Spring 1978

AN INVESTIGATION OF THE INTERACTIONS OF STUDENT ABILITY PROFILES AND INSTRUCTION IN HEURISTIC STRATEGIES WITH PROBLEM SOLVING PERFORMANCE AND PROBLEM SORTING SCHEMES

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UNIVERSITY OF NEW HAMPSHIRE, PH.D., 1978

University Microfilms International 300 N. ZEEB ROAD, ANN ARBOR, MI 48106

309

AN INVESTIGATION OF THE INTERACTIONS OF STUDENT ABILITY PROFILES AND INSTRUCTION IN HEURISTIC STRATEGIES WITH PROBLEM SOLVING PERFORMANCE AND PROBLEM SORTING SCHEMES

A DISSERTATION

SUBMITTED TO THE UNIVERSITY OF NEW HAMPSHIRE

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

DEPARTMENT OF MATHEMATICS

APRIL, 1978

by

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Acknowledgements

I wish to sincerely thank the members of my committee who contributed much to the present work. I am especially grateful to Dr. William Geeslin for his knowledge, interest, and careful attention to the progress of this dissertation. I am also indebted to Dr. Richard Balomenos, Dr. Alfred Forsyth and Dr. Philip Smith for their many comments and suggestions.

Thanks are due also to Dr. James Cerney for computer assistance and to Ms. Lisa DeGraan for her help in the preparation of this manuscript.

In addition, I would like to express gratitude to the students who served as subjects for this study and to the members of the Department of Mathematics and Physics at the Massachusetts College of Pharmacy, Professor George Matelli, Professor Lila Siddons, and Dr. Frederick Parmenter for their cooperation.

Finally, I would like to thank my family who supported me in my work.

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The study investigated differences in problem solving performance between experimental subjects who followed a course of instruction in heuristic problem solving and control subjects who followed a course of instruction in college algebra. Other aims of the study were to explore relationships between student ability profiles and problem solving performance, to look for changes in problem sorting schemes following instruction in heuristic strategies, and to compare experimental and control students with respect to performance in algebra and trigonometry.

The subjects, 84 freshmen enrolled in a college of pharmacy in New England, were divided randomly into experimental (n = 37) and control (n = 47) groups and each group was provided with ten weeks of instruction. Experimental students received instruction in problem solving and the use of both general and specific heuristic strategies, whereas, the control group received instruction in college algebra and trigonometry. All students took five ability pretests: Hidden Figures Test, Scrambled Words Test, Nonsense Syllogism Test, Deciphering Languages Test, and the Toothpicks Test. A Solomon four group design provided pretest data on problem solving performance and problem sorting schemes for approximately half of the experimental and control group students. At the end of the instructional period posttest measures of problem solving performance, algebra and trigonometry performance, and problem sorting schemes were ob-

tained for all students. The nine item problem solving test was designed to include problems solved by three heuristic strategies (algebraic symbolism, contradiction, and pattern generation) as well as incorporating three contextual cues (triangle problems, number problems, and word problems). The problem sorting scheme information was gathered by means of a problem similarity questionnaire that required students to rate each pair of the nine problems on a continuous similarity scale.

Analyses of posttest performance measures revealed that whereas experimental students significantly outperformed control students on the problem solving posttest ($p < .01$), students receiving instruction in college algebra and trigonometry significantly outperformed those students receiving instruction in heuristic problem solving on the algebra and trigonometry posttest ($p < .001$).

Complete link hierarchical clustering analyses indicated that heuristic instruction did not greatly alter the dominant problem sorting schemes of the experimental students although students receiving heuristic instruction did show evidence of being more attentive to heuristic cues than control students. Additionally, it was shown that recognition of heuristic cues was significantly correlated ($p < .04$) with superior performance in problem solving, however, several individual students provided evidence that it was possible to achieve high scores on the problem solving test without sorting heuristically.

The ability test data was subjected to Ward's hierarchical clustering analysis and four homogeneous ability profile groups were isolated. It was found that the degree to which a student sorted heuristically was related to the student's ability profile type with students scoring low on a semantic-divergent thinking factor receiving lower heuristic sorting scores than students scoring significantly higher on this factor.

The problem solving posttest was divided into subtests corresponding to the three heuristic strategies covered in the experimental course (algebraic symbolism, contradiction, and pattern generation). Results showed that of the four ability profile groups isolated in this study only one performed significantly better across all three subtests following heuristic instruction. This result gives evidence that heuristic instruction may be more beneficial for certain ability types.

CHAPTER I

THE RESEARCH PROBLEM

Psychologists, educators and mathematicians have long agreed that increasing problem solving performance is an important goal of mathematics instruction. Much of the recent research on problem solving has explored the relationship between individual differences (such as field independence, sex, anxiety) and problem solving performance. This includes the research done by Leach and Marshall (1970), Kagan (1964) and Witkin (1975) on cognitive style as it relates to problem solving as well as the work of Russell and Sarason (1966) on anxiety and the investigations of Milton (1957) on the relationship of sex to problem solving and mathematical ability. Recent studies by Chartoff (1976) and Silver (1977) have opened the way for the investigation of a new aspect of individual difference research, namely, sorting schemes employed by individual problem solvers. The present investigation is designed to explore the effects of a freshman college level course in heuristic problem solving on student performance and to look for changes in problem sorting schemes as a result of such instruction. Moreover, this study will attempt to determine what relationship, if any, exists between student ability profiles and problem solving performance and will consider the differential problem solving performances of students with different problem sorting schemes.

Although few educators would disagree as to the importance of problem solving and its place in the mathematics curriculum, an examination of the current commercial texts and standardized tests indicates that very little genuine problem solving is being fostered in most high school and introductory college mathematics courses. In fact a much stronger statement can be made. If genuine problem solving is defined to be the ability to devise your own plans and create strategies for solving problems that may not be familiar ones and if we distinguish this kind of problem solving from one that simply calls for the application of an algorithmic procedure, then the evidence we have from present commercial texts is that not only is genuine problem solving not a primary component but even algorithmic problem solving is presented almost as an afterthought. One of the reasons for this observed neglect is that it is difficult to devise effective strategies for teaching problem solving. Another reason is that much controversy exists as to exactly what the importance of problem solving is relative to the other goals of the mathematics curriculum such as building up the students' knowledge of specific content and developing necessary mathematical skills. This controversy goes beyond the specific issue of mathematical problem solving and has its roots in educational psychology. The Gagne (1965) school with its hierachical view of learning sees problem solving as an advanced skill to be mastered only when the prerequisite content knowledge has been mastered. This view is to be contrasted with that held by Bruner (1960) and his followers. Bruner would be willing to start with problem solving, believing that the student will learn necessary prerequisite skills as they are needed, feeling that an interesting problem will provide the motivation to learn and discover prerequisite skills. An interpretation of Bruner's view could result in teaching strategies that are designed not to teach prerequisite subject matter but rather to suggest heuristics that can be applied to solve newly presented problems. It is this investigator's opinion that overemphasis on mathematical structure and content may contribute to the view that mathematics is static and already fully developed, whereas an increased emphasis on problem solving could likely change the impression many students have of mathematics as facts and rules to be memorized to that of a creative activity.

So the situation as it stands is; many researchers, educational psychologists, and mathematics educators agree that problem solving is the heart of the mathematics curriculum but disagree as to how to best bring about improvements in problem solving performance. The structure (referred to here as the "traditional structure") that is used in most mathematics texts at the high school and introductory college level has been criticized for its restrictiveness and could be described as follows. Problems are categorized with respect to either content (mixture problem, age problem, etc.) or algebraic type (quadratic equation problem, system of linear equations problem, etc.). Students are then taught to look for cues in the problem which will enable them to place it in one of the above categories. Problem solving then becomes a rote translation process followed by an imitation process as students attempt to solve their "mixture problem" in a fashion similar to the way the instructor solved the problem in class. The major criticism that can be leveled at this method of teaching problem solving is that no instruction is given on how to conceptualize or approach problems that do not closely resemble problems previously encountered. In fact, the unmotivated presentation of complex problems gives problem solving an almost magical quality that leaves the student with the impression that certain problems have "trick" solutions that one just has to stumble upon by chance. Instead of viewing problem solving as a clever, persistent task, the student sees it as a chancy, haphazard adventure. This claim regarding students' impressions and reactions to challenging, nontraditional problems is this investigator's analysis based on her experience while trying to teach problem solving within the context of high school and college calculus and algebra courses and on her communications with other faculty members faced with similar tasks.

A primary research concern in the future should be the determination of which methods of teaching problem solving reduce this perceived view of problem solving as a chancy adventure while at the same time fostering genuine problem solving.

What may be needed is a method of teaching problem solving that emphasizes not specific problem types but rather problem solving strategies. This approach is known as heuristic problem solving and the most thorough account of both the theoretical and practical aspects of heuristic teaching can be found in the works of George Polya (1957, 1962, 1965). The word "heuristic" has its origin in the Greek verb "heuriskein", "to discover." Heuristic is the science of discovery, and the plural, heuristics, denotes a collection of techniques for discovering ways to solve problems. We could define heuristic teaching as an instructional method whereby students are taught to solve problems in a non-algorithmic way. A key difference between the heuristic method and the traditional method is that the heuristic approach does not rely on already existing algorithms or mechanistic procedures for the solution to problems. The beauty of modern heuristics is that it suggests plans to be followed should your problem not be a familiar one. These include: trying to arrive at a contradiction, working the problem backwards, specializing, generalizing, establishing patterns, etc. So whereas the traditional method specializes in teaching solutions to a fixed class of problems, the heuristic method is more ambitious in that it attempts to teach general problem solving techniques applicable to a wide variety of situations.

The purpose of this study is two-fold. One of the objectives is to determine the effectiveness of the proposed course on improving problem solving performance. The second area of investigation, however, will be a determination of whether or not the instructional sequence produces any changes in the way students categorize problems, and whether such changes are related to problem solving performance. A major difference between the heurisitc method and the traditional method of problem solving is in how students are encouraged to think about problems. Basically, the traditional method organizes according to content and mathematical structure[1] whereas the heuristic method organizes according to the method of solution. Specifically, the study will attempt to answer the following questions:

1. Does the proposed course in heuristic problem solving improve problem solving performance?
2. Does the proposed course in heuristic problem solving effect changes in the way students categorize problems?
3. What is the relationship between the type of problem sorting scheme used and problem solving performance? That is, is a tendency to sort heuristically correlated with superior problem solving performance?

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[1]Geeslin and Shavelson (1975) define mathematical structure as a "set of interrelated, abstract, symbolic systems."

4. What is the role played by individual differences in this study? That is, given prior measures of mathematical "ability," which students are likely to benefit most from the proposed course and which will benefit least? The concern of this study is not what "abilities" are related to efficient problem solving but rather what interaction, if any, can be found between student ability profiles and the instructional approach one follows.
5. What is the relationship between a student's ability profile type and the sorting scheme employed by that student. Do students with certain ability profiles sort less heuristically than others?
6. Does the problem solving course result in increased or comparable performance on basic mathematical skills? That is, can mathematical content be taught via problem solving as well as by the traditional method?

The next chapter discusses research related to the questions outlined above. Much research has been done in isolating the individual difference factors that influence problem solving performance and the role of instructional variables in increasing such performance. In addition, the recent work on sorting scheme usage will be presented.

CHAPTER II

RELATED RESEARCH

Since this investigation involves (a) individual differences in problem solving ability, (b) instructional variables, and (c) differences in sorting schemes, research closely related to each of these three areas will be reviewed. Since this study is concerned with investigating those factors that influence and contribute to genuine solving (i.e. heuristic problem solving) as opposed to purely algorithmic problem solving only those studies dealing with this kind of problem solving performance will be discussed in this chapter. Problem solving will then be taken to mean more than an automatic, unconscious application of a well studied procedure and will be characterized by at least some conscious deliberation of the aspects of the problem situation and the method to be used in the solution. That is not to say that a student who uses a well known algorithm is not solving the problem heuristically. The important consideration is whether or not the algorithm was mechanically applied or applied with thought and deliberation.

Individual Difference Research

It is well established in the research literature that individual differences constitute much of the variability observed in problem solving performance and it is this research that will be discussed in this section. The purpose of these studies is to discover which characteristics of the problem solver seem to exert the most influence on problem solving performance and then use this information to help explain why the studied factor results in differential problem solving performance.

One of the most studied correlates of problem solving is general intelligence. Since tests of general intelligence sample a wide variety of skills and knowledge, it is not surprising that they correlate well with many types of problem solving. Burke and Maire (1965), however, report a study of the relationship between solving insight problems and general intelligence that failed to produce any significant results.

The obvious drawback of the general intelligence tests is that they were designed to predict academic success, not necessarily problem solving performance and as a result fail to isolate specific correlates of problem solving efficiency. The factor analytic studies of Merriefield, Guilford, Christensen,and Frick (1962) are specifically designed to detect general factors needed to solve problems. Based on Guilford's structure of the intellect model, this group rejected the notion that problem solving was a single ability and developed a battery of tests that would test for certain abilities hypothesized as necessary for at least some problem solving situations. Their results show that problem solving ability is a composite of several abilities, such as verbal comprehension, numerical facility, perceptual speed, visualization and spacial orientation.

Krutetskii (1976), a leader in Russian research on the psychological bases for mathematical ability, worked with school children over a period of twelve years and, based on logic rather than factor analysis, was able to isolate several components of mathematical ability. Among them are: (a) formalized perception of mathematical material, (b) generalization of mathematical material, (c) "curtailment" of thought, (d) flexibility of thought, (e) mathematical memory, and (f) special concepts.

In addition to factor analytic tests, a variety of "creativity" tests have surfaced that claim to single out those individuals capable of original "insightful" problem solving. These include the Torrance Test for Creative Thinking and the Wallach and Kogan Creativity Test. Crockenberg (1972) has provided a useful description and evaluation of these two popular creativity tests. The Torrance battery consists of 7 verbal and 3 figural tests and is aimed at identifying children who are creative in their approach to problem situations. Tasks include thinking of unusual uses for common objects and finishing an incomplete figure to form an unusual picture. Children are scored for fluency, flexibility, and originality. The Wallach and Kogan tests are similar in format to the Torrance tests except that the scoring procedures for the Wallach and Kogan tests take into account only the number of different responses given to a particular item and not the quality or originality of the response. Research has shown that it is possible to score high on the Wallach and Kogan test without having a high I.Q. score. One drawback of these tests is their somewhat inferior reliability as compared with many general intelligence tests (Crockenberg, 1972).

Another, often quoted, correlate of differences in problem solving is sex. Several studies have shown that males outperform females in certain problem solving tasks (Hoffman and Maier, 1966). A possible explanation for this observed difference is given by Milton (1957) who suggests that it is not sex but rather the degree to which someone associates with the traditional masculine-feminine role that accounts for these findings. His study reveals that problem solving success is strongly related to masculine role identification. Since those women who identify more closely with the traditional female role in society tend to be inferior problem solvers, problem solving is thought by some (perhaps less today than in the past) to be a masculine trait.

Anxiety has been posed as another reason for individual differences in problem solving behavior. Research has shown that test-anxious subjects perform more poorly than low-anxious subjects (Russell and Sarason, 1966). Related to this finding is the research on the adverse effect of previous failure in problem solving situations (Feather,1966).

This is consistent with the anxiety finding since previous failure is likely to induce an anxious response to future similar situations.

Finally, there are those researchers interested in the personality correlates of problem solving. Most of the research of this nature is concerned with the concept of "cognitive style" or "perceptual style." Cognitive style is a term that refers to a collection of individual differences that describe a preferred way of perceiving, thinking, solving problems, learning, and relating to others (Witkin, 1975). Some of the cognitive styles investigated with respect to problem solving are flexidity-rigidity, reflection-impulsivity, global-analytic and convergent-divergent. The flexible-rigid dimension investigated by Leach and Marshall (1970) is a measure of the ability to overcome persevative behavior. The more flexible the individual, the easier he will find it to overcome the problem of "fixation" that the Gestalt psychologists investigate. The reflex-impulsive aspect discussed by Kagan (1964) describes the amount of thought given to a problem. Does the individual proceed to work on the problem immediately with little forethought or does he begin by carefully deciding on a reasonable strategy? Witkin's (1975) global-analytic (or field dependent-independent) polarity analyzes how individuals differ with respect to the degree to which they view problems or situations globally as opposed to analytically. The analytic or field independent individual will isolate individual features of a problem or stimulus field, whereas, the field dependent or global individual will tend to leave the problem or field "as is." Finally, Hudson (1966) has studied the convergent-divergent cognitive style. Divergent individuals are characterized as being able to produce or think of a variety of responses or solutions, whereas, convergent individuals tend to see a problem in only one way — their thinking channels the information so that it leads to one correct solution or the most conventional solution.

Some psychologists reject the notion of cognitive style. Among them is Cronbach (1960) who argues that tests of cognitive style are nothing more than tests of general ability. These tests he claims, have failed to uncover any new ability factors. A report by Sherman (1967) argues that Witkin's test for field-independence is nothing more than spacial ability. Another valid criticism of these tests of cognitive style is that they often fail to differentiate between semantic, figural, and symbolic abilities (Davis, 1971). Witkin has claimed that the global-analytic dimension cuts across the cognitive domain. The research seems to indicate, however, that cognitive style cannot be used to predict behavior across a wide variety of circumstances or problem situations.

Two recent research studies on the relationship of cognitive style and success in problem solving seem to indicate that one's approach or success is related not to cognitive style _per se_, but rather is task dependent (Forsyth, 1976 and Marshall, 1973). An individual might be reflective in one problem solving situation and impulsive in another. The consistency in style is task related.

Instructional Variables

One of the questions concerning the influence of instruction on problem solving performance has to do with the amount and kind of guidance that is provided. This reverts back to the much debated distinction between expository and discovery methods of instruction. In a survey of the literature Mayer (1974) drew the following conclusions. First, that minimum direction instructional procedures take more time, result in less initial learning, transfer, and retention than other instructional methods. Secondly, that method direction instructional procedures, as compared with answer direction methods, result in equivalent initial learning, equivalent or inferior short term retention, and superior long term retention. The problem with minimum direction seems to be that unless the problem is sufficiently simple, the subjects may never "discover" the to be learned rule and as a result learn the solution in a rote manner. Method direction, on the other hand, provides the subject with enough clues to enable him to acquire the to be learned rule and yet still take part in the problem solving activity in that he must apply the rule correctly. Finally, answer direction seems to foster the learning or memorizing of specific rote responses.

Mayer goes on to review the research comparing deductive instruction (rule followed by example) to inductive instruction (illustrative example followed by discovery or statement of the rule) and draws the following conclusions from the present research findings. For the teaching of problems with simple solution rules, deductive methods result in superior learning ease and retention, but inductive methods result in superior near and far transfer. For the teaching of a complex, non-intuitive problem, deductive methods result in superior learning ease, retention and near transfer whereas inductive methods result in superior far transfer. Deductive subjects see problem solving as learning how to apply a rule, whereas, inductive subjects view problem solving as learning how to generate rules.

Polya's (1957) heuristic approach to problem solving is perhaps the finest example of inductive method direction as applied to mathematical problem solving. Unfortunately, the research on training in heuristic methods is not decisive. Covington and Crutchfield (1965) report a successful study aimed at increasing problem solving ability, creative thinking, and attitude towards problem solving. Fifth and sixth grade students, using self-instructional materials, employed heuristic techniques in solving a variety of interesting problems. Another successful study is reported by Lucas (1972) who found that students who had received heuristic instruction in calculus were superior to control students in devising workable plans, analytic deduction, using methods or results of related problems, organizing data, and introducing mnemonic notation. Lucas' instructional program lasted for eight weeks during which time the control group was given only expository treatment of problem solutions whereas the experimental group considered the same problems but with the emphasis on applying heuristic strategies. Less encouraging results are reported by Goldberg (1974) who investigated the effects of heuristic instruction on the ability of college students to construct proofs in number theory. Three instructional strategies were contrasted: a reinforced heuristic strategy, a non-reinforced heuristic strategy, and a non-heuristic strategy. Goldberg found some evidence, although not statistically significant, that reinforced heuristic instruction was superior. Also reported was a tendency, again not statistically significant, for high ability students to do best under reinforced heuristic instruction. Finally, it was found that non-heuristic teaching made for a more positive attitude toward problem solving than either of the other two teaching methods. Goldberg's study casts doubt as to whether heuristic teaching does, in fact, foster better problem solving and improved attitude toward mathematics for all students.

Related research is reported by Smith (1973) who contrasts the effects of task specific versus general heuristic instruction in three different task environments on 176 college students. The study revealed that task specific instruction was significantly superior to general heuristic instruction in solving the test problems. Furthermore, subjects receiving general heuristic instruction did not solve the transfer problem any better than the task-specific subjects, nor did they appear to use heuristic advice when attempting to solve transfer problems. It should be noted that Smith's study was only of three weeks duration and was a programmed instruction format. It may be that since heuristic problem solving and the ability to transfer heuristic techniques to novel situations is such a high order task that a program of instruction aimed at improving transfer ability must be of longer duration. It is also possible that in-class instruction may be more conducive to the teaching of heuristic techniques than programmed instruction. Nevertheless, Smith's research provides another piece of evidence indicating that heuristic techniques are not as easily generalizable as one might hope.

In addition to this rather negative research on the success of teaching heuristic strategies, personal comments from educators confirm this. One mathematics educator reports the following:

> It came as a shock when I learned that few people responsible for training students in mathematical problem solving at the college level actually use Polya's work as a foundation for their instruction in problem solving. A colleague who has very successfully coached his university's team for competition in the nationwide W.L. Putnam Mathematics Competition told me that his students did not find Polya's works useful. They enjoyed the books a great deal, but they neither seemed to solve problems more effectively, nor perceived themselves as having a greater array of useful techniques for solving problems, than before they had read them. The faculty member who coached the team that won the Putnam Competition that year told me much the same thing. (Schoenfeld, 1977, p.3)

Perhaps what is needed to begin to make some sense out of what seems to be contradictory research on the success of heuristic teaching are studies that address themselves to the question of the relationship between specific individual differences and beneficial results due to instruction in heuristic methods. Do students with specific ability profiles benefit more from instruction in heuristic problem solving whereas students with other ability profiles do not? Another question to be dealt with is the question of a student's perception of which problems are related and which are not. One of Polya's most powerful heuristics is "think of a related problem." Research described in the next section will show that students view "relatedness" or "similarity" in more than one way. Problems can be viewed as structurally related, contextually related, etc. So the manner in which a student organizes problems, i.e., the way in which he perceives them as being related, is a dimension of individual differences that may well be associated with beneficial effects of heuristic problem solving. What needs to be considered then are possible aptitude-treatment interactions (ATI).

Sorting Scheme Research

The research studies of Chartoff (1976) and Silver (1977) were the first to deal with problem sorting schemes as a dimension of individual differences in problem solving. Chartoff's study involved a total of 506 subjects taken from urban secondary schools (grades 7 to 12). The experiment involved having these students rate 66 pairs of algebra word problems on a continuous similarity scale on day 1; on day 2 students were shown the solution to the problems they had rated on day 1; on day 3 the students re-rated the same 66 pairs they rated on day 1. The algebra word problems were chosen so as to incorporate several of the ideas of Polya (1957): specialization, abstraction, insufficient data, and reversal. Chartoff's research had two purposes. He wanted to determine what criteria students used when they initially decide where to place a new problem in their existing cognitive structure. As a second goal, Chartoff was evaluating the usefulness of INDSCAL, (INdividual Difference SCALing) (Cohen, 1976) a multidimensional scaling procedure, to gain information about students' sorting schemes. Chartoff's study is one of the first to investigate sorting schemes as a dimension of individual differences in problem solving. Most studies previous to Chartoff's had used student introspection as a means to understand individual differences in problem solving perception (Kilpatrick, 1969 and Lucas, 1972). Many investigators feel somewhat uncomfortable with introspection studies because of their subjective interpretation and multidimensional scaling had never been investigated as a reliable alternative to such studies. Multidimensional scaling is a statistical procedure similar to factor analysis. Like factor analysis, multidimensional scaling attempts to establish a geometric representation of a set of data of minimum dimensionality. Whereas the data set for factor analysis is a set of N observations on a set on n variables, the data set for multidimensional scaling is an n by n similarity matrix. An excellent review and discussion of the relationship between factor analysis and multidimensional scaling is given by MacCullum (1974). When Chartoff used INDSCAL to analyze students' similarity ratings, he found that four sorting dimensions could be identified.

1. Recognition of the Polya variations, i.e. recognition of how the problems are solved.
2. Contextual setting: for example, in a particular problem set three of the problems referred to kindergarten children and as a result some students rated these as similar.

3. Generic comparison: students classified problems as either specializations or generalizations.
4. Classification based on the goal or question of the problem: does the question ask "how much?", "how far?", or "how many?"

Chartoff concluded that students use the above four criteria to decide which questions are similar but that students used these criteria with different emphasis. His study leaves open the question as to which sorting schemes are related to problem solving performance.

Silver's research study also was concerned with the manner in which students categorized problems but his study included information concerning individual difference measures and changes in similarity ratings before and after they had seen the solutions to the problems. The study consisted of two phases. During phase one, 98 eighth grade students were asked to sort 24 mathematics word problems displayed on cards into groups of similar problems. The basis for the selection of these 24 problems was the work of Krutetskii (1976). Krutetskii's studies had led him to the conclusion that two of the abilities that mathematically gifted students displayed were the ability to distinguish relevant data from the contextual details of a problem and the ability to recognize the formal structure of the problem. As a result, Silver constructed his 24 problems so that they varied along two dimensions: the problem structure and contextual details. Silver's students also were given a variety of tests of individual differences: verbal and non-verbal intelligence; numerical, verbal and abstract reasoning tests; computational ability; field dependence-independence, and problem solving ability. And finally, the card sorting task was administered twice, the first time prior to any discussion of how the problems could be solved and a second time after the students had seen the solution to the problems. The second phase of the study involved an additional 58 eighth grade students. It closely resembled phase one, except this time a revised card-sorting task was administered to better identify the sorting dimensions uncovered in phase one. The primary results of the study can be summarized as:

1. Four problem similarity dimensions were uncovered:
   (a) Context: grouping together problems measuring the same quantities such as age, weight, or time.
   (b) Mathematical structure: grouping together problems requiring similar methods of solution.
   (c) Question posed: grouping together problems with the same question form such as "find the....", or "how far....."
   (d) Pseudostructure: forming categories such as "age problem" or "distance problem." Such problems have both a contextual and a structural component.

2. Students who tend to sort according to mathematical structure tended not to sort along other dimensions. Students who tended to sort according to pseudostructure also tended to sort according to the question posed.
3. Students who tended to sort according to structure scored high on the various measures of mathematical ability, whereas students who tended to sort contextually or according to questions posed scored low on measures of mathematical ability. The sorting according to pseudo-structure was not related to mathematical ability measures.
4. When general ability variables were controlled, mathematical structure sorting was correlated significantly with problem solving ability.
5. In general, students sorted more on the basis of structure after having seen the problem solution than before and sorted less on the basis of the other three categories on the second sort.

Several interesting and important remarks can be made about these two studies. Chartoff constructed his problem set using Polya's "vary the problem" heuristic, whereas, Silver's problem sets were based on Krutetskii's analysis of mathematical ability. Yet, in spite of this, the sorting schemes arrived at by these two investigators are remarkably similar. In particular, both studies isolate a structure and a contextual dimension. Neither study involved an instructional component, but Silver's study did indicate that sorting schemes can be manipulated by exposure to the problem solution. This observation, together with the fact that Silver's research indicated that there appears to be a preferred dimension of problem sorting, suggests that instruction aimed at inducing students to reclassify problems along those preferred dimensions might be effective in improving problem solving performance. Silver's research implies that attention to contextual details or even pseudo-structure is not conducive to efficient problem solving.

The present study expands on the work of Chartoff and Silver by introducing an instructional component. Its design allows for the measurement of initial and posttreatment sorting schemes in an attempt to determine whether the course in heuristic problem solving effected changes in sorting schemes.

CHAPTER III

RESEARCH DESIGN AND METHODOLOGY

The subjects for this study were college freshmen (35 females and 49 males) attending a pharmaceutical college in New England. These students were primarily middle income, Caucasian students of average mathematical ability (Math SAT scores ranged from approximately 450 to 600) residing chiefly in the New England area.

Design

The subjects were divided randomly into two sections. One section (n_1 = 37) received the course in heuristic problem solving while the other section (n_2 = 47) served as a control group receiving instruction in college algebra and trigonometry. Unequal n's were due to the smaller seating capacity of the room available for the course in heuristic problem solving and the fact that seven subjects dropped out during the study, four students from the control course and three students from the problem solving course. These seven students were deleted from all data analyses.

Preinstruction ability measures were obtained for all students. Information regarding preinstruction problem solving performance and sorting schemes also were gathered and to control for a possible interaction between treatment and testing procedures, a Solomon four group design was employed (Campbell and Stanley, 1969). Each of the treatment groups was divided randomly into two subgroups for testing purposes (See Table 1). As a result, pretest measures of problem solving performance and problem sorting schemes were obtained for 19 experimental students and 22 control students. These pretest measures took the form of a nine item problem solving test and accompanying problem similarity questionnaire. All students then followed a ten week instructional sequence corresponding to the fall quarter of the academic year 1977-1978. The control students were enrolled in a course entitled College Algebra, a required freshman course while the experimental students were assigned to take a new elective course proposed by the investigator, Problem Solving. Those students taking Problem Solving in the fall went on to take College Algebra in the winter term, and those taking College Algebra in the fall went on to take Problem Solving in the winter term. At the end of this instructional period, both an algebra-trigonometry posttest and a problem solving posttest were administered to all students. The porblem solving posttest was followed then by a problem similarity questionnaire.

Table 1

Design of the Study

Group	N	Ability Pretests	Problem Solving Pretests	Posttests
Experimental	18	Yes	No	Yes
Experimental	19	Yes	Yes	Yes
Control	25	Yes	No	Yes
Control	22	Yes	Yes	Yes

Instrumentation

Ability

All students involved in the study took five tests in the "Kit of Factor-Referenced Cognitive Tests" (Ekstrom, French, Harman and Dermen, 1976): Hidden Figures Test (CF-1), Scrambled Words Test (CV-1), Nonsense Syllogism Test (RL-1), Deciphering Languages Test (R1-4) and the Toothpicks Test (XF-1). The Hidden Figures Test and the Scrambled Words Test were chosen because they furnish measures of convergent thinking, the first within a figural context and the second within a semantic context. The task in the Hidden Figures Test is to decide which of 5 geometric figures is embedded in a complex design and the authors suggest that this test is suitable for grades 8-16. For the Scrambled Words Test the subject is asked to write a common English word from a group of scrambled letters (suitable for grades 8-16). The Nonsense Syllogism Test and the Deciphering Languages Tests were chosen because they provide measures of logical or deductive reasoning ability, the first involving semantic content whereas the second uses symbolic content. In the Nonsense Syllogism Test the subjects are presented with a formal syllogism using nonsensical content and the task is to decide whether or not the conclusion is logically correct (suitable for grades 11-16). The Deciphering Languages Test requires that the subject determine the English translation of artifical languages (suitable for grades 11-16). Finally, the Toothpicks Test served as a measure of divergent thinking within a figural context. Here the subjects are asked to give up to five different arrangements of toothpicks according to specified rules (suitable for grades 11-16). These abilities, namely, convergent thinking, divergent thinking,and deductive reasoning, have been identified by Guilford (1965) and others as predictors of mathematical performance. All subjects were given these five tests in one two-hour session prior to the beginning of their course.

Problem Solving Pretest

To obtain information concerning both the initial problem solving performance and the initial sorting schemes employed by the subjects, a set of nine problems (PS1) was constructed (See Appendix A). The basis for the construction of these problems was both heuristic and contextual. The problems were chosen so that if analyzed according to a heuristic sorting scheme, three problems (1,6,8) would naturally be grouped together because they are most easily solved using algebraic symbolism; another three porblems (3,5,7) would be grouped together since they are most easily solved using pattern generation techniques and, finally, three problems (2,4,9) would be classified as problems solved by the method of contradiction. These represent the three heuristic techniques that were covered in the experimental course. If, however, a contextual sorting scheme was used, then different groups would arise. Certain problems (2,5,8) would be grouped because they are "verbal problems," other problems (1,4,7) because they are "geometric problems," and finally three problems (3,6,9) would be grouped because they are "number problems." Problems one to nine were ordered randomly on the problem solving pretest. The research of Chartoff (1976) and Silver (1977) was the basis for the selection of these two sorting schemes. Note that all questions had multiple choice answers so as to make the grading easier and more objective. The students were given 45 minutes to work on this nine item test and although this may not have been sufficient time for all students to arrive at correct solutions for all problems, it provided adequate time for them to read, understand, and attempt one or more solutions to each problem.

Problem Solving Posttest

To obtain a posttest measurement of problem solving ability, a 24 item problem solving examination (PS2) was constructed by the investigator (See Appendix A). The first nine items of the posttest were identical to the nine items on the problem solving pretest (PS1). Thus changes in problem solving performance could be analyzed for the two subgroups who took the problem solving pretest. Item selection was based on the content of the experimental course. The test can be broken into three subtests corresponding to the three problem solving strategies covered in the problem solving course as shown in Table 2.

Notice that more items calling for the student to set-up and solve the correct algebraic equation(s) (subtest 1 items) were included than were problems dealing with the contradiction or pattern generation heurisitc. The test was designed in this way so as to supply data with which to compare the two groups on the ability to solve problems using the techniques of algebra and trigonometry. This comparison is important to the study since both groups received instruction related to this type of problem. The algebra (control) group received extensive instruction on how to <u>solve</u> algebraic and trigonometric equations but only minimal instruction on how to <u>set-up</u> such equations given a problem situation. The experimental group, on the other hand, was instructed carefully on the methods of setting-up equations to be solved and only reviewed the various algebraic techniques as they were needed. The eleven test items on the problem solving posttest that come under subtest 1 can then be used to compare the effectiveness of the two methods of instruction in teaching students how to solve problems using algebraic and trigonometric techniques.

The problem solving posttest was administered to all subjects at the end of the instructional period during the scheduled examination period. The time limit was two hours and about eighty percent of the students handed their exams in before time was clled. Calculators were allowed to be used on this exam and students were told that the examination would count for ten percent of their final grade.

Table 2

Subtests of the Problem Solving Posttest

Subtests	Item Numbers
1. Use of Algebraic Symbolism	2, 4, 8, 10, 12, 13, 17, 20, 22, 23, 24
2. Use of the Contradiction Heuristic	5, 6, 9, 11, 14, 16, 18
3. Use of the Pattern Generation Heuristic	1, 3, 7, 15, 19, 21

Initial and Final Problem Sorting Schemes

Following administration of both the problem solving pretest (PS1) and the problem solving postest (PS2), students were required to complete a problem similarity questionnaire (PSQ). A copy of the questionnaire appears in Appendix A. The questionnaire was designed to provide information regarding student similarity judgements on the nine items of the problem solving pretest which were also the first nine items of the problem solving posttest. Results could then be examined for pre-post differences in sorting schemes. Students were asked to judge the similarity of all possible pairs of these nine items for a total of 36 comparisons. In imitation of Chartoff's (1976) questionnaire, students were told to rate each pair of problems on a continuous similaity scale ranging from extremely dissimilar to extremely similar. The similarity scale measured 15 cm and each pair of problems was given a similarity score based on the distance the slash mark was from the left end of the line. So the responses to each similarity questionnaire could be translated into a nine by nine similarity matirx.

Students were given the following instructions by the investigator:

> I will ask you to refer to your test and direct your attention to two problems at a time. When I do, re-read those problems and then record your first impression as to how similar or dissimilar they are by placing a vertical slash mark on that part of the line that best describes how similar you feel they are. For example, if I ask you to compare problems 1 and 2 and you feel they are very much alike you could indicate that by:

Extremely Dissimilar	Moderately Similar	Extremely Similar

If you felt they were only somewhat alike you could indicate that by:

Extremely Dissimilar	Moderately Similar	Extremely Similar

The investigator used an overhead projector to demonstrate exactly how the questionnaire was to completed.

Notice that in this study, as opposed to those conducted by Chartoff and Silver, the students are required to solve (or at least attempt to solve) the problems before they are asked to judge their similarity. It was felt that students are more likely to record a high similarity rating based on a heuristic judgement after they have considered how the problem should be solved, than would be the case if they had simply read the problem through once or twice.

Algebra and Trigonometry Performance

Since one of the aims of the study was to compare the experimental and control groups with respect to performance in basic algebra and trigonometry, a 24 item posttest in algebra and trigonometry (AT) was constructed by the investigator. The selection of problems and topics covered by the examination was determined by those areas covered in the control course in algebra. The test, which appears in Appendix A, can be subdivided on the basis of the algebraic technique needed to solve each item (Table 3).

Treatments

In order to fulfill a distribution requirement, all students at the college where this study was conducted are required to take nine quarter hours (three, 3 credit courses) in mathematics. Because most entering freshmen are deficient in a number of algebraic skills and since the pharmacy curriculum is such that these skills are extremely necessary for future courses in physics, chemistry, and pharmacology, all freshmen (with the exception of four or five advanced placement students) are required to take a course in College

Table 3

Subdivisions of the Algebra and Trigonometry Posttest

Algebraic Technique	Item Numbers
Simplifying algebraic expressions involving exponents and radicals	1, 2, 3, 4, 5, 6, 7
Solving quadratic equations	8, 9, 10
Adding and multiplying algebraic fractions	11, 12
Graphing the equation of a straight line	13
Working with logarithms and exponential equations	14, 15, 16, 17
Solving triangles (law of sines and cosines)	18, 19, 20, 21, 22
Solving linear systems	23, 24

Algebra in the fall quarter. This course in College Algebra accounts for three of the nine quarter hours in mathematics and the remaining six quarter hours are then chosen from elective offerings. The experimental course in Problem Solving was a new elective course first offered in the fall of 1977. For the purposes of the study, this course was only open to those freshmen randomly assigned to take it in the fall of 1977.

An attempt was made to keep several of the instructional variables constant in both experimental and control classes. The instructor variable was held fixed by having the investigator teach both the control and the experimental courses. Both classes met three times a week for 50 minutes and a 90 minute "help session" was conducted weekly. During these sessions, the investigator was available to discuss assignments and help students review for upcoming tests. In the course of the quarter, students were required to hand in four assignments which were graded and returned to them. Both groups of students took four in-class examinations. The number and length of the problem assignments and examinations as well as the grading system was the same for both groups. However, the method and clarity of the solution was considered more important in grading the assignmnets of the experimental group since the method of solution rather than the specific answer was the focal interest of the problem solving course.

Both experimental and control subjects were encouraged to participate in the solution(s) to all problems presented in class. The method of instruction differed with respect to problem solving. The control group had as the goal of the instruction the application of mathematical and algebraic techniques, whereas, the experimental group focused on heuristic strategies. This difference in instructional strategy arose because of a difference in objectives. The control group studied problems so as to reinforce newly learned algebraic concepts whereas the experimental group studied problems for the purpose of learning more general techniques and strategies to be applied in problem solving situations. As a result problems were discussed in much fuller detail and for a longer period of time in the experimental class. Students in the problem solving class were reminded often of such strategies as using methods and results of previously solved problems, organizing information, devising and using suitable notation, making use of diagrams, working problems out carefully and in complete detail on paper as opposed to working them in their heads, avoidance of guessing, and checking results. A more flexible approach to problem solving was encouraged in the experimental class as students were encouraged to look for multiple solutions to a problem, to generate new problems from old problems, and to test their intuition about what the solution should look like.

Control Course

The textbook used in the control course was the popular Algebra and Trigonometry: A Functions Approach by Mervin Keedy and Marvin Bittinger (1974). The investigator has taught this particular course for three years (1974-1976). No essential changes in either material or emphasis were made when the course was presented in the fall of 1977 as the control for this study. Topics covered in the course included a three week review of basic algebra (exponents, radicals, factoring polynomials, operations with algebraic fractions, and solving linear and quadratic equations). Two weeks were devoted to a study of logarithms, three weeks to trigonometry, and two weeks to linear algebra topics. The sections on linear and quadratic equations, logarithms, trigonometry, and linear algebra all concluded with a subsection entitled "applications." In the case of logarithms these applications included compound interest, bacteria growth, and radioactive decay. Vectors were studied as an application of trigonometric techniques and a variety of traditional "word problems" provided applications of solving systems of linear equations.

The major thrust of the course was the teaching of algebraic skills. As each new topic was introduced an overview of the algebraic techniques to be covered in that section was given. The motivation provided the students was that learning these basic techniques allowed them to perform more advanced algebraic processes. For example, students were told that learning to factor polynomials was necessary in order to add polynomial fractions or to solve quadratic equations. Similarly, one should learn how to deal with logarithms because logarithms are used to solve exponential equations. Hence, the emphasis and the motivation for the course were algebraic techniques — solving quadratic equations, logarithmic equations, exponential equations, solving triangles, and solving systems of linear equations. Recall that each section contained a subsection on "applications" and that following Keedy and Bittinger (and, in fact, most commercial texts in college algebra), this subsection appeared at the end of the discussion. Word problems calling for students to set-up and solve quadratic and exponential equation problems, triangle problems, and systems of linear equations problems were presented in the algebra control course, but only after the needed algebraic techniques had been fully developed. Only about fifteen percent of the total calss time was spent on such applications.

Experimental Course

The experimental course, entitled Problem Solving, was an elective course first offered in the fall of 1977. The textbook for the course was an 85 page set of notes prepared by the investigator. Rather than including the entire text as an appendix, a detailed outline together with a sampling of the problems discussed in class will be provided here. The text itself is available from the author upon request.[1] The course was organized aroud three heurisitc strategies: the use of algebraic symbolism, the contradiction heuristicm and the pattern generation heuristic.

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[1]Available from: Prof. Martha L. Hunt, Mathematics Dept., Massachusetts College of Pharmacy, Boston, Mass. 02115

Each of the three chapters in the text treated only one of these three heuristics.

The first chapter, entitled "The Symbolic Language of Mathematics," discusses how to extract pertinent information from a problem and record it symbolically. Students are encouraged to sketch diagrams whenever they could be useful, to write down all the facts on paper before beginning to solve the problem, to be neat and label explicitly, and to check their solution at the end. They are taught how to choose symbols wisely - so as to remind them of the mathematical concept - and to keep the number of different symbols used in solving a problem to a minimum. Finally, emphasis is put on the procedure used to arrive at the answer not the answer itself. A problem is solved by presenting a clearly stated arguement that would convince anyone reading it that the problem had in fact been solved correctly. The examples presented and discussed in the first chapter sample from a variety of techniques ordinarily studied in high school algebra abd trigonometry. In particular, solving exponential and logarithmic equations, solving first and second degree equations, solving systems of linear equations, and solving problems using trigonometry in their solution. To assist the reader in understanding the nature of these problems, the four examples given in Chapter 1 will be presented.

Example 1: Tom, Dick, and Harry mow lawns in the summer to earn money. They each have a lawn mower and one Saturday they decided to mow a 5,900 sq.ft. lawn together, using all three mowers. Tom mows 70 sq.ft. per minute, Dick 50, and Harry 40. Dick and Harry start mowing the lawn at the same time, but Tom has trouble starting his mower and is delayed for 30 minutes. All three boys stop mowing at the same time, when the lawn is finished. How long does Tom mow?

Example 2: A zoologist, in an experiment involving mice, finds he needs a food mix that contains, among other things, 23 grams of protein, 6.6 grams of fat, and 16 grams of moisture. He has on hand mixes of the following compositions:

	Protein %	Fat %	Moisture %
Mix A	20	2	15
Mix B	10	6	10
Mix C	15	5	5

How many grams of each mix should be used to obtain the desired diet mix?

Example 3: Two cars start at the same time from an intersection of two highways. The car on one highway averages 32 miles per hour while the car on the other highway is driven steadily at 44 mph. If the highways are straight and the angle of intersection is 28°, how far apart are the cars at the end of 1 hour and 15 minutes?

Example 4: The radioactive substance Stronium 90 has a half life of 25 years. If we start with 36 grams, how long will it take for there to be only 2.25 grams?

At the end of Chapter 1 are 136 problems of a similar nature ranging in difficulty from easy to challenging. The four weeks devoted to this chapter were spent discussing the four examples presented above as well as some 18 additional problems from the exercises at the end of Chapter 1. Each problem was discussed by having the investigator ask the class to take an active part in solving the problems by suggesting ways to represent them symbolically, proper equations representing the givens, methods for solving these equations, and finally, ways to check the reasonableness of the answer. As each problem was presented, the students were asked if they recalled a similar problem(s) and after each problem was solved time was taken to reconsider the solution in light of other problems they had solved - to look for similarities and differences. Finally, whenever possible, alternative solutions were given and the relative merits of each were discussed.

Cahpter II is "The Method of Contradiction" which presented students with a variety of problems requiring them to arrive at a contradiction(s) as a means of solution. Examples discussed in class include how to work a multiple choice exam question backwards, problems in logic, alphametrics, magic squares, and a number of miscellaneous problems. The works of both Polya (1957, 1962, 1965) and Wickelgren (1974) were used as problem sources for this chapter as well as for suggestions as to how these problems should be discussed and presented to students. Again, the nine examples given in the text will be presented as an illustration of the material in this chapter.

Example 1: The solution of $\sqrt{7x - 3} + \sqrt{x - 1} = 2$ is:

A. $x = 3$ B. $x = 3/7$ C. $x = 2$ D. $x = 1$ E. $x = 0$

Example 2: The Nelsons have gone out for the evening leaving their four children with a new babysitter, Nancy Wiggens. Among the many instructions the Nelsons gave Nancy before they left was that of their children, three were consistent liars and only one of them consistently told the truth, and told her which one. But in the course of receiving so much information, Nancy forgot which child was the truar. As she was preparing dinner for the children, one of them broke a vase in the next room. Nancy rushed in and asked who broke the vase. These were the childrens' statements:

Betty: Steve broke the vase.

Steve: John broke it.

Laura: I didn't break it.

John: Steve lied when he said I broke it.

Knowing that only one of these statements was true, Nancy quickly determined which child broke the vase. Who was it?

<u>Example 3</u>: Alpha, Beta, Gamma, and Omega were four young ladies of ancient Greece who were training to become oracles; in fact, only one of them actually did and she got a post at Delphi. Of the other three, one became a professional dancer, another a lady in waiting, and the third a harp player. During their training, when they were practicing predictions one day, Alpha forecast that whatever else Beta did she would never become a professional dancer; Beta forecast that Gamma would end up as a Delphic oracle; Gamma forecast that Omega would not become a harp player; and Omega predicted she would marry a man called Arataxerxes. The only prediction, that in fact came true, was that made by the Delphic oracle. Who became what? Did Omega marry Arataxerxes?

<u>Example 4</u>: On the Island of Perfection there are four political parties; the Free Food, the Pay Later, the Perfect Parity, and the Greater Glory. Smith, Brown, and Jones were speculating about which one of them would win the forthcoming election.

Smith thought it would be either the Free Food Party or the Pay Later Party. Brown felt quite confident that it would certainly not be the Free Food Party. And Jones expressed the opinion that neither the Pay Later Party nor the Greater Glory Party stood a chance. Only one of them was right. Which party won the election?

<u>Example 5</u>: Charlie told me the other day that he had not been first in a race he had with Alf, Bert, Duggie and Ernie. Duggie, he also informed me, was two places below Ernie, who was not second, and Alf was neither first nor last. I heard later from Bert that he was one place below Charlie. Find the order in which they finished the race (no ties).

<u>Example 6</u>: Replace the letters by digits from 0 to 9 (each letter stands for a different digit) so that the following will be a valid multiplication. (BE) (BE) = MOB

<u>Example 7</u>: Replace the letters by digits from 0 to 9 (each letter stands for a different digit) so that the following will be a valid addition:

```
   F O O D
+    F A D
---------
 D I E T S
```

<u>Example 8</u>: Complete the following 3 by 3 magic square:

```
8 . .
. . 9
. . .
```

<u>Example 9</u>: In numbering the pages of a book, a printer used 3289 digits. How many pages were there in the book, assuming that the first page in the book was numbered 1?

There were 27 additional problems at the end of Chapter II that served as assignment problems and further examples to be discussed in class. Again, students were taught to look for similarities between problems and their solutions and most of the steps in the solution to each problem were supplied by students in the class.

In Chapter III, "Searching for Patterns," the heuristic of discovering and continuing patterns was explained. The first part of the chapter discussed Pascal's triangle, the oblong numbers, the triangular numbers, and the series $1 + 4 + 9 + \ldots + n^2$. Students were encouraged to write down the first several cases and to look for some sort of emerging pattern. The second part of the chapter was devoted to a development of the method of finite differences. It was felt that since some of the more complicated patterns required a great deal of intuition to discover the correct rule, the method of finite differences would provide a more general technique for handling such problems. The following six examples and their solutions using finite differences were discussed in the text.

Example 1: Find the sum $S = 1 + 2 + 3 + 4 + \ldots + n$

Example 2: Find the sum $S = 1 + 4 + 9 + \ldots + n^2$

Example 3: What is the maximum number of regions into which 5 circles of arbitrary radii divide the plane? What about 10 circles?

Example 4: How many rectangles with integral sides are contained within an N by N rectangle? In particular, how many smaller rectangles are contained within a 5 by 6 rectangle?

Example 5: Obtain a formula for the nth octagonal number and use that formula to find the first five octagonal numbers.

Example 6: Find a formula for the sum of the face angles of a polyhedron.

Twenty three additional problems were provided at the end of the chapter. These served as assignment problems.

CHAPTER IV

RESULTS

The analyses presented in this chapter are directed at uncovering possible (1) pre-post and treatment differences in problem solving performance, (2) treatment effects in algebra and trigonometry performance, (3) pre-post differences in problem sorting schemes, (4) interaction effects between ability profiles and treatment variables, and (5) ability and treatment effects in problem sorting schemes.

Since the problem solving posttest can be subdivided into three subtests (as given in Table 2 of Chapter III) corresponding to the three heuristic strategies taught in the problem solving course, separate scores on each of these three subtests were calculated. So in addition to obtaining information on how treatment variables relate to posttest problem solving performance, additional information as to how treatment variables relate to specific subtest scores also was obtained.

The problem similarity questionnaires were used to gain insight into the sorting schemes employed by the subjects. Using these problem similarity questionnaires, it was possible to derive a measure of the degree to which a student sorted heuristically. This measure is referred to as the heurisitc sorting score (HSS) and will be defined precisely later in the chapter. Basically, high heuristic sorting scores are indicative of a tendency to sort heuristically.

Posttest measures of problem solving performance, algebra and trigonometry performance, and heuristic sorting scores are summarized in Table 4 to provide the reader with an overview of the results.

Treatment - Pretest Interaction

This study employed the Solomon four group design to check for a possible pretesting effect due to the administration of the problem solving pretest and problem similarity questionnaire. A two factor, treatment by pretest, analysis of variance was performed and the results indicated that only the treatment factor was significant ($F = 40.028$, $df = 1/80$, $p < .001$). Hence no pretesting effect was introduced and the pretest data can be used as a covariate in performing the analysis of covariance on the postest problem solving scores. Table 5 gives the mean scores on the problem solving posttest for each of the four groups of students in the study.

Table 4

Posttest Means for Instructional Groups

Group	n	PS2	S1	S2	S3	AT	HSS
Control	47	48.8	47.7	43.1	46.4	60.2	-1.9
Experimental	37	68.2	70.2	54.4	80.1	49.2	0.6
Pooled	84	57.3	57.6	48.1	61.3	55.4	-0.9

PS2 = the 24 item problem solving posttest

S1 = algebraic symbolism heuristic subtest of the problem solving posttest

S2 = contradiction heuristic subtest of the problem solving posttest

S3 = pattern generation heuristic subtest of the problem solving posttest

AT = algebra and trigonometry posttest

HSS = posttest heuristic sorting score

Table 5

Problem Solving Posttest (PS2) Means by Treatment

	Experimental	Control
Problem Solving Pretest	67.0 (n = 19)	49.1 (n = 22)
No Pretest	69.5 (n = 18)	48.5 (n = 25)
Pooled	68.2 (n = 37)	48.8 (n = 47)

Changes in Problem Solving Performance

An analysis of covariance was performed using the data obtained from the 19 experimental and the 22 control group students who took the nine item problem solving pretest. This analysis compared scores on the problem solving pretest with scores on only the first nine items of the problem solving posttest. Recall that these two sets of items were identical. The analysis of covariance revealed that when pretest scores were taken into consideration, the experimental group significantly outperformed the control group on items one to nine of the problem solving posttest ($F = 8.627$, $df = 1/38$, $p < .01$). Furthermore, this analysis revealed that the pretest scores were not correlated significantly with the posttest scores ($F = 0.52$, $df = 1/38$). Mean scores on items 1 to 9 for both experimental and control groups are given in Table 6.

Table 6

Mean Scores for the Problem Solving Test
Items One to Nine

	Control	Experimental
Pretest	33.7	30.8
Posttest	45.9	63.9
n	22	19

A second analysis was performed using the entire group of 84 students. Here the 24 item posttest was partitioned into the three subtests described in Table 2 of the previous chapter. These subtests correspond to the three heuristics taught in the problem solving course: algebraic symbolism, contradiction, and pattern generation. Mean scores on these three subtests appear in Table 4. To analyze the relationship between treatment group and problem solving scores, a two by three factorial analysis with repeated measures on the subtest factor was carried out (Winer, 1962). The experimental group significantly outperformed the control group ($F = 46.07$, $df = 1/82$, $p < .01$), there was a difference in performance across the three subtests ($F = 11.07$, $df = 2/164$, $p < .01$), and there was a treatment across subtests interaction ($F = 7.60$, $df = 2/164$, $p < .01$). A Newman-Keuls procedure was used to test for differences between all possible subtests means.

It was found that subtest 1 (algebraic symbolism) scores were significantly higher than subtest 2 (contradiction) scores ($p < .01$) and that subtest 3 (pattern generation) scores were significantly higher than subtest 2 scores ($p < .01$) but no significant differences existed between subtests 1 and 3.

To explain the interaction effect, tests on the simple main effects of the subtest factor at each of the two treatment levels were performed. The F ratio for subtests at the control level was nonsignificant ($F = .7479$, $df = 1$) whereas the F ratio for subtests at the experimental level was significant ($F = 17.7$, $df = 1$, $p < .01$). This analysis indicated that students in the control group performed the same across all three subtests, whereas students in the experimental group did not. In the experimental group, students performed best on pattern generation and poorest on contradiction.

Differences in Algebra and Trigonometry Skills

In addition to studying changes in problem solving performance, this study also was designed to test for differences in performance on basic algebra and trigonometry skills. The control group received a thorough review of these concepts, whereas the experimental group reviewed only those aspects of algebra and trigonometry needed to solve the problems they encountered. Comparison of the scores on the algebra and trigonometry posttest revealed that the control group did significantly better than the experimental group on basic algebra skills ($t = 2.905$, $df = 82$, $p < .001$) See Table 4 for the mean posttest scores for the algebra and trigonometry test.

Changes in Sorting Schemes

To analyze the sorting scheme data, the pretest and posttest similarity questionnaires were subjected to a completelink hierarchical clustering analysis, a statistical procedure described by Anderbery (1973) and Hubert and Baker (1976). The complete link method operates on a similarity matrix to produce clusters by starting with clusters consisting of single objects (problems) and fusing clusters which are closest or most similar. To illustrate the complete link clustering technique consider the following matrix taken to represent the similarity measures among five objects.

$$S = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} x & 9.0 & 5.0 & 1.0 & 2.0 \\ & x & 6.0 & 2.0 & 3.0 \\ & & x & 7.0 & 6.0 \\ & & & x & 8.0 \\ & & & & x \end{bmatrix} \end{array}$$

At stage one of the clustering, objects one and two are fused to form a cluster since 9.0 is the largest entry in the matrix. The similarity between the cluster (1,2) and the remaining objects is obtained as follows:

$$s_{(12)3} = \min (s_{13}, s_{23}) = 5.0$$

$$s_{(12)4} = \min (s_{14}, s_{24}) = 1.0$$

$$s_{(12)5} = \min (s_{15}, s_{25}) = 2.0$$

A new similarity matrix results:

$$S' = \begin{array}{c} \\ (12) \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccc} (12) & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} x & 5.0 & 1.0 & 2.0 \\ & x & 7.0 & 6.0 \\ & & x & 8.0 \\ & & & x \end{bmatrix} \end{array}$$

The complete link method dictates that the next cluster should be (4,5) since 8.0 is the largest entry of S'. Again, new similarity measures are computed:

$$s_{(12)(45)} = \min (s_{12}, s_{14}, s_{15}, s_{24}, s_{25}) = 1.0$$

$$s_{(45)3} = \min (s_{34}, s_{35}) = 6.0$$

These new similarity measures can be used to create S".

$$S'' = \begin{array}{c} \\ (12) \\ 3 \\ (45) \end{array} \begin{array}{c} \begin{array}{ccc} (12) & 3 & (45) \end{array} \\ \begin{bmatrix} x & 5.0 & 1.0 \\ & x & 6.0 \\ & & x \end{bmatrix} \end{array}$$

The largest entry is now 6.0, hence the next cluster is (3,4,5). So the complete link clustering scheme for the given matrix is:

Level 0: (1) (2) (3) (4) (5)
Level 1: (1, 2) (3) (4) (5)
Level 2: (1, 2) (3) (4, 5)
Level 3: (1, 2) (3, 4, 5)
Level 4: (1, 2, 3, 4, 5)

The reader should be aware of the fact that the complete link method used here is but one of several widely used clustering procedures that operate on a similarity matrix to yield a hierarchical partition of a set of objects. These procedures differ with respect to the manner in which they define the distance between an object and a group containing several objects, or between two groups of objects. In addition to the complete link method other well known methods include the single link method, the centroid method, the median method, the group average method, Ward's method and the simple average method. Cunningham and Ogilvie (1971) compared the seven hierarchical grouping techniques mentioned above on four artificially constructed data sets. They computed the rank correlation (Kendall's tau) between the elements of the given distance matrix d_{ij} and the output distance $d_{ij}*$ taken as the value associated with the partition in which i and j first appear in the same subset. These authors found that for most data sets the group average method and the complete link method were at least as good a grouping method as any other and that the single link method was the most dependent on the type of input data. Based on the results of this report, it was decided to analyze the problem solving questionnaires using the complete link method.

Tables 7, 8 and 9 give the similarity matrices that were used in the complete link analyses. Recall that the problem similarity questionnaire required students to compare items one to nine on the problem solving test for a total of 36 comparisons and similarity measurements were obtained by measuring the distance from the end of the line to the slash mark on a continuous similarity scale. Each entry in the similarity matrix is the average similarity rating over all students in that group. The complete link clustering results for these similarity matrices are given in Tables 10, 11 and 12.

Table 7

Similarity Ratings Derived from the Pretest Problem Solving Questionnaire (Both Groups Combined n = 41)

Problem Number	Problem Number 1	2	2	4	5	6	7	8	9
1	-	3.46	7.53	4.88	8.18	5.07	7.17	8.07	4.92
2		-	3.92	3.23	4.67	4.56	6.01	4.57	3.86
3			-	4.01	9.88	4.14	9.03	11.46	6.80
4				-	3.50	10.43	3.78	3.72	3.45
5					-	3.32	10.68	8.03	5.38
6						-	3.47	3.58	3.32
7							-	7.16	5.77
8								-	7.51
9									-

Table 8

Similarity Ratings Derived from the Posttest Problem Solving Questionnaire (Control group only, n = 47)

Problem Number	Problem Number 1	2	3	4	5	6	7	8	9
1	-	3.29	7.02	4.00	5.69	5.12	5.72	6.40	4.20
2		-	3.91	3.95	4.69	3.55	5.18	4.35	3.39
3			-	3.19	8.87	3.61	7.90	9.66	6.60
4				-	3.59	9.95	3.84	3.32	3.36
5					-	3.32	9.38	8.52	6.11
6						-	3.32	3.41	3.56
7							-	7.56	6.33
8								-	8.91
9									-

Table 9

Similarity Ratings Derived from the Posttest Problem Solving Questionnaire (Experimental Group only, n = 37)

Problem Number	1	2	3	4	5	6	7	8	9
1	-	2.93	8.52	3.98	5.99	3.94	6.44	5.81	3.61
2		-	2.93	2.69	3.41	2.74	3.49	3.75	2.64
3			-	2.69	7.46	3.14	8.43	7.91	4.46
4				-	2.90	10.13	3.83	3.27	3.02
5					-	2.92	8.09	6.77	4.09
6						-	2.82	2.85	2.74
7							-	7.11	4.98
8								-	5.29
9									-

Table 10

Complete Link Clustering for the Pretest Similarity Matrix

Level	Clusters Generated
1	(3,8)
2	(3,8) (5,7)
3	(3,8) (5,7) (4,6)
4	(3,5,7,8) (4,6)
5	(1,3,5,7,8) (4,6)
6	(1,3,5,7,8,9) (4,6)
7	(1,2,3,5,7,8,9) (4,6)
8	(1,2,3,4,5,6,7,8,9)

Table 11

Complete Link Clustering for Control Group Posttest Similarity Matrix

Level	Clusters Generated
1	(4,6)
2	(4,6) (3,8)
3	(4,6) (3,8) (5,7)
4	(4,6) (3,5,7,8)
5	(4,6) (3,5,7,8,9)
6	(4,6) (1,3,5,7,8,9)
7	(2,4,6) (1,3,5,7,8,9)
8	(1,2,3,4,5,6,7,8,9)

Table 12

Complete Link Clustering for Experimental Group Posttest Similarity Matrix

Level	Clusters Generated
1	(4,6)
2	(4,6) (1,3)
3	(4,6) (1,3) (5,7)
4	(4,6) (1,3) (5,7,8)
5	(4,6) (1,3,5,7,8)
6	(4,6) (1,3,5,7,8,9)
7	(1,3,4,5,6,7,8,9) (2)
8	(1,2,3,4,5,6,7,8,9)

An examination of the pretest clusters (Table 10) reveals many instances of contextual sorting. The first three clusters generated, clusters (3,8), (5,7), and (4,6), are all based on contextual cues. Problems 3 and 8 both present a number series, yet the heuristic best used to solve problem 3 is pattern generation and the heuristic best used to solve problem 8 is algebraic symbolism. Problems 5 and 7 are short "algebra word problems," but problem 5 is classified heuristically as a problem solved by contradiction and problem 7 is a problem solved by pattern generation. The clearest indication of contextual sorting on the pretest is the cluster (4,6). Both problems 4 and 6 depict a triangle. Problem 4 is best solved algebraically and problem 6 by contradiction.

The same remarks apply to the posttest clusters for the control group (Table 11) since the clusters that appear in level 3 are the same as those that appeared on the pretest. Only the order in which the clusters are generated is different. On the posttest, control students found problems 4 and 6 to be the most similar as opposed to problems 3 and 8 on the pretest.

A different level 3 clustering scheme can be observed in the experimental posttest groupings. Again we find the clusters (4,6) and (5,7), both contextual in nature, but instead of problems 3 and 8 being grouped together, we have instead the cluster (1,3). This is interesting since problem 1 deals with determining the maximum number of regions into which ten lines divide a triangle and problem 3 is a number series - the contextual cues appear to be very weak. Furthermore both problems are examples of the heuristic of pattern generation. So whereas no evidence of heuristic sorting could be observed in the level 3 clusterings of the pretest or control groups, a cluster did arise in the level 3 grouping of the experimental group that was heuristic in nature.

Looking at level 6, no differences either between the two posttest groups or between the pretest and the posttest groups are observable. The level 6 clusters are: (1, 3, 5, 7, 8, 9); (4, 6); (2). A possible explanation for the cluster (1, 3, 5, 7, 8, 9) is that none of the problems in that cluster are traditional "algebra word problems." Even though problem number 8 can be solved algebraically, it is not of the type typically encountered in high school algebra texts and as a result may not be thought of as an algebra problem. Problems 2 and 4 remain outside the cluster perhaps because they are recognizable as problems familiar to the subjects from previous algebra courses.

Hence the complete link analyses presented here indicates that with the exception of one heuristic cluster observed in the sorting scheme of the experimental group, subjects were not overly attentive to heuristic cues and tenden to sort more on the basis of contextual cues.

As a final comparison of the three sets of clusters generated by the pretest and posttest similarity questionnaires, the Spearman correlation coefficients (Table 13) relating the three similarity matrices (Tables 7, 8 and 9) were computed. The significance of each of the three correlation coefficients indicates that few, if any, real differences in sorting schemes occurred as a result of the problem solving course.

Table 13

Spearman Correlation Coefficients for the Pretest and Posttest Similarity Matrices

	M_1	M_2	M_3
M_1	1.0000	0.8975*	0.9040*
M_2		1.0000	0.8940*
M_3			1.0000

*$p < .001$

M_1 = Pretest Similarity Matrix (Table 7).

M_2 = Control Posttest Similarity Matrix (Table 8).

M_3 = Experimental Posttest Similarity Matrix (Table 9).

Heuristic Sorting and Problem Solving Performance

The clusters that would have arisen if the students had sorted purely heuristically are (1, 3, 7), (2, 4, 8), and (5, 6, 9). To test whether or not a tendency to sort heuristically is related to problem solving performance each student in the study was assigned a heuristic sorting score on the basis of the problem sorting questionnaire. This score was computed by taking the mean similarity rating for all pairs of problems that were related heuristically and subtracting the mean similarity rating for all pairs of problems that were not related heuristically. As a result the higher the heuristic sorting score, the more the student tended to recognize heuristic cues as opposed to nonheuristic cues. Next a simple linear regression analysis was done to relate problem solving performance (as measured by the posttest in problem solving) and heuristic sorting (as measured by the heuristic sorting score). These sorting scores, together with the complete set of data on each student, appear in Appendix B. The regression analysis gave a correlation coefficient $r = .204$ ($p < .04$) indicating that heuristic sorting and problem solving performance are correlated positively. The scatter diagram for this analysis is given in Figure 1.

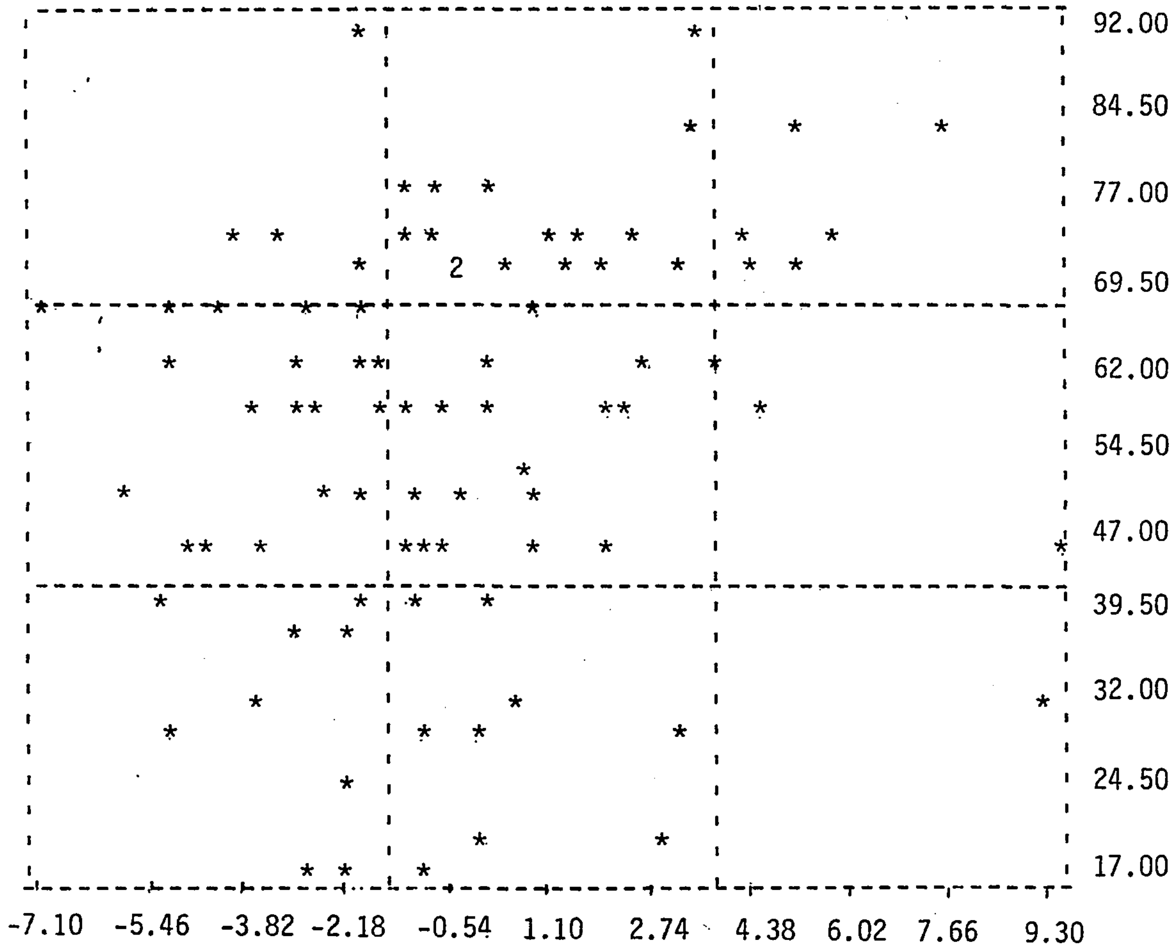


FIGURE 1

Scattergram of Posttest Problem Solving Score Versus Heuristic Sorting Score

Analysis of the Ability Test Data

The five ability tests administered to both groups of students at the beginning of the study were subjected to a principal components factor analysis. Two factors were isolated accounting for 72% and 28% of the total variability. A varimax orthogonal rotation was performed next to provide a theoretical definition for the two ios-lated factors. The rotated factor matrix appears in Table 14. This analysis indicated that the Hidden Figures Test, the Deciphering Languages Test and the Toothpicks Test all loaded high on facotr 1 whereas only the Scrambled Words Test loaded high on factor 2. The Nonsense Syllogism Test has essentially a zero loading on factor 2 and a low loading on factor 1. In fact, the communality of only 0.0806 indicated that only eight percent of the variability in the Nonsense Syllogism Test scores was accounted for by factors 1 and 2. Factor 1 was labelled Disembedding-Logical Reasoning and factor 2 was labelled Semantic-Divergent Thinking. The Hidden Figures, Deciphering Languages, and Toothpicks Tests all demand that the subject be able to locate either a figure or symbol in a complex pattern and then decide how that figure or symbol is to be used to answer the question. Since the Hidden Figures Test loads highest on factor 1, it is felt that the disembedding aspect is the most important descriptor of this factor. Note that the Nonsense Syllogism Test has a loading of

0.28206 on factor 1, indicating that logical reasoning has some place in the definition of this factor but it is not nearly as important as the disembedding aspect. Factor 2 was labelled Semantic Divergent Thinking since the only test that loaded high on factor 2 was the Scrambled Words Test. Since each of the words presented in this test was short (five to seven letters), divergent production of letter combinations would yield the solution to each problem rather quickly. The only other test with a significant loading on factor 2 was the Hidden Figures Test. Here too, divergent production of possible locations for the to-be-found figure would yield a solution.

Table 14

Varimax Rotated Factor Matrix for Ability Test Data

Ability Test	Factor 1	Factor 2	Communality
Scrambled Words	0.06572	0.78120	0.61459
Hidden Figures	0.65457	0.37928	0.57232
Deciphering Languages	0.54601	0.22981	0.35084
Nonsense Syllogism	0.28206	0.03234	0.08060
Toothpicks	0.53963	-0.06989	0.29609

The next step in the analysis was to compute factor scores for each student to represent their strength on both of the factors and then use these factor scores to form homogeneous subgroups of students with similar profiles on both factors 1 and 2. Ward's hierarchical clustering analysis was the method used to form such groups (Anderbery, 1973).

Ward's method, another of the hierarchical grouping procedures, starts with n objects measured on p orthogonal variables. Initially each of the n objects is its own cluster. At each further stage of the analysis, the error sum of squares (ESS), defined to be the total sum of squared deviations of every point from the mean of the cluster to which it belongs, is specified and clusters are fused so as to create the minimum increase in the ESS. It is important to note that the variables that one uses as the basis for Ward's method must be orthogonal and it was to this end that a varimax (orthogonal) factor analysis of the five ability tests was performed. It would have been improper to form clusters of students with respect to five nonorthogonal measures of ability, however, the two derived ability factors isolated in this section are appropriate variables on which to cluster students using Ward's procedure. The reader interested in reports concerned with the goodness of fit of Ward's procedure is referred to Cunningham and Ogilvie (1971) as well as to Gross (1972).

Both studies indicate that Ward's procedure can be used with confidence.

Ward's procedure was carried out on the individual factor scores of each student and a decision was made to perform the analysis of ability profiles at the stage where four groups of students had been specified. This decision was based on an analysis of the ESS generated at each stage of the clustering. As each new group is formed this ESS increases. In this study, five groups gave an ESS = 7.1802, four groups gave an ESS = 8.9332 and three groups gave an ESS = 15.4490. Since only a small increase in the ESS occurred as a result of creating four groups whereas a much larger increase in the ESS occurred when three groups were created, it was decided to use four groups of students in further analyses. Average factor scores on each of factors 1 and 2 were computed for each of the four groups as well as 95% confidence intervals for each of these mean scores. These scores appear in Table 15.

Table 15

Mean Scores and 95% Confidence Intervals on Factor 1 and Factor 2 for the Four Groups of Students Given by Ward's Hierarchical Clustering

	n	Factor 1	Factor 2
Group 1	35	$\bar{x} = 9.19$ $8.08 < \mu < 10.31$	$\bar{x} = 35.64$ $34.7 < \mu < 36.6$
Group 2	16	$\bar{x} = 1.64$ $0.21 < \mu < 3.10$	$\bar{x} = 28.24$ $26.9 < \mu < 29.6$
Group 3	29	$\bar{x} = 0.27$ $-0.40 < \mu < 0.95$	$\bar{x} = 34.44$ $33.8 < \mu < 35.1$
Group 4	4	$\bar{x} = 5.03$ $-1.10 < \mu < 11.10$	$\bar{x} = 16.09$ $12.5 < \mu < 19.7$

Group 4 provides little information since the sample size is only 4. As a result it is impossible to discriminate members of group 4 on the basis of factor 1, however, even given such a small sample size it can be determined that group 4 students do significantly worse on factor 2 than do members of groups 1, 2 or 3. Unfortunately, all members of group 4 were in the experimental group and for this reason they were dropped from the analysis comparing the problem solving performance of students in experimental and control groups. Comparing groups 1, 2 and 3 the following observations can be made:

1. Group 1 students do significantly better than group 2 students on both factors 1 and 2.
2. Group 1 students do significantly better than group 3 students on factor 1 but not factor 2.
3. Group 3 students do significantly better than group 2 students on factor 2 but not factor 1.

The next stage of the analysis is to determine which of the three groups of students benefited from the problem solving course and which did not. To this end a 3 x 2 x 3 (student profile group x treatment group x problem solving subtest) analysis of variance with repeated measures on the third factor was performed.

The analysis is given in Table 16 and as noted earlier it was performed on only 80 of the 84 students involved in the study and on only three of the four subgroups isolated by the Ward's analysis. Results include the following:

1. The student profile group is not a significant main effect.
2. There is no significant interaction between student profile group and treatment group.
3. There is no significant interaction between student profile group and problem solving subtest.
4. There is a significant three factor interaction amongst treatment group, student profile group, and problem solving subtest.

As was the case in the analysis presented in the second section of this chapter, problem solving subtest, treatment group and treatment group cross problem solving subtest effects are all significant. The reader is referred to the previous section on Changes in Problem Solving Performance for an interpretation of these significant differences (See Table 16).

To further investigate the three way interaction uncovered here, a three way summing over table was constructed with entries representing the mean scores of each group of students on the three subtests of the problem solving posttest. The summing over table is presented in Table 17. Tests on the simple-simple main effects of the treatment factor at each of the nine levels of profile group cross subtest were performed. The F ratios for these tests are given in Table 18.

Combining the analyses given in Table 18 with the summing over table given in Table 17, the following conclusions can be drawn about each of the three profile groups.

Profile Group 1: No significant differences in performance exists between experimental and control group students on subtest 2. On subtests 1 and 3, however, the experimental group outperformed the control group ($p < .05$ and $p < .01$, respectively).

Profile Group 2: No significant differences in performance exist between experimental and control groups on any of the

Table 16

Treatment (T) versus Student Profile Group (G) versus Problem Solving Subtest (S) Analysis of Variance with Repeated Measures on Problem Solving Subtest (S)

Source of Variation	Sum of Squares	D.F.	Mean Squares	F
Between Subjects	93153.6291	79		
G	3314.1743	2	1657.0872	1.97
T	26356.9218	1	26356.9218	31.37***
G x T	1312.1942	2	656.0971	.78
Subjects within groups	62170.3388	74	840.1397	
Within Subjects	52261.6667	160		
S	5595.1583	2	2792.5792	10.79**
G x S	810.8346	4	202.7987	.77
T x S	3885.7512	2	1942.8761	7.43**
T x S x G	3257.2830	4	814.3208	3.11 *
S x subjects within groups	38712.6387	148	261.5719	

*p < .05, **p < .01, ***p < .001

three subtests.

Profile Group 3: Profile group 3 students exhibit statistically significant differences between experimental and control groups across all three subtests. That is students enrolled in the course in heuristic problem solving did significantly better than control students on all three subtests.

Table 17

Mean Scores on the Subtests of the Problem Solving Posttest for the Three Student Profile Groups and the Two Treatment Groups

		Subtest 1	Subtest 2	Subtest 3
Profile Group 1	Control	51.2	51.7	42.9
	Experimental	66.1	58.1	86.4
Profile Group 2	Control	52.6	41.3	57.4
	Experimental	63.9	46.9	64.3
Profile Group 3	Control	41.9	35.4	44.7
	Experimental	73.9	55.7	74.9

Table 18

F Ratios for the Simple-Simple Main Effects of Treatment Group at Profile Group Cross Subtests

Profile Group x Subtest	F
Group 1 x Subtest 1	4.24*
Group 1 x Subtest 2	0.78
Group 1 x Subtest 3	36.17**
Group 2 x Subtest 1	1.11
Group 2 x Subtest 2	0.27
Group 2 x Subtest 3	0.37
Group 3 x Subtest 1	14.76**
Group 3 x Subtest 2	5.94*
Group 3 x Subtest 3	13.15**

*$p < .05$, **$p < .01$

As a final way of examining the rather complex three factor interaction encountered here the profile of means for each of the three student profile groups is depicted in Figure 2.

The summary statistics for the four students that comprised group 4 are given below. Recall that these students were all in the experimental group. On subtest 1 scores were 91, 100, 82 and 82, yielding an average score of 89. On subtest 2 scores were 43, 71, 43, and 43 giving an average score of 50. On subtest 3 scores were 100, 100,

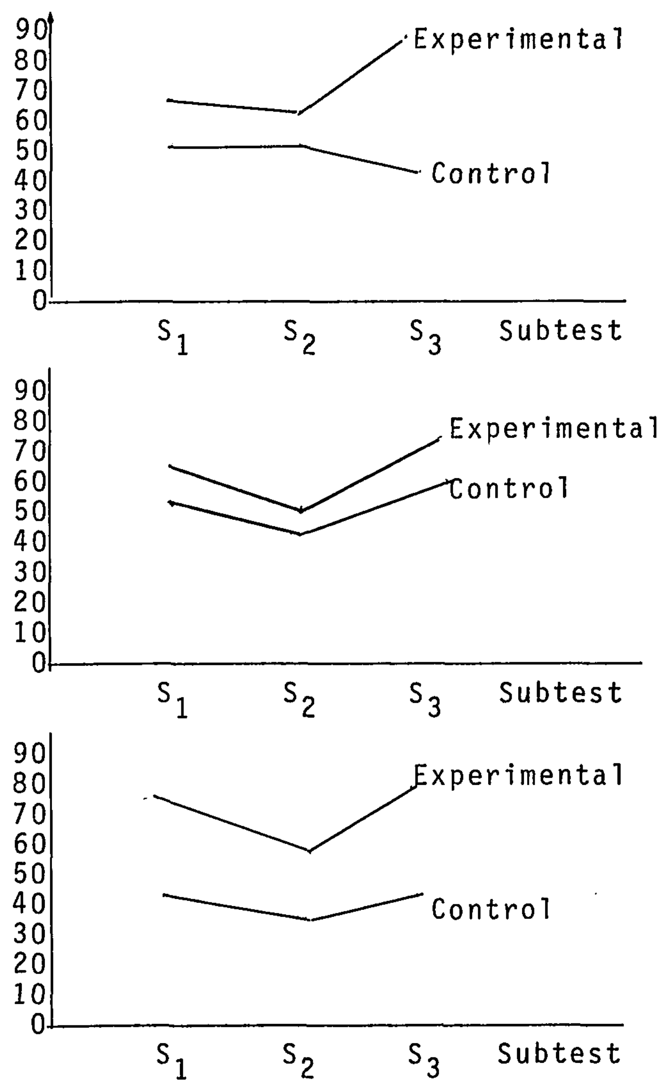
Figure 2

Profile of Means (Mean Scores of the Problem Solving Subtests) For Each of the Three Student Profile Groups

Profile Group 1:

Profile Group 2:

Profile Group 3:



100, and 83 for an average of 96. The mean scores for the other 33 students in the experimental group were 68, 55, and 78, respectively. So these four students performed better than average (as compared to other students in the experimental group) on subtests 1 and 3 and approximately the same on subtest 2.

Relationship Between Ability Profiles and Sorting Schemes

The last question addressed in this study involved the relationship between student ability profiles and sorting scheme usage. That is, do certain ability profiles types tend to sort more heuristically than other profile types? To answer this question a two factor analysis of variance was performed with treatment group and ability profile group as the independent variables and the derived heuristic sorting score as the dependent variable. Again, since profile group 4 consisted of only experimental group students, these four students were not part of this analysis. Mean heurisitc sorting scores are presented in Table 19 and the analysis of variance is given in Table 20.

This analysis indicates that both treatment group and profile group are significant main effects and that no interaction effect existed. Referring to Table 19, it can be concluded that experimental group students had significantly higher heuristic sorting scores than control group students ($p < .001$). A Newman-Keuls analysis was performed on the profile group means (See Table 21).

The results of the Newman-Keuls procedure indicate that profile group 2 students had significantly lower heuristic sorting scores than students in either profile group 1 or 3 and that no significant differences exist between the heuristic sorting scores of profile groups 1 and 3. Profile group 2 students are characterized by lower scores on factor 2 (semantic-divergent thinking) and so it appears that low scores on this factor correspond to a tendency to sort nonheuristically.

In and earlier section the complete link clusterings for the posttest problem similarity questionnaires were given (Tables 11 and 12) and only a few differences in clustering schemes were observed. These differences involved the first few levels of the clustering where it was observed that control group students focused primarily on contextual cues whereas experimental group students showed evidence of attending to both contextual and heuristic cues. It should be noted that hierarchical clustering methods are one dimensional in nature and as a result they only give information regarding the dominant clustering scheme employed. The analysis presented in this section suggests that even though the dominant sorting schemes of the experimental and control groups following instruction have much in common these sorting schemes are not equivalent. The sorting scheme employed by the experimental group recognized heuristic cues to a greater degree than did those of the control group.

Table 19

Mean Heuristic Sorting Scores for Profile Groups 1, 2, and 3

	Profile Group 1	Profile Group 2	Profile Group 3	Pooled
Control	-1.4368 (n = 19)	-4.8888 (n = 9)	-1.0388 (n = 19)	-1.9369 (n = 47)
Experi-mental	1.2938 (n = 16)	-0.6857 (n = 7)	0.4400 (n = 10)	0.5971 (n = 33)
Pooled	-0.1885 (n = 35)	-3.0499 (n = 16)	-0.5289 (n = 29)	-0.8916 (n = 80)

Table 20

Treatment By Student Profile Group Analysis of Variance on Heuristic Sorting Scores

Source	Sum of Squares	D.F.	Mean Squares	F
Profile Group (G)	112.6661	2	56.8331	7.14*
Treatment Group (T)	126.6903	1	126.6903	15.91**
G x T	3.8284	2	1,9142	0,24
Residual	589.2472	74	7.9627	
Total	833.4420	79		

*p < .01, **p < .001

Table 21

Test on the Ordered Mean Heuristic Sorting Scores Using Newman-Keuls Procedure

	Group 2	Group 3	Group 1	
Group 2	-------	2.5210*	2.8614*	Observed Difference
		1.76	2.05	Critical Difference
Group 3		-------	0.3404	Observed Difference
			1.42	Critical Difference
Group 1			------	

*$p < .05$

CHAPTER V

INTERPRETATION AND DISCUSSION

Research cited in Chapter II indicates that student ability profiles (Merriefield, Guilford, Christensen and Frick, 1962) and problem sorting schemes (Chartoff, 1976 and Silver, 1977) are factors that should be considered when analyzing problem solving performance. The research concerning the instructional variables and procedures that best promote efficient problem solving is much more ambigi-ous. Research was outlined in Chapter II that suggested heuristic instruction in problem solving was superior to expository instruction (Lucas, 1972) as was research that cast doubt on the transfer to all subjects of heuristic instruction (Goldberg, 1974 and Smith, 1973). The study presented here extends these investigations by looking for possible interactions between instructional variables, student ability profiles, and sorting scheme usage. Perhaps the best way of stating the overall objective of the present investigation is an attempt to isolate an aptitude-teat-ment interaction (ATI) for a course in heuristic problem solving. Such an ATI would account for some of the indeci-siveness concerning the usefulness of heuristic strategies. Data also was gathered that allowed for comparisons between experimental and control groups with respect to problem solving performance in basic algebra and trigonometry.

Sorting scheme data was obtained and this provided for analyses and comparisons of the sorting schemes employed.

Primary Findings

Ability Profiles

A principal components factor analysis of the five ability tests administered to the students in this study isolated two orthogonal ability factors labeled Disembedding - Logical Reasoning (Factor 1) and Semantic Divergent Thinking (Factor 2). Ward's hierarchical clustering procedure then was used to create four homogeneous groups of students with similar profiles across these two ability factors. Further analysis uncovered a significant three way (treatment by problem solving posttest by student ability profile group) interaction ($p < .05$).

Students in profile group 1 were characterized by high scores on both ability factors. A comparison of experimental and control subgroups uncovered significant differences in performance on subtests 1 and 3 of the problem solving posttest. Since the items of subtests 1 and 3 both make use of algorithmic solutions, this result does not indicate any beneficial effects of heuristic instruction, but rather suggests that only mastery of a variety of very specific algorithms separates the two treatment groups. A possible conclusion is that for students with high scores on both ability factors, instruction in algorithmic procedures is more beneficial than heuristic instruction.

Profile group 2 students score low on both ability factors 1 and 2 (lower than group 1 on factor 1 and lower than both groups 1 and 2 on factor 2). No statistical differences between experimental and control subgroups were found for this profile group.

Profile group 3 students scored low on ability factor 1 and high on ability factor 2 (lower than group 1 on factor 1, and higher than group 2 on factor 2). For this profile group there is an indication that heuristic instruction is beneficial. Comparison of experimental and control subgroups revealed significant differences in performance across all three subtests. Particularly interesting was a significant difference in performance on subtest 2. Experimental group students significantly outperforming control group students on the subtest requiring the greatest amount of transfer of heuristic strategies indicated that for profile group 3 students, heuristic instruction may have transfer properties it does not have for students of other ability profiles.

Finally profile group 4 is best described as consisting of students with low scores on factor 2 (lower factor 2 scores than any of the other three profile groups). The small sample size (n = 4) made any statistical analyses impossible but mean subtest scores indicated that these four experimental group students did exceptionally well on subtest 1 (mean = 89) and subtest 3 (mean = 96) and poorer on subtest 2 (mean = 50). The high scores on subtests calling for algorithmic solutions and the low score on the subtest calling for transfer of heuristic strategies, suggested that for students with low scores on factor 2, heuristic instruction may not be the most productive instructional strategy.

In summary, this section describes an aptitude treatment interaction (ATI) between student ability profile and heuristic instruction in problem solving versus instruction in algebraic techniques. It was shown that heuristic instruction provided for more transfer for profile type 3 students than for the other three profile types described in this study.

Sorting Scheme Usage

Another major purpose of this study was to determine whether the course in heuristic problem solving caused students to attend to heuristics in addition to contextual cues. The complete link hierarchical analysis presented in Chapter IV indicates that few, if any, differences in sorting scheme usage can be observed either between the control group and the experimental group after instruction or between the pretest sorting schemes and the posttest sorting schemes of the experimental group. Data analyses revealed that both groups of students pay close attention to contextual cues and tend to distinguish between traditional algebra problems and nontraditional problems. So in spite of the superior performance on the problem solving posttest, the experimental group appears to have attended to the same features of the problem statement as the control group. This is further evidence that the superior performance of the experimental group may be due in part to effective use of specific algorithms rather than use of general heuristic strategies. It should be noted, however, that contextual cues were built very strongly into the problem solving test. If these contextual cues had been somewhat less obvious it may be that differences in sorting schemes would have arisen.

On the other hand a simple linear regression relating a derived heuristic sorting score to the problem solving posttest score showed that a significant positive correlation existed between these two variables ($r^2 = .04174$, $p < .04$). Here we have an indication that the tendency to recognize heuristic cues is related to efficient problem solving. Even though the coefficient of determination proved to be statistically significant, its value reveals that just slightly more than 4% of the variability in the problem solving posttest scores can be attributed to heuristic sorting scores. An examination of the scatter diagram for this analysis (Figure 1) gives some additional information in terms of certain outliers. Heuristic sorting scores ranged from a low of -7.1 to a high of 9.3.

Although for most students high heuristic sorting scores correspond to relatively high posttest problem solving scores, certain students were exceptions to this pattern. For example, four students in the experimental group scored relatively high on the problem solving posttest while at the same time receiving a low heuristic sorting score. The cases in question are the following four sets of scores received by treatment group students:

Problem Solving Score	Heuristic Sorting Score
92	-2.2
92	-3.4
75	-3.5
67	-7.1

The two students scoring 92 on the problem solving posttest represent the two highest scoring students in the study. Their relatively low heuristic sorting scores seem to suggest that it is possible to perform well in certain problem solving situations without being overly attentive to heuristic cues. The same comment holds for the students who received problem solving posttest scores of 75 and 67. Both of these scores are above the mean, yet the corresponding heuristic sorting scores are among the lowest of all students in the study. In fact, the heuristic sorting score of -7.1 <u>was</u> the lowest obtained.

Two other outliers should be mentioned. The first is the experimental group student who received a score of 46 on the problem solving posttest and yet also received the highest heuristic sorting score of all students involved in the study, a 9.3. The second is a control group student who scored 33 on the problem solving posttest and received a heuristic sorting score of 9.0, the second highest heuristic sorting score found. These students present evidence that the ability to recognize heuristic cues is not sufficient to insure effective problem solving.

The regression analysis indicated that a tendency to recognize heuristic cues is useful in terms of improving problem solving performance. On the other hand, there was also evidence that at least for the types of problem solving tasks used in this study, it is not necessary to attend carefully to such cues in order to solve such problems correctly.

This study additionally considered the question as to whether the tendency to attend to heuristic cues also was related to ability profile type. The results of a two-factor profile group by treatment analysis of variance presented in Chapter IV indicated that both treatment group and ability profile group were significant factors in determining the attention paid to heuristic cues ($p < .001$ and $p < .01$, respectively). Further analysis revealed that profile group 2 students (those scoring low on factor 2, semantic-divergent thinking) recognized heuristic cues to a significantly lesser degree than students in profile groups 1 and 3. This result is interesting since profile group 2 was also the ability group that showed the least amount of benefit from heuristic instruction (i.e., control and experimental groups performed the same across all three subtests of the problem solving posttest).

As noted above the analysis of variance uncovered a significant treatment effect. The complete link clustering technique employed in this study provided insight into the dominant clustering schemes employed by experimental and control students. The signficiant treatment effect indicated that experimental group students recognize heuristic cues more than control group students, even though their dominant sorting schemes are somewhat alike. Recall that the major difference between the sorting schemes of the two groups was the clustering of items 1 and 3 by the experimental group. These items are both solved by pattern generation (or finite differences). Recall also that items of this type comprised subtest 3, the subtest on which the experimental group performed the best. Thus recognition of an heuristic cue by experimental group students is coupled with superior performance on items requiring that heuristic.

Secondary Findings

The secondary findings of this study involve comparisons between experimental and control subjects with respect to posttest problem solving performance and algebra and trigonometry performance. Once it was established that no pretesting effect existed, an analysis of covariance revealed that after taking into account pretest scores, students enrolled in the course in heuristic problem solving significantly outperformed control students on the problem solving posttest ($p < .01$). In a second analysis the problem solving posttest was subdivided into three subtests corresponding to the three heuristic strategies covered in the problem solving course (algebraic symbolism, contradiction, and pattern generation). This treatment by subtest analysis uncovered a significant subtest effect ($p < .01$) as well as a significant treatment by subtest interaction ($p < .01$). It was found that students in the control group performed the same across all three subtests whereas the students in the experimental group performed better on subtests 1 and 3 than they did on subtest 2. It is interesting to note that the control group, who received ten weeks of instruction in algebra and trigonometry, performed the same on subtest 1 as on subtests 2 and 3, even though subtest 1 contained only those problems requiring the heuristic of setting up and solving an algebraic equation. This result seems to indicate that time spent learning the mechanics of algebra and trigonometry does not necessarily transfer to similar problem solving situations. In fact, the control group did significantly better than the experimental group on the algebra and trigonometry posttest ($p < .001$). Yet in spite of this superior performance on basic skills, the control group was apparently not able to transfer this information to a problem solving situation requiring these same skills. Further, with respect to the experimental group, we find students performing significantly better on subtests 1 and 3 than on subtest 2. The highest mean score (80.1) was on subtest 3. The items on subtest 3 all required the searching for a pattern heuristic. When this particular heuristic was discussed in class (and in the text available to the experimental group), two methods for solving the given problems were presented. The first method followed the procedures of Polya (1957), namely, looking for some ingenious way of rewriting the given sequence or series so as to make the pattern obvious and then using these ingenious devices over again when new problems were encountered. Students found this method extremely challenging, perhaps because of the special insights necessary to solve some of the more difficult problems. The second method presented to the experimental group was the method of finite differences. This is a rather mechanistic procedure whereby a table of values is constructed and differences taken until these differences result in a string of constants. The number of differences that must be taken determines the degree of the polynomial relationship that describes the pattern and the problem reduces to solving a system of linear equations. Students much preferred the method of finite differences and once a review of the algebra necessary to solve a system of linear equations was given, they began to use finite differences with confidence. As a result, the superior performance on subtest 3 probably reflects a mastery of the method of finite differences and of solving systems of linear equations rather than mastery of the heuristics of Polya.

The second highest mean subtest score (70.2) for the experimental group was on subtest 1. Since four weeks were devoted to problems of the type encountered on subtest 1, it was possible to cover a wide variety of such problems in class. Even though no problem on the problem solving posttest was either covered in class, given on an assignment, or appeared as a test item on an examination taken during the quarter, it is fair to say that items "similar" to those on the posttest were encountered by the experimental group at sometime during the course of instruction. By "similar" it is meant that problems using the Pythagorean Theorem, or dealing with radioactive decay, compound interest, or using the laws of sines and cosines were encountered. Hence superior performance on subtest 1 could be interpreted as an ability to recognize and correctly apply well studied algorithms for solving such problems. Since a wide variety of algorithms are useful to solve the items on subtest 1, whereas only one algorithm (finite differences) is needed for subtest 3, this could explain the higher mean score on subtest 3.

Finally, students in the problem solving group performed least well on subtest 2 with a mean score of only 54.4. These items all required the method of contradiction for their solution and even though posttest items resembled items presented both in class and on assignments, these items differed from items on subtests 1 and 3 in that they did not readily lend themselves to any algorithmic solution. Of the three subtests, subtest 2 required the most transfer. No algorithmic procedures for solving these problems were presented in class or in the text, rather heuristic suggestions were given as to how to organize a list or table of possible alternatives and then systematically begin to eliminate those alternatives that led to a contradiction. Superior performance on subtest 2 could be interpreted as the ability to apply the contradiction heuristic.

The findings presented here seem to support those of Smith (1973) regarding the effects of task specific versus general heuristic instruction in that experimental group students performed best on test items for which they had been given a specific algorithm (subtest 3) and least well on test items requiring the use of the contradiction heuristic (subtest 2). These findings appear to cast doubt as to whether general heuristics have the broad transfer potential some mathematicians claim they have.

Another of the secondary goals of this study was to test the hypothesis that experience at solving problems requiring certain algebraic skills would produce the same or perhaps even superior performance in these skills as would concentrated instruction in algebra and trigonometry. Comparison of the scores on the algebra and trigonometry posttest revealed that control students significantly outperformed experimental students ($p < .001$) on basic skills. So whereas control students had difficulty transferring algebraic skills to a problem situation, experimental students were less adept at solving straight forward algebraic and trigonometric relationships outside of a problem solving situation. These results can be interpreted in terms of the review of the research comparing deductive and inductive instruction provided by Mayer (1974). Those procedures, such as the solving of standard algebra and trigonometry equations, that have simple solution rules appear to be best taught by deductive methods (rule followed by example). Control group students were taught the precise algorithm needed to solve each of the problems on the algebra and trigonometry posttest and spent more time practicing how to solve such problems than the experimental group. Therefore, if the goal of instruction is limited to having students learn specific algebraic procedures, then instruction in the methods and algorithms necessary to perform these procedures results in superior learning over instruction in how to apply a general class of such procedures. The corresponding conclusion to be drawn from the analysis of the problem solving posttest data is that when the goal of instruction is the teaching of specific problem types providing instruction in the algorithms necessary to solve such problems results in superior learning compared to instruction in algebraic procedures.

CHAPTER VI

FUTURE RESEARCH AND PRACTICE

Summary

The purposes of this study were (1) to test for changes in problem solving performance following an instructional sequence in heuristic problem solving. (2) to look for changes in sorting scheme usage following this heuristic instruction, (3) to investigate what relationship, if any, exists between problem sorting scheme usage and problem solving performance, (4) to compare the performance on basic algebra and trigonometry skills of control students who followed an instructional sequence in algebra and trigonometry and experimental students who followed an instructional sequence in heuristic problem solving, (5) to look for relationships between student ability profiles and problem solving performance, and (6) to check for connections between student ability profiles and problem sorting schemes.

Eighty four freshmen students enrolled in a college of pharmacy in New England served as subjects for this study. These students were divided randomly into experimental (n = 37) and control (n = 47) groups and each participated in a ten week course of instruction. The control group took a course in college algebra while the experimental group received instruction in heuristic problem solving.

Prior to instruction all students took five tests of mathematical ability: Hidden Figures, Scrambled Words, Nonsense Syllogism, Deciphering Languages and Toothpicks. A Solomon four group design was utilized whereby approximately half of the experimental and control groups took a specially designed problem solving pretest and accompanying problem similarity questionnaire. At the conclusion of the study all subjects responded to posttests in algebra and trigonometry, problem solving, and problem similarity questionnaire.

Data analyses revealed that whereas control students outperformed experimental students on the algebra and trigonometry posttests, experimental students performed significantly better on the problem solving posttest. Results indicated that heuristic instruction in problem solving did not greatly alter the dominant problem sorting scheme of experimental students although students receiving heuristic instruction did show evidence of being more attentive to heuristic cues than control students. Additionally it was shown that recognition of heuristic cues was correlated significantly with superior performance in problem solving. However, several individual students provided evidence that it was possible to achieve high scores on the problem solving test without sorting problems heuristically. Four ability profile groups were specified using Word's hierarchical clustering analysis and the degree to which a student sorted heuristically was found to be related to the student's ability profile. Students scoring low on a semantic-divergent thinking factor were shown to have lower heuristic sorting scores than students scoring significantly higher in this factor. A major finding of this study was an aptitude treatment interaction suggesting that heuristic instruction is more beneficial for certain ability profile types. The problem solving posttest was subdivided into 3 subtests corresponding to the 3 heuristic strategies taught in the experimental course (algebraic symbolism, contradiction, pattern generation). Analyses revealed that significant differences in performance across the 3 subtests existed for only one of the ability profile types described in the study.

Recommendations For Future Research

Recommendations for future research include further exploration of the relationship between ability profiles and heuristic instruction as well as the related question of other personality correlates of problem solving performance such as cognitive style, anxiety and attitude towards problem solving. Sorting scheme research was also a major concern of this investigation and suggestions for additional studies along these lines are also given. Finally, although many of the instructional variables were held fixed in this study, more closely controlled investigations could be designed that would permit a more thorough analysis of which aspects of instruction promote superior problem solving performance.

Attention should be paid to the aptitude treatment interaction uncovered in this study. Future investigations could, by isolating more orthogonal ability factors, look at more detailed ability profile types and their relationships to increased problem solving performance following a course in heuristic problem solving. In addition to expanding the battery of aptitude tests administered, varying the heuristics taught in the problem solving course should be considered, too. In this study students in the problem solving course showed the greatest increases in performance on those problems that could be solved algorithmically. Future studies then could investigate the relationship between profile types and algorithmic versus nonalgorithmic problem solving to determine if there is an "algorithmic mode of thought." That is, do certain ability profile type subjects tend to think and approach problems best algorithmically whereas others approach problems in a nonalgorithmic manner. The pattern generation heuristic was taught both algorithmically and nonalgorithmically in this study. A future study could develop and compare two instructional procedures, a Polya procedure which treats problems that call for the uncovering of a pattern in an insightful, nonalgorithmic manner, and a finite differences procedure that covers these same problems algorithmically. Along the lines of algorithmic versus nonalgorithmic problem solving, is the suggestion that studies be conducted on students at an earlier age to test for "algorithmic familiarity." The ATI uncovered here suggests that there may be an "algorithmic state of mind." Students in this study were all college age and it would be interesting to know whether younger students (or which profile type younger students) do best on algorithmically solved problems. It would be desirable to conduct such a study on students with little or no prior experience with algorithmic problem solving since the purpose of such a study would be to determine whether or not algorithms are somehow "natural" ways of approaching problems or if instead they result from the curriculum. Since it is unlikely that a group of students who had not been exposed to algorithmic procedures could be found, this study would have to include some sort of pretest of algorithmic familiarity. If no strong evidence existed to support the hypothesis that most students have a natural tendency to solve problems algorithmically then it might be wise for educators to provide training in nonalgorithmic heuristics at an early age in an attempt to promote transfer.

In addition to investigating the impact of ability factors on performance following heuristic instruction, other individual difference factors such as attitude towards problem solving, math anxiety and cognitive style should be studied. Research questions include the following. Which "attitude type" students benefit most from heuristic instruction? Does heuristic instruction improve a student's attitude towards problem solving? Does heuristic instruction work best for low or high math anxious students? Does heuristic instruction reduce or increase math anxiety? Which cognitive styles respond best to heuristic instruction? Which cognitive styles are most algorithmic in their approach to problem solving and which are most flexible? Hence measures of attitude towards problem solving, math anxiety, and cognitive style would provide further insight into the impact of heuristic instruction and its usefulness for certain students.

With respect to the sorting scheme aspect of this investigation, two rather negative results were uncovered. A low correlation between heuristic sorting scores and problem solving performance and few differences in the hierarchical clusterings of the experimental and control groups gave evidence that the perceived notions of problem similarity are quite rigid and that it is possible to perform well in problem solving tasks without careful attention to heuristic cues. These results are somewhat in disagreement with those reported by Silver (1977) who found that students tended to sort more on the basis of structure and less on the basis of context, pseudostructure, and question posed after they had seen the problem solution.

Recall, however, that the subjects in Silver's study were eighth grade students. These results may indicate that notions of problem similarity are more flexible in younger students and that the curriculum has much to do with shaping and fixating similarity notions. Further studies using subjects of varying age levels would help clarify the role of the curriculum in shaping a student's perception of problem similarity. This study also uncovered a significant relationship between heuristic sorting score and ability profile type. Suggestions for future research along these lines include exploring the relationship between cognitive style and heuristic sorting as well as the relationship between more detailed ability profile types and heuristic sorting.

Finally research on the impact of heuristic instruction might include a closer examination of the instructional variables in an attempt to discover exactly what features of the course in heuristic problem solving caused superior performance. Some of the instructional variables that could be investigated are the text, the quantity of assigned problems, the number of tests, the amount of explicit directions, the number of questions asked, the amount of criticism, or the use of student ideas. Specifically, it would be worthwhile to investigate what ability types or cognitive styles learn best from deductive as opposed to inductive instruction. Also of interest would be a determination of which type of instructional materials are best taught by deductive versus inductive instruction.

Implications for Educational Practice

The findings of this study which offer the most in terms of suggestions for educational practice are those relating to the aptitude treatment interaction and the stability of perceived problem similarities. The aptitude treatment interaction focuses attention on the current approach to teaching problem solving which is almost totally algorithmic and as such neglects those ability type students who could benefit from heuristic or nonalgorithmic instruction. The problem of sticking to one method of teaching problem solving is that it inhibits the development of other skills necessary for superior problem solving. Also the relative stability of problem similarity schemes suggests that the curriculum has crystallized notions of problem similarity. These results taken together seem to suggest a much more varied approach to teaching problem solving. In the early years (grades K-6) especially this would involve exposing students to a multitude of problem situations calling for the use of many different heuristics. Another suggestion would be to incorporate into the curriculum the notion of "problem posing." Walter and Brown (1977) describe a "what if not?" strategy that encourages students to choose an attribute of a theorem or phenomenon, vary the attribute, pose a problem about the varied attribute and then try to solve the problem. They see this "what if not?" strategy as a means of interrelating problem solving and problem posing and maintain that "generating questions is as important as answering them."

In addition to not being taught how to pose questions, Geeslin (1977) expresses concern over the inability of students to talk or write about mathematical concepts. In a study involving students of all ages and levels of ability who were asked to write about mathematics, and explain the relationship between two concepts, Geeslin observed that

> When asked to explain how two concepts are related or to write a sentence containing both words, students almost never write the definition or any mathematically correct statement. Based on past experience and observation of mathematics classes, it seems reasonable that this poor performance is partly due to the small amount of experience that students have in writing about mathematics; primarily, they are asked to "get the right answer" or "prove the theorem"...as students become more precise in their mathematical ideas, performance on more traditional tasks should imporve.... Can we expect students to apply mathematics if they cannot deal with basic mathematical concepts?

In summary, the problem solving curriculum should be modified to include instruction in a variety of algorithmic and nonalgorithmic heuristics, should focus not only on problem solving but also on problem posing and should include opportunities for students to discuss and write about mathematical concepts.

This study was peculiar in the respect that the investigator was also the instructor for both the experimental and control courses. A few personal observations also might be relevant for educators interested in problem solving instruction. Recall that the study design called for a cross over whereby both groups received each of the two instructional sequences, either in the fall or winter quarter. As a result, the investigator had an opportunity to observe both groups of students as they progressed through the course in heuristic problem solving. In each case students seemed to be most interested and participate most in class when the topic was problems solved by contradiction. This is interesting since performance on problems of this type was poorer than on problems using other heuristics. So it seems that even though students had more difficulty solving these nonalgorithmic problems, their enthusiasm was sparked perhaps because of their nontraditional nature. An alternative explanation would be that most of these problems are very easily posed (alphametrics, magic squares, logic problems) and as such appear quite unthreatening. Even though the solution to a particular problem might be difficult, since the students had not had experience with such problems before they were unaware of the complexity of the problem and tackled it without fear. Also the problems that students seemed to solve with the most confidence (although the enthusiasm for these problems was less than for contradiction problems) were those using the method of finite differences. This was a new algorithm for all students and perhaps because they were familiar with an algorithmic approach to problem solving they felt comfortable applying this not-too-complicated method. Finally, students seemed least enthusiastic and confident with traditional algebra word problems. It could be that such problems evoked memories of previous failure or that they lacked interest because they presented no new challenges. It would appear that students at the college level show most interest and are most comfortable with problems that are somewhat unfamiliar to them or that require the use of new algorithms.

One of the most successful features of the problem solving course were the four required problem assignments. Students were instructed to give complete detailed solutions to all assigned problems and their grade reflected not only the correctness of their final asnwer but also the clarity of the solution. This investigator's observations and recommendations follow those of Geeslin in that forcing students to write out complete solutions fosters awareness of their own mental processes. It appeared not to be so much the quantity of assigned problems that made for superior problem solving but rather the insistance that students think about what they are doing that made a difference. In fact the most useful aspect of Polya's heuristic advice in terms of instructional methods was the suggestion that teachers help their students to reflect on and evaluate their approaches to problem solving. This investigator came away from the present study feeling that her students had learnt more than how to solve a special class or classes of problems but that problem solving had become less magical and more intuitive and well reasoned.

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APPENDIX A

INSTRUMENTATION

PROBLEM SIMILARITY QUESTIONNAIRE

Name:____________

I.D.____________

1. EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR
2. EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR
3. EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR
4. EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR
5. EXTREMELY DISSIMILAR	MODERATLEY SIMILAR	EXTREMELY SIMILAR
6. EXTREMELY DISSIMILAR	MODERATLEY SIMILAR	EXTREMELY SIMILAR
7. EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR
8. EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR
9. EXTEMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR
10. EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR
11. EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR
12. EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR

13.
EXTREMLEY DISSIMILAR — MODERATELY SIMILAR — EXTREMELY SIMILAR

14.
EXTREMELY DISSIMILAR — MODERATELY SIMILAR — EXTREMELY SIMILAR

15.
EXTREMELY DISSIMILAR — MODERATELY SIMILAR — EXTREMELY SIMILAR

16.
EXTREMELY DISSIMILAR — MODERATELY SIMILAR — EXTREMELY SIMILAR

17.
EXTREMELY DISSIMILAR — MODERATELY SIMILAR — EXTREMELY SIMILAR

18.
EXTREMELY DISSIMILAR — MODERATELY SIMILAR — EXTREMELY SIMILAR

19.
EXTREMELY DISSIMILAR — MODERATELY SIMILAR — EXTREMELY SIMILAR

20.
EXTREMELY DISSIMILAR — MODERATELY SIMILAR — EXTREMELY SIMILAR

21.
EXTREMELY DISSIMILAR — MODERATELY SIMILAR — EXTREMELY SIMILAR

22.
EXTREMELY DISSIMILAR — MODERATELY SIMILAR — EXTREMELY SIMILAR

23.
EXTREMELY DISSIMILAR — MODERATELY SIMILAR — EXTREMELY SIMILAR

24.
EXTREMELY DISSIMILAR — MODERATELY SIMILAR — EXTREMELY SIMILAR

25.
EXTREMELY DISSIMILAR — MODERATELY SIMILAR — EXTREMELY SIMILAR

26. ____

EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR

27. ____

EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR

28. ____

EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR

29. ____

EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR

30. ____

EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR

31. ____

EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR

32. ____

EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR

33. ____

EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR

34. ____

EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR

35. ____

EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR

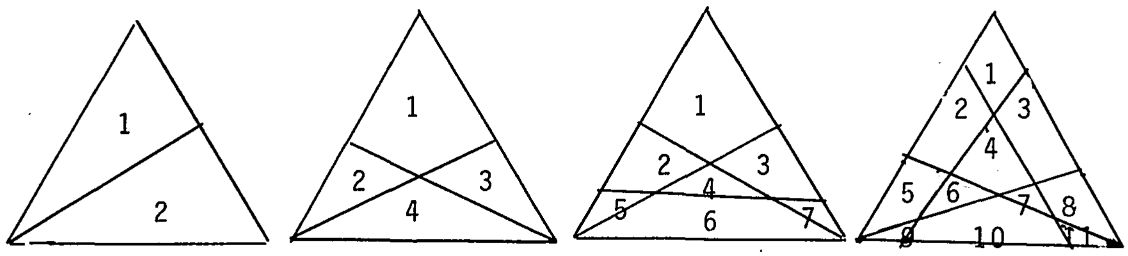
36. ____

EXTREMELY DISSIMILAR	MODERATELY SIMILAR	EXTREMELY SIMILAR

Final Examination

Part 1

1) Observe that 1 (one) line divides a triangle into at most 2 regions; 2 (two) lines divide a triangle into at most 4 regions; 3 (three) lines divide a triangle into at most 7 regions; 4 (four) lines divide a triangle into at most 11 regions. What is the maximum number of regions into which 10 (ten) lines divide a triangle?



a) 44 b) 51 c) 56 d) 67 e) 72

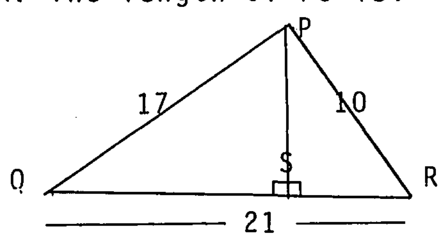
2) Ten pounds of a salt water solution is 20% salt. How much water must be evaporated to strengthen it to a 25% solution?

a) .5 lb. b) 1 lb. c) 1.5 lb. d) 2 lb. e) 2.5 lb.

3) Find the sum: $1 + 5 + 9 + \ldots\ldots\ldots + 197 = ?$

a) 785 b) 4802 c) 4950 d) 6841 e) 77421

4) In triangle PQR, the sides are 10, 17 and 21 and PS is perpendicular to QR. The length of PS is:

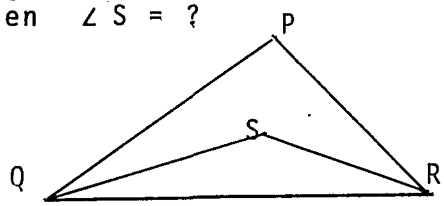


a) $6\sqrt{3}$ b) 6.4 c) $6\sqrt{2}$ d) $\sqrt{63}$ e) 8

5) To number the pages of a bulky volume, the printer used 1890 digits. How many pages has the volume?

a) 598 b) 623 c) 666 d) 689 e) 702

6) In triangle PQR, $\angle P = 70^0$, QS bisects $\angle$ Q and RS bisects $\angle$ R. Then $\angle S = ?$



a) 110^0 b) 115^0 c) 120^0 d) 125^0 e) 130^0

7) Not uncommon are athletic events in which each team plays the other exactly once. How many games would be necessary if there were 20 teams?

a) 150 b) 160 c) 170 d) 180 e) 190

8) An arthmetic progression is a sequence of numbers in which each number is gotten from the previous number by adding a constant. For example:

1, 3, 5, 7, 9 and

1, 3/2, 2, 5/2

are arithmetic progressions since the first is gotten by adding 2 to each term and the second is gotten by adding 1/2 to each term.

A particular arithmetic progression has 5 terms. The sum of all five terms is 100; the sum of the three largest terms is seven times the sum of the two smallest terms. What is one term of this sequence?

a) 4/5 b) 5/4 c) 3/4 d) 4/3 e) 5/3

9) A 3 by 3 magic square is an arrangement of the digits 1 to 9 in the form of a square so that the sum of the numbers in each row, column and diagonal is exactly the same. The following is an example of such a square:

8	1	6
3	5	7
4	9	2

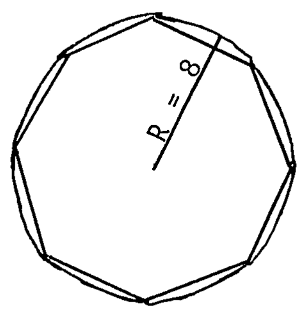
Complete the following square so as to make it a magic square.

?→⊙	,	.
9	,	.
.	.	8

Then the entry in the first row and first column (i.e. the circled entry) is?

a) 1   b) 2   c) 3   d) 4   e) 5

10. A regular octagon is inscribed in a circle of radius 8. What is the perimeter of the octagon?



(a) 49
(b) 51
(c) 53
(d) 55
(e) 57

11. Replace the *'s in the following multiplication problem so as to make a valid multiplication.

```
      *  *  5
         4  *
   -----------
      3  *  *
   * 2 *  *
   -----------
   1 * *  *  *
```

The last line should read:

a) 13840   b) 13965   c) 12845   d) 12970   e) 12915

12. A ladder 100 ft. long is leaning against a building so that it reaches a window ledge 80 ft. high. How many feet must the ladder be moved away from the building so that the top of the ladder will reach a ledge of 8 ft. lower down?

a) 6.2 ft. b) 7.4 ft. c) 8.6 ft. d) 9.4 ft. e) 10.5 ft.

13. A gasoline distributor has two pumps. The main pump can fill the tank of a delivery truck in 30 minutes, whereas the second, a smaller auxiliary unit, requires 45 minutes to fill the same truck. How long would it take if both pumps were used simultaneously?

    a) 16 min. b) 17 min. c) 18 min. d) 19 min.

    e) 20 min.

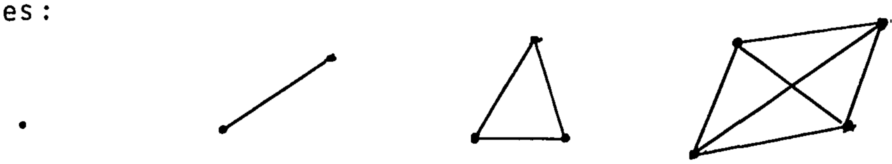
14. In the following letter arithmetic problem each letter stands for a different digit from 0 to 9. Replace the letters so as to make a proper addition:

$$\begin{array}{rcccc} & d & c & d & b \\ + & d & a & b & c \\ \hline & a & b & c & b \end{array}$$

    The letter "a" stands for:

    a) 4 b) 5 c) 6 d) 7 e) 8

15. The problem is to find the number of line segments connecting a given number of points. Consider the following cases:



1 point	2 points	3 points	4 points
no line	1 line	3 lines	6 lines

    How many lines could be drawn connecting 12 points?

    a) 66 b) 68 c) 70 d) 72 e) 74

16. "We've all remarked on it many times before," observed Mr. Bankes, "but it still gets me. Where else in the world would you find a lawyer, author, dentist and banker at one table, and bearing the names of Law, Penn, Banks and Tooth?"

    "It's worth putting in Ripley," replied the dentist, whose surname corresponded to Mr. Penn's profession, "especially as our names do not agree with our respective occupations."

Which of the following statements is true?

a) Mr. Law is a banker

b) Mr. Law is a dentist

c) Mr. Bankes is a lawyer

d) Mr. Bankes is an author

e) Mr. Penn is a dentist

17. How long (to the nearest year) will it take for a sum of money to double if invested at 5% compounded quarterly?

a) 12 yrs. b) 14 yrs. c) 16 yrs. d) 18 yrs.

e) 20 yrs.

18. Three starters lined up for the race. Mike, who was known to pick winners in the past, stated with confidence, "Fay's Folly must win." Steve wasn't so sure. "Anyway she won't finish second." Stan had been studying his card and trying to decide between Kimono and Satan.. "Satan will be either first or second," he declared.

Only one of the 3 friends had been a true prophet. What were the final placings?

a) Fay's Folly is first; Satan is second; Kimono is third.
b) Fay's Folly is first; Satan is third; Kimono is second.
c) Fay's Folly is second; Satan is first; Kimono is third.
d) Fay's Folly is second; Satan is third; Kimono is first.
e) Fay's Folly is third; Satan is second; Kimono is first.

19. What is the 49th term in the following sequence?

3, 7, 13, 21, 31,............

a) 2451 b) 2453 c) 2455 d) 2457 e) 2459

20. Radioactive Stronium 90 has a half life of 3,385 years. How long (to the nearest year) will it take for Stronium 90 to decompose so that only 1 percent of the original mass remains?

a) 22 yrs. b) 24 yrs. c) 26 yrs. d) 28 yrs.

e) 30 yrs.

21. If n represents the number of points on one side of a pentagonal lattice and L = the number of non-overlapping line segments joining points in the network, then find the number of non-overlapping line segments in a lattice with 10 points on one side. Use the following diagrams to get started.

$n = 1$	$n = 2$	$n = 3$	$n = 4$
$L = 0$	$L = 5$	$L = 13$	$L = 24$

a) 162 b) 164 c) 166 d) 168 e) 170

22. Two points A and B on one bank of a river are 95 ft. apart. A point C across the river is located so that angle CAB is 75° and angle CBA is 80°. How far is C from A?

a) 191 ft. b) 203 ft. c) 214 ft. d) 221 ft.

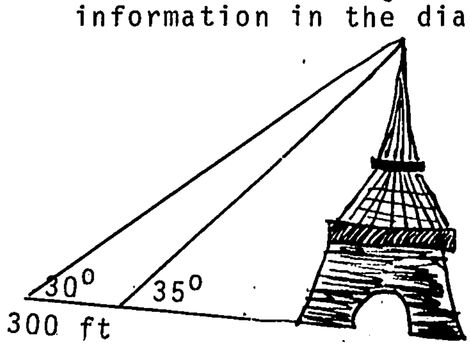
e) 237 ft.

23. A canoeist paddles at a constant rate. He finds it takes him 2 hours longer to make a 12 mile trip upstream than it does downstream. If the current is 3 mph, how fast is the canoeist paddling?

a) 3 mph b) 4 mph c) 5 mph d) 6 mph

e) 7 mph

24. Determine the height of the Eiffel Tower given the information in the diagram below.



Find h:

a) 921 ft.

b) 997 ft.

c) 1022 ft.

d) 1051 ft.

e) 1129 ft.

Final Examination

Part 2

1. $(2\sqrt{3} + 2)(\sqrt{3} - 2\sqrt{2}) = ?$

a) $2 - 3\sqrt{6}$ b) $6 + 5\sqrt{6}$ c) $2\sqrt{3} - 3\sqrt{2}$

d) $-\sqrt{6}$ e) $3 - 2\sqrt{6}$

2. Simplify: $\sqrt{x\sqrt{x^6}}$

a) x^8 b) $x^{3/2}$ c) x^3 d) x^2 e) x

3. Simplify: $3\sqrt{2} - \dfrac{4}{\sqrt{2}}$

a) $\sqrt{2}$ b) $-\dfrac{\sqrt{2}}{2}$ c) $\dfrac{7\sqrt{2}}{2}$ d) $\dfrac{-1}{\sqrt{2}}$ e) -1

4. Combine and simplify: $5\sqrt{75} + 7\sqrt{108} - 6\sqrt{245}$

a) $12\sqrt{102}$ b) $6\sqrt{62}$ c) $25\sqrt{2}$ d) 25

e) $67\sqrt{3} - 42\sqrt{5}$

5. If $7^{-x} = 10$, then 7^{2x} is equal to

a) 1/100 b) 1/20 c) 30 d) 100 e) 50

6. $(x - x^{-1})^{-1} = ?$

a) $x + \dfrac{1}{x}$ b) $\dfrac{x^2-1}{x}$ c) $\dfrac{x}{x^2-1}$ d) $\dfrac{x}{x+1}$

e) $\dfrac{x}{x-1}$

7. If $4^x = \sqrt{2^{3y}}$ then,

a) $x = \frac{3y}{4}$ b) $x = 3y$ c) $y = \frac{3x}{4}$

d) $x = \frac{1}{3}y$ e) $y = \frac{2}{3}x$

8. The roots of the equatio $\sqrt{12 + x} = x$ are

a) - 3 and 4 b) 4 only c) -3 only

d) 3 and -4 e) 3 only

9. For what values of K will the equation $x^2 + 4x - 2K = 0$ not have any real solutions?

a) $K < -2$ b) $K < 0$ c) $K < -2$ d) $-5 < K < -2$

e) $-2 < K < 4$

10. Solve for x: $\frac{x}{x - 1} - \frac{x}{x + 1} = \frac{4}{3}$

a) $x = 2$ and $-\frac{1}{2}$ b) $x = -2$ and $\frac{1}{2}$ c) $x = 2$ and -2

d) $x = -\frac{1}{2}$ and $\frac{1}{2}$ e) $x = 1$ and $\frac{1}{2}$

11. $\frac{2x}{x^2 + x - 2} - \frac{2}{x - 1} = ?$

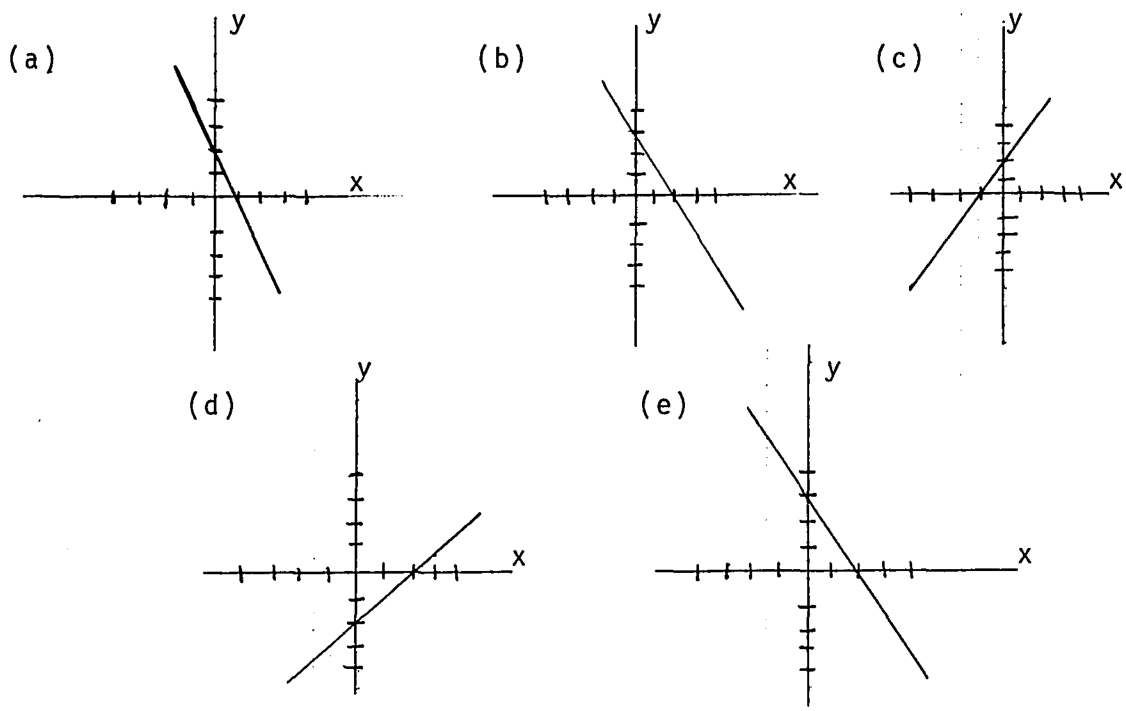
a) $\frac{2x - 2}{x^2 + 2x - 3}$ b) $\frac{4x - 4}{x^2 + x - 2}$ c) $\frac{-4}{x^2 + x - 2}$

d) $\frac{4}{x^2 + x - 2}$ e) $\frac{-4x^2}{x^2 + 2x - 3}$

12. $\left(\frac{1}{x} + \frac{1}{y}\right)\left(\frac{x}{x + y}\right) = ?$

a) $1/y$ b) $1/x$ c) x/y d) y/x e) x^2/y

13. The graph of $2y - 3x = 4$ is:



14. If log 7 = p and log 5 = q, then log 175 = ?

a) p + 2q   b) p 2q   c) 2p + q   d) 2p q
e) p + q

15. $\log_3 \frac{1}{81} = ?$

a) 27   b) -4   c) 1/27   d) 4   e) -3

16. Solve for x: $3 \log_7 x + \log_7 1 - \log_{13} 13 = 2$

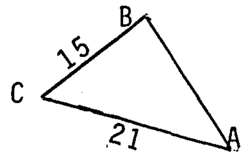
a) -7 and 7   b) 7   c) 3   d) 3 and -3   e) 1

17. Solve for x: $2^{3x} = 90$

a) $\frac{\log 90}{3 \log 2}$ b) $\frac{3 \log 90}{\log 2}$ c) $\frac{3 \log 2}{\log 90}$ d) $\frac{\log 2}{3 \log 90}$

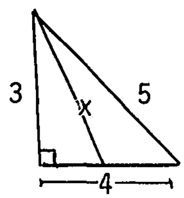
e) $\frac{\log 90}{\log 6}$

18. In triangle ABC, $\angle A = 37^o$. Find $\angle B$



a) 21^o b) 31^o c) 44^o d) 53^o e) 57^o

19. In a 3-4-5 right triangle, an angle bisector is drawn to the longer leg. Find its length.



Find x

a) 9/2 b) $3/2\sqrt{5}$ c) 15/4 d) $3\sqrt{3}$

e) cannot be determined from information given.

20. Two angles of a triangle are 30^o and 135^o. The ratio of the longest side to the side opposite the 30^o angle is:

a) 2:1 b) $\sqrt{3}:1$ c) $\sqrt{2}:1$ d) $\sqrt{3}:2$ e) $\sqrt{2}:2$

21. The sides of a triangle are 3,5 and 7. The value of the cosine of the smallest angle of the triangle is:

a) 5/7 b) 13/14 c) 11/13 d) 8/15 e) 12/13

22. If cos x = m and x is an acute angle, then tan x = ?

a) $\dfrac{\sqrt{1+m^2}}{m}$ b) $\dfrac{m}{\sqrt{1-m^2}}$ c) $\dfrac{\sqrt{1+m^2}}{m}$

d) $\dfrac{m}{\sqrt{1+m^2}}$ e) $\dfrac{\sqrt{1-m^2}}{m}$

23. Solve for y:

$$\begin{aligned} x + y + 2Z &= 3 \\ 2x - y + 4Z &= 0 \\ x \qquad + 3Z &= 2 \end{aligned}$$

a) 0 b) 1 c) 2 d) 3 e) 4

24. The value of K for which the system

$$\begin{aligned} Kx + 3y &= 7 \\ 2x - 5y &= 3 \end{aligned}$$

has no solution is:

a) 2 b) 5 c) 6/5 d) -6/5 e) -3/5

Appendix B

The following pages represent the raw data used in the study. Below is a description of the items in each column.

Column Number:

1. Treatment group: 0 = Control, 1 = Experimental
2. Score on Scrambled Words Test
3. Score on Hidden Figures Test
4. Score on Deciphering Languages Test
5. Score on Nonsense Syllogism Test
6. Score on Toothpicks Test
7. Problem solving pretest score (if the subject took the pretest)
8. Score on the problem solving posttest (items 1-24)
9. Score on the problem solving posttest (items 1-9 only)
10. Score on the algebra posttest
11. Score on subtest 1 of the problem solving posttest
12. Score on subtest 2 of the problem solving posttest
13. Score on subtest 3 of the problem solving posttest
14. Heuristic sorting score on the posttest

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
0	50	12.8	21.6	3	1	33	46	44	33	5	4	2	-4.7
0	41	3.2	8.0	-6	4	0	75	78	58	9	4	5	-3.5
0	49	7.4	12.4	4	13		33	33	46	4	1	3	0.5
0	46	5.6	7.2	8	4	11	63	56	54	10	3	2	-2.2
0	50	3.4	9.4	5	15		50	44	67	7	3	2	-2.0
0	35	5.8	13.8	4	5		33	33	50	3	1	3	9.0
0	45	2.4	9.0	4	7		42	44	58	3	2	4	-1.3
0	45	6.8	6.6	8	6	22	46	44	71	5	2	3	-3.7
0	45	20.0	6.6	8	14		54	56	71	7	4	2	0.6
0	48	10.8	8.6	6	7		67	67	75	9	2	5	-4.5
0	48	10.2	10.8	4	2		58	67	58	5	4	5	4.4
0	44	6.4	7.2	-3	5	44	46	67	67	4	4	3	-1.1
0	45	14.8	8.0	11	2		46	56	29	5	4	2	-0.8
0	50	11.8	23.0	0	17	44	63	44	88	8	3	3	-3.2
0	50	16.4	14.4	2	9	44	58	56	71	8	4	2	1.9
0	48	16.8	22.8	18	4		63	56	75	6	5	4	-5.2
0	47	16.6	7.0	3	11		58	67	54	6	5	3	-1.4
0	50	3.4	6.6	7	3		17	11	54	3	1	0	-3.0
0	46	7.0	5.4	6	8	33	21	22	58	1	2	2	-0.1
0	47	0.8	9.6	-2	12	33	58	44	75	7	3	4	2.2
0	46	21.2	19.2	2	9		50	56	58	4	3	4	0.7
0	50	24.0	20.4	13	12		42	22	58	4	5	1	-5.3
0	50	8.0	6.8	-2	6		46	56	46	4	4	3	

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
0	40	1.6	6.0	6	5	33	42	44	58	5	2	3	-0.1
0	35	0.8	7.6	9	10		58	56	50	7	2	5	-1.8
0	46	8.6	7.6	4	4	11	29	22	79	5	0	1	-5.2
0	48	20.0	16.4	9	17	78	79	78	79	9	5	5	-1.4
0	42	5.8	13.4	10	3	56	46	56	63	3	4	4	-1.3
0	48	5.8	18.0	8	1		38	33	63	3	3	3	-2.3
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0	29	14.4	8.0	-6	7		50	56	25	7	1	4	-2.8
0	45	7.4	9.8	-6	8	33	46	44	58	3	3	5	0.8
0	44	8.6	16.8	6	6		67	67	67	9	5	2	-5.1
0	49	20.8	5.6	4	12		29	33	67	4	1	2	-1.0
0	44	6.6	10.4	2	10	56	17	22	33	1	3	0	-1.0
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0	44	13.0	12.2	1	0		29	22	54	3	2	1	3.0
0	47	26.0	21.8	0	9		50	56	79	8	2	1	-0.6
0	47	10.8	9.4	0	4		33	44	29	6	6	3	-3.9
0	40	14.8	8.4	22	9	33	63	44	50	6	4	4	3.7
0	37	13.0	4.2	4	12	22	58	44	50	10	6	3	-0.8
0	50	13.0	16.8	22	12	56	83	78	75	6	2	3	7.3
0	50	13.2	22.8	3	11		50	56	54	2	5	2	-6.0
0	39	22.0	16.8	8	13		38	37	56	5	1	4	-3.1
0	40	2.0	7.2	-4	2	44	42	33	71	7	3	4	-2.1

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
0	50	18.0	17.2	14	8	33	58	56	79	2	5	1	-3.9
1	49	24.2	24.0	22	20		83	100	83	8	6	6	4.8
1	22	-0.8	7.2	5	15		79	100	83	10	3	6	-0.1
1	50	11.0	21.6	16	4	11	75	89	58	9	4	5	-4.2
1	49	17.0	13.4	16	7	44	71	78	58	7	5	5	1.3
1	49	20.0	14.8	-5	16	11	71	67	79	7	4	6	3.1
1	39	19.4	15.6	11	15		75	67	67	9	4	5	-1.1
1	47	5.4	7.8	-6	7	22	75	78	33	9	3	6	-1.4
1	49	0.8	16.2	9	1	22	63	56	50	6	3	5	-1.9
1	40	3.6	7.2	-1	7		71	78	42	7	4	3	4.8
1	47	15.6	7.2	5	8		58	33	63	8	3	6	0.0
1	47	18.8	9.8	0	9		71	67	50	10	6	6	4.2
1	48	11.2	19.8	18	5		92	89	67	6	6	6	3.4
1	47	10.8	10.8	-3	5	44	75	78	13	6	4	1	0.9
1	45	1	11	8	4		46	56	46	8	3	6	9.3
1	50	12.8	7.6	-4	4	56	71	78	63	7	3	2	-0.6
1	47	0.8	7.6	6	2	33	50	56	33	8	4	5	-1.3
1	39	11.8	13.6	-1	3	22	71	67	50	11	4	4	-0.6
1	21	2.0	17.6	5	15		79	78	33	11	5	6	5.4
1	24	11.0	15.0	13	7		92	89	54	9	3	6	-2.2
1	38	18.0	12.8	-2	7		75	78	67	7	3	6	4.0
1	50	25.0	24.0	-9	15		67	67	54	11	2	6	0.7
1	50	13.8	6.6	3	3		79	89	79	10	3	3	-0.9

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
1	41	7.4	17.0	25	10	11	67	44	75	9	4	5	-2.2
1	50	26.6	13.0	6	11	33	75	67	25	8	3	5	5.5
1	36	8.8	13.4	7	5	33	67	56	46	8	2	5	-3.0
1	42	6.0	5.8	-3	9	22	63	67	25	6	4	4	0.0
1	48	12.6	24.0	5	11	33	58	42	33	8	3	5	-2.8
1	50	5.8	6.4	-5	4		67	78	29	8	5	5	-7.1
1	50	15.8	24.0	5	9		75	100	50	4	6	1	2.3
1	40	3.2	6.4	7	11	56	46	33	17	6	5	4	-4.8
1	50	15.8	22.8	14	15		63	56	33	7	5	6	2.6
1	46	11.8	24.0	-8	9	44	75	78	50	6	4	4	1.4
1	37	8.2	3.6	10	9		58	67	42	3	0	2	-3.0
1	47	9.0	7.0	6	9	22	83	89	25	9	5	6	3.2
1	50	18.6	14.6	11	14	56	46	44	63	2	3	6	1.9
1	44	6.8	12.2	4	10	11	71	56	38	8	4	5	1.8
1	25	1.0	8.0	0	7		21	33	46	9	3	5	2.8