Nonlinear Evolution of Mirror Instability in the Earth's Magnetosheath in PIC Simulations

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NONLINEAR EVOLUTION OF MIRROR INSTABILITY IN THE EARTH’S MAGNETOSHEATH IN PIC SIMULATIONS

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DEDICATION

To my parents, Fatemeh and Shokrollah
whose love has always been with me.
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Abstract

Nonlinear Evolution of Mirror Instability in the Earth’s Magnetosheath in PIC Simulations

by

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Mirror modes are large amplitude non-propagating structures frequently observed in the magnetosheath and they are generated in space plasma environments with proton temperature anisotropy of larger than one. The proton temperature anisotropy also drives the proton cyclotron instability which has larger linear growth rate than that of the mirror instability. Linear dispersion theory predicts that electron temperature anisotropy can enhance the mirror instability growth rate while leaving the proton cyclotron instability largely unaffected. Contrary to the hypothesis, electron temperature anisotropy leads to excitement of the electron whistler instability. Our results show that the electron whistler instability grows much faster than the mirror instability and quickly consumes the electron free energy, so that there is not enough electron temperature anisotropy left to significantly impact the evolution of the mirror instability. Observational studies have shown that the shape of mirror structures is related to local plasma parameters and distance to the mirror instability threshold. Mirror structures in the form of magnetic holes are observed when plasma is mirror stable or marginally mirror unstable and magnetic peaks are observed when plasma is mirror unstable. Mirror structures are created downstream of the quasi-perpendicular bow shock and they are convected toward the magnetopause. In the middle magnetosheath, where plasma is mirror unstable, mirror structures are dominated by magnetic peaks. Close to the magnetopause, plasma expansion makes the region mirror stable and magnetic peaks

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evolve to magnetic holes. We investigate the nonlinear evolution of mirror instability using expanding box Particle-in-Cell simulations. We change the plasma conditions by artificially enlarging the simulation box over time to make the plasma mirror stable and investigate the final nonlinear state of the mirror structures. We show that the direct nonlinear evolution of the mirror instability leads to magnetic peaks while in expanding box simulations, mirror structures evolve to deep magnetic holes.
1.1 Overview of Plasma Environment Surrounding Earth

For the development of the human activity in space, we must understand the electromagnetic environment surrounding the Earth. This space is filled with plasma. A plasma is an ionized gas consisting of positively and negatively charged particles with approximately equal charge densities. It is an interesting fact that most of the material in the visible universe, as much as 99%, is in the plasma state. This includes the Sun, most stars, and a significant fraction of the interstellar medium. Thus, plasma plays a major role in the universe. The atmosphere of the sun produces a flow of plasma toward the space named the “solar wind”. The typical speed of the solar wind near the Earth is \( \sim 400 - 500 \) km/s, which is larger than the magnetosonic speed (a few tens km/s). Therefore, the solar wind is a supersonic flow. The solar wind plasma interacts with Earth’s dipole magnetic field, deforming the electromagnetic environment around the Earth. Figure 1-1 shows the schematic illustration of the Earth’s magnetosphere and surrounding electromagnetic environment. The magnetosphere consists of various regions characterized by different plasma and magnetic field parameters. In the present thesis, we focus on the region called magnetosheath. In this region, the temperature of the protons perpendicular to the magnetic field line larger than the temperature parallel to the background field are observed.

The Earth’s magnetic field is a magnetic obstacle for the supersonic solar wind. Therefore, a shock structure called “bow shock” is generated on the dayside of the Earth. As
the solar wind moves past our planet, the magnetosphere trails away in the same direction, creating an elongated tail shape which extends outward to around a few hundred Earth radii. The resulting shape deviates significantly from the magnetic dipole model, starting with a distance of a few Earth radii, mostly due to external currents as shown in Figure 1-1.

The solar wind has mass and momentum which exerts pressure on the magnetosphere. In equilibrium, the force of solar wind against the magnetosphere and the force of the magnetosphere against the solar wind are in balance. The location of this equilibrium is the outer boundary of the magnetosphere and it is named “magnetopause”. The magnetopause location is pressure-sensitive. The magnetopause is essentially a current sheet and, in its simplest model, the currents close in on themselves and satisfy a momentum equation such that the force $j \times B$ acts to deflect the incoming solar wind plasma. The magnetopause has been measured to be approximately a thousand kilometers in thickness [39]. As shown in Figure 1-2, the magnetosheath region is between the bow shock and Earth’s magnetopause. In the magnetosheath, the solar wind plasma becomes subsonic with a typical bulk velocity
∼ 250 km/s. The Earth’s magnetic field becomes weak and irregular in the magnetosheath due to interaction with the incoming solar wind. In this region, the density of the particles is lower than what is found beyond the bow shock, but greater than within the magnetopause. The magnetosheath structure depends strongly on the properties of the bow shock and upstream solar wind conditions. Farrugia et al. [18, 19] have shown that the magnetic shear across the magnetopause and solar wind Alfvén Mach number ($M_A = u/v_A$) play a central role in determining the structure of the magnetosheath where $u$ is solar wind speed and $v_A$ is Alfvén speed defined by $v_A = (B^2/\mu_0 nm)^{1/2}$. The magnetosheath is an entirely open system with large influx of energy from solar wind and this makes it a magnetically turbulent region. The turbulence is mainly the expression of the ways that the plasma dissipates the energy which is carried into the system by the solar wind.

When interplanetary magnetic field (IMF) is northward, the field lines tend to pile up near the magnetopause, and the plasma pressure, density, and/or temperature decrease to keep the total pressure in balance. The resulting layer is called the plasma depletion layer. Its main characteristics are lower plasma density and higher magnetic field values compared to their corresponding upstream magnetosheath values [19]. The plasma depletion layer can also form during periods of southward IMF with high solar wind dynamic pressure.
conditions, and could be related to the limitations of reconnection flows for high solar wind Mach numbers [2].

The bow shock is mainly separated into quasi-parallel shock and quasi-perpendicular shock, as shown in Figure 1-3. In quasi-parallel shock, the angle between the IMF and the shock normal vector is less than 45°, and for quasi-perpendicular shock the angle between the IMF and the shock normal vector is larger than 45°. Downstream of the quasi-perpendicular shock, the ions in the solar wind are strongly heated in the direction perpendicular to the background magnetic field. This heating causes the temperature along the magnetic field lines to be less than the temperature perpendicular to it. This temperature anisotropy leads to generation of plasma instabilities.

Figure 1-3: Schematic illustration of quasi-perpendicular region (red region) downstream of the bow shock by Shoji et al. [66].
1.2 Temperature Anisotropy Instabilities

In an ordinary gas like air, the number density is about $10^{25}$ particles/m$^3$ and the mean free path of the particles is approximately 68 nm. Mean free path is the average distance traveled by a moving particle without collisions that modifies its direction, energy and other properties. Therefore, the gas dynamics is dominated by collisions between particles. In the case of space plasmas like solar wind, the number density is about $10^6$ particles/m$^3$ which is much smaller than the air number density. The mean free path of the solar wind plasma is $\sim 1$ AU, which is the distance between the Sun and the Earth. Comparing the mean free path of the solar wind plasma with the scale of regions around Earth, the space plasma environment can be treated as collisionless. Without collisions, the kinetic energies of the particles are exchanged through interaction with electrostatic or electromagnetic plasma waves.

In statistical mechanics, a plasma is in thermodynamic equilibrium when it has a Maxwellian distribution. When the plasma is not in thermodynamic equilibrium (for example, when the plasma does not have a Maxwellian distribution in velocity space), an instability can grow to take the plasma toward stable conditions. When plasma is not in equilibrium, it means the system has free energy. The free energy is the amount of work that a thermodynamic system can perform. A plasma instability occurs by conversion of steady free energy in plasma into fluctuating field energy. Particles accelerate or decelerate and this leads to generation of electric and magnetic field perturbations. Plasma instabilities that originate from the non-equilibrium velocity-space are primarily due to an anisotropic distribution with respect to the ambient magnetic field or the direction of the wave propagation. This characteristic of plasma waves and their dispersion relations has been investigated by Stix [72].

As we discussed above, the dayside magnetosheath behind the quasi-perpendicular portion of the bow shock is characterized by the proton temperature anisotropy $T_{\perp} > T_{\parallel}$ [49, 86] and is dominated by locally generated low-frequency waves. The $T_{\perp}$ and $T_{\parallel}$ are the temperatures perpendicular and parallel with respect to the background magnetic field. The
proton temperature anisotropy \((T_{p\perp}/T_{p\parallel}) > 1\) leads to generation of two instabilities: proton cyclotron instability and mirror instability.

The proton cyclotron instability \([25, 38, 62, 83]\) is a resonant instability and it propagates parallel to the background magnetic field with frequencies less than the proton gyrofrequency \((\omega < \Omega_p)\) while the proton mirror instability \([12, 29, 30]\) has zero frequency \((\omega = 0)\) in the plasma frame in a homogeneous plasma and its wave vector is oblique to the background magnetic field. Here, \(\Omega_p\) denotes the proton gyrofrequency. In a resonant instability, resonant particles exchange energy with the wave. For the resonant particles, the Doppler shifted wave frequency in the frame that moves with the particle along the magnetic field is an integer \((n)\) multiple of the particle’s gyrofrequency,

\[
\omega - k_{\parallel}v_{\parallel} = n\Omega
\]  

(1.1)

If \(n\) is zero, the resonance is a Landau resonance and if \(n\) is nonzero, it is called a cyclotron resonance. The mirror instability creates magnetic depressions or magnetic mirrors in the plasma which can trap particles and in this way, particles exchange their kinetic energy to the wave and instability grows \([46–48, 71]\). Electron temperature anisotropy has also been observed in the magnetosheath \([45, 54]\). Electron temperature anisotropy \((T_{e\perp} > T_{e\parallel})\) generates electron whistler instability \([38, 64]\) and electron mirror instability \([22]\). Electron whistler instability propagates along the field lines with frequencies smaller than electron gyrofrequency and larger than proton gyrofrequency \((\Omega_p < \omega < \Omega_e)\). The electron mirror instability is similar to the proton mirror instability, but its wavelength is of the order of electron inertial length \((d_e = c/\omega_{pe})\). Figure 1-4 shows our kinetic simulations of each instability and its properties.

In an electron-proton plasma, the proton cyclotron instability has a lower threshold and larger maximum growth rate than the mirror instability under many space plasma conditions. According to linear Vlasov theory for bi-Maxwellian electron-proton plasma \([20]\), the proton
Electron whistler instability

Proton cyclotron instability

Mirror instability

Figure 1-4: Temperature anisotropy instabilities in the quasi-perpendicular region downstream of the bow shock. The plots show the magnetic field perturbations caused by these instabilities in our fully kinetic simulations.
cyclotron instability grows faster for low proton beta plasma \((\beta_p \parallel \leq 6)\), while for high proton beta plasmas, the mirror instability dominates. Plasma beta, \(\beta_{\parallel}\), is the ratio of parallel particle pressure to magnetic field pressure \((\beta_{\parallel} = n k_B T_{\parallel} / (B^2 / 2\mu_0))\).

There are frequent observations of proton mirror mode structures in the Earth’s magnetosheath [37, 77]. Proton mirror modes have also been observed in the solar wind [82], at comets [61], in the magnetosheaths of other planets like Jupiter and Saturn [11, 15, 36], and in the heliosheath [9]. Proton mirror modes have been observed in regions with low proton beta \(\beta_{p\parallel}\), although the linear dispersion theory predicts that proton cyclotron mode should be the dominant mode in these regions [20]. Price et al. [59] have shown that the presence of a small density of heavy ions could reduce the linear growth rate of the proton cyclotron mode, while leaving the mirror mode unchanged [21]. Shoji et al. [66] performed two and three-dimensional hybrid simulations to study the competition between these two modes. They suggested that in three dimensional simulations, proton mirror modes consume most of the free energy of the system which it stops the growth of the proton cyclotron waves. Seough and Yoon [65] suggested that temporal or spatial variations in the magnetic field strength, which affect the resonance condition for the proton cyclotron instability but do not affect the resonance condition for the proton mirror instability, may also suppress the proton cyclotron instability.

A part of this work is to study the effects of electron temperature anisotropy on the evolution of proton mirror instability. Linear dispersion theory shows that the electron temperature anisotropy enhances the proton mirror instability growth rate but does not affect the proton cyclotron instability growth rate significantly [20]. Since we need to consider electron dynamics, we use particle-in-cell simulations to include kinetic effects of both protons and electrons.

The nonlinear evolution of the mirror instability is a subject of current research which aims to answer how the amplitude of the mirror instability saturates and what governs the development of the saturated magnetic structures [10, 33, 41, 51, 52, 58]. The interest
in these questions is motivated by satellite observations, which consistently find stationary magnetic structures with mirror mode properties in planetary magnetosheaths and the solar wind [3, 36, 70, 82]. Mirror modes are typically observed as trains of quasi-periodic large amplitude structures of two types: local enhancements of magnetic field intensity (magnetic peaks) with depression in plasma density and opposite structures (magnetic holes), identified by a magnetic field decrease and an increase in plasma density. Figure 1-5 shows Cluster observations of magnetic peaks and magnetic holes in Earth’s magnetosheath.

![Figure 1-5: Cluster observations of magnetic peaks and magnetic holes in the magnetosheath. (Taken from [70])](image)

Theoretical studies and simulations have shown that the direct nonlinear saturation of the mirror instability leads to magnetic peaks [10, 42]. Therefore, the question is under what conditions the mirror instability evolves to magnetic holes in its nonlinear stage of saturation. Several observational studies have shown that the shape of mirror structures is related to local plasma parameters [27, 70]. Specifically, low $\beta_{p||}$ conditions ($\beta_{p||} < 2$) are associated with observations of holes while peaks are usually observed in higher $\beta_{p||}$ plasma. Soucek et al. [70] have shown that peaks are usually observed when plasma is mirror unstable and mirror mode structures observed in the mirror stable region are likely to have the form of magnetic holes. Mirror structures created behind the bow shock are convected downstream to reach nonlinear saturation in the middle magnetosheath. At this stage, plasma is kept in a marginally unstable state above the threshold. As the plasma flow reaches the plasma depletion layer, it undergoes an expansion, forcing plasma into mirror stable state. Mirror structures survive this transition, but they assume the form of holes under such conditions. Therefore, we developed an expanding box simulation in order to resemble the convection
of the mirror mode in the magnetosheath. As the plasma expands we can control how to change plasma parameters according to the direction of the background magnetic field and direction of expansion. This allows us to increase or decrease anisotropy \((T_\perp/T_\parallel)\) and \(\beta\) in the plasma, so we can investigate how the mirror instability structures will evolve in a mirror stable plasma.

### 1.3 Significance of Computer Simulations

For studying the space plasma environments, three approaches exist: ground and spacecraft observations, theoretical analysis, and computer simulations. Spacecraft and ground observations have found many varieties of plasma waves around the Earth, other planets, and in the solar wind. With statistical studies of the observations, some aspects of the physical characters of the plasma waves has been clarified. However, observations provide only limited data at a point or several points, so we are not able to understand the spatial evolutions of the waves.

To explain the observed wave phenomena, theoretical approaches have been investigated. Using linear approximations, we neglect second or higher order fluctuations in Maxwell’s equations and the momentum equations of the particles and derive the linear dispersion relations of the plasma waves. This treatment of the waves is limited in the theoretical approach, in that it is quite difficult to solve the evolution of the distribution functions given by Vlasov equation. Another limitation of the theory arises when solving nonlinear equations with the assumption of large amplitude electromagnetic waves.

The third approach for investigating the space plasma is the use of computer simulations. We must solve the equations for the plasma dynamic and Maxwell’s equations. In order to describe the space plasma behavior self-consistently using computer simulations, two approaches exist: we can treat the plasmas as fluids or particles. If we assume that the plasmas are fluid, we need to solve Maxwell’s equations and the fluid equations of the plasma.
In the fluid description of the plasma, the kinetic effect are neglected. Fluid models describe plasma behavior very well at large scales. The magnetohydrodynamic (MHD) model is widely used as one of the fluid models for global simulations of planetary magnetosphere. In the MHD model, the electrons and ions are treated as one fluid.

In order to consider the kinetic effects, we need to solve the evolution of the distribution function $f$ for the protons and electrons. There are two methods to do this: (1) Vlasov simulations [78, 84] and (2) particle-in-cell simulations [6, 14]. The kinetic description of the plasma, while computationally very expensive, overcomes many limitations of fluid equations like MHD models.

Vlasov simulations solve the Vlasov equation (equation 2.10), which is the collisionless Boltzmann equation. By solving this equation, we obtain the time evolution of the distribution function in phase space. The Vlasov simulation is a more noise-less method than the particle-in-cell simulations. However, since the dimension of the distribution function in the phase space is six, huge memory space is needed for these type of simulations.

The particle-in-cell simulations were developed in 1980’s. The basic equations for the particle-in-cell simulations are Maxwell’s equations and equations of motions for quasi-particles. The plasma simulations using the particle-in-cell approach is categorized into two groups: a full-particle scheme in which all the species in plasmas are treated as particles, and hybrid scheme in which electrons are treated as a fluid and protons are described as particles. In the present study, we use the full-particle simulations to study the nonlinear evolution of the mirror instability and the electron temperature anisotropy effects.

1.4 Contribution of the Present Study

The motivation for current study is to understand the nonlinear evolution of the mirror instability. In this dissertation, we investigate the mirror instability, driven by the proton
temperature anisotropy $T_{p\perp}/T_{p\parallel} > 1$. The present thesis describes the nonlinear characteristics of the mirror instability by self-consistent particle-in-cell simulations, comparing with spacecraft observations. Specifically, we focus on the problems in the Earth’s magnetosheath, where this instability is dominant.

In chapter 2, we explain numerical techniques and models of particle-in-cell method. We explain the basic equations and algorithm used in our plasma simulation code. We also numerically solve the linear dispersion theory for bi-Maxwellian distributions in a homogeneous magnetized plasma.

In chapter 3, we review the mirror instability linear and nonlinear growth mechanisms. We describe the main theories for nonlinear saturation of the mirror instability.

In chapter 4, we investigate the effects of electron temperature anisotropy on mirror instability evolution. We show that the electron whistler instability grows much faster than the proton mirror instability and quickly consumes the electron free energy.

In chapter 5, we explain the particle-in-cell expanding box method implemented in our plasma simulation code (psc) to mimic the magnetosheath plasma expansion and compression.

In chapter 6, we simulate the deep magnetic holes observed in the magnetosheath. We investigate the nonlinear saturation mechanism of mirror instability with direct and expanding box simulations and the evolution of mirror structures.

In chapter 7, we summarize the present study and give conclusions obtained in the our computer simulations. We also present suggestions for future studies.
Chapter 2

Plasma Simulation Code and Linear Dispersion Solver

In order to study the nonlinear evolution of the mirror instability, we use particle-in-cell simulations. Mirror instability is a kinetic instability and it grows because of wave-particle interactions. Therefore, we need to use particle-in-cell simulations to investigate its evolution and its nonlinear saturation mechanisms. We also numerically solved the linear dispersion relation for bi-Maxwellian distributions in a magnetized plasma to measure the maximum growth rate and wavenumbers of the temperature anisotropy instabilities. This helps us to resolve the wavenumber of the maximum growth rate in our simulations.

In this chapter, we describe the particle-in-cell model used by our plasma simulation code (psc). We briefly explain the basic equations, the algorithm, and the models. We then review the linear dispersion relation that we use for linear analysis. These two discussions form the basis of the simulation and analytic work in this thesis.

2.1 Plasma Simulation Code (psc)

The description of the natural world is based on describing the interaction of particles of matter via force fields. In the case of a plasma, the system is composed of charged particles (electrons and ions) interacting via electric and magnetic forces. The full particle-in-cell simulations [6], which assumes both the electrons and the ions as particles, is the most accurate method to solve the interactions between the space plasmas and the electromagnetic fields.
The particle-in-cell method [6, 34, 43] solves equations of motion for particles and Maxwell’s equations to find forces between those particles, which is similar to the description of a plasma as a system of charged particles. We use plasma simulation code (psc) to study the kinetic mirror instability. psc is a state of the art electromagnetic particle-in-cell simulations code with advanced features like load-balancing and GPU support described by Germaschewski et al. [28].

2.1.1 Basic Equations

If we identify each particle with a label $p$ and their charge with $q_p$, position with $x_p$, velocity with $v_p$, the force acting on the particles is the combination of the electric and magnetic force (Lorentz force),

$$F_p = q_p(E_p(x_p) + v_p \times B_p(x_p))$$ (2.1)

The force acting on the particles is calculated from the electric and magnetic fields evaluated at the particle location. The electric and magnetic fields are created by the particles in the system and by additional sources outside the system (for example, Earth’s magnetic field). The electromagnetic fields $E$ and $B$ are self-consistently evolved using Maxwell’s equations:

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$ (2.2)

$$\nabla \cdot B = 0$$ (2.3)

$$\frac{\partial E}{\partial t} = c^2\nabla \times B - \frac{j}{\epsilon_0}$$ (2.4)

$$\frac{\partial B}{\partial t} = -\nabla \times E$$ (2.5)
where charge density $\rho$ and current density $\mathbf{j}$ are obtained from the particle distribution functions:

$$\rho = \sum_s q_s \int f_s(x, \mathbf{p}, t) \, d^3p \quad (2.6)$$

$$\mathbf{j} = \sum_s q_s \int \mathbf{v}_s f_s(x, \mathbf{p}, t) \, d^3p \quad (2.7)$$

where $f_s(x, \mathbf{p}, t)$ is the distribution function of species $s$. The distribution function gives the probability of finding particles in a given volume of phase space. The interaction between the particles and electromagnetic fields is enabled through the current $\mathbf{j}$ in the Ampère’s Law (2.4).

The divergence equations (2.2), (2.3) in Maxwell’s equations can be considered as initial conditions. If they are satisfied at some initial time, it is easy to show from Ampère’s Law (2.4) and Faraday’s Law (2.5) that they will remain satisfied at all times provided that the charge continuity equation also holds:

$$\partial_t \rho + \nabla \cdot \mathbf{j} = 0. \quad (2.8)$$

For numerical calculation, normalized quantities are used. We normalize the equations of motions and Maxwell’s equations with the help of the ion plasma frequency $\omega_{pi}$, the speed of light $c$, and the ion inertial length $d_i = c/\omega_{pi}$. The following dimensionless parameters are used

$$\tilde{t} = \omega_{pi} t, \quad \tilde{x} = \frac{x}{d_i}, \quad \tilde{p} = \frac{p_j}{m_j c} \quad (2.9)$$
2.1.2 Finite Size Particles

The number of charged particles in a real plasma is very large and it is not feasible to iterate through all of them. Therefore, an artificially reduced number of quasi-particles are introduced. These quasi-particles have finite extent in space and they behave like point particles until they start to overlap. When two quasi-particle overlap, the overlapped area is neutralized, hence they only interact weakly with each other. Particle-in-cell method models the collective behavior between particles rather than individual particle-particle forces.

Using finite-size quasi-particles is computationally cheaper, because it allows one to solve the field equations on a mesh, rather than directly calculating the interaction of each particle with all others. Particle-in-cell scales linearly in the number of particles \( N \), as opposed to exact interaction approach that scales like \( O(N^2) \). Therefore, particle-in-cell method is better understood as a numerical method to solve the Vlasov-Maxwell system of equations that describes the time evolution of the particle distribution function \( f_s(x, p, t) \):

\[
\frac{\partial f_s}{\partial t} + v \cdot \frac{\partial f_s}{\partial x} + q_s(E + v \times B) \cdot \frac{\partial f_s}{\partial p} = 0 \tag{2.10}
\]

The distribution function \( f_s \) is approximated using quasi-particles with finite extent in configuration space:

\[
f_s(x, p, t) = \sum_{i=1}^{N_s} N_i^s \phi(x - x_i^s(t)) \delta^3(p - p_i^s(t)) \tag{2.11}
\]

where \( \phi \) is the shape function for the quasi-particles and \( N_i^s \) is the number of physical particles of type \( s \) that are present in the element of phase space represented by the quasi-particles. Using the \( \delta \)-function in velocity space ensures that if all particles within the element of phase space described by the quasi-particle have the same speed, they remain closer in phase space during the subsequent evolution, and the spatial extent of each quasi-particle remains constant in time. The selection of the shape function \( \phi \) determines properties of the
Equations of motions for the quasi-particles are derived by taking moments of the Vlasov equation (2.10):

\[
\frac{dN_s^i}{dt} = 0, \quad \frac{dx_s^i}{dt} = v_s^i, \quad \frac{dp_s^i}{dt} = q_s^i (E_i + v_s^i \times B_i)
\]  

The first equation expresses that the number of physical particles \(N_s^i\) represented by each quasi-particle \(i\) remains constant. The other two equations are the usual equations of motion for a point particle with the modification that the electromagnetic fields \(E_i, B_i\) acting on the particle are given by

\[
E_i = \int E \phi(x - x_s^i) d^3x, \quad B_i = \int B \phi(x - x_s^i) d^3x
\]  

which means that the electromagnetic fields are averaged over the extent of the particle.

### 2.1.3 Algorithm of the Particle-in-Cell Code

The finite-difference time domain (FDTD) method has been used for computationally solving Maxwell’s equations [85]. The FDTD method employs the staggered Yee grid, as shown in Fig. 2-1, to represent magnetic fields on faces, electric fields and current densities on edges, and charge densities on corners of the computational mesh.

We define the following discrete curl operators:

\[
(\nabla^+ \times E)_{x,i,j+1/2,k+1/2} = \frac{E_{z,i,j+1/2,k+1/2} - E_{z,i,j,k+1/2}}{\Delta y} - \frac{E_{y,i,j+1/2,k+1} - E_{y,i,j+1/2,k}}{\Delta z}
\]

\[
(\nabla^- \times B)_{x,i+1/2,j,k} = \frac{B_{z,i+1/2,j,k+1/2} - B_{z,i+1/2,j,k-1/2}}{\Delta y} - \frac{B_{y,i+1/2,j,k+1/2} - B_{y,i+1/2,j,k-1/2}}{\Delta z}
\]

where the \(y\) and \(z\) components are obtained by cyclic permutation.
Maxwell’s equations are discretized using these operators, and we employ a leap-frog scheme staggered in time (see also Fig. 2-2):

\[
\frac{E_{ijk}^{n+1/2} - E_{ijk}^{n-1/2}}{\Delta t} = c^2 \nabla \times B_{ijk}^n - \frac{j_{ijk}^n}{\epsilon_0} \tag{2.14}
\]

\[
\frac{B_{ijk}^{n+1} - B_{ijk}^n}{\Delta t} = -\nabla \cdot E_{ijk}^{n+1/2} \tag{2.15}
\]

where

\[
E_{ijk} = (E_{x,i+1/2,j,k}, E_{y,i,j+1/2,k}, E_{z,i,j,k+1/2}) \tag{2.16}
\]

\[
B_{ijk} = (B_{x,i+1/2,j,k+1/2}, B_{y,i+1/2,j,k+1/2}, B_{z,i,j+1/2,k+1/2}) \tag{2.17}
\]

We use a standard leap-frog method to advance quasi-particles in time, see also Fig. 2-2.

\[
\frac{x_{i}^{n+1/2} - x_{i}^{n-1/2}}{\Delta t} = v_{i}^{n} \tag{2.18}
\]

\[
\frac{p_{i}^{n+1} - p_{i}^{n}}{\Delta t} = q_{s} \left( E_{i}^{n+1/2} + v_{i}^{n+1/2} \times B_{i}^{n+1/2} \right) \tag{2.19}
\]
where $v^n_i = p^n_i/(m_s \gamma_i^n)$. We follow Boris [7] in choosing

$$v^{n+1/2}_i = \frac{p^n_i + p^{n+1}_i}{2m_s \gamma_i^{n+1/2}}$$  \hspace{1cm} (2.20)

and splitting the momentum update into a half step acceleration by $E$, a rotation by $B$, and another half step acceleration by $E$.

### 2.1.4 Time integration

The particle-in-cell method advances both electromagnetic fields and quasi-particles self-consistently. The time integration scheme used in psc is sketched out in Fig. 2-2. The figure shows the FDTD scheme (blue), and particle integrator (red), and also their interactions: To update the momentum, the electric and magnetic fields are needed to find the force on a given quasi-particle (black arrows). $E^{n+1/2}$ exists at the proper centered time to do so, while $B^{n+1/2}$ is in principle found by averaging $B^n$ and $B^{n+1}$; however, in practice, we rather split the $B^n \rightarrow B^{n+1}$ update into two half steps.

Particle motion feeds back into Maxwell’s equations by providing the source term $j^n$. The current density is computed from the particles to exactly satisfy the discrete charge continuity equation, which requires knowing particle positions at the naturally existing $x^{n-1/2}$ and $x^{n+1/2}$, and is fed back into Maxwell’s equations (green arrows).

psc uses two methods to satisfy charge continuity: For 1st-order particles we use the scheme by Villasenor-Buneman [80], while for 2nd-order particles we follow the method by Esirkepov [16]. psc also implements some alternating-order interpolation schemes from [69] for improved energy conservation. For a discussion of conservation properties of particle-in-cell codes, see also [17].
2.2 Linear Dispersion Relation

We numerically solved the linear dispersion theory for bi-Maxwellian distributions in a magnetized plasma to calculate the maximum growth rate and wavenumber of the temperature anisotropy instabilities. This helps us to resolve the maximum growth rate wavelength for the chosen plasma parameters in our particle-in-cell simulations.

The solution of the Vlasov equation is obtained by calculating the first-order perturbation to the velocity distribution function in the Lagrangian system of coordinates, that is, in coordinates that follow the zeroth-order trajectory of the particles [72]. Knowledge of the perturbed velocity distribution in terms of the first-order electric field allows one, by taking moments, to calculate the macroscopic charge and current density and also the susceptibility tensors. Substituting into Maxwell’s equation then gives the dispersion relation.

2.2.1 Linearization of the Vlasov Equation

We consider a uniform, nonrelativistic, collisionless plasma immersed in a steady, uniform background magnetic field $\mathbf{B}_0 = B_0 \hat{z}$. Since the wave amplitudes are assumed to be small, the distribution function consists of a constant homogeneous term, $f^{(0)}_s(p)$, plus a small perturbation, $f^{(1)}_s(x,p,t)$, for particles of kind $s$, so that

$$f_s(p) = f^{(0)}_s(p) + f^{(1)}_s(x,p,t) \quad (2.21)$$

We derive dispersion equation using the coupled system of the Vlasov equation (2.10) and the Maxwell equations. We follow the derivation method given by Stix [72]. The standard procedure starts with the linearized Vlasov equation. We see that the zero-order kinetic equation is given by
\[
\left( \frac{df_s^{(0)}(p)}{dt} \right)_0 = 0 \tag{2.22}
\]

The first order Vlasov equation is given by

\[
\left( \frac{df_s^{(1)}(x, p, t)}{dt} \right)_0 = \frac{\partial f_s^{(1)}(x, p, t)}{\partial t} + v \cdot \frac{\partial f_s^{(1)}(x, p, t)}{\partial x} + \frac{q_s}{c} \left( \frac{v \times B_0}{c} \right) \cdot \frac{\partial f_s^{(1)}(x, p, t)}{\partial p} = -q_s \left( E_s^{(1)}(x, t) + \frac{v \times B_s^{(1)}(x, t)}{c} \right) \cdot \frac{\partial f_s^{(0)}(p)}{\partial p} \tag{2.23}
\]

where \( B = B_0 + B^{(1)} \) and \( E = E^{(1)} \). We derive the fully electromagnetic dispersion equation for instabilities at an arbitrary direction of propagation in a plasma with bi-Maxwellian zeroth order distribution for each species \( s \):

\[
f_s^{(0)}(v||, v_\perp) = \frac{1}{\pi^{3/2} v_{th,\perp s} v_{th,|| s}} \frac{1}{v_{th,\perp s} v_{th,|| s}} \exp \left( -\frac{v_\perp^2}{v_{th,\perp s}^2} - \frac{(v|| - V_s)^2}{v_{th,|| s}^2} \right) \tag{2.24}
\]

where \( v_{th,\perp s} = \sqrt{2K_B T_{\perp s}/m_s} \), \( v_{th,|| s} = \sqrt{2K_B T_{|| s}/m_s} \), and \( V_s = \langle v|| \rangle_s \). No further approximations are made so that our results are valid at arbitrary wavenumbers and frequencies and for the full range of nonrelativistic plasma parameters. Now the problem is to solve for \( f_s^{(1)} \),

\[
f_s^{(1)}(x, p, t) = -q_s \int_{-\infty}^{t} dt' \left[ E_s^{(1)}(x', t') + \frac{v'}{c} \times B_s^{(1)}(x', t') \right] \cdot \frac{\partial f_s^{(0)}(p')}{\partial p'} \tag{2.25}
\]

This integral is solved by substituting asymptotic field \( E_s^{(1)}(x', t') = \mathbf{E} \mathbf{e}^{i(k \mathbf{x} - \omega t')} \) and use of Maxwell’s Faraday equation, \( B^{(1)} = (k \mathbf{c}/\omega) \times \mathbf{E}^{(1)} \), to replace \( B^{(1)} \).

We can solve Maxwell’s equations for plane waves. We have Ampère equation (2.4) and Faraday’s equation (2.5) in CGS units,
\[ \nabla \times \mathbf{B} = \frac{4\pi j}{c} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \]  
(2.26)

\[ \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \]  
(2.27)

The wave equation for a homogenous plasma is derived by assuming solutions with harmonic time dependent \( e^{-i\omega t} \),

\[ \nabla \times \mathbf{B} = \frac{4\pi j}{c} - \frac{i\omega}{c} \mathbf{E} \]  
(2.28)

\[ \nabla \times \mathbf{E} = \frac{i\omega}{c} \mathbf{B} \]  
(2.29)

Substituting for \( \mathbf{B} \) in Ampère equation from Faraday equation gives,

\[ \nabla \times (\nabla \times \mathbf{E}) = \frac{4\pi i\omega}{c^2} \mathbf{j} + \frac{\omega^2}{c^2} \mathbf{E} \]  
(2.30)

It is necessary to express the plasma current density \( \mathbf{j} \) in terms of the electric field \( \mathbf{E} \). One may make this replacement using a conductivity tensor in a dielectric medium and introduce a dielectric tensor. The electric displacement \( \mathbf{D} \) includes the vacuum displacement plus the plasma current according to the Ampère equation,

\[ \nabla \times \mathbf{B} = \frac{4\pi j}{c} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} \]  
(2.31)

and after Fourier analysis in time

\[ \mathbf{D}(\omega, \mathbf{k}) = \epsilon(\omega, \mathbf{k}) \cdot \mathbf{E}(\omega, \mathbf{k}) = \mathbf{E}(\omega, \mathbf{k}) + \frac{4\pi i}{\omega} \mathbf{j}(\omega, \mathbf{k}) \]  
(2.32)

where \( \epsilon(\omega, \mathbf{k}) \) is the dielectric tensor and it is additive in its components. A representation
that emphasizes this additive property is the susceptibility. The susceptibility $\chi_s$ of the $s$-the plasma component is its contribution to the dielectric tensor,

$$\epsilon(\omega, k) = 1 + \sum_s \chi_s(\omega, k) \quad (2.33)$$

where $1$ is the unit dyadic and the sum is over all plasma species. The contribution to the plasma current due to a susceptibility $\chi_s$ is $j_s$ and it is given through the linear relation

$$j(\omega, k) = \sum_s j_s = \sum_s q_s \int d^3pv_s f_s^{(1)}(x, p, t) = -\frac{i\omega}{4\pi} \sum_s \chi_s(\omega, k) \cdot E(\omega, k) \quad (2.34)$$

After solving for $f_s^{(1)}(x, p, t)$ for a hot magnetized plasma, we are now able to calculate velocity moment to find the contributions to the first order plasma current $j(\omega, k)$ for each species.

### 2.2.2 Susceptibility for $f_s(p_\perp, p_\parallel)$

To evaluate $\chi_s(\omega, k)$, the first velocity moment of the distribution function equation (2.34) is calculated and it results in,

$$\chi_s(\omega, k) = \left[ \hat{e}_\parallel \hat{e}_\parallel \frac{2\omega_p^2}{\omega^2 k_\parallel^2 v_{th\perp}^2} \langle v_\parallel \rangle + \frac{\omega_p^2}{\omega^2} \sum_{n=-\infty}^{\infty} e^{-\lambda Y_n(\lambda)} \right]_s \quad (2.35)$$

where

$$\lambda = \frac{k_\perp^2 v_{th\perp}^2}{2\Omega^2}, \quad \Omega = \frac{qB_0}{\gamma m_0c}, \quad \gamma^2 = \frac{p^2}{m_0^2 c^2} + 1$$

and

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\[
Y_n(\lambda) = \begin{pmatrix}
\frac{n^2 I_n}{\lambda} A_n & -in(I_n - I'_n)A_n & \frac{k_n n I_n}{\lambda} B_n \\
\frac{\omega^2 I_n}{\lambda} A_n & (\frac{n^2 I_n}{\lambda} + 2\lambda I_n - 2\lambda I'_n)A_n & \frac{ik_n}{\Omega} (I_n - I'_n)B_n \\
\frac{k_n n I_n}{\lambda} B_n & -\frac{ik_n}{\Omega} (I_n - I'_n)B_n & \frac{2(\omega - n\Omega)}{k_n v_{th}} I_n B_n
\end{pmatrix}
\] (2.36)

\[I_n = I_n(\lambda)\] is the modified Bessel function with argument \(\lambda\) and

\[
A_n = \left(\frac{T_\perp}{T_\parallel} - 1\right) + \frac{1}{k_{\parallel} v_{th}} \frac{(\omega - k_{\parallel} V - n\Omega)T_\perp + n\Omega T_\parallel}{T_\parallel} Z_0
\]

\[
B_n = \frac{1}{k_{\parallel}} \frac{(\omega - n\Omega)T_\perp - (k_{\parallel} V - n\Omega)T_\parallel}{T_\parallel} + \frac{1}{k_{\parallel} v_{th}} \frac{\omega - n\Omega}{k_{\parallel} v_{th}} \frac{(\omega - k_{\parallel} V - n\Omega)T_\perp + n\Omega T_\parallel}{T_\parallel} Z_0
\] (2.37)

where,

\[Z_0 = Z_0(\xi_n), \quad \xi_n = \frac{\omega - k_{\parallel} V - n\Omega}{k_{\parallel} v_{th}}, \quad \frac{Z_0(\xi_n)}{d\xi_n} = -2[1 + \xi_n Z_0(\xi_n)]\]

\(Z_0\) is the plasma dispersion function. Now we can write components of the dielectric tensor,

\[
\epsilon_{xx} = 1 + \sum_s \chi_{s,xx} = 1 + \sum_s \frac{\omega^2 p,s}{\omega^2} \sum_{n=-\infty}^{\infty} e^{-\lambda_s} Y_{n,xx}(\lambda_s)
\] (2.38)

\[
\epsilon_{xy} = \sum_s \chi_{s,xy} = 1 + \sum_s \frac{\omega^2 p,s}{\omega^2} \sum_{n=-\infty}^{\infty} e^{-\lambda_s} Y_{n,xy}(\lambda_s)
\] (2.39)

\[
\epsilon_{xz} = \sum_s \chi_{s,zx} = 1 + \sum_s \frac{\omega^2 p,s}{\omega^2} \sum_{n=-\infty}^{\infty} e^{-\lambda_s} Y_{n,zx}(\lambda_s)
\] (2.40)
\[ \epsilon_{yx} = -\epsilon_{xy} \]  

(2.41)

\[ \epsilon_{yy} = \sum_s \chi_{s,yy} = 1 + \sum_s \frac{\omega_{p,s}^2}{\omega^2} \sum_{n=-\infty}^{\infty} e^{-\lambda_s} Y_{n,yy}(\lambda_s) \]  

(2.42)

\[ \epsilon_{yz} = \sum_s \chi_{s,yz} = 1 + \sum_s \frac{\omega_{p,s}^2}{\omega^2} \sum_{n=-\infty}^{\infty} e^{-\lambda_s} Y_{n,yz}(\lambda_s) \]  

(2.43)

\[ \epsilon_{zx} = \epsilon_{xz} \]  

(2.44)

\[ \epsilon_{zy} = -\epsilon_{yz} \]  

(2.45)

\[ \epsilon_{zz} = 1 + \sum_s \chi_{s,zz} = 1 + \sum_s \left[ \frac{2\omega_{p,s}^2}{\omega k_{\parallel} v_{th,s}} < v_{\parallel,s} > + \frac{\omega_{p,s}^2}{\omega^2} \sum_{n=-\infty}^{\infty} e^{-\lambda_s} Y_{n,zz}(\lambda_s) \right] \]  

(2.46)

After Fourier analysis, equation (2.30) gives the homogeneous plasma wave equation,

\[ \mathbf{k} \times (\mathbf{k} \times \mathbf{E}) + \frac{\omega^2}{c^2} \epsilon \mathbf{E} = 0 \]  

(2.47)

or in matrix form with \( \mathbf{k} = k_x \hat{x} + k_z \hat{z} \) and \( \mathbf{n} = \mathbf{k} c/\omega \),
This vector equation has nontrivial solutions when the determinant of the $3 \times 3$ matrix is zero. That condition provides the dispersion relation for the homogenous plasma system.

\[ \begin{pmatrix}
  \epsilon_{xx} - n_z^2 & \epsilon_{xy} & \epsilon_{xz} + n_x n_z \\
  \epsilon_{yx} & \epsilon_{yy} - n_x^2 - n_z^2 & \epsilon_{yz} \\
  \epsilon_{zx} + n_x n_x & \epsilon_{zy} & \epsilon_{zz} - n_x^2
\end{pmatrix}
\begin{pmatrix}
  E_x \\
  E_y \\
  E_z
\end{pmatrix} = 0 \quad (2.48) \]

### 2.3 Verification of psc by comparison to linear dispersion theory

In this section, we compare the psc results with linear dispersion theory. In order to show that psc can capture temperature anisotropy instabilities correctly, we measured the growth rate of the instabilities from simulation in the linear regime for selected plasma parameters and compare with linear theory predictions. We start with bi-Maxwellian protons and Maxwellian electrons. We choose $T_{p\perp} / T_{p\parallel} = 2.5$, $T_{e\perp} / T_{e\parallel} = 1$, $\beta_p = \beta_e = 1$. We perform two-dimensional simulations with $L_y = L_z = 128d_p$ where $L_y$ and $L_z$ are the length of the simulation box in $y$ and $z$ directions, $\omega_{pi}$ is the proton plasma frequency and $d_p = c / \omega_{pi}$ is the proton inertial length. The number of grid points ($n_y \times n_z$) is $4096 \times 4096$. Periodic boundary conditions are used in both dimensions. A constant background magnetic field $B_0$ is assumed in the $z$ direction.

With anisotropic protons ($T_{p\perp} / T_{p\parallel} > 1$), proton cyclotron and proton mirror instabilities will grow. From numerical evaluation of linear theory dispersion relation, we expect the maximum growth rate of proton cyclotron instability to be $\gamma_m = 0.10\Omega_p$ at $k_m d_p = 0.54$ and $\theta = 0^\circ$ while the proton mirror instability maximum growth rate is $\gamma_m = 0.039\Omega_p$ with $k_m d_p = 0.50$ at $\theta = 63^\circ$ as shown in Figure 2-3. $\theta$ is the angle between the wave number vector $k$ and $B_0$. Figure 2-4 shows the temperature anisotropy evolution of both protons.
and electrons. As proton cyclotron and proton mirror instabilities start growing, the proton temperature anisotropy decreases. The linear regime of proton temperature anisotropy instabilities extends through about $\Omega_p t = 70$ in this case. Figures 2-5 and 2-6 compare the measured maximum growth rate of proton cyclotron and proton mirror instabilities from simulation with linear dispersion theory predictions. The simulation results are in good agreement with predictions from linear theory.

We perform similar benchmarking simulation for electron whistler and electron mirror instabilities to show that we are resolving the electron scale in our simulations. Here, we choose $T_{p\perp}/T_{p\parallel} = 1$, $T_{e\perp}/T_{e\parallel} = 2$, $\beta_p = \beta_e = 1$. We use similar simulation parameters as in the previous case. Now, with anisotropic electrons ($T_{e\perp}/T_{e\parallel} > 1$), electron whistler instability and electron mirror instability grow. Figure 2-7 shows the growth rate of electron whistler and electron mirror instabilities as a function of wave number $k$.

For the given plasma parameters, linear dispersion theory predicts that the maximum growth rate of the electron whistler instability is $\gamma_m = 0.10\Omega_e$ at $k_m d_e = 0.64$ and $\theta = 0^\circ$ while electron mirror instability has a maximum growth rate of $\gamma_m = 0.006\Omega_e$ at $k_m d_e = 0.37$ and $\theta = 73^\circ$. Figure 2-8 shows the temperature anisotropy evolution. The proton
Figure 2-5: Measured maximum growth rate of proton cyclotron instability from simulation in the linear regime. The measured growth rate is in agreement with linear dispersion theory prediction.

Figure 2-6: Measured maximum growth rate of proton mirror instability from simulation in the linear regime. The measured growth rate is in good agreement with linear dispersion theory prediction.

Figure 2-7: The growth rate as a function of wavenumber for electron whistler and electron mirror instability. Solid line shows the electron whistler instability growth rate at $\theta_m = 0$ while the dashed line shows the growth rate of electron mirror instability at $\theta_m = 73$. The maximum growth rate of electron whistler instability is $\gamma_m/\Omega_e = 0.1$ at $k_m d_e = 0.64$ while the electron mirror instability maximum growth rate is $\gamma_m/\Omega_e = 0.006$ with $k_m d_e = 0.37$.

Figure 2-8: Proton and electron temperature anisotropy evolution as a function of time in 2D particle-in-cell simulation. Initial parameters are $T_{p\perp}/T_{p\parallel} = 1$, $T_{e\perp}/T_{e\parallel} = 2$ and $\beta_p = \beta_e = 1$. The linear regime of proton temperature anisotropy instabilities is about $\Omega_e t = 100$. 

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distribution stays in equilibrium and isotropic. Electron temperature anisotropy instabilities use the electron free energy and isotropize the electrons. The linear regime of electron whistler instability is about $\Omega_e t = 20$, while the linear regime of electron mirror instability is $\Omega_e t = 250$ since electron mirror instability maximum growth rate is about 17 times smaller than the electron whistler instability maximum growth rate. Figures 2-9 and 2-10 show the comparison of the measured growth rates from simulation with linear dispersion theory predictions. We see that the results are in a good agreement with the predictions.

2.4 Discussions

In this chapter we have first introduced the particle-in-cell model that we will use for simulation work in this thesis. The psc accurately represents the plasma dynamics at the characteristic scale of both ions and electron motions. We will study the effects of electrons on the mirror instability evolution in chapter 4. We also reviewed the linear dispersion theory for a homogenous plasma with bi-Maxwellian distributions in a magnetized plasma and numerically solved the linear dispersion equation. We can find the growth rate of both proton and electron temperature anisotropy instabilities for any given plasma parameters.
We have benchmarked PSC against numerically solved linear dispersion equation for both proton and electron temperature anisotropy instabilities and they are in excellent agreement. In chapter 3, we will review the linear and nonlinear theory of growth mechanisms of mirror instability.
CHAPTER 3

MIRROR INSTABILITY LINEAR AND NONLINEAR MECHANISMS

Mirror instability was originally derived from magnetohydrodynamic fluid theory, but later works showed that there were significant differences between the fluid theory and a kinetic approach. In this chapter, we review both fluid theory and kinetic approach. Nonlinear evolution of mirror instability is topic of current research and there are different theories trying to explain its nonlinear evolution. The purpose of this chapter is to describe the different analysis presented for nonlinear evolution of the mirror instability in order to help answer the question. We also review the observations of mirror instability structures in planetary magnetosheath and their properties and their relation to plasma parameters.

3.1 Linear mechanism

Mirror instability was identified theoretically by Chandrasekhar [12] and Vedenov and Sagdeev [79] and Thompson [74] as one of the two magnetohydrodynamic instabilities that occur in presence of velocity space anisotropy in a uniform plasma, the other instability being firehose. Later, Tajiri [73] derived a kinetic description that shows the actual instability is not correctly described as a fluid instability. Hasegawa [29, 30] also greatly clarified the work and developed a theory for a medium in which the plasma is nonuniform.

For mirror instability excitation, the phase space anisotropy of the bulk of the hot plasma distribution serves as the source of energy. This was the reason that mirror instability is referred to as a fluid instability. But kinetic treatment shows that the instability grows because of a subtle coupling between a group of particles with small parallel velocity and the rest of the population. We describe these in the following.
3.1.1 Fluid description

Hasegawa [30] gives a description of the mirror instability based on fluid magnetohydrodynamic approach. Mirror instability results from antiphase response of the perpendicular pressure $\delta p_\perp$ to the compressional change in the magnetic field $\delta B$ at very low frequencies,

$$\delta p_\perp = 2p_\perp(1 - \frac{T_{p\perp}}{T_{p\parallel}})\frac{\delta B}{B}$$  \hspace{1cm} (3.1)

for a bi-Maxwellian distribution with $T_{p\perp} > T_{p\parallel}$. This equation shows as field strength increases ($\delta B > 0$), particle pressure decreases. The perturbed plasma pressure is proportional to unperturbed plasma pressure and if the unperturbed plasma pressure is large enough, the change in total pressure produced by field change may be the opposite of the change in magnetic pressure. For a bi-Maxwellian distribution,

$$\frac{B\delta B}{\mu_0} + \delta p_\perp = \frac{B\delta B}{\mu_0} + 2p_\perp(1 - \frac{T_{p\perp}}{T_{p\parallel}})\frac{\delta B}{B} < 0$$  \hspace{1cm} (3.2)

therefore,

$$1 + \beta_{p\perp}(1 - \frac{T_{p\perp}}{T_{p\parallel}}) < 0$$  \hspace{1cm} (3.3)

This is the instability condition in cold electron limit in fluid description for the mirror instability. The antiphase plasma pressure response in equation (3.1) derives the instability. When the pressure anisotropy is large enough, an increase in magnetic field leads to a local decrease in total pressure which in turn causes the field lines to move closer together. This effect causes the magnetic field to continue to increase and therefore the instability grows.

In the linear regime of the instability, first adiabatic invariant (magnetic moment $\mu = mv_\perp^2/2B$) and kinetic energy of the particles are conserved. As $B$ increases, the perpendicular velocity of the particles increases to keep the magnetic moment conserved. Therefore,
the perpendicular particle energy changes in phase with changes in field because of conservation of magnetic moment. There is an opposite change in parallel energy compared to perpendicular energy to conserve the particle energy. Therefore, there is an exchange of energy between parallel and perpendicular degrees of freedom of a particle moving in a spatially varying magnetic field when magnetic moment is conserved. This leads to squeezing the plasma out of high field regions and into weak field regions. Because of that, the density of particles and the magnetic field are anticorrelated and this also leads to anticorrelation between the plasma pressure and the magnetic field pressure. Figure 3-1 shows the anticorrelation between the plasma density and magnetic field.

3.1.2 Kinetic description

Mirror instability is a hot plasma instability and it has to be treated in the kinetic limit. A major difference introduced by the kinetic approach is that not all of the plasma particles respond to the field changes in the same way. The plasma pressure variation given by equation (3.1) is applicable to particles with small pitch angles (large parallel velocities),
but particles with large pitch angles (small parallel velocities) respond differently because they are not affected by the spatially changing magnetic field to the same degree.

Southwood and Kivelson [71] used kinetic description to derive the mirror instability growth rate. In this section, we follow the Southwood and Kivelson [71] derivation of mirror instability condition. The distribution function is bi-Maxwellian and gyrotropic. We keep the explicit form of the distribution function $F$ but we can write it as a function of two quantities: $W_\parallel$ and $W_\perp$, the parallel and perpendicular energies. At low frequencies, the first adiabatic invariant $\mu$ is conserved. Therefore, the perpendicular energy is related to the local magnetic field strength, $W_\perp = \mu B$. Total energy is $W = W_\perp + W_\parallel$. We can write the changes in $W_\perp$, $W_\parallel$, $\delta W_\perp$, $\delta W_\parallel$ as

$$\delta W_\parallel = \delta W - \mu \delta B \quad \text{and} \quad \delta W_\perp = \mu \delta B \quad (3.4)$$

The change in distribution function is given by [40]

$$\delta F = -\delta W_\parallel \frac{\partial F}{\partial W_\parallel} - \delta W_\perp \frac{\partial F}{\partial W_\perp} \quad (3.5)$$

Thus

$$\delta F = -\delta W \frac{\partial F}{\partial W_\parallel} - \mu \delta B \left( \frac{\partial F}{\partial W_\perp} - \frac{\partial F}{\partial W_\parallel} \right) \quad (3.6)$$

and for a bi-Maxwellian distribution,

$$\delta F = \left[ \frac{\delta W}{T_\parallel} + \frac{\mu \delta B}{T_\perp} \left( 1 - \frac{T_\perp}{T_\parallel} \right) \right] F \quad (3.7)$$
The change in energy due to change in field strength in the low frequency limit is given by the adiabatic expression [50]

\[
\frac{dW}{dt} = \mu \frac{\partial B}{\partial t}
\]  

(3.8)

By integrating equation (3.8) for a disturbance varying as \( \delta B \propto e^{(ik \cdot r + \gamma t)} \),

\[
\delta W = \frac{\gamma}{\gamma + ik \parallel v} \mu \delta B
\]  

(3.9)

we derive an expression for \( \delta W \). Now we can write the distribution function in low frequency limit as

\[
\delta F = \left[ \frac{\mu \delta B}{T_{\perp}} (1 - \frac{T_{\perp}}{T_{\parallel}}) \right] F + \left( \frac{\gamma \mu \delta B}{\gamma + ik \parallel v} \right) \frac{F}{T_{\parallel}}
\]  

(3.10)

The last term on the right-hand side is a contribution that arises from the kinetic approach. This term is important for particles with \( v_{\parallel} = 0 \). For these particles, the term is of the same order and potentially of larger magnitude than the preceding mirror term. It shows that particles with small \( v_{\parallel} \) behave differently to the rest of the particle distribution. Southwood and Kivelson [71] refer to these particles as resonant particles. The presence of resonant particles causes the difference in dispersion relation between kinetic and fluid mirror instability treatments.

Taking the second moment of \( \delta F \) in equation (3.10) and applying the pressure balance condition gives the marginal instability condition,

\[
\frac{B \delta B}{\mu_0} + 2p_{\perp} (1 - \frac{T_{\perp}}{T_{\parallel}}) \frac{\delta B}{B} + 2 \left( \int dv_{\parallel} \frac{\gamma^2}{\gamma^2 + k_{\parallel}^2 v_{\parallel}^2} F_{\parallel} \right) \frac{T_{\parallel}^2}{T_{\parallel}} \frac{\delta B}{B} = 0
\]  

(3.11)
where \( F_\parallel \) means the distribution of parallel velocities after the integral over perpendicular velocities has been performed. \( F_\parallel \) is assumed to be a symmetric function of \( v_\parallel \).

For small growth rates \( \gamma \to 0 \), the integral approximates to

\[
\lim_{\gamma \to 0} \frac{\gamma}{\gamma^2 + x^2} = \pi \delta(x)
\] (3.12)

where \( \delta(x) \) is the Dirac delta function. Therefore,

\[
\frac{B \delta B}{\mu_0} + 2p_\perp (1 - \frac{T_\perp}{T_\parallel}) \frac{\delta B}{B} + 2\frac{\gamma}{k_\parallel} \left( \int dv_\parallel \pi \delta(v_\parallel) F_\parallel \right) \frac{T_\parallel^2}{T_\perp} \delta B = 0
\] (3.13)

The third term contains the resonant distribution. We rewrite the expression for any distribution function

\[
\frac{\gamma}{k_\parallel} \int dv_\parallel \pi \delta(v_\parallel) F(v_\parallel) = \frac{\gamma}{k_\parallel} \pi F(0) = \frac{\gamma}{k_\parallel} F_{res}
\] (3.14)

and so

\[
\gamma = -k_\parallel \frac{B^2}{\mu_0} \frac{1 + \beta_\perp (1 - T_\perp/T_\parallel)}{2\pi (T_\perp^2/T_\parallel) F_{res}}
\] (3.15)

From equation (3.15), it is clear that resonant particle play a different role compared to other resonant instabilities. Here, the linear growth rate is inversely proportional to the number of resonant particles or the resonant pressure.

Equation (3.16) represents the total pressure balance. First term is the change in magnetic pressure, the second is the mirrorlike response of the bulk of the plasma and the third term shows the response of resonant particles with close to zero parallel velocity. The energy of the particles in the bulk of the plasma is conserved. In the bulk of the plasma, particles undergo betatron acceleration and energy is exchanged between perpendicular and parallel velocities.
degrees of freedom. But the energy of the resonant particles change as instability develops. Because of close to zero parallel velocity, resonant particles do not move a significant distance along the field as instability grows. Therefore, the change in field that a resonant particle detects is due to local temporal changes in field. On the other hand, the bulk of the plasma moves through the field and the change in field experienced by these particles is through the spatial variation of the field perturbations. Figure 3-2 illustrates the difference between resonant and nonresonant particle behavior in the perturbed magnetic field of the mirror instability in its linear phase.

Based on the discussions, the linear mirror instability develops in the following manner. An increase/decrease in the magnetic field causes a pressure decrease/increase in the bulk
of the plasma that leads to a total pressure deficit/surplus. The resonant particles balance the pressure deficit/surplus by being accelerated/decelerated by the field in the increasing/decreasing field regions. Therefore, if there are fewer particles at small parallel velocity, the growth rate needs to be higher to balance the pressure imbalance generated by the bulk of the plasma distribution.

3.2 Nonlinear mechanism

There are different theories proposed for nonlinear evolution and saturation of mirror instability. Pantellini et al. [52] used quasi-linear theory to study the nonlinear evolution of the mirror instability. They suggest that the main mechanism that ends the linear phase of the mirror instability is particle trapping. Porazik et al. [58] use gyrokinetic simulations and show that magnetic holes saturate at lower amplitude and earlier than the magnetic peaks and this results in a peaked saturated structures. The final structures are wide shallow troughs and peaked narrow crests. The saturation in troughs is due to trapping but at the location of crests, saturation is the results of the reduction in $\beta_\perp$ because of the decrease in density and increase in the magnetic field. Hasegawa [29] interprets the nonlinear limit of the mirror instability as a patchwork of higher than background and lower than background field regions with spatially sinusoidal variations. Kivelson and Southwood [41] suggest that a sinusoidal spatial structure does not lead to stability in the nonlinear regime. They argue that the stability condition can be achieved in the magnetic peaks with field compression but in magnetic holes, the magnetic field has to decrease substantially to make plasma marginally stable.

The final evolved state should satisfy the pressure balance condition,

$$\frac{\partial}{\partial s} \left( p_\perp + \frac{B^2}{2\mu_0} \right) = 0 \quad (3.16)$$
where the derivative is along the flux tube. The question is how the spatial structure development modifies the plasma distribution so that the stability condition is satisfied everywhere along the flux tube. As mirror structures develop, the field and plasma distribution change to balance the pressure through the plasma. The expectation is that these structures evolve until the inequality in equation (3.2) becomes an equality. Observations show that the signatures of anticorrelated field and particle pressure changes can last for very extended periods of time [15] and it provides empirical support for a model of the fully evolved structures as stable and the final state of the mirror instability. These fully evolved structures should satisfy equation (3.16).

The physics of the nonlinear regime may differ from the linear regime physics. Because of the formation of magnetic peaks and magnetic holes, the particle distribution separate into trapped and untrapped components that respond differently to the changing field. A particle with \( v_\perp = v_{\perp 0} \) and \( v_\parallel = v_{\parallel 0} \) at the field strength \( B_0 \) will have \( v_\perp = v'_\perp \) and \( v_\parallel = 0 \) at its turning point with field strength \( B' \). Then the invariance of \( \mu \) yields

\[
\frac{mv^2_{\perp 0}}{2B_0} = \frac{mv'^2_{\perp}}{2B'}
\]  

(3.17)

Conservation of kinetic energy requires

\[
v'^2_{\perp} = v^2_{\perp 0} + v^2_{\parallel 0} = v^2_0
\]  

(3.18)

Combining equations (3.17) and (3.18) gives

\[
\frac{B_0}{B'} = \frac{v^2_{\perp 0}}{v'^2_{\perp}} = \frac{v^2_{\perp 0}}{v^2_0} = \sin^2 \theta
\]  

(3.19)

where \( \theta \) is the pitch angle of the particle in the weak field region. Particles with smaller \( \theta \) will reflect in regions of higher \( B \). Therefore, at any point where the field strength is \( B \), the
trapped population are those particles with pitch angle \( \theta \), where

\[
\frac{\pi}{2} > |\theta| > \sin^{-1} \left( \left( \frac{B}{B_{\text{max}}} \right)^{1/2} \right)
\]

Figure 3-3 illustrates the distinction in behavior between trapped and untrapped particles. It shows that in a mirror structure, the spatially varying pressure of the trapped particles along the flux tube creates conditions even more unstable compared to the initial uniform state. Trapped particles are excluded from maximum field (mirror point) regions. In the magnetic holes, the trapped particles cannot achieve marginal stability without cooling. Kivelson and Southwood [41] proposed the cooling process is caused by Fermi acceleration as the magnetic wells become deep and mirror points move apart.

We review the proposed mechanism by Kivelson and Southwood [41]. They argue particles of different pitch angles respond in very different ways to the evolution of the instability and this causes the final distribution to depend on magnetic moment and pitch angle. There are two different saturation mechanism, one acts in magnetic peaks and one that operates in magnetic holes. Both work to suppress the instability. In the magnetic peaks, as the field increases, the density and pressure decrease because particles with large pitch angles are excluded from rising field region by reflecting particles at field maxima and onset of trapping. This effect reduces \( \beta_\perp \) and stability condition can be achieved locally and lead to suppression of growth at the mirror location.

This mechanism cannot work in the magnetic holes. In the center of the hole, particles are trapped and if \( \mu \) and energy are conserved, the density and thermal pressure rise. As the magnetic field decreases, the field pressure gets smaller but total pressure continues to rise. In order to achieve stability in the magnetic holes, trapped particles need to cool down. The cooling can be produced by moving the mirror points. Trapped particles decelerate if the magnetic mirrors move apart. Figure 3-4 shows the Fermi acceleration and decrease of the particles in a time sequence of the wave amplitude. Near the center of the well,
Figure 3-3: An illustration of the distinction between the orbits of untrapped (upper panel) and trapped (lower panel) particles in a mirror geometry. Local velocity, density, and perpendicular pressure perturbations for adiabatic responses are characterized below each panel. (Taken from [41])
the trapped particles lose perpendicular energy and total energy. However, not all trapped particles lose energy during the instability development. Particles whose mirror point move closer together, as the field is growing, may have gained energy. These particles will have mirror points on the edge of the well. If the proposed stabilizing mechanism is correct, it can explain the observed feature of the mirror instability that the mirror events correspond to the development of magnetic holes. We will investigate the proposed mechanism in chapter 6.

3.3 Properties of the Magnetosheath Mirror Mode Structures

There are many observations of mirror structures in the magnetosheath from Cluster and Time History of Events and Macroscale Interactions during Substorms (THEMIS) missions and recently Multiscale Magnetospheric (MMS) mission. People have studied the properties of mirror modes in this region extensively. Mirror mode structures have been observed in both form of magnetic peaks and magnetic holes.

It is shown that the character of mirror structures is related to the local degree of instability of the plasma with respect to the mirror instability threshold: magnetic peaks are typically observed in a mirror unstable plasma, while mirror structures observed deep within stable region appear almost exclusively as holes. An abrupt transition of mirror structures from peaks to holes close to the magnetopause was identified by multi-spacecraft analysis and Soucek et al. [70] interpret this effect as a consequence of plasma expansion in the vicinity of the magnetopause locally changing the plasma condition toward a more stable state. Figure 3-5 shows the distribution of mirror mode structures in an anisotropy-beta plane measured by Cluster observations of mirror structures.

Cattaneo et al. [11] using Saturn magnetosheath data and Joy et al. [36] using Jupiter data suggested that mirror modes are generated close to the bow shock and in this early stage
Figure 3-4: Changing amplitude of a sinusoidally varying magnetic field causes Fermi acceleration of part of the distribution and decceleration of part of distribution. The time sequence of the wave amplitude is from solid curve to dashed curve (increasing amplitude). Blue line shows bounce orbit of a particle that is trapped close to the field minimum and its mirror points move apart as field amplitude has grown. The red line shows bounce orbit of a particle that is mirroring close to the field maximum and as field grows, its mirror points move closer. (Idea taken from [41])
Figure 3-5: Distribution of mirror modes of different types in the anisotropy-beta plane. Red triangles denote peaks, green filled circles holes and the remaining ambiguous mirror mode events are marked by grey stars. Solid blue line shows the theoretical mirror threshold (equation (3.3)), dashed-dotted blue line the empirical marginal stability relation ($T_{\perp}/T_{\parallel} = 1 + 0.83/\beta_{\parallel}^{0.58}$) and the black dashed line is the fitted boundary between peaks and holes ($T_{\perp}/T_{\parallel} = 2.15/\beta_{\parallel}^{0.39}$). (Taken from [70])
of development they appear as quasi-sinusoidal waves. Further downstream in the magnetosheath they approach nonlinear saturation and change into non-periodic large amplitude structures of either type; both peaks and holes are observed in the middle magnetosheath. As the mirror modes are convected towards the magnetopause, notably in the plasma depletion layer, plasma becomes mirror-stable and mirror structures start to collapse and decay away. The experimental studies [36] also show that the character of observed mirror modes depend on plasma beta: low beta plasma ($\beta_p < 1$) is usually populated by holes, while peaks are mostly observed in high beta plasma ($\beta_p > 5$).

Because magnetosheath plasma in the presence of mirror modes tends to follow the marginal stability path, where the instability growth rate is kept close to zero, it is natural to investigate how the character of mirror modes changes as plasma conditions change between a region of stability to an unstable state. Soucek et al. [70] suggest that mirror mode properties change abruptly due to plasma expansion in the plasma depletion layer and the magnetopause distance is likely an important determining factor of their shape. They propose that the character (peakness) of mirror modes is largely determined by spatial variation of plasma parameters. Mirror structures created behind the bow shock are convected downstream to reach nonlinear saturation in the middle magnetosheath. At this stage, plasma is kept in a marginally unstable state above the mirror threshold. As the plasma flow reaches the plasma depletion layer, it undergoes a rapid expansion forcing plasma into a mirror stable state. Mirror structures survive this transition, but they assume the form of holes under such conditions. In the next section, we test the survival of magnetic structures in the form of peaks and holes in a mirror stable plasma.

3.4 Bi-Stability

Califano et al. [10] conclude that magnetic holes do not result from direct nonlinear saturation of the mirror instability and that leads to magnetic peaks. Initial condition
in the form of large amplitude magnetic holes survive during their hybrid simulations both when plasma is linearly mirror stable and unstable. This indicates the existence of a bistable regime. Kuznetsov et al. [42] proposed an alternative model based on perturbative expansion of Vlasov-Maxwell equations including the effects of finite ion Larmor radius. This theory also predicts that holes can exist deep in the mirror stable region, while peaks will be dissipated rapidly under such conditions.

We performed one-dimensional simulations using PSC to test the bi-stability theory. We assume a homogenous electron-proton plasma with $\beta_p = 2.5$, $T_{p\perp}/T_{p\parallel} = 1$ and $\beta_e = 1$ with isotropic electron temperature. These plasma parameters generate a mirror stable plasma according to instability condition equation (3.3). We start the simulation with a magnetic peak with an amplitude about 50 percent of the background field. Figure 3-6 shows the evolution of the magnetic peak in a mirror stable plasma. Each solid line shows the time that matches its color. The black solid line shows the start of the simulation and the green line shows the last timestep. We see that the magnetic peak damps quickly and it is not able to survive in this condition. Figure 3-7 shows the evolution of a magnetic hole in a mirror stable plasma. The magnetic hole amplitude is about 50 percent of the background field at the start of the simulation. We see that the magnetic hole can survive in a mirror stable plasma. Its amplitude decreases but it doesn’t damp completely while the magnetic peak was completely damped at $\Omega_p t = 306$.

### 3.5 Discussions

We reviewed the physical description of the mirror instability for linear and nonlinear growth mechanisms. The linear analysis shows that the instability results from pressure imbalance between the bulk of the plasma and the magnetic field. The resonant particles produce a pressure perturbation in phase with the field pressure change while the bulk of the plasma responds in antiphase to the changes in magnetic field pressure. In the linear regime,
Figure 3-6: Evolution of a magnetic peak in a mirror stable plasma.

Figure 3-7: Evolution of a magnetic hole in a mirror stable plasma.
unlike the bulk of the plasma particles, the resonant particles experience energy changes as the instability develops.

In the nonlinear regime of the mirror instability, the evolution of the instability depends on the process of particle trapping. Particles with small parallel velocities are excluded from high field regions through trapping. This produces a local net decrease in particle pressure in the high field regions and allows the marginally stable state to be reached. In contrast, in magnetic wells, the particle pressure increases due to trapping and marginal stability condition cannot be attained without cooling of the trapped distribution. We investigate this proposed mechanism in chapter 6.

Observations show that the character of mirror structures is related to the local degree of instability of the plasma with respect to the mirror instability threshold. Peaks are typically observed in an unstable plasma, while mirror structures observed deep within stable region appear almost always as holes. A transition of mirror structures from peaks to holes was identified by multi-spacecraft analysis close to the magnetopause and this is interpreted as a consequence of plasma expansion locally changing the plasma condition toward a more stable state. We developed an expanding box technique in the PSC to resemble the magnetosheath expansion. We describe it in chapter 5.
It has been suggested that electron temperature anisotropy can enhance the proton mirror instability growth rate while leaving the proton cyclotron instability largely unaffected, therefore causing the proton mirror instability to dominate the proton cyclotron instability in Earth’s magnetosheath. Here, we use particle-in-cell simulations to investigate the electron temperature anisotropy effects on proton mirror instability evolution.

Proton temperature anisotropy with $T_{p,\perp}/T_{p,||} > 1$ leads to generation of proton cyclotron instability and proton mirror instability while presence of electron temperature anisotropy with $T_{e,\perp}/T_{e,||} > 1$ generates electron whistler instability and electron mirror instability. All of these instabilities compete with each other to consume the available free energy of the system which is contained in the temperature anisotropies. Proton cyclotron and proton mirror instabilities compete with each other for the available free energy in proton temperature anisotropy while electron whistler and electron mirror instabilities compete for consuming the electron temperature anisotropy. But there is also a competition between proton mirror instability and electron whistler instability to consume the available electron free energy.

The purpose of this chapter is to study the effects of electron temperature anisotropy on the evolution of proton mirror instability. Linear dispersion theory shows that the electron temperature anisotropy enhances the proton mirror instability growth rate but it doesn’t affect the proton cyclotron instability growth rate significantly [1, 20]. Since we need to consider electron dynamics, we use particle-in-cell simulations to include kinetic effects of both protons and electrons. Electrons get anisotropically heated in the shock layer similar
to protons [8]. Some previous studies have assumed electrons to be isotropic, since they performed hybrid simulations which treats electrons as a fluid [31, 66]. However, Tsurutani et al. [77] have shown that the electron temperature anisotropy is generally larger than 1 in Earth’s magnetosheath.

Masood et al. [45] analyzed Cluster data in Earth’s magnetosheath and found that electrons exhibit significant temperature anisotropy in the deep magnetosheath due to magnetic field line draping while being isotropic downstream of the quasi-perpendicular bow shock. Pokhotelov et al. [55–57] developed a linear theory to study the effects of finite electron temperature on proton mirror instability threshold and they confirmed that for sufficiently hot electrons, the proton mirror instability growth rate is enhanced. Remya et al. [60] used linear theory to study the role of electron temperature anisotropy on the proton cyclotron and proton mirror instabilities and they conclude that an inclusion of anisotropic electrons with $T_{e\perp}/T_{e\parallel} \geq 1.2$ reduces the proton cyclotron growth rate substantially and increases the proton mirror instability growth rate. However, they did not consider the presence of the electron whistler instability. In this chapter, we present the simulation results for different proton to electron mass ratios and how electron anisotropy affects the growth of the proton mirror instability.

## 4.1 Linear Analysis

We solved the linear dispersion relation for a homogeneous, collisionless plasma with bi-Maxwellian distributions to measure the growth rates of the temperature anisotropy instabilities for typical magnetosheath plasma parameters as we described in chapter 2. We consider two species: protons and electrons. We assume charge neutrality $n_p = n_e$ and zero relative drift between the electrons and protons [72]. Solutions of the linear dispersion equation are typically expressed in terms of dimensionless variables. It is natural to use electron inertial length and electron gyrofrequency as normalizing factors for electrons and proton
inertial length and proton gyrofrequency for normalizing proton related instabilities.

In Earth’s magnetosheath, the distributions become anisotropic because of heating of the particles across the quasi-perpendicular bow shock and field line draping. The time scale of the heating through the shock is about one proton gyroperiod. This time scale is very fast and does not allow the proton instabilities to grow in the shock layer. Therefore a considerable amount of proton temperature anisotropy is left downstream of the quasi-perpendicular shock in the magnetosheath. For electrons, on the other hand, one proton gyroperiod equals 1836 electron gyroperiods. Thus, electron instabilities have sufficient time to grow and isotropize the electron distributions. Therefore, we consider high proton temperature anisotropies and lower electron temperature anisotropies to resemble the magnetosheath plasma conditions downstream of the quasi-perpendicular shock [8].

4.1.1 Competition between electron whistler and electron mirror instability

Figure 4-1a shows the instability thresholds for electron whistler and electron mirror instabilities. We keep \( T_{p\perp}/T_{p\parallel} = 1 \) and \( \beta_{p\parallel} = 1 \). The instability thresholds \( (\gamma_m/\Omega_e = 0.01) \) are measured using linear dispersion theory. \( \gamma_m \) refers to maximum growth rate. Comparing the electron whistler and electron mirror instability growth rates in Figure 4-1a, we clearly see that the electron whistler instability has a lower instability threshold than the electron mirror instability and it may therefore suppress the electron mirror mode. Observations show that electrons follow the marginal stability path of the electron whistler instability in Earth’s magnetosheath which indicates that electron whistler instability is the dominant instability [26].

Gary and Wang [24] provided an analytical instability threshold for electron whistler instability. The threshold condition for \( \gamma_m/\Omega_e = 0.01 \) is
\[ R_w = \beta_e^{0.55} \left( \frac{T_{e\|}}{T_{e\perp}} - 1 \right) \geq 0.36 \] (4.1)

\( R_w \geq 0.36 \) means plasma is unstable relative to electron whistler instability and for \( R_w < 0.36 \) plasma is electron whistler stable.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{instability_thresholds.png}
\caption{(a) Electron temperature anisotropy at the \( \gamma_m/\Omega_e = 0.01 \) thresholds of electron whistler and electron mirror instabilities as function of \( \beta_e\perp \). The solid line shows the instability threshold of electron whistler instability and the dashed line shows the electron mirror instability threshold. If the plasma parameters lie below the threshold line, the instabilities won’t be able to grow. (b) Proton temperature anisotropy at the \( \gamma_m/\Omega_p = 0.01 \) thresholds of proton cyclotron and proton mirror instabilities as function of \( \beta_p\perp \). The solid line shows the instability threshold for proton cyclotron instability and the dashed line shows the proton mirror instability threshold. If the plasma parameters lie below the threshold line, the instabilities won’t be able to grow.}
\end{figure}

4.1.2 Competition between proton cyclotron and proton mirror instability

In the case of the proton temperature anisotropy instabilities, the proton cyclotron instability has larger growth rate compared to the proton mirror instability for low proton plasma beta \( \beta_p\| \) and it should be the dominant instability in the magnetosheath as shown in Figure 4-1b.

The analytical threshold condition for the proton mirror instability in a homogeneous plasma with warm anisotropic electrons is given by Pantellini et al. [53] and Pokhotelov et
al. [57],

\[
R_m = \beta_p (\frac{T_{p\perp}}{T_{p\parallel}} - 1) + \beta_e (\frac{T_{e\perp}}{T_{e\parallel}} - 1) - \frac{\beta_e (T_{p\perp} - T_{e\perp})}{2} (\frac{T_{p\parallel} - T_{e\parallel}}{1 + \frac{T_{e\parallel}}{T_{p\parallel}}})^2 \geq 1
\]  

(4.2)

and for proton cyclotron instability the analytical threshold of the instability is given by Gary and Lee [23],

\[
R_p = \beta_p^{0.42} (\frac{T_{p\perp}}{T_{p\parallel}} - 1) \geq 0.43
\]  

(4.3)

We use these threshold conditions to determine which instability is dominant in our simulations. In Figure 4-1b, we keep electrons isotropic and measure the proton cyclotron and mirror instability thresholds \((\gamma_m / \Omega_p = 0.01)\) using linear dispersion theory. It is clear that the proton cyclotron instability has larger growth rate compared to mirror instability for low \(\beta_{p\parallel}\) and high \(T_{p\perp} / T_{p\parallel}\). But observations show that in regions where we expect the dominance of the proton cyclotron instability, mirror instability has grown and it is the dominant mode. So the question is what helps the proton mirror instability to grow faster than the proton cyclotron instability in low \(\beta_{p\parallel}\) regions?

One possibility is the effects of electron temperature anisotropy on the proton mirror instability growth rate. Figure 4-2 shows that by increasing the electron temperature anisotropy, mirror instability growth rate increases while leaving the proton cyclotron instability only slightly affected. The reason is that proton cyclotron instability is a resonant instability and electrons do not resonate with proton cyclotron mode, but they can get trapped in the magnetic bottles of mirror instability and exchange energy with the wave.

In order to study the nonlinear effects of electron dynamics on the evolution of the proton mirror instability, we use particle-in-cell simulations.

### 4.2 Nonlinear Evolution Simulation Results

We use \texttt{psc} to obtain the results of this section. First, we start with bi-Maxwellian distributions for both protons and electrons. We choose parameters that are characteristic of
Figure 4-2: Electron temperature anisotropy effects on mirror instability and proton cyclotron maximum growth rates. Solid line shows the maximum growth rate of the proton cyclotron instability as a function of electron temperature anisotropy and dashed line shows the maximum growth rate of proton mirror instability. $T_{p\perp}/T_{p\parallel} = 2.5$ and $\beta_{p\parallel} = \beta_{e\parallel} = 1$ are fixed.

The magnetosheath. In particular, the plasma parameters are $T_{p\perp}/T_{p\parallel} = 2.5$, $T_{e\perp}/T_{e\parallel} = 1.5$, $\beta_{p\parallel} = 2$ and $\beta_{e\parallel} = 0.5$. In the magnetosheath, electrons are about 10 times colder than protons. We choose electron temperature to be 4 times colder because of the limitations of particle-in-cell simulations. We need to resolve the electron Debye length and colder electrons means smaller electron Debye length which needs finer grid resolutions. We perform two-dimensional particle-in-cell simulations. A constant background magnetic field $B_0 = v_A/c = 0.025$ is assumed in the $z$ direction where $v_A$ is the proton Alfven speed and $c$ is speed of light. In the magnetosheath, $v_A/c$ is about $10^{-4}$ which leads to very small time steps in particle-in-cell simulations. Therefore, we select a larger $v_A/c$ to avoid the computationally expensive simulations. The number of grid points ($n_y \times n_z$) are $2048 \times 2048$. Periodic boundaries are used in each dimension. The number of particles used is on average 200 particles/cell. The size of the grid cells is taken to be $\Delta y = \Delta z = 0.015d_p$.

For these parameters, linear theory predicts, the maximum growth rate of proton cyclotron instability to be $\gamma_m = 0.14\Omega_p$ at $k_mD_p = 0.47$, while the proton mirror instability maximum growth rate is $\gamma_m = 0.10\Omega_p$ with $k_mD_p = 0.53$ at $\theta = 57^\circ$. The electron whistler instability maximum growth rate is $\gamma_m = 0.008\Omega_e$ with $kMd_e = 0.6$.  

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(a) Electron temperature anisotropy evolution for different $m_p/m_e$. As we increase the mass ratio, the linear $m_p/m_e$. We only show the linear regime of proton regime of electron whistler instability becomes smaller and electrons quickly isotropize.

Since there is an electron temperature anisotropy ($T_e \perp /T_e \parallel > 1$), the electron whistler instability grows and rapidly isotropizes the electron distribution. Also, the proton cyclotron and the proton mirror instability grow due to the presence of the proton temperature anisotropy ($T_p \perp /T_p \parallel > 1$).

We choose different mass ratios $m_p/m_e = (25, 100, 400, 1836)$ and examine the electron temperature anisotropy evolution compared to the proton temperature anisotropy changes. Figure 4-3a shows the dependence of electron temperature anisotropy evolution as a function of proton to electron mass ratio ($m_p/m_e$) in particle-in-cell simulations. We only show the linear regime of the proton instabilities which lasts to about $\Omega_p t = 50$ in this case, because we want to see how much electron temperature anisotropy is left when proton instabilities start to grow nonlinearly. Figure 4-3b shows the proton temperature anisotropy as a function of time for different $m_p/m_e$. Since we are keeping the $\omega_p/\Omega_p$ as a constant in all simulations, we expect the same linear regime for proton temperature anisotropy instabilities. Figure 4-3a shows that as we increase the mass ratio, the linear regime of the electron whistler instability becomes smaller since we are making the electrons faster and more close to reality. For $m_p/m_e = 1836$, at the end of proton instabilities linear regime, when proton instabilities start growing nonlinearly, there is no electron temperature anisotropy left for proton mirror instability to take advantage of and to win the competition.
with the proton cyclotron instability. Unless, there is a mechanism that constantly drives the electron temperature anisotropy in the magnetosheath. The adiabatic expansion close to the magnetopause, in the plasma depletion layer, could be a continuous driver of the temperature anisotropies. Although, electron distribution becomes isotropic more slowly with $m_p/m_e = 25$ compared to larger mass ratios, but it still leads to relatively isotropic distribution at the end of proton instabilities linear regime.

Figure 4-4: Time evolution of magnetic field components. First column shows $B_x$, second column $B_y$ and third column is $\delta B_z$.

In order to examine the effects of electron temperature anisotropy on proton mirror
instability in more detail, we perform two simulations with similar parameters and different electron temperature anisotropies. In one simulation we keep electrons isotropic and in another one, we start with \( T_{e\perp}/T_{e\parallel} = 2 \). The simulation parameters are \( T_{p\perp}/T_{p\parallel} = 2.5 \), \( \beta_{p\parallel} = 1 \), \( \beta_{e\parallel} = 1 \), \( m_p/m_e = 25 \) and \( B_0 = v_A/c = 0.1 \). We use \( m_p/m_e = 25 \) to keep the computational cost manageable. While we have shown that by the end of the proton linear phase, the electrons have essentially isotropized both at this as well as at the real mass ratio, the artificially lowered mass ratio exaggerated the effects of the electron anisotropy. This is actually helpful as it allows us to more clearly identify the impact on the proton instabilities.

Figure 4-4 shows the components of the magnetic field at different timesteps from the simulation with anisotropic electrons \( (T_{e\perp}/T_{e\parallel} = 2) \). We can see that at early timesteps, electron whistler waves gets excited and are propagating along the background magnetic field. As time goes on, the electron whistler instability saturates and both the proton cyclotron and the proton mirror instability start growing. It is clear that proton cyclotron waves are propagating along the background magnetic field while proton mirror waves are present in the direction oblique to the background magnetic field.

Figure 4-5 shows the spectrum of the total magnetic field in wavenumber space at different times. Each instability has been marked in the spectrum in the Figure 4-5. The electron mirror instability is about 20 times weaker than the electron whistler instability. At early times, the electron whistler instability is the dominant mode. At later times, the proton cyclotron and the proton mirror instability start growing while the electron whistler instability is still present.

We make cuts in the \( B_y \) along \( z \) direction at \( y = 64d_p \) and in the \( \delta B_z \) along \( y \) direction at \( z = 64d_p \) from Figure 4-4. The \( B_y \) and \( \delta B_z \) cuts are shown in Figure 4-6. These cuts resemble the satellite crossings at the locations where these instabilities would typically be present. In Figure 4-6, we see the electron scale wavelengths that are electron whistler waves and later, proton scale wavelength structures grow which are a combination of proton cyclotron and proton mirror mode waves. In the \( \delta B_z \) cuts in, the proton scale structures are
Figure 4-5: Total magnetic field spectrum. At early time, electron whistler instability is present. Later on, proton cyclotron and proton mirror instabilities grow. As each mode grows nonlinearly, their wavelength becomes larger and their spectrum moves to smaller wavenumbers.
proton mirror waves since proton cyclotron waves cannot have perturbations in $z$ direction in two-dimensional simulations.

![Magnetic field cuts](image)

**Figure 4-6:** Magnetic field cuts of $B_y$ along $z$ direction at $y = 64d_p$ and $\delta B_z$ along $y$ direction at $z = 64d_p$ at different times. Black solid line shows $B_y$ and red dashed line is $\delta B_z$ cut.

In Figure 4-7, the evolution of proton and electron temperature anisotropy is shown. The proton instabilities start growing nonlinearly around $\Omega_p t = 75$. At this time, the electron temperature anisotropy is still $T_{e,\perp}/T_{e,\parallel} = 1.62$. For plasma parameters at this timestep, the proton mirror instability is stronger than the proton cyclotron instability. The proton cyclotron maximum growth rate is $\gamma_m/\Omega_p = 0.07$ at $k_m d_p = 0.48$ while proton mirror
instability maximum growth rate is \( \gamma_m / \Omega_p = 0.10 \) with \( k_m d_p = 0.79 \) at \( \theta = 62^\circ \). Then, in
the nonlinear regime, both instabilities are present as shown in Figure 4-5.

\[
\frac{m_p}{m_e} = 25, \beta_p = \beta_e = 1.0, v_A / c = 0.1
\]

**Figure 4-7:** Temperature anisotropy evolution of protons and electrons with \( m_p/m_e = 25 \).

Figure 4-9 shows the time evolution of the magnetic energy density of proton cyclotron, proton mirror mode and electron whistler waves. We measure the magnetic energy density of each wave by filtering the wave spectra for each mode. The wave spectra shows three ranges for wave number vector space as seen in Figure 4-8. We define the proton cyclotron instability range to be \( 0 \leq \theta \leq 30^\circ \) and proton mirror instability range is \( 30^\circ \leq \theta \leq 80^\circ \) for \( 0 < k_{\perp,||} \leq 1 \). For electron whistler instability, we choose \( 0 \leq \theta \leq 30^\circ \) but the wave number range is \( 0 < k_{\perp} \leq 1 \) and \( 1 < k_{||} \leq 4 \). We find a significant difference between the saturation levels of the proton cyclotron and the proton mirror instabilities for the isotropic and anisotropic electron cases, respectively. In the isotropic case, shown in Figure 4-9 with dashed lines, the magnetic energy density of the proton cyclotron instability is much larger than that of the proton mirror instability. With isotropic electrons, the proton cyclotron instability maximum growth rate is about 3 times stronger than the proton mirror instability, and we expect proton cyclotron instability to consume most of the available free energy.

In the anisotropic electrons case, the proton mirror instability maximum growth rate is larger than that of the proton cyclotron instability, but we see that the magnetic energy
Figure 4-8: Energy spectrum regions for each instability. Electron whistler instability exist in large $k_\parallel$ region and proton cyclotron instability in small $k_\parallel$. Proton mirror instability is present in oblique directions with $k_\perp > k_\parallel$. Electron mirror instability is very weak and it doesn’t contribute in energy density consumption.

Figure 4-9: Energy density evolution for different $T_{e\perp}/T_{e\parallel}$. Solid lines show the energy density of the instabilities with $T_{e\perp}/T_{e\parallel} = 2$ and dashed lines show the energy density of instabilities with $T_{e\perp}/T_{e\parallel} = 1$. Solid black line shows the electron whistler instability.
density of the proton cyclotron instability is still more than that of the proton mirror instability. Also, the proton mirror instability gains more magnetic energy density compared to the isotropic electron case, which shows that the electron anisotropy affects the proton mirror instability evolution. At late times, when electrons become isotropic, the instabilities in both simulations saturate at roughly the same magnetic energy density levels. Also, we see that the proton cyclotron instability starts growing at a slightly different time when an electron temperature anisotropy is present, since the presence of an electron temperature anisotropy decreases the proton cyclotron instability growth rate. The proton mirror instability starts growing earlier in the anisotropic electron case, because the electron anisotropy enhances the proton mirror instability growth rate. We see that choosing $m_p/m_e = 25$ has some impacts on the evolution of proton mirror and proton cyclotron instabilities compared to choosing real mass ratio when electron temperature anisotropy is present.

4.3 Discussions

In this chapter, we have investigated the effects of electron temperature anisotropy on the proton mirror instability evolution. Linear theory predicts that presence of an electron temperature anisotropy can enhance the proton mirror instability growth rate, and if it is large enough, it can make the proton mirror instability stronger than the proton cyclotron instability. We showed that anisotropic electrons, however, primarily drive the electron whistler instability. We performed two-dimensional particle-in-cell simulations with different electron to proton mass ratios. We studied how varying the mass ratio affects the electron whistler instability evolution and how it impacts the proton cyclotron and proton mirror instability growth rates. We find that the electron whistler instability consumes the electron free energy before the proton mirror instability grows into the nonlinear regime, because it grows much faster than the proton temperature anisotropy instabilities. Therefore, all the electron free energy is gone quickly and has little impact on the much slower proton
mirror instability that has barely started growing by that time. Our results show that
temperature anisotropy instabilities are sensitive to the chosen mass ratio $m_p/m_e$ in particle-
in-cell simulations, since an artificial mass ratio can affect the growth and dynamics of the
instabilities.

If there is a mechanism in the magnetosheath that keeps $T_{e\perp} > T_{e\parallel}$, it can enhance the
proton mirror instability growth rate. For example, the adiabatic expansion in the plasma
depletion layer close to the magnetopause makes $T_{e\perp} > T_{e\parallel}$
Chapter 5

The PIC Method in an Expanding or Compressing Box

I investigate the nonlinear evolution of the mirror instability in the magnetosheath by means of fully-kinetic particle-in-cell (PIC) simulations. I have modified the three-dimensional electromagnetic particle-in-cell code psc to account for the compression or expansion of the system. The modified code is following the expanding box method given by Sironi et al. [68]. The physical motivation behind the expanding box code is the assumption that the typical scales of macroscopic processes as expansion and compression are usually much larger than the kinetic ion and electron scales.

In this chapter, I describe the implementation of the expanding and compressing box method into psc. I derive the Maxwell’s equations and Lorentz force in the moving frame and apply the required approximations. After describing the method, I verify it by testing Chew-Goldberger-Low (CGL) predictions [13].

5.1 The Expanding Box Method

Here, I first need to describe the basic equations of the particle-in-cell method in a compressing or expanding box. I review the derived equations for expanding box method given by Sironi et al. [68]. I will solve the Maxwell’s and equations of motion in the fluid comoving frame. The fluid comoving frame is related to the laboratory frame by a Lorentz boost, with velocity $U$. It is reasonable to only consider the non-relativistic limit ($|U|/c \ll 1$) since the compression and expansion velocities in the magnetosheath are non-relativistic as we justify it later.
In the fluid comoving frame, we choose two sets of spatial coordinates, primed and un-primed coordinate systems. Both primed and unprimed coordinate systems are related to the same reference frame (the comoving frame), so they share the same time coordinate \((t' = t)\). The unprimed coordinate system in the fluid comoving frame has a basis of unit vectors, therefore it is the appropriate coordinate set for measuring all physical quantities. In the primed coordinate system, we redefine the unit length of the spatial axes such that a particle subject only to compression or expansion stays at fixed coordinates. This helps us to avoid the additional time derivatives in the Maxwell’s equations in the primed coordinate system in the comoving frame. Quantities measured in the laboratory frame will be labeled with the subscript \(L\).

The particle location in the laboratory frame is related to its position in the primed coordinate system of the fluid comoving frame by

\[
x_L = Lx',
\]

where compression or expansion are described by the diagonal matrix

\[
L = \frac{\partial x}{\partial x'} = \begin{pmatrix} a_x & 0 & 0 \\ 0 & a_y & 0 \\ 0 & 0 & a_z \end{pmatrix}, \quad l = \det(L)
\]

where \(l\) is the determinant. In general, \(a_x\), \(a_y\) and \(a_z\) are functions of time, but not of the spatial coordinates. As we mentioned both primed and unprimed coordinate systems exist in the same reference frame (the comoving frame), so they have the same time coordinate \(t' = t\) and \(dt' = dt\). By differentiating \(x_L = Lx'\), we find
\[ d\mathbf{x}_L = \mathbf{L}d\mathbf{x}' + \dot{\mathbf{L}}\mathbf{x}'dt' \]  

(5.3)

where \( \dot{\mathbf{L}} = \frac{d\mathbf{L}}{dt'} \).

The relation between the primed and unprimed coordinate systems is such that \( d\mathbf{x} = \mathbf{L}d\mathbf{x}' \). By defining \( \mathbf{U} = \dot{\mathbf{L}}\mathbf{x}' = \dot{\mathbf{L}}\mathbf{L}^{-1}\mathbf{x}_L \), we can rewrite equation (5.3) as

\[ d\mathbf{x}_L = d\mathbf{x} + \mathbf{U}dt' \]  

(5.4)

which describes the Lorentz transformation from the comoving frame to the laboratory frame, to first order in the boost velocity \( |\mathbf{U}|/c \ll 1 \). In the magnetosphere concept, the velocity \( \mathbf{U} \) is the expansion or compression velocity of the magnetosheath which depends on solar wind pressure. For example for an expansion parallel to the background magnetic field aligned in the \( z \) direction,

\[ a_x = 1, \quad a_y = 1, \quad a_z = 1 + q_z t \]

where \( 1/q_z \) is the expansion characteristic time. Therefore, in our setup, \( U_x = U_y = 0 \), whereas \( U_z = q_z z' \). The typical scale of our simulations is the proton Larmor radius \( \rho_p \) and this gives us \( U_z/c \sim q_z \rho_p/c \sim (q_z/\omega_p)(v_{thp}/c) \), where \( v_{thp} \) is the proton thermal velocity and \( \omega_p \) refers to the proton plasma frequency. In the magnetosheath regime, we expect non-relativistic protons \( (v_{thp}/c \ll 1) \) and slow expansions \( (q_z/\omega_p \ll 1) \), so our assumption \( \mathbf{U}/c \ll 1 \) is easily satisfied.

Now we derive the Lorentz transformation of the time coordinate between the laboratory frame and the comoving frame to the first order in the boost velocity \( \mathbf{U}/c \ll 1 \),
\[ dt_L = dt + (U/c^2) \cdot dx = dt' + (U/c^2) \cdot (Ldx') \]  \hspace{1cm} (5.5)

Using equations (5.4)-(5.5), we obtain the relation between the momentum of a particle in the laboratory frame and in the comoving frame. We define \( p_L = mdx_L/d\tau \) and \( p' = mdx'/d\tau = L^{-1}p \), where \( \tau \) is the proper time and \( p = mdx/d\tau \) is the physical momentum in the unprimed coordinate system. Proper time is the time measured in the frame in which the particle is at rest. Therefore,

\[ p_L = Lp' + U\gamma'm = p + U\gamma m \]  \hspace{1cm} (5.6)

while \( \gamma \) and \( \gamma' \) are the particle Lorentz factors in unprimed and primed coordinates, respectively. The Lorentz factor in primed coordinate is defined by \( \gamma' = dt'/d\tau \) and in unprimed coordinate, it is \( \gamma = dt/d\tau \). Since \( dt' = dt \), it results in \( \gamma' = \gamma \). By defining the Lorentz factor in laboratory frame \( \gamma_L = dt_L/d\tau \), we find

\[ \gamma_L = \gamma' + (U/c^2) \cdot (Lp'/m) = \gamma + (U/c^2) \cdot (p/m) \]  \hspace{1cm} (5.7)

We show this is consistent with a Lorentz transformation at first order in \( U/c \ll 1 \). Without any approximation, \( \gamma_L = \sqrt{1+(p_L/mc)^2} \) and \( \gamma = \gamma' = \sqrt{1+(p/mc)^2} \). With rewriting \( \gamma_L \) in terms of \( p \) and \( \gamma \),
\[
\gamma_L = \sqrt{1 + \left(\frac{p + U \gamma m}{mc}\right)^2}
\]
\[
= \sqrt{1 + \left(\frac{p}{mc}\right)^2 + \frac{2U}{c^2} \cdot \frac{p}{m} \gamma + \frac{U^2}{c^2} \gamma^2}
\]
\[
= \sqrt{\gamma^2 + 2 \frac{U}{c^2} \cdot \frac{p}{m} \gamma + \frac{U^2}{c^2} \gamma^2},
\]

therefore,

\[
\gamma_L = \gamma \sqrt{1 + 2 \frac{U}{c^2} \cdot \frac{p}{m} \gamma + \frac{U^2}{c^2}}
\]

By using the first order approximation in \(U/c \ll 1\),

\[
\gamma_L = \gamma (1 + \frac{U}{c^2} \cdot \frac{p}{m\gamma} + \frac{1}{2} \frac{U^2}{c^2}) = \gamma + \frac{U}{c^2} \cdot \frac{p}{m}
\]

which is in agreement with equation (5.7). Equation (5.7) will be used to obtain the Lorentz force in the fluid comoving frame.

We can also derive the relation between the temporal and spatial derivatives, when transforming from the laboratory frame to the comoving frame in the primed coordinates. We obtain,

\[
\frac{\partial}{\partial x_L} = L^{-1} \frac{\partial}{\partial x'} - \frac{U}{c^2} \frac{\partial}{\partial t'} \quad (5.8)
\]
\[
\frac{\partial}{\partial t_L} = \frac{\partial}{\partial t'} - U \cdot (L^{-1} \frac{\partial}{\partial x'}) \quad (5.9)
\]
The two equations above will be used to find the form of Maxwell’s equations in the comoving frame.

### 5.1.1 Maxwell’s Equations

There are some assumptions that we need to mention before deriving the Maxwell’s equations. As we said before, we assume that the compression and expansion velocity is non-relativistic, i.e., $U/c \ll 1$. So, we only keep the terms that linearly depend on $U/c$, neglecting higher order terms. For the same reason, we neglect terms containing $\ddot{\mathbf{L}}$, since this is the temporal derivative of $\dot{\mathbf{L}}^2/2$, which we consistently discard.

In all the circumstances related to this work, the rate of expansion or compression of the system does not change with time, i.e., $\ddot{\mathbf{L}} = 0$. Therefore, we neglect acceleration terms proportional to $\ddot{\mathbf{L}}$ in the equations.

The Lorentz force that we obtain in the fluid comoving frame, holds for any particle momentum, i.e., for non-relativistic, trans-relativistic or ultra-relativistic particles.

Maxwell’s equations in the comoving frame, to first order in $|U|/c \ll 1$:

\begin{align*}
\nabla' \cdot (l\mathbf{L}^{-1}\mathbf{E}) &= 4\pi l \rho' \\
\nabla' \cdot (l\mathbf{L}^{-1}\mathbf{B}) &= 0 \\
\n\nabla' \times (l\mathbf{E}) &= -\frac{1}{c} \frac{\partial}{\partial t'} (l\mathbf{L}^{-1}\mathbf{B}) \\
\n\nabla' \times (l\mathbf{B}) &= \frac{1}{c} \frac{\partial}{\partial t'} (l\mathbf{L}^{-1}\mathbf{E}) + \frac{4\pi}{c} l J' 
\end{align*}
where the temporal and spatial derivatives are in the primed coordinate system. The primed and unprimed systems share the same time coordinate, so $\partial/\partial t' = \partial/\partial t$, whereas spatial derivatives differ, $\nabla' = L\nabla$. In the above equations, $E$ and $B$ are the physical electromagnetic fields measured in the unprimed coordinate system.

The transformation of the electromagnetic fields between inertial frames is given by,

$$E = \gamma L (E_L + \frac{U}{c} \times B_L) - (\gamma L - 1)(E \cdot \hat{U})\hat{U}$$

$$B = \gamma L (B_L - \frac{U}{c} \times E_L) - (\gamma L - 1)(B \cdot \hat{U})\hat{U}$$

After approximating $\gamma L \approx 1$, we get

$$E = E_L + \frac{U}{c} \times B_L \simeq E_L + \frac{U}{c} \times B \quad (5.14)$$

$$B = B_L - \frac{U}{c} \times E_L \simeq B_L + \frac{U}{c} \times E \quad (5.15)$$

These are consistent with a Lorentz transformation at first order in $|U|/c \ll 1$.

The charge and current density Lorentz transformation between inertial frames gives,

$$\rho_L = \gamma(\rho' + (U/c^2) \cdot J)$$

$$J_L = J + \gamma U \rho - (\gamma - 1)(J \cdot \hat{U})\hat{U}$$

After approximation, the charge and current density transform as
\[
\rho_L = \rho' + \left( \frac{U}{c^2} \right) \cdot (LJ') \quad (5.16)
\]

\[
J_L = LJ' + U\rho' \quad (5.17)
\]

with \( \rho = \rho' \) and \( J = LJ' \) in the unprimed coordinate system.

In our PIC algorithm, we solve the two evolutionary equations (5.12)-(5.13).

The charge-conserving algorithm implemented in PSC ensures that continuity equation is satisfied at all times,

\[
\frac{\partial}{\partial t'} (l\rho'c) + \nabla' \cdot (lJ') = 0, \quad (5.18)
\]

The time-dependent terms in expansion and compression matrix \( L \) affect the Courant-Friedrichs-Lewy (CFL) condition for numerical stability. The CFL condition for a three dimensional box with \( \Delta x = \Delta y = \Delta z \) is

\[
c \leq \frac{a\Delta x}{\sqrt{3}\Delta t} \quad (5.19)
\]

with \( a_x = a_y = a_z = a \) for the case of an isotropic compression or expansion. For compression, \( a = 1 - qt \), the simulation eventually runs into stability issues. In compressing box simulation, we compress the domain to half of its initial size and choose the CFL condition that is satisfied during the whole simulation. For expanding box simulations, \( \Delta x \) is increasing therefore the CFL condition is satisfied at all times.

It is convenient to define the electromagnetic fields in the primed coordinate system,
\[ B' = lL^{-1}B \quad E' = lL^{-1}E \]  \hspace{1cm} (5.20)

so that the Ampere’s law and Faraday law in the primed coordinate become

\[
\nabla' \times (l^{-1}L^2E') = -\frac{1}{c} \frac{\partial B'}{\partial t'} \hspace{1cm} (5.21)
\]

\[
\nabla' \times (l^{-1}L^2B') = \frac{1}{c} \frac{\partial E'}{\partial t'} + \frac{4\pi}{c} lJ' \hspace{1cm} (5.22)
\]

In the numerical algorithm, the term \( l^{-1}L^2 \) is evaluated at the same time as \( E' \) in equation (5.21) and \( B' \) in equation (5.22).

### 5.1.2 The Lorentz Force

The Lorentz force in the comoving frame can be obtained from the Lorentz force in the laboratory frame by differentiating equation (5.6) with respect to time. At first order in \( |U|/c \ll 1 \), we find

\[
\frac{dp'}{dt'} = -2\dot{L}L^{-1}p' + qL^{-1}(E + \frac{Lv'}{c} \times B) \hspace{1cm} (5.23)
\]

where \( q \) is the particle charge and \( v' = dx'/dt' = p'/\gamma'm \).

It is simpler to implement the momentum equation in the unprimed coordinate system and the particle push in the primed coordinate system in the algorithm. The evolution of the particle orbits is solved with these set of equations,
\[
\frac{dp}{dt'} = -\dot{L}L^{-1}p + q(E + \frac{v}{c} \times B) \quad (5.24)
\]

\[
\frac{dx'}{dt'} = v' \quad (5.25)
\]

In summary, we solve equation (5.24) in unprimed coordinates and equation (5.25) in primed coordinates for particle push and equations (5.21) and (5.22) for the electromagnetic fields in the primed coordinate system.

The standard Boris pusher is implemented in our particle-in-cell code that updates the particle momentum in three steps: (i) acceleration via the electric field for half timestep; (ii) rotation by the magnetic field for a full timestep; (iii) acceleration via electric field for half a timestep. The rotation by the magnetic field is still \( v \times B \) and it is unchanged by expansion or compression of the box. We need modify the acceleration via electric field parts to preserve the second order accuracy of the numerical discretization of the equations.

Assume \( P_n \) and \( P_{n+1} \) are the particle momenta at timestep \( n \) and \( n+1 \), respectively, and let \( P_{n^-} \) and \( P_{n^+} \) be the momenta respectively before and after the magnetic rotation. The equations are then discretized as follows,

\[
\frac{p_{i,n^-} - p_{i,n}}{\Delta t/2} = -\left(\frac{\dot{a}_i}{a_i}\right)_{n+\frac{1}{2}} p_{i,n} + p_{i,n^-} + qE_{i,n+\frac{1}{2}} \quad (5.26)
\]

\[
\frac{p_{i,n^+} - p_{i,n+1}}{\Delta t/2} = -\left(\frac{\dot{a}_i}{a_i}\right)_{n+\frac{1}{2}} p_{i,n^+} + p_{i,n+1} + qE_{i,n+\frac{1}{2}} \quad (5.27)
\]

where \( i = x, y, z \).
5.1.3 Charge and Current densities

The electric current density $J'$ in the primed coordinate system is calculated as a sum of the contributions of individual particles. In the unprimed coordinate system of the fluid comoving frame, the current density is

$$J(x,t) = \sum_{\alpha} q_{\alpha} v_{\alpha} S[x - x_{\alpha}(t)]$$  \hspace{1cm} (5.28)

where the summation is over the particles $\alpha$. $S$ is the weighting function that depends on the particle shape function. The current density in the primed coordinate is given by

$$J' = L^{-1}J = l^{-1} \sum_{\alpha} q_{\alpha} v'_{\alpha} S[x' - x'_{\alpha}(t')]$$  \hspace{1cm} (5.29)

Also, the charge density in the primed coordinate system is given by

$$\rho' = l^{-1} \sum_{\alpha} q_{\alpha} S[x - x_{\alpha}(t)]$$  \hspace{1cm} (5.30)

From equation (5.29) and equation (5.30), the charge conservation in equation (5.18) follows.

5.2 Verifying Expanding and Compressing Box Simulations

To verify the accuracy of the implemented expanding and compressing box method, we perform different benchmarking tests. A test is done by using the Chew-Goldberger-Low (CGL) predictions [13] of the plasma properties when plasma is expanding or compressing adiabatically.
5.2.1 CGL predictions

In an adiabatic expansion or compression, when there are no waves present in the system, first and second adiabatic invariants are conserved based on the CGL condition and these invariants tell us how the parallel and the perpendicular temperatures change.

First adiabatic invariant: \[
\frac{d}{dt} \left( \frac{p_{\perp}}{nB} \right) = 0 \quad \rightarrow \quad T_{\perp} \propto B \quad (5.31)
\]

Second adiabatic invariant: \[
\frac{d}{dt} \left( \frac{p_{\parallel}}{n^3} \right) = 0 \quad \rightarrow \quad T_{\parallel} \propto \frac{n^2}{B^2} \quad (5.32)
\]

In order to test our method, we perform tests with the expanding and compressing box in directions parallel or perpendicular to the background magnetic field and compare the results with the theoretical predictions. The plasma properties depend on the expansion or compression geometry. The expansion along the background magnetic field \(B\) and the compression in a direction perpendicular to \(B\) lead to \(T_{\perp} > T_{\parallel}\), whereas an expansion perpendicular to \(B\) and compression along \(B\) lead to \(T_{\perp} < T_{\parallel}\). There are four tests that we perform: expansions parallel and perpendicular to the background field and compressions parallel and perpendicular to the background field. In each case, plasma parameters change differently and we can verify the simulation results by comparing to the theoretical predictions. The evolution of plasma parameters are ideal as long as a wave activity in the simulations is negligible. Nonideal effects such as heat flux and wave activity are expected to break the invariants in the magnetosheath and lead to a different behavior.

First we need to know how the density and magnetic field change in each type of expansion or compression. The density and magnetic field evolve as:
Here, $L_j$'s are the elements of the expansion matrix.

**Parallel expansion**

For an expansion parallel to the background field, we assume the background magnetic field to be in $z$ direction with $\mathbf{B} = B_0 \hat{z}$. We expand the simulation box in $z$ direction with $L_z = (1 + q_z t)$ and $L_x = L_y = 1$. The changes in the density and the magnetic field are:

$$n(t) = \frac{n(0)}{L_z(t)}, \quad \mathbf{B}(t) = \frac{\sum_j L_j(t) [\mathbf{B}(0) \cdot \hat{e}_j] \hat{e}_j}{L_x(t) L_y(t) L_z(t)} \tag{5.33}$$

Therefore, first and second adiabatic invariants from equations (5.31) and (5.32) results in:

$$T_\perp \propto \text{constant}, \quad T_\parallel \propto \frac{1}{L_z^2} \tag{5.35}$$

Now we can predict the plasma parameters in a parallel expansion:

$$\frac{T_\perp}{T_\parallel} \propto L_z^2, \quad \beta_\perp \propto \frac{1}{L_z}, \quad \beta_\parallel \propto \frac{1}{L_z^3} \tag{5.36}$$

Therefore, a parallel expansion increases anisotropy and reduces the plasma $\beta$.

Figure 5-1 shows the comparison between simulation and the CGL predictions for a parallel expansion. Figure 5-1a shows the evolution of the temperature anisotropy. The red dotted line is the expected adiabatic evolution and the black solid line shows the simulation
result. We show the comparison of $\beta_\perp$ and $\beta_\parallel$ with predictions in Figure 5-1b. Our results are in agreement with CGL predictions.

**Perpendicular expansion**

For an expansion perpendicular to the background magnetic field, we expand the simulation box in $y$ direction while keeping the background magnetic field in $z$ direction. This means that $L_y = (1 + q_y t)$ while $L_x = L_z = 1$. Therefore, for the density and the magnetic field, we get:

$$n(t) = \frac{n(0)}{L_y}, \quad B(t) = \frac{B(0)}{L_y} \hat{z}$$

In this case, both the density and the magnetic field decrease as the box expands. So,

$$T_\perp \propto \frac{1}{L_y}, \quad T_\parallel \propto \text{constant}$$

The plasma parameters for a perpendicular expansion are:
\[ \frac{T_\perp}{T_\parallel} \propto \frac{1}{L_y}, \quad \beta_\perp \propto \text{constant}, \quad \beta_\parallel \propto L_y \quad (5.39) \]

In a perpendicular expansion, temperature anisotropy decreases while plasma $\beta_\parallel$ increases. We show the comparison between simulation and the CGL predictions for a perpendicular expansion in Figure 5-2. Figure 5-2a displays the evolution of temperature anisotropy (black solid line). Also, we show the evolution of plasma $\beta_\perp$ and $\beta_\parallel$ in Figure 5-2b and compare them with prediction given by equation (5.39).

**Parallel compression**

For compression parallel to the background magnetic field, we assume the background field in $z$ direction and compress the simulation box in $z$ direction with $L_z = (1 - q_z t)$ and $L_x = L_y = 1$.

\[ n(t) = \frac{n(0)}{L_z}, \quad B(t) = B(0) \hat{z} \quad (5.40) \]
Figure 5-3: Adiabatic evolution of temperature anisotropy and plasma beta in a parallel compressing box.

\[ T_\perp \propto \text{constant}, \quad T_\parallel \propto \frac{1}{L_z^2} \]  \hspace{1cm} (5.41)

Now we can predict the plasma parameters in a parallel compression. The evolution of plasma parameters is similar to equation (5.36). The only difference is that \( L_z \) is decreasing.

In a parallel compression, the temperature anisotropy decreases while the plasma beta increases as shown in Figure 5-3. Our results are in agreement with CGL predictions.

**Perpendicular compression**

Compression in \( y \) direction perpendicular to the background magnetic field in \( z \) direction with \( L_y = (1 - q_y t) \) and \( L_x = L_z = 1 \) gives:

\[ n(t) = \frac{n(0)}{L_y}, \quad B(t) = \frac{B(0)}{L_y} \hat{z} \]  \hspace{1cm} (5.42)

In this case, both the density and the magnetic field increase as the box compresses. So,
5.3 Discussions

In this chapter, we described a modified version of psc, a particle-in-cell expanding box code. The modified code is an implementation of the expanding box model used by Sironi et al. [68] to study the effects of electron heating by proton cyclotron waves in accretion flows. The expanding box code models the expansion as a linearly driven evolution where the physical lengths varies with time. We use the expanding box simulations in next chapter to investigate the effects of a slow expansion on the mirror instability evolution.
Magnetic holes have been observed in the Earth’s magnetosheath and solar wind. It is believed that these structures are the result of nonlinear saturation of the mirror instability. In this chapter, we investigate the nonlinear evolution of the mirror instability in direct particle-in-cell simulations and also expanding box particle-in-cell simulations. By direct particle-in-cell simulations, we mean that we initialize the plasma by bi-Maxwellian distributions which are unstable to mirror instability. In expanding box simulations, we start with isotropic distributions and let the expansion drive the anisotropy and make the plasma mirror unstable.

The objective of this chapter is to investigate how the mirror structures look like, how they form and what is their relation to the plasma parameters. Multi-spacecraft analysis has shown that mirror modes are extended in a direction oblique to the ambient magnetic field [35, 44, 63, 81]. The magnetic fluctuations within these structures are often observed to be far from sinusoidal and are displayed as trains of large-amplitude holes or peaks. Magnetic fluctuations mostly appear as holes in regions where plasma is marginally unstable with respect to the linear instability. Cattaneo et al. [11] reported the Voyager observations of mirror structures on the dayside of Saturn. The authors track the evolution of mirror structures from a quasi-perpendicular bow shock to the magnetopause. The observed structures evolve from quasi-sinusoidal waves to non-periodic structures, consisting of both magnetic holes and magnetic peaks, and finally they evolve to magnetic holes in the plasma depletion layer close to the magnetopause. Soucek et al. [70] studied the mirror modes observed by Cluster spacecraft and their response to changes in plasma parameters. They report that
magnetic peaks are typically observed in a mirror unstable plasma while mirror structures observed deep within the mirror stable region appear as magnetic holes. Using multi-spacecraft analysis, they observe an abrupt transition of mirror structures from peaks to holes at an approximate distance of 2 Earth radii from the magnetopause and they interpret this effect as a consequence of plasma expansion in the vicinity of the magnetopause where plasma conditions are locally changing towards a more mirror stable state.

Several models in connection with observations have been proposed [27, 36] to explain the evolution of mirror modes from the bow shock to the magnetopause. Computational [4, 5, 27, 32, 51, 75] and theoretical [10, 41, 42, 58] works investigated how these structures could be formed. We show that direct simulation (without expansion) of the mirror instability leads to dominance of magnetic peaks while in the expanding box simulations, mirror instability leads to magnetic holes. In direct simulation of mirror instability, the maximum amplitude of magnetic field perturbations is about 20% of the background field while in expanding box simulations, we generate mirror structures with $\delta B/B \sim 0.5$.

### 6.1 Nonlinear evolution of mirror instability in direct simulations

In order to study the direct nonlinear evolution of mirror instability, we perform 2-dimensional particle-in-cell simulations initialized with bi-Maxwellian distributions. We start with the background field in the $z$ direction, anisotropic protons and isotropic electrons. The simulation parameters are $n_y = n_z = 4096$, $L_y = L_z = 128d_i$, 200 particles per cell, $m_p/m_e = 25$, $v_A/c = 0.1$. The plasma parameters are $T_{p\perp}/T_{p\parallel} = 2.5$, $T_{e\perp}/T_{e\parallel} = 1$, $\beta_{p\parallel} = 1$ and $\beta_{e\parallel} = 1$. We choose plasma parameters that resemble magnetosheath plasma properties. For these parameters, linear theory predicts, the maximum growth rate of proton cyclotron instability to be $\gamma_m = 0.10\Omega_p$ at $k_md_p = 0.54$, while the proton mirror instability maximum growth rate is $\gamma_m = 0.039\Omega_p$ with $k_md_p = 0.50$ at $\theta = 63^\circ$. Proton cyclotron instability
is stronger in this simulation and it dominates the field perturbations perpendicular to the background field. Parallel field perturbation are generated solely by mirror instability since proton cyclotron can only have perpendicular perturbations because of parallel propagation relative to the background field. Therefore, for investigating the mirror instability structures, we only consider perturbations parallel to the direction of the magnetic field (the $z$ direction).

$$m_p/m_e = 25, \beta_p || = \beta_e || = 1.0, v_A/c = 0.1$$

Figure 6-1: Evolution of the proton and electron temperature anisotropies in 2D particle-in-cell simulations. The solid line shows the proton temperature anisotropy and the dashed line shows the electron temperature anisotropy. Electrons stay isotropic during the simulation except for getting slightly anisotropic at nonlinear regime of proton temperature anisotropy instabilities. Proton temperature anisotropy evolves to $T_{\perp p}/T_{|| p} = 1.5$ where the instabilities saturate.

Figure 6-1 shows the evolution of the proton and electron temperature anisotropies. Electrons stay isotropic during the simulation expect for getting slightly anisotropic at nonlinear regime of proton temperature anisotropy instabilities. Electrons can get heated by proton cyclotron instability [67, 68]. It is clear in Figure 6-1 that electrons get anisotropic when proton instabilities are going nonlinear. Proton temperature anisotropy evolves to $T_{\perp p}/T_{|| p} = 1.5$ where the instabilities saturate.

Figure 6-2 shows the instability thresholds for proton cyclotron instability and mirror instability. The dashed line in Figure 6-2 are the analytical threshold conditions given in chapter 4 by equations (4.2) and (4.3). It is expected that protons follow the threshold of the proton cyclotron instability since it had a larger growth rate. At $\Omega_p t = 300$, plasma
is marginally unstable for both mirror instability and proton cyclotron instability but it is closer to stability threshold for mirror instability.

![Graph showing thresholds of proton cyclotron instability and mirror instability.](image)

Figure 6-2: Thresholds of proton cyclotron instability and mirror instability in 2D particle-in-cell simulations with isotropic electrons. The black dotted line shows the marginal stability threshold for mirror instability $R_m = 1$ and the red dotted line shows the proton cyclotron threshold with $R_p = 0.43$. The solid lines show the measured values of $R_m$ and $R_p$ from the simulation.

Figure 6-3 shows the magnetic field perturbations created by proton cyclotron instability and mirror instability. Perturbations in $\delta B_x$ and $\delta B_y$ are propagating in parallel direction. These are the proton cyclotron instability perturbations. The field perturbations in $\delta B_z$ are due to mirror instability. They are stationary structures with an oblique wave vector. The maximum amplitude of mirror instability is about 10% of the background field. This amplitude is small compared to observed mirror mode structures with amplitudes of $\delta B/B \sim 1$.

Figure 6-4 shows the energy exchange between magnetic field and particles as instabilities are growing and saturating. The red solid line shows the changes in magnetic field energy while the black solid line shows the changes in particles (electrons and protons) kinetic energy. As particles are losing the kinetic energy, magnetic field fluctuations are growing. There is about 0.25% numerical heating in this simulation.
Figure 6-3: The perturbations in magnetic field components. The first and second columns show the perpendicular field perturbations $\delta B_x$ and $\delta B_y$ in magnetic field. These perturbations are generated by proton cyclotron instability. The field perturbations in $\delta B_z$ are due to mirror instability.
To make mirror instability stronger than the proton cyclotron instability, we perform another run with the same set of parameters, but with additional electron temperature anisotropy. As we discussed in chapter 4, the presence of electron temperature anisotropy enhances the mirror instability growth rate but electron whistler instability quickly grows and consumes the electron free energy before the mirror instability starts growing. We choose a small proton to electron mass ratio $m_p/m_e = 25$ to exaggerate the effects of the electron temperature anisotropy and keep the computational cost manageable. For $m_p/m_e = 25$, electron whistler instability grows slower compared to real mass ratio $m_p/m_e = 1836$. Therefore, when mirror instability starts growing, electron are still anisotropic and they can slightly impact the mirror instability growth rate. Now, the plasma parameters are $T_{p\perp}/T_{p\parallel} = 2.5$, $T_{e\perp}/T_{e\parallel} = 2$, $\beta_{p\parallel} = 1$ and $\beta_{e\parallel} = 1$. With the presence of anisotropic electrons, electron whistler instability grows and consumes the electron free energy until it saturates around $\Omega_p t = 50$ as shown in Figure 6-5. Proton temperature anisotropy instabilities saturates close to $T_{p\perp}/T_{p\parallel} = 1.5$. Figure 6-6 shows the evolution of instability thresholds for all of the present instabilities. The electron temperature anisotropy evolves to values below the electron whistler instability threshold given by equation (4.1). This may be the result of
mirror instability presence which is isotropizing electrons further below the threshold.

![Graph showing the evolution of the proton and electron temperature anisotropies.](image)

**Figure 6-5:** Evolution of the proton and electron temperature anisotropies in 2D particle-in-cell simulations. The solid line shows the proton temperature anisotropy and the dashed line shows the electron temperature anisotropy. Proton temperature anisotropy evolves to $\frac{T_{\perp p}}{T_{\parallel p}} = 1.5$ where the instabilities saturate.

With stronger mirror instability compared to previous simulation, the maximum amplitude of mirror structures reaches to $\delta B/B \sim 0.2$ as shown in Figure 6-7. Now, the mirror instability perturbations are also visible in perpendicular magnetic field perturbations $\delta B_x$ and $\delta B_y$. In Figure 6-7, the first and second columns show the perpendicular field perturbations $\delta B_x$ and $\delta B_y$ in magnetic field. These perturbations are generated by proton cyclotron instability, mirror instability and electron whistler instability. The field perturbations in $\delta B_z$ are due to mirror instability. At early times, the electron whistler instability is only present in perpendicular perturbations and at late times, the perturbations are dominated by proton cyclotron instability.

Figure 6-8 shows the energy exchange between magnetic field and particles as instabilities are growing and saturating. As we see, at early times, $\Omega_p t = 4$, electron whistler instability is growing and around $\Omega_p t = 75$, the proton cyclotron instability and mirror instability start growing. The numerical heating in this simulation is about 0.8%
Figure 6-6: Thresholds of proton cyclotron instability and mirror instability and electron whistler instability in 2D particle-in-cell simulations with anisotropic electrons. The black dotted line shows the marginal stability threshold for mirror instability $R_m = 1$, the red dotted line shows the proton cyclotron threshold with $R_p = 0.43$, and the blue line shows the electron whistler instability threshold with $R_w = 0.36$. The solid lines show the measured values of $R_m$, $R_p$, and $R_w$ from the simulation. Electrons are isotropized to values below the electron whistler instability threshold.

6.1.1 Relation between skewness and mirror instability threshold

In order to measure the dominance of peaks or holes, we use an statistical value called skewness. Skewness measures the asymmetry of a distribution of a real value variable about its mean. If skewness is positive, it means that the asymmetry of the distribution is dominated by values larger than mean and if skewness is negative, the distribution asymmetry is toward values smaller than mean value. Extending the meaning of skewness to magnetic field perturbations, positive skewness means magnetic peaks are dominant and negative skewness means the perturbations is dominated by magnetic holes.

For a sample of $n$ values, the sample skewness is,

$$S = \frac{\frac{1}{n} \sum_{i=1}^{n} (B_i - \bar{B})^3}{\left(\frac{1}{n} \sum_{i=1}^{n} (B_i - \bar{B})^2\right)^{3/2}}$$  \hspace{1cm} (6.1)$$

where $\bar{B}$ is sample mean. We measure the skewness in $B_z$ for previous simulations. Figures 6-9 and 6-10 show the measured skewness from simulations and distance to mirror instability.
Figure 6-7: The perturbations in magnetic field components. The first and second columns show the perpendicular field perturbations $\delta B_x$ and $\delta B_y$ in magnetic field. These perturbations are generated by proton cyclotron instability, mirror instability and electron whistler instability. The field perturbations in $\delta B_z$ are due to mirror instability. At early times, the electron whistler instability is only present in perpendicular perturbations and in late times, it is dominated by proton cyclotron instability.
Figure 6-8: The red solid line shows the changes in magnetic energy and the black solid line shows the variation in particles energy. As particles are losing energy, the magnetic field is gaining energy and the instabilities grow. This shows the conservation of energy. In late times, there is 0.8% numerical heating.

threshold \( R_m - 1 \). The measured skewness in \( B_z \) for the simulation with isotropic electrons (first performed simulation) is shown in Figure 6-9. Also, Figure 6-10 displays the results of the simulation starting with anisotropic electrons (second performed simulation). According to observations, when magnetic peaks are observed, plasma is mirror unstable and magnetic holes are observed in mirror stable plasma.

In Figure 6-9, skewness is zero until mirror fluctuations grow in amplitude and start shaping as periodic structures with magnetic peaks being dominant. Then, skewness is positive until to the point the instability saturates and skewness becomes slightly negative. At the saturation level, plasma is marginally unstable to mirror instability and skewness becomes negative. Although, the skewness is negative but the amplitude of the magnetic field fluctuations is about 6% of the background field. The reason for plasma getting closer to mirror instability threshold in the first run can be the presence of a stronger proton cyclotron instability. Proton cyclotron instability has lower threshold than the mirror instability and it can make the plasma mirror stable if protons follow the proton cyclotron threshold. In Figure 6-10, skewness is positive and magnetic peaks are dominant structures. In both Figures 6-9 and 6-10, skewness is positive while plasma is mirror unstable.
Figure 6-9: Skewness of $B_z$ and distance to mirror instability threshold for a simulation with isotropic electrons. Skewness is slightly negative at saturation level. At the saturation level, plasma is marginally mirror unstable.

Figure 6-10: Skewness of $B_z$ and distance to mirror instability threshold for a simulation with anisotropic electrons. In this simulation, magnetic peaks are dominant structure since skewness is positive.
Our simulation results generally show that peaks are dominant in a direct simulation of the mirror instability. The amplitude of the mirror instability structures grows to values about 20% of the background field. Figure 6-11 shows the evolution of mirror mode structures as a function of time in the simulation with anisotropic electrons. We make a line cut at $z/d_i = 64$ in $B_z$ at different timesteps and stack them on top of each other to show the evolution of the magnetic structures as a function of time. Mirror structures grow to high amplitude peaks and holes at nonlinear regime. At later times, the amplitude of both peaks and holes decreases. Also, skewness decreases at saturation level of the mirror instability but magnetic peaks are slightly dominant.

![Figure 6-11: Evolution of $B_z$ fluctuations as a function of time. We make a line cut at $z/d_i = 64$ in $B_z$ at different timesteps and stack them on top of each other.](image)

6.1.2 Saturation in Magnetic Peaks and Magnetic Holes

In the nonlinear regime, plasma becomes relatively mirror stable in the magnetic peaks but it stays mirror unstable in the magnetic holes. Figure 6-12 shows the plasma parameters in the simulations with anisotropic electrons. Figure 6-12a shows mirror instability structures and Figure 6-12b shows the distance to mirror instability threshold at the nonlinear regime for previous simulation. We clearly see that although mirror instability has saturated, there
are regions of mirror unstable plasma. We have shown the anticorrelation between magnetic field perturbations and the mirror instability threshold in Figure 6-13 at different times during the instability evolution. We made a line cut at $z \omega_{pe}/c = 64$ along $y$ direction. The reason for anticorrelation is the formation of magnetic bottles and particle trapping. Particles with small parallel velocity get trapped near the center of the well. This leads to the distributions in the center of the well that are warmer in perpendicular direction and it causes higher temperature anisotropy and high beta (since magnetic field is minimum). Figure 6-12c shows the $\beta_{\perp p}$ and also $T_{\perp p}/T_{\parallel p}$ is shown in Figure 6-12f. High beta makes plasma mirror unstable in magnetic holes. But in magnetic peaks, beta is small (maximum field) and density of the particles is minimum since most of the particles reflect at lower magnetic field amplitudes. The population that is present at magnetic peaks has high parallel velocity and a broad range of perpendicular velocities and this leads to a more isotropic distribution in magnetic peaks and plasma becomes mirror stable. But on average the whole plasma is close to mirror instability threshold although there are patches of mirror unstable plasma.

The interesting point from Figures 6-13 and 6-14 is that although at late times, plasma is still mirror unstable in magnetic holes but the degree of instability has decreased considerably compared to earlier times. Also, the amplitude of magnetic holes is decreasing as plasma is becoming more mirror stable. This contradicts the predicted nonlinear saturation mechanism by Kivelson and Southwood [41] as we described in chapter 3. Kivelson and Southwood proposed that plasma becomes mirror stable in the magnetic holes if magnetic holes get deeper. Also, we show the temperature anisotropy cuts in Figure 6-15. The plasma is slightly more anisotropic in magnetic holes compared to magnetic peaks. We can conclude that there is another saturation mechanism in magnetic holes.
Figure 6-12: Plasma parameters in simulation with anisotropic electrons at nonlinear stage of mirror instability. Plasma is mirror unstable in magnetic holes and it is mirror stable in magnetic peaks.
Figure 6-13: Distance to threshold in simulation with anisotropic electrons at nonlinear stage of mirror instability at different timesteps. Black solid line shows the parallel magnetic field fluctuations divided by background field. Red solid line shows the distance to mirror instability threshold. Plasma is mirror unstable in magnetic holes and it is more stable relative to mirror instability in magnetic peaks.
Figure 6-14: Plasma parameter $\beta_{p,\perp}$ in simulation with anisotropic electrons at nonlinear stage of mirror instability at different timesteps. Black solid line shows the parallel magnetic field fluctuations divided by background field. Blue solid line shows the $\beta_{p,\perp}$. The value of $\beta_{p,\perp}$ is higher in magnetic holes.
Figure 6-15: Plasma parameter $T_{p\perp}/T_{p\parallel}$ in simulation with anisotropic electrons at nonlinear stage of mirror instability. Black solid line shows the parallel magnetic field fluctuations divided by background field. Green solid line shows the $T_{p\perp}/T_{p\parallel}$ cuts.
6.2 Nonlinear evolution in expanding box simulations

In previous section, we showed that direct nonlinear saturation of mirror instability generates magnetic peaks. In this section, we use the implemented expanding box method into our particle-in-cell code to study mirror instability structures in a mirror stable plasma. As we mentioned earlier, we want to mimic the plasma expansion in the magnetosheath. The expanding box simulation models an evolution of a small fraction of the plasma which expands under the effect of the global magnetosheath flow around the magnetospheric cavity. This model neglects the global inhomogeneities and replaces the spatial dependence by a temporal one. We perform 2-dimensional expanding box simulation with an expansion along the background field in z direction (parallel expansion). According to CGL conditions mentioned in the previous chapter, for an expansion parallel to the background field, temperature anisotropy increases and $\beta$ decreases. The reduction in $\beta$ helps to make plasma mirror stable for long enough simulations.

We start with isotropic electrons and slightly anisotropic protons. The simulation parameters are $n_y = n_z = 2048, L_y = L_z = 32, 200$ particles per cell, $m_p/m_e = 25, v_A/c = 0.1, a_z = (1+q_z t')$ with $q_z = 10^{-4}$. The initial plasma parameters are $T_{p\perp}/T_{p\parallel} = 1.1, T_{e\perp}/T_{e\parallel} = 1, \beta_{p\parallel} = 13$ and $\beta_{e\parallel} = 1$. The conservation of first and second adiabatic invariants lead to temperature anisotropies and generation of proton cyclotron and proton mirror instabilities. Protons and electrons follow the adiabatic path until the anisotropy is large enough for the instabilities to grow.

Figure 6-16 shows the evolution of the plasma parameters. In order to compare the simulation results with the Vlasov linear prediction, we also plot the isocontours of the maximum growth rate as a function of $\beta_{p\parallel}$ and $T_{p\perp}/T_{p\parallel}$ for the mirror and proton cyclotron instabilities in the corresponding homogeneous plasma. Figure 6-16a shows that $\beta_{p\parallel}$ decreases with time. Initially system evolves adiabatically and after a transition, the system follows the proton cyclotron instability path. After following the proton cyclotron threshold, the system slightly departs from the proton cyclotron instability threshold. Again, the instabilities overtake the
expansion and transit the system toward more isotropic proton distribution. We see that plasma becomes very unstable before proton cyclotron and mirror instabilities can grow. The reason is the limitation in choosing the simulation box size in particle-in-cell simulations. Therefore, we are not resolving the maximum growth rate wavelengths for small temperature anisotropies. Also, the expansion rate can affect the growth of the instabilities. Since we start the simulation with a small electron beta ($\beta_{e||} = 1$), plasma is stable to electron whistler instability when we average the plasma parameters in the entire simulation domain. This leads to high electron temperature anisotropy since electron whistler instability is not able to grow. This behavior can be seen in Figure 6-16b. Now, the presence of the electron temperature anisotropy can enhance the mirror instability growth rate and help the mirror instability grow faster than the proton cyclotron instability.

### 6.2.1 Magnetic holes

Figure 6-17 displays the evolution of mirror structures by measuring skewness for magnetic fluctuations in parallel direction. As expansion proceeds, the anisotropy increases and plasma becomes mirror unstable. Mirror mode fluctuations remains mainly sinusoidal until
\( \Omega_p t = 240 \) since skewness is zero. After this time, mirror fluctuations grow in amplitude and start shaping as peaks. The growth of mirror fluctuations reduces the distance to threshold. About \( \Omega_p t = 580 \), the magnetic peaks start collapsing and magnetic holes become dominant structures. As time goes on, the magnetic fluctuations transit to periodic structures. At \( \Omega_p t = 1180 \), magnetic holes get deeper as skewness is becoming more negative.

![Graph showing skewness of magnetic field and distance to mirror instability threshold](image)

**Figure 6-17:** Skewness of \( B_z \) and distance to mirror instability threshold in expanding box simulation. At early times, magnetic peaks are dominant but when plasma approaches the marginal stability path, magnetic holes become dominant.

We also plotted the skewness as a function of distance to threshold and \( \beta_p || \) in Figure 6-18. In Figure 6-18a, the skewness becomes negative for \( R_m < 1.75 \) and in Figure 6-18b, skewness is negative for \( \beta_p || < 2.7 \).

To evaluate the evolution of the plasma parameters in magnetic peaks and magnetic holes, we have plotted the plasma parameters at different timesteps in our simulation in Figures 6-19 and 6-20. We clearly see the anticorrelation between the density and magnetic field fluctuations in Figures 6-19b and 6-20b which is a signature of mirror instability as we explained in chapter 3. Figures 6-19e and 6-20e show that the plasma is mirror stable in magnetic peaks while it is very unstable in magnetic holes and this behavior persists at very late stages of the nonlinear evolution. The distance to threshold \( (R_m) \) strongly depends on \( \beta_p \perp \) and \( T_p \perp /T_p || \) as shown in equation (4.2). Therefore, we have also plotted the \( \beta_p \perp \)
in Figures 6-19d and 6-20d and $T_{p\perp}/T_{p\parallel}$ in Figures 6-19f and 6-20f. The values of $\beta_p$ is much higher in magnetic holes compared to magnetic peaks. Also, the proton temperature anisotropy is vary large in magnetic holes but the proton distribution is isotropic in magnetic peaks. This leads to plasma becoming very unstable in magnetic holes and mirror stable in magnetic peaks.

To show that the magnetic field fluctuations get very deep, we have made cuts through the parallel magnetic field fluctuations in Figure 6-21. We show the magnetic field fluctuations at $\Omega_p t = 866$ and $\Omega_p t = 1949$. At $\Omega_p t = 866$, the magnetic field perturbations look periodic with same amplitudes in magnetic peaks and magnetic holes in Figures 6-21c and 6-21e. Later at $\Omega_p t = 1949$, the magnetic holes have grown to larger amplitudes with $\delta B_z/B \sim 0.5$, while the magnetic peaks have remained at the same amplitudes.

6.3 Discussions

In this chapter, we performed two-dimensional particle-in-cell simulations to study the nonlinear evolution of the mirror instability. The motivation for this study originates from
Figure 6-19: Illustration of plasma parameters in expanding box simulation. Plasma is mirror unstable in magnetic holes and it is mirror stable in magnetic peaks.
Figure 6-20: Illustration of plasma parameters in expanding box simulation.

(a) Magnetic field perturbations
(b) Proton density
(c) $\beta_p^{||}$
(d) $\beta_p^{\perp}$
(e) Distance to threshold
(f) $T_p^{\perp}/T_p^{||}$
Figure 6-21: Deep magnetis holes.
frequent observations of magnetic holes in planetary magnetosheaths. It is believed that
the observed deep magnetic holes are the result of nonlinear saturation of mirror instability
fluctuations.

In a direct simulation of mirror instability, we see that the periodic structures are formed
with same amplitude in peaks and holes. In late nonlinear regime, both magnetic peaks
and magnetic holes amplitude gets smaller and plasma gets closer to marginal stability. The
skewness in $B_z$ is close to zero which means structures are periodic. Our results contra-
dicts the saturation mechanism proposed by Kivelson and Southwood [41]. They proposed
that direct saturation of mirror instability leads to deep magnetic holes while in our direct
simulations, magnetic peaks are dominant.

In expanding box simulations, initially the plasma is stable with respect to proton cy-
clotron and mirror instabilities, and it evolves adiabatically in agreement with CGL pre-
dictions. This evolution leads to development of temperature anisotropies. When the tem-
perature anisotropy becomes stronger than the threshold for the instabilities, the proton
cyclotron and mirror instabilities are generated and the evolution departs from the adiabatic
path. The system remains near the marginal stability for proton cyclotron instability. Later,
it departs from the proton cyclotron instability threshold.

In expanding box simulation, magnetic peaks are dominant when plasma parameters are
far from mirror instability threshold and mirror fluctuations evolved to deep magnetic holes
when plasma is marginally mirror unstable. Although, the averaged plasma parameters
are close to mirror instability threshold, the plasma in magnetic holes is highly unstable to
mirror instability. The proton plasma beta and proton temperature anisotropy are very high
in magnetic holes. The survival of the magnetic holes in a marginally mirror unstable plasma
agrees with bi-stability theory proposed by Califano et al. [10] and Kuznetsov et al. [42].
Chapter 7

Conclusions

To understand the electromagnetic environment surrounding the Earth, we have investigated microscopic nonlinear evolution of mirror instability. There are frequent observations of magnetic hole structures in the Earth’s magnetosheath and solar wind. These structures are believed to be the result of nonlinear saturation of mirror instability. The plasma in these environment has bi-Maxwellian distributions which leads to the generation of mirror instability and proton cyclotron instabilities. There is a competition between proton cyclotron and mirror instability in downstream of the quasi-perpendicular bow shock in Earth’s magnetosheath. We investigated the effects of electron temperature anisotropy on mirror instability growth rate using particle-in-cell simulations. We implemented an expanding box technique in our plasma simulation code to resemble the magnetosheath expansion to study the evolution of mirror instability structures in a mirror stable plasma.

7.1 Summary of important results

In chapter 1, we introduced the magnetospheric environment around Earth and the regions with proton temperature anisotropy larger than one. We described different instabilities that arise from the proton and electron temperature anisotropies of larger than one. We reviewed different computer simulation techniques for studying space plasmas.

In chapter 2, we introduced the particle-in-cell model that we use for simulation work in this thesis. The \texttt{psc} accurately represents the plasma dynamics at the characteristic scale of both ions and electron motions. We also reviewed the linear dispersion theory for a homogeneous plasma with bi-Maxwellian distributions in a magnetized plasma and
numerically solved the linear dispersion equation. We can find the growth rate of both proton and electron temperature anisotropy instabilities for any given plasma parameters. We verified the particle-in-cell simulation results with linear dispersion theory predictions and we showed we can capture both electrons and protons dynamics correctly.

We reviewed the physical description of the mirror instability for linear and nonlinear growth mechanisms in chapter 3. The linear analysis shows that the instability results from the antiphase response of the bulk of the plasma pressure to the changes in the magnetic field pressure. The resonant particles (particles with small $v_{||}$) produce a pressure perturbation in phase with the field pressure change. In the nonlinear regime of the mirror instability, the evolution of the instability depends on the process of particle trapping. Particles with small parallel velocities are excluded from high field regions through trapping. This produces a local net decrease in particle pressure in the high field regions and allows the marginally stable state to be reached. In contrast, in magnetic wells, the particle pressure increases due to trapping and marginal stability condition cannot be attained. The proposed mechanism by Kivelson and Southwood [41] showed that plasma becomes marginally stable in magnetic holes by cooling of the trapped distribution. Many observational studies suggested that mirror modes get created close to the bow shock and at this early stage they appear as quasi-sinusoidal waves. Further downstream in the magnetosheath, they approach nonlinear saturation and change into non-periodic large amplitude structures of peaks or holes. As the mirror modes are convected towards the magnetopause, notably in plasma depletion layer, the plasma becomes mirror stable and mirror structures start to collapse and decay away. Observations showed that the character of mirror structures is related to the local degree of instability of the plasma with respect to the mirror instability threshold. Peaks are typically observed in an unstable plasma, while mirror structures observed deep within stable region appear almost always as holes. A transition of mirror structures from peaks to holes was identified by multi-spacecraft analysis and this is interpreted as a consequence of plasma expansion in the vicinity of the magnetopause locally changing the plasma condition toward
a more stable state.

In chapter 4, we investigated the effects of electron temperature anisotropy on the proton mirror instability evolution. Linear theory predicts that presence of an electron temperature anisotropy can enhance the proton mirror instability growth rate, and if it is large enough, it can make the proton mirror instability stronger than the proton cyclotron instability. We show that anisotropic electrons primarily drive the electron whistler instability. We performed two-dimensional particle-in-cell simulations with different electron to proton mass ratio. We studied how different mass ratios affect the electron whistler instability evolution and how it impacts the proton cyclotron and proton mirror instability growth rates. We found that the electron whistler instability consumes the electron free energy before proton mirror instability grows into the nonlinear regime, because it grows much faster than the proton temperature anisotropy instabilities. Therefore, all the electron free energy is gone before proton mirror instability starts growing. Our results showed that temperature anisotropy instabilities are sensitive to chosen mass ratio $m_p/m_e$ in particle-in-cell simulations, since an artificial mass ratio can affect the growth and dynamics of the instabilities.

In chapter 5, we described the implementation of the expanding and compressing box method into psc. I derived the Maxwell’s equations and Lorentz force in the moving frame and applied the required approximations. After describing the method, I verify it by testing CGL predictions.

In chapter 6, we investigate the nonlinear evolution of the mirror instability in direct particle-in-cell simulations and also expanding box simulations. In direct particle-in-cell simulations, we initialized the plasma by bi-Maxwellian distributions which is unstable to mirror instability. In expanding box simulations, we started with isotropic plasma and let expansion drive the anisotropy and make plasma mirror unstable. We showed that direct nonlinear saturation of the mirror instability leads to dominance of magnetic peaks while in the expanding box simulations, mirror instability leads to magnetic holes. In direct simulation of mirror instability, the maximum amplitude of magnetic field perturbations is
about 20% of the background field while in expanding box simulations, we generated mirror structures with $\delta B/B \sim 0.5$.

In a direct simulation of mirror instability, we showed that in late nonlinear regime, both magnetic peaks and magnetic holes amplitude gets smaller and plasma gets closer to marginal stability. Our results contradicted the saturation mechanism proposed by Kivelson and Southwood [41]. In expanding box simulation, magnetic peaks were dominant when plasma parameters were far from mirror instability threshold and mirror fluctuations evolved to deep magnetic holes when plasma was marginally mirror unstable. The survival of the magnetic holes in a marginally mirror unstable plasma agrees with bi-stability theory.

### 7.2 Future directions

For the future investigations on the mirror instability, we can study the effects of the expansion rate on the evolution of the mirror structures. A fast expansion quickly creates an anisotropic plasma and instabilities have to grow faster to overtake the expansion. This can impact the evolution of the magnetic structures. Also, we can study mirror instability in compressing box simulations. In a perpendicular compressing box simulation, we can also create temperature anisotropy of larger than one and let the instabilities grow. A perpendicular compressing box can resemble the quasi-perpendicular shock layer which creates large temperature anisotropies.

One interesting feature that we are observing in our expanding box simulations, is the electron heating by proton cyclotron instability which was reported by Sironi et al. [68] in compressing box simulations. This is an interesting subject that can be studied further. We also observe the electron whistler signatures at the gradients of the magnetic holes which agrees with recent MMS observations. The simulations can help us to explain how the electron temperature anisotropy is enhanced at the gradients of the magnetic holes while proton temperature anisotropy is anticorrelated with the magnetic field.
The development of the three-dimensional models are needed for studying the competition between mirror instability and proton cyclotron instability in more details. With increasing computational powers, the three-dimensional particle-in-cell simulations can be feasible in the near future.
List of References


[56] O. A. Pokhotelov *et al.*, “Linear theory of the mirror instability in non-maxwellian space plasmas,” *J. Geophys. Res.*, vol. 107,


