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The Vertical Structure of Shallow Water Flow in the Surf Zone and Inner Shelf

Thomas C. Lippmann
University of New Hampshire, Durham, t.lippmann@unh.edu

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Dr. Thomas C. Lippmann
Civil and Environmental Engineering & Geodetic Science, Byrd Polar Research Center
1090 Carmack Rd., Ohio State University, Columbus, OH 43210-1002
phone: (614) 688-0080 fax: (614) 292-4697 email: lippmann.2@osu.edu

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LONG-TERM GOALS

The long-term goals of this research are to understand the three-dimensional structure of wave- and tidally-driven shallow water flows in the shallow depths of the inner shelf and surf zone.

OBJECTIVES

1. Theoretical investigations of the vertical structure of mean currents, tidal flows, and low frequency (infragravity) oscillations on the inner shelf and inside the surf zone. This work expands on recent theoretical developments for shear instabilities that follow methodology developed for tidal flows on the continental shelf. Results will be compared with field data examined as part of the second objective.

2. Ongoing examination of the spatial and temporal variations in wave breaking patterns and its relationship to surface mean and oscillatory flow inside the surf zone. This objective will be accomplished using video-based field observations of wave breaking distributions (from timestack imagery) and surface flow measurements (from PIV analysis) obtained as part of the 2003 NCEX field experiment.

APPROACH

The vertical structure of the three-dimensional mean and low frequency oscillatory flow field in shallow marine environments is examined with a combination of field data analysis and theoretical development. Theoretical solutions to the forced (*e.g.*, including surface shear stresses) horizontal momentum equations in the presence of a bottom boundary layer will be compared with field observations to examine the local vertical structure of mean and oscillatory flows in and very near the surf zone. The theoretical development follows work on tidal flows on the continental shelf (Prandle, 1982) and shear instabilities of the longshore current in the surf zone (Lippmann, Thornton, and Stanton, 2006; Lippmann and Bowen, 2006). The theory has the attractive attribute that it allows estimation of the sub-surface flow fields to be made from remotely observed surface flow (obtained from airborne or land-based instrumentation) in a variety of shallow marine environments (including the surf zone, inner shelf, and estuaries). Solutions to the depth-varying equations of motion have been obtained relative to the water surface (Lippmann and Bowen, 2006) that qualitatively predict the observed structure in low frequency flows associated with shear instabilities of the alongshore current (Lippmann, Thornton, and Stanton, 2006). In this work, we will extend the theoretical development to include (1) depth varying mean alongshore flows, (2) cross-shore mean flows with assumed vertical structure, and (3) tidally modulated mean flows with unknown horizontal and vertical structure. The plan is to systematically extend the present theory using in sequence (1)-(3) above. The over-arching goal is to derive a unified three-dimensional solution for mean and slowly varying surf zone circulation within the limits of simple but realistic parameterizations that allow analytic solutions. Solutions derived in such a manner are invaluable in furthering our understanding of the relevant physical behavior of the circulation. Additionally, this methodology has the attractive benefit of being applicable to other Navy or DOD relevant regions where three-dimensional effects are important.

Integral to this research is the development and exploitation of particle image velocimetry (PIV) video techniques to measure the large scale surface flow fields in and around the surf zone. In conjunction with estimates of wave breaking distributions, surf zone width, and run-up oscillations, qualitative examination of the forcing for rip currents and nearshore circulation can be done. These data can be utilized in numerical models to improve now-casting and fore-casting capabilities. Continued comparison of the surface flow fields with sub-surface instrumentation will lead to better understanding of the vertical variation in mean and oscillatory flows as well as the dynamics of the nearshore surface layer.

WORK COMPLETED

Observations of the vertical variation in low frequency motions (frequencies about 0.005 Hz) show significant vertical structure across the surf zone (Lippmann, *et al.*, 2006). In particular, nearly linear phase shifts of up to 45 degrees are observed in both the cross-shore and alongshore components of the flows. The bottom sometimes leads and lags the surface depending on the location in the surf zone relative the maximum location of the mean alongshore current as well as other factors not fully understood (including the strength of the alongshore current and shear magnitude, eddy mixing, and bottom drag). Additionally, rotary analysis shows that the ellipse orientation changes as a function of depth, and that the rotary coefficient is non-zero and can change sign in the vertical indicating that the surface flow of these long period motions can be opposite to that of the near bottom flow. The vertical variation in phase and rotation leads to a sharp drop-off in the coherence, even for co-located sensors separated vertically by as little as 1 meter. The observed vertical structure adds significant complexity to the behavior of low frequency motions in the surf zone not previously observed in the field or considered in depth-averaged models. The turning and rotational changes in the currents have implications to surf zone mixing and sediment transport that should be considered in numerical model formulations.

Following the work of Prandle (1982), simple boundary layer theory with a vertically uniform eddy viscosity, ν_t, and quadratic bottom drag formulation qualitatively predicts the observed structure (Lippmann and Bowen, 2006). The formulation is similar to the that found for tidal flows on the continental shelf, except that the Coriolis parameter does not enter into the solutions. Instead, the solutions depend on the parameter $p = \left(i(kV - \sigma + \partial V/\partial x)/\nu_t\right)$, where σ and k are the radian frequency and wavenumber of the oscillatory wave motion, V and $\partial V/\partial x$ are the mean alongshore current and its cross-shore shear, and ν_t is eddy viscosity. Theoretical vertical phase variations using typical values have similar structure as the observations, with up to 45 degree phase shifts over the water column for a range of infragravity frequencies. The sign of the phase shift depends on the Doppler shifted frequency relative to the mean current shear. Consideration of the rotary components shows that the phase shifts and amplitude structure leads to turning of the currents with depth and a rotational sign change across the vertical, also similar to that observed in the field.

Longshore surface currents within the surf zone were measured using two complementary remote sensing techniques: microwave Doppler radar and optical video. Doppler radar relies on small scale surface roughness that scatters the incident electromagnetic radiation so that velocities are obtained from the Doppler shift of the backscattered radiation. Video relies on texture and contrast of scattered sunlight from the sea surface, and velocity estimates are determined using Particle Imaging Velocimetry (PIV). This study compares video PIV-derived and Doppler radar surface velocities over a 1 km alongshore by 0.5 km cross-shore area in the surf zone of a natural beach. The two surface velocity estimates are strongly correlated ($R^2 \geq 0.79$) over much of the surf zone. Estimates differ at the outer edge of the surf where strong breaking is prevalent, with radar velocities as much as 50% below the video data. The radar and PIV velocities at particular locations in the surf zone track each other well over a 6 hour period, showing strong modulations in the mean alongshore flow occurring on 10-20 minute time intervals. The good spatial and temporal agreement between the two remote measurement techniques which rely on very different mechanisms, suggests that both are reasonably approximating the true surface velocity.

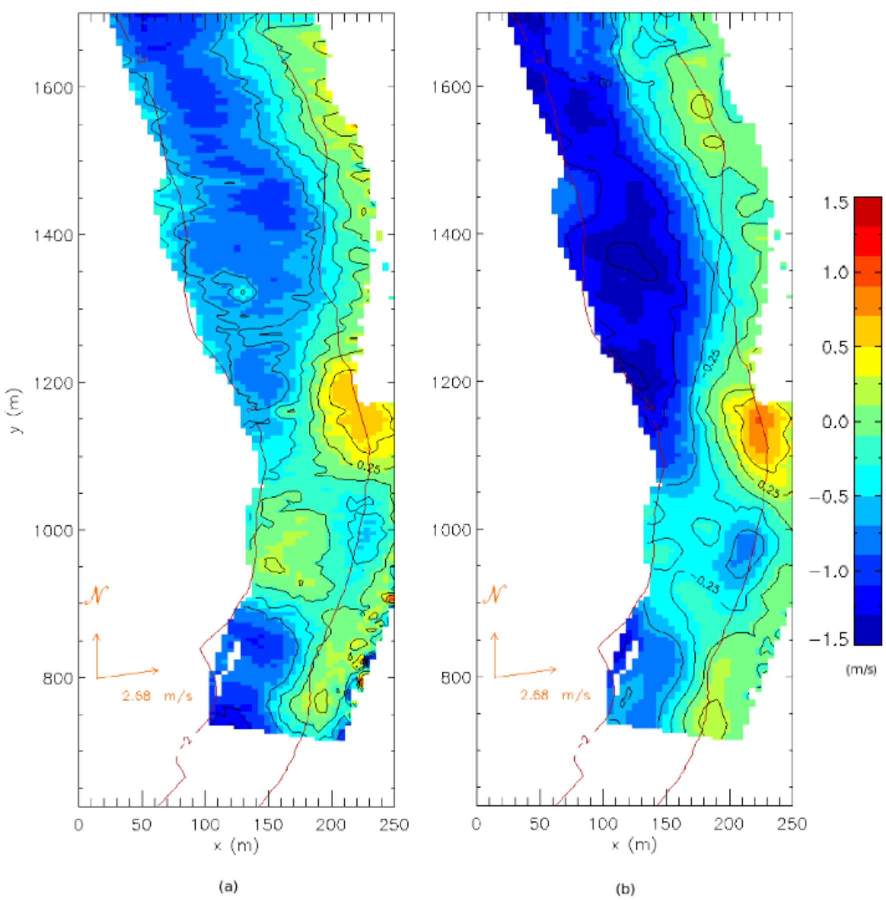

Figure 1. Comparison of Doppler radar radial velocity (left panel) and video PIV-derived surface currents (right panel) for 30 min mean flow observed at NCEX on 31 October between 0600-0630 Hrs. PST. The coordinates are in local UTM Northings and Eastings, with the shoreline to the right, offshore to the left, and the 0 and -2 m contours are indicated by the red lines. The flow magnitudes (m/sec) are shown by the color contours with scale on the right-hand-side. The direction and speed of the wind is shown with red arrows relative to true north.

RESULTS

Example comparison between Doppler radar and video PIV surface currents observed during the 2003 Nearshore Canyon Experiment (NCEX) experiment are shown in Figure 1 (Perkovic, *et al.*, 2007). The velocity scales are the same for both images and range +/- 1.5 m/sec. As the radar only measures the radial velocity component, the bi-directional PIV velocity estimates were projected onto the radar's radial direction for comparison. The location of the radar and video cameras was such that the radial velocity is very nearly alongshore at the NCEX field site (Blacks Beach near La Jolla, CA); thus the velocities shown are essentially longshore currents. Good agreement between the radar and video surface flows are clearly evident, including close similarities in the alongshore variability of current features such as the reversal of the longshore current at about the 1100 m alongshore coordinate suggestive of a eddy-like structure at that location. In general, the radar and video observations compare very well, with squared correlations greater than about 0.79 and with rms errors of about 0.18 m/sec suggesting that both remote sensing techniques – which rely on very different mechanisms – are reasonably accurately measuring the mean longshore surface current.

Analytical solutions to the forced horizontal momentum equations including the presence of a bottom boundary layer are found for the local vertical structure of mean flow within a saturated surf zone. Similar to the theoretical development of flows within the wave bottom boundary layer driven by arbitrary free stream wave motions (Foster, *et al*., 1999), the oscillating surface boundary condition is distributed through the water column allowing for local analytical solutions of the vertical structure of time-averaged currents to be found without explicitly defining wave trough levels or surface mass fluxes. The solutions have the attractive attribute that they allow estimation of sub-surface flow from remotely observed surface currents (obtained from airborne or land-based instrumentation) in a variety of shallow marine environments in which water depths are known.

The forced horizontal momentum equations for the two-dimensional cross-shore flow are given by

$$\frac{\partial}{\partial x}\left(P+\rho\hat{u}^2\right)+\frac{\partial}{\partial z}\rho\hat{u}\hat{w}=0 \tag{1}$$

(Stive and Wind,1982), where P is the wave pressure, $\hat{u}$ and $\hat{w}$ are the cross-shore and vertical velocities, ρ is density, and x and z are the horizontal and vertical Cartesian coordinates with the z positive upward from the still water level. The velocities are assumed to be composed of mean (U), wave ($\tilde{u},\tilde{w}$), and turbulent (u',w') components, such that $\hat{u}=U+\tilde{u}+u'$ and $\hat{w}=\tilde{w}+w'$. Time-averaging (1) leads to a governing equation for the mean flow field. The surface boundary conditions (at $z=\eta$) are functions of space and time, $u(\eta)=u(x,t)$ and $w(\eta)=w(x,t)$, precluding direct analytic solutions for the mean flow structure. By separating the vertical flow structure into two layers, above and below the wave trough level, and conserving mass over the vertical whereby the onshore mass flux above the trough level is balanced by an imposed depth uniform mean return flow, U_r, below the trough (*e.g.*, Garcez Faria, *et al.*,2000), an equation for the mean flow below the wave trough is found

$$\frac{\partial}{\partial z}\left(\rho\nu\frac{\partial U}{\partial z}\right)=\frac{1}{2}\frac{\partial}{\partial x}\left[\rho\overline{\left(\tilde{u}^2-\tilde{w}^2\right)}\right]+\rho g\frac{\partial\eta}{\partial x}+\frac{\partial\rho U_r{}^2}{\partial x} \tag{2}$$

where the over-bar indicates time averaging over the wave period, and it has been assumed that the time-averaged turbulent shear stresses are determined by $-\rho\overline{u'w'} = \rho\nu\left(\partial U/\partial z\right)$ with ν an eddy viscosity. Numerical solutions for the mean flow structure below the trough level, $U(z)$, are determined by the form for ν, and various methods for specifying U_r from the vertical mass balance. In these methods, no form for the structure of the mean flow above the trough level is found, limiting the solution to subsurface flow field.

In this work, we show that a complete analytical solution spanning the water column from the bottom to the surface ($-h < z < \eta$) can be found. We follow the oscillatory wave bottom boundary layer model of Foster, *et al.* (1999) in which they transform the vertical coordinate to allow for constant (and zero) boundary conditions at the free stream and at the bottom. The coordinate transformation takes the form $z' = z + h/\eta + h$, so that the velocities in the transformed system are given by $u(z') = \hat{u} - z'\hat{u}_o$ and $w(z') = \hat{w} - z'\hat{w}_o$, where the simplified notation $\hat{u}(\eta) = \hat{u}_o$ and $\hat{w}(\eta) = \hat{w}_o$ has been used. The boundary conditions become zero both at the surface ($z' = 1$) and at the bottom ($z' = 0$). After significant simplification, the transformed time-averaged governing equation becomes

$$\frac{\partial}{\partial x}\overline{P} - \overline{\frac{\rho u_o w_o}{\eta + h}} - \overline{\frac{w_o}{\eta + h}}U(z') = \overline{\frac{\nu}{(\eta + h)^2}}\frac{\partial^2}{\partial z'^2}U(z') \tag{3}$$

Equation (3) has form $(G(x) - \partial^2/\partial z'^2)U(z') - H(x) = 0$, where G and H are complicated functions of the surface wave field independent of depth, with hyperbolic solutions that are found using the surface and bottom boundary conditions,

$$\hat{U} = \overline{\frac{z+h}{\eta + h}}\hat{U}_o + \frac{H}{G\sinh\sqrt{G}}\left[\sinh\sqrt{G} - \sinh\sqrt{G}\,\overline{\frac{z+h}{\eta+h}} - \sinh\left(\sqrt{G}(1 - \overline{\frac{z+h}{\eta+h}})\right)\right] \tag{4}$$

after transformation back to the original coordinate system. Assuming mass conservation over the vertical, $\int_{-h}^{\eta}\hat{U}dz = 0$, forces $\hat{U}_o = -(2H/G)\left(1 - 2\left(\cosh\sqrt{G} - 1\right)/\sqrt{G}\sinh\sqrt{G}\right)$.

Solutions are shown in Figure 2 for a saturated surf zone with wave amplitude to water depth ratio $a/h = 0.17$. The solutions are as expected, with surface flow onshore above the trough with maximum value at the elevation of the wave crest, and subsurface flow (*i.e.* the undertow) with maximum below the wave trough level. The form for $\hat{U}(z)$ is determined by the relative magnitudes of G and H. With H/G held constant (*e.g.*, a particular forcing condition independent of ν), and then choosing a range of G (which varies inversely with ν) allows an examination of the flow structure in response to changes in the eddy viscosity. As the eddy viscosity increases, the vertical mixing is stronger and the vertical variation in mean flow is reduced (*i.e.*, more uniform throughout the water column), as expected. By holding G constant and varying the ratio H/G, the impact of stronger forcing is qualitatively examined; for higher wave forcing, both the onshore surface flow and subsurface return flows are more pronounced.

When waves pass by a given location in the surf zone, the elevation of the water level changes with wave amplitude. Thus an observing system that follows the water surface measures the mean and oscillatory surface flow field that depends on the elevation, or phase, at which the measurement is made. A time-average over the wave period is equivalent to a time average over the range of elevations spanned, that is, from the wave trough to the wave surface. Hence, an observing system following the surface flow can be time averaged (such as those shown in Figure 1) and compared with the solutions given by (4) and shown in Figure 2.

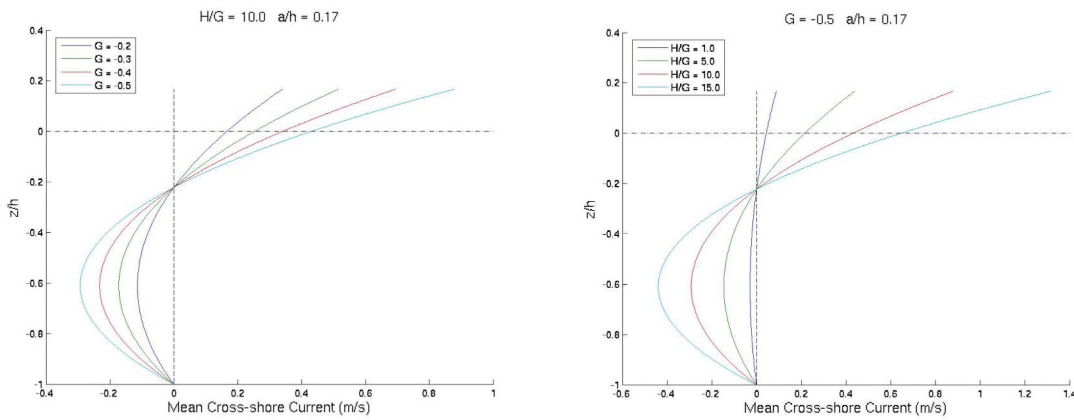

Figure 2. Analytical solutions for the vertical structure of the mean cross-shore flow, $\hat{U}(z)$, given by equation (8). (left panel) Solutions for a range of G, showing the effect of stronger/weaker eddy mixing (corresponding to smaller/larger G, respectively). (right panel) Solutions for a range of H/G, showing the effect of larger forcing (corresponding to larger H/G).

IMPACT/APPLICATIONS

Improvements in the sampling and modeling of wave breaking have lead to improved models for ensemble-averaged wave transformation and the forcing for mean flow. Development of remote sensing methods for measuring surface currents over large areas of the surf zone can be used to verify circulation models in the nearshore where *in situ* instrumentation is difficult to deploy. Field verification of the relationship between surface and subsurface flow fields will allow estimates of subsurface flows to be made along potentially any coastline if surface flows are obtained by other reliable remote sensing systems on a variety of platforms (*e.g.*, airborne sensing systems). Analytic solutions for the vertical structure of mean flows can be verified with field data and subsequently utilized in models for shallow water circulation.

TRANSITIONS

Many of the surf zone characterization techniques relating to this effort are being transitioned via collaborators (PI Holland) under the NRL Littoral Environmental Nowcasting System program for eventual Naval operational use.

RELATED PROJECTS

Video data analysis of the 1990 Delilah, 1994 Duck94, 1996 MBBE, 1997 SandyDuck, 2001 RIPEX, and 2003 NCEX experiments are being examined in collaboration with other ONR-funded scientists making *in situ* observations of wave and current properties.

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