INTERPRETATION OF NARROW BEAM SONAR ECHOS USING A VARIABLE IMPEDANCE SUBBOTTOM MODEL

ROBERT LEICESTER BOLUS
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Abstract
An underwater antenna consisting of an acoustic paraboloid reflector with a 12 kHz transducer mounted at its focus was designed and built for remote sensing of the subbottom [1]. Surveys taken with the 3 degree half-power beamwidth projector operating in a 1/2 ms pulsed mode indicated horizontal and vertical resolutions of less than 1 m with penetration into the sediments of up to 20 m at 20 m of depth. The image detail was substantially increased over that taken with a conventional 30 degree profiler.

The data was interpreted using a physical model of the subbottom consisting of plane parallel layers of variable impedance overlying those of monotonically increasing impedance to a basement of granite [2,3]. A signal processing technique, extracting both phase and amplitude information of the acoustic echos, was used to simulate the data. The magnitude and phase angle of the boundary reflection coefficients were calculated allowing determination of positive or negative changes in the characteristic acoustic impedance of the respective layers. Relative impedance profiles, plotted over a region where facsimile graphs indicated a continuous subbottom layer, showed a consistent decrease of impedance at that depth before increasing again as expected.

Keywords
Physics, Acoustics
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University of New Hampshire

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INTERPRETATION OF NARROW BEAM SONAR ECHOS
USING A VARIABLE IMPEDANCE SUBBOTTOM MODEL

by

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A DISSERTATION

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In Partial Fulfillment of
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This dissertation has been examined and approved.

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December 8, 1980
Date
Dedicated to

my wife Margaret

and

children Shelly and Alec
Frontispiece - Relative impedance profiles showing a detectable decrease in 8 out of 10 at the same depth
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ABSTRACT

INTERPRETATION OF NARROW BEAM SONAR ECHOS USING A VARIABLE IMPEDANCE SUBBOTTOM MODEL

by

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University of New Hampshire, December 1980

An underwater antenna consisting of an acoustic paraboloid reflector with a 12 kHz transducer mounted at its focus was designed and built for remote sensing of the subbottom [1]. Surveys taken with the 3 degree half-power beamwidth projector operating in a 1/2 ms pulsed mode indicated horizontal and vertical resolutions of less than 1 m with penetration into the sediments of up to 20 m at 20 m of depth. The image detail was substantially increased over that taken with a conventional 30 degree profiler.

The data was interpreted using a physical model of the subbottom consisting of plane parallel layers of variable impedance overlying those of monotonically increasing impedance to a basement of granite [2,3]. A signal processing technique, extracting both phase and amplitude information of the acoustic echos, was used to simulate the data. The magnitude and phase angle of the boundary reflection coefficients were calculated allowing determination of positive or negative changes in the characteristic acoustic impedance of the respective layers. Relative impedance profiles, plotted over a region where facsimile graphs indicated a continuous subbottom layer, showed a consistent decrease of impedance at that depth before increasing again as expected.
CHAPTER 1

INTRODUCTION

1.1 Purpose

The goal of this dissertation is to obtain better information about the ocean floor and sub-floor than is currently possible through acoustic remote sensing by increasing the angular resolution of low frequency sound sources necessary for penetration into the sediments. If the resolution can be increased to the point where the beam lies completely within the bounds of a remotely insonated target, then there is useful information in the phase of the echo signal as well as in its amplitude. This information can be used to define the boundaries of two vertically or horizontally adjacent targets as well as to indicate positive and negative changes in the relative acoustic impedance. A high resolution acoustic projector and a signal processing scheme using both phase and amplitude information are investigated to attain the goal.

1.2 Background

The need for detailed subbottom information exists by the many professional enterprises and federal agencies who are actively engaged in ocean ventures. Among these are geologists, oil and ore prospectors, coast and geodetic surveyors, archaeologists, the National Oceanic and Atmospheric Agency, and the Environmental Protection Agency. Conventional methods of sensing the subbottom structure with the use of sound waves are described by Hersey [4] in the chapter on continuous reflection profiling in "The Sea". He lists the general characteristics
of several acoustic profiler sources which are: approximately 1 ms long output pulses of near cavitation-level sound pressure, 1\% conversion of thousands of joules of stored energy into sound energy, and sounding repetition rates on the order of 1 s. Since the projector apertures are no more than 1/3 m and the signal frequency spectrums are from 1 to 1000 Hz, the acoustic radiation patterns for these sources are omnidirectional. Knott and Bunce [5] attempted to increase the directivity by using two simultaneously timed sources towed from either side of a ship and Rona et al. [6] in a departure from conventional profiler sources increased the directivity by using an array of linear-1000Hz-tunned transducers to achieve a 1 by 1.5 radian sound beam. Because the amount of information about the subbottom structure is limited by the angular resolution of the sound beam, these are considered as important first steps taken toward achieving a better "picture" than has previously been obtainable.

1.3 Improvement of Subbottom Information Detail

The mapping of sounding echos into reflection images has been done by modulating the gray scale of a facsimile plotter by the envelope amplitude of the received signal [7]. A way to improve the detail of a subbottom profile taken along a line is to use a narrower sound beam such that the size of the lateral area from which the echo returns is smaller at any point, and to gather more information points along the line. Not only will the detail improve from the increased resolution, but also the signal-to-noise ratio of vertically returned information will improve as the angle through which diffracted interference signals return becomes smaller [3].

The extraction of subbottom structure information from the echos has been one of the prime problems addressed by analysts using digital
signal processing techniques [8,9,10]. Efforts to date have been made to deconvolve the multiple round-trip reflections generated within the water column layer from the series of reflections generated from the deep subbottom structures. This works well where the topography of the subbottom is configured in nearly flat gently-varying layers, however in regions above ridges, underwater mountains, scarps and other rapidly changing structures the results are questionable. With an improvement of the directivity of the sound source, not only is it possible to do this sort of signal processing over rougher topography where several beam widths would insonate a linear segment before a structural change was encountered, but also valid attempts may be made to use the phase shift information of carrier modulated echos which here-to-fore has had a random sort of character.

1.4 The Subbottom Surveyor

Shaping of sound beams by reflectors has been tried and written about [11,12,13]. In particular the use of large paraboloidal reflectors for transmission of sound through the sea was investigated by Gregory [14] and used by Kronengold and Toulis [15]. In order to consider the use of reflectors for subbottom sounding, both the properties of the propagation of sound waves in matter and the directivity of reflection antennas were taken into consideration. The nature of the propagation of sound waves into the sediments is such that they are attenuated about as the first power of frequency. On the other hand to derive good directivity from a reflector, the diameter must be several wavelengths across. There is, therefore a tradeoff to be made between the physical size of the reflector and the frequency of operation of the sound source. The advantage
of using a reflector over a non-linear "parametric" device is the high
directivity of the reflector in both transmit and receive modes, whereas
current applications of parametric devices achieve high directivity only
in the transmit mode. The advantage over using an array of sources or
receivers is that only one transducer element is needed. Since no
application of reflectors had been made to subbottom sounding and the
above advantages exist over other methods, the "surveyor" was built as a
3 m paraboloidal reflector underwater antenna, operating at 12 kHz and
was used to sense the subbottom structure. The experimental data, when
displayed in real time on a facsimile plotter in comparison with data
from a conventional profiler operating at the same frequency, showed a
substantial increase in resolvable detail.

1.5 Model Development

With the improvement of the vertical information content in the
received echo signals, it was necessary to be able to extract structural
information from them and to display it in a meaningful way. The ex­
perimental data of the facsimile plots was viewed to theorize what may
have caused the patterns of light and dark seen on them, to construct
mathematical models of the assumed structure, and to synthesize signals
from them for comparison with the experimental signals. The experimental
plots showed in part a vertical time series composed of several low
amplitude single pulse width echos and several high amplitude, unresolved,
greater than single pulse width echos. The low amplitude pulses were
assumed to be reflections from discrete boundaries and the unresolved
echos were assumed to be from a region where the impedance changed in a
nearly continuous way as a function of depth. A combined discrete and
continuous vertical model, based on the Riccati differential equations,
was used to simulate the experimental data, where the continuous part of the model approximated the echo by assuming higher than first order multiple reflections as negligible. Because of this and the fact that digital computers are particularly well suited to do discrete analysis, a discrete plane-parallel-layer model which counted all multiple reflections was used for analysis.

Further observations made of the facsimile plots showed semi-randomly positioned reflectors in the sediment column with high definition of subbottom features when taken with the 3° surveyor, while those taken with the 30° profiler over the same track showed layers and loss of images. To demonstrate that this was consistent with the beamwidths used, a lateral mathematical model was used to synthesize reflection signals from a raised 20 m thrust plate and an equal laterally-spaced sequence of 2 m disks embedded in a constant impedance medium at the same depth. The results of this simulation showed that in the case of insonation with the 3° beam, the disks and edges of the thrust plate were well defined, whereas in the case of insonation with the 30° beam the image of the disks was smeared into a layer and the image of the thrust plate was nearly lost. These results were in close agreement with the experimental data.

1.6 Analysis and Interpretation

The data was interpreted in a way to satisfy the assumed final layer boundary condition. The analytic model was found by inverting the selected synthesis model where the output of the synthesis model became the analytic model input and the analytic model outputs became the estimation of the subbottom parameters [16,17]. A digital signal processing scheme, implementing the discrete model of plane-parallel-
layers of constant real impedance, used both the echo envelope amplitude information and the carrier phase shift information to extract the boundary reflection coefficient values. The results of this processing pointed out the need, especially in the regions where the echo pulses were unresolvable, to interpret the physical situation in these regions in a way which would lead to an acceptable solution. The interpretation took the form of adjusting the boundary reflection coefficients within the model to all be positive below a certain depth where the last high-amplitude unresolved region of echo returns were located. It was assumed that this was the region just prior to the granite bedrock where the materials should show a monotonic increase in characteristic impedance because of compacting due to the overburden pressure. To achieve the proper impedance value at the final boundary, the scale factor which was used to account for the amount of spherical spreading loss in the water column, was adjusted.
2.1 Antenna

With an acoustic antenna it is possible to increase the lateral resolution and signal to noise ratio over that of conventional profilers for remote sensing of the relative acoustic impedance of the bottom and subbottom. Possible configurations are: (1) an array of active elements, (2) one active element and a reflector, and (3) one active element operated in a non-linear mode to achieve a pseudo array [20,21, 22]. The first configuration was not used because of the number of active elements needed at 1/2 wavelength spacing for an antenna with narrow beam width. For example, 100 elements are needed in a 5 wavelength diameter array for an angular resolution of 12 degrees. The third configuration was not used because of the low efficiency (less than 1%) of mixing the two primary frequencies in water which form the pseudo array at the difference frequency. The narrow beam is lost completely at reception because the signal to noise ratio is at too low a level to recover the signal after low efficiency mixing, less than 1%. In order to project to and receive from a small spot in the acoustic field with as few elements as possible, a single paraboloid reflector with a sound transducer mounted at its focus was chosen as shown in Figure 2.1. A photograph of the working antenna mounted on the research vehicle at the test site is shown in Figure 2.2.

An approximate mathematical analysis may be used for the determination of the radiated acoustic field from the antenna. Huygen's
Figure 2.1 - Paraboloid with Transducer at Focus

Figure 2.2 - Antenna at Test Site
Principle states that the disturbance at a point P in a radiation field can be found by summing the spherical wave contributions from all secondary sources located on a surface σ which surrounds the primary source S [23]. By a Huygen's construction, as in Figure 2.3, it can be shown that a parabolic reflector converts a spherical wave emitted from its focus to a plane wave leaving the paraboloid within the region bounded by the reflector's surface and a plane across its opening. The aperture at the opening can then be considered as a circular piston radiator with a source distribution determined from the beam pattern of the transducer insonating the paraboloid.

The mathematical application of Huygen's Principle, Kirchoff's Integral Formula, is used to find the field radiation pattern,

\[ p(r) \sim \frac{j}{\lambda} \int_S b(r') (e^{-jks/s}) q(\theta) r' \, dr' \quad \text{....} \quad (2.1) \]

where

- \( q(\theta) = (1 + \cos \theta)/2 \) is the obliquity factor of the secondary source,
- \( b(r') \) is the beam pattern of the primary source, and \( e^{-jks/s} \)
  is the spherical propagation factor for the secondary sources [24].

Figure 2.4 shows the geometry of the field calculation and defines the space variables. The field may be divided into two parts. The Fresnel or near-field is found where the phase angle of the radiated wave from any secondary source varies over many cycles as the field point moves over one aperture distance perpendicular to the radial axis and the Fraunhoffer or far-field is found where the phase angle varies less than one cycle under the same conditions. The resultant summation of the radiated waves
TRANSMITTED SPHERICAL WAVE
DIRECTRIX
PARABOLOID REFLECTOR
REFLECTED PLANE WAVE
SPHERICAL SECONDARY WAVE
SPHERICAL PRIMARY SOURCE, S
FOCAL POINT

Figure 2.3 - Huygen's Construction for Paraboloid Reflector

PARABOLOID REFLECTOR
FIELD POINT
CENTRAL AXIS
TRANSDUCER AT FOCAL POINT
APERTURE OF DIAMETER, D

Figure 2.4 - Geometry for Radiation Field Calculation
from all secondary sources at a point in the Fresnel field fluctuates rapidly from high to low intensity over one aperture distance from the radial axis, whereas the resultant summation in the Fraunhoffer field varies smoothly in intensity over the same distance with a maximum on the radial axis. The phase angle is determined by $k_s$, where

$$s = \left[ r^2 + r'^2 - 2rr'\cos (r, r') \right]^{1/2}$$

(2.2)

and the amplitude is proportional to $1/s$, where

$$1/s \sim 1/r$$

is an adequate approximation. The near-field is found by numerical integration of Kirchoff's Formula using the binomial expansion of $s$ in the phase expression,

$$s \sim r + (r'^2 - 2rr'\cos \theta)/2r$$

(2.3)

which is quadratic in $r'$. The far-field is found by making the further approximation in the phase expression,

$$s \sim r - r'\sin \theta \cos \theta$$

(2.4)

which is linear in $r'$. Since the sediments attenuate sound roughly proportional to the first power of frequency, low frequencies on the order of 200 to 6000 Hz are conventionally used in subbottom sounding [4]. This means that efforts to achieve high directivity result in physically large radiating devices. For example, the sonic wavelength at 6000 Hz in water is 25 cm., where

$$\lambda = c/f$$

(2.5)
is the wavelength. For a 1 degree half-power beamwidth, the aperture
diameter of the radiating source becomes 15 m at 6000 Hz and larger at
the lower frequencies,

\[ \phi = \lambda / D \]  

(2.6)
is the -3 dB beamwidth of the far-field piston source. As high
directivity is achieved, the boundary between near and far fields moves
to large distances from the source,

\[ r_{\text{bound}} \]  

(2.7)
is commonly given as the boundary distance [25]. Using the numbers in
the example considered above, \( r_{\text{bound}} \) is 900 m. It becomes important to
find the exact distance from the radiating aperture to the point in the
field where the sound is focused for optimum design of shallow water sub-
bottom sounders.

A numerical study of the near field of underwater piston sources
has been made by Zemanek [26] for the range of values \( 2 < D / \lambda < 40 \).
The results of this study are that for that range of values and \( D / \lambda \)
extrapolated to larger values, there is a minimum - 3 db spot size
located at,

\[ r_{\text{min}} = 3/16 \ D^2 / \lambda \]  

(2.8)
in the radiation field with a diameter of,

\[ d = 1/8 \ D \]  

(2.9)
This distance is appreciably less than the near-far boundary distance
and the spot size there is less than the near-far spot size.

Equations 2.5, 2.6 and 2.8 can be used in the design of the
antenna. By specifying any two variables, the other three are then determined. For example, in the current design a 3 m reflector was chosen at an operating frequency of 12 kHz. The angular resolution was then determined to be approximately 3 deg. and the minimum depth of operation was fixed at 13.5 m. An alternative method of design would be to use the nomogram relating equation 2.5, 2.6, 2.8 shown in Figure 2.5. As an example as one lowers the frequency to 1 kHz for better sediment penetration, a large reflector, 30 m, is required for the same angular resolution of 3 degrees. The minimum depth of operation is then fixed at 116.5 meters, which no longer allows operation in shallow water depths, i.e. less than 100 feet.

The location of the focal point of the paraboloid with respect to its opening is related to the F number of the reflector, which is the ratio of focal distance to diameter [13,27],

\[ F_{\text{number}} = \frac{\lambda}{D}. \]  

The F number is chosen on the basis that most of the central lobe of the field pattern of the transducer mounted at the focus be intercepted by the paraboloidal surface. By referring to the nomograph in Figure 2.6 for a piston source, the total intercepted angle for the -10 dB beamwidth of the MASSA TR-63 transducer used, is found to be 60 degrees. Since

\[ \tan \phi \approx \frac{D/2}{\lambda} \]  

for a shallow paraboloid, and \( \tan 30 \text{ deg} \) is approximately .5, then \( \lambda \) is equal to \( D \) and an F number of 1 is sufficient for reflection of the major radiation lobe.

A case study of the radiation field for the experimental configuration was done on the digital computer with the program PLANER.FOR
Figure 2.5 - Nomograph for Antenna Aperture

Figure 2.6 - Nomograph for Piston Transducer [20]
found in Appendix A. A 25λ aperture diameter and a uniform source function which approximated the real distribution were used. Radiation patterns along the axis in the vicinity of the last near field null and maximum are plotted in Figure 2.7. Cross axis plots are shown in Figure 2.8 for the uniform distribution at the last maximum where,

$$r_{\text{max}} = \frac{D^2}{4\lambda}.$$

(2.12)

The -3 dB spot diameter at the maximum, determined to be 7λ from the plot, can be reduced to 3λ by moving 40λ closer to the source, refer to equations 2.8 and 2.9. When this effect is taken into account, the minimum depth of operation for the antenna is approximately 15 meters.

An acoustic coating was applied to the surface of the paraboloid to increase its reflectivity to near unity. The reflection coefficient at a boundary between two media is:

$$C_1 = \frac{(z_2-z_1)}{(z_2^*+z_1)},$$

(2.14)

where

$$z = \rho c$$

(2.15)

is the characteristic acoustic impedance,

ρ is the density, and

c is the velocity of propagation [25].

The reflection coefficient for an air-water boundary can be compared with that for an aluminum-water boundary when densities and propagation velocities are known. The information in Table 2.1 shows that the air-water interface reflects over 90% of the incident wave as compared with about 80% for the aluminum-water interface. With this as the goal, a 1 cm layer of an air foamed neoprene, Rubatex, was used as the acoustic coating as shown in Figure 2.9.
Figure 2.7 - Antenna Axial Radiation Plot

Figure 2.8 - Antenna Cross Axis Radiation Plot
<table>
<thead>
<tr>
<th>Medium</th>
<th>Density (kg/m³)</th>
<th>Velocity (m/s)</th>
<th>Acoustic Impedance (rayls)</th>
<th>Reflection Coefficient (re H₂O)</th>
</tr>
</thead>
<tbody>
<tr>
<td>air</td>
<td>1.293</td>
<td>331.6</td>
<td>428</td>
<td>0.93</td>
</tr>
<tr>
<td>water</td>
<td>1000</td>
<td>1500</td>
<td>1.5x10⁶</td>
<td>0.0</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2700</td>
<td>6300</td>
<td>17x10⁶</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Table 2.1 - Acoustic Properties of Air, Water, Al [25]
2.2 Electronics

The following considerations were used in designing the electronic system:

1. portable operation,
2. maximizing the electronic signal to noise ratio,
3. minimizing power consumption,
4. minimizing acoustic sources of noise,
5. several hours of operation before recharging batteries, and
6. utilizing as far as possible existing equipment and designs available [28].

Following these principles, lead-acid cell 12 volt batteries were used for power, the driver and receiver pre-amplifier of conventional design were located in close proximity to the sound transducer, all solid-state design was used throughout, a crystal controlled synchronous timer was used for exact reproduction of wave-forms, and a standard facsimile graphics recorder and magnetic tape recorder were used. The electronics are described as two subsystems:

1. an underwater transmitter and receiver interfacing with the transducer, and
2. a control and recording surface system under operator control [1].

A block diagram of the underwater electronics is shown in Figure 2.10. The transducer, a MASSA TR63 was energized at a 1 to 4 Hz rate with a .5 to 2 ms pulse width of a 12 kHz square wave carrier. To minimize cross-coupling a low level drive signal was sent from the surface along with the transmit/receive (T/R) enable signal, DC power, and echo signals in a single cable. The T/R switch protected the receiver
RUBATEX foamed neoprene

ALUMINUM REFLECTOR

$3/8"$

Figure 2.9 - Acoustic Coating on Reflector

DC/DC CONVERTER

ENERGY STORAGE CAPACITOR

MASSA TR-63 TRANSUDER

RECEIVER

DRIVER

T/R SWITCH

TRANSFORMER

Figure 2.10 - Block Diagram of Underwater Electronics
by switching 60 dB of attenuation into the receiver path when the transducer was energized. The driver amplified the drive voltage to a 150 volt square wave level and a 2100 μf capacitor supplied the energy needed to pulse the 25 ohm AC load at .8 kW. A 1:8 step-up transformer was used to increase the voltage to the maximum drive level consistent with the lead zirconite crystals in the transducer which had a nominal load resistance of 1600 Ω at 12 kHz. After 8 ms the transmit enable signal went low and the T/R switch connected the transducer directly to the receiver to maximize the echo signal reception. The receiver pre-amp was a low noise differential instrumentation amplifier which provided gain at the transducer before the signal was sent to the surface. A photograph of the transducer with its electronics and waterproof compartment is shown in Figure 2.11.

A block diagram of the control and recording electronics is shown in Figure 2.12. Two 12 volt, 72 amp-hour batteries supplied the power. A crystal clock oscillating at 144 kHz provided stable and synchronous timing to the system. The clock frequency was counted down to provide the 12 kHz signal carrier, a continuous 12 kHz reference signal, the time intervals for pulse widths, T/R enable, repetition rates, and a 60 Hz signal used to synchronize a power inverter that supplied power to the facsimile and tape recorders. The echo was further amplified in the surface electronics by adjusting the signal gain in a continuous or discrete step fashion before going to the recorders. A portable oscilloscope allowed visual observation of the echo in real time for level setting to 1 volt pk-pk into the tape recorder. Several control modes allowed automatic switching sequences of drive and received signals. They were:
Figure 2.11 - Transducer, Electronics and Waterproof Housing

Figure 2.12 - Block Diagram of Control & Recording Electronics
(1) alternating carrier frequency between 10 and 12 kHz,
(2) alternating receiver gain from low to high, and
(3) combinations of those two.

The 12 kHz reference was frequency multiplexed with two inclinometer signals, to indicate the orientation of the antenna, at 600 Hz and 2400 Hz before going to the tape recorder. A photograph of the control and recording electronics at the test site are shown in Figure 2.13.

2.3 Mechanical

The research vessel and mechanical configuration of the experimental equipment is shown in Figure 2.14 at the test site. The research vessel is a double pontooned, decked craft with A-frame winch and hoist, and provisions for self propulsion. The reflector antenna is shown being deployed over the stern of the vessel. It pivoted around a specially designed double hinged frame as it was winched into the water to a vertically down directed position [29]. A conventional 30 degree profiler was attached to the vessel by an aluminum support member shown outboard at the bow. In order to move external sources of acoustic noise away from the antenna and profiler, a Boston Whaler, seen in the left background, was used to tow the vessel through the water. It was found that the antenna maintained a stable orientation in the water at speeds up to .5 m/s.
Figure 2.13 - Control & Recording Electronics at Test Site

Figure 2.14 - Mechanical Configuration at Test Site
CHAPTER 3

EXPERIMENTAL RESULTS

3.1 Methods

Experimental data was taken during the summers of 1977 and 1978 at the UNH Ocean Modeling Facility at Lake Winnipesaukee, N.H. A navigation chart of the test range with marked cruise courses is shown in Figure 3.1. The water depths ranged from 13 to 30 meters with observed sediment depths from 1 to 16 meters. Two types of data were taken:

(1) fixed position data where the research vessel was anchored and the antenna system, surveyor, was tested for repeatability, and

(2) constant speed data over straight line transects between marker bouys where subbottom surveys of interesting structure and maximum depth of sedimentation were investigated [1].

Several different experimental configurations were used:

(1) operating the surveyor with gain switching between low and high,

(2) operating a conventional 30 degree profiler over the same course, and

(3) operating both surveyor and profiler over the same course at the same time by time multiplexing transmit and echo signals.

In all cases the echo data was input to a facsimile recorder for real time observation and to a magnetic tape recorder for further signal processing.
3.2 Data

Figure 3.2 shows data taken at a fixed position test site. There are three echo amplitude envelopes taken consecutively while anchored in calm weather conditions. These were generated from the tape recorded data by sampling, digitizing, and demodulating them with a digital signal processing scheme. They show good repeatability from echo to echo over a 1 second time interval. This verified the operation of the surveyor for consistent insonation of a small area of the bottom and subbottom.

Data was taken over several transects, but the most interesting appear to be over courses C, D and E, between Diamond Island and Rattlesnake Island. The data in Figures 3.3 and 3.4 were taken over C and D respectively with the surveyor. The transmitter pulse occurs at the top of the figure followed by a striated area which is due to reflector to transducer reverberations. A light area follows over which no sound returns to the receiver as the sound transits the water column followed by indicated bottom and subbottom echos. The vertical scale between marker lines is 6 ms one-way travel time in water and the horizontal scale is 6 m for the length of the marker line corresponding to a tow speed of .5 m/s. Note the image clarity and the existence of both horizontal and vertical structures in the subbottom. For instance, Figure 3.3 shows a vertical structure approximately at a quarter of the way through the transect. Both profiles clearly show the existence of two major discontinuous layers near the bedrock. Above these the subbottom echos form a pattern indicating an aggregation of scatterers which is different from a model of smoothly layered media [30]. Cores were taken on another project were found to indicate only the presence of a vegetational ooze in the subbottom with no definite layered structure.
Figure 3.2 - Fixed Position Repeatable Echos

Figure 3.3 - Profile with High resolution 3 Deg. Beam Antenna
Figure 3.5 shows data taken over the D transect with the 30 deg. transducer, MASA TR63, pointed down and no reflector. The data show the hyperbolic arcs formed by diffracted waves from sharp edge objects and show striations in the sediment column, indicating that the scattering aggregations of the previous graphs have been averaged by the wide beam into pseudo-layers. The identity of structures within the sediment column or at the basement rock level is lost in the long reverberatory tail caused by wide angle reflected and diffracted waves. This tail is evident below the basement rock in Figure 3.5 when compared to the sharp cut off below the basement rock in Figure 3.4.

A time multiplexing scheme was used to operate both the surveyor and a 30 deg. profiler, ORE Model 1032 profiling system, simultaneously over the same transect. A comparison of narrow beam to wide beam data was made for a better interpretation of sediment formations. Data was taken in the following places:

1. regions showing sedimentary layers with the wide beam and thin scattering aggregations with the narrow beam,
2. regions showing structural images with the narrow beam, and
3. regions showing thick scattering aggregations in the sediment column with the narrow beam.

Field data, taken in real time over course transect E is shown in Figures 3.6 and 3.7. Each figure represents two complete displays, where:

1. the top half of the graph is data taken with the 3 deg. surveyor, and
2. the bottom half is data taken simultaneously with the 30 deg. profiler.
Figure 3.4 - Profile with High Resolution 3 Deg. Beam Antenna

Figure 3.5 - Profile with Conventional 30 Deg. Beam Transducer
Looking at any of the displays, the transmitter pulse occurs at the top followed by a striated region indicating reverberations in the system. These are due to multiple reflections between the air water interface and the profiler, and multiple reflections between the reflector antenna and its transducer. A clear area follows where no sound has returned followed by indicated bottom and subbottom reflections. The vertical and horizontal scale is the same as before, 6 ms vertical and 6 m horizontal.

Figure 3.6 (top) shows a region in the left-half of the graph where the narrow beam echos from a thin aggregation of irregular shapes in the sediment column. Figure 3.6 (bottom) shows apparent layers indicated by the wide beam throughout the sediment column. Since the wide beam collects information over a large area, a segmented aggregation of scatterers could be averaged into a layer, whereas the narrow beam would indicate each scatterer independently within its resolution.

Figure 3.7 (top) shows a discontinuity halfway through the track in the basement area, where 20 meters of it is thrust up above the surrounding floor. In the right half of the graph, there is a thick aggregation of scatterers indicated above a continuous floor. Figure 3.7 (bottom) does not show the presence of these structures, but is masked by the long reverberatory tail due to wide beam angular reflected and diffracted waves which are received after the vertical echo for that depth.

3.3 Digital Demodulation

Although the facsimile graph presents a real time profile along the cruise track, some of the signal is lost or distorted because of the limited number of gray scale shades and the demodulation technique. A
Figure 3.6 - Transect E Profile: 3 Deg. Beam (top), 30 Deg. Beam (bottom)
Figure 3.7 - Transect E Profile: 3 Deg. Beam (top),
30 Deg. Beam (bottom)
single diode, used to demodulate the signal, results in the loss of low level signals and an RC filter, following it, causes back edge fill-in due to the time constant. There are about 10 distinct shades of gray allowing a 10:1 amplitude quantization, whereas by observing the raw echo signal as in Figures 3.8 and 3.9, a ratio of 100:1 is possible. Figure 3.8 is a single modulated echo from the 3 deg. surveyor and Figure 3.9 is a single modulated echo from the 30 deg. profiler. A digital demodulation scheme was used to find the modulating signal from these echos with increased detail over that achieved by analog methods.

In order to use the available analog to digital converter interfacing with the DEC-10 computer, it was necessary to translate the max. signal frequency present of 16 kHz to less than twice the maximum sampling rate available. Since this was approximately 12 kHz per channel for the two channels being digitized, the analog data tapes were reproduced at a 4:1 speed reduction which lowered the maximum signal frequency present from 16 kHz to 4 kHz. The signals were prefiltered with a 5 kHz, 90 dB rolloff low pass filter to avoid aliasing the data by limiting the input spectrum. The data were sampled, passed through the analog to digital converter and stored on 9 track magnetic tape in a high density packing format [31].

A series of computer programs were used for handling the digitized data. The process was to unpack the data, find the transmit pulse, remove intervening zeros between it and the echo, and sort the echo signals into an orderly numbered computer file. A 1024 point record of each echo frame was used in the digital demodulation scheme to obtain the envelope of the signal over 20 ms which was sufficient to completely display the echo time series. The programs used for unpacking, synchronizing,
Figure 3.8 - Single Echo Taken with 3 Deg. Beam Surveyor
Figure 3.9 - Single Echo Taken with 30 Deg. Beam Profiler
The following digital demodulation process was used. A 1024 point echo record was transformed from the time domain into the frequency domain by a Fast Fourier Transform (FFT),

$$S(\omega_i) = \sum_k s(t_k) e^{-j\omega_i t_k}.$$  \hspace{1cm} (3.1)

The Fourier Transform of the analytic signal was found by,

$$A(\omega) = 2S(\omega), \quad \omega \geq 0$$

$$= 0, \quad \omega < 0.$$  \hspace{1cm} (3.2)

The inverse FFT of the analytic signal spectrum was taken to get the analytic time signal,

$$a(t) = s(t) + jh(t),$$  \hspace{1cm} (3.3)

where

$$h(t)$$ is the Hilbert Transform of $$s(t).$$

The envelope of the time series was found by taking the magnitude of the analytic signal, [32, 33, 34]

$$|a(t)| = [s(t)^2 + h(t)^2]^{1/2}.$$  \hspace{1cm} (3.4)

The digital demodulation programs are found in Appendix C.

The results of digital demodulation on a single frame of a wide and narrow time series echo is shown in Figures 3.10 and 3.11. These allow a more detailed examination of subbottom echo structure than the facsimile graphs do. For example, low amplitude pulses due to reflections in the sedimentary column are seen and the wide black regions at the basement are now seen as a number of resolvable pulses in the wide beam.
Figure 3.10 - Envelope Magnitude of Wide Beam Echo

Figure 3.11 - Envelope Magnitude of Narrow Beam Echo
series and several unresolvable pulses in the narrow beam series. Reverberatory tails are seen following the basement echos of about 1 ms in the narrow beam series and 6 ms in the wide beam series.

3.4 Discussion

From observation of the facsimile graphs of Figures 3.6 and 3.7, an improved picture of the subbottom is given from surveyor data over profiler data. Images which are clearly defined in the narrow beam graphs are lost in the wide beam graphs. The trajectory of echo returns from sharp edge objects form hyperbolas as the beam moves horizontally over them which interfere with the vertical echos from new targets.

Some analytical calculations can be made of the excess two-way travel time and distance for a 30 deg. half-beam angle source and shown to agree with co-ordinate measurements of the hyperbolas in the wide beam data. Consider that the velocity of propagation through the sediment is near that in water. Then for a 30 deg. triangle and a depth to the bedrock of 40 meters, as shown in Figure 3.12, the excess distance is:

\[ S - d \sim \frac{1}{7} d = 5.7 \text{ m} \quad (3.5) \]

This distance would show up as the maximum vertical displacement co-ordinate on the hyperbolas and the corresponding horizontal co-ordinate is:

\[ \frac{S}{2} \sim \frac{4}{7} d = 23 \text{ m} \quad (3.6) \]

From a few measurements on the series of hyperbolas which pass through the edge of the basement thrust plate in Figure 3.7, one can verify that the above calculated co-ordinates are reasonably close. The reverberatory tail following the bedrock in the wide-beam data may also be explained by calculating the excess two-way travel time,
Figure 3.12 - Distance Relations for a 30 Deg. Triangle

\[ s^2 = d^2 + \frac{s^2}{4} \]
\[ d^2 = \frac{3}{4} s^2 \]
\[ s = \frac{2}{\sqrt{3}} d \]
\[ \sqrt{3} \approx 1.72 \approx \frac{7}{4} \]
\[ s \approx \frac{8}{7} d \]
\[ \Delta t = \frac{2(S-d)}{c} = 7.6 \text{ ms.} \quad (3.7) \]

This agrees reasonably closely to the measurement of 6 ms made from Figure 3.10.

Within the sediment column the narrow beam data shows thin aggregations of scatterers, while the wide beam indicates layers. To explain this phenomena, it is necessary to model the subbottom and simulate the data. The digital demodulation process provides envelopes of sufficient detail for comparison with modeling work [2,3]. These envelopes should provide improved graphical detail over that provided by the standard facsimile graphs.

The experiment shows that for high resolution subbottom sensing and better model interpretation, it is necessary to use two dimensional lateral information in depth. The difficulties of precise navigation (within a fraction of a wavelength) at sea along with the presence of wind and wave interference preclude successful two dimensional synthetic aperture methods. Therefore, it is necessary to increase the lateral resolution of acoustic devices to be used in subbottom sounding over conventional systems.
CHAPTER 4

MATHEMATICAL MODELS

4.1 Derivation Methods

Mathematical models were derived and used in the analysis of the experimental data. The derivations followed a process where the data was first looked at and theories for causal mechanisms were postulated. Deciding on a topographic structure and a subbottom layered model, the physical laws for acoustic wave propagation, reflection, and diffraction were applied to synthesize output echographs [41,42,43,44]. A discrete element approach was used to simulate the echographs from the structures model and a combined discrete element and continuous function approach were used to simulate the echographs from the layered model [45,46]. Because certain approximations had to be made in order to evaluate a non-linear integral equation for the continuous function, the simulated echographs did not include all multiple reflection terms. To incorporate all multiple reflection terms into the mathematical analysis of the data, it was decided that the discrete element description should be used exclusively.

In order to extract information from the acoustic echos which had been recorded as experimental data, the subbottom layered model was used. The analytic procedures were found by inverting the synthesis model to an analysis model such that the experimental data from an echograph was used as the input and subbottom parameters were the output. The model was expanded to include the determination of boundary
phase angle, non-uniform layer spacing, and nominal attenuation from up-to-date empirical laws for subbottom materials [47]. A method of deconvolving the boundary reflections from multiple reflections and the input pulse, where both magnitude and phase shift of the input signal were used, was derived. With this information, both the two-way travel times for the layers and the relative impedance profile were found and plotted for analyzed data.

4.2 Source Pulse

The excitation signal to the surveyor is a rectangular pulse modulated with 5 cycles of a 12kHz sine wave carrier. It is shaped after passing through the transfer function of the underwater acoustic transducer into a smoothly rising and falling pulse of about 1 ms duration. The source pulse, considered to be that which enters the underwater medium at the top of the water column, is modeled closely after the experimentally observed 1/2 ms pulse. Echographs, synthesized later in the chapter to show the ability of the models to qualitatively match the experimental data, require discrete samples of the source pulse spectrum. The Fourier Transform of the echograph.

\[ M(f) = H(o, f) I(f) \]  

(4.1)

where

\[ H(o, f) \] is the underwater system function, and
\[ I(f) \] is the source pulse spectrum, is calculated on a digital computer by finding the system function for discrete values of frequency. Since this involves either an iterative procedure over many boundaries in the layered subbottom model or a double integral over many secondary field sources in the geometrically
structured bottom model for each value of frequency, it is necessary to use a finite number of discrete frequencies to minimize the computation time [46, 48].

The spectral function with the minimum discrete frequency range which can be analytically transformed to a 1/2 ms pulse in the time domain is the rectangular function. For spectral width 2/T, the transform has a pulse width between first zero crossings of a sinc(t/T) function of T seconds. By taking a frequency range of 4kHz around the carrier frequency of 12kHz, a 1/2 ms pulse with side lobes is realized. This is the sort of spectral function which is desired, but the time domain side lobes have too large an amplitude compared with the major lobe which will lead to distortion of the model time series. To overcome this, a progressive % of raised cosine shaping of the rectangular edges was used while simultaneously widening the spectral frequency range until an acceptable result was obtained [49]. A 50% raised cosine spectrum shown in figure 4.1 over a frequency range of 6kHz transformed to a 1/2 ms wide pulse in the time domain with the first set of side lobes no more than 2% of the amplitude of the major lobe as shown in figure 4.2. A 1024 point FFT algorithm was used to transform the spectrum to the time domain. At a real time sampling rate of 51.2kHz, this allowed a 20 ms time window which was ample to model the echograph. The savings in computation time due to using a limited range spectrum function for the input pulse model is the ratio of the number of discrete frequency points not used to 1024. For both the geometrical and layered modeling, the CPU time to process a model was about 1 minute. Without the 90% savings, the CPU time would have been 10 minutes, which is prohibitively long.
Figure 4.1 - Frequency Spectrum of Assumed Input Pulse

Figure 4.2 - Time Domain Envelope of Assumed Input Pulse
4.3 **Echograph Synthesis from Subbottom Layers**

A facsimile plot of the vertical profile of the subbottom reflection series over a cruise track with the narrow beam 3 degree surveyor is shown in Figure 3.3. The transmitter pulse occurs at the top of the figure with several parallel lines following due to reflector and surface reverberation. The signal from the subbottom, shown below, reveals an accumulation of semi-random single pulse width reflections and a wide unresolvable signal both in the sediment column as well as across the basement. A model for synthesis of echograms is proposed, based on the form of the data and possible causal mechanisms. Since the material in the column is under increasing pressure as a function of depth, it is considered that the material in the main unresolvable region may be assumed to have continuous density and propagation velocity profiles as a function of depth [50,51,52]. Therefore the vertical impedance is modeled in the simplest case as several discrete layers above an inhomogeneous medium with an assumed variation in impedance as a function of depth [2]. The front edge reflection coefficient is then synthesized for an input pulse by solving for the reflection coefficient from the inhomogeneous medium and substituting it into the discrete layer reflection polynomial [45,46]. The echogram amplitude envelope for several different situations was obtained and compared with the experimental data shown in Figure 3.11.

Reflection signals are synthesized by simulating the acoustic input pulse, assuming the propagating field to be a plane wave, and modeling the subbottom as several plane parallel layers of constant acoustic impedance and a final layer of varying impedance as a function of depth. The synthesis of output signals is carried out on a digital computer by
Applying discrete sample theory to signals and spectra for information input to the computer. Linear system theory is used to find the output spectrum of a system composed of the propagating field and the reflecting medium with an input which is the simulated pulse. The output spectrum is then transformed to the time domain and operated on in such a way as to digitally demodulate the envelope time series from the carrier.

The reflection coefficient for \( n \) homogeneous discrete layers all of finite equal two-way travel times is \([46]\),

\[
R_n = \frac{C_0 P_n - Q_n}{(P_n - C_0 Q_n)}
\]  

(4.2)

where

\( C_0 \) is the first boundary reflection coefficient,

\( Z = e^{-2j\beta z} \) is the round-trip space propagation factor,

\( P_n = P_{n-1} - C_n Z Q_{n-1} \text{ }^R \) and

\( Q_n = Q_{n-1} - C_n Z P_{n-1} \text{ }^R \) are polynomials in \( Z \) called generating functions with initial values of \( P_0 = 1 \) and \( Q_0 = 0 \), and

\( P_n^R, Q_n^R \) are the reverse polynomials.

The generating functions are the numerator and denominator of the reflection coefficient \((-Q_n/P_n)\) evaluated at the first boundary. When that reflection coefficient is converted to impedance and combined with the characteristic impedance of the zeroth transmission medium, the overall reflection coefficient for the system of layers is obtained. The derivation is shown in Appendix H. For computations, the reflection series polynomial was approximated by neglecting terms involving a product of three prime coefficients compared with one in the numerator and a product of four prime coefficients compared with two in the denominator.
For an inhomogeneous layer with a continuously varying impedance as a function of depth, the reflection coefficient, $V$, satisfies a non-linear Riccati differential equation

$$\frac{dV}{dz} + 2jkV + \gamma(1 - V^2) = 0 \quad (4.5)$$

where

$$\gamma = \frac{1}{2\rho c}(d(\rho c)/dz) \quad (4.6)$$

is a function of the acoustic impedance of the layer,

$\rho c$ is the acoustic impedance of the layer which is a function of $z$,

$z$ is the down directed space variable, and

$k$ is the wavenumber.

The derivation of the Riccati equation for a layer which has an inhomogeneous impedance as a function of $z$ is shown in Appendix I.

Equation 4.5 does not have a closed form solution. However, a numerical iterative procedure can be used to solve for $V(\omega, \tau)$ where $\tau$ is the two way travel time to any point in the layer while $T$ is the total two way travel time for the layer \([44,45]\),

$$V(\omega, \tau) = e^{j\omega \tau} \int_{T}^{T} \gamma(s) e^{-j\omega s}[1 - V_2(\omega, s)] ds. \quad (4.7)$$

An inspection of the data in one region of interest suggests the subbottom may be assumed to consist of a combination of three discrete layers followed by a continuous layer. The approximate reflection coefficient for such a combination is then obtained by substituting the continuous reflection coefficient, $V$, for the last prime coefficient in the discrete reflection polynomial. The combined front edge reflection coefficient is approximated by,
\[
R = \frac{C_0^2 + C_1^2 + C_2^2 + v^2}{1 + (C_0 C_1 + C_1 C_2 + C_2 v) Z + (C_0 C_2 + C_1 v) Z^2 + C_0 v Z^3}
\]

(4.8)

where \( C_i \) are boundary reflection coefficients, \( V \) is the continuous layer reflection coefficient, and \( Z = e^{-2j\beta z} \).

The computer programs used to synthesize signals from the subbottom models are found in Appendix D.

4.3.1 Computer Simulation of an Inhomogeneous Layer

In the simplest case where compressional velocity and density vary linearly over depth for a particular sediment layer [50,51,52], the acoustic impedance varies as the square of depth over the layer. Assuming that the layer ends at a rock basement interface, there must be a limiting value to the impedance. A function which can approximate this behavior in a smoothed way is a shaped rising exponential function whose slope goes to zero at both ends. The gamma function for the shaped exponential layer (Eq. 4.6) is constant over most of the region of the exponential dropping to zero at each end by using a 10% raised cosine, (see Appendix J). Results from synthesis with this impedance function for a 3 m.s. two way travel time are shown in Fig. 4.3. The results show, for small variations in impedance, a reflected pulse during the beginning cosine function, another at the start of the exponential function, another at the end of the exponential function, and one more during the ending cosine function.
Figure 4.3 - Simulated Time Series Response from 3-ms Two-Way Travel Time Inhomogeneous Layer
4.3.2 Computer Simulation of an Inhomogeneous Layer Followed by a Basement Rock

The inhomogeneous layer is assumed to have a shaped exponential of 2 m.s., while the hard basement rock is simulated by means of a faster rising shaped exponential of 1 m.s. Results from this synthesis are shown in Fig. 4.4. There is a reflected pulse at the beginning and ending of each exponential function and during the beginning and ending cosine functions, resulting in six observable reflected pulses. The pulse for the middle cosine function is non-resolvable.

4.3.3 Simulation of the Experiment - Combined Discrete and Continuous Layer

From an inspection of the experimental results the subbottom is modeled to consist of three discrete layers combined with an inhomogeneous layer bounded by hard basement rock. Fig. 4.5 shows the impedance profile used for simulation of reflection signals. The three discrete layer reflection coefficients are combined with the reflection coefficient of the inhomogeneous layer followed by rock weighted by .75. Fig. 4.6 shows reflections from the three discrete layers separated by 1 m.s. two way travel time from each other and from the continuous function, and with a set of reflection coefficients equal to/.03, -0.1, +0.1, .75/. This combination resulted in observable reflected pulses from all layer boundaries, the expected pulses from the continuous function and the appearance of multiple reflected pulses within the time series and trailing the time series.

Comparing the synthesized reflection time series in Fig. 4.6 with a reflection series taken with the narrow beam surveyor (Fig. 3.11),
Figure 4.4 - Simulated Time Series Response for 2-ms Two-Way Travel Time Inhomogeneous Layer Followed by Basement Rock

Figure 4.5 - Impedance Profile for Simulation of Vertical Reflection Series
one notes that both the figures have a general qualitative similarity, viz, they have separated leading pulses, a wide unresolvable central series with a rapid decay at its back edge, and multiple reflected signals within and after the unresolvable series. In the experimental data the two way travel time for the unresolved region is greater than in the synthesized data, indicating a wider region of continuous impedance than that which was synthesized.

4.4 **Echograph Synthesis from Subbottom Structures**

In order to compare narrow beam and wide beam systems, data was simultaneously taken over the same track with a conventional 30 degree profiler. Facsimile plots of the subbottom signals taken simultaneously with a 30 degree profiler (bottom) and the 3 degree surveyor (top) are shown in Figs. 3.6 and 3.7. Signals in the sediment column of the left half of the surveyor graph (Fig. 3.6) form a pattern of semi-randomly sized and spaced reflectors, while those in the profiler graph form continuous multiple layers throughout. Signals in the basement trace of Fig. 3.7 indicate a large thrust up section in the surveyor plot which tends to merge into and disappear into the basement in the profiler plot.

To verify the edge effects from the wide beam, reflections from a large circular plate thrust up from the rock basement were simulated. Then a lateral model consisting of small circular disks of similar size and spacing, suspended in the sediment column was proposed to explain layer effects. Theoretical signals composed of waves reflected and diffracted from the lateral boundaries were synthesized with Kirchoff's Integral Formula which has also been used in surface topography estimation [53,54]. The effectiveness of the model to explain the experimental
data became evident as the results of source motion over the model produced a layer effect in the wide beam case and individual detected reflectors in the narrow beam case.

Reflections are simulated from a system consisting of the input pulse, a highly directive projected acoustic field, and circular disks of unity reflection coefficient embedded in the sediment column. The reflection spectrum is found from linear system theory by computing the spectrum of the lateral reflector model on a digital computer and multiplying it by the spectrum of the input pulse. The envelope of the reflection time series is found by the following digital demodulation technique. The spectrum of the analytic time signal is found and transformed by Fast Fourier Transform into the time domain. The envelope is found by taking the magnitude of the analytic time signal.

A mathematical model for the reflection coefficient of an arbitrarily shaped surface, S, insonated and received by the same transducer is [25]:

$$H(\omega) = \frac{-1}{\lambda} \oint_S f^2(\theta)cose \frac{e^{2jkr}}{r^2} r'dr'd\theta'$$  \hspace{1cm} (4.10)

where

- $\lambda$ is the wavelength
- $f(\theta)$ is the source beam pattern
- $cose$ is the obliquity factor
- $r', \theta'$ are coordinates on $S$
- $r, \theta$ are coordinates from source center to $S$
- $k$ is the wavenumber.

For a circular piston source the beam pattern is [25]:
\[ f(\theta) = \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \]  
\( (4.11) \)

where
\( a \) is the radius of the piston.

The reflection coefficient for both wide and narrow beams is found at each discrete frequency in the set by numerical integration either over the area \( S \) or over an area bounded by the zero of the first major radiation lobe of radius:

\[ B = \frac{Z_0}{D (1 - 1/D^2)^{1/2}} \]  
\( (4.12) \)

where
\( B \) is the radiation radius in wavelengths
\( Z_0 \) is the depth of \( S \) below the source
\( D \) is the ratio \( (2a/1.22 \lambda) \),

with the constraint of staying within the first major radiation lobe. This is a good approximation for the reflection coefficient as 84% of the energy passes through the major radiation lobe (see Appendix K).

The transfer function is programmed numerically as a single summation, where the double integration has been reduced to a line integral from inner disk edge to outer edge with the included polar angle calculated at each step. This increases the calculation speed over finding the transfer function from summing elemental areas. Fig. 4.7 illustrates the geometry of the reflector from which the transfer function is calculated:

\[ H(\omega) = 2 \sum_{n=1}^{N} f(r'_n, \theta'_n, \theta(r'_n)) \]  
\( (4.13) \)

where
Figure 4.6 - Simulated Time Series Response for Several Discrete Layers Followed by the Inhomogeneous Layer of Fig. 4.4

Figure 4.7 - Geometry for Calculation of Transfer Function with Kirchoff's Integral Equation
\( f(r') \) contains phase and amplitude information

\( r' \) is the radial coordinate of integration

and \( \theta'(r') \) is the included polar angle.

The included polar angle is:

\[
\theta'(r') = \cos^{-1} \left[ \frac{(r'-r_o)^2 + (r'_1-r_o)^2 - A^2}{2 (r'-r_o)(r'_1-r_o)} \right]
\]  

(4.14)

where

- \( A \) is the radius of the disk
- \( r_o \) is the beam center coordinate
- and \( r'_1 \) is the disk center coordinate.

The computer programs used to synthesize signals from bottom structures are found in Appendix E.

4.4.1 Simulation From a 20 Meter Thrust Plate

The thrust up region which appears in Fig. 3.7 with good definition in the narrow beam graph, scaled directly from the graph, is modeled as a 20 meter circular plate thrust up 4 meters from the surrounding basement rock. Reflection and boundary wave responses are found by computer simulation for both wide and narrow beams for the thrust plate and the surrounding basement below. Computations are simplified by the approximation that the vertical walls from the rim to the basement are non-reflective. The simulation is worked in two parts, first for the 20 meter plate with no surrounding basement, then for an infinite extent basement with a 20 meter hole in it. In this way the pulse trajectories may be found over lateral distance even though pulse interaction which occurs when trajectories from the plate cross those from the basement has been neglected. Fig. 4.8 shows reflected and boundary wave pulses for the 20 meter plate insonated by the 30 degree beam width projector.
as it moves from a coincident centered position outward to the rim and beyond. Several amplitude and arrival time series are plotted against a third dimension, distance, which is the displacement of the projector from the plate center. At 0 meters of displacement the first pulse (8a) is reflected and the second pulse (8b) comes from the rim which is simultaneously reached around its circumference by the sound wave. At 5 meters of displacement the first pulse (8c) is reflected from the surface, the second pulse (8d) comes from the near rim and the third pulse (8e) from the far rim. At 10 meters and beyond the series are composed of two pulses, one from the near rim (8f) and the other from the far rim (8g). The increasing distance of the projector from the plate shows up as an increasing time delay before the first pulse occurs and the amplitude decreases as less of the beam energy returns. Fig. 4.9 shows the results of insonating the same plate with a 3 degree beam width projector. The beam width is so narrow that at 0 meters distance the beam does not even extend as far as the rim, so only a reflected pulse (Sa) is seen. As the projector moves so that the rim is insonated, the beam is so narrow that reflected and rim pulses (9b) and (9c) merge together unresolvably. At 10 and 12.5 meters the near rim pulse (9d) and (9e) is seen and again the beam is so narrow that the far rim is not insonated. At 15 meters (9f) the beam is entirely outside of the plate and no boundary pulse is seen. The narrow beam gives a better indication of the shape of the plate over the translational direction of the projector than the wide beam.

The second part of the simulation, the infinite extent basement with a 20 meter hole, is worked in a similar manner. Those results have been added to the plate simulation results and appear plotted in
Figure 4.8 - Simulated 3-D Time Series Response Generated by Moving the 30 Deg. Beam (in 5-m intervals) Laterally Over the 20-m Reflective Disk

Figure 4.9 - Simulated 3-D Time Series Response Generated by Moving the 3 Deg. Beam (in 2.5-m intervals) Over the 20-m Reflective Disk
Fig. 4.10 for comparison with the experimental data in Fig. 3.7. Fig. 4.10 is a distance versus arrival time plot drawn in the style of the experimental facsimile plot. The trajectories are drawn for all points representing a range of amplitudes from maximum down to approximately 1% of maximum. This demonstrates that the effect of diffraction of acoustic waves can cause the thrust plate image to be lost in the wide beam case. Further, because of the slopes of the basement on either side of the thrust plate in the experiment, it appears to be hidden in the basement clutter.

4.4.2 Simulation of the Experiment from a Sequence of 2 Meter Disks

The situation of narrow beam indication of semi-random reflectors in the sediment column and wide beam indication of layers in the sediment column as shown in Fig. 3.6, can be resolved in the following way. The response from a collinear sequence of 2 meter disks, separated by 9 meters spacing each was simulated using a digital computer. The results show that the narrow beam surveyor indicates them all independently while the wide beam profiler transforms them into a layer. But consider the case where the collinear axis through the center of the disks is displaced from the center of the narrow beam such that it does not insonate them. Then the narrow beam surveyor indicates nothing, while the wide beam profiler still indicates a layer. Geologically, it is accepted that the sediments accumulate in layers and yet it is not surprising that either the layers are connectively discontinuous in one case or are made up of unconnected reflectors in another.

The simulation was worked in two parts. First, the response for a single 2 meter disk was found as a function of the lateral displacement distance of the source from its center for both wide and
narrow beam case until the returns were no greater than 1% of the return amplitude found when centered. The results of these numerical computations made with Kirchoff's Integral Formula (Eq. 4.10) are shown in Tables 4.1 and 4.2.

Then the responses from all disks, neglecting pulse interaction, were summed to find the trajectories shown in Fig. 4.11. Fig. 4.11 shows a pictorial view of the disk sequence data drawn in the style of a facsimile plot for comparison with the experimental data in Fig. 3.6. The wide beam combines them well enough to pick up the continuous representation of layers while the narrow beam sees them as discrete reflectors.

4.5 Synthesis Conclusions

The sequence model is only an approximation as the experimental narrow beam data shows semi-random positioned reflectors where the size and spacing of the disks is random, but the vertical position of the disks in the sediment column is not.

The experimental data was used to choose the parameters, such as the layer spacing, pulse shape and disc sizes and separation, for the synthesis of signals in the models. The effects of reflection from vertical layers and the reflection and the diffraction from lateral boundaries were worked separately. The simulations demonstrate the effect of several possible mechanisms which may have generated the experimental data.

4.6 Echograph Analysis by Synthesis Inversion

Consider a plane wave normally incident upon a set of plane parallel layers and focus upon the \( n + 1 \) st layer as shown in Figure
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<th>diffraction [rel. amp.]</th>
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Table 4.1 Wide Beam Case
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Table 4.2 Narrow Beam Case
Figure 4.10 - Simulated Response to the 20-m Thrust Plate Plotted in the Style of a Facsimile Plot for the 3 Deg. Beam (top) and The 30 Deg. Beam (bottom)

Figure 4.11 - Simulated Response to the Series of 2-m Disks Plotted in the Style of a Facsimile Plot for the 3 Deg. Beam (top) and the 20-Deg. Beam (bottom)
4.12. The wave is entering the layer from the left with unity amplitude. By counting the number of multiple reflections within the layer, each of which contributes to the reflected and transmitted wave, the total reflected wave can be written as a series, [55]

\[ V_n = C_n + (1 - C_n) V_{n+1} (1 + C_n) Z_{n+1} + (1 - C_n) V_{n+1}^2 (-C_n) (1 + C_n) Z_{n+1}^2 + \ldots \]

(4.15)

where \( C_n \) is the boundary reflection coefficient at the nth boundary, \( V_n \) is the total reflection coefficient at the nth boundary, and \( Z_n = e^{-2jkn+1} \) for a lossless material. (4.16)

By factoring the above expression and realizing that the remaining terms are a summation which converges, the reflection coefficient becomes,

\[ V_n = C_n + (1 - C_n^2) V_{n+1} Z_{n+1} \frac{1}{1 - C_n V_{n+1} Z_{n+1}} \]

(4.17)

A collection of terms over a common denominator yields,

\[ V_n = \frac{C_n + V_{n+1} Z_{n+1}}{1 + C_n V_{n+1} Z_{n+1}} \]

(4.18)

which is the formula for the reflection coefficient of an imbedded layer.

Front-end deconvolution of the echograph signal from an assumed number of discrete non-equally spaced plane-parallel layers proceeds as follows. The first pulse out of the layers is proportional to \( C_0 \) and the second one, by simply ray tracing is proportional to \( (1 - C_0^2) C_1 \). After that there are several different possibilities for the next pulse out. Therefore, \( C_0, 1 - C_0^2, \) and \( C_1 \) are now solved for from the amplitudes for the first two pulses in the time signal to within a proportionality constant. The boundary coefficient phase angle is found from the time
Figure 4.12 - Multiple Reflections from the n+1st Embedded Layer
series, assuming that the characteristic impedances are real. The phase shift for the time series is found as a function of time and from it is subtracted the time phase shift,

$$\phi(t) = \theta(t) - \omega t,$$

(4.19)

where $\theta(t)$ is the unwrapped continuous signal phase shift, and $\omega t$ is the time phase shift.

Then at the peaks of the first two pulses, the phase shift difference is taken and the two-way travel time phase angle is removed which leaves the difference in boundary phase angles,

$$\phi_1 - \phi_0 + \omega(\tau_1 - \tau_0) = \phi_{b1} - \phi_{b0},$$

(4.20)

where $\phi_b$ is the boundary phase angle, and $\omega(\tau_1 - \tau_0)$ is the two-way travel time phase angle.

By working through the signal time series from the front end, where it is assumed the $\phi_{b0} = 0^\circ$, the next boundary phase angle is found,

$$\phi_{b1} = \phi_1 - \phi_0 + \omega(\tau_1 - \tau_0) + \phi_{b0}.$$

(4.21)

A model is then used to find the reflection coefficient for the first layer alone,

$$V_{0,1} = \frac{C_0 + C_1 Z_1}{1 + C_0 C_1 Z_1},$$

(4.22)

where $V_{0,1}$ is the reflection coefficient at the 0th boundary for 1 layer, and all of the internal multiple reflections for that layer may be generated.

Before generating those reflection pulses, the reflection coefficient for the first one from the back boundary is removed. This is done so that when the remaining reflection coefficient is transformed to time for
subtraction from the echograph signal,
\[ r(t) - \frac{1}{2\pi} \int_{-\infty}^{\infty} [V_{0,1} - C_1(1 - C_0^2)Z_1] I(\omega) Z_0 e^{i\omega t} d\omega, \] (4.23)

the first pulse from the back boundary of layer 1 will be available for phase shift comparison with the first pulse of the back boundary of layer 2.

The process is then repeated for layer 2, i.e. the first pulse reflected from the front boundary of layer 2 is proportional to \((1 - C_0^2)\) \(C_1\) and the second pulse from the back boundary is proportional to \((1 - C_1^2)(1 - C_0^2)C_2\). These will be the next two pulses seen in the echo signal in time order. The boundary phase angle for \(C_2\) is found in a way similar to that used before and then using a model the reflection coefficient for the first two layers is found,

\[ V_{1,2} = \frac{C_1 + C_2 Z_2}{1 + C_1 Z_2} \quad \text{and} \quad V_{0,2} = \frac{C_0 + V_1 Z_1}{1 + C_0 V_1 Z_1} \] (4.24)

The reflection coefficient for the first pulse from the back boundary of layer 2 is removed in a similar way as before and after transforming to time, the model time series is subtracted from the echograph signal,

\[ r(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [V_{0,2} - C_2(1 - C_1^2)(1 - C_0^2)Z_1Z_2] I(\omega) Z_0 e^{i\omega t} d\omega. \] (4.25)

The process is repeated until \(r(t) - m(t) < \epsilon\), where the \(\epsilon\) is an acceptable remaining error or the end of the series occurs.

For front-end deconvolution, where the subbottom materials are considered lossy, the following procedure is used. First, the best
empirical data on subbottom materials has fit the $\alpha$ in the $e^{-\alpha X}$, attenuation factor, to [47,56]

$$8.686\alpha = hf,$$ \hspace{1cm} (4.26)

where $h$ is a constant depending on the material type. This is similar to the Coulomb-friction loss [57] where the boundary reflection coefficients, $C_1$, are pure real with either $a(+)$ or $(-)$ sign depending on whether the characteristic impedance of the next layer is greater than or less than that of the layer before. With attenuation included, the phase propagation factor becomes,

$$Z_n = e^{-2\alpha n X_n} e^{-2jk n X_n}.$$ \hspace{1cm} (4.27)

and the first pulse out of the first layer is proportional to $C_0$, while the second is proportional to $C_1(1 - C_0^2)e^{-2\alpha_1 X_1}$. These are the only kind of changes which must be made in the front-end deconvolution procedure which is otherwise the same.

To complete the discussion the scale factor, impedance profile and basement boundary condition are considered. Since the reflection pulse train is received at the transducer after passing through the water column, there is a proportionality constant between the pulse amplitudes and the value of the reflection coefficients. This has been called the scale factor, SF. All the $C_n$ after determination from the pulse amplitudes are scaled by the scale factor,

$$C_n = (SF) C_n.$$ \hspace{1cm} (4.28)

To set a value for SF, the $C_n$ is related to the characteristic impedances of the nth and n+1 st layers,
An impedance profile relative to the impedance of water taken as unity is generated by,

\[ C_n = \frac{(\rho c)_{n+1} - (\rho c)_n}{(\rho c)_{n+1} + (\rho c)_n}, \]  

(4.29)

The scale factor, \( SF \), is picked such that \( (\rho c)_{\text{base}} \) is that of granite, which is assumed to underlie the subbottom materials where the experimental data was taken [58].
CHAPTER 5

SIGNAL PROCESSING ANALYSIS

5.1 Description

Parameter estimation signal processing, shown in Figure 5.1, was used to analyze the experimental data. The data were the recorded output signals from the subbottom which was remotely excited by an acoustic source pulse. The data were sampled and digitized by an A/D converter and stored in a file in a digital computer. Subbottom layer parameters were estimated from the data for one layer at a time and used in a mathematical model of the subbottom to simulate first boundary and multiple reflections. These simulated signals were removed from the experimental signal in order to deconvolve first boundary reflections from source pulse and multiple boundary reflections. The process was then continued one layer at a time until the entire time series was analyzed. When all parameters had been estimated, they were transformed to relative impedance of the corresponding layers.

The input to the model is described by its waveshape, the transducer Q and bandwidth, and the projected and received waveshapes from the transducer. The input waveshape is 5 cycles of 12 kHz which are gated on at zero deg. phase angle with a positive going slope. The transducer has a Q ~ 1.5 with ±1 slope rolloff on either side of the center frequency. If it is assumed that the reflective boundaries of the sediments do not change the waveshape, then the received signal is similar to one which is output from a circuit with a Q ~ 3. It
Figure 5.1 - Parameter Estimation Signal Processing

Figure 5.2 - Received Waveform from the Transducer
takes $Q$ cycles for the transient to go 95.5% of the way towards its final value and 1.5 $Q$ cycles to reach 99% of the way [59]. Figure 5.2 shows the approximate received waveshape from the transducer. The transient shows that the envelope peak occurs at zero deg. of the carrier with a positive going slope.

The physics of wave propagation in water and sediments along with empirical data [60,61,62,63,64,65,66] are used to derive the mathematical model used. At the distance that the bottom is from the source and with the increment of sediment compared to the depth, plane wave propagation may be assumed. The bottom and sediments are assumed to be a series of plane parallel layers over the beamwidth of the antenna with normal incidence of the input wave. The geometry and the subbottom media are assumed to be such that the frequencies in the spectrum of the input pulse are not changed or non-dispersive. Geometrically, even though the velocity of propagation does vary as a function of depth, the input wave is propagated parallel to this variation and so is not refracted. The material parameters such as shear modulus, $\mu$, bulk modulus, $\lambda + 2\mu$, and coefficient of viscosity, $\eta$ are assumed to be non-frequency dependent. Therefore the velocity of propagation is non-frequency dependent or non-dispersive as a result of the material parameters. A further consequence of these assumed conditions is that the characteristic impedance of any subbottom layer is real and the reflection coefficient at any boundary may only have a zero deg. or 180 deg. phase angle. Nominal attenuation coefficients are assumed for the subbottom layers ranging from that for vegetational ooze, clayey silt, sand-silt-clay, sand, to that for granite which are all proportional to frequency to the first power [47,56]. A single factor scales all boundary reflection
coefficients to account for the spherical spreading loss through the water column.

The estimation of the system parameters involves the use of the above information in combination with digital signal processing techniques. The fundamental process of these techniques is the Fast Fourier Transform (FFT) which is an algorithm for the rapid computation of the Discrete Fourier Transform of a properly pre-filtered, sampled, and digitized signal. By proper shaping or windowing the spectra of an FFT, digital filtering is done. From the Hilbert Transform of the experimental data, the envelope magnitude and carrier phase shift of the modulated signal are calculated [32,33,34]. Peak detectors and phase unwrapping algorithms are used on the envelope magnitude and carrier phase respectively to remove the parameter information from the digitized signal. The mathematical model is used in the reconstruction of the signal to be feedback and compared with the original to close the loop. An open loop algorithm takes the parameter information and transforms it into relative impedance profiles for further interpretation.

5.2 Program Plan

An interactive signal processing program, Boundary Coefficients Analysis, was written for analysis of the experimental data, shown in block diagram in Figure 5.3. The software version of it, called BCOEFA.FOR is found in Appendix F.

The first block in the diagram is the FFT. This is an algorithm for calculating the Discrete Fourier Transform (DFT) in a particularly efficient way using both the properties of symmetry and periodicity. The DFT for a finite number of equi-time interval samples of a real
Figure 5.3 - Block Diagram of Boundary Coefficients Analysis
The time function is found by writing an expression for the sampled function,

$$x(t) = \sum_{n=0}^{N-1} x(n\Delta t) \delta(t-n\Delta t)$$  \hfill (5.1)

where

$\Delta t$ is time sample-interval, and

$\delta$ is the unit impulse function.

The Fourier Transform of both sides is taken,

$$X(f) = \sum_{n=0}^{N-1} x(nT) e^{-j2\pi fnT}$$  \hfill (5.2)

The frequency is made to be a discrete variable, such that,

$$X(f) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$$  \hfill (5.3)

where

$f = k/NT$.

Equation 5.3 is the DFT. To get the inverse DFT, IDFT, take Eq. 5.3 and sum both sides with a complex exponential,

$$\sum_{k=0}^{N-1} X(k) e^{j2\pi rk/N} = \sum_{k=0}^{N-1} \left( \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \right) e^{j2\pi rk/N}$$  \hfill (5.4)

By changing the order of the summations, the right hand side becomes

$$\sum_{n=0}^{N-1} x(n) \sum_{k=0}^{N-1} e^{-j2\pi (n-r)k/N}$$  \hfill (5.5)

The inner summation is,

$$\sum_{k=0}^{N-1} e^{-j2\pi (n-r)k/N} = N, \text{ for } n=r$$  \hfill (5.6)

$$= 0, \text{ otherwise}$$

The right hand side becomes equal to $N x(r)$. Therefore,
\[ x(r) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{j2\pi r k / N} \]  

(5.7)

which is the IDFT. the DFT pair may be written as,

\[ X(k) = \sum_{n=0}^{N-1} x(n) w^{-nk} \]  

(5.9)

\[ x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) w^{nk} \]

\[ w^{nk} = e^{j2\pi nk / N} \]

A four point FFT algorithm is realized as follows,

\[ X(k) = \sum_{n} x(n) w^{nk} + \sum_{n} x(n) w^{-nk} \]  

(5.10)

even \hspace{1cm} odd

Substitute 2r for even n and 2r+1 for odd n,

\[ X(k) = \sum_{r=0}^{N/2-1} x(2r) w^{-2rk} + \sum_{r=0}^{N/2-1} x(2r+1) w^{-2rk} \]  

(5.11)

Since, \[ w^{2nk} = e^{j2\pi nk / N} = e^{j2\pi nk / N/2} = w^{nk} \]  

(5.12)

equation 5.11 may be rewritten as follows,

\[ X(k) = \sum_{r=0}^{N/2-1} x(2r) w^{-2rk} + w^{k} \sum_{r=0}^{N/2-1} x(2r+1) w^{-2rk} \]  

(5.13)

= G(k) + w^{k} H(k).

The signal flow graph for equation 5.12 is shown in figure 5.4, where the following substitutions have been made using the periodicity property,
\[ G(2) = G(0) \] \hfill (5.14)

\[ H(2) = H(0), \text{ since} \]
\[ W_2^0 = e^{j2\pi0/2} = e^{j2\pi2/2} = W_2^0. \]

Also
\[ G(3) = G(1) \]
\[ H(3) = H(1). \]

The signal flow graph for the 2 point DFT is found by writing down the expressions for \( G(k) \),
\[ G(0) = x(0) W_2^0 + x(2) W_2^0 \]
\[ G(1) = x(0) W_2^0 + x(2) W_2^1 \] \hfill (5.15)

Then by using equation 5.12 and substituting into the flow graph of figure 5.4, the explicit signal flow graph for the 4 point FFT is found as shown in figure 5.5. The purpose of the FFT is to reduce the number of complex additions and multiplications from \( N^2 \), required for the DFT, to a smaller number. From the flow graph, there are \( N \) complex multiplications and additions per stage and there are \( \log_2 N \) stages. The number of equations have been reduced from \( N^2 \) to \( N \log_2 N \) which becomes appreciable as \( N \) becomes larger [32].

A graphical description showing how the discrete sample points relate to the time and frequency domain will aid in the detailed description of the filter implementation. There are \( N \) discrete data points taken to describe a truncated sample of continuous time function, \( T \) seconds in length. The sampling interval is \( \Delta t \) and \( n \) is an integer variable representing the \((n+1)\)st sample at the time \( n\Delta t \), as shown in figure 5.6a,
\[ t_n = n\Delta t, \quad n = 0, 1, 2, \ldots N - 1. \] \hfill (5.16)
Figure 5.4 - Signal Flow Graph for 4 Point FFT

Figure 5.5 - Explicit Signal Flow Graph for 4 Point FFT
The sampling rate $1/\Delta t$ is chosen to be greater than twice the highest frequency component in the continuous time function, satisfying Nyquist's sampling theorem. There are also $N$ discrete data points in the frequency domain describing the DFT of the time function. The total frequency range from 0 to $F$ includes both positive and negative frequencies. The frequency interval is $\Delta f$ and $k$ is an integer variable representing the $(k+1)$st sample of positive frequency of $k\Delta f$ for $k \leq N/2$ and the $(N-k)$th sample of negative frequency for $k > N/2$,

$$f_k = k\Delta f, \quad k = 0, 1, 2, \ldots N/2$$

$$= -(N-k)\Delta f, \quad k = N/2 + 1, \ldots N-1,$$

as shown in figure 5.6-b. The relation between the number of samples, frequency resolution, time sample interval, total time range and total frequency range are

$$\Delta t = 1/F,$$  \hspace{1cm} (5.18)

$$\Delta f = 1/T,$$

and

$$N = T/\Delta t = F/\Delta f$$

Although the data was reduced to four times slower than real time so that the maximum available sampling rate of 12.8 kHz would not be exceeded, the equivalent sampling rate for the real time data was 51.2 kHz. For analysis of the experimental data the equivalent sample rate of,

$$F = 51.2 \text{ kHz},$$

was used which allowed proper sampling of the highest signal frequency component of 16 kHz with ease. The number of points was chosen to be,

$$N = 1024,$$
to allow a display time,

\[ T = N \Delta t = N/F = 20 \text{ ms}, \]

of continuous signal which was adequate for all echoes analyzed.

The second block in the figure 5.3 is the filter which is rectangular with a unity pass-band from 9 kHz to 15 kHz. The filter is implemented in the frequency domain such that,

\[ F(k) = H(k) X(k) \]  \hspace{1cm} (5.19)

where

\[ H(k) = 1, \quad 9 \text{ kHz} \leq |k \Delta f| \leq 15 \text{ kHz} \]

\[ = 0, \quad \text{otherwise}. \]

Since,

\[ \Delta f = 50 \text{ Hz}, \]

the range of \( k \) for positive frequencies is from 180 to 300, and the range for negative frequencies is from 844 to 724, as shown in figure 5.7a. It is informative to examine the impulse response of the filter in the discrete sampled time domain. The Fourier Transform of a rectangular function is a \( \sin x/x \) function [67]. The first zero of that function in the time domain occurs at \( 1/f \), where \( f \) is the width of the rectangle in the frequency domain,

\[ T_z = 1/f = 1/6 \text{ ms}. \]

In terms of the number of discrete samples in the time domain, this is,

\[ (T_z/T)N = 8.5 \]

Two facts may be stated about this function, shown plotted in figure 5.7b, which are:
Figure 5.6 - Location of Discrete (a) Time and (b) Frequency Samples

Figure 5.7 - Filter in Discrete (a) Frequency and (b) Time Domain
(1) that the total zero-cross over width is 1/3 ms, which is close to the pulse width of the transmitted signal, and

(2) that the response is non-causal.

Fact 1 means that in effect the processing is doing a cross-correlation of the echo with the assumed transmitted pulse, which has the best possible signal-to-noise ratio, S/N, for extracting returning boundary pulses from the echo. Since the processing is not done in real-time, fact 2 only means that there is no delay introduced into the processed signal at this point in the process.

The next two blocks in figure 5.3 are an IFFT and a summing node which operate on the signal in that respective order. The IFFT transforms the discrete filtered spectrum back to the time domain where it is compared with the discrete-time experimental signal. At this point the computation loop is closed and an overview of the way in which the loop functions becomes appropriate. Each pass through the loop extracts the magnitude, two-way travel time, and phase shift information of the next detectable peak in time order from the experimental signal. This information is used to estimate the boundary reflection coefficient magnitude and phase for the next layer boundary in increasing depth order. The space propagation factor, found from two-way travel time and assumed attenuation coefficient, along with the boundary reflection coefficient is inserted into the recursive parallel-layer mathematical model algorithm. The algorithm is used first to compute the reflection coefficient at the top edge of the new depth layer. Then in a recursive way, using the stored boundary coefficients of the overburden layers, it is used to compute the reflection coefficients, $V(m, k) \ldots V(1, k)$, of the boundary at $m$, back to the first at the water-
sediment interface. Since it is desirable to leave the pulse from the last detected boundary in the experimental signal for phase comparison purposes with the pulse to be detected in the next loop pass, its Fourier Transform is removed from $V(1, k)$ while the transform of its multiple-reflections is left in. The resulting transfer function, $R(k)$, is multiplied by the spectrum, $I(k)$, of the assumed input pulse and an IFFT, is taken to get the model signal for comparison with the experimental signal at the feedback summing node. Refer to Appendix G for an example of the numerical comparison of $m(n)$ with $f(n)$ at that node. The loop process is repeated until all large amplitude pulses have been processed and the remaining ones are assumed to have been generated by multiple-reflections.

Inside the computation loop, the Hilbert Transform, analytic signal, and envelope magnitude blocks are used for digital demodulation of the input signal as described in detail in Chapter 3, equations 3.1 through 3.4. Once the analytic signal has been obtained, its components may also be used to extract the phase shift of the signal with respect to zero degrees of a cosine function in the following way. The discrete analytic signal is

$$a(n) = s(n) + jh(n)$$  \hspace{1cm} (5.20)

The signal phase shift is found,

$$\psi(n) = \tan^{-1} \frac{h(n)}{s(n)}.$$  \hspace{1cm} (5.21)

Since the arctan function is a two quadrant function, a phase unwrapping algorithm was used to generate the phase shift as a sampled continuous function of time.
The software program called UNWRAP is found in Appendix F. The computed phase shift $\psi(n)$ at the first point is compared with the polarity of the discrete sample from the modulated signal as in figure 5.8. This comparison determines which quadrant $\psi(n)$ is in, as shown by the possibilities and outcomes in table 5.1.

The angle of $\psi(n)$ with respect to zero degrees on the positive cosine function is determined and stored. The process is repeated for the next sample $\psi(n+1)$ which is differenced with $\psi(n)$ to find the incremental increase,

$$d\theta = \psi(n+1) - \psi(n). \quad (5.22)$$

If the increase is positive, then it is summed with $\theta(n)$, and if it is negative, then $(360-d\theta)$ is set equal to $d\theta$ and summed with $\theta(n)$,

$$\theta(n+1) = \theta(n) + d\theta. \quad (5.23)$$

The unwrapping is continued until all 1024 samples have been done and $\theta(n)$ as a discrete variable of a sampled function has been found.

The time phase is removed from $\theta(n)$ by subtraction,

$$\phi(n) = \theta(n) - \omega_c n\Delta t, \quad (5.24)$$

over the entire function. Plots of the envelope magnitude $|a(n)|$, and the unwrapped phase shift minus time phase, $\phi(n)$, for one of the first echos of figure 3.3 are shown in figures 5.9 and 5.10.

It is observed from these plots that the phase shift, $\phi$, takes on a constant value over the same points where there are well-resolved reflected pulse shapes indicated in the magnitude plot. This is to be expected for a reflected pulse from a well defined boundary of a non-dispersive media. It remains only to remove the space phase accumulated...
Figure 5.8 - Two Quadrant Phase Angle Values wrt Cosine
<table>
<thead>
<tr>
<th>polarity</th>
<th>+</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td>angle</td>
<td>+</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>IV</td>
</tr>
</tbody>
</table>

Table 5.1 Quadrant Determination for Phase Unwrapping
Figure 5.9 - Envelope Magnitude of Experimental Echo

Figure 5.10 - Carrier Phase Shift of Experimental Echo Minus Time Phase
from one pulse to the next and the calculated boundary phase of the top edge of the layer, to find the boundary phase of the bottom edge,

\[ \phi_b(m+1) = \phi(m) + \omega \tau(m+1) + \phi_b(m) \]  (5.25)

Here,

\[ \Delta \tau(m+1) = \tau(m+1) - \tau(m) \]

is the two-way travel time which is used in the propagation argument instead of \( z \), since it is directly measurable, where as \( z \) cannot be found without knowledge of the propagation velocity \( c(z) \);

\[ \tau = 2 \int_0^z \frac{dz}{c(z)}. \]

The index \( m \),

\[ m = 1, 2, 3, \ldots M \]

is the space index of the boundary layers. The principle part of \( \phi_b(m) \) is found and an estimated value for the boundary phase of the reflection coefficient for non-dispersive media is made,

\[ \hat{\phi}_b(m) = 0^\circ \quad \text{for } \phi_b(m) < 90^\circ \]
\[ \hat{\phi}_b(m) = 180^\circ \quad \text{for } \phi_b(m) > 90^\circ. \]

This procedure is carried out in figure 5.3 at the second summation node following the unwrap block.

The next three blocks consisting of the reflection coefficient, space propagator, and mathematical model make up the analysis model which has been discussed in detail in Chapter 4, Mathematical Models. The magnitude of the \( J \)th reflection coefficient, \( C(J) \), is found by removing the multiple reflections from the time series in front of where its reflected pulse appears. This is done by processing the series from the
front, one coefficient at a time by calculating multiple reflections from that boundary with all others in front and then subtracting them from the time series. The next pulse to be processed needs only to be corrected for transmission and attenuation effects,

$$|C(J)| = (SF) \frac{A(J)}{A_0 \prod_{m=1}^{J-1} [1 - C^2(m)] \prod_{m=1}^{J} e^{-\alpha(m)c_w C_r(m) \Delta \tau_d(m)}}$$

(5.26)

where

- SF is a scale factor to account for spherical spreading loss through the water,
- $A(J)$ is the non-interactive amplitude of the pulse at $J$,
- $A$ is the amplitude of the highest pulse in the series,
- $\prod_{m=1}^{J-1} [1 - C^2(m)]$ is the product of the transmission terms at the boundary of $J-1$ through the first,
- $\prod_{m=1}^{J} e^{-\alpha(m)c_w C_r(m) \Delta \tau_d(m)}$ is the product of the attenuation factors through the layers before $J$ through the first.

Here, $8.686 \alpha(m)$ is the attenuation coefficient of the layer before $m$ in dB/m,
- $c_w$ is the propagation velocity in water,
- $C_r(m)$ is the ratio of the propagation velocity in the layer before $m$ to that in water, and

$$\Delta \tau_d(m+1) = \Delta \tau(m+1) + (\phi_z(m+1) - \phi_z(m))/(360f_c)$$

where

- the second term is a correction to bring the pulse at $J$ in the model time series in phase with the pulse in the experimental time series.
The space propagator is the exponential function expressing both two-way time delay and attenuation,

\[ Z(m) = e^{\frac{h k \Delta \phi_c}{w_c \Delta r} (m)} e^{-j2\pi k \Delta \phi_d (m)} \]  

(5.27)

where

the attenuation coefficient is proportional to the first power of frequency,

\[ 8.686\alpha = hf. \]

The mathematical model used is the discrete version where the reflection coefficient at the top boundary of the subbottom is calculated for layers up to J by applying equation 5.28,

\[ V(J-1, k) = \frac{C(J-1) + C(J) Z(J)}{1 + C(J-1) C(J) Z(J)} \]  

(5.28)

in a recursive way until \( V(1, k) \) is obtained. The pulse at J is then removed from the model for phase comparison purposes with the next processed pulse and the transfer function becomes

\[ R(k) = V(1, k) Z(1) - C(J) \prod_{m=1}^{J-1} [1 - C^2(m)] \prod_{m=1}^{J} Z(m). \]  

(5.29)

The transfer function is an estimate of the experimental time series where the input pulse signal has been deconvolved from it. In order to compare model data with experimental data the transfer function is multiplied by the spectrum of the assumed input pulse,

\[ M(k) = R(k) I(k), \]  

(5.30)

where

\[ I(k) \] is a 50% raised cosine spectrum and, its frequency range is \( 9\text{kHz} \leq |f| \leq 15\text{kHz}. \)

Its implementation in the discrete frequency domain is,
\[ I(k) = 0.5[1 - \cos 4\pi(k - k_1)/(k_2 - k_1)], \quad (5.31) \]

where
\[ k_1 = 180 \quad \text{and} \quad k_2 = 300 \quad \text{and}, \]
\[ 180 \leq k \leq 300. \]

This only takes care of the positive discrete frequencies as seen in graphs 5.6 and 5.7 which were part of the rectangular discrete frequency filter described earlier in the chapter. In order to multiply the negative frequencies in \( R(k) \) by \( I(k) \) properly,
\[ \begin{align*}
\text{Re}[M(N-k)] &= \text{Re}[M(k)] \\
\text{Im}[M(N-k)] &= -\text{Im}[M(k)], 
\end{align*} \quad (5.32) \]

for the same range of \( k \), which makes the negative discrete spectrum equal to the conjugate of the positive spectrum. An IFFT is taken of \( M(k) \) to get \( m(n) \), the model time series, and the analysis loop is closed.

A description of the transformation of the estimated parameters into other variables as shown in figure 5.1 follows. This is done in the impedance profile block shown in figure 5.3, after all the boundary reflection coefficients have been determined. The results are relative impedances of the layers normalized to that of water, where
\[ \rho_c(1) = 1. \]

Since the boundary reflection coefficient at the \( J \)th boundary is,
\[ C(J) = [\rho_c(J) - \rho_c(J-1)]/[\rho_c(J) + \rho_c(J-1)], \quad (5.33) \]

the ratio of characteristic impedances is found to be,
\[ \rho_c(J)/\rho_c(J-1) = [1+C(J)]/[1 - C(J)]. \quad (5.34) \]

Solving for the characteristic impedance of the layer before the \( J \)th boundary,
\[ \rho_c(j) = \prod_{m=1}^{J} \frac{\rho_c(m)}{\rho_c(m-1)} \]  

(5.35)

This process can be implemented from the top of the sediments through to the bedrock by using,

\[ \rho_c(m) = \rho_c(m-1) \frac{(1 + C(m))}{(1 - C(m))} \]  

(5.36)

in a recursive way. The characteristic impedance values are stored and then plotted into a relative impedance profile for further interpretation of the subbottom structure.

5.3 Analysis

The echo frames analyzed were taken from transect C shown in Figure 3.1 and located on the survey profile shown in Figure 5.11.

5.3.1 Lossless Case - Echo #1

Signal processing analyses were made using BCOEFA on echos taken from the profile shown in figure 3.3. The first analysis was worked for the lossless case where all attenuation coefficients were set equal to zero. The experimental magnitude of the envelope for echo #1 is shown in figure 5.12. This should be compared with the simulated magnitude shown in figure 5.13. The degree of fit of the simulation to the experiment, the residual energy left in the signal after the simulated series has been removed from it, is shown in figure 5.14. This shows that 75% of the signal energy has been removed. The remaining energy is due to three causes,

(1) pulses not found because they were below the threshold level,

(2) unresolved pulses which were not found because the loop
Figure 5.11 - Location of Echos Analyzed from Experimental Data
Figure 5.12 - Experimental Envelope Magnitude for Echo #1

Figure 5.13 - Simulated Envelope Magnitude for Echo #1, Lossless Case, Reflection Coefficient Scale Factor = .12, Threshold = .033
calculations proceeded in a forward direction without backing up, and

(3) multiple reflected pulses which did not always coincide with the experimental end-of-series pulses.

For this simulation the threshold level was set at .033 which means that only pulses greater than 3% of the largest pulses in the series were fit. The scale factor which,

(1) accounts for the spherical spreading loss of the acoustic wave through the water column,

(2) effects the scale of the relative impedance profile, and

(3) effects the size of the multiple reflections of the sound pulse between the layer boundaries,

was set at .12. This was done in an interactive way by running the program, observing the impedance scale and multiple-reflection size, and adjusting the scale factor accordingly. The multiple-reflection size is the amplitude of the pulses after the last large amplitude pulse of the series that has been fit, and the impedance is scaled for a maximum value equal to that of granite, as given in table 5.2.

It was assumed that the basement was granite and so the maximum value of the relative impedance was set near 11. Since the large amplitude pulses near the end of the series were largely unresolved, it was also assumed that the boundary phase should always be positive in that part of the time series such that the relative impedance values would increase in a monotonic way to that of granite. Figure 5.15 shows the relative impedance profile for the lossless case.
<table>
<thead>
<tr>
<th>material</th>
<th>density [g/cm³]</th>
<th>velocity ratio</th>
<th>relative impedance coefficient</th>
<th>reflection</th>
<th>attenuation @ 12 kHz [dB/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ooze</td>
<td>-</td>
<td>-</td>
<td>1.440</td>
<td>-0.180</td>
<td>0.70</td>
</tr>
<tr>
<td>silty clay</td>
<td>1.421</td>
<td>0.994</td>
<td>1.412</td>
<td>-0.171</td>
<td>0.84</td>
</tr>
<tr>
<td>clayey silt</td>
<td>1.488</td>
<td>1.014</td>
<td>1.509</td>
<td>-0.203</td>
<td>1.08</td>
</tr>
<tr>
<td>sand silt clay</td>
<td>1.590</td>
<td>1.033</td>
<td>1.642</td>
<td>-0.243</td>
<td>1.80</td>
</tr>
<tr>
<td>silt</td>
<td>1.767</td>
<td>1.062</td>
<td>1.877</td>
<td>-0.305</td>
<td>2.76</td>
</tr>
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<td>sandy silt</td>
<td>1.787</td>
<td>1.088</td>
<td>1.944</td>
<td>-0.321</td>
<td>5.40</td>
</tr>
<tr>
<td>silty sand</td>
<td>1.806</td>
<td>1.091</td>
<td>1.970</td>
<td>-0.327</td>
<td>8.28</td>
</tr>
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<td>very fine sand</td>
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<td>1.111</td>
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<td>-0.349</td>
<td>8.04</td>
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<tr>
<td>granite</td>
<td>-</td>
<td>-</td>
<td>10.760</td>
<td>0.830</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.2 Subbottom Material Parameters [47,64,65]
Figure 5.14 - Residual Signal Energy for Echo #1, Lossless Case

Figure 5.15 - Relative Impedance Profile for Echo #1, Lossless Case
5.3.2 Lossy Case - Echo #1

An analysis for echo #1 was worked for the lossy case where attenuation coefficients for real materials were put into the BCOEFA program. Figure 5.16 shows the experimental magnitude plot for same page comparison with the simulated time series shown in figure 5.17. The fit is the same as for the lossless case as shown by the residual energy plot in figure 5.18, i.e. a 75% fit. Table 5.3 shows the assumed order of materials layered in the subbottom with their attenuation coefficients.

The vegetational ooze had been detected at the water-bottom interface during experimental surveying. The attenuating materials, starting with the clayey-silt are put into the analysis program at the depth where the positive boundary phase constraint is introduced, i.e. the impedance changes are forced to be positive. The attenuation increases in the same order as the materials are listed in the table and then decreases again after the sand for materials which have been assumed to have characteristic impedances greater than 2.5. This behavior has been shown to be a general characteristic trait of subbottom material layering by Hamilton [65].

A print-out for this analysis, run interactively by,

1. reading the tape with READTP, IROT3
2. analyzing the signal with BCOEFA, and
3. storing the estimated parameters in data files

is shown in Appendix G. The print-out of the executed BCOEFA program shows that the threshold was set at .032 as before, but the SF had to be decreased to .045 from the value of .12 used before. The SF value was arrived at in an interactive way by requiring the final impedance value
<table>
<thead>
<tr>
<th>Material</th>
<th>Attenuation [dB/m]</th>
<th>Relative Impedance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Vegetational ooze</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Clayey-silt</td>
<td>1</td>
<td>1.0-1.5</td>
</tr>
<tr>
<td>Sand-silt-clay</td>
<td>2</td>
<td>1.5-2.0</td>
</tr>
<tr>
<td>Sand</td>
<td>6</td>
<td>2.0-2.5</td>
</tr>
<tr>
<td>.</td>
<td>2</td>
<td>2.5-</td>
</tr>
<tr>
<td>.</td>
<td>1</td>
<td>1.0-</td>
</tr>
<tr>
<td>Granite</td>
<td>-</td>
<td>11.0</td>
</tr>
</tbody>
</table>

Table 5.3 Assumed Layering of Attenuating Materials
Figure 5.16 - Experimental Envelope Magnitude for Echo #1

Figure 5.17 - Simulated Envelope Magnitude for Echo #1, Lossy Case, Reflection Coefficient Scale Factor = .045, Threshold = .033
in the profile to be near that of granite. The fact that the SF is less than before indicates that,

(1) there is more loss within the subbottom because of the assumed attenuating layers, and

(2) that this extra loss is accounted for by less loss due to spherical spreading in the water column.

This analysis indicates that the lower SF is more nearly correct and that if it had also been used in the lossless case, that the final value of the impedance profile would have been considerably less than that for granite.

The relative impedance profile for the lossy case is shown in figure 5.19. In comparison with the lossless case, it appears to be more symmetric about the center point in the monotonically increasing section. It is interesting to point out how well the shape of this curve agrees with the model for a continuously increasing impedance due to compaction proposed in chapter 4 on Mathematical Models.

5.3.3 Consistent Depth Layer - Echos #177-195

The next region of interest was where there appeared to be consistent sharp high amplitude echo returns within the sediment column as shown from echo frames 177 to 195. Since the gain was alternately switched from low to high during the transient, every other echo in the series was processed for a total of 10. The experimental magnitude of those is shown in figure 5.20, where there is no evidence of a pattern between echo frames. A simulation of the echos was run with BCOEFA for each echo frame by unstacking them one at a time from a random-access file with the program STACUN given in Appendix F, processing that one,
Figure 5.18 - Residual Signal Energy for Echo #1, Lossy Case

Figure 5.19 - Relative Impedance Profile for Echo #1, Lossy Case
Figure 5.20 - Experimental Envelope Magnitudes for Alternate Echos #177-195
Figure 5.21 - Simulated Envelope Magnitudes for Alternate Echos #177-195, Reflection Coefficient Scale Factor = .045, Threshold = .132
Figure 5.22 - Relative Impedance Profiles for Alternate Echos #177-195, Reflection Coefficient Scale Factor = .045, Threshold = .033
and restacking the output parameter files with STACUN. The threshold for the simulated echo magnitudes, shown in figure 5.21, was raised to .132 or up to 13% of the maximum peak. This was an attempt to look for the patterns which showed up in the facsimile graph by eliminating the low amplitude pulses which interfere with the picture. Next the BCOEFA program was rerun for all 10 echos using a threshold value of .033 and a SF of .045 which worked satisfactorily in processing echo #1 with attenuation losses. The impedance profiles are shown for the 10 frames in figure 5.22. In order to examine the impedance in the region before the monotonic increase, these profiles were truncated at a value, but did actually extend up to the final impedance value of granite. The results were a decrease in the impedance profiles of 80% of the echos at the depth where a layer image was indicated on the facsimile plot. The interpretation of these events is that a layer of gas bubbles from vegetational decomposition has been detected in the sedimentary material.

5.3.4 Wide High Intensity Regions - Echo #923

The experimental and simulated magnitude for echo envelope #923 are shown in figures 5.23 and 5.24 respectively. By looking on the facsimile plot of figure 5.19, it is seen that this is in the region where there appear to be two groups of high-amplitude wide-travel-time pulse series. Figure 5.25 shows that the simulated series fits the experimental up to 75%, the same as with the other echos analyzed. The interpretation of the structure will influence how the impedance profile looks and there are at least two interpretations. The one used for the profile shown in figure 5.26 is that the structure is the usual positive and negative impedance change layering up to the second or basement high amplitude region. There the impedance is forced to increase monotonically to the
Figure 5.23 - Experimental Envelope Magnitude for Echo #923

Figure 5.24 - Simulated Envelope Magnitude for Echo #923, Reflection Coefficient Scale Factor = .08 Threshold = .033
Figure 5.25 - Residual Signal Energy for Echo #923

Figure 5.26 - Relative Impedance Profile for Echo #923
final value of that for granite. A second interpretation might be that the impedance of the first high amplitude region should also be forced positive, followed by positive and negative changes, and ending in the positive increase again at the basement. Which ever one is correct can only be determined by some ground truth information, i.e. a core! After that was determined, the surveyor could provide detailed tracking data which could be analyzed and transformed into impedance profiles.
CHAPTER 6

CONCLUSIONS

6.1 Engineering and Signal Processing Results

The conclusions and results achieved for this engineering and signal processing endeavor are the following:

1. A substantial increase in resolvable detail is seen in sub-bottom images on facsimile graphs taken with the 3 degree acoustic parabolic antenna over those taken with the 30 degree transducer, as seen in graphs 3.6 and 3.7.

2. The phase shift information may be used to supplement the amplitude information of the echo carrier in a digital signal processing technique to find the relative impedance profile of the subbottom, under the following conditions:
   a. the projector lateral resolution must be within the target lateral area or diffracted pulses will also be present as seen in graph 3.5.
   b. the vertical resolution must be finer than the layer spacing or reflected pulses will become unresolvable as seen at the end of data time series in graph 3.11.
   c. the acoustic transmitted signal must be modulated by a carrier frequency from which the phase shift is extracted as seen in graph 3.8 and,
   d. the transmitted waveform must have a known shape from which the two-way travel time is found to within a few degrees of one cycle of the carrier.
3. A digital processing program has been used to analyze the echo time series, by using an impedance model and a signal processor in a digital feedback loop with the experimental data as the input, where:

a. a plane parallel layer impedance model was assumed with real characteristic impedances and attenuations proportional to the first power of frequency as determined from empirical observations reported by others in the literature [47,68,69].

b. the signal processor estimates reflection coefficients on the basis of resolved peak amplitudes, carrier phase shift, and time delay; and feeds back first and multiple internal boundary reflection signals for each embedded layer starting with the top and working in a stepwise fashion through the sedimentary layers to the basement rock, and

c. the outputs of the digital computation feedback loop include the simulated magnitude of the time series, the phase shift of the carrier of the time series, the residual power in the echo after removal of the simulated series, and the relative impedance profile of the sub-bottom layers as in graphs 5.15, 5.19, 5.22 and 5.26.

4. A 75% power fit has been achieved between modeled time series and experimental time series as shown in graph 5.14, where the remaining 25% is due to pulses below the noise threshold and unresolved pulses at the end of the time series which were not detected.
5. Gas bubbles from decaying vegetation is one interpretation for the decrease in the relative impedance in 80% of the adjacent profiles over a cruise track at the same depth where the facsimile plot indicates a sediment layer, as seen in graphs 5.22 and 5.11 respectively.

6.2 Recommendations for Further Study

Further study could be directed toward the following:

1. use of a surveyor-type device in an offshore area where known petroleum deposits exist for further evaluation of the ideas and procedures presented,

2. development of a complete 3-dimensional mathematical model for analysis of subbottom data,

3. the investigation of non-linear devices for narrow beam formation, signal level and beam steering capability for possible development of a scanning surveyor,

4. development of better graphical techniques than presently exist for rapid presentation of highly detailed processed data, and

5. the incorporation of the signal processing routines into fast algorithms for real time processing.

6.3 Future Directions

The projected directions for high resolution narrow beam sound devices are unlimited in both fields of endeavor and the results to be achieved by phase processing. For example, when the beam is narrower than the object to be insonated, phase processing is possible and when both amplitude and phase of the reflected signal are present as good
(signal to noise) ratio deterministic signals, then image reconstruction is possible. To reconstruct an image field, the beam must be scanned over the field. This has applications in medical diagnosis, subbottom surveying, biomass estimation in the seas, man-made offshore structure integrity examination, etc.
APPENDIX A

PROGRAM TO COMPUTE ACOUSTIC FIELD PATTERN FOR A PISTON SOURCE
TYPE PLANER.F4

COMPLEX PRES,C1,C2,C3,C4,C5,C6,C4NEXT,C5NEXT
DIMENSION PRESAB(400),PRESAN(400)
DATA GAMA,ELMT,RAD,XMAX/0.,25,12,5,25./
DATA DX,ZMAX,ZMIN,DZ/1.,312.,25.,1./
PRINT 550
PI=3.1415926
NMAX=RAD/ELMT
MMAX=NMAX*PI
I=0
JMIN=ZMIN
JMAX=ZMAX
DO 700 J=JMIN,JMAX
Z=J
IF(J-JMAX/2)50,100,50
50
X=0
K=0
R=Z
TH=0
GO TO 200
100 LMAX=2*XMAX+1
DO 700 L=1,LMAX
K=L-XMAX-1
X=K
R=SQR(T(X**2+Z**2))
TH=ATAN(X/Z)
200 DRPRM=RAD/NMAX
DTHPRM=PI/MMAX
C6=CMPLX(0.,-2*PI*R)
C1=CMPLX(0.,(COS(TH)+COS(GAMA))*(DRPRM**2*DTHPRM/R))**C
C2=CMPLX(0.,-PI*DRPRM**2/R)
C3=CMPLX(0.,-2*PI*DRPRM*(SIN(GAMA)-SIN(TH)))
C4=CMPLX(0.,0.)
DO 500 N=1,NMAX
C5=CMPLX(0.,0.)
DO 400 M=1,MMAX
C5NEXT=EXP(C3*(N-.5)*COS(DTHPRM*(M-.5)))
400 C5=C5+C5NEXT
C4NEXT=(N-.5)*EXP(C2*(N-.5)**2)*C5
500 C4=C4+C4NEXT
PRES=C1*C4
I=I+1
PRESAB(I)=ABS(PRES)/2
DEN=REAL(PRES)
IF(DEN.EQ.0) GO TO 510
PRESAN(I)=ATAN(IMAG(PRES)/DEN)*180/PI
GO TO 520
510 PRESAN(I)=90
520 PRINT 600,J,K,PRESAB(I),PRESAN(I)
550 FORMAT(3X,'Z',3X,'X',12X,'PRESSURE',12X,'ANGLE')
600 FORMAT(I5,I4,11X,2(E15.7,5X))
700 CONTINUE
END
APPENDIX B

PROGRAMS TO SYNCHRONIZE, UNPACK, SORT, AND STORE
SUBBOTTOM ECHOS FROM DIGITIZED DATA
ROBERT BOLUS

THIS PROGRAM USES AS AN INPUT, DIGITALIZED DATA
AQUIRED FROM EXPERIMENT, IT SEPERATES AND WRITES AS
OUTPUT THREE PORTIONS OF THE INPUT DATA: NOISE SAMPLE
TRANSIMITION SAMPLES, AND ECHO SAMPLES. INTERWOVEN
WITH THE EXPERIMENTAL DATA IS SAMPLES OF A 12 K HERTZ
REFERENCE SIGNAL. THE DATA IS PACKED THREE SAMPLES TO
A 36 BIT WORD.

TIME CONSTANTS - ALL TIME UNITS IN SAMPLE TIME

ISLO, ISHI - MINIMUM AND MAXIMUM DURATION FOR 8 MIL
PULSE.
NTRAHI, NTRALO - MINIMUM AND MAXIMUM LENGTH OF 58 MILLI
GAP BETWEEN END OF 8 MSEC PULSE AND 1/ TONE.
NOISE - AMOUNT OF TIME AFTER 8 MSEC PULSE THE 
IS BEGUN TO BE STORED.
KHOP - TIME SKIPPED AFTER 1/2 MSEC TONE TO SK 
OVER REVERBERATION.
IECHO - LENGTH OF TIME SAMPLED FOR ECHO 
KPFWD - AMOUNT OF TIME SKIPPED AFTER ECHO SAMP 
TO APPROCH NEXT 8 MSEC PULSE.
JANGE - AMOUNT OF TIME THAT AN AMPLITUDE IS ME 
BEFORE IT IS CONCLUDED THAT THE WRONG 
IS BEING ANALYSED.

DIMENSION CHA(3), ISKIP(4000) 
DIMENSION NAT(2000), ING(6), JPAC(2) 
DATA IYES/‘YES’/ 
DATA NDAT, JANGE, ISHI, ISLO/12500,4000,420,380/ 
DATA KHOP, IECHO, KPFWD/534,834,4000/ 
DATA NOISE, NTRAHI, NTRALO, IFMAX/1650,3044,2756,1000/ 
SKIP=0. 
ICNT=4001 
AMPMAX=2048 
AMPTH=.060*AMPMAX
IF (IFRAME EQ 10) GO TO 200

OPEN (UNIT=19, DEVICE='MTA', MODE='IMAGE', FILE='PING', I BUFFER COUNT=10)
OPEN (UNIT=11, DEVICE='DSKS', MODE='IMAGE', FILE='PING', I BUFFER COUNT=10)

700 WRITE (5, 1000)
1000 FORMAT('HOW MANY FRAMES DO YOU WISH TO JUMP?')
READ (5, 1002) JFRAME
1002 FORMAT (I4)
    IF (JFRAME LE 0) GO TO 305
    DO 400 J=1, JFRAME
400 READ (11) ISKIP
305 READ (11) ISKIP (1)
    WRITE (5, 310) ISKIP (1)
310 FORMAT ('WORD IS ', 12)
    WRITE (5, 320)
320 FORMAT ('DO YOU WISH TO CHANGE CHANNELS?')
READ (5, 330) YES
330 FORMAT (A3)
    IF (YES .NE. YES) GO TO 340
    READ (11) ISKIP
340 WRITE (5, 350)
350 FORMAT ('DO YOU WISH TO ENTER PROGRAM?')
READ (5, 360) YES
360 FORMAT (A3)
    IF (YES .NE. YES) GO TO 700
    WRITE (5, 2) IFRAME
2 FORMAT ('FRAME NUMBER IS', I4, ' ')!
    IF (IFRAME EQ 10) GO TO 200
K=0
3 NATSET=0
4 IBSET=0
5 MPSET=0
10 JCNT=0
20 IF (ICNT LE 4000) GO TO 25
ICNT=1
READ (11) ISKIP
25 JFAC (1) = ISKIP (ICNT)
    JPAC (2) = ISKIP (ICNT + 1)
ICNT=ICNT+2
DO 30 IO=1, 2
I1=I0+3-2
    CALL IROT (JFAC (IO), ING (I1), ING (I1+1), ING (I1+2))
30 CONTINUE
    JCNT=JCNT+3
DO 35 I=1, 3
J=2*I-1
35 CHA (I) = ABS (ING (J))!
    WRITE (5, 37) JCNT, CHA (1), CHA (2), CHA (3)
37 FORMAT (2X, I6, 3(2X, F10.3))
    IF (NATSET EQ 1) GO TO 118
IF(IBSET.EQ.1)GO TO 80
IF(MPSET.EQ.1)GO TO 60
DO 40 I=1,3
IF(CHA(I).GT.AMPTH)GO TO 50
IF(JCNT.GT.NDAT)GO TO 150
40 CONTINUE
GO TO 20
50 MPSET=1
WRITE(5,55)
55 FORMAT(' AMPLITUDE THRESHOLD ACQUIRED. ')
GO TO 10
60 CHAD=CHA(1)+CHA(2)+CHA(3)
IF(CHAD.LT.AMPTH) GO TO 70
IF(JCNT.GT.JANGE) GO TO 65
GO TO 20
65 READ(11)SKIP
ICNT=4001
WRITE(5,67)
67 FORMAT(' CHANGE CHANNELS')
GO TO 5
70 IF((JCNT.LE.ISHI).AND.(JCNT.GE.ISLO))GO TO 74
WRITE(5,72)
72 FORMAT(1X,'SYNC LOST WHILE SEARCHING FOR 8 MSEC TONE')
GO TO 5
74 WRITE(5,75)
75 FORMAT(' 8 MS TONE DETECTED.')
IBSET=1
GO TO 5
80 DO 90 FOR I=1,3
IF(CHA(I).GT.AMPTH)GO TO 100
90 CONTINUE
IF(JCNT.GT.NOISE)GO TO 110
GO TO 20
100 MPSET=1
IBSET=0
WRITE(5,105)
105 FORMAT(1X,'SYNC LOST WHILE WAITING TO TAKE NOISE SA')
GO TO 10
110 NATSET=1
WRITE(5,115)
115 FORMAT(1X,'STORING NOISE SAMPLE.')
118 DO 119 I=1,3
IF(CHA(I).GT.AMPTH)GO TO 120
119 CONTINUE
K=K+2
NAT(K-1)=JPA C(1)
NAT(K)=JPA C(2)
GO TO 20
120 IF((JCNT.LE.NTRAHI).AND.(JCNT.GE.NTRALO))GO TO 121
WRITE(5,121)
121 FORMAT(1X,'SYNC LOST WHILE EXPECTING TRANSMITTER SA')
GO TO 10
GO TO 3
ICNT=ICNT-2
DO 510 J=K+1,K+60
IF(ICNT.LE.4000)GO TO 530
ICNT=1
READ(11)ISKIP
530 NAT(J)=ISKIP(ICNT)
510 ICNT=ICNT+1
WRITE(5,125)
125 FORMAT(‘STORING TRANSMITTER SAMPLE’)
129 KNOISE=3*IFRAME-2
WRITE(19)KNOISE
WRITE(19)(NAT(I),I=1,K-36)
KTRANS=3*IFRAME-1
WRITE(19)KTRANS
WRITE(19)(NAT(I),I=K-35,K+60)
DO 130 J=1,KECHO
IF(ICNT.LE.4000)GO TO 540
ICNT=1
READ(11)ISKIP
540 KIP=ISKIP(ICNT)
130 ICNT=ICNT+1
KECHO=3*IFRAME
WRITE(19)KECHO
WRITE(5,140)
140 FORMAT(‘STORING ECHO SAMPLE.’)
DO 550 J=1,IECHO
IF(ICNT.LE.4000)GO TO 560
ICNT=1
READ(11)ISKIP
560 NAT(J)=ISKIP(ICNT)
550 ICNT=ICNT+1
WRITE(19)(NAT(I),I=1,IECHO)
DO 570 J=1,KPFWD
IF(ICNT.LE.4000)GO TO 580
ICNT=1
READ(11)ISKIP
580 KIP=ISKIP(ICNT)
570 ICNT=ICNT+1
IFRAME=IFRAME+1
GO TO 1
580 IF(IFRAME.GT.IFMAX)GO TO 200
GO TO 10
200 STOP
END
TYPE SORT.FOR

C

C   THIS PROGRAM TAKES THE PACKED DATA FROM SYNC.FOR AND
C   SEPERATES THE THREE TYPES OF DATA, LEAVING IT IN ITS
C   PACKED FORM, FOR USE BY THE FFT.FOR PROGRAM.

C

C   DECLARATION

C   INTEGER WORD
C      - THE DATA WORD READ FROM THE INPUT FI
C   INTEGER WRDCNT,BLKCNT
C      - WORD AND DATA BLOCK COUNTERS
C   INTEGER TYPE
C      - INDICATES WHICH TYPE OF DATA IS EXAM
C      (0 FOR ECHO, 1 FOR NOISE, 2 FOR TRAN
C   INTEGER TOTWRD(3)
C      - TOTAL NUMBER OF WORDS IN EACH OUTPUT
C   LOGICAL DONE
C      - LOGICAL FLAG SET WHEN ALL DATA HAS B
C   INTEGER ANSWER,NIL,YES,ZEROS,TTY
C   DOUBLE PRECISION FILENM
C      - THE NAME OF THE INPUT FILE

C

C   MAIN PROGRAM

C   DONE=.FALSE.
C   WRDCNT=0
C   BLKCNT=0
C   DATA TOTWRD/0,0,0/
C   DATA YES/'Y'/
C   DATA NIL/0/
C   DATA TTY/5/

C   INPUT FILE INFORMATION
C   WRITE(TTY,1)
C   1 FORMAT('1','WHAT IS THE NAME OF YOUR INPUT FILE ? ','$
C   READ(TTY,2) FILENM
C   2 FORMAT(A10)
C   WRITE(TTY,3) FILENM
C   3 FORMAT(1X,'ON WHAT DEVICE IS ','A10,' LOCATED ? ','$
C   READ(TTY,4) INDEV
C   4 FORMAT(A5)
C   WRITE(TTY,5)
C   5 FORMAT(1X,'ON WHAT DEVICE DO YOU WANT YOUR OUTPUT FILE
C   READ(TTY,6) OUTDEV
C   6 FORMAT(A5)
C
OPEN DATA FILES
OPEN(UNIT=19, DEVICE=INDEV, MODE='IMAGE', FILE=FILENAME)
OPEN(UNIT=10, DEVICE=OUTDEV, MODE='IMAGE', FILE='NOIS')
OPEN(UNIT=13, DEVICE=OUTDEV, MODE='IMAGE', FILE='TRAN')
OPEN(UNIT=12, DEVICE=OUTDEV, MODE='IMAGE', FILE='ECHO')
C
C
INPUT CALCULATION INFORMATION

WRITE(TTY,10)
10 FORMAT(1X,'DO YOU WANT TO ADD ZEROS TO THE ECHO DATA')
READ(TTY,20) ANSWER
20 FORMAT(A1)
IF(ANSWER='NE, YES) GO TO 70
30 CONTINUE
WRITE(TTY,40)
40 FORMAT(1X,'HOW MANY ZEROS DO YOU WANT TO ADD')
READ(TTY,*ZEROS)
IF (INT(ZEROS/2)*2 .EQ. ZEROS) GO TO 60
WRITE(TTY,50)
50 FORMAT(1X,'YOU MUST USE AN EVEN NUMBER')
GO TO 30
60 CONTINUE
70 CONTINUE

WRITE(TTY,80)
80 FORMAT(1X,'WHAT MAXIMUM NUMBER OF DATA BLOCKS DO YOU')
READ(TTY,*NUMBKS)

C
READ DATA WORD

100 READ(19,END=125) WORD
125 CONTINUE
DONE=.TRUE.
GO TO 175
150 CONTINUE
IF(ABS(WORD) .GT. 1000) GO TO 300
C
DATA WORD IS BLOCK HEADING
WRITE(TTY,31) WORD
31 FORMAT(1X,'THE BLOCK HEADING IS',I6)
IF(WORD .GT. NUMBKS) GO TO 800
IF (BLKCNT .EQ. 0) GO TO 275
175 WRITE(TTY,200) BLKCNT, WRDCNT
200 FORMAT(1X,'DATA BLOCK #',I5,' CONTAINS ',F6,' WORDS.'
TOTWRD(TYPE+1)=TOTWRD(TYPE+1)+WRDCNT
IF(TYPE .NE. 0 .OR. ANSWER .NE. YES) GO TO 275
DO 250 I=1,ZEROS
WRITE(12) NIL
250 CONTINUE
TOTWRD(1)=TOTWRD(1)+ZEROS
275 CONTINUE

BLKCNT=BLKCNT+1
TYPE=MOD(BLKCNT,3)
WRDCNT=0
IF(DONE) GO TO 725
GO TO 100
300 CONTINUE
WRDCNT=WRDCNT+1
IF(TYPE .NE. 0) GO TO 400
C DATA IS ECHO SAMPLE
WRITE(12) WORD
GO TO 100
400 CONTINUE
IF(TYPE .NE. 1) GO TO 500
C DATA IS NOISE SAMPLE
WRITE(10) WORD
GO TO 100
500 CONTINUE
IF(TYPE .NE. 2) GO TO 600
C DATA IS TRANSMISSION DATA
WRITE(13) WORD
GO TO 100
600 CONTINUE
WRITE(TTY,700) TYPE
700 FORMAT(1X,'ERROR MOD SUBROUTINE FAILURE=',I3)
725 WRITE(TTY,750)(TOTWRD(I),I=1,3)
750 FORMAT(/,1X,'NUMBER OF WORDS IN ECHO SAMPLE: ',I6,/
+ 1X,'NUMBER OF WORDS IN NOISE SAMPLE: ',I6,/ 
+ 1X,'NUMBER OF WORDS IN TRANSMISSION SAMPLE: ',I6,/ 
800 WRITE(TTY,900)
900 FORMAT(1X,'END OF PROGRAM',/)
STOP
END
DATA STORED ON TAPE #000437 IS PIECES OF DIGITALIZED DATA. THE DATA IS A BINARY FILE WRITTEN AS AN INTEGER. THE DATA ON THE TAPE IS STORED WITH THREE SAMPLES PACKED TO A WORD. THE SAMPLES ALTERNATE DATA AND REFERENCE DATA. SO THAT ANY TWO WORDS FROM THE TAPE CONTAIN THREE PIECES OF DIGITALIZED DATA. EACH BLOCK HAS AN INTEGER BLOCK HEADING. BLOCK #1 IS NOISE DATA, BLOCK #2 IS TRANSMISSION DATA, BLOCK #3 IS ECHO DATA, BLOCK #4 IS NOISE DATA FROM THE SECOND FRAME, ETC. SO IF YOU WANT SEVERAL FRAMES OF ECHO DATA, YOU WANT BLOCKS: 3, 6, 9, 12, 15, ....

TO RUN THIS PROGRAM, THE FOLLOWING IS DONE:

.COUNT MTA:19/REELID:000437/RONLY

REQUEST QUEUED

WAITING...2 "C'S TO EXIT
"C
"C

"C.23 J

"C.MOUNT/C

CHECK YOUR COMMAND IN QUEUE

"C.MOUNT/C

NO COMMANDS IN QUEUE

"C.SYS/BUSY

FIND OUT WHAT

BUSY DEVICES:

DEVICE JOB WHY LOGICAL

TTY45 19 AS

MTAO11 43 AS+INIT

MTAO12 10 AS 19 <----- THAT'S YOU: MTA01

MTAO13 14 AS

EXECUTE READTP.FOR, IROT3.MAC

...... ANSWER QUESTIONS ......

.DISMOUNT MTA012

<AT THIS POINT THE DATA YOU REQUESTED IS IN FILE: SIG.\n
\n
C AND THE CORRESPONDING REFERENCE DATA IS IN FILE: S
C---------------------------------------------------------------------------------
C---------------------------------------------------------------------------------
C---------------------------------------------------------------------------------
C
INTEGER HEAD, OLDHD, WORD
DIMENSION JPAC(2)*ING(6)
DATA YES/'YES'/

OPEN(UNIT=19, DEVICE='MTA', MODE='IMAGE', FILE='PIN'
OPEN(UNIT=11, DEVICE='DSKC', MODE='IMAGE', FILE='SIG
OPEN(UNIT=12, DEVICE='DSKC', MODE='IMAGE', FILE='SIG

OLDHD=0
REWIND 19

WRITE(5,10)
10 FORMAT(1X,'FROM WHICH DATA BLOCK NUMBER DO YOU WANT
+ DATA ?'/,1X,'NOISE: 1,4,7... TRANS: 2,5,8... ECHO
READ(5,**)HEAD
IF(HEAD .NE. JPAC(1)) GO TO 13
   WORD=JPAC(1)
   GO TO 30
13 CONTINUE
IF(HEAD .GT. OLDHD) GO TO 20
   REWIND 19
   WRITE(5,15) OLDHD,HEAD
15 FORMAT(1X,'NOW POSITIONED AFTER BLOCK #',I6,/
   1X,'REWINDING TAPE TO FIND BLOCK #',I6)
   CONTINUE
   OLDHD=HEAD

25 READ(19,END=100) WORD
20 IF(WORD .EQ. HEAD) GO TO 30
   GO TO 25
30 WRITE(5,40) WORD
40 FORMAT(1X,'THE BLOCK HEADING IS',I6)
45 ICNT=0
40 JCNT=0
45 READ(19,END=60) JPAC(1)
   IF(ABS(JPAC(1)),LT,3000) GO TO 60
   READ(19) JPAC(2)
C
   DO 50 IO=1,2
      I1=IO*3-2
      CALL IROT(JPAC(IO),ING(I1),ING(I1+1),ING(I1+2))
50 CONTINUE
   ICNT=ICNT+3
   IF(ICNT.LE.1) GO TO 45
   JCNT=JCNT+3
   ...
IF(JCNT.GT.1024)GO TO 55
WRITE(11) ING(1), ING(3), ING(5).
WRITE(12) ING(2), ING(4), ING(6)
GO TO 45
55 WRITE(11)ING(1)
WRITE(12)ING(2)
JCNT=JCNT-2
57 READ(19) JPAC(1)
IF(ABS(JPAC(1)).LT.3000)GO TO 60
GO TO 57
C
60 CONTINUE
WRITE(5,70) JCNT, HEAD
70 FORMAT(1X,'THERE ARE ',I6,' SAMPLES IN BLOCK #',I6)
75 WRITE(5,80)
80 FORMAT(1X,'DO YOU WANT ANOTHER BLOCK ? ',$)
READ(5,90) JYES
90 FORMAT(A3)
IF (JYES .EQ. IYES) GO TO 1
GO TO 110
100 WRITE(5,105) HEAD
105 FORMAT(1X,'DATA BLOCK #',I6,' NOT FOUND. ')
REWIND 19
OLDHD=0
GO TO 75
110 STOP
END
.TYPE IROT3.MAC

00100        ENTRY IROT
00200  IROT:  MOVE  1, @ (16)
00250        MOVEI 0, 0
00300        LSHC 0, 
00302        LSH 0, 
00304        ASH 0, 
00308        MOVEM 0, @1(16)
00400        MOVEI 0, 0
00500        LSHC 0, 
00502        LSH 0, 
00504        ASH 0, 
00508        MOVEM 0, @1(16)
00600        MOVEI 0, 0
00700        LSHC 0, 
00702        LSH 0, 
00704        ASH 0, 
00708        MOVEM 0, @1(16)
00800        MOVEI 0, 0
00900        MOVEM 0, @1(16)
01000        POPJ 17
01100        ENTRY ALT
01200  ALT:  MOVE  0, @ (16)
01300        AND 0, 
01400        MOVEM 0, @ (16)
01450        POPJ 17
01475        LIT
01500        END
APPENDIX C

PROGRAMS FOR DIGITAL DEMODULATION OF ECHOS AND PLOTTING ROUTINE
This program takes the Hilbert transform of data from disk. The data can be either packed, in which case it is assumed that two tracks are interwoven and packed three samples to the word, or unpacked, in which case it is assumed that there is only one channel stored one sample to the word. The data is assumed to be stored in frames and the Hilbert transform is taken of each frame individually. The program processes the first integral power of two samples and discards the rest of each frame. The output of this program is a disk file (two disk files in the case of packed data) which is ready to be plotted by the program 'PLOT3FOR'. The maximum amplitude of each channel is in this program so it will not have to be found in the program.

---

INTEGER DATA
INTEGER TTY, ANSWER, UNPACK, SIZE(14), NUM, DISCRD, DATA2
INTEGER FRAMES, HOLD1, HOLD2, HOLD3, INDEV

REAL XR(8192), XI(8192), XR2(8192), XI2(8192)

LOGICAL PACK

DOUBLE PRECISION FILENM

DATA TTY/5/
DATA UNPACK/'UN'/
DATA SIZE/1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096,
PACK=.FALSE.

---

INPUT INFORMATION

WRITE(TTY, 20)
20 FORMAT(1X, 'WHAT IS YOUR INPUT DEVICE? ', A$)
READ(TTY, 30) INDEV
30 FORMAT(A5)
WRITE(TTY, 50)

FORMAT (1X, 'WHAT IS THE NAME OF YOUR INPUT FILE ? ',
READ (TTY, 75) FILENM
75 FORMAT (A10)
WRITE (TTY, 100)
100 FORMAT (1X, 'IS THE DATA PACKED OR UNPACKED ? ', $)
READ (TTY, 200) ANSWER
200 FORMAT (A2)
IF (ANSWER .EQ. UNPACK ) GO TO 300
C FALSE-THE DATA IS PACKED, SET FLAG
PACK=.TRUE.
300 CONTINUE
C
IF (PACK) GO TO 400
WRITE (TTY, 350)
350 FORMAT (1X, 'HOW MANY SAMPLES ARE IN EACH FRAME ? ', $)
READ (TTY, *) NUM
GO TO 500
400 CONTINUE
WRITE (TTY, 425)
425 FORMAT (1X, 'HOW MANY SAMPLES PER CHANNEL ARE IN EACH F
READ (TTY, *) NUM
IF ((NUM/3)*3 .EQ. NUM) GO TO 475
WRITE (TTY, 450) NUM
450 + FORMAT (1X, 'NUMBER OF PACKED SAMPLES MUST BE A
'MULTIPLE OF THREE. ', I4, ' IS NOT. ')
GO TO 400
475 CONTINUE
500 CONTINUE
C
C
C
I=0
600 CONTINUE
IF (SIZE(I).EQ.NUM) GO TO 700
IF (SIZE(I+1).GT.NUM) GO TO 700
I=I+1
GO TO 600
700 CONTINUE
DISCRD=NUM-SIZE(I)
IF (DISCRD .EQ. 0) GO TO 900
WRITE (TTY, 800) SIZE(I), NUM, DISCRD
800 + FORMAT (1X, 'WE WILL ONLY ANALYSE', I6, ' OF THE'
+ 'SAMPLES AVAILABLE,'I6, ' WILL BE DISCARDED.')
+ NUM=SIZE(I)
900 CONTINUE
C
C
C
WRITE (TTY, 1000)
1000 FORMAT (1X, 'HOW MANY FRAMES DO YOU WANT TRANSFORMED ?')
READ(TTY*)FRAMES

OPEN FILES

OPEN(UNIT=10,DEVICE='DSKC',MODE='IMAGE',FILE='PHASE.DAT')
OPEN(UNIT=14,DEVICE='DSKC',MODE='IMAGE',FILE='FFT.DAT')
OPEN(UNIT=11,DEVICE='INDEV',MODE='IMAGE',FILE='FILENAME')
OPEN(UNIT=12,DEVICE='DSKC',MODE='IMAGE',FILE='GRAPH.DAT')
OPEN(UNIT=13,DEVICE='DSKC',MODE='IMAGE',FILE='GRAPH.DAT')
OPEN(UNIT=15,DEVICE='DSKC',MODE='IMAGE',FILE='FFT.DAT')

CONTINUE

MAIN BODY OF PROGRAM

AMAX=0.
IF(PACK) GO TO 2000
DO 1900 J=1 FRAMES
  DO 1200 I=1 NUM
    READ(11) DATA
    XR(I)=FLOAT(DATA)
    XI(I)=0.
  CONTINUE
  IF(DISCRD.EQ.0) GO TO 1300
  DO 1300 I=1 DISCRD
    READ(11) DATA
    CONTINUE
  CALL COOLER(XR(1),XI(1),NUM,1)
  TEMP=0.
  WRITE(14) TEMP
  DO 1333 I=2 NUM
    TEMP=(XR(I)**2+XI(I)**2)/10000.
  WRITE(14) TEMP
  DO 1370 I=1,179
    XR(I)=0.
    XI(I)=0.
  CONTINUE
  DO 1400 I=180 NUM/2-212
    XR(I)=2*XR(I)
    XI(I)=2*XI(I)
  CONTINUE
  DO 1500 I=NUM/2-211 NUM
XK(I)=0.
XI(I)=0.

CONTINUE

CALL COOLER(XR(I),XI(I),NUM,-1)
PI=3.1415926
DO 1550 I=1,NUM
IF(XR(I).EQ.0.)GO TO 1510
PHASE=ATAN(XI(I)/XR(I))*180./PI
GO TO 1550

1510 IF(XI(I).GE.0.)GO TO 1520
PHASE=-90.
GO TO 1550

1520 PHASE=+90.

CONTINUE

WRITE(10) PHASE
DO 1700 I=1,NUM
XR(I)=SQRT(XR(I)**2+XI(I)**2)
IF(XR(I).LT.AMAX)GO TO 1600
AMAX=XR(I)

1600 CONTINUE

WRITE(12)XR(I)
CONTINUE

WRITE(TTY,1800) I
FORMAT(1X,'COMPLETED FRAME #',I4)
CONTINUE

GO TO 3000

PACKED DATA

CONTINUE

AMAX2=0.
DISCRD=(2*(DISCRD+NUM)/3)-((2*INT(NUM/3.0))+2)
DO 2900 J=1,FRAMES
CONTINUE

WRITE(11)DATA1,DATA2
CALL IROT(DATA1,HOLD1,HOLD2,HOLD3)
XR(3*I-2)=FLOAT(HOLD1)
XR(3*I-2)=FLOAT(HOLD2)
XR(3*I-2)=FLOAT(HOLD3)
CALL IROT(DATA2,HOLD1,HOLD2,HOLD3)
XR(3*I-1)=FLOAT(HOLD1)
XR(3*I)=FLOAT(HOLD2)
XR(3*I)=FLOAT(HOLD3)
CONTINUE

READ(11)DATA1,DATA2
CALL IROT(DATA1,HOLD1,HOLD2,HOLD3)
XR(3*INT(NUM/3.0)+1)=FLOAT(HOLD1)
XR2(3*INT(NUM/3.0)+1)=FLOAT(HOLD2)
IF(MOD(NUM,3).LT.2)GO TO 2200
XR(NUM)=FLOAT(HOLD3)
CALL IROT(DATA2,HOLD1,HOLD2,HOLD3)
XR2(NUM)=FLOAT(HOLD1)

2200 CONTINUE
DO 2300 I=1,DISCRD
READ(1)DATA
CONTINUE

2300 DO 2350 I=1,NUM
XI(I)=0.
XI2(I)=0.
CONTINUE

2350 CONTINUE
CALL COOLER(XR(1),XI(1),NUM,1)
CALL COOLER(XR2(1),XI2(1),NUM,1)
TEMP=0.
WRITE(14)TEMP
WRITE(15)TEMP
DO 2333 I=2,NUM
TEMP=(XR(I)**2+XI(I)**2)/10000.
TEMP2=(XR2(I)**2+XI2(I)**2)/10000.
WRITE(15)TEMP2

2333 WRITE(14)TEMP
DO 2370 I=1,25
XI(I)=0.
XR(I)=0.
XI2(I)=0.
XR2(I)=0.
CONTINUE

2370 CONTINUE
DO 2400 I=26,NUM/2-25
XI(I)=2*XI(I)
XR(I)=2*XR(I)
XI2(I)=2*XI2(I)
XR2(I)=2*XR2(I)
CONTINUE

2400 DO 2500 I=NUM/2-24,NUM
XI(I)=0
XR(I)=0
XI2(I)=0
XR2(I)=0
CONTINUE

2500 CALL COOLER(XR(1),XI(1),NUM,-1)
CALL COOLER(XR2(1),XI2(1),NUM,-1)
DO 2800 I=1,NUM
XR(I)=SORT(XR(I)**2+XI(I)**2)
XR2(I)=SORT(XR2(I)**2+XI2(I)**2)
IF(XR(I).LT.AMAX)GO TO 2600
AMAX=XR(I)
CONTINUE

2600 IF(XR2(I).LT.AMAX2)GO TO 2700
AMAX2=XR2(I)
CONTINUE

2700 WRITE(12)XR(I)
WRITE(13)XR2(I)
CONTINUE
2900 WRITE(TTY,1800)J
CONTINUE
3000 WRITE(TTY,3100)FRAMES,NUM
3100 FORMAT(1X,'INFORMATION FOR PLOTTING PROGRAM:/',
+ 5X,'NUMBER OF FRAMES: ',I5/,
+ 5X,'NUMBER OF SAMPLES PER FRAME: ',I5)
IF(PACK) GO TO 3300
WRITE(TTY,3200)AMAX
3200 FORMAT(5X,'MAXIMUM AMPLITUDE: ',F10.3)
GO TO 3500
3300 CONTINUE
WRITE(TTY,3400)AMAX,AMAX2
3400 FORMAT(5X,'MAXIMUM AMPLITUDE FOR FIRST CHANNEL: ',F10.3)
+ 5X,'MAXIMUM AMPLITUDE FOR SECOND CHANNEL: ',F10.3)
3500 CONTINUE
C
C
STOP
END

!RIFE WINDOW IN TIME (DATA WINDOW)
SUBROUTINE RIFWIN(SERIES,LTIME)
DIMENSION SERIES(1)
DATA D1,D2/-1.19685,0.19685/
X=LTIME
TPI=2.0*3.14159265/X
FPI=2.0*TPI
DO 1 J=1,LTIME
X=J-1
SERIES(J)=SERIES(J)*(1.+D1*COS(TPI*X)+D2*COS(FPI*X))
1 RETURN
END
.TYPE PLOT3 FOR

* THIS PROGRAM PlOTS A THREE DIMENSIONAL CURVE
* SPECIFIED BY A FUNCTION OF Z WITH RESPECT TO X AND Y
* THE FUNCTION IS NAMED *FUNCT*

PROGRAM INITIALIZATION

COMMON N,PEN

INTEGER TTY
INTEGER BLACK,BLUE,RED,PENNOW
INTEGER TITLE(7),ANSWER,BLANK,Y Es
REAL INCX,INCZ,INCX,INCZ
INTEGER PEN(3)
REAL DELTAX,DELTAY,DELTAZ
REAL BEGIN,END,AXISDE
REAL POINT(3)
DOUBLE PRECISION I LAB
DOUBLE PRECISION INAME,ILABEL

LOGICAL LIQUID,POLAR
DATA TTY/5/
DATA BLACK,BLUE,RED/1,2,3/
DATA YES/'Y'/
DATA BLANK/' '/

AXLEN=6.0
FACTR=1.0
IND='I'
DATA PEN/3,3,3/
N=1

POLAR=.FALSE.
LIQUID=.FALSE.
WRITE(TTY,1)
1 FORMAT(1X,'DO YOU WANT A LIQUID PLOT?',$)
READ(TTY,2)ANSWER
2 FORMAT(A1)
IF(ANSWER.NE.YES)GO TO 3
LIQUID=.TRUE.
3 WRITE(TTY,4)
4 FORMAT(1X,'DO YOU WANT A MAGNITUDE PLOT?',$)
READ(TTY,5)ANSWER
5 FORMAT(A1)
IF (ANSWER .EQ. YES) GO TO 6
POLAR = .TRUE.

C

INITIALIZE THE PLOTTER

CALL PLOTS (IND, 20.0, 11.0)
CALL FACTOR (FACTR)
CALL PLOT (3.667, 3.667, -13)

C

INPUT INFORMATION

IF (LIQUID) GO TO 20
WRITE (TTY, 10)
10 FORMAT ('1', 'PROGRAM SET UP FOR MULTICOLORED PEN
GO TO 40
20 CONTINUE
WRITE (TTY, 30)
30 FORMAT ('1', 'PROGRAM SET UP FOR LIQUID INK (ALL
40 CONTINUE
WRITE (TTY, 50)
50 FORMAT (1X, 'WHAT IS THE NAME OF YOUR INPUT FILE ?')
READ (TTY, 75) INAME
75 FORMAT (A10)
OPEN (UNIT = 19, DEVICE = 'DSKC', MODE = 'IMAGE', FILE = INAME)
REWIND 19

C

WRITE (TTY, 100)
100 FORMAT (' ', 'HOW MANY FRAMES ARE TO BE PLOTTED ?')
READ (TTY, *) XMAX
XMIN = 1.0

C

WRITE (TTY, 200)
200 FORMAT (' ', 'HOW MANY SAMPLES PER FRAME ARE THERE ?')
READ (TTY, *) ZMAX
ZMIN = 1

C

YMIN = 0.0
WRITE (TTY, 210)
210 FORMAT (1X, 'DO YOU KNOW THE MAXIMUM AMPLITUDE ?')
READ (TTY, 220) ANSWER
220 FORMAT (A1)
IF (ANSWER .EQ. YES) GO TO 280
C
FALSE - READ THROUGH DATA TO FIND MAXIMUM
YMAX = 0.0
230 CONTINUE
READ (19, END = 260) AWORD
IF (YMAX .GT. AWORD) GO TO 240
YMAX = AWORD
CONTINUE
GO TO 230
REWIND 19
WRITE(TTY,270)YMAX
FORMAT(1X,'MAXIMUM AMPLITUDE=',F12.3)
GO TO 300
C TRUE-INPUT MAXIMUM AMPLITUDE
CONTINUE
WRITE(TTY,290)
FORMAT(1X,'WHAT IS THE MAXIMUM AMPLITUDE ?')
READ(TTY,*)YMAX
C C C C C
WRITE(TTY,600)
FORMAT( 'DO YOU WANT A TITLE ?')
READ(TTY,700)ANSWER
FORMAT(A1)
IF(ANSWER .NE. YES)GO TO 930
C FALSE-INPUT AND PLOT THE TITLE
WRITE(TTY,800)
FORMAT( 'TYPE IN THE TITLE. ',5X,'UP TO 25 CHAR
+','1234567890123456789012345')
READ(TTY,900)(TITLE(I),I=1,7)
FORMAT(7A5)
C FIND THE NUMBER OF CHARACTORS
NUM=0
CONTINUE
IF(TITLE(NUM/5) .EQ. BLANK) GO TO 920
NUM=NUM+5
GO TO 910
CONTINUE
CALL SYMBOL(-3.0,6.5,0.4,TITLE,0.0,NUM)
C C C
WRITE(TTY,921)
FORMAT(1X,'WHAT LABEL DO YOU WANT CORRESP. TO FRAME #')
READ(TTY,922)ILAB
FORMAT(A10)
WRITE(TTY,923)ILAB
FORMAT(1X,'WHAT ARE THE BEGIN. AND END VALUES OF ',A10)
READ(TTY,*)BEG,EN
C WRITE(TTY,931)
FORMAT(1X,'WHAT LABEL DO YOU WANT CORRESPONDING TO SAM
READ(TTY,932)ILABEL
FORMAT(A10)
WRITE(TTY,933)ILABEL
FORMAT(1X,'WHAT ARE THE BEGINNING AND ENDING VALUES OF
READ(TTY,*), BEGIN, END

WRITE(TTY,940)

940 FORMAT(‘ ’,’/’,/’,’,’,'DOING CALCULATIONS.....PLEASE

CALCULATE SEVERAL VALUES THAT WE'LL NEED

INCZIN=AXLEN/(ZMAX-1)
INCXIN=AXLEN/(XMAX-1)

INCZ=(ZMAX-ZMIN)/(AXLEN/INCZIN)
INCX=(XMAX-XMIN)/(AXLEN/INCXIN)

DELTAX=(XMAX-XMIN)/AXLEN
DELTAY=(YMAX-YMIN)/AXLEN
DELTAZ=(ZMAX-ZMIN)/AXLEN
AXISDE=(END-BEGIN)/AXLEN
AXISD=(END-BEGIN)/AXLEN

DRAW AXIS

CALL NEWPEN(BLACK)
X-AXIS
CALL SETSZ(0.14,.105,.105,.7071)
CALL AXIS(0.0,0.0,ILABEL,-10,AXLEN,225.0,BEG,AXISD)
CALL SETSZ(0.14,.105,.105,1.0)

Y-AXIS
CALL AXIS(0.0,0.0,9HAMPLITUBE,9,AXLEN,0.0,0.0,YMIN,DELTAY

Z-AXIS
CALL AXIS(0.0,6.0,ILABEL,-10,AXLEN,-90.0,BEG,AXISD

DRAW THE THREE DIMENSIONAL CURVE

PENNOW=BLACK
X=XMIN

LOOP—INCREMENT X FROM LOWER TO UPPER LIMIT
1000 CONTINUE
IF(ISAV.NE.1)GO TO 1105
CALL READ(POINT(1),POINT(2),POINT(3),POLAR)

C MOVE PEN BACK TO BEGINNING OF PLOT
1105 IF( LIQUID ) GO TO 1050
   PENNOW= PENNOW+1
   IF( PENNOW .LE. RED ) GO TO 1030
   PENNOW= BLACK
1030   CONTINUE
   CALL NEWPEN(PENNOW)
1050   CONTINUE
   YZERO= (( ZMAX-ZMIN )/ DELTAZ )-(( X-XMIN )/( 2* DELTAX ))
   XZERO= (( POINT(ISAV)-YMIN )/ DELTAY )-(( X-XMIN )/( 2* DELTAX ))
   CALL PLOT(XZERO,YZERO,3)

C  Z=0.

C LOOP-INCREMENT Z FROM LOWER TO UPPER LIMIT
1100    CONTINUE
    DO 1110 I=ISAV,3
       XPLT=(( POINT(I)-YMIN )/ DELTAY )-(( X-XMIN )/( 2* DELTAX ))
       YPLT=(( ZMAX-Z )/ DELTAZ )-(( X-XMIN )/( 2* DELTAX ))
       CALL PLOT(XPLT,YPLT,PEN(I))
       Z=Z+ INCZ
    1110   IF( Z.EQ.ZMAX ) GO TO 1200
1100    CONTINUE
    CALL READ(POINT(1),POINT(2),POINT(3),POLAR)
    ISAV=1

C EXIT-IF MAXIMUM IS REACHED
1200    CONTINUE
    ISAV=I+1
    IF( ISAV.GT.3 ) ISAV=1
    WRITE( TTY, 1250 ) X
    1250 FORMAT( 1X, 'COMPLETED FRAME # ', F4.0 )
    X= X+ INCX
C EXIT IF MAXIMUM IS REACHED
    IF( X.GT. XMAX ) GO TO 1300
1200    CONTINUE

C

C

C STOP
END

FUNCTION TO BE GRAPHED

SUBROUTINE READ(POINT1,POINT2,POINT3,POLAR)
COMMON N,PEN

INTEGER UP,DOWN,PEN(3)
LOGICAL PACKED,POLAR

DATA UP,DOWN/3,2/

PACKED=.FALSE.

PEN(1)=DOWN
PEN(2)=DOWN
PEN(3)=DOWN

IF (.NOT. PACKED) GO TO 50

CONTINUE ! DATA IS UNPACKED
READ(19,END=400)POINT1,POINT2,POINT3
IF(N .LT. 0) GO TO 400
90 CONTINUE

IF(POLAR)GO TO 300
IF (POINT1 .GE. 0) GO TO 100
   PEN(1)=UP
   POINT1=0
100 CONTINUE
IF (POINT2 .GE. 0) GO TO 200
   PEN(2)=UP
   POINT2=0
200 CONTINUE
IF (POINT3 .GE. 0) GO TO 300
   PEN(3)=UP
   POINT3=0
300 CONTINUE
GO TO 500

CONTINUE
IF END OF FILE, FILL WITH ZEROS
POINT1=0
POINT2=0
POINT3=0
PEN(1)=UP
PEN(2)=UP
PEN(3)=UP
IF (N .LT. 0) GO TO 475

WRITE(5,450)
450 FORMAT(1X,'-----ALL DATA COMPLETE------')
CONTINUE
N=-1
500  CONTINUE
C
RETURN
END
APPENDIX D

PROGRAMS FOR SIMULATING ECHOS FROM LAYERED SUBBOTTOM MEDIA
TYPE RICAT2.FOR

COMPLEX ONE,T,RCOEQ

COMPLEX K,SUM,TERM,K1,HTOT,H

DIMENSION K(121,160),GAM(121),HTOT(160)

DATA C1,C2/1000.,7000./

DATA N,DF/160.,121.,50./

DATA T,SCALE./003.,10000./

ONE=CMPLX(1.,0.)

DT=T/(N-1)

PI=3.1415926

CALL GAMMA(GAM,N,DT,C1,C2)

CALL COSTP2(GAM,N,C1,C2)

WRITE(5,*)GAM

DO 5 1=1,N

DO 6 J=1,M

K(I,J)=CMPLX(0.,0.)

CONTINUE

5

ESS=0.

F=7950.

DO 600 J=1,M

F=F+DF

SUM=CMPLX(0.,0.)

DO 500 I=N,1,-1

TAU=(I-1)*DT

IF(I.EQ.1)GO TO 15

TRCOEF=ONE-K(I-1,J)**2

GO TO 30

15

TRCOEF=CMPLX(1.,1.)

30

TERM=GAM(I)*CEXP(CMPLX(0.,-2*PI*F*TAU))*DT*TRCOEF

K1=K(I,J)

K(I,J)=CEXP(CMPLX(0.,2*PI*F*TAU))*(SUM+TERM)

IF(I.NE.1)GO TO 500

IF(K(I,J).EQ.0.)GO TO 500

ESS=ESS+CABS(K1-K(I,J))**2/CABS(K(I,J))**2

500

SUM=SUM+TERM

600

CONTINUE

WRITE(5,*)ESS

IF(ESS.GT.01)GO TO 10

OPEN(UNIT=11,DEVICE='DSKC',MODE='IMAGE',FILE='SPEC.DH

H=CMPLX(0.,0.)

DO 1 I=1,160

WRITE(11)H

1

CONTINUE

1=1

DO 3 J=1,M

HTOT(J)=SCALE*K(I,J)

LEN=M

DO 700 I=1,M

700 WRITE(11)HTOT(I)

DO 2 I=1,704
SUBROUTINE GAMMA(GAM,N,DT,C1,C2)
DIMENSION GAM(1)
DO 1 I=1,N
   GAM(I)=C1
1 CONTINUE
DO 2 I=1,N
   GAM(I)=C2
2 RETURN
END

SUBROUTINE COSTAP(HTOT,LEN)
COMPLEX HTOT
DIMENSION HTOT(1)
ILEN = LEN / 2
HTOT(1) = CMPLX(0.,0.)
HTOT(LEN) = CMPLX(0.,0.)
FAC = 3.1415926/FLOAT(ILEN)
DO 400 J=2,ILEN
   WFAC = COS(FAC*FLOAT(J-1))
   WFAC = (1.-WFAC)**.5
   HTOT(J) = WFAC*HTOT(J)
J = LEN-J+1
400 HTOT(JJ) = WFAC*HTOT(JJ)
RETURN
END

SUBROUTINE COSTP1(GAM,N,C1,C2)
DIMENSION GAM(1)
IN = N/10
GAM(1)=0.
GAM(N)=0.
FAC = 3.1415926/FLOAT(IN)
DO 400 J=2,IN
   WFAC = COS(FAC*FLOAT(J-1))
   WFAC = (1.-WFAC)**.5
   GAM(J) = WFAC*GAM(J)
J = N-J+1
400 GAM(II) = WFAC*GAM(II)
RETURN
END

SUBROUTINE COSTP2(GAM,N,C1,C2)
DIMENSION GAM(1)
IN = N/10
GAM(1)=0.
GAM(N)=0.
GAM(81)=C1
FAC = 3.1415926/FLOAT(IN)
DO 400 J=2,IN
   WFAC = COS(FAC*FLOAT(J-1))
   WFAC = (1.-WFAC)**.5
GAM(J) = W FAC * GAM(J)
II = N - J + 1
GAM(II) = W FAC * GAM(II)
JJ = 80 + J
400 GAM(JJ) = C1 + W FAC * (C2 - C1)
RETURN
END
TYPE POLY3.FOR

COMPLEX Z,NUM,DEN,HTOT,H,R3
DIMENSION HTOT(160)
DATA C0,C1,C2/0.03,-1.1,1/
DATA T,SCALE/.003,5000./
DATA DF,M,N/50.,160,3/
DT=T/N
PI=3.1415926
F=7950.
OPEN UNIT=11,DEVICE='DSKC',MODE='IMAGE',FILE='INPUT'.
AMAX=0.
5 READ(11,END=7) H
AMP=CABS(H)
IF(AMP.LT.AMAX)GO TO 5
AMAX=AMP
GO TO 5
7 REWIND 11
DO 11 I=1,160
11 READ(11) H
DO 600 J=1,M
F=F+DF
Z=CEXP(CMPLX(0.,-2*PI*F*DT))
READ(11) R3
IF(CABS(R3).EQ.0)GO TO 12
R3=.75*R3/AMAX
12 CONTINUE
NUM=C0+C1*Z+C2*Z**2+R3*Z**3
DEN=CMPLX(1.,0.)+(C0*C1+C1*C2+C2*R3)*Z+(C1*R3+C0*C2)
+ C0*R3*Z**2
HTOT(J)=NUM/DEN
CHECK=CABS(HTOT(J))
IF(CHECK.LT.1)GO TO 600
WRITE(5,10)
10 FORMAT(1X,'REFL. COEF. MAG. EXCEEDS 1 !')
600 CONTINUE
OPEN UNIT=12,DEVICE='DSKC',MODE='IMAGE',FILE='SPEC.D'
H=CMPLX(0.,0.)
DO 1 I=1,160
1 WRITE(12) H
DO 3 I=1,M
3 HTOT(I)=SCALE*HTOT(I)
LEN=M
CALL COSTAP(HTOT,LEN)
DO 700 I=1,M
700 WRITE(12) HTOT(I)
DO 2 I=1,704
2 WRITE(12) H
STOP
END
SUBROUTINE COSTAP(HTOT,LEN)
COMPLEX HTOT
DIMENSION HTOT(1)
ILEN=LEN/2
HTOT(1)=CMPLX(0.,0.)
HTOT(LEN)=CMPLX(0.,0.)
FAC=3.1415926/FLOAT(ILEN)
DO 400 J=2,ILEN
  WFAC=COS(FAC*FLOAT(J-1))
  WFAC=(1.-WFAC)*.5
  HTOT(J)=WFAC*HTOT(J)
JJ=LEN-J+1
400 HTOT(JJ)=WFAC*HTOT(JJ)
RETURN
END
APPENDIX E

PROGRAMS FOR SIMULATING ECHOS FROM BOTTOM STRUCTURES
TYPE SCATTR.FOR

COMPLEX JAY,H,DH,PHASE,HTOT
DIMENSION HTOT(160)
REAL L,NORM,MOV
INTEGER FLAG
FLAG=0
DATA SCALE/1000000/
DATA DIA,XO,MOV/1.5,20,1./
DATA Z0,Z1/33.,45./
DATA UNIT,DF,BTOA/,25,50.,.0416256/
OPEN(UNIT=11,DEVICE='DSKC',MODE='IMAGE',FILE='SPEC.D'
DO 5 I=1,160
5 HTOT(I)=CMPLX(0.,0.)
DO 10 J=1,17
H=CMPLX(0.,0.)
DO 1 I=1,180
WRITE(11)H
1 CONTINUE
PI=3.1415926
C=1500.
F=8950.
DO 600 I=1,120
F=F+DF
WVL=C/F
D=DIA/(1.22*WVL)
B=Z0/(D*SQR(1.-1./(D**2)))
DX=WVL*UNIT
H=CMPLX(0.,0.)
X1=MOV*(J-1)+X0
AA=20.
X=X1-AA-DX/2.
N=2./DX
IF(B.GT.X1-X0+AA)GO TO 50
N=(B-X1+X0+AA)/DX
IF(N.LE.0)GO TO 550
50 IF(J.GT.1)GO TO 100
X=X1+DX/2.
N=(X1-X0)/DX
IF(B.GT.X1-X0)GO TO 60
X=X0+B+DX/2.
N=B/DX
60 IF(J.GT.1)GO TO 100
X=X1+AA+DX/2.
N=1./DX
IF(B.GT.X1-X0+AA)GO TO 100
X=X0+B+DX/2.
N=B/DX
100 DO 200 JJ=1,N
IF(J.GT.1)GO TO 101
IF(FLAG.EQ.0)GO TO 110
101  X=X+DX
    GO TO 120
110  X=X-DX
120  IF(J.EQ.1) GO TO 30
    IF(J.EQ.1.AND.X.LT.X1) GO TO 30
121  PHI=ACOS(((X-XO)**2+(X1-X0)**2-AA**2)/(2*(X-X0)*(X1-
30  FX=20
    Z=FX
    L=SQRT((X-XO)**2+Z**2)
    SINTH=(X-XO)/L
    COSTH=Z/L
    A=DIA/WVL
    W=PI*A*SINTH
    WSQ=W**2
    W4TH=W4TH**2
    W6TH=W6TH*W4TH
    W8TH=W8TH**2
    PHASE=CEXP(CMPLX(0.,-4.*PI*L/WVL))
    JAY=CMPLX(0., -1.)
    DH=JAY*(F/C)*FTH**2*PHASE*COSTH*(X-XO)*DX/L**2*
    DH=2.*PI*DH
    IF(J.EQ.1) GO TO 200
    IF(J.EQ.1.AND.X.LT.X1) GO TO 200
    DH=(PHI*DH)/(2*PI)
200  H=H+DH
    GO TO 210
    IF(FLAG.EQ.1) GO TO 205
    IF(J.NE.4) GO TO 205
    FLAG=1
    X=X1-DX/2.
    N=AA/DX
    IF(B.GT.X1-X0+AA) GO TO 100
    N=(B-X1+X0)/DX
    IF(N.LE.0) GO TO 205
    GO TO 100
205  FLAG=0
210  H=BT0A*SCALE*H
    HTOT(I)=H
    HMAG=CABS(HTOT(I))
    WRITE(5,*HMAG
    DEN=REAL(HTOT(I))
    IF(DEN.EQ.0) GO TO 300
    HANG=ATAN(AIMAG(HTOT(I))/DEN)*180/PI
    GO TO 600
300  HANG=90.
    GO TO 600
550  HTOT(I)=CMPLX(0.,0.)
600  CONTINUE
    LEN=120
    CALL COSTAP(HTOT,LEN)
```
800  DO 700 I=1,120
700  WRITE(11)HTOT(I)
     H=CMPLX(0.,0.)
     DO 2 I=1,724
     WRITE(11)H
     CONTINUE
2     WRITE(5,90)J
90    FORMAT(1X,'COMPLETED TARGET *',I4)
10    CONTINUE
     STOP
     END
SUBROUTINE COSTAP(HTOT,LEN)
COMPLEX HTOT
DIMENSION HTOT(1)
ILEN=LEN/2
HTOT(1)=CMPLX(0.,0.)
HTOT(LEN)=CMPLX(0.,0.)
FAC=3.1415926*2./FLOAT(LEN)
DO 400 J=2,ILEN
     WFAC=COS(FAC*FLOAT(J-1))
     WFAC=(1.-WFAC)*.5
     HTOT(J)=WFAC*HTOT(J)
     JJ=LEN-J+1
400    HTOT(JJ)=WFAC*HTOT(JJ)
     RETURN
     END
```
APPENDIX F

PROGRAMS FOR DISCRETE LAYER MODEL ANALYSIS OF EXPERIMENTAL DATA
DEclarations
COMPLEX C(100),DEN,ATDEN
REAL XR(8192),XI(8192),XR1(8192),PHASE(8192),TAU(100)
REAL ALPHA(5),CRATIO(5),KA(100),CR(100)
LOGICAL FOUND
LOGICAL ATTENDONE
INTEGER TTY,DATA,ANSWER,NO,TM,TN,BIT
DOUBLE PRECISION FILENM
DATA TTY,NO/5,'N'/
DATA ALPHA/1.2,6,2,1./
DATA CRATIO/1.014,1.033,1.150,1.300,1.600/

OPEN FILES
WRITE(TTY,10)
10 FORMAT(1X,'WHAT IS THE NAME OF YOUR INPUT FILE?',$)
READ(TTY,20)FILENM
20 FORMAT(1X,'HOW MANY DATA SAMPLES ARE TO BE ANALYZED?
READ(TTY,*)NUM
DO 40 I=1,NUM
READ(10)DATA
XR(I)=FLOAT(DATA)
40 XI(I)=0.

FFT, THEN RECT FILTER (9-15 KHZ), IFFT
CALL COOLER(XR(1),XI(1),NUM,1)
DO 50 I=1,179
XR(I)=0.
50 XI(I)=0.
DO 60 I=301,NUM-299
XR(I)=0.
60 XI(I)=0.
DO 70 I=NUM-177,NUM
XR(I)=0.
CALL COOLER(XR(I),XI(I),NUM,-1).

OPTION TO OUTPUT EXPERIMENTAL FILTERED TIME SERIES
WRITE(TTY,80)
FORMAT(1X,'DO YOU WISH TO OUTPUT EXP.FILTERED TIME S
READ(TTY,90)ANSWER
FORMAT(A1)
IF(ANSWER.EQ.NO) GO TO 100
DO 95 I=1,NUM
95 WRITE(11)XR(I)
100 CONTINUE

INITIALIZATION BEFORE ANALYSIS LOOP
DO 110 I=1,NUM
XR1(I)=XR(I)
XR(I)=0.
110 XI(I)=0.
AMAX=0.
EPHIBM=0.
ATDEN=CMPLX(1.,0.)
DEN=CMPLX(1.,0.)
TIME=1.
PHIBM=0.
T=.020
LASTPK=25
TM=0
TN=-5
ATTEN=.FALSE.
DONE=.FALSE.
CW=1500.
KA(1)=0.
CR(1)=0.
ID=1

ANALYSIS LOOP
DO 500 L=1,100
DO 120 I=1,NUM
120 XR(I)=XR1(I)-XR(I)

CALCULATE ANALYTIC SIGNAL, OUTPUT RESIDUAL SIGNAL FOR
CALL COOLER(XR(I),XI(I),NUM,1)
TEMP=0.
DO 180 I=1,NUM/2
XR(I)=2*XR(I)
XI(I)=2*XI(I)
180 TEMP=TEMP+(XR(I)**2+XI(I)**2)/2.
IF(L.GT.1)GO TO 185
FZERO=TEMP
185 TEMP=TEMP/FZERO*100.
IF(L.EQ.1)GO TO 189
IF(L.GT.2)GO TO 187

DT=T/NUM
JJ=TAU(1)/DT
DO 186 I=1,JJ
STEMP=100.
186 WRITE(12)STEMP
187 K=TAU(L)/DT
DO 188 I=1,K
JJ=JJ+1
188 WRITE(12)TEMP
189 CONTINUE
DO 190 I=NUM/2+1,NUM
XR(I)=0.
190 XI(I)=0.
CALL COOLER(XR(1),XI(1),NUM,-1)

OPTION TO OUTPUT ERROR SIGNAL MAGNITUDE
WRITE(TTY,*)TEMP
WRITE(TTY,200)
200 FORMAT(1X,'DO YOU WISH TO OUTPUT ERROR SIGNAL MAGNITUDE?
READ(TTY,210)ANSWER
210 FORMAT(A1)
IF(ANSWER.EQ.'NO')GO TO 230
DO 215 I=1,5
A=0.
215 WRITE(13)A
DO 220 I=6,NUM
A=SQRT(XR(I)**2+XI(I)**2)
220 WRITE(13)A
230 CONTINUE

FIND AMAX,SET ATHRESH, PICK SCALE FACTOR-PASS 1
IF(L.GT.1)GO TO 260
WRITE(TTY,235)
235 FORMAT(1X,'GIVE VALUE FOR PEAK DET. THRESHOLD!','$)
READ(TTY,**VALUE
DO 240 I=1,NUM
A=SQRT(XR(I)**2+XI(I)**2)
240 IF(A.GT.AMAX)AMAX=A
AT=VALUE*AMAX
WRITE(TTY,250)
250 FORMAT(1X,'GIVE REF.COEF. SCALE FACTOR<1','$)
READ(TTY,**SF
260 CONTINUE

FIND MTH AND NTH PEAKS IN TIME SERIES
FOUNDM=.FALSE.
AMAXM=0.
AMAXN=0.
ICOUNT=0
DO 280 I=LASTPK,NUM
A = SQRT(XR(I)**2 + XI(I)**2)
IF (L.GT.1.AND.FOUNDM.GE.0) GO TO 265
IF (A.LT.AT) GO TO 280
265 IF (FOUND M) GO TO 270
IF (A.GT.AMAXM) AMAXM = A
TM = I - 1
IF (TM.EQ.TN) FOUNDM = .TRUE.
IF (A.LT.AMAXM) FOUNDM = .TRUE.
GO TO 280
270 ICOUNT = ICOUNT + 1
IF (ICOUNT.LT.9) GO TO 280
IF (A.LT.ALAST) GO TO 275
IF (A.GT.AMAXN) AMAXN = A
TN = I
275 IF (A.LT.AMAXN) GO TO 290
280 ALAST = A
290 CONTINUE
WRITE (TTY, *) TM, AMAXM, TN, AMAXN
IF (AMAXN.EQ.0) GO TO 505
C
C FIND CARRIER PHASE SHIFT
PI = 3.1415926
DO 320 I = 1, NUM
IF (XR(I).EQ.0.) GO TO 300
PHASE(I) = ATAN(XI(I)/XR(I)) * 180./PI
GO TO 320
300 IF (XI(I).GE.0.) GO TO 310
PHASE(I) = -90.
GO TO 320
310 PHASE(I) = +90.
320 CONTINUE
C
C UNWRAP CARRIER PHASE, FIND PHASE AT PEAKS WRT. COS R
CALL UNWRAP(XR(1), PHASE(1), TM, TN, PHIZM, PHIZN, NUM)
C
C REMOVE TIME PHASE, FIND PHIM PHIN
C OPTION TO OUTPUT ERROR SPACE PHASE
WRITE (TTY, 330)
330 FORMAT (IX, 'DO YOU WISH TO OUTPUT ERROR SPACE PHASE?'
READ (TTY, 340) ANSWER
340 FORMAT (A1)
BIT = 512
ANGLE = 173.2
DO 350 I = 1, 4
ANGLE = ANGLE/2
BIT = 2*BIT
IF (NUM.EQ.BIT) GO TO 360
350 CONTINUE
360 DO 370 I = 1, NUM
PHI = PHASE(I) - I*ANGLE - 960.8755
IF (I.EQ.TM) PHIM = PHI
IF (I.EQ.TN) PHIN = PHI
IF (ANSWER.EQ.NO) GO TO 370
WRITE (14) PHI
CONTINUE

370

PRINCIPLE PART OF NTH BOUND. PHASE AND EST. BOUND. PH
PHIBN = PHIN - PHIM + ANGLE*(TN-TM)+PHIBM+720.

380

PHIBN = PHIBN-360.
IF (PHIBN.GT.0.) GO TO 380
PHIBNP = PHIBN+360.
IF (PHIBNP.GT.ABS(PHIBN)) GO TO 385
PHIBN = PHIBNP

385

REAL(POSITIVE OR NEGATIVE) IMPEDANCE LAYER CONSTRAINT
IF (ABS(PHIBN).GT.90.) GO TO 400
EPHBN = 0.
GO TO 410
EPHBN = 180.
CONTINUE
WRITE (TTY, *) EPHBN, EPHBN

C

DECISION TO TERMINATE & CLOSE RESID. POWER FILE
WRITE (TTY, 415)
FORMAT (1X, 'DO YOU WISH TO TERMINATE SIMULATION?', $)
READ (TTY, 416) ANSWER
FORMAT (A1)
IF (ANSWER.EQ.NO) GO TO 417
JJ = JJ + 1
WRITE (12) TEMP
IF (JJ.LT.NUM) GO TO 414
LL = L - 1
GO TO 429

417
CONTINUE

C

CALCULATE BOUND. REF. COEF. AND 2-WAY TRAVEL TIME
IF (L.GT.1) GO TO 420
C(1) = SF*AMAXM/AMAX*CEXP(CMPLX(0., PI*PHIBM/180.))
FC = ANGLE/360.**NUM/T
TAU(1) = (ANGLE*(TM-1.)+0.-PHIZM)/360.**1./FC
DEN = (1-C(L)**2)*DEN
ATDEN = (1-C(L)**2)*ATDEN
IF (EPHBN.EQ.0.) GO TO 426
IF (PHIZN.GT.0.) GO TO 425
PHIZN = PHIZN+180.
GO TO 426
PHIZN = PHIZN-180.
425
IF (EPHBN.EQ.0.) GO TO 428
IF (PHIZM.GT.0.) GO TO 427
PHIZM = PHIZM+180.
GO TO 428
PHIZM = PHIZM-180.
427
CONTINUE
428
C(L+1)=SF*AMAXN/(AMAX*ATDEN)*CEXP(CMPLX(0.,PI*EPHIBN/TAU(L+1)=(ANGLE*(TN-TM)+PHIZM-PHIZN)/360.*1./FC
C
PUT IN NARROW BAND ATTENUATION COEF.
IF(DONE)GO TO 435
IF(ATTEM)GO TO 431
IF(AMAXN.LT.AMAX/4.6.OR.TN.LT.776)GO TO 432
ATTEM=.TRUE.
LATTEN=L+1
ATDEN=ATDEN*EXP(-ALPHA(ID)/8.686*CRATIO(ID)*TAU(L+)
C(L+1)=SF*AMAXN/(AMAX*ATDEN)*CEXP(CMPLX(0.,PI*EPHIBN/
KA(L+1)=ALPHA(ID)/(8.686*FC)
CR(L+1)=CRATIO(ID)
ID=ID+1
IF(ID.GT.5)DONE=.TRUE.
GO TO 433
431
WRITE(TTY,434)
434
FORMAT(IX,'ATTENUATION LAYERS ARE FINISHED!')
432
KA(L+1)=0.
CR(L+1)=0.
433
TIME=TIME+TAU(L)*NUM/T
WRITE(TTY,'PHIZM-PHIZM TIME
WRITE(TTY,'C(L),C(L+1),TAU(L),TAU(L+1)
C
C
CALCULATE MODEL REFLE. COEF. AS A FUNC. OF FREQ.
LL=L
429
CALL MODEL(LL,DEN,C(1),TAU(1),CR(1),KA(1),XR(1),XI(1)
C
C
SHAPE MODEL SPECTRUM WITH RAISED COSINE
LEN=120
CALL COSTAP(XR(1),XI(1),LEN,NUM)
C
C
FOLD FREQ. OVER INTO NEG. SPECTRUM
DO 430 I=1,NUM/2
XR(NUM-I+1)=XR(I+1)
430
XI(NUM-I+1)=-XI(I+1)
DO 440 I=1,NUM
GF=3450./SF
XR(I)=GF*XR(I)
440
XI(I)=GF*XI(I)
C
C
OUTPUT MODEL T-SERIES MAGNITUDE ON SIM. TERMINATION
IF(ANSWER.EQ.NO)GO TO 444
DO 441 I=1,NUM/2
XR(I)=2*XR(I)
441
XI(I)=2*XI(I)
DO 442 I=NUM/2+1,NUM
XR(I)=0.
442
XI(I)=0.
CALL COOLER(XR(1),XI(1),NUM,-1)
DO 443 I=1,NUM
SIM=SQRT(XR(I)**2+XI(I)**2)
WRITE(16)'SIM
GO TO 505
CONTINUE

C
IFFT TO MODEL TIME SERIES & UPDATE BOUNDARY PHASE
CALL COOLER(XR(1),XI(1),NUM,-1)
WRITE(TTY,446)
FORMAT(1X,'DO YOU WISH TO EXAM. TIME SERIES? ',$)
READ(TTY,447)ANSWER

FORMAT(A1)
IF(ANSWER.EQ.NO)GO TO 448
DO 445 I=TM-12,TM+12
WRITE(TTY,*I,XR(I),XR1(I))
CONTINUE
LASTPK=TN
PHIBM=PHIBN

500 EPHIBM=EPHIBM
C
CAL. REL. IMP. PROFILE AND FILE IT FOR OUTPUT
505 DT=T/NUM
RHOC=1.
J=TAU(1)/DT
DO 510 I=1,J
WRITE(15)RHOC
DO 530 I=1,100
IF(C(I+1).EQ.0.)GO TO 540
BEDROCK CONSTRAINT
IF(I.GE.LATTEN)C(I)=CABS(C(I))
RHOC=RHOC*CABS((1+C(I))/(1-C(I)))
K=TAU(I+1)/DT
DO 520 M=1,K
WRITE(15)RHOC
J=J+1
IF(J.EQ.NUM)GO TO 550
CONTINUE
530 CONTINUE
540 RHOC=-1.
WRITE(15)RHOC
J=J+1
IF(J.LT.NUM)GO TO 540
WRITE(TTY,560)
560 FORMAT(1X,'INFORMATION FOR PLOTING PROGRAM'
WRITE(TTY,570)JJ,NUM
570 FORMAT(1X,I6,'POINTS ARE IN RES POWER FILE',I6,'IN DT
STOP
END
C
PROGRAM SUBROUTINES
C
UNWRAP CARRIER PHASE ANGLE
SUBROUTINE UNWRAP(XR,PHASE,TM,TN,PHIZM,PHIZN,NUM)
INTEGER TM,TN
REAL XR(1),PHASE(1),NEXT,LAST
LAST=0.
THETA=0.
DO 100 J=1,NUM
NEXT=PHASE(J)
IF(NEXT GT 0 .AND. XR(J) GT 0 .)GO TO 10
IF(NEXT LT 0 .AND. XR(J) LT 0 .)GO TO 20
IF(NEXT GT 0 .AND. XR(J) LT 0 .)GO TO 30
IF(NEXT LT 0 .AND. XR(J) GT 0 .)GO TO 40
10 NEXT=NEXT
GO TO 50
20 NEXT=90.+(90.-ABS(NEXT))
GO TO 50
30 NEXT=180.+NEXT
GO TO 50
40 NEXT=270.+(90.-ABS(NEXT))
50 IF(J.NE.TM)GO TO 60
IF(NEXT.GT.180.)GO TO 55
PHIZM=NEXT
GO TO 60
55 PHIZM=NEXT-360.
60 IF(J.NE.TN)GO TO 70
IF(NEXT.GT.180.)GO TO 65
PHIZN=NEXT
GO TO 70
65 PHIZN=NEXT-360.
70 IF(NEXT.GT.LAST)GO TO 80
DTHETA=360.-(LAST-NEXT)
GO TO 90
80 DTHETA=NEXT-LAST
90 THETA=THETA+DTHETA
PHASE(J)=THETA
LAST=NEXT
100 CONTINUE
RETURN
END

C
MODEL TO CAL.REF.COEFF.AS A FUNC.OF FREQ.
SUBROUTINE MODEL(LL,DEN,C,TAU,CR,KA,XR,XI,SF)
REAL SF
REAL CR(1),KA(1)
REAL XR(1),XI(1),TAU(1)
COMPLEX V,Z,TRANS,DEN,C(1)
COMPLEX U
INTEGER TTY
DATA PI,DF,TTY/3.1415926,50.5/
F=8950.
CW=1500.
DO 20 I=181,301
TRANS=DEN
F = F + DF
V = C(LL + 1)
DO 10 J = LL + 1, -1

Z = CEXP(CMPLX(-KA(J + 1) * F * CW*CR(J + 1) * TAU(J + 1), -2*PI*F*T)
TRANS = TRANS*Z
V = (C(J) + V*Z) / (1. + C(J) * V*Z)
Z = CEXP(CMPLX(0., -2*PI*F*TAU(1)))
CHECK = CABS(V*Z)
IF(CHECK, LT, 1.) GO TO 15
WRITE(TTY, 14) SF
14 FORMAT(1X, 'REFLECTION COEF. MAG. EXCEEDS 1! MAKE SF<', F4.3)
15 V = Z*TRANS*C(LL + 1)
XR(I) = REAL(V)
XI(I) = AIMAG(V)
RETURN
END

C
C  COSINE TAPERED SHAPING OF SPECTRA
C
SUBROUTINE COSAPE(XR, XI, LEN, NUM)
REAL XR(I), XI(I)
ILEN = LEN / 2
DO 5 I = 1, 180
XR(I) = 0.

5 XI(I) = 0.
FAC = 3.1415926 * 2. / FLOAT(LEN)
DO 10 J = 1, ILEN
WFAC = COS(FAC * FLOAT(J - 1))
WFAC = (1. - WFACT) * .5
XR(180 + J) = WFACT * XR(180 + J)
XI(180 + J) = WFACT * XI(180 + J)
JJ = 180 + LEN - J + 2
XR(JJ) = WFACT * XR(JJ)
XI(JJ) = WFACT * XI(JJ)
DO 10 I = 302, NUM
XR(I) = 0.

15 XI(I) = 0.
RETURN
END

C
C  FAST FOURIER TRANSFORM AND INVERSE TRANSFORM
C
SUBROUTINE COOLER(XR, XI, NUM, ISIGN)
INTEGER BIT(14)
REAL XR(I), XI(I)
EQUIVALENCE (OR, TEMP, FLX)(I2, NW1)
DATA BIT/8192, 4096, 2048, 1024, 512, 256, 128, 64, 32, 16, 8, 4
DO 1 I = 1, 14
IF(NUM, EQ, BIT(I)) GO TO 10
CONTINUE
10 N = 14 - I
I1 = 15 - N
I2 = I1 + 1
LX=BIT(I1)*2
LX4=BIT(I2)
FACTOR=2.*314159265/FLOAT(LX)
DO 4 I=1,N
NBLOCK=BIT(-I*LAY+15)
LBLOCK=LX/NBLOCK
LHALF=LBLOCK/2
NW=0
DO 4 I=1,NBLOCK
FMW=NW
WR=COS(FMW*FACTOR)
WI=SIN(FMW*FACTOR)
IF(ISIGN.GT.0)WI=-WI
LSTART=1+LBLOCK*(IBLOCK-1)
LEN=LSTART+LHALF-1
DO 2 I=LSTART,LEN
ILB=I+LHALF
GR=XR(ILB)*WR-XI(ILB)*WI
QI=XR(ILB)*WI+XI(ILB)*WR
XR(ILB)=XR(I)-GR
XI(ILB)=XI(I)-QI
DO 3 I=I2,I4
IF(NW.LT.BIT(I))GOTO 4
NW=NW-BIT(I)
I=14
NW=NW+BIT(I)
NW=0
DO 7 K=1,LX
NW1=NW+1
IF(NW1.LE.K)GOTO 5
TEMP=XR(NW1)
XR(NW1)=XR(K)
XR(K)=TEMP
TEMP=XI(NW1)
XI(NW1)=XI(K)
XI(K)=TEMP
DO 6 I=I1,I4
IF(NW.LT.BIT(I))GOTO 7
NW=NW-BIT(I)
I=14
NW=NW+BIT(I)
IF(ISIGN.GT.0)GOTO 9
FLX=LX
DO 8 J=1,LX
XR(J)=XR(J)/FLX
XI(J)=XI(J)/FLX
RETURN
END
C

TYPE STACUN.FOR

INTEGER RECORD(1024)
INTEGER TTY, ANSWER, UN, FRAME
DOUBLE PRECISION FILENM
LOGICAL STACK

DATA TTY, UN / 'U' /
STACK = .TRUE.
WRITE(TTY, 5)

5 FORMAT(1X, 'WHAT IS THE NAME OF THE FILE TO BE STACKED?')
READ(TTY, 7) FILENM

7 FORMAT(A10)
OPEN(UNIT=11, DEVICE = 'DSKC', MODE= 'IMAGE', ACCESS = 'RANDOM

+ RECORD SIZE=1024, FILE = 'STACK.DAT')
OPEN(UNIT=12, DEVICE = 'DSKC', MODE= 'IMAGE', FILE = FILENM)
WRITE(TTY, 10)

10 FORMAT(1X, 'DO YOU WISH TO STACK OR UNSTACK?')
READ(TTY, 20) ANSWER

20 FORMAT(A1)
IF(ANSWER.EQ.UN) STACK = .FALSE.
WRITE(TTY, 30)

30 FORMAT(1X, 'HOW MANY SAMPLES PER FRAME?')
READ(TTY, *) NUM
WRITE(TTY, 40)

40 FORMAT(1X, 'WHICH FRAME NUMBER DO YOU WISH TO ACCESS?')
READ(TTY, *) FRAME
IF(STACK) GO TO 70
READ(11#FRAME) RECORD
WRITE(12)(RECORD(J), J=1, 1024)
GO TO 90

70 READ(12)(RECORD(J), J=1, 1024)
WRITE(11#FRAME) RECORD

90 WRITE(TTY, 100)(NUM, FRAME)

100 FORMAT(1X, I6, ' SAMPLES OF FRAME #', I6, ' HAVE BEEN TRAN
STOP
END
APPENDIX G

SAMPLE EXECUTION OF BOUNDARY COEFFICIENTS ANALYSIS INTERACTIVE PROGRAM
.EX READTP, FOR, IROT3, MAC

LINK:  Loading

[LNKXCT READTP Execution]

FROM WHICH DATA BLOCK NUMBER DO YOU WANT TO ACCESS DATA?
NOISE: 1, 4, 7... TRANS: 2, 5, 8... ECHO: 3, 6, 9...

3

THE BLOCK HEADING IS 3

THERE ARE 1024 SAMPLES IN BLOCK 3

DO YOU WANT ANOTHER BLOCK? NO

STOP

END OF EXECUTION

CPU TIME: 0.83  ELAPSED TIME: 13.40

EXIT
"C

.EX BCOEFA.FOR

FORTRAN: BCOEFA

MAIN.

UNWRAP

MODEL

COSTAP

COOLER

LINK:  Loading

[LNKXCT BCOEFA Execution]

WHAT IS THE NAME OF YOUR INPUT FILE? SIGNAL

HOW MANY DATA SAMPLES ARE TO BE ANALYZED? 1024

DO YOU WISH TO OUTPUT EXP.FILTERED TIME SERIES? NO

100.000

DO YOU WISH TO OUTPUT ERROR SIGNAL MAGNITUDE? YES

GIVE VALUE FOR PEAK DET. THRESHOLD! .033

GIVE REF.COEF. SCALE FACTOR < 1.045

278, 23.87715, 306, 22.71500

DO YOU WISH TO OUTPUT ERROR SPACE PHASE? YES

0.0000000, 0.0000000

DO YOU WISH TO TERMINATE SIMULATION? NO

-129.6543, -157.2119, 279.4972

(0.0025975, 0.0000000), (0.0024711, 0.0000000), 0.5439398

0.5530902E-03

DO YOU WISH TO EXAM. TIME SERIES? YES

266, 1.565911, 5.126769

267, 3.972511, 8.845724

268, -2.044778, -4.760254

269, -7.543343, -11.94619

270, 1.465962, 3.042398

271, 11.72590, 15.71294

272, 0.6194026, 0.5786400

273, -15.65504, -18.89000

274, -4.234296, -5.700241

275, 18.33570, 20.29569

276, 8.863560, 11.06844

277, -18.98255, -19.49355

278, -13.55083, -15.23731

279, 17.31880, 16.94745

280, 17.18705, 17.31331

281, -13.70254, -13.59742

282, -18.98275, -17.27018

283, 9.018557, 10.23343

284, 18.27227, 15.67417

285, -4.375616, -7.169752

286, -15.62451, -13.12303

287, 0.7341368, 4.435530

288, 11.71906, 9.903213

289, 1.383899, -2.237829
DO YOU WISH TO OUTPUT ERROR SPACE PHASE? NO
  180.0000 , 0.0000000 ,
DO YOU WISH TO TERMINATE SIMULATION? NO
  175.3648 , -164.751 , 810.2510 ,
  (-0.0329331, 0.), (-0.0546652, -0.), 0.222264E-03, 0.31108

DO YOU WISH TO EXAM. TIME SERIES? NO
  36.05732 ,
DO YOU WISH TO OUTPUT ERROR SIGNAL MAGNITUDE? NO
  824, 243.6625 , 842, 312.8622 ,
DO YOU WISH TO OUTPUT ERROR SPACE PHASE? NO
  0.000000 , 180.0000 ,
DO YOU WISH TO TERMINATE SIMULATION? NO
  -188.7202 , -120.8436 , 826.1785 ,
  (-0.0546652, -0.), (-0.0740343, 0.0000000), 0.3110836E-03,
  0.3407647E-03,
DO YOU WISH TO EXAM. TIME SERIES? NO
  36.10574 ,
DO YOU WISH TO OUTPUT ERROR SIGNAL MAGNITUDE? NO
  842, 321.0939 , 860, 190.8485 ,
DO YOU WISH TO OUTPUT ERROR SPACE PHASE? NO
  180.0000 , 0.0000000 ,
DO YOU WISH TO TERMINATE SIMULATION? NO
ATTENUATION LAYERS ARE FINISHED!
  -121.0275 , -115.3004 , 843.6257 ,
  (-0.0740343, 0.0000000), (-0.0454104, -0.), 0.3407647E-03,
  0.3502709E-03,
DO YOU WISH TO EXAM. TIME SERIES? NO
  31.42335 ,
DO YOU WISH TO OUTPUT ERROR SIGNAL MAGNITUDE? NO
  860, 196.6324 , 870, 189.3614 ,
DO YOU WISH TO OUTPUT ERROR SPACE PHASE? NO
  0.0000000 , 0.0000000 ,
DO YOU WISH TO TERMINATE SIMULATION? YES
INFORMATION FOR PLOTTING PROGRAM
  1024 POINTS ARE IN RES POWER FILE 1024 IN OTHERS
STOP

END OF EXECUTION
CPU TIME: 59.22 ELAPSED TIME: 9:54.37
EXIT
.DIR *.DAT

XDATA DAT 0 <077> 24-Aug-80 DSKC: [2000,32051]
POWER DAT 8 <077> 24-Aug-80
EMAG DAT 8 <077> 24-Aug-80
EPHI DAT 8 <077> 24-Aug-80
PROFIL DAT 8 <077> 24-Aug-80
SIMAG DAT 8 <077> 24-Aug-80

Total of 40 blocks in 6 files on DSKC: [2000,32051]
APPENDIX H

DERIVATION OF REFLECTION COEFFICIENT FOR N PLANE-PARALLEL, EQUAL TWO-WAY TRAVEL-TIME LAYERS [46]

Consider a set of N plane-parallel layers where each has a constant specific acoustic impedance $p_c$ and all are separated by equal two-way travel times $\tau$. At the nth boundary an expression for the total up and down going plane wave propagating normal to the layers may be written as:

\[
D_{n+1} = z^{1/2} \left( 1 + c_n \right) D_n - c_n U_{n+1} \\
U_n = z^{1/2} \left( 1 - c_n \right) U_{n+1} + c_n D_n.
\]

Simultaneously solving these for the $n+1$ components gives:

\[
\begin{bmatrix}
D_{n+1} \\
U_{n+1}
\end{bmatrix} = z^{-1/2} \begin{bmatrix}
z & -c_n \\
1-c_n & 1
\end{bmatrix} \begin{bmatrix}
D_n \\
U_n
\end{bmatrix}.
\]

By iterating this solution all the way back to the 1st boundary, the following polynomials in $Z$ may be defined:

\[
\begin{bmatrix}
D_{n+1} \\
U_{n+1}
\end{bmatrix} = (1-Z) \ldots (1-c_1) \begin{bmatrix}
P_n & Q_n \\
Q_n & P_n
\end{bmatrix} \begin{bmatrix}
D_1 \\
U_1
\end{bmatrix}
\]

When a similar expression is found at the $n-1$ st boundary for $D_n$ and $U_n$ and substituted into equation 2, a recursive relation for the polynomials is found:

\[
P_n = P_{n-1} - c_n Z Q_{n-1} \\
Q_n = Q_{n-1} - c_n Z P_{n-1}
\]
At the 1st boundary for the condition that there is no up-going wave in the n+1st layer, \( U_{n+1} = 0 \), the following is true:

\[
Q_n D_1 + P_n U_1 = 0. \tag{H.5}
\]

The wave propagation across the 0th boundary is:

\[
\begin{bmatrix}
D_1 \\
U_1
\end{bmatrix} = \begin{bmatrix}
1 & -C_0 \\
-C_0 & 1
\end{bmatrix}
\begin{bmatrix}
D_0 \\
U_0
\end{bmatrix} \tag{H.6}
\]

By substituting (6) into (5) and taking the ratio of \( U_0/D_0 \), which is the reflection coefficient at the 0th boundary,

\[
R = \frac{U_0}{D_0} = \frac{C_0 P_n Q_n}{P_n C_0 Q_n} \tag{H.7}
\]

However, it seems that the equation for \( R \) is incomplete or incorrectly formatted as it stands.
APPENDIX I

DERIVATION OF RICCATI EQUATION FOR REFLECTION COEFFICIENT OF AN INHOMOGENEOUS LAYER [44]

Consider a layer between two plane-parallel boundaries where the specific acoustic impedance varies as a function of the depth variable \( z \),

\[
\rho c = \rho c(z). \tag{I.1}
\]

The equations of physics for the medium are:

\[
\begin{align*}
\rho \frac{\partial v_z}{\partial t} &= -\frac{\partial p}{\partial z}, \quad dp = \frac{1}{c^2} \frac{dp}{dt} \quad \rho \frac{\partial v_z}{\partial z} = -\frac{\partial p}{\partial t} = -\frac{1}{c^2} \frac{\partial p}{\partial t}. \tag{I.2}
\end{align*}
\]

At a depth \( z \) in the layer, assume a solution of up and down-going waves:

\[
\begin{align*}
P(z) &= [D(z)+R(z)] e^{j(\omega t-kz)} \\
V(z) &= \frac{[D(z)-R(z)]}{\rho c} e^{-j(\omega t-kz)} \tag{I.3}
\end{align*}
\]

for the pressure and velocity. By assuming \( k \) to be constant in the increment \( \Delta z \) around \( z \) and by substituting equations 3 into equations 2 gives:

\[
\begin{align*}
&D - (D-R) \gamma = 0 \quad \text{R} \\
&R - (D-R) \gamma + 2jkR = 0 \quad \text{D}, \tag{I.4}
\end{align*}
\]

where

\[
\gamma = \frac{1}{2\rho c} \frac{\partial (\rho c)}{\partial z} \tag{.5}
\]

The reflection coefficient is defined as the ratio of reflected, \( R \), to direct, \( D \), waves.
\[ V = \frac{R}{D} \]  

By combining equations 4, a Riccati differential equation is found for the reflection coefficient:

\[ V' + 2jkV + \gamma(1-V^2) = 0. \]
APPENDIX J

FORM OF THE $\gamma$ FUNCTION FOR A SHAPED EXPONENTIAL IMPEDANCE LAYER

From the derivation of the Riccati Equation for the reflection coefficient, the $\gamma$ function is defined as:

$$\gamma = \frac{1}{2pc(z)} \frac{dpc(z)}{dz}$$  \hspace{1cm} (J.1)

When the specific acoustic impedance of the inhomogeneous layer is assumed to have an exponential functional form:

$$pc(z) = \rho c_0 e^{2kz}, \hspace{.3cm} (0<z<H)$$
$$= \rho c_0, \hspace{.3cm} (z<0)$$
$$= \rho c_0 e^{2kH}, \hspace{.3cm} (z>H),$$  \hspace{1cm} (J.2)

where $H$ is the thickness of the layer.

The form $\gamma$ function has the form:

$$\gamma = K, \hspace{.3cm} (0<z<H)$$
$$= 0, \hspace{.3cm} \text{elsewhere.}$$  \hspace{1cm} (J.3)

Since it was found that the rectangular $\gamma$ function as found in equation 3 was causing aliasing in the iterative integral used to solve for the reflection coefficient, $V$, a 10% raised cosine function was used to shape both edges and reduce the side lobes of the transformed rectangular $\gamma$ function.
APPENDIX K

DERIVATION OF THE REFLECTION COEFFICIENT FOR ARBITRARILY-SHAPED INSONATED SURFACE

The radiation pattern for a circular piston radiator is:

\[
\frac{2J_1(ka \sin \theta)}{ka \sin \theta}
\]  

(K.1)

The reflection coefficient for this radiation pattern can be approximated by summing the secondary sources of an insonated object over the area covered by the main lobe. The first null of the Bessel function is found at:

\[ka \sin \theta = 3.83,\]  

(K.2)

where \(a\) is the radius of the piston,

\(k\) is the wavenumber \(= 2\pi/\lambda\), and

\(\theta\) is the angle from the normal to the piston.

Therefore,

\[
\sin \theta = \frac{b}{\sqrt{b^2 + z_0^2}} = \frac{\frac{1}{2} \frac{22\lambda}{2a} \pm \frac{1}{D}}{D(1-1/D^2)^{1/2}}
\]  

(K.3)

where \(b\) is the radius of the beam at the insonated object, and

\(z_0\) is the depth from the piston to the insonated object.

Solving for \(b\) gives:

\[
b = \frac{z_0}{D(1-1/D^2)^{1/2}}
\]  

(K.4)
REFERENCES


