A more accurate approach is to evaluate \(\Delta(\omega, k)\) with the complex frequency. Thus

\[
| \Delta(\omega, k) | = 0 = \text{Re} | \Delta(\omega_R + i\omega_\|, k) | + i \text{Im} | \Delta(\omega_R + i\omega_\|, k) | = 0
\]

in which case a mode with frequency \(\omega_R + i\omega_\|\) would exist where the real and imaginary parts of \(|\Delta|\) are simultaneously zero. A first guess at \(\omega_R\) and \(\omega_\|\) could be obtained from the present theory used in this study. The dispersion in \(k\)-space using this method would be most accurate. This has important consequences with respect to quasilinear diffusion because the restriction \(\omega_\| < \omega_R\) is relaxed somewhat. The fastest growing mode in \(k\)-space could be identified with complete certainty. The regions in \(k\)-space where \(\omega_\| (k)\) diverges would indicate the nonlinear development of the wave modes. In the present theory one can get only an estimate of where nonlinear processes are taking over, since the theory breaks down when \(\omega_\|/\omega_R\) increases.
With this in mind, it is still correct to say that the ion beams proved to be weakly unstable at satellite altitudes, a result that came out of the use of linear kinetic theory. The growth region in k-space was small, indicating marginal stability. Other theoretical formulations might give erroneous results. The instability is definitely a resonant process, involving resonant particles in the \( \text{H}^+ \) and \( \text{O}^+ \) distributions to provide growth. Thus, a fluid treatment might be inadequate as it ignores this microscopic effect. The application of a nonlinear treatment is a possibility one could consider. The first step is to do a quasilinear treatment of the \( \text{H}^+ \) and \( \text{O}^+ \) ion beams. This would involve modeling the \( \text{H}^+ \) and \( \text{O}^+ \) ion beams to what they would look like in velocity space at lower altitudes, and then allowing the beams to evolve by quasilinear diffusion. One would have to identify the instability with the highest growth rate as the process to do the diffusion. At different plasma parameters characteristic of lower altitudes, other modes might be excited, like the parallel ion acoustic instability or ion cyclotron modes, for example.

Another possibility is to follow the quasilinear evolution of the beams to higher altitude with the instability generated at the satellite. This could provide one with a definite idea of how the instability develops. With the diffusion times scales in figures 43-46, one can get a definite idea as to how the resonant ions will diffuse. Oxygen ions at small perpendicular velocities diffuse faster. For hydrogen the diffusion time is essentially a constant as a function of the perpendicular velocity of the beam ions. Rather, for hydrogen ions the crucial velocity is the parallel velocity of the resonant ion. As the beams flow to higher altitudes the instability will produce weak per-
perpendicular heating on the high velocity side of the $O^+$ beam, and an equally weak parallel heating of $H^+$ ions on the low velocity side of the beam, depending upon the value of the wave amplitude. This heating should take place over a distance on the order of an earth radius and the instability might be damped by this time if the electron temperature remains constant. This damping will be due to the increased thermal velocity of hydrogen in the parallel direction. The increased $T_\parallel$ in oxygen will not damp the wave because the instability depends on the parallel temperature of the ion beams. It is likely that the electron temperature will increase as the beams move upward. This will enhance the instability and counteract any damping. As the beams flow into weaker magnetic field regions the adiabatic cooling that will take place will also increase growth rates. I support this latter conclusion where the beams are unstable all the way to the plasma sheet and continue to undergo weak heating.

Finally, one can do a numerical simulation to see how the beams evolve under various forms of turbulence as they flow out of the acceleration region. Several workers are presently investigating this approach.

Aside from theoretical work, the advances made in the acquisition of plasma and wave measurements through the use of satellites are crucial in our understanding of space. The measurements come first, and the theory second. Some experimental efforts could include obtaining higher energy resolution, which would give better distribution functions. This would make the identification of the instabilities a more accurate process. The accompaniment of simultaneous wave measurements verifies this analysis. A satellite at low altitudes, such as
DE-2, can give simultaneous particle measurements with a high altitude satellite. This could identify roughly the same beam ions in the two regions.

The auroral acceleration region has been studied extensively over the past two decades. There are still problems to be worked out, such as the nature of the ion heating mechanism at low altitudes. This study provides a clue as to the nature of this mechanism, making the task of other workers less formidable.
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APPENDIX A

THE DIELECTRIC TENSOR
AND THE ELECTROSTATIC APPROXIMATION

The dielectric tensor contains all the information on electromagnetic waves in a homogeneous plasma immersed in a uniform magnetic field. No restriction is placed on the direction of \( \vec{k} \) or \( \vec{E} \). Some useful relations can be obtained when the dielectric tensor is evaluated in certain limits, with respect to the magnitude and direction of \( \vec{k} \) and \( \vec{E} \).

If the wave equation is written in terms of \( \vec{n} = \frac{\vec{k} c}{\omega} \), one obtains

\[
\vec{n} \times (\vec{n} \times \vec{E}) + \mathbb{E} \cdot \vec{E} = 0
\]

\[
(\hat{n} \hat{n} - \mathbb{E} + \mathbb{E}) \cdot \vec{E} = \Delta \cdot \vec{E} = 0
\]  \hspace{1cm} (A1)

Defining \( \vec{E} = \vec{E}_{||} + \vec{E}_{\perp} \) where \( \vec{E}_{||} \parallel \vec{k} \), and \( \vec{E}_{\perp} \perp \vec{k} \), and also taking the scalar product with \( \vec{n} \), one finds

\[
\vec{n} \cdot \mathbb{E} \cdot (\vec{E}_{||} + \vec{E}_{\perp}) = 0
\]

If \( \vec{E}_{||} \gg \vec{E}_{\perp} \), the wave is electrostatic, and the dispersion relation becomes

\[
\vec{n} \cdot \mathbb{E} \cdot \vec{n} = 0
\]  \hspace{1cm} (A2)

A further condition for electrostatic waves can be seen by rewriting the dispersion relation in terms of \( \vec{E}_{||} \) and \( \vec{E}_{\perp} \). One has

\[
(\hat{n} \mathbb{I} - \mathbb{E}) \vec{E}_{\perp} = \mathbb{E} \cdot \vec{E}_{||}
\]  \hspace{1cm} (A3)

This says that \( \vec{E}_{||} \gg \vec{E}_{\perp} \) when \( n^{2} \gg |\mathbb{E}_{i}| \) for all \( \mathbb{E}_{i} \).  \hspace{1cm} (A4)

In this limit, the dispersion relation for electrostatic waves, from
(A2), is
\[ k_{\perp}^2 \varepsilon_{xx} + 2 k_{\perp} k_{\parallel} \varepsilon_{xz} + k_{\parallel}^2 \varepsilon_{zz} = 0. \]  

(A5)

The condition (A4) for electrostatic waves implies that the phase velocity of the wave is much less than the velocity of light \( c \). Magnetic perturbations propagate at the Alfvén velocity \( V_A = B_0/\sqrt{4\pi \rho} \). If an electrostatic wave perturbs the magnetic field, these perturbations will be damped quickly if the phase velocity of the wave is less than \( V_A \).

At a resonance \( |\hat{n}| \), the refractive index, becomes infinite. At the lower hybrid and upper hybrid resonance \( \varepsilon_{ij} \) is finite. Thus, these waves are electrostatic. At the cyclotron frequencies \( \varepsilon_{ij} \) has a possibility of becoming infinite because of terms proportional to \( (\omega - k_{\parallel} v_{\parallel} - n\omega_{ce})^{-1} \). Thus, electrostatic electron and ion cyclotron waves will become electromagnetic at certain values of \( k \).
APPENDIX B

PROGRAM DISP3

Program DISP3 is the computer program which solves for the determinant of the electromagnetic tensor, $\Delta$. The roots of $|\Delta|$ are the normal modes of the plasma. $\Delta$ is given by

$$\Delta = \vec{n}\cdot\vec{n} - n^2 I + \varepsilon$$

$$\vec{n} = \frac{\vec{k} \cdot \vec{c}}{\omega}$$

$$(B1)$$

$$\begin{bmatrix}
-n^2 \cos^2\theta + \varepsilon_{xx} & \varepsilon_{xy} & n^2 \sin\theta \cos\theta + \varepsilon_{xz} \\
\varepsilon_{xy} & -n^2 + \varepsilon_{yy} & \varepsilon_{yz} \\
n^2 \sin\theta \cos\theta + \varepsilon_{xz} & -\varepsilon_{yz} & -n^2 \sin^2\theta + \varepsilon_{zz}
\end{bmatrix}$$

The program is written in CGS units. The package consists of the main program DISP3.F77 and submodules IDSB3.F77 and DISSB3.F77, which contain the subroutines. DISSB3.F77 contains subroutines PEMIN3, DTICK3, and EFUNCT. IDSB3 contains subroutines WREAL, ATSE, ALI, E1, PROD1,BESI1, MATCH, LIMITS, and BORDER.

The main program DISP3.F77 calls subroutine PEMIN3.F77, the input routine, which asks the user for all input parameters. The main program then calls subroutine EFUNCT.F77, which calculates $\varepsilon$ and takes the
determinant of $\Delta$. The real and imaginary parts of $\Delta$ are plotted vs. frequency $\omega_k$, so that one can identify the roots.

Subroutine PEMIN3 is the input routine for program DISP3. The user specifies the magnetic field strength, the electron and ion species with corresponding densities, temperatures, streaming velocities and the number of harmonic terms to keep in the summations over $n$. The program can contain a maximum of 10 species. The species $E^-$, $H^+$, $O^+$, $He^+$, and $NO^+$ are contained in the code along with their corresponding masses and charge states. These species can be changed to accommodate different plasmas.

The user specifies two of the following k-space parameters; $k$, $k_\perp$, $k_\parallel$, and $\Theta$. The x-axis options are, in addition to real $\omega$; $T_\parallel$, $T_\perp$, $B_0$, $Re(\omega/\omega_{\omega})$, a single $Re(\omega/\omega_{\omega})$, a single $Re\omega$, a single $Re(\omega/2\pi)$. The y-axis options are; the real or imaginary determinant, any real or imaginary element of $\Delta$, the real or imaginary determinant of $\Delta$ rotated through $\Theta$ so that $k \parallel \hat{z}$, any real or imaginary element of rotated $\Delta$, and the real or imaginary longitudinal dielectric function $\varepsilon(\omega,k)$. The plotting can be log or linear on both axes. A scale factor for the y-axis is asked for so as to include a linear scale around zero in log y.

Subroutine EFUNCT, called by DISP3.F77, evaluates the real and imaginary elements of $\xi$. The common area EREI contains the elements of the tensor for a single species. The arrays SER, SEI contain the real and imaginary elements of the tensor summed over species and the arrays SEROT, SEIROT contain the real and imaginary elements of the rotated tensor. Element 3,4 in the arrays SER, SEI is the longitudinal dielectric function and is identical to element 3,3 in SEROT, SEIROT. EFUNCT returns the determinant of $\Delta$, or any one of its elements.
Subroutine El, called by EFUNCT, calculates the elements of $\mathbf{\xi}$ for a single species, performing the summation over $n$. The routine is split into the $n=0$ terms, and a do loop over $n$ for the $n>0$ terms. El also calculates three limits of $\mathbf{\xi}$. These are: $T_u = T_L$, $u_o = 0$; $k_u = 0$, $k_L = 0$; and the cold plasma relations.

Subroutine WREAL, called by El, calculates the real part of the $W$ function, which is related to the plasma dispersion function $Z$. Three methods are used; interpolation, power series, and asymptotic series. If the argument of $W(z)$ is $> 0.5$ and $< 8.0$, the answer is obtained by interpolating from a table of tabulated values (Fried and Conte, 1961). If $z > 8.0$, the asymptotic series is used. If $z < 0.5$, the power series is used. The imaginary part of $W(z)$ is calculated directly in subroutine El.

Subroutine BESII1, called by El, calculates the Lambda function by series and asymptotic approximation, where $\Lambda_{\eta}(\lambda_\alpha) = I_\eta(\lambda_\alpha) e^{-\lambda_\alpha}$.

Subroutine PROD1, called by EFUNCT, calculates the determinant of $\Delta$ by direct product of its elements.

Subroutines ATSE and ALI, called by WREAL, are mathematical routines which perform the interpolation of the table of values in WREAL.

Subroutine MATCH, called by PEMIN3, matches a given ion supplied by the user to the list of ions contained in the program.

Subroutine LIMITS, called by PEMIN3, scales the x-y limits supplied by the user to values used by the plot package.

Subroutine BORDER, called by PEMIN3, draws a border and evenly spaced tick marks.

Subroutine DTICK3, called by PEMIN3, plots additional tick marks at the cyclotron and plasma frequencies of each species and at the lower and upper hybrid frequencies.