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University of New Hampshire

PH.D. 1986

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**ANALYSIS AND SIMULATION OF THE PNEUMATIC
BRAKING SYSTEM OF FREIGHT TRAINS**

BY

Khaled Sayed Abdol-Hamid

B.Sc, Faculty of Engineering, Ain Shams University, 1979

M.ENG, Concordia University, 1983

DISSERTATION

**Submitted to the University of New Hampshire
in Partial Fulfillment of
the Requirements for the Degree**

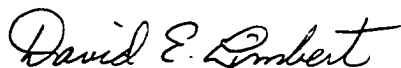
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NOMENCLATURE

$a_{i,1}$	Leakage cross-sectional area at the i^{th} car (section), $\text{m}^2(\text{in}^2)$.
$a_{i,2}^j$	ABD/ABDW vent valve flow area as a function of time step j , and car location i , $\text{m}^2(\text{in}^2)$.
$a_{i,3}^j$	ABD/ABDW flow area, connecting the i^{th} brake pipe to the inshot valve at the j^{th} time step, $\text{m}^2(\text{in}^2)$.
$a_{i,4}^j$	ABD/ABDW flow area, connecting the brake pipe to the auxiliary reservoir at the i^{th} car and j^{th} time step, $\text{m}^2(\text{in}^2)$.
$a_{i,5}^j$	ABD/ABDW flow area, connecting the brake pipe to the emergency reservoir at the i^{th} car and j^{th} time step, $\text{m}^2(\text{in}^2)$.
$a_{i,6}^j$	ABD/ABDW flow area, connecting the brake pipe to the quick service volume at the i^{th} car and j^{th} time step, $\text{m}^2(\text{in}^2)$.
$a_{i,7}^j$	ABD/ABDW flow area, connecting the brake pipe to the quick action chamber at the i^{th} car and j^{th} time step, $\text{m}^2(\text{in}^2)$.
$a_{i,8}^j$	ABDW flow area, connecting the brake pipe to the bulb volume of the AAV at the i^{th} car and j^{th} time step, $\text{m}^2(\text{in}^2)$.
A_i	Cross-sectional of the brake pipe in the i^{th} car (section), $\text{m}^2(\text{in}^2)$.
$A_{1,1}$	Feedback orifice cross-sectional area of the 26C relay valve, $\text{m}^2(\text{in}^2)$.

$A_{1,2}$	Area of the 26C relay valve diaphragm, $m^2(in^2)$.
$A_{1,3}$	Cross-sectional area of the 26C relay valve inner chamber orifice, $m^2(in^2)$.
$A_{1,4}^j$	Exhaust valve flow area of the 26C relay valve, $m^2(in^2)$.
$A_{1,5}$	Cross-sectional area of the 26C relay valve exhaust orifice, $m^2(in^2)$.
$A_{1,6}^j$	Supply valve flow area of the 26C relay valve, $m^2(in^2)$.
$A_{2,1}$	Area of the regulating valve diaphragm, $m^2(in^2)$.
$A_{2,2}^j$	Exhaust valve flow area of the regulating valve, $m^2(in^2)$.
$A_{2,3}^j$	Supply valve flow area of the regulating valve, $m^2(in^2)$.
$A_{3,1}$	Outer area of the brake pipe cut-off valve piston, $m^2(in^2)$.
$A_{3,2}$	Inner area of the brake pipe cut-off valve piston, $m^2(in^2)$.
$A_{3,3}^j$	Brake pipe cut-off valve flow area, $m^2(in^2)$.
$A_{i,3}^j$	ABD/ABDW flow area, connection the auxiliary reservoir with the inshot valve at the i^{th} car and j^{th} time step, $m^2(in^2)$.
$A_{i,5}^j$	ABD/ABDW flow area, connection the auxiliary reservoir with the emergency reservoir at the i^{th} car and j^{th} time step, $m^2(in^2)$.
A_{AAV1}^j	AAV flow area of the 278 control exhaust choke, $m^2(in^2)$.

A_{AAV2}	AAV flow area of the by pass choke, $m^2 (in^2)$.
A_{AAV3}^j	AAV check valve flow area, $m^2 (in^2)$.
A_{AAV4}	AAV flow area of the 278 breather choke, $m^2 (in^2)$.
A_{AVT}^j	AAV vent area, connecting bulb volume to atmosphere, $m^2 (in^2)$.
A_{BC}	Area of brake cylinder piston, $m^2 (in^2)$.
A_{BCV}^j	Brake cylinder vent area, $m^2 (in^2)$.
A_{EQVE}	Equivalent fixed restriction area of the regulating valve during recharge, $m^2 (in^2)$.
A_{EQVS}	Equivalent fixed restriction area of the regulating valve during application, $m^2 (in^2)$.
A_{INS}^j	Inshot effective flow area, $m^2 (in^2)$.
A_{INSP}	Inshot piston cross-sectional area, $m^2 (in^2)$.
A_{INS1}	Inshot fixed restriction flow area, $m^2 (in^2)$.
A_{INS2}^j	Inshot variable restriction flow area, $m^2 (in^2)$.
A_{QAV}^j	Equivalent quick action chamber vent flow area, $m^2 (in^2)$.
A_{QAV1}^j	Variable restriction of the quick action vent flow area, $m^2 (in^2)$.
A_{QAV2}	fixed restriction of the quick action vent flow area, $m^2 (in^2)$.
A_{QSV}	Quick service vent flow area, $m^2 (in^2)$.
A_{SS1}	A19 fixed restriction flow area, $m^2 (in^2)$.
A_{SS2}^j	A19 variable restriction flow area, $m^2 (in^2)$.
A_{P19}	Area of the A19 piston, $m^2 (in^2)$.
B	Coefficient matrix.
$B_{i,j}$	Coefficient of tridiagonal matrix.

$b_{i,j,k}$	Coefficient of the block matrix $B_{i,j}$.
c	Speed of sound, m/sec (ft/sec).
D_{EXP}^j/d_{exp}^j	Diameter for the corresponding A_{EXP}^j/a_{EXP}^j at the j^{th} time step, m (in).
D_{EXP}/d_{EXP}	Diameter for the corresponding A_{EXP}/a_{EXP} , m (in).
f	Friction factor.
$F_{1,1}^j$	Pressure force on the outer relay valve diaphragm, N (lb).
$F_{1,2}^j$	Pressure force on the inner relay valve diaphragm, N (lb).
i	Space index (car number).
j	Time index.
$K_{1,1}$	Exhaust valve spring constant of the 26C relay valve, N/m (lb/in).
$K_{1,2}$	Spring constant of the 26C relay valve diaphragm rod, N/m (lb/in).
$K_{1,3}$	Supply valve spring constant of the 26C relay valve, N/m (lb/in).
$K_{2,1}$	Supply/Exhaust valve spring constant of the regulating valve, N/m (lb/in).
$K_{3,1}$	Brake pipe cut-off valve spring constant, N/m (lb/in).
K_{CV}	Accelerated emergency release check valve spring constant, N/m (lb/in).
K_{BC}	Brake cylinder spring constant, N/m (lb/in).
K_{INS}	Inshot valve equivalent spring constant, N/m (lb/in).

K_{P19}	A19 spring constant, N/m (lb/in).
K_{SV}	Emergency slide valve spring constant, N/m (lb/in).
L_{EXP}	Spring preload for the corresponding spring constant, K_{EXP} , N (lb).
λ	Total length of the train (brake pipe), m (ft).
\dot{m}_{EXP}^j	Normalized mass flow rate through the corresponding flow area, A_{EXP} , A_{EXP}^j , a_{EXP} or a_{EXP}^j .
$\dot{m}_{1,1}$	Mass flow rate through $A_{1,1}$, kg/sec (lb/sec).
$\dot{m}_{1,2}$	Mass flow rate through $A_{1,3}$, kg/sec (lb/sec).
$\dot{m}_{1,3}$	Mass flow rate through $A_{1,6}^j$ or A^j ($A_{1,4}^j, A_{1,5}^j$), kg/sec (lb/sec).
$\dot{m}_{2,1}$	Mass flow rate through A_{REG}^j ($(A_{EQVS}, A_{2,3})$ or $(A_{EQVE}, A_{2,2})$), kg/sec (lb/sec).
$\dot{m}_{3,1}$	Mass flow rate through $A_{3,1}^j$, kg/sec (lb/sec).
N	Number of car ($\lambda = \sum_{i=1}^N \Delta x_i$).
$P_{i,BC}^j$	Normalized brake cylinder pressure at the i^{th} car.
P_{EXP}^j/P_{EXP}	Normalized pressure downstream of A_{EXP}^j or a_{EXP}^j/A_{EXP} or a_{EXP} .
P_S	Supply pressure, N/m ² (lb/in ²).
P_{mr}^j	Pressure of the A_{19} volume, N/m ² (lb/in ²).
P_A	Atmospheric pressure, N/m ² (lb/in ²).
P_{eq}	Equalizing reservoir pressure, N/m ² (lb/in ²).
$P_{1,1}$	Relay valve outer diaphragm pressure, N/m ² (lb/in ²).

$P_{1,2}$	Relay valve inner diaphragm pressure, N/m^2 (lb/in ²).
$P_{1,3}$	Relay valve intermediate volume pressure, N/m^2 (lb/in ²).
$P_{3,1}$	$=P_{1,3}$, N/m^2 (lb/in ²).
Q	Right hand side array of equation (2.43).
$r_{\beta,1}, r_{\beta,2}$	Flags used in equation (5.3).
R_g	Gas constant, $J/kg \cdot ^\circ K$ (in ² /sec ² $^\circ R$).
t	Time, sec.
T	$= \frac{tC}{\lambda}$, normalized time.
U	$U(X,T) = \frac{M(X,T)}{P(X,T)} = \frac{u}{c}$, normalized velocity at distance X and time T .
V_{EXP}	Normalized volume for the corresponding P_{EXP} .
$V_{i,j}$	Volume for the corresponding pressure $P_{i,j}$, m ³ (in ³).
v	Branch pipe volume per unit length, m ² (in ²).
W_{SV}	Emergency slide valve weight, N (lb).
x	Distance along the brake pipe from the head end, m (ft).
X	Normalized spacial coordinate, $\frac{x}{\lambda}$.
x_{BC}^j	Brake cylinder piston movement, m (in).
x_{CV}^j	Accelerated emergency release check valve movement, m (in).
x_{ECV}	AAV exhaust check valve movement, m (in).
x_{INS}	Inshot valve piston movement, m (in).
x_{SV}	Emergency valve piston movement, m (in).
x_{SS2}	A19 check valve movement, m (in).

$X_{1,1}$	Relay valve diaphragm movement, m (in).
$X_{2,1}$	Regulating valve diaphragm movement, m (in).
$X_{3,1}$	Brake pipe cut-off valve piston movement, m (in).
Y	Array contains the dependent variables at the $j+1$ time step.
Γ	Ratio of specific heats.
z	$z(X,T)$, Normalized leakage flow per normalized unit length.
z_i^j	Normalized leakage/branch pipe flow per normalized unit length from the i^{th} car.
α_i^j	$= z_i^j \Delta X_i = \sum_{K=1}^{L_i} \alpha_{i,k}^j$
$\alpha_{i,k}^j$	Normalized flow rate from the brake pipe through $a_{i,k}^j$.
μ	Kinematic viscosity of the air m^2/sec (in^2/sec).
ρ_{EXP}	Air density for the corresponding P_{EXP} , kg/m^3 (lb/in^3).

ABSTRACT

ANALYSIS AND SIMULATION OF THE PNEUMATIC BRAKING SYSTEM OF FREIGHT TRAINS

by

Khaled Sayed Abdol-Hamid

University of New Hampshire, May, 1986

In North America, and in many other parts of the world, freight trains still employ pneumatic equipment for braking. The 26C locomotive valve and control valves of the cars are connected by a brake pipe which serves the dual purpose of conducting pneumatic power and transmitting the control signal (brake signal) from the locomotive to each car of the train. The development of the air brake system has been largely empirical and experimental, which may take years to develop a new designs.

In this thesis, a new brake pipe mathematical model is developed, including the interaction of the branch pipe. Two different numerical methods (finite difference and finite element techniques) are developed to provide transient and steady state solutions for the new brake pipe model with leakage. Two mathematical models are developed to describe and study the dynamic behaviour of the 26C

locomotive valve parts, namely the complete and modified models. The combination of the implicit finite difference solution technique and the modified model of the 26C valve are found to be better than any other of the present developed combinations for providing an accurate and fast simulation.

Mathematical models for the ABD/ABDW subvalves are developed and incorporated with the air brake system model. The new mathematical models for the ABD/ABDW are capable of describing the dynamic behaviour of these valves for any of the several application modes (service, emergency, recharge, dry charge and recharge after emergency).

These models and numerical techniques were used to develop a simulation program for the air brake system. This Fortran program is a very good tool to analyze and simulate the transient and steady state behaviour of the air brake system.

CHAPTER 1

INTRODUCTION

1.1 Air Brake System

Most trains in North America still employ pneumatic brakes. A schematic illustration of a freight train brake system is shown in figure (1.1). An automatic air brake system of a freight train typically consists of a locomotive control unit situated at the head-end of the train, car control unit located at each car, and a pipe (brake pipe) connecting the locomotive control unit to car control units.

In some locomotives, the 26C locomotive valve (figure 1.2), is used for the following purposes:-

1. charging the auxiliary and emergency reservoirs of the car control unit and the auxiliary reservoir of the locomotive control unit.
2. varying the brake pipe pressure to cause the automatic train brakes to apply or release.

After the brake pipe is charged with the 26C locomotive valve handle in release position, movement to the right will cause a reduction in the equalizing reservoir pressure. This pressure reduction will cause a reduction in the brake pipe pressure. Figure (1.3) shows the 26C locomotive valve handle positions:

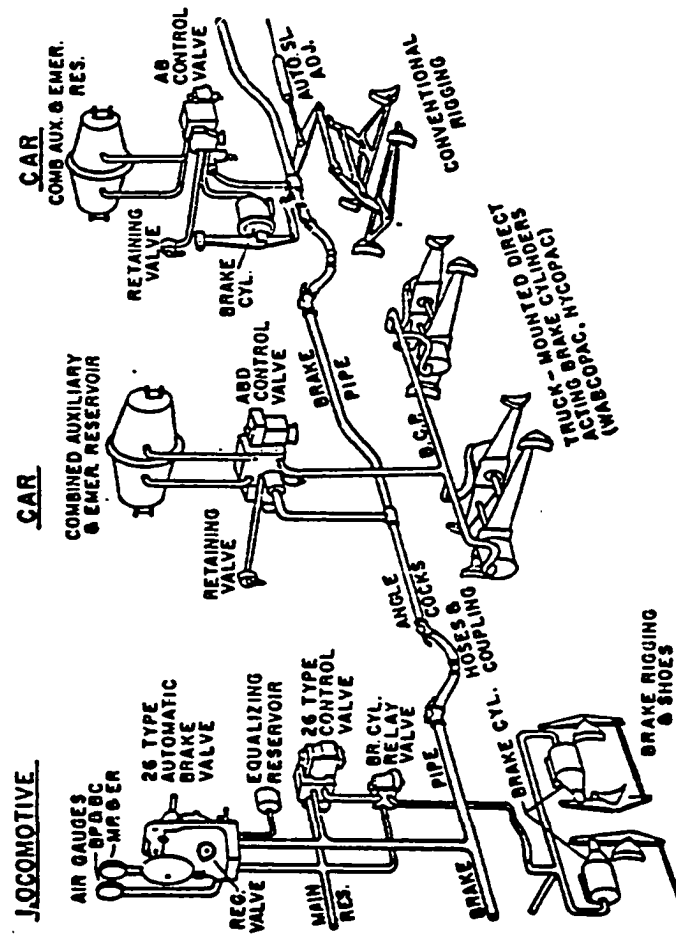


Figure 1.1: Schematic of the Air Brake System

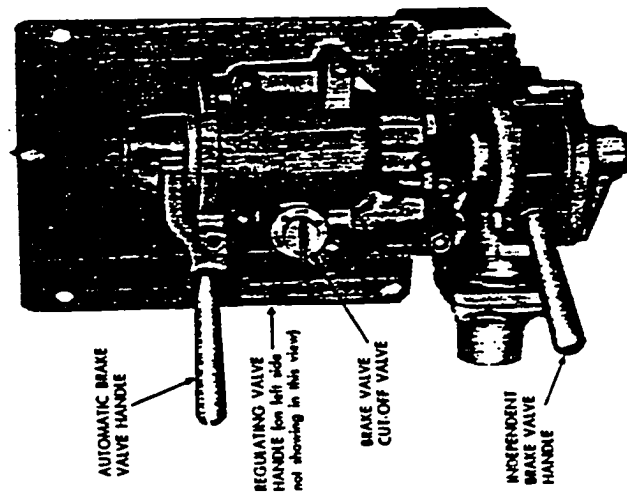


Figure 1.2: 26C Automatic Locomotive Valve

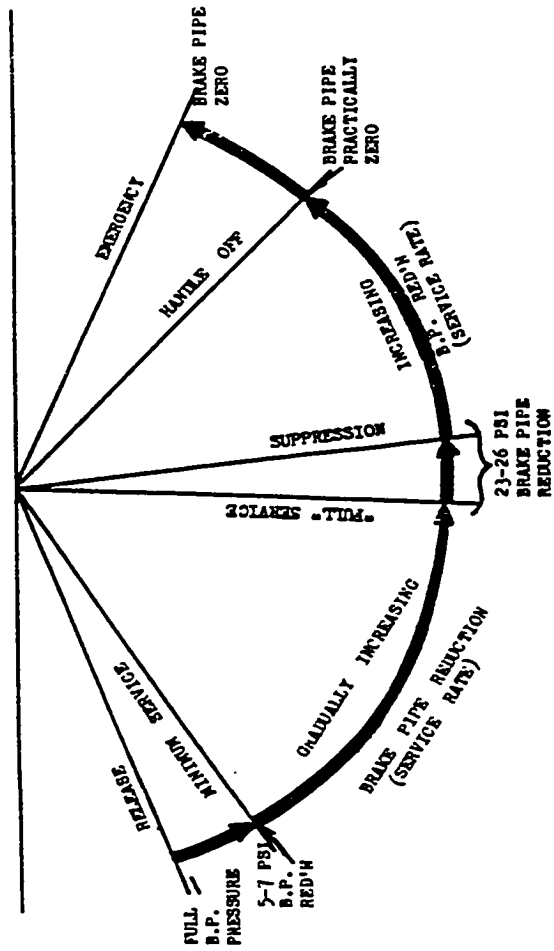


Figure 1.3: 26C Locomotive Valve Handle Positions

1. Release position, the equalizing reservoir and brake pipe pressure correspond to the regulating valve knob setting. The pressure in the brake pipe is usually a little less than that in the equalizing reservoir. A small pressure differences are needed to actuate the valve and usually there is some flow through the valve because of leakages in the train.
2. Minimum service position: 5-7 psi reduction in equalizing and brake pipe pressures. A further reduction can be achieved by moving the handle progressively towards the right.
3. Full service position: 23-26 psi reduction of the equalizing and brake pipe pressures.
4. Suppression position: 23-26 psi reduction and air flows into "Suppression Pipe" to permanently suppress safety control.
5. Handle off position: brake pipe pressure is further reduced to approximately 30 psig because of the action of the brake pipe cut-off valve, and equalizing reservoir pressure to approximately 0 psig. Leakage will eventually drops the brake pipe pressure to 0 psig.
6. Emergency position: brake pipe pressure is quickly vented through a large opening to atmosphere.

The car control unit consists of a control valve (ABD/ABDW), emergency reservoir (3500 in³), auxiliary reservoir (2500 in³), brake cylinder and shoes. A train

could consist of as few as one control unit (1-car) or could contain many more. A long train could contain up to 200 car control units.

The locomotive and car control units are connected together in chain like manner by the brake pipe (train-line). The brake pipe is connected by hoses and glad hands at the end of each car and locomotive unit. The brake pipe supplies pneumatic power to operate the brake cylinders and transmits the control signal (locomotive handle signal) from the locomotive to the last car of the train.

One of the factors that most affects air brake system performance is leakage. There are two classifications of leakage:

1. Brake pipe leakage: the leakage in the hose assemblies, angle cocks and branch pipes.
2. System leakage: brake pipe leakage plus leakage in the control valves (ABD/ABDW) and other types of control valves.

Brake pipe leakage occurs at pipe fittings, threaded joints, valves and gaskets. A typical freight car has from 40 to as many as 100 joints and seals [1]. This leakage is due to extreme environmental conditions and long maintenance intervals. Both are common in the railroad environment. As a result of leakage, there is a considerable difference between locomotive pressure and last car pressure, this difference is referred to as the pressure gradient by the railroad industry [1]. The amount of the pressure gradient

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depends on the location and size of the leakage [2]. As the leakage increases, resulting in an increased pressure gradient, the braking capability and response of the brake system is degraded. It is therefore necessary for the locomotive engineer to know the leakage present in the brake system.

A computer simulation program for the air brake system is needed to understand its different functions. This program is needed as a useful tool for the railroad industry. Among its several applications, it may be used to,

- a. study the effects of leakage on steady state and transient behaviours of the air brake systems,
- b. provide for brake cylinder pressures, which can be used by a different computer program to study the forces which develop at the coupler between cars, during normal service operations,
- c. develop new criteria for evaluating brake system performance,
- d. design new and simpler control valves,
- e. detect and test the causes of existing field failure, and
- f. determine the sensitivity of the air brake system to the valve characteristics.

In order to achieve these goals, one may need to develop an accurate and simple model for each of the following air brake system components:

1. Brake pipe model that is capable of representing a pipe with different cross-section areas and lengths.
2. 26C locomotive valve.
3. A19 flow indicator adaptor.
4. Control valves (ABD/ABDW).
5. Brake cylinder.

The main objective of this research work is to develop a computer simulation program for the pneumatic braking system. This program should be designed for easy modifications such that the user can:-

- a. add modules for the new control valves,
- b. change in the dimensions and configurations of the brake pipe, the 26C locomotive and the control valves (up to five),
- c. have more than one control unit designs with unlimited numbers of each unit, at each car of the train, and
- d. have different configurations for the brake cylinder, auxiliary reservoir and emergency reservoirs.

1.2 Review of Previous Work

During the last 10 years, the University of New Hampshire and New York Air Brake Company cooperated in the development of a Fortran Program, the main purpose of which is to simulate the Air Brake System of the freight train. Many theses (Gauthier [7], Wright [8], Banister [9] and

Kreel [10]) and Technical Reports were the result of this project under the supervision of Dr. Charles K. Taft, at University of New Hampshire. They have developed a mathematical model for the brake pipe, several functions of the control valves (ABD and ABDW), brake cylinder, and reservoirs as well as a simple model for the 26C locomotive valve. Others have also made efforts toward modeling various elements of the air brake system. This section reviews some of the development of all these researchers.

Many researchers have attempted to develop a mathematical model to represent the brake pipe pressure and mass flow rate. In the case of the brake pipe problem, the inside diameter of the pipe is small as compared with its length, which causes high frictional losses. Velocity change in the radial direction may be neglected. In these studies, the flow inside the brake pipe is assumed to be one dimensional as suggested by Abdol-Hamid [2], Funk and Robe [3], Shute et. al. [4], Cheng, Katz and Abdol-Hamid [5], Ho [6], Gauthier [7], Wright [8], Banister [9], Kreel [10], Abdol-Hamid, Limbert and Chapman [11] among others. Many of the brake pipe mathematical models fail to incorporate the branch pipe with the brake pipe governing equations. However, Refs [4,7,8,9,10] have incorporated the effect of the branch pipe by increasing the diameter of the pipe to give it an equivalent volume.

The brake pipe is basically a fluid transmission line. The transient behaviour of fluid transmission lines has been

the subject of many investigations. Many researchers developed closed form solutions to a linearized form of the transmission line governing equations. One of the earliest investigations into the transient response is that of Schuder and Binder [12]. They developed a theory and performed experiments for the step response of long pneumatic transmission lines terminated by a volume or blockage. Their theory does not consider heat transfer effects and assumes a constant laminar friction. Nichols [13] included the effects of heat transfer and the change in the laminar velocity profile shape. Simultaneously, Brown [14 & 15] developed the propagation operator in terms of Laplace transforms. Cheng, Katz and Abdol-Hamid [5] showed that the semi-infinite line solution can be used to predict the transient response of the blocked line. For a matched line (semi-infinite line), several solutions have been presented. Kantola [16] obtained an infinite series representation for the impulse and step response and Karm [17] transformed a simplified frequency domain model into the time domain. Brown and Nelson [18] found the step response solution of liquid lines from the frequency response. Katz [19] developed the direct conversion of the exact frequency response for terminated transmission lines into transient response.

Most of the numerical techniques and solutions, mentioned above, lack the ability to present the steady state solution of the brake pipe problem including leakage.

Also, a mathematical model is needed to study the effect of leakage in the brake pipe steady state behaviour.

It is also difficult to employ any of the methods mentioned above to predict the behaviour of a real brake pipe, because of the complex boundary conditions (the presence of the locomotive and control valves, brake cylinder, and auxiliary and emergency reservoirs at each car), in addition to leakage. Refs. [4,7,8,9,10] did include the complex boundary condition, using the method of characteristics.

The method of characteristics has been used by many researchers to solve the transients of fluid transmission lines. Zielke [20] and Brown [21] have presented a quasi-method of characteristics that takes into account frequency dependent shear and heat transfer. This method has been applied by Krishner and Katz [22] to find the step and pulse responses of blocked line. Abdol-Hamid [2] used the finite difference techniques (explicit and implicit) to find the transient response of the brake pipe problem. Funk and Robe [3] developed the method of lines for solving the brake pipe governing equations. They used the finite difference technique in the space coordinate, x , to reduce the governing partial differential equations into a set of differential-difference equations, which are solved by Kutta-Fehlberg algorithm [23]. Ho [6], Gauthier [7], Wright [8], Banister [9] and Kreel [10] have used the method of characteristics to find the transient response of the brake

pipe as a pneumatic transmission line and combined it with locomotive and car control valves models. Good correlation with the experimental results were obtained for many types of brake operation. The computer program, as was developed, could not be used for trains containing cars of various lengths, brake pipe diameter, and valve types. In order to handle the case of various length, a small uniform length has to be used, where several element must be used to represent one car. This will cause the use of longer solution time.

Many researchers developed mathematical models to study the steady state behaviour of the brake pipe with leakage. In 1977, Katz and Cheng [23] first proposed a ladder network model for the train-line for the purpose of studying the pressure distribution in steady state conditions. Then, Cheng, Abdol-Hamid and Katz [24] used the ladder network to develop two methods for locating multiple faulty leaks. Abdol-Hamid [2] proposed a mathematical model to study the behaviour of the train-line and developed two fault locating methods for the train-line with leakage. Shute, Wright, Taft and Banister in [4] used their mathematical model to study the effect of the leakage on brake pipe steady state performance. They have used the method of characteristics to solve the mathematical model equations, which is better suited for transient calculations than for strictly steady state behaviour because it requires a large computer time.

The 26C locomotive valve plays the important role of initiating the pressure control signal to the air brake system. Ho [6], Gauthier [7] and Wright [8] have attempted to develop a simplified model for the 26C relay valve (a part of the 26C valve), which is used to represent only the service mode of the valve. The relay valve was represented by a fixed restriction (.25 (in) in diameter) through which the brake pipe was vented to atmosphere. Banister [9] developed a mathematical model for the relay valve, which may represent both the service and release modes. This model does not accurately represent the dynamic behaviour of the 26C locomotive valve. Kreel [10] developed a mathematical model for the A19 flow indicator adaptor, and he used the same 26C model developed by Banister [9]. The author of this dissertation is not aware of the existence of a complete mathematical model, which describes accurately the functions of all the main parts of the 26C locomotive valve. Models for the regulating valve and brake pipe cut-off valve are needed for a more complete 26C model.

The control valve (ABD, ABDW,... etc.) is basically a pressure sensor device, which is activated by the local brake pipe pressure time rate of change (amount and direction).

During the service mode (pressure in the brake pipe is dropped), a brake sequence is activated, which allows air to flow from the auxiliary reservoir to the brake cylinder. Gauthier [7] developed two mathematical models to represent

the service mode of the ABD control valve and brake cylinder filling process respectively. Gauthier [7] was also the first to use the input-output diagram (quasi-steady assumption) to model the control valve. Wright [8] developed a mathematical model for the Accelerated Application Valve (AAV), which may be combined with the ABD model to construct a mathematical model for the ABDW control valve.

During the release mode (pressure in the brake pipe is increased), the control valve connects the auxiliary and emergency reservoirs to the brake pipe. Banister [9] developed a mathematical model to represent the release after application mode. In order to model the release after emergency mode, the function of the accelerated emergency release check valve should be incorporated in the brake valve model.

During the emergency mode (large and rapid pressure decrease in the brake pipe pressure), auxiliary and emergency reservoirs are connected to the brake cylinder. Kreel [10] developed a mathematical model to represent the emergency mode. In order to have a complete model for the emergency mode, the functions of the inshot and high pressure spool valves needed to be added to the mathematical model developed by Kreel [10]. Gauthier [7] developed the input-output diagrams for each of the inshot valve and the high pressure spool valve.

This section concludes that, there is a need for a complete model/program for the air brake system. This requires the following:

1. A brake pipe mathematical model should be developed, which includes a separate flow contribution due to the interaction with the branch pipe.
2. A new numerical technique should be used to solve the brake pipe governing equations. The numerical technique should be capable of providing efficient (accurate and fast) transient and steady state solutions for a brake pipe. It should be capable of modeling cars with various lengths brake pipe diameters, and amount of leakage.
3. An accurate and complete mathematical model for the main working parts of the 26C locomotive valve. Then, the model should be modified to be an integral part of the air brake system program to provide the functions of the valve while allowing an efficient solutions.
4. The ABD/ABDW should include the modeling of the following sub-valves:-
 - a. accelerated emergency release check valve,
 - b. inshot valve,
 - c. high pressure spool valve.
5. The Fortran Program needs to be designed for easy modification, such as allowing the user to add models of new or different components of the air brake system.

6. The air brake system mathematical model needs to be validated. This implies that the form of the models used for the components and sub-valves of the system should be correct, and proper values should be used. The model should be compared to experimental data as did [4,7,8,9,10] for all types of operations to be simulated.

1.3 Scope of the Dissertation

This dissertation attempts to develop a complete model/program (computer simulation) for the air brake system satisfying the needs listed above.

Chapter 2 describes two numerical techniques that can provide the solutions of the brake pipe governing equations. The brake pipe mathematical model is developed in Appendix A. The two numerical techniques are: the finite difference (implicit and explicit) and the implicit finite element.

Chapter 3, begins by explaining the operation principle of the 26C locomotive valve and the functions of its parts. Then, the complete model of the 26C relay, regulating, and brake pipe cut-off valves are developed. Then a simplified model of the 26C relay and brake pipe cut-off valves is developed to be incorporated with the rest of the air brake system components.

Chapter 4, explains the functions of each of the air brake system components. The corresponding models of the control valves are developed in appendices G and H. The

mathematical models of accelerated emergency release check, inshot and high pressure spool valves are developed in Appendix G. The same Appendix shows how these models are incorporated with the mathematical model of the ABD, developed by Gauthier [7] (service mode), Banister [9] (release mode), Kreel [10] (emergency mode). Appendix H describes the correct formulation of the AAV mathematical equations, proposed by Wright [8]. Appendix I describes the mathematical model of the A19 flow indicator adaptor, developed by Kreel [10].

Verification of the models is also demonstrated in this chapter by comparison with experimental results. Three different experimental set-ups are used to validate the mathematical models of the 26C locomotive valve, brake pipe, and the complete model of the entire air brake system respectively.

Chapter 5, investigates the effect of leakage on the steady state behaviour of the brake pipe. The implicit finite difference scheme is used to provide the steady state solution of the brake pipe mathematical model.

Chapter 6 briefly summarizes this dissertation results and conclusion.

Finally, Appendix L describes and lists the air brake system Fortran Program

CHAPTER 2
NUMERICAL SOLUTION
OF
THE BRAKE PIPE MATHEMATICAL MODEL

2.1 Introduction

The analysis presented in this chapter is for a brake pipe with leakage or\and branch pipe flow, but it may be applied to any pneumatic transmission line. The one-dimensional formulation of the governing equations has been assumed [2,3,4,6,7,8,8,10,11]. We begin with modeling of the brake pipe and formulating the system equations in Section 2.2. The full derivations of the brake pipe governing equations is explained in detail in Appendix A. In Section 2.3, two finite difference formulations are presented to solve the brake pipe (train-line) governing equations. Section 2.4 describes the finite element formulation which is used to solve the governing equations. Finally, section 2.5 summarizes the numerical development made in this chapter, and also describes one of the techniques which may be used to solve the final algebraic system of equations.

2.2 Brake Pipe Mathematical Model

The laws of conservation of mass, momentum, and energy and the equations of state for air are used to develop the

governing equations (see Appendix A for further derivation details). The governing equations for the brake pipe with mass flow removed per unit length, s (leakage flow or/and flow to the brake valve unit through a branch pipe volume per unit length, v) are described as follows:-

Continuity Equation

$$(A+v) \frac{\partial \rho}{\partial t} + \frac{\partial \rho u A}{\partial x} + s = 0.$$

or, $\frac{\partial \rho}{\partial t} + \frac{1}{(A+v)} \frac{\partial \rho u A}{\partial x} + \omega = 0.$ (2.1)

(for car section that have no branch pipe, $v=0$)

x-Momentum Equation

$$\frac{\partial \rho u}{\partial t} + \frac{1}{A} \frac{\partial \rho u^2 A}{\partial x} + \frac{\partial p}{\partial x} + f_{\text{avg}} \frac{\rho u^2}{2d} \left[\frac{u}{|u|} \right] = 0. \quad (2.2)$$

where,

$A = \frac{\pi}{4} d^2(x)$, cross-sectional area of the brake pipe as a function of distance, $x(\text{m})$, m^2 .

$p = p(x,t)$ is the air pressure inside the brake pipe as a function of distance, $x(\text{m})$, and time, $t(\text{sec})$, kPa .

$u = u(x,t)$ is the air velocity inside the brake pipe as a function of displacement, x , and time, t , m/sec .

$v = v(x)$ is the branch pipe volume per unit length (Δx) as a function of displacement, x , m^2 .

$s = \text{leakage per unit length, kg/(sec.m)}$.

$\rho = \rho(x,t)$ is the air density inside the brake pipe as a function of distance, x , and time, t , kg/m^3 .

$\omega = \frac{s}{A}$,

$$\Lambda = \frac{A}{A+v},$$

$$\Lambda_{avg} = \frac{1}{\Delta x} \int_x^{x+\Delta x} \Lambda dx \quad (2.3a)$$

and, the average hydraulic diameter is:

$$\bar{d} = \sqrt{\frac{4}{\pi} \Lambda_{avg}} = \sqrt{\frac{4}{\pi \Delta x} \int_x^{x+\Delta x} \Lambda dx} \quad (2.3b)$$

$\frac{u}{|u|}$ in equation (2.2) is used to account for the "opposing motion" quality of friction. f is the wall friction factor, which is a function of the local Reynolds number R_e .

$$R_e = \frac{u \bar{d}}{\mu} \quad (2.4)$$

where,

$$\mu = \text{the kinematic viscosity of the air m}^2/\text{sec.}$$

$$\propto \frac{1}{p}$$

The basic functional form used for the friction factor, f , is

$$f = a R_e^b \quad (2.5)$$

where a and b are selected to give a good fit to data for different flow regimes, laminar, transition and turbulent, (see Appendix K).

Isothermal flow is assumed as suggested by Abdol-hamid [2], Funk and Robe [3], and Gauthier [7], among others. Therefore, the density can be related to pressure as in the:

Equation of State

$$\rho = \frac{p}{R_g \theta} \quad (2.6)$$

where,

R_g = gas constant, J/(kg. °K).

θ = local temperature (assumed to be constant), °K.

These equations are now normalized by introducing the variables as:

$$M = \frac{\rho u}{\rho_A c} \quad P = \frac{p}{p_A} \quad \xi = \frac{s \lambda}{\rho_A c A} \quad (2.7a)$$

$$X = \frac{x}{\lambda} \quad T = \frac{t}{\lambda} \quad (2.7b)$$

Where,

c = the local speed of sound, m/sec.

p_A = local atmospheric pressure, N/m².

λ = the total length of the brake pipe, m.

ρ_A = local atmospheric density, kg/m³.

When those relations are substituted into (2.1) and (2.2) and we substitute for ρ using equation (2.6), the following normalized equations of motion are obtained:

$$\frac{\partial P}{\partial T} + \frac{1}{A+v} \frac{\partial MA}{\partial X} + \Lambda \xi = 0 \quad (2.8)$$

$$\frac{\partial M}{\partial T} + \frac{1}{A} \frac{\partial MUA}{\partial X} + G_1 \frac{\partial P}{\partial X} + G_2 M = 0 \quad (2.9)$$

where,

$$U = \frac{M}{P} \quad (2.10a)$$

$$G_1 = \frac{R_g \theta}{c^2} \quad (2.10b)$$

$$\text{and, } G_2 = f \frac{A_{avg}}{A} \frac{\lambda}{2 \bar{d}} \frac{U^2}{|U|} \quad (2.10c)$$

The continuity equation (2.8) and momentum equation (2.9) form a pair of nonlinear partial differential

equations in terms of two dependent variables, mass velocity M (or U) and pressure P , and two independent variables, distance along the pipe line X and time T .

At this point, let us describe the brake pipe mathematical model as shown in figure (2.1). The brake pipe is a continuous network model, consisting of N sections of lengths (ΔX_i) . Let P_i^j and M_i^j be defined as the dependent variable pressure and velocity at grid points $X=X_i (0 \leq X_i \leq 1)$ and $T=T_j (0 \leq T_j \leq \infty)$. The leakage flow, Σ , is lumped at each node as α_i^j . where,

$$\alpha_i^j = \Sigma_i^j \Delta X_i \quad (2.11)$$

$$\text{and, } \alpha_i^j = \sum_{k=1}^{L_i} \alpha_{i,k}^j$$

where, $\alpha_{i,k}^j$ is the k^{th} mass flow out of the brake pipe (through leakage or brake pipe valve) at the i^{th} section in the j^{th} integration time step. The index k varies from 1 to L_i , the number of leakage flows at the node.

The brake pipe (train-line) behaves as a long transmission line with a significant damping (friction, flow through branch pipe..etc) affecting the transient behaviour of the pressure and mass transfer inside the line. Thus, for simplicity in the following analysis, all air leakages or/and brake valves are assumed to be lumped at the end of each space element. There are two ways to calculate the mass flow rate $\alpha_{i,k}^j$, the isotropic approach and the average density approach. First the isotropic approach, here $\alpha_{i,k}^j$ can be presented as a nonlinear function of the leakage up

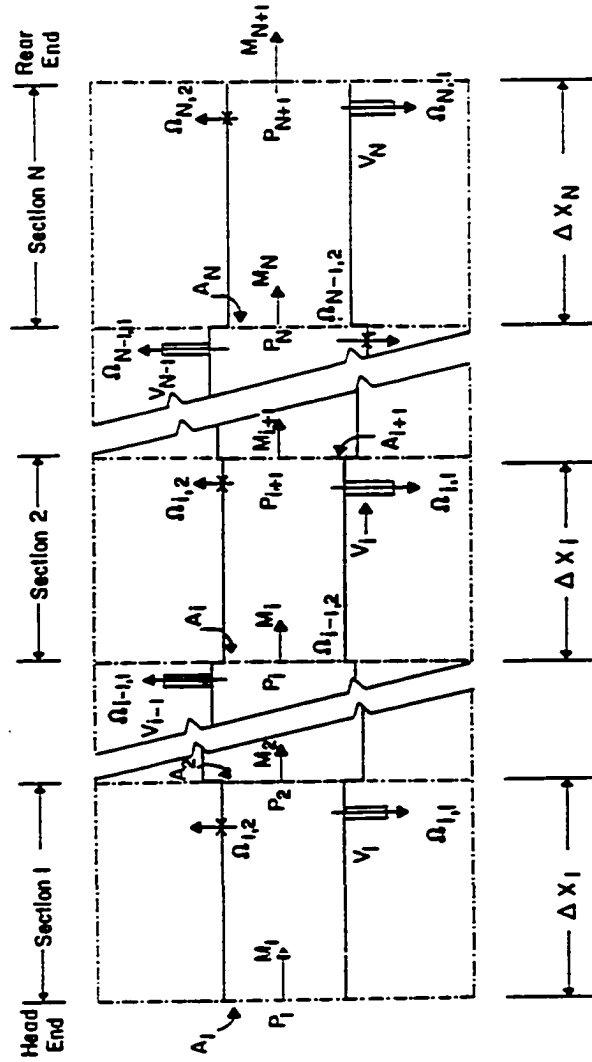


Figure 2.1: Brake Pipe Mathematical Model

stream and down stream pressure (P_i^j and $P_{i,k}^j$) for subsonic flow at the i -th node, and as a linear function of the P_i^j for sonic flow, as follows:

subsonic, for $P_i^j \leq 1.893P_{i,k}^j$

$$\dot{m}_{i,k}^j = \frac{a_{i,k}}{A_i} P_i^j \left[\frac{2}{\gamma-1} \left[R_{i,k}^{-\frac{2}{\gamma}} - R_{i,k}^{-\frac{\gamma+1}{\gamma}} \right] \right] \quad (2.12a)$$

where,

$$R_{i,k} = \frac{P_i^j}{P_{i,k}^j} \quad (2.12b)$$

sonic, for $P_i^j > 1.893P_{i,k}^j$

$$= 0.58 \frac{a_{i,k}}{A_i} P_i^j \quad (2.12c)$$

where $a_{i,k}$ is the k^{th} restriction (leakage or brake valve flow area) at the i^{th} node, A_i is the brake pipe cross-sectional area for the i^{th} section. Second, the average density approach suggested by Gauthier [7] and Banister [9]. The mass flow rate is related to the upstream and downstream pressure and flow area as

$$\dot{m}_{i,k}^j = 0.6 \frac{a_{i,k}}{A_i} P_i^j \sqrt{\frac{|R_{i,k}^2 - 1|}{\gamma} \frac{|R_{i,k} - 1|}{R_{i,k} - 1}} \quad (2.12d)$$

So, we see the general form for equations (2.12a), (2.12c) and (2.12d) is

$$\dot{m}_{i,k}^j = K_i^j P_i^j \frac{a_{i,k}}{A_i} \quad (2.12e)$$

Where, K_i^j may represent the nonlinear resistance per unit area of the leakage at the i -th node. K_i^j is evaluated at each node and time from the sonic and subsonic relation

when using the isotropic approach, or from terms in equation (2.12d) when using the average density approach.

Gauthier [7] and Banister [9] show that, the difference between using equation (2.12a-c) and (2.12d) is always less than 10%. For computational efficiency with reasonable accuracy, equations (2.12d) will be used to calculate the leakage or/and brake valve mass flow rate.

Equations (2.8) and (2.9) are coupled together through the two dependent variables $M(X,T)$ and $P(X,T)$. They have been defined to be hyperbolic equations. Thus, two boundary conditions need to be specified before proceeding to solve equations (2.8) and (2.9). At the rear-end of the pipe, the value of $M(l,T)$ during the entire integration period is:

$$M(l,T) = 0 \quad (2.13a)$$

because there is no flow out of the end of the pipe, and, at the head-end of the pipe, the boundary condition is

$$P(0,T) = f(T) \quad (2.13b)$$

where $f(T)$ is the pressure provided by the functions of the 26C locomotive valve, which is modeled in Chapter 3. Finally, the initial conditions are:

$$\begin{aligned} M_{i-1} &= 0 \\ P_i^0 &= 0 \end{aligned} \quad (2.13c)$$

for an initially empty brake system. Equations (2.8) and (2.9) have two characteristic curves. The slopes of these curves are known to be the eigenvalues of the governing equations (2.8) and (2.9), defined as λ :

$$\lambda = \frac{dX}{dT} = -U \pm \sqrt{\frac{1}{A+v} [U^2 v + G_1 A]}$$

In order to approximate the solution of equations (2.8) and (2.9) by most numerical techniques, a network of grid points is first established throughout the region $0 \leq X_i \leq 1$ and $0 \leq T_j \leq \infty$, as shown in figure (2.2) with grid spacing ΔX_i and ΔT , where

ΔX_i = the normalized distance step size at the i^{th} section, and

ΔT = the normalized time step size.

In figure (2.2), the line $X = X_i$ and $T = T_j$ are called the grid lines, and their intersection is called the grid point (mesh point).

In order to find the largest time increment ΔT which may be used to provide an accurate transient solution, the largest slope of the two characteristic curves, λ , has to be found. The largest slope is associated with the maximum value of the normalized velocity, U , through the brake pipe. The largest value U , through the transient response (for the brake pipe application), will not exceed a value of .2 (established by experiment for 10 or more 50 ft cars) and $G_1 = 1./1.4$ for air. Assume that $v = 0$, thus,

$$.645 < |\lambda| < 1.045$$

then, $\lambda_{\text{max}} \approx 1$.

With nonzero values of $(v < A)$ the maximum eigen values will decrease. In order to find an accurate transient solution,

a) DIFFERENTIAL

$$\frac{\partial P}{\partial T} + \frac{1}{A+V} \frac{\partial MA}{\partial X} + \Lambda \Sigma = 0 \quad (2.8)$$

$$T \quad \frac{\partial M}{\partial T} + \frac{1}{A} \frac{\partial MUA}{\partial X} + G_1 \frac{\partial P}{\partial X} + G_2 M = 0 \quad (2.9)$$

O X I

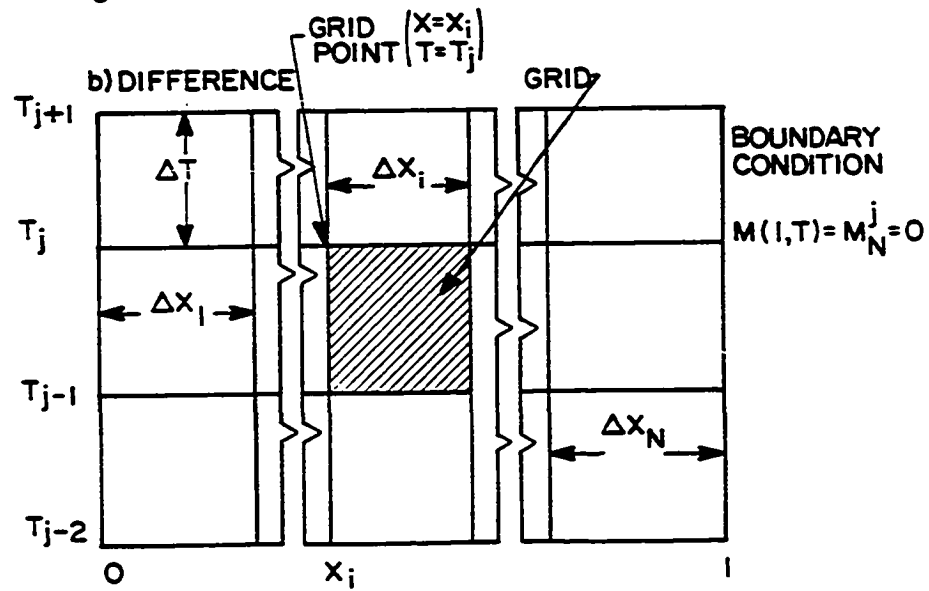


Figure 2.2: Numerical Simulation of Equations (2.8) and (2.9)

ΔT should be evaluated using the maximum η and minimum ΔX_i , as,

$$\Delta T = \frac{\text{Min}(\Delta X_i)}{\eta_{\max}} \quad (2.14)$$

thus, $\Delta T = \text{Min}(\Delta X_i)$

This is, also, based on the fact that the pressure disturbance inside the pipe is transmitted with the local speed of sound (c) [25], provided that the effect of the continuum speed, $U(X,T)$, has no effect on the solution stability during the entire transient.

2.3 Finite Difference Formulation

The finite difference schemes, among the many numerical techniques used to solve a differential equation, are the explicit and the implicit schemes. Both schemes can provide accurate and stable solutions under some considerations and conditions. These schemes will be compared with one another, as well as the solution provided by the finite element formulation in chapter 4.

Using finite differences to solve equations (2.8) and (2.9) requires, that the finite difference in space has to be either backward difference or central difference [25], because the pressure wave (information) is always transmitted from the head-end to the rear-end of the train line (brake pipe).

In general, the finite difference procedure consists of approximating a set of partial differential equations

(2.8) and (2.9) by a system of nonlinear algebraic equations.

2.3.1 Implicit Scheme

In chapter 5, the effect of leakage on the steady state behaviour of the brake pipe is investigated. In order to complete the investigation, it is required to develop an accurate, stable and fast numerical scheme to provide a steady state solution for equations (2.8) and (2.9). The only technique which proves to be accurate and unconditionally stable is the implicit finite difference scheme [25]. This scheme also allows the use of nonuniform displacement steps (ΔX_i) without causing any instability problem. The technique described hereafter can be used to provide either a transient solution or steady state solution of equations (2.8) and (2.9). Fortunately, as shown by references [2,6], the main characteristic of the brake pipe problem is that the displacement rate of change of the pressure and velocity are very small as compared with the time rate of change of any of these variables. That means that a semi-implicit finite difference scheme may be used instead of fully implicit scheme. This choice will make it possible to reduce the number of unknowns (three instead of five) in each equation and avoid the nonlinearity of these equations. The finite difference formulation of the terms appearing in equations (2.8) and (2.9) within the i -th

section (car) and between $j, j+1$ integration time steps may be written as follows:

$$\begin{aligned} \left[\frac{\partial P}{\partial T} \right]_i^{j+1} &= \frac{P_i^{j+1} - P_i^j}{\Delta T} \\ \left[\frac{\partial M}{\partial T} \right]_{i-1}^{j+1} &= \frac{M_{i-1}^{j+1} - M_{i-1}^j}{\Delta T} \\ \left[\frac{1}{A+v} \frac{\partial MA}{\partial X} \right]_i^{j+1} &= \frac{M_i^{j+1} A_i - M_{i-1}^{j+1} A_{i-1}}{(A+v)_i \Delta X_i} \quad (2.15) \\ \left[\frac{\partial P}{\partial X} \right]_i^{j+1} &= \frac{P_i^{j+1} - P_{i-1}^{j+1}}{\Delta X_i} \\ \left[\frac{1}{A} \frac{\partial MUA}{\partial X} \right]_i^j &= \frac{M_i^j U_i^j A_i - M_{i-1}^j U_{i-1}^j A_{i-1}}{A_i \Delta X_i} \\ \left[G_2^{MU} \right]_{i-1}^{j+1} &\approx \left[G_2^U \right]_i^j M_{i-1}^{j+1} \end{aligned}$$

Substituting equation (2.15) into equations (2.8) and (2.9), the following set of algebraic equations completely describe the semi-implicit finite difference formulation for equations (2.8) and (2.9)

$$B_{\alpha,1} P_{i-1}^{j+1} + B_{\alpha,2} M_{i-1}^{j+1} + B_{\alpha,3} P_i^{j+1} = B_{\alpha,4} \quad (2.16a)$$

$$B_{\beta,1} M_{i-1}^{j+1} + B_{\beta,2} P_i^{j+1} + B_{\beta,3} M_i^{j+1} = B_{\beta,4} \quad (2.16b)$$

where, $\alpha = 2i-3$

$$\beta = 2i-1 \quad i=2,3,\dots,N+1$$

$$B_{\alpha,1} = -B_{\alpha,3} = -G_1$$

$$B_{\beta,1} = - \frac{A_{i-1}}{(A+v)_{i-1}}$$

$$B_{\beta,3} = \frac{A_i}{(A+v)_{i-1}}$$

$$B_{\alpha,2} = C_i \left[1 + f_i^j \left[\Lambda_{avg} \right]_i \frac{P}{2 \bar{d}_i} \frac{\left[U_{i-1}^j \right]^2}{|U_{i-1}^j|} \Delta T \right]$$

$$B_{\beta,2} = C_i$$

$$B_{\alpha,4} = C_i M_{i-1}^j - \frac{M_i^j U_i^j A_i - M_{i-1}^j U_{i-1}^j A_{i-1}}{A_i}$$

$$B_{\beta,4} = C_i P_i^j - \Lambda_i \Omega_i^j$$

$$C_i = \frac{\Delta T}{\Delta X_i} \quad (2.17)$$

for $i=2$, $P_1^{j+1} = P(0,T) = f(T, M_0, \dots \text{etc}) = P(0,T)$, as

$$P_1^{j+1} = P(0,T) = P_1^j + \frac{M(\text{source}) - M_1^j}{C_1} \quad (2.18)$$

thus equation (2.16a) may be rewritten in this form:

$$B_{2,4} = B_{2,4} - B_{2,1} P(0,T) \quad (2.19)$$

and for $i=N+1$, $M_{N+1}^{j+1} = M(1,T) = 0$, thus equation (2.16b) can be rewritten in this form

$$B_{2N,1} M_N^{j+1} + B_{2N,2} P_{N+1}^{j+1} = B_{2N,4} \quad (2.20)$$

Using the above boundary conditions ($P(0,T)$ and $M(1,T)$), reduced the number of equations to be solved simultaneously to $2N$ equations. In general, these equations form a tri-diagonal set of simultaneous linear algebraic equations, which may be solved using the algorithm described in Appendix B.

2.3.2 Explicit Scheme

Many researchers who have solved a set of partial differential equations similar to equations (2.8) and (2.9), Refs.[4,6,7,8,9,10] suggested the use of the explicit scheme over the full implicit to solve equations (2.8) and (2.9). Equations (2.16a) and (2.17b) can be easily converted from its implicit formulation into a simple explicit form, following these procedures:-

1- Change the terms P_i^{j+1} and P_{i-1}^{j+1} in the left hand side of equation (2.16a) to P_i^j and P_{i-1}^j respectively, and

2- Change the terms M_i^{j+1} and M_{i-1}^{j+1} in the left hand side of equation (2.16b) to M_i^j and M_{i-1}^j respectively,

then rewrite equations (2.16a) and (2.16b) in these forms

$$M_{i-1}^{j+1} = \frac{B_{\alpha,4} - B_{\alpha,1} P_{i-1}^{j+1} - B_{\alpha,3} P_i^{j+1}}{B_{\alpha,2}} \quad (2.21a)$$

$$P_{i-1}^{j+1} = \frac{B_{\beta,4} - B_{\beta,1} M_{i-1}^{j+1} - B_{\beta,3} M_i^{j+1}}{B_{\beta,2}} \quad (2.21b)$$

but, before solving equations (2.21a) and (2.21b), it is important to consider the stability limitation of this formulation (explicit scheme). It is difficult to do a stability analysis to equations (2.8) and (2.9) in their nonlinear forms. Instead, linear forms of these equations can be used to represent both equations as suggested by Ref. [27] as follows:

$$\frac{\partial P}{\partial T} + \frac{\bar{U}A}{A+v} \frac{\partial P}{\partial X} \approx 0 \quad (2.8a)$$

$$\frac{\partial M}{\partial T} + \bar{U} \frac{\partial M}{\partial X} + G_1 \frac{\partial P}{\partial X} \approx 0 \quad (2.9a)$$

Using von Neumann stability analysis, one may find that

$$C_{2.8a} = \frac{A}{A+v} |\bar{U}| \frac{\Delta T}{\Delta X_i} \leq 1$$

is the stability limitation of equation (2.8a), which is known as the Courant number restriction. For equation (2.9a), first by ignoring the pressure gradient term and using the von Neumann stability analysis, the stability limitation of equation (2.9a) may be found to be as

$$C_{2.9a} = |\bar{U}| \frac{\Delta T}{\Delta X_i} \leq 1$$

But, the pressure gradient term in the momentum equations affects the continuum information speed, \bar{U} . However, a small pressure disturbance travels at the local speed of sound, c ,

in all the directions, so only $c > 0$ need to be considered. Thus the magnitude of the continuum speed is now $|\tilde{U}| + 1$, and the applicable courant number restriction is

$$C = [|\tilde{U}| + 1] \frac{\Delta T}{\Delta X_i} \leq 1 \quad (2.22)$$

This is the Courant, Friedrichs, and Lewy [32] or CFL restriction. However, for any fixed value of ΔT , the CFL number of equation (2.22) will be larger than any of $C_{2.8a}$ and $C_{2.9a}$. So, equation (2.22) may be used as the stability limitation of equation (2.8) and (2.9). But, the value of \tilde{U} can go as high as unity, thus, the integration time step should follow the following inequality

$$\Delta T \leq .5 \text{Min}(\Delta X_i)$$

In the course of this project investigation, ΔT will take a value of $.5 \text{Min}(\Delta X_i)$.

$$\Delta T = .5 \text{Min}(\Delta X_i) \quad (2.23)$$

The implicit or the explicit scheme formulated in this section has a local truncation error of order $O(\max(\Delta X_i) + \Delta T)$. In chapter 4, both schemes will be compared with the experimental result provided by NYAB.

2.4 Finite Element Formulation

The finite element method, which was first used as a procedure for structural analysis, has come to be recognized as an effective analysis tool for a wide range of physical problems. Among these are problems in the field of flow analysis. In this section, the finite element technique is

used to formulate the brake pipe system equations derived from the general form equations (2.8) and (2.9). This technique allows the use of nonuniform displacement steps (ΔX_i) as shown in the equation development. The technique can be used to provide both transient and steady state formulations for the governing equations.

In general, the finite element process may be defined as:

1. the behaviour of the whole system is approximated to by a finite number, $N_{ND} = N + 1$, of parameters, P_i^e and M_i^e , may be replaced by a_i^e , for which
2. the N -equations governing the behaviour of the whole system,

$$F_k(a_i^j) = 0, \quad i \text{ \& } e = 1, 2, \dots, N_{ND}$$

can be assembled by adding the contribution of each term from all elements which divide the system into physically identifiable entities. Thus

$$F_k = \sum F_k^e \quad e=1, 2, \dots, N$$

where F_k^e is the element contribution to the quantity under consideration.

Assuming that the brake pipe can be divided into N elements (sections). Each section may represent one car. Figure (2.3) shows the finite element representation of a section of the brake pipe. This section consists of two nodes and may have one leakage located at the second node. Because the displacement rate of changes of the pressure and velocity are very small as compared with any other term in

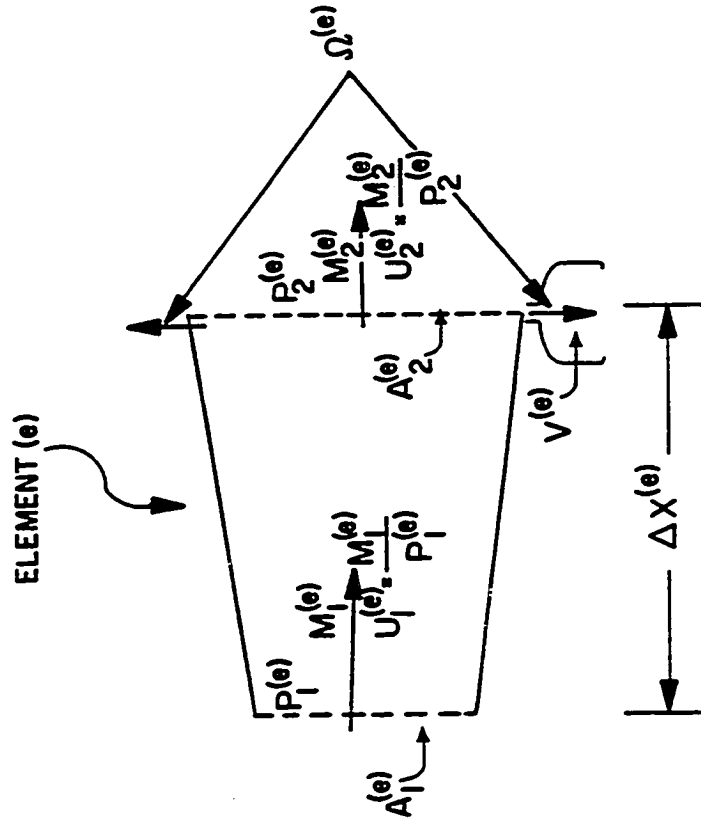


Figure 2.3: Finite Element Representation

equations (2.8) and (2.9) [2,3,4,6,7,8,9,10,11], the linear element is used to represent each section within the solution domain. Thus, the pressure and velocity within each element may be prescribed as:

$$P^{(e)}(X,T) = N_1^{(e)} P_1^{(e)}(T) + N_2^{(e)} P_2^{(e)}(T)$$

$$M^{(e)}(X,T) = N_1^{(e)} M_1^{(e)}(T) + N_2^{(e)} M_2^{(e)}(T) \quad (2.24)$$

where,

$$N_1^{(e)} = \frac{X_2^{(e)} - X^{(e)}}{\Delta X^{(e)}}, \quad \frac{\partial N_1^{(e)}}{\partial X} = -\frac{1}{\Delta X^{(e)}}$$

and,

$$N_2^{(e)} = \frac{X^{(e)} - X_1^{(e)}}{\Delta X^{(e)}}, \quad \frac{\partial N_2^{(e)}}{\partial X} = \frac{1}{\Delta X^{(e)}} \quad (2.25)$$

At this point, let's rewrite equations (2.8) and (2.9) in the following form for each element

$$\frac{\partial \Gamma}{\partial T} + \bar{\alpha}_\Gamma^{(e)} \frac{\partial \Gamma \beta_\Gamma}{\partial X} + \bar{\sigma}_\Gamma^{(e)} \frac{\partial \lambda \delta_\Gamma}{\partial X} + \bar{\Xi}_\Gamma^{(e)} = 0 \quad (2.26)$$

where, Γ may represent the pressure, P , or the mass velocity, M , for $\Gamma = M$, $\lambda = P$, $\bar{\sigma}_M^{(e)} = G_1$,

$$\bar{\alpha}_M^{(e)} = \frac{A_1^{(e)} + A_2^{(e)}}{2A_1^{(e)} A_2^{(e)}},$$

$$\beta_M = \begin{Bmatrix} U_1^{(e)} A_1^{(e)} & 0 \\ 0 & U_2^{(e)} A_2^{(e)} \end{Bmatrix}$$

$$\delta_M = \begin{Bmatrix} 1 & 0 \\ 0 & 1 \end{Bmatrix}$$

$$\begin{aligned} \text{and } \Xi(e) &= f \frac{A_{\text{avg}}^{(e)}}{A^{(e)}} \frac{1}{2\bar{d}(e)} \frac{[U_1^{(e)}]^2}{|U_1^{(e)}|} M_1^{(e)} \\ &= g_2^{(e)} U_1^{(e)} \end{aligned} \quad (2.27)$$

for $\Gamma = P$, $\lambda = M$, $\alpha_P^{(e)} = 0$,

$$\bar{\sigma}_P^{(e)} = \frac{1}{(A+v)_1^{(e)}} \quad ,$$

$$\delta_P = \begin{Bmatrix} A_1^{(e)} & 0 \\ 0 & A_2^{(e)} \end{Bmatrix},$$

$$\text{and, } \Xi(e) = \frac{\Lambda^{(e)} \Omega^{(e)}(T)}{\Delta X^{(e)}}$$

Now, applying the weighted residual method (Galerkin Method [26]) to equation (2.26), we obtain for an element

$$\int_{X_1^{(e)}}^{X_2^{(e)}} \left[\frac{\partial \Gamma}{\partial T} + \bar{\alpha}_\Gamma^{(e)} \frac{\partial \Gamma \beta}{\partial X} \Gamma + \bar{\sigma}_\Gamma^{(e)} \frac{\partial \lambda \delta}{\partial X} \Gamma + \Xi_\Gamma^{(e)} \right] N_i dX = 0 \quad (2.28)$$

Equation (2.28) with the use of equations (2.25) can now be written in this form

$$C^{(e)} \Gamma^{(e)} + K_{\Gamma}^{(e)} \Gamma^{(e)} + K_{\lambda}^{(e)} \lambda^{(e)} + S^{(e)} = 0 \quad (2.29)$$

where,

$$C^{(e)} = \int_{X_1^{(e)}}^{X_2^{(e)}} \{N\}^T \{N\} dX$$

$$K_{\Gamma}^{(e)} = \int_{X_1^{(e)}}^{X_2^{(e)}} \frac{\alpha_{\Gamma}^{(e)}}{2} \{N\}^T \frac{\partial \{N\}}{\partial X} \{\beta\} dX$$

$$K_{\lambda}^{(e)} = \int_{X_1^{(e)}}^{X_2^{(e)}} \frac{\sigma_{\Gamma}^{(e)}}{2} \{N\}^T \frac{\partial \{N\}}{\partial X} \{\beta\} dX \quad (2.30)$$

$$\text{and } S^{(e)} = \int_{X_1^{(e)}}^{X_2^{(e)}} \Xi_{\Gamma}^{(e)} dX$$

Integrating equation (2.30) with the use of equation (2.25), $C^{(e)}$, $K_{\Gamma}^{(e)}$, $K_{\lambda}^{(e)}$ and $S^{(e)}$ may be written as:

$$C^{(e)} = \frac{\Delta X^{(e)}}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad K_{\Gamma}^{(e)} = \frac{\alpha_{\Gamma}^{(e)}}{2} \begin{bmatrix} -\beta(1) & \beta(2) \\ -\beta(1) & \beta(2) \end{bmatrix}, \quad (2.31)$$

$$K_{\lambda}^{(e)} = \frac{\sigma_r^{(e)}}{2} \begin{bmatrix} -s(1) & s(2) \\ -s(1) & s(2) \end{bmatrix} \text{ and } S_r^{(e)} = \frac{\Delta X^{(e)} \Xi^{(e)}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We Also use the forward finite difference to replace the time rate of changes of $r^{(e)}$:

$$\dot{r} = \frac{r^{j+1} - r^j}{\Delta T} \quad (2.32)$$

In order to have an accurate transient, the integration time step must be less than or equal to the propagation time [27]. That means the normalized integration time step is

$$\Delta T \leq \min \left[\Delta X_i^{(e)} \right] \quad (2.33)$$

Substituting for \dot{r} from equation (2.32) and assembling the global matrices, the i -th system equation may be written in this form

$$B_{i,1} Y_{i-1}^{j+1} + B_{i,2} Y_i^{j+1} + B_{i,3} Y_{i+1}^{j+1} = B_{i,4} \quad (2.34a)$$

where,

$$B_{i,L} = \begin{bmatrix} b_{1,i,L} & b_{3,i,L} \\ b_{2,i,L} & b_{4,i,L} \end{bmatrix}, \quad (2.34b)$$

$$B_{i,4} = \begin{bmatrix} b_{1,i,4} \\ b_{2,i,4} \end{bmatrix} \quad \text{and } Y_i = \begin{bmatrix} P_i \\ M_i \end{bmatrix},$$

for $L=1$, $\nu = -.5$ and for $L=3$, $\nu = .5$ Thus,

$$b_{1,i,L} = \frac{\Delta X^{(i,L)}}{6\Delta T}$$

$$b_{2,i,L} = r_{\sigma_M}^{(i,L)} s_M(i,L)$$

$$b_{3,i,L} = r_{\sigma_P}^{(i,L)} s_P(i,L) \quad (2.35a)$$

$$b_{4,i,L} = r_{\alpha_M}^{(i,L)} \beta_M(i,L) + \frac{\Delta X^{(i,L)}}{6\Delta T}$$

$$b_{4,i,1} = b_{4,i,1} + \frac{G_2^{(i-1)}}{2} \Delta X^{(i-1)}$$

where, $(i,2) \rightarrow (i,3) \rightarrow (i)$

and $(i,1) \rightarrow (i-1)$

for $L=2$

$$b_{1,i,2} = \frac{\Delta X^{(i-1)} + \Delta X^{(i)}}{3\Delta T}$$

$$b_{4,i,2} = .5 \left[\bar{\alpha}_M^{(i-1)} - \bar{\alpha}_M^{(i)} \right] \beta_M(i) + \frac{\Delta X^{(i-1)} + \Delta X^{(i)}}{3\Delta T}$$

$$b_{2,i,2} = .5 \left[\bar{\sigma}_M^{(i-1)} - \bar{\sigma}_M^{(i)} \right] s_M(i) \quad (2.35b)$$

$$= 0 \quad \text{because, } \sigma_M^{(i-1)} = \sigma_M^{(i)} = G_1$$

$$b_{3,i,2} = .5 \left[\bar{\sigma}_P^{(i-1)} - \bar{\sigma}_P^{(i)} \right] s_P(i)$$

$$b_{4,i,2} = b_{4,i,2} + \frac{G_2^{(i)} \Delta X^{(i)}}{2}$$

$$b_{1,i,4} = \frac{\Delta X^{(i-1)}}{6\Delta T} P_{i-1}^j + \frac{\Delta X^{(i-1)} + \Delta X^{(i)}}{3\Delta T} P_i^j + \frac{\Delta X^{(i-1)}}{6\Delta T} P_{i+1}^j - .5 \left[\Xi_P^{(i-1)} \Delta X^{(i-1)} + \Xi_P^{(i)} \Delta X^{(i)} \right] \quad (2.35c)$$

$$b_{2,i,4} = \frac{\Delta X^{(i-1)}}{6\Delta T} M_{i-1}^j + \frac{\Delta X^{(i-1)} + \Delta X^{(i)}}{3\Delta T} M_i^j + \frac{\Delta X^{(i-1)}}{6\Delta T} M_{i+1}^j$$

for $i=N+1$, equation (2.34) is written in this form

$$B_{N+1,1} Y_N^{j+1} + B_{N+1,2} Y_{N+1}^{j+1} = B_{N,4} \quad (2.36)$$

where, for $L=1$, $r_1 = -.5$ and $r_2 = 1$, and for $L=2$, $r_1 = .5$ and $r_2 = 2$

$$\begin{aligned} b_{1,N+1,L} &= r_2 \frac{\Delta X^{(N,L)}}{6\Delta T} \\ b_{2,N+1,L} &= r_1 \sigma_M^{(N,L)} s_M^{(N,L)} \\ b_{3,N+1,L} &= r_1 \sigma_P^{(N,L)} s_P^{(N,L)} \\ b_{4,N+1,L} &= r_1 \bar{\alpha}_M^{(N,L)} \rho_M^{(N,L)} + r_2 \frac{\Delta X^{(N,L)}}{6\Delta T}, \\ b_{4,N+1,1} &= b_{4,N+1,1} + \frac{G_2^{(N)} \Delta X^{(N)}}{2} \end{aligned} \quad (2.37)$$

$$b_{1,N+1,4} = \frac{\Delta X^{(N)}}{3\Delta T} P_{N+1}^j + \frac{\Delta X^{(N)}}{6\Delta T} P_N^j - .5 \Xi_P^{(N)} \Delta X^{(N)}$$

$$b_{2,N+1,4} = \frac{\Delta X^{(N)}}{6\Delta T} U_N^j$$

$$U_{N+1}^j = 0 \quad \text{for } j=0,1,2,\dots*$$

for $i=1$, equation (2.26) is written in this form

$$B_{1,2} Y_1^{j+1} + B_{1,3} Y_2^{j+1} = B_{1,4} \quad (2.38)$$

where, for $L=2$, $r_1 = -.5$ and $r_2 = 2$, and for $L=3$, $r_1 = .5$ and $r_2 = 1$

$$b_{1,1,L} = r_2 \frac{\Delta X^{(1)}}{6\Delta T}$$

$$b_{2,1,L} = r_1 \sigma_M^{(1)} \delta_M^{(1)}$$

$$b_{3,1,L} = r_1 \sigma_P^{(1)} \delta_P^{(1)}$$

$$b_{4,1,L} = r_1 \bar{\alpha}_M^{(1)} \beta_M^{(1)} + r_2 \frac{\Delta X^{(1)}}{6\Delta T}$$

$$b_{4,1,1} = b_{4,1,1} + \frac{G_2^{(1)} \Delta X^{(1)}}{2} \quad (2.39)$$

$$b_{1,1,4} = \frac{\Delta X^{(1)}}{3\Delta T} P_1^j + \frac{\Delta X^{(1)}}{6\Delta T} P_2^j - .5 \Xi_P^{(1)} \Delta X^{(1)}$$

$$b_{2,1,4} = \frac{\Delta X^{(1)}}{3\Delta T} U_1^j + \frac{\Delta X^{(1)}}{6\Delta T} U_2^j$$

Notice that, the value of P_1^{j+1} (head-end boundary condition) can be evaluated using equation (2.38). In order to keep the

equation in its present form, the coefficients $b_{1,1,1}$ and $b_{1,1,4}$ need to be replaced with the following values

$$b_{1,1,1} = H \quad \text{and} \quad b_{1,1,4} = H P_1^{j+1} \quad (2.40)$$

where, H is a very large value as compared with the rest of the coefficients in the same equation ($\text{Ex. } 10^5$). In such a way, the final solution of equation (2.38) will provide the same answer given for P_1^{j+1} in the right hand side of the equation. In the course of the present analysis, H is set to be equal to 10^5 .

Finally, the finite element formulation has a local truncation error order higher than the two finite difference schemes, developed earlier in section (2.3), which is of $O(\max(\Delta X_i^2) + \Delta T)$.

2.5 Summary

In this chapter, two numerical techniques were proposed to provide a solution for the brake pipe mathematical model developed earlier in section (2.2). The first technique was the finite difference with its two of the possible formulations:

1. Implicit finite difference (section (2.3.1))
2. Explicit finite difference (section (2.3.2))

The implicit formulation is the one which may be used to provide a stable solution to solve for the transient or steady state solution of the brake pipe equations regardless of the integration time step (ΔT). The transient solution

accuracy can be achieved, with the implicit formulation, by using a smaller ΔT or/and ΔX_i , because this formulation has a local truncation of order $O(\max(\Delta X_i), \Delta T)$. The explicit formulation is a favorable scheme, used by many researchers, to solve a problem similar to the present case. But, this explicit form has a stability limitation for the selection of ΔT , based on the CFL restriction (2.22). We used

$$\Delta T = .5\text{Min}(\Delta X_i) \quad (2.23)$$

This scheme has the same local truncation error as the one provided by the implicit scheme. The second form of numerical technique was the finite element formulation. In this form of finite element method, a linear element is chosen to represent each section of the brake pipe model. Section (2.4) describes the formulation of the system equation using a finite element. The final form of the system equation in its general form (2.34), makes this scheme similar to a formulation that may be made with the use of an Implicit Forward-Time and Centered-Space (IFTCS) with a very simple modification. The IFTCS is a one of the implicit finite difference forms. Using this analogous method, it can be shown that the present finite element formulation has a local truncation error of $O(\max(\Delta X_i^2), \Delta T)$, and because it is implicit, there is no stability restriction on ΔT . If the explicit FTCS is used, instead of IFTCS, the solution of equations (2.8) and (2.9) will be unstable no matter what value of ΔT is used. It has been shown by the stability analysis made in Ref-26, that the

error grows monotonically as long as there is no diffusion term present in equation (2.9). This error growth, which is called a static instability, can not be removed by lowering the time step ΔT and it can only be removed by using some other finite difference formulation. The diffusion term usually appears in a form similar to this form

$$\text{Diffusion} = \alpha \frac{\partial^2 U}{\partial x^2} \quad (2.41)$$

and the stability restriction should follow

$$\Delta T \leq \frac{2\alpha}{c\lambda U^2} \quad (2.42)$$

The final set of equations, formulated using either the implicit finite difference or finite element, has to be solved simultaneously. Both sets of equations (2.16) and (2.34) can be rewritten in this matrix form

$$\{B\} \cdot \{Y\}^{j+1} = \{Q\} \quad (2.43)$$

where $\{B\}$ is the system matrix

$$\{B\} = \begin{bmatrix} B_{1,2} & B_{1,3} & 0 & \dots & 0 & 0 & 0 \\ B_{2,1} & B_{2,2} & B_{2,3} & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & B_{\alpha-1,1} & B_{\alpha-1,2} & B_{\alpha-1,3} \\ 0 & 0 & 0 & \dots & 0 & B_{\alpha,1} & B_{\alpha,2} \end{bmatrix}$$

and

$$\{Q\} = \left\{ B_{1,4}, B_{2,4}, \dots, B_{\alpha-1,4}, B_{\alpha,4} \right\}^T \quad (2.44b)$$

$$\{Y\}^{j+1} = \left\{ P_1^{j+1}, U_1^{j+1}, P_2^{j+1}, \dots, U_N^{j+1}, P_{N+1}^{j+1} \right\}^T$$

In the case of finite difference formulation

a) $\alpha = 2N-1$

b) the size of $B_{\alpha,L}$ is 1×1 matrix

In the case of finite element formulation

a) $\alpha = N$

b) the size of $B_{\alpha,L}$ ($L = 1, 2$ or 3) is 2×2

c) the size of $B_{\alpha,4}$ is 2×1

In order to solve the set of equations (2.43), a modified Thomas method is used [see Appendix B] instead of Gauss elimination or any other technique because there is only one restriction associated with this technique. This restriction is that the determinant of the diagonal element of each row has to be greater than the summation of the other two elements within the same row [27]. Either equation (2.16) or equation (2.34) shows that this condition is always satisfied.

In chapter 4 the NYAB laboratory brake model is used for the purpose of comparisons between both numerical techniques (finite element and finite difference in its

explicit and implicit forms). These comparisons provide, in general, the validation of the three numerical techniques to accurately solve the brake pipe governing equation. Then, in chapter 5, the finite difference technique is used to study the effect of leakage in the steady state behaviour of the brake pipe.

CHAPTER 3

MATHEMATICAL MODEL

OF

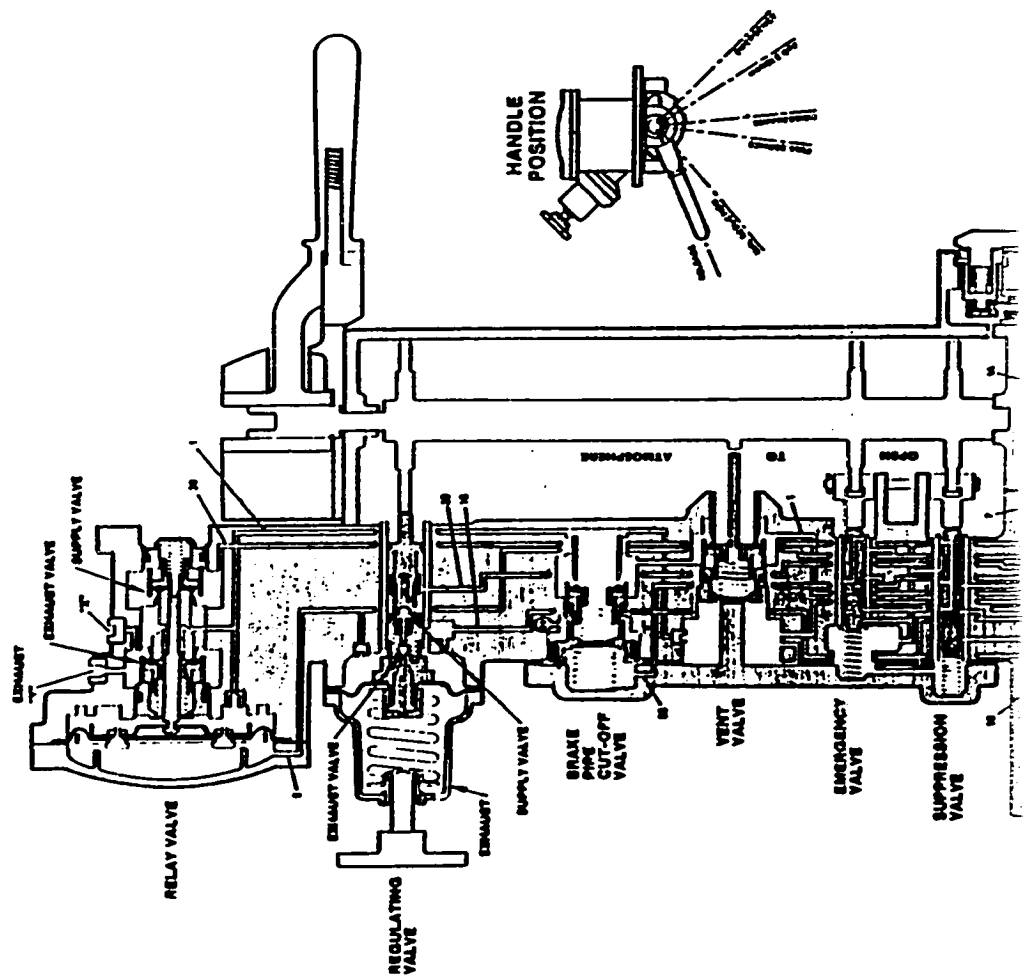
THE 26C LOCOMOTIVE VALVE

3.1 Introduction

The ultimate objective of the entire study of the behaviour of the trainline is to develop a dynamic model of the brake system including the locomotive valve, car brake valves, and interconnecting pipes.

In the present chapter, the mathematical model of the 26C locomotive valve is developed. In the previous chapter, the mathematical model of the trainline (brake pipe) was developed. These two numerical methods are used to provide the solution of the trainline model equations viz. the finite element and the finite difference. Also, the possibility of using the implicit and explicit schemes to represent the finite difference equations are explored through out the previous chapter.

One of the most important parts of the air brake system is the 26C valve, figure (3.1), which is located in the locomotive of the train, generally at the head-end of the trainline. The primary function of the 26C valve is to supply regulated air pressure through the trainline to the



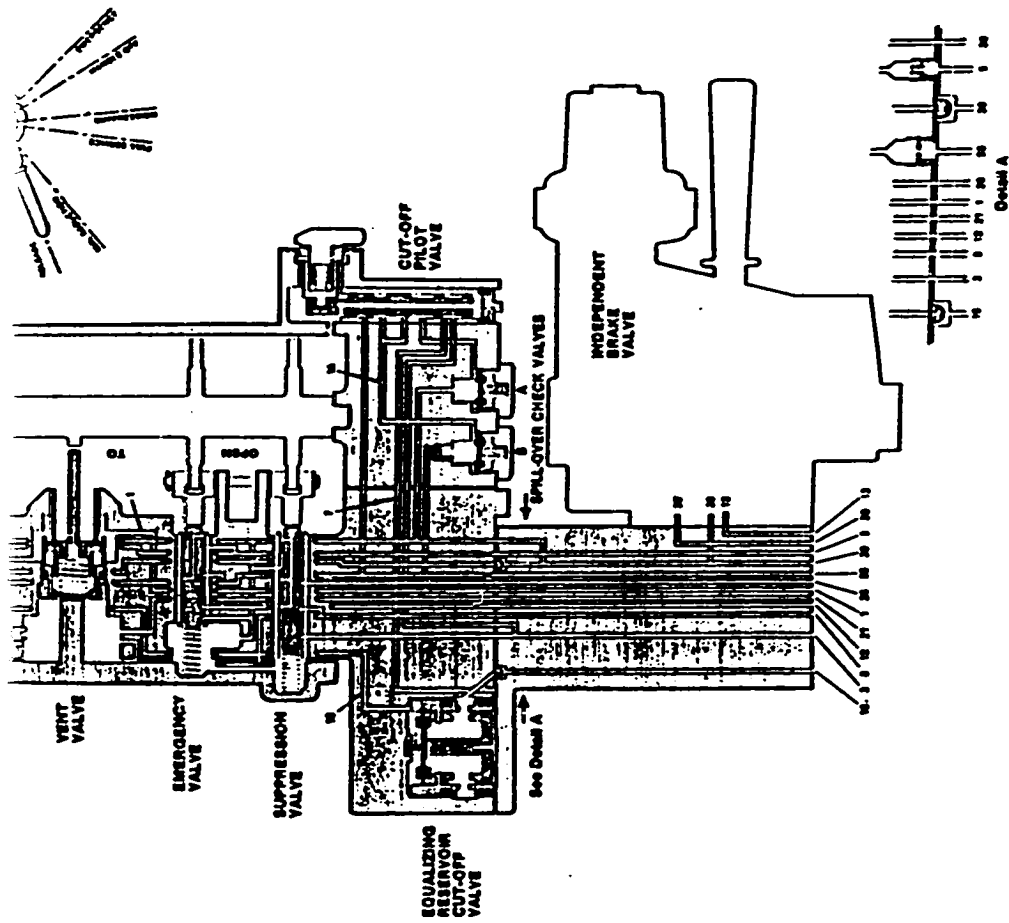


Figure 3.1: Schematic Drawing of 26C Locomotive Valve

automatic air brake valves in the cars of the train. By manipulation of the brake valve handle of the 26C, trainline air pressure can be reduced at a controlled rate through the 26C valve to apply the brakes. The trainline air pressure can also be increased for system charging (dry charge) and recharge and brake release. The 26C would attempt to hold the pre-selected pressure value regardless of leakage changes.

Incorporated in the 26C is a cam-operated self-lapping regulating valve. The function of the regulating valve is to increase or decrease equalizing reservoir pressure which depends upon the brake valve handle position. In turn, trainline pressure is increased or decreased by a self-lapping type of relay valve, which is controlled by equalizing reservoir pressure to maintain in the trainline a pressure approximately equal to equalizing reservoir pressure. The brake pipe cut-off valve is operated with a pneumatic-mechanical means. The valve starts to open when the head-end brake pressure is greater than 42 psi. The brake pipe cut-off valve interrupts the flow of air from the relay valve to the brake pipe during an emergency brake application and if the cut-off pilot valve is in "OUT" position. The vent and emergency valves are only controlled by the 26C cam (mechanically operated). Both valves are activated only during an emergency operation. The emergency valve is used to vent the brake pipe and the vent valve is used to vent the equalizing reservoir to atmosphere. The

suppression valve is used to guide a part of the main reservoir air to open the equalizing reservoir cut-off valve. When the 26C handle is in release position, and the cut-off pilot valve in "FRT" position, the equalizing reservoir cut-off valve will be held open during the release mode of the 26C locomotive valve. However, with the cut-off pilot valve in "PASS" position, the equalizing reservoir cut-off valve is held open in all 26C handle position. Based on these discussions, one can find that is very important to develop a mathematical model for the pneumatic-mechanical valve as:

1. regulating valve,
2. relay valve,
3. brake pipe cut-off valve.

In section 3.2, a complete description of the operation principles for the regulating valve, relay valve and the brake pipe cut-off valve are given. Section 3.3 describes the complete mathematical model for the 26C brake valve (relay valve, regulating valve and brake pipe cut-off valve). Then, section 3.4 discusses the possibility of simplifying the combination of the relay valve and brake pipe cut-off valve mathematical models. Finally, section 3.5 briefly summarizes the developments made in this chapter.

3.2 Operation Principles

In this section, the operation principle of the 26C brake valve (shown in figure (3.1)) is described for brake

applications (partial and full services), emergency, brake release and charging of the air brake system components. In order to assist the reader in locating the components in the diagram and provide a better tool for visualizing the dynamic changes during the many different modes. All the 26C parts are given section identification (ID) numbers (1) through (9). Each number has no significance to the function or the name of the components. The following is the list of major components by section numbers:

- | | |
|--|---------------------|
| (1) Relay Valve | <i>Figure (3.5)</i> |
| (2) Regulating Valve | <i>Figure (3.2)</i> |
| (3) Brake Pipe Cut-off Valve | <i>Figure (3.7)</i> |
| (4) Brake Valve Handle | |
| (5) Vent Valve | |
| (6) Emergency valve | |
| (7) Suppression Valve | |
| (8) Equalizing Reservoir Cut-Off Valve | |
| (9) Cut-Off Pilot Valve | |

An alphanumeric designation procedure has been adopted for some components. As an example, $K_{1,1}$ refers to spring number 1 to be found in section 1 (relay valve) of the drawing. The main character indicates the type of component, the first subscript is the section ID, the second subscript is the component number within the defined section. The following are the main characters used in modeling the 26C brake valve:-

A = represents flow area opening, diaphragm area,...
etc, m^2 .

D = represents equivalent diameter of A , m .

F = represents force due to pressures on areas, N .

I = represents inertial mass of the moving parts, kg .

K = represents spring constant, N/m .

L = represents spring preload, N .

m = represents equivalent mass, kg .

\dot{m} = represents mass flow rate, kg/sec .

p = represents local pressure, N/m^2 .

S = represents spring force, N .

V = represents volume, m^3 .

θ = represents local temperature, $^{\circ}K$.

The regulating valve shown in figure (3.1), is controlled by one of the four cams which are mounted on the shaft of the automatic brake valve handle. The regulating valve is a self-lapping type of valve. The self-lapping feature of the regulating portion automatically maintains equalizing reservoir pressure against overcharges and leakages (especially when the cut-off pilot valve is in "PASS" position). When the cut-off pilot valve is in "FRT" position, the regulating valve can maintain the equalizing reservoir pressure against leakages only when the handle is in the release position.

In the case of release or recharge (dry charge), the handle is moved to the release position, the regulating

valve cam will rotate to a higher position and allow only the supply valve seat of $A_{2,3}$ to move away (left) from the handle, opening the supply valve ($A_{2,3}$) of the regulating valve and sealing the exhaust valve ($A_{2,2}$). The air flows through the supply valve and supply orifices (A_{EQVS}) [it is not shown in figure (3.1)] to the inner diaphragm chamber ($V_{2,1}$) through the equalizing reservoir cut-off valve, where air pressure from the main reservoir raises the piston of the valve and unseats its check valve, and finally to the equalizing reservoir (V_{eq}). The equalizing reservoir pressure (p_{eq}) and $p_{2,1}$ increases until the pressure reaches a value 20 psi less than the pre-adjusted pressure level set by the regulating valve handle. Then the spool will start moving away (to the left) from the handle, closing the valve as p_{eq} increases.

Movement of the brake valve handle from release into the service sector causes the regulating valve to reduce equalizing reservoir pressure in proportion to the amount of handle movement towards the full service position. As the handle is moved towards the full service position, the regulating valve cam will rotate to a lower position and allow the regulating valve spool to move toward (to the right) the handle. But, at the same time, the air pressure from the main reservoir holding up the check valve of the equalizing reservoir cut-off valve will be removed and the check will be sealed. This action will cause the air to flow only from the equalizing reservoir to the regulating valve,

which will prevent any increase of the pressure in the equalizing reservoir during the entire application until the handle is moved to a release position in the case of "FRT" application, or in the case of "PASS" application, to any position higher than the present location of the handle (clockwise direction). Within the regulating valve, the exhaust valve ($A_{2,2}$) will almost move the same displacement as that made by the supply valve seat, this action will open the exhaust valve and seal the supply valve. Then, the air flows from the equalizing reservoir through the cut-off valve, $V_{2,1}$, $A_{2,2}$ and A_{EQVE} (it is not shown in figure (3.1)) to the atmosphere. The equalizing reservoir pressure (p_{eq}) and $p_{2,1}$ decreases, the exhaust valve seat moves toward the handle closing the exhaust valve.

The pneumatic pressure relay valve shown in figure (3.1) functions as a "pneumatic -pneumatic" servo-valve with static gain of unity. This means that the outlet pressure, $p_{1,2}$, is a reproduction of the control pressure, $p_{1,1}$, in the diaphragm chamber $V_{1,1}$. The main working elements of the relay valve are a supply valve ($A_{1,6}$) and exhaust valve ($A_{1,4}$), which are controlled by a diaphragm rod movement. There is always a total gap, X_0 , between the diaphragm rod and one or both valves ($A_{1,4}$, $A_{1,6}$), where X_0 is the gap between the diaphragm rod and the supply valve when the relay valve seated against the exhaust valve (when it is closed). We use $X_{1,1}$ to denote the displacement of the diaphragm; its equilibrium position, $X_{1,1} = 0$. The diaphragm

should travel a distance X_I away from the diaphragm rod before the exhaust valve can be opened. This observation was made by the author through the course of the experimentation on the relay valve dynamic behaviour (see chapter 4 for more detail).

Air enters or leaves the outer diaphragm chamber through the feedback orifice of the equalizing reservoir ($A_{1,1}$) and causes the pressure $p_{1,1}$ in the outer chamber of the diaphragm to increase or decrease. This pressure exerts a force $F_{1,1}$ on the outer face of the diaphragm causing it to move. Motion of the diaphragm causes a change in the volumes of the outer and inner chambers, $V_{1,1}$ and $V_{1,2}$. Charging of the volumes affects the respective pressure. The pressure $p_{1,2}$ in the inner chamber of the diaphragm exerts a force $F_{1,2}$, which opposes $F_{1,1}$. If the net force ($F_{1,1} - F_{1,2}$) is greater than the combined force ($S_{1,2} + S_{1,3}$) of the diaphragm rod spring and the supply valve spring, the supply valve opens to allow air to flow from the supply valve to the intermediate volume, $V_{1,3}$, located between the brake pipe cut-off valve and the inner chamber of the diaphragm, $V_{1,2}$. If the force, $F_{1,3} = F_{3,1}$, acting on the brake pipe cut-off valve is greater than the valve spring preload, $L_{3,1}$, the brake pipe cut-off valve, $A_{3,1}$, opens permitting the air to flow to or from the brake pipe. On the other hand, if the net force ($F_{1,2} - F_{1,1}$) is greater than the combined force ($S_{1,1} - S_{1,2}$), where $S_{1,1}$ is the force exerted by the exhaust valve spring, the exhaust valve opens and the

air flows from the brake pipe and the intermediate volume, $V_{1,3}$ to the atmosphere through the exhaust valve opening ($A_{1,4}$) and the exhaust orifice ($A_{1,5}$). Because there is always a gap between the diaphragm rod and at least one of the valves (supply and exhaust), only one valve opens at a time.

In the steady state case, pressure $p_{1,1}$, $p_{1,2}$ and brake pipe pressure, ($p_{bp} = p_A p_1$), are approximately equal to equalizing reservoir pressure (p_{eq}). Since the diaphragm effective area is same on both sides, the relay valve has a static gain of unity. The steady state position of the diaphragm rod, X_L , will always be between $-X_I$ and X_O (ie $-X_I \geq X_L \geq X_O$); this position is referred to as the lap position of the relay valve. In this position, both the supply valve and the exhaust valve are closed. Note, that the maximum openings of the exhaust valve, $A_{1,4}$, and supply valve, $A_{1,6}$ are controlled by stops. The diaphragm can travel a maximum distance, $X_{1,1} = X_S$, when opening the supply valve, and a maximum distance, $X_{1,1} = -X_E$, when opening the exhaust valve.

In the case of emergency, the 26C handle will move the cams of the locomotive valve opening the vent and emergency valves. The vent valve will open the brake pipe passage to atmosphere through the vent cavity of the valve. A sharp reduction in the brake pipe pressure will activate the brake pipe vent valves (see Appendix G for detail) along the train to propagate the emergency rate brake pipe pressure reduction. The vent valve will move to the left to direct

the the equilizing reservoir air to atmosphere through passage 5 causing quick venting of the equilizing reservoir pressure. This action prevents the relay valve from going into the release mode, as explained earlier in this section, in response to the reduction in the brake pipe pressure.

3.3 26C Locomotive Valve Mathematical Model

Relay, regulating and brake pipe cut-off valves are partially or fully controlled pneumatically. On the other hand, the rest of the valves are fully mechanically operated. The mathematical models of emergency, vent, equalizing reservoir cut-off and suppression valves are similar to the mathematical model, developed in appendix G, for the vent valve of the ABD/ABDW control valve. So, this section is only concerned with the development of a mathematical model for each of the relay, regulating and brake pipe cut-off valve.

At present, a universal technique for modeling the dynamic characteristic of a pneumatic system does not exist. The thermodynamic process of charging or discharging of these systems can be extremely complex, since it involves highly nonlinear flow processes. However, the situation is not hopeless, since the basic laws of thermodynamics may be applied to develop equations describing these processes. Solutions to these equations can be obtained using available computational techniques. The ideal process may be isothermal if sufficient time is available for heat transfer

to occur, or it may be adiabatic if the process is rapid. In fact actual processes are neither isothermal nor adiabatic, but somewhere in between these extremes, and are called polytropic processes. It is known for dry air, the value of the polytropic constant is between 1.0 and 1.4, which are the values for isothermal and adiabatic processes respectively. In this analysis, it is assumed that most of the processes are close to being isothermal ($n=1$).

The universal law of gases for a volume may be written as:-

$$pV = mR_g\theta \quad (3.1)$$

where,

p = is the pressure inside the volume, N/m^2 .

V = is the volume, m^3 .

m = is the mass inside the volume, kg.

R_g = is the gas constant, $J/(kg^\circ K)$.

θ = is the temprature inside the volume, $^\circ K$.

Differentiating equation (3.1), yields the general dynamic equation for an open isothermal volume, which is used for volumes within the relay valve ($V_{1,1}$, $V_{1,2}$ and $V_{1,3}$) and equalizing reservoir as described in sections (3.3.1) and (3.3.2):-

$$\frac{dp}{dt} = \frac{1}{V} \left[R_g\theta \frac{dm}{dt} - p \frac{dV}{dt} \right] \quad (3.2)$$

It is well known that the shape and the kind of the restrictions ($A_{1,1}$, $A_{1,3}$, $A_{1,4}$, $A_{1,5}$, $A_{1,6}$, $A_{2,2}$, $A_{2,3}$ and $A_{3,3}$) play an important role in evaluating the mass flow

rates using either the compressible or the incompressible formula. In the case of a square-edged-orifice, Grace and Lapple [28] show that the compressible equation can be used to evaluate the mass flow rate as

$$\dot{m} = C A p_u \quad (3.3)$$

for subsonic flow

$$C = 0.8 \sqrt{\frac{2(B^{-2/\Gamma} - B^{-(\Gamma+1)/\Gamma})}{(\Gamma - 1) R_g \theta}} \quad B < 1.89 \quad (3.4a)$$

and for sonic

$$K = 2.367 \quad B \geq 1.89 \quad (3.4b)$$

where $B = \frac{p_u}{p_d}$

and p_u and p_d are the pressure upstream and down stream of the restriction. In the case of a sharp-edged-orifice and the poppet valves such as the supply and the exhaust valves, Perry [29] recommends using the following formula

$$\dot{m} = .6 A p_d \sqrt{\frac{|B^2 - 1|}{R_g \theta} \frac{|B-1|}{B-1}} \quad (3.5)$$

Gauthier (pp. 195-196) [7] and Banister [9] show that, the difference between using equation (3.4) or (3.5) is always less than 10%, which may be less than the variation in the geometric area of the restriction. Anderson [30] shows that actual flows tend to fall somewhere between these values. Equation (3.5) is much simpler to use in general. In a given flow situation, equation (3.5) can be used regardless of the value of B (sonic or subsonic), whereas,

with equation (3.4) one must always check the value of B against the critical pressure ratio (1.89) to determine which of (3.4a) or (3.4b) to be used. So, for computational efficiency with reasonable accuracy, equation (3.5) is chosen to calculate the mass flow rate through any of the restrictions in any of the air brake components.

3.3.1 Regulating Valve Mathematical Model

To evaluate the time rate of change of $p_{1,1}$, one needs to calculate the mass flow rate $\dot{m}_{1,1}$ through the feedback orifice $A_{1,2}$. But, $\dot{m}_{1,1}$ is controlled by the change of the equalizing reservoir, p_{eq} . General form (3.2) applies, except that there is no volume change (so $dV_{eq}/dt = 0$). The following equation shows that the time rate of change of p_{eq} is only due to the net of the flow rate $\dot{m}_{1,1}$ through $A_{1,2}$ and $\dot{m}_{2,1}$ through the regulating valve restrictions (A_{REG}):

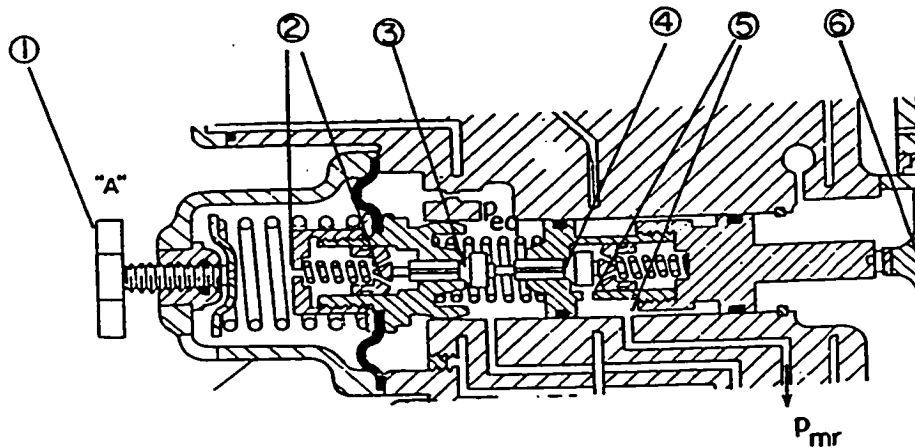
$$\frac{dp_{eq}}{dt} = \frac{R_g \theta}{V_{eq}} \left[\frac{d\dot{m}_{2,1}}{dt} - \frac{d\dot{m}_{1,1}}{dt} \right] \quad (3.6)$$

and,

$$\dot{m}_{2,1} = .6 A_{REG} P_{REG} \sqrt{\frac{|B^2 - 1|}{R_g \theta}} \frac{|B-1|}{B-1} \quad (3.7)$$

where, $B = \frac{P_{REG}}{P_{eq}}$,

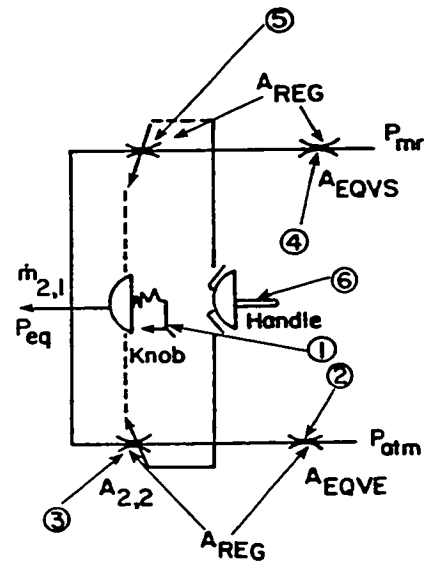
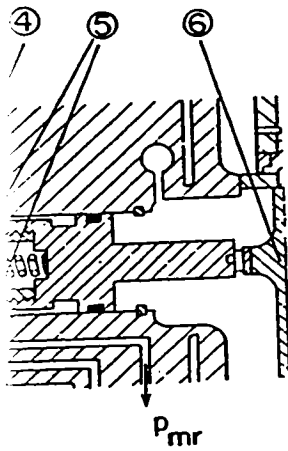
The values of P_{REG} and A_{REG} are function of the position of the 26C locomotive handle. Figure (3.2) shows the fluid network for the regulating valve and the equalizing reservoir. The network consists of two variable



a) Schematic Diagram

1. Valve Adjustment Knob
2. A_{EQVE}
3. Exhaust Valve, $A_{2,2}$

Figure 3.2: 26C Regulating V



b) Fluid Network

Knob

4. Supply Valve, $A_{2,3}$

5. A_{EQVS}

6. 26C Handle

Figure 3.2: 26C Regulating Valve

restrictions ($A_{2,2}$ and $A_{2,3}$), two fixed restrictions (A_{EQVE} and A_{EQVS}) and the equalizing reservoir volume, V_{eq} . A_{EQVE} and A_{EQVS} represent the equivalent restriction for all the fixed restrictions air will flow through in the case of brake application and release (recharge or dry charge) respectively. Fortunately, all these restrictions are in series, so the following simple formula is used to evaluate an equivalent restriction (A_{EQV}) for any n restrictions in series (See Appendix D for more details).

$$A_{EQV} = \frac{1}{\sqrt{\sum_{i=1}^n \frac{1}{A_i^2}}} \quad (3.8)$$

Using equation (3.8), It is easy to find A_{REG} as

(a) Brake Application:

$$A_{REG} = \frac{A_{EQVE} A_{2,2}}{\sqrt{A_{EQVE}^2 + A_{2,2}^2}} \quad (3.9a)$$

and $p = p_A$

(b) Brake Release:

$$A_{REG} = \frac{A_{EQVS} A_{2,3}}{\sqrt{A_{EQVS}^2 + A_{2,3}^2}} \quad (3.9b)$$

and $p = p_{mr}$

Fortunately, the exhaust valve and the supply valve have the same geometry and opening one of these valves will cause the closing of the other. Thus, in the following analysis, we

will only consider one of these valves. Let's try to evaluate the opening area of the supply valve ($A_{2,3}$) as a function of the valve displacement ($x_{2,1}$). Figure (3.3) shows the supply valve in an opening position for a valve displacement $x_{2,1}$. Assume that there are only two fluid restrictions in series created by the relative movement of the valve with respect to its seat. One of these is the restriction perpendicular to the valve movement direction (r-o plane), called A_x , and may be calculated using this formula:-

$$A_x = \left[A_0 - \pi (r_0 - x_{2,1})^2 \right] \quad (3.10)$$

for, $x_{2,1} \leq r_0 - r_I = x_C$

where, A_0 is the outside area of the valve seat and r_0 is the diameter of this area. The other area is the fluid area tangent to the valve movement (x-o plane), called A_r , and it may be evaluated using this formula:-

$$A_r = \pi D_I x_{2,1} \quad (3.11)$$

where, D_I is the diameter of the inside area of the valve seat. A_x and A_r are also in series, then equation (3.8) can be used to evaluate $A_{2,3}$ as

$$A_{2,3} = \frac{A_x A_r}{\sqrt{A_x^2 + A_r^2}} \quad (3.12)$$

In order to complete the mathematical model of the 26C locomotive valve, the regulating valve displacement, $x_{2,1}$, needs to be evaluated. It is known that the equalizing reservoir, p_{eq} , plays the role of the commanding signal for

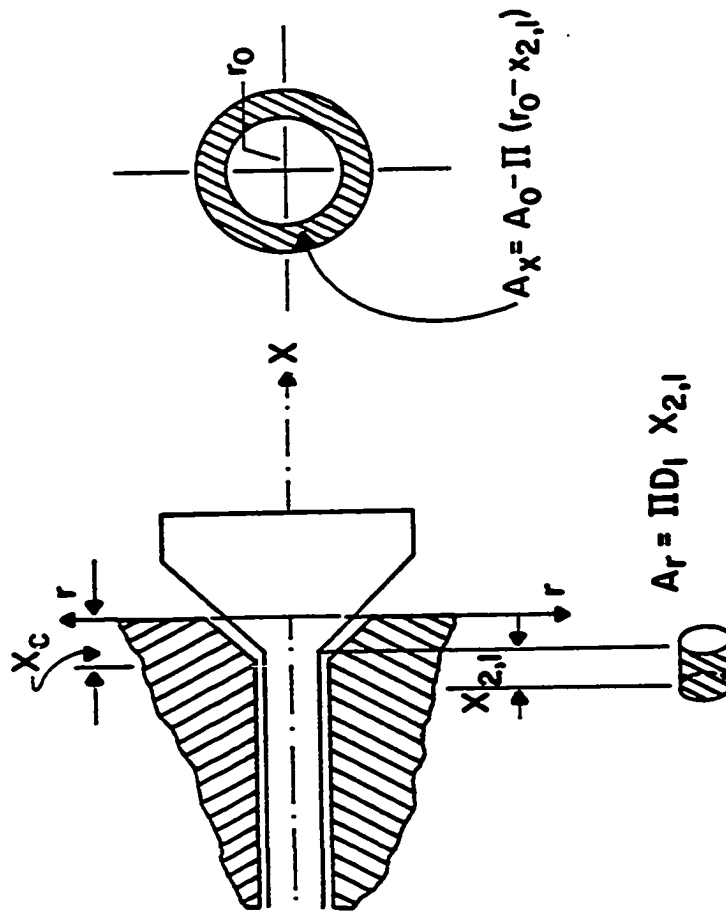


Figure 3.3: Supply Valve Opening Diagram

the 26C valve. However, the rate of change of p_{eq} is always faster than the rate of change of any of the pressure in the relay valve as well as the locomotive car (brake pipe) pressure (p_{bp}). Thus, one may not need to consider the exact modeling of the regulating valve. Instead, the quasistatic approximation will be used in the modeling. Figure (3.4) shows the free body diagram of the regulating valve diaphragm. Simply, there are only two forces acting on the diaphragm, first a pressure force, $p_{eq}A_{2,1}$, acting on the inner face of the diaphragm, and a spring force, $K_{2,1}x_{REG}$ and the preload, $L_{2,1}$ (made by the adjustment of the regulating valve knob), acting on the outer face of the diaphragm. Equating these two forces, the valve displacement can be found as:

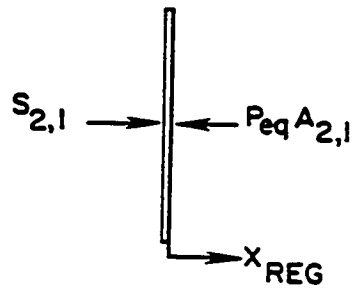
$$x_{REG} = \frac{p_{eq}A_{2,1} - L_{2,1}}{K_{2,1}} \quad (3.13)$$

In fact, any of the valves (supply or exhaust) will start closing when x_{REG} is less than a value x_C (as shown in figure (3.3)), until the equalizing reservoir reaches a cut-off pressure value p_C . x_C is the maximum effective opening made by the valves, and p_C is the final steady state value of the equalizing pressure. Now, instead of using equation (3.13), renaming x_{REG} to $x_{2,1}$ with the condition that x_{REG} is less than x_C , the following equation is used to calculate $x_{2,1}$

$$x_{2,1} = \frac{|p_{eq} - p_C| A_{2,1}}{K_{2,1}} \quad (3.13)$$

providing that,

a) Actual free body diagram



b) Simplified free body diagram

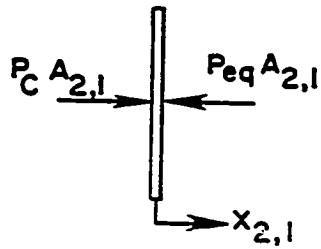


Figure 3.4: 26C Regulating Valve Free Body Diagram

$$\left| p_{eq} - p_C \right| \leq \frac{K_{2,1} X_C}{A_{2,1}} \quad (3.14)$$

and,

$$p_C A_{2,1} = L_{2,1} + K_{2,1}(X_{REG} - X_{2,1}) \quad (3.15)$$

3.3.2 Relay Valve Mathematical Model

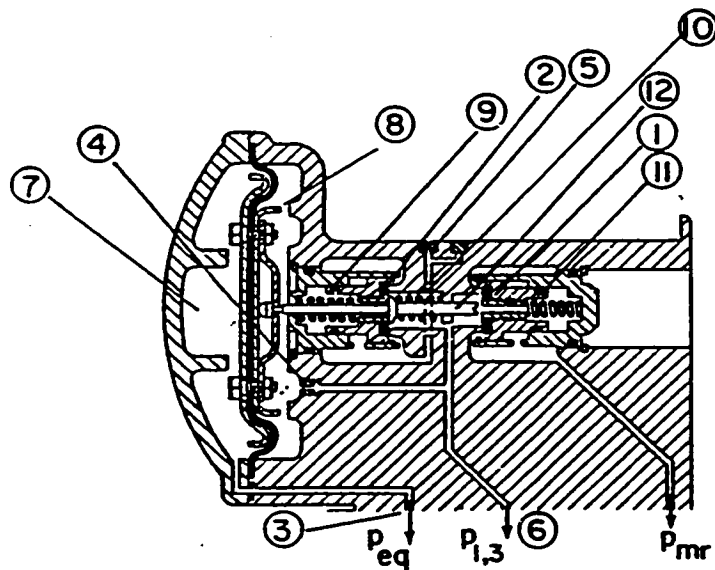
Figure (3.5b) shows the equivalent pneumatic network of the relay valve. This network consists of three chambers ($V_{1,1}$, $V_{1,2}$ and $V_{1,3}$), two variable restriction ($A_{1,4}$ and $A_{1,6}$) and three fixed restriction ($A_{1,1}$, $A_{1,3}$ and $A_{1,5}$). Starting with $V_{1,1}$, the time rate of change of $p_{1,1}$ is due to the mass flow rate $\dot{m}_{1,1} = \frac{dm}{dt}$ through the feedback orifice, $A_{1,1}$, and the time rate of change of $V_{1,1}$:

$$\frac{dp_{1,1}}{dt} = \frac{1}{V_{1,1}} \left[R_g \theta \frac{dm_{1,1}}{dt} - p_{1,1} \frac{dV_{1,1}}{dt} \right] \quad (3.16)$$

Similarly, the time rate of change of $p_{1,2}$ is due to the total mass flow rate through the inner chamber orifice $A_{1,3}$ and the time rate of change of volume $V_{1,2}$:

$$\frac{dp_{1,2}}{dt} = \frac{1}{V_{1,2}} \left[R_g \theta \frac{dm_{1,2}}{dt} - p_{1,2} \frac{dV_{1,2}}{dt} \right] \quad (3.17)$$

However, the time rate of change of $p_{1,3}$ is only due to the sum of the flow rate $\dot{m}_{1,2}$ through $A_{1,3}$, $\dot{m}_{1,3}$ through either the supply valve $A_{1,6}$ or the combination of the exhaust valve $A_{1,4}$ and the exhaust orifice $A_{1,5}$, and the mass flow rate $\dot{m}_{1,4} = \dot{m}_{3,1}$ through the brake pipe cut-off valve $A_{3,1}$:



a) Schematic Diagram

b) F

- | | |
|--------------------------|--------------------------|
| 1. Supply Valve | 5. To Brake Pipe Cut-off |
| 2. Exhaust Valve | 6. Exhaust Orifice |
| 3. Feed Back Orifice | 7. Outer Chamber |
| 4. Inner Chamber Orifice | 8. Inner Chamber |

Figure 3.5: 26C Relay Valve

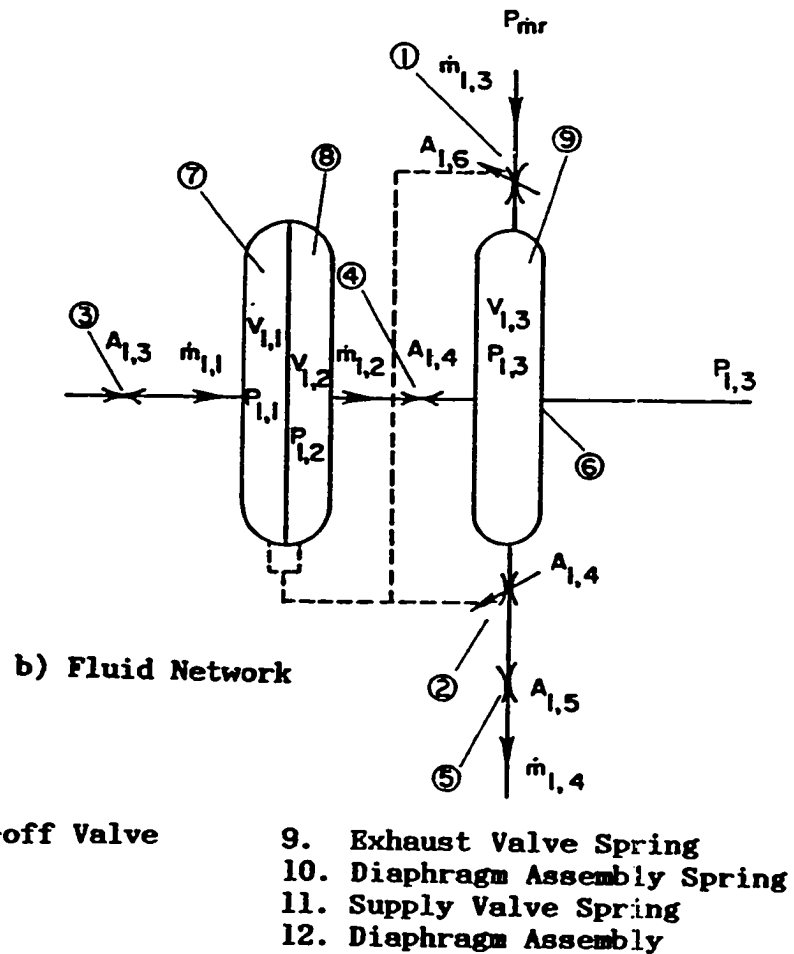


Figure 3.5: 26C Relay Valve

$$\frac{dp_{1,3}}{dt} = \frac{R_g \theta}{v_{a,3}} \left[\frac{dm_{1,3}}{dt} + \frac{dm_{1,4}}{dt} - \frac{dm_{1,2}}{dt} \right] \quad (3.18)$$

The time rate of changes of $v_{1,1}$ and $v_{1,2}$ are modulated by the diaphragm motion. Therefore,

$$\frac{dv_{1,1}}{dt} = A_{1,2} U \quad (3.19a)$$

$$\frac{dv_{1,2}}{dt} = A_{1,2} U \quad (3.19b)$$

where $U_{1,1} = dx_{1,1}/dt$ is the velocity of the diaphragm.

Figure (3.6) shows the free body diagram for the relay valve diaphragm, including the backlash, X_I , between the diaphragm rod and the diaphragm, as well as, the gap, X_0 , between the diaphragm rod and the supply valve $A_{1,6}$. The equation of motion of the diaphragm may be written as follows:

$$I \frac{dU_{1,1}}{dt} = F_{1,1} - F_{1,2} - S_{1,1} - S_{1,2} - S_{1,3} - K_D X_{1,1} \quad (3.20)$$

where K_D is the spring constant of the relay valve diaphragm, and I is the equivalent mass of the relay valve moving parts. But, I is a function of the displacement of the diaphragm, X .

$$\begin{aligned} I &= I_{1,1} + I_{1,2} + I_{1,3} & -X_E \leq X_{1,1} \leq -X_I \\ &= I_{1,1} & -X_I < X_{1,1} < 0 \\ &= I_{1,1} + I_{1,3} & 0 \leq X_{1,1} < X_0 \\ &= I_{1,1} + I_{1,3} + I_{1,4} & X_0 \leq X_{1,1} \leq X_S \end{aligned} \quad (3.21)$$

The external forces exerted by the diaphragm are $F_{1,1}$ and $F_{1,2}$; spring forces are $S_{1,1}$ of the exhaust valve spring,

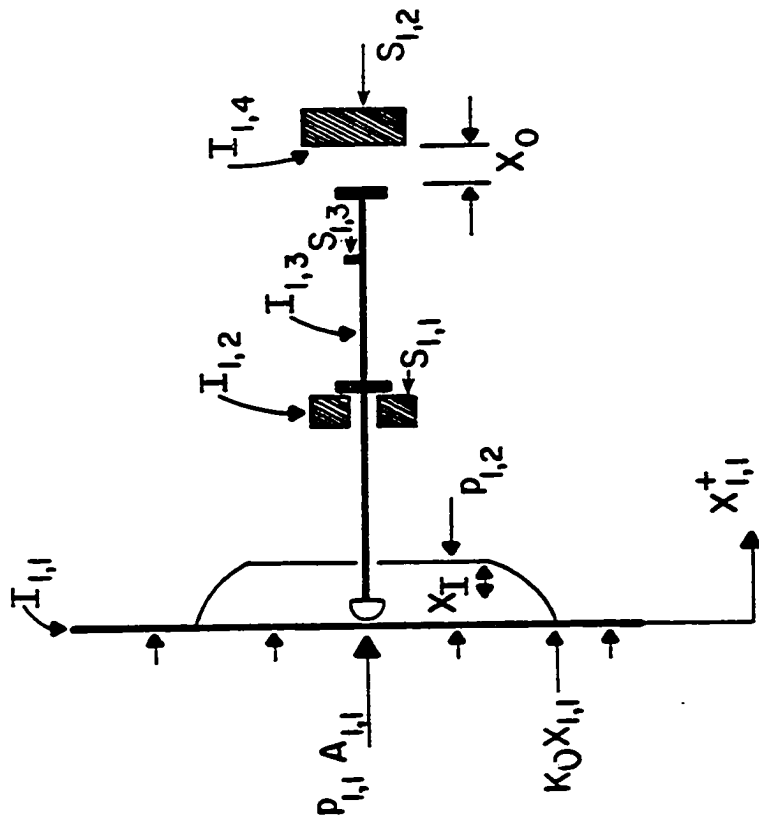


Figure 3.6: 26C Relay Valve Free Body Diagram

$S_{1,2}$ of the diaphragm rod spring and $S_{1,3}$ of the supply valve spring. Now, the forces acting on the diaphragm can be described as follows:

always

$$F_{1,1} = P_{1,1} A_{1,2} \quad (3.22a)$$

$$F_{1,2} = P_{1,2} A_{1,2} \quad (3.22b)$$

$$\begin{aligned} S_{1,2} &= L_{1,2} + K_{1,2} X_{1,1} & 0 \leq X_{1,1} \leq X_S \\ &= L_{1,2} + K_{1,2} (X_{1,1} + X_I) & -X_E \leq X_{1,1} \leq X_I \\ &= 0 & \text{Elsewhere} \end{aligned} \quad (3.22c)$$

Supply open

$$\begin{aligned} S_{1,1} &= 0 \\ S_{1,3} &= L_{1,3} + K_{1,3} (X_{1,1} - X_0) & X_0 \leq X_{1,1} \leq X_S \end{aligned} \quad (3.22d)$$

Exhaust open

$$\begin{aligned} S_{1,3} &= 0 \\ S_{1,1} &= -L_{1,1} + K_{1,1} (X_I + X_{1,1}) & -X_E \leq X_{1,1} \leq -X_I \end{aligned} \quad (3.22e)$$

where: $K_{1,1}$, $K_{1,2}$ and $K_{1,3}$ are the respective spring constants, and $L_{1,1}$, $L_{1,2}$ and $L_{1,3}$ are the spring preloads.

In order to solve equations (3.16), (3.17), (3.18) and (3.20), one needs to find the mass flow rates $\dot{m}_{1,1}$, $\dot{m}_{1,2}$, the mass flow rate entering from the supply valve or the flow rate leaving the exhaust valve, $\dot{m}_{1,3}$ and the flow leaving or entering the brake pipe cut-off valve, $\dot{m}_{1,4}$. Utilizing equation (3.5), the mass flow rate $\dot{m}_{1,1}$, $\dot{m}_{1,2}$, $\dot{m}_{1,3}$ and $\dot{m}_{1,4}$ are evaluated as follows:

$$\dot{m}_{1,1} = .6 A_{1,1} P_{1,1} \sqrt{\frac{|B^2 - 1|}{R_g \theta}} \frac{|B-1|}{B-1} \quad (3.23)$$

where, $B = \frac{P_{eq}}{P_{1,1}}$,

$$\dot{m}_{1,2} = .6 A_{1,3} P_{1,2} \sqrt{\frac{|B^2 - 1|}{R_g \theta}} \frac{|B-1|}{B-1} \quad (3.24)$$

where, $B = \frac{P_{1,3}}{P_{1,2}}$,

$$\dot{m}_{1,3} = .6 A_{1,6} P_{1,3} \sqrt{\frac{|B^2 - 1|}{R_g \theta}} \frac{|B-1|}{B-1} \quad (3.25)$$

where, $B = \frac{P_{mr}}{P_{1,3}}$,

$$X_0 \leq X_{1,1} \leq X_S$$

$$\dot{m}_{1,3} = .6 A_{EX} P_{1,3} \sqrt{\frac{|B^2 - 1|}{R_g \theta}} \frac{|B-1|}{B-1} \quad (3.26)$$

where, $B = \frac{P_A}{P_{1,3}}$

$$\text{and } A_{EX} = \frac{A_{1,4} A_{1,5}}{\sqrt{A_{1,4}^2 + A_{1,5}^2}}$$

$$\dot{m}_{1,4} = .6 A_{3,3} P_{1,3} \sqrt{\frac{|B^2 - 1|}{R_g \theta}} \frac{|B-1|}{B-1} \quad (3.27)$$

where, $B = \frac{P_{bp}}{P_{1,3}}$

The supply valve and exhaust valve areas are modulated by the diaphragm movement as:

$$\begin{aligned} A_{1,6} &= 0 & X_{1,1} < X_0 \\ &= \pi D_{1,6} (X_{1,1} - X_0) & X_0 \leq X_{1,1} \leq X_S \end{aligned} \quad (3.28)$$

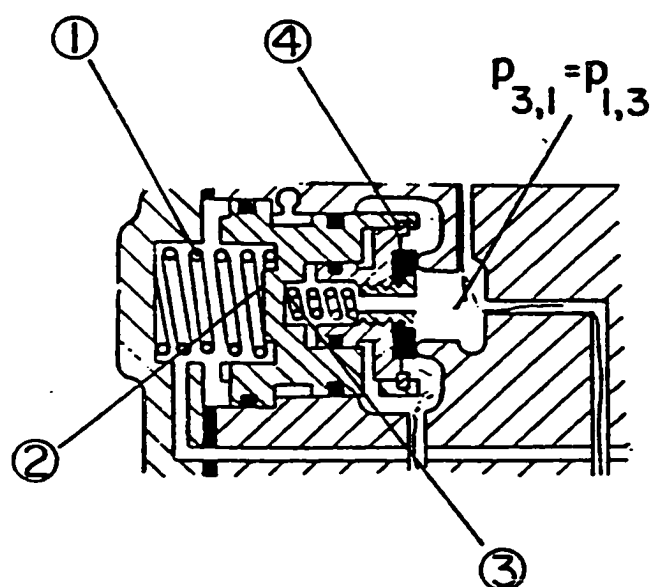
$$\begin{aligned} A_{1,4} &= 0 & X_{1,1} > -X_I \\ &= -\pi D_{1,4} (X_I + X_{1,1}) & -X_E \leq X_{1,1} \leq -X_I \end{aligned} \quad (3.29)$$

3.3.3 Brake Pipe Cut-off Valve Mathematical Model

In order to calculate the mass flow rate through the brake pipe cut-off valve, $m_{3,1}$, the flow area $A_{3,3}$ needs to be evaluated. Figure (3.7) shows the the brake pipe cut-off valve network, which includes only one restriction, $A_{3,3}$, controlled by the pressure $p_{3,1} = p_{1,3}$. Brake pipe cut-off piston has a very small mass, $I_{3,1}$, which does not play any important role in controlling the time rate of change of the brake pipe pressure and does not affect the dynamics of the relay valve, especially in the case of applying the brake. So, the inertial effects can be neglected, and the quasi-steady condition is assumed. A free body diagram of the brake pipe cut-off valve is shown in figure (3.8). Summing the forces acting on the brake pipe cut-off valve piston gives:

$$p_{3,1}A_{3,2} - p_A A_{3,1} - K_{3,1}x_{3,1} - L_{3,1} = 0 \quad (3.30)$$

Equation (3.19) says that the sum of the forces acting on both sides of the piston is equal to zero. These forces are; the pressure $p_{3,1}$ acting on the upper side of the piston times $A_{3,2}$ minus the atmospheric pressure, p_A , acting on the lower side of the piston times $A_{3,1}$ minus the brake pipe cut-off valve spring rate, $K_{3,1}$, times the piston displacement minus the spring preload $L_{3,1}$. In order to open the valve, the pressure $p_{3,1}$, needs to over come the forces exerted from the spring preload, $L_{3,1}$, and pressure p_A on $A_{3,1}$, which can be expressed as:



a) Schematic Diagram

b) Flow

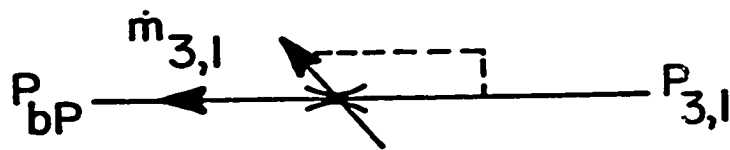
1. Piston Spring, $K_{3,1}$

3. Inner

2. Outer Side of the Piston, $A_{3,1}$

4. Flow

Figure 3.7: 26C Brake Pipe Cut-off



A_{3,1}

3. Inner Side of the Piston, $A_{3,3}$
4. Flow Area, $A_{3,3}$

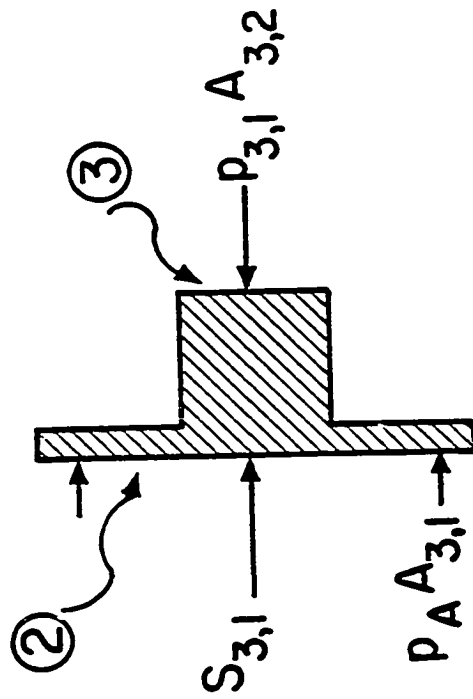


Figure 3.8: Brake Pipe Cut-off Valve Free Body Diagram

$$p_{3,1} > p_B \quad (3.31)$$

where,

$$p_B = \frac{p_A A_{3,1} + L_{3,1}}{A_{3,2}}$$

Based on the dimension given in the NYAB and WABCO blue prints, and using equation (3.31), the author found that that the pressure $p_{3,1}$ should exceed a value of 42.0 psi (27.5 psig) before the brake pipe cut-off valve starts moving. However, Ref.[31], which describes the operation of the 26C, says that "Air pressure in the brake pipe charging passage flows to the brake pipe cut-off valve which will open when the charging air exceeds about 10 psig.....". Experiments (chapter 4) verify the statement made by the author for the 27.5 psig.

Using equation (3.30), it is possible to calculate $A_{3,3}$, first solving for $x_{3,1}$

$$x_{3,1} = A_{3,2} \frac{p_{3,1} - p_B}{K_{3,1}} \quad p_B \leq p_{3,1} \leq p_B + E_1 \quad (3.32)$$

$$x_{3,1} = x_B \quad p_{3,1} > p_B + E_1$$

where, x_B and E_1 are dependent on the brake pipe cut-off valve dimension. The area of the brake pipe cut-off valve is equal the annular ring created by the displacement $x_{3,1}$ times the seat circumference as:

$$A_{3,3} = \pi D_{3,3} x_{3,1} \quad (3.33)$$

This equation completes the 26C mathematical model, which has been explored through out this entire section. Appendix

E lists all the equations used to model the 26C locomotive valve. In Chapter 4, the present mathematical model will be compared with the experiment data from UNH and NYAB. These comparisons will verify the capability of the 26C valve mathematical model to predict the dynamic behaviour of the 26C locomotive valve and its interaction with the brake pipe in different modes (release and brake application).

3.4 Simplified Model of the 26C Relay and Brake Pipe Cut-off Valves

More than one third of the previous section was devoted to the description of a detailed mathematical model for the 26C relay valve. The model completely describes the dynamic behaviour of the pressures within the small volumes ($V_{1,1}$, $V_{1,2}$ and $V_{1,3}$). This model may be used to aid in the understanding of the dynamic behaviour of the 26C relay valve, and how redesign and modification of the relay valve may produce different performance. However, one of the main objectives of this research project is to develop a simple mathematical model for the 26C locomotive valve to be incorporated with the rest of the mathematical models developed for the following air brake system components;

- 1) Brake pipe mathematical model and solution procedures (Chapter 2),
- 2) ABD control valve mathematical model (Appendix G),
- 3) ABDW control valve mathematical model (Appendix H),
- and 4) A19 Flow adapter mathematical model (Appendix I).

The 26C relay valve mathematical model which was developed earlier is a more complex model than one may need to use to study the transient behaviour of the air brake system. Including the complete model will also cause a tremendous increase in the computer simulation time because of the small integration time step required by the small volume within the 26C relay valve.

Hence, there is a need for an accurate and simplified model which may be used to represent the 26C relay valve. Fortunately, the rate of change of the locomotive pressure (p_{bp}) is slower than any of the pressures associated with the relay valve volumes. This behaviour makes the locomotive pressure control the dynamic behaviour of these volumes. So, the following assumption may be made based on a quasi-static approximation:

- 1) the inertial force is very small as compared with any of the forces associated with dynamics of the diaphragm movement,

$$I \frac{dU_{1,1}}{dt} \approx 0 \quad (3.34)$$

- 2) the pressure rate of change of $p_{1,1}$ is equivalent to the pressure rate of change of the equalizing reservoir, p_{eq} ,

$$p_{1,1} \approx p_{eq} \quad (3.35)$$

- 3) the pressure rate of change of $p_{1,3}$ is equivalent to the pressure rate of change of $p_{1,2}$,

$$p_{1,3} \approx p_{1,2} \quad (3.36)$$

4) the volume $V_{1,3}$ is assumed to be equal to zero,

$$V_{1,3} \approx 0 \quad (3.37)$$

Utilizing the above assumptions, the relay valve pneumatic network (figure (3.2a)) combined with the brake pipe cut-off valve network (figure (3.4b)) may be replaced by the network shown in figure (3.9). Using the first three assumptions, equations (3.20) and (3.22) may be rewritten in the following forms:

$$X_{1,1} = X_S \quad H_0 < Dp \quad (3.38a)$$

$$= B_0(Dp - H_1) + X_0 \quad H_1 \leq Dp \leq H_0 \quad (3.38b)$$

$$= X_0 \quad H_2 \leq Dp \leq H_1 \quad (3.38c)$$

$$= B_1(Dp - H_3) \quad H_3 \leq Dp \leq H_2 \quad (3.38d)$$

$$= 0.0 \quad 0 \leq Dp \leq H_3 \quad (3.38e)$$

$$= B_2 Dp \quad H_4 \leq Dp \leq 0 \quad (3.38f)$$

$$= -X_I \quad H_5 \leq Dp \leq H_4 \quad (3.38g)$$

$$= B_3(Dp - H_5) - X_I \quad H_6 \leq Dp \leq H_5 \quad (3.38h)$$

$$= -X_E \quad H_6 > Dp \quad (3.38i)$$

where,

$$B_0 = \frac{A_{1,1}}{[K_{1,2} + K_{1,3} + K_D]}$$

$$B_1 = \frac{A_{1,2}}{[K_{1,2} + K_D]}$$

$$B_2 = \frac{A_{1,2}}{[K_D]}$$

$$B_3 = \frac{A_{1,2}}{[K_{1,1} + K_{1,2} + K_D]}$$

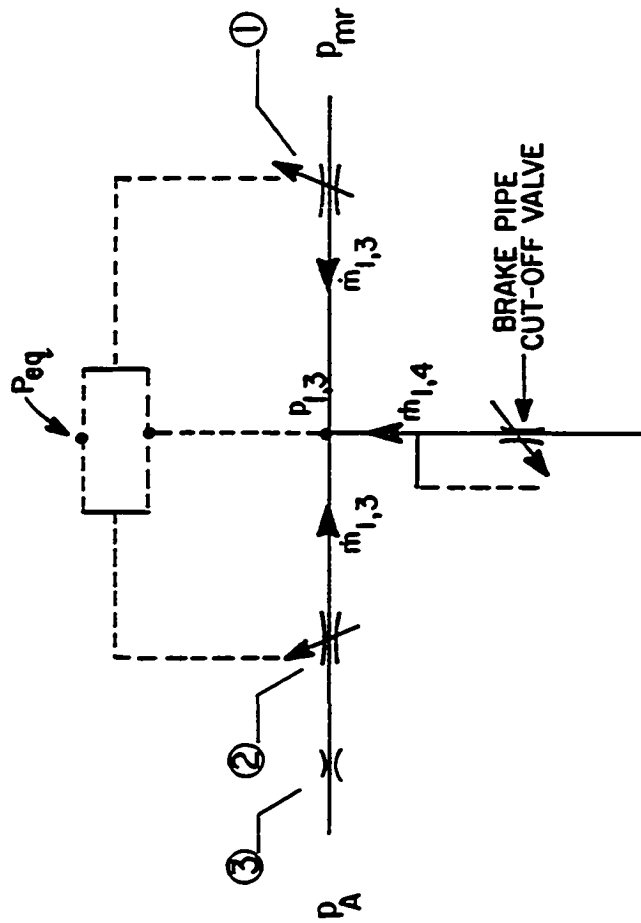


Figure 3.9: Relay and Brake Pipe Cut-off Valve Modified Network

$$\begin{aligned}
 H_0 &= \frac{[X_S - X_0]}{B_0} + H_1 & (3.39) \\
 H_1 &= \frac{L_{1,3}}{A_{1,2}} + H_2 \\
 H_2 &= \frac{X_0}{B_1} + H_3 \\
 H_3 &= \frac{L_{1,2}}{A_{1,2}} \\
 H_4 &= -\frac{X_I}{B_2} \\
 H_5 &= \frac{L_{1,2} - L_{1,1}}{A_{1,2}} + H_4 \\
 H_6 &= \frac{[X_I - X_E]}{B_3} + H_5
 \end{aligned}$$

and, approximate the pressure difference across the relay valve diaphragm as,

$$Dp = p_{eq}^j - p_{1,3}^j$$

Note that,

$$p_{1,3}^j = p_{bp}^j \quad -X_I \leq X_{1,1} \leq X_0$$

In order to solve equation (3.38), one needs to find the value of the pressure $p_{1,3}$ at each time step j . Figure (3.9) shows the network, which will be used to solve for $p_{1,3}$, as

$$p_{1,3} = \sqrt{\frac{p_{bp}^2 A_{3,3}^2 + p^2 A^2}{A_{3,3}^2 + A^2}} \quad (3.40)$$

Note that, j will be omitted from any equation as long as there is no $j-1$ terms. In equation (3.40), p and A are dependent on the state in which the relay valve is operating. In the following subsection, the solution of

equation (3.40) with the use of equations (3.38) and (3.39) will be discussed for the three different states of the relay valve operations.

3.4.1 Relay Valve States

During any of the brake pipe modes, the relay valve is operating at one or more states at different times. Any of these states is a function of the difference between the equalizing reservoir pressure and $p_{1,3}$. Figure (3.10) shows the relay valve different states (a,b and c), which are represented using the input-output diagram of the 26C relay valve. These states are;

- a) Supply State
- b) Intermediate State
- c) Exhaust State.

3.4.1.1 Supply State

This state represents the opening of the supply valve of the 26C relay valve. This state can be activated during any of the brake pipe modes (application, release/recharge and emergency). In order for the relay valve to maintain the pre-selected pressure value against the brake pipe leakage, the lapping position can be located within the region of this state, for any of the above modes.

During this state the value of p and A used in equation (3.40) will take the following forms:

$$p = p_{mr}$$

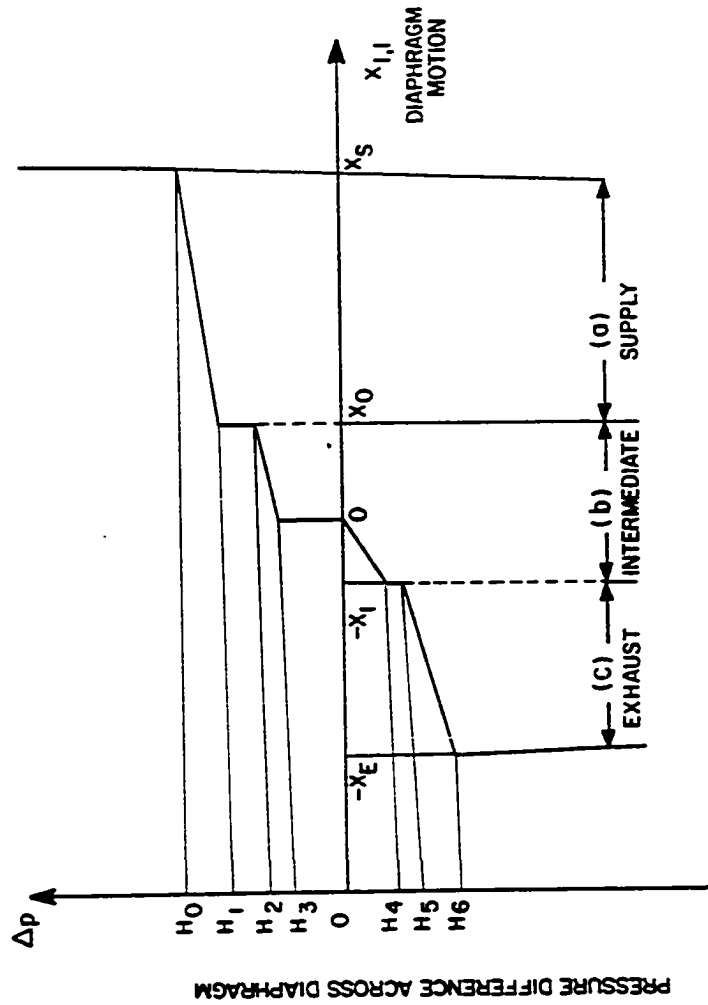


Figure 3.10: Relay Valve Input Output Diagram

(3.41)

and $A = A_{1,6}$

But $A_{1,6}$ is a function of $X_{1,1}$ (see equation (3.28)) which in turn is a function of $p_{1,3}$ (see equation (3.38b)). Thus A may be written in the following form as a function of $p_{1,3}$ (see figure (3.11a)):

for $p_{eq} - H_0 < p_{1,3} < p_{eq} - H_1$

$$A = \alpha_1 p_{1,3} + \alpha_2 \quad (3.42a)$$

for $p_{1,3} \leq p_{eq} - H_0$

$$A = C_1 = \pi D_{1,6} (X_S - X_0) \quad (3.42b)$$

for $p_{1,3} \geq p_{eq} - H_1$

$$A = 0 \quad (3.42c)$$

where,

$$\alpha_1 = -\pi D_{1,6} B_0, \text{ and}$$

$$\alpha_2 = \pi D_{1,6} B_0 \left[p_{eq} - H_1 \right] \quad (3.43)$$

Also, the brake pipe cut-off valve area, $A_{3,3}$, is a function of $p_{1,3}$ (see figure (3.11b)) as follows:

$$A_{3,3} = \alpha_3 p_{1,3} + \alpha_4 \quad p_{1,3} > p_B \quad (3.44a)$$

$$A_{3,3} = C_2 \quad p_{1,3} > p_B + E_1 \quad (3.44b)$$

$$= \pi D_{3,3} X_B$$

$$A_{3,3} = 0 \quad p_{1,3} \leq p_B \quad (3.44c)$$

comparing equation (3.44a) with equations (3.32) and (3.33),

the values of α_3 and α_4 can be evaluated as:

$$\alpha_3 = \pi D_{3,3} \frac{A_{3,2}}{K_{3,1}}, \text{ and}$$

$$\alpha_4 = -\pi D_{3,3} \frac{A_{3,1}}{K_{3,1}} p_B \quad (3.45)$$

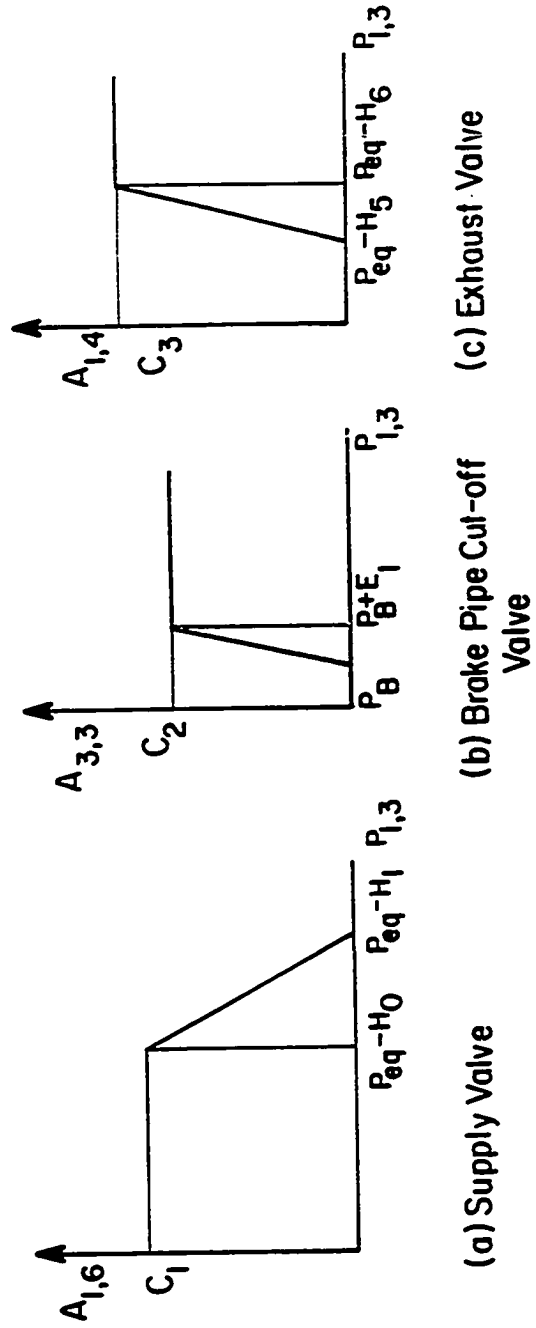


Figure 3.11: Input Output Diagrams

where,

$$P_B = \frac{P_A A_{3,1} + L_{3,1}}{A_{3,2}} \quad (3.20)$$

Substituting for p , A and $A_{3,3}$ using equations (3.41)–(3.44) into equation (3.40), the resultant equation may take the following general form:

$$\sum_{n=0}^4 \beta_n \left[P_{1,3} \right]^n = 0 \quad (3.46)$$

In order to solve the above equality, one may use an iterative technique to find the value of $P_{1,3}$, which satisfies equation (3.46). One of the possible technique can be used is the Newton-Raphson method, this technique is utilized in Appendix D to solve the star network problem. In order to use the Newton-Raphson method, one needs to define the function to be solved, in this case the function is simply,

$$\text{FUNCTION} = \sum_{n=0}^4 \beta_n \left[P_{1,3} \right]^n \quad (3.47)$$

Figure (3.12) shows how the FUNCTION varies with respect to $P_{1,3}$. You may find from figure (3.12) that this is not a well behaved function because of the discontinuities at locations (a), (b) and (c). This bad behaviour is caused by the nonlinearity of the coefficients (β_0 , β_1 , β_2 , β_3 and β_4). These coefficients are a nonlinear function of $P_{1,3}$ and P_{eq} , they have a different value for a different region as follows (figure (3.12) showing four of these regions):

Region I

$$P_B \leq P_{1,3} < P_B + E_1 \quad \text{and} \quad P_{1,3} < P_{eq} - H_0$$

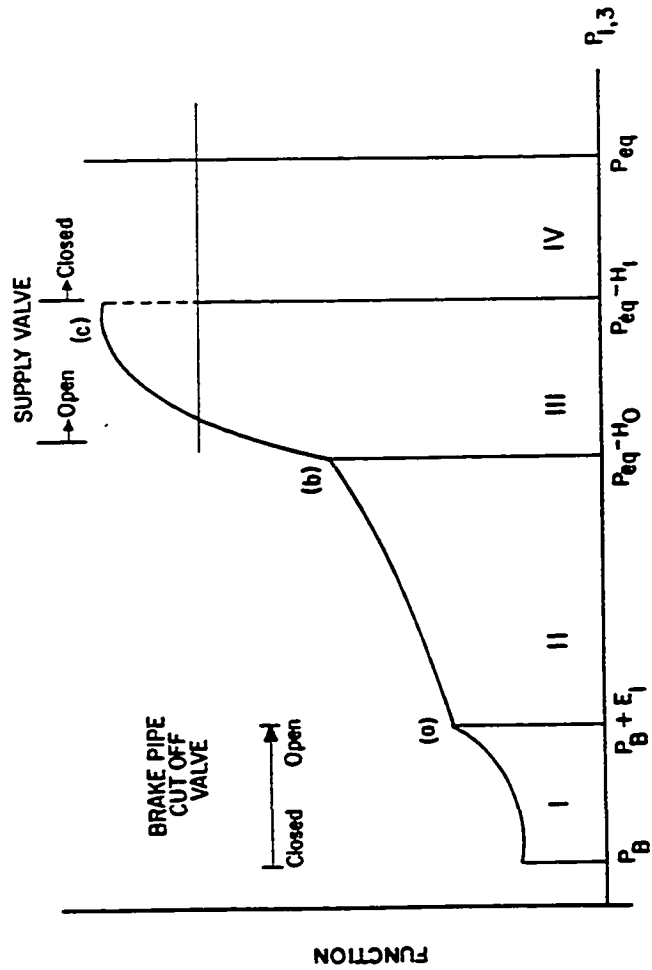


Figure 3.12: FUNCTION Versus Brake Pipe Cut-off Valve Pressure

Simply in this region

Supply valve fully open and Brake pipe cut-off valve varies

$$A = A_{1,6} = C_1 \quad \text{and} \quad A_{3,3} = \alpha_3 p_{1,3} + \alpha_4$$

Now,

$$\begin{aligned} \beta_0 &= -\alpha_4^2 p_{bp}^2 - C_1^2 p_{mr}^2 \\ \beta_1 &= -2\alpha_3 \alpha_4 p_{bp} \\ \beta_2 &= C_1^2 + \alpha_4^2 - \alpha_3^2 p_{bp}^2 \\ \beta_3 &= 2\alpha_3 \alpha_4 \\ \beta_4 &= \alpha_3^2 \end{aligned} \quad (3.48)$$

Region II $p_B + E_1 \leq p_{1,3}$ and $p_{1,3} < p_{eq} - H_0$

Simply in this region,

Supply valve fully open

$$A = A_{1,6} = C_1$$

and,

Brake pipe cut-off valve fully open

$$A_{3,3} = C_2$$

Now,

$$\begin{aligned} \beta_0 &= -C_1^2 p_{mr}^2 - C_2^2 p_{bp}^2 \\ \beta_1 &= \beta_3 = \beta_4 = 0 \\ \beta_2 &= C_1^2 + C_2^2 \end{aligned} \quad (3.49)$$

Region III $p_{1,3} > p_B + E_1$ and $p_{eq} - H_0 \leq p_{1,3} < p_{eq} - H_1$

Simply in this region

Supply valve varies

$$A = A_{1,6} = \alpha_1 p_{1,3} + \alpha_2$$

and,

Brake pipe cut-off valve fully open

$$A_{3,3} = C_2$$

Now,

$$\begin{aligned}\beta_0 &= -\alpha_2^2 p_{mr}^2 - C_2^2 p_{bp}^2 \\ \beta_1 &= -2\alpha_1 \alpha_2 p_{mr}^2 \\ \beta_2 &= C_2^2 + \alpha_2^2 - \alpha_1^2 p_{mr}^2 \\ \beta_3 &= 2\alpha_1 \alpha_2 \\ \beta_4 &= \alpha_1^2\end{aligned}\quad (3.50)$$

Region IV $p_{1,3} > p_B + E_1$ and $p_{1,3} \geq p_{eq} - H_1$

FUNCTION=0.

Brake pipe cut-off valve fully open, supply and exhaust valves fully closed

Region V (is not shown in figure (3.12))

$$p_B \leq p_{1,3} < p_B + E_1 \quad \text{and} \quad p_{eq} - H_0 \leq p_{1,3} < p_{eq} - H_1$$

Simply in this region,

Supply valve varies

$$A = A_{1,6} = \alpha_1 p_{1,3} + \alpha_2$$

and,

Brake pipe cut-off valve varies

$$A_{3,3} = \alpha_3 p_{1,3} + \alpha_4$$

Now,

$$\begin{aligned}\beta_0 &= -\alpha_2^2 p_{mr}^2 - \alpha_4^2 p_{bp}^2 \\ \beta_1 &= -2\alpha_1 \alpha_2 p_{mr}^2 - 2\alpha_3 \alpha_4 p_{bp}^2 \\ \beta_2 &= \alpha_2^2 + \alpha_4^2 - \alpha_1^2 p_{mr}^2 - \alpha_3^2 p_{bp}^2 \\ \beta_3 &= 2\alpha_1 \alpha_2 + 2\alpha_3 \alpha_4\end{aligned}\quad (3.51)$$

$$\beta_4 = \alpha_1^2 + \alpha_3^2$$

Now using the above values of the coefficients within different regions, it is possible to solve for $p_{1,3}$, using the Newton-Raphson method, given the values of p_{mr} , p_{bp} and p_{eq} at any j time step. Care should be taken, when solving the FUNCTION, especially as $p_{1,3}$ approaches the equalizing reservoir pressure value.

3.4.1.2 Intermediate State

This could be activated during any of the brake pipe modes (application, release or recharge), and the lapping position may be located within its region as long as there is no leakage within the entire air brake system. During this state, both the supply valve and exhaust valve are completely closed,

$$A_{1,6} = A_{1,3} = A = 0. \quad (3.52)$$

Hence,

$$p_{1,3} = p_{bp} \quad (3.53)$$

3.4.1.3 Exhaust State

This state can only be activated during any of the brake pipe service reduction (application). The exhaust area, A_{EX} is calculated using the following equation:

$$A_{EX} = \frac{A_{1,4} A_{1,5}}{\sqrt{A_{1,4}^2 + A_{1,5}^2}} \quad (3.54a)$$

and the exhaust valve area, $A_{1,4}$, is a function of $p_{1,3}$ as

for $p_{eq}^{-H_5} < p_{1,3} < p_{eq}^{-H_6}$

$$A_{1,4} = \alpha_5 p_{1,3}^{\alpha_6} \quad (3.54b)$$

for $p_{1,3} \geq p_{eq}^{-H_6}$

$$A_{1,4} = C_3 = \pi D_{1,4} (X_E - X_I) \quad (3.54c)$$

for $p_{1,3} = p_{eq}^{-H_5}$

$$A_{1,4} = 0 \quad (3.54d)$$

where,

$$\alpha_5 = \pi D_{1,4} B_3 \quad (3.55)$$

$$\text{and } \alpha_6 = \pi D_{1,4} B_3 [H_5 - p_{eq}]$$

Fortunately, during this state, the brake pipe cut-off valve is always fully open. However, the value of $A_{1,5}$ is very small as compared with $A_{1,4}$, which makes A_{EX} a very small value as compared with the brake pipe cut-off area. This will make the pressure $p_{1,3}$ very close to the brake pipe pressure. Because of the above, one may use a simpler approach to solve equations (3.40) and (3.54) than the one used during the supply state. Simply, solve equations (3.40) and (3.54) at different time steps,

$$p_{1,3}^j = \left[\frac{p_{bp}^2 C_2^2 + p_{mr}^2 A_{EX}^2}{C_2^2 + A_{EX}^2} \right]^{j-1} \quad (3.56)$$

for $p_{eq}^j - H_5 < p_{1,3}^j < p_{eq}^j - H_6$

$$A_{1,4}^j = \alpha_5^j p_{1,3}^j{}^{\alpha_6^j} \quad (3.57a)$$

for $p_{1,3}^j \geq p_{eq}^j - H_6$

$$A_{1,4}^j = C_3 = \pi D_{1,4} (X_E - X_I) \quad (3.57b)$$

Now, solve equation (3.56) for $p_{1,3}^j$ using the values of p_{mr} , p_{bp} , p_{eq} and A_{EX} at the $j-1$ time step. Then, solve equation (3.57) to solve for $A_{1,4}$ at the j time step. In the case of $p_{1,3}^j = p_{eq} - H_5$, the value of A_{EX} will be equal to zero, hence, the value of $p_{1,3}$ will be equal to the value of the locomotive pressure, p_{bp} .

This section completes the development of a simplified mathematical model for the 26C relay valve, which may be used with the rest of the air brake system components to study the dynamic behaviour of the entire system. A part of Chapter 4 will describe the experimental investigation made to compare the 26C complete model results with the new simplified model results. It will be shown, that the simplified (modified) model can tremendously reduce the computer time to at least one fifth of the that time needed for the complete model simulation.

3.5 Summary

This chapter was completely devoted to find a suitable mathematical model which may be used to represent the 26C locomotive valve. Our first goal was to formulate a mathematical model describing most of the internal design of the valve (ie volumes, restrictions... etc). Secondly, we need a model which can be used efficiently (fairly accurate dynamic behaviour and less computational time) to describe the dynamic interaction between the 26C and the rest of the air brake components.

In order to achieve these goals, one needs to understand the operation principle of each of the 26C locomotive valve components (relay valve, regulating valve and brake pipe cut-off valve). Section 3.2 described completely the main roles played by the above components and gives a clear picture about their operation modes.

The first goal was achieved in Section 3.3, where each of the main components of the 26C locomotive valve is completely modeled including the small volumes in the relay valve and all the restrictions within these components. This section provided us with a complete descriptive model, which may be used in a future redesigning of the 26C relay valve.

Section 3.4 composed a new simplified (modified) model for the 26C relay and brake pipe cut-off valves which may be used in conjunction with the other mathematical models developed for the rest of the air brake system components. The modified model is very complex logically as compared to the complete model. The next chapter will prove that this model accurately presents the functions of the combination of the 26C locomotive valve, and the computer simulation time is much less than the time needed for simulating the complete model.

CHAPTER 4

EXPERIMENTAL RESULTS AND DISCUSSIONS

4.1 Introduction

The objective of this chapter is to provide the validations of the mathematical models of the air brake system components. This chapter also will attempt to have a close look at the interactions between each of the air brake system components during the different modes of operation of the system. These modes are; the dry charge mode, recharge after application, full application, and emergency.

The brake pipe mathematical model is developed in Appendix A. Two different numerical techniques have been formulated in Chapter 2, namely, the finite difference and finite element techniques. Two of the possible formulations for the finite difference are also presented. They are, the semi-implicit and fully explicit schemes.

Chapter 3, describes the formulations of the complete model for the 26C locomotive valve. The model completely represents the different components of the valve. Then, an attempt has been made to simplify/modify the 26C model to give an efficient (accurate and fast) representation of the valve. The modified model, eventually, will be a part of the complete simulation of the air brake system.

Finally, Appendices G, H and I, describe the formulations of the mathematical models for the ABD and ABDW control valves, and the A19 flow indicator adaptor respectively.

In this chapter, several experimental set-ups are utilized to achieve the main goals of this chapter. Section 4.2, describes the 26C experimental set-up and results, used to

1. give a close look to the different functions of the 26C locomotive valve components,
- and, 2. compare and validate the complete and modified mathematical model of the valve.

Section 4.3, describes the brake pipe experimental set-up and results, used to

1. understand the interaction between the 26C locomotive valve and the brake pipe,
2. compare between the experimental results and the different possible combinations of the 26C mathematical models and the brake pipe numerical techniques,
- and 3. choose the best combination to be used as a part of the air brake system components computer simulation.

Section 4.4, describes the air brake system components experimental set-ups and discuss the results. The results are used to

1. understand the interaction between the different components as a part of the air brake system

and, 2. validate the air brake system computer simulation, developed and formulated in this research project. Finally, section 4.5 summarizes the results and discussions, presented in this chapter.

4.2 26C Experimental Set-up, Results and Model Comparisons

Figure (4.1) shows the 26C experimental set-up, which consists of the following:-

1. 26C locomotive valve.
2. Brake pipe reservoir (1400 in³).
3. Main reservoir (4500 in³).
4. Equalizing reservoir (220 in³)
5. 4 (0-100 psi) pressure transducers, used to monitor;
 - a. brake pipe pressure,
 - b. main reservoir pressure,
 - c. equalizing reservoir,
 - and d. inner diaphragm of the 26C relay valve.
6. Differential transducer (0-5 psi), used to monitor the pressure difference, developed across the 26C relay valve diaphragm.
7. LVDT displacement transducer, used to monitor the diaphragm displacement of the 26C relay valve,
8. PDP 11/34 minicomputer, used to collect the experimental data from the above mentioned transducers.

The experiment has been performed in the following sequence:-

P.T. PRESSURE TRANSDUCER

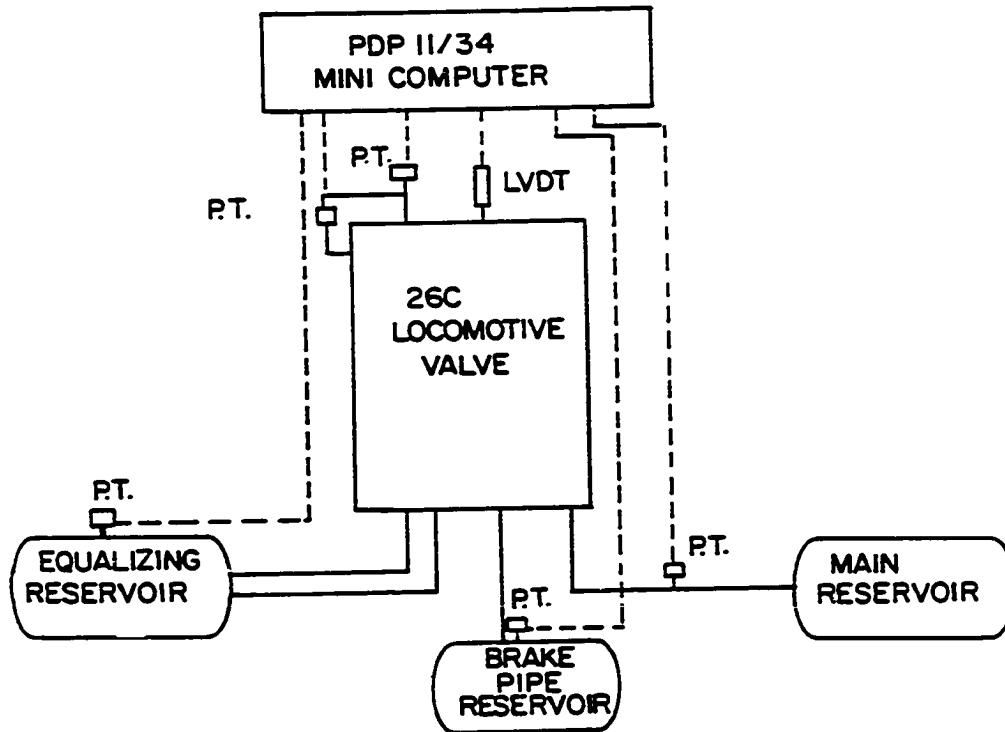


Figure 4.1: 26C Experimental Set-up

1. the regulating valve adjustment knob is set to a 60 (psig) pressure level,
2. the 26C handle is moved to the release position (dry charge to 60 psig) for a period of 12 seconds,
3. then, the 26C handle is moved to its full service position (23 psi reduction) for the rest of the experiment.

4.2.1 Experimental Results

During the dry charge period, the equalizing reservoir pressure, figure (4.2a), increases with a rate controlled by the 26C regulation valve. Air from the equalizing reservoir enters the outer diaphragm chamber of the 26C relay valve through the feed back orifice, $A_{1,1}$, causes the increase in the $P_{1,1}$, shown in figure (4.2b). As a result, the pressure difference, across the relay valve diaphragm, starts to increase, shown in figure (4.2c), which causes the movement of the diaphragm, $X_{1,1}$, (figure (4.2d)). When $X_{1,1}$ overcomes the gap, $X_{1,1} = .042$ in, between the diaphragm rod and the supply valve of the relay valve, the supply valve opens, allowing air to flow from the main reservoir to the inner diaphragm chamber, through the intermediate volume $V_{1,3}$. Now, the pressure in both sides of the diaphragm and the equalizing reservoir increase with almost the same pressure rate. When, the intermediate volume pressure, $P_{1,3} = P_{3,1}$, overcomes the spring preload, $L_{3,1}$, of the brake pipe cut-off valve, the valve starts to open. This opening causes a

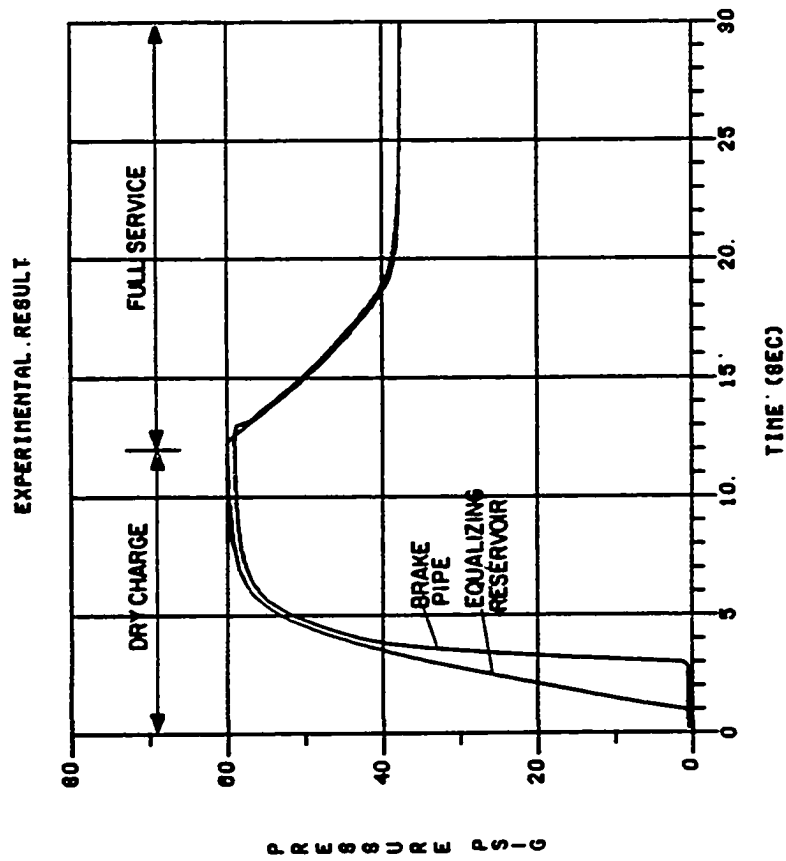


Figure 4.2a: Transient Response of Brake Pipe and Equalizing Reservoir Pressures (Experimental Data)

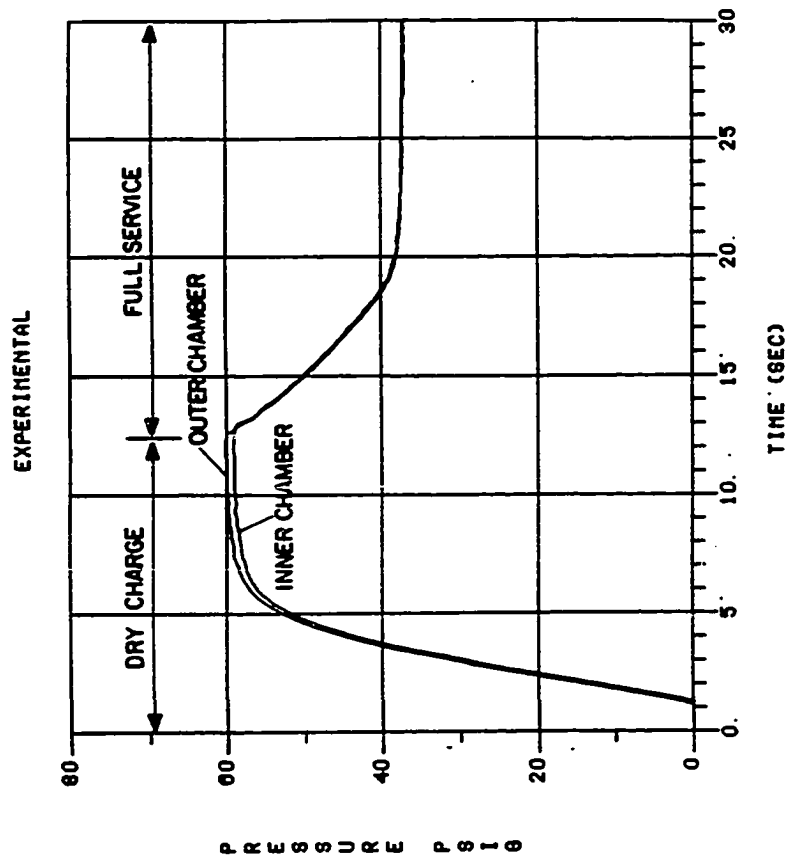
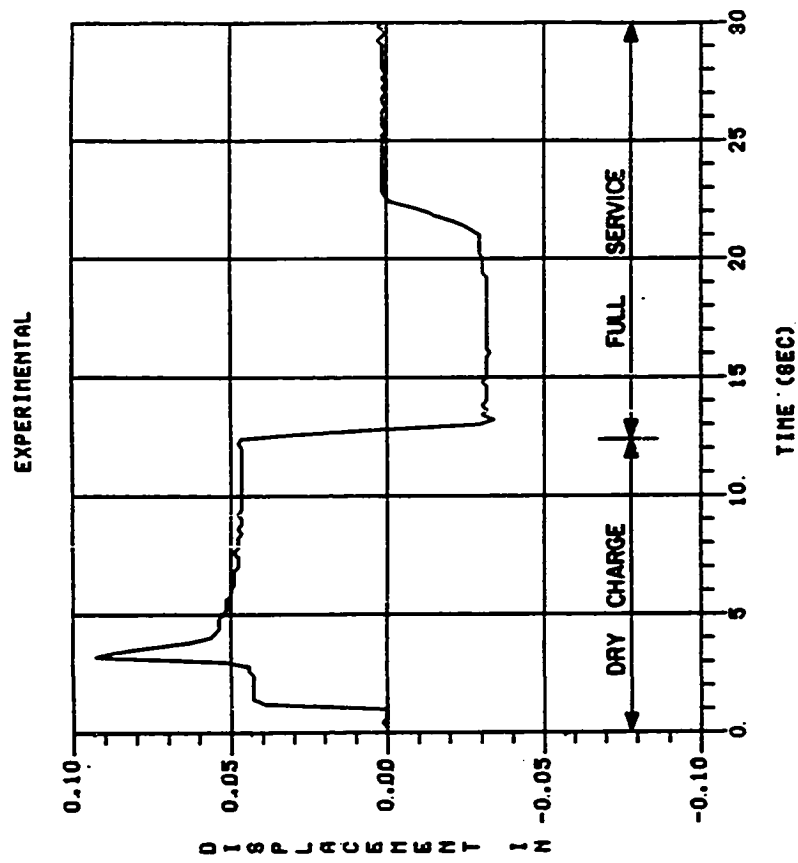


Figure 4.2b: Transient Response of the Inner and Outer Chamber Pressures
(Experimental Data)



**Figure 4.2c: Transient Response of 26C Relay Valve Diaphragm Displacement
(Experimental Data)**

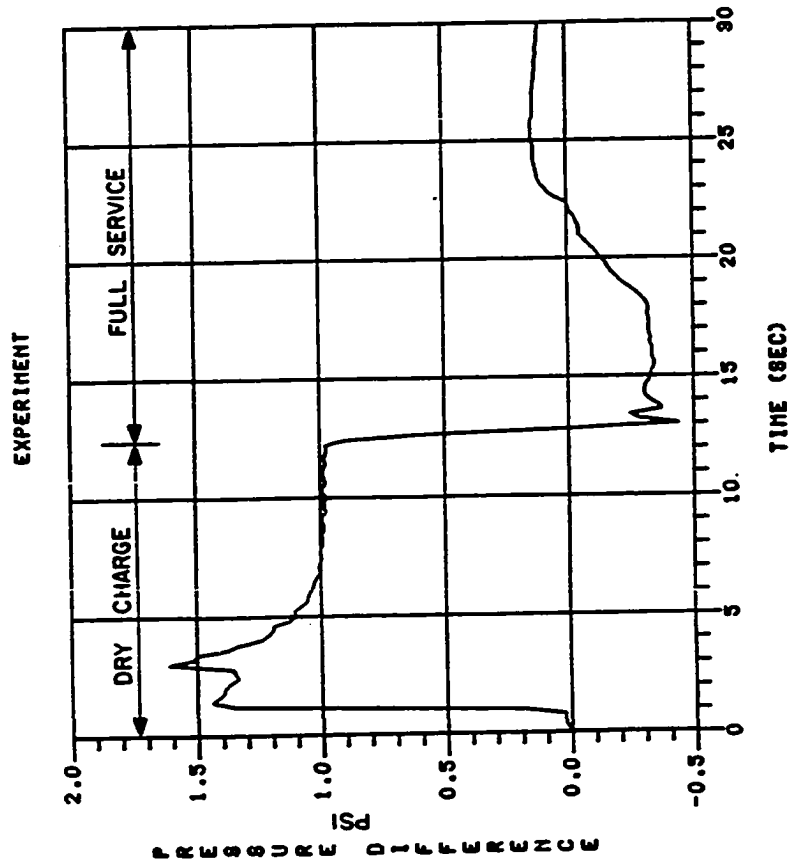


Figure 4.2d: Transient Response of Pressure Difference Across 26C Relay Valve Diaphragm (Experimental Data)

sudden drop in the intermediate volume and inner chamber pressures. Then, the diaphragm rod moves, causing a larger opening in the supply valve to compensate for the pressure difference existing across the diaphragm. Now, more air flows from the main reservoir to the brake pipe, causing a rapid increase in the brake pipe pressure, as shown in figure (4.2a), and decrease in the main reservoir pressure. As the brake pipe, intermediate volume and inner diaphragm chamber pressures increase, approaching the equalizing reservoir and outer chamber pressures, the supply valve starts to close. Then, the relay valve diaphragm stays at a lap position ($X_{1,1} = .042$ in). This completes the dry charge test.

During the full service mode, the brake pipe cut-off valve is in a full open position. First equalizing reservoir pressure, figure (4.2a), drops with a pressure rate controlled by the regulating valve variable restriction. The outer diaphragm chamber pressure drops following the equalizing reservoir pressure. This action causes a sudden drop in the pressure difference, which forces the diaphragm rod to move towards the exhaust valve. Then the diaphragm moves without its rod for $-.03 < X_{1,1} < 0$. The exhaust valve starts to open as the diaphragm valve overcomes the $-.03$ (in) gap. Now, the inner chamber, intermediate volume and the brake pipe pressures drop with a rate controlled by the exhaust area, A_{EX} . As the brake pipe approaches the equalizing reservoir pressure, the exhaust valve starts to

close, terminating the reduction mode. Then, the diaphragm stays at a lap position, $X_{1,1} \approx 0$. This completes the description of the full service mode for the 26C locomotive valve.

4.2.2 26C Complete Model Results

Figures (4.3a-4.3d) show the corresponding computational results, using the complete model of the 26C locomotive valve. These figures show that, the complete model is able to duplicate the experimental data in detail, shown earlier in figures (4.2a-4.2d). The only disadvantage using the complete model, is the expensive need to use a very small integration time step, to overcome the following obstacles:-

1. The integration instability, which may arise because of the fact that the ordinary differential equations, described earlier in Chapter 3, are nonlinear stiff equations,
2. The need to have a very close agreement with the experimental data.

However, the complete mathematical model for the 26C locomotive valve is a very useful tool to help one understand the dynamic behaviour of the valve; redesign and modification of the valve design may produce different performance.

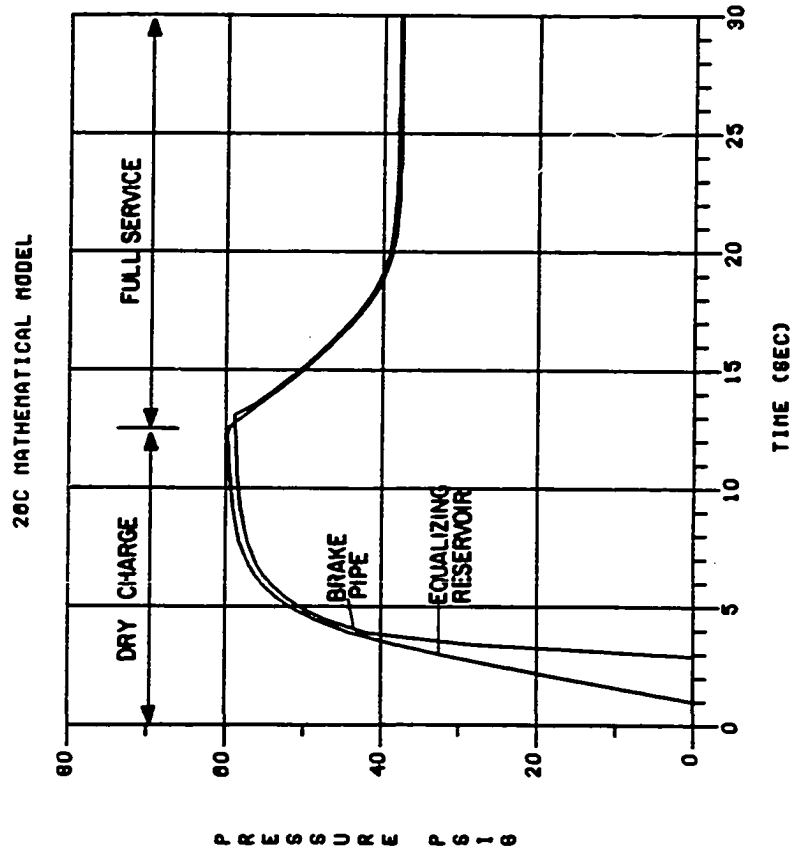
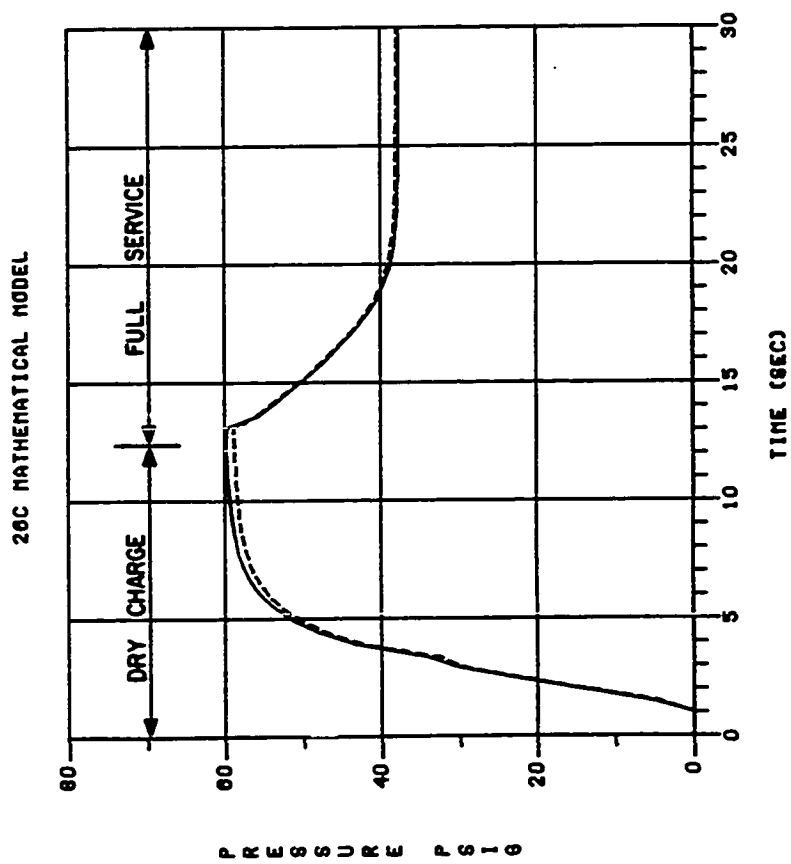


Figure 4.3a: Transient Response of Brake Pipe and Equalizing Reservoir Pressures (Complete Model)



**Figure 4.3b: Transient Response of the Inner and Outer Chamber Pressures
(Complete Model)**

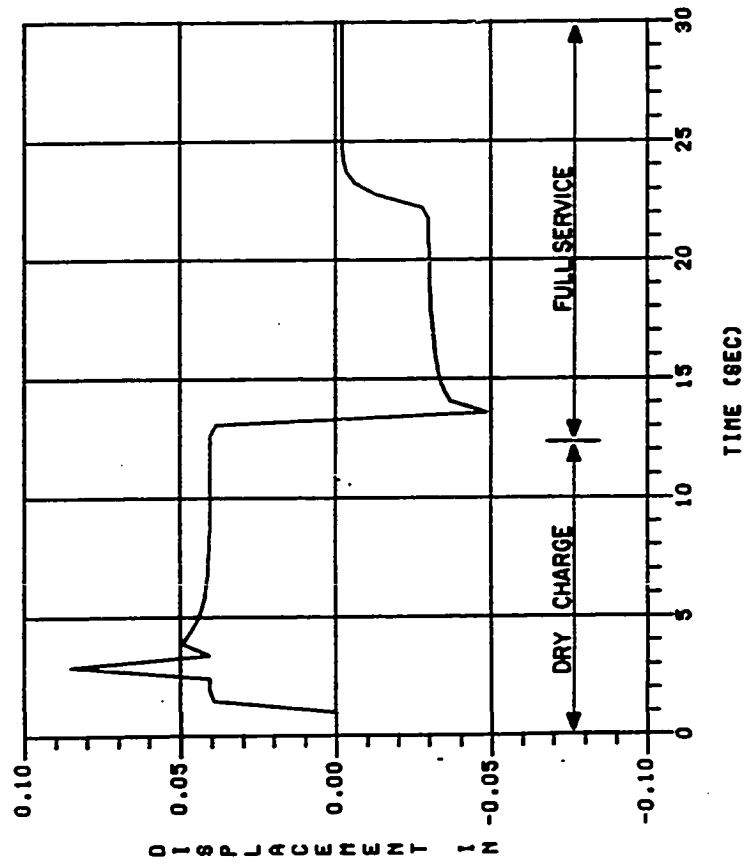


Figure 4.3c: Transient Response of 26C Relay Valve Diaphragm Displacement
(Complete Model)

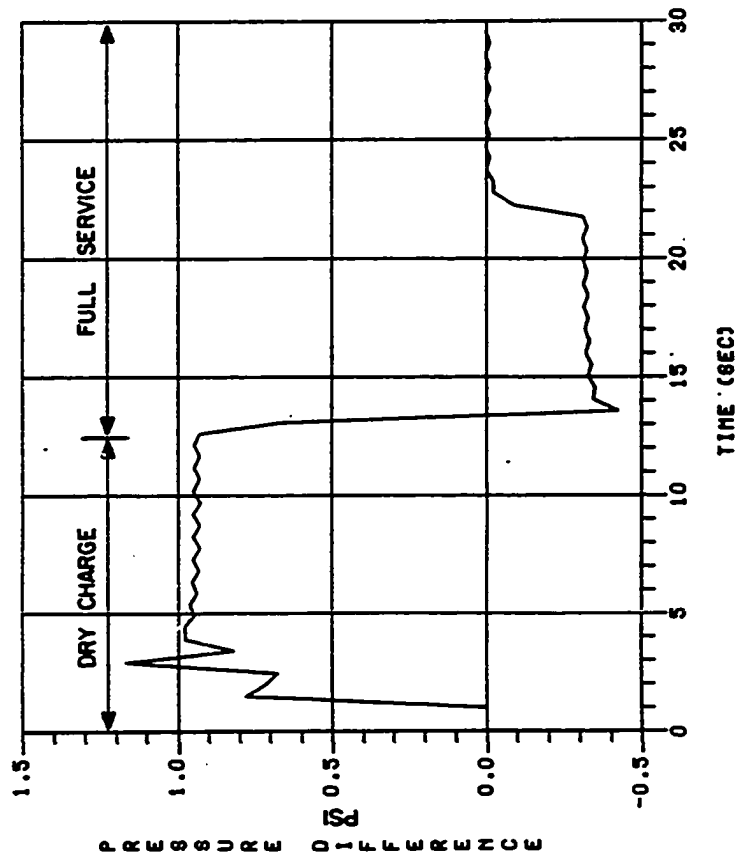


Figure 4.3d: Transient Response of Pressure Difference Across 26C Relay Valve Diaphragm (Complete Model)

4.2.3 26C Modified Model Results

Figure (4.4a) shows the computational results, using the 26C modified mathematical model, for the equalizing and brake pipe reservoirs pressures. The modified model results are in very good agreement with the experimental results, shown earlier in figure (4.2a). Figures (4.4b) and (4.4c). They show that the modified model gives a good overall picture for the diaphragm displacement and the pressure difference across the diaphragm. The modified model fails to predict the pressure difference and diaphragm displacement peaks, shown in figures (4.2c) and (4.2d). However, these peaks last for a very small time as compared with the total transient time, and have no effect on the transient behaviour of the brake pipe pressure. Section 4.3, will show that the ability of the modified model to predict the transient response of the pressure difference and displacement of the relay valve diaphragm is sufficient for good brake pipe pressure distribution simulation. As a conclusion, the modified model can predict the dynamic behaviour of the 26C locomotive valve with a reasonable accuracy, using a relatively small computer simulation time as compared with the time taken by the complete model (1:8 time ratio). That means, the modified model may be used, instead of a very expensive complete model, to represent the 26C locomotive valve as a part of the air brake system components program.

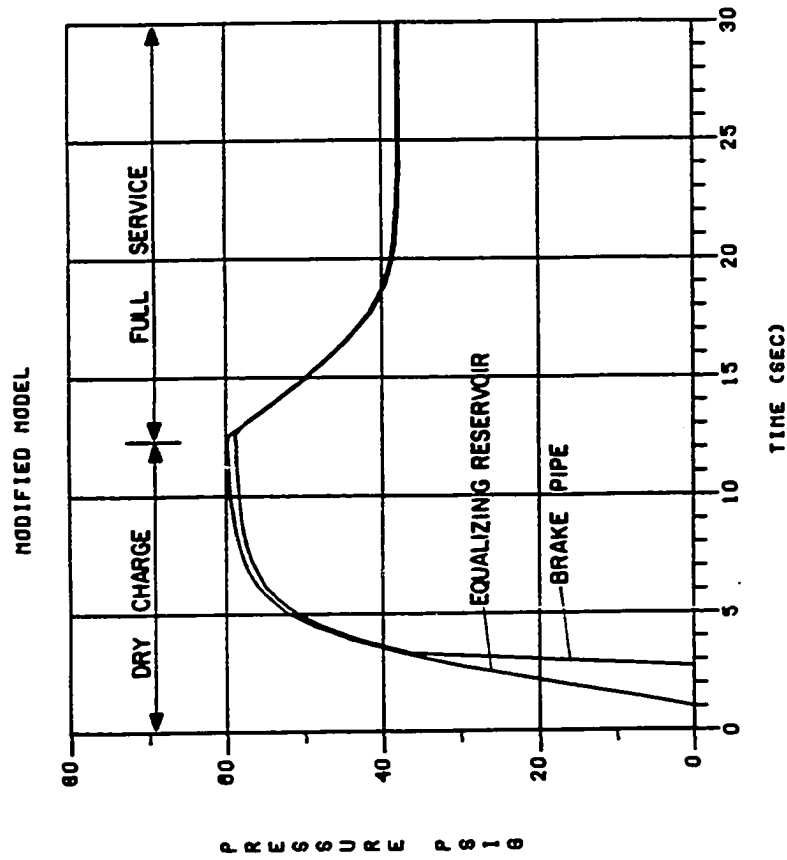


Figure 4.4a: Transient Response of Brake Pipe and Equalizing Reservoir Pressures (Modified Model)

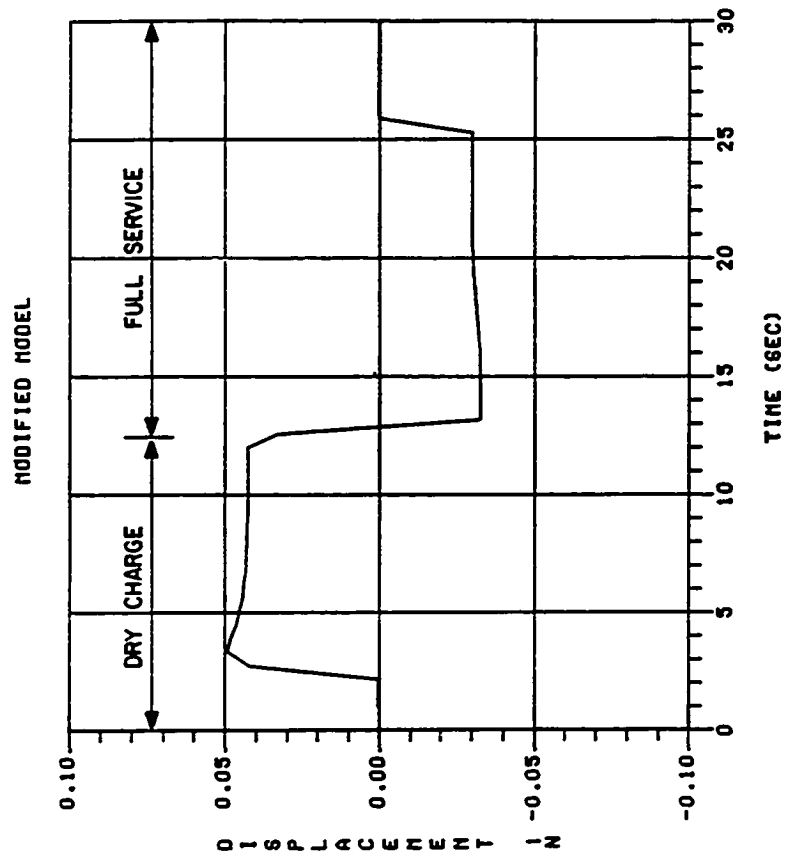


Figure 4.4b: Transient Response of 26C Relay Valve Diaphragm Displacement
(Modified Model)

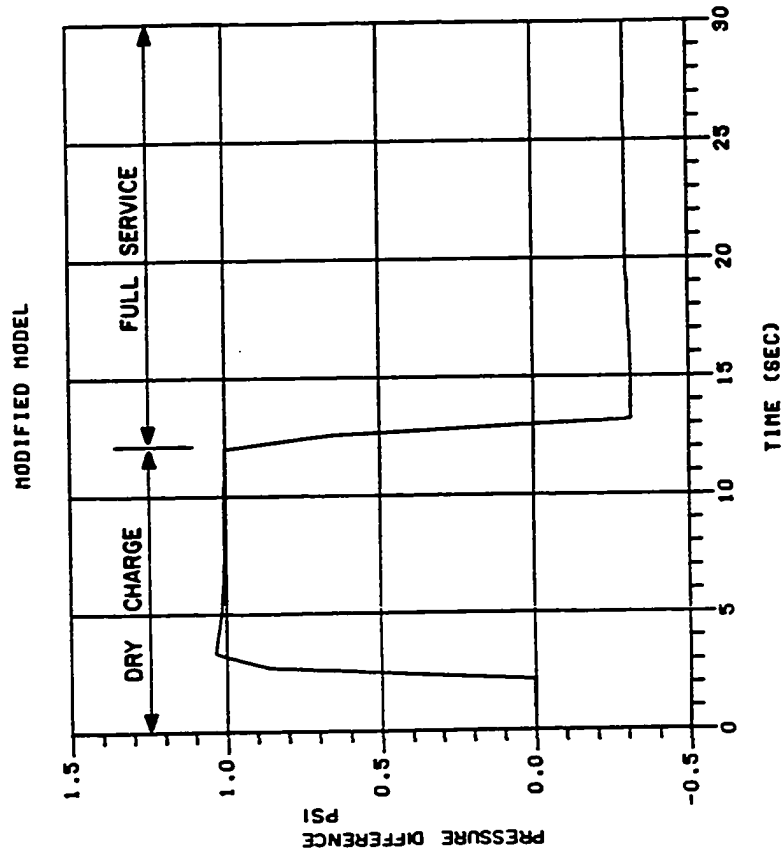


Figure 4.4c: Transient Response of Pressure Difference Across 26C Relay Valve Diaphragm (Modified Model)

4.3 Brake Pipe Experimental Set-up Results and Model Comparisons

Figure (4.5) shows the brake pipe experimental set-up, used to study the dynamic interaction between the 26C locomotive valve and a 20-car brake pipe (train), with the control valves cut-out. This experimental set-up consists of the following components:-

1. 26C locomotive valve.
2. 20-car train (50 (foot) length and 1.25 (in) diameter each).
3. Main reservoir (55,000 in³).
4. Equalizing reservoir (220 in³).
5. 4 pressure transducers (0-100 psig), to monitor the pressures at:-
 - a. 26C inlet,
 - b. equalizing reservoir,
 - c. head-end of the 20-car train,
 - and, d. rear-end of the 20-car train.
6. LVDT to measure the 26C diaphragm displacement.
7. Differential pressure transducer to monitor the pressure difference across the diaphragm of the 26C relay valve.
8. HP minicomputer, used to collect the experimental data from the pressure transducers and LVDT.

Three different experiments were performed, using the brake pipe experimental set-up, as follows:-

1. full service (23 psi reduction from 60 psig),

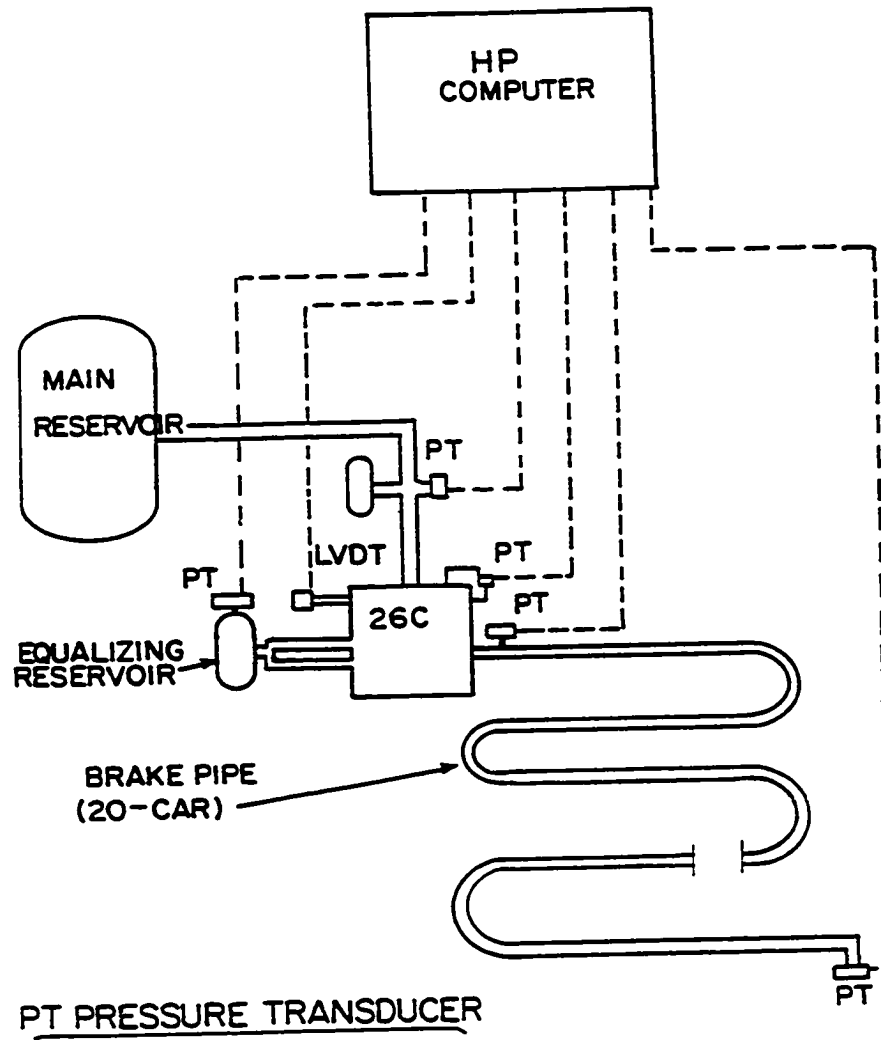


Figure 4.5: Brake Pipe Experimental Set-up

2. Recharge after full service (23 psi recharge),
- and 3. Dry charge (60 psi recharge from 0 psig).

Additional tests were performed to determine the variation of the brake pipe friction factor with respect the flow regimes. Appendix K describes the experimental set-ups used to determine these variations.

4.3.1 Full Service

4.3.1.1 Experimental Results

Figures (4.6a) and (4.6b) show the experimental results of the full service test (23 psi reduction). The equalizing reservoir pressure, shown in figure (4.6a), drops with a pressure rate controlled by the regulating valve. Following the same operation procedure, discussed in the previous section, the pressure difference across the diaphragm drops, opening the exhaust valve, as shown in figure (4.6b). This opening allows the brake pipe air to flow to atmosphere through a very small restriction (.25 (in) in diameter), which causes a decrease in the brake pipe pressure. Then, the equalizing reservoir pressure reaches its final value, in approximately 6 seconds. As the head-end brake pipe pressure approaches the constant equalizing reservoir pressure, the exhaust valve starts to close, metering the flow of the brake pipe to atmosphere. Then, the relay valve closes and the 26C locomotive goes to the lapping mode ($X_{1,1}=0$).

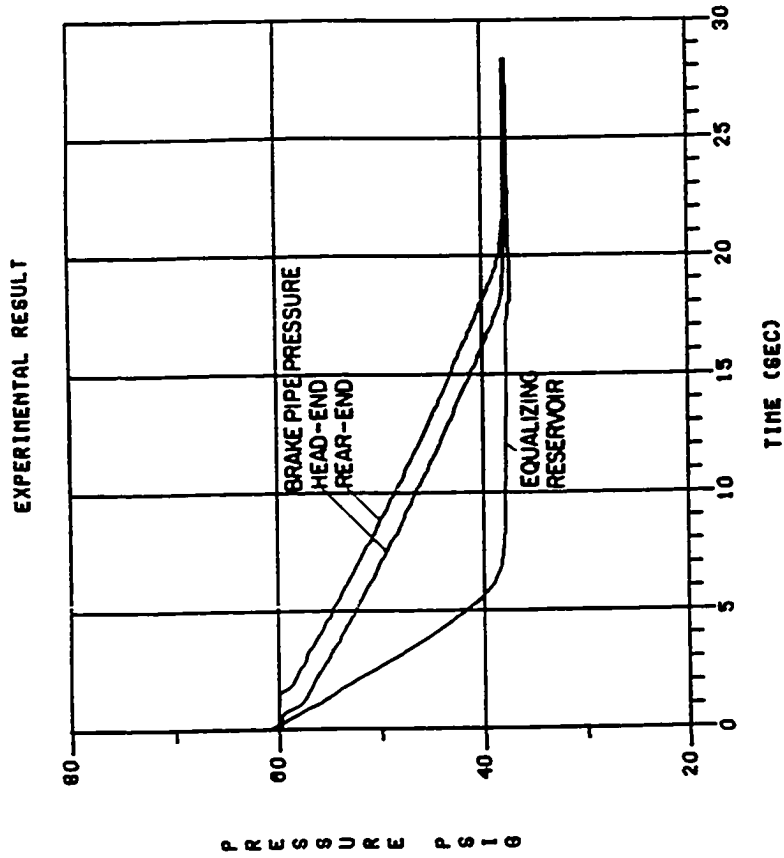


Figure 4.6a: Brake Pipe and Equalizing Reservoir Pressures Vs. Time (Full Service Experimental Data)

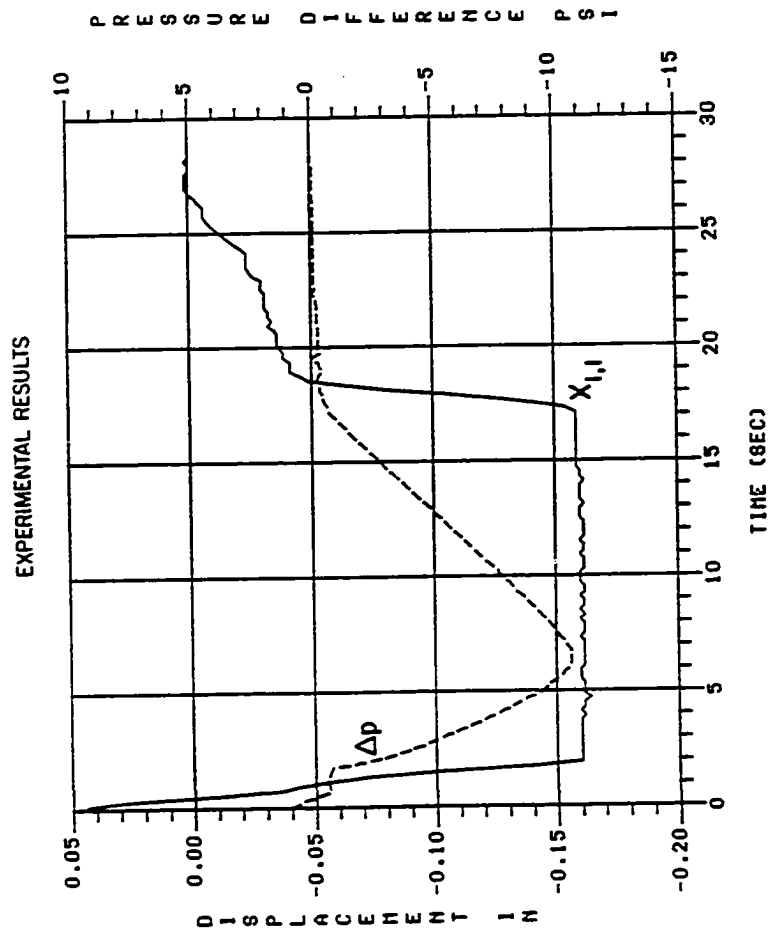


Figure 4.6b: Diaphragm Displacement and Pressure Difference Vs. Time (Full Service Experimental Data)

It is noticed, during the course of the full service test, that the brake pipe pressure transient behaviour, at any location along the 20-car train, is a duplicate of the pressure at the head-end of the train. This phenomena is believed to be caused by the fact that, the exhaust restriction, R_E , is smaller than the brake pipe frictional restriction, R_f , thus, the exhaust restriction controls the pressure dynamic behaviour of the brake pipe, where,

$$R_E = \left[\frac{A_{bp}}{A_{EX}} \right]^2 \quad (4.1)$$

$$R_f = f \frac{l}{d} \quad (4.2)$$

A_{bp} = Brake pipe cross-sectional area

A_{EX} = 26C relay valve exhaust area (see equation (3.16))

f = Brake pipe frictional factor,

l = Total length of the 20-car train,

and d = Brake pipe diameter.

However, this phenomena exists, only, for a relatively small train.

4.3.1.2 Model Comparisons

Now, the experimental results may be compared with the computational results, made possible from the combinations of the following 26C mathematical models and the numerical techniques, used to provide the solution for the brake pipe mathematical model,

1. 26C mathematical models;

a. Complete model

- b. modified model
- 2. Numerical techniques;
 - a. finite difference
 - 1. Implicit scheme
 - 2. Explicit scheme
 - b. implicit finite element

Figures (4.7a-4.7c) show the variations of the pressure difference across the diaphragm and the displacement of the diaphragm for the combinations of the 26C complete model with the following numerical techniques:-

- 1. Implicit finite difference (figure (4.7a),
- 2. Explicit finite difference (figure (4.7b),
- and 3. Implicit finite element (figure (4.7c).

These figures, demonstrates again the ability of the 26C complete model to closely predict the dynamic behaviour of the 26C locomotive valve. The combination between the complete model and the finite element, gives the best correlation with the experimental results, shown in figure (4.6a), as compared with the others. The modified model results, shown in figure (4.7d), are in very good agreement with the experimental result (figure (4.6a)). However, the modified model lacks the ability to predict the initial delay, caused by the 26C relay valve small volumes ($V_{1,1}$, $V_{1,2}$ and $V_{1,3}$) which have been neglected in formulating the 26C modified mathematical model.

Figures (4.8a-4.8d) show the brake pipe pressure dynamic behaviour using, a- complete model with implicit

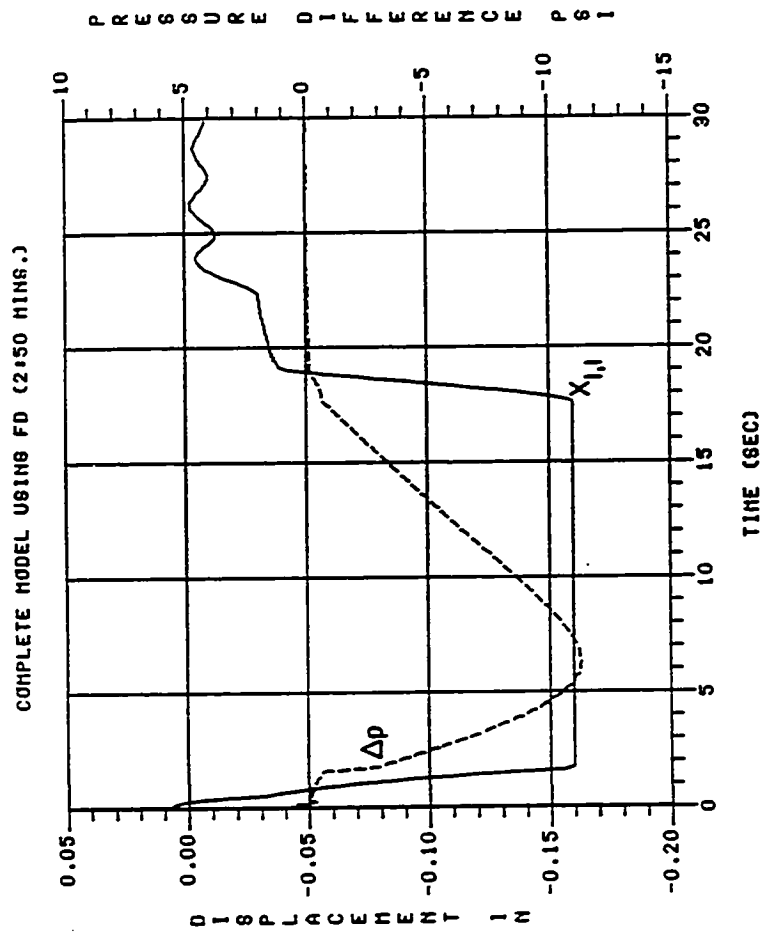


Figure 4.7a: Diaphragm Displacement and Pressure Difference Vs. Time (Full Service IFD with Complete Model)

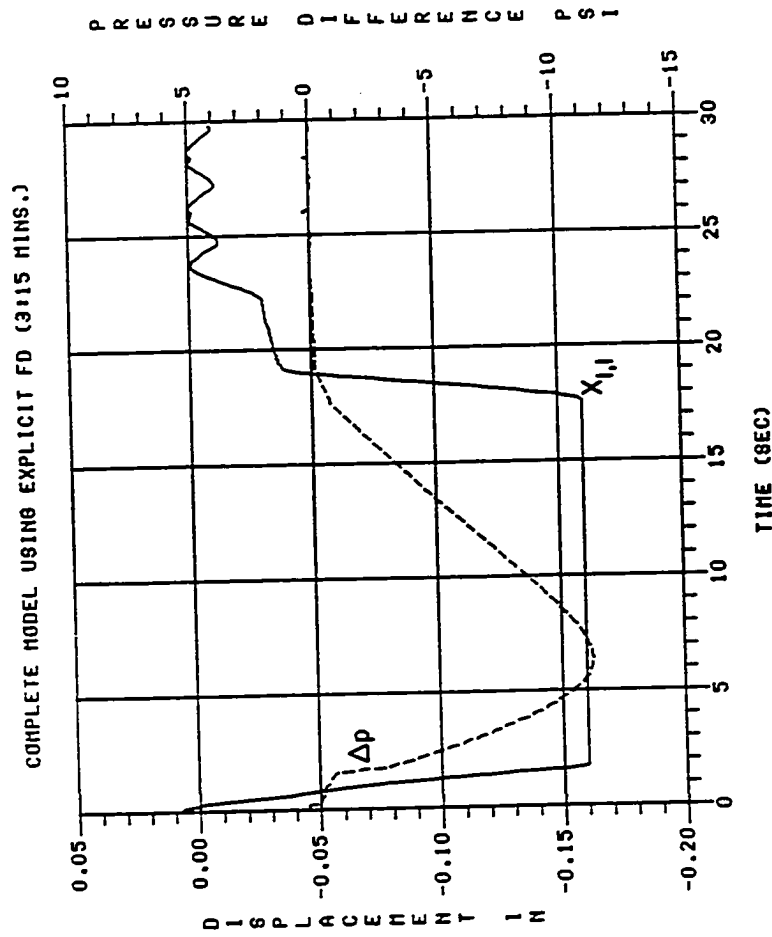


Figure 4.7b: Diaphragm Displacement and Pressure Difference Vs. Time (Full Service BFD with Complete Model)

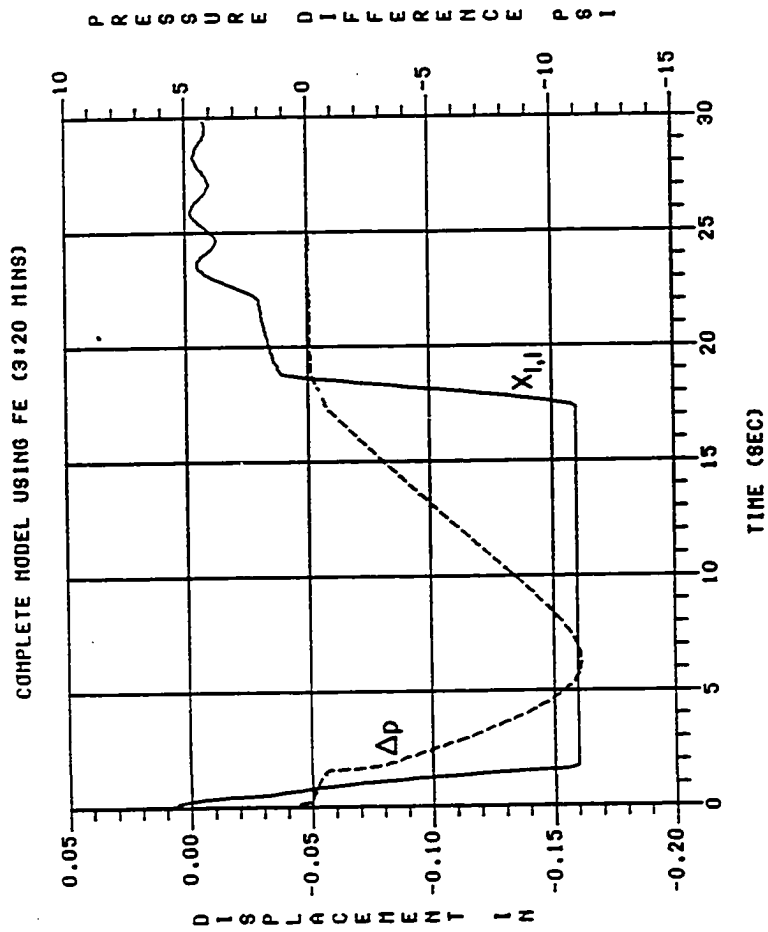


Figure 4.7c: Diaphragm Displacement and Pressure Difference Vs. Time (Full Service IFB with Complete Model)

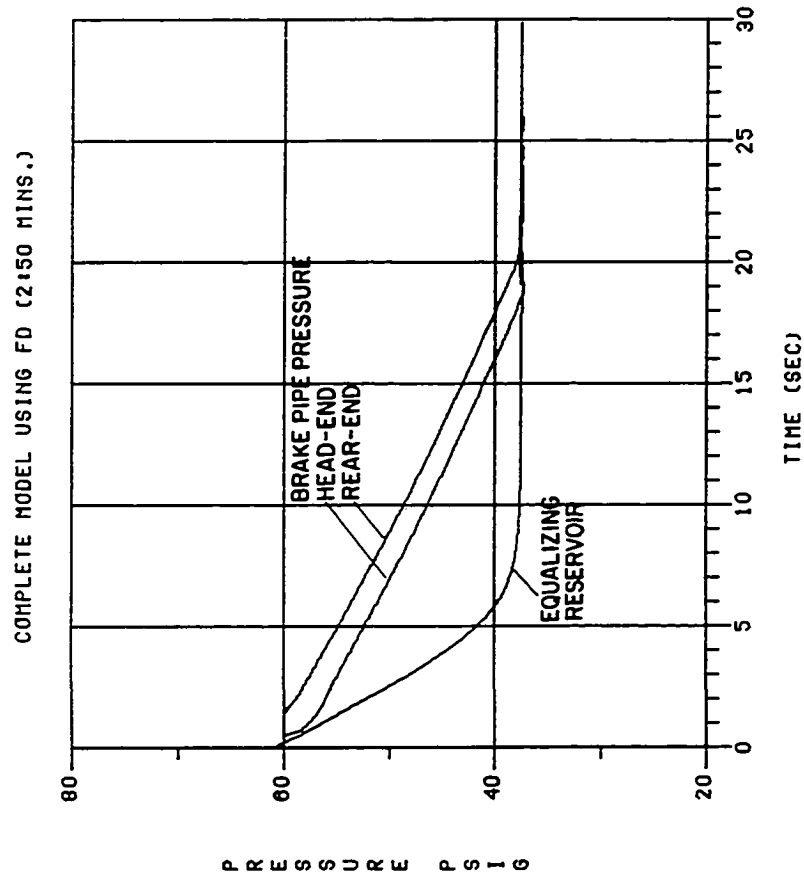


Figure 4.8a: Brake Pipe and Equalizing Reservoir Pressures Vs. Time (Full Service IFD with Complete Model)

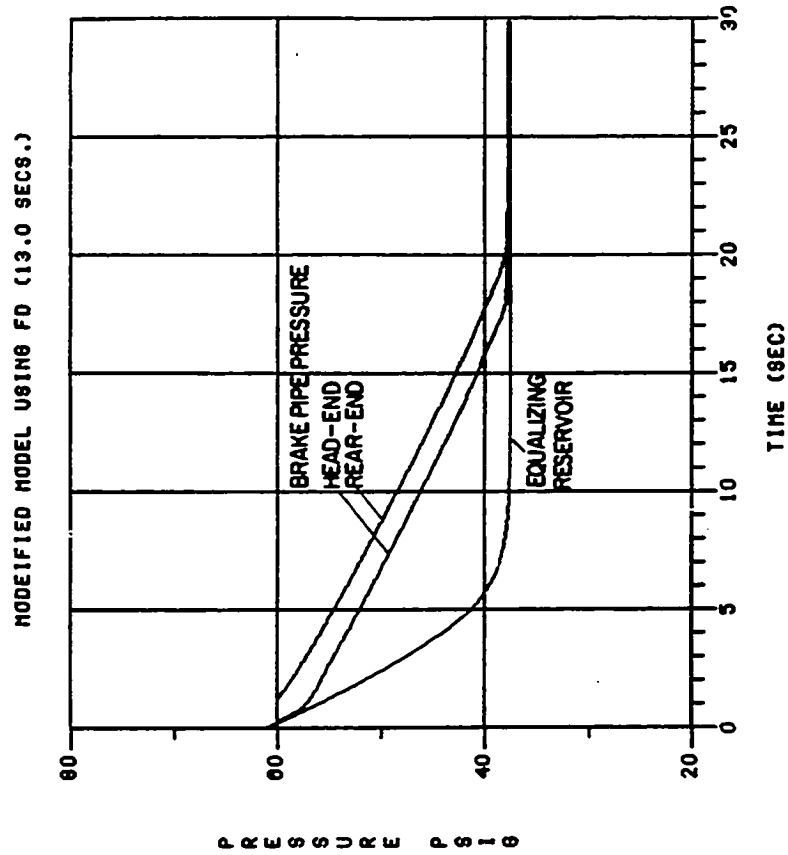


Figure 4.8b: Brake Pipe and Equalizing Reservoir Pressures Vs. Time (Full Service IFD with Modified Model)

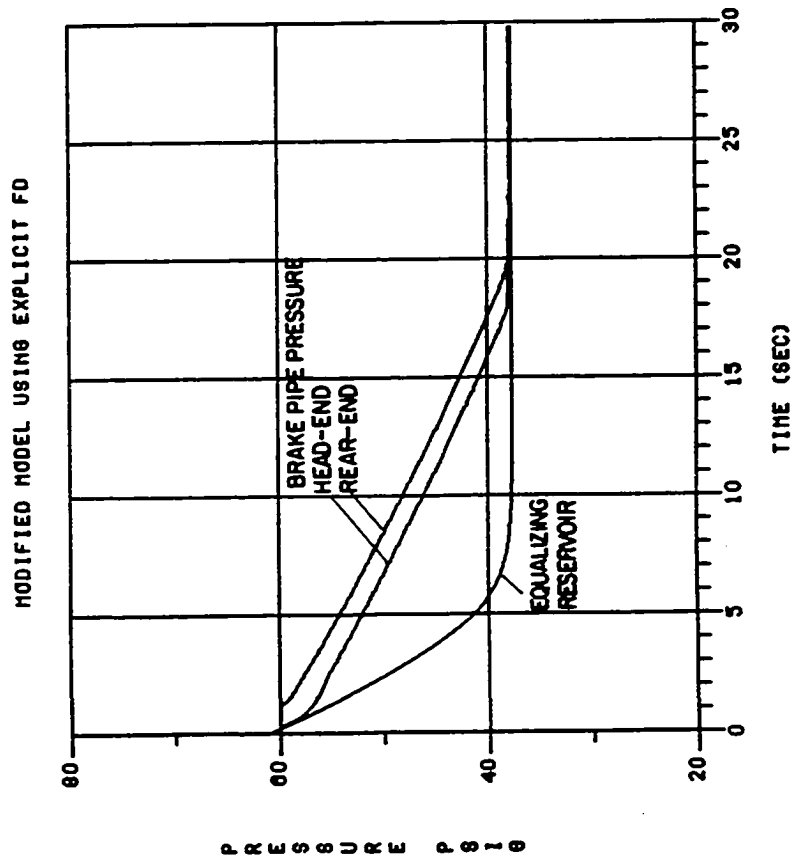


Figure 4.8c: Brake Pipe and Equalizing Reservoir Pressures Vs. Time (Full Service BFD with Modified Model)

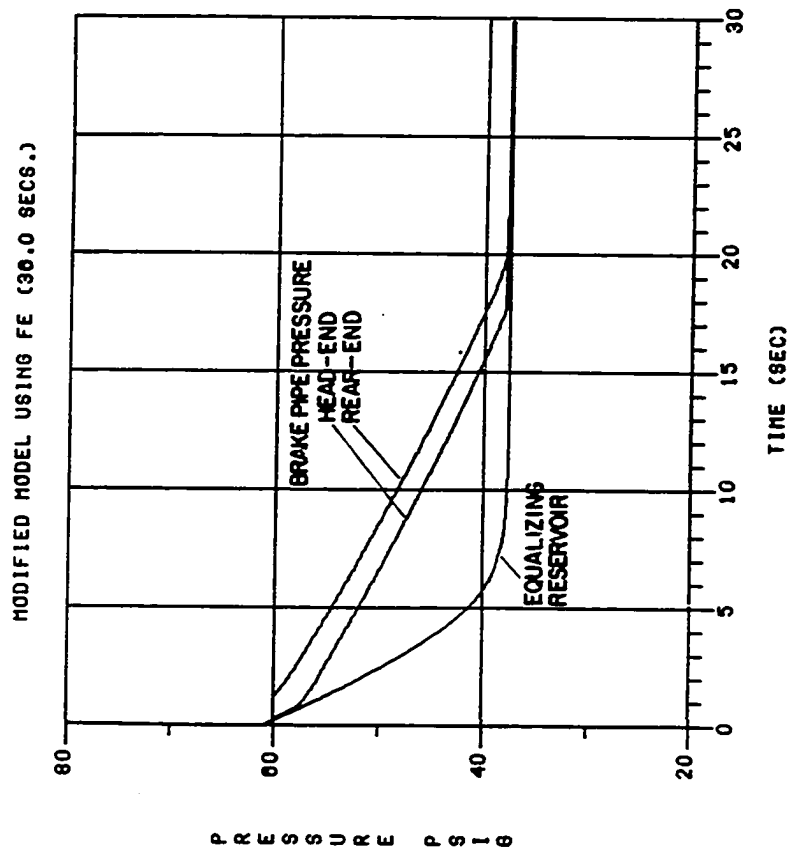


Figure 4.8d: Brake Pipe and Equalizing Reservoir Pressures Vs. Time (Full Service (FB with Modified Model))

finite difference, and b,c,d for the combinations of the modified model with each of the implicit finite difference, explicit finite difference and the implicit finite element respectively. All the three numerical techniques show very good agreement with the experimental results, shown in figure (4.6a). However, the two implicit techniques (finite difference and finite element) fail to predict the initial sudden drop in the rear-end pressure, because of the artificial damping added by using the implicit formulation in general [25,27]. The explicit finite difference formulation has successfully predicted the brake pipe pressure dynamic behaviour, including the initial drop on the rear-end pressure (compare rear end pressure of 4.8b and 4.8c at time = 1.5-2 second and note that 4.2c drops a bit faster over this time period). Figures (4.8a-4.8c) show, that the modified model fails also to predict the initial delay, seen in the head-end brake pipe pressure drop, caused by the small volumes in the 26C locomotive valve. However, combining the complete model with any of the numerical techniques, makes it possible to capture this initial delay, see figure 4.8a and compare with the experimental result, shown in figure (4.6a).

The above combinations demonstrates the following:-

1. The ability of the 26C complete mathematical model to accurately represent the dynamic behaviour of the 26C locomotive valve. But, this model is a very expensive/long computer simulation time.

2. The ability of the 26C modified model to give a reasonably accurate representation of the 26C dynamic behaviour, with much less computer time than that needed by the 26C complete model.
3. The finite element technique, formulated in section 2.4, gives the most accurate (best overall fit) solution for the brake pipe mathematical model as compared with both finite difference formulations.
4. The explicit finite difference scheme, developed in section 2.3.2, was able to capture the rear-end pressure drop, during the first 2 seconds.
5. The implicit finite difference technique, developed in section 2.3.1, gives a reasonably accurate solution, with shorter computer time, as compared with the other two techniques.

Based on the above conclusion, and the need for a numerical technique, which may be used in calculating the steady and dynamic states of the brake pipe, one may choose the combination of the modified model and the implicit finite difference to be a part of the air brake system simulation tools. The computer results, which are presented hereafter, are the result of combining the modified model with the implicit finite difference technique.

4.3.2. Experimental and Model Results -Recharge

The experimental results of the recharge after full service test, are shown in figures (4.9a) and (4.9b). The

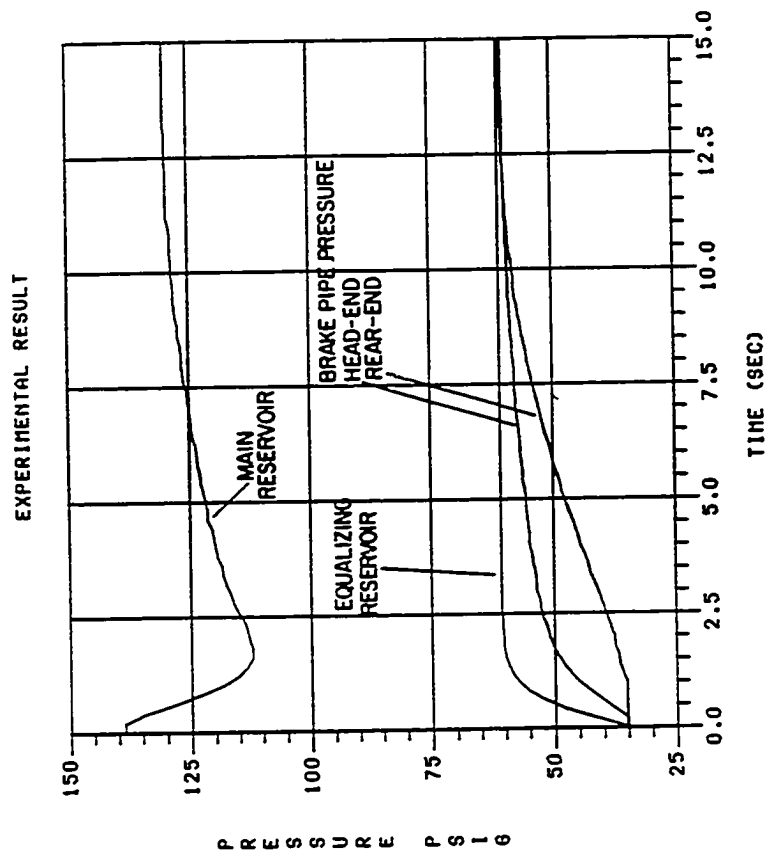


Figure 4.9a: Brake Pipe and Equalizing Reservoir Pressures Vs. Time
(Recharge Experimental Data)

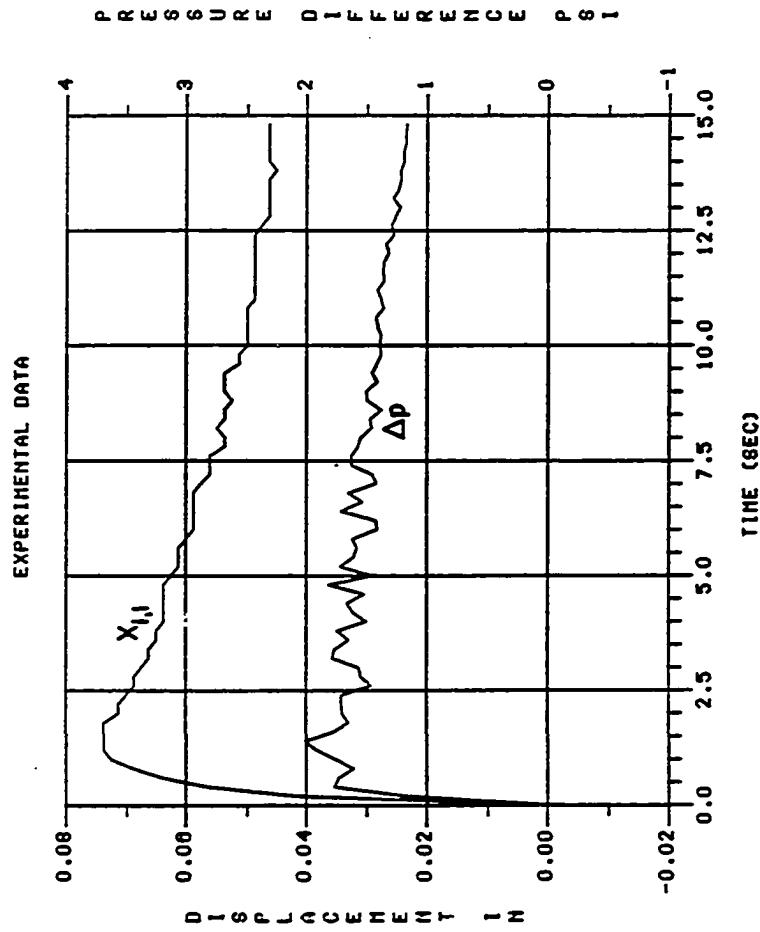


Figure 4.9b: Diaphragm Displacement and Pressure Difference Vs. Time
(Recharge Experimental Data)

equalizing reservoir, shown in figure (4.9a), rises with a pressure rate determined by the regulating valve. This pressure causes a positive pressure difference, shown in figure (4.9b), across the diaphragm, forcing the supply valve to open ($X_{1,1} > .042$ in), which allows air to flow from the main reservoir to the brake pipe through the brake pipe cut-off valve. This results in an increase of the brake pipe pressure and a decrease of the main reservoir pressure. Then, the equalizing reservoir pressure approaches its steady state condition. As the pressure at the head-end of the train approaches the equalizing reservoir pressure, the supply valve starts to close, metering the flow of air from the main reservoir.

Figure (4.10) shows the computational results, which are in very good agreement with the experimental results shown in figures (4.9a) and (4.9b).

4.3.3 Experimental and Model Results -Dry Charge

Finally, figure (4.11) shows the experimental results of the dry charge test. It should be noticed, that the rear-end pressure dynamic behaviour is not a duplication of the head-end pressure. This is caused by the fact that the friction resistance controls the dynamic of the brake pipe, and it is much larger than the flow resistance caused by the the brake pipe cut-off valve opening. However, the 26C locomotive valve goes through the same dynamic operation principles, discussed in section 4.2. Note, at the beginning

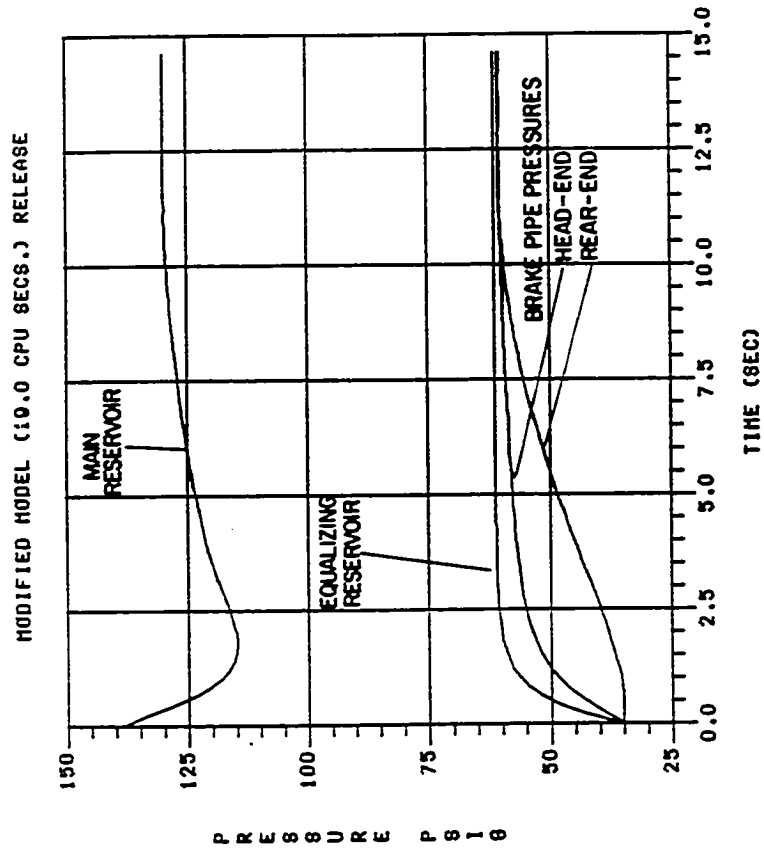


Figure 4.10a: Brake Pipe and Equalizing Reservoir Pressures Vs. Time
(Recharge IFD with Modified Model)

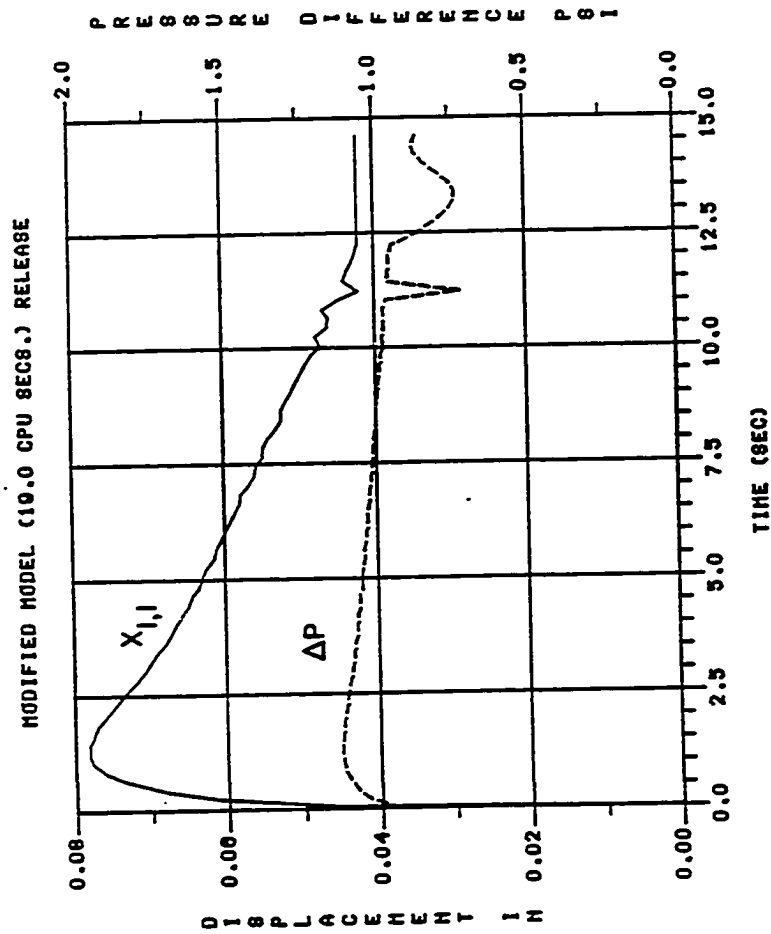


Figure 4.10b: Diaphragm Displacement and Pressure Difference Vs. Time
(Recharge IFD with Modified Model)

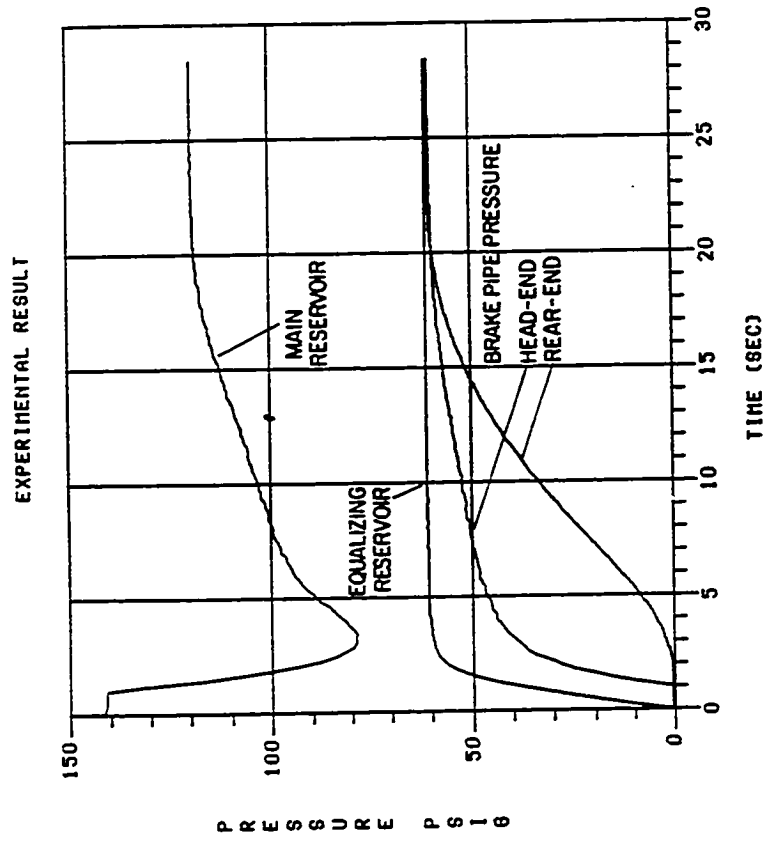


Figure 4.11a: Brake Pipe and Equalizing Reservoir Pressures Vs. Time (Dry Charge Experimental Data)

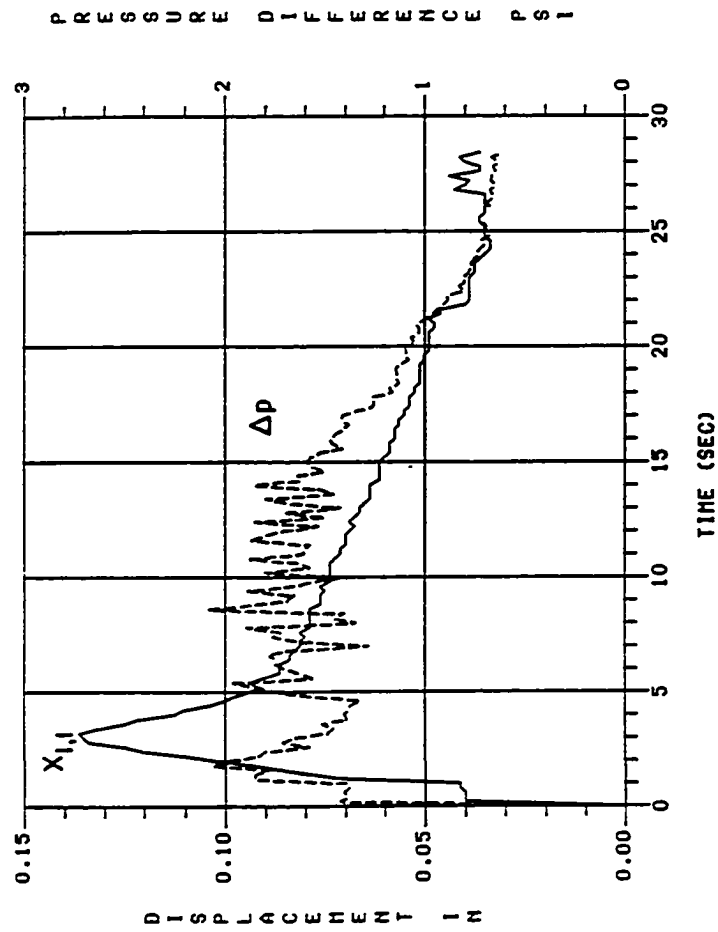


Figure 4.11b: Diaphragm Displacement and Pressure Difference Vs. Time (Dry Charge Experimental Data)

of the dry charge test, the brake pipe cut-off valve is fully closed. Thus, it takes the head-end pressure almost one second to start to rise, because the brake pipe cut-off valve starts to open only when the intermediate volume pressure is larger than 42 psi.

The computational results, shown in figure (4.12), are in very good agreement with the corresponding experimental results. However, the brake pressure computational results, at the beginning of the test, rise faster than the experimental results. The brake pipe mathematical model, assumes that the pipe follows the isothermal process. However, at the beginning of the dry charge, it is believed that the brake pipe may follow the polytropic process instead of the isothermal. The computational result, shown in figure (4.12b), shows an oscillation as the supply valve starts to close, caused by the function discontinuity representing the relay and brake pipe cut-off valves. Figure (4.12c) shows the computational results using the complete model for the dry charge. The complete model was able to predict most of the 26C dynamic behaviour, but it demonstrates a very wild oscillation (stable), caused by the dynamics of the small volumes as the supply valve starts to close. The oscillation problem appears only when the diaphragm moves from the charge mode to the no-leakage lap position ($X_{1,1} = .042$ in). One of the possible solutions to this problem is to add a very small leakage (artificial leakage) to the brake pipe to prevent the diaphragm from

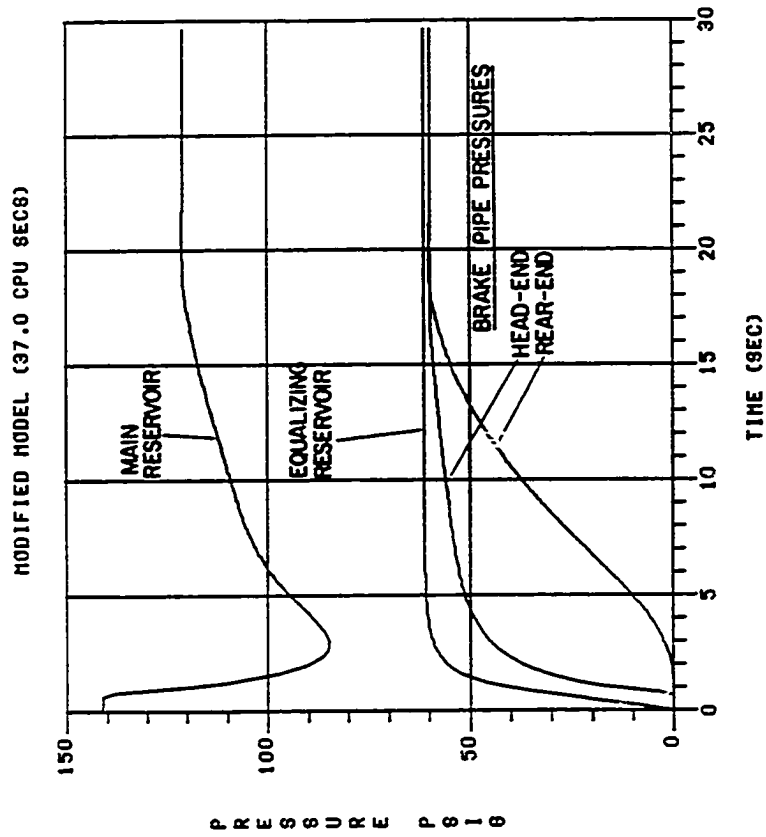


Figure 4.12a: Brake Pipe and Equalizing Reservoir Pressures Vs. Time (Dry Charge IFD with Modified Model)

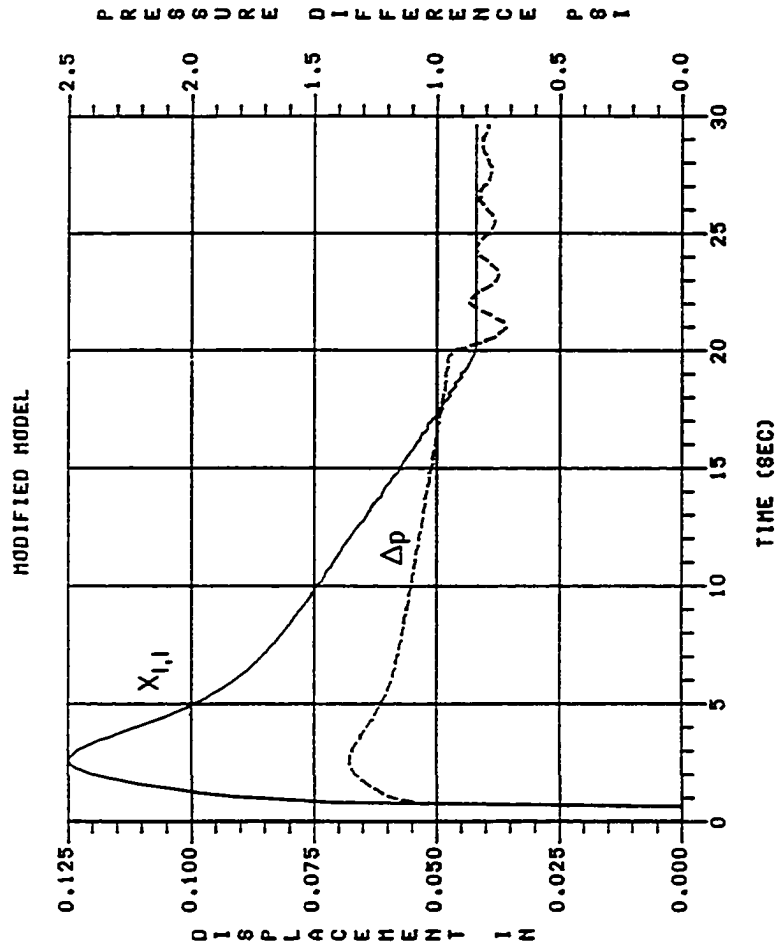


Figure 4.12b: Diaphragm Displacement and Pressure Difference Vs. Time (Dry Charge 1FD with Modified Model)

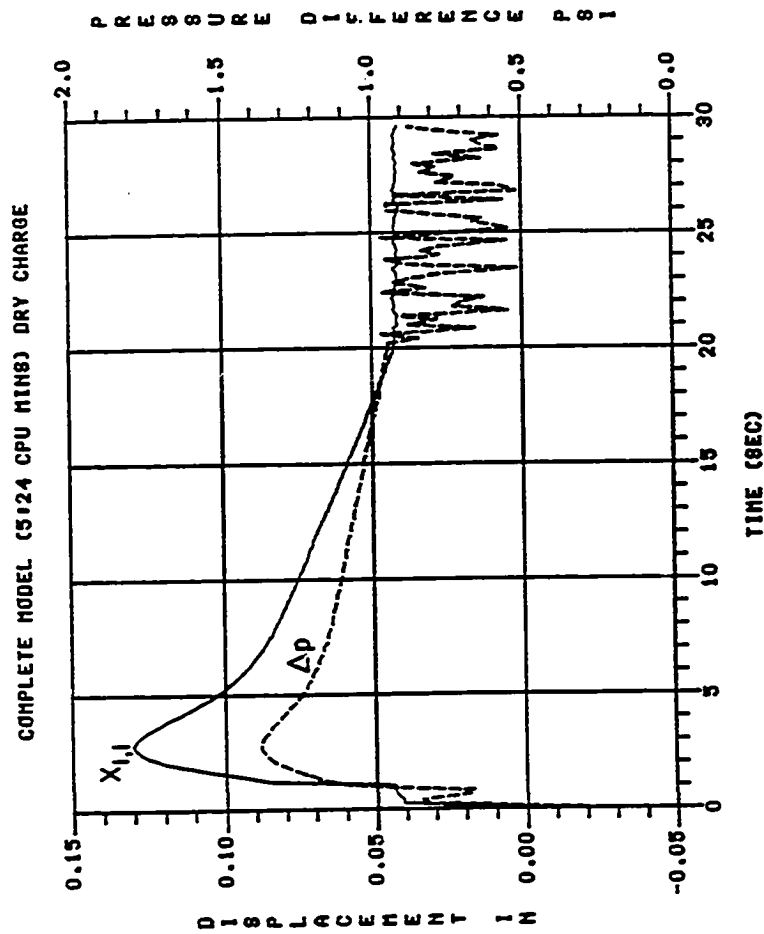


Figure 4.12c: Diaphragm Displacement and Pressure Difference Vs. Time (Dry Charge IFD with Complete Model)

having the lap position at $X_{1,1} = .042$ in. The results of using this suggestion are shown in figure (4.13a), for the modified model, and figure (4.13b), for the complete model using a time integration step three times less than the results shown in figure (4.12c). The artificial leakage is able to solve the oscillation problem.

4.4 Air Brake System Experimental Set-ups, Results and Model Comparisons

This section has utilized two different experimental set-ups to achieve its goals. The first set-up is similar to the one used earlier in section (4.3), except the control valves (ABDW) are cut-in and a constant pressure source is feeding the main reservoir. The second experimental set-up is a complete representation of the air brake system, which may include the following components:-

1. A19 flow indicator adaptor
2. Control valves (ABD/ABDW)
2. 130,000 in³ main reservoir
3. 50-car, 75-car and 100-car trains
4. 200 psig pressure supply (compressor)

Two tests have been performed, using the first experimental set-up. The full service (application) test of 20-car train, which contains ABDW control valves, is used to understand the interaction between the 26C locomotive valve and the rest of the air brake system components. Figure (4.14a) shows, the variation of the displacement of

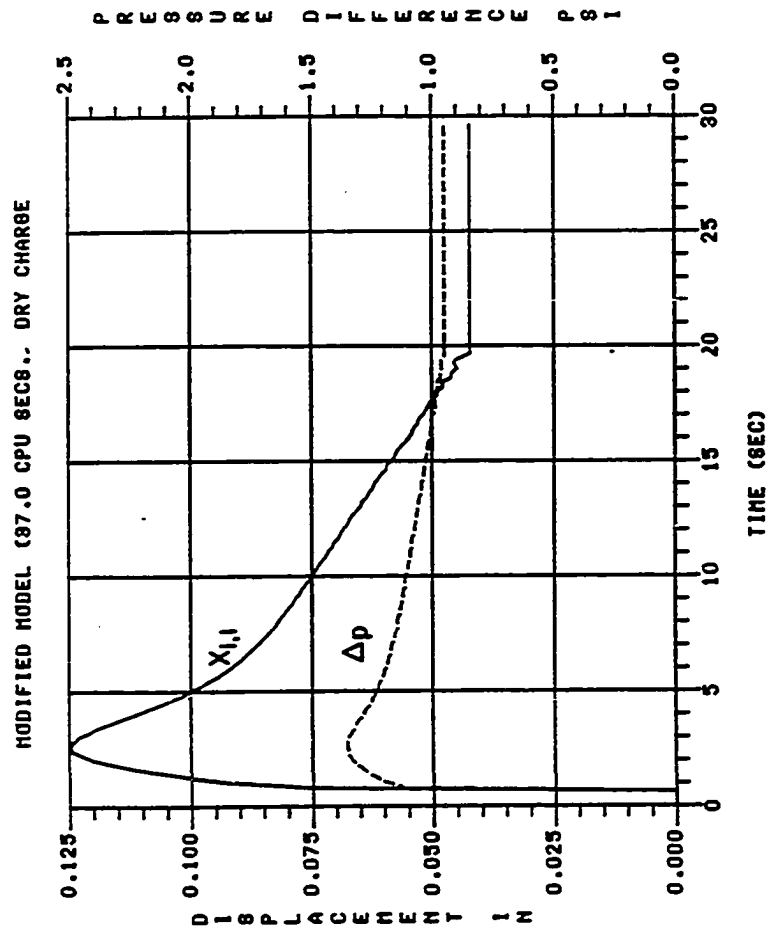


Figure 4.13a: Diaphragm Displacement and Pressure Difference Vs. Time
(IPD, Modified Model and Leakage)

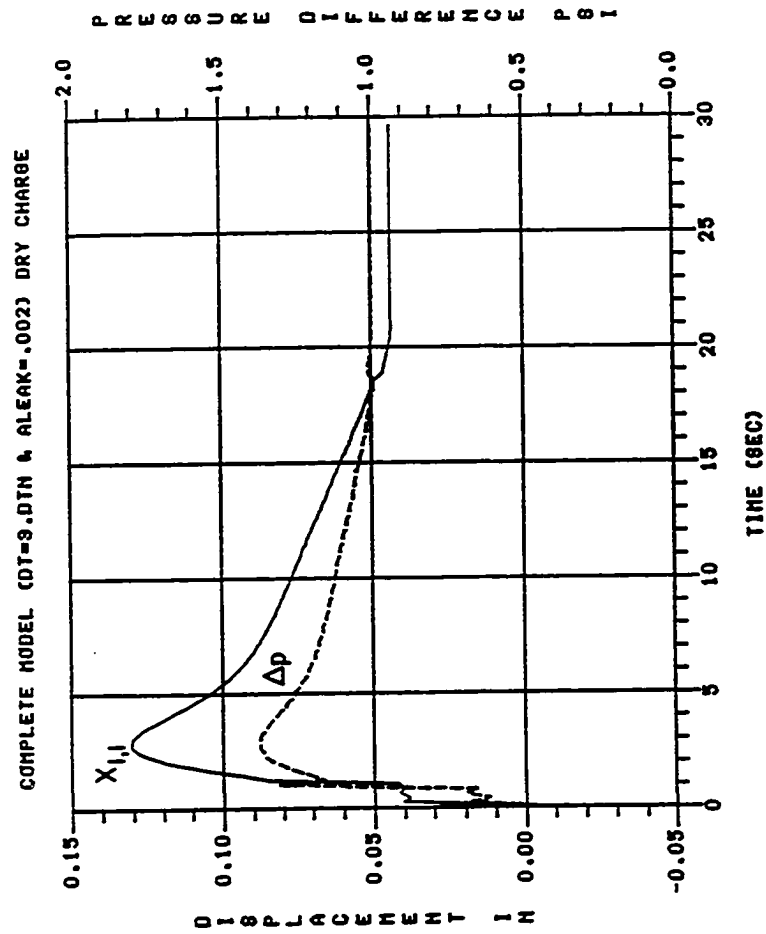


Figure 4.13b: Diaphragm Displacement and Pressure Difference Vs. Time
(IFD, Complete Model and Leakage)

FULL APPLICATION OF 20-CAR TRAIN

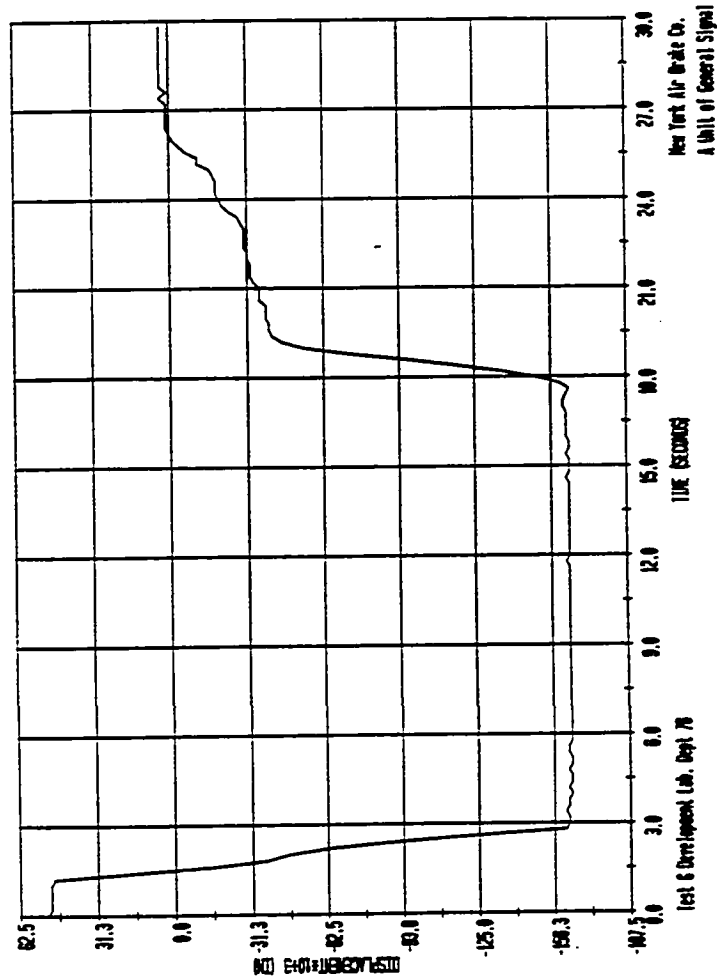


Figure 4.14a: Diaphragm Displacement Vs. Time (Full Service)/ABDW
Experimental Data)

the 26C relay valve diaphragm with respect to time. It has been noticed that the result, shown in figure (4.14a), is just a duplication of the experimental result, shown in figure (4.6b), except the exhaust valve of the relay valve opens for a little shorter time than figure (4.6b) experimental result. This is caused by the fact that the brake pipe loses some air, during the service mode, to the following elements:-

1. quick action chamber (part of the emergency portion of the ABDW control valve),
 2. quick service volume (a part of the service portion of the ABDW control valve),
- and, 3. brake cylinder.

Figure (4.14b) shows the variation of the pressure difference across the 26C relay valve with respect to time. There is no significant different between figure (4.14b) and the result displayed in figure (4.6b).

Figure (4.14c) shows the corresponding results, produced by the computer simulation of the air brake system. The modified model was, again, able to predict the dynamic behaviour of the 26C locomotive valve.

The second test case is to recharge the 20-car train after full application (release). Figure (4.15a) shows the experimental results for the 26C relay valve inner chamber, main reservoir, equalizing reservoir, and brake pipe head-end and rear-end pressures. Main reservoir, inner chamber, equalizing reservoir and head-end pressures behave similarly

FULL APPLICATION 20-CAR TRAIN

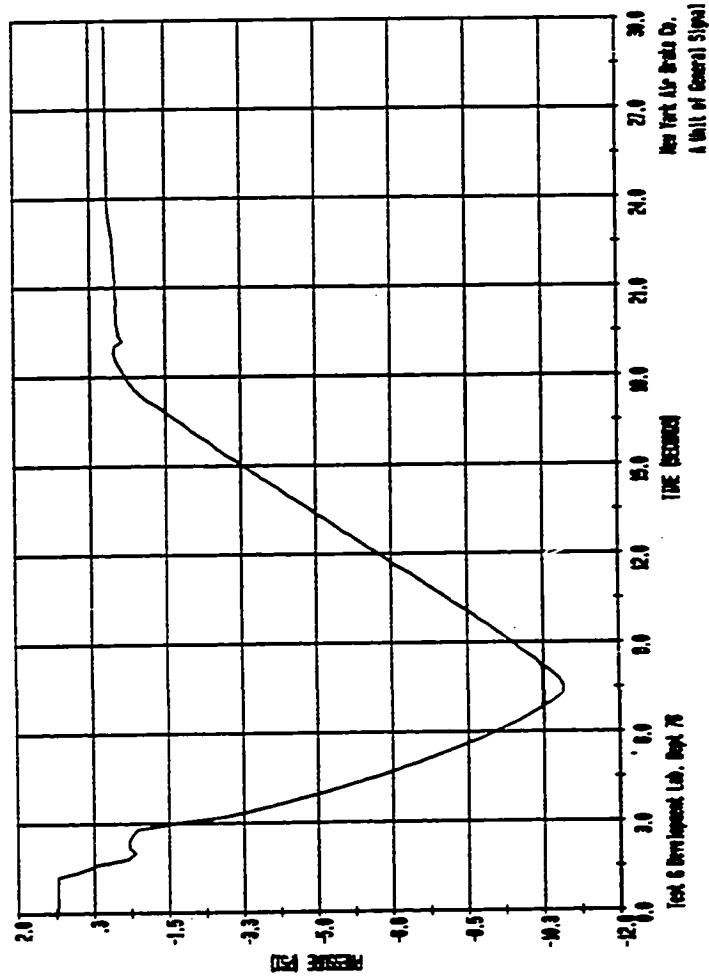


Figure 4.14b: Pressure Difference Across Relay Valve Diaphragm Vs. Time
(Full Service/ABDW Experimental Data)

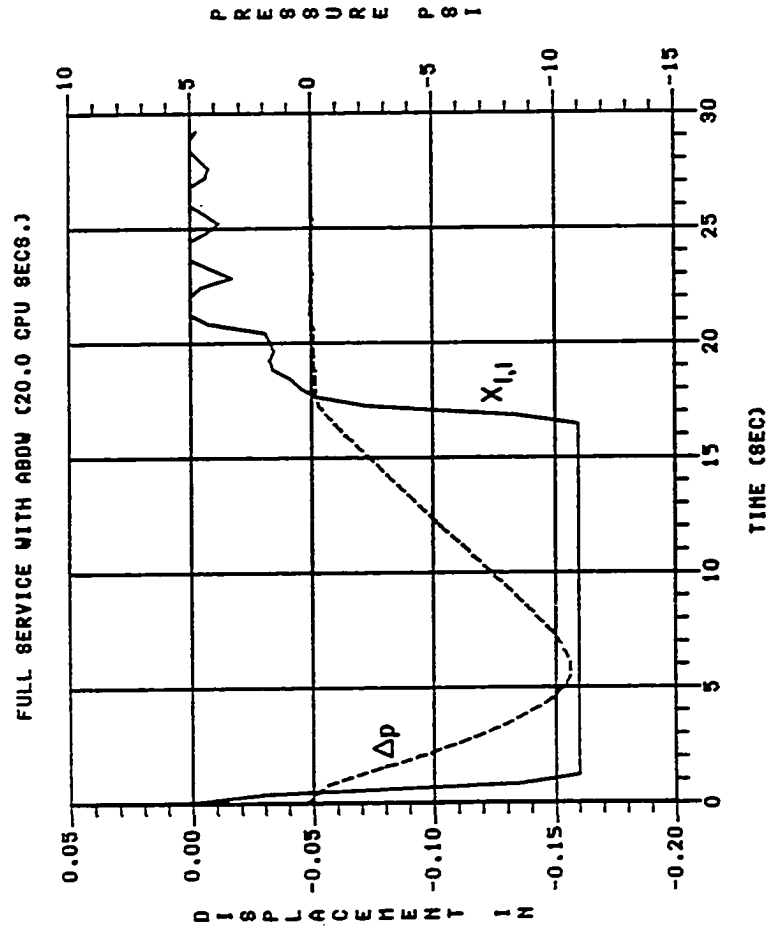


Figure 4.14c: Diaphragm Displacement and Pressure Difference Vs. Time
(Full Service/ABDW IFD with Modified Model)

RELEASE OF 20-CAR TRAIN

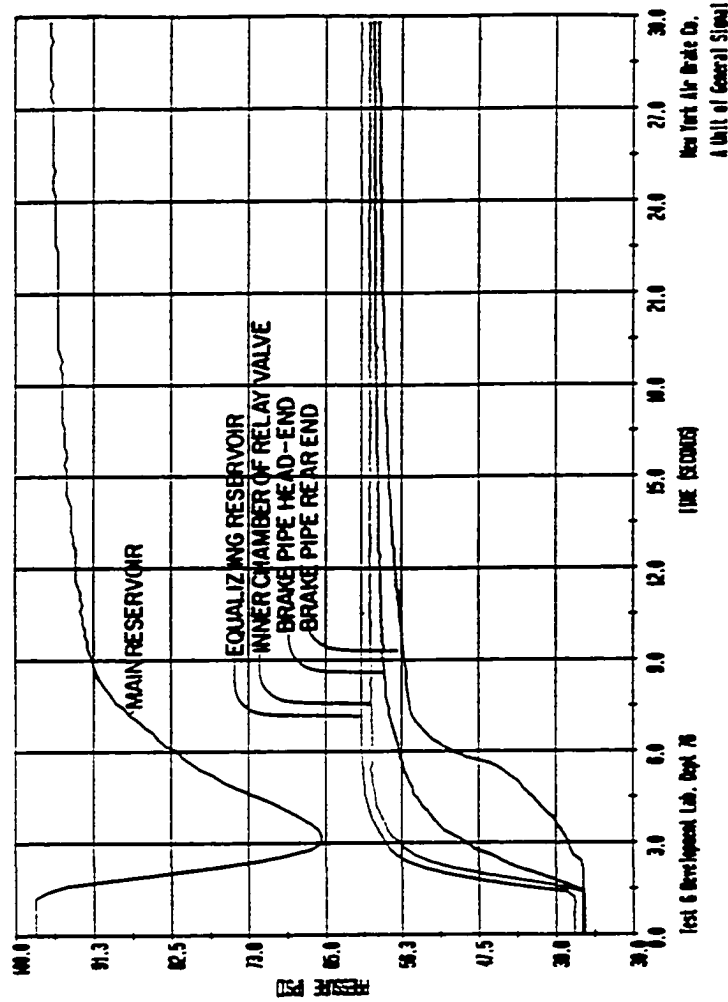


Figure 4.16a: Brake Pipe, Relay Valve and Equalizing Reservoir Pressures
Vs. Time (Recharge/ABDW Experimental Data)

to the experimental results produced with the control valves cut-out (see figure (4.9a)). However, the rear-end pressure behaves differently than that shown in figure (4.9a). This is caused by activating the accelerated service release valve (a part of the service portion of the ABD/ABDW) during the release of the 20-car train. If the recharge rate of the local brake pipe over the local auxiliary reservoir pressure is high enough to create a 1.5-2 psi pressure difference, the accelerated release valve can be activated. This valve allows the air to flow from the emergency reservoir to the brake pipe, providing that the emergency reservoir pressure is higher than the brake pipe pressure.

Figure (4.15b) shows the computer simulation results for the release of the 20-car train. The results are in very good agreement with the corresponding experimental results, shown in figure (4.15a).

Figure (4.16a) shows the experimental results, using the second experimental set-up, of the recharge after a 15 psi reduction of 75-car train. It is, again, clear that the accelerated release service valve, at different locations along the brake pipe, is activated during the release operation. This figure, also, shows the release of the brake cylinders at the 1st, 25th, 50th and 75th car along the train. The release of the brake cylinder is only activated when the local brake pipe pressure is 1 psi higher than the local auxiliary reservoir. Figure (4.16b) shows the computational results of the brake pipe pressures at the

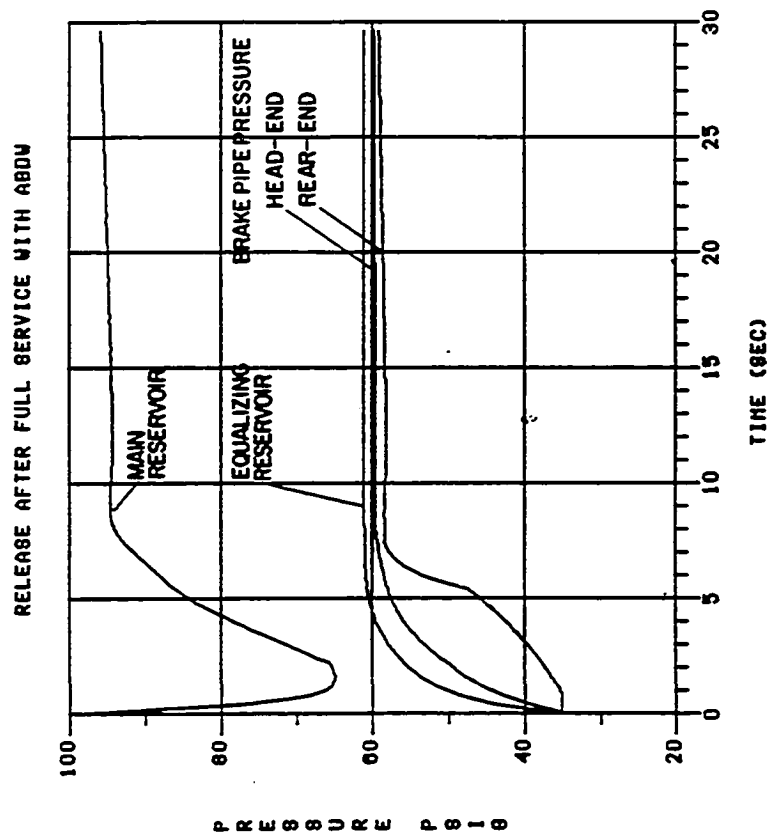


Figure 4.15b: Brake Pipe and Equalizing Reservoir Pressures Vs. Time
(Recharge/ABDW IFD with Modified Model)

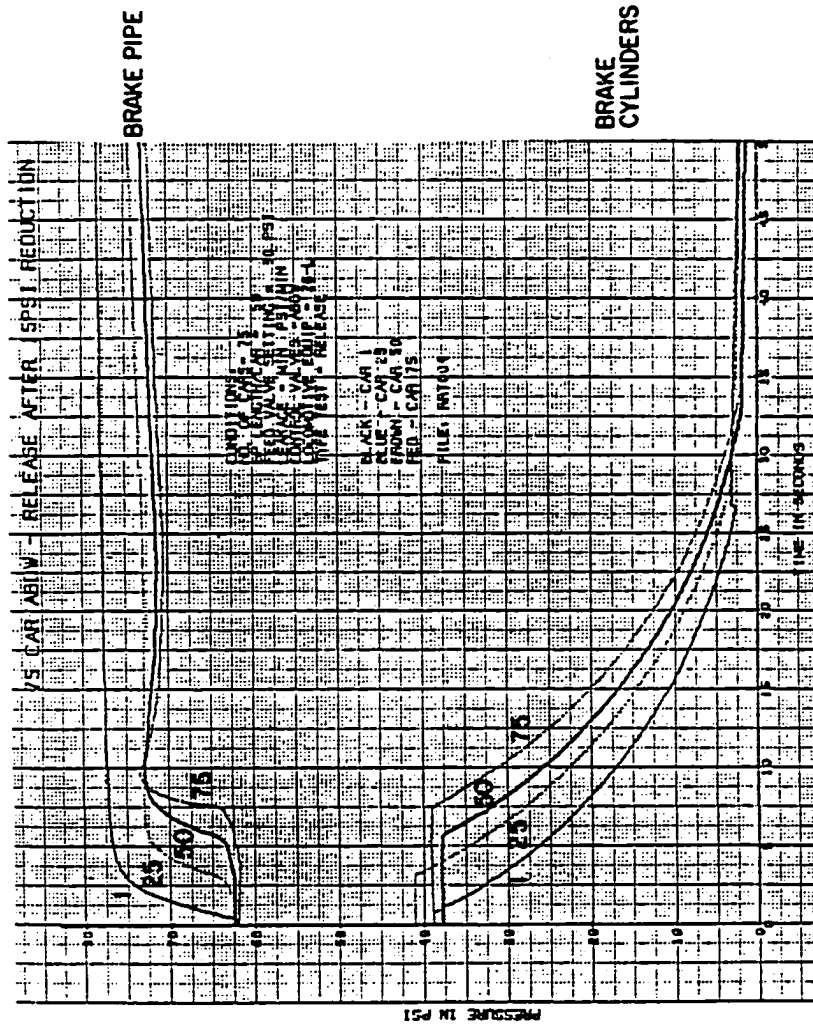


Figure 4.16a: Brake Pipe Pressures Vs. Time (75-Car Recharge/ARD
Experimental Data)

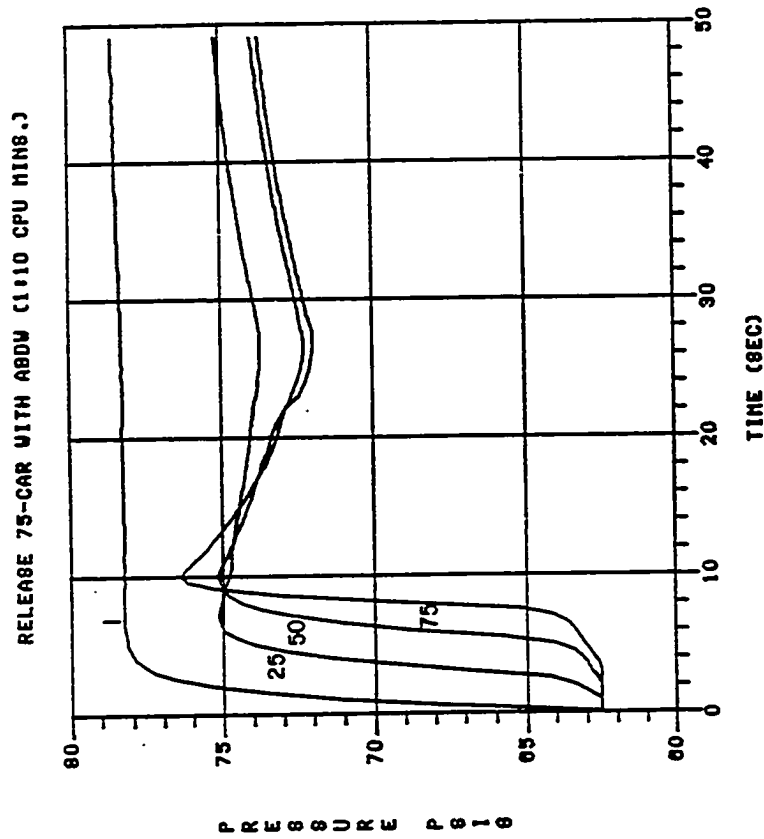


Figure 4.16b: Brake Pipe Pressures Vs. Time (75-Car Recharge/ABD IFD with Modified Model)

1st, 25th, 50th and 75th car. The simulation results closely duplicate the experimental results, shown in figure (4.16a). It has been noticed, from the experimental and computational results, that the brake pipe pressure, at several locations along the brake pipe, suddenly rises, then drops and finally increases to approach the steady state condition. This phenomena may be explained with the help of the computational results of the emergency reservoir pressures, shown in figure (4.16c). The local accelerated release service valve is activated, allowing the air to flow from the emergency reservoir to the brake pipe. This causes a pressure drop in the emergency reservoir and a sudden increase in the brake pipe pressure. When the brake pipe pressure reaches a value higher than the emergency reservoir pressure, the accelerated release mode is then terminated, causing the drop in the brake pipe pressure (note that the brake pipe always supplies air to the auxiliary reservoir). As the emergency reservoir pressure approaches the auxiliary reservoir pressure, the brake pipe, emergency and auxiliary reservoirs pressures slowly increase, depending on the main reservoir air as the only flow source.

Figure (4.17a) shows the experimental results of the dry charge test. In this case, the pressures of the air brake system volumes (equalizing, emergency and auxiliary reservoirs) and brake pipe have initial pressure values of 2.5 psig. Dry charge operation is a very slow process, which may take a lot of time to complete (the system reaches the

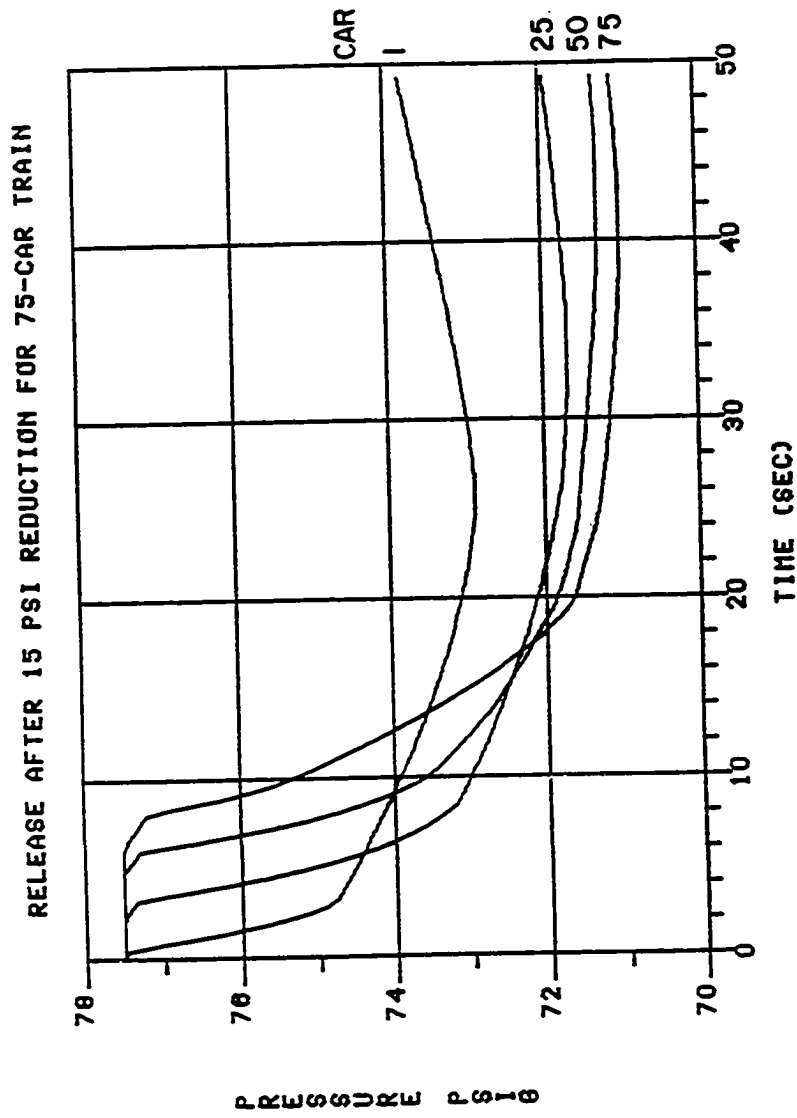
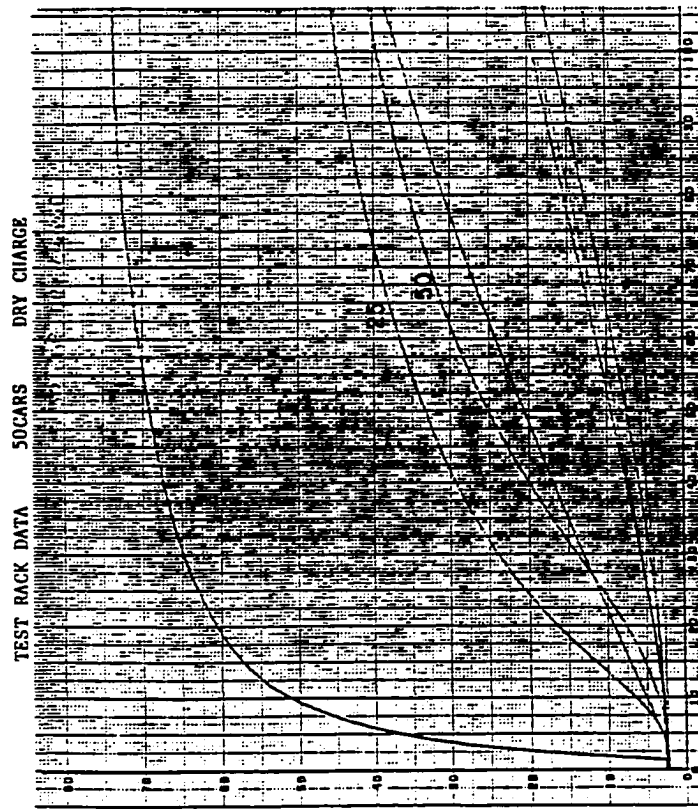


Figure 4.16c: Emergency Reservoir Pressures Vs. Time (75-Car Recharge/ABD
IFD with Modified Model)



**Figure 4.17a: Brake Pipe Pressure Vs. Time (50-Car Dry Charge/ABD
Experimental Data)**

steady state condition). This experiment is quite similar to the experiment described in section (4.3), except that this experiment involves the charging of the auxiliary and emergency reservoirs. Figure (4.17b) shows the computer simulation results. The results are, in general, in very good agreement with the experimental results, shown in figure (4.17a). However, an obvious discrepancy can be seen. The computational results show a faster pressure time rate of change, during the first 30 seconds of the experiment, than the experimental results. Two reasons may contribute to this discrepancy:

1. the main reservoir pressure, actually, varies with time (2-6 psi pressure variations),
- and, 2. the brake pipe may follow an polytropic process rather than isothermal.

Figure (4.18a) shows the experimental results of 50-car train, equipped with ABD control valves. The 26C was disconnected and the brake pipe was vented to atmosphere through an 'F' drill orifice. The maximum opening of the variable restriction, C_1 connecting the auxiliary reservoir and brake cylinder, is a function of the clearance between the slide valve piston and cover of the ABD valve service portion (See figure G.1). Figure (4.18b-d), shows the results, produced by the air brake system simulation program, of the brake pipe and brake cylinder pressures versus time for $C_1 = .009, .007$ and $.005$ respectively. The simulation and experimental data show remarkably good

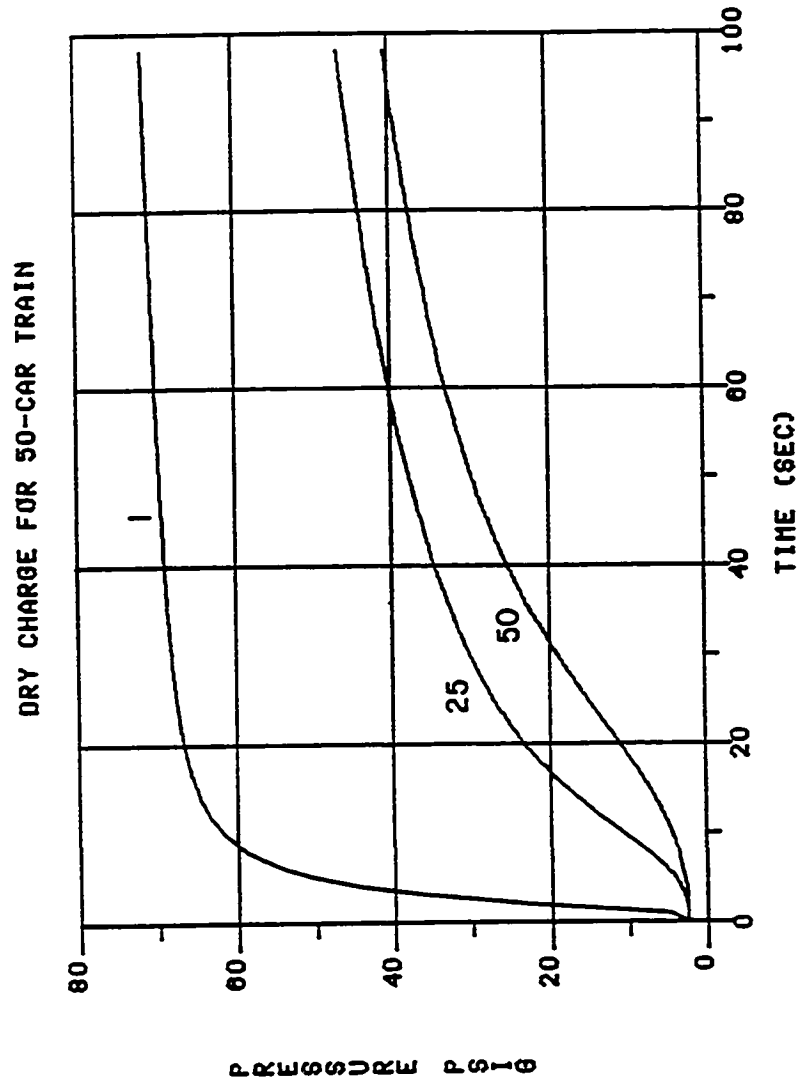


Figure 4.17b: Brake Pipe Pressure Vs. Time (50-Car Dry Charge/ABD IFD with Modified Model)

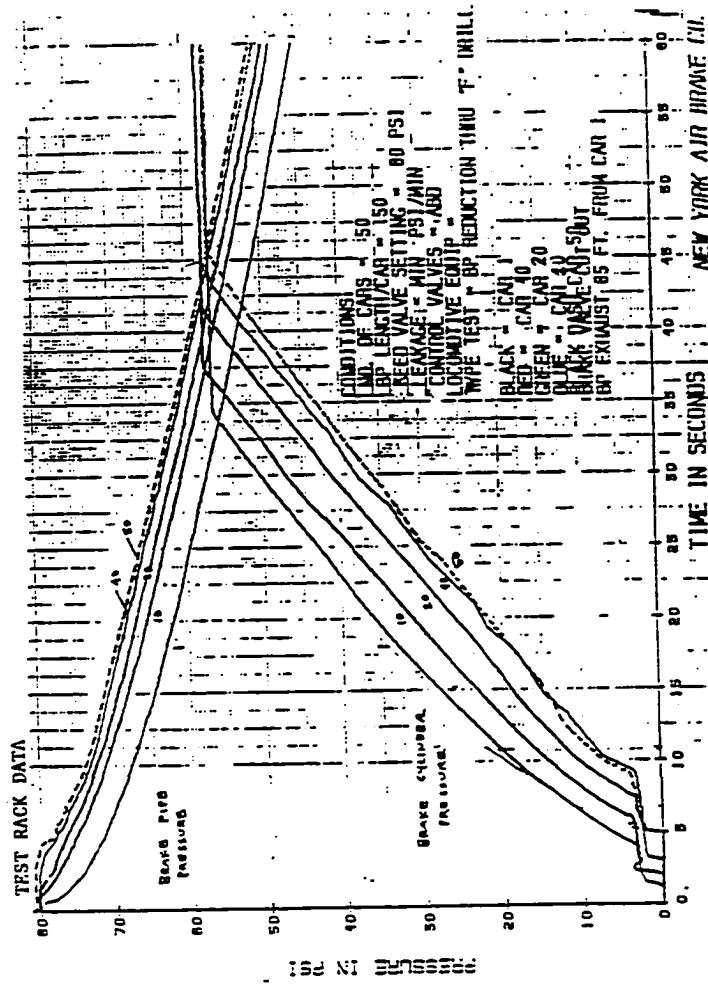


Figure 4.18a: Brake Pipe and Brake Cylinder Pressures Vs. Time (50-Car Pressure Reduction/ABD Experimental Data)

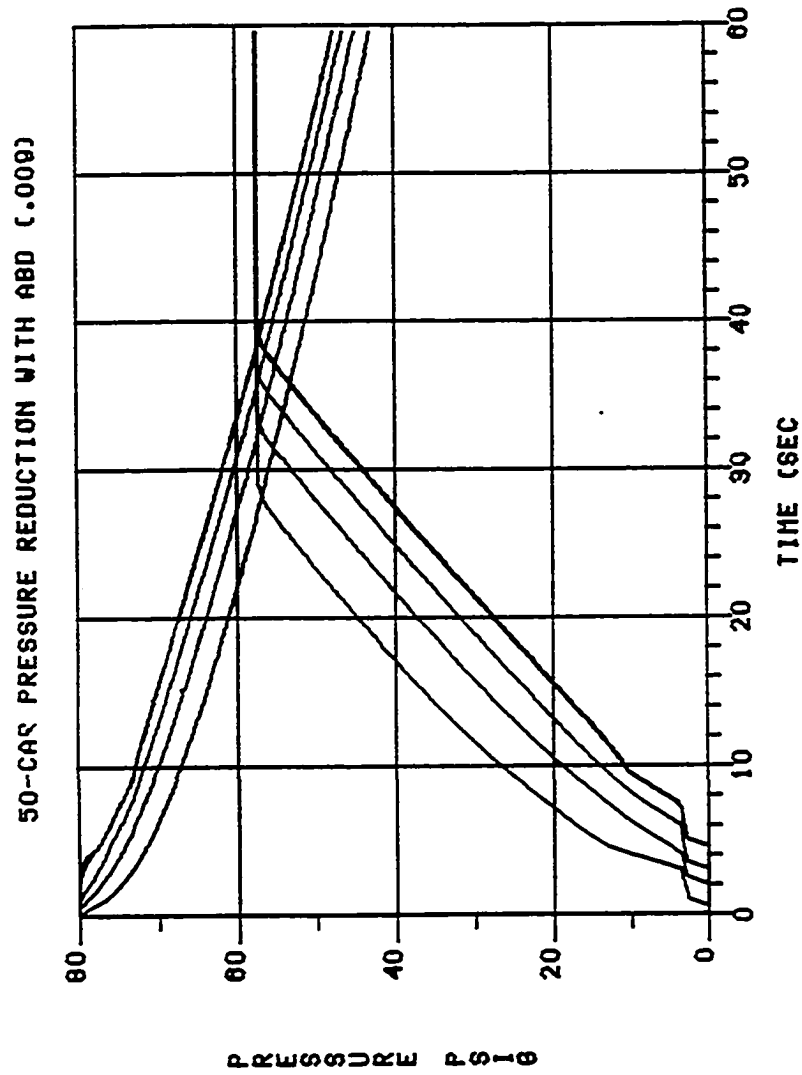


Figure 4.18b: Brake Pipe and Brake Cylinder Pressures Vs. Time (50-Car Pressure Reduction/ABD IFD with $C_1 = .009$)

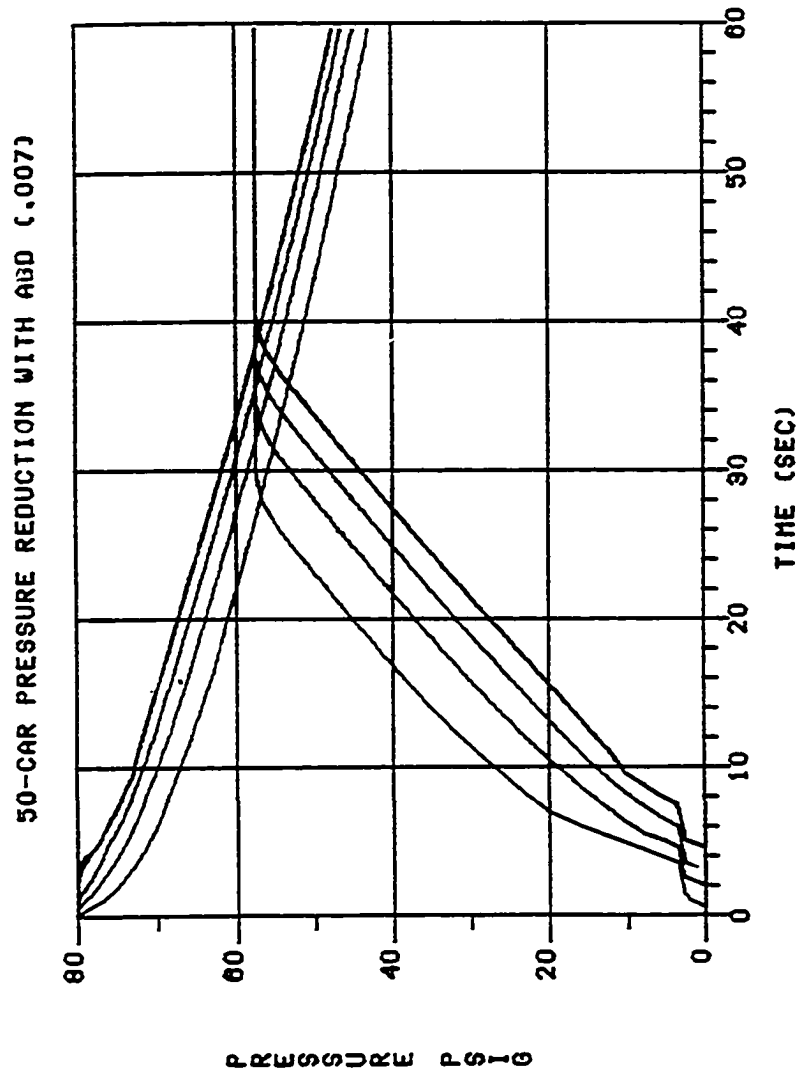


Figure 4.18c: Brake Pipe and Brake Cylinder Pressures Vs. Time (50-Car Pressure Reduction/ABD IFD with $C_1 = .007$)

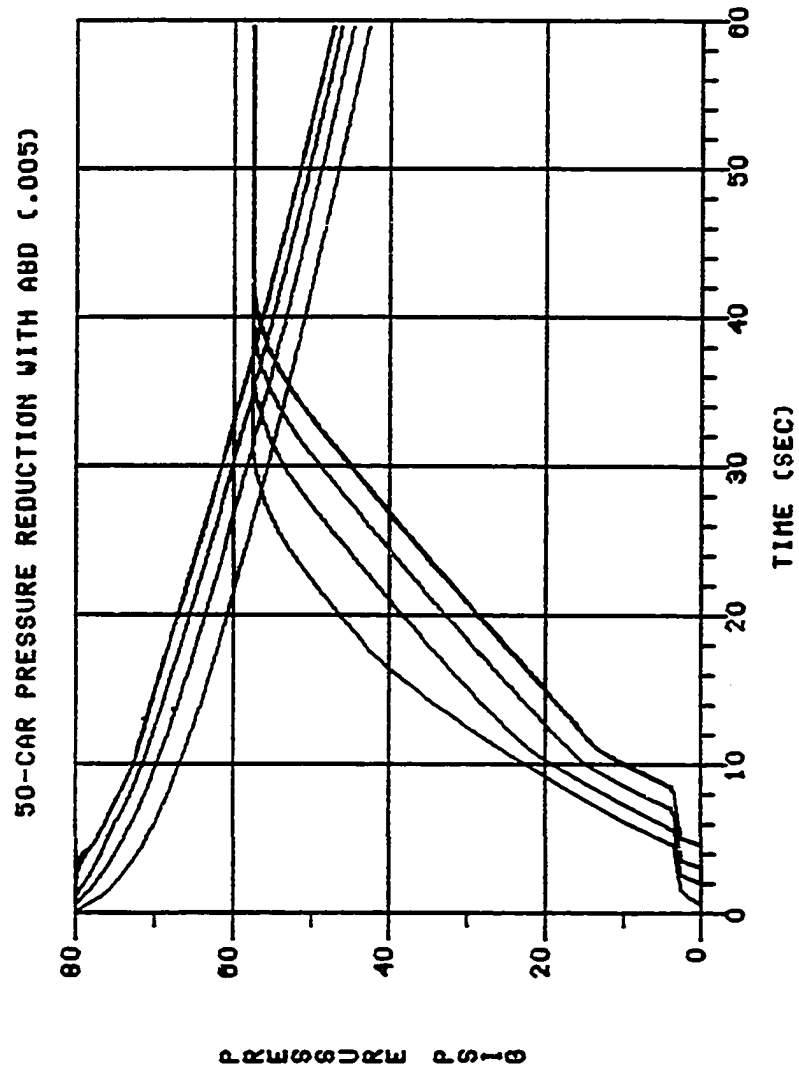


Figure 4.18d: Brake Pipe and Brake Cylinder Pressures Vs. Time (50-Car Pressure Reduction/ABD IFD with $C_1 = .005$)

correlation, validating the ability of the brake pipe mathematical model to predict the dynamic behaviour of the brake pipe. Note that, the value of C_1 has no significant effect on the transient behaviour of the brake pipe pressures. It has been noticed, that the larger the value of C_1 ; the faster the brake cylinder built up during the third stage of the brake filling for the first car. It should be mentioned that, variation of C_1 has no significant effect on the brake cylinder pressure transient behaviour of car 20, 40 and 50 and there are minor changes for 1st and 10th car. In general, the brake cylinder mathematical model (see Appendix G) was able to predict the dynamic behaviour of the brake cylinders through the different filling stages.

Figure (4.19a) shows the experimental data from a similar experimental set-up as the one used to produce the results, shown in figure (4.18a), except the ABD control valves are replaced by the ABDW. The ABDW is an ABD type valve with the addition of the continuous quick service. The continuous quick service is provided by the Accelerated Application Valve (AAV). The AAV used the exhausted quick action air to further decrease the local brake pipe pressure. This function will basically cause a more rapid service application than the one that may be provided by the ABD control valve. Figure (4.19b) shows that the computer simulation results, for the brake pipe and brake cylinder pressures at the head-end and the rear-end of the train are

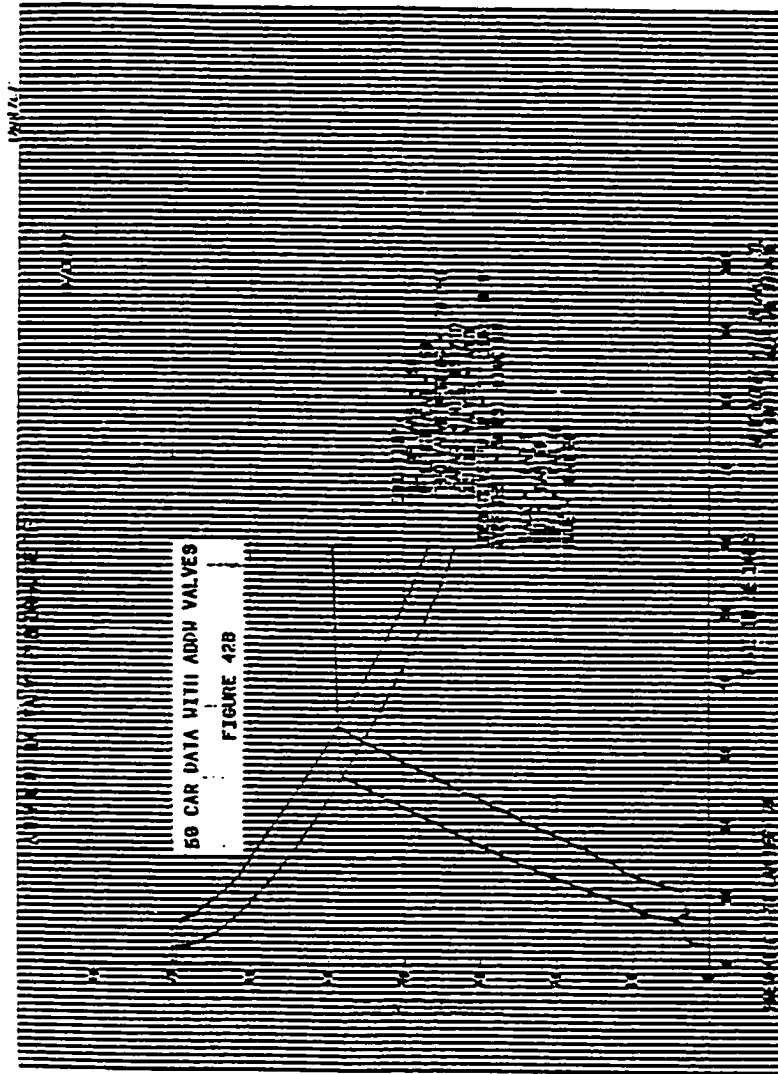


Figure 4.19a: Brake Pipe and Brake Cylinder Pressures Vs. Time (50-Car Pressure Reduction/ABDW Experimental Data)

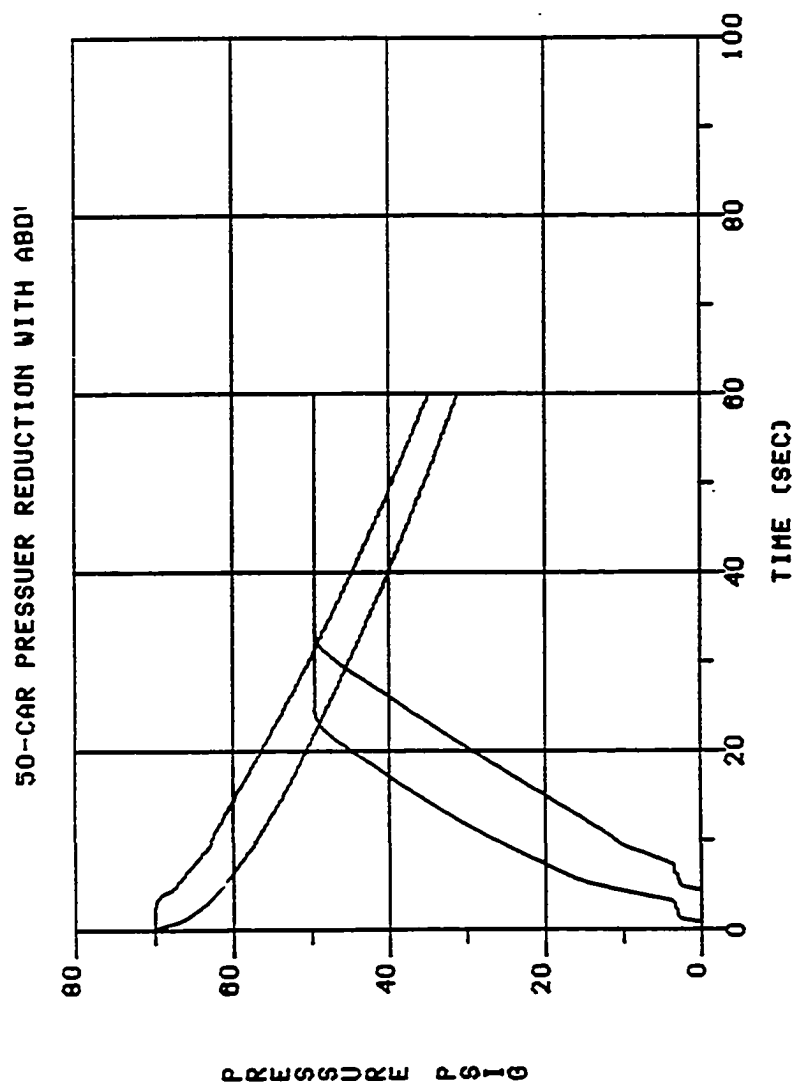


Figure 4.19b: Brake Pipe and Brake Cylinder Pressures Vs. Time (50-Car
Pressure Reduction/ABDW IFD with $C_F = 0.007$)

in very good agreement with the experimental results shown in figure (4.19a).

Figure (4.20a) shows the experimental results of the brake pipe and brake cylinder pressures, for a 50-car train, during an emergency brake application. The emergency mode is activated, in general, when the pressure rate of reduction of the local brake pipe is higher than the local quick action chamber. Then, the local vent valve (a part of the emergency portion of the ABD/ABDW control valve) is unseated, opening a large and direct passage for the local brake pipe to atmosphere. A rapid venting of the brake pipe causes an emergency reduction pressure rate to pass serially and rapidly through the entire train due to the same operation principle of the ABD/ABDW control valves on adjoining cars. As a result, the auxiliary reservoir is connected, through the service piston of the control valve, to the brake cylinder. When the pressure difference between the local quick action chamber and brake pipe exceeds a value close to 25 psi, the high pressure spool valve is unseated, opening a passage between the emergency reservoir and the brake cylinder. The air from the combined auxiliary and emergency reservoirs flows through the inshot valve openings, causing a rapid increase in the brake cylinder pressure. When the brake cylinder pressure approaches, approximately, 15 psig, the inshot piston moves to the left, closing the large opening in the inshot valve. Final brake cylinder build up continues through the inshot by-pass choke

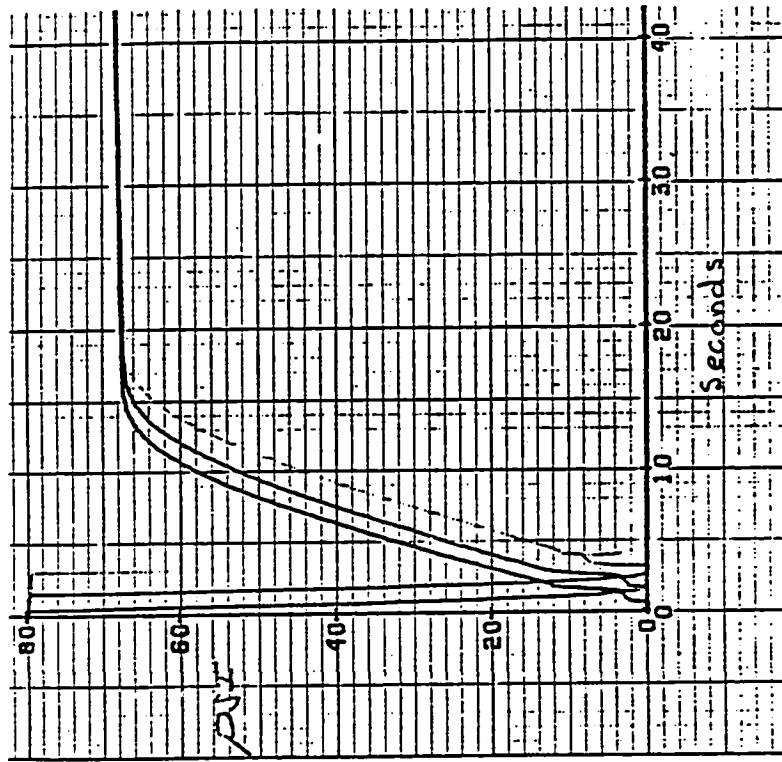


Figure 4.20a: Brake Pipe and Brake Cylinder Pressures Vs. Time (60-Car Emergency Mode/ABSW Experimental Data)

until the brake cylinder, auxiliary, and emergency reservoirs pressures reach the equilibrium state. Figures (4.20b) and (4.20c) show the simulation results for the brake pipe and brake cylinder pressures respectively. The results are in very good agreement with the corresponding experimental results.

Finally, figure (4.21a) shows the experimental results of the A19 upstream and downstream pressures, during a dry charge test of 50-car train with the control valves are cut-out. Figure (4.21b) shows the A19 downstream pressure, produced using the air brake system simulation program. The simulation result is in very good agreement with the corresponding experimental results. The differences are primarily due to assumption in the model of a constant supply pressure of 130 psig, giving a large pressure drop and greater flow rate.

4.5 Summary

Different experimental set-ups have been utilized to provide a good understanding of the functions of each of the air brake system components:-

1. 26C locomotive valve (section 4.2)
2. Brake pipe and 26C locomotive valve (section 4.3)
3. 26C locomotive valve, brake pipe, ABD and ABDW control valves, A19 flow indicator adaptor, auxiliary and emergency reservoirs, and brake cylinder (section 4.4).

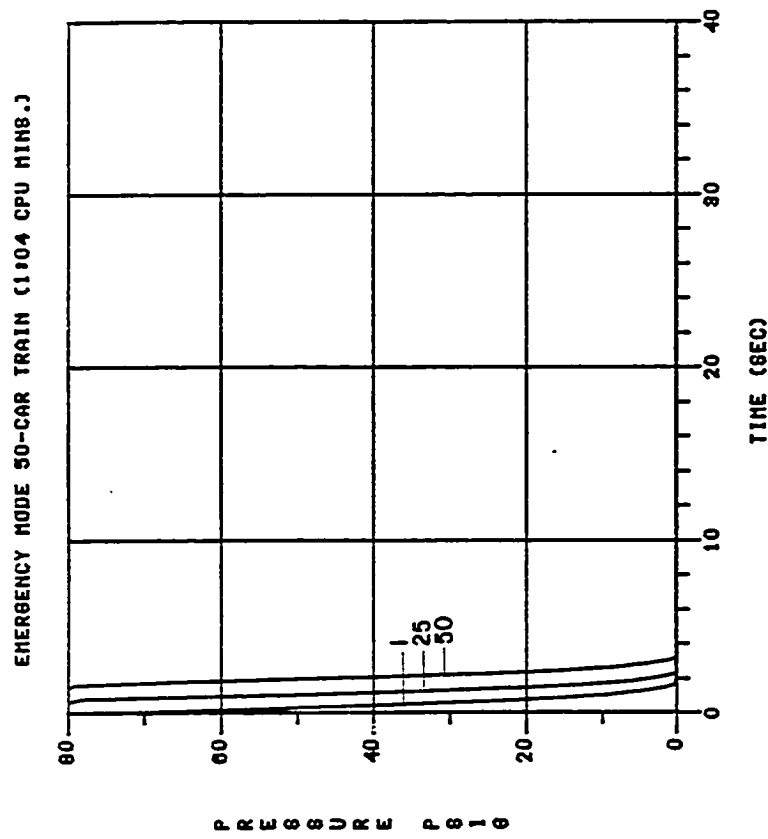


Figure 4.20b: Brake Pipe Pressures Vs. Time (50-Car Emergency Mode/ABDW
IFD with Modified Model)

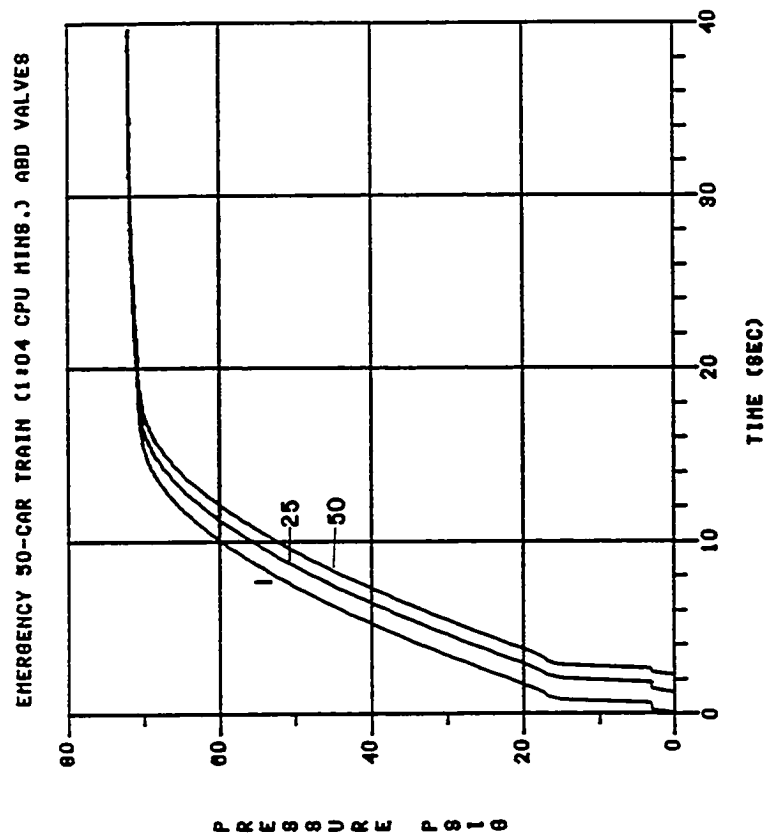


Figure 4.20c: Brake Cylinder Pressures Vs. Time (50-Car Emergency Mode/ABDW IFD with Modified Mode1)

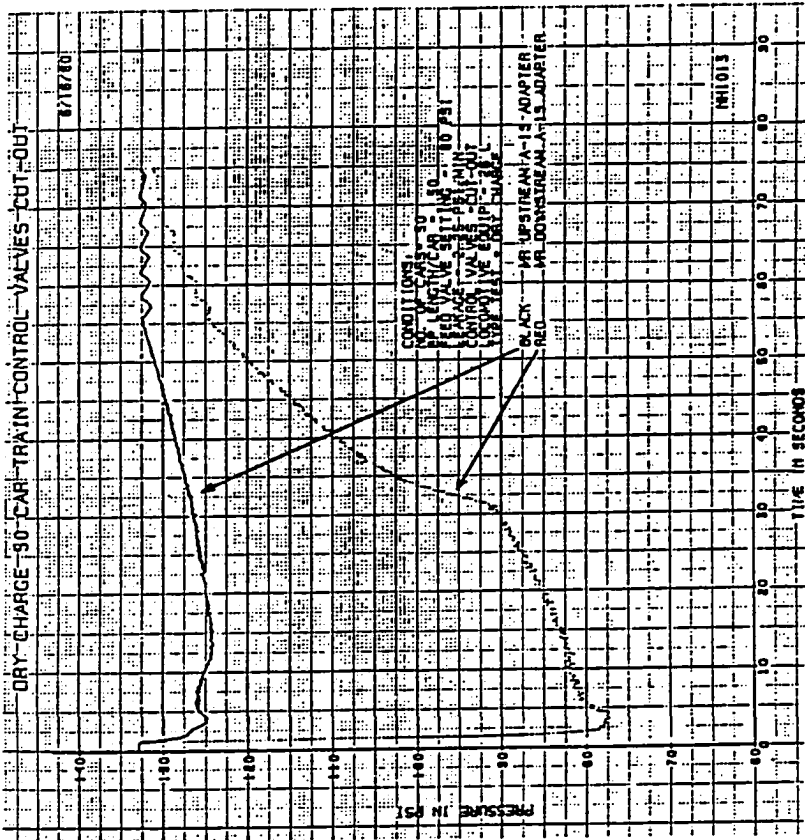


Figure 4.21a: A19 Upstream and Downstream Pressures Vs. Time (50-Car Dry Charge Experimental Data)

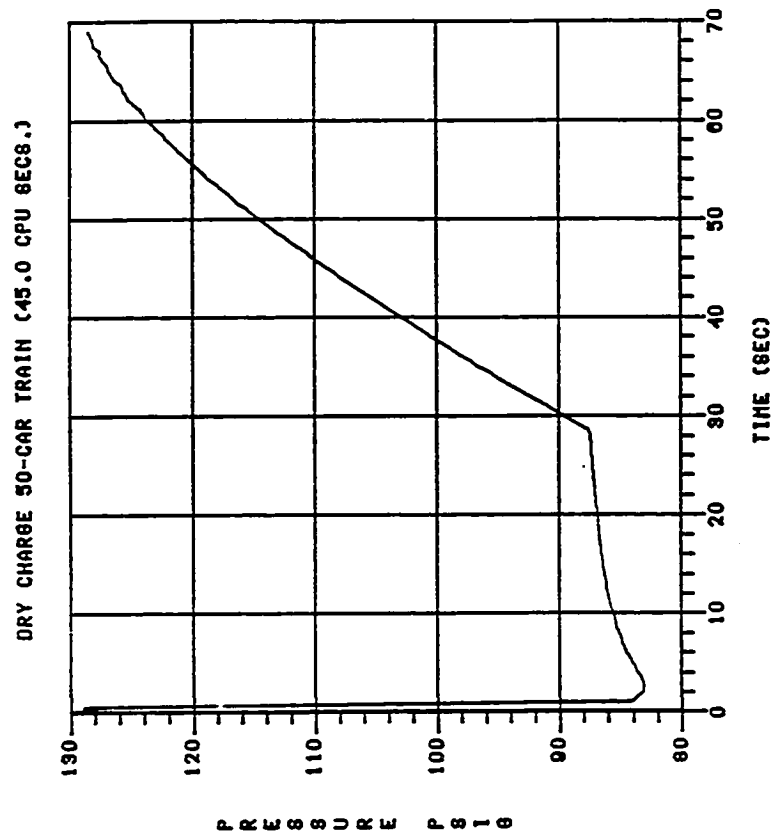


Figure 4.21b: A19 Downstream Pressure Vs. Time (50-Car Dry Charge IFD with Modified Model)

Then, the experimental results are used to compare between and validate the following models and numerical techniques:-

1. 26C mathematical (Chapter 3)
 - a. Complete model (section 3.3)
 - b. Modified model (section 3.4)
2. Brake pipe mathematical model (Appendix A)
3. ABD control valve mathematical model (Appendix G)
4. ABDW control valve mathematical model (Appendix H)
5. A19 flow indicator adaptor (Appendix I)
6. Finite difference techniques (section 2.3)
 - a. Implicit scheme (section 2.3.1)
 - b. Explicit Scheme (section 2.3.2)
7. Implicit finite element technique (section 2.4)

As general conclusions:-

1. The complete mathematical model of the air brake system components is able to closely predict and simulate the dynamic behaviour of each of the air brake system components.
2. The combination of the 26C modified model and the implicit finite difference scheme, proves to be the most efficient (accurate and faster) combination.
3. The complete mathematical model of the 26C locomotive valve provides an excellent representation for the dynamic behaviour of the valve. However, this model is very expensive to be a part of the air brake system model.

CHAPTER 5

THE EFFECT OF LEAKAGE ON BRAKE PIPE STEADY STATE BEHAVIOUR

5.1 Introduction

One of the factors that most affects brake pipe performance is leakage. There are two classifications of leakage:-

- 1) brake pipe leakage: the leakage in the hose assemblies, angle cocks and branch pipes.
- 2) system leakage: brake pipe leakage plus leakage in the AB, ABD and other types of control valves.

In practice, brake pipe leakage is unavoidable and occurs at pipe fittings, threaded joints, valves and gaskets. As a result of leakage, there is a considerable difference between locomotive pressure and last car pressure, this difference is referred to as the pressure gradient [1]. The amount of the pressure gradient depends on the location and size of the leakage [2].

The main objective of this chapter is to study the effects of leakage on the steady state behaviour of the brake pipe. The implicit finite difference technique, developed earlier in section 2.3.1, is employed here to solve the governing equations (steady state solution). The same technique has been used, in the previous chapter, to

solve for the transient case by choosing the proper integration time step.

The analysis presented in this chapter is for a brake pipe with leakage, but it may be applied to any pneumatic transmission line. We begin the finite difference technique which is used to solve the governing equations for two special cases: first, to predict the pressure distribution along the brake pipe, knowing the leakage size and distribution; secondly, to predict the leakage size along the brake pipe, knowing the pressure gradient. Section 5.3 describes the test set-ups of the laboratory scaled down brake pipe used for purposes of model validation (first case). Then Section 5.4 shows the comparisons between the present model, the model proposed by Ref.[4] and the full scale industrial test racks available (New York Air Brake [4] and Westinghouse Air Brake [1]). These comparisons provide the validation of the model for the second case. Section 5.5 describes some of the steady state behaviours of the pneumatic transmission line (brake pipe as special case). Finally, section 5.6 summarizes the result of this work.

5.2 Steady State Solution

The implicit finite difference technique is the most appropriate technique to provide the steady state solution for equations (2.8) and (2.9). In the case of the steady state solution of those equations, we need a stable and

accurate technique for a large time step ΔT . In this case, a value of ΔT_s fifty times ΔT_t , the value which may be used to provide an accurate transient solution, can be used; so we will use:

$$\Delta T_s = 50 \Delta T_t \quad (5.1)$$

In general, the finite difference procedure consists of approximating a set of partial differential equations (2.8) and (2.9) by a system of nonlinear algebraic equations. Substituting equation (2.15) into equations (2.8) and (2.9), a set of algebraic equations may be written in this matrix form:

$$\{B\} \cdot \{Y\}^{j+1} = \{Q\} \quad (5.2)$$

where $\{B\}$ is the system matrix

$$\{B\} = \begin{bmatrix} B_{1,2} & B_{1,3} & 0 & \dots & 0 & 0 & F_1 \\ B_{2,1} & B_{2,2} & B_{2,3} & \dots & 0 & 0 & F_2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & B_{\alpha-1,1} & B_{\alpha-1,2} & B_{\alpha-1,3} \\ 0 & 0 & 0 & \dots & 0 & B_{\alpha,1} & B_{\alpha,2} \end{bmatrix}$$

and

$$\{Q\} = \left\{ B_{1,4}, B_{2,4}, \dots, B_{\alpha-1,4}, B_{\alpha,4} \right\}^T \quad (5.2b)$$

$$\{Y\}^{j+1} = \left\{ M_1^{j+1}, P_2^{j+1}, M_2^{j+1}, \dots, M_N^{j+1}, D_{N+1}^{j+1} \right\}^T$$

Note that, $\{B\}$ and $\{Y\}$ take different forms than equation (2.44).

The coefficients of the {B} and {Q} matrices can be written in these forms:-

where, $\alpha = 2i-3$

$$\beta = 2i-1 \quad i=2,3,\dots,N+1$$

$$B_{\alpha,1} = -B_{\alpha,3} = -G_1$$

$$B_{\beta,1} = -\frac{A_{i-1}}{(A+v)_{i-1}}$$

$$B_{\beta,3} = \frac{A_i}{(A+v)_{i-1}}$$

$$B_{\alpha,2} = C_i \left[1 + f_i^j \left[\Lambda_{avg} \right]_i \frac{1}{2} \frac{1}{d_i} \frac{[U_{i-1}^j]^2}{|U_{i-1}^j|} \Delta T \right]$$

$$B_{\beta,2} = C_i$$

$$B_{\alpha,4} = C_i M_{i-1}^j - \frac{M_i^j U_i^j A_i - M_{i-1}^j U_{i-1}^j A_{i-1}}{A_i}$$

$$B_{\beta,4} = C_i P_i^j \left\{ r_{\beta,1} - K_i^j \left[\frac{d_i}{d_p} \right]^2 [1 - r_{\beta,2}] \right\}$$

$$F_{\alpha} = 0.$$

$$F_{\beta} = C_i r_{\beta,2} K_i^j P_i^j \frac{d_i^j}{d_p}$$

$$C_i = \frac{\Delta X_i}{\Delta T} \quad (5.3a)$$

for $i=1$

$$B_{1,4} = B_{1,4} - B_{1,1} P(0) \quad (5.3b)$$

If the pressure distribution solution is required (knowing the leakage distribution and size) the following parameters are set as follows:

$$\begin{aligned} r_{\beta,1} &= 1. & \beta &= 2,4,\dots,2N \\ r_{\beta,2} &= 0. \\ D_N^{j+1} &= P_N^j \end{aligned} \quad (5.4)$$

On the other hand, if the leakage size is desired (knowing leakage distribution and pressure gradient), the value of $r_{\beta,1}$, $r_{2N,1}$, $r_{\beta,2}$ and D_N^{j+1} are set to be as follows:

$$\begin{aligned} r_{\beta,1} &= 1. & \beta &= 2,4,\dots,2(N-1) \\ r_{2N,1} &= 0. \\ r_{\beta,2} &= 1.0 \\ D_N^{j+1} &= d_i^j \end{aligned} \quad (5.5)$$

The solution proceeds by starting with any arbitrary initial condition and solving equation (5.2) iteratively until the desired accuracy is achieved.

In order to solve the set of equations (5.2), a modified Thomas method is used [see Appendix C] instead of Gauss elimination or any other technique because there is only one restriction associated with this technique. The diagonal element of each row has to be greater than the summation of the other two elements within the same row [25]. Equation (5.3a) shows that this condition is always satisfied. After the pressure at each node is evaluated, by

solving equation (5.2), the bound error is calculated using this variance form:

$$\sigma = \frac{\sum (p_i^{j+1} - p_i^j)^2}{\sum p_i^j} \quad (5.6)$$

If e is within a desirable value, the integration (iteration) is terminated, otherwise, we go back and solve equation (5.2) again using the most recent values obtained. In this paper, we have assigned a value of 10^{-6} for e .

5.3 Scaled Down Test Set-up and Results

The scaled down brake pipe laboratory model (figure (5.1)) consisted of twelve sections of internal diameter 6.35×10^{-3} m (1/4 inch) and each 17.15 m (56.25 ft) long. Each section consisted of five pipes which were connected to each other by short lengths of flexible plastic tube. A cross was fitted at the end of each section so that a toggle valve and an orifice could be located there. The purpose of the toggle was to permit the measurement of static pressure at the end of each section (node). The leakage in each section was simulated by an orifice. A regulated supply of air was provided at the head-end of the brake pipe. This arrangement gave an approximate scaled down ratio of 5:1 in length dimension (diameter and pipe length).

In equation (2.5), f depends on the value of the Reynolds number, R_e , and we use 3 functions for the flow regimes (laminar, turbulent and transition), all of the general form

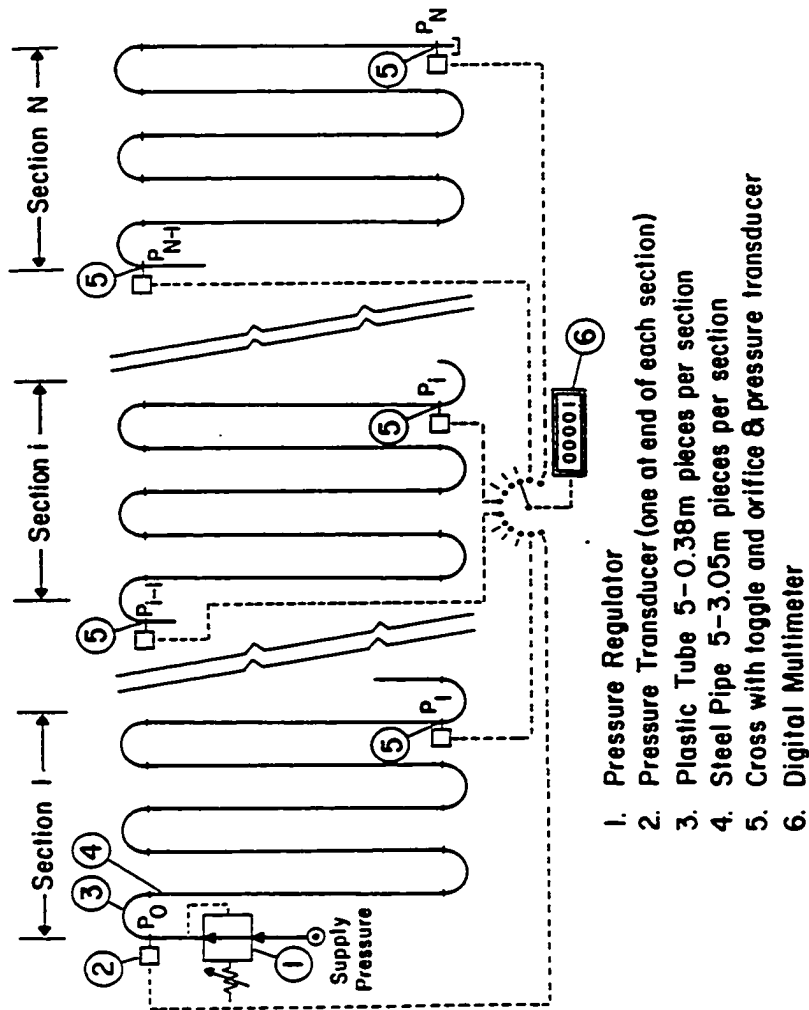


Figure 5.1: Schematic of the Scaled Down Laboratory Model for the Brake Pipe

$$f_i^j = a(R_e^b)_i^j \quad (2.5)$$

Appendix K, lists the values of a and b for the different flow regimes.

The scaled down brake pipe laboratory model was used to evaluate how closely the mathematical model represented the pressure distribution for different leakage sizes and locations. To perform these tasks, two different types of leakage configurations were used.

The first configuration had a fixed total leakage size concentrated, for different experiments, as follows:

- 1- The head-end region (I) of the brake pipe (nodes 1, 2, 3, and 4).
- 2- The middle region (II) of the brake pipe (nodes 5, 6, 7, and 8).
- 3- The rear-end region (III) of the brake pipe (nodes 9, 10, 11, and 12).

Within each region, the total, normalized leakage area, S, was distributed equally to the four nodes of the region. This total, normalized leakage area is expressed as a ratio of the total leakage area within the region with respect to the brake pipe cross-sectional area A:

$$S = 4 \left(\frac{d}{d_p} \right)^2 \quad (5.7)$$

The leakage diameter used was $d=0.5$ mm giving $S=2.5\%$.

On the other hand, the second configuration had a fixed leakage location (region III), for four different leakage

sizes. The values of leakage diameter and normalized leakage area for this region were:

experiment	d, mm	S
1	.3	.009
2	.4	.016
3	.5	.025
4	.6	.0357

In these experiments, the head-end pressure $P(0)$ was maintained at 580 kPa (70 psig). Pressures were measured at each node using differential pressure transducers. The transducers had an accuracy of 0.5% and a digital multimeter with accuracy estimated at .04%.

For comparison, the pressure at each node was calculated using the set of equations (5.2). Figures (5.2) and (5.3) show the experimental and theoretical results of the pressure at the node locations. These show the calculated results are in good agreement with the experimental results within 5%. The calculations were performed on a Digital Equipment Corporation DEC-10 computer with a computation time of approximately 0.3 seconds for each curve.

5.4 Present Solution and Refs. [1 & 4] Experimental Results

Shute, Wright, Taft and Banister [4] proposed a mathematical model to describe the dynamic behaviour of the air brake system. They used the method of characteristics to

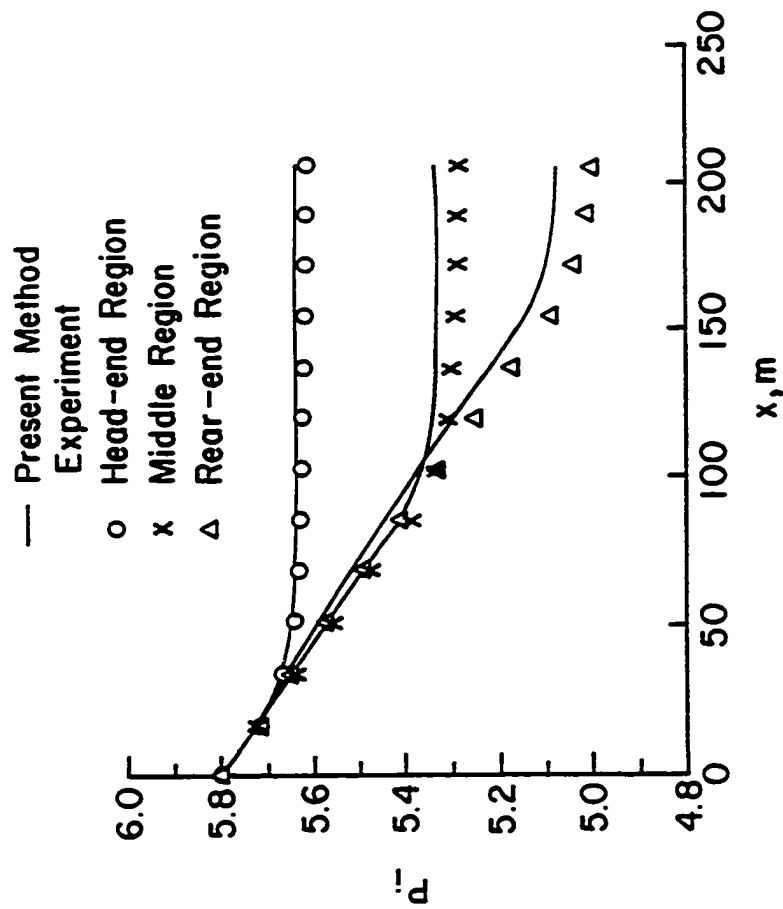


Figure 5.2: Different Leakage Locations Pressure Distribution (Scaled Down Model Experimental and Theoretical)

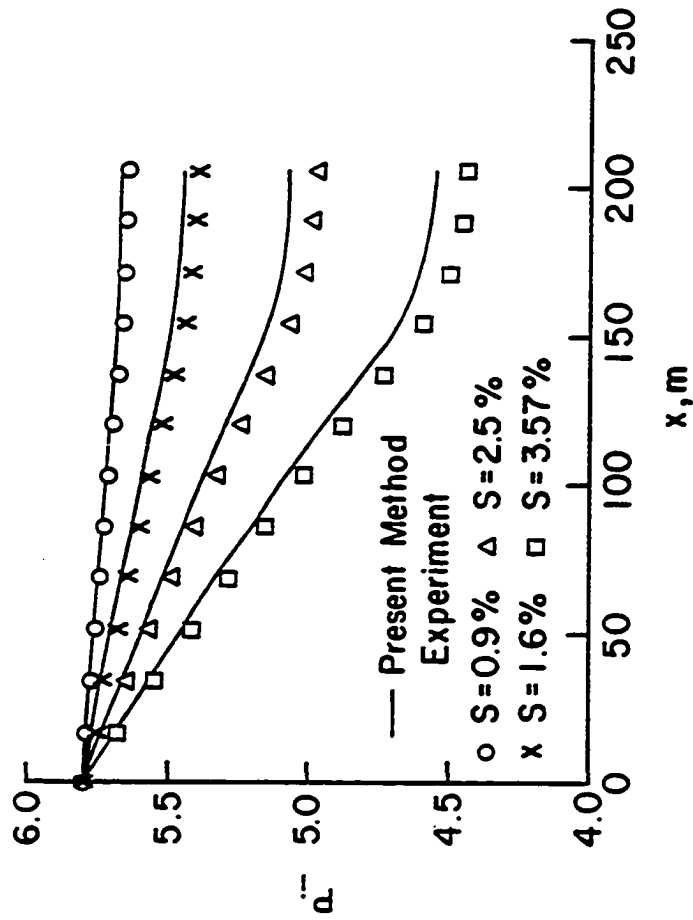


Figure 5.3: Different Leakage Size Pressure Distribution (Scaled Down Model Experimental and Theoretical)

solve their model equations. Their model has been used to predict the locomotive flow rate for a given pressure gradient with different leakage distributions and different numbers of cars (Fig. 12 of Ref. [4]) Their calculations were in good agreement with New York Air Brake (NYAB) data, generally within 10 percent. Their model calculations required, however, a large memory space and each simulation took considerable computer time. As an example, to simulate a 100 car train, the computation time was approximately 1000 CPU seconds using a DEC 10 computer. For comparison, using the present technique, the brake pipe was simulated. It has a larger internal diameter than the lab model. Gautheir [7], Wright [8] Banister [4 & 9] and Kreel [10] have suggested that the friction factor f_i could take the following forms. Laminar flow, as before and Transition flow $2000 \leq (R_e)_i^j \leq 4000$

$$f_i^j = 2.76 \cdot 10^{-3} (R_e^{0.332})_i^j \quad (5.8)$$

and Turbulent flow $(R_e)_i^j > 4000$

$$f_i^j = 0.042 \quad (5.9)$$

Figure (5.4) shows the comparison between the present model prediction and NYAB data, (Ref. [4]). The calculated results are in good agreement with the NYAB data within 5%. Also, the present method is compared with Westinghouse Air Brake data (Ref. [1]) for locomotive flow rate (table (5.1)) for different numbers of cars and pressure gradients. The present model calculations are also in good agreement with Ref. [2] data within 5%. The present method takes less

Table (5.1): Locomotive Valve Flow Rate SCFM Vs. Pressure Gradient for N=50, N=100 and N=150

Pressure Gradient (psi)	Locomotive Flow Rate SCFM					
	N=50		N=100		N=150	
	Ref[1]	F.D.*	Ref[1]	F.D.	Ref[1]	F.D.
1	22	21.3	13	13.9	9	11.6
3	43	42.6	27	29	19	21.
5	58	57	36	39.7	27	31
10	87	87	54	58	41	40
15	110	107	67	72	53	54

*F.D. → present finite difference technique.

computer time and smaller memory size as compared with the method proposed by Ref. [4]. As an example, to solve set of equations (14) for 100 car train ($N=100$), the computation time was 2.0 CPU seconds using DEC 10 Computer. Figure (5.4) and Table (5.1) show that for constant pressure gradient, the flow rate is indirectly proportional to the length of the train (l or N).

The difference between using equations (5.8) and (5.9) or equations (K.9) and (K.12) is very small to be considered (less than 2%).

5.5 Leakage and Brake Pipe Steady State Behaviour

In this section the brake pipe mathematical model is used to quantitatively investigate the effects of leakage on the inlet velocity ($U(0) = M(0)/P(0)$) and the pressure gradient ($\Delta P = P(0) - P(N)$) of the brake pipe (for $d_p/l = .003\%$ representing a 70 car train). This investigation is only applicable to pipe flow in general if the pipe can be treated as a one dimensional flow problem. In Section 5.3, different leakage locations were used to verify the applicability of the mathematical model. In order to study the effect of the leakage locations, two additional leakage locations have been chosen,

- 1- a single leakage is located at the rear-end of the brake pipe (i.e at the end of the last section of the pipe).

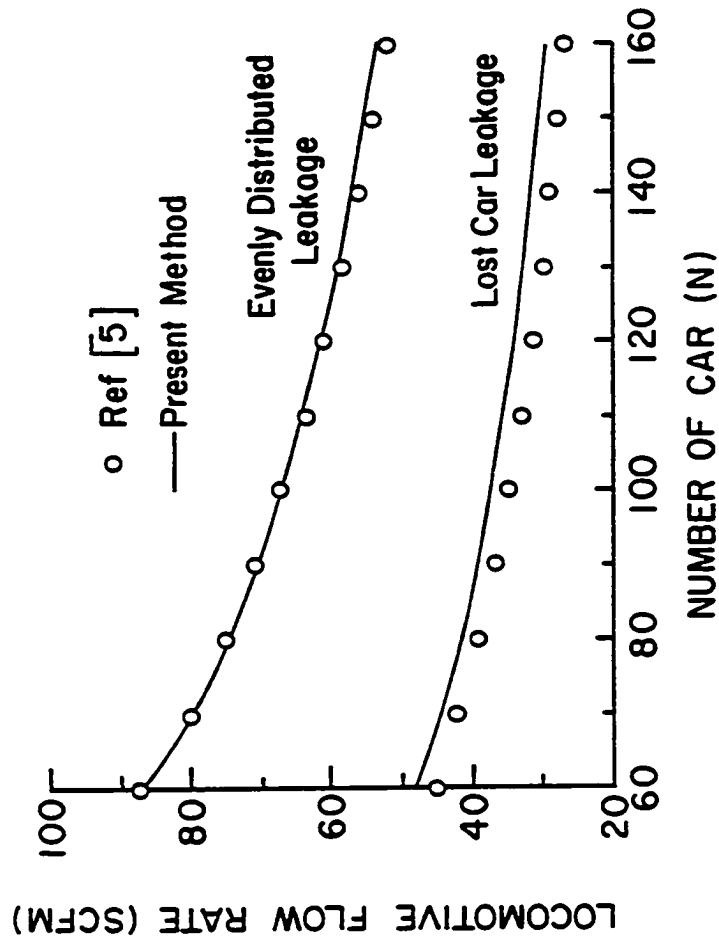


Figure 5.4: Locomotive Flow Rate as a Function of Number of Cars
(Experimental and Theoretical)

2- a single leakage is located at the head-end of the brake pipe (i.e at the end of the first section of the pipe).

Figure (5.5) shows a plot of the brake pipe pressure gradient and the inlet velocity versus leakage size, for different leakage locations. This figure also shows the railroad application zone as specified by AAR [7]. One of the rules (Ref. [1]) is that, "rear end brake pipe pressure must be within 15 psi of locomotive feed valve (but not less than 60 psig) before making the application and leakage test.". For this investigation, it means that the pressure gradient must not be greater than $\Delta P = 0.68$ (69. kPa or 10 psi) for a head-end pressure $P(0) = 5.8$ [Note the horizontal line $\frac{\Delta P}{P(0)} = 0.12$ in figure (5.5a)]. In this case, figure (5.5b) shows that for $P = 0.68$, as the leakage moves towards the head-end (locomotive), the inlet velocity increases.

In figure (5.5b), it can be observed that, if the leakage size is less than 2.0%, the inlet velocity varies insignificantly with the change of the leakage locations, whereas it varies significantly with the change of the leakage size (the same observations have been made by Shute, Taft and Banister [4]). When the leakage area is large, for example $S > 8.0\%$, the inlet flow varies significantly as a location of the leakage is changed. Figure (5.5a) shows that as a constant leakage is moved towards the rear-end of the train, the pressure gradient increases, and the inlet

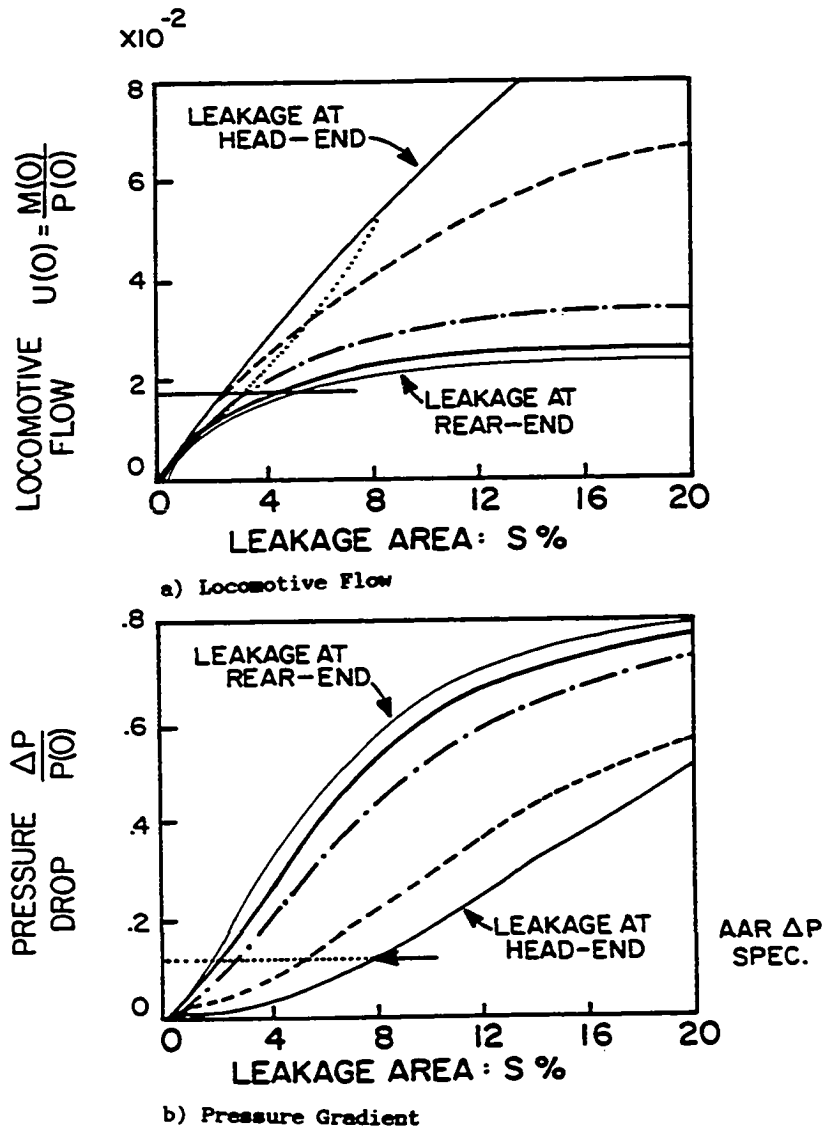


Figure 5.5 : Steady State Behaviour of the Brake Pipe as a Function of Leakage Size and Distribution

velocity decreases. This is caused by the fact that, when the leakage is concentrated at the rear-end, the in-line resistance will increase which will cause a large increase in the pressure drop and a decrease in the inlet flow. Finally, for a fixed leakage location, as leakage size increases, the pressure gradient and the inlet velocity increases.

5.6 Summary

The main features of this work, which concerns the use of the implicit finite difference technique to solve the brake pipe mathematical model equations are

1. The model is capable of predicting closely the steady state pressure, leakage, and velocity distribution.
2. Computation time and memory needed using the implicit finite difference technique are very small as compared with the previous technique proposed by Ref. [4].
3. We conclude that, the implicit finite difference scheme is a more suitable technique to solve the brake pipe governing equations for steady state solutions. It can also be used to provide transient solutions by only changing the integration time step.

The brake pipe mathematical model has been used to investigate the effect of leakage size and location on the

steady state behaviour of the brake pipe. The investigation results may be summarized as follows:

1. The leakage location has a significant effect on the pressure distribution (i.e. as the leakage is moved towards the rear-end, the larger the pressure gradient). On the other hand, it does not have any significant effect on the inlet flow rate for the small leakage size ($S < 2\%$). For a large leakage size ($S > 8\%$), the inlet flow rate increases as the leakage moves towards the head-end.
2. The leakage size also has a great effect on the pressure distribution (i.e. the larger the leakage size, the larger pressure gradient). On the other hand, the leakage size does not have a great effect on the inlet flow rate for larger leakage size ($S > 8\%$), because the friction of the pipe then tends to control the flow rate.
3. The fitness of the brake pipe can not be determined based on the pressure gradient measurement or on the flow test alone, but it can be determined using both methods.

CHAPTER 6

CONCLUSIONS

Since 1870, most freight trains in North America have been equipped with the automatic air brakes. By the middle of this century, the air brake system started to take its present form.

The back-bone of the air brake system is the the brake pipe, which is known as the nerve line or the train line. The new brake pipe mathematical model is developed, in Appendix A, which describes the interaction of the branch pipe with the brake pipe pressure and flow equations. In chapter 2, two numerical techniques are presented to provide the numerical solutions of the brake pipe governing equations. The solution methods allow for trains consisting of cars of different lengths, branch pipe volumes, brake pipe cross-sectional areas, leakage openings, and valve equipment. These numerical techniques, allow the addition of the air brake system control units (locomotive and car unit) to the brake pipe at any location along the pipe.

In chapter 3, the 26C locomotive valve is, for the first time in the history of railroad industry, completely modeled (complete and modified models), describing the functions of the locomotive valve sub-valves (relay, regulating and brake pipe cut-off valves). The complete model of the 26C

locomotive valve includes, as a part of the model, the dynamic effects of the small volumes of the relay valve. On the other hand, the simplified (modified) model uses the quasi-steady approach, thus neglecting the dynamics of the small volumes. However, both models successfully duplicate (especially the complete model), in detail, the dynamic behaviour of the 26C locomotive valve during its operation modes (service and release), when connected to a brake pipe or equivalent volume.

The mathematical models of the car control unit components (ABD and ABDW control valves, brake cylinder, auxiliary reservoir and emergency reservoirs) are formulated in Appendix G and H (AAV model). These models, accurately, simulate the service (partial or full), release after service, release after emergency and dry charge modes of the the car control unit.

The main features of this work, may be summarized in the following points:

1. A new brake pipe mathematical model is developed, and includes the presence of the branch pipe.
2. The implicit finite difference scheme, developed in the present research work, can be used to provide accurate and faster solutions (steady and transient) for the brake pipe governing equations. It allows different car pipe lengths and cross sectional areas.

3. The modified model of the 26C locomotive valve is developed, and accurately represents the dynamic behaviour of the locomotive valve.
4. The 26C complete model is a very good tool for investigating the effect of the change in the locomotive valve configuration (volumes, spring constant,... etc.) in the performance of the valve. However, it is a very expensive model to be an integral part of the air brake system simulation program.
5. The control valves (ABD and ADBW) are completely modeled after including the functions of the following sub-valves:-
 - a. accelerated emergency release check valve,
 - b. inshot valve,
 and, c. high pressure spool valve.

The control valves models, accurately, represent the different modes of the valves.

6. The Fortran (air brake simulation) Program is designed for easy modifications (see Appendix L for more detail), such that the user can:-
 - a. add modules for the new control valves,
 - b. change in the dimensions and configurations of the brake pipe, the 26C locomotive and the control valves (up to five),

- c. have up to two different control unit designs with unlimited numbers of each unit, at each car of the train, and
- d. have different configurations for the brake cylinder, auxiliary reservoir and emergency reservoirs (up to ten).

This program can be a very useful tool for the railroad industry. Among its several applications, it can be used to,

- a. study the effects of leakage on the steady state and dynamic behaviours of the air brake system components,
 - b. provide for brake cylinder pressures, which can be used by a different computer program to study the forces which develop at the coupler between cars, during normal service operation,
 - c. develop new criteria for evaluating brake system performance,
 - d. design new and simpler control valves,
 - e. detect and test the causes of existing field failure,
- and, f. determine the sensitivity of the air brake system to the valve characteristics.

In this dissertation, the Fortran Program is used to study the effect of leakage (size and location) on the brake pipe steady state behaviour. The results of study may be summarized as follows:-

7. The leakage location has a significant effect on the pressure distribution (i.e. as the leakage is moved towards the rear-end, the larger the pressure gradient). On the other hand, it does not have any significant effect on the inlet flow rate for the small leakage size ($S < 2\%$). For a large leakage size ($S > 8\%$), the inlet flow rate increases as the leakage moves towards the head-end.
8. The leakage size also has a great effect on the pressure distribution (i.e. the larger the leakage size, the larger pressure gradient). On the other hand, the leakage size does not have a great effect on the inlet flow rate for larger leakage size ($S > 8\%$), because the friction of the pipe then tends to control the flow rate.
9. The fitness of the brake pipe can not be determined based on the pressure gradient measurement or on the flow test alone, but, it can be determined using both methods.

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APPENDIX A

QUASI-ONE-DIMENSIONAL FLOW EQUATIONS

A.1 The Continuity Equation

The assumption of quasi-one-dimensional flow idealizes a flow in the following manners:

1. it ignores the two smaller velocity components (u_y and u_z) and assumes that the flow direction at any cross section is parallel to one axis (x-axis);
2. It assumes that the magnitude of the flow velocity is uniform at any cross section.

The continuity equation shows the principle of mass conservation. Figure (A.1) shows the control volume for developing the appropriate continuity equation for quasi-one-dimensional flow. Consider a volume element of the duct $(A+v)\Delta x$ (Shaded area in figure A.1), where A is the local cross-sectional area of the duct and v is the local volume per unit length. In this analysis, v assumed to be very small as compared to the total volume per unit length of the element. The time rate of change of the mass within the control volume is

$$\frac{\partial}{\partial t} \int_{C.V} \rho_x dV = \frac{\partial}{\partial t} \int_{C.V_1} \left(\rho + \frac{\partial \rho}{\partial x} x \right) \left(A + \frac{\partial A}{\partial x} x \right) dx$$

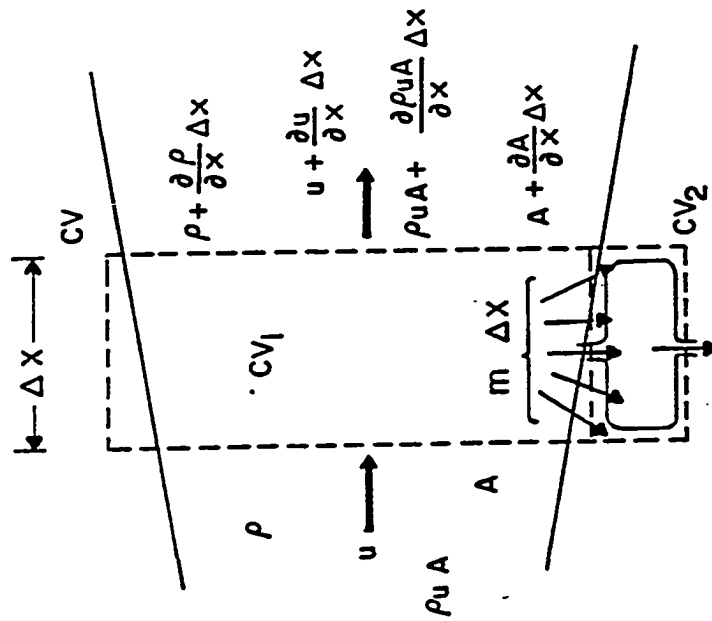


Figure A.1: Continuity Equation Control Volume

$$+ \frac{\partial}{\partial t} \int_{C.V_2} \left(\rho + \frac{\partial \rho}{\partial x} \Delta x \right) v dx \quad (A.1)$$

Expanding the right hand terms of equation (A.1) and integrating, and then ignoring the higher order terms (Δx^2 and higher), the time rate of change of the mass in the control volume can be written in this final form

$$\frac{\partial}{\partial t} \int_{C.V} \rho dV = (A+v) \frac{\partial \rho}{\partial t} \Delta x \quad (A.2)$$

Fluid is entering and leaving this element through its ends. Fluid may also enter and leave the duct through an opening in the volume v and other openings in the duct (eg. leakage), where μ is the mass flow leaving the element per unit length through these openings. The rate of flow of masses which enter and leave the element during the time interval dt are shown in figure A.1. The excess of the inflows over the outflows, Ξ , is

$$\begin{aligned} \Xi = \rho u A - \left(\rho + \frac{\partial \rho}{\partial x} \Delta x \right) \left(u + \frac{\partial u}{\partial x} \Delta x \right) \left(A + \frac{\partial A}{\partial x} \Delta x \right) \\ + \mu \Delta x \end{aligned} \quad (A.3)$$

Expanding the first term in the right hand side of equation (A.3), rearranging and then eliminate the higher order term, the excess flow, Ξ , can be written in this form

$$\Xi = \left[\frac{\partial \rho A u}{\partial x} + \mu \right] \Delta x \quad (A.4)$$

But, the excess flow, Ξ (equation (A.4)), must be equal to the increase of the mass contained in the element (equation (A.2)) and the continuity equation thus becomes.

$$(A+v) \frac{\partial p}{\partial t} + \frac{\partial p u A}{\partial x} + \mu = 0 \quad (\text{A.5})$$

$$\text{or} \quad \frac{\partial p}{\partial t} + \frac{1}{A+v} \frac{\partial p u A}{\partial x} + \Lambda \omega = 0 \quad (\text{A.6})$$

where,

$$\omega = \frac{\mu}{A} \quad (\text{A.7})$$

$$\text{and, } \Lambda = \frac{A}{(A+v)}$$

A.2 The Momentum Equation

Section (A.1) discussed quasi-one-dimensional flow approximation and developed the appropriate continuity equation (A.6) for the present flow problem. The Euler momentum equation for this flow will now be developed. The pressure and the shear forces will be considered in this development. Figure (A.2) shows a duct of gradually varying cross-sectional area. The flows of the linear momentum entering and leaving the control volume are shown in this figure. Neglecting the velocity in the x-direction within the C.V₁, the inertial terms of the momentum control-volume equation in the x-direction can be written in this straightforward form

$$\begin{aligned} L_m &= \frac{\partial}{\partial t} \int_{C.V} \rho_x u_x \, dV + \int_{C.S} \rho_x u_x^2 \, dA \\ &= \frac{\partial}{\partial t} \int_0^{\Delta x} \left(\rho + \frac{\partial \rho}{\partial x} x \right) \left(u + \frac{\partial u}{\partial x} x \right) \left(A + \frac{\partial A}{\partial x} x \right) dx \\ &\quad + \left(\rho + \frac{\partial \rho}{\partial x} \Delta x \right) \left(u + \frac{\partial u}{\partial x} \Delta x \right)^2 \left(A + \frac{\partial A}{\partial x} \Delta x \right) - \rho u^2 A \end{aligned} \quad (\text{A.8})$$

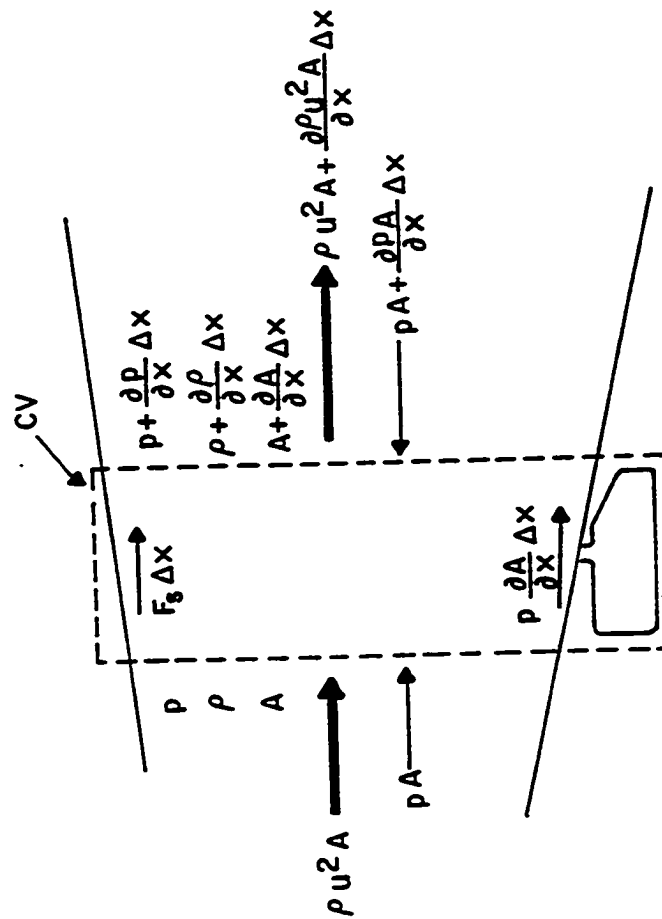


Figure A.2: Momentum Equation Control Volume

Integrating, and rearranging equation (A.8), the final form of the linear momentum can be written as:

$$L_m = \left[A \frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2 A}{\partial x} \right] \Delta x \quad (A.9)$$

Figure (A.2) shows also the forces acting on the control surface. These forces are the pressure and the shear stress forces, and can be written in this form

$$\begin{aligned} F_T &= pA - (p + \frac{\partial p}{\partial x} \Delta x)(A + \frac{\partial A}{\partial x} \Delta x) - p(A + \frac{\partial A}{\partial x} \Delta x) + F_S \Delta x \\ &= - \left[A \frac{\partial p}{\partial x} + F_S \right] \Delta x + O(\Delta x^2) \end{aligned} \quad (A.10)$$

Neglecting the higher order term ($O(\Delta x^2)$), and putting both sides of the momentum equation (equations (A.9) and (A.10)) together results in

$$L_m = F_T$$

or,

$$A \frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2 A}{\partial x} + A \frac{\partial p}{\partial x} + F_S = 0 \quad (A.11)$$

In the case of duct flow, one may assume that the force F_S is only related to the wall shear stress as:

$$F_S = -\tau_w d \quad (A.12)$$

where, the hydraulic diameter, d , is given as:

$$d = \sqrt{\frac{4}{\pi} A_{avg}} = \sqrt{\frac{4}{\pi \Delta x} \int_x^{x+\Delta x} A dx} \quad (A.12)$$

However, in the case of one-dimensional flow, the wall shear stress can be related to the fluid velocity and density as:-

$$\tau = f \frac{\rho u^2}{8} \left[\frac{u}{|u|} \right] \quad (\text{A.13})$$

where f is the wall friction factor and the term $u/|u|$ is included to account for the "opposing motion" quality of the shear stress. Now, with the use of equations (A.11) and (A.13) the momentum equation may be written as:-

$$\frac{\partial \rho u}{\partial t} + \frac{1}{A} \frac{\partial \rho u^2 A}{\partial x} + \frac{\partial p}{\partial x} + f \frac{A_{avg}}{A} \frac{\rho u^2}{2d} \left[\frac{u}{|u|} \right] = 0 \quad (\text{A.14})$$

where,

$$A_{avg} = \frac{A_{avg}}{A} \quad (\text{A.15})$$

APPENDIX B

SOLUTION PROCEDURE FOR A TRI-DIAGONAL SYSTEM USING THOMAS ALGORITHM

In this appendix, we present a method to solve the set of simultaneous equations presented in this text (Chapter 2). This full description of Thomas algorithm can be found in Ref. [27]. In this appendix, we extend the use of Thomas algorithm to solve a set of simultaneous equations with a source term $D_i X_N$. Assume that the system equations may be written in this form:

$$A_i X_{i-1} + B_i X_i + C_i X_{i+1} + D_i X_N = E_i \quad (B.1)$$

where, A_i , B_i , C_i and D_i are matrices each having size $(m \times m)$, and X_i and E_i each having size $(m \times 1)$ each.

$$A_1 = C_N = D_N = D_{N-1} = 0 \quad (B.2)$$

and where, X_i is the dependent variable at node i , and the number of unknowns is equal to $(N)(m)$. We postulate the existence of three matrices F , G and H . Such that, for any X_i

$$X_i = F_i X_{i+1} + G_i X_N + H_i \quad (B.3)$$

The existence of such matrices F , G and H will become evident. We now index equation (B.3) down in i to obtain

$$X_{i-1} = F_{i-1} X_i + G_{i-1} X_N + H_{i-1} \quad (B.4)$$

Substituting for X_{i-1} from equation (B.4) into equation (B.1) and solving for X_i gives

$$X_i = \left[B_i + A_i F_{i-1} \right]^{-1} \left[-C_i X_{i+1} + (A_i G_{i-1} + D_i) X_N + E_i - A_i H_{i-1} \right] \quad (B.5)$$

Comparing equations (B.5) and (B.3), and noting that both equations hold for all X_i , we must have

$$F_i = - \left[B_i + A_i F_{i-1} \right]^{-1} C_i \quad (B.6)$$

$$G_i = - \left[B_i + A_i F_{i-1} \right]^{-1} \left[A_i G_{i-1} + D_i \right]$$

$$H_i = - \left[B_i + A_i F_{i-1} \right]^{-1} \left[E_i - A_i H_i \right]$$

For $i=1$

$$F_1 = -B_1^{-1} C_1, \quad G_1 = -B_1^{-1} D_1, \quad \text{and } H_1 = B_1^{-1} E_1 \quad (B.7)$$

The solution proceeds, starting with (B.7). Then we use the recursion relations (B.6) to calculate all F_i , G_i and H_i up to $i=N$. Then, equation (B.3) can be used to find all X_i up to $i=N$. To see the efficiency of this method, note that it requires only three multiplications and two divisions per space point, per time, in addition to the computation of the coefficients.

APPENDIX C

NUMERICAL FORMULATIONS

OF

BRAKE PIPE MATHEMATICAL MODEL

C.1 Finite Difference Formulation

C.1.1 Implicit Scheme

$$\begin{aligned}
 \left[\frac{\partial P}{\partial T} \right]_i^{j+1} &= \frac{P_i^{j+1} - P_i^j}{\Delta T} \\
 \left[\frac{\partial M}{\partial T} \right]_{i-1}^{j+1} &= \frac{M_{i-1}^{j+1} - M_{i-1}^j}{\Delta T} \\
 \left[\frac{1}{A+v} \frac{\partial MA}{\partial X} \right]_i^{j+1} &= \frac{M_i^{j+1} A_i - M_{i-1}^{j+1} A_{i-1}}{(A+v)_i \Delta X_i} \\
 \left[\frac{\partial P}{\partial X} \right]_i^{j+1} &= \frac{P_i^{j+1} - P_{i-1}^{j+1}}{\Delta X_i} \\
 \left[\frac{1}{A} \frac{\partial MUA}{\partial X} \right]_i^j &= \frac{M_i^j U_i^j A_i - M_{i-1}^j U_{i-1}^j A_{i-1}}{A_i \Delta X_i} \\
 \left[G_2 MU \right]_{i-1}^{j+1} &\approx \left[G_2 U \right]_i^j M_{i-1}^{j+1}
 \end{aligned} \tag{C.1}$$

Substituting equation (C.1) into equations (2.8) and (2.9), the following set of algebraic equations completely describe the semi-implicit finite difference formulation for equations (2.8) and (2.9)

$$B_{\alpha,1} P_{i-1}^{j+1} + B_{\alpha,2} M_{i-1}^{j+1} + B_{\alpha,3} P_i^{j+1} = B_{\alpha,4} \tag{C.2a}$$

$$B_{\beta,1} M_{i-1}^{j+1} + B_{\beta,2} P_i^{j+1} + B_{\beta,3} M_i^{j+1} = B_{\beta,4} \tag{C.2b}$$

where, $\alpha = 2i-3$

$$\beta = 2i-1 \quad i=2,3,\dots,N+1$$

$$\begin{aligned} B_{\alpha,1} &= -B_{\alpha,3} = -G_1 \\ B_{\beta,1} &= -\frac{A_{i-1}}{(A+v)_{i-1}} \\ B_{\beta,3} &= \frac{A_i}{(A+v)_{i-1}} \\ B_{\alpha,2} &= C_i \left[1 + f_i^j \left[\Lambda_{avg} \right]_i \frac{1}{2} \frac{[U_{i-1}^j]^2}{|U_{i-1}^j|} \Delta T \right] \\ B_{\beta,2} &= C_i \\ B_{\alpha,4} &= C_i M_{i-1}^j - \frac{M_i^j U_i^j A_i - M_{i-1}^j U_{i-1}^j A_{i-1}}{A_i} \\ B_{\beta,4} &= C_i P_i^j - \Lambda_i \Omega_i^j \\ C_i &= \frac{\Delta T}{\Delta X_i} \end{aligned} \quad (C.3)$$

BOUNDARY EQUATIONS

for $i=2$, $P_1^{j+1} = P(0,T) = f(T, M_0, \dots \text{etc}) = P(0,T)$, as

$$P_1^{j+1} = P(0,T) = P_1^j + \frac{M(\text{source}) - M_1^j}{C_1} \quad (C.4)$$

thus equation (C.5a) may be rewritten in this form:

$$B_{2,4} = B_{2,4} - B_{2,1} P(0,T) \quad (C.5)$$

and for $i=N+1$, $M_{N+1}^{j+1} = M(1,T) = 0$, thus equation (C.2b) can be rewritten in this form

$$B_{2N,1} M_N^{j+1} + B_{2N,2} P_{N+1}^{j+1} = B_{2N,4} \quad (C.7)$$

Using the above boundary conditions ($P(0,T)$ and $M(1,T)$), reduced the number of equations to be solved

simultaneously to $2N$ equations. In general, these equations form a tri-diagonal set of simultaneous linear algebraic equations, which may be solved using the algorithm described in Appendix B.

C.1.2 Explicit Scheme

- 1- Change the terms P_i^{j+1} and P_{i-1}^{j+1} in the left hand side of equation (C.2a) to P_i^j and P_{i-1}^j respectively, and
- 2- Change the terms M_i^{j+1} and M_{i-1}^{j+1} in the left hand side of equation (C.2b) to M_i^j and M_{i-1}^j respectively,

then rewrite equations (C.2a) and (C.2b) in these forms

$$M_{i-1}^{j+1} = \frac{B_{\alpha,4} - B_{\alpha,1}P_{i-1}^{j+1} - B_{\alpha,3}P_i^{j+1}}{B_{\alpha,2}} \quad (C.8a)$$

$$P_{i-1}^{j+1} = \frac{B_{\beta,4} - B_{\beta,1}M_{i-1}^{j+1} - B_{\beta,3}M_i^{j+1}}{B_{\beta,2}} \quad (C.8b)$$

But, before solving equations (C.8a) and (C.8b), it is important to consider the stability limitation of this formulation as an explicit scheme (see chapter 2). Note, that the implicit or the explicit scheme formulated in this section has a local truncation error of order $O(\max(\Delta X_i) + \Delta T)$.

C.2 Finite Element Formulation

$$B_{i,1}Y_{i-1}^{j+1} + B_{i,2}Y_i^{j+1} + B_{i,3}Y_{i+1}^{j+1} = B_{i,4} \quad (C.9)$$

where,

$$B_{i,L} = \begin{bmatrix} b_{1,i,L} & b_{3,i,L} \\ b_{2,i,L} & b_{4,i,L} \end{bmatrix}, \quad (C.10)$$

$$B_{i,4} = \begin{bmatrix} b_{1,i,4} \\ b_{2,i,4} \end{bmatrix} \quad \text{and} \quad Y_i = \begin{bmatrix} P_i \\ M_i \end{bmatrix},$$

for $L=1$, $\gamma = -.5$ and for $L=3$, $\gamma = .5$ Thus,

$$\begin{aligned} b_{1,i,L} &= \frac{\Delta X(i,L)}{6\Delta T} \\ b_{2,i,L} &= \gamma \sigma_M^{(i,L)} \delta_M(i,L) \\ b_{3,i,L} &= \gamma \sigma_P^{(i,L)} \delta_P(i,L) \\ b_{4,i,L} &= \gamma \alpha_M^{(i,L)} \beta_M(i,L) + \frac{\Delta X(i,L)}{6\Delta T} \end{aligned} \quad (C.11a)$$

$$b_{4,i,1} = b_{4,i,1} + \frac{G_2^{(i-1)}}{2} \Delta X^{(i-1)}$$

where, $(i,2) \rightarrow (i,3) \rightarrow (i)$

and $(i,1) \rightarrow (i-1)$

for $L=2$

$$\begin{aligned} b_{1,i,2} &= \frac{\Delta X^{(i-1)} + \Delta X^{(i)}}{3\Delta T} \\ b_{4,i,2} &= .5 \left[\bar{\alpha}_M^{(i-1)} - \bar{\alpha}_M^{(i)} \right] \beta_M(i) + \frac{\Delta X^{(i-1)} + \Delta X^{(i)}}{3\Delta T} \\ b_{2,i,2} &= .5 \left[\bar{\sigma}_M^{(i-1)} - \bar{\sigma}_M^{(i)} \right] \delta_M(i) \quad (C.11b) \\ &= 0 \quad \text{because, } \sigma_M^{(i-1)} = \sigma_M^{(i)} = G_1 \\ b_{3,i,2} &= .5 \left[\bar{\sigma}_P^{(i-1)} - \bar{\sigma}_P^{(i)} \right] \delta_P(i) \\ b_{4,i,2} &= b_{4,i,2} + \frac{G_2^{(i)} \Delta X^{(i)}}{2} \end{aligned}$$

$$b_{1,i,4} = \frac{\Delta X^{(i-1)}}{6\Delta T} P_{i-1}^j + \frac{\Delta X^{(i-1)} + \Delta X^{(i)}}{3\Delta T} P_i^j + \frac{\Delta X^{(i-1)}}{6\Delta T} P_{i+1}^j - .5 \left[\Xi_P^{(i-1)} \Delta X^{(i-1)} + \Xi_P^{(i)} \Delta X^{(i)} \right] \quad (C.11c)$$

$$b_{2,i,4} = \frac{\Delta X^{(i-1)}}{6\Delta T} M_{i-1}^j + \frac{\Delta X^{(i-1)} + \Delta X^{(i)}}{3\Delta T} M_i^j + \frac{\Delta X^{(i-1)}}{6\Delta T} M_{i+1}^j$$

BOUNDARY EQUATIONS

for $i=N+1$, equation (C.9) is written in this form

$$B_{N+1,1} Y_N^{j+1} + B_{N+1,2} Y_{N+1}^{j+1} = B_{N,4} \quad (C.12)$$

where, for $L=1$, $\gamma_1 = -.5$ and $\gamma_2 = 1$, and for $L=2$, $\gamma_1 = .5$ and $\gamma_2 = 2$

$$\begin{aligned} b_{1,N+1,L} &= \gamma_2 \frac{\Delta X^{(N,L)}}{6\Delta T} \\ b_{2,N+1,L} &= \gamma_1 \sigma_M^{(N,L)} \delta_M^{(N,L)} \\ b_{3,N+1,L} &= \gamma_1 \sigma_P^{(N,L)} \delta_P^{(N,L)} \\ b_{4,N+1,L} &= \gamma_1 \alpha_M^{(N,L)} \beta_M^{(N,L)} + \gamma_2 \frac{\Delta X^{(N,L)}}{6\Delta T}, \\ b_{4,N+1,1} &= b_{4,N+1,1} + \frac{G_2^{(N)} \Delta X^{(N)}}{2} \\ b_{1,N+1,4} &= \frac{\Delta X^{(N)}}{3\Delta T} P_{N+1}^j + \frac{\Delta X^{(N)}}{6\Delta T} P_N^j - .5 \Xi_P^{(N)} \Delta X^{(N)} \\ b_{2,N+1,4} &= \frac{\Delta X^{(N)}}{6\Delta T} U_N^j \\ U_{N+1}^j &= 0 \quad \text{for } j=0,1,2,\dots \end{aligned} \quad (C.13)$$

for $i=1$, equation (C.9) is written in this form

$$B_{1,2} Y_1^{j+1} + B_{1,3} Y_2^{j+1} = B_{1,4} \quad (C.14)$$

where, for $L=2$, $\gamma_1 = -.5$ and $\gamma_2 = 2$, and for $L=3$, $\gamma_1 = .5$ and $\gamma_2 = 1$

$$\begin{aligned}
b_{1,1,L} &= r_2 \frac{\Delta X^{(1)}}{6\Delta T} \\
b_{2,1,L} &= r_1 \sigma_M^{(1)} s_M^{(1)} \\
b_{3,1,L} &= r_1 \sigma_P^{(1)} s_P^{(1)} \\
b_{4,1,L} &= r_1 \bar{\alpha}_M^{(1)} \beta_M^{(1)} + r_2 \frac{\Delta X^{(1)}}{6\Delta T} \\
b_{4,1,1} &= b_{4,1,L} + \frac{G_2^{(1)} \Delta X^{(1)}}{2} \\
b_{1,1,4} &= \frac{\Delta X^{(1)}}{3\Delta T} P_1^j + \frac{\Delta X^{(1)}}{6\Delta T} P_2^j - .5 \bar{\alpha}_P^{(1)} \Delta X^{(1)} \\
b_{2,1,4} &= \frac{\Delta X^{(1)}}{3\Delta T} U_1^j + \frac{\Delta X^{(1)}}{6\Delta T} U_2^j
\end{aligned} \tag{C.15}$$

Notice that, the value of P_1^{j+1} (head-end boundary condition) can be evaluated using equation (C.14). In order to keep the equation in its present form, the coefficients $b_{1,1,1}$ and $b_{1,1,4}$ need to be replaced with the following values

$$b_{1,1,1} = H \text{ and } b_{1,1,4} = H P_1^{j+1} \tag{C.16}$$

where, H is any value larger than the rest of the coefficients in the same equation. In such a way, the final solution of equation (C.14) will provide the same answer given for P_1^{j+1} in the right hand side of the equation. In the course of the present analysis, H is set to be equal to 10^5 .

Finally, the finite element formulation has a local truncation error order higher than the two finite difference schemes, developed earlier in section (C.1), which is of $O(\max(\Delta X_i^2) + \Delta T)$.

APPENDIX D

SOLUTION PROCEDURES FOR A STAR AND SERIES FLUID NETWORKS

D.1 Star Fluid Network

In the case of a star configuration as shown in figure (D.1), the terminal (source or destination) pressures (P_i for $i=1, n$) control the direction and amount of the mass flow rate (\dot{m}_i) through each of the restrictions (A_i). The relation between \dot{m}_i , P_i , A_i and P_m , using the average density equation [30], is

$$\dot{m}_i = 0.6 A_i \left[\frac{|P_i^2 - P_m^2|}{R_g \theta} \frac{|P_i - P_m|}{P_i - P_m} \right] \quad (D.1)$$

At steady state condition, we know that the total mass flow rate, \dot{m} , which enters the network through the n restrictions must be equal to zero.

$$\begin{aligned} \dot{m} &= \sum_{i=1}^n \dot{m}_i = \sum_{i=1}^n \dot{m}_i \\ &= \sum_{i=1}^n 0.6 A_i \left[\frac{|P_i^2 - P_m^2|}{R_g \theta} \frac{|P_i - P_m|}{P_i - P_m} \right] \end{aligned} \quad (D.2)$$

In order to find the direction and amount of any of \dot{m}_i , one may need to evaluate P_m from equation (D.2). To do this, first rearrange equation (D.2) and write it as

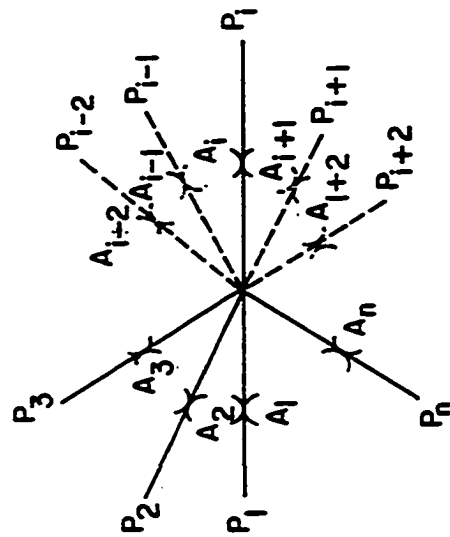


Figure D.1: Star Fluid Network

$$F(P_m) = \frac{\sqrt{R_g \theta}}{0.6} \sum_{i=1}^n 0.6 A_i \sqrt{|P_i^2 - P_m^2|} \frac{|P_i - P_m|}{P_i - P_m} \quad (D.3)$$

and try to find the value of P_m that makes $F(P_m) = 0$ by using the Newton-Raphson method. For that, we need to differentiate equation (D.3), to obtain:

$$F'(P_m) = \frac{dF(P_m)}{dP_m} = - \sum_{i=1}^n \frac{A_i P_m}{\sqrt{|P_i^2 - P_m^2|}} \quad (D.4)$$

The Newton-Raphson method proceeds from the initial guess $P_{m,0}$ as follows:

1. Approximate the function by a straight line tangent to the curve of $F(P_m)$ at $P_{m,k}$ (k^{th} iteration step) by
 - a) calculating $F(P_m)$ to find a point in the curve.
 - b) calculate $F'(P_m)$ to find the slope of the tangent line.
2. Then get a new estimate of P_m , where $F(P_m) = 0$ by finding where the tangent line will intersect with $F(P_m) = 0$ by using:

$$P_{m,k+1} = P_{m,k} - \frac{F(P_m)}{F'(P_m)} \quad (D.5)$$

3. Evaluate the bounded error (E) as

$$E = \frac{P_{m,k+1} - P_{m,k}}{P_{m,k}} \quad (D.6)$$

If E is less than a desired value, then terminate the calculation procedure, otherwise repeat steps 1, 2 and 3.

In order to reduce the number of iteration steps, the initial value of the unknown pressure (P_m) should be chosen as close as to the actual value. It can be shown, using equation (D.2), that the best initial value for P_m is related to the terminal pressures (P_i) and restrictions (A_i) as:

$$P_{m,0} = \left[\frac{\sum_{i=1}^n A_i^2 P_i^2}{\sum_{i=1}^n A_i^2} \right]^{1/2} \quad (D.7)$$

D.2 Series Fluid Network

Assume, there are n restrictions in series (as shown in figure (D.2)) the mass flow rate enters or leaves any of these restrictions is

$$\dot{m}_i = 0.6 A_i \sqrt{\frac{|P_i^2 - P_{i+1}^2|}{R_g \theta}} \frac{|P_i - P_{i+1}|}{P_i - P_{i+1}} \quad \text{for } i=1, \dots, n \quad (D.3)$$

For all the n restrictions we obtain the system of equations:

$$P_i^2 - P_{i+1}^2 = \frac{\dot{m}_i^2 R_g \theta}{0.36 A_i^2} \quad (D.8)$$

adding together the right and left-hand sides respectively of all equations (D.7), we get



Figure D.2: Series Fluid Network

$$P_1^2 - P_{n+1}^2 = \frac{\sum_{i=1}^n \frac{m_i^2 R_{g\theta}}{0.36}}{\sum_{i=1}^n \frac{1}{A_i^2}} \quad (\text{D.9})$$

Introducing the equivalent area A_{EQV} , characterized as follows:

$$P_1^2 - P_{n+1}^2 = \frac{\sum_{i=1}^n \frac{m_i^2 R_{g\theta}}{0.36}}{A_{\text{EQV}}^2} \quad (\text{D.10})$$

from equations (D.9) and (D.10) we get

$$A_{\text{EQV}} = \frac{1}{\sqrt{\sum_{i=1}^n \frac{1}{A_i^2}}} \quad (\text{D.11})$$

APPENDIX E

26C MATHEMATICAL MODELS FORMULATIONS

E.1 Relay Valve Complete Model

E.1.1 Pressure Equations

$$\frac{dp_{1,1}}{dt} = \frac{1.}{v_{1,1}} \left[R_g \theta \frac{dm_{1,1}}{dt} - p_{1,1} \frac{dv_{1,1}}{dt} \right] \quad (E.1)$$

$$\frac{dp_{1,2}}{dt} = \frac{1.}{v_{1,2}} \left[R_g \theta \frac{dm_{1,2}}{dt} - p_{1,2} \frac{dv_{1,2}}{dt} \right] \quad (E.2)$$

$$\frac{dp_{1,3}}{dt} = \frac{R_g \theta}{v_{a,3}} \left[\frac{dm_{1,3}}{dt} + \frac{dm_{1,4}}{dt} - \frac{dm_{1,2}}{dt} \right] \quad (E.3)$$

E.1.2 Volume Equations

$$\frac{dv_{1,1}}{dt} = A_{1,2} U \quad (E.4a)$$

$$\frac{dv_{1,2}}{dt} = A_{1,2} U \quad (E.4b)$$

where,

$$U = \frac{dx_{1,1}}{dt}$$

E.1.3 Displacement Equation

$$I \frac{dU}{dt} = F_{1,1} - F_{1,2} - S_{1,1} - S_{1,2} - S_{1,3} - K_D X_{1,1} \quad (E.5)$$

where,

$$\begin{aligned} I &= I_{1,1} + I_{1,2} + I_{1,3} & -X_E \leq X_{1,1} \leq -X_I \\ &= I_{1,1} & -X_I < X_{1,1} < 0 \end{aligned}$$

$$\begin{aligned}
 &= I_{1,1} + I_{1,3} & 0 \leq X_{1,1} < X_0 \\
 &= I_{1,1} + I_{1,3} + I_{1,4} & X_0 \leq X_{1,1} \leq X_S
 \end{aligned} \tag{E.6}$$

E.1.4 Force Equations

$$\text{always} \quad F_{1,1} = P_{1,1} A_{1,2} \tag{E.7a}$$

$$\text{"} \quad F_{1,2} = P_{1,2} A_{1,2} \tag{E.7b}$$

$$\begin{aligned}
 S_{1,2} &= L_{1,2} + K_{1,2} X_{1,1} & 0 \leq X_{1,1} \leq X_S \\
 &= L_{1,2} + K_{1,2} (X_{1,1} + X_I) & -X_E \leq X_{1,1} \leq X_I \\
 &= 0 & \text{Elsewhere}
 \end{aligned} \tag{E.7c}$$

Supply open

$$S_{1,3} = L_{1,3} + K_{1,3} (X_{1,1} - X_0) \quad X_0 \leq X_{1,1} \leq X_S \tag{E.7d}$$

Exhaust open

$$S_{1,1} = L_{1,1} - K_{1,1} (X_I + X_{1,1}) \quad -X_E \leq X_{1,1} \leq -X_I \tag{E.7e}$$

E.1.5 Flow Rate Equations

$$\dot{m}_{1,1} = .6 A_{1,1} P_{1,1} \sqrt{\frac{|B^2 - 1|}{R_g \theta}} \frac{|B-1|}{B-1} \tag{E.8}$$

where, $B = \frac{P_{eq}}{P_{1,1}}$,

$$\dot{m}_{1,2} = .6 A_{1,3} P_{1,2} \sqrt{\frac{|B^2 - 1|}{R_g \theta}} \frac{|B-1|}{B-1} \tag{E.9}$$

where, $B = \frac{P_{1,3}}{P_{1,2}}$,

$$\dot{m}_{1,3} = .6 A_{1,6} P_{1,3} \sqrt{\frac{|B^2 - 1|}{R_g \theta}} \frac{|B-1|}{B-1} \tag{E.10}$$

where, $B = \frac{P_{mr}}{P_{1,3}}$,

$$X_0 \leq X_{1,1} \leq X_S$$

$$\dot{m}_{1,3} = .6 A P_{1,3} \sqrt{\frac{|B^2 - 1|}{R_g \theta}} \frac{|B-1|}{B-1} \quad (E.11)$$

where, $B = \frac{P_{atm}}{P_{1,3}}$

and $A = \frac{A_{1,4} A_{1,5}}{\sqrt{A_{1,4}^2 + A_{1,5}^2}}$

$$\dot{m}_{1,4} = .6 A_{3,3} P_{1,3} \sqrt{\frac{|B^2 - 1|}{R_g \theta}} \frac{|B-1|}{B-1} \quad (E.12)$$

where, $B = \frac{P_{bp}}{P_{1,3}}$

E.1.6 Variable Restriction Equations

$$A_{1,6} = \pi D_{1,6} (X_{1,1} - X_0) \quad X_0 \leq X_{1,1} \leq X_S \quad (E.13)$$

$$A_{1,4} = -\pi D_{1,4} (X_I + X_{1,1}) \quad -X_E \leq X_{1,1} \leq -X_I \quad (E.14)$$

E.2 Brake Pipe Cut-off Valve Model

E.2.1 Displacement Equation

$$X_{3,1} = A_{3,2} \frac{P_{3,1} - P_B}{K_{3,1}} \quad (E.15)$$

where $P_B = \frac{P_A A_{3,1} + L_{3,1}}{A_{3,1}}$ and $P_{3,1} > P_B$

E.2.2 Variable Restriction Equation

$$A_{3,3} = \pi D_{3,3} X_{3,1} \quad (E.16)$$

E.3 Regulating Valve and Equalizing Reservoir Models

E.3.1 Pressure Equation

$$\frac{dp_{eq}}{dt} = \frac{R_g \theta}{V_{eq}} \left[\frac{dm_{2,1}}{dt} - \frac{dm_{1,1}}{dt} \right] \quad (E.17)$$

E.3.2 Flow Rate Equation

$$\dot{m}_{2,1} = .6 A_{REG} P \sqrt{\frac{|B^2 - 1|}{R_g \theta}} \frac{|B-1|}{B-1} \quad (E.18)$$

where, $B = \frac{P}{P_{eq}}$,

$$A_{REG} = \frac{A_{EQVE} A_{2,2}}{\sqrt{A_{EQVE}^2 + A_{2,2}^2}} \quad (E.19a)$$

and $P = P_A$

(b) Brake Release:

$$A_{REG} = \frac{A_{EQVS} A_{2,3}}{\sqrt{A_{EQVS}^2 + A_{2,3}^2}} \quad (E.20b)$$

and $P = P_{mr}$

E.3.3 Displacement Equation

$$x_{2,1} = \frac{P_{eq} A_{2,1}}{K_{2,1}} \quad (E.21)$$

providing that,

$$\left| P_{eq} - P_C \right| \leq \frac{K_{2,1} x_C}{A_{2,1}} \quad (E.22)$$

E.3.4 Variable Restriction Equations

$$A_{2,3} = \frac{A_x A_r}{\sqrt{A_x^2 + A_r^2}} \quad (\text{E.23})$$

where,

$$A_x = \left[A_0 - \pi (r_0 - x_{2,1})^2 \right] \quad (\text{E.24})$$

providing that,

$$\begin{aligned} x_{2,1} &\leq r_0 - r_I = x_C \\ A_r &= \pi D_I x_{2,1} \end{aligned} \quad (\text{E.25})$$

E.4 Simplified Model of the 26C Relay and Brake Pipe Cut-off Valves

The following assumption may be made based on a quasi-static approximation are:

- 1) the inertial force is very small as compared with any of the forces associated with dynamics of the diaphragm movement,

$$I \frac{dU_{1,1}}{dt} \approx 0 \quad (\text{E.26})$$

- 2) the pressure rate of change of $p_{1,1}$ is equivalent to the pressure rate of change of the equalizing reservoir, p_{eq} ,

$$p_{1,1} \approx p_{eq} \quad (\text{E.27})$$

- 3) the pressure rate of change of $p_{1,3}$ is equivalent to the pressure rate of change of $p_{1,2}$,

$$p_{1,3} \approx p_{1,2} \quad (\text{E.28})$$

- 4) the volume $V_{1,3}$ is assumed to be equal to zero,

$$V_{1,3} \approx 0 \quad (\text{E.29})$$

.

Using the first three assumptions, equations (E.5) and (E.7) may be rewritten in the following forms:

$$X_{1,1} = X_S \quad H_0 < D_P \quad (E.29a)$$

$$= B_0(D_P - H_1) + X_0 \quad H_1 \leq D_P \leq H_0 \quad (E.29b)$$

$$= X_0 \quad H_2 \leq D_P \leq H_1 \quad (E.29c)$$

$$= B_1(D_P - H_3) \quad H_3 \leq D_P \leq H_2 \quad (E.29d)$$

$$= 0.0 \quad 0 \leq D_P \leq H_3 \quad (E.29e)$$

$$= B_2 D_P \quad H_4 \leq D_P \leq 0 \quad (E.29f)$$

$$= -X_I \quad H_5 \leq D_P \leq H_4 \quad (E.29g)$$

$$= B_3(D_P - H_5) - X_I \quad H_6 \leq D_P \leq H_5 \quad (E.29h)$$

$$= -X_E \quad H_6 > D_P \quad (E.29i)$$

where,

$$B_0 = \frac{A_{1,1}}{[K_{1,2} + K_{1,3} + K_D]}$$

$$B_1 = \frac{A_{1,2}}{[K_{1,2} + K_D]}$$

$$B_2 = \frac{A_{1,2}}{[K_D]}$$

$$B_3 = \frac{A_{1,2}}{[K_{1,1} + K_{1,2} + K_D]}$$

$$H_0 = \frac{[X_S - X_0]}{B_0} + H_1 \quad (E.30)$$

$$H_1 = \frac{L_{1,3}}{A_{1,2}} + H_2$$

$$H_2 = \frac{X_0}{B_1} + H_3$$

$$H_3 = \frac{L_{1,2}}{A_{1,2}}$$

$$\begin{aligned}
 H_4 &= -\frac{X_I}{B_2} \\
 H_5 &= \frac{L_{1,2} - L_{1,1}}{A_{1,2}} + H_4 \\
 H_6 &= \frac{[X_I - X_E]}{B_3} + H_5
 \end{aligned}$$

and, approximate the pressure difference across the relay valve diaphragm as,

$$Dp = p_{eq}^j - p_{1,3}^j$$

Note that,

$$\begin{aligned}
 p_{1,3}^j &= p_{bp}^j & -X_I \leq X_{1,1} \leq X_0 \\
 p_{1,3} &= \sqrt{\frac{p_{bp}^2 A_{3,3}^2 + p^2 A^2}{A_{3,3}^2 + A^2}} & (E.31)
 \end{aligned}$$

E.4.1 Relay Valve States

During any of the brake pipe modes, the relay valve is operating at one or more states at different times. Any of these states is a function of the difference between the equalizing reservoir pressure and $p_{1,3}$.

- a) Supply State
- b) Intermediate State
- c) Exhaust State.

E.4.1.1 Supply State

$$P = P_{mr} \quad (E.32)$$

and $A = A_{1,6}$

for $p_{eq} - H_0 < p_{1,3} < p_{eq} - H_1$

$$A = \alpha_1 P_{1,3} + \alpha_2 \quad (\text{E.33a})$$

for $P_{1,3} \leq P_{eq} - H_0$

$$A = C_1 = \pi D_{1,6} (X_S - X_0) \quad (\text{E.33b})$$

for $P_{1,3} \geq P_{eq} - H_1$

$$A = 0 \quad (\text{E.33c})$$

where,

$$\alpha_1 = -\pi D_{1,6} B_0, \text{ and} \quad (\text{E.34})$$

$$\alpha_2 = \pi D_{1,6} B_0 \left[P_{eq} - H_1 \right]$$

$$A_{3,3} = \alpha_3 P_{1,3} + \alpha_4 \quad P_{1,3} > P_B \quad (\text{E.35a})$$

$$A_{3,3} = C_2 \quad P_{1,3} > P_B + E_1 \quad (\text{E.35b})$$

$$= \pi D_{3,3} X_B$$

$$A_{3,3} = 0 \quad P_{1,3} \leq P_B \quad (\text{E.35c})$$

$$\alpha_3 = \pi D_{3,3} \frac{A_{3,2}}{K_{3,1}}, \text{ and} \quad (\text{E.36})$$

$$\alpha_4 = -\pi D_{3,3} \frac{A_{3,1}}{K_{3,1}} P_B$$

where,

$$P_B = \frac{P_A A_{3,1} + L_{3,1}}{A_{3,2}} \quad (\text{E.37})$$

Thus,

$$\sum_{n=0}^4 \beta_n \left[P_{1,3} \right]^n = 0 \quad (\text{E.38})$$

$$\text{FUNCTION} = \sum_{n=0}^4 \beta_n \left[P_{1,3} \right]^n \quad (\text{E.39})$$

REGION I

$$P_B \leq P_{1,3} < P_B + E_1 \quad \text{and} \quad P_{1,3} < P_{eq} - H_0$$

Simply in this region

Supply valve fully open and Brake pipe cut-off valve varies

$$A = A_{1,6} = C_1 \quad \text{and} \quad A_{3,3} = \alpha_3 P_{1,3} + \alpha_4$$

Now,

$$\begin{aligned} \beta_0 &= -\alpha_4^2 P_{bp}^2 - C_1^2 P_{mr}^2 \\ \beta_1 &= -2\alpha_3 \alpha_4 P_{bp} \\ \beta_2 &= C_1^2 + \alpha_4^2 - \alpha_3^2 P_{bp}^2 \\ \beta_3 &= 2\alpha_3 \alpha_4 \\ \beta_4 &= \alpha_3^2 \end{aligned} \quad (E.40)$$

REGION II $P_B + E_1 \leq P_{1,3}$ and $P_{1,3} < P_{eq} - H_0$

Simply in this region,

Supply valve fully open

$$A = A_{1,6} = C_1$$

and,

Brake pipe cut-off valve fully open

$$A_{3,3} = C_2$$

Now,

$$\begin{aligned} \beta_0 &= -C_1^2 P_{mr}^2 - C_2^2 P_{bp}^2 \\ \beta_1 &= \beta_3 = \beta_4 = 0 \\ \beta_2 &= C_1^2 + C_2^2 \end{aligned} \quad (E.42)$$

REGION III $P_{1,3} > P_B + E_1$ and $P_{eq} - H_0 \leq P_{1,3} < P_{eq} - H_1$

Simply in this region

Supply valve varies

$$A = A_{1,6} = \alpha_1 P_{1,3} + \alpha_2$$

and,

Brake pipe cut-off valve fully open

$$A_{3,3} = C_2$$

Now,

$$\begin{aligned}
 \beta_0 &= -\alpha_2^2 p_{mr}^2 - C_2^2 p_{bp}^2 \\
 \beta_1 &= -2\alpha_1 \alpha_2 p_{mr}^2 \\
 \beta_2 &= C_2^2 + \alpha_2^2 - \alpha_1^2 p_{mr}^2 \\
 \beta_3 &= 2\alpha_1 \alpha_2 \\
 \beta_4 &= \alpha_1^2
 \end{aligned} \tag{E.43}$$

REGION IV $p_{1,3} > p_B + E_1$ and $p_{1,3} > p_{eq} - H_1$
 FUNCTION=0.

Brake pipe cut-off valve fully open, supply and exhaust
 valves fully closed

REGION V (is not shown in figure (3.12))

$$p_B \leq p_{1,3} < p_B + E_1 \quad \text{and} \quad p_{eq} - H_0 \leq p_{1,3} < p_{eq} - H_1$$

Simply in this region,

Supply valve varies

$$A = A_{1,6} = \alpha_1 p_{1,3} + \alpha_2$$

and,

Brake pipe cut-off valve varies

$$A_{3,3} = \alpha_3 p_{1,3} + \alpha_4$$

Now,

$$\begin{aligned}
 \beta_0 &= -\alpha_2^2 p_{mr}^2 - \alpha_4^2 p_{bp}^2 \\
 \beta_1 &= -2\alpha_1 \alpha_2 p_{mr}^2 - 2\alpha_3 \alpha_4 p_{bp}^2 \\
 \beta_2 &= \alpha_2^2 + \alpha_4^2 - \alpha_1^2 p_{mr}^2 - \alpha_3^2 p_{bp}^2 \\
 \beta_3 &= 2\alpha_1 \alpha_2 + 2\alpha_3 \alpha_4 \\
 \beta_4 &= \alpha_1^2 + \alpha_3^2
 \end{aligned} \tag{E.44}$$

E.4.1.2 Intermediate State

$$A_{1,6} = A_{1,3} = A = 0. \quad (\text{E.45})$$

Hence,

$$P_{1,3} = P_{bp} \quad (\text{E.46})$$

E.4.1.3 Exhaust State

$$A_{EX} = \frac{A_{1,4} A_{1,5}}{\sqrt{A_{1,4}^2 + A_{1,5}^2}} \quad (\text{E.47a})$$

for $P_{eq}^{-H_5} < P_{1,3} < P_{eq}^{-H_6}$

$$A_{1,4} = \alpha_5 P_{1,3} + \alpha_6 \quad (\text{E.46b})$$

for $P_{1,3} > P_{eq}^{-H_6}$

$$A_{1,4} = C_3 = \pi D_{1,4} (X_E - X_I) \quad (\text{E.46c})$$

for $P_{1,3} = P_{eq}^{-H_5}$

$$A_{1,4} = 0 \quad (\text{E.46d})$$

where,

$$\alpha_5 = \pi D_{1,4} B_3 \quad (\text{E.47})$$

and $\alpha_6 = \pi D_{1,4} B_3 [H_5 - P_{eq}]$

$$P_{1,3}^j = \left[\frac{P_{bp}^2 C_2^2 + P_{mr}^2 A_{EX}^2}{C_2^2 + A_{EX}^2} \right]^{j-1} \quad (\text{E.48})$$

for $P_{eq}^j - H_5 < P_{1,3}^j < P_{eq}^j - H_6$

$$A_{1,4}^j = \alpha_5^j P_{1,3}^j + \alpha_6^j \quad (\text{E.49a})$$

for $P_{1,3}^j > P_{eq}^j - H_6$

$$A_{1,4}^j = C_3 = \pi D_{1,4} (X_E - X_I) \quad (\text{E.50b})$$

Notice: To assist the reader understand the 26C Model, see Figures (2.1) through (2.12)

APPENDIX F

FOURTH ORDER RUNGE-KUTTA INTEGRATION METHOD

The general form of a system of n first order ordinary differential equations may be represented as follows:

$$\frac{dP_i}{dt} = f_i(t, P_1, P_2, P_3, \dots, P_i, \dots, P_n) \quad i=1, \dots, n \quad (F.1)$$

The fourth order Runge-Kutta method [23] is one of the most popular technique to integrate a wide range of n first order differential equations. Let h be the integration step size and m be the integration step. Evaluate the following parameters, four for each equation

$$\begin{aligned} K_{i,1} &= h f_i(t_m, P_{i,m}) \\ K_{i,2} &= h f_i\left[t_m + \frac{h}{2}, P_{i,m} + \frac{K_{i,1}}{2}\right] \\ K_{i,3} &= h f_i\left[t_m + \frac{h}{2}, P_{i,m} + \frac{K_{i,2}}{2}\right] \\ K_{i,4} &= h f_i(t_m + h, P_{i,m} + K_{i,3}) \end{aligned} \quad (F.2)$$

Based on these parameters, the next values of P_1, P_2, \dots, P_i can be evaluated as:

$$P_{i,m+1} = P_{i,m} + \frac{K_{i,1} + 2K_{i,2} + 2K_{i,3} + K_{i,4}}{6} \quad (F.3)$$

$$t_{m+1} = t_m \quad (F.4)$$

These procedures are repeated to cover the required period of time that is of interest. In general, as the integration step h is reduced, the accuracy of the computation

tends to increase at the expense of increasing the computation time.

The Runge-Kutta methods for numerical integration are quite popular, and they are self starting (explicit techniques), i.e., only the initial conditions have to be specified.

APPENDIX G

ABD CONTROL VALVE MATHEMATICAL MODEL

G.1 Introduction

This Appendix briefly describes the mathematical model of the ABD control valve as has been developed by Gauthier [7], Wright [8], Bansiter [9] and Kreel [10]. The ABD control valve (Figure G.1) consists of a complicated network of springs, diaphragms, pistons, subvalves and orifices. A complete development of the detailed mathematical model can be found in Refs. [7,8,9,10]. They have developed a mathematical model which describes the two application modes (service & emergency modes) and charge (recharge/release) modes. The valve detects the time rate of changes in the brake pipe pressure, and based on the amount and direction (increase/decrease) of the local brake pipe pressure, the subvalves open and close some of the control valve orifices. These openings may connect any of the air brake system components (brake pipe, auxiliary reservoir, emergency reservoir and brake cylinder) together (Figure (G.2)). Section G.2 describes the equations of mass flow and pressure for each of the air brake system components and ABD volumes associated with the ABD control valves operation. Then, section G.3 describes how to evaluate the openings of all the subvalves and restrictions used to calculate the

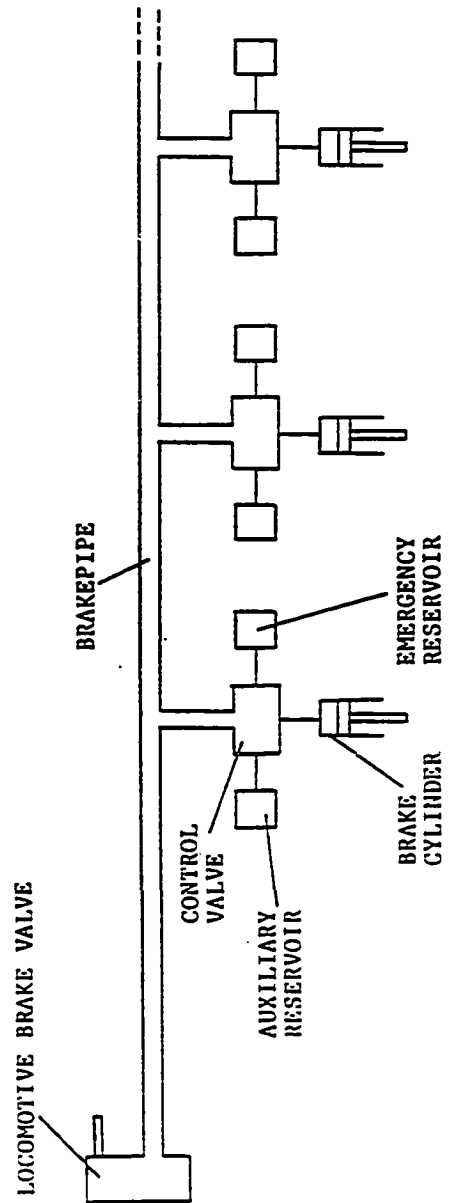


Figure G.2: Schematic of the Air Brake System

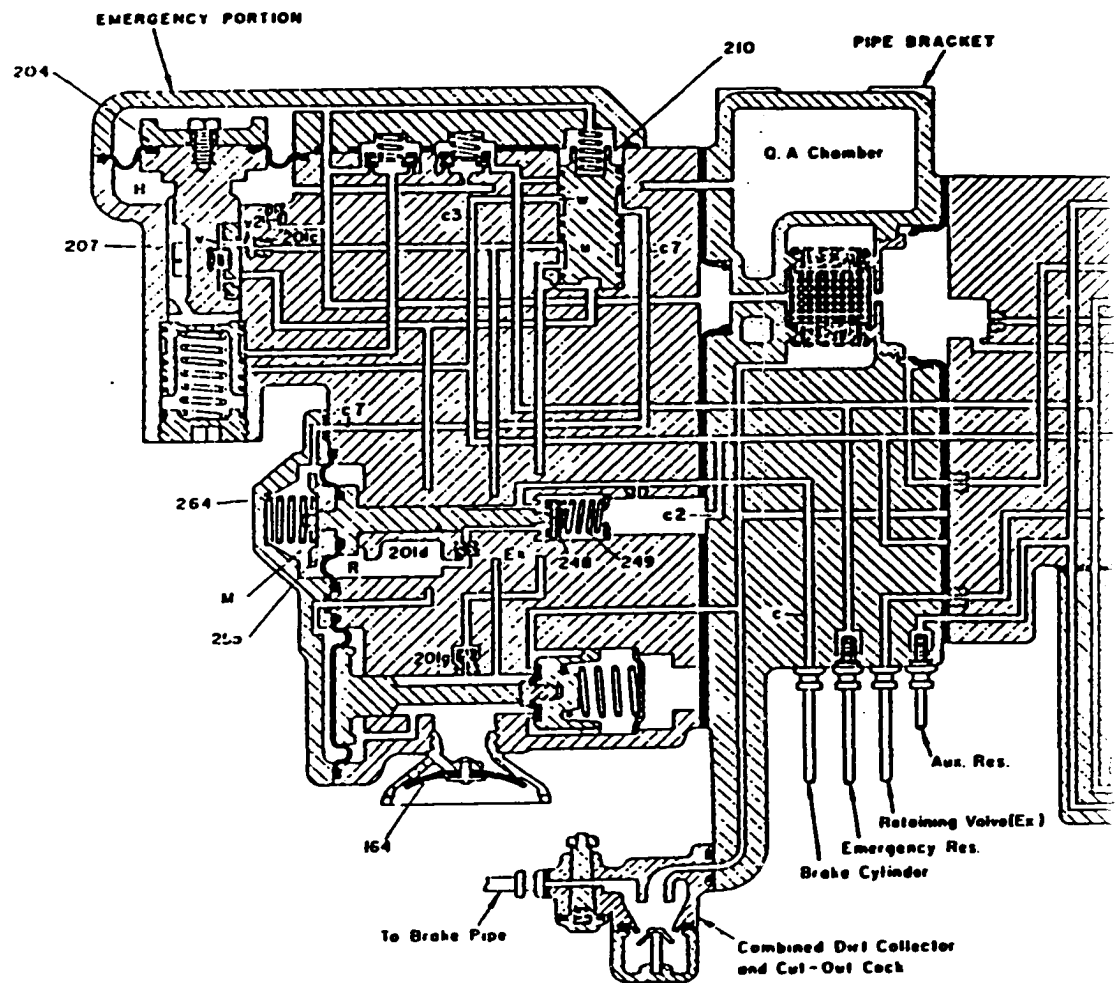


Figure G.1: ABD Control Valve Schematic

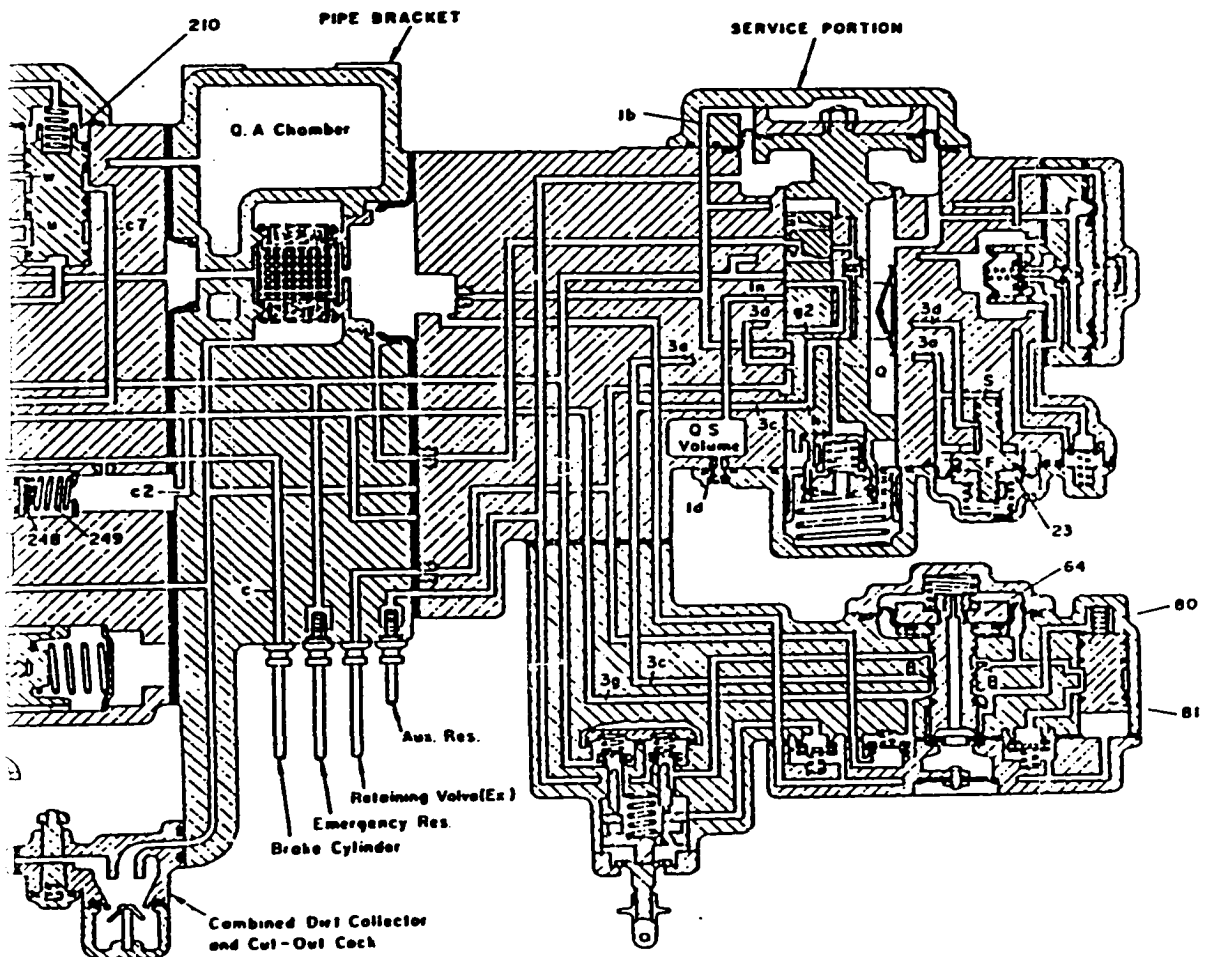


Fig. G.1: ABD Control Valve Schematic Diagram

mass flow rate in section G.2, for service, emergency and recharge modes respectively.

G.2 ABD Flow and Pressure Equations

Four of the air brake system components may be connected to each other through the ABD control valve during any of its three modes. Those four components are:

1. local brake pipe,
2. local auxiliary reservoir,
3. local emergency reservoir, and
4. local brake cylinder.

There are also two ABD valve internal volumes which may be connected to the local brake pipe through some of the ABD control valve subvalves/restrictions:

5. quick action volume, and
6. quick service volume.

Figure G.3 shows the complete network of restrictions, which may connect any of the above six elements together. In order to have a complete mathematical model for the air brake system, we need to evaluate the flow from or to any of the above components or volumes and the local pressure at each of these. The following subsections describe the equations developed by Refs. [7,8,9,10] to evaluate these variables. The general flow equation used hereafter is described in Appendix J.

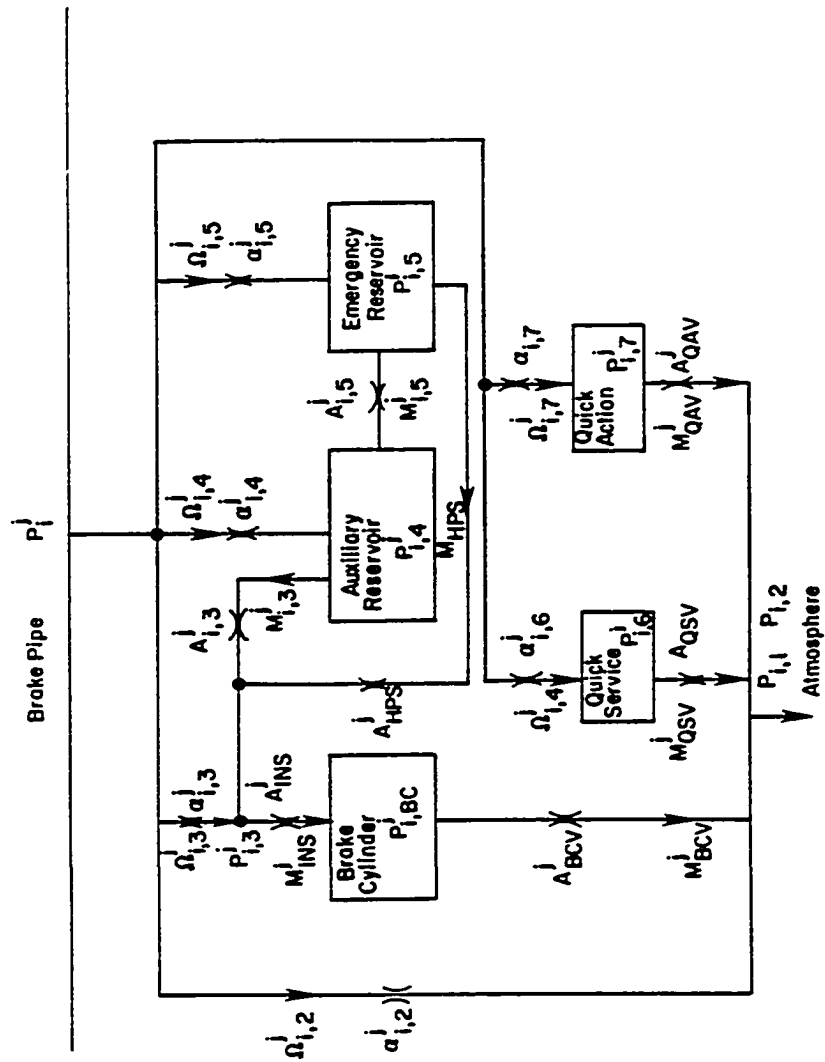


Figure G.3: Fluid Network of the ABD Control Valve

G.2.1 Brake Pipe Flow Equation

In order to complete the mathematical model of the brake pipe developed earlier in Chapter 2, one needs to evaluate the mass flow rate from the brake pipe through the ABD/ABDW control valve to one of the air brake system componets or the atmosphere as follows:

1. Through the vent valve restriction, $a_{i,2}^j$ [see vent valve located in the emergency portion of the control valve, ABD/ABDW, shown in figures (G.1) and (G.3)] to the atmosphere, the mass flow rate $\dot{m}_{i,2}^j$ may be evaluated as follow:

ONLY EMERGENCY

$$\dot{m}_{i,2}^j = 0.6 \frac{a_{i,2}^j}{A_i} P_i^j \sqrt{\frac{|R_{i,2}^2 - 1|}{\Gamma} \frac{|1 - R_{i,2}|}{1 - R_{i,2}}} \quad (G.1)$$

Evaluation of $a_{i,2}^j$ is described in Section G.3.2.

2. The mass flow rate $\dot{m}_{i,3}^j$ leaves/enters the brake pipe through $a_{i,3}^j$ may be calculated as follows:

CHARGE & SERVICE

$$\dot{m}_{i,3}^j = 0.6 \frac{a_{i,3}^j}{A_i} P_i^j \sqrt{\frac{|R_{i,3}^2 - 1|}{\Gamma} \frac{|1 - R_{i,3}|}{1 - R_{i,3}}} \quad (G.2)$$

$a_{i,3}^j$ may take on different forms during the charge and service modes. These forms may be explained as follows:

CHARGE MODE

In the case of release after emergency, the brake pipe is connected to the inshot valve through the emergency accelerated release check valve $a_{i,3}^j$,

shown in figures (G.1) and (G.3). The calculation of this restriction is described in Section G.3.

SERVICE MODE

During this mode, the brake pipe is connected to the inshot valve through the quick service choke (fixed restriction), located on the service portion of the control valve, $a_{i,3}^j$.

3. Through a restriction, $a_{i,4}^j$, connecting the brake pipe with the auxiliary reservoir, the flow rate $q_{i,4}^j$ may be calculated as follows:

CHARGE & SERVICE

$$q_{i,4}^j = 0.6 \frac{a_{i,4}^j}{A_i} p_i^j \sqrt{\frac{|R_{i,4}^2 - 1|}{r} \frac{|1 - R_{i,4}|}{1 - R_{i,4}}} \quad (G.3)$$

$a_{i,4}^j$ is a function of the service slide movement as described later in sections G.3.1 and G.3.3.

4. During the release mode, the emergence reservoir is connected to the brake pipe, Through choke lb [$a_{i,5}^j$ shown in figures (G.1) and (G.3)], provided that the accelerated service release valve [located in the service portion of the control valve shown in figure (G.1)] is in operation. This connection is interrupted by the back flow check valve 29 [located between choke lb and the accelerated service release valve shown in figure (G.1)] only if the emergency reservoir pressure is greater than the brake pipe pressure.

The flow rate $q_{i,5}^j$ through $a_{i,5}^j$ may be calculated as follows:

ONLY CHARGE

$$\dot{Q}_{i,5}^j = 0.6 \frac{a_{i,5}^j}{A_i} P_i^j \sqrt{\frac{|R_{i,5}^2 - 1|}{\Gamma} \frac{|1 - R_{i,5}|}{1 - R_{i,5}}} \quad (G.4)$$

5. Through the quick service choke [located on the service slide valve passage g_2 shown in figure (G.1)], $a_{i,6}^j$ (fixed restriction), connecting the brake pipe with the quick service, the flow rate $\dot{Q}_{i,6}^j$ may be calculated as follows:

ONLY SERVICE

$$\dot{Q}_{i,6}^j = 0.6 \frac{a_{i,6}^j}{A_i} P_i^j \sqrt{\frac{|R_{i,6}^2 - 1|}{\Gamma} \frac{|1 - R_{i,6}|}{1 - R_{i,6}}} \quad (G.5)$$

6. Through choke 201b located in the emergency portion of the control valve shown in figure (G.1), $a_{i,7}$, connecting the brake pipe all the time with the quick action chamber, the flow rate $\dot{Q}_{i,7}^j$ may be calculated as follows:

ALL THREE MODES

$$\dot{Q}_{i,7}^j = 0.6 \frac{a_{i,7}}{A_i} P_i^j \sqrt{\frac{|R_{i,7}^2 - 1|}{\Gamma} \frac{|1 - R_{i,7}|}{1 - R_{i,7}}} \quad (G.6)$$

$$\text{where, } R_{i,k} = \frac{P_{i,k}^j}{P_i^j} \quad (G.7)$$

$P_{i,2}^j$ & $P_{i,1}^j$ is the normalized atmospheric pressure,
 $P_{i,3}^j$ is the normalized intermediate pressure located at the center of the star connecting the brake pipe with brake cylinder and auxiliary reservoir during the service application mode as shown in figure (G.4)

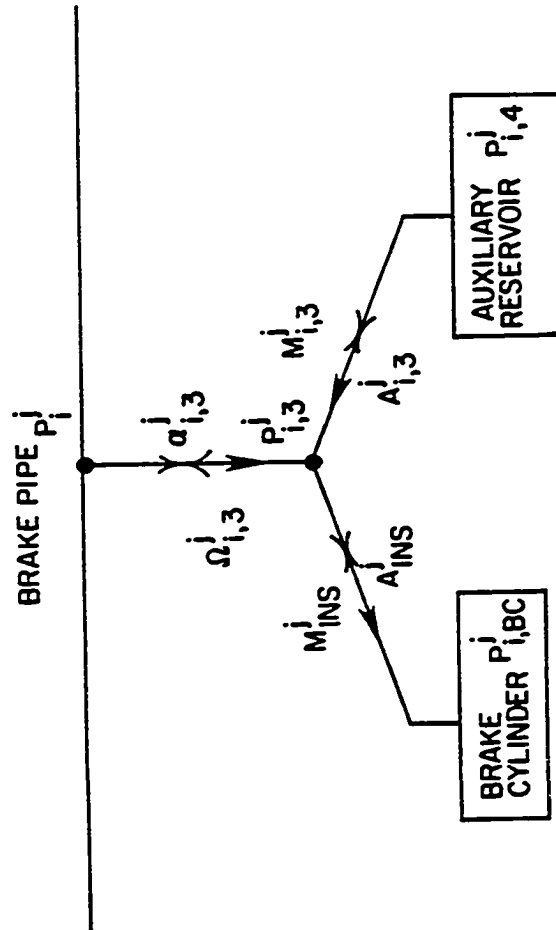


Figure G.4: Service Mode Fluid Star Network

(Appendix D describes a method to solve for the flow rate direction and amount through a star network),

$P_{i,4}^j$ = the i^{th} auxiliary reservoir normalized pressure,

$P_{i,5}^j$ = the i^{th} emergency reservoir normalized pressure,

$P_{i,6}^j$ = the i^{th} quick service volume normalized pressure,

and $P_{i,7}^j$ = the i^{th} quick action chamber normalized pressure.

G.2.2 Auxiliary Reservoir Flow and Pressure Equations

The net mass flow rate M_{AUX}^j leaves or enters the auxiliary reservoir may be evaluated as follows:

$$M_{\text{AUX}}^j = \dot{m}_{i,4}^j - M_{i,3}^j - M_{i,5}^j \quad (\text{G.8})$$

where,

$$M_{i,3}^j = 0.6 \frac{A_{i,3}^j}{A_i} P_{i,4}^j \sqrt{\frac{|R_{i,3}^2 - 1|}{\Gamma} \frac{|1 - R_{i,3}|}{1 - R_{i,3}}} \quad (\text{G.9})$$

$$M_{i,5}^j = 0.6 \frac{A_{i,5}^j}{A_i} P_{i,4}^j \sqrt{\frac{|R_{i,5}^2 - 1|}{\Gamma} \frac{|1 - R_{i,5}|}{1 - R_{i,5}}} \quad (\text{G.10})$$

$$R_{i,3} = \frac{P_{i,3}^j}{P_{i,4}^j}, \quad \text{and} \quad R_{i,5} = \frac{P_{i,5}^j}{P_{i,4}^j}$$

$A_{i,3}^j$ is a variable restriction (see Section G.3.1), which is a function of the service graduating valve movement, connecting the auxiliary reservoir with the inshot valve through passage 3c during all three modes, and $A_{i,5}^j$ is a fixed restriction, located in the service slide valve

passage o, connecting the auxiliary reservoir with emergency reservoir during the charge (release/recharge) mode.

The time rate of change auxiliary reservoir pressure is a function of the mass flow rate M_{AUX}^j and V_{AUX} as:

$$\frac{dP_{i,4}}{dT} = \frac{M_{AUX}^j}{V_{AUX}} \quad (G.11)$$

G.2.3 Emergency Reservoir Flow and Pressure Equations

The net mass flow rate M_{EMR}^j leaves or enters the emergency reservoir may be evaluated as follows:

$$M_{EMR}^j = \alpha_{i,5}^j + M_{i,5}^j + (\alpha_{i,3}^j + M_{i,3}^j - M_{INS}^j) \quad (G.12)$$

where, M_{INS}^j is the mass flow rate enters/leaves the inshot valve, located on the emergency portion of the control valve, described latter in Section (G.2.6).

The time rate of change emergency reservoir pressure is a function of the mass flow rate M_{EMR}^j and V_{EMR} as:

$$\frac{dP_{i,5}^j}{dT} = \frac{M_{EMR}^j}{V_{EMR}} \quad (G.13)$$

G.2.4 Quick Service Flow and Pressure Equations

The net mass flow rate M_{QS}^j leaves or enters the quick service volume may be evaluated as follows:

$$M_{QS}^j = \alpha_{i,6}^j - M_{QSV}^j \quad (G.14)$$

where,

$$M_{QSV}^j = 0.6 \frac{A_{QSV}}{A_i} P_{i,6}^j \sqrt{\frac{|R_{QSV}^2 - 1|}{\Gamma} \frac{|1 - R_{QSV}|}{1 - R_{QSV}}} \quad (G.15)$$

$$R_{QSV} = \frac{P_{i,2}^j}{P_{i,6}^j}$$

A_{QSV} is a fixed restriction ld as shown in figure (G.1).

The time rate of change quick service volume pressure is a function of the mass flow rate M_{QS}^j and V_{QS} as:

$$\frac{dP_{i,6}}{dT} = \frac{M_{QS}}{V_{QS}} \quad (G.16)$$

G.2.5 Quick Action Chamber Flow and pressure equations

The net mass flow rate M_{QA}^j leaves or enters the quick action chamber may be evaluated as follows:

$$M_{QA}^j = \dot{m}_{i,7}^j - M_{QAV}^j \quad (G.17)$$

where,

$$M_{QAV}^j = 0.6 \frac{A_{QAV}}{A_i} P_{i,7}^j \sqrt{\frac{|R_{QAV}^2 - 1|}{\Gamma} \frac{|1 - R_{QAV}|}{1 - R_{QAV}}} \quad (G.18)$$

$$R_{QAV} = \frac{P_{i,2}^j}{P_{i,7}^j}$$

M_{QAV}^j is the air leaves the quick action chamber through A_{QAV}^j (see section G.3.1 for the calculation of A_{QAV}^j).

The time rate of change quick action volume pressure is a function of the mass flow rate M_{QA}^j and V_{QA} as:

$$\frac{dP_{i,7}}{dT} = \frac{M_{QA}^j}{V_{QA}} \quad (G.19)$$

G.2.6 Brake Cylinder Flow and Pressure Equations

The net mass flow rate M_{BC}^j leaves or enters the brake cylinder volume may be evaluated as follows:

$$M_{BC}^j = M_{INS}^j - M_{BCV}^j \quad (G.20)$$

where,

$$M_{INS}^j = 0.6 \frac{A_{INS}^j}{A_i} P_{i,3}^j \sqrt{\frac{|R_{INS}^2 - 1|}{\Gamma} \frac{|1 - R_{INS}|}{1 - R_{INS}}} \quad (G.21)$$

$$M_{BCV}^j = 0.6 \frac{A_{BCV}^j}{A_i} P_{i,BC}^j \sqrt{\frac{|R_{BCV}^2 - 1|}{\Gamma} \frac{|1 - R_{BCV}|}{1 - R_{BCV}}} \quad (G.22)$$

$$R_{BCV} = \frac{P_{i,2}^j}{P_{i,BC}^j}, \quad P_{INS}^j = P_{i,3}^j, \text{ and } R_{INS} = \frac{P_{BC}^j}{P_{INS}^j} = \frac{P_{BC}^j}{P_{i,3}^j}$$

During the charge mode, the brake cylinder is vented through the release valve 60, shown in figure (G.1), the flow restriction area is referred to in equation (G.22) as A_{BCV}^j . The value of A_{INS}^j (inshot valve equivalent restriction) consisted of a fixed restriction A_{INS1} and a variable opening restriction A_{INS2}^j ,

$$A_{INS}^j = A_{INS1} + A_{INS2}^j \quad (G.23)$$

But A_{INS2}^j is a function of the brake cylinder pressure and

the ABD valve operating mode as follows,

$$A_{INS2}^j = \pi D_{INS2} X_{INS} \quad (G.24)$$

Figure (G.5) shows the free body diagram for the piston of the inshot valve. Neglecting the inertial term and solve for x_{INS}^j as long as $0 \leq x_{INS}^j \leq x_{INM}$,

$$x_{INS}^j = \frac{F_{OUT} - F_{BC} + L_{INS}}{K_{INS}} \quad (G.25)$$

where,

$$F_{BC} = P_A P_{i,BC}^j A_{INSP},$$

SERVICE MODE

$$F_{OUT} = P_A P_{i,BC}^j A_{INSP},$$

and, A_{INSP} is the inshot piston cross-sectional area (see figure (G.5), thus,

$$x_{INS} = x_{INM}$$

and, $A_{INS2} = \pi D_{INS2} x_{INM}$ (this based on the inshot dimension)

EMERGENCY MODE

$$F_{OUT} = P_A A_{INSP}$$

and, x_{INS} is calculated using equation (G.25).

The time rate of change brake cylinder volume pressure is a function of the mass flow rate M_{BC}^j and v_{BC}^j and may be written in this general form:

$$\frac{dP_{i,BC}^j}{dT_j} = \frac{M_{BC}^j}{v_{BC}^j} \quad (G.25)$$

where $v_{i,BC}$ is a function of the pressure and the geometry of the brake cylinder as follows (see figure (G.6)):

$$\begin{aligned} v_{BC}^j &= v_{BCI} & P_{i,BC}^j &\leq P_{BCI}, \\ v_{BC}^j &= v_{BCF} & P_{BCF} &\leq P_{i,BC}^j, \end{aligned} \quad (G.26)$$

and for $P_{BCI} \leq P_{i,BC}^j \leq P_{BCF}$

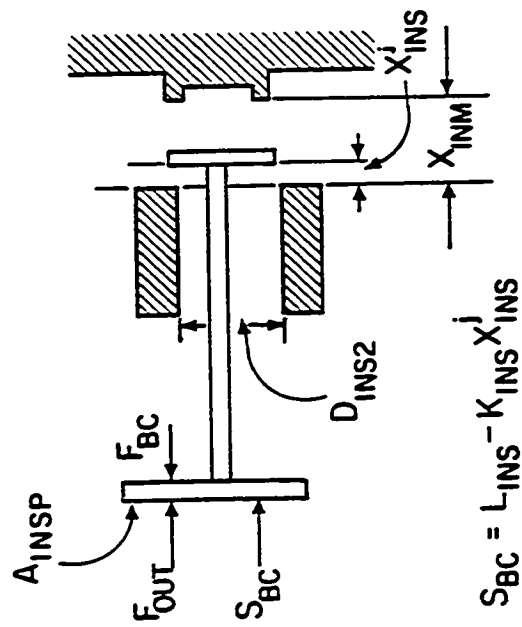


Figure G.6: Inshot Valve Free Body Diagram

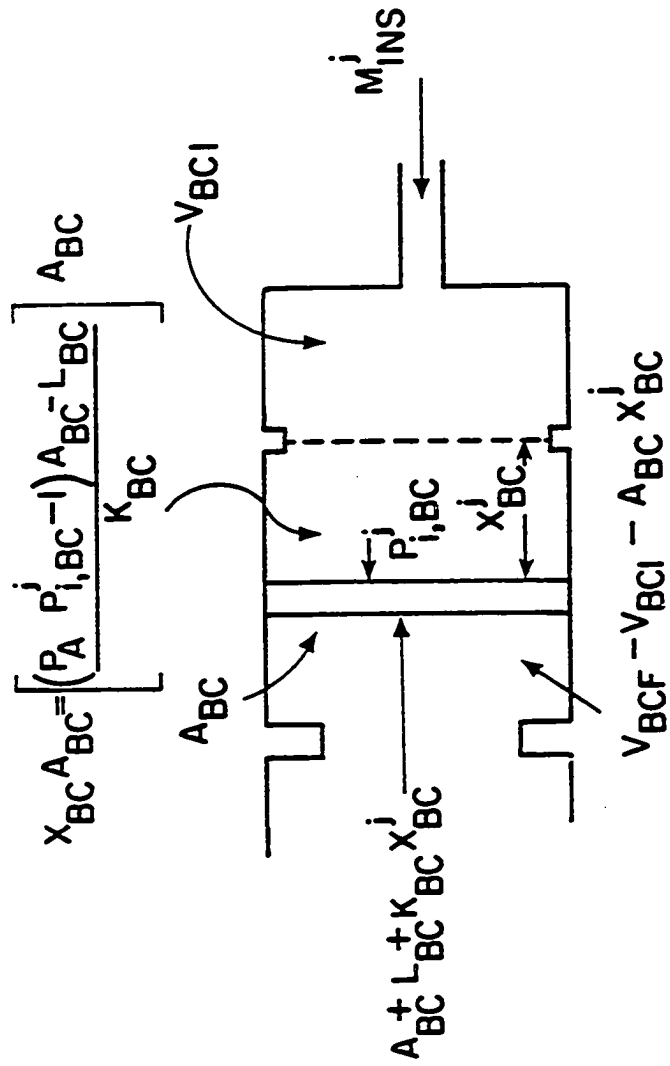


Figure Q.6: Brake Cylinder Free Body Diagram

$$v_{BC}^j = v_{BCI} + \left[\frac{p_A (2P_{i,BC}^j - 1.) A_{BC} - L_{BC}}{K_{BC}} \right] A_{BC}$$

$$\text{where, } P_{BCI} = \frac{L_{BC}}{A_{BC} p_A} + 1. , P_{BCF} = P_{BCI} + \frac{K_{BC} x_{MAX}}{A_{BC} p_A} ,$$

$$v_{BCF} = v_{BCI} + A_{BC} x_{MAX} ,$$

K_{BC} and L_{BC} are the brake cylinder spring constant and preload respectively, A_{BC} is the cross-sectional area, x_{MAX} is the maximum brake cylinder piston displacement, and p_A is the atmospheric pressure.

G.3 ABD Operation Modes

The openings of the flow areas, listed through out equations (G.1) - (G.24) are controled by the different operation modes of the ABD. Refs. [7-10] basically used the pressure difference, v , between the local brake pipe and auxiliary reservoir, in the case of sevice and charge modes, and used the pressure difference, Δ , between the local brake pipe pressure and quick action chamber, in the case of emergency, where,

$$v = p_A [P_{i,4}^j - P_i^j]$$

and,

(G.27)

$$\Delta = p_A [P_{i,7}^j - P_i^j]$$

to distinguish between the different modes for each of the ABD control valve. These modes are:

1. service mode
2. emergency mode, and
3. recharge/release mode.

In order to help the reader understand the different stages during the service and charge modes, figure (G.7) shows the input output diagram for the ABD functions operated during those modes.

G.3.1 Service Mode

- a) First stage: $0 \leq v \leq H_3$

The auxiliary reservoir is connected both to the brake pipe and the emergency reservoir, through $a_{i,4}^j$ and $A_{i,5}^j$ respectively, where,

$$a_{i,4}^j = A_{AUX} \text{ (fixed restriction)}$$

and $A_{i,5}^j = A_{EMR} \text{ (fixed restriction)}$

- b) Second stage: $v > H_3$

The auxiliary reservoir is disconnected from both the brake pipe and the emergency reservoir, so

$$a_{i,4}^j = A_{i,5}^j = 0 \quad (G.28)$$

- c) Third stage: $H_3 \leq v \leq H_4$

The brake pipe is connected to the quick service volume through $a_{i,6}^j$.

- d) Fourth stage: $v > H_4 \text{ \& } p_{i,3}^j \leq H_6$

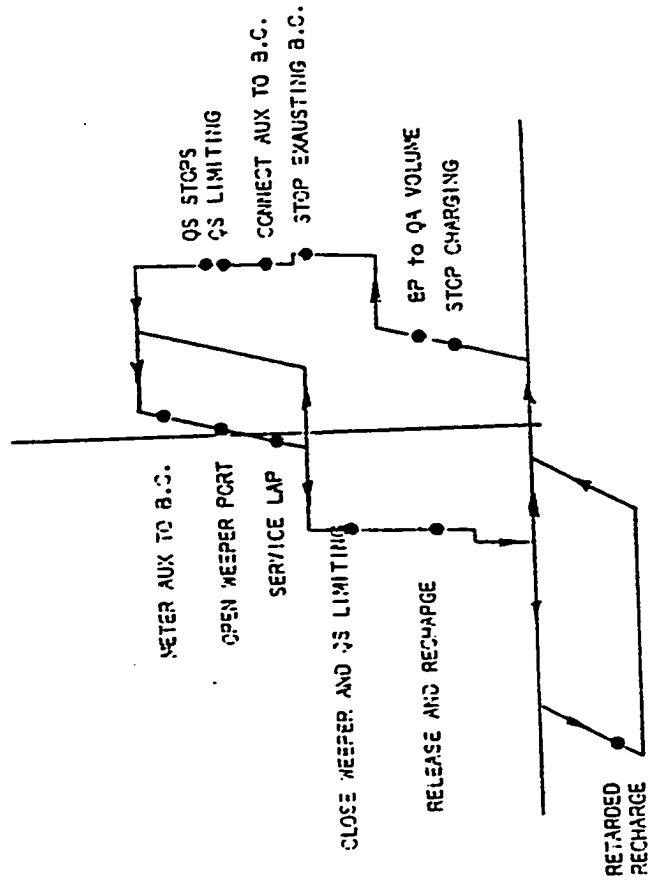


Figure G.7: ABD Input Output Diagram

The auxiliary reservoir, brake pipe and brake cylinder form a star network, shown in figure (G.4) with $A_{HPS}^j = 0$. Instead of solving a complex set of equations, one may be able to simplify the star network to the network shown in figure (G.8). This simplification is based on the assumption that the inshot flow area, A_{INS} , is very large as compared to the other two restrictions, $a_{i,3}^j$ and $A_{i,3}^j$, existing within the same network. Thus the pressure $P_{i,3}^j$ may be approximated by the brake cylinder pressure $P_{i,BC}^j$. In this case, the mass flow rate entering the brake cylinder may be calculated using the following form,

$$M_{INS}^j = a_{i,3}^j + M_{i,3}^j \quad \text{and} \quad P_{i,3}^j = P_{BC}^j \quad (G.29)$$

Also during this stage, the brake pipe is disconnected from the quick service volume,

$$a_{i,6}^j = 0 \quad (G.30)$$

However, $A_{i,3}^j$ is a function of brake pipe and auxiliary reservoir pressures as well as the dimension of the ABD control valve as:

$$\begin{aligned} A_{i,3}^j &= 0 & v &\leq H_4 \\ A_{i,3}^j &= C_0 \left[v - H_0 \right] & 0 &\leq A_{i,3}^j \leq C_1 \\ A_{i,3}^j &= C_1 & C_1 &\leq A_{i,3}^j \end{aligned} \quad (G.31)$$

e) Fifth stage: $v > H_4$ & $P_{i,3}^j > H_7$

This is the final stage during the service mode. The brake pipe is disconnected from the network shown in figure (G.8), so,

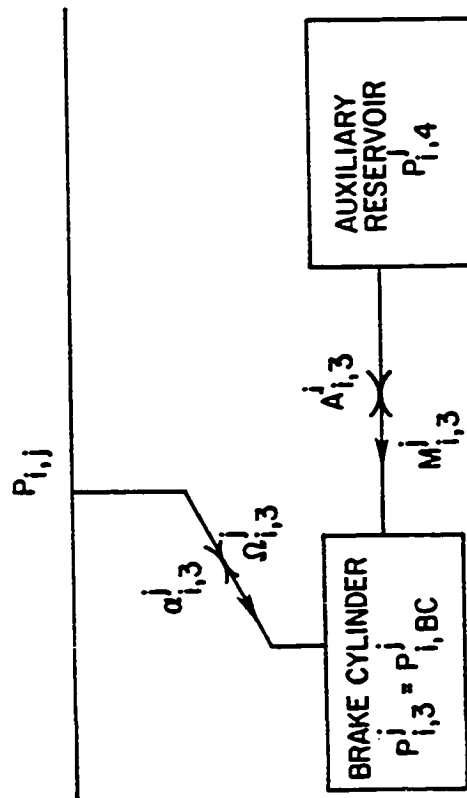


Figure Q.8: Simplified Service Fluid Star Network

$$a_{i,3}^j = 0 \quad \text{and} \quad M_{INS}^j = M_{i,3}^j \quad (G.32)$$

g) Service lap position

As the air continues to flow from the auxiliary reservoir to the brake cylinder, the auxiliary reservoir pressure drops until it reaches a slightly lower value than the brake pipe pressure. Then the restriction area connecting the auxiliary reservoir to the brake cylinder starts to close. When this restriction is completely closed, this is the lap position, thus

$$A_{i,3}^j = 0 \quad \text{and} \quad v \approx H_1 \quad (G.33)$$

During the entire service mode, the quick action volume is connected to the brake pipe through $a_{i,7}$ and to the atmosphere through A_{QAV}^j . Figure (G.9) shows that A_{QAV}^j is a combination of two restrictions in series, and it may be evaluated as following:

$$A_{QAV}^j = \frac{A_{QAV1}^j A_{QAV2}^j}{\sqrt{[A_{QAV1}^j]^2 + [A_{QAV2}^j]^2}} \quad (G.34)$$

A_{QAV2}^j is a fixed restriction, called 201c and it is shown in figure (G.1) in the emergency portion of the ABD valve. On the other hand, A_{QAV1}^j is a variable restriction and it is a function of the movement of the emergency slide valve of the ABD control valve, x_{SV}^j shown in figure G.10, as follows:

$$A_{QAV1}^j = D_4 F(x_{SV}^j) \quad (G.35)$$

where,

$$F(x_{SV}^j) = 0 \quad x_{SV}^j < D_0$$

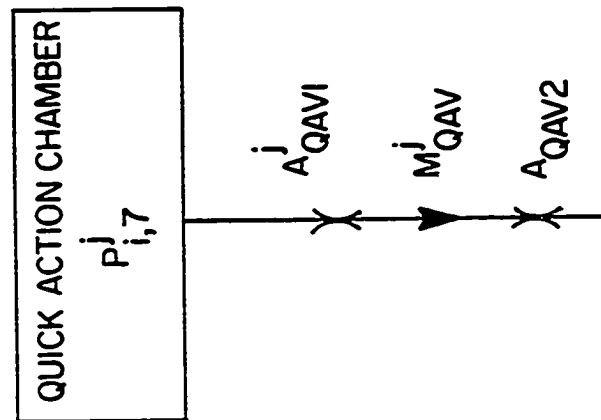


Figure G.9: Vent Restrictions of Quick Action Chamber

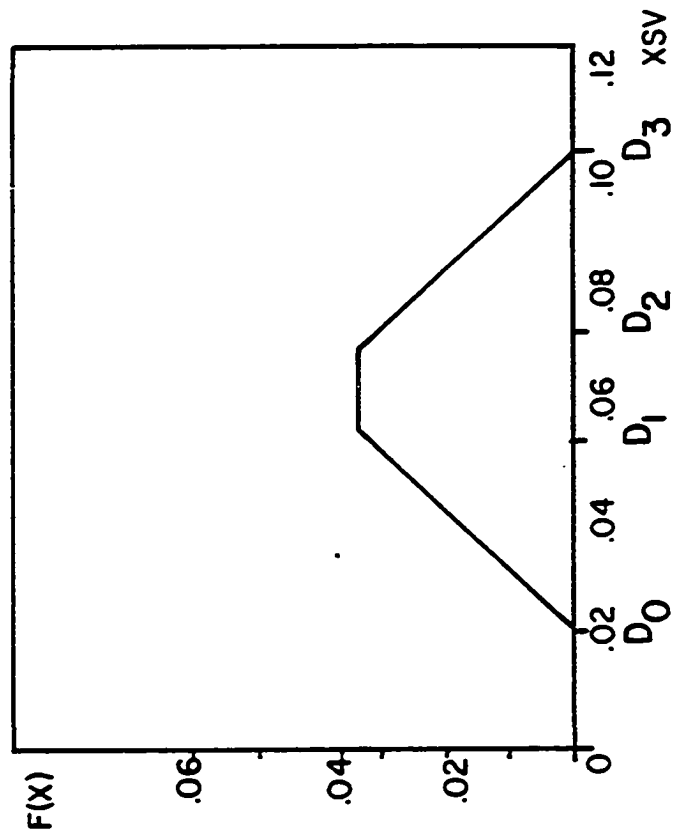


Figure G.10: Input Output Diagram for Quick Action Variable Restriction

$$\begin{array}{ll}
 F(X_{SV}^j) = X_{SV}^j - F_0 & D_0 \leq X_{SV}^j \\
 F(X_{SV}^j) = F_1 & D_1 \leq X_{SV}^j < D_2 \\
 F(X_{SV}^j) = D_3 - X_{SV}^j & D_2 \leq X_{SV}^j \leq D_3 \\
 F(X_{SV}^j) = 0 & D_3 < X_{SV}^j
 \end{array}$$

where X_{SV}^j is related to the pressure difference between the quick action chamber volume and the brake pipe pressure as,

$$X_{SV}^j = \frac{[P_{i,7}^j - P_i^j] P_A A_{SV} - L_{SV} - W_{SV}}{K_{SV}} \quad (G.36)$$

where,

A_{SV} = is the cross-sectional area of the service valve,

L_{SV} = is the slide valve spring preload,

K_{SV} = is the slide valve spring constant,

and, W_{SV} = is the weight of the slide valve moving parts,

Note that, the constants $C_0, C_1, D_0, D_1, D_2, D_3, H_1, H_2, H_3, H_4, H_5, H_6$ and H_7 are evaluated from the ABD control valve dimensions, spring constants and preloads, and may vary from valve to another. A complete list of these constant may be extract from Refs. [32-35].

G.3.2 Emergency Mode

This mode is activated by the emergency slide valve in the emergency portion of the ABD control valve, based on the pressure difference between the quick action chamber pressure and the prake pipe pressure as,

$$\Delta > H_2 \quad (G.37)$$

During this mode;

..

1. the brake pipe is connected to the atmosphere through $a_{i,2}^j$. The quick action air flows to chamber X at the left of the vent valve piston 262 (see figure (G.1)) unseats the vent valve 238, opening a large and direct passage, $a_{i,2}^j$, from the local brake pipe to atmosphere. Figure (G.11) shows the free body diagram of the vent valve during the emergency mode, $a_{i,2}^j$ may take this form,

$$a_{i,2}^j = \pi D_{\text{VENT}}^X X_{\text{VENTM}} \quad (\text{G.38})$$

2. the auxiliary reservoir, brake cylinder and emergency reservoir form a star network as shown in figure (G.4) (with $a_{i,3}^j = 0$). During the first stage of the emergency;

- a- A_{INS}^j varies with the brake cylinder pressure (see Section G.2.6), until the brake cylinder pressure reaches a value close to H_7 , and
- b- the emergency reservoir is connected to the star network, through A_{HPS}^j , when the value of Δ is greater than H_8

Because of the above actions (a- and b-), it is not possible to find a simplification for this star network as it has been done before in Section G.3.1. Instead of solving a star network problem (as explained in Appendix D), it is possible to modify the network by introducing a small volume, V_{INT} (as compared with the volumes associated with this network), at the center of the star network, as shown in figure (G.12). This modification reduces the amount of computer time, required to solve a star network problem.

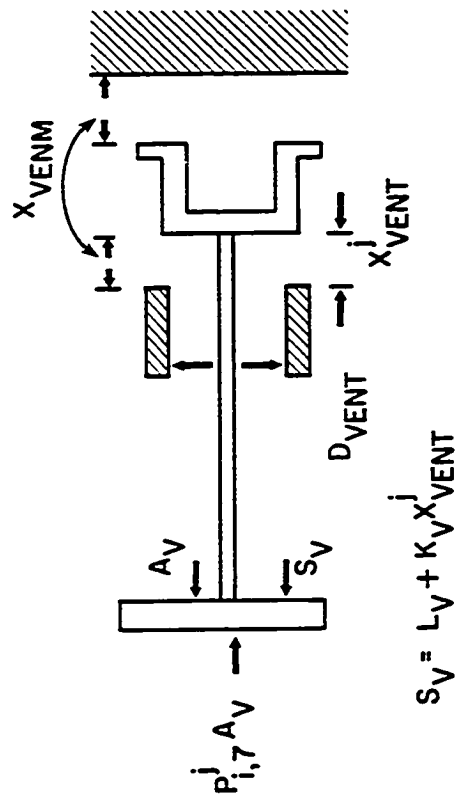


Figure G.11: Vent Valve Free Body Diagram

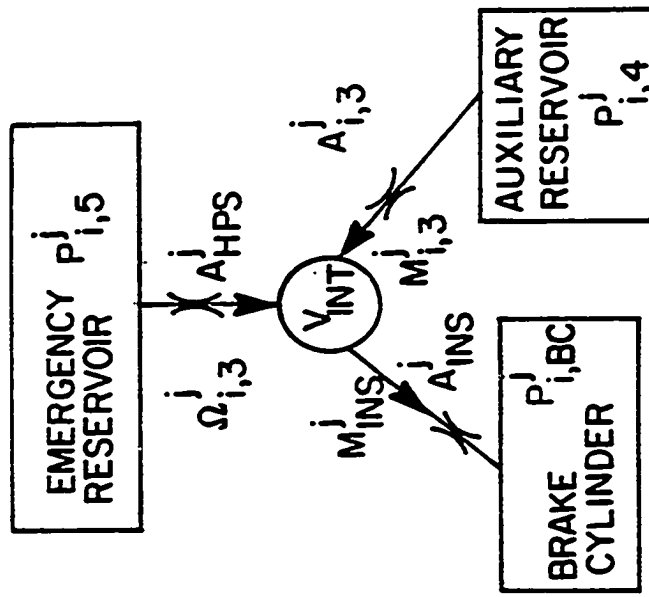


Figure G.12: Emergency Mode Star Fluid Network

G.3.3 Recharge/Release Mode

a) First stage: $H_3 \leq -v$

The brake cylinder is connected to atmosphere through A_{BCV}^j (fixed restriction shown as choke 20lg in figure (G.1)).

b) Second stage: $H_3 \leq -v \leq H_5$

The auxiliary reservoir is connected to the brake pipe through $a_{i,4}^j = A_{AUX}$, and through $a_{i,5}^j$ (see Section G.2.1) to emergency reservoir.

b) Third stage: $H_4 \leq -v$

The emergency reservoir is connected to the brake pipe.

c) Fourth stage: $H_5 \leq -v$

This is the final stage of this mode. The auxiliary reservoir is connected through another restriction to the brake pipe, $a_{i,4}^j = A_{AUX1}$.

However, in the case of release after emergency, the valve will go through a different stage before reaching the first stage described in this mode. This stage is called the accelerated emergency stage. This stage is started when the following condition is satisfied,

$$\Delta > H_6 \quad (G.39)$$

At this stage, the brake cylinder pressure is equilized with the auxiliary reservoir pressure and $A_{i,3}^j$ and A_{INS}^j are fully opened, then brake pipe is connected to $P_{i,3}^j$ through $a_{i,3}^j$

(this opening is called the accelerated emergency release check valve), which can be evaluated based on the following formulations (see figure (G.13)).

$$a_{i,3}^j = \pi D_{i,3} x_{CV}^j \quad 0 \leq x_{CV}^j \leq x_{CVM}^j$$

$$x_{CV}^j = \frac{[P_{i,3}^j - P_i^j] P_A A_{CV} - L_{CV}}{K_{CV}} \quad (G.40)$$

Instead of solving the star network shown in figure (G.4) (with $A_{HPS}^j = 0$), it is possible to simplify this network to the one shown in figure (G.8). Then, as the brake pipe pressure builds up to a value assigned in stage 1, the brake valve will go through the rest of stages as mentioned above. This completes the ABD valve mathematical model and its functions during different modes.

Figure G.13: Accelerated Emergency Release Check Valve Free Body Diagram

APPENDIX H

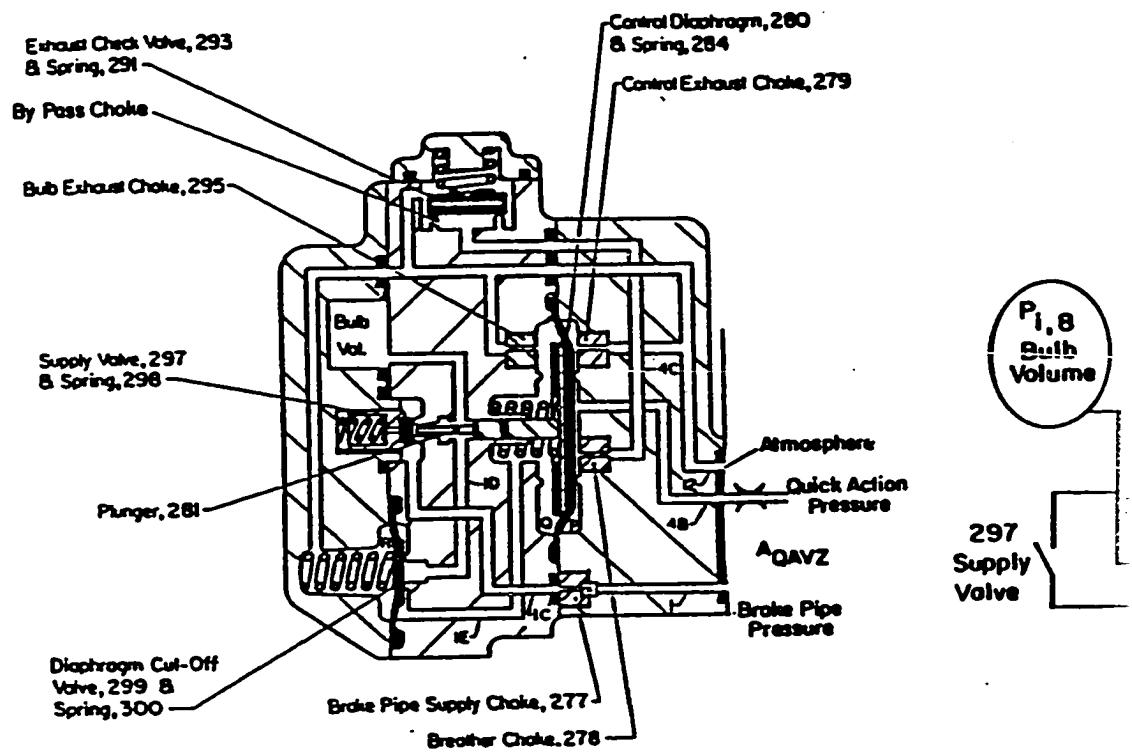
ABDW CONTROL VALVE MATHEMATICAL MODEL

H.1 Introduction

Appendix G briefly describes the mathematical model equations for ABD control valve as developed by Refs. [7-10]. In order to have a mathematical model for the ABDW, Wright [8] shows that you may only need to model the W portion of the valve and incorporate it with the ABD mathematical model ($ABDW \rightarrow ABD + W$). This W is known as the Accelerated Application Valve (AAV). In this appendix, the development of the flow equations based on the simplified restriction network given by Refs. [8] (the complete model is also given in Ref. [8]) is carried out, and a list of the pressure equations as developed by Wright [8] is presented.

H.2 AAV Flow and Pressure Equations

The AAV was added to the emergency portion of the ABD to continuously vent the local brake pipe to atmosphere at each car, $\alpha_{i,g}^j$. This results in faster brake applications, especially service application after service Lap. Figure H.1 shows a schematic drawing of the AAV and figure H.2 is the schematic diagram of the AAV fluid network as developed by Refs. [8].

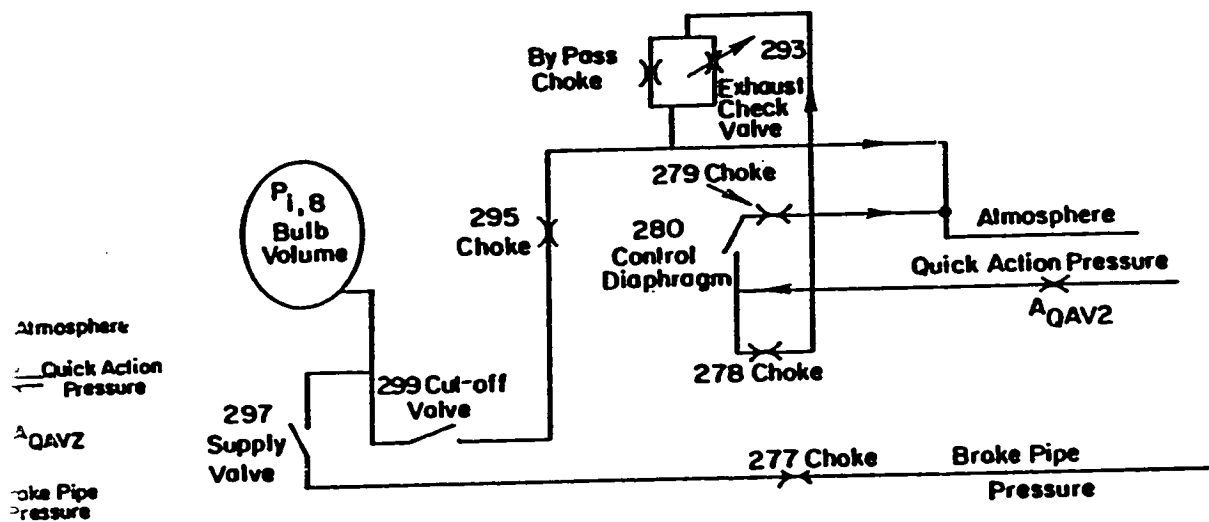


a) Schematic Diagram

H.1 Accelerated Application

280

Choke, 279



b) Fluid Network

elerated Application Valve

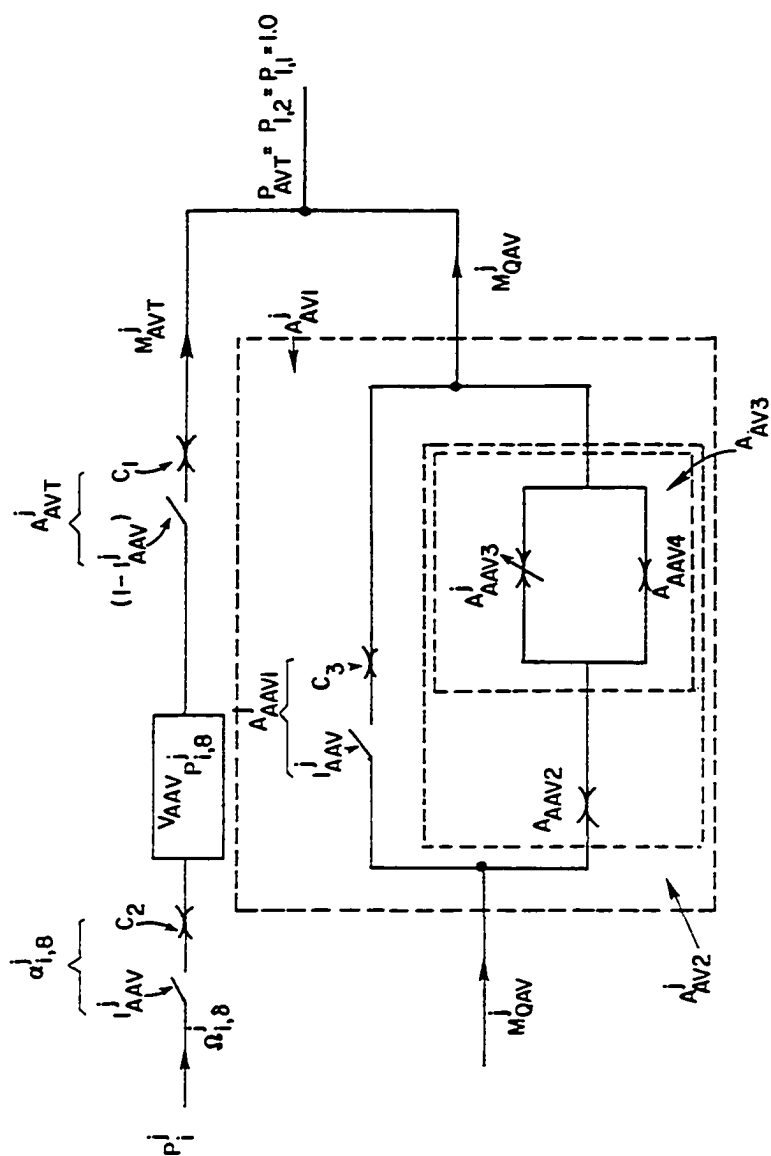


Figure H.2: AAV Restrictions Diagram

Appendix j, describes the flow equation and the normalization reference variables as used in this entire research project.

The mass flow rate through $a_{i,8}^i$, using the average density equation, is a function of the 297 valve opening as follows:

a. 297 valve is opened

$$\dot{n}_{i,8}^j = 0.6 \frac{a_{i,8}^j}{A_i} P_i^j \sqrt{\frac{|R_{i,8}^2 - 1|}{\Gamma} \frac{|1 - R_{i,8}|}{1 - R_{i,8}}} \quad (\text{H.1a})$$

b. 297 valve is closed

$$\dot{n}_{i,8}^j = 0 \quad (\text{H.1b})$$

where, $R_{i,8} = \frac{P_{i,8}^j}{P_i^j}$

$a_{i,8}^j$ is the brake pipe supply choke 277 in series with the large supply valve 297 opening, shown in figure (H.1), connects the brake pipe with the bulb volume of the AAV portion.

The time rate of change of the pressure $P_{i,8}^j$ is a function of the mass flow rate M_{AAV} , and V_{AAV} as

$$\frac{dP_{i,8}^j}{dT} = \frac{M_{AAV}^j}{V_{AAV}} \quad (\text{H.2})$$

Where, $M_{AAV}^j = \dot{n}_{i,8}^j - M_{AVT}^j$.

However, M_{AVT} is a function of the opening of the 299 cut-off valve as follows:

a. 299 cut-off valve is opened

$$M_{AVT}^j = 0.6 \frac{A_{AVT}}{A_i} P_{AVT}^j \sqrt{\frac{|R_{AVT}^2 - 1|}{\Gamma} \frac{|1 - R_{AVT}|}{1 - R_{AVT}}} \quad (H.3a)$$

b. 299 cut-off valve is closed

$$M_{AVT}^j = 0 \quad (H.3b)$$

where,

$$R_{AVT} = \frac{P_{AVT}^j}{P_{i,8}^j}$$

Note that (see figure (G.1)):

1. A_{AVT} is made up of bulb exhaust choke 297 and a large opening controlled by the diaphragm cut-off valve 299, in series.
2. P_{AVT} is downstream of A_{AVT} , control exhaust chock 279 and hence is atmospheric; it is equivalent to $P_{i,2} = 1$. (these pressures are all normalized with respect to atmospheric pressure, p_A).

H.3 AAV Flow Restrictions Equations

The flow restriction, $a_{i,8}^j$, connecting the local brake pipe with the bulb volume is evaluated as,

$$a_{i,8}^j = I_{AAV}^j C_2 \quad (H.4)$$

where,

C_2 is the area of 277 brake pipe supply choke,

$$\text{and, } I_{AAV}^j = 1, \quad P_{AAV3}^j - 1 > C_4 (P_{i,8}^j - 1) + C_5$$

$$I_{AAV}^j = 0, \quad P_{AAV3}^j - 1 \leq C_6 (P_{i,8}^j - 1) + C_7$$

C_4 , C_5 , C_6 and C_7 are evaluated and described in Ref. [9].

The flow restriction, A_{AVT}^j , connecting the bulb volume to atmosphere, may be evaluated as,

$$A_{AVT}^j = (1 - I_{AAV}^j) C_1 \quad (H.5)$$

where, C_1 is flow area of the 295 bulb exhaust choke.

There is a slight modification that should be made to evaluate the flow restriction area, A_{QAV}^j , mentioned in equation (G.34), as follows:

$$A_{QAV}^j = \frac{A_{QAV1}^j A_{AV1}^j}{\sqrt{[A_{QAV1}^j]^2 + [A_{AV1}^j]^2}} \quad (H.6)$$

where,

$$A_{QAV1}^j = D_4 F(X_{SV}^j), \quad [\text{see equation (G.35)}]$$

$$A_{AV1}^j = A_{AAV1}^j + A_{AV2}^j,$$

$A_{AAV1}^j = I_{AAV}^j C_3$, C_3 is the flow area of 279 control exhaust choke,

$$A_{AV2}^j = \frac{A_{AAV2} A_{AV3}^j}{\sqrt{A_{AAV2}^2 + [A_{AV3}^j]^2}},$$

A_{AAV2} is the flow area of by pass choke,

$$A_{AV3}^j = A_{AAV3}^j + A_{AAV4}^j,$$

A_{AAV4} is the flow area of the 278 breather choke,

$$A_{AAV3}^j = C_4 X_{ECV}^j,$$

$$C_3 = \pi D_3,$$

D_3 = the diameter of the exhaust check valve, 293,

$$X_{ECV} = \frac{[P_{ECV}^j - 1] P_A A_{ECV} - L_{ECV}}{K_{ECV}},$$

$$\text{and, } P_{ECV} = \sqrt{\frac{P_{AAV3} A_{AAV1}^2 + A_{AV1}^2}{A_{AAV1}^2 + A_{AV1}^2}}$$

$$P_{AAV3} = \sqrt{\frac{P_{i,7} A_{QAV1}^2 + A_{AV1}^2}{A_{QAV1}^2 + A_{AV1}^2}} \quad (H.7)$$

where, P_{AAV3} is the pressure down stream of the A_{QAV1} choke. This completes the mathematical model of the AAV, which makes it possible to have a mathematical model for the ABDW by incorporating the AAV (W portion) with the ABD mathematical model described in Appendix G.

APPENDIX I

A19 FLOW INDICATOR MATHEMATICAL MODEL

I.1 Operation Principle

The equivalent fluid network for the A19 flow indicator adaptor is shown in figure (I.1). This component is located between the supply reservoir and the input to the air brake system through passage 30 of the 26C locomotive valve (see figure (3.1) in Chapter 3). This component will act as a pressure difference regulating valve which tries to maintain the pressure difference, ΔP^j , across the adaptor. There are two parallel restrictions; one is a fixed restriction (A_{SS1}), whereas the other restriction area (A_{SS2}^j) varies with pressure difference across the adaptor to maintain the difference within a desired limit. Figure (I.2) shows the pressure upstream and downstream of the A19 flow indicator adaptor as recorded from the NYAB (New York Air Brake company) experiment set-up during a recharge of 50-car train, each car has a length of 50 feet, equipped with ABDW valves, auxiliary and emergency reservoir.

During a recharge, the initial pressure downstream of the A19, $P_{mr}^j = P_{mr}^j / P_A$ (normalized pressure with respect to atmospheric pressure), is equal to the supply pressure, P_S . As the air flows out of the volume v_{19} , its pressure, P_{mr}^j , decreases and causes air to flow out from the supply

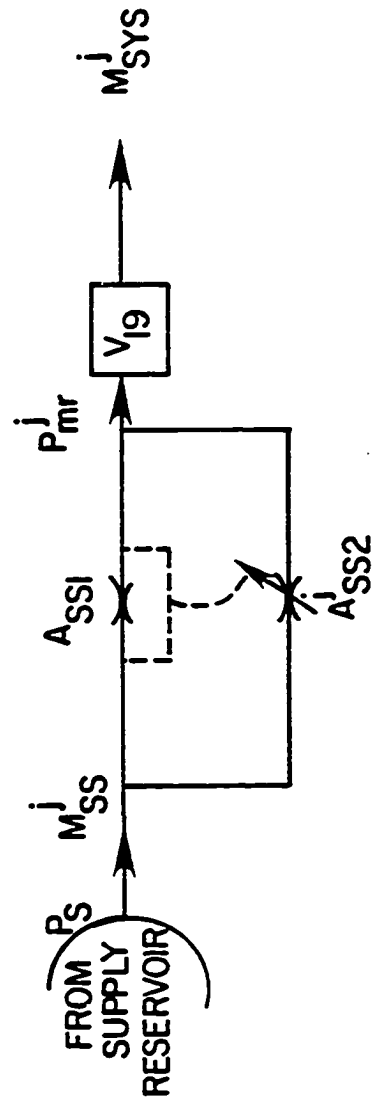


Figure I.1: A19 Fluid Network

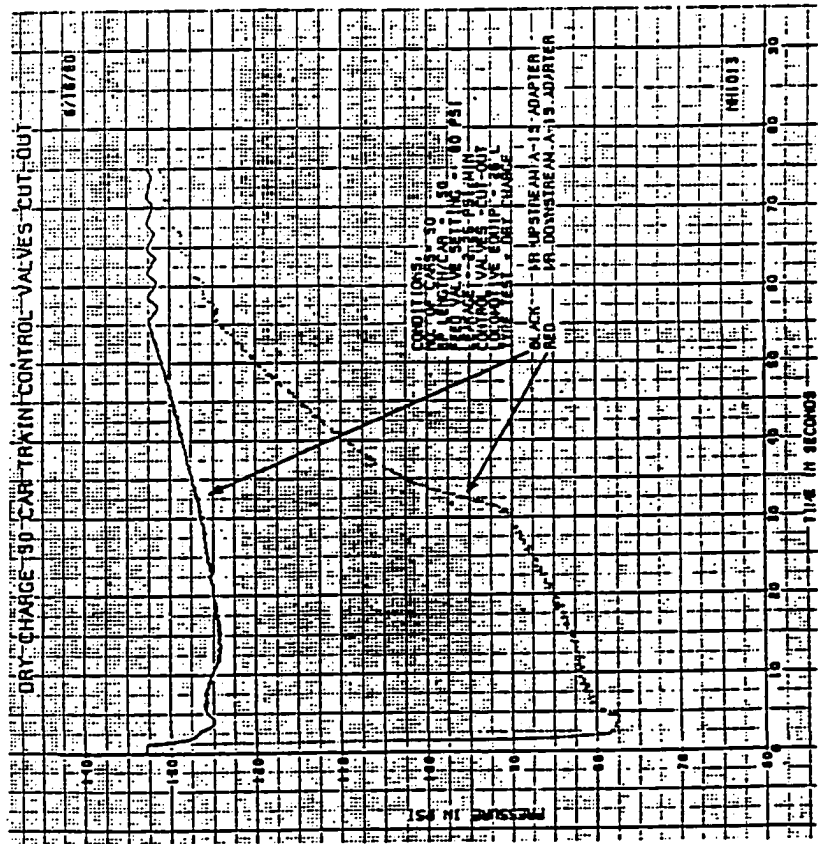


Figure I.2: A19 Upstream and Downstream Pressures Vs. Time (50-Car Dry Charge Experimental Data)

reservoir (which is filled by an external compressor) to the volume v_{19} through the fixed restriction, A_{SS1} . As the pressure drop across the A19 becomes large enough, the variable restriction opens, and more air flows to v_{19} . The flow of air through the two restrictions prevents P_{mr}^j from dropping more than a predetermined value away from the supply pressure, and produce a relative constant pressure drop across the A19.

As the brake pipe (head-end) pressure increases, pressure P_{mr}^j increases, and the pressure drop across the A19 becomes smaller. Then the variable restriction will begin to close.

I.2 Mathematical Model

Kreel [10] has developed a mathematical model describing the function of the A19 flow indicator adaptor. This section summarizes the development of this model and lists the equations used to present the A19.

Figure (I.3) shows the freebody diagram of the A19. Summing the forces acting on the variable restriction piston, ignoring the inertial term and rearranging the terms, the displacement of A_{SS2}^j can be solved for as follows:

$$\begin{aligned} &\text{for } 0 \leq x_{SS2}^j \leq x_{SSM} \\ x_{SS2}^j &= \frac{[P_S - P_{mr}^j] P_A A_{P19} - L_{P19}}{K_{P19}} \end{aligned} \quad (I.1)$$

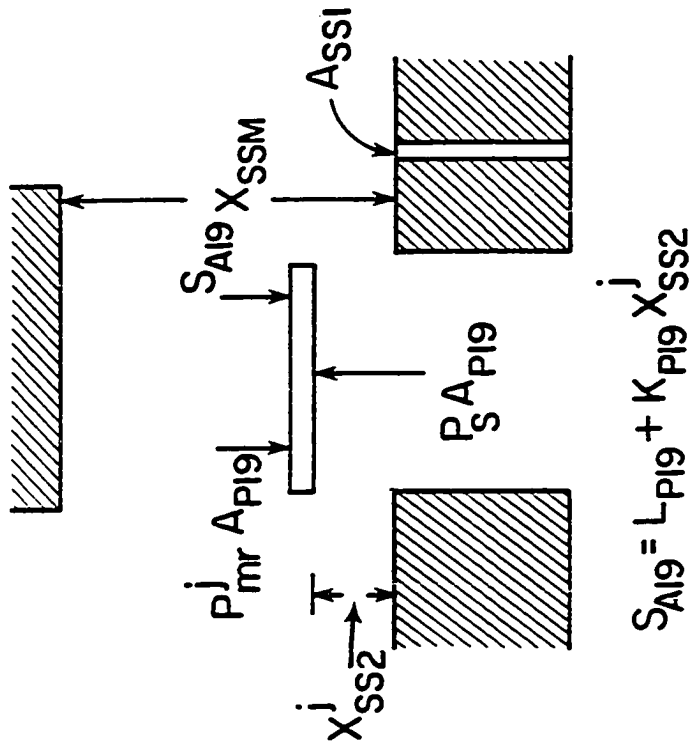


Figure I.3: A19 Free Body Diagram

where, X_{SSM} is the maximum displacement of the A19 piston calculated from the its dimension drawing. Once the displacement is known, the area of the variable restriction can be evaluated as:

$$A_{SS2}^j = \pi D_{P19} X_{SS2}^j \quad (I.2)$$

Referring to figure (I.1), P_{mr}^j can be calculated as a function of the mass flow of air into and out of the volume v_{19} as follows

$$\frac{dP_{mr}^j}{dT} = \frac{1}{v_{19}} \left[\dot{M}_{SS}^j - \dot{M}_{SYS}^j \right] \quad (I.3)$$

where,

$$v_{19} = \frac{v_{19}}{A_1 \lambda}, \text{ normalized volume for the actual } v_{19}.$$

A_1 = is the cross-sectional area of the locomotive car (brake pipe).

λ = is the total length of the train-line (brake pipe).

$$\dot{M}_{SS}^j = \frac{A_{SS1} + A_{SS2}^j}{A_1} \left[\frac{|R_{SS}^2 - 1|}{r} \right] \frac{|1 - R_{SS}|}{1 - R_{SS}} \quad (I.4)$$

$$\dot{M}_{SYS}^j = \frac{[\dot{m}_{1,3}^j + \dot{m}_{2,1}^j]}{\rho_A A_1 c} \quad (I.3)$$

where, $\dot{m}_{1,3}^j$ is the mass flow rate is entering the relay valve system when the equalizing reservoir pressure is higher than the brake pipe pressure, $\dot{m}_{2,1}^j$ is the mass flow rate entering the regulating valve during a release/recharge

mode, c is the local speed of sound and ρ_A is the density of air at atmospheric conditions.

This section completes the A19 flow indicator adaptor mathematical model as well as the mathematical model for the last element that has been simulated and modeled during this entire research project.

APPENDIX J

AVERAGE DENSITY FLOW EQUATION

This Appendix describes the flow equation through a restriction, and the normalization reference variables, used in this research project.

Banister [9] has suggested the average density equation to be used to evaluate the mass flow rate through all the restrictions associated with the ABD/ABDW. He also assumed that all the restrictions have a constant discharge coefficient of .6. This value implies that each of those restrictions behaves as a sharp-edged-orifice. Hence, the normalized mass flow rate, M_{EXP} , through a restriction, A_{EXP} , is related to the normalized upstream and down stream pressure (P_{UP} , $P_{DOWN} \rightarrow P_{EXP}$) as,

$$M_{EXP}^j = 0.6 \frac{A_{EXP}}{A_i} P_{UP}^j \sqrt{\frac{|R_{EXP}^2 - 1|}{\Gamma} \frac{|1 - R_{EXP}|}{1 - R_{EXP}}}$$

where,

$$R_{i,k} = \frac{P_{EXP}^j}{P_{UP}^j}$$

A_i → is the local brake pipe cross-sectional area,

i → represents the car/brake pipe section location,

j → will be present only if the term associated with it changes with time,

EXP → could be any alphanumeric expression which represents the location of this restriction,

M → represents the mass flow rate and it may be changed to \dot{m} in the case of directly entering or leaving the brake pipe, and

A_{EXP} → represents the flow restriction and it may be changed into a_{EXP} in the case of being directly connected to the brake pipe.

Normalization of pressure, mass flow rate, and volumes are given by:

$$P_{EXP} = \frac{P_{EXP}}{P_A}, \quad \dot{M}_{EXP} = \frac{\dot{m}_{EXP}}{\rho_A c A_i} \quad \text{and} \quad V_{EXP} = \frac{v_{EXP}}{A_i \ell}$$

where,

P_{EXP} = is the pressure at destination EXP, N/m^2 ,

\dot{m}_{EXP} = is the flow rate through EXP, kg/sec,

v_{EXP} = is the volume of EXP, m^3 ,

P_A = is the atmospheric pressure, N/m^2 ,

ρ_A = is the density at atmospheric conditions, kg/m^3 ,

c = is the local speed of sound, m/sec,

A_i = is the cross-sectional area of the i^{th} car, m^2 ,

and, ℓ = is the total length of the brake pipe, m.

APPENDIX K

EVALUATIONS OF THE BRAKE PIPE FRICTION FACTOR

K.1 Introduction

A mathematical model for the brake pipe has been developed in Appendix A. In order to complete the model, it is necessary to find the friction factor as a function of the flow regimes. The flow regimes may be represented by the Reynold Number, where,

$$R_e = \frac{\rho u D_{bp}}{\mu} \quad (K.1)$$

ρ = is the local brake pipe air density, kg/m^3 ,

u = is the local brake pipe air velocity, m/sec ,

D_{bp} = is the local brake pipe diameter, m ,

and, μ = is the air dynamic viscosity, kg/(m.sec) .

The objective of this appendix is to determine experimentally and theoretically, the variations of the friction factor with respect to the Reynold Number, for the full scale and scaled down brake pipe.

K.2 Full Scale Brake Pipe Friction Factor

The Experimental set-up used to determine the friction factor consists of the following instruments and components:-

1. Hewlett Packard (HP) 86A micro-computer,

2. HP 3421 A data acquisition system,
 3. 1 Setra (0-1 psi) pressure transducer,
 4. 2 Setra (± 5 psi) pressure transducer,
 5. 2 Sharp-edged orifice plates (0.3 and .6 in), that have been calibrated using a standard ASME flow nozzle (.5 in),
- and, 6. Water and Mercury pressure manometers.

Figure (K.1) shows the experimental set-up used to calibrate the sharp-edged orifice. The sharp-edged orifice plates were designed to fit into the brake pipe glad hand, shown in figure (K.2). The experimental data and the discharge coefficient for each orifice are listed in table (K.1). The sharp-edged orifice discharge coefficient is evaluated using the following equation

$$C_d = \frac{M_N}{M_T} \quad (K.2)$$

where,

M_N = is the ASME nozzle mass flow rate, kg/sec

$$= 1.11 \frac{C Y d_N^2}{\sqrt{1 - \beta^2}} \sqrt{P_2 [P_1 - P_2]} \quad (K.3)$$

$$C = .99622 + .00059 D_N - \frac{6.36 - .13 D_N - .24 \beta^2}{\sqrt{R_e}} \quad (K.4)$$

$$Y = \left[R^{2/\Gamma} \left[\frac{\Gamma}{\Gamma-1} \right] \left[\frac{1-R}{1-R} \right] \left[\frac{1-\beta^4}{1-\beta^4 R^{2/\Gamma}} \right] \right]^2 \quad (K.5)$$

$$R = \frac{P_2}{P_1},$$

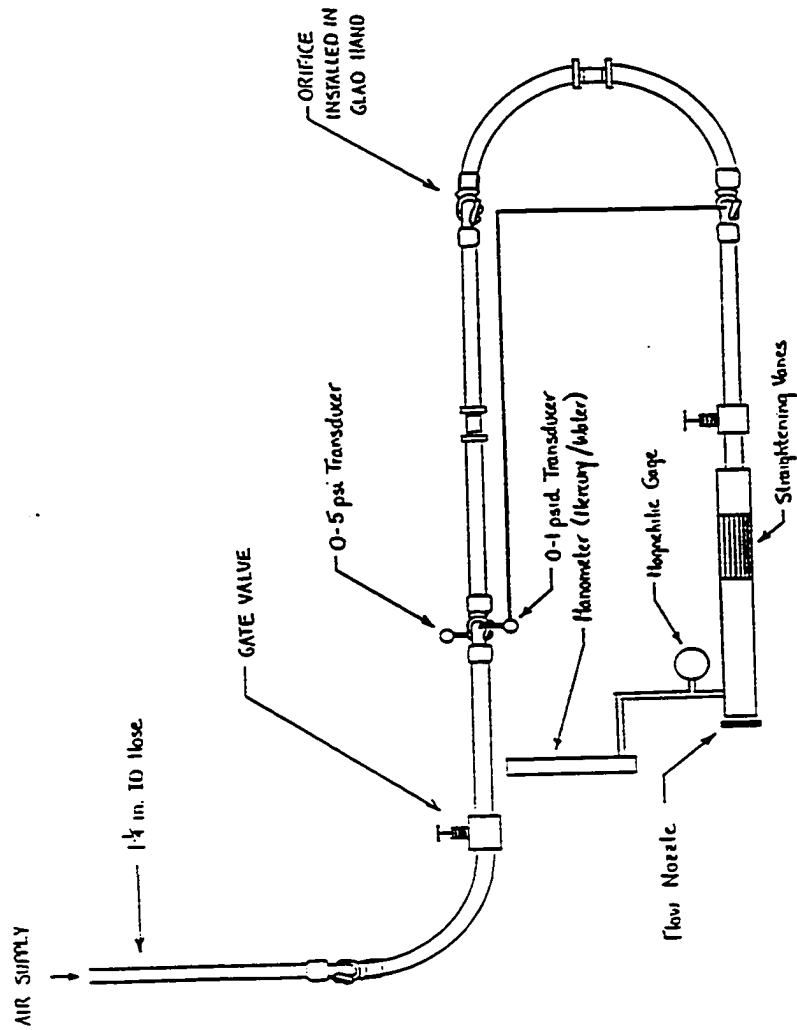


Figure K.1: Sharp-edged Orifice Calibration Set-up

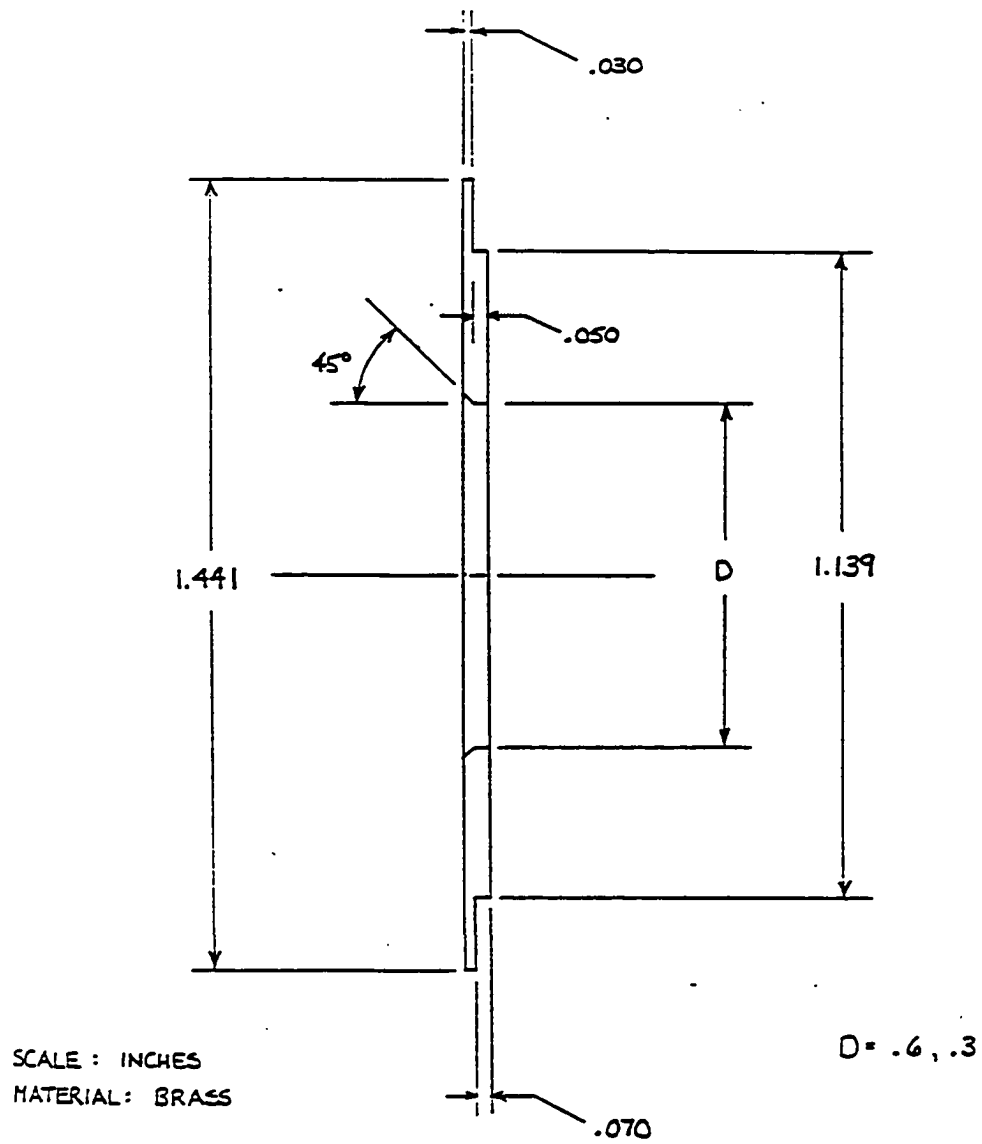


Figure K.2: Sharp-edged Orifice Drawing

Table K.1: Sharp-edged Orifice Calibration Data

Orifice Diameter - .6 inch					
Test No.	Upstream Orifice Pressure (PSI)	Orifice Differential Pressure (PSI)	Upstream Nozzle Pressure (PSI)	Discharge Coefficient	
1	7.78	.162	.227	.628	
2	7.09	.294	.415	.645	
3	5.40	.384	.487	.639	
4	6.64	.458	.605	.635	
5	8.73	.450	.686	.651	
6	5.18	.820	.956	.627	
Average Coefficient				.638	

Orifice Diameter - .3 inch					
Test No.	Upstream Orifice Pressure (PSI)	Orifice Differential Pressure (PSI)	Upstream Nozzle Pressure (PSI)	Discharge Coefficient	
7	9.57	.289	.029	.650	
8	5.01	.622	.045	.623	
9	6.97	.980	.082	.649	
10	7.37	1.49	.109	.610	
11	7.06	2.06	.146	.616	
12	7.27	3.01	.199	.610	
13	8.48	3.76	.236	.589	
Average Coefficient				.621	

$$\beta = \frac{d_N}{D_N},$$

$$R_e = \frac{4M_N}{\pi d_N \mu},$$

d_N = Nozzle exit diameter, m,

D_N = Nozzle inlet diameter, m,

p_1 = upstream nozzle air pressure, N/m^2 ,

p_2 = downstream nozzle air pressure, N/m^2 ,

ρ_2 = downstream nozzle air density, kg/m^3 ,

(Note that equations (K.3), (K.4) and (K.5) are taken from Ref. [32])

M_T = Theoretical air mass flow rate through the sharp-edged orifice, kg/sec,

$$= .78 d_o \sqrt{\bar{p} [p_0 - p_1]} \quad (K.6)$$

d_o = orifice diameter, m,

p_0 = orifice upstream pressure, N/m^2 ,

and, $\bar{p} = \frac{p_1 + p_2}{2}$

Figure (K.3) shows the experimental set-up used to evaluate the brake pipe friction factor. The brake pipe in New York Air Brake 200-car freight test rack was used. The following information was recorded for various experimental configurations (10-car, 20-car and 50-car brake pipe lengths):-

1. Upstream pressure of the sharp-edged orifice.
2. Differential pressure across the orifice.

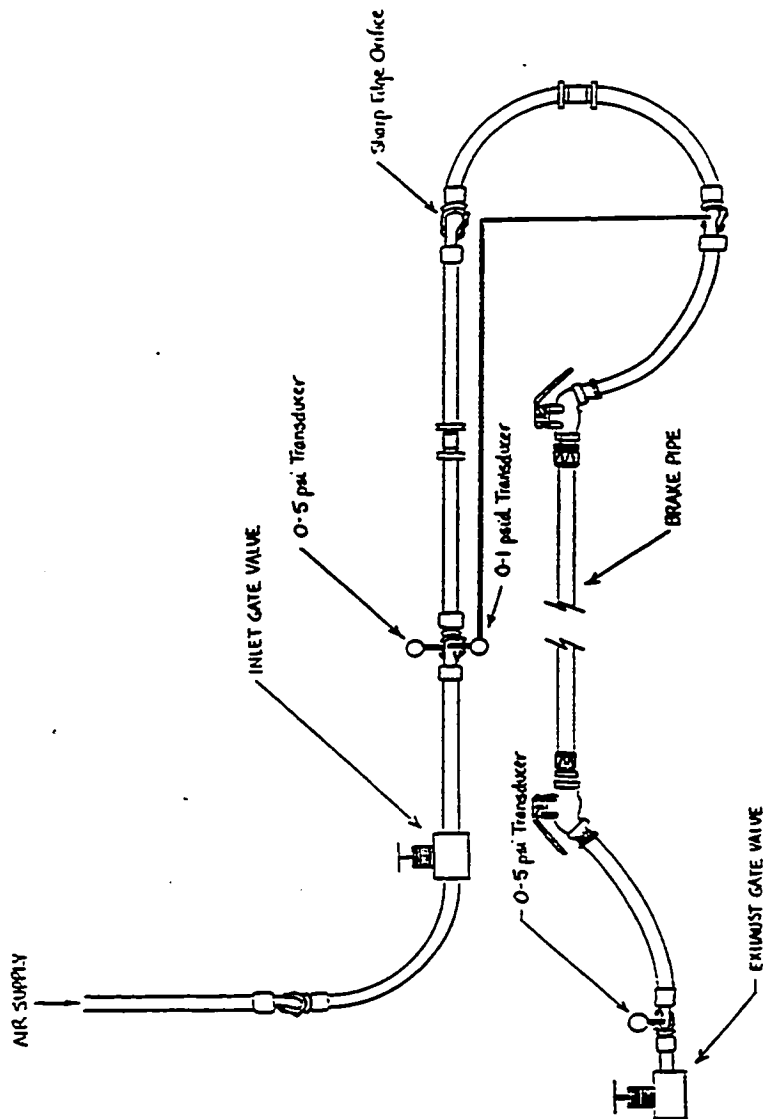


Figure K.3: Brake Pipe Friction Factor Experimental Set-up

3. Pressure at the last car (brake pipe) in the experimental set-up.

The experimental data and the corresponding friction factor and the Reynold Number are listed through out tables (K.2-6).

The friction factor and the Reynold Number were computed using the following equations:-

$$f = P_R A_R^2 \left[\frac{D_{bp}}{\lambda} \right]^2 \quad (K.7)$$

$$R_e = \frac{4C_d M_T}{\pi D_{bp} \mu} \quad (K.8)$$

where,

P_R = pressure ratio,

$$= \frac{\frac{P_1^2}{2} - \frac{P_2^2}{2}}{\frac{P_0^2}{2} - \frac{P_1^2}{2}},$$

A_R = area ratio,

$$= \left[\frac{D_{bp}}{d_o} \right]^2,$$

λ = brake pipe total length, m,

P_2 = brake pipe pressure at the last car, N/m^2 .

Figure (K.4) is a plot of the experimental results of the friction factor versus the Reynold Number. Assuming that the experimental data is related as

$$f = a R_e^b \quad (K.8)$$

Using the least squares method, one may find the values of a and b in the turbulent region, $R_e > 4000$ as:-

for $4000 < R_e < 40000$

Table K.2: 50 Car Brake Pipe Friction Data (Cars 1 - 50)

Test No.	Upstream Orifice Pressure (psig)	Differential Orifice Pressure (psig)	Downstream Orifice Pressure (psig)	Pressure @ Car 50 (psig)	Reynold No.	Brake Pipe Friction Factor
1	4.190	.024	4.166	4.132	1450	.0465
2	4.939	.056	4.883	4.814	2256	.0397
3	5.027	.097	4.930	4.780	2973	.0499
4	5.038	.133	4.905	4.694	3479	.0510
5	4.973	.166	4.807	4.544	3876	.0508
6	4.991	.244	4.747	4.362	4692	.0504
7	4.948	.276	4.672	4.232	4980	.0508
8	4.955	.319	4.636	4.135	5349	.0500
9	5.017	.371	4.646	4.069	5769	.0492
10	5.001	.407	4.594	3.960	6034	.0492
11	4.990	.441	4.549	3.856	6273	.0495
12	5.122	.532	4.590	3.772	6896	.0482
13	4.925	.542	4.383	3.546	6923	.0483
14	5.050	.653	4.397	3.374	7601	.0487
15	4.974	.728	4.246	3.119	7992	.0478
16	5.321	.761	4.560	3.393	8239	.0473
17	4.959	.789	4.170	2.950	8303	.0476
18	4.962	.848	4.114	2.819	8594	.0468
19	4.816	.858	3.958	2.642	8608	.0469
20	5.460	1.047	4.413	2.941	9621	.0427
21	5.403	1.558	3.845	1.443	11550	.0449
22	5.074	1.956	3.118	.017	12674	.0445

Sharp Edge Orifice Diameter = .300 in.
Discharge Coef. = .621

Table K.3: 20 Car Brake Pipe Friction Data (Cars 1 - 20)

Test No.	Upstream Orifice Pressure (psig)	Differential Orifice Pressure (psig)	Downstream Orifice Pressure (psig)	Pressure @ Car 20 (psig)	Reynold No.	Brake Pipe Friction Factor
1	10.099	.111	9.988	9.900	3567	.0641
2	10.066	.148	9.918	9.806	4112	.0613
3	10.103	.289	9.814	9.613	5733	.0558
4	9.999	.436	9.563	9.280	7004	.0519
5	9.840	.626	9.214	8.826	8330	.0492
6	9.931	.766	9.165	8.713	9207	.0488
7	10.012	.957	9.055	8.476	10260	.0476
8	9.920	1.372	8.548	7.729	12145	.0462

Sharp Edge Orifice Diameter = .300 in.
 Discharge Coef. = .621

Table K.4: 20 Car Brake Pipe Friction Data (Cars 1 - 20)

Test No.	Upstream		Differential		Downstream		Pressure @ Car 20 (psig)	Reynold No.	Brake Pipe Friction Factor
	Orifice Pressure (psig)	Orifice Pressure (psig)	Orifice Pressure (psig)	Orifice Pressure (psig)	Orifice Pressure (psig)				
1	10.073		.044		10.029	9.575	9476	.0491	
2	9.959		.035		9.904	9.341	10567	.0486	
3	10.017		.067		9.950	9.271	11673	.0479	
4	10.017		.102		9.915	8.916	14391	.0460	
5	9.951		.139		9.812	8.458	16763	.0454	
6	9.998		.201		9.797	7.868	20148	.0441	
7	9.932		.221		9.711	7.617	21088	.0434	
8	10.013		.249		9.764	7.396	22407	.0433	
9	9.978		.289		9.689	6.945	24100	.0428	
10	10.023		.339		9.684	6.470	26095	.0423	
11	9.938		.401		9.537	5.718	28290	.0418	
12	9.324		.421		8.903	4.884	28603	.0416	
13	10.048		.458		9.590	5.251	30262	.0411	
14	10.005		.497		9.508	4.709	31467	.0414	
15	10.058		.588		9.470	3.718	34191	.0409	
16	10.038		.654		9.384	2.916	35987	.0406	
17	10.083		.755		9.328	1.729	38610	.0401	
18	9.901		.789		9.112	1.041	39288	.0402	
19	10.249		.879		9.370	.147	41682	.0400	

Sharp Edge Orifice Diameter = .600 in.
Discharge Coef. = .638

Table K.5: 10 Car Brake Pipe Friction Data (Cars 1 - 10)

Test No.	Upstream Orifice Pressure (psig)	Differential Orifice Pressure (psig)	Downstream Orifice Pressure (psig)	Pressure at Car 10 (psig)	Reynold No.	Brake Pipe Friction Factor
1	9.764	1.544	8.220	.262	53796	.0396

Sharp Edge Orifice Diameter = .600 in.
Discharge Coef. = .638

Table K.6: 20 Car Brake Pipe Friction Data (Cars 160-169)

Test No.	Upstream Orifice Pressure (psig)	Differential Orifice Pressure (psig)	Downstream Orifice Pressure (psig)	Pressure @Car 169 (psig)	Reynold No.	Brake Pipe Friction Factor
1	7.990	.017	7.973	7.810	5640	.0439
2	7.360	.075	7.285	6.580	11663	.0444
3	6.450	.181	6.269	4.530	17688	.0441
4	7.040	.240	6.800	4.510	20621	.0432
5	6.830	.350	6.480	3.070	24707	.0427
6	6.590	.410	6.180	2.170	26545	.0421
7	6.680	.478	6.202	1.450	28670	.0419

Sharp Edge Orifice Diameter = .600 in.
Discharge Coef = .638

BRAKE PIPE FRICTION EXPERIMENT

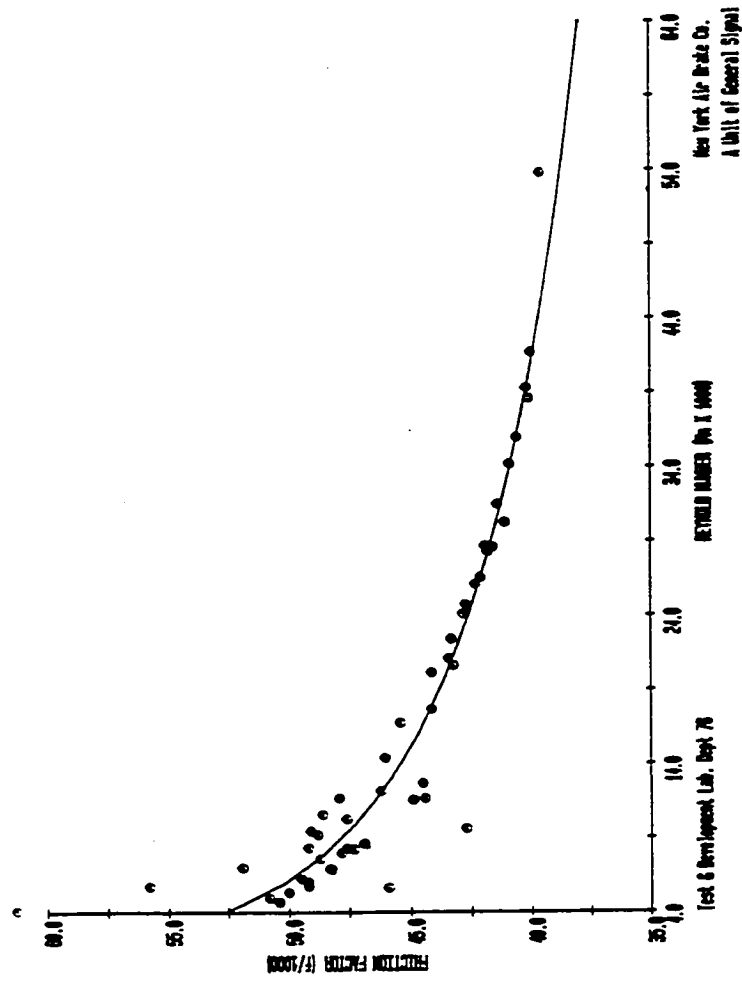


Figure K.4: Brake Pipe Friction Vs. The Reynolds Number (Experimental Data)

$$a = .13977 \quad (K.9)$$

$$b = -.11781$$

for $R_e > 40000$

$$f = .04 \quad (K.10)$$

Within the laminar region ($R_e < 2000$), the friction factor will follow the poiseuille law as

$$f = \frac{64}{R_e} \quad (K.11)$$

Finally, in the transition region, the friction factor may follow equation (K.8), where

$$b = .717 \quad (K.12)$$

$$a = 1.375 \times 10^{-4}$$

The above values of b and a are obtained by connecting the friction factor value at $R_e=2000$ to that at $R_e=4000$, with a straight line on the Log-Log scale. Figure (K.5) shows, the theoretical values of the friction factor, which will be used as a part of the air brake system simulation.

K.3 Scaled Down Brake Pipe Friction Factor

Ref. [2] describes the experimental results used to determine the friction factor of the scaled down brake pipe (.25 (in) in diameter). Figure (K.6) shows, the theoretical results of the friction factor as a function of the Reynold Number. These values are used as a part of the brake pipe program, used in chapter 5.

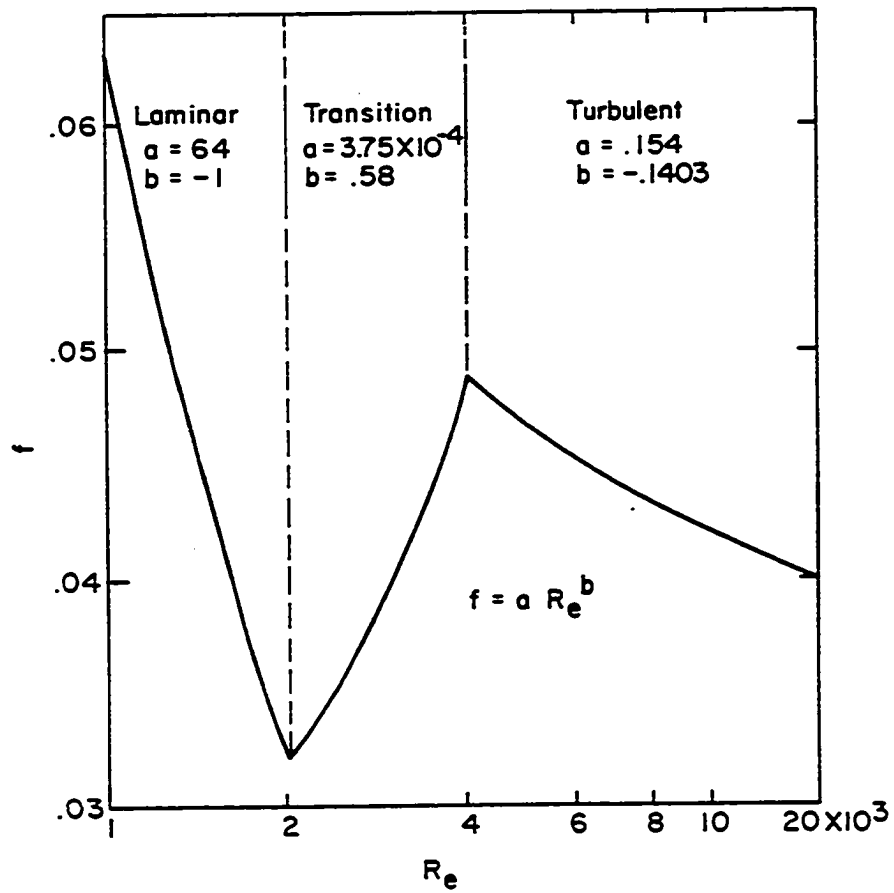


Figure K.5: Brake Pipe Friction Vs. The Reynolds Number (Theoretical Result)

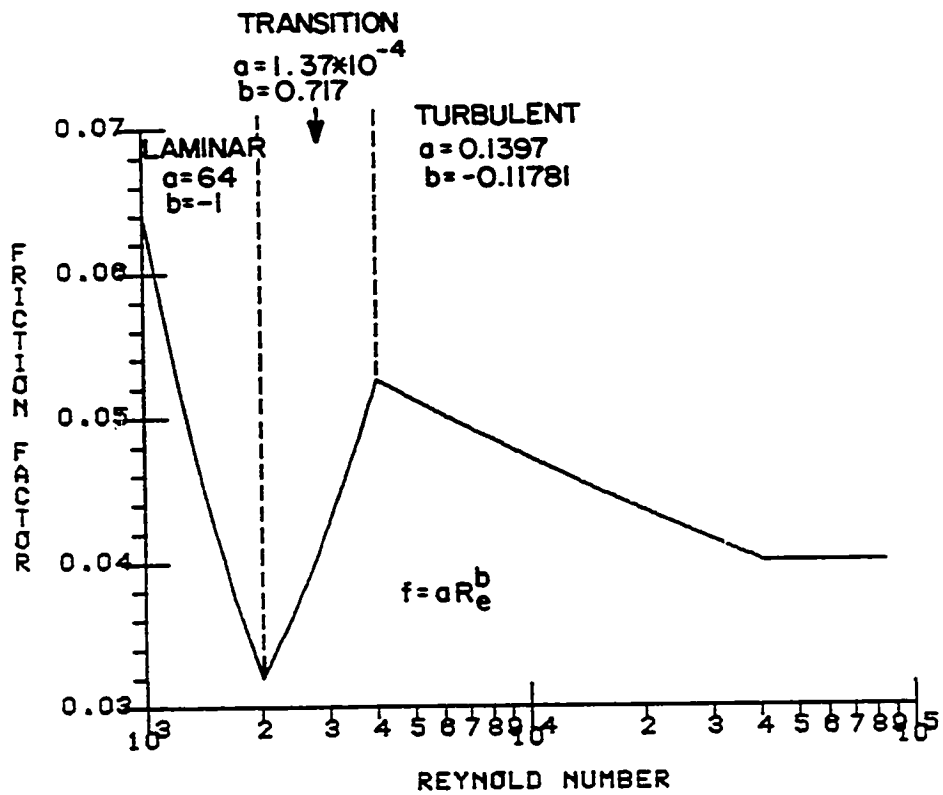


Figure K.6: Scaled Down Brake Pipe Friction Vs. The Reynolds Number
(Theoretical Result)

a and b are taken the following values:-

a. laminar region

a = 64. and b=-1.

b. Transition region

$a=3.75 \times 10^{-4}$ and b = .58

c. turbulent region

a = .154 and b=-.1403

APPENDIX L

AIR BRAKE SYSTEM SIMULATION PROGRAM

L.1 General Aspects and Program Structure

The Air Brake System Simulation Program (ABSSP) is a combination of several mathematical models, representing the brake system components. The program was designed for easy modification to accept further air brake system components. Right now, the following models are included in the simulation program:

1. Brake pipe model, which allows a different number of cars for each simulation run and, different lengths, leakage sizes and brake pipe cross sectional areas for each car in the train.
2. 26C locomotive models;
 - a- complete,and, b- modified.
3. Control valves
 - a- ABDand, b- ABDW models.

The program accepts upto five different valves with five different design configurations for each of them. Each car can have upto two groups of control valves with any number of valves for each group.

4. Brake cylinder, and auxiliary and emergency reservoirs. The program can accept upto ten different configurations for them.

5. A19 flow indicator adaptor model.

One of two different numerical techniques may be used to provide the solutions for the brake pipe model equations (partial differential). These techniques are:

1. Finite difference;
 - a. explicit
 - and, b. implicit formulations.
2. Implicit finite element formulation.

Runge Kutta method is used to solve the ordinary differential equations, which may be part of 26C locomotive valve, control valves, A19 valve and reservoirs mathematical models.

The program consists of several segments, named as follows (see figure (L.1)):

- TABLES SET
- BP NUMERICAL
- 26C VALVE
- CONTROL VALVES
- RESBCLP
- A19
- LEAK

RUNG subroutine and FLOW function are called by many of the program segments. RUNG is used to solve N first order

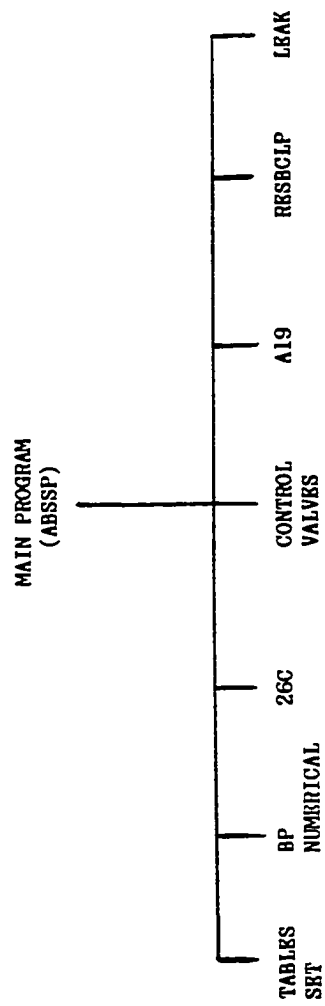


Figure L.1: Air Brake System Simulation Program Segments

differential equations, which may take one of the following forms:

Pressure Form:

$$\frac{dY(i)}{dT} = \frac{1}{V(i)} \left\{ AM(i) - Y(i)R(i)X1 \right\} \quad (L.1)$$

Acceleration Form:

$$\dot{Y}(1) = -R1(1) \quad (L.2a)$$

$$\dot{Y}(2) = Y(1) \quad (L.2b)$$

The dummy variable used in RUNG are:

RUNG(A,B,NI,Y,V,YPRIME,N)

A → T, for most problems, set this to 0.0

B → T + DDT/K

DDT → the normalized integration time step used by the main program.

K → the number of integration steps used by the module, which is using RUNG.

= min(ΔX_i), implicit techniques,

= .5*min(ΔX_i), explicit techniques.

ΔX_i → the normalized length of the i^{th} car by the total length of the train.

NI → the number of integration steps used by RUNG.

= 1 normally for most problems.

Y → the dependent variable (say normalized pressure).

V → independent variable (say volume per train unit length with m^2 as a unit). It is not used in the

case of Acceleration Form solution, and it may take any value.

YPRIME it may take one of the following function forms:

PREL or DIS (you need to specify the EXTERNAL PREL,DIS at the beginning of the module uses RUNG). PREL is used in the case of solving Pressure Form. DIS is used to solve Acceleration Form.

N → Number of equations to be integrated. In the case of Pressure Form solution, N can have a value less than or equal 6. In the case of Acceleration Form, N takes a value of 2.

In the case of Pressure Form solution, the PRFUN common block is used:

```
COMMON/PRFUN/R(6),AM(6),X1
```

R(i) → 1, for variable volume,
2, for fixed volume.

AM(i) → net flow rate, which may be a result of the FLOW function. It has the area unit (m^2).

X1 → the multiplication of the normalized diaphragm or piston velocity {normalized by the total length of the train (TL)} by the diaphragm or piston area. It has the unit of area (m^2).

In the case of Acceleration Form solution, the DISP common is used:

COMMON/DISP/AL(2),R1(2)

where,

AL(1) = 0., AL(2) = 1.,

R1(1) = $-\dot{Y}(1)$ and R1(2) = 0.0

FLOW function is used to evaluate the normalized flow rate through a restriction (fixed or variable) using the average density equation formulation. The mass flow (say AM(1)) through a restriction A is evaluated using this form:

AM(1) = .507*FLOW

$$\text{FLOW} = \text{AB} \sqrt{\left| 1. - \left[\frac{\text{C}}{\text{B}} \right]^2 \right|} \frac{|\text{B} - \text{C}|}{\text{B} - \text{C}} \quad (\text{L.3})$$

The function is called in this manner:

FLOW(A,B,C)

where,

A + restriction area in m².

B + restriction upstream pressure, normalized by
the atmospheric pressure.

C + restriction downstream pressure, normalized by
the atmospheric pressure.

L.2 Program Segments and Modules Outlines

Most of the ABSSP segments consist of several modules. The following sub-sections describe these modules under their abbreviated names.

L.2.1 TABLES SET

This segment constructs the air brake system, sets the configuration of the 26C locomotive valve and control valves and initializes pressure and mass flow rate for different components of the system. It achieves these goals through three different modules and five subroutines. Figure (L.2) shows the organization chart of the TABLES SET segment. The modules are TABS, T26C and VALVES.

-TABS constructs the air brake system and evaluates the initial brake pipe pressure and mass flow rate along the entire train (using the implicit finite difference formulation). These goals are controlled by the user data file, "BP.DAT" (Table (L.1) shows a sample of the "BP.DAT" file). The "BP.DAT" may be constructed as follows:-

Line 1

specifies the numerical technique to be used to solve the brake pipe mathematical model ('FDMETHOD' or 'FEMETHOD').

Line 2

specifies the solution formulation ('EXPLICIT' or 'IMPLICIT'). Note that an EXPLICIT finite element method is not available; if the user specifies FEMETHOD, the program will use IMPLICIT FEMETHOD instead.

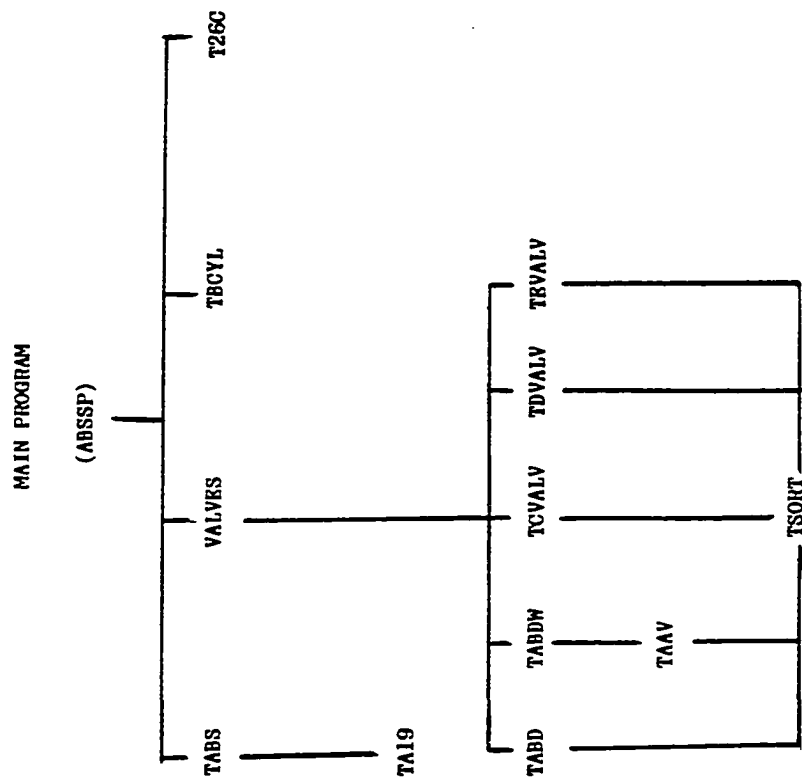


Figure L.2: TABLES SRT Organization Chart

Table: L.1 "BP.DAT" Data File Sample

n=2 and Nfile=2

Line 1 FDMETHOD
Line 2 IMPLICIT
Line 3 MODIFIED
Line 4 20, 1.25, 1000., 5., 60., 1, 20
Line 5 1, 10, 50., 0., 1.25, 60., 2, 1, 1, 1, 0, 0, 0, 0
Line 6 (4+2):
 11, 20, 50., 0., 1.25, 60., 1, 2, 2, 1, 0, 0, 0, 0
Line 7 (5+n):
 1, 30.
Line 8 (6+n):
 .2
Line 9 (7+n):
 2
Line 10 (8+n):
 PBCYL1
Line 11 (8+n+1):
 4, 1, 5, 10, 20
Line 12 (6+n+2Nfile):
 PBP
Line 13 (6+n+2Nfile+1):
 4, 1, 5, 10, 20
Line 14 (8+n+2Nfile):
 61., 37.5, 130., 1
Line 15 (9+n+2Nfile):
 .298, .625, .305, 32., 13., .12, 50.

Line 3

specifies which 26C model to be used ('COMPLETE' or 'MODIFIED').

Line 4 contains:

number of cars, diameter of the brake pipe in the locomotive (in), total length of the train (feet), leakage (psi/min), pressure at which the psi/min is measured (psig), first car that has a leakage, last car that has a leakage.

Line 5 contains:

1, last car in this group, length of the brake pipe per section (feet), leakage diameter {is non-zero only if psi/min was equal to zero} (in), brake pipe diameter (in), branch pipe volume (in³), integer flags of the first control valve group [IV1, IV2, IV3, IV4], flags for the second control valve group [IV5, IV6, IV7, IV8]. Note that;

IV1, IV5 specifies the type of the control valve as:

0 → no valve is used, 1 → ABDW, 2 → ABD, 3 → CVALV, 4 → DVALV, 5 → EVALV, 6 → LOCMOD. Note that if IV1 or IV5 is equal to 6, the rest of the flags will not be used by the program.

IV2, IV6 specifies the design of chosen valve Ex.

1 → ABD, 2 → ABD1, 3 → ABD2, 4 → ABD3, 5 → ABD4

IV3, IV7 specifies the number of valves in each group

IV4, IV8 specifies the design configuration of the brake cylinder, auxiliary and emergency reservoirs

(one of 10 choices listed in BCYL data file. These flags are stored in VALVEF(I,8) matrix as:

VALVEF(I,j) = IVj,

where, I is the brake pipe (section) location.

Line 4+n contains:

information similiar to the one mentioned in Line 5 for the n^{th} group of brake pipe sections (cars).

Line 5+n contains:

number of air brake system operations to be simulated, simulation time required for the first operation,....., simulation time required for the last operation (seconds). See line 8+n+2Nfile+1 for equalizing reservoir pressure commands to be used for each operation.

Line 6+n contains the sampling time for the output files (seconds).

Line 7+n contains the number of output files (Nfile).

Line 8+n contains the name of the first data file

Line 8+n+(1) contains

the number of data (Ndata), $I_1, I_2, \dots, I_{Ndata}$, where I_i specifies the car number.

Note that the data file name may be chosen from the followings:

- PBP brake pipe pressure output file
- PBCYL1 brake cylinder pressure output file for the first control valve group.
- PAUX1 auxiliary reservoir pressure output file for the first control valve group.

- PEMR1 emergency reservoir pressure output file for the first control valve group.
- PBCYL2 brake cylinder pressure output file for the second control valve group.
- PAUX2 auxiliary reservoir pressure output file for the second control valve group.
- PEMR2 emergency reservoir pressure output file for the second control valve group.
- RELAY relay valve output data file. The program does not expect the line following the output file name "RELAY". So, Line 9+n is not expected. The output file will contain, the pressure difference across the diaphragm, diaphragm displacement, equalizing reservoir, main reservoir, inner chamber, and output chamber pressures.

line 7+n+2Nfile+(1) contains:

initial equalizing reservoir pressure, first, ...,
last operation equalizing reservoir pressure, main
reservoir pressure (psig), IDRY;

IDRY = 0 if the first operation is a dry charge.

IDRY = 1 if the first operation is a recharge after
emergency.

The following modules data files may contain one or
more of these characters:

d_{EXP} is a restriction diameter, connected to the brake
pipe, (in),

D_{EXP} is a restriction diameter, not connected to the
brake pipe, (in),

A_{EXP} is piston or diaphragm cross-sectional area,
(in²),

V_{EXP} is a volume, (in³),

K_{EXP} is a spring constant, (lb/in),

L_{EXP} is a spring preload, (lb),

I is the mass of the relay diaphragm, (lb mass)

and, x_{EXP} is diaphragm or piston displacement, (in).

EXP indicates that some subscript expression will be used to
indicate the location or use of part parameters. See Chapter
3 for more detail concerning the above characters.

-TA19 is used to set the design configuration of the A19
flow indicator adaptor. The last line of "BP.DAT"
contains these configurations;

Line 8+n+2Nfile+(1) contains values for A19 flow indicator
adaptor:

D_{SS1} , D_{SS2} , A_{P19} , K_{P19} , L_{P19} , x_{SSM} , V_{19} (see Appendix
I for detailed explanations).

-T26C is used to set the 26C valve configuration for both
the modified and complete models. It reads the "26C"
data file (Table (L.2) shows a sample of the "26C"
file). The data file is constructed as follows:-

Line 1 $D_{1,4}$, $D_{1,6}$, $D_{1,1}$, $D_{1,3}$, $D_{1,5}$.

Line 2 $A_{1,2}$, V_{EQ} , $V_{1,1}$, $V_{1,2}$, $V_{1,3}$.

Line 3 K_D , $K_{1,3}$, $K_{1,1}$, $K_{1,2}$, $L_{1,3}$, $L_{1,1}$, $L_{1,2}$, I .

Table L.2: "26C" Data File Sample

<u>Line 1</u>	.93, .73, .046, .0935, .25
<u>Line 2</u>	12.96, 220., 8., 15., 8.
<u>Line 3</u>	33., 10.1, 9.9, 20.54, 3.7, 6.47, 6.33, 1.
<u>Line 4</u>	.042, .25, -.03, -.16
<u>Line 5</u>	.4, 12., 34.6, .25, .785

Line 4 $X_0, X_S, -X_I, -X_E$

Line 5 $D_{3,3}, K_{3,1}, L_{3,1}, X_B, A_{3,1}$

-*TBCYL* is used to set the design configurations of the brake cylinder and auxiliary and emergency reservoir. *TBCYL* reads the "BCYL" data file, shown in Table (L.3), where,

Line 1 $V_{BI}, D_{BC}, X_{BCM}, L_{BC}, K_{BC}$ (for the 1st brake cylinder configuration).

Line 2 V_{AUX}, V_{EMR} (for the 1st configuration of the emergency and auxiliary reservoirs).

.
.
.

Line 19 $V_{BI}, D_{BC}, X_{BCM}, L_{BC}, K_{BC}$ (for the 10th brake cylinder configuration).

Line 20 V_{AUX}, V_{EMR} (for the 10th configuration of the emergency and auxiliary reservoirs).

Based on the design configurations of brake cylinder and emergency and auxiliary reservoir, and the brake pipe initial condition (pressure distribution along the train), *TBCYL* sets the following variables:

PB(I,3+IDV) brake cylinder pressure at the ith brake pipe section,

PB(I,4+IDV) auxiliary reservoir pressure at the ith brake pipe section, and

PB(I,5+IDV) emergency reservoir pressure at the ith brake pipe section.

Table L.3: "BCYL" Data File Sample

<u>Line 1</u>	170., 10., 8., 200., 10.
<u>Line 2</u>	2500, 3500
<u>Line 3</u>	170., 10., 7., 200., 10.
<u>Line 4</u>	2500, 3500
<u>Line 5</u>	170., 10., 7., 200., 10.
<u>Line 6</u>	2500, 3500
<u>Line 7</u>	170., 10., 7., 200., 10.
<u>Line 8</u>	2500, 3500
<u>Line 9</u>	170., 10., 7., 200., 10.
<u>Line 10</u>	2500, 3500
<u>Line 11</u>	170., 10., 7., 200., 10.
<u>Line 12</u>	2500, 3500
<u>Line 13</u>	170., 10., 7., 200., 10.
<u>Line 14</u>	2500, 3500
<u>Line 15</u>	170., 10., 7., 200., 10.
<u>Line 16</u>	2500, 3500
<u>Line 17</u>	170., 10., 7., 200., 10.
<u>Line 18</u>	2500, 3500
<u>Line 19</u>	170., 10., 7., 200., 10.
<u>Line 20</u>	2500, 3500

where,

IDV sets to 0 if $1 \leq IV1 \leq 5$, first group of valves,

and

IDV sets to 8 if $1 \leq IV5 \leq 5$, second group of valves.

-**VALVES** calls TABDW, TABD, TCVALV, TDVALV or TEVALV, based on the value of (IV1 and IV2) or (IV5 and IV6).

TABDW calls TABD and TAAV to construct the ABDW control valve. TAAV reads the "AAV" data file (see Table (L.4)), which contains the following:

Line 1 $C_8, X_{ECVM}, L_{ECV}, K_{ECV}$.

Line 2 D_{AAV4}, C_3, C_2, C_1 .

Line 3 C_4, C_5, C_6, C_7 .

Line 4 $V_{i,8}$.

(see Appendix H for more detailed explanations for the AAV functions).

-**TABD** reads the "ABD" or "ABDW" data file (see Table (L.5)),

which may contain the following lines;

Line 1 $d_{i,2}, d_{i,3}, d_{i,5}, d_{i,6}, d_{i,7}, D_{QSV}, D_{AUX}, D_{i,5}, D_{BCV}$.

Line 2 $D_{i,3}, X_{CVM}, L_{CV}, K_{CV}, A_{CV}$.

Line 3 $D_0, D_1, D_2, D_3, D_4, D_{QAV2}, A_{SV}, L_{SV}, K_{SV}, W_{SV}$.

Line 4 $D_{INS1}, D_{INS2}, L_{INS}, K_{INS}, A_{INSO}, A_{INSI}, X_{INM}, D_{HP1}, D_{HP2}$.

Line 5 C_0, C_1, H_1 .

Line 6 $H_2, H_3, H_4, H_5, H_6, H_7, H_8$.

Line 7 V_{QA}, V_{QS} .

Table L.4: "AAV" Data File Sample

<u>Line 1</u>	.65, .1, 5.3, 5.21
<u>Line 2</u>	.02, .0156, .055, .031
<u>Line 3</u>	4.5, 8.4, .225, 2.27
<u>Line 4</u>	5.

Table L.5: "ABD" Data File Sample

<u>Line 1</u>	.5, .025, .15, .04, .02, .03125, .063, .0465, .0938
<u>Line 2</u>	.625, .125, 5.25, 12., .6
<u>Line 3</u>	.03, .065, .08, .115, .28, .081, 8.95, .25, 70., .97
<u>Line 4</u>	.096, .625, 27.67, 21.5, 1.85, 1.917, .125, .143, .18
<u>Line 5</u>	.07, .0032, .2
<u>Line 6</u>	.8, .9, 2.4, 3., 11., 24.7, 25.
<u>Line 7</u>	160., 13.6

TCVALV, TDVALV and TEVALV are user defined subroutines, which are used to construct the CVALV, DVALV and EVALV respectively. Each of TABDW, TABD, TCVALV, TDVALV and TEVALV calls SORT subroutine to sort one of the five design for the corresponding valve.

PB(I,16) matrix may be utilized by the user to store time dependent variables (pressure normalized by the atmospheric pressure). Note that, the six locations PB(I,3+IDV), PB(I,4+IDV) and PB(I,5+IDV) are used by TBCYL subroutine for brake cylinder, auxiliary and emergency pressures, where IDV =0 for the first valve group and 8 for the second valve group of a car (section). The other 10 locations may be used in any way desired.

The user should use VALVEF(I,1) and/or VALVEF(I,5) to define the location (brake pipe section) and control valve group for this user valve.

L.2.2 BP NUMERICAL

Figure (L.3) shows the modules (FDMETH and FEMETH), which are used to construct the BP NUMERICAL segment.

-FDMETH is the fortran code used to formulate either explicit or implicit scheme of the finite difference to solve the brake pipe equations. In the case of explicit, FDMETH calls the SLMEXP subroutine to find the pressure and mass flow rate at each car. In the case of implicit, FDMETH calls SLMAT to find the pressure and mass flow rate at each car node.

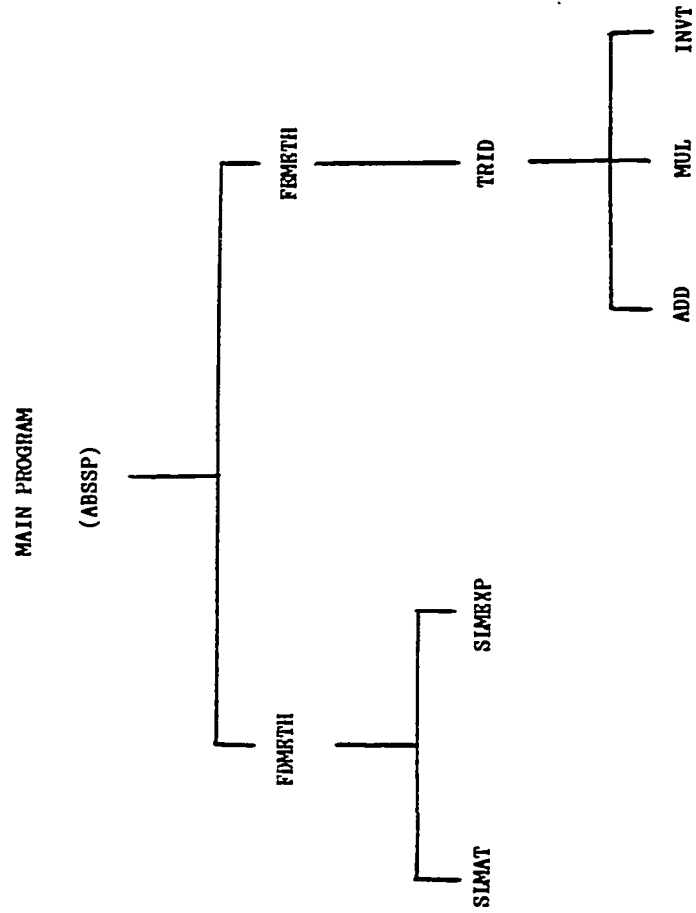


Figure L.3: BP NUMERICAL Organization Chart

-FEMETH constructs the algebraic equations for the implicit finite element formulation. It calls TRID subroutine to solve the block tri-diagonal matrix. TRID calls ADD (adds two matrices), MUL (multiplies two matrices) and INVT (inverts an nXn matrix).

L.2.3 26C VALVE

This segment contains two different models; the 26C complete model (LOCCMP) and the 26C modified model (LOCMOD). Figure (L.4) shows the modules, used to construct 26C VALVE.

-LOCCMP simulates the complete model for the 26C locomotive, described earlier in Section 3.3. It calls RELCMP and REGV. RELCMP is used to solve the relay valve model equations, described in Section 3.3.2 and brake pipe cut-off valve model, described in Section 3.3.3. It utilizes the FLOW (average flow equation) function and RUNG (Runge Kutta method) subroutine. REGV simulates the regulating valve equations, described in Section 3.3.1.

-LOCMOD simulates the modified model of the 26C relay and brake pipe cut-off valve, described in Section 3.4. It calls RELMOD, SOLV, RUNG, and REGV subroutines and FLOW function. RELMOD represents the relay valve displacement equations (3.38) and (3.39). SOLV is used to find the solution for the FUNCTION, described in Section 3.4.1.1.

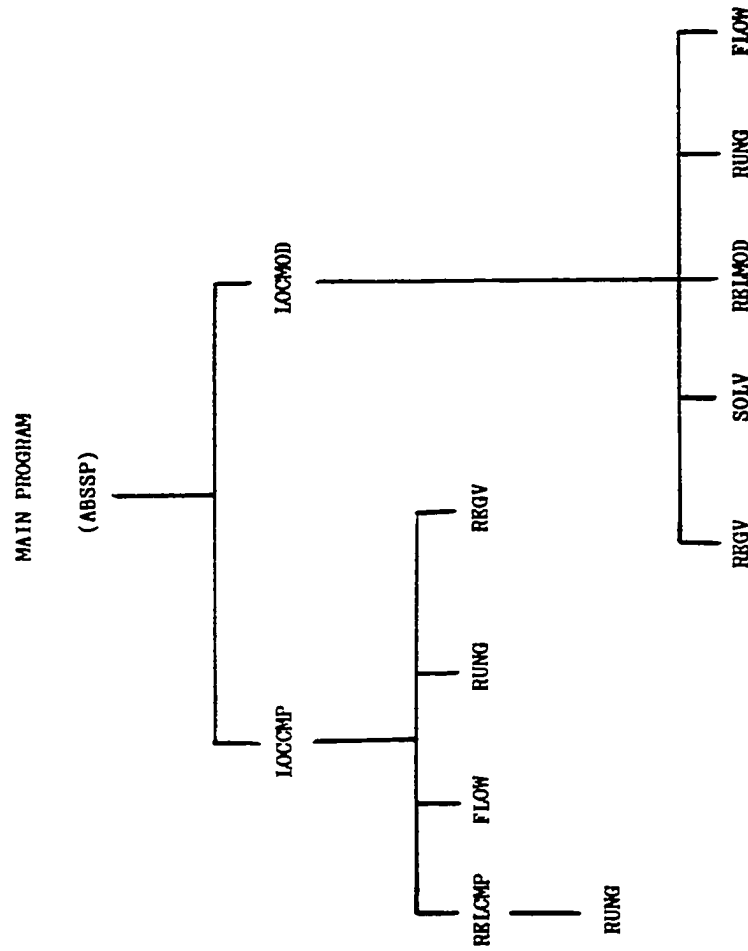


Figure L.4: 26C VALVE Organization Chart

L.2.4 CONTROL VALVES

This segment contains five different modules (ABD, ABDW, CVALV, DVALV and EVALV). Figure (L.5) shows the organization chart of this segment. In this research work, the ABD and ABDW control valve modules were constructed. The CVALV, DVALV and EVALV can be constructed and defined by the user of the ABSSEP. The main program calls a certain control valve module and passes (I,J) to it;

I is the brake pipe (section) location, and
J is valve design configuration number, defined by
 VALVEF(I,2) or VALVEF(I,6).

The main program also uses the ABDV and WRBPS1 common blocks to transfer data to and from the control valve modules;

COMMON/ABDV/PB,CVM

CVM is a four element array;

CVM(1) is the mass flow rates that leaving the brake
 pipe {upstream pressure B = P(I+1) using equation (L.3)}

CVM(2) is the mass flow rates that entering the brake
 cylinder {upstream pressure C = PB(I,3+IDV)
 using equation (L.3)}

CVM(3) is the mass flow rates that entering the
 auxiliary reservoir {upstream pressure C =
 PB(I,4+IDV) using equation (L.3)}

CVM(4) is the mass flow rates that entering the emer-
 gency reservoir {upstream pressure C =
 PB(I,5+IDV) using equation (L.3)}

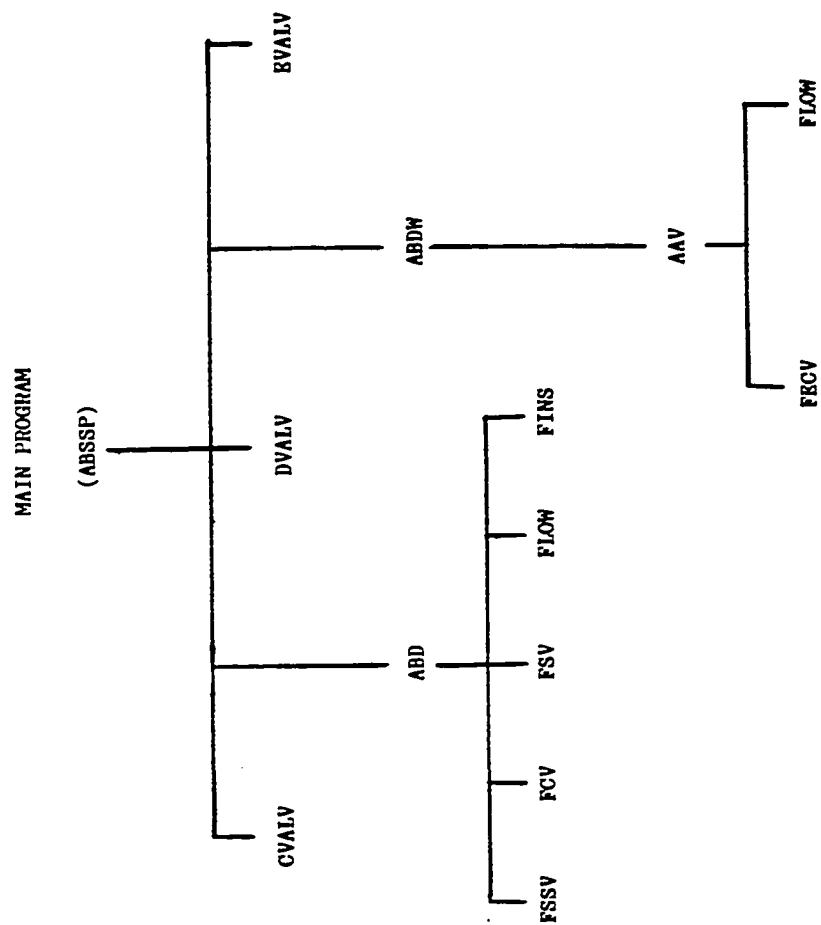


Figure L.5: CONTROL VALVES Organization Chart

CVM(I) should be evaluated using equation (L.3), with the proper dimensions. PB array is defined in TABLES SET section L.2.1.

COMMON/WRBPS/P

P is an array with 201 element long, where, P(I) is the brake pipe pressure at the Ith node (I-1)th section.

The user may need to use the integration time step DDT, which is stored in BPSMP common block, and the total length of the train TLBP, which is stored in BPCOMM common block.

-ABD simulates the different modes of the ABD control valves (Appendix G). ABD calls FSSV, FCV, FSV and FINS functions. These functions represents some of the ABD valve subvalves. FSSV represents the service slide valve, located in the service portion of the ABD control valve. FCV represents the accelerated emergency release check valve, located in the emergency portion of the ABD control valve. FSV simulates the function of the emergency service valve, located in the emergency portion of the ABD control valve. FINS models the inshot valve, located in the emergency portion of the ABD control valve. This segment calls also the FLOW function.

-ABDW represents the ABDW control valve. ABDW calls ABD and AAV modules to construct an ABDW control valve. AAV represents the accelerated application valve. AAV evaluates the A_{QAV2} variable restriction, $P_{i,8}$ and

calls FECV. FECV represents the exhaust check valve of the AAV, which meters the amount of air leaving the quick action chamber.

L.2.5 A19

Figure (L.6) shows the construction of the A19 segment. A19 models the function of the A19 flow indicator adaptor valve. It calls A2A19 function to calculate A_{SS2} variable restriction and the FLOW function to calculate the air flow through the A19 valve.

L.2.6 RESBCLP

RESBCLP segment consists of the BCYL and RESVP modules, as shown in figure (L.7). The BCYL simulates the brake cylinder model. BCYL calculates the brake cylinder pressure, knowing the net air flow rate. RESVP simulates the auxiliary and emergency reservoirs. It calculates the pressures in each of the reservoirs, knowing the respective net flow rates.

L.2.7 LEAK

LEAK simulates the air flow through the leakage at each node along the brake pipe, knowing the pressure at the respective node (car).

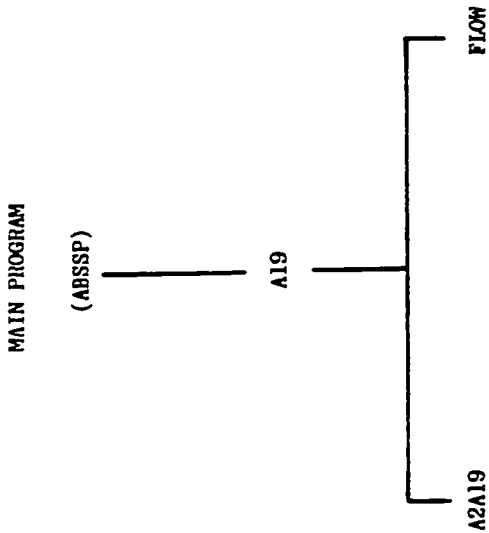


Figure L.6: A19 Organization Chart

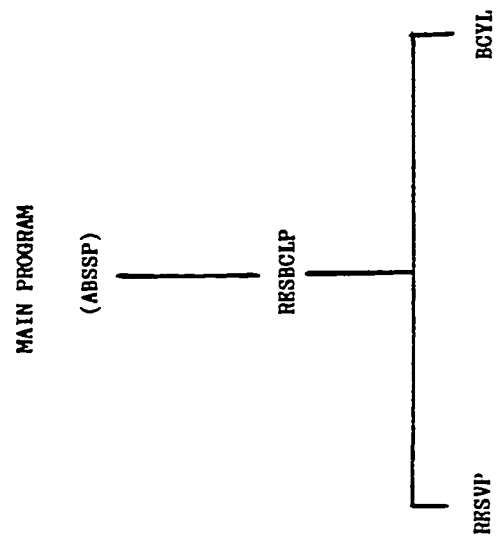


Figure L.7: RESBCLP Organization Chart

```

C*****
PROGRAM ABSP
C*****
C
COMMON/WRBPS/M,FL,DBP,ABP,AV
COMMON/ABDWV/IDFV,AAAV
COMMON/ALVALV/IDV
COMMON/ACDT/DDTA
COMMON/DREGV/AST,AEX,ABPRE
COMMON/WRBPS1/P
COMMON/COEF1/IFLAG
COMMON/BPBCY/DPRES,IDRY
COMMON/COEF/B1,B2,B3,B4,C4,C7,PMAX,DPMAX,PDMAXI,B22
COMMON/ABDV/PB,CVM
COMMON/VALVE/VALVEF,NBV
COMMON/TECH/IMETHD,ITECH,IMODEL
COMMON/BPCOMM/CC,TLBP,P0,IREL,DX(201),RL(201)
COMMON/BPSMP/DDT,NNODE,XR1,INC,REL
COMMON/MPROG/P1(3),PF,TR,N,AC,NE,NOPR,AVTB,DP1,X
COMMON/NEW3/P6,P4
COMMON/ALL/DT1
COMMON/US/TT
COMMON/LOCM/PL
CHARACTER*9 IMETHD,ITECH,IMODEL
REAL AM(3),PT(201),P(201),M(201),DBP(201),ABP(201)
REAL FL(201),AV(201),PT1(201),PF(10),PL(201,4)
REAL PB(201,16),CVM(4)
INTEGER VALVEF(201,8),NE(10)

```

LEGEND

```
C  
C  
C  
C      ABP(I)   Brake pipe cross-sectional area  
C      M(I)     normalized air mass flow rate through the  
C               ith car.  
C      FL(I)    normalized air leakage flow rate from the  
C               ith leakage.  
C      DBP(I)   brake pipe diameter of the ith car.  
C      AV(I)    branch volume per unit length of the ith  
C               control valve.  
C      NOPR     number of air brake system operations to be  
C               simulated  
C      NFILE    number of output data files  
C      FLNM(J)  the jth output file name.  
C      IDATA(J) the number of data variables to be sent  
C               to the jth output data file.  
C      D2A19    diameter of the A19 fixed restriction.  
C      KA19     spring constant of the A19.  
C      LA19     spring preload of the A19  
C      XA19M    maximum displacement of the A19 check valve.  
C      AA19     A19 check valve piston area.  
C      DA19     diameter of the A19 variable restriction.  
C      VA19     volume of the upstream reservoir of the A19.
```

```

C      PSUP      supply pressure (compressor)
C      DDTA      acoustic normalized time step.
C      DX(I)     Distance between two leakage
C      NBV       Number of brake valve (CAR)
C      P(I)      Pressure at node I
C      PB(I,3)   brake cylinder pressure for the first control
C               valve group
C      PB(I,4)   auxiliary reservoir pressure for the first control
C               valve group at the ith car.
C      PB(I,5)   emergency reservoir pressure for the first control
C               Valve group at the ith car.
C      PB(I,6)   quick service pressure for the first control valve
C               group at the ith car.
C      PB(I,7)   quick action pressure for the first control valve
C               group.
C      PB(I,8)   bulb volume pressure for the first group.
C      PB(I,11)  brake cylinder pressure for the second group.
C      PB(I,12)  auxiliary reservoir pressure for the second group.
C      PB(I,13)  emergency " " " " " "
C      PB(I,14)  quick service pressure for the second control valve
C               group at the ith car.
C      PB(I,15)  quick action pressure for the second control valve
C               group.
C      PB(I,16)  bulb volume pressure for the second group.
C      The rest of the PB(I,16) array are user defined pressure
C      variables.
C      CVM(1)    air mass flow rate leaves or enters the brake
C               pipe from the control valve.
C      CVM(2)    air mass flow rate leaves or enters the brake cylinder,
C      CVM(3)    air mass flow rate leaves or enters the auxiliary
C               reservoir
C      CVM(4)    air mass flow rate leaves or enters the emergency
C               reservoir.
C      TLBP     total length of the brake pipe.
C      RL(I)    leakage flow restriction at the ith car.
C      DDT      brake pipe integration time step.
C      NNODE    NBV+1
C      Pl(1)    equalizing reservoir pressure.
C      Pl(2)    main reservoir pressure.
C      Pl(3)    head-end brake pipe pressure.
C      DT1      26C locomotive valve integration time step.
C      DP1      pressure difference across the 26C relay valve
C               diaphragm
C      X        26C relay valve diaphragm displacement.
C
C
C
C=====
C      This program is a fortran simulation of the air brake
C      system. This program is developed by UNH and NYAB
C      company. At UNH, K.S. Abdol-Hamid and D.E. Limbert
C      developed the mathematical models of the air brake
C      system components and wrote the fortran code. At
C      NYAB, L. Vaughn and G. Chapman helped UNH to perform

```

```

C      different experiments on the NYAB experimental set-ups.
C=====
C
C\////////////////////////////////////
C
C      The following are the set of modules used to construct
C      the air brake system for the computer simulation. These
C      modules are referred to as the TABELS SET segment.
C
C      CALL TABS
C      CALL T26C
C      CALL VALVES
C      CALL TBCYL
C\////////////////////////////////////
111 DO 111 I=1,4
      CVM(I)=0.
111 CONTINUE
      XX=0.
      IF(IREL.EQ.1) THEN
        X=.042
        DPl=1.
      END IF
      DT1=.0006666666
      ITER=DDT/DT1
      IF(IMODEL.EQ.'MODIFIED') THEN
        DT1=.00238
        ITER=DDT/DT1
      END IF
      IF(ITER.LT.1) THEN
        ITER=1
        DT1=DDT
      END IF
      ITER1=DDT/.00238
      DDT1=.00238
      T=0.
      DT11=DT1
      DO 1000 KO=1,NOPR
        DT1=DT11
        IF(PF(KO).EQ.0.) DT1=DDT
        DO 30 I=1,NE(KO)
          T=T+DDT
          TT=T*(TLBP/AC)
          IF(PF(KO).EQ.0.) THEN
            Pl(1)=1.
            PL(1,1)=1.
            IF(P(1).LE..01) GOTO 300
            AM3=-.507*FLOW(AVTB,P(1),1.)
            CALL HEAD(AM3)
          ELSE
C\////////////////////////////////////
C
C      The following is the 26C VALVE segment of the air
C      Brake system program.
C

```



```

DO 102 J=1,ITER
  IF(IMODEL.EQ.'COMPLETE') THEN
    CALL LOCCMP(AM,P1,PF(KO),X,DPI)
  END IF
  IF(IMODEL.EQ.'MODIFIED') THEN
    PL(1,3)=P(1)
    CALL LOCMOD(AM,1,PF(KO),X,DPI)
    P1(2)=PL(1,2)
  END IF
  AM3=AM(3)
C
C//////////
  CALL HEAD(AM3)
C
C\//////////
C
C      The following is the A19 segment of the program.
C
C      CALL A19(AM(2),P1(2))
C
C//////////
  P1(3)=P(1)
102  CONTINUE
  END IF
C
C\//////////
C
C      The following is the LEAK segment of the program
C
C      CALL LEAK(NBV)
C
C//////////
  DTA1=DT1
C
DO 200 IV=2,NBV+1
  IIV=IV-1
  IF(PF(KO).EQ.0.) THEN
    TEMR=FLOAT(I)*DDT
    DTEST=FLOAT(IIV)*DDTA
    IF(TEMR.LT.DTEST) GOTO 200
  END IF
  JV=VALVEF(IIV,1)
  IF(JV.EQ.6.AND.PF(KO).NE.0.) THEN
    PL(IV,3)=P(IV)
    DT1=DDTL1
    IVI=IV
DO 103 J=1,ITER1
  CALL LOCMOD(AM,IVI,PF(KO),XX1,DPP1)
C
C
C      The following is the A19 segment of the program.
C
C      CALL A19(AM(2),PL(IV,2))
C

```

```

C////////////////////////////////////
103  CONTINUE
      FL(IIV)=FL(IIV)-AM(3)
      END IF
      IF(JV.EQ.1) IDEF=0
      JJV=VALVEF(IIV,2)
      IF(JV.EQ.0) GOTO 250
      IDV=0
      IV4=VALVEF(IIV,4)
C////////////////////////////////////
C
C      The following is the BRAKE VALVES segment
C
      IF(JV.EQ.1) CALL ABD(IIV,JJV)
      IF(JV.EQ.2) CALL ABDW(IIV,JJV)
      IF(JV.EQ.3) CALL CVALV(IIV,JJV)
      IF(JV.EQ.4) CALL DVALV(IIV,JJV)
      IF(JV.EQ.5) CALL EVALV(IIV,JJV)
      FL(IIV)=FL(IIV)+VALVEF(IIV,3)*CVM(1)
C
C////////////////////////////////////
C
C////////////////////////////////////
C
C      The following is the RESBCLP SEGMENT of the program.
C
      IF(CVM(3).NE.0.0.OR.CVM(4).NE.0.0) CALL RESVP(IIV,IV4)
      IF(CVM(2).NE.0.0) CALL PBCYL(IIV,IV4)
C
C////////////////////////////////////
250  JV=VALVEF(IIV,5)
      JJV=VALVEF(IIV,6)
      IF(JV.EQ.0) GOTO 200
      IF(JV.EQ.1) IDEF=0
      IDV=8
      IV8=VALVEF(IIV,8)
C////////////////////////////////////
C
C      The following is the BRAKE VALVES segment
C
      IF(JV.EQ.1) CALL ABD(IIV,JJV)
      IF(JV.EQ.2) CALL ABDW(IIV,JJV)
      IF(JV.EQ.3) CALL CVALV(IIV,JJV)
      IF(JV.EQ.4) CALL DVALV(IIV,JJV)
      IF(JV.EQ.5) CALL EVALV(IIV,JJV)
C
C////////////////////////////////////
C
C////////////////////////////////////
C
C      The following is the RESBCLP SEGMENT of the program.
C
      IF(CVM(3).NE.0.0.OR.CVM(4).NE.0.0) CALL RESVP(IIV,IV8)
      IF(CVM(2).NE.0.0) CALL PBCYL(IIV,IV8)

```

```

C
C///////////////////////////////////////////////////////////////////
C
      FL(IIV)=FL(IIV)+VALVEF(IIV,7)*CVM(1)
200  CONTINUE
      DT1=DTA1
C
C///////////////////////////////////////////////////////////////////
C
C      The following is the BP NUMERICAL segment of the
C      program.
C
900  IF(IMETHD.EQ.'FEMETHOD') THEN
      CALL FEMETH
    END IF
    IF(IMETHD.EQ.'FDMETHOD') THEN
      CALL FDMETH
    END IF
C
C///////////////////////////////////////////////////////////////////
      DO 101 IJ=1,NNODE
        IF(P(IJ).LT.1) P(IJ)=1.
101  CONTINUE
      AB=0.
      IF(T.GE.XX) THEN
        XX=XX+XR1
        CALL OUTPUT
      END IF
      M(NNODE)=0.
30   CONTINUE
      IFLAG=0
      PDMAX=.0648
      IDRY=1
      IF(PF(KO).EQ.0.) THEN
        DO 1200 JK=1,NBV
          PB(JK,7)=P(JK+1)
          PB(JK,15)=P(JK+1)
1200  CONTINUE
        IDRY=0.
      END IF
1000 CONTINUE
      STOP
      END
C
C*****
      SUBROUTINE TABS
C
C*****
      COMMON/WEBPS/M,FL,DBP,ABP,AV
      COMMON/STS/IST
      COMMON/OUT/NEP,NFILE,IDATA
      COMMON/OUTFL/FLNM
      COMMON/A19AD/D2A19,K19,L19,XA19M,PA19,PA19M,AA19,D1A19
1      ,VA19,PSUP

```

```

COMMON/ACDT/DDTA
COMMON/BPBCY/DPRES, IDRY
COMMON/WRBPS1/P
COMMON/VALVE/VALVEF, NBV
COMMON/BPCOMM/CC, TLBP, PO, IREL, DX(201), RL(201)
COMMON/BPSMP/DDT, NNODE, XRL, INC, REL
COMMON/ALL/DT
COMMON/MPROG/Pl(3), PF, TR, N, AC, NE, NOPR, AVTB, DPl, X
COMMON/LOCSUB/A5, V1
COMMON/MRS/V2
COMMON/BPS/V3
COMMON/DREGV/AST, AEX, ABPRE
COMMON/TEST1/A4, A6, V4, V5, V6, P4, P5, P6, AD, TL, V44, V66, XX1
COMMON/NEW1/XEX, XO, X2, XF
COMMON/NEW3/PU, PD
COMMON/TECH/IMETHD, ITECH, IMODEL
COMMON/ABDV/PB, CVM
COMMON/LOCM/PL
CHARACTER*9 IMETHD, ITECH, IMODEL, ITECH1
REAL D(201), P(201), M(201), DBP(201), ABP(201), AV(201), FL(201)
1      , PB(201,16), CVM(4), PF(10), TF(10), PS(201), PL(201,4)
REAL KA19, LA19
INTEGER VALVEF(201,8), IDATA(21,8), NE(10)
CHARACTER*9 FLNM(8)

C
C
C*****
C      Input the set of data to be used during the program calculation
C      The data file is called BP.DAT.
      OPEN(UNIT=20, FILE='BP.DAT')
      READ(20,30) IMETHD
      READ(20,30) ITECH
      READ(20,30) IMODEL
30     FORMAT(9A)
C      READ NUMBER OF BRAKE VALVE, BRAKE PIPE DIAMETER AND TOTAL LENGTH
      READ(20,*) NBV, DBP1, TLBP, DPlM, PLEAK, ICAR, LCAR
      IF(DPlM.NE.0.) THEN
        NLEAK=LCAR-ICAR+1
        PLEAK=14.7+PLEAK
        FLEAK=.6*FLOW(1., PLEAK, 14.7)
        VLBP=1.*DBP1*DBP1*TLBP*12.
        CLBP=SQRT(286.*293.)/.0254
        DLEAK=NLEAK*FLEAK*CLBP/VLBP
        DLEAK=SQRT(DPlM/(60.*DLEAK))
C        DFRIC=DBP1*DBP1*SQRT(DBP1/(.04*TLBP*12.))
C        DFRIC=SQRT(DFRIC)
C        DLEAK=DLEAK*DFRIC/SQRT(DFRIC*DFRIC-DLEAK*DLEAK)
      END IF
      ALEAK=0.
      NNODE=NBV+1
      NBV1=0
      CC=.714
      AC=342.5
      DMINX=1.

```

```

C      Read the lengths and diameter of each leakage
10     READ(20,*) NBVF,NBVL,DXJ,DJ,DBPJ,AVJ,IV1,IV2,IV3,IV4,IV5
      1,      IV6,IV7,IV8
      NBVT=NBVL-NBVF+1
      DO 11 I=NBVF,NBVL
        AV(I)=AVJ
        VALVEF(I,1)=IV1
        IF(IV1.EQ.1) THEN
          VALVEF(I,2)=IV2+5
        ELSE
          VALVEF(I,2)=IV2
        END IF
        VALVEF(I,3)=IV3
        VALVEF(I,4)=IV4
        VALVEF(I,5)=IV5
        IF(IV5.EQ.1) THEN
          VALVEF(I,6)=IV6+5
        ELSE
          VALVEF(I,6)=IV6
        END IF
        VALVEF(I,7)=IV7
        VALVEF(I,8)=IV8
        DBP(I)=DBPJ
        D(I)=DJ
        DX(I)=DXJ
        AV(I)=(AV(I)/(12.*DX(I)))*.0254*.0254
        DX(I)=DX(I)/TLBP
        DBP(I)=DBP(I)*2.54/100.
        ABP(I)=22.*DBP(I)*DBP(I)/28.
        AV(I)=AV(I)+ABP(I)
        IF(DPLM.NE.0.) THEN
          IF(I.GE.ICAR.AND.I.LE.LCAR) THEN
            D(I)=DLEAK*.0254
          END IF
        ELSE
          D(I)=D(I)*2.54/100.
        END IF
        IF(D(I).NE.0.) THEN
          RL(I)=22.*D(I)*D(I)/28.
        ELSE
          RL(I)=0.
        END IF
        ALEAK=ALEAK+RL(I)
        IF(DX(I).LT.DMINX) DMINX=DX(I)
11     CONTINUE
      NBV1=NBV1+NBVT
      IF(NBV1.LT.NBV) GOTO 10
      AV(NBV+1)=AV(NBV)
      ABP(NBV+1)=ABP(NBV)
      TLBP=TLBP*30.5/100.
      TL=TLBP
      AT=0.
      READ(20,*) NOPR,(TF(K),K=1,NOPR)
      READ(20,*) XR

```

```

      READ(20,*) NFILE
      DO 350 J=1,NFILE
        READ(20,30) FLNM(J)
        IF(FLNM(J).EQ.'RELAY') GOTO 350
        READ(20,*) IDATA(1,J),(IDATA(K,J),K=2,IDATA(1,J)+1)
350    CONTINUE
        DO 40 J=1,NOPR
          AT=AT+TF(J)
40    CONTINUE
        NEP=AT/XR
        DDTA=DMINX
        DDT=DMINX
        IF(IMETHD.EQ.'FEMETHOD') THEN
          ITECH='IMPLICIT'
        END IF
        IF(ITECH.EQ.'EXPLICIT') DDT=.5*DDT
        DT=DDT
        REL=(2./1.77)*100000.*50000.0/(CC*AC)
        XR1=XR*AC/TLBP
        XX=XR1
        READ(20,*) PI,(PF(J),J=1,NOPR),P1(2),IDRY
        P0=PF(1)
        DPRS=(P0-PI)/14.7
        IREL=0
        IF(P0.LT.PI) IREL=1
        P0=(P0+14.7)/14.7
        P1(2)=(P1(2)+14.7)/14.7
        PSUP=P1(2)
        PI=(PI+14.7)/14.7
        P1(1)=PI
        P6=P1(1)
        P1(3)=PI-1./14.7
        IF(IREL.EQ.0) P1(3)=PI
        P4=P1(3)
        PU=P6
        PD=P4
        P5=P1(3)
        DO 20 I=1,NNODE
          PL(I,1)=P1(1)
          PL(I,2)=P1(2)
          PL(I,3)=P1(3)
          PL(I,4)=P5
          M(I)=0.
          P(I)=PI-1./14.7
          IF(IREL.EQ.0) P(I)=PI
20    CONTINUE
          P(1)=PI-1./14.7
          IF(IREL.EQ.0) P(1)=PI
          DO 50 J=1,NOPR
            NE(J)=TF(J)*AC/(DDT*TLBP)
50    CONTINUE
          T=0.
          DDTS=DDT
          ITECH1=ITECH

```

```

      ITECH='IMPLICIT'
      IF(ALBAK.NE.0.) THEN
        IST=1
        DDT=50.*DDTS
        DO 60 I=1,1000
          CALL LEAK(NBV)
          CALL FDMETH
          AA=0.
          AB=0.
          DO 70 J=1,NNODE
            AA=AA+ABS(P(J)-PS(J))
            AB=AB+P(J)
            PS(J)=P(J)
70      CONTINUE
          ERR=AA/AB
          IF(ERR.LE..00001) THEN
            GOTO 80
          END IF
60      CONTINUE
80      DDT=DDTS
      WRITE(5,*) I
      DO 111 J=1,NNODE
C      WRITE(22,*) J,14.7*(P(J)-1.)
C111 CONTINUE
C      STOP
      ITECH=ITECH1
      END IF
      TR=AC/TLBP
      IST=0
      DT=DDT/TR
      TIME=1./TR
      CALL TA19
C      CLOSE(UNIT=15)
      DO 370 I=1,NFILE
        OPEN(UNIT=19+I,FILE=FLNM(I))
        IF(FLNM(I).EQ.'RELAY') THEN
          WRITE(19+I,*) NEP,7
          GOTO 370
        END IF
        WRITE(19+I,*) NEP, IDATA(1,I)+1
370    CONTINUE
      AVTB=(22./(28.))*52*52
      AVTB=AVTB*.0254*.0254
      RETURN
      END
C
C
C*****
      SUBROUTINE T26C
C*****
C
      COMMON/WRBPS/M,FL,DBP,ABP,AV
      COMMON/RELPAR/B0,B1,B2,B3,H0,H1,H2,H3,H4,H5,H6
      COMMON/RELIMT/AXEX,AX0,AX2,AXF

```

```

COMMON/WRBPS1/P
COMMON/COEF/BB1, BB2, BB3, BB4, C4, C7, FMAX, DFMAX, PDMAX, BB22
COMMON/COEF1/IFLAG
COMMON/MRSUP/PP(6), VV(5), AA(6)
COMMON/BPCOMM/CC, TLBP, P0, IREL, DX(201), RL(201)
COMMON/TEMP/ABPR
COMMON/MPROG/P1(3), PF, TR, N, AC, NE, NOPR, AVTB, DPl, X
COMMON/LOCSUB/A5, V1
COMMON/MRS/V2
COMMON/BPS/V3
COMMON/DRELAY/DEX, DCH, AO, ABPRL
COMMON/DREGV/AST, AEX, ABPRE
COMMON/TEST1/A4, A6, V4, V5, V6, P4, P5, P6, AD, TL, V44, V66, XX1
COMMON/NEW/RT, AKD, AK69, AK71, AK74, AL69, AL71, AL74
COMMON/NEW1/XEX, XO, X2, XF
COMMON/NEW3/PU, PD
COMMON/TECH/IMETHD, ITECH, IMODEL
CHARACTER*9 IMETHD, ITECH, IMODEL
REAL D(201), P(201), M(201), DBP(201), ABP(201), AV(201), FL(201)
REAL PF(10), NE(10)
VCONT=.0254*.0254*.0254
PI=22./7.
AST=.0085
AEX=.6014
OPEN(UNIT=29, FILE='26C')
READ(29,*) DEX, DCH, DERF, D0257, D15
READ(29,*) AD, VEQ, V4, V5, V6
READ(29,*) AKD, AK69, AK71, AK74, AL69, AL71, AL74, AIM
AIM=AIM/2.25
READ(29,*) AX2, AX0, AXEX, AXF
READ(29,*) DBPC, AKBPC, ALBPC, XBPCM, ABPCA
A5=PI*DBPC
B0=AD/(AK74+AK69+AKD)
B1=AD/(AK74+AKD)
B2=AD/AKD
B3=AD/(AKD+AK71+AK74)
H3=AL74/AD
H4=AXEX/B2
H2=H3+AX2/B1
H1=H2+AL69/AD
H0=H1+(AX0-AX2)/B0
H5=H4+(AL74-AL71)/AD
H6=H5+(AXF-AXEX)/B3
BB1=-.6*PI*DCH*B0*14.7
BB2=-.6*PI*DCH*B0*H1
BB3=-.6*14.7*DBPC*ABPCA*PI/AKBPC
BB4=-.6*PI*DBPC*ALBPC/AKBPC
C4=.6*PI*DCH*(AX0-AX2)
C7=.6*PI*DBPC*XBPCM
A4=PI*D0257*D0257/4.
A6=PI*DERF*DERF/4.
D15=.6*PI*D15*D15/4.
V1=VCONT*VEQ
IF(IMODEL.EQ.'MODIFIED') V1=VCONT*(VEQ+V4)

```



```

V4=VCONT*V4
V5=VCONT*V5
V6=VCONT*V6
ABPR=1.
ABPR=ABPR/(.0254*.0254)
AST=AST/ABPR
AEX=AEX/ABPR
A5=A5/ABPR
A6=A6/ABPR
A4=A4/ABPR
AO=D15/ABPR
AD=AD/ABPR
ABPRL=ABPR
ABPRE=ABPR
V1=V1/(TLBP)
V4=V4/(TLBP)
V2=V2/(TLBP)
V3=V3/(TLBP)
V5=V5/(TLBP)
V6=V6/(TLBP)
V44=V4
V66=V6
AK69=AK69*172.8
AK71=AK71*172.8
AK74=AK74*172.8
AL69=AL69*4.39
AL71=AL71*4.39
AL74=AL74*4.39
AKD=AKD*172.8
XEX=AXEX*.0254/TLBP
XO=AXO*.0254/TLBP
X2=AX2*.0254/TLBP
XF=AXF*.0254/TLBP
RT=TL/(AIM*1.4*288*293)
IFLAG=0
FMAX=(AKBPC*XBPCM+ALBPC)/(14.7*ABPCA)
DPMAX=.13385
PDMAX=.0648
DCH=.6*DCH
DEX=.6*DEX
CLOSE(UNIT=29)
RETURN
END

```

```

C
C*****
C      SUBROUTINE VALVES
C*****
C
C      COMMON/VALVE/VALVEF,NBV
C      INTEGER VALVEF(201,8)
C      INTEGER TFLAG(5)
C
C
C

```

```

C      This routine is used to select one of the following
C      five valves
C          1. ABD
C          2. ABDW
C          3. CVAL
C          4. DVAL
C          5. EVAL
C
C-----
C
      DO 100 I=1,NBV
        DO 200 K=1,5
          IF(TFLAG(K).EQ.K) GOTO 200
          IF(VALUEF(I,1).EQ.K) TFLAG(K)=K
          IF(VALUEF(I,5).EQ.K) TFLAG(K)=K
200      CONTINUE
100      CONTINUE
        WRITE(5,*) (TFLAG(K),K=1,5)
          IF(TFLAG(1).EQ.1) CALL TABD
          IF(TFLAG(2).EQ.2) CALL TABDW
          IF(TFLAG(3).EQ.3) CALL TCVALV
          IF(TFLAG(4).EQ.4) CALL TDVALV
          IF(TFLAG(5).EQ.5) CALL TEVALV
400      CONTINUE
          RETURN
          END
C*****
      SUBROUTINE SORT(IV)
C*****
C
      COMMON/SORTF/TFLAG
      COMMON/VALVE/VALUEF,NBV
      INTEGER VALUEF(201,8)
      INTEGER TFLAG(10)
C
C-----
C
C-----
C
      DO 100 I=1,10
        TFLAG(I)=0
100      CONTINUE
        IF(IV.EQ.1) THEN
          KI=6
          KF=10
        ELSE
          KI=1
          KF=5
        END IF
        DO 200 I=1,NBV
          IF(VALUEF(I,1).NE.IV.AND.VALUEF(I,5).NE.IV) GOTO 200
          DO 300 J=KI,KF
            IF(TFLAG(J).EQ.J) GOTO 300
            IF(VALUEF(I,1).EQ.IV.AND.VALUEF(I,2).EQ.J) TFLAG(J)=J
          300
        200
      100

```

```

      IF (VALVEF(I,5).EQ.IV.AND.VALVEF(I,6).EQ.J) TFLAG(J)=J
300  CONTINUE
200  CONTINUE
      RETURN
      END
C
C*****
      SUBROUTINE TA19
C*****
C
      COMMON/BPCOMM/CC,TLBP,P0,IHEL,DX(201),RL(201)
      COMMON/A19AD/D2A19,KA19,LA19,XA19M,PA19,PA19M,AA19,D1A19
1      ,VA19,PSUP
      REAL KA19,LA19
C
C-----
C
C      This sybroutine is used to read the data file for
C      A19 flow adapter.
C-----
C
      PI=22./7.
      ACONT=.0254*.0254
      VCONT=ACONT*.0254
      READ(20,*) D1A19,D2A19,AA19,KA19,LA19,XA19M,VA19
      D1A19=PI*ACONT*D1A19*D1A19/4.
      D2A19=PI*ACONT*D2A19
      VA19=VCONT*VA19/TLBP
      PA19=LA19/AA19
      PA19M=(LA19+KA19*XA19M)/AA19
      CLOSE(UNIT=20)
      RETURN
      END
C
C
C*****
      SUBROUTINE A19(AMS,PMR)
C*****
C
      COMMON/A19AD/D2A19,KA19,LA19,XA19M,PA19,PA19M,AA19,D1A19
1      ,VA19,PSUP
      COMMON/ALL/DT1
      REAL KA19,LA19
C
C-----
C
C      This routine is used to simulate the A19 flow adapter.
C-----
C
      IF(AMS.EQ.0.) THEN
          PMR=PSUP
          RETURN
      END IF

```

```

      DP=14.7*(PSUP-PMR)
      A19A=D1A19+A2A19(DP)
      AMM=.507*FLOW(A19A,PSUP,PMR)
      AM=AMM+AMS
      PMR=PMR+AM*DT1/VA19
      RETURN
      END
C
C
      FUNCTION A2A19(DP)
C*****
C
      COMMON/A19AD/D2A19,K19,L19,XA19M,PA19,PA19M,AA19,D1A19
1      ,VA19,PSUP
      REAL K19,L19
C
C
      IF(DP.GT.PA19.AND.DP.LE.PA19M) THEN
        A2A19=D2A19*(DP*AA19-L19)/K19
      ELSE
        A2A19=0.
        IF(DP.GT.PA19M) A2A19=D2A19*XA19M
      END IF
      RETURN
      END
C
C
C*****
      SUBROUTINE OUTPUT
C*****
C
      COMMON/MPROG/P1(3),PF,TR,N,AC,NE,NOPR,AVTB,DPL,X
      COMMON/US/TT
      COMMON/NEW3/P6,P4
      COMMON/ABDV/PB,CVM
      COMMON/WRBPS1/P
      COMMON/OUT/NEP,NFILE,IDATA
      COMMON/OUTFL/FLNM
      REAL PT(20),P(201),M(201),DBP(201),ABP(201)
      REAL FL(201),AV(201),PT1(201),PF(10)
      REAL PB(201,16),CVM(4)
      INTEGER IDATA(21,8),NE(10)
      CHARACTER*9 FLNM(8)
C
C
      DO 200 I=1,NFILE
        IF(FLNM(I).EQ.'PBCYL1') THEN
          DO 210 J=1,IDATA(1,I)
            PT(J)=14.7*(PB(IDATA(J+1,I),3)-1.)
210          CONTINUE
            WRITE(19+I,*) TT,(PT(J),J=1,IDATA(1,I))
          END IF
          IF(FLNM(I).EQ.'PBCYL2') THEN
            DO 220 J=1,IDATA(1,I)

```

```

        PT(J)=14.7*(PB(IDATA(J+1,I),11)-1.)
220    CONTINUE
        WRITE(19+I,*) TT,(PT(J),J=1,IDATA(1,I))
    END IF
        IF(FLNM(I).EQ.'PAUX1') THEN
            DO 230 J=1,IDATA(1,I)
                PT(J)=14.7*(PB(IDATA(J+1,I),4)-1.)
230    CONTINUE
                WRITE(19+I,*) TT,(PT(J),J=1,IDATA(1,I))
            END IF
            IF(FLNM(I).EQ.'PAUX2') THEN
                DO 240 J=1,IDATA(1,I)
                    PT(J)=14.7*(PB(IDATA(J+1,I),12)-1.)
240    CONTINUE
                    WRITE(19+I,*) TT,(PT(J),J=1,IDATA(1,I))
                END IF
                IF(FLNM(I).EQ.'PEMR1') THEN
                    DO 250 J=1,IDATA(1,I)
                        PT(J)=14.7*(PB(IDATA(J+1,I),5)-1.)
250    CONTINUE
                        WRITE(19+I,*) TT,(PT(J),J=1,IDATA(1,I))
                    END IF
                    IF(FLNM(I).EQ.'PEMR2') THEN
                        DO 260 J=1,IDATA(1,I)
                            PT(J)=14.7*(PB(IDATA(J+1,I),13)-1.)
260    CONTINUE
                            WRITE(19+I,*) TT,(PT(J),J=1,IDATA(1,I))
                        END IF
                        IF(FLNM(I).EQ.'PBP') THEN
                            DO 270 J=1,IDATA(1,I)
                                PT(J)=14.7*(P(IDATA(J+1,I))-1.)
270    CONTINUE
                                WRITE(19+I,*) TT,(PT(J),J=1,IDATA(1,I))
                            END IF
                            IF(FLNM(I).EQ.'RELAY') THEN
                                P11=14.7*(P1(1)-1.)
                                P12=14.7*(P1(2)-1.)
                                WRITE(19+I,*) TT,DP1,X,P11,P12,P4,P6
                            END IF
200    CONTINUE
        RETURN
    END

C
C
C*****
    SUBROUTINE TABDW
C*****
    COMMON/ABDWV/IDEF,AAAV
C    This routine will call ABD and AAV to construct an
C    ABDW valve.
C
    IDEF=2
    CALL TAAV
    CALL TABD

```

```

      RETURN
      END
C
C
C*****
      SUBROUTINE TABD
C*****
C
      COMMON/ABDWV/IDF,AAAV
      COMMON/INTP/PINT(201,2)
      COMMON/SORTF/TVALV
      COMMON/VALVE/VALVEF,NBV
      COMMON/WRBPS1/P
      COMMON/BPCOMM/CC,TLBP,PO,IREL,DX(201),RL(201)
      COMMON/ABDV/PB,CVM
      COMMON/RESTR/DAVENT,DABR,DAPERF,DQAS,D201B,DQSV,DAUX
1      ,DEMR,D1C
      COMMON/CV/DCV,KCV,LCV,ACV,XCVM,PCV,PCM
      COMMON/QAC/DO,D1,D2,D3,D4,ASV,LSV,KSV,WSV,PSV,PSVM
      COMMON/QAAR/DQAV2
      COMMON/INS/DINS1,DINS2,LINS,KINS,AINSO,AINSI,XINSM,PINS,PINSM
      COMMON/SSV/C0,C1,H1,PSSVM
      COMMON/LIMIT/H2,H3,H4,H5,H6,H7,H8,DHP1
      COMMON/VOLMS/VQA,VQS,VINT
C
C-----
C
C      This subroutine is used to read the data files
C      for the ABD/ABDW control valve. It can read up
C      to five different designs for the ABD/ABDW valve
C      and it will read all the restrictions, springs,
C      preloads and volumes associated with the dynamics
C      of the ABD/ABDW valve.
C
C      The five different files are:
C          1. ABD
C          2. ABD1
C          3. ABD2
C          4. ABD3
C          5. ABD4
C      Where, ABD is the default valve.
C-----
C
      REAL DAVENT(10),DABR(10),DAPERF(10),DQAS(10),D201B(10)
1      ,DCV(10),LCV(10),KCV(10),ACV(10),XCVM(10),DO(10),D1(10)
2      ,D2(10),D3(10),D4(10),DQAV2(10),ASV(10),LSV(10)
3      ,KSV(10),WSV(10),DINS1(10),DINS2(10),LINS(10),KINS(10)
4      ,AINSO(10),AINSI(10),XINSM(10),C0(10),C1(10),H1(10),H2(10)
5      ,H3(10),H4(10),H5(10),H6(6),H7(10),VQA(10),VQS(10)
6      ,PCV(10),PCVM(10),PSV(10),PSVM(10),PINS(10),PINSM(10)
7      ,PSSVM(10),DQSV(10),DAUX(10),DHP1(10),DEF2(10),H8(10)
8      ,DEMR(10),D1C(10)
      REAL PB(201,16),P(201),CVM(4)

```

```

C
INTEGER TVALV(10), VALVEF(201,8)
CHARACTER*6 TSORT(10)

PI=22./7.
VCONT=.0254*.0254*.0254
ACONT=.0254*.0254
ALCONT=.0254
TSORT(1)='ABDW'
TSORT(2)='ABDW1'
TSORT(3)='ABDW2'
TSORT(4)='ABDW3'
TSORT(5)='ABDW4'
TSORT(6)='ABD'
TSORT(7)='ABD1'
TSORT(8)='ABD2'
TSORT(9)='ABD3'
TSORT(10)='ABD4'
IF(IDEF.NE.2) THEN
CALL SORT(1)
  KI=6
  KF=10
ELSE
  KI=1
  KF=5
END IF
DO 100 I=KI,KF
  IF(TVALV(I).EQ.I) THEN
    OPEN(UNIT=29,FILE=TSORT(I))
  ELSE
    GOTO 100
  END IF
  READ(29,*) DAVENT(I),DABR(I),DAPERP(I),DQAS(I),D201B(I)
1  ,DQSV(I),DAUX(I),DEMR(I),D1C(I)
  READ(29,*) DCV(I),XCV(I),LCV(I),KCV(I),ACV(I)
  READ(29,*) D0(I),D1(I),D2(I),D3(I),D4(I),DQAV2(I),ASV(I)
1  ,LSV(I),KSV(I),WSV(I)
  READ(29,*) DINS1(I),DINS2(I),LINS(I),KINS(I),AINSO(I),AINSI(I)
1  ,XINSM(I),DHP1(I),DHP2(I)
  READ(29,*) C0(I),C1(I),H1(I)
  READ(29,*) H2(I),H3(I),H4(I),H5(I),H6(I),H7(I),H8(I)
  READ(29,*) VQA(I),VQS(I)
  CLOSE(UNIT=29)
  DAVENT(I)=PI*ACONT*DAVENT(I)*DAVENT(I)/4.
  DABR(I)=PI*ACONT*DABR(I)*DABR(I)/4.
  DHP1(I)=PI*ACONT*DHP1(I)*DHP1(I)/4.
  DHP2(I)=PI*ACONT*DHP2(I)*DHP2(I)/4.
  DHP2(I)=DHP2(I)*DHP1(I)/SQRT(DHP2(I)*DHP2(I)+DHP1(I)*DHP1(I))
  DAPERP(I)=PI*ACONT*DAPERP(I)*DAPERP(I)/4.
  DQAS(I)=PI*ACONT*DQAS(I)*DQAS(I)/4.
  DQSV(I)=PI*ACONT*DQSV(I)*DQSV(I)/4.
  DAUX(I)=PI*ACONT*DAUX(I)*DAUX(I)/4.
  DEMR(I)=PI*ACONT*DEMR(I)*DEMR(I)/4.
  D1C(I)=PI*ACONT*D1C(I)*D1C(I)/4.
  D201B(I)=PI*ACONT*D201B(I)*D201B(I)/4.

```

```

DCV(I)=PI*ACONT*DCV(I)
DQAV2(I)=PI*ACONT*DQAV2(I)*DQAV2(I)/4.
D4(I)=ACONT*D4(I)
DINS1(I)=PI*ACONT*DINS1(I)*DINS1(I)/4.
DINS2(I)=PI*ACONT*DINS2(I)
C0(I)=ACONT*C0(I)
C1(I)=ACONT*C1(I)
VQA(I)=VCONT*VQA(I)/TLBP
VQS(I)=VCONT*VQS(I)/TLBP
PSSVM(I)=H1(I)+(C1(I)/C0(I))
PCVM(I)=(ECV(I)*XCV(I)+LCV(I))/ACV(I)
PCV(I)=LCV(I)/ACV(I)
PSVM(I)=(LSV(I)+WSV(I)+KSV(I)*D3(I))/ASV(I)
PSV(I)=(LSV(I)+WSV(I)+KSV(I)*D0(I))/ASV(I)
PINS(I)=(14.7*AINSI(I)+LINS(I)-KINS(I)*XINSM(I))/AINSO(I)
PINSM(I)=(14.7*AINSI(I)+LINS(I))/AINSO(I)
100 CONTINUE
VINT=VCONT*120./TLBP
DO 200 I=1,NBV
  IF (VALVEF(I,1).EQ.1.OR.VALVEF(I,1).EQ.2) THEN
    DO 300 J=1,8
      IF (J.EQ.3.OR.J.EQ.4.OR.J.EQ.5) GOTO 300
      PB(I,J)=1.
300    CONTINUE
      PB(I,7)=P(I+1)
    END IF
    IF (VALVEF(I,5).EQ.1.OR.VALVEF(I,5).EQ.2) THEN
      DO 400 J=1,8
        IF (J.EQ.3.OR.J.EQ.4.OR.J.EQ.5) GOTO 400
        PB(I,J+8)=1.
400    CONTINUE
        PB(I,15)=P(I+1)
      END IF
200    CONTINUE
    DO 500 I=1,NBV
      PINT(I,1)=1.
      PINT(I,2)=1.
500    CONTINUE
  RETURN
END

C
C
C*****
SUBROUTINE TAAV
C*****
C
C
COMMON/SORTF/TVALV
COMMON/WRBPS1/P
COMMON/BPCOMM/CC,TLBP,P0,IHEL,DX(201),RL(201)
COMMON/RCV/DECV,XECVM,LECV,KECV,PECV,PECVM,ARCV
COMMON/DAV/DAAV5,DAAV3,DAVT,D031
COMMON/CC/C4,C5,C6,C7
COMMON/VOL/VAAV

```



```

C
C
C-----
C
C      This routine with the TABD routine are used to
C      read the data files for the ABDW valve. It can read up
C      to five different designs for the ABDW valve
C      and it will read all the restrictions, springs,
C      preloads and volumes associated with the dynamics
C      of the AAV portion.
C
C      The five different files are:
C          1. AAV
C          2. AAV1
C          3. AAV2
C          4. AAV3
C          5. AAV4
C      Where, AAV is the default file for the AAV portion.
C
C-----
C      REAL DECV(5),XECVM(5),LECV(5),KECV(5),DAAV5(5)
1      ,DAAV3(5),DAVT(5),D031(5),C4(5),C5(5),C6(5)
2      ,C7(5),VAAV(5),PECV(5),PECVM(5),AECV(5)
C      INTEGER TVALV(10)
C      CHARACTER*5 TSORT(10)
C
C      PI=22./7.
C      VCONT=.0254*.0254*.0254
C      ACONT=.0254*.0254
C      TSORT(1)='AAV'
C      TSORT(2)='AAV1'
C      TSORT(3)='AAV2'
C      TSORT(4)='AAV3'
C      TSORT(5)='AAV4'
C      CALL SORT(2)
C      DO 100 I=1,5
C          IF(TVALV(I).EQ.I) THEN
C              OPEN(UNIT=29,FILE=TSORT(I))
C          ELSE
C              GOTO 100
C          END IF
C      READ(29,*) DECV(I),XECVM(I),LECV(I),KECV(I)
C      READ(29,*) DAAV5(I),DAAV3(I),DAVT(I),D031(I)
C      READ(29,*) C4(I),C5(I),C6(I),C7(I)
C      READ(29,*) VAAV(I)
C      CLOSE (UNIT=29)
C      AECV(I)=PI*DECV(I)*DECV(I)/4.
C      DECV(I)=PI*ACONT*DECV(I)
C      PECV(I)=LECV(I)/AECV(I)
C      PECVM(I)=(LECV(I)+KECV(I)*XECVM(I))/AECV(I)
C      D031(I)=PI*ACONT*D031(I)*D031(I)/4.
C      DAAV5(I)=PI*ACONT*DAAV5(I)*DAAV5(I)/4.
C      DAAV3(I)=PI*ACONT*DAAV3(I)*DAAV3(I)/4.
C      DAVT(I)=PI*ACONT*DAVT(I)*DAVT(I)/4.

```

```

      VAAV(I)=VCONT*VAAV(I)/TLBP
100  CONTINUE
      RETURN
      END
C
C*****
      SUBROUTINE TCVALV
C*****
C
      COMMON/SORTF/TVALV
      COMMON/VALVE/VALVEF,NBV
      COMMON/WRBPS1/P
      COMMON/BPCOMM/CC,TLBP,P0,IREL,DX(201),RL(201)
      COMMON/ABDV/PB,CVM
      REAL PB(201,16),P(201),CVM(4)
      INTEGER TVALV(10),VALVEF(201,8)
C  P U T   Y O U R   C O M M O N   B L O C K
C      H       E       R       E
C-----
C
C      This subroutine is used to read the data files
C      for the CVALV control valve. It can read up
C      to five different designs for the CVALV valve
C      and it will read all the restrictions, springs,
C      preloads and volumes associated with the dynamics
C      of the CVALV valve.
C
C      The five different files are:
C          1. CVALV
C          2. CVALV1
C          3. CVALV2
C          4. CVALV3
C          5. CVALV4
C      Where, CVALV is the default valve.
C-----
C
C      CHARACTER*6 TSORT(10)
C
C      PUT YOUR REAL/INTEGER STATEMENT HERE AS FOLLOWING
C      REAL/INTEGER   EXP1(5),EXP2(5),.....EXPN(5)
C
      TSORT(1)='CVALV'
      TSORT(2)='CVALV1'
      TSORT(3)='CVALV2'
      TSORT(4)='CVALV3'
      TSORT(5)='CVALV4'
      CALL SORT(3)
      DO 100 I=1,5
        IF(TVALV(I).EQ.I) THEN
          OPEN(UNIT=29,FILE=TSORT(I))
        ELSE
          GOTO 100
        END IF
      END IF

```

```

C
C      PUT YOUR READING STATEMENTS HERE AS FOLLOWING
C      READ(29,*) EXP1(I),EXP2(I),.....EXPN(I)
C
100    CONTINUE
      RETURN
      END
C
C*****
C      SUBROUTINE TDVALV
C*****
C      COMMON/SORTF/TVALV
C      COMMON/VALVE/VALVEF,NBV
C      COMMON/BPCOMM/CC,TLBP,P0,IRRL,DX(201),RL(201)
C      COMMON/ABDV/PB,CVM
C      COMMON/WRBPS1/P
C      REAL PB(201,16),P(201),CVM(4)
C      INTEGER TVALV(10),VALVEF(201,8)
C
C      P U T   Y O U R   C O M M O N   B L O C K
C          H       E       R       E
C-----
C
C      This subroutine is used to read the data files
C      for the DVALV control valve. It can read up
C      to five different designs for the DVALV valve
C      and it will read all the restrictions, springs,
C      preloads and volumes associated with the dynamics
C      of the DVALV valve.
C
C      The five different files are:
C          1. DVALV
C          2. DVALV1
C          3. DVALV2
C          4. DVALV3
C          5. DVALV4
C      Where, DVALV is the default valve.
C-----
C
C      CHARACTER*6 TSORT(10)
C
C      PUT YOUR REAL/INTEGER STATEMENT HERE AS FOLLOWING
C      REAL/INTEGER   EXP1(5),EXP2(5),.....EXPN(5)
C
C      TSORT(1)='DVALV'
C      TSORT(2)='DVALV1'
C      TSORT(3)='DVALV2'
C      TSORT(4)='DVALV3'
C      TSORT(5)='DVALV4'
C      CALL SORT(4)
C      DO 100 I=1,5
C          IF(TVALV(I).EQ.I) THEN
C              OPEN(UNIT=29,FILE=TSORT(I))

```

```

      ELSE
        GOTO 100
      END IF

C
C      PUT YOUR READING STATEMENTS HERE AS FOLLOWING
C      READ(29,*) EXP1(I),EXP2(I),.....EXPN(I)
C
100  CONTINUE
      RETURN
      END

C
C
C*****
      SUBROUTINE TEVALV
C*****
      COMMON/SORTF/TVALV
      COMMON/VALVE/VALVEF,NBV
      COMMON/WEBPS1/P
      COMMON/BPCOMM/CC,TLBP,PO,IREL,DX(201),RL(201)
      COMMON/ABDV/PB,CVM
      REAL PB(201,16),P(201),CVM(4)
      INTEGER TVALV(10),VALVEF(201,8)

C
C      P U T   Y O U R   C O M M O N   B L O C K
C      H       E       R       E
C-----
C
C      This subroutine is used to read the data files
C      for the EVALV control valve. It can read up
C      to five different designs for the EVALV valve
C      and it will read all the restrictions, springs,
C      preloads and volumes associated with the dynamics
C      of the EVALV valve.
C
C      The five different files are:
C          1. EVALV
C          2. EVALV1
C          3. EVALV2
C          4. EVALV3
C          5. EVALV4
C      Where, EVALV is the default valve.
C-----
C
C      CHARACTER*6 TSORT(10)

C
C      PUT YOUR REAL/INTEGER STATEMENT HERE AS FOLLOWING
C      REAL/INTEGER EXP1(5),EXP2(5),.....EXPN(5)
C
      TSORT(1)='EVALV'
      TSORT(2)='EVALV1'
      TSORT(3)='EVALV2'
      TSORT(4)='EVALV3'
      TSORT(5)='EVALV4'

```

```

      CALL SORT(5)
      DO 100 I=1,5
        IF(TVALV(I).EQ.I) THEN
          OPEN(UNIT=29,FILE=TSORT(I))
        ELSE
          GOTO 100
        END IF
C
C      PUT YOUR READING STATEMENTS HERE AS FOLLOWING
C      READ(29,*) EXP1(I),EXP2(I),.....EXPN(I)
C
100   CONTINUE
      RETURN
      END
C
C
C*****
      SUBROUTINE TBCYL
C*****
C      This routine is used to read the brake cylinder
C      and emergency and auxiliary reservoirs data file
C
C-----
      COMMON/BPCOMM/CC,TLBP,P0,IREL,DX(201),RL(201)
      COMMON/VALVE/VALVEF,NBV
      COMMON/AUX/VAUX
      COMMON/EMR/VEMR
      COMMON/ABDV/PB,CVM
      COMMON/WRBPS1/P
      COMMON/BCYL/DBC,VBI,VBF,ABC,LBC,KBC,PBCM,PBC,XBCM
      COMMON/BPBCY/DPRES,IDRY
      REAL P(201),PB(201,16),CVM(4)
      REAL VAUX(10),VEMR(10),DBC(10),VBI(10),VBF(10),ABC(10)
1     ,LBC(10),KBC(10),PBCM(10),PBC(10),XBCM(10)
      INTEGER VALVEF(201,8)
C
      PI=22./7.
      VCONT=.0254*.0254*.0254
      ACONT=VCONT/.0254
      OPEN(UNIT=29,FILE='BCYL')
      DO 10 I=1,10
        READ(29,*) VBI(I),DBC(I),XBCM(I),LBC(I),KBC(I)
        READ(29,*) VAUX(I),VEMR(I)
        VEMR(I)=VEMR(I)*VCONT/TLBP
        VAUX(I)=VAUX(I)*VCONT/TLBP
        ABC(I)=PI*DBC(I)*DBC(I)/4.
        DBC(I)=VCONT*ABC(I)/TLBP
        VBI(I)=VBI(I)*VCONT/TLBP
        VBF(I)=VBI(I)+DBC(I)*XBCM(I)
        PBC(I)=LBC(I)/ABC(I)
        PBCM(I)=(KBC(I)*XBCM(I)+LBC(I))/ABC(I)
10    CONTINUE
      DO 100 J=1,NBV

```

```

IV4=VALVEF(J,4)
IV8=VALVEF(J,8)
VEQ=VBF(IV4)+VAUX(IV4)+VEMR(IV4)
VEQ2=VBF(IV8)+VAUX(IV8)+VEMR(IV8)
IF(IV4.EQ.0) THEN
  PEQ=1.
ELSE
  PEQ=((1.+DPRES)*(VAUX(IV4)+VEMR(IV4))+VBI(IV4))/VEQ
END IF
IF(IV8.EQ.0) THEN
  PEQ2=1.
ELSE
  PEQ2=((1.+DPRES)*(VAUX(IV8)+VEMR(IV8))+VBI(IV8))/VEQ2
END IF
VEQ1=VBF(IV4)+VAUX(IV4)
VEQ12=VBF(IV8)+VAUX(IV8)
IF(DPRES.LT.0.) THEN
  IF(IV4.NE.0) THEN
    PB(J,3)=1.
    PB(J,4)=P(J+1)
    PB(J,5)=P(J+1)
  END IF
  IF(IV8.NE.0) THEN
    PB(J,11)=1.
    PB(J,12)=P(J+1)
    PB(J,13)=P(J+1)
  END IF
ELSE
  IF(P(J+1).LE.1.5) THEN
    IF(IDRY.EQ.0) THEN
      IF(IV4.NE.0) THEN
        PB(J,3)=PEQ
        PB(J,4)=PEQ
        PB(J,5)=PEQ
      END IF
      IF(IV8.NE.0) THEN
        PB(J,11)=PEQ2
        PB(J,12)=PEQ2
        PB(J,13)=PEQ2
      END IF
    ELSE
      PB(J,3)=1.
      PB(J,4)=P(J+1)
      PB(J,5)=P(J+1)
      PB(J,11)=1.
      PB(J,12)=P(J+1)
      PB(J,13)=P(J+1)
    END IF
  ELSE
    IF(IV4.NE.0) PB(J,5)=P(J+1)+DPRES
    IF(IV8.NE.0) PB(J,13)=P(J+1)+DPRES
    IF(IV4.NE.0) THEN
      PEQ1=((P(J+1)+DPRES)*VAUX(IV4)+VBI(IV4))/VEQ1
    END IF
  END IF

```

```

      IF(IV8.NE.0) THEN
        PEQ12=((P(J+1)+DPRES)*VAUX(IV8)+VBI(IV8))/VEQ12
      END IF
      IF(PEQ1.GT.P(J+1)) THEN
        PB(J,3)=PEQ1
        PB(J,4)=PEQ1
      ELSE
        IF(IV4.NE.0) THEN
          PB(J,4)=P(J+1)
          PB(J,3)=(VBI(IV4)+DPRES*VAUX(IV4))/VBF(IV4)
        END IF
      END IF
      IF(PEQ12.GT.P(J+1)) THEN
        PB(J,11)=PEQ12
        PB(J,12)=PEQ12
      ELSE
        IF(IV8.NE.0) THEN
          PB(J,12)=P(J+1)
          PB(J,11)=(VBI(IV8)+DPRES*VAUX(IV8))/VBF(IV8)
        END IF
      END IF
    END IF
  END IF
100 CONTINUE
  CLOSE(UNIT=29)
  RETURN
END

C
C
C*****
C      SUBROUTINE FDMETH
C*****
C
C      THIS SUBROUTINE IS USED TO FORMULATE THE FINITE
C      ELEMENT GLOBAL MATRIX ASS(NNODE-1,4)
C
COMMON/WRBPS/M,FL,DBP,ABP,AV
COMMON/STS/IST
COMMON/VALVE/VALVEF,NBV
COMMON/WRBPS1/P
COMMON/BPCOMM/CC,TLBP,P0,IREL,DX(201),RL(201)
COMMON/BPSMP/DDT,NNODE,XR1,INC,REL
COMMON/TECH/IMETHD,ITECH,IMODEL
CHARACTER*9 IMETHD,ITECH,IMODEL
REAL A1(402),A2(402),A3(402),P(201),M(201),FL(201)
1, A4(402),PU(402),ABP(201),AV(201),DBP(201)
INTEGER VALVEF(201,8)
FL(NNODE)=0.
IC=2
DO 10 I=2,NNODE
  IF(I.EQ.NNODE) THEN
    D3=P(I)*DX(I-1)/DDT
    A2(2*I-1)=DX(I-1)/DDT
    GOTO 11
  
```

```

      END IF
      D3=P(I)*DX(I)/DDT
      A2(2*I-1)=DX(I)/DDT
11    A3(2*I-1)=ABP(I)/AV(I-1)
      A3(2*I)=CC
      A1(2*I-1)=-ABP(I-1)/AV(I-1)
      A1(2*I)=-CC
      D2=-FL(I-1)/AV(I-1)
      A4(2*I-1)=D3+D2
      IF(VALVEF(I-1,1).EQ.6.AND.IST.EQ.1) THEN
        A4(2*I-1)=P(I)*1000000.
        A2(2*I-1)=1000000.
      END IF
      IF(I.NE.NNODE) THEN
        D11=M(I-1)*M(I-1)/P(I-1)
        D12=M(I)*M(I)*ABP(I)/(ABP(I-1)*P(I))
        D1=D11-D12
        RE=REL*DBP(I-1)*ABS(M(I-1))
        US=0.
        IF(M(I-1).NE.0.) US=M(I-1)/ABS(M(I-1))
        D3=M(I)*DX(I)/DDT
        IF(RE.GE.4000.) R=.0924/(RE**.078)
        IF(RE.LT.2000..AND.RE.GT.0.) R=64./RE
        IF(RE.LT.4000..AND.RE.GE.2000.) R=.0001375*RE**.717
        IF(RE.GE.4000.) THEN
          IF(R.LE..04125) R=.04125
        END IF
        G22=US*(M(I-1))*R*DX(I-1)*TLBP/(2.*DBP(I-1)*P(I-1))
12    A2(2*I)=(DX(I-1)/DDT)+G22
      A4(2*I)=D1+D3
      END IF
10    CONTINUE
      A3(2)=CC
      A1(2)=-CC
      D1=DX(1)*M(1)/DDT
      D11=M(1)*M(1)/P(1)
      D12=M(2)*M(2)*ABP(2)/(ABP(1)*P(2))
      D2=D11-D12
      RE=REL*M(1)*DBP(1)
      RE=ABS(RE)
      IF(RE.GE.4000.) R=.0924/(RE**.078)
      IF(RE.LT.2000..AND.RE.GT.0.) R=64./RE
      IF(RE.LT.4000..AND.RE.GE.2000.) R=.0001375*RE**.717
      IF(RE.GE.4000.) THEN
        IF(R.LE..04125) R=.04125
      END IF
      US=0.
      IF(M(1).NE.0.) US=M(1)/ABS(M(1))
      G21=US*(M(1))*R*DX(1)*TLBP/(2.*DBP(1)*P(1))
      A2(2)=G21+(DX(1)/DDT)
      A4(2)=D1+D2
      A4(2)=A4(2)-A1(2)*P(1)
      A1(1)=0.
      A1(2)=0.

```



```

      A3(2*NNODE-1)=0.
      NEQ=2*(NNODE-1)
      IF(ITECH.EQ.'EXPLICIT') THEN
      DO 33 I=1,NNODE-1
        PU(2*I+1)=P(I+1)
        PU(2*I)=M(I)
33    CONTINUE
      CALL SIMEXP(A1(IC),A2(IC),A3(IC),A4(IC),PU(IC),NEQ)
      END IF
      IF(ITECH.EQ.'IMPLICIT') THEN
      CALL SIMAT(A1(IC),A2(IC),A3(IC),A4(IC),PU(IC),NEQ)
C    CALL TRID(A1(IC),A2(IC),A3(IC),A4(IC),PU(IC),NEQ,1)
      END IF
      DO 30 I=1,NNODE-1
        P(I+1)=PU(2*I+1)
        M(I)=PU(2*I)
30    CONTINUE
      PU(NNODE)=0.
      RETURN
      END
C*****
      SUBROUTINE FEMETH
C*****
C
C    THIS SUBROUTINE IS USED TO FORMULATE THE FINITE
C    ELEMENT GLOBAL MATRIX ASS(NNODE-1,4)
C
      COMMON/WRBPS/M,FL,DBP,ABP,AV
      COMMON/WRBPS1/P
      COMMON/BPCOMM/CC,TLBP,P0,IREL,DX(201),RL(201)
      COMMON/BPSMP/DDT,NNODE,XR1,INC,REL
      REAL A1(808),A2(808),A3(808),P(201),M(201),AV(201)
      1, A4(808),PU(201),ABP(201),DBP(201),FL(201)
C-----
C
      NEQS=2*NNODE
      NEQ2=NEQS*NEQS
      A2(1)=2.*DX(1)/(3.*DDT)
      A2(2)=-CC
      A2(3)=ABP(1)/AV(1)
      A2(4)=2.*DX(1)/(3.*DDT)
      A3(1)=DX(1)/(3.*DDT)
      A3(2)=CC
      A3(3)=ABP(1)/AV(1)
      A3(4)=DX(1)/(3.*DDT)
      A4(2)=DX(1)*(M(2)+2.*M(1))/(3.*DDT)
      A4(1)=DX(1)*(P(2)+2.*P(1))/(3.*DDT)
C    FL(1)=(FL(1)-PU1)
      A4(1)=A4(1)-FL(1)/AV(1)
      A4(2)=A4(2)+M(1)*(M(1)/P(1))-ABP(2)*M(2)*M(2)/(ABP(1)*P(2))
      RE=REL*M(1)*DBP(1)
      RE=ABS(RE)
      IF(RE.GE.4000.) R=.0924/(RE*.078)
      IF(RE.LT.2000..AND.RE.GT.0.) R=64./RE

```

```

IF(RE.LT.4000..AND.RE.GE.2000.) R=.0001375*RE**.717
IF(RE.GE.4000.) THEN
  IF(R.LE..04125) R=.04125
END IF
IF(M(1).NE.0.) US=M(1)/ABS(M(1))
G21=US*M(1)*R*DX(1)*TLBP/(2.*DBP(1)*P(1))
A2(4)=A2(4)+G21
DO 10 I=2,NNODE-1
  IL=4*(I-1)
  IL1=2*(I-1)
  A1(1+IL)=(DX(I-1)/(3.*DDT))
  A1(2+IL)=-CC
  A1(3+IL)=-ABP(I-1)/AV(I-1)
  A1(4+IL)=(DX(I-1)/(3.*DDT))
  A2(1+IL)=2.*(DX(I-1)+DX(I))/(3.*DDT)
  A2(2+IL)=0.
  A2(3+IL)=0.
  A2(4+IL)=2.*(DX(I-1)+DX(I))/(3.*DDT)
  A3(1+IL)=(DX(I)/(3.*DDT))
  A3(2+IL)=CC
  A3(3+IL)=ABP(I+1)/AV(I)
  A3(4+IL)=(DX(I)/(3.*DDT))
  D2=(1./AV(I-1))*FL(I-1)+(1./AV(I))*FL(I)
  AI1=.5*((1./ABP(I-1))+(1./ABP(I)))
  AI2=.5*((1./ABP(I+1))+(1./ABP(I)))
  D11=AI1*ABP(I-1)*M(I-1)*M(I-1)/P(I-1)
  D13=(AI1-AI2)*ABP(I)*M(I)*M(I)/P(I)
  D12=AI2*ABP(I+1)*M(I+1)*M(I+1)/P(I+1)
  D1=D11-D12-D13
  RE=REL*M(I-1)*DBP(I-1)
  RE=ABS(RE)
  IF(RE.GE.4000.) R=.0924/(RE**.078)
  IF(RE.LT.2000..AND.RE.GT.0.) R=64./RE
  IF(RE.LT.4000..AND.RE.GE.2000.) R=.0001375*RE**.717
  IF(RE.GE.4000.) THEN
    IF(R.LE..04125) R=.04125
  END IF
  IF(M(I-1).NE.0.) US=M(I-1)/ABS(M(I-1))
  G21=US*M(I-1)*R*DX(I-1)*TLBP/(2.*DBP(I-1)*P(I-1))
  RE=ABS(REL*M(I)*DBP(I))
  IF(RE.GE.4000.) R=.0924/(RE**.078)
  IF(RE.LT.2000..AND.RE.GT.0.) R=64./RE
  IF(RE.LT.4000..AND.RE.GE.2000.) R=.0001375*RE**.717
  IF(RE.GE.4000.) THEN
    IF(R.LE..04125) R=.04125
  END IF
  IF(M(I).NE.0.) US=M(I)/ABS(M(I))
  G22=US*M(I)*R*DX(I)*TLBP/(2.*DBP(I)*P(I))
  A1(4+IL)=A1(4+IL)+G21
  A2(4+IL)=A2(4+IL)+G22
  D3=(P(I-1)*DX(I-1)+P(I+1)*DX(I))/(3.*DDT)
  D4=2.*P(I)*(DX(I-1)+DX(I))/(3.*DDT)
  A4(1+IL1)=-D2+D3+D4
  D3=(M(I-1)*DX(I-1)+M(I+1)*DX(I))/(3.*DDT)

```

```

      D4=2.*M(I)*(DX(I-1)+DX(I))/(3.*DDT)
      A4(2+IL1)=D3+D4+D1
10  CONTINUE
      IL=4*(NNODE-1)
      IL1=2*(NNODE-1)
      N1=NNODE-1
      A1(IL+1)=(DX(N1))/(3.*DDT)
      A1(2+IL)=-CC
      A1(3+IL)=-ABP(N1)/AV(N1)
      A1(IL+4)=(DX(N1))/(3.*DDT)
      A2(1+IL)=2.*(DX(N1))/(3.*DDT)
      A2(2+IL)=CC
      A2(3+IL)=ABP(N1)/AV(N1)
      A2(4+IL)=2.*(DX(N1))/(3.*DDT)
      D2=DX(N1)*(P(N1)+2.*P(NNODE))/(3.*DDT)
      A4(1+IL1)=D2-FL(N1)/AV(N1)
      RE=REL*M(N1)*DBP(N1)
      RE=ABS(RE)
      IF(RE.GE.4000.) R=.0924/(RE*.078)
      IF(RE.LT.2000..AND.RE.GT.0.) R=64./RE
      IF(RE.LT.4000..AND.RE.GE.2000.) R=.0001375*RE*.717
      IF(RE.GE.4000.) THEN
        IF(R.LE..04125) R=.04125
      END IF
      IF(M(N1).NE.0.) US=M(N1)/ABS(M(N1))
      G21=US*M(N1)*R*DX(N1)*TLBP/(2.*DBP(N1)*P(N1))
      A1(4+IL)=A1(4+IL)+G21
      A4(2+IL1)=0.
      A2(4+IL)=10.**10.
      A2(1)=10.**10.
      A4(1)=P(1)*10.**10.
      CALL TRID(A1,A2,A3,A4,PU,NNODE,2)
      DO 30 I=1,NNODE
        P(I)=PU(2*I-1)
        M(I)=PU(2*I)
30  CONTINUE
      M(NNODE)=0.
      RETURN
      END

C
C=====
C
C
C      SUBROUTINE TRID(A1,A2,A3,A4,X,N,L)
C
C
C=====
C
      REAL A1(1),A2(1),A3(1),A4(1)
1,      BETA(808),X(1)
      REAL GAMMA(808),ALPH(808),A(9),B(9),C(9)
C
C      This routine is used to solve a tridiagonal system.
C      Each of the diagonal elements could be a matrix LxL

```

```

C      and the size of the global matrix is N*LxN*L. The
C      algorithm used here is based on Thomas algorithm with
C      some minor modifications.
C
C      A1(L,L,I) is the sub-diagonal element at the I node
C      A2(L,L,I) is the diagonal element at the I node
C      A3(L,L,I) is the super-diagonal element at the I node
C      A4(L,L,I) is the last column element at the I node
C
C      A4(L,I) is the right hand side of the system equations
C      at the I node
C
C      Inialized the dummy matrices A,B,C,D and set C to Unity Matrix
C      DO 1 I=1,L
C      DO 2 J=1,L
C      A(I+(J-1)*L)=0.0
C      B(I+(J-1)*L)=0.0
C      C(I+(J-1)*L)=0.0
2      CONTINUE
1      CONTINUE
C      DO 3 I=1,L
C      C(I+(I-1)*L)=1.
3      CONTINUE
C
C      Set BETA to be equal to the inverse of A2(L,L,1)
C
C      DO 10 I=1,L
C      DO 20 J=1,L
C      BETA(I+(J-1)*L)=A2(I+(J-1)*L)
20      CONTINUE
10      CONTINUE
C
C      CALL INVT(BETA(1),B(1),L)
C      CALL ADD(B(1),A(1),BETA(1),L,L,1.0,0.0)
C
C      CALL MUL(BETA(1),A4(1),GAMMA(1),L,L,1,1,1.)
C
C      DO 30 I=2,N
C      LK=L*L*(I-1)
C      LK1=L*L*(I-2)
C      LK1=LK1+1
C      LK=LK+1
C      LK3=L*(I-1)
C      LK3=LK3+1
C      LK4=L*(I-2)
C      LK4=LK4+1
C
C      CALL MUL(A1(LK),BETA(LK1),ALPH(LK),L,L,L,1.,1.)
C      CALL MUL(ALPH(LK),A3(LK1),B(1),L,L,L,1.,1.)
C      CALL ADD(A2(LK),B(1),BETA(LK),L,L,1.,-1.)

```

```

C
C:.....
      CALL INVT(BETA(LK),A(1),L)
      CALL ADD(A(1),B(1),BETA(LK),L,L,1.,0.)
C:.....
C
C-----
      CALL MUL(A1(LK),GAMMA(LK4),ALPH(LK),L,L,1.,1.)
      CALL ADD(A4(LK3),ALPH(LK),B(1),L,1,1.,-1.)
      CALL MUL(BETA(LK),B(1),GAMMA(LK3),L,L,1,1.,1.)
C-----
C
C
30  CONTINUE
    LN=L*(N-1)
    LN=LN+1
    Solve for the last node array X(I,N) I=1,...,L
C
C-----
      CALL MUL(C(1),GAMMA(LN),X(LN),L,L,1,1.,1.)
C-----
    LAST=N-1
C
C    Using the back substituting, solve for X(I,J)
C    I=1,...,L and J=1,...,N
C
C-----
    DO 50 K=1, LAST
      I=N-K
      LK2=L*I
      LK2=LK2+1
      LK=L*L*(I-1)
      LK1=L*(I-1)
      LK=LK+1
      LK1=LK1+1
      CALL MUL(BETA(LK),A3(LK),B(1),L,L,L,1.,1.)
      CALL MUL(B(1),X(LK2),A(1),L,L,1,1.,1.)
      CALL ADD(GAMMA(LK1),A(1),X(LK1),L,1,1.,-1.)
50  CONTINUE
C-----
    RETURN
    END
C
C
C=====
      SUBROUTINE ADD(X,Y,Z,M,N,F1,F2)
C=====
C
C    This routine can be used to add two matrices together
C    X(M,N) and Y(M,N) and the result is stored in matrix
C    Z(M,N).
C    Z = F1*X + F2*Y  F1 and F2 are scalar
C

```

```

C      REAL X(1),Y(1),Z(1)
C
C      DO 10 I=1, N
C          DO 20 J=1,M
C              Z(J+(I-1)*M)=F1*X(J+(I-1)*M)+F2*Y(J+(I-1)*M)
20      CONTINUE
10      CONTINUE
        RETURN
        END

C
C=====
C      SUBROUTINE MUL(X,Y,Z,M,N,K,F1,F2)
C=====
C
C      This routine can be used to multiply two matrices
C      X(M,N) and Y(N,K) and the result is Z(M,K)
C      Z = F2*X   if F1=0
C      Z = F2*X*Y  and F2 is a scalar
C
C      REAL X(1),Y(1),Z(1)
C
C      IF(F1.EQ.0.) THEN
C          DO 5 I=1,M
C              DO 6 J=1,N
C                  X(I+(J-1)*M)=F2*X(I+(J-1)*M)
6          CONTINUE
5          CONTINUE
          RETURN
      END IF
      DO 10 I=1,K
C          DO 20 J=1,M
C              Z(J+(I-1)*K)=0.0
C              DO 30 L=1,N
C                  Z(J+(I-1)*K)=Z(J+(I-1)*K)+F2*X(J+M*(L-1))*
1          Y(L+(I-1)*K)
30      CONTINUE
20      CONTINUE
10      CONTINUE
        RETURN
        END

C
C
C=====
C      SUBROUTINE INVT(A,B,N)
C=====
C
C      This routine can be used to provide the inverse
C      of A matrix and return in B
C      N is limited to 1 or 2

```

```

C
C
      REAL A(1),B(1)
      IF(N.EQ.1) THEN
        B(1)=1./A(1)
        RETURN
      END IF
      DDTA=A(1)*A(4)-A(2)*A(3)
      B(1)=A(4)/DDTA
      B(2)=-A(2)/DDTA
      B(3)=-A(3)/DDTA
      B(4)=A(1)/DDTA
      RETURN
      END
C*****
      SUBROUTINE LEAK(N)
C*****
C
      COMMON/WRBPS/M,FL,DBP,ABP,AV
      COMMON/WRBPS1/P
      COMMON/BPCOMM/CC,TLBP,PO,IREL,DX(201),RL(201)
      REAL P(201),M(201),FL(201)
      1,      ABP(201),AV(201),DBP(201)
C
      DO 10 I=1,N
        IF(RL(I).EQ.0.) THEN
          FL(I)=0.
          GOTO 10
        END IF
        FL(I)=.6*FLOW(RL(I),P(I+1),I.)/SQRT(1.4)
10      CONTINUE
        IF(FL(2).EQ.0.) THEN
          FL(2)=.002*(P(2)-1.)*ABP(2)
        END IF
      RETURN
      END
C*****
C
      SUBROUTINE HEAD(AM)
C
C*****
C
C
      COMMON/WRBPS/M,FL,DBP,ABP,AV
      COMMON/WRBPS1/P
      COMMON/BPCOMM/CC,TLBP,PO,IREL,DX(201),RL(201)
      COMMON/BPSMP/DDT,NNODE,XR1,INC
      COMMON/ALL/DT1
      REAL D(201),P(201),M(201),DBP(201),ABP(201),AV(201),FL(201)
      A2=DX(1)/DT1
      A3=P(1)
      D3=P(1)*DX(1)/DT1
      A4=D3+(AM/ABP(1))
      P(1)=(A4-M(1))/A2

```

```

      RETURN
      END
C*****
      SUBROUTINE RUNG(A,B,NI,Y,V,YPRIME,N)
      REAL Y(1),V(1)
      REAL G1(6),G2(6),G3(6),G4(6),YY(6)
      R=(B-A)/NI
      S=A
      DO 50 I=1,NI
      DO 1 IK=1,N
      YY(IK)=Y(IK)
      J=IK
      G1(IK)=YPRIME(YY(IK),Y(1),V,J)
      YY(IK)=Y(IK)+(G1(IK)*R/2.)
1      CONTINUE
      DO 2 IL=1,N
      J=IL
      G2(IL)=YPRIME(YY(IL),Y(1),V,J)
      YY(IL)=Y(IL)+(G2(IL)*R/2.)
2      CONTINUE
      DO 3 IM=1,N
      J=IM
      G3(IM)=YPRIME(YY(IM),Y(1),V,J)
      YY(IM)=Y(IM)+(G3(IM)*R)
3      CONTINUE
      DO 4 IN=1,N
      J=IN
      G4(IN)=YPRIME(YY(IN),Y(1),V,J)
      Y(IN)=Y(IN)+R*(G1(IN)+2.*G2(IN)+2.*G3(IN)+G4(IN))/6.
4      CONTINUE
50     CONTINUE
      RETURN
      END
      FUNCTION PREL(Y,DUM,V,I)
      REAL V(1)
      COMMON/PRFUN/R(6),AM(6),X1
      PREL=(AM(I)-X1*Y*R(I))/V(I)
      RETURN
      END
C
      FUNCTION DIS(Y,V,DUM,I)
      COMMON/DISPL/AL(2),R1(2)
      DIS=-R1(I)+V*AL(I)
      RETURN
      END
C
C*****
      SUBROUTINE SIMAT(A1,A2,A3,A4,X,N)
C*****
      REAL X(1),A1(1),R,BETA(402),GAMMA(402)
      I, A2(1),A3(1),A4(1)
      BETA(1)=A2(1)
      GAMMA(1)=A4(1)/BETA(1)
      DO 10 I=2,N

```



```

      BETA(I)=A2(I)-A1(I)*A3(I-1)/BETA(I-1)
      GAMMA(I)=(A4(I)-A1(I)*GAMMA(I-1))/BETA(I)
10    CONTINUE
      X(N)=GAMMA(N)
      LAST=N-1
      DO 20 K=1, LAST
        I=N-K
        X(I)=GAMMA(I)-A3(I)*X(I+1)/BETA(I)
20    CONTINUE
      RETURN
      END
C*****
      SUBROUTINE SIMEXP(A1,A2,A3,A4,X,N)
C*****
      REAL X(1),A1(1),R,BETA(402),GAMMA(402)
      1,      A2(1),A3(1),A4(1)
      X(1)=(A4(1)-A3(1)*X(2))/A2(1)
      DO 1 I=2,N-1
        X(I)=(A4(I)-A1(I)*X(I-1)-A3(I)*X(I+1))/A2(I)
      1 CONTINUE
      X(N)=(A4(N)-A1(N)*X(N-1))/A2(N)
      RETURN
      END
C
C
C*****
C
      SUBROUTINE LOCMOD(AM1, KK, PF, X, DP1)
C
C*****
      REAL AM1(1), PL(201,4), P1IN(201)
      COMMON/LOCSUB/A5,V1
      COMMON/LOCM/PL
      COMMON/RELLMT/AXEX,AXO,AX2,AXF
      COMMON/TEST1/A4,A6,V4,V5,V6,P4,P5,P6,AD,TL,V44,V66,XX1
      COMMON/COEF/B1,B2,B3,B4,C4,C7,PMAX,DPMAX,PDMAXI,B22
      COMMON/COEF1/IFLAG
      COMMON/DREL/ABPC
      COMMON/PRFUN/R(6),AM(6),X1
      COMMON/PRELAY/PHREL,PBP
      COMMON/PREGV/PHREG,PEQ
      COMMON/ALL/DT
      COMMON/MRSUP/PP(6),VV(5),AA(6)
      EXTERNAL PREL
      DP=0.
      R(1)=0.
      PHREL=PL(KK,2)
      PHREG=PL(KK,2)
      PBP=PL(KK,3)
      IF(PL(KK,1).LT.3.) PL(KK,4)=PL(KK,1)
      ARR=ABS(PL(KK,1)-P1IN(KK))
      XRR=ABS(AX2-X)
      IF(IFLAG.EQ.0) THEN
        IF(ARR.LE..00001.AND.XRR.LE..00003) THEN

```

```

      PDMAX=.1
      IFLAG=1
    ELSE
      PDMAX=PDMAXI
    END IF
  END IF
  IF(PL(KK,1).GT.3.1.OR.PBP.GT.3.1) THEN
    IF((PBP+PDMAX).LE.PL(KK,1)) THEN
      PDMAX=.0645
      B2=-B1*PL(KK,1)+B22
      CALL SOLV(PHREL,PBP,PL(KK,1),PL(KK,4))
      GOTO 100
    END IF
    IF((PBP+.005).LE.PL(KK,1).AND.(PBP+PDMAX).GE.PL(KK,1)) THEN
      IF((PL(KK,1)-PBP).LT.0.) PL(KK,4)=PBP-.002
      IF((PL(KK,1)-PBP).GE.0.) PL(KK,4)=PBP+.002
      GOTO 100
    END IF
    IF((PBP+.005).GE.PL(KK,1)) THEN
      P2=1.
      ABPC=0.6*A5
      PRT=(P2*P2-PL(KK,3)*PL(KK,3))*AREL*AREL/(ABPC*ABPC)
      PL(KK,4)=SQRT(PRT+PL(KK,3)*PL(KK,3))
    C    P5=P(3)
      GOTO 200
    END IF
  100  XB=(PL(KK,4)-3.)*14.7*22./(28.*12.)
      ABPC=.6*.25*A5
      IF(XB.LE..25) THEN
        ABPC=.6*A5*XB
      END IF
  200  DP=(PL(KK,1)-PL(KK,4))*14.7
      DPl=DP
      CALL REIMOD(DP,AM(3),X,AREL)
    ELSE
      AM(3)=0.
    END IF
    DP=PF-(PL(KK,1)-1.)*14.7
    PBP=PL(KK,3)
    PEQ=PL(KK,1)
    CALL REGV(AM(1),DP)
    IF(X.GT.AX2) THEN
      AM(2)=-AM(3)
    ELSE
      AM(2)=0.
    END IF
    IF(DP.GT.0.) THEN
      AM(2)=AM(2)-AM(1)
    END IF
    DO 10 I=1,3
      AM1(I)=AM(I)
  10  CONTINUE
      PLIN(KK)=PL(KK,1)
      PL(KK,1)=PL(KK,1)+AM(1)*DT/V1

```

```

      RETURN
      END
C
C
C*****
C
      SUBROUTINE MRSUB(AM1,PMR)
C
C*****
C
      COMMON/PRFUN/R(6),AM(6),X1
      COMMON/MRSUP/PP(6),VV(5),AA(6)
      COMMON/MRS/V2
      COMMON/ALL/DT
      REAL AMM(6)
      EXTERNAL PREL
C
      PP(5)=PMR
      DO 10 I=1,4
      AMM(I)=FLOW(AA(I),PP(I+1),PP(I))/SQRT(1.4)
10    CONTINUE
      AM(1)=AMM(1)
      DO 20 J=1,4
      CALL RUNG(0.,0.+DT,1,PP(J),VV(J),PREL,1)
C      PP(J)=PP(J)+AM(1)*DT/VV(J)
      AM(1)=AMM(J+1)-AMM(J)
20    CONTINUE
C      AM(1)=.2*FLOW(AA(5),PMR,PP(6))
C      CALL RUNG(0.,0.+DT,1,PP(6),VV(4),PREL,1)
      AM(1)=AM1-AMM(4)
      CALL RUNG(0.,0.+DT,1,PMR,V2,PREL,1)
C      PMR=PMR+AM(1)*DT/V2
      RETURN
      END
C
C*****
C
      SUBROUTINE REIMOD(DP,AM,X,AREL)
C
C*****
C
      COMMON/DRELAY/DEX,DCH,AO,ABP
      COMMON/DREL/ABPC
      COMMON/RELPAR/B0,B1,B2,B3,H0,H1,H2,H3,H4,H5,H6
      COMMON/RELIMT/AXEX,AX0,AX2,AXF
      COMMON/PRELAY/PMR,PBP
      COMMON/MRSUP/PP(6),VV(5),AA(6)
C
      PI=22./7.
      AOUT=ABPC*AO/SQRT(AO*AO+ABPC*ABPC)
C
      IF(DP.LT.H6) X=AXF
      IF(DP.GE.H6.AND.DP.LT.H5) X=B3*(DP-H5)+AXEX
      IF(DP.GE.H5.AND.DP.LT.H4) X=AXEX

```

```

      IF(DP.LT.0..AND.DP.GE.H4) X=B2*DP
      IF(DP.GE.0..AND.DP.LT.H3) X=0.
      IF(DP.GE.H3.AND.DP.LT.H2) X=B1*(DP-H3)
      IF(DP.GE.H2.AND.DP.LT.H1) X=AX2
      IF(DP.GE.H1.AND.DP.LT.H0) X=B0*(DP-H1)+AX2
      IF(DP.GE.H0) X=AX0
C
      AM=0.
      AREL=0.
C
      IF(X.GT.AX2.AND.X.LE.AX0) THEN
        AREL=PI*DCH*(X-AX2)/ABP
        AREL=AREL*ABPC/(SQRT(AREL*AREL+ABPC*ABPC))
        AM=FLOW(AREL,FMR,PBP)/SQRT(1.4)
      END IF
C
C
      IF(X.GE.AXF.AND.X.LE.AXEX) THEN
        AREL=-PI*DEX*(X-AXEX)/ABP
        AREL=AREL*AOUT/SQRT(AOUT*AOUT+AREL*AREL)
        AM=FLOW(AREL,1.,PBP)/SQRT(1.4)
      END IF
C
C
      RETURN
      END
C
C
C*****
C
      SUBROUTINE REGV(AM,DP)
C
C*****
C
      COMMON/PREGV/FMR,PEQ
      COMMON/DREGV/AST,AEX,ABP
C
      PI=22./7.
      IF(ABS(DP).LT.24) THEN
        XV=.00378*ABS(DP)
        XV1=.00378*ABS(DP)
      ELSE
        XV=.00378*24.
        XV1=.00378*24.
      END IF
C
      AV1=(.0547-PI*(.132-XV1)**2.)/ABP
      AV=.075*XV/ABP
C
      AV=.12*XV/ABP
      AV=AV*AV1/SQRT(AV*AV+AV1*AV1)
C
      IF(DP.GT.0.) THEN
        AO=AST*AV/SQRT(AST*AST+AV*AV)
        AM=FLOW(AO,FMR,PEQ)/SQRT(1.4)

```

```

      END IF
C
      IF(DP.LT.0.) THEN
        AO=AEX*AV/SQRT(AEX*AEX+AV*AV)
        AM=FLOW(AO,1.,PEQ)/SQRT(1.4)
      END IF
C
      IF(DP.EQ.0.) AM=0.
C
      RETURN
      END
C
C+++++
      FUNCTION FLOW(A,B,C)
C+++++
C
      BB=B-C
      BB1=ABS(BB)
      IF(BB1.LE..01) THEN
        FLOW=0.0
        RETURN
      END IF
      FLOW=A*B*SQRT(ABS(1.-(C/B)**2.))*SIGN(1.,BB)
      RETURN
      END
C
      SUBROUTINE LOCCMP(AM1,P,PF,X,DP1)
C
C*****
      REAL P(1),AM1(1)
      COMMON/LOCSUB/A5,V1
      COMMON/MRSUP/PP(6),VV(5),AA(6)
      COMMON/DREL/ABPC
      COMMON/NEW3/PU,PD
      COMMON/PRFUN/R(6),AM(6),X1
      COMMON/PHRELAY/PHREL,PBP
      COMMON/PREGV/PHREG,PEQ
      COMMON/ALL/DT
      COMMON/TEST1/A4,A6,V4,V5,V6,P4,P5,P6,AD,TL,V44,V66,XX1
      EXTERNAL PREL
      DP=0.
      R(1)=0.
      PHREL=P(2)
      PHREG=P(2)
      PBP=P5
      AM16=.6*FLOW(A6,P(1),P6)/SQRT(1.4)
      X1=XX1
      ARC=0.
      IF(P5.GT.3.) THEN
        XB=(P5-3.)*14.7*22./(28.*12.)
        ARC=.25*A5
        IF(XB.LE..25) THEN
          ARC=A5*XB
        END IF
      END IF

```

```

      END IF
      ABPC=ABC
      AM54=.6*FLOW(A4,P4,P5)/SQRT(1.4)
      AM5B=.6*FLOW(ABPC,P(3),P5)/SQRT(1.4)
      DP=PF-(P(1)-1.)*14.7
      PBP=P5
      PEQ=P(1)
      CALL REGV(AM(1),DP)
      IF(X.GT..042) THEN
        AM(2)=-AM3
      ELSE
        AM(2)=0.
      END IF
      IF(DP.GT.0.) THEN
        AM(2)=AM(2)-AM(1)
      END IF
      AM5=AM54+AM5B+AM3
      DO 10 I=1,2
        AM1(I)=AM(I)
10    CONTINUE
      AM(1)=AM(1)-AM16
      P(1)=P(1)+AM(1)*DT/V1
      AM(1)=AM16
      V6=V66+AD*X*.0254/TL
      R(1)=1.
      CALL RUNG(0.,DT,1,P6,V6,PREL,1)
      AM(1)=-AM54
      V4=V44-AD*X*.0254/TL
      R(1)=-1.
      CALL RUNG(0.,DT,1,P4,V4,PREL,1)
      AM(1)=AM5
      R(1)=0.
      CALL RUNG(0.,DT,1,P5,V5,PREL,1)
      AM1(3)=-AM5B
      DP=(P6-P4)*14.7
      DP1=DP
      CALL RELCMP(DP,AM3,X,AREL)
      PU=14.7*(P6-1.)
      PD=14.7*(P4-1.)
      RETURN
      END

C
C
C*****
C
      SUBROUTINE RELCMP(DP,AM,X,AREL)
C
C*****
C
      REAL XX(2)
      COMMON/DRELAY/DEX,DCH,AO,ABP
      COMMON/DREL/ABPC
      COMMON/PRELAY/FMR,PBP
      COMMON/NEW/RT,AKD,AK69,AK71,AK74,AL69,AL71,AL74

```

```

COMMON/NEW1/XEX,XO,X2,XF
COMMON/TEST1/A4,A6,V4,V5,V6,P4,P5,P6,AD,TL,V44,V66,XX1
COMMON/DISPL/AL(2),R1(2)
COMMON/ALL/DT
EXTERNAL DIS

C
PI=22./7.
AOUT=AO
AL(1)=0.
AL(2)=1.
R1(2)=0.0
DF=12.96*4.388844*RT*DP
I1=0
I2=0
I3=0
IF (XX(2).LT.XEX) THEN
  F71=AL71-TL*AK71*(XX(2)-XEX)
  I1=1
END IF
IF (XX(2).LE.XO.AND.XX(2).GE.XF) THEN
  F74=AL74+TL*AK74*XX(2)
  IF (XX(2).LE.0.0) F74=0.
  IF (XX(2).LE.XEX) F74=AL74+TL*AK74*(XX(2)-XEX)
  I2=1
END IF
IF (XX(2).LE.XO.AND.XX(2).GT.X2) THEN
  F69=AL69+TL*AK69*(XX(2)-X2)
  I3=1
END IF
IF (XX(2).GE.XEX) F71=0.0
IF (XX(2).LE.X2) F69=0.0
IF (XX(2).EQ.XEX) F74=0.0
F71=F71*RT
F69=F69*RT
F74=F74*RT
BETA1=1.5*SQR(TL*(I2*AK74+I3*AK69-I1*AK71+AKD))
IF (XX(2).GT.XEX.AND.XX(2).LE.0.) THEN
  R1(1)=-DF+AKD*RT*XX(2)*TL
  R1(1)=BETA1*XX(1)+R1(1)
ELSE
  R1(1)=BETA1*XX(1)+F74+F69-DF-F71
  R1(1)=R1(1)+AKD*RT*TL*XX(2)
END IF
C IF (XX(2).GT.0.AND.XX(2).LT.X2) R1(1)=1.4*R1(1)
C IF (XX(2).GE.XEX.AND.XX(2).LE.0.) R1(1)=2.5*R1(1)
X22=X
CALL RUNG(0.,DT,1,XX,DUM,DIS,2)
IF (XX(2).LE.XF) THEN
  XX(2)=XF
END IF
IF (XX(2).GE.XO) THEN
  XX(2)=XO
END IF
IF (XX(2).LE.XF.OR.XX(2).GE.XO) THEN

```

```

      XX(1)=0.
      END IF
      AREL=0.
      XX1=AD*XX(1)
      X=XX(2)*TL/.0254
      AM=0.
C
      IF(X.GE..042.AND.X.LE..25) THEN
        AREL=PI*DCH*(X-.042)/ABP
C        AREL=AREL*ABPC/(SQRT(AREL*AREL+ABPC*ABPC))
        AM=FLOW(AREL,PMR,PBP)/SQRT(1.4)
      END IF
C
C
      IF(X.GE.-.16.AND.X.LE.-.03) THEN
        AREL=-PI*DEX*(X+.03)/ABP
        AREL=AREL*AOUT/SQRT(AOUT*AOUT+AREL*AREL)
        AM=FLOW(AREL,1.,PBP)/SQRT(1.4)
      END IF
C
C
      RETURN
      END
C
C
C
C*****
      SUBROUTINE SOLV(PMR,PBP,PEQ,PM)
C*****
C
      COMMON/COEF/B1,B2,B3,B4,C4,C7,FMAX,DPMAX,PDMAX
      COMMON/EXT/FUN,J
      REAL A(5)
      J=0
1      A(1)=-(B2*B2*PMR*PMR+B4*B4*PBP*PBP)
      J=J+1
      A(2)=-2.*(B1*B2*PMR*PMR+B3*B4*PBP*PBP)
      FUN=0.
      FUN1=0.
      IF(PM.GE.FMAX.AND.(PM+DPMAX).LE.PEQ) THEN
        A(5)=0.
        A(4)=0.
        A(3)=C4*C4+C7*C7
        A(2)=0.
        A(1)=-(C4*C4*PMR*PMR+C7*C7*PBP*PBP)
      GOTO 100
      END IF
      IF(PM.GE.FMAX) THEN
        A(5)=B1*B1
        A(4)=2*B1*B2
        A(3)=C7*C7+B2*B2-B1*B1*PMR*PMR
        A(2)=-2.*B1*B2*PMR*PMR
        A(1)=-PBP*PBP*C7*C7-B2*B2*PMR*PMR
      GOTO 100

```



```

      END IF
      IF((PM+DPMAX).LE.PEQ) THEN
        A(5)=B3*B3
        A(4)=2*B3*B4
        A(3)=C4*C4+B4*B4-B3*B3*PBP*PBP
        A(2)=-2.*B3*B4*PBP*PBP
        A(1)=-PMR*PMR*C4*C4-B4*B4*PBP*PBP
        GOTO 100
      END IF
      A(5)=B1*B1+B3*B3
      A(4)=2*(B1*B2+B3*B4)
      A(3)=B2*B2+B4*B4-B1*B1*PMR*PMR-B3*B3*PBP*PBP
100   DO 10 I=1,4
        FUN=FUN+A(I+1)*PM**I
        FUN1=FUN1+I*A(I+1)*PM**(I-1)
10    CONTINUE
      FUN=FUN+A(1)
      PM=PM-(FUN/FUN1)
      IF((PM+PDMAX).GE.PEQ) THEN
        PM=PEQ-.1
        RETURN
      END IF
      IF(PM.LT.3..OR.PM.GT.PEQ) THEN
        IF(PM.LT.PBP) PM=PBP
        IF(PM.GT.PEQ) PM=(PEQ+PBP)/2.
        IF(PM.LT.3.) PM=(3.+PEQ)/2.
        GOTO 1
      END IF
      ERR=ABS(FUN/(PM*FUN1))
      IF(ERR.LE..0001.OR.J.GE.10) RETURN
      GOTO 1
    END
C*****
      SUBROUTINE ABDW(I,J)
C*****
C
      COMMON/ABDV/PB,CVM
      COMMON/ALVALV/IDV
      COMMON/BPSMP/DDT,NNODE,XR1,INC,RRL
      COMMON/AAVF/IAAV,AVT
      COMMON/VOL/VAAV
      COMMON/WRBPS1/P
C
C-----
C
C      This routine is used to simulate the dynamic of the
C      ABDW. It will call AAV portion of the ABDW and then
C      call the ABD control valve.
C-----
C
      REAL PB(201,16),P(201),CVM(4),VAAV(5)
      INTEGER IAAV(201)
C
      CVM(1)=0.

```

```

C      IF(PB(I,8).LT.1.) PB(I,8)=1.
C      IF(PB(I,8).GT.P(I+1)) PB(I,8)=P(I+1)
      CALL AAV(AI8,I,J)
      CALL ABD(I,J)
      II=I+1
      IF(IAAV(I).EQ.2) RETURN
      IF(IAAV(I).NE.0) THEN
        AM=.507*FLOW(AI8,P(II),PB(I,8+IDV))
        CVM(1)=CVM(1)+AM
      ELSE
        AM=-.507*FLOW(AVT,PB(I,8+IDV),1.)
      END IF
200    PB(I,8+IDV)=PB(I,8+IDV)+AM*DDT/VAAV(J)
      RETURN
      END

C
C
C*****
      SUBROUTINE ABD(I,J)
C*****
C
C
      COMMON/ABDFL/NQS,NAUX,NBC
      COMMON/ALVALV/IDV
      COMMON/BPBCY/DPRES,IDRY
      COMMON/INTP/PINT(201,2)
      COMMON/ABDWV/IDF,AAAV
      COMMON/BPSMP/DET,NNODE,XR1,INC,REL
      COMMON/WRBPS1/P
      COMMON/ABDV/PB,CVM
      COMMON/RESTR/DAVENT,DABR,DAPERP,DQAS,D201B,DQSV,DAUX
1      ,DEMR,D1C
      COMMON/CV/DCV,KCV,LCV,ACV,XCVM,PCV,PCM
      COMMON/QAC/D0,D1,D2,D3,D4,ASV,LSV,KSV,WSV,PSV,PSVM
      COMMON/QAAR/DQAV2
      COMMON/INS/DINS1,DINS2,LINS,KINS,AINSO,AINSI,XINSM,PINS,PINSM
      COMMON/SSV/C0,C1,H1,PSSVM
      COMMON/LIMIT/H2,H3,H4,H5,H6,H7,H8,DHP1
      COMMON/VOIMS/VQA,VQS,VINT

C
C-----
C
C      This subroutine is used to model the dynamic of both
C      the ABD and ABDW control valve (after calling the AAV
C      portion subroutine.
C-----
C
      REAL DAVENT(10),DABR(10),DAPERP(10),DQAS(10),D201B(10)
1      ,DCV(10),LCV(10),KCV(10),ACV(10),XCVM(10),D0(10),D1(10)
2      ,D2(10),D3(10),D4(10),DQAV2(10),ASV(10),LSV(10)
3      ,KSV(10),WSV(10),DINS1(10),DINS2(10),LINS(10),KINS(10)
4      ,AINSO(10),AINSI(10),XINSM(10),C0(10),C1(10),H1(10),H2(10)
5      ,H3(10),H4(10),H5(10),H6(6),H7(10),H8(10),VQA(10),VQS(10)
6      ,PCV(10),PCVM(10),PSV(10),PSVM(10),PINS(10),PINSM(10)

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```

7      ,FSSVM(10),DQSV(10),DAUX(10),DHP1(10),HB(10)
8      ,DEMR(10),D1C(10)
      REAL PB(201,16),P(201),CVM(4)
      INTEGER NQS(201),NAUX(201),NBC(201)
C
      DO 10 KK=1,4
          CVM(KK)=0.
10     CONTINUE
          II=I+1
          DPS=14.7*(PB(I,4+IDV)-P(II))
          DPE=14.7*(PB(I,7+IDV)-P(II))
C
C=====
C
C      Evaluate the brake pipe flow rate CVM(1) to the quick
C      action chamber, and then calculate the quick action
C      chamber pressure. The net flow rate within the quick
C      action chamber is QA.
C
C      If the simulation is for the ABDW, do not calculate
C      the quick action vent area AAAV, and use the one previously
C      has been calculated using AAV subroutine.
C
      IF(IDEF.NE.2) THEN
          AAAV1=FSV(DPE,J)
          AAAV=AAAV1*DQAV2(J)/SQRT(AAAV1*AAAV1+DQAV2(J)*DQAV2(J))
      END IF
      CVM(1)=.507*FLOW(D201B(J),P(II),PB(I,7+IDV))
      IF(AAAV.NE.0.) THEN
          AM=CVM(1)-.507*FLOW(AAAV,PB(I,7+IDV),1.)
      ELSE
          AM=CVM(1)
      END IF
      PB(I,7+IDV)=PB(I,7+IDV)+AM*DDT/VQA(J)
      IF(IDRY.EQ.0) THEN
          IF(DPS.LE.H1(J)) GOTO 100
          CVM(3)=.507*FLOW(FSSV(DPS,J),PB(I,3+IDV),PB(I,4+IDV))
          CVM(2)=-CVM(3)
          IF(DPE.LE.(-H6(J))) THEN
              DPB=14.7*(PB(I,3+IDV)-P(II))
              CV=.507*FLOW(FCV(DPB,J),P(II),PB(I,3+IDV))
              CVM(2)=CV-CVM(3)
              CVM(1)=CVM(1)+CV
              RETURN
          END IF
      END IF
C=====
C
C=====
C
100    IF(DPS.GT.0.) THEN
          IF(DPS.GT.0..AND.DPS.LT.H3(J)) THEN
              IF(NAUX(I).NE.2) THEN

```

```

      CVM(3)=.507*FLOW(DAUX(J),P(II),PB(I,4+IDV))
      CVM(1)=CVM(1)+CVM(3)
    END IF
  ELSE
    IF(DPS.GE.H3(J)) THEN
      NAUX(I)=2
    END IF
  END IF
  IF(DPS.GE.H3(J)) THEN
    IF(NQS(I).NE.2) THEN
      IF(DPS.GE.H3(J).AND.DPS.LT.H4(J)) THEN
        CV=.507*FLOW(DQAS(J),P(II),PB(I,6+IDV))
        AM=CV-.507*FLOW(DQSV(J),PB(I,6+IDV),1.)
        CVM(1)=CVM(1)+CV
        PB(I,6+IDV)=PB(I,6+IDV)+AM*DDT/VQS(J)
      ELSE
        IF(DPS.GT.H4(J)) THEN
          NQS(I)=2
          PB(I,6+IDV)=1.
        END IF
      END IF
    END IF
  END IF
  PBC=14.7*PB(I,3+IDV)
  IF(DPE.LE.H2(J)) THEN
    IF(DPS.GE.H4(J).OR.NBC(I).EQ.2) THEN
      AI4=FSSV(DPS,J)
      NBC(I)=2
      QABC=.507*FLOW(AI4,PB(I,4+IDV),PB(I,3+IDV))
      IF(PBC.LE.H7(J)) THEN
        CV=.507*FLOW(DABR(J),P(II),PB(I,3+IDV))
      ELSE
        CV=0.
      END IF
      CVM(1)=CVM(1)+CV
      CVM(2)=CV+QABC
      CVM(3)=-QABC
    END IF
  ELSE
    AINS=FINS(PBC,J)+DINS1(J)
    IF(IDV.EQ.0) THEN
      PPINT=PINT(I,1)
    ELSE
      PPINT=PINT(I,2)
    END IF
    CVM(2)=.507*FLOW(AINS,PPINT,PB(I,3+IDV))
    IF(DPE.GE.H8(J)) THEN
      AREAA=DHP1(J)*AINS/SQRT(DHP1(J)*DHP1(J)+AINS*AINS)
      CVM(4)=-.507*FLOW(AREAA,PB(I,5+IDV),PB(I,3+IDV))
    ELSE
      CVM(4)=0.
    END IF
    AI4=FSSV(DPS,J)
    CVM(3)=.507*FLOW(AI4,PB(I,3+IDV),PB(I,4+IDV))
  END IF

```

```

C      AM=-(CVM(4)+CVM(3)+CVM(2))
C      CVM(2)=- (CVM(3)+CVM(4))
C      IF(IDV.EQ.0) THEN
C        PINT(I,1)=PINT(I,1)+AM*DDT/VINT
C        IF(PINT(I,1).LE.1.) PINT(I,1)=1.
C      ELSE
C        PINT(I,2)=PINT(I,2)+AM*DDT/VINT
C        IF(PINT(I,2).LE.1.) PINT(I,2)=1.
C      END IF
C      CVM(1)=CVM(1)+.507*FLOW(DAVENT(J),P(II),1.)
C      END IF
C      ELSE
C      IF(DPS.LE.(-H3(J))) THEN
C        CVM(1)=.507*FLOW(DAUX(J),P(II),PB(I,4+IDV))
C        CVM(4)=.507*FLOW(DEMR(J),PB(I,4+IDV),PB(I,5+IDV))
C        CVM(3)=CVM(1)-CVM(4)
C        CVM(2)=.504*FLOW(D1C(J),1.,PB(I,3+IDV))
C      END IF
C      IF(DPS.LE.(-H5(J)).AND.PB(I,5+IDV).GT.P(II)) THEN
C        CV=.507*FLOW(DAPERP(J),P(II),PB(I,5+IDV))
C        CVM(4)=CVM(4)+CV
C        CVM(1)=CVM(1)+CV
C      END IF
C      END IF
C      RETURN
C      END
C
C*****
C      SUBROUTINE PBCYL(I,J)
C*****
C
C      COMMON/ABDV/PB,CVM
C      COMMON/ALVALV/IDV
C      COMMON/BPSMP/DDT,NODE,XR1,INC,REL
C      COMMON/BCYL/DBC,VBI,VBF,ABC,LBC,KBC,PBCU,PBCL,XBCM
C      REAL PB(201,16),CVM(4)
C      REAL DBC(10),VBI(10),VBF(10),ABC(10)
C      1      ,LBC(10),KBC(10),PBCL(10),PBCU(10),XBCM(10)
C
C
C
C      This subroutine is used to evaluate the brake
C      cylinder pressure, PINT(I), as a function of time.
C
C
C
C
C      DP=14.7*(PB(I,3+IDV)-1.)
C      DTP=14.7*(CVM(2)*DDT/VBI(J))
C      PBCB=14.7*PB(I,3+IDV)+DTP-14.7
C      IF(PBCB.LE.PBCL(J)) THEN
C        VBC=VBI(J)
C        PB(I,3+IDV)=PB(I,3+IDV)+DTP/14.7
C        IF(PB(I,3+IDV).LE.1.) PB(I,3+IDV)=1.
C      RETURN

```

```

      END IF
      DIF=PBCL(J)-DP
      VBC=DBC(J)*((ABC(J)*(14.7*PB(I,3+IDV)+DP)-LBC(J))/KBC(J))
      VBC=VBC+VBI(J)
      IF(DIF.GT.0.) THEN
        DTP=(CVM(2)*DDT*(1.-DIF/DTP)/VBC)+DIF/14.7
        PB(I,3+IDV)=PB(I,3+IDV)+DTP
        IF(PB(I,3+IDV).LE.1.) PB(I,3+IDV)=1.
        RETURN
      END IF
      IF(DP.LE.PBCL(J)) VBC=VBI(J)
      IF(DP.GE.PBCU(J)) VBC=VBF(J)
      IF(DP.GT.PBCL(J).AND.DP.LT.PBCU(J)) THEN
        VBC=DBC(J)*((ABC(J)*(14.7*PB(I,3+IDV)+DP)-LBC(J))/KBC(J))
        VBC=VBC+VBI(J)
      END IF
      PB(I,3+IDV)=PB(I,3+IDV)+CVM(2)*DDT/VBC
      IF(PB(I,3+IDV).LE.1.) PB(I,3+IDV)=1.
      RETURN
    END

C
C*****
C      SUBROUTINE RESVP(I,J)
C*****
C
C      COMMON/ABDV/PB,CVM
C      COMMON/ALVALV/IDV
C      COMMON/BPSMP/DDT,NNODE,XR1,INC,REL
C      COMMON/AUX/VAUX
C      COMMON/EMR/VEMR
C
C      REAL PB(201,16),CVM(4),VAUX(10),VEMR(10)
C
C-----
C      This subroutine is used to calculate both the auxi-
C      liary and emergency reservoirs pressures, PB(I,4) and
C      PB(I,5) respectively, as a function of time.
C-----
C
C      IF(CVM(3).NE.0.) THEN
C        PB(I,4+IDV)=PB(I,4+IDV)+CVM(3)*DDT/VAUX(J)
C      END IF
C      IF(CVM(4).NE.0.) THEN
C        PB(I,5+IDV)=PB(I,5+IDV)+CVM(4)*DDT/VEMR(J)
C      END IF
C      RETURN
C      END
C
C*****
C      SUBROUTINE CVALV(I,J)
C*****
C
C      COMMON/WRBPS1/P

```

```

COMMON/ALVALV/IDV
COMMON/ABDV/PB,CVM
COMMON/BPSMP/DDT,NNODE,XR1,INC,REL
C PUT YOUR COMMON BLOCK
C H E R E
REAL P(201),PB(201,16),CVM(4)
C
C-----
C This subroutine is a user defined routine for the
C CVALV (C-Valve). You need to return back to the main
C program the following variables
C
C a) CVM(1) is the mass flow rate that leaving the
C brake pipe.
C b) CVM(2) is the mass flow rate that entering the
C brake cylinder
C c) CVM(3) is the mass flow rate that entering the
C auxiliary resrvoir
C d) CVM(4) is the mass flow rate that entering the
C emergency reservoir
C Note, do not modify the following variables
C 1. P(I+1) is brake pipe pressure at the end of the i-th
C section.
C 2. PB(I,3+IDV) is reserved for the brake cylinder pressure.
C 3. PB(I,4+IDV) is reserved for the auxiliary reservoir
C pressure.
C 4. PB(I,5+IDV) is reserved for the emergency reservoir pressure.
C
C The PB(201,16) matrix is used in the form of PB(I,K+IDV).
C I is the section number, and J is the valve design number,
C defined by VALVEF(I,2) or VALLVEF(I,6).
C-----
C
RETURN
END
C
C
C*****
SUBROUTINE DVALV(I,J)
C*****
C
COMMON/WRBPS1/P
COMMON/ALVALV/IDV
COMMON/ABDV/PB,CVM
COMMON/BPSMP/DDT,NNODE,XR1,INC,REL
C PUT YOUR COMMON BLOCK
C H E R E
REAL P(201),PB(201,16),CVM(4)
C
C-----
C This subroutine is a user defined routine for the
C CVALV (C-Valve). You need to return back to the main
C program the following variables
C

```

```

C          a) CVM(1) is the mass flow rate that leaving the
C              brake pipe.
C          b) CVM(2) is the mass flow rate that entering the
C              brake cylinder
C          c) CVM(3) is the mass flow rate that entering the
C              auxiliary reservoir
C          d) CVM(4) is the mass flow rate that entering the
C              emergency reservoir
C      Note, do not modify the following variables
C      1. P(I+1) is brake pipe pressure at the end of the i-th
C          section.
C      2. PB(I,3+IDV) is reserved for the brake cylinder pressure.
C      3. PB(I,4+IDV) is reserved for the auxiliary reservoir
C          pressure.
C      4. PB(I,5+IDV) is reserved for the emergency reservoir pressure.
C
C      The PB(201,16) matrix is used in the form of PB(I,K+IDV).
C      I is the section number, and J is the valve design number,
C      defined by VALVEF(I,2) or VALVEF(I,6).
C-----
C
C      RETURN
C      END
C
C
C*****
C      SUBROUTINE EVALV(I,J)
C*****
C
C      COMMON/WRBPS1/P
C      COMMON/ALVALV/IDV
C      COMMON/ABDV/PB,CVM
C      COMMON/BPSMP/DDT,NNODE,XRL,INC,REL
C      P U T   Y O U R   C O M M O N   B L O C K
C          H       E       R       E
C      REAL P(201),PB(201,16),CVM(4)
C
C-----
C
C      This subroutine is a user defined routine for the
C      CVALV (C-Valve). You need to return back to the main
C      program the following variables
C
C          a) CVM(1) is the mass flow rate that leaving the
C              brake pipe.
C          b) CVM(2) is the mass flow rate that entering the
C              brake cylinder
C          c) CVM(3) is the mass flow rate that entering the
C              auxiliary reservoir
C          d) CVM(4) is the mass flow rate that entering the
C              emergency reservoir
C      Note, do not modify the following variables
C      1. P(I+1) is brake pipe pressure at the end of the i-th
C          section.
C      2. PB(I,3+IDV) is reserved for the brake cylinder pressure.

```


C 3. PB(I,4+IDV) is reserved for the auxiliary reservoir
 C pressure.
 C 4. PB(I,5+IDV) is reserved for the emergency reservoir pressure.
 C
 C The PB(201,16) matrix is used in the form of PB(I,K+IDV).
 C I is the section number, and J is the valve design number,
 C defined by VALVEF(I,2) or VALLVEF(I,6).

C

RETURN
 END

C
 C
 C

C*****
 C SUBROUTINE AAV(AI8,I,J)
 C*****

C
 C

COMMON/QAAR/DQAV2
 COMMON/ALVALV/IDV
 COMMON/ABDWV/IDEF,AAAV
 COMMON/DAV/DAAV5,DAAV3,DAVT,D031
 COMMON/CC/C4,C5,C6,C7
 COMMON/WRBPS1/P
 COMMON/ABDV/PB,CVM
 COMMON/AAVF/IAAV,AVT
 1 REAL DAAV5(5),DAAV3(5),DAVT(5),D031(5),C4(5),C5(5),C6(5)
 ,C7(5)
 REAL PB(201,16),P(201),DQAV2(10)
 INTEGER IAAV(201)

C
 C
 C
 C
 C
 C
 C
 C
 C

This subroutine is used to represent the dynamic of
 the Accelerated Application Valve (AAV) of the ABDW.
 the equivalent area of the AAV, AAAV, and the area
 AI8 are evaluated within this routine.

IDEF=2
 DPE=14.7*(PB(I,7+IDV)-P(I+1))
 AAAV1=FSV(DPE,J)
 IF(AAAV1.EQ.0.) THEN
 IAAV(I)=2
 AAAV=0.
 RETURN
 END IF
 DP3=14.7*(PB(I,1+IDV)-1.)
 DP8=14.7*(PB(I,8+IDV)-1.)
 DPE=14.7*(PB(I,7+IDV)-P(I+1))
 IF(DP3.GT.(C5(J)+C4(J)*DP8)) IAAV(I)=1
 IF(DP3.LE.(C7(J)+C6(J)*DP8)) IAAV(I)=0

```

AAAV3=IAAV(I)*DAAV3(J)
AI8=IAAV(I)*LC31(J)
AVT=(1-IAAV(I))*DAVT(J)
AAAV2=DQAV2(J)
AAAV4=FECV(PB(I,2+IDV),J)
AAAV5=DAAV5(J)
AAV4=AAAV4+AAAV5
AAV3=AAAV2*AAV4/SQRT(AAAV2*AAAV2+AAV4*AAV4)
AAV2=AAAV3+AAV3
AAAV=AAV2*AAAV1/SQRT(AAAV1*AAAV1+AAV2*AAV2)
PI7=PB(I,7+IDV)*PB(I,7+IDV)*AAAV1*AAAV1+AAV2*AAV2
PB(I,1+IDV)=SQRT(PI7/(AAAV1*AAAV1+AAV2*AAV2))
PI1=PB(I,1+IDV)*PB(I,1+IDV)*AAAV2*AAAV2+AAV4*AAV4
PB(I,2+IDV)=SQRT(PI1/(AAAV2*AAAV2+AAV4*AAV4))
RETURN
END

```

```

C
C*****
C      FUNCTION FINS(P,J)
C*****
C
C      COMMON/INS/DINS1,DINS2,LINS,KINS,AINSO,AINSI,XINSM,PINS,PINSM
C
C      REAL DINS1(10),DINS2(10),LINS(10),KINS(10),AINSO(10),AINSI(10)
1      ,XINSM(10),PINS(10),PINSM(10)
C
C-----
C
C      This function will be used to simulate the inshot valve
C      in the case of emergency.
C
C-----
C
C      DP=14.7*AINSI(J)-P*AINSO(J)+LINS(J)
C      IF(P.GT.PINS(J).AND.P.LT.PINSM(J)) THEN
C          FINS=DINS2(J)*DP/KINS(J)
C      ELSE
C          FINS=0.
C          IF(P.LE.PINS(J)) FINS=DINS2(J)*XINSM(J)
C      END IF
C      RETURN
C      END

```

```

C
C*****
C      FUNCTION FSSV(P,J)
C*****
C
C      COMMON/SSV/C0,C1,H1,PSSVM
C      REAL C0(10),C1(10),H1(10),PSSVM(10)
C
C-----
C
C      This function is used to evaluate the flow area between
C      the auxiliary reservoir and the brake cylinder through
C      the service slide valve of the ABD/ABDW.
C
C

```

```

C-----
      IF(P.GE.H1(J).AND.P.LE.PSSVM(J)) THEN
        FSSV=C0(J)*(P-H1(J))
      ELSE
        IF(P.GT.PSSVM(J)) FSSV=C1(J)
        IF(P.LT.H1(J)) FSSV=0.
      END IF
      RETURN
      END

C
C*****
      FUNCTION FCV(P,J)
C*****
C
      COMMON/ CV/DCV,KCV,LCV,ACV,XCVM,PCV,PCM
      REAL DCV(10),KCV(10),LCV(10),ACV(10),XCVM(10),PCV(10),PCVM(10)

C-----
C      This function is used to evaluate the flow area between
C      the brake pipe and the brake cylinder through
C      the accelerated release check valve of the ABD/ABDW.
C-----
      IF(P.GE.PCV(J).AND.P.LE.PCVM(J)) THEN
        FCV=DCV(J)*(P*ACV(J)-LCV(J))/KCV(J)
      ELSE
        IF(P.GT.PCVM(J)) FCV=DCV(J)*XCVM(J)
        IF(P.LT.PCV(J)) FCV=0.
      END IF
      RETURN
      END

C
C
C
C*****
      FUNCTION FECV(P,J)
C*****
C
      COMMON/ ECV/DECV,XECVM,LECV,KECV,PECV,PECVM,AECV
      REAL DECV(5),XECVM(5),LECV(5),KECV(5),DAAV5(5),PECVM(5)
      1      ,PECV(5),AECV(5)

C-----
C      This subroutine is used to calculate the flow area
C      of the Exhaust check Valve (ECV) located in the
C      AAV potion of the ABDW.
C-----
      DP=(P-1.)*14.7
      IF(DP.GT.PECV(J).AND.DP.LT.PECVM(J)) THEN
        XECV=(DP*AECV(J)-LECV(J))/KECV(J)
        FECV=DECV(J)*XECV
      ELSE

```

```

      FECV=0.
      IF(DP.GE.FECVM(J)) FECV=DECV(J)*XECVM(J)
    END IF
    FECV=DECV(J)*XECVM(J)
  C
  RETURN
  END

  C
  C
  C*****
  FUNCTION FSV(P,J)
  C*****
  C
  COMMON/QAC/D0,D1,D2,D3,D4,ASV,LSV,KSV,WSV,PSV,PSVM
  REAL D0(10),D1(10),D2(10),D3(10),D4(10),ASV(10),LSV(10),KSV(10)
  1      ,WSV(10),PSV(10),PSVM(10)

  C
  C-----
  C
  C      This function is used to evaluate the area of the
  C      emergency slide valve opening (AAAV1).
  C-----
  C
  IF(P.GT.PSV(J).AND.P.LT.PSVM(J)) THEN
    XSV=(P*ASV(J)-LSV(J)-WSV(J))/KSV(J)
    IF(XSV.LT.D1(J).AND.XSV.GE.D0(J)) DSV=XSV-D0(J)
    IF(XSV.LT.D2(J).AND.XSV.GE.D1(J)) DSV=D1(J)-D0(J)
    IF(XSV.LT.D3(J).AND.XSV.GE.D2(J)) DSV=D3(J)-XSV
    FSV=D4(J)*DSV
  ELSE
    FSV=0.
  END IF
  RETURN
  END

  C
  C

```