Fall 2016

"Smoking-Gun" Observables of Magnetic Reconnection: Spatiotemporal Evolution of Electron Characteristics Throughout the Diffusion Region

Jason Shuster
University of New Hampshire, Durham

Follow this and additional works at: https://scholars.unh.edu/dissertation

Recommended Citation
https://scholars.unh.edu/dissertation/1363

This Dissertation is brought to you for free and open access by the Student Scholarship at University of New Hampshire Scholars' Repository. It has been accepted for inclusion in Doctoral Dissertations by an authorized administrator of University of New Hampshire Scholars' Repository. For more information, please contact nicole.hentz@unh.edu.
"Smoking-Gun" Observables of Magnetic Reconnection: Spatiotemporal Evolution of Electron Characteristics Throughout the Diffusion Region

Abstract
How does magnetic reconnection happen in a collisionless plasma? Knowledge of electron-scale dynamics is necessary to answer this outstanding question of plasma physics. Based on fully kinetic particle-in-cell (PIC) simulations of symmetric reconnection, the spatiotemporal evolution of velocity distribution functions in and around the electron diffusion region (EDR) elucidates how electrons are accelerated and heated by the cooperating reconnection electric and normal magnetic fields. The discrete, triangular structures characteristic of EDR distributions rotate and gyrotropize in velocity space as electrons remagnetize, forming multicomponent arc and ring structures. Further downstream, exhaust electrons are found to exhibit highly structured, time-dependent anisotropies that can be used to infer the temporal stage of reconnection. Cluster spacecraft measurements from a magnetotail reconnection exhaust region agree with these simulation predictions. In PIC simulations of asymmetric reconnection, EDR distributions acquire crescent-shaped populations, indicative of accelerated magnetosheath electrons mixing with electrons of magnetospheric origin. NASA's successfully launched Magnetospheric Multiscale (MMS) mission caught an EDR at the magnetopause and confirmed the signature crescent electron populations. A virtual spacecraft trajectory through the PIC domain is determined quantitatively by inputting MMS magnetic field measurements into an algorithm that outputs a trajectory along which the input measurements are matched. The crescent structures observed by MMS in the EDR are consistent with the simulation distributions at the corresponding time along the computed trajectory. This work demonstrates that electron characteristics can serve as "smoking-gun" observables of the EDR at the heart of the magnetic reconnection mystery.

Keywords
electron diffusion region, magnetic reconnection, magnetosphere, Magnetospheric Multiscale (MMS), plasma, Physics, Applied mathematics, Plasma physics
“Smoking-Gun” Observables of Magnetic Reconnection: Spatiotemporal Evolution of Electron Characteristics Throughout the Diffusion Region

by

JASON R. SHUSTER

B.S. in Physics, University of New Hampshire, 2012

Dissertation

Submitted to the University of New Hampshire in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics

September, 2016
This dissertation has been examined and approved in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics by:

Dissertation Director, Li-Jen Chen
Affiliate Research Associate Professor, University of New Hampshire
Associate Research Scientist, NASA Goddard Space Flight Center and Department of Astronomy, University of Maryland at College Park

Roy B. Torbert
Professor of Physics
Associate Director of EOS, University of New Hampshire
Director of SwRI-EOS, Southwest Research Institute

Charles J. Farrugia
Research Professor of Physics
Research Associate Professor, Space Science Center (EOS)

Mark McConnell
Professor and Chair, Department of Physics
Professor, Space Science Center (EOS)

Karsten Pohl
Professor of Physics
Professor, Materials Science Program

On May 24, 2016

Original approval signatures are on file with the University of New Hampshire Graduate School.
DEDICATION

For my mom, Kathy Shuster.

“ARILU!”
Acknowledgments

My doctoral journey came with many unexpected trials, some of which include: the death of my father before I had a chance to know him, a broken heart, and a near-suicidal depression that I unwisely tried to hide. I have also experienced many joys, some of which include: becoming reunited with my brother and his kind family, a restored faith, and the promise of eternal life through the love of Jesus Christ. I am richly blessed to know this love that surpasses knowledge, and to be encouraged by so many inspiring people who I wish to thank with gratitude indescribable:

Li-Jen Chen, with her infinite passion for life and discovery, her unstoppable determination, and her inspiring faith. While leading cutting-edge research and being a powerfully motivating advisor, I also witnessed Li-Jen’s great love and devotion for her family. I had the honor of watching her two beautiful children, Arthur and Iris, grow over the years. I will always remember Iris curiously exploring our offices, checking to ensure we were not working too hard. I first met Li-Jen and her then little companion Arthur at the UNH Chess Club that I founded. Arthur is now taller than I am, and I can no longer beat him in a game of chess! Not long after that first meeting at the chess club, I began asking Li-Jen more about her work in space plasma physics studying electron distribution functions. Her contagious enthusiasm sparked a passion of my own for space physics that brought me on this graduate research expedition, culminating in this dissertation. Li-Jen taught me what it means to forgive, and encouraged me to “keep going” no matter the circumstances. Often when I
became too engrossed in physics problems, I would recall Li-Jen’s perspective: “You think physics is hard? Try being a mom!”

Roy Torbert, for his unwavering leadership, support, and confident guidance. Amid preparing NASA’s Magnetospheric Multiscale (MMS) mission for launch, Roy taught my sophomore undergraduate classical mechanics course – my first exposure to the formalism underlying the calculus of variations, spinning tops, and Lagrangians, to name a few highlights. At a research group meeting, I distinctly remember one of Roy’s challenges for us: “I want you all to be thinking about this: what constitutes smoking-gun evidence of magnetic reconnection?”, a question which motivated the title of this dissertation! In the first chapter, I enjoyed describing Roy’s exploding-wire demonstration that he crafted as an analogy to motivate the search for reconnection diffusion regions in space. I am thankful for the opportunity Roy gave me to work closely with Hans Vaith on calibrating MMS Electron Drift Instrument (EDI) data, results which we presented at the Chapman Conference in Fairbanks, Alaska. I often still think back to Roy’s guest lectures on ‘imaginary’ numbers, which helped me see the ‘complex’ numbers as complete numbers.

Charles Farrugia, for his expert enthusiasm, smile-inducing sense of humor, and for our joint-adventures in geospace – may the saga always continue! Mark McConnell, for introducing me to the physics department at UNH as an undergraduate freshman, for teaching me about scattering in scintillator detectors, and how to find water on Mars! Karsten Pohl, for equipping me with foundational freshman-level physics knowledge, directing our undergraduate Society of Physics Students (SPS), and teaching me graduate mathematical and statistical physics concepts (including such remarkable results as Cauchy’s integral formula and Liouville’s theorem).
I thank William Daughton, for sharing some of his expertise with me and for performing
the kinetic simulations featured in this dissertation. I am grateful for my colleagues Shan
Wang and Naoki Bessho, who helped me answer any physics question I sent them. Matthew
Argall, for helpful advice, leadership, and teamwork. Fathima Muzamil, for her collaboration,
her ever-positive spirit, and for sharing our research together through outreach in local high
school physics classrooms. I also wish to thank Lou C. Lee and Kun Han Lee, Jongsoo Yoo,
Daniel Gershman, Guanlai Li, and Christopher Mouikis for instructive collaborations.

James Harper, for entrusting me with his counter-intuitive physics toy: a little green
rattleback (which still fascinates me!), for encouraging me to continue sharing chess with
youngsters and my friends, and for teaching me that: “the more at peace we can be with
uncertainty, the more at peace we can be.” Dawn Meredith, for training me to think quan-
tum mechanically, for walks in college woods, and for challenging me to explain angular
momentum to a 6-year-old. I am grateful for all my UNH physics and math professors who
were willing to share their knowledge and motivation with me, including: Eberhard Möbius,
Adam Boucher, Kevin Short, Lynn Kistler, Kai Germaschewski, and Martin Lee.

My mom, Kathy Shuster, receives my greatest thanks from the deepest place in my heart.
I can sincerely say, as Abraham Lincoln did:

All that I am or ever hope to be, I owe to my angel mother.

Mom’s life is full of miracles: when she was only one year old, she survived a tragic fire
and had to spend her second birthday in the hospital; she is a two-time cancer survivor; she
raised and provided for me on her own; she became an internationally renown artist through
her position as Director and Head Illustrator of the Scientific Illustration and Photography
Unit of the Biology Department at Purdue University. I inherited her passion for scientific
visualization, and her appreciation for education and the sciences. For the remaining days of life I receive, I hope to learn how to love and encourage so unconditionally as mom does.

My aunt, Nancy Kraus (mom’s sister), for her patience, her beautiful music, for sharing life with mom and me, and for teaching us how to ‘call the old things lost’. My uncle, George F. Shuster III, who has become like a father to me over the years. I cannot thank him enough for faithfully loving and caring for mom and me, for coaching me through many academic quandaries, and for still letting me win the occasional chess game. My uncle, John R. Shuster (aka “Uncle Bert”), for his one-of-a-kind word constructions and expressions, and for his love and support of mom and me throughout my life. My cousins, George and Brenda Bisbort, their children, and their children’s children, who have faithfully and consistently comforted and encouraged my family, especially mom and me, for as long as I can remember. I am immeasurably blessed to have a life full of loving and supporting family, cousins, and relatives.

My life is blessed even more by my dear and fantastic friends: Cristian Ferradas, Timothy Schroeder, David Fifty, Toby DeWhurst, Joseph Hansalik, Keith and Ginny Teeter, Iulia Barbu, Keatyn Bergsten, Kate Luksha, Rebecca Jacobson, Katherine Norwood, Shona Ort, Jin Lee, Matthew Young, Gen Li, Sambid Wasti, Girma Moges and Mirkat Oshone, Eunsang Cho, and my Graduate Christian Fellowship (GCF) Family. I thank my church family for helping to nurture and strengthen my faith, including: Pastor Dave Barber, Don and June Barber, Bill and Sharon Black, and the Stuart and Watson families. For my entire graduate career, I had the privilege of living with Barry and Claire Reinhold, who provided me with an ever-welcoming home away from home where I could always come to join their family for games and meals (I’ll never forget Claire’s famous chili!), contribute to snow-
shoveling parties, and share in many facets of daily life together. Ellen O'Keefe, for her guidance, friendship, and exemplary example of what it means to be a mathematics educator, her selflessness, and her hugs. Hanna Landers, for her down-to-earth, genuine, profound, and never-ending questions about life that she kindly shared with me throughout my time at UNH. I could always count on Mark Chutter and Barbara Briggs for fun and witty discussions. I thank Michael Routhier, for being willing to print our posters late into the night! I also thank Norm and Atida, for working faithfully to keep Morse Hall looking the way it does.

Mr. Lovall Morrison, my high school calculus teacher, for instilling a deep-rooted mathematical curiosity inside of me, for all the chess games, for being always willing to invite, listen, and share, and for being my friend. Mr. and Mrs. Bevin, and their hope-filled family, for their generosity, kindness, and supportive hearts, attending all of my concerts, and for sharing their love with mom and me. The young Philip Bevin, who displayed unfathomable bravery and courage while battling brain cancer, deeply inspires me to press onward toward our heavenly destination, where we are promised that mourning, crying, pain, and even death itself, shall be no more.

This dissertation work was supported in part by a UNH College of Engineering and Physical Sciences (CEPS) Fellowship, a James M. E. Harper Fellowship, NASA Earth and Space Science Fellowship (NESSF), NASA grants to the Theory and Modeling Program, FIELDS team, and the Fast Plasma Investigation (FPI) of the MMS mission, and National Science Foundation (NSF) grants AGS-1543598, AGS-1202537, and AGS-1552142.
FOREWORD

I lift up my eyes to the mountains— where does my help come from?

My help comes from the LORD, the Maker of heaven and earth.

He will not let your foot slip— he who watches over you will not slumber;
indeed, he who watches over Israel will neither slumber nor sleep.

The LORD watches over you— the LORD is your shade at your right hand;
the sun will not harm you by day, nor the moon by night.

The LORD will keep you from all harm— he will watch over your life;
the LORD will watch over your coming and going both now and forevermore.

Psalm 121

The heavens declare the glory of God;
the skies proclaim the work of his hands.
Day after day they pour forth speech;
night after night they reveal knowledge.
They have no speech, they use no words;
no sound is heard from them.
Yet their voice goes out into all the earth,
their words to the ends of the world...

Psalm 19
# Table of Contents

Dedication ........................................................................................................ iii
Acknowledgments ............................................................................................... iv
Foreword ............................................................................................................. ix
List of Tables ....................................................................................................... xiii
List of Figures ..................................................................................................... xiv
Abstract ............................................................................................................. xvi

## 1 Introduction

1.1 Seventy-Year History of Collisionless Reconnection ................................. 1
1.2 Kinetic Theory: Collisionless Boltzmann Equation ................................. 5
1.3 Particle-in-Cell Simulations ..................................................................... 11
   1.3.1 Discreteness in PIC: The Meaning of E and B .............................. 12
   1.3.2 Insensitivity of the Reconnection Rate ........................................... 13
   1.3.3 Five Simulation Studies ............................................................... 14
1.4 Spacecraft Instrumentation .................................................................... 19
   1.4.1 Cluster ..................................................................................... 19
   1.4.2 MMS ....................................................................................... 20
1.5 Movie and Animation Repository .............................................................. 23

## 2 Highly Structured Electron Anisotropy in the Exhaust ......................... 25
2.1 Introduction ............................................................................................... 25
2.1.1 Previous Knowledge .................................................. 25
2.1.2 Two Types of Anisotropy ............................................. 27
2.2 Kinetic Anisotropy in the Exhaust ................................. 29
  2.2.1 Rings, Wings, and Arcs ........................................... 31
  2.2.2 Inside the Secondary Island ...................................... 35
  2.2.3 Electron Anisotropy in Earth’s Magnetotail: Confirmation from Cluster 39
2.3 Summary and Discussion .............................................. 42

3 Spatiotemporal Evolution of Electron Characteristics in the Electron Diffusion Region .............................................. 46
  3.1 Introduction ............................................................. 46
  3.2 Previous Knowledge ................................................... 47
  3.3 Evolution of Electron Velocity Distributions:
      Implications for Acceleration and Heating ......................... 48
      3.3.1 Gyrotropization of the X-line Distribution .................... 50
      3.3.2 Visualizing Collisionless Dissipation .......................... 53
      3.3.3 Time Dependence ............................................... 58
      3.3.4 Development of Nongyrotropy ................................ 60
      3.3.5 Fragmentation of Inflow Distributions: Electron Phase Space Holes
          Along the Separatrix ............................................. 63
  3.4 Electron Encyclopedia: Establishing Maps of PIC Distributions for Orienting
      Spacecraft Measurements ............................................ 67
      3.4.1 Robustness of Triangular EDR Distributions .................. 69
  3.5 Summary and Conclusions ............................................ 74
1.1 Parameters for the five particle-in-cell simulations considered in this dissertation. 16
# List of Figures

1.1 Example phase space representations for the evolution of the forced, damped pendulum. .................................................. 10
1.2 Time evolution of the reconnection rates in the 2D simulations considered in this dissertation. ............................................. 15
1.3 Beautiful launch of the MMS spacecraft on an Atlas V rocket. ............. 22

2.1 Distinction between fluid anisotropy and kinetic anisotropy. ................. 28
2.2 Evolution of electron temperature anisotropy $T_e/\parallel/T_e/\perp$ near peak reconnection. 30
2.3 Three highly structured, anisotropic, ring distributions in the exhaust. ...... 32
2.4 Instability of ring distributions to whistler waves. ............................ 36
2.5 Electron distributions taken from inside a secondary island. .................. 37
2.6 Crescent structure of secondary island electron populations revealed in 3D velocity space. .................................................... 38
2.7 Cluster spacecraft measurements confirm predicted exhaust anisotropic structures. ......................................................... 40

3.1 Spatiotemporal evolution of the electron temperature $T_e$ during Run #1. .. 49
3.2 Rotation and gyrotropization of electron velocity distributions downstream of the X-line. ....................................................... 51
3.3 Electron trajectories illustrating collisionless dissipation throughout the electron outflow jet. ................................................... 54
3.4 Position, phase, and velocity space trajectories of 2,256 test-electrons passing through the EDR. ............................................. 57
3.5 Spatial and temporal evolution of distributions from the X-line to the end of
the electron jet. .......................................................... 59
3.6 Development of electron nongyrotropy as electrons meander in the EDR. . 61
3.7 Fragmentation of inflow distributions in association with electron holes. . . 64
3.8 Trajectory of a test inflow electron in position and phase space, and \( x-v_x \)
electron holes that form along separatrices. .................................. 66
3.9 Cluster multi-spacecraft reconstruction of a reconnection X-line in the mag-
etotail. ................................................................. 68
3.10 Comprehensive map of \( v_x-v_y \) distributions for Run #1. ............... 70
3.11 Map of \( v_x-v_y \) distributions for Run #2 with a small guide field. ....... 71
3.12 Map of \( v_x-v_y \) distributions for Run #3 for realistic mass ratio. ........ 72

4.1 Schematic illustrating how the trajectory-finding algorithm works. ....... 80
4.2 Comparison of MMS measurements and simulation quantities along the com-
puted trajectory. ................................................................. 85
4.3 Errors for the trajectory computed by the algorithm featured in Figure 4.2. . 88
4.4 Electron energy and pitch angle spectrograms near the EDR encountered by
MMS and computed in PIC. .............................................. 91
4.5 MMS observations of electron crescent structures in the EDR consistent with
simulation predictions. ..................................................... 93
4.6 Comparisons of \( z-v_x \), \( z-v_y \), and \( z-v_z \) phase space for Runs #1, #2, #3, and #4. 96
4.7 Map of \( v_{||}-v_{\perp 1} \) “leaf” distributions in the asymmetric configuration of Run #4. 98
4.8 Map of \( v_{\perp 1}-v_{\perp 2} \) crescent and horse-shoe distributions in Run #4. .... 99
4.9 Map of \( v_x-v_y \) distributions in a 3D, asymmetric run with strong guide field
(Run #5). ................................................................. 100

5.1 Summary of triangular distributions characteristic of the X-line region in Runs
#1, #2, and #4. ......................................................... 108
ABSTRACT

“Smoking-Gun” Observables of Magnetic Reconnection: Spatiotemporal Evolution of Electron Characteristics Throughout the Diffusion Region

by

Jason R. Shuster

University of New Hampshire, September, 2016

How does magnetic reconnection happen in a collisionless plasma? Knowledge of electron-scale dynamics is necessary to answer this outstanding question of plasma physics. Based on fully kinetic particle-in-cell (PIC) simulations of symmetric reconnection, the spatiotemporal evolution of velocity distribution functions in and around the electron diffusion region (EDR) elucidates how electrons are accelerated and heated by the cooperating reconnection electric and normal magnetic fields. The discrete, triangular structures characteristic of EDR distributions rotate and gyrotropize in velocity space as electrons remagnetize, forming multicomponent arc and ring structures. Further downstream, exhaust electrons are found to exhibit highly structured, time-dependent anisotropies that can be used to infer the temporal stage of reconnection. Cluster spacecraft measurements from a magnetotail reconnection exhaust region agree with these simulation predictions. In PIC simulations of asymmetric reconnection, EDR distributions acquire crescent-shaped populations, indicative of accelerated magnetosheath electrons mixing with electrons of magnetospheric origin. NASA’s
successfully launched Magnetospheric Multiscale (MMS) mission caught an EDR at the magnetopause and confirmed the signature crescent electron populations. A virtual spacecraft trajectory through the PIC domain is determined quantitatively by inputting MMS magnetic field measurements into an algorithm that outputs a trajectory along which the input measurements are matched. The crescent structures observed by MMS in the EDR are consistent with the simulation distributions at the corresponding time along the computed trajectory. This work demonstrates that electron characteristics can serve as “smoking-gun” observables of the EDR at the heart of the magnetic reconnection mystery.
Magnetic reconnection is a fundamental plasma physics process by which magnetic energy can be explosively converted to plasma kinetic energy. Though these ‘magnetic explosions’ occur ubiquitously throughout our universe, the reconnection process itself remains unexplained [Burch et al., 2009; Phillips, 2009; Cassak, 2016]. Reconnection is understood in the collisional regime: frequent plasma particle collisions effect a resistivity analogous to a resistor in a lightbulb filament, which produces heat and dissipates electromagnetic energy. This mechanism cannot operate in the absence of particle collisions, thus it would seem impossible for the dissipative process of reconnection to operate in this ‘collisionless’ regime. The mystery is that magnetic reconnection is known to occur even in collisionless plasmas! This outstanding puzzle is the focus of the following dissertation.

1.1 Seventy-Year History of Collisionless Reconnection

Seventy years ago in 1946, two momentous plasma physics discoveries were made: Ronald G. Giovanelli introduced the concept of magnetic “neutral points” to explain his solar flare observations [Giovanelli, 1946], while Lev D. Landau presented a theory explaining how waves in a plasma could be damped without collisions [Landau, 1946]. Giovanelli’s external
examiner, Fred Hoyle, was interested in applying the new theory of neutral points to the aurora, and so Hoyle encouraged his graduate student, James W. Dungey, to pursue these ideas further [Hones, 1984; Cassak, 2008]. It was Dungey who first coined the phrase “magnetic reconnection” in describing what is now referred to as the Dungey cycle, a process by which plasmas originating from the Sun penetrate into Earth’s magnetosphere once it is opened via topological changes in the magnetic field surrounding the neutral points [Dungey, 1961]. Amusingly, “Jim Dungey mentioned that the idea of the ‘open’ magnetosphere came to him in a sidewalk café in Montparnasse, France” [Stern, 1986].

Meanwhile, Landau considered electron oscillations using equations governing the kinetic theory of plasmas. Circumnavigating the mathematical difficulties first encountered by Anatoly A. Vlasov [Vlasov, 1938] by means of clever analytic continuations, Landau discovered a collisionless mechanism for damping plasma waves [Landau, 1946]. Describing this mechanism, now referred to as Landau damping, Francis Chen writes: “The theoretical discovery of wave damping without energy dissipation by collisions is perhaps the most astounding result of plasma physics research. That this is a real effect has been demonstrated in the laboratory. Although a simple physical explanation for this damping is now available, it is a triumph of applied mathematics that this unexpected effect was first discovered purely mathematically in the course of a careful analysis of a contour integral” [Chen, 1974]. Thus in the same year, both the foundational ideas of magnetic reconnection and collisionless dissipation were born.

In the seventy years that passed since these discoveries, reconnection has become a ubiquitous process in plasma physics. Several motivating applications are listed here with the hope that they inspire the reader as they have the author:
1. On Earth’s sunward side (dayside), reconnection enables the Sun’s solar wind plasma to enter Earth’s magnetic environment. Auroral phenomena can be produced when this plasma is further accelerated toward Earth’s polar regions by reconnection occurring in Earth’s magnetotail (nightside) [Dungey, 1961].

2. Predicting when reconnection will trigger coronal mass ejections (CMEs) and Solar flares is imperative for the safety of astronauts, communication satellites, and electrical power grids [Odenwald, 2009; Shuster, 2013; Hydro Québec, 2016; Cassak, 2016].

3. In the laboratory, reconnection is known to cause so-called “saw-tooth crashes” in thermonuclear fusion devices, inhibiting the goal of harnessing fusion as a viable energy source [Yamada et al., 1997; Phillips, 2009].

4. In astrophysical contexts, reconnection has been invoked to explain the massive flare activity associated with the accretion disks of black holes and pulsars [Burch et al., 2009].

5. Reconnection could even explain the generation of magnetic fields themselves in dynamo theory [Biskamp, 1996; Priest and Forbes, 2000].

This list is by no means complete; there have been many thousands of reconnection publications since the phenomenon was discovered. Nevertheless, despite decades of focused research, the problem of collisionless reconnection has remained unsolved. Particularly, the electron-scale kinetic physics of the electron diffusion region (EDR) at the heart of the reconnection process has remained elusive, challenging to study observationally, experimentally, and numerically in part because of the small spatial scales (on the order of the electron
inertial length [Vasyliunas, 1975]) of the EDR that are embedded in large-scale magnetic structures.

The fundamental importance of reconnection research is reflected by NASA’s investment of over one billion dollars into its successfully launched flagship mission, Magnetospheric Multiscale (MMS), which exploits Earth’s magnetosphere as a natural laboratory to probe and understand collisionless magnetic reconnection at the unprecedented electron scale [Burch et al., 2015]. For the NASA Mission Science Briefing given two days before MMS launch, Roy Torbert performed a demonstration as an analogy to visualize the reconnection diffusion regions sought by MMS: Torbert sent increasing amounts of current through a thin wire under tension until the wire heated up enough that the tension ripped the wire apart in a mini-explosion. Amid the still-smoking wire fragments, Torbert explained: “I understand why this wire heats up and would cause this dissipation. [...] MMS is investigating reconnection because we do not understand at all what causes the dissipation in [collisionless] plasmas. If you look at current theory, it should never happen, because a plasma is almost a superconductor...” [Torbert, 2015].

This dissertation is focused on elucidating the kinetic physics that fuels the collisionless dissipation mechanisms of reconnection diffusion regions. Electron distribution functions throughout the diffusion region are investigated, and are found to exhibit such diverse and remarkable deviations from an isotropic, Maxwellian distribution that their velocity-space structures can serve as “smoking-gun” observables of EDR physics, as will be demonstrated in the results chapters.
1.2 Kinetic Theory: Collisionless Boltzmann Equation

Here we review the plasma kinetic theory upon which the simulation results presented in this dissertation are based. We begin by describing a system of $N$ plasma particles with a single-particle distribution function, $f_s$, for each species, $s$ (where $s = i$ for positively charged ions, and $s = e$ for electrons). We define $f$ to be the number of particles per unit phase space volume. Thus, $f$ is a measure of the plasma’s phase space density, in units of #/([length]$^3$·[velocity]$^3$) = $s^3/m^6$. This statistical description is valid provided the number of particles in the system is sufficiently large. In three spatial dimensions, a distribution function, $f$, is a function in six dimensional phase space: $f = f(x, v, t) = f(x, y, z, v_x, v_y, v_z, t)$. By definition, integrating the phase space density, $f$, over the entire phase space volume will yield the total number of particles in the system:

$$N = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, v, t) \, dx \, dy \, dz \, dv_x \, dv_y \, dv_z = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f \, d^3x \, d^3v. \quad (1.1)$$

This is analogous to the usual 3D configuration space, where the total number of particles in a volume is simply equal to the integral of the particle density, $\rho$, over that volume: $N = \iiint \rho \, d^3x$. Continuing with the analogy, we can construct an analogous continuity equation for phase space. There are two ways for the number of particles in a given volume to change: (1) particles leave or enter the volume across the boundary, and (2) particles are somehow created or lost within the volume. If we assume there are no sources of ionization or recombination (i.e. no mechanisms to add or subtract plasma particles from the system),
then we can write the following equivalent statements of the continuity equation:

\[ \frac{dN}{dt} = - \iint \mathbf{J} \cdot dS = - \iiint \rho \mathbf{v} \cdot dS \]  \hspace{1cm} (1.2)

\[ \frac{\partial \rho}{\partial t} = - \nabla \cdot (\rho \mathbf{v}) \]  \hspace{1cm} (1.3)

\[ \frac{d\rho}{dt} = - \rho (\nabla \cdot \mathbf{v}) \]  \hspace{1cm} (1.4)

where \( \mathbf{J} = \rho \mathbf{v} \) is the particle flux. If the flow is incompressible, then \( \nabla \cdot \mathbf{v} = 0 \) and thus the total derivative \( d\rho/dt = 0 \) (equation 1.4).

In phase space, the argument follows the same reasoning: without sources or sinks of particles, the only way for the phase space density in a volume to change in time is for particles to enter or leave the phase space volume across a surface bounding that volume. The density \( \rho \) now becomes phase space density \( f \), with the only difference being the inclusion of the three additional velocity space dimensions in the formulation. We can think of this step as letting the original three-component velocity \( \mathbf{v} = \{\dot{x}, \dot{y}, \dot{z}\} \) become the six-component phase space “velocity” \( \mathbf{V} = \{\dot{x}, \dot{y}, \dot{z}, \dot{v}_x, \dot{v}_y, \dot{v}_z\} = \{\mathbf{v}, \mathbf{a}\} \) (where \( \mathbf{a} = \dot{\mathbf{v}} \) is the acceleration), representing the change in each of the independent phase space coordinates. Substituting this \( \mathbf{V} \) for \( \mathbf{v} \) and \( f \) for \( \rho \) in the above equations, we obtain continuity equations in phase space:

\[ \frac{dN}{dt} = - \iiint_{S_v} f \mathbf{V} \cdot dS_v = - \iiint_{S_p} f \mathbf{v} \cdot dS_p - \iiint_{S_v} f \mathbf{a} \cdot dS_v \]  \hspace{1cm} (1.5)

\[ \frac{\partial f}{\partial t} = - \nabla_v \cdot (f \mathbf{V}) = - \nabla \cdot (f \mathbf{v}) - \nabla_v \cdot (f \mathbf{a}) \]  \hspace{1cm} (1.6)

\[ \frac{df}{dt} = - f (\nabla_v \cdot \mathbf{V}) = - f (\nabla \cdot \mathbf{v}) - f (\nabla_v \cdot \mathbf{a}) \]  \hspace{1cm} (1.7)
Working with equation 1.7, we can immediately make two simplifications. The first is that \( \nabla \cdot \mathbf{v} = 0 \) automatically, since \( \mathbf{x} \) and \( \mathbf{v} \) are treated as independent coordinates in phase space.

The second is that with the Lorentz force, we have \( \mathbf{a} = \mathbf{F}/m = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})/m \). Computing the velocity-space divergence \( \nabla_{\mathbf{v}} \cdot \mathbf{a} \), the electric field term vanishes since \( \mathbf{E} \) is independent of \( \mathbf{v} \), and the divergence of the magnetic force also vanishes since the cross product \( \mathbf{v} \times \mathbf{B} \) is perpendicular to \( \mathbf{v} \). Thus our plasma system flows incompressibly throughout phase space, since:

\[
\frac{df}{dt} = \frac{\partial f}{\partial t} + (\mathbf{v} \cdot \nabla)f + \frac{q}{m} [ (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}}] f = 0. \tag{1.8}
\]

This result, known as the Vlasov equation [Vlasov, 1938], is an expression of Liouville’s theorem for the collisionless plasma, which can be equivalently stated in two ways: (1) phase space density is constant along particle trajectories, and (2) the phase space volume occupied by an ensemble of particles is conserved in time. With the addition of a collision term, \( (\delta f/\delta t)_c \), accounting for the nearly instantaneous changes in particle velocities due to abrupt, short-range forces, we arrive at Boltzmann’s more general equation:

\[
\frac{\partial f}{\partial t} + (\mathbf{v} \cdot \nabla)f + \left( \frac{\mathbf{F}}{m} \cdot \nabla_{\mathbf{v}} \right) f = \left( \frac{\delta f}{\delta t} \right)_c. \tag{1.9}
\]

Collisions can be reasonably neglected if the number of electrons per Debye cube is large, i.e. \( n \lambda_D^3 \gg 1 \), where \( \lambda_D \) is the Debye length: \( \lambda_D = \sqrt{\epsilon_0 kT_e/\epsilon^2 n} \).

The collisionless Boltzmann equation, namely the Vlasov equation (equation 1.8), stems from a conservation law similar to the one used to obtain Liouville’s theorem for the N-particle distribution function: \( dF_N/dt = 0 \), where \( F_N \) is a function in \( 6N + 1 \) dimensional \( \Gamma \)-space that even accounts for short-range collisions between particles [Huang, 1963]. However,
working in $6N + 1$ dimensions, while extremely fundamental, is often impractical, and so the single-particle distribution function, $f$, is used. This simplified distribution represents a continuous “probability fluid”, describing the probability of finding any one particle in a given region of phase space. For a more comprehensive discussion of these concepts, the reader is referred to Chen [1974] and Gurnett and Bhattacharjee [2005].

A characteristic of systems governed by the Vlasov equation is that they preserve entropy. As pointed out by Cassak [2016], this presents a conceptual concern for the collisionless reconnection problem: “... in the purely collisionless limit in which the kinetic physics is described exactly by the Vlasov equation, one can show rigorously that the entropy of the system is conserved. Thus, there is no irreversible heating in the Vlasov model. Formally, it is not a well-posed question to ask what causes dissipation allowing [collisionless] reconnection...” Cassak continues by emphasizing the importance of non-Maxwellian distribution functions that are known to develop in collisionless plasmas: “In some cases, distribution functions are far from Maxwellian. In such systems, physics that was previously considered ignorable can become important, including kinetic physics ultimately related to collisions. Therefore, in addition to the importance of determining which fluid term describes the dissipation, it is also important to understand what – kinetically – allows field lines to break. This requires understanding what gives distribution functions their structure at and near the X line and what collisional physics governs the evolution of plasmas with non-Maxwellian distributions” [Cassak, 2016].

To make some of these abstract ideas more concrete, Figure 1.1 illustrates an example of a one-dimensional $\theta-\dot{\theta}$ phase space occupied by a pendulum ensemble. Movies 1.1 and 1.2 are animations from which Figure 1.1 is derived. Each pendulum obeys the nonlinear,
damped (e.g. due to friction) and driven (e.g. by a motor) oscillator equation:

\[ \ddot{\theta} = -\omega_0 \cdot \sin(\theta) - b \cdot \dot{\theta} + F \cdot \cos(\omega_d \cdot t), \]

where \( \theta \) is the angle, \( \omega_0 \) is the frequency of small oscillations neglecting the damping and driving terms, and \( b, F, \) and \( \omega_d \) are parameters to control the strength of the damping, driving force, and driving frequency, respectively. The upper, multi-color panels show the evolution of 100 pendulums each starting with only slightly different initial conditions (indicated by the different colors), while the bottom two panels (blue dots) show the evolution of 10,000 pendulums initially spread out over the entire phase space (bottom left panel). Though each of the 100 pendulums start with almost the same initial position and angular velocity, their paths soon diverge from one another (upper right, multi-color panels). At an early time (upper left panels), the pendulums has begun to spread out in position space, though they follow qualitatively similar trajectories in phase space. At a later time (right panels), the pendulums have diverged from one another and sampled nearly the entire phase space. The bottom right panel shows that even when particle (or pendulum) trajectories appear hopelessly complicated, the entire ensemble of points in phase space develops into and preserves an intricate structure, in this case resembling the swirling patterns of cream in a coffee cup, or the rolling ocean waves. Movie 1.2 shows the development and evolution of these mesmerizing structures, demonstrating how seeming disorder in configuration space can be highly structured in phase (or velocity) space. Throughout this dissertation, velocity space distributions are examined to elucidate collisionless magnetic reconnection processes. Chapter 3 features an animation in position, phase, and velocity space of electrons passing by the X-line, and in Chapter 4 we compare the phase space structures characteristic of several reconnection simulation configurations.
Figure 1.1: Evolution of the forced, driven pendulum system, providing a tangible example of the types of intricate structures that can form in phase space even when the configuration space appears intractably complicated. This figure is assembled from frames of Movies 1.1 and 1.2, which show the simultaneous movement of the pendulum systems in both position and phase space.
1.3 Particle-in-Cell Simulations

Vlasov’s equation can be solved numerically by implementing a particle-in-cell (PIC) algorithm, a common way to model self-consistent plasma dynamics [Birdsall and Langdon, 1985; Daughton et al., 2006]. Working directly from Maxwell’s equations of electromagnetism and Newton’s equations of motion, ordinarily there are too many computations required to model a realistic plasma. In the PIC approach, “cells” are used to compute the average electromagnetic fields and charge densities throughout the simulation domain. This averaging introduces the concept of “superparticles”, which represent anywhere from $10^4$ to $10^6$ actual plasma particles [Piel, 2010].

Recent reconnection PIC simulation configurations can be broadly sorted into three categories based on: (1) symmetry, (2) guide field strength, and (3) dimensionality. Reconnection is symmetric when the upstream plasma parameters are the same on both sides of the current sheet, which is the case for reconnection occurring in Earth’s magnetotail. When these upstream conditions (e.g. temperature, density, magnetic field strength) are different, such as at Earth’s magnetopause, reconnection is asymmetric. A guide magnetic field, $B_g$, refers to the out-of-plane component of the field which controls the magnetic shear across the layer. When there is no guide field ($B_g = 0$), reconnection is said to be antiparallel (since the reconnecting fields initially point in opposite directions on either side of the current layer). With a guide field, reconnection is referred to as component reconnection. Reconnection is commonly studied in 2D, assuming spatial symmetry in the out-of-plane direction, though recent computational advances have enabled 3D PIC simulations. The initial setup for reconnection usually begins with a current sheet in Harris equilibrium [Harris, 1962; Birn et al., 2001]. Other important parameters to consider are the ratios of the ion to electron mass,
\( m_i/m_e \), electron plasma frequency to cyclotron frequency, \( \omega_{pe}/\Omega_{ce} \), and electron skin depth to Debye length, \( d_e/\lambda_D \), all of which are often artificially small in PIC simulations to avoid the computational cost of using realistic values. Many PIC simulation studies address aspects of reconnection in these different regimes (e.g. Daughton et al. [2006, 2009, 2011, 2014]; Bessho and Bhattacharjee [2007]; Karimabadi et al. [2007]; Bowers et al. [2008]; Pritchett [2010]; Hesse et al. [2011, 2014]; Germaschewski et al. [2016]).

### 1.3.1 Discreteness in PIC: The Meaning of \( E \) and \( B \)

There is a subtlety in the PIC formalism. The discreteness of individual particles and their interactions described by the \( N \)-particle distribution function, \( F_N \), are removed in the single-particle distribution, \( f \), of Vlasov theory. Nevertheless, the distribution function in PIC is approximated as follows:

\[
f = \sum_{j=1}^{N} w_j \cdot \delta(x - x_j(t))\delta(v - v_j(t)), \tag{1.10}
\]

where \( w_j \) represents the statistical weight for the \( j^{th} \) particle. What then do we gain by following so many (\( N \sim 10^9 \) to \( 10^{12} \)) discrete superparticles?

Following are excerpts from a lecture prepared by William Daughton [Daughton et al., 2007] discussing this seeming paradox: “Notice that if we let \( w_j = 1 \) and take ‘\( N \)’ to be the number of particles in the real system, these equations [Vlasov’s equation, Newton’s 2nd Law with the Lorentz force, and Maxwell’s equations] reduce to the exact classical description of the \( N \)-body problem. This may seem strange since the original Vlasov equation neglected collisions entirely! We have in fact introduced a subtle change by what we mean in the fields
In the Vlasov description, the discreteness of the particles was eliminated, but in the PIC approach we have re-introduced discreteness. We must be careful here, since we had good reasons to eliminate this in the first place. We want to eliminate the short-range discrete interactions since these are very expensive to compute and dynamically far less important in high temperature plasmas.” To do this, Daughton explains the use of a spatial grid in the PIC algorithm: “The charge and current density is interpolated to a spatial mesh using a variety of possible schemes. These smoothed sources are used to solve Maxwell’s equations on the same spatial mesh. Computationally the approach is order $N$ vs. $N^2$ for the exact force calculation. The same interpolation scheme used to deposit the moments must also be used to compute the force on the particles from $E$ and $B$.”

From this discussion, we see that the “cell” in particle-in-cell refers to this spatial grid, over which the characteristics of discrete particles are averaged. In this way, short-range interactions (i.e. “collisions”) are averaged away, and the charge and current density can be more efficiently computed and used to solve for $E$ and $B$. The theoretical subtleties involved using both the entropy-conserving Vlasov equation (which eliminates discreteness) and the ensemble of superparticles (which reintroduces discreteness) as is done in PIC is important to consider conceptually, and is discussed more extensively by Birdsall and Langdon [1985].

1.3.2 Insensitivity of the Reconnection Rate

As demonstrated by Birn et al. [2001] and pointed out by Cassak [2016], reconnection models routinely achieve similar reconnection rates that appear to be somewhat independent of the dissipation mechanism included to violate the frozen-in condition. This rate is about $0.1v_{Ai}B_0$, where $v_{Ai}$ is the upstream ion Alfvén speed, and $B_0$ is the upstream reconnecting
magnetic field strength. Essentially all of the simulation models discussed by Birn et al. [2001] produced the same reconnection rate profile, having a peak of $0.24v_{Ai,0}B_0$, where $v_{Ai,0}$ was based on the Harris sheet density, $n_0$, rather than the upstream density, $n_b$ (referred to as $n_\infty$ in Birn et al. [2001]). When normalized to the upstream $v_{Ai}$, the peak rates in Birn et al. [2001] become about $0.11v_{Ai}B_0$, since their $\sqrt{n_\infty/n_0} = \sqrt{0.2} \approx 0.45$.

In Figure 1.2, we plot the reconnection rate profiles for the four 2D PIC simulation runs of this dissertation (we omit Run #5 because of the difficulty in determining the reconnection rate in 3D [Daughton et al., 2014]). While there are some differences between the runs and which measure of the reconnection rate is computed, overall each of the runs exhibit a reconnection rate profile that is qualitatively similar to the rate found previously: about $0.1v_{Ai}B_0$. When the reconnection rate is normalized to upstream magnetosheath (MSH) values, even asymmetric configurations (e.g. Run #4, and the simulation reported in Bessho et al. [2016]) exhibit this reconnection rate peak of about $0.1v_{Ai}B_0$. For Run #1, two additional measures for the rate of reconnection are shown. The maximum ion inflow velocity, $u_{iz,max}$, peaks at about $0.21v_{Ai}$ around $t\Omega_{ci} = 22$ at the time when a secondary island forms, which is about $4\Omega_{ci}^{-1}$ after the peaks in $<E_y>$ and $\partial\Delta A_y/\partial t$. The reconnection rate in the asymmetric run (Run #4) shows a more gradual increase to its peak compared to the symmetric runs.

1.3.3 Five Simulation Studies

Table 1.1 lists the five PIC simulations considered in this dissertation with a few initial parameters for comparing the configurations of each run. The 2D PIC runs in this dissertation model reconnection in two spatial and three velocity dimensions. The symmetric simula-
Figure 1.2: Comparison of the reconnection rate time profiles for the four 2D simulations (Runs #1, #2, #3, and #4) discussed in this dissertation. The black curves in each panel represent the reconnection electric field, $E_y$, normalized to $v_{Ai}B_0$ and averaged over a $2d_i \times 2d_i$ region centered at the dominant X-line. Two additional measures of the reconnection rate are plotted in the panel for Run #1: (blue) the time derivative of the reconnected flux $\partial \Delta A_y / \partial t$ is shown in blue, where $\Delta A_y$ is equal to the difference $\max(A_y) - \min(A_y)$ taken along $z = 0$, and (red) the maximum ion inflow bulk velocity, $u_{iz,\text{max}}$, normalized to the upstream Alfvén speed (red axis). In each of these runs, $\langle E_y \rangle$ starts around 0.0, increases rapidly to a peak reconnection rate, and eventually settles to a value close to $0.1v_{Ai}B_0$ (dotted line in all panels).
### Table 1.1: Parameters for the five particle-in-cell simulations considered in this dissertation.

These simulations were performed by William Daughton of Los Alamos National Laboratory (LANL).

<table>
<thead>
<tr>
<th>Run #</th>
<th>Dimension</th>
<th>Configuration</th>
<th>Mass Ratio $m_i/m_e$</th>
<th>Guide Field $B_g/B_0$</th>
<th>Upstream Electron Beta $\beta_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2D</td>
<td>symmetric</td>
<td>400</td>
<td>0</td>
<td>0.0028</td>
</tr>
<tr>
<td>2</td>
<td>2D</td>
<td>symmetric</td>
<td>400</td>
<td>0.03</td>
<td>0.0028</td>
</tr>
<tr>
<td>3</td>
<td>2D</td>
<td>symmetric</td>
<td>1836</td>
<td>0</td>
<td>0.0289</td>
</tr>
<tr>
<td>4</td>
<td>2D</td>
<td>asymmetric</td>
<td>100</td>
<td>0</td>
<td>0.333 (MSH), 0.022 (MSP)</td>
</tr>
<tr>
<td>5</td>
<td>3D</td>
<td>asymmetric</td>
<td>100</td>
<td>1.0</td>
<td>0.333 (MSH), 0.022 (MSP)</td>
</tr>
</tbody>
</table>

(Runs #1, #2, and #3) model undriven reconnection with open boundary conditions [Daughton et al., 2006]. The only parameter of Run #2 that differs from Run #1 is a 3% guide field. Each of these runs begins with an initial Harris current sheet. More information regarding how the equations of motion for the particles and Maxwell’s equations are solved self-consistently are given by Bowers et al. [2008]. The initial magnetic field is given by $B_x = B_0 \cdot \tanh(z/L)$, where $B_0$ is the asymptotic value of the magnetic field and $L$ is the half-width of the current sheet. The number density is given by $n(z) = n_0 \cdot \text{sech}^2(z/L) + n_b$, where $n_0$ is the initial peak density in the current sheet and $n_b$ is the background density.

The simulation domain for Runs #1 and #2 is $L_x \times L_z = 80d_i \times 20d_i$ resolved into 10240 × 2560 cells with 600 particles per cell, where $d_i = c/(4\pi n_0 e^2/m_i)^{1/2}$ is an ion skin depth based on $n_0$. Additional simulation parameters for Runs #1 and #2 are: the electron plasma to cyclotron frequency ratio $\omega_{pe}/\Omega_{ce} = 2$, ion to electron mass ratio $m_i/m_e = 400$, ion to electron temperature ratio $T_i/T_e = 5$, background to current sheet temperature ratio $T_b/T_0 = 0.333$, $L/d_i = 0.5$, $n_b/n_0 = 0.05$, upstream electron beta $\beta_e = 0.0028$ (a value comparable to $\beta_e$ in the magnetotail lobe), with $3.1 \times 10^{10}$ particles. Reconnection was initiated by adding a magnetic perturbation [Daughton et al., 2009].
Run #3 was performed with a real mass ratio, \( m_i/m_e = 1836 \). The simulation domain size for Run #3 is: \( L_x \times L_z = 20d_i \times 20d_i \) resolved into \( 5120 \times 5120 \) cells with 400 particles per cell. Additional parameters are: \( \omega_{pe}/\Omega_{ce} = 2 \), \( T_i/T_e = 5 \), \( T_b/T_0 = 0.76 \), \( L/d_i = 0.5 \), \( n_b/n_0 = 0.228 \), \( \beta_e = 0.0289 \), with about \( 2.1 \times 10^{10} \) particles.

For the 2D asymmetric run (Run #4), the boundary conditions are periodic in the outflow direction for fields and particles, while conducting for fields and reflecting for particles in the current sheet normal direction. Choosing the upstream magnetosphere (MSP) to magnetosheath (MSH) density ratio \( n_{MSP}/n_{MSH} = 1/8 \) and the initial MSH plasma beta \( \beta_{MSH} = 1 \), the ratio of upstream magnetic field strengths is constrained to be: \( B_{MSP}/B_{MSH} \approx 1.37 \). The initial magnetosheath (MSH) total (ion + electron) plasma beta is chosen to be unity: \( \beta_{MSH} = 2\mu_0 \cdot n_{MSH} \cdot (T_i + T_e)/B_{MSH}^2 = 1 \) (where \( B_{MSH} \) is the asymptotic magnetic field strength, \( n_{MSH} \) is the initial upstream density, and \( T_{i,e} \) are uniform initial ion and electron temperatures, respectively). We can write the initial reconnecting magnetic field component as:

\[
\mathbf{B}(z) = \frac{1}{2} \cdot [(B_{MSP} - B_{MSH}) + (B_{MSP} + B_{MSH}) \cdot \tanh (z/\lambda)] \hat{x} + B_g \hat{y},
\]

where \( \lambda = 1d_i \) is the half-thickness of the current layer, \( d_i \) is the initial ion skin depth based on \( n_{MSH} \), and \( B_g \) is a uniform guide field. For the algorithm presented in Chapter 4, we focus mainly on the zero guide field case \( B_g = 0 \), but we also compare our results to a simulation with small guide field \( B_g = 0.1B_{MSH} \). The density profile across the layer is given by:

\[
n(z) = n_b(z) + n_h(z) = \frac{1}{2} \cdot [(n_{MSP} + n_{MSH}) + (n_{MSP} - n_{MSH}) \cdot \tanh (z/\lambda)] + n_c \cdot \text{sech}^2(z/\lambda),
\]

which is the sum of a non-drifting Maxwellian population \( n_b(z) \) and a Harris component \( n_h(z) \).
with \( n_c \) being the density at the center of the layer. The domain is \( L_x \times L_z = 75d_i \times 25d_i \) with 3072 \( \times \) 2048 cells and an average of 3,000 particles per cell. Additional parameters include the ion to electron mass ratio \( m_i/m_e = 100 \), temperature ratio \( T_i/T_e = 2 \), and initial MSH electron plasma to cyclotron frequency ratio \( \omega_{pe}/\Omega_{ce} = 2 \). Reconnection is initiated by adding a perturbation to the magnetic field [Daughton et al., 2014]. For more details regarding the simulation setup, please consult Chen et al. [2016] and references therein.

For the 3D asymmetric run (Run #5), several of the parameters are actually the same as the 2D asymmetric run, except the following: the dimensions are 85\( d_i \times 85d_i \times 35d_i \) resolved into 2920 \( \times \) 2920 \( \times \) 1200 cells with 100 particles per cell, giving a total number of about 2.046 \( \times \) 10^{12} particles, and the guide field is \( B_g = 1.0B_0 \). More details regarding this 3D run are provided in Daughton et al. [2014].

Throughout this dissertation, simulation velocities are normalized to either to the speed of light, \( c \), or to a relevant Alfvén speed, lengths to the electron skin depth \( d_e = c/(4\pi n_0 e^2/m_e)^{1/2} \), magnetic field strengths to \( B_0 \), and times are reported in units of \( \Omega_{ci}^{-1} \) (where \( \Omega_{ci} \) is an ion cyclotron frequency). Except where noted, the simulation electron velocity distributions represent electrons from spatial bins of about \( 2d_e \times 2d_e \) in area. Chapter 2 focuses on results from Run #1, while Chapter 3 considers all of the symmetric runs (Runs #1, #2, and #3). The last results chapter, Chapter 4, considers the two asymmetric runs (Run #4 and #5) and comparisons between all of the 2D runs.
1.4 Spacecraft Instrumentation

This dissertation presents spacecraft data measured by the Cluster mission (in Chapters 2 and 3) and the MMS mission (in Chapter 4). Here, we summarize several of the instrument suites onboard each of these four-spacecraft fleets which gathered the data used in this study.

1.4.1 Cluster

Cluster was the first multi-spacecraft mission of its kind equipped to probe the small-scale (∼100 km) structures of Earth’s magnetosphere. As explained by Escoubet et al. [2001]: “The main goal of the Cluster mission is to study the small-scale plasma structures in three dimensions in key plasma regions, such as the solar wind, bow shock, magnetopause, polar cusps, magnetotail and the auroral zones.” The four Cluster spacecraft were launched in July and August of 2000 into a polar orbit with a 4 $R_E$ perigee and 19.6 $R_E$ apogee (where $R_E = 6,371$ km is an Earth radius). A host of multi-spacecraft analysis techniques are possible with the four Cluster observatories [Paschmann and Daly, 1998].

The Cluster Magnetic Field Investigation (FGM) measures the magnetic field with an accuracy of about 0.1 nT [Balogh et al., 2001]. Each spacecraft has a fully redundant FGM instrument, consisting of two triaxial fluxgate magnetometers. We performed minimum variance analysis on the magnetic field vector (MVAB) to determine a boundary-normal coordinate system for the magnetotail reconnection event discussed in Chapter 2.

Ions are detected by the Cluster Ion Spectrometry (CIS) experiment, which measures the full, three-dimensional distribution function of four ion species: $H^+$, $He^+$, $He^{++}$, and $O^+$ [Rème et al., 2001]. Two analyzers, the hot ion analyzer (HIA), and the ion compo-
sition and distribution function analyzer (CODIF), are mounted on opposite sides of the spacecraft parallel and tangential to the body of the spacecraft. HIA is an electrostatic analyzer with symmetric quadrisphere ("top-hat") geometry, and CODIF is a high-sensitivity, mass-resolving spectrometer capable of distinguishing between ion species. Both detectors measure a complete distribution within one spin period of the spacecraft (∼4 seconds).

The Plasma Electron and Current Experiment (PEACE) consists of two top-hat electrostatic analyzers that can measure the full electron distribution in half a spin period (∼4 seconds), though partial angular distributions can be detected in as low as 7.8 milliseconds [Johnstone et al., 1997]. The two analyzers can detect electrons from the same energy range of 0.59 eV to 26.4 keV, but a difference in geometric factor allows the two sensors to concentrate on a particular energy range. The Low Energy Electron Analyzer (LEEA) has a smaller geometric factor preferable for measuring the high fluxes of low energy (0.59 eV to 9.45 eV) electrons, while the High Energy Electron Analyzer (HEEA) is designed for detecting higher energy electrons. Since the two analyzers are mounted on opposite sides of the spacecraft, together they can measure the full electron distribution in half a spin period. Chapters 2 and 3 present and discuss PEACE electron distribution data.

1.4.2 MMS

The successful launch of the Magnetospheric Multiscale (MMS) mission with its pioneering instrument suites marks the arrival of the “MMS Era” of reconnection research. The purpose of the MMS mission is to “understand the microphysics of magnetic reconnection by determining the kinetic processes occurring in the electron diffusion region that are responsible for collisionless magnetic reconnection” [Burch et al., 2015].
On 12 March 2015, MMS launched on an Atlas V rocket into Phase 1 of its mission to study reconnection at the dayside magnetopause (Figure 1.3). The orbit has a low inclination ($28^\circ$) with a $1.2\ R_E$ perigee and $12\ R_E$ apogee during this phase. The first encounter with a magnetopause electron diffusion region has already been published by [Burch et al., 2016] and is addressed in Chapter 4 of this dissertation. After MMS passes by the region of interest at the magnetopause once more, Phase 2 of the science mission will begin, starting with maneuvers to adjust the orbit’s apogee to $25\ R_E$ so the MMS tetrahedron can ‘catch’ reconnection happening in Earth’s magnetotail. The spacecraft separation in the region of interest can be made remarkably small: $\lesssim 10\ km$ (considerations to reduce this separation even further are currently being discussed), which, along with the unprecedented time resolution achieved by the instrument suites, enables MMS to measure the microscale structures of reconnection.

The FIELDS suite refers to the instrumentation onboard each MMS spacecraft responsible for measuring magnetic and electric fields [Torbert et al., 2016a]. These instruments include: (1) redundant digital and analog fluxgate magnetometers (DFG and AFG) [Russell et al., 2016], (2) a search coil magnetometer (SCM) for measuring AC fields [Le Contel et al., 2016], (3) an electron drift instrument (EDI) that can calibrate magnetometer offsets [Torbert, 2016b], and (4) three-axis electric field measurements by the spin plane double probes (SDP) [Lindqvist et al., 2016] and axial probes (ADP) [Ergun et al., 2016]. EDI can also measure ambient electron flux for some energies with millisecond resolution, which motivates the use of EDI data to infer field line connectivity [Shuster et al., 2015c] and electron nongyrotropy [Argall et al., 2016].

The Fast Plasma Instrument (FPI) is indeed fast: its duel electron and ion spectrometers
Figure 1.3: Successful launch of the MMS spacecraft on 12 March 2015, 10:44pm (EDT). (Photo courtesy of John Macri.)
(DES and DIS) onboard each MMS spacecraft measure electron and ion distribution functions faster than any instrument of its kind. This accomplishment is made possible primarily because of the innovative strategy to distribute multiple sensors around the spacecraft, thus enabling FPI to sample the full distributions rapidly without being limited by the spin period of the spacecraft. This improvement in temporal resolution required the construction and coordination of 64 spectrometers in total for the mission, in order to have 16 operational spectrometers per spacecraft (8 individual electron spectrometers per spacecraft + 8 individual ion spectrometers per spacecraft).

The endeavor paid off: FPI successfully measures full plasma distribution functions every 30 milliseconds for electrons and every 150 milliseconds for ions. Compared to a previous mission which may have accumulated a single electron distribution over a time period of about 3 seconds, MMS resolves 100 distributions in this 3-second interval! An example is shown in Movie 2 of Burch et al. [2016] (direct link: http://bcove.me/9fkcpfn1). Some of these highly structured distributions and their moments (density $n_e$, bulk velocity $U_e$, and temperature $T_e$) are compared to PIC simulation results in Chapter 4 of this dissertation.

As a final note on how promising MMS observations are so far, Torbert recently noted: “This dataset is so revolutionary that I think we’ll be mining it for 50 years” [Potier, 2016].

1.5 Movie and Animation Repository

The chapters in this dissertation include animated movie content. These movies are stored remotely as *.mp4 and *.gif files in a repository which can be accessed via the following link: https://drive.google.com/open?id=0BwUFZvYO52UUS29jVEJIY2NSTUE
Additionally, the end of each chapter provides a list of all the movies referenced in the chapter with a link that leads directly to that movie. A few moments after clicking the link, the movie should load and play automatically. If the movie does not play, it can be downloaded and played locally. The movies of this introductory chapter are presented here:

Movies

**Movie 1.1:** An ensemble of 100 pendulums in phase space governed by the nonlinear, damped, driven oscillator equation, each starting from slightly different initial conditions to exhibit the phenomenon of deterministic chaos readily visible in the phase-space representations derived in the chapter.

Direct link:
https://drive.google.com/open?id=0BwUFZvYO52UUQldOX2pqMEVFMWc

**Movie 1.2:** Phase space visualization of 10,000 pendulums initially mapping the entire phase space, illustrating how particle trajectories in phase space cannot intersect, and demonstrating the rich structure that such Poincaré surfaces of section can acquire.

Direct link:
https://drive.google.com/open?id=0BwUFZvYO52UUX2JDNG5pYXZRZmc
Chapter 2

Highly Structured Electron Anisotropy in the Exhaust

2.1 Introduction

Results from the first simulation listed in Table 1.1 reveal that around the time when the reconnection rate peaks, electron velocity distributions become highly structured in magnetic islands and open exhausts \cite{Shuster et al., 2014}. Rings, arcs, and counter-streaming beams are generic and lasting components of the exhaust electron distributions. The temporal dependence of electron distributions provides a perspective to explain an outstanding discrepancy concerning the degree of electron anisotropy in reconnection exhausts, and enables inference of the reconnection phase based on observed anisotropic electron distributions. Some of the structures predicted by this simulation are confirmed by measurements from the Cluster spacecraft during its encounter with reconnection exhausts in the magnetotail.

2.1.1 Previous Knowledge

Electron characteristics in distinct magnetic regions during reconnection with negligible guide fields have been reported based on spacecraft observations and particle-in-cell (PIC) simulations. However, a comprehensive picture has not yet been established. \textit{Hoshino et al.}
[2001a,b] reported Geotail observations and a PIC simulation of non-Maxwellian electron distributions that are anisotropic with different types of $T_{e\|} > T_{e\perp}$ structures and that become more isotropic closer to the current sheet, where $T_{e\|}$ and $T_{e\perp}$ are electron temperatures parallel and perpendicular to the magnetic field. Asano et al. [2008] reported Cluster observations of isotropic flat-top distributions frequently observed near the X-line in the exhaust before reaching the magnetic field pileup region, and occasional strong field-aligned beams moving toward the X-line concurrent with the flat-top distributions. Fujimoto and Sydora [2008] performed a PIC simulation and showed that exhaust electrons exhibit $T_{e\|} < T_{e\perp}$ in the magnetic field pileup region, which they attribute to adiabatic heating and the escaping of high energy electrons along field lines. Chen et al. [2008a, 2009] showed that electrons in the exhaust were relatively isotropic and hot, while electrons in the inflow exhibit $T_{e\|} > T_{e\perp}$ and are colder than exhaust electrons, based on Cluster measurements and a PIC simulation. The picture of roughly isotropic exhaust and anisotropic inflow electrons was used to delineate the reconnection exhaust and inflow regions, and to distinguish between the interior and exterior regions of magnetic islands [Chen et al., 2008b]. Egedal et al. [2010] proposed a parallel potential model to explain the anisotropy of Cluster electron distributions in the inflow and the isotropic flat-top distributions in the exhaust. Egedal et al. [2012] reported anisotropic electron distributions from reconnection exhaust mainly near the separatrix region from a PIC simulation, and suggested pitch-angle scattering in the weak magnetic field regions as well as wave instabilities to isotropize and flatten exhaust distributions. Le et al. [2013] further supported the picture of nearly isotropic exhaust electrons based on PIC simulations of reconnection with weak to zero guide fields. The above studies provide an important observational and theoretical basis for understanding electron characteristics in
inflow and exhaust regions. However, for regimes with negligible guide fields, whether exhaust electrons are isotropic or anisotropic remains an open question. The PIC simulation results reported thus far were based on single time snapshots; none have addressed whether and how electron characteristics vary as reconnection evolves.

The absence of knowledge concerning evolution of electron characteristics is the primary motivation for our study. We find that electron velocity distributions become highly structured in exhausts starting around the time of peak reconnection with the emergence of arcs, rings, and field-aligned beams. Our result indicates that the seeming discrepancy concerning the degree of electron anisotropy in exhausts can be explained by considering time evolution, and suggests a technique to infer the evolution stage of reconnection based on measurements of anisotropic electron distributions.

2.1.2 Two Types of Anisotropy

Throughout this chapter, the distinction is made between temperature anisotropy and kinetic anisotropy. Temperature anisotropy refers to the bulk fluid property: \( T_{e\perp}/T_{e\parallel} = P_{e\perp}/P_{e\parallel} \neq 1 \), where \( P_{e\parallel} \) and \( P_{e\perp} \) are the parallel and perpendicular pressures, respectively. The temperatures are related to the distribution function, \( f_e(x, v, t) \), as follows:

\[
T_{e\parallel} = T_{e\parallel}(x, t) \propto \frac{P_{e\parallel}(x, t)}{n_e(x, t)} = \frac{m_e \int (v_{e\parallel} - U_{e\parallel})^2 f_e(x, v, t) \, d^3v}{\int f_e(x, v, t) \, d^3v} \tag{2.1}
\]

\[
T_{e\perp} = T_{e\perp}(x, t) \propto \frac{P_{e\perp}(x, t)}{n_e(x, t)} = \frac{m_e \int (v_{e\perp} - U_{e\perp})^2 f_e(x, v, t) \, d^3v}{2 \int f_e(x, v, t) \, d^3v} \tag{2.2}
\]

where \( n_e \) is the number density, and \( U_{e\parallel} \) and \( U_{e\perp} \) denote the parallel and perpendicular bulk velocities, respectively.
Figure 2.1: Isotropy in the fluid measure ($T_{e\perp}/T_{e\parallel} \approx 1$) does not necessarily imply that kinetic distributions are Maxwellian, as this example shows. The bulk plasma is isotropic in the vicinity of the featured distribution taken at $(x, z) = (1175, 0)$, with $T_{e\perp}/T_{e\parallel} \approx 1.1$ (top panel with 1D cut below). However, the distribution is certainly not “isotropic” in a kinetic sense. There are four distinct, non-Maxwellian populations of the distribution shown in $v_{\parallel}$-$v_{\perp}$ (bottom right) and $v_{x}$-$v_{z}$ (middle left) space: two counter-streaming beams, a faint ring population elongated in the $v_{\perp} \sim v_{x}$ direction, and a colder core. In this case, the counter-streaming beams enhance $T_{e\parallel}$ while the faint energetic ring population in $v_{\perp}$ leads to a balancing increase in $T_{e\perp}$, with the net effect being that $T_{e\perp}/T_{e\parallel} \approx 1$. 

28
On the other hand, “anisotropy” in a kinetic sense refers to the non-Maxwellian structure (i.e. departure from isotropy in velocity space) of the distribution function itself. The subtlety here is that a distribution can have a non-Maxwellian structure even in regions where the plasma is isotropic in a fluid sense with $T_{e\perp}/T_{e\parallel} \approx 1$. In other words, kinetic anisotropy is necessary but not sufficient for temperature anisotropy, as illustrated in the example shown in Figure 2.1. The reason for this difference is that the integrations in equations 2.1 and 2.2 average over any anisotropic structures of the distribution function, which can result in $T_{e\perp}/T_{e\parallel} \approx 1$.

2.2 Kinetic Anisotropy in the Exhaust

We begin by considering the evolution of electron temperature anisotropy $T_{e\perp}/T_{e\parallel}$ around the time of peak reconnection for the entire reconnection region shown in Figure 2.2. Up to $t\Omega_{ci} = 17$, the temperature is mostly isotropic ($T_{e\perp} \approx T_{e\parallel}$) in the leftmost ($x \sim 700$) and rightmost ($x \sim 880$) exhausts, and electron distributions are roughly isotropic, consistent with several previous results [Asano et al., 2008; Chen et al., 2008a, 2009; Egedal et al., 2010, 2012; Le et al., 2013]. However, there is a temperature anisotropy $T_{e\perp}/T_{e\parallel} \approx 3$ on the magnetic island-side of the exhaust ($x \sim 815$) generated by the dominant X-line at $(x, z) \sim (840, 0)$ (Figure 2.2a,b). This island formed before the explosive growth phase of reconnection, and will be referred to as a ‘primary’ island hereafter (see Chen et al. [2012] for more in-depth discussions of the island properties). When the reconnection rate maximizes at approximately $t\Omega_{ci} = 18$, the regions at the end of every electron outflow jet have $T_{e\perp}/T_{e\parallel} \gtrsim 2$, as high as 8 on the island-side of the exhaust near $x \sim 810$ (Figure 2.2c,d,e).
Figure 2.2: (a-d) Electron temperature anisotropy $T_{e\perp}/T_{e\parallel}$ near peak reconnection, showing that $T_{e\perp}/T_{e\parallel} \gtrsim 2$ near the end of every outflow jet and is particularly enhanced in the exhaust involving a primary island (centered at $x \sim 785$). Black curves in (a) and (c) are contours of the magnetic flux function. One-dimensional cuts of $T_{e\perp}/T_{e\parallel}$ in (a) and (c) along $z = 0$ are shown in (b) and (d), respectively. (e) Cut of $U_{ex}$ along $z = 0$ showing that the local peaks of $T_{e\perp}/T_{e\parallel}$ occur near the end of every electron outflow jet for $t\Omega_{ci} = 18$, a feature which lasts until the end of the run.
2.2.1 Rings, Wings, and Arcs

The strong temperature anisotropy develops in association with ring structures that form in electron velocity distributions. Three electron distributions each with a ring population are presented in Figure 2.3 for \( t\Omega_{ci} = 19 \) to show the spatial evolution of the distributions from the edge of the electron outflow jet (Distribution 1) to just inside the closed field lines of the primary island (Distribution 3). Figure 2.3b shows how the electron temperature anisotropy \( T_{e\perp}/T_{e\parallel} \) developed past the time of peak reconnection, and shows the relative locations of Distributions 1, 2, and 3. Each of the simulation distributions shown in this chapter represent electrons from the same size spatial bin: \( \Delta x \times \Delta z = 2d_e \times 2d_e \). The maximum electron temperature anisotropy \( T_{e\perp}/T_{e\parallel} \) occurs at approximately \( x = 810 \), near the boundary of the closed island field lines and the open exhaust field lines. The open exhaust field lines just outside of the primary island have been reconnected at the dominant X-line (\( x \approx 850 \)) though not yet reconnected at the X-line to the left of the island (\( x \approx 715 \)). The primary island is ejected in the \(-x\) direction because of the stronger reconnection rate at the dominant X-line.

Distribution 1 (rightmost of the three distributions) is taken at \( (x, z) = (821, 0) \) from slightly beyond the end of the electron outflow jet, and has a tree-shape structure in \( v_{\parallel}-v_{\perp} \) space. The \( v_{x}-v_{y} \) representation shows a colder core, two overlapping ring populations, and an arc. The arc is non-gyrotropic, having non-uniform counts which span about 180° in gyrophase in the positive \( v_{y} \) domain with a maximum \( v_{y} \) of about 0.7. The enhanced counts on the \(+v_{y}\) side of the rings around \( v_{y} \sim 0.45 \) are projections of the wing-resembling populations with \( |v_{\parallel}| > 0.4 \). The \( v_{x}-v_{z} \) representation shows another projection of the wings which extend to their highest \( |v_{z}| \) values near \( v_{x} \sim -0.5 \). These arc, wing, and ring structures
Figure 2.3: Three highly structured, anisotropic electron distributions in $v_\parallel$-$v_\perp$, $v_x$-$v_y$, and $v_x$-$v_z$ space from an exhaust associated with the primary island at $t\Omega_{ci} = 19$. Distribution 1 (rightmost panels) exhibits arc and ring components as well as a low-energy core, and is dissected in Movie 2.1. Distribution 2 has a ring structure with no low-energy core. Movie 2.2 further explores the origin of some of these electrons. Distribution 3 consists of ring and core populations. To provide the temporal and spatial context for the distributions, the reconnection rate profile (a) and the electron temperature anisotropy $T_{e\perp}/T_{e\parallel}$ with black curves showing contours of the magnetic flux function (b) are also shown. The white curves in (b) are field lines that go through the center of the bins from which distributions 2 and 3 are sampled. B at the center of each distribution bin is given above each $v_\parallel$-$v_\perp$ distribution panel.
have fully developed by $t\Omega_{ci} = 18$, and are found in distributions from all exhausts near the end of electron outflow jets. The presence of a secondary island that has formed by $t\Omega_{ci} = 23$ (see Figure 2.5) greatly disrupts the structure and magnitude of the outflow jet [Li et al., 2012], resulting in less outstanding ring and arc structures.

Rotating views of this distribution in three-dimensional (3D) velocity space are presented in Movie 2.1, which first shows the electrons as black points and then colors the distribution by phase space density. The end of the movie shows consecutive slices of this distribution in each of the $v_x$-$v_y$, $v_x$-$v_z$, and $v_y$-$v_z$ planes to give a more thorough visualization of distribution’s internal structure, which can be somewhat buried by 2D velocity space projections. The next chapter discusses the even more intricate structures of distributions at and just downstream of the X-line, and how and their evolution leads to these kinds of complicated velocity space populations.

Distribution 2 is a ring distribution located at $(x, z) = (811, 0)$, representative of distributions occurring along the open exhaust field lines on the island side up to about $z = \pm 10$ and only on the $+x$ side of the island. The low-energy electrons ($v_{\perp} < 0.3$) are almost entirely absent while the majority of electrons are centered around $v_{\perp} = 0.6$, giving the gyrotropic distribution a toroidal shape in 3D velocity space. The low energy electrons with $|v_x|$ and $|v_y|$ less than 0.3 start to vanish within $1 \Omega_{ci}^{-1}$ after the maximum reconnection rate occurs. The depletion lasts for $\sim 5 \Omega_{ci}^{-1}$ until about $t\Omega_{ci} = 23$ when a low-energy population returns, forming a ring and core structure (as in Distribution 3). No other exhaust region features such a depletion of the low-energy electrons as does this open exhaust region adjacent to the primary island. All other exhausts, such as inside the primary island (e.g. Distribution 3), and the open exhausts near $x = 670$ and $x = 900$, retain the low-energy population.
To better understand how such a pronounced ring distribution arises, Movie 2.2 shows test-particle orbits traced backward in time for some of the lower energy electrons in Distribution 2. Electrons with \(v_x^2 + v_y^2 < (0.25c)^2\) are colored red and marked by ‘+’ symbols, while electrons which are part of the inner ring with 
\[(0.25c)^2 < v_x^2 + v_y^2 < (0.35c)^2\]
are marked with blue dots. The majority of the inner ring is comprised mostly of electrons which pass through or near the dominant X-line at \(x \sim 850\). The fewer number of low energy (red) electrons originate mainly from the left side of the domain \((x < 600)\), entering the ring distribution region via open field lines which have been reconnected by the dominant X-line and wrap around the closed primary island field lines. At Frame -400, these red low energy electrons have \(v_x > 0,\) and \(v_\parallel > 0 (< 0)\) for the \(z > 0 (< 0)\) population. The open field lines are pulled inward toward \(z = 0\) by the inflowing plasma at the X-line to the left of the island, and thus magnetically link the region of ring distributions to the inflow region near \(x = 715\). As a result of this magnetic connectivity, the parallel potential \(\Phi_\parallel\) known to trap electrons and generate \(T_{e\parallel} > T_{e\perp}\) temperature anisotropy in the inflow [Egedal et al., 2010, 2012] appears to act as a low-energy electron filter for these field lines. Thus from the \(-x\) side, only electrons with high enough velocities to overcome this potential well near \(x = 715\) will be able to reach the ring region near \((x, z) = (811, 0)\). This leaves mainly the electrons that have been accelerated in the vicinity of the EDR near \(x \sim 850\) which subsequently remagnetize and mirror in the stronger \(B\) regions of the island to form the notable ring structure. The higher energy ring electrons (results not shown) have similar trajectories as the inner ring (blue) electrons: they mostly originate from the inflow on the \(+x\) side of the island, become energized in and around the electron diffusion region, and then mirror near the ring distribution locations while \(E \times B\) drifting in the \(-x\) direction.
To assess the stability of the ring distribution, we have performed analysis using an additional PIC simulation in which electrons were initialized to a ring distribution like Distribution 2 in Figure 2.3 with $\omega_{pe}/\Omega_{ce} = 2$ and with immobile ions similar to the simulation employed by Lee et al. [2009, 2011] (see Omura [2007] for more details). We find the ring distribution to be unstable to whistler wave generation. Within $16 \Omega_{ce}^{-1}$ the toroidal distribution diffuses into a shell, and by about $500 \Omega_{ce}^{-1}$ the distribution fully isotropizes, as shown in Figure 2.4. The ring distribution we observe from the reconnection simulation lasts for about $2000 \Omega_{ce}^{-1}$, implying that the reconnection process sustains the ring distribution.

Distribution 3, taken at $(x, z) = (804, 0)$ from inside the island, exhibits multiple components: one ring population centered at $v_{\perp} = 0.6$, and two field-aligned populations. The corresponding distribution function in $v_x-v_y$ space shows a gap separating the ring from the lower energy populations. This type of ring and core distribution first becomes discernible at $t\Omega_{ci} = 17$ in the region where $T_{e\perp}/T_{e\parallel} \approx 3$ (Figure 2.2a), then develops in every exhaust at around the time of the peak reconnection, and lasts for the rest of the run.

2.2.2 Inside the Secondary Island

Two examples of the structured distributions in the secondary island centered at $(x, z) \sim (863, 0)$ are shown in Figure 2.5. An overview of the temperature anisotropy $T_{e\perp}/T_{e\parallel}$ at the time of the distributions is shown in Figure 2.5a. The data are from $t\Omega_{ci} = 23$ when the island has grown to $d_i$-scales ($1d_i = 20d_e$), about $1 \Omega_{ci}^{-1}$ after the secondary island was born in the electron current layer. Like the primary island, this secondary island is ejected in the $-x$ direction and later merges with the open exhaust.

Distribution 1 in Figure 2.5 is from $(x, z) = (863, 12)$ and consists of three components:
Figure 2.4: The initial ring distribution shown in $v_{\parallel}-v_{\perp}$ space (upper left panel) quickly evolves into a shell (middle left panel at $16\Omega_{ce}^{-1}$, where $\Omega_{ce}$ is the electron cyclotron frequency), and then isotropizes (bottom left panel) due predominantly to whistler waves. The right three panels show $\omega-k$ spectrograms for components of the electric (top two panels) and magnetic (bottom panel) fields. The parallel-propagating whistler mode is the most unstable. The Langmuir-, R-, and L-mode waves are on the order of thermal noise. Black ‘V’-lines indicate the speed of light, while the horizontal dashed lines indicate the electron plasma frequency.
Figure 2.5: (a) The temperature anisotropy $T_{e\perp}/T_{e\parallel}$ at about $1 \Omega_{ci}^{-1}$ after the birth of the secondary island with black contours of the magnetic flux function. The two electron velocity distributions from the locations marked as 1 and 2 in (a) highlight the types of highly structured distributions that develop inside the secondary island exhaust at $t\Omega_{ci} = 23$.

A colder core population and two counter-streaming beam-like populations centered at $v_\parallel = \pm0.5$ and extended in the $v_\perp$-direction. Distribution 1 is characteristic of the spatial region: $[858, 870]$ in $x$ and $[8, 16]$ in $z$, toward the $\pm z$ edges of the secondary island. The structures in $v_x$-$v_y$ and $v_x$-$v_z$ (not shown) closely resemble the distribution in $v_\parallel$-$v_\perp$ space, since in this region the magnetic field is mainly along $x$. The two extended beams further elongate in gyrophase to form a shell deeper in the island, shown in Distribution 2. Close to the island core, electrons appear dominantly in the $v_\perp$ direction (data not shown) which supports the strong out-of-plane electron flow, characteristic of secondary island formation reported by Chen et al. [2012].

There is an important feature seen in the 3D velocity space representation provided in Movie 2.3 that is missed by the 2D projections of Distribution 1. The populations centered at $v_\parallel \sim \pm0.5$ in fact wrap around the colder core population like a hemispherical shell shifted...
toward the $-v_y$ side of the velocity space. That is, these populations are not gyro trope

like the shell in Figure 2.4: rather than exhibiting gyrotropy in the plane perpendicular to

$\mathbf{B}$ (which is $v_y$-$v_z$ since $\mathbf{B} \approx B\hat{x}$ at this location), they form crescent shapes in the $v_y$-$v_z$
slices. Figure 2.6 is a snapshot from the end of Movie 2.3 of the slice at $v_x = 0.1$ with a
green highlighted box in the $v_y$-$v_z$ plane that encloses most of the crescent piece comprised
of electrons mostly with $|v_y| > 0.5$. This understanding gives a kinetic perspective into the
characteristic bulk velocity $|U_{ey}|$ feature which secondary islands inherit as they form inside
the electron current layer.

Figure 2.6: Snapshot from the end of Movie 2.3 showing the crescent-shaped cross section
of the hemispherical shell populations hidden by the 2D projections. The green box encom-
passes the region with $v_y < -0.5$ in the $v_y$-$v_z$ slice at $v_x = 0.1$. 
2.2.3 Electron Anisotropy in Earth’s Magnetotail: Confirmation from Cluster

The structure of kinetic anisotropy in the open exhaust near the end of the simulation at $t\Omega_{ci} = 29$ is explored in Figure 2.7 by constructing an array of distributions from the magnetic field pileup region to compare with measurements from the Cluster mission. These anisotropic structures first appear in the simulation near the time of peak reconnection. Figure 2.7a shows the ion outflow, $U_{ix}$, with nine white boxes marking the locations far from the separatrix and deep within the exhaust near peak ion outflow where the distributions in Figure 4b are taken.

Similar to the distribution featured in Figure 2.1 from an earlier time, Distributions 1a, 1b, 2a, and 2b in Figure 2.7b consist of four populations: two counter-streaming beams parallel to $B$ with $v_\parallel \approx \pm 0.7$, one population exhibiting $T_{e\perp} > T_{e\parallel}$, and a colder core population. The counter-streaming beams and population with $T_{e\perp} > T_{e\parallel}$ develop after the peak reconnection rate and last until the end of the run. The population with $T_{e\perp} > T_{e\parallel}$ is gyrotropic and appears as a ring in $v_x$-$v_y$ space similar to the ring of Distribution 3 in Figure 2.3. For Distribution 1a, the ring population is centered at about $v_\perp = 0.8$ in $v_\parallel$-$v_\perp$ space (not shown). The $v_\perp$ population occurs within $15d_e$ of the $z = 0$ plane and is most pronounced along $z = 0$ from $x = 1300$ to 1320, close to the local maximum in $B_z$.

The field-aligned counter-streaming beams off the $z = 0$ plane give rise to anisotropies with $T_{e\perp}/T_{e\parallel} < 1$ (e.g. $T_{e\perp}/T_{e\parallel} = 0.78$ for Distribution 1a, and $T_{e\perp}/T_{e\parallel} = 0.22$ for Distribution 3c), consistent with the temperature anisotropy from the PIC and Geotail observation results reported by Hoshino et al. [2001a]. However, note that the temperature anisotropy
Figure 2.7: Simulation and observation examples of electron kinetic anisotropies in the deep exhaust. (a) Ion outflow velocity $U_{ix}$ in the simulation at $t\Omega = 29$ with nine white distribution bins taken in the vicinity of peak $U_{ix}$. (b) Electron distributions in $v_x$-$v_z$ space from the bins in the $U_{ix}$ plot, showing the anisotropic structures characteristic of the open exhaust in the magnetic field pileup region. The black line in each panel indicates the in-plane magnetic field direction. (c) The $v_\parallel$-$v_\perp$ representation of Distribution 3c for comparison with distributions observed by Cluster (e). The light blue bar in the plots of magnetic field and H$^+$ velocity (d) marks the time near the maximum ion outflow at which Cluster 4 captured distribution featured in (e) on 17 August 2001. Numbers inside the Cluster distribution indicate start end times of the data accumulation (about 1/8 s) in seconds since 16:34:29UT, and the dashed lines indicate where the cuts (f) are taken.
alone does not capture the counter-streaming and ring populations. The counter-streaming beams occur throughout the entire exhaust downstream of the electron outflow jet (except for \( x > 1350 \) in the region near the mid-plane beyond the \( \mathbf{B} \)-pileup region where distributions resemble Distribution 1c. Along \( z = 0 \), the beam velocity is largest just before the local maximum in \(|\mathbf{B}|\) in the \( \mathbf{B} \)-pileup region. The beam velocity decreases both further away from the \( z = 0 \) plane along field lines and in time, and thus does not appear to be a simple function of either the maximum out-of-plane electron speed in the electron current layer nor the maximum electron outflow jet speed.

We present an observation example for Distribution 3c from a magnetotail reconnection event observed by the Cluster spacecraft on 17 August 2001. To aid the comparison, distribution 3c is replotted in \( v_{\parallel}-v_{\perp} \) space in Figure 2.7c. The magnetic field and ion velocity measured by Cluster 4 are rotated to a suitable boundary-normal “LMN” frame found from minimum variance analysis on the magnetic field vector (MVAB) and shown in Figure 2.7d (where N gives the approximate current-sheet normal direction, L is directed along the outflow, and M completes an orthogonal coordinate system). The Cluster distribution in Figure 2.7e was detected near the maximum ion flow marked by a vertical blue bar in Figure 2.7d. The out-of-plane magnetic field \( B_M \) reverses sign at about the same time as the ion outflow \( V_L \) reversal, supporting the interpretation that the guide field is negligible. The Cluster distribution in Figure 2.7e exhibits two counter-streaming field-aligned beams and a population with \( T_{e\perp} > T_{e\parallel} \). The one-dimensional cuts of the distribution in Figure 2.7f illustrate quantitatively that the parallel and antiparallel beams have approximately the same speed at \( 4 \times 10^4 \) km/s, and that the phase space density along cuts perpendicular to the magnetic field is larger than that of parallel cuts for velocities higher than \( 6 \times 10^4 \) km/s, evidence for
the $T_{e\perp} > T_{e\parallel}$ population. The upstream electron beta ($\beta_e = 8\pi n T_e / B^2$) for this event is approximately 0.01. The agreement between the simulation distribution and the Cluster distribution implies that the exhaust region Cluster encountered was from reconnection which had developed beyond the peak rate of reconnection.

### 2.3 Summary and Discussion

Based on results from simulation #1 in Table 1.1 with zero guide field, we found that electron velocity distributions in magnetic reconnection exhausts become highly structured near the time when the reconnection rate achieves its maximum. The anisotropic structures include ring, arc, and counter-streaming populations in every exhaust. Once developed, most of the anisotropies are sustained through the course of reconnection. The discrete arc and ring structures are a result of the remagnetization (mainly by $B_z$) of the electrons accelerated in the electron diffusion region, and are related to the discrete striations in distributions at the X-line first reported by Ng et al. [2011] and studied in depth by Bessho et al. [2014] and Shuster et al. [2015a], as will be addressed in the next chapter. Arc and ring populations form near the end of the electron outflow jet in all exhausts, giving rise to large electron temperature anisotropies $T_{e\perp}/T_{e\parallel} > 1$ (in contrast with the inflow temperature anisotropy $T_{e\perp}/T_{e\parallel} < 1$). Test particle tracing suggests that the depletion of low energy electrons in the ring distributions next to the primary island results because of this region’s magnetic connectivity to a trapping region and its parallel potential well characteristic of the inflow [Egedal et al., 2010]. The counter-streaming beams along the magnetic field yield temperature anisotropies $T_{e\perp}/T_{e\parallel} < 1$ above and below the $z = 0$ plane deep in the exhaust;
the pronounced phase-space density peaks at the beam speeds distinguish these exhaust
distributions from inflow distributions which are also known to exhibit $T_\perp/T_\parallel < 1$.

The structured anisotropies presented in this paper are in contrast to the isotropic ex-
hauist electrons predicted in earlier PIC simulation studies [Chen et al., 2008a; Egedal et
al., 2010, 2012; Le et al., 2013]. The differences can likely be explained by considering the
time dependence and full velocity space structure of the distributions in addition to the
temperature anisotropy $T_{e\perp}/T_{e\parallel}$. Our results show counter-streaming beams parallel to the
magnetic field for exhausts off the $z = 0$ plane, giving a temperature anisotropy $T_{e\perp} < T_{e\parallel}$
consistent with the findings of Hoshino et al. [2001a]. The temperature anisotropy $T_{e\perp} > T_{e\parallel}$
in the vicinity of the magnetic field pileup region reported in this paper is consistent with
the results of Fujimoto and Sydora [2008]. However, we found counter-streaming electron
beams co-existing with the $T_{e\perp} > T_{e\parallel}$ population. Furthermore, our results indicate that the
$T_{e\perp} > T_{e\parallel}$ anisotropy is largely due to ring populations which are unstable to whistler wave
generation, whereas Fujimoto and Sydora [2008] attributed the whistler-driving anisotropy
to adiabatic heating from enhanced $|B|$ and the escaping of electrons with high parallel
velocities.

By considering the time evolution of the distribution functions resulting from recon-
nection, the discrepancy between reports of isotropic electron distributions observed in the
exhaust by Cluster [Asano et al., 2008; Chen et al., 2008a,b, 2009; Egedal et al., 2010, 2012]
and the anisotropic electron distributions measured by Geotail [Hoshino et al., 2001a] can
be resolved. We suggest that for reconnection with a negligible guide field, the observations
of anisotropic electron distributions were from reconnection occurring at a time near or after
the peak reconnection rate.
Understanding why exhaust electron anisotropy develops at around the time of peak reconnection requires further investigation. We have performed analyses of additional PIC runs with different parameters (including: $m_i/m_e = 100, 1836; T_i/T_e = 1, 2; n_b/n_0 = 0.0375, 0.2; \text{upstream } \beta_e = 0.01875, 0.06$), and the results confirmed that exhaust electrons develop structured anisotropies at around the time of maximum reconnection rate.

In conclusion, our results indicate that for reconnection with negligible guide fields, electrons in the exhausts exhibit highly structured anisotropies. The representative anisotropies reported here are most discernible after the explosive growth phase of reconnection. These results are immediately relevant for interpreting data from NASA’s successfully launched Magnetospheric Multiscale mission, which offers the unprecedented time resolution needed to resolve the types of highly structured electron distributions reported in this work.

Movies

**Movie 2.1:** This animation presents Distribution 1 of Figure 2.3 in 3D velocity space. The “wing” populations are the spiraling branches extending outward from the core toward larger $v_z \sim v_\parallel$. The outermost arc is the most energetic population and is contained near $v_z = 0$.

Direct link: https://drive.google.com/open?id=0BwUFZvYO52UeijRIdU9HRndldkE

**Movie 2.2:** Test-particle tracing of the inner-ring electrons’ trajectories, showing connectivity to the X-line on the $-x$ side of the primary island via open field lines.

Direct link:
Movie 2.3: Dissection of Distribution 1 of Figure 2.5 from inside the secondary island, in the same style as Movie 2.1. The distribution is notably heated, filling most of the available velocity space volume, and appears roughly isotropic from some viewpoints; however, the slices toward the end of the movie reveal the complicated internal structure.

Direct link:

https://drive.google.com/open?id=0BwUFZvYO52UURzUzWGFpV3NmTmM
Chapter 3

Spatiotemporal Evolution of Electron Characteristics in the Electron Diffusion Region

3.1 Introduction

How are electrons accelerated and heated in the electron diffusion region (EDR)? In the last chapter, we glimpsed the kinetic effects of EDR electron energization mechanisms, which produced discrete ring, arc, and other nongyrotropic structures near the end of the electron outflow jet, though we were focused mainly on properties of the exhaust regions. In this chapter, we now direct our attention to the heart of the reconnection puzzle: the EDR, where we investigate the spatiotemporal evolution of electron distributions to elucidate EDR energization processes as reported by Shuster et al. [2015a]. Most of the analyses pertain to results from simulation #1, and runs #2 and #3 listed in Table 1.1 are also considered to address the dependence of these results on the guide field strength $B_g$ and the ion to electron mass ratio $m_i/m_e$. 

46
3.2 Previous Knowledge

Previously, the reconnection electric field has been assumed to heat inflowing plasma and drive intense currents in the EDR [Hesse et al., 2009]. Several studies have shown that for antiparallel reconnection, transport of the out-of-plane ($y$) electron momentum to the outflow ($x$) direction is the main mechanism that limits the current density and provides the resistivity for reconnection to occur [Lyons and Pridmore-Brown, 1990; Cai et al., 1994; Fujimoto et al., 2009; Hesse et al., 2009; Chen et al., 2011], supporting the concept of inertia resistivity [Speiser, 1970; Hesse et al., 2011]. Anomalous resistivity due to scattering of electrons by wave fluctuations has also been proposed for limiting the out-of-plane current density as well as heating electrons (see the review paper by Hesse et al. [2011] and references therein). A recent mechanism that has received much attention in simulation and space observation communities involves a parallel electric potential that accelerates electrons and controls the ‘shoulder’ energy of flat-top electron distributions, and hence the amount of electron heating in the exhaust [Egedal et al., 2010, 2012]. Through examination of the spatial and temporal evolution of electron distributions, we show in this chapter that heating in the EDR is mainly accomplished by the combination of electric field acceleration and cyclotron turning.

The electron velocity distributions within a few electron skin depths of the X-line have been shown to exhibit a bifurcated, triangular structure with discrete striations by using test particle tracing and Liouville mapping given a prescribed inflow distribution [Ng et al., 2011, 2012], and by employing PIC simulations [Ng et al., 2012; Bessho et al., 2014]. The EDR and the exhaust are thought to be nearly isotropic for weak guide fields ($B_g < 0.1B_0$) based on the bulk quantity $P_e///P_e\perp$, the ratio of the pressure components parallel and perpendicular
to the magnetic field [Le et al., 2013]. As discussed in the previous chapter, Shuster et al. [2014] show highly structured, anisotropic electron distributions in the exhaust, including arc and ring populations, which develop at around the time when the peak reconnection rate occurs. The discrete arc and ring structures are formed by electrons accelerated in the EDR and turned by the normal component of the magnetic field, $B_z$ [Bessho et al., 2014], which increases downstream of the X-line. In the reconnection layer, the increasing $|B_z|$ turns the $y$-momenta of accelerated electrons to the $x$ (outflow) direction to form jets and limit the out-of-plane current [Hesse et al., 2009; Chen et al., 2011], transfers part of the incoming parallel energy to perpendicular energy [Ng et al., 2011], and converts bulk kinetic energy into thermal energy by gyrotropizing distributions, as will be discussed here.

### 3.3 Evolution of Electron Velocity Distributions: Implications for Acceleration and Heating

Here we discuss the spatiotemporal evolution of electron velocity distributions, and what we can learn about electron acceleration and heating in the EDR. Throughout this chapter, “heating” refers to increases of the electron temperature $T_e$ defined as $Tr(P_e)/3n_e$, where $Tr(P_e)$ is the trace of the electron pressure tensor and $n_e$ the electron density. Movie 3.1 illustrates where and when electron bulk heating occurs during this run: as reconnection proceeds, the evolution of $T_e$ is shown in color along with 1D cuts along $z = 0$ (left of color panel) and $x = 850$ (below color panel). Additionally, the vertical blue bar in the reconnection rate plot (bottom left) is updated during the movie to indicate the temporal stage of the reconnection process. Figure 3.1 shows three frames of Movie 3.1.
Figure 3.1: Three frames of Movie 3.1 from before (top), at (middle), and after (bottom) peak reconnection, showing both the spatial and temporal evolution of the electron temperature $T_e$ during Run #1.
3.3.1 Gyrotropization of the X-line Distribution

The spatial evolution of distributions upstream from, within, and downstream of the EDR at $t \Omega_{ci} = 19$ is presented in Figure 3.2, with velocities in units of $v_A$, the initial ion Alfvén speed in the current sheet defined by $B_0$ and $n_0$. An overview of the outflow jet $U_{ex}$ and its 1D cut along $z = 0$ are displayed in Figure 3.2a. Five electron distributions are taken from the EDR locations marked with white boxes in the $U_{ex}$ panel, and displayed in Figure 3.2b in $v_x-v_y$ (Distributions 1A-5A), $v_x-v_z$ (1B-5B), and $v_z-v_y$ (1C-5C) space, respectively. The electron distribution at the X-line exhibits a distinct triangular shape with discrete striations shifted in $-v_y$ (Distribution 1A). This feature is consistent with previous reports [Ng et al., 2011, 2012; Bessho et al., 2014], indicating the robustness of the triangular shape. However, the number of discernible striations differ, reflecting the structure’s sensitivity to initial plasma parameters, such as those of the Harris sheet. The discrete striations in Distribution 1(A-C) result from the oscillatory out-of-plane force $F_y = -eE_y - ev_zB_x$ on meandering electrons with oscillatory $v_z$ [Bessho et al., 2014]. The oscillations in $F_y$ lead to steps in $v_y$ and hence discrete regions of enhanced phase-space densities in the $v_y$ dimension of the distributions, such as Distributions 1A and 1C.

The formation of the triangular distribution highlights the cause of EDR heating from upstream to the X-line. The first striation in Distribution 1A is formed by electrons entering the EDR and arriving at the X-line without any bounces in $z$ [Ng et al., 2011; Bessho et al., 2014]. Comparing this first striation to the inflow distribution shown in grey in the same panel (taken from the grey box $6d_e$ above the X-line), we observe a shift in $-v_y$ and a slight decrease in the $v_x$ width. Electrons in the higher order striations spend more time meandering in the EDR, allowing them to experience more $E_y$ acceleration and turning by
Figure 3.2: Electron velocity distributions in and near the EDR revealing how electrons are heated downstream as the discrete, striated populations at the X-line rotate and spread out in gyrophase. (a) Electron outflow jet $U_{ex}$ and its 1D cut along $z = 0$ providing the spatial context for the distributions. (b) Projections in $v_x$-$v_y$ (1-5)A, $v_x$-$v_z$ (1-5)B, and $v_z$-$v_y$ (1-5)C space of the EDR distributions taken from the five white bins in (a). The upstream inflow distribution taken from they grey bin at $z = 6$ in (a) is overlaid in grey on the X-line distribution 1(A-C) to visually illustrate the amount of acceleration and heating achieved in the EDR. Distribution 1(A-C) at the X-line consists of two triangular lobes, each with discrete striations. (c) The $v_x$-$v_y$ distribution taken from the red bin at $x = 883$ ($7d_e$ downstream from distribution 5) is almost fully gyrotropized and has a $T_e$ larger than distribution 5. All distributions are made using a spatial bin size of $2d_e \times 1d_e$. 
As a result, the incoming $v_x$ of these electrons is converted almost completely to $v_y$, whereas electrons in lower-order striations retain more of their spread in $v_x$. A triangular distribution is thus formed in the vicinity of the X-line, since electrons in higher order striations have larger $|v_y|$ and less spread in $v_x$. The mixing of EDR electrons that are accelerated preferentially in this way leads to an abrupt increase in $T_e$, which will be further discussed and shown in Figures 3.3 and 3.6.

The two main populations in Distributions 1B-3B and 1C-3C reside at $v_z > 0$ and $v_z < 0$, consistent with the electron $z$-$v_z$ phase-space hole that forms due to the meandering oscillations of electrons under finite reconnection inflow reported by Chen et al. [2011]. The average $|v_z|$ of both populations is larger than the maximum electron inflow speed (roughly the center $v_z$ of the grey inflow distribution shown in panels 1B and 1C) because the electrons have been accelerated by the inversion electric field $E_z$ that self-consistently supports the electron phase-space hole structure. When the inversion $E_z$ is insignificant (such as at the edge of the EDR or at later phases of reconnection), each of the two $v_z$ populations is roughly centered at the respective maximum inflow electron fluid velocity (see distribution J in Figure 3.6b for example), resulting in an increase of $T_{ezz} = P_{ezz}/n_e$.

Heating in the EDR downstream of the X-line can be seen by comparing Distributions 1A-5A and the distribution in Figure 3.2c (taken from the red bin shown in Figure 3.2a). The populations accelerated in $-y$ acquire $+v_x$ (2A-4A), and then further rotate toward $+v_y$ and $-v_x$ (4A-5A) as electrons perform partial cyclotron orbits due to $B_z$. Arc and spiral structures are formed (best seen in $v_x$-$v_y$, distributions 3A-5A) before the distribution is subsequently gyrotropized (Figure 3.2c). The $v_x$-$v_y$ spread (which is approximately $T_{e\perp} = (P_{exx} + P_{eyy})/2n_e$ since the magnetic field in the $z = 0$ plane is mainly along $z$) increases from
the X-line (Distribution 1A) to just beyond the end of the jet (Figure 1c). This increase of \( T_{e\perp} \) will be shown quantitatively in Figure 3.6a.

### 3.3.2 Visualizing Collisionless Dissipation

Cyclotron turning by \( B_z \) converts the electron’s bulk velocity (directed mainly in the \(-y\) direction at the X-line) into thermal energy by spreading out the electrons’ gyrophases, as shown in the \( v_x-v_y \) distributions of Figure 3.2. To further illustrate this process, we present results from particle tracing using the \( E \) and \( B \) fields from \( t\Omega_{ci} = 19 \) in Figure 3.3. Representative electrons from each striation are traced forward in time starting from the initial velocities marked with small black diamonds shown in Figure 3.3a and 3.3b. The trajectories of the traced electrons in \( v_x-v_y \) space are displayed and color-coded to distinguish their originating striations. Figure 3.3a shows the tracing results up to \( \bar{t}\Omega_{ci} = 0.09 \) (\( \bar{t}\omega_{pe} = 76.16 \)), and Figure 3.3b up to \( \bar{t}\Omega_{ci} = 0.14 \) (\( \bar{t}\omega_{pe} = 112.83 \)), where \( \bar{t} \) is time in the particle tracing simulation. The large colored diamonds in Figure 3.3a (3.3b) indicate the electron velocities at \( \bar{t}\Omega_{ci} = 0.09 \) (\( \bar{t}\Omega_{ci} = 0.14 \)).

The selected electrons spread out further in gyrophase as \( \bar{t} \) advances. Electrons starting with larger \( v_x \) (such as the electron marked with a blue diamond from the first striation that has already completed \( \sim 3/4 \) of a gyration in Figure 3.3a) travel faster downstream and encounter the stronger \( B_z \) before electrons that started with smaller \( v_x \). Electrons starting with \( v_x = 0 \) (leftmost of the initial black diamonds) remain in the vicinity of the X-line the longest, and thus have been accelerated by \( E_y \) the most, which can be seen visually in Figure 3.3a: the \( v_x = 0 \) electrons in each striation have gained the most \( |v_y| \) compared to the other electrons at this time. Both the blue and green electrons have acquired more
Figure 3.3: Electron trajectories illustrating how electrons are heated downstream from the X-line, and the electron temperature profile showing quantitatively the amount of heating. (a,b) $v_x-v_y$ trajectories of electrons from the first, second, third, and fourth striations in distribution 1A (Figure 3.2b) are colored blue, green, orange, and red, respectively. Small black diamonds mark the initial velocities of the traced particles, while the large colored diamonds mark the velocities of electrons at the elapsed particle tracing time, $\bar{t}$, where $\bar{t}\Omega_{ci} = 0.09$ in (a), and $\bar{t}\Omega_{ci} = 0.14$ in (b). The trajectories show how the electrons spread out in gyrophase as they flow downstream and perform partial cyclotron orbits in the increasing $B_z$. Thus, the initial bulk velocity in $-y$ is converted to the thermal energy, leading to an increase of $T_{e\perp}$. (c) The electron temperature, $T_e$, increases by approximately $4T_{e0}$ going from just upstream of the EDR ($z \sim 6$) to the X-line ($z \sim 0$), and increases again by almost $12T_{e0}$ going from the X-line ($x \sim 848$) to the end of the outflow jet ($x \sim 880$), where $T_{e0}$ is the electron temperature far upstream at $z = 40$. Movie 3.2 provides an animation of panels (a) and (b) with an evolving distribution in the background of velocity space. Movie 3.3 shows both backward and forward tracing results for 2,256 electrons, convincingly illustrating the heating that takes place throughout the EDR.
|v_y| after completing their first gyration (Figure 3.3b), as $E_y$ acceleration continues to occur during the cyclotron turning until electrons are remagnetized. The large bulk velocity $-v_y$ near the X-line is converted into thermal velocity perpendicular to $B_z$ as electrons travel downstream, resulting in an increased $T_{e\perp}$. Consequently, $T_e$ is increased, since $T_{e\parallel} \sim T_{ezz}$ does not change appreciably from the X-line to the end of the electron jet, which can be seen in the $v_z$ spreads of Distributions (1-5)B and (1-5)C.

Movie 3.2 is an animation of these test electron trajectories in the $x$-$z$, $v_x$-$v_y$, $v_z$-$v_y$, and $v_x$-$v_z$ spaces. The same $U_{ex}$ panel from Figure 3.2a is shown in the background of the top panel to orient the particle positions in $x$-$z$. Additionally, the PIC electron distribution is updated behind the particles in the three velocity space panels following the path of the electron which starts in the first striation with the smallest initial $v_{x0}$ and $v_{y0}$ (i.e. the leftmost blue electron). The evolving distribution illustrates how the bulk velocity $|v_y|$ acquired by the accelerated, unmagnetized electrons near the X-line is converted to thermal energy as the distribution gyrotropizes along electron trajectories through the outflow jet. “Wing” structures similar to those of Distribution 1 in Figure 2.3 from the previous chapter are seen to form as electrons are diverted either to $\pm z$ depending on the sign of their $v_z$-bounce motion as they exit the outflow jet.

Movie 3.3 shows both backward and forward time tracing of 2,256 electrons chosen from the two $\pm v_z$-edges of the phase-space-hole structure shown in the upper right panel. (Note that $z$-$v_z$ space for this simulation corresponds to $x$-$v_x$ in Chen et al. [2011].) These electrons represent a sample of Distribution 1A in Figure 3.2, taken from a $2d_e \times 2d_e$ spatial bin centered at $(x, z) = (848, 0)$. The subsequent evolution in configuration, velocity, and phase space shown in Movie 3.3 is the simultaneous visualization of an ensemble of Speiser-type orbits.
[Speiser, 1965] that pass close to the X-line. Comparing the beginning of Movie 3.3 (Frame -100) to the end (Frame +100), the increase in temperature (i.e. spread in velocity space) of this ensemble of particles is visually obvious. Figure 3.4 shows these frames from Movie 3.3.

These animations are useful for understanding the time histories and futures of the particles comprising the X-line distribution at Frame 0, however the other frames do not necessarily represent the structures that would be observed by a single spacecraft at one time since the particles originate from and move to different spatial locations. Negative times (in $\omega_{pe}^{-1}$ and $\Omega_{ci}^{-1}$, the electron plasma frequency and ion gyrofrequency, respectively) and frame numbers are for tracing steps that were performed backward in time. For this run, $\omega_{pe}/\Omega_{ci} = (\omega_{pe}/\Omega_{ce}) \cdot (m_i/m_e) = 800$.

The amount of temperature increase from the inflow to the X-line and end of the electron jet is shown quantitatively in Figure 3.3c where $T_e$ in units of $m_e v_A^2$ is displayed. Taking $T_{e0} = T_e(z = 40) = 10.3$ upstream of the ion diffusion region, $T_e$ rises to $3.3T_{e0}$ upstream and just outside the EDR over a few tens of $d_e$. Then, $T_e$ abruptly increases to $7.4T_{e0}$ at $(x, z) = (848, 0)$ at the X-line, and further grows to $19.3T_{e0}$ near $(x, z) = (880, 0)$ just beyond the end of the electron jet. The $T_e$ increase of $16T_{e0}$ from just upstream to downstream of the EDR is a combined effect of $E_y$ acceleration and cyclotron turning by $B_z$, while the $T_e$ increase of $2.3T_{e0}$ upstream from the EDR is mainly due to acceleration by the parallel electric field [Chen et al., 2009; Egedal et al., 2010]. Further downstream, $T_e$ does not exceed $T_e$ near the end of the electron outflow jet ($x \sim 880$). Note that the sharp rise in $T_e$ on the $-x$ side of the X-line ($x < 820$) is associated with highly anisotropic ($T_{e\perp} > T_{e||}$) ring distributions discussed in the previous chapter.
Figure 3.4: First (top) and last (bottom) frames of Movie 3.3, showing the increased temperature (spread in velocity space) of this ensemble of test-electrons after they pass through the EDR.
3.3.3 Time Dependence

The time evolution of the distributions in the \( v_x - v_y \) space from the X-line to the end of the electron outflow jet is shown in Figure 3.5. The time range is \( t\Omega_{ci} = 17 \) (one \( \Omega_{ci}^{-1} \) before the time of peak reconnection) to \( t\Omega_{ci} = 29 \). During this time, the distance from the X-line to the end of the electron jet increases from 1 to 8 \( d_i \) (see Movie 3.1) and is denoted by the number \( d \) (in \( d_i \)) given in Figure 3.5 for each time. The first 1(D-G) and last 5(D-G) distributions are taken from the X-line (at \( x/d_e = x_0 \)) and the end of the jet (\( x/d_e = x_0 + \sqrt{m_i/m_e}d \)), respectively. Distributions 2-4 are evenly spaced between the locations of Distributions 1 and 5.

Distributions 1(D-G) from the X-line in Figure 3.5 all exhibit the characteristic triangular structure. Distributions 1(D-F) show discrete striations in the triangle. After \( t\Omega_{ci} = 21 \), discrete striations are less visible. The striations of distribution 1F (especially on the \( -v_x \) side) are more fragmented, comprised of multiple populations within each striation, while Distribution 1G is much less coherent than the X-line distribution from earlier times, no longer exhibiting distinct striations.

Common to all reconnection phases shown in Figure 3.5, the distributions downstream from the X-line within the electron jet show swirling structures, signatures of EDR accelerated electrons executing partial cyclotron orbits around \( B_z \). The asymmetry in both \( v_x \) and \( v_y \) of Distributions 2 through 4 gives rise to a finite \( P_{exy} \) whose gradient \( \partial P_{exy}/\partial x \) contributes to balancing \( E_y \) in the EDR. As the EDR lengthens with time, \( |\partial P_{exy}/\partial x| \) decreases. The magnitude of \( B_z \) increases gradually with the distance from the X-line. At the end of the electron jet, the majority of electrons are magnetized by \( B_z \), as indicated by the mostly gyrotropic distributions in panels 5(D-G).
Figure 3.5: Spatial and temporal evolution of electron distributions in $v_x$-$v_y$ space from the X-line to the end of the electron outflow jet (horizontal) for $t\Omega_{ci} = 17, 18, 20, \text{and } 29$ (vertical). All distributions are made using a spatial bin size of $2d_e \times 2d_e$. As the electron current layer lengthens, the end of the electron outflow jet is stretched farther away from the X-line, and is given by the number $d$ (in $d_e$), which denotes the distance from the X-line to the location where Distribution 5 is taken. For $t\Omega_{ci} = 17$ and 18, the spatial evolution of the distributions is similar to those shown in Figure 3.2b. Later in time, the triangular shape of the X-line distributions remains, though the striations become fragmented. Arcs are found near the end of the electron jet for $t\Omega_{ci} \geq 17$. 

59
The nongyrotropic components of Distributions 5(D-G) in Figure 3.5 at the end of the electron jet manifest themselves as arc populations. The arc electrons are the most energetic electrons that have been accelerated in the EDR. The maximum speed of the arc electrons increases with time (compare distribution 5D to 5G). Multiple thin and discrete arcs are observed in distribution 5G, likely because the accelerated populations of Distribution 1G are fragmented.

3.3.4 Development of Nongyrotropy

The degree of electron nongyrotropy \( (D_{ng}) \), vertical cuts (taken at \( x = 848 \)) of \( D_{ng} \) (black), the electron density, \( n_e \) (red), and \( T_{e\perp} \) (blue) on the left, and a horizontal cut of \( T_{e\perp} \) (black) are shown in Figure 3.6a to demonstrate the correlations between these quantities relevant to the EDR heating mechanisms discussed thus far. Because \( \mathbf{B} \sim B_x \hat{x} \) along the vertical cut through the X-line, \( T_{e\perp} \sim (P_{eyy} + P_{ezz})/2n_e \). \( T_{e\perp} \) increases by 20 times within 2 \( d_e \) (blue dashed cut), reaching its maximum at the same location as the \( n_e \) and \( D_{ng} \) peaks (see vertical cuts in the left panel of Figure 3.6a), while \( T_{e\parallel} \) (\( \sim T_{exx} \)) decreases by about a factor of three (data not shown). From the X-line to the end of the electron jet, the horizontal cut shows that \( T_{e\perp} \sim (P_{exx} + P_{eyy})/2n_e \) (since for \( z = 0 \), \( \mathbf{B} \sim B_z \hat{z} \)) further increases by a factor of about 2.4 (from \( \sim 100 \) to \( \sim 240 \) \( m_e V_A^2 \)).

The most probable turning points of the meandering electrons in the EDR give rise to the two \( n_e \) maxima that are co-located with the extrema of \( D_{ng} \), which measures the departure of a distribution (in the bulk velocity frame) from rotational symmetry around the local magnetic field, and is defined as the Frobenius norm of the nongyrotropic part of the pressure tensor normalized by the local thermal energy density [Aunai et al., 2013].
Figure 3.6: (a) 2D plot of the degree of electron nongyrotropy, $D_{ng}$, with vertical cuts (at $x = 848$) of the perpendicular temperature, $T_{\perp}$ (blue dashes), electron density, $n_e$ (red), and $D_{ng}$ (black), and a horizontal cut (at $z = 0$) of $T_{\perp}$. Entering the EDR from upstream, $T_{\perp}$, $n_e$, and $D_{ng}$ rise sharply in unison. $T_{\perp}$ increases by a factor of 20 within $2d_e$ from $|z| = 3$ to $|z| = 1$, and grows by another factor of three from the X-line ($x = 848$) to the outflow jet’s end ($x \sim 880$). (b) Electron velocity distribution from $z = 1$ at the location of peak $T_{\perp}$, $n_e$, and $D_{ng}$ just above the X-line. The distribution in $v_z-v_y$ (H) reveals that the sharp rise in $T_{\perp}$ and $D_{ng}$ is due to the slanted, gyrophase-bunched populations of accelerated meandering electrons in addition to the zero-bounce population. The distribution in $v_x-v_y$ (I) exhibits the triangular structure characteristic of the X-line distribution, though without striations because the slanted populations overlap when projected into the $v_x-v_y$ plane. The two $v_z$ populations in $v_x-v_z$ (J) are roughly centered at the maximum inflow velocities. (c) Time evolution of the gyrobunched populations from $t\Omega_{ci} = 17$, 18, and 29, showing that consistent acceleration and heating processes persist as time advances. All distributions are made using a spatial bin size of $3d_e \times 1d_e$. 

61
Increased $D_{ng}$ not only occurs in the EDR, but also extends along the exhaust side of the separatrices as shown in the 2D $D_{ng}$ plot; this extension of enhanced $D_{ng}$ continues for tens of $d_i$ away from the X-line (data not shown). For the reconnection configurations considered in this paper, enhanced $D_{ng}$ identifies approximately the same spatial regions as the agyrotropy parameter defined previously by Scudder et al. [2008]. Nongyrotropic electrons are important observables of diffusion region processes, and have been reported in several spacecraft observation studies [Chen et al., 2008a; Scudder et al., 2012].

The electron distribution from the peak $D_{ng}$ and $n_e$ region right above the X-line (marked by the white bin in the $D_{ng}$ plot) is projected onto the $v_z$-$v_y$ (H), $v_x$-$v_y$ (I), and $v_x$-$v_z$ (J) planes in Figure 3.6b. The $v_z$-$v_y$ plane is approximately perpendicular to the magnetic field at this location. All of the discrete populations in distribution H are accelerated in the $-y$ direction compared to the greyscale inflow distribution shown in Figure 3.2b, and they are bunched in gyrophase. The asymmetry of the distribution in $v_y$ and $v_z$ at the peak $D_{ng}$ location gives rise to a nonzero $P_{eyz}$ whose gradient along $z$ is important for balancing the reconnection electric field in the EDR [Hesse et al., 2011; Chen et al., 2011].

The detailed structure of the distribution can be readily understood by considering the physics behind the discrete striations at the X-line. The population with the smallest $|v_y|$ in distribution H (Figure 3.6b) consists of electrons that have not undergone any bounces in $z$; since these electrons are inflowing electrons, the population is centered at a negative $v_z$. The higher order striations are approximately centered at $v_z = 0$, formed by electrons with more than one $z$-bounce in the vicinity of their turning points. The slant structure of the populations centered at $v_z = 0$ (electrons with positive $v_z$ have lower $|v_y|$ than those with negative $v_z$) develops because electrons with negative $v_z$ are reflected and spent more...
time in the EDR, thus gaining more $|v_y|$ than the electrons with positive $v_z$ that have not yet been reflected. Note that since the location of the distribution is above the X-line, the electrons with $v_z > 0$ are approaching their turning points, while those with $v_z < 0$ are leaving. For the location of peak $D_{ng}$ below the X-line, the slanting slope reverses sign, and the population with the smallest $v_y$ is centered at $v_z > 0$ (not shown).

A characteristic triangular shape is observed in distribution I. The lack of discrete striations is due to the overlapping in $v_y$ of the slanted populations in distribution H. Compared to the grey inflow distribution of Figure 3.2, distribution J shows the increase of $v_z$ spread without acceleration by the inversion electric field $E_z$ (since $E_z$ is insignificant at the edge of the EDR).

The time evolution of the gyrophase-bunched $v_z$-$v_y$ distribution is shown in Figure 3.6c. All distributions (K-M) exhibit similar structures as distribution H with higher order bounce populations at larger $|v_y|$ than the zero-bounce population, indicating consistent acceleration and heating processes operating even for later phases of reconnection such as at $t \Omega_{ci} = 29$. All populations acquire larger $v_y$ at later times. The discrete populations of distribution M, like distribution 1G (Figure 3.5), are more fragmented.

3.3.5 Fragmentation of Inflow Distributions: Electron Phase Space Holes Along the Separatrix

The fragmentation of the distributions starting after $t \Omega_{ci} = 18$ in Figures 3.5 and 3.6 is a feature associated with likewise fragmented inflow distributions upstream of the EDR, as described in Figure 3.7. These multi-component, disjointed inflow distributions are correlated with nonlinear wave signatures in $E_\parallel$ and the presence of electron phase space holes along
Figure 3.7: Development of fragmented, caterpillar-resembling inflow distributions in Run #1 and Run #2, related to nonlinear wave signatures in the parallel electric field, $E_\parallel$, and electron $x$-$v_x$ phase space holes along the separatrices (see Movies 3.4 and 3.5). The left three and middle three panels are from Run #1, showing when the $v_\parallel$-$v_\perp$ distributions become disjointed. The 2D panels above the distributions show the out-of-plane bulk velocity $U_{ey}$ and white bins indicating where the distributions were taken. At $t\Omega_{ci} = 18$ just before peak reconnection (left panels), distributions are simpler and exhibit a temperature anisotropy $T_e\parallel > T_e\perp$ characteristic of the inflow region. At $t\Omega_{ci} = 29$, the distributions have split into multiple populations bunched along $v_\parallel$, and the structure of $U_{ey}$ has become more extended in $x$ and broader in $z$. The rightmost two panels are from Run #2 with a small guide field $B_g = 0.03B_0$, demonstrating that the fragmentation is not unique to the antiparallel configuration. The multi-component distribution (bottom right) is taken from $(x, z) = (787, 7)$, marked by the small white bin in the upper right panel of $E_\parallel$ (in natural PIC units of $\omega_{pe}m_e/c/e$). The 1D cut of $E_\parallel$ at $z = 7$ shows strong amplitude signatures of electric field solitary waves (electron holes) in the region of fragmented inflow distributions. (Figure adapted from Shuster et al. [2015b].)
the separatrices that develop around the time of peak reconnection.

Movie 3.4 shows particle tracing results for an example inflow electron and the evolving distribution following the particle’s trajectory through some of these electron holes in Run #1. The intricate, caterpillar-like transformations that the distributions undergo along the electron’s path are indicative of wave-particle interactions, and likely temporal effects not captured by this test-particle orbit which was computed using $E$ and $B$ fields from one point in time at $t\Omega_{ci} = 22$. Movie 3.5 shows some of these $x$-$v_x$ phase space holes that develop along the separatrices at $t\Omega_{ci} = 22$. The animation shows a portion of the phase space centered at $x = 975$ (labeled “x = 0” in the animation) and spanning $100d_e$ in $x$ for consecutive $z$-slices going from $z = 30$ to $z = 50$. As the $z$-slice increases, the phase space animation reveals hole structures adorning the separatrix which intersects with the $z$-slice at increasing values of $x$. Figure 3.8 features a frame from Movie 3.4 and Movie 3.5.

Cluster spacecraft measurements of electron holes (electron solitary waves) have been reported in several studies [Cattell et al., 2005; Khotyaintsev et al., 2010], and have been used to identify separatrix and diffusion region structures when found in association with counter-streaming electron distributions and other high frequency waves [Viberg et al., 2013]. The complicated structure of inflow distributions after peak reconnection (Figure 3.7) implies that the assumed analytical form of the inflow distribution function employed by Egedal et al. [2010] and Ng et al. [2011] may only be valid for early stages of reconnection.
Figure 3.8: Frames from Movies 3.4 and 3.5, showing a portion of an inflow electron’s trajectory in position and velocity space (top panels), and a few of the electron hole structures in $x-v_x$ phase space that form along the separatrix at this time (bottom panel).


3.4 Electron Encyclopedia: Establishing Maps of PIC Distributions for Orienting Spacecraft Measurements

This section is motivated by the “picture-puzzle” approach established by Chen et al. [2008a, 2009] for organizing Cluster multi-spacecraft measurements of electron characteristics during reconnection. Figure 3.9 shows some of the same Cluster distributions shown in Figures 1 and 3 of Chen et al. [2009], only in $v_{\|}-v_{\perp}$ velocity space rather than energy space for comparison to the PIC simulations presented in this dissertation. Each full distribution panel represents about four seconds of data.

On 1 October 2001 between 09:48:24.296UT and 09:48:26.414UT (7th of the 10 distribution panels shown), Cluster 2 encountered an electron current layer. Cluster 1 was above while Cluster 3 and 4 were below the current sheet at this time, enabling the reconstruction shown in Figure 3.9. Such an assembling of the four spacecraft measurements permits efficient determination of the colder, inflow distributions exhibiting a temperature anisotropy $T_{e\|} > T_{e\perp}$, and the exhaust region which is comprised of hotter, more isotropic electrons. The layout even suggests the separatrix boundary between the inflow and exhaust (sketched in Figure 1 of Chen et al. [2009]).

In the spirit of this approach employed by Chen et al. [2009], Figure 3.10 shows PIC distributions from Run #1 at $t\Omega_{ci} = 19$ assembled analogously to the Cluster distribution array. All the distributions shown are in $v_x-v_y$ with axes labels and color bars suppressed to allow the distributions to display larger. The top panel is $U_{ex}$ with an array of to-scale white bins corresponding to the locations used for the distribution map below. This analogous “multi-spacecraft” kinetic electron reconstruction of the simulated reconnection
Figure 3.9: Cluster measurements of electron velocity distributions observed on 1 October 2001, illustrating the picture-puzzle approach adopted by Chen et al. [2008a, 2009] for organizing multi-spacecraft data in order to reconstruct the topology of the reconnection site.

region catalogs most of the structures reported in this dissertation so far, including: (1) triangular X-line distributions that gyrotropize toward the end of the electron jet, (2) rings with $T_{e\perp} > T_{e\parallel}$ and nongyrotropic arcs that develop beyond the jet in open and island exhausts, (3) counter-streaming beams in exhaust distributions with $v_\perp$ rings close to the mid-plane, (4) multi-component inflow and separatrix distributions associated with electron holes, (5) overall colder inflow distributions exhibiting $T_{e\parallel} > T_{e\perp}$, and (6) generally hotter, more isotropic distributions deeper into the exhaust.

Movie 3.6 shows several PIC distribution arrays in the style of Figure 3.10 to capture the temporal dependence of these distribution structures as reconnection proceeds. The first frame is from $t\Omega_{ci} = 17$ before peak reconnection. At this time, the EDR, inflow, and exhaust distributions can be delineated, though most of the structures are relatively isotropic compared to later times. By $t\Omega_{ci} = 19$, nearly all the distributions (except for those far
upstream and downstream from the X-line) exhibit distinguishing, non-Maxwellian features. At \( t\Omega_{ci} = 21 \), the reconnection structure has grown relative to the spatial region sampled by the fixed distribution array, and fragmentation of inflow and thus EDR distributions has developed. In the last frame of the movie at \( t\Omega_{ci} = 23 \), nongyrotropic distributions appear inside the secondary island that has grown to \( d_r \)-scale.

This type of comprehensive spatial and temporal mapping of the simulation distributions can serve as an encyclopedic guide for orienting spacecraft measurements of distribution functions and assessing the validity of our kinetic models. If spacecraft distributions are observed from a reconnection diffusion region that do not ‘fit’ anywhere in these maps, then the simulation parameters chosen are likely missing the essential physics needed to understand the observations (e.g. in 3D reconnection events which do not exhibit 2D symmetry).

### 3.4.1 Robustness of Triangular EDR Distributions

Catalogs of PIC distributions in the style of Figure 3.10 can be generated for any kinetic simulation. Figures 3.11 and 3.12 show example \( v_x-v_y \) arrays from Runs #2 (\( B_g = 0.03B_0 \)) and #3 (\( m_i/m_e = 1836 \)). The kinetic profile for Run #2 (Figure 3.11) is similar to Run #1, though there are also some notable differences between the two simulations that were not yet addressed in \( Shuster et al. \) [2015a]. The X-line distributions still exhibit a distinct triangular structure, though the guide field of Run #2 has disrupted the discreteness of the striations. Instead of defined striations (as in Distribution 1A of Figure 3.2), the X-line distribution in Run #2 is more centralized around \( v_x = 0 \) with enhancements along both sides of the triangle extending toward increasing \( |v_y| \). Part of the reason for these differences can be understood by comparing the phase spaces of each run, which will be discussed in
Figure 3.10: Electron $v_x-v_y$ distribution map from $t\Omega_{ci} = 19$ (one of the frames in Movie 3.6) of Run #1, showcasing most of the distribution structures presented in Chapters 2 and 3.
Figure 3.11: Map of $v_x - v_y$ distributions for Run #2 (with the small guide field $B_g = 0.03B_0$).
Figure 3.12: Map of $v_x-v_y$ distributions for the real mass ratio simulation (Run #3).
the next chapter (see Figure 4.6). Additionally, the double peak structure in \( n_e \) and \( D_{ng} \) does not develop, suggesting that the meandering motion is characteristically different in the presence of a weak guide field. Nevertheless, the overall heating mechanisms appear to be similar since the X-line structures still gyrotropize in a similar fashion to those of Run #1 from the X-line to beyond the electron jet, forming arcs and rings. The ring with depleted core shown in Distribution 2 of Figure 2.3 does not develop in Run #2, even though there is still strong temperature anisotropy \( T_{e\perp} > T_{e\parallel} \) along the open field lines adjacent to the primary island as in Run #1 (Figure 2.3b).

Considering the realistic mass ratio simulation, the EDR distributions in Run #3 are less structured overall even after peak reconnection (Figure 3.12), which is likely a result of several other parameter changes: the initial upstream electron beta, \( \beta_e \), is 10 times larger than for Runs #1 and #2, the background density is about 4.5 times larger, and the background temperature is also greater by a factor of about 2.3. At the time shown in Figure 3.12, the outflow jets span a larger distance in \( x \) and are narrower in \( z \), while both fluid (data not shown) and kinetic anisotropy is small in the exhaust. In spite of these differences, there is a double peak structure in \( n_e \) and \( D_{ng} \) as well as a \( z-v_z \) phase space hole (which will be shown later in Figure 4.6), indicating that the meandering motion of electrons is similar to Run #1. The \( v_x-v_y \) distribution map (Figure 3.12) does show spatial evolution that is reminiscent of the rotating triangular distributions in Runs #1 and #2, and there are faint, though less discernible, arc structures that develop toward the end of the jets.
3.5 Summary and Conclusions

In this chapter, we discussed the spatiotemporal evolution of electron velocity distributions from the X-line to beyond the end of the electron outflow jet applicable to reconnection with symmetric upstream conditions and weak guide fields. Comparison of the EDR distributions with the distributions just upstream and downstream of the EDR enables us to address the open question of electron heating. We demonstrate that the $T_e$ increase is mainly achieved by the cooperation of $E_y$ and $B_z$ forces on meandering EDR electrons. The EDR distributions in Run #1 and #2 are highly structured, and the structures indicate consistent acceleration and heating processes occurring throughout the reconnection phases we have examined, implying that wave fluctuations and instabilities only play a minor role, if any, in determining electron heating in the EDR. For Run #1, the amount of electron heating throughout the EDR is about seven times larger than that due to the parallel potential ($\Phi_\parallel$) upstream from the EDR as shown in Figure 3.3; for Run #2 with a 3% guide field, the EDR heating was about five times larger than that due to $\Phi_\parallel$ (data not shown).

In Runs #1, #2, and #3, there are triangular shaped distributions in the EDR that gyrotropize downstream of the X-line throughout the electron outflow jet. If observed by spacecraft, the multi-component, nongyrotropic distributions would be ‘smoking-gun’ observables for identifying EDR acceleration and heating mechanisms of reconnection. As we will see in the next chapter and as been reported recently by Chen et al. [2016], the triangular distribution is found even in the asymmetric configuration (Run #4), though with additional structures that arise due to the mixing of the magnetosheath and magnetospheric plasmas on either side of the reconnection layer. NASA’s successfully launched Magnetospheric Multiscale mission, with unprecedented time resolution, has already resolved some of
these structures during its encounter with an EDR at the magnetopause, which is the focus of the next chapter.

**Movies**

**Movie 3.1:** Temporal and spatial evolution of the electron temperature during Run #1, showing where and when (but not how) electrons are heated by the reconnection process.

Direct link:

https://drive.google.com/open?id=0BwUFZvYO52UULULaDc4aGdDdFE

**Movie 3.2:** Test particle tracing forward in time for each of the discrete striations of the triangular X-line distribution. The color panel at the top is the same $U_{ex}$ panel from Figure 3.2. As in Figure 3.3, electron trajectories from the first, second, third, and fourth striations are colored blue, green, orange, and red, respectively, and small black diamonds mark the initial velocities of the traced particles. Distributions in the background of the three velocity space panels below $U_{ex}$ are updated at the spatial location of the electron in the first striation with the smallest initial $v_{x0}$ (initially the leftmost blue electron).

Direct link:

https://drive.google.com/open?id=0BwUFZvYO52USEhEOHZLh5Tlk

**Movie 3.3:** Backward and forward test particle tracing for 2,256 electrons of the X-line distribution in the same format as Movie 3.2 with the addition of $z-v_z$ phase space shown in the upper right, and without $U_{ex}$ in the background of the $x-z$ panel in order to see the individual electrons on their journey through the EDR.

Direct link:
Movie 3.4: Fragmentation and wave-like spatial evolution of the inflow distributions at $t\Omega_{ci} = 22$, providing evidence of wave-particle interactions in the inflow and along separatrices due to the development of electron holes (see Movie 3.5).

Direct link:
https://drive.google.com/open?id=0BwUFZvY052UUTV95bFFnVF3rT1U

Movie 3.5: Electron $x-v_x$ phase space holes along the separatrices of Run #1 at $t\Omega_{ci} = 22$ associated with nonlinear wave signatures in the parallel electric field $E_{\parallel}$.

Direct link:
https://drive.google.com/open?id=0BwUFZvYO52UUQkFvckFtOVpMZFk

Movie 3.6: While Movie 3.1 represents the evolution of the bulk temperature $T_e$, this movie shows the evolution from a kinetic perspective.

Direct link:
https://drive.google.com/open?id=0BwUFZvYO52UUmUFnajhBUFlOeE
4.1 Introduction

How can we infer a spacecraft’s trajectory through the geometry of magnetic reconnection? Answering this question is crucial for interpreting satellite observations of reconnection in Earth’s magnetosphere, and is the focus of Shuster et al. [2016]. Viewed from the frame of a reconnection X-line at the magnetopause, the spacecraft’s motion is often some complex, nonlinear path through the various reconnection regions.

The simplest attempt to orient spacecraft with respect to a theoretical model is to compare the spacecraft’s measurements to a one-dimensional (1D), linear slice through a simulation. This approach is reasonable provided the magnetopause can be modeled as a 1D boundary whose velocity is large relative to the spacecraft, so that the satellite’s path through the boundary is approximately straight. Many enlightening studies have employed this technique to interpret reconnection events using two-dimensional (2D) particle-in-cell (PIC) simulations of reconnection (e.g. Mozer et al. [2008], Eastwood et al. [2010]), and recently three-dimensional (3D) simulations (e.g. Chen et al. [2012], Liu et al. [2013]), where
considering the spatial variation of quantities along a 1D cut offers insight into the reconnection structure. A more realistic 1D trajectory can be constructed using a nonlinear axis scaled by a local plasma parameter, as in Mozer and Pritchett [2009]. Cattell et al. [2005] studied the electron density cavities and bipolar parallel electric field structures along a path which followed a magnetic field line in a 2D PIC simulation as a framework for examining Cluster observations of electron holes during magnetotail reconnection with a guide field. Considering the plasma density, magnetic field direction, and ion bulk velocity, Muzamil et al. [2014] inferred the Polar spacecraft’s traversal of guide field reconnection structures in a new regime of extreme density asymmetry poleward of the cusp. Motivated by the structure of electron temperature anisotropy found in profiles through a PIC simulation of asymmetric reconnection, Scudder et al. [2012] reordered temporal measurements into a nonuniform spatial coordinate to interpret Polar’s magnetopause electron diffusion region (EDR) crossing. As discussed in Section 3.4 of the last chapter, comparing maps of electron distribution functions measured by the four Cluster spacecraft to arrays of PIC distributions is another established technique for elucidating reconnection structures and processes, including magnetic islands and spatially extended electron current layers [Chen et al., 2008a, 2009], the temporal evolution stage of reconnection [Shuster et al., 2014], and electron heating mechanisms in the reconnection exhaust [Wang et al., 2016]. Most recently, Burch et al. [2016] and Torbert et al. [2016c] reported an EDR encountered by the Magnetospheric Multiscale (MMS) spacecraft; using plasma and fields measurements, both studies included a sketch of the MMS tetrahedron’s trajectory through the EDR of a 2D PIC domain, and Torbert et al. [2016c] interpreted MMS signatures of energy dissipation using 1D slices through the simulation of analogous quantities.
In each of these studies, spacecraft trajectories were inferred qualitatively by comparing bulk quantities and sometimes maps of electron distribution functions measured by the spacecraft to simulation predictions. However, even after careful comparisons of this nature, these qualitative methods can leave significant uncertainty regarding where the spacecraft were located in the reconnection structure at a particular time. In this paper, we approach the problem quantitatively by inputting spacecraft measurements to an algorithm which outputs a realistic trajectory through the domain of Run #4 that matches the input data. We apply this method to acquire the MMS fleet’s trajectory through the EDR encountered at the magnetopause on 16 October 2015 \cite{Burch et al., 2016; Torbert et al., 2016c}. We find crescent-shaped electron velocity distributions (studied in depth by Bessho et al. [2016], Chen et al. [2016], and Shay et al. [2016]) in the simulation at the location along the trajectory corresponding to the time at which MMS measured crescent structures.

4.2 Trajectory-Finding Algorithm

In this section, we explain how the algorithm operates. First, we determine the direction normal to the current layer via minimum variance analysis on the magnetic field (MVAB), where the rotation matrix is explicitly listed in Figure 1 of Torbert et al. [2016c]. Here, we use the same transformation to these “LMN” coordinates, where $+\hat{L}$ is directed along the outflow in the ‘northward’ sense, $+\hat{N}$ points normal to the current sheet from MSP to MSH, and $+\hat{M}$ points in the ‘dawnward’ sense. Considering projections of the magnetic field vector in the $L-N$ plane, called magnetic hodograms \cite{Sonnerup and Scheible, 1998}, the algorithm takes in as input the observed $B_L-B_N$ magnetic field hodogram and returns a...
Figure 4.1: Schematic illustrating how the algorithm iteratively determines the spacecraft’s location in the simulation domain. The dotted blue and red curves indicate contours of $B_{L}^{\text{PIC}}$ and $B_{N}^{\text{PIC}}$, respectively, corresponding to $B_{L}^{\text{MMS}}$ and $B_{N}^{\text{MMS}}$ at time $t_n$, while the solid contours correspond to $t_{n+1}$. The intersection of these contours at time $t_n$, marked with green circles, gives the $n^{th}$ location $(x_n, z_n)$ of the spacecraft’s trajectory. The background black lines indicate contours of the magnetic flux function. (Movie 4.1 shows the algorithm in action as it computes the trajectory featured in Figure 4.2.)

path in the simulation which will reproduce this hodogram. Because of the symmetry of 2D reconnection, the signs of $B_L$ and $B_N$ roughly determine the quadrant of the $L-N$ plane in which the trajectory location will reside.

Figure 4.1 shows a schematic illustrating the simple, iterative mechanism by which two normalized components of the magnetic field observed by MMS are mapped into the PIC domain in order to reconstruct the hodogram. The blue lines are contours of the PIC reconnecting component $B_{L}^{\text{PIC}}(x, z, t_{\text{PIC}})$, indicating all of the points in the domain at time $t_{\text{PIC}}$ for which $B_{L}^{\text{PIC}}$ is equal in magnitude to $\hat{B}_{L}^{\text{MMS}}(t)$, the normalized reconnecting component observed by MMS at a particular time $t$. Likewise, the red lines are contours of the PIC normal component $B_{N}^{\text{PIC}}(x, z, t_{\text{PIC}})$ corresponding to $\hat{B}_{N}^{\text{MMS}}(t)$. As shown in the diagram,
together these two contours can constrain the possible locations to a single point (green circle), namely the intersection of the \(B^\text{LPI}C\) (blue) and \(B^\text{PIC}N\) (red) contours. Choosing this location for the PIC spacecraft ensures both \(B^\text{LPI}C\) and \(B^\text{PIC}N\) will agree exactly with the observed \(B^\text{LMM}S\) and \(B^\text{MMS}\). Figure 4.1 includes contours from two times, \(t_n\) (dotted) and \(t_{n+1}\) (solid), showing how data from consecutive times are used to trace a path through the simulation: spacecraft data at time \(t_n\) yield the intersection \((x_n, z_n)\); repeating this procedure for \(t_{n+1}, t_{n+2}, \text{etc.}\), for each measurement during an entire time interval, the resulting trajectory will have the property that both \(B^\text{LPI}C(x_n, z_n)\) and \(B^\text{PIC}N(x_n, z_n)\) match the observations at each \(t_n\) for \(n = \{1, 2, \ldots\}\). Because of this property, this trajectory is more realistic and accurate than one picked manually or by qualitative methods.

4.2.1 Caveats of the Procedure

Before proceeding to the application, here we consider cases where the trajectory-finding algorithm can fail and how to address them. In practice, the procedure described in the main text may not return a single intersection – sometimes no intersection exists, while at other times multiple intersections are possible. Thus, further constraining the algorithm is sometimes necessary to handle these situations. We consider both cases here:

Case 1: no intersection

In order for a valid location to be found, the contours must (1) exist and (2) intersect. Condition (1) will be satisfied as long as the range of normalized spacecraft input values are contained within the range of normalized PIC data. Provided condition (1) holds, an additional normalization factor \(S > 0\) exists which will guarantee condition (2), as shown in the Appendix. This somewhat subtle result means that we can always find an \(S\) for which the
contours given by $\hat{B}_{\text{PIC}}^L(x, z) \equiv B_{\text{PIC}}^L(x, z)/S = \hat{B}_{\text{MMS}}^L$ and $\hat{B}_{\text{PIC}}^N(x, z) \equiv B_{\text{PIC}}^N(x, z)/S = \hat{B}_{\text{MMS}}^N$ will intersect at some $(x_0, z_0)$, a point constrained to lie somewhere along these contours and the contour given by $R(x, z) = B_{\text{MMS}}^N/B_{\text{MMS}}^L$, where $R(x, z)$ is defined to be the ratio $B_{\text{PIC}}^N(x, z)/B_{\text{PIC}}^L(x, z)$. As a check for consistency, the factor should result in a physically plausible normalization. Such a renormalization may limit the region in the simulation domain that is accessible to the trajectory, which is valid as long as the spacecraft is believed to remain close to a particular structure (e.g. the diffusion region) rather than traversing to the asymptotic regions of the simulation (i.e. the edge of the domain) where the original upstream normalization was taken. For the trajectory featured in Figure 2 of the main text, we chose $S = B_{\text{PIC}}^0 = 0.8B_{\text{MSH}}$ following the procedure described more formally in the Appendix.

**Case 2: multiple intersections**

Multiple intersections can occur for a variety of reasons, including dipolarization fronts and magnetic islands that introduce more complicated structures to the $B_N$ contours. For example, one intersection may exist on the X-line side of the dipolarization front, while at the same time another intersection occurs on the downstream-edge of the dipolarization structure. Another example occurs just after 13:06:49UT in Movie 4.1, where the red $B_{\text{PIC}}^N$ contour consists of two pieces: one going fairly straight across the EDR, and the other making a closed loop inside a small magnetic island that is starting to form. Because of these situations, there can be times at which the spacecraft location computed using the $B_L$-$B_N$ hodogram may not be unique. Some of these intersections can be ruled out if following them eventually results in nonphysical, discontinuous jumps in the trajectory. Additionally, intersections far from the X-line (i.e. on the downstream side of the dipolarization fronts)
often have much larger overall errors, sometimes 3× larger than the intersection closest to the X-line. For these reasons, manually constraining the domain accessible to the algorithm, and specifying a threshold region around the previous trajectory location inside of which to prioritize intersections helps to ensure the trajectory’s continuity.

4.3 Magnetopause Electron Diffusion Region Encounter

In this section, we apply the algorithm described above to gain insight into how the MMS tetrahedron may have traversed the EDR ‘caught’ on 16 October 2015 around 13:07:02UT [Burch et al., 2016; Torbert et al., 2016c]. MMS observed the reconnecting fields and particle distributions with unprecedented accuracy and time resolution: the FIELDS instrument suite measured a magnetic field vector 128 times per second [Russell et al., 2016; Torbert et al., 2016a], and the Fast Plasma Investigation (FPI) measured a full plasma distribution function every 150 ms for ions and 30 ms for electrons [Collinson et al., 2012; Pollock et al., 2016].

4.3.1 Trajectory Determination

Figure 4.2 exhibits MMS 2 measurements and simulation quantities along the computed trajectory. Figure 4.2a shows the magnetic hodogram observed by MMS (white points) during the approximately 30 second interval from 13:06:43.5UT to 13:07:10.75UT, and the matched simulation hodogram (colored points, mostly covering the MMS points) corresponding to the trajectory in Figure 4.2b. The points in Figure 4.2a,b (small circles) are colored according to the color bar at the top of Figure 4.2c to indicate the passage of time. The virtual
spacecraft starts in the MSH on the \(-L\) side of the X-line near \((L, N) \approx (350, 5) d_e\) (large green circle), samples the EDR on the MSP-edge of the layer, and eventually leaves the EDR as \(|B_L|\) increases near \((L, N) \approx (385, 24) d_e\) (large red circle). Movie 4.1 illustrates how the magnetic field contours were used to determine the spacecraft’s location at each time throughout this interval. The gold stars indicate the time at which MMS 2, 3, and 4 observed crescent-shaped electron velocity distributions [Burch et al., 2016], and will be addressed in the discussion for Figure 4.5. By construction, the algorithm uses \(B_L\) and \(B_N\) to ensure that the simulation hodogram will match the observed hodogram, which is why there is almost no difference between MMS and simulation quantities in Figure 4.2a,c,e.

The remaining panels Figure 4.2d,f,g,h,i show five other MMS quantities (black, left axis) compared to corresponding simulation quantities (color, right axis) taken along the computed trajectory. Each of these simulation quantities has some features which are consistent with the spacecraft data, and some which are different. The M-component of the magnetic field (Figure 4.2d) somewhat agrees with the MMS data, except for two times at about 13:06:48UT and 13:07:08UT where the simulation trace (green) deviates most noticeably from MMS. The electron density (gold trace in Figure 4.2f) agrees well with MMS during 13:06:55UT to 13:07:08UT even following the dip in density near 13:06:58UT, but does not follow MMS as closely elsewhere. The electron and ion outflow velocities (Figure 4.2g and h, respectively) show the largest deviations from MMS, but share some important consistencies with the spacecraft data: the electron velocity (light blue trace) exhibits jets in the negative L-direction around 13:07:00UT as does MMS, and the ion jet (magenta trace) reverses at this time along with MMS. The PIC electron and ion velocities are normalized to their maximum jet values, about 0.15\(c\) for electrons and 0.03\(c\) for ions. Lastly, in Figure 4.2i the electron
Figure 4.2: MMS 2 measurements and simulation quantities along the computed trajectory through the EDR. (a) MMS magnetic field hodogram in the $B_L-B_N$ plane with matched simulation hodogram (colored by time) over-plotted. (b) Determined trajectory of MMS 2 in the simulation $L-N$ plane (colored by time) which corresponds to the hodogram matched in (a). (c-i) Seven quantities measured by MMS 2 (black) compared to simulation quantities (color) along the trajectory shown in (b): (c) L-component, (d) M-component, and (e) N-component of the magnetic field; (f) electron density; L-component of the (g) electron and (h) ion velocity; and (i) electron temperature parallel to the magnetic field. The larger green and red circles and the gold star in (a) and (b) correspond to the times shown at the top of (c), and the colored bar is shown to indicate time along the trajectory. The gold star indicates the time when MMS 2 observed crescent structures in electron velocity distributions.
temperature parallel to the magnetic field $T_{e\parallel}$ (orange trace, normalized to $0.15m_e c^2$) is consistent with MMS especially at the two peaks a few seconds before 13:07:00UT associated with the density dip, but did not capture the peaks seen on MMS just after this time.

4.3.2 Accuracy and Robustness

As a measure of the trajectory’s overall accuracy, we calculate normalized differences between each simulation quantity shown in Figure 4.2c-i and the corresponding MMS data, and average these relative differences over the time interval shown. The error analysis results are shown in Figure 4.3. At a given time, the error is computed by first normalizing each MMS and PIC quantity by its maximum absolute value achieved during the interval (so that each ranges from -1 to 1, or 0 to 1, depending on the quantity), and then taking the difference $Q_{MMS} - Q_{PIC}$ (black traces), where $Q_{MMS}$ (blue curves) are the normalized MMS quantities and $Q_{PIC}$ (green) are the corresponding renormalized PIC quantities. For each quantity, the average of the absolute value of this difference over the whole interval is reported as a percentage of the normalization used in red beside each panel. The last panel is an average of the absolute value of each quantity’s differences, representing the total error at a given time based on the quantities considered. The average of this total error over the interval represents the total average error for the trajectory, and is a measure that can be used to compare the accuracy of different trajectories.

For both $B_L$ and $B_N$, this average error is less than 1.3% in normalized units (about 0.5nT). For the trajectory featured in Figure 4.2, the overall error based on all seven quantities used is about 17.9% (last panel of Figure 4.3). During just the interval including the EDR from 13:06:53UT to 13:07:05UT the average error improved to 16.5%. At the time
when MMS observed the crescent distributions (gold star), the error was about 10%. As can be seen in Movie 4.1, this was a time when the $B_L$ and $B_N$ contours had multiple intersections due to the $B_N$ contour’s distortion associated with the formation of a small magnetic island. At this time, the algorithm chose the intersection which had the least total error. As a basis for interpreting these numbers, the error of a trajectory chosen by eye (without the algorithm) and interpolated to match the resolution of the MMS data was about 40%.

To further quantify the robustness of the algorithm, we explored how the output trajectory and its error depend on:

1. simulation normalization $B_0^{PIC}$ (from $0.7B_{MSH}$ to $1.0B_{MSH}$),
2. spacecraft data (MMS 2 vs. MMS 3),
3. simulation time (from after peak reconnection at $t\Omega_{ci} = 68$ to before at $t\Omega_{ci} = 56$),
4. guide field (from $B_g = 0$ to $B_g = 0.1B_{MSH}$), and
5. using $U_{iL}$ in place of $B_N$ for determining contour intersections.

For (1)-(3), the trajectories were qualitatively similar and their errors remained near 22%. For (4), the error decreased to 19.7%, indicating that the 10% guide field run is slightly better for modeling the event. This is consistent with the MMS data: the average $B_M$ during the approximately 2.5 minute interval from 13:05:24UT to 13:07:43UT (data not shown) was about 6nT, about 15% of the upstream $B_L \approx 40$nT near 13:05:40UT. For (5), we modified the algorithm to search for contour intersections using $B_L$ and $U_{iL}$ so that the resulting trajectory matched the MMS $B_L$ and $U_{iL}$ curves rather than the $B_L$-$B_N$ hodogram. The overall error of the resulting trajectory was 20.6%.

Increasing the normalization factor $B_0^{PIC}$ increased the accessible domain for the algorithm and consequently the spread of the resulting trajectory about the EDR, though the
Figure 4.3: Error analysis for the trajectory featured in Figure 4.2. The red line in the last panel represents the average error (about 16.5%) over a reduced time interval as described in the text.
structure and shape of the trajectory remained similar to that shown in Figure 2b. The overall error for this trajectory was about 22%. Using data from MMS 3 rather than MMS 2, the resulting trajectory followed almost the same path as before with a similar overall error (about 20%) but with a different timing. Changing the simulation time from \( t\Omega_{ci} = 68 \) (after peak reconnection) to \( t\Omega_{ci} = 56 \) (before peak reconnection) reduced the spread of the trajectory (with an error of about 22%) since the reconnection structure was less developed at that time. Using the 10% guide field simulation, the resulting trajectory in the PIC domain exhibited qualitative similarities, except the X-line in the guide field run moved by about 30\( d_e \) to \( L \approx 400d_e \) (rather than about 370\( d_e \) in Figure 2b). When the algorithm searched for contour intersections using \( B_L \) and \( U_{iL} \), the error in \( U_{iL} \) became about 1% (about 2 km/s) with an overall trajectory error of 20.6%, which could likely be further improved if the velocity data is shifted to a frame where the ion flow reversal is centered about \( v = 0 \). The output trajectory was predominately on the \(-L\) side of the X-line since \( U_{iL} < 0 \) for most of the interval.

## 4.3.3 Electron Spectrograms

After finding the trajectory (Figure 4.2b), we are equipped to compare kinetic aspects of the MMS observations and simulations. In Figure 4.4, we compute simulation electron energy vs. time and pitch angle vs. time spectrograms to compare with FPI’s duel electron spectrometers’ (DES) data on MMS 2. To generate the PIC spectrograms, we select electrons from a bin of \( 1d_e \times 1d_e \) centered at each location along the trajectory. Figure 4.4a and b show the MMS and PIC omnidirectional energy-time spectrograms, respectively, while Figure 4.4c-f show MMS and PIC pitch angle (PA) spectrograms: Figure 4.4c,d show low energy
electrons (0 to 200eV for MMS, and 0 to 0.05$m_c$c$^2$ for PIC), while Figure 4.4e,f show a middle energy range (0.2 to 2keV for MMS, and 0.05 to 0.5$m_c$c$^2$ for PIC).

From 13:06:55UT to 13:07:03UT (marked by black bars at the bottom of the MMS panels), the electron energy spectrogram measured by MMS (Figure 4.4a) shows significant electron energization up to a few keV (color in the MMS panels in Figure 4.4a,c,e show energy flux in keV/[cm$^2$·s·str·keV]). This feature is seen as a drop in flux of low-energy electrons for all PAs except close to 0° and 180°. The energized electrons appear in the mid-energy PA spectrogram (Figure 4.4e) especially at 0° and 180°, which explains the $T_{e∥}$ peaks around this time (Figure 4.2i). Also of note are several discrete structures of increasing electron flux extending toward more perpendicular PAs. Comparing these observations with the PIC spectrograms, as with the bulk quantities along this trajectory, there are both similarities and differences. The PIC energy spectrogram (Figure 4.4b) shows significantly increased counts throughout energies ranging from about 0.05 to 0.5$m_c$c$^2$, in qualitative agreement with MMS. However, the time interval of energization is longer, starting at near 13:06:47UT and lasting until about 13:07:09UT (marked by the magenta horizontal bars below the PIC panels). The decrease in electron counts for the PIC low-energy PA spectrogram (Figure 3d) appears as increased counts in the mid-energy PA spectrogram (Figure 3f) during this extended interval as was the case for MMS during the shorter interval. The distribution of PIC pitch angles, however, is more intricate than MMS observed: much of the PIC mid-energy electron PAs are predominantly centered in the range of 50° to 150° (e.g. from 13:06:51UT to 13:07:03UT), except a few times where parallel and antiparallel populations accompany the complicated perpendicular populations (e.g. from 13:06:56UT to 13:06:59UT).
Figure 4.4: Electron spectrograms observed by MMS 2 with analogous PIC spectrograms calculated along the trajectory shown in Figure 4.2b: (a,b) omnidirectional energy spectrograms; (c,d) low-energy pitch angle spectrograms; (e,f) mid-energy pitch angle spectrograms. The red and green circles, gold star, and colored time bar at the top of (a) indicate the same times as in Figure 4.2. (Data was not available for the gap near 13:07:11UT in b,d,f.)
4.3.4 Electron Velocity Distributions: Crescent Structures

The gold star was shown in Figures 4.2 and 4.4 because at this time MMS 2, 3, and 4 observed crescent-shaped electron velocity distributions, one of which from MMS 2 is displayed in Figure 4.5a-c (reproduced in part from Figure 3 in Burch et al. [2016]) and compared to the PIC distribution in Figure 4.5d-h taken at the corresponding location along the computed trajectory: \((x, z) = (358.27, 1.17)d_e\) (roughly \(10d_e\) downstream from the X-line). The complicated, energized perpendicular electrons of the PIC mid-energy PA spectrogram correspond to these highly nongyrotropic crescent populations. At this time, the PIC magnetic field was mainly along \(+\hat{L}\) (see Figure 4.2c-e), so \(v^\parallel_{MMS} \to v^\parallel_{PIC}\). Additionally, the electric field was mostly along \(+\hat{N}\) (data not shown), so \(v^\perp_{MMS} \to v^\perp_{PIC}\) since \(v^\perp_1\) was defined to point in the \(\hat{E} \times \hat{B}\) direction. Thus, \(v^\perp_{MMS} \to v^\perp_{PIC}\).

The MMS \(v^\perp_1-v^\perp_2\) distribution in Figure 4.5a shows a distinct crescent structure characteristic of accelerated, meandering MSH electrons measured on the MSP side of the EDR [Hesse et al., 2014; Bessho et al., 2016; Chen et al., 2016]. The corresponding PIC distribution in \(v_M-v_N\) (Figure 4.5d) has higher counts at the ends of its crescent around \(\pm v_N \approx 0.3m_e c^2\) and \(v_M \approx 0\), whereas MMS measured the highest PSD values near \(v^\perp_2 \approx 0\) and \(v^\perp_1 \approx 0.5 \times 10^4\) km/s. Taking a slice of the PIC distribution in 3D velocity space (Figure 4.5g), the crescent is more readily apparent. In the \(v_L-v_M\) projection (Figure 4.5e) the PIC crescent appears as a population centered around \(v_L \approx -0.1 c\) and \(v_M \approx 0.5 c\), qualitatively consistent with the MMS distribution (Figure 4.5b) whose main population is centered at \(v^\parallel \approx -0.1 \times 10^4\) km/s and \(v^\perp_1 \approx 0.5 \times 10^4\) km/s. The PIC distribution resolves numerous discrete structures resembling a “leaf” shape as described by Chen et al. [2016], while the MMS distribution has a weaker, counter-streaming component near \(v^\perp_1 \approx -0.4 \times 10^4\) km/s.
Figure 4.5: Crescent-shaped electron velocity distributions observed by MMS 2 and found in the simulation in the EDR at the time indicated by the gold star in Figures 4.2 and 4.4: MMS 2 distribution in the (a) $v_{\perp 1}-v_{\perp 2}$, (b) $v_{\parallel}-v_{\perp 1}$, and (c) $v_{\parallel}-v_{\perp 2}$ planes, where $v_{\perp 1}$ is the $\mathbf{E}\times\mathbf{B}$ direction (adapted from Figure 3 of Burch et al. [2016]); simulation distribution in the analogous (d) $v_M-v_N$, (e) $v_L-v_M$, and (f) $v_L-v_N$ planes; (g) $v_y-v_z$ and (h) $v_x-v_y$ slices of the distribution shown in (d-f). (Movie 4.2 shows a more complete visualization of this 3D velocity space structure.)
absent from the PIC distribution. Fewer discrete structures are contained in a slice through one of the $v_N$ lobes (Figure 4.5h). Both the MMS $v_{\parallel}-v_{\perp2}$ distribution (Figure 4.5c) and the corresponding PIC $v_{L}-v_N$ distribution (Figure 4.5f) exhibit counter-streaming electrons in $\pm v_N$ ($\pm v_{\perp2}$) and a faint background population heated in $v_L$ ($v_{\parallel}$). Movie 4.2, from which Figure 4.5g and h were derived, thoroughly “dissects” the distribution offering an illuminating visualization of the multiple, embedded structures.

### 4.4 Electron Encyclopedia: Mapping the Asymmetric Configuration

These discrete populations are reminiscent of the swirling striations in the triangular EDR distributions studied by Bessho et al. [2014] and Shuster et al. [2015a] for symmetric reconnection that were discussed in the last chapter. Here in the asymmetric case, the bifurcated structure in $v_N$ ($v_{\perp2}$) is connected via the crescents, which results in a “filled-in” $N$-$v_N$ phase space rather than the phase space hole structure that can form in the symmetric configuration [Chen et al., 2011]. Comparisons of phase space for Runs #1 through #4 are presented in Figure 4.6. From top to bottom, the rows of distributions are in $z$-$v_x$, $z$-$v_y$, and $z$-$v_z$ space, respectively.

Starting with the leftmost panels for Run #1 which has $\omega_{pe}/\Omega_{ce} = 2$, the phase space structures are consistent with Figures 1, 2, and 4 of Chen et al. [2011]. The phase space hole structure can develop provided the ratio $\omega_{pe}/\Omega_{ce}$ is small enough so that Debye-scale turbulence does not hinder its formation [Jara-Almonte et al., 2014]. The bottom panel shows this prominent $z$-$v_z$ feature (also shown in Movie 3.3), and the $z$-$v_y$ panel of Figure
4.6 resolves the discrete structures corresponding to the $v_x$-$v_y$ striations of Distribution 1A in Figure 3.2. In this $z$-$v_y$ representation, the energized striations appear like rungs of a ladder or ‘shoelaces’ increasing in $|v_y|$. With the addition of a small guide field (see phase space panels for Run #2), the $z$-$v_z$ phase space hole structure vanishes, which explains why the X-line distributions in this run did not exhibit distinct striations as in Run #1. Even with different initial parameters and real mass ratio, Run #3 still exhibits the $z$-$v_z$ hole, though without discreteness in $z$-$v_y$. Lastly for the asymmetric configuration (rightmost panels for Run #4), even with zero guide field the $z$-$v_z$ hole is populated by incoming MSP electrons, resulting in a ‘ball-and-socket’ or ‘tuning fork’ structure as the higher density MSH electrons energize and mix with MSP electrons. Electrons are energized from $z \approx -2d_e$ to $z \approx 4d_e$, though most of the discreteness is hidden from this viewpoint – except for crescent electrons which appear as an energetic protrusion toward the MSP side of the layer near $z \approx 3.5d_e$.

Figures 4.7 and 4.8 show comprehensive maps in the same layout as those presented for symmetric runs in the last chapter. The maps are presented in $v_\parallel$-$v_{\perp 1}$-$v_{\perp 2}$ space to facilitate comparison to the MMS observations. In the $v_\parallel$-$v_{\perp 1}$ array (Figure 4.7), the triangular “leaf” distributions rotate downstream of the X-line region, though the overall structure of the electron outflow jet is different than for the symmetric case where the jet was laminar and centered around the mid-plane at $z = 0$. Here the jet follows the MSP separatrix, extending in the $+z$ direction. The distributions in the exhaust are complicated with multiple embedded structures. Inflow distributions on both the MSP and MSH side exhibit parallel heating with $T_{e\parallel} > T_{e\perp}$, though the anisotropy is significantly larger on the MSP (low density) side. The $v_{\perp 1}$-$v_{\perp 2}$ array in Figure 4.8 shows how crescent structures surround the X-line region and extend along the electron jets. Going from the MSP to MSH, the distinct crescent
Figure 4.6: Comparisons of $z-v_x$, $z-v_y$, and $z-v_z$ phase space for Runs #1, #2, #3, and #4.
shape stretches into a “horse-shoe” structure, wrapping around the colder core population. Elsewhere, distributions are mostly gyrotropic in this $v_{\perp 1}$-$v_{\perp 2}$ space.

As a step toward future work, Figure 4.9 shows a $v_x$-$v_y$ electron distribution array from a 3D, asymmetric PIC simulation (Run #5). The structure of $U_{ex}$ is more turbulent, showing intricate regions of enhanced flows. Overall, there are still two dominant outflows in the $x$ directions, but there are also inflowing streams along what would be separatrices and finer-scale structures especially along the outflow’s edge. The separatrix surfaces in 3D can be difficult to find, but there are other ways to identify the topological boundary, such as the electron-mixing parameter [Daughton et al., 2014]. Close to $z = 0$, distributions split into multiple, counter-streaming components. On the $+z$ inflow side, distributions exhibit $T_{e||} > T_{e\perp}$ anisotropy. The strong guide field in this run, $B_g = 1.0B_0$, limits the demagnetization of electrons, and thus mostly prevent nongyrotropy from developing in the distribution structures.

4.5 Discussion and Conclusions

We develop an algorithm to find realistic spacecraft trajectories through simulation domains. Inputting the magnetic field $B_L$-$B_N$ hodogram observed by MMS 2 during a magnetopause EDR crossing, and using 2D PIC simulations of asymmetric reconnection, we compute trajectories that match the input MMS magnetic field components. Considering the ability of the algorithm to, in general, find contour intersections in 2D required to locate the virtual spacecraft, we conclude that for an appropriate renormalization of the simulation data such intersections are guaranteed, provided that a reasonable correspondence can be found be-
Figure 4.7: Array of $\eta_v-\nu_{11}$ distributions from Run #4, showing the spatial evolution of the triangular, “leaf” structures.
Figure 4.8: Array of $v_{\perp 1}$-$v_{\perp 2}$ distributions in Run #4 from the same locations as Figure 4.7, showing corresponding crescent and horseshoe resembling shapes.
Figure 4.9: Array of $v_x - v_y$ distributions from a $y$-slice of the 3D, asymmetric simulation (Run #5).
tween the ranges of spacecraft and simulation data. We tested the algorithm’s sensitivity to the simulation normalization, time, guide field, spacecraft number, and the quantities used to specify contour intersections.

Applying the algorithm to MMS data during the time interval closest to the EDR crossing, the virtual spacecraft made the following observations consistent with MMS measurements: (1) crescent-shaped electron distributions with qualitatively similar features in three projections of the $v_{\parallel}-v_{\perp 1}-v_{\perp 2}$ velocity space, (2) omnidirectional energy spectrograms exhibiting electron energization throughout the EDR, (3) pitch angle spectrograms showing increased flux of energized, counter-streaming electrons at $0^\circ$ and $180^\circ$ with discrete populations appearing in the perpendicular directions, (4) qualitative agreement in several other quantities, including a relatively steady $B_M$ component, density dip near 13:06:58UT correlated with peaks in $T_{e\parallel}$, electron jets from 13:06:57UT to 13:07:03UT, and an ion jet reversal around 13:07:02UT. While there are discrepancies along the trajectory, the overall error is about 18%, and only 10% at the time when MMS observed the crescent distributions in the EDR. Slices of the PIC distributions in 3D velocity space are somewhat more consistent with MMS measurements, possibly because FPI also measures ‘slices’ of the total plasma population.

We speculate that the new “parallel crescents” reported in Burch et al. [2016] are related to the “swirling” spatial evolution of the distribution function downstream of the X-line where the $v_L$ electron jets enhance, a region which MMS could have sampled after passing by the magnetic null. This evolution is studied in detail by Shuster et al. [2015a] for symmetric reconnection and Chen et al. [2016] in the asymmetric case. A recent study reports the parallel crescent and horseshoe structures forming in association with parallel electric fields that develop in the region of finite $B_N$ and strong $E_N$ downstream of the X-line [Shay et al.,
There are several ways to improve the algorithm we developed. One is to extend the procedure to 3D simulations, where ‘contours’ would become surfaces, and at least three quantities (e.g. the full magnetic field vector $B_L$, $B_M$, and $B_N$) would be needed to identify a finite number of virtual spacecraft locations. The 3D version of this algorithm would in principle resemble the technique developed by Komar et al. [2013] for efficiently tracing separators in 3D global MHD simulations, only rather than searching for nulls along a separator, the algorithm would search for regions of the magnetic topology which correspond to input spacecraft data. Another improvement would be to relax the assumption that the reconnection structure does not change in time, allowing the simulation to evolve in the course of the trajectory determination. If in this process we find a particular time evolution which reduces the total error considerably, we could use this information to infer the reconnection rate of the event observed by the spacecraft. Such improvements are underway in anticipation of continued MMS discoveries that will further strengthen our understanding of the electron-scale phenomena fueling magnetic reconnection.

Movies

Movie 4.1: This movie is an animated version of Figure 4.2, illustrating how the algorithm maps two components of the MMS magnetic field data into the PIC domain. The moving blue curve is the contour of $\hat{B}_L^{PIC}$ corresponding to $\hat{B}_L^{MMS}$, and the moving red curve is the contour of $\hat{B}_N^{PIC}$ corresponding to $\hat{B}_N^{MMS}$. The intersection of these two contours at each time is taken to be the location of the virtual spacecraft, marked by a yellow diamond. For reference, the contour of $\hat{B}_M^{PIC}$ is also animated (green moving curve in Figure 4.2b), but
this contour is not used for determining the virtual spacecraft’s location in this study. Each panel (a-i) is animated simultaneously to communicate the relative correspondence between the hodogram (Figure 4.2a) in $B_L-B_N$ space, the trajectory (in simulation $L-N$ space), and the MMS data shown in time (Figure 4.2c-i).

Direct link:
https://drive.google.com/open?id=0BwUFZvYO52UZmgtQlhRsRnBpOWc

**Movie 4.2:** The slices of the distribution shown in Figure 4.5g,h offer a revealing perspective ‘inside’ the distribution projections shown in Figure 4.5d-f. These representations are created analogously to the 2D distributions by dividing the 3D velocity space into cubes of equal volume $\Delta v_x \times \Delta v_y \times \Delta v_z$, and coloring each cube according to the number of electrons in that bin. This movie is a thorough “dissection” in 3D velocity space of this distribution in the same format as Movie 2.1, showing the full 3D velocity-space structure which can be somewhat buried by the reduced nature of the 2D projections. The first part of the movie shows the velocities of all electrons in the $1d_e \times 1d_e$ spatial bin described in the paper centered at $(x, z) = (358.27, 1.17)$. The second part of the movie shows this distribution with velocity space divided into $40 \times 40 \times 40$ bins colored by counts (red corresponds to a maximum count of about 140 electrons). The grey shadows are the same projections as Figure 4d-f. The end of the movie zooms toward the distribution and shows slices in each of the $v_x-v_y$, $v_x-v_z$, and $v_y-v_z$ planes.

Note the relationship between these three sets of coordinate systems in use: $x$-$y$-$z$ for PIC, $L$-$M$-$N$ for MMS fields, and $v_{||}$-$v_{\perp 1}$-$v_{\perp 2}$ for FPI electron distributions. At the time when crescent distributions were observed on MMS (gold star in Figures 2 and 3), the three
systems were related as follows:

\[ +\hat{x}_{PIC} = +\hat{L} \approx +\hat{v}_\parallel \]
\[ +\hat{y}_{PIC} = -\hat{M} \approx -\hat{v}_{\perp 1} \]
\[ +\hat{z}_{PIC} = -\hat{N} \approx -\hat{v}_{\perp 2} \]

Direct link:

https://drive.google.com/open?id=0BwUFZvYO52UUY2lQaDdZajljeckE
In this dissertation, we elucidate how collisionless magnetic reconnection energizes electrons in the electron diffusion region (EDR) for both symmetric and asymmetric configurations with weak guide fields. Traveling through the EDR modeled by fully kinetic, particle-in-cell simulations from a kinetic electron’s perspective, we demonstrate how electron distribution functions can offer ‘smoking-gun’ evidence necessary for identifying the EDR in spacecraft measurements. This final chapter summarizes our conclusions. Since we have not yet ‘solved’ magnetic reconnection, we also address open questions for guiding future investigations.

5.1 Highly Structured Electron Anisotropies

In Chapter 2, we discussed the discovery that electron distributions become highly structured after the explosive growth phase of reconnection. Ring, ‘wing’, and arc structures are found in every exhaust. Magnetic island exhausts give rise to more intricate structures, including an unstable ring distribution with depleted low energy core that develops in the exhaust modified by a primary island, and the hemispherical shell population that forms inside the secondary island. Counterstreaming beams and energetic ring populations exhibiting
$T_{e\perp} > T_{e\parallel}$ anisotropy are predicted to develop throughout the exhaust especially near the magnetic field pileup region, and are confirmed by Cluster spacecraft measurements. This work suggests a method for inferring the temporal evolution stage of reconnection based on measurements of anisotropic distribution functions, and helps to clarify the discrepancy regarding the degree of electron anisotropy in the exhaust.

5.2 Spatiotemporal Evolution of EDR Electrons

As discussed in Chapter 3, electron heating in the EDR is accomplished mainly by the cooperation of the preferential acceleration by the reconnection electric field ($E_y$) and cyclotron turning by the normal magnetic field ($B_z$), which converts the electrons’ acquired bulk $y$-momentum to thermal energy. The triangular X-line distribution thus gyrotropizes downstream of the X-line, resulting in the arc and ring structures that develop in the exhaust. The swirling triangular distributions are found in Run #1, Run #2 (with the addition of a small, 3% guide field), Run #3 (real ion to electron mass ratio), and Run #4 in the asymmetric configuration, and thus are a robust, predicted signature of the EDR. The weak guide field seems to eliminate the distinct striations of the X-line distribution from forming, in part because of the absence of the phase space hole structure. However, new structures develop in the presence of the guide field, including enhanced ‘stripe’-like edges along the triangle. The phase space hole is still supported in the real mass ratio run, though distributions are much less structured overall likely because of other parameter changes. Fragmentation of the inflow distributions in Runs #1 and #2 associated with parallel electric field signatures and electron holes helps to explain why the X-line distributions become likewise fragmented at
later times. The fragmentation also implies that the analytical form of the inflow distribution that was assumed in previous studies may only be valid for early stages of reconnection.

5.3 Hodographic Trajectory-Finding Algorithm Applied to MMS Observations at the Magnetopause

Analyzing MMS measurements from an EDR encountered at the magnetopause, we develop and implement a trajectory-finding algorithm to quantitatively determine a realistic trajectory of the MMS tetrahedron through the simulation domain. Inputting the outflow and normal magnetic field components observed by MMS into the algorithm, we obtain trajectories along which this input data is matched to within about 1%. Based on five other quantities considered \( B_M, n_e, U_{eL}, U_{iL}, \) and \( T_{e||} \), the overall average, relative error of the trajectory is about 18%. We checked the algorithm’s sensitivity to several parameters, including the simulation time, normalization, and guide field, as well as which spacecraft data and simulation quantities are used to obtain contour intersections. Crescent-shaped distributions are found in the simulation along the trajectory corresponding to the time when MMS measured EDR crescent structures in \( v_{\parallel}-v_{\perp1}-v_{\perp2} \) space, and simulation electron energy and pitch angle spectrograms also agree qualitatively with the MMS observations.

Throughout these chapters, we assemble comprehensive maps of electron distributions in an encyclopedic fashion to catalog the diverse types of highly structured, non-Maxwellian features predicted by the simulations. The structures most characteristic of the X-line region at the heart of the EDR are summarized here in a final figure (Figure 5.1).
Figure 5.1: Summary figure highlighting the most distinctive structures that form in the vicinity of the X-line region for Runs #1, #2, and #4.
5.4 Open Questions

As a direction for future investigation, here we ask several open questions motivated by this dissertation:

1. Why does kinetic anisotropy develop after peak reconnection? Is there some analytical relationship between the distribution function, \( f_e(x, v, t) \), and the reconnection rate, \( <E> \sim V_{in}/V_{out} \), that can be established?

2. What types of distribution structures develop when the guide field is stronger in 2D (e.g. in the extended current layers reported by Le et al. [2013], or in the nongyrotropic regions studied by Wendel et al. [2016])? In 3D, can the distribution evolution help to explain the enhanced energetic electron production in the stochastic fields reported in Dahlin et al. [2015]? Also, is there some threshold guide field strength, \( B_g \), which keeps electrons sufficiently magnetized to limit these kinds of highly structured distributions from developing, as seemed to occur in the 3D, asymmetric simulation (Run #5)?

3. Why do the striations and phase space hole (and hence inversion electric field signature) vanish in the 3% guide field case, and what leads to the formation of enhanced ‘stripes’ along the triangular X-line distribution’s edges? Also, why doesn’t the ring distribution form, and does this mean that whistler waves are also suppressed in the weak guide field case?

4. How does varying the parameter \( \omega_{pe}/\Omega_{ce} \) affect the structure and evolution of EDR, exhaust, and inflow distributions? Jara-Almonte et al. [2014] has showed that increasing \( c/v_{the} \) (which is the same as varying \( \omega_{pe}/\Omega_{ce} \) as long as \( \beta_e \) and \( T_i/T_e \) remain fixed)
to values that exceed $\sim 30$ can introduce Debye-scale turbulence to the phase space, but the structure of velocity space distributions in these regimes has not yet been explored. As shown by EDR distributions in the weak guide field simulation (Run #2), even when there is no phase space hole, distributions can still be intricate, highly structured indicators of EDR mechanisms.

5. What is the nature of the wave particle interactions that lead to fragmentation of inflow and thus EDR distributions? Are there others besides the whistler mode at work, such as Langmuir and electron cyclotron waves as reported in Cluster observations by Viberg et al. [2013]?

6. Are electrons less structured in the real mass ratio run because of upstream parameters (e.g. $n_b/n_0$ and $T_{eb}/T_{e0}$), or is there something fundamentally different (e.g. inertial effects) in the realistic mass ratio regime that reduces the distribution structures?

7. Do triangular distributions form in the EDR for 3D simulations, or is this structure only inherent to the 2D geometry?

As we address these open questions in the future, the author plans to continue expanding our knowledge of the ‘electron encyclopedia’ with kinetic catalogs of velocity and phase space for additional simulations and parameter regimes, in hopes of bringing us closer to answering the still outstanding question:

*How does magnetic reconnection happen in a collisionless plasma?*
Guaranteeing Contour Intersections in Two Dimensions

The two dimensional problem may be posed as follows: Does there exist some constant normalization factor \( S > 0 \) such that contours given by \( \hat{B}^{PIC}_L(x, z) = B^{PIC}_L(x, z) / S = \hat{B}^{MMS}_L \) and \( \hat{B}^{PIC}_N(x, z) = B^{PIC}_N(x, z) / S = \hat{B}^{MMS}_N \) can be made to intersect at least once at some point \((x_0, z_0)\)? The answer is yes, provided the functions \( B^{PIC}_L(x, z) \) and \( B^{PIC}_N(x, z) \) are continuous and that the full range of normalized MMS data is contained within the PIC range.

To understand this result, it is instructive to formulate an analogous problem algebraically in one dimension: Consider two continuous functions \( f(x) \) and \( g(x) \) and the points \( x_1 \) and \( x_2 \) defined such that \( f(x_1) = a \) and \( g(x_2) = b \) for reasonable values of \( a \) and \( b \). (The points \( x_1 \) and \( x_2 \) are 1D analogs to contours in 2D, and the points \( a \) and \( b \) play the role of \( B^{MMS}_L \) and \( B^{MMS}_N \).) Does there exist some constant factor \( S > 0 \) for which \( \hat{f}(x^*) = f(x^*) / S = a \) and \( \hat{g}(x^*) = g(x^*) / S = b \) at the point \( x^* \)? Solving for \( S \), we find:

\[
S = \frac{f(x^*)}{a} = \frac{g(x^*)}{b},
\]

which reduces the problem to finding \( x^* \). Solving for \( x^* \) analytically is difficult unless we know the inverses of \( f \) and \( g \), which are nontrivial especially in 2D where the ‘inverse’ is a path in the \( x-z \) plane. We can determine \( x^* \) numerically by defining the function \( R \) as the ratio of \( f \) and \( g \), which is independent of \( S \). We then search for the \( x^* \) which satisfies the following equation:

\[
R(x^*) \equiv \frac{f(x^*)}{g(x^*)} = \frac{a}{b}.
\]

Thus, for a given \( x^* \) that satisfies \( R(x^*) = a/b \), then we can find an \( S \) which will ensure that
\( \hat{f}(x^*) = a \) and \( \hat{g}(x^*) = b \), as desired.

Returning to the 2D case with this insight, we can construct the ratio function, which does not depend on \( S \):

\[
R(x, z) \equiv \frac{B^\text{PIC}_N(x, z)}{B^\text{PIC}_L(x, z)}. \tag{3}
\]

As with the 1D case, we assert analogously that provided there exists at least one point \((x_0, z_0)\) for which \( R(x_0, z_0) = \hat{B}^\text{MMS}_N/\hat{B}^\text{MMS}_L \), then we can find an \( S \) such that contours given by \( \hat{B}^\text{PIC}_L(x, z) \equiv B^\text{PIC}_L(x, z)/S = \hat{B}^\text{MMS}_L \) and \( \hat{B}^\text{PIC}_N(x, z)/S \equiv B^\text{PIC}_N(x, z) \) can be made to intersect at \((x_0, z_0)\). Solving for \( S \), we have:

\[
S = \frac{B^\text{PIC}_L(x_0, z_0)}{\hat{B}^\text{MMS}_L} = \frac{B^\text{PIC}_N(x_0, z_0)}{\hat{B}^\text{MMS}_N}. \tag{4}
\]

As before, we have reduced the problem to finding a suitable point \((x_0, z_0)\), which is constrained to lie along a new contour given by \( R(x, z) = \hat{B}^\text{MMS}_N/\hat{B}^\text{MMS}_L \). Unlike the 1D case where \( x^* \) and \( S \) were uniquely determined (2 equations, 2 unknowns), here in the 2D case \( x_0, z_0, \) and \( S \) are not uniquely determined (2 equations, 3 unknowns). This implies that any location along the ratio contour (excluding locations where \( \hat{B}^\text{MMS}_L = 0 \)) can be used as \((x_0, z_0)\), which in turn determines \( S \) in equation 4.
Bibliography


Li, G., et al. (2012), How electron plasma flows in the reconnection diffusion region are modified by the presence of magnetic islands, American Geophysical Union, Fall Meeting, Abstract: SM21B-2264.


