A NEW AND PRECISE CONSTRUCTION OF THE LOCAL INTERSTELLAR ELECTRON SPECTRUM FROM THE RADIO BACKGROUND AND AN APPLICATION TO THE SOLAR MODULATION OF COSMIC RAYS SHOWING AN INCOMPATIBILITY OF THE ELECTRON AND NUCLEI MODULATION USING THE SPHERICALLY SYMMETRIC FOKKER-PLANCK EQUATION

JOHN MICHAEL ROCKSTROH

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SYMMETRIC FOKKER-PLANCK EQUATION

by

JOHN MICHAEL ROCKSTROH
B. Math., University of Minnesota, 1966

A THESIS

Submitted to the University of New Hampshire
In Partial Fulfillment of
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Graduate School
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September, 1977
This thesis has been examined and approved.

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__________________________
August 10, 1977
Date
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>SECTION</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>vii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>x</td>
</tr>
<tr>
<td>I.  INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. COMPILATION OF A DATA BASE CONTAINING THE RADIO FREQUENCY BACKGROUND AND THE COSMIC RAY ELECTRON FLUXES</td>
<td>5</td>
</tr>
<tr>
<td>1. Synchrotron Radiation Formulas</td>
<td>5</td>
</tr>
<tr>
<td>2. Intensity Of The Diffuse Radio Frequency Background</td>
<td>11</td>
</tr>
<tr>
<td>3. Cosmic-Ray Electron Spectra</td>
<td>17</td>
</tr>
<tr>
<td>III. A BRIEF SUMMARY OF SOLAR MODULATION</td>
<td>28</td>
</tr>
<tr>
<td>IV. DERIVATION OF THE LOCAL INTERSTELLAR ELECTRON SPECTRUM</td>
<td>41</td>
</tr>
<tr>
<td>1. Previous Estimates Of The Local Interstellar Electron Spectrum</td>
<td>43</td>
</tr>
<tr>
<td>2. Our calculation Of The Interstellar Electron Spectrum</td>
<td>50</td>
</tr>
<tr>
<td>A. Effect Of The Magnetic Field Distribution</td>
<td>51</td>
</tr>
<tr>
<td>B. The Local Emissivity As Deduced From H II Region Surveys And The Integration Path Length L</td>
<td>56</td>
</tr>
<tr>
<td>C. Our Construction Of The Interstellar Electron Spectrum</td>
<td>61</td>
</tr>
<tr>
<td>3. The Interstellar Electron Spectrum Deduced From The Positron Ratio</td>
<td>66</td>
</tr>
<tr>
<td>V. DETAILS OF THE SOLAR MODULATION PROBLEM</td>
<td>77</td>
</tr>
<tr>
<td>1. Selection Of The Diffusion Coefficient</td>
<td>77</td>
</tr>
<tr>
<td>2. The Electron Modulation</td>
<td>83</td>
</tr>
<tr>
<td>3. The Proton And Helium Nuclei Modulation</td>
<td>99</td>
</tr>
<tr>
<td>4. Discussion Of The Modulation Reluctance Of The Electrons</td>
<td>115</td>
</tr>
<tr>
<td>5. The Effect Of The Diffusion Coefficient On The Various Species</td>
<td>131</td>
</tr>
<tr>
<td>VI. DISCUSSION OF THE DIFFERENCES IN MAGNITUDE OF THE ELECTRON AND NUCLEI MODULATION</td>
<td>140</td>
</tr>
<tr>
<td>VII. UNIQUENESS OF THE DEPTH OF MODULATION</td>
<td>151</td>
</tr>
</tbody>
</table>
1. A Comment On The Interstellar Proton And Helium Spectra .......................... 152

VIII. SUMMARY................................................................. 154

BIBLIOGRAPHY ............................................................ 157
LIST OF TABLES

<table>
<thead>
<tr>
<th></th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Non-Thermal Radio Intensities In The Anti-Center Direction</td>
<td>15</td>
</tr>
<tr>
<td>4.1</td>
<td>Emissivities Deduced In The Direction Of H II Regions</td>
<td>60</td>
</tr>
<tr>
<td>5.1</td>
<td>The Solar Cycle's Depth Of Modulation</td>
<td>98</td>
</tr>
</tbody>
</table>
LIST OF ILLUSTRATIONS

CHAPTER II

2.1 Frequency Spectrum of Magnetic Bremsstrahlung .. 8
2.2 Measured Radio Frequency Spectra in the
Anti-Center and Polar Directions ................. 14
2.3 The 1965-1969 Electron Data ..................... 20
2.4 The 1969-1974 Electron Data ..................... 22
2.5 Electron Measurements at High Energies ....... 26

CHAPTER IV

4.1 Cummings' Interstellar Electron Spectrum ....... 47
4.2 Cummings' Deduced Emissivity Compared With
Measured Emissivities in the Direction of H II
Regions ......................................................... 49
4.3 Mariani's 1972 Interplanetary B-Field
Distribution and a Possible Galactic B-Field
Distribution .................................................. 54
4.4 Emissivities in the direction of H II Regions in
the Galaxy .................................................... 59
4.5 The Interstellar Electron Spectrum We Deduce ... 65
4.6 Electron and Positron Production Spectrum in the
Galaxy ........................................................ 70
4.7 The Measured Positron Ratio and the Interstellar
Electron Spectrum That is Deduced From the Positron Ratio .................................................. 74

CHAPTER V

5.1 Zwickl and Webber's Mean Free Path Determined From Solar Particle Propagation .................. 81
5.2 The Rigidity Dependence of the Diffusion Coefficient Which We Use to Study the Modulation .................. 85
5.3 1965-66 Electron data and Calculation ............. 87
5.4 1968 Electron Data and Calculation ................. 89
5.5 1969 Electron Data and Calculation ................. 91
5.6 1970 Electron Data and Calculation ................. 93
5.7 1971 Electron Data and Calculation ................. 95
5.8 1972-73-74 Electron Data and Calculation ........ 97
5.9 The Depth of Modulation ................................ 101
5.10 1965-68-69 Proton Data with Calculation ........... 104
5.11 1970-71-72 Proton Data with Calculation ........... 106
5.12 1972-74-75 Proton Data with Calculation ........... 108
5.13 1965-68-69 Helium Data with Calculation .......... 110
5.14 1970-71-72 Helium Data with Calculation .......... 112
5.15 1972-74-75 Helium Data with Calculation .......... 114
5.16 Proton Differential Modulation ..................... 118
5.17 Helium Differential Modulation ..................... 120
5.18 Electron Differential Modulation .................... 122
5.19 Predictions For The Modulation Ratio ............. 125
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.20</td>
<td>Helium-to-Proton Modulation Ratio</td>
<td>127</td>
</tr>
<tr>
<td>5.21</td>
<td>The Electron-to-Proton Modulation Ratio</td>
<td>129</td>
</tr>
<tr>
<td>5.22</td>
<td>Diffusion, Convection, and Energy Loss Fractions</td>
<td>134</td>
</tr>
<tr>
<td></td>
<td>For Electrons</td>
<td></td>
</tr>
<tr>
<td>5.23</td>
<td>Diffusion, Convection, and Energy Loss Fractions</td>
<td>137</td>
</tr>
<tr>
<td></td>
<td>For Protons</td>
<td></td>
</tr>
<tr>
<td>5.24</td>
<td>Diffusion, Convection, and Energy Loss Fractions</td>
<td>139</td>
</tr>
<tr>
<td></td>
<td>For Helium</td>
<td></td>
</tr>
</tbody>
</table>
ABSTRACT

A NEW AND PRECISE CONSTRUCTION OF THE LOCAL INTERSTELLAR ELECTRON SPECTRUM FROM THE RADIO BACKGROUND AND ITS APPLICATION TO THE SOLAR MODULATION OF COSMIC RAYS SHOWING AN INCOMPATABILITY OF THE ELECTRON AND NUCLEI MODULATION USING THE SPHERICALLY SYMMETRIC FOKKER-PLANCK EQUATION

by

JOHN MICHAEL ROCKSTROH

Cosmic-ray electrons generate the observed radio-frequency background. Previous attempts in the literature to reconcile quantitatively the measured radio-frequency intensity with the intensity deduced from the electron spectrum measured at earth have culminated in the problem that to get the respective emissivities to agree, an unacceptably high interstellar B field must be chosen. In the light of new experimental data on the emissivity as deduced from H II region studies and on the functional dependence of the diffusion coefficient with solar radius and particle rigidity, we have re-examined closely the assumptions under which the electron emissivity comparison
has been made. We resolve the paradox between predicted and measured emissivity by ascribing to the magnetic fields of the galaxy a distribution of magnetic field strengths. From modified synchrotron formulas, the interstellar electron spectrum has been constructed from the radio frequency emission data with greatly improved precision.

Our interstellar electron spectrum has been determined independently of the solar modulation and provides, therefore, an estimate of the absolute depth of the electron modulation. We then systematically compare the measured electron, proton, and helium-nuclei fluxes to the predictions of the spherically symmetric Fokker-Planck equation using the electron modulation as a base. We observe a previously un-noticed non-tracking of the modulation parameters during the recent recovery that did not occur during the 1965-69 period. Although our argument could be presented just as well by attributing the anomaly to the nuclei, we have arbitrarily tailored our discussion to the electrons, and we name this new phenomenon, the modulation reluctance of the cosmic-ray electrons. The electrons have not only failed to return to their 1965-66 level, but also exhibit much smaller modulation parameters than would be based on either the 1968-69 electron fluxes or on the extremely-well measured nuclei fluxes. The modulation reluctance is compared with and distinguished from the hysteresis effects that have also been discovered during the recent recovery. The differences in the relative
tracking of the electron and nuclei modulation parameters observed between 1965-69 period and the 1969-74 recovery period cannot be explained by current forms of the spherically symmetric Fokker-Planck equation. We discuss a few of the possibilities for understanding this effect.
CHAPTER I

INTRODUCTION

Many efforts have been made in the past to deduce the residual cosmic-ray modulation near the sun by comparing the maximum measured electron spectrum at earth with that deduced to exist in nearby interstellar space using measurements of the non-thermal galactic radio spectrum. Once the residual modulation has been deduced, standard spherically symmetric modulation theory is then applied to study the time variations of electrons, protons and helium nuclei at the earth. Typical recent values for the residual modulation parameter, phi\(^1\), obtained at a time of sunspot minimum, are 300-350 MV (Garrard, 1973; Fisk, 1971; Urch and Gleeson, 1972), although some earlier values were much lower (Lezniak and Webber, 1971).

The question arises as to why, after so many earlier efforts, it is necessary to examine this topic again. Several new observational facts related to different aspects of this overall issue are what prompt this current study. First of all we have the question of the interstellar electron spectrum itself. Previous estimates of this spectrum have started from the absolute radio emissivity in

\(^1\)Greek letters will be spelled in the text but will appear in the equations as symbols.
the anti-center direction, and have assumed: (1) a constant B field—typically 5 microG, and (2) a uniform distribution of radiating electrons; both of which extend over a distance of perhaps 5 Kpc to the outer limit of the galactic spiral structure; and proceeded to deduce an average electron spectrum, which is then assumed to be the one present outside of the solar cavity. We shall show later in this thesis that several assumptions in this line of reasoning are probably suspect. Indeed several calculations in the past as well as recent ones, all in a somewhat different context, have shown that such an approach generally leads to what is, in effect, much too high an electron flux outside the solar cavity. These calculations have been interpreted in terms of an inconsistency between cosmic-ray electron and non-thermal radio emission data (See e.g. Badhwar, et al., 1977.). Alternatively, if the B-field were to have greater strength, fewer electrons would be needed to account for the observed radio emissivity, but the absence of an observable Zeeman splitting for the 21-cm line apparently precludes an average B field of sufficient strength.

We present a simple spectral normalization argument which utilizes new radio data and enables us to bypass the details of the above comparison between the cosmic-ray electrons and the non-thermal radio emission, and to determine the electron spectrum present just outside the solar cavity. This electron spectrum is found to be in good
agreement with that deduced using an argument based on the observed \( e^+/(e^- + e^+) \) ratio at earth. It is noted that the typical interstellar electron spectra previously estimated from the radio emissivity have not always been in agreement with those deduced using arguments based on positron measurements.

The second new set of observational data relates the interplanetary diffusion coefficient and its radial dependence. Recent measurements of solar cosmic rays have enabled this important parameter to be measured at several times in the solar cycle over a rigidity range from 1 MV to 2 GV and for radii from 1-5 AU from the sun. This solar cosmic-ray data, when combined with new radial gradient data for cosmic-ray nuclei out to 12 AU, puts strong constraint on a combination of quantities including the extent of the modulating cavity as defined by a boundary, \( r_B \), the total residual modulation, \( \phi \), and the overall radial dependence of the modulation.

The cosmic-ray data itself includes the measured spectra of electrons, protons and helium nuclei in the period from 1965 to 1969, when the level of modulation was increasing, but all energies appeared to track along unique regression curves; and similar data for these charge components from 1969-1975 when the overall modulation was decreasing and several instances of nontracking between changes at various energies were observed. The estimation
of the residual modulation existing in the 1965 and 1975 periods is one important aspect of this analysis. It is found that, whereas the electron modulation and the nuclei modulation are consistent during the 1965-1969 period, they are not consistent during the 1970-1975 period. The modulation level that is predicted by the electron measurements is greater than the actually observed nuclei modulation. The relationship between this new effect which we call the electron reluctance and the well-known "hysteresis" effect between nuclei of different energies is discussed.

It is possible to relate the individual terms in the transport equation for the modulation to specific features of the modulation and thereby to more fully understand the specific limitations of the spherically symmetric models used in modulation theory.

Finally we have tried to examine the uniqueness of our result vis-a-vis the question of a high or low value of the residual modulation. This study, along with limitations set by a study of recent gradient measurements enables new confidence levels to be put on the interstellar proton and helium nuclei spectra at low and intermediate energies.
CHAPTER II

COMPILATION OF A DATA BASE CONTAINING
THE RADIO FREQUENCY BACKGROUND AND
THE COSMIC-RAY ELECTRON FLUXES

The synchrotron mechanism is generally considered to be responsible for most of the non-thermal radio emission from our galaxy. Several authors have discussed the theory of synchrotron emission from electrons and have presented the relevant formulas. We present here a summary of this discussion as it relates to the present analysis.

II.1 Synchrotron Radiation Formulas

Synchrotron emission is a bremsstrahlung process resulting from the acceleration necessary to maintain a particle in its helical orbit along an external magnetic field. The elementary physical processes describing the radiation of a single particle were derived by Schwinger (1949). The total power radiated per unit frequency interval is

\[ P(\nu)dv = \frac{\sqrt{3}}{2} \frac{e^3}{mc^2} B \frac{\nu}{\nu_c} \int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta) d\eta \ dv \]  

\[ \nu_c (\text{MHz}) = \frac{3e B_1}{4\pi mc} \gamma^2 = 16.1 \ B_1 (\mu\text{Gauss}) E^2 (\text{GeV}) \]
where
\begin{align*}
e & \text{ is particle charge} \\
m & \text{ is particle mass} \\
c & \text{ is speed of light} \\
K_{5/3} & \text{ is the MacDonald function} \\
B_1 & \text{ is magnetic field strength, perpendicular to particle velocity, in vacuum; we do not distinguish } B_1 \text{ and } H_1 \\
\gamma & \text{ is the Lorentz factor} \\
E & \text{ is particle energy}
\end{align*}

The frequency distribution, \( P(\nu) \), for a single electron, shown in Figure 2.1 (Westfold, 1959), exhibits a maximum of power radiated at .299 times the critical frequency of the classical description. An interesting observation concerns the behavior of this function as \( \nu \) approaches 0, where it follows a \( \nu^{3.33} \) dependence. Therefore, a steeper dependence than \( 1/3 \) in a synchrotron spectrum must necessarily be due to absorption effects.

The appreciation of the dependence of \( P(\nu) \) on magnetic field strength cannot be directly seen in eqn. 2.1 because \( B \) is implicitly present in the lower limit of the integral. Let us consider the total energy loss rate which is (Schwinger, 1949)

\[
\frac{dE}{dt} \left( \text{erg/ sec} \right) = - \int \nu P(\nu) d\nu = -6 \cdot 10^{-28} E^2 \text{(MeV)} B_1^2 \text{(\mu G)}
\]  \[2.3\]
Figure 2.1 The frequency spectrum of the magnetic bremsstrahlung is shown as a function of frequency. It is assumed that the particle orbit is circular.
$F(X) = X \int_{x}^{\infty} K_{5/3}(\eta) \, d\eta$
The power in the synchrotron emission is now seen to depend quadratically on the perpendicular component of the instantaneous B field.

The next step in the cosmic application of synchrotron radiation is the derivation of the resultant synchrotron emission spectrum from an ensemble of energetic electrons distributed according to a power law in energy,

\[ N(E) \frac{dE}{E} = KE^{-\gamma} \frac{dE}{E}; \quad N(E) = \frac{4\pi}{c} \frac{3A}{3E} \]

K in units of ergs\(^{-1}\)/cm\(^3\)

The volume emissivity of such an ensemble is mathematically given by

\[ \varepsilon(v) = \int_0^\infty P(v) N(E) \frac{dE}{E} \]

The integration is far from straightforward; fortunately, Ginzburg and Syrovat-skii (1964) include the result without reference in their compendium of synchrotron formulas.

\[ \varepsilon(v) = \frac{\sqrt{3}}{\gamma+1} \frac{\Gamma(3\gamma-1)}{\Gamma(3\gamma+1/2)} \frac{3}{12} \frac{e^3}{4\pi mc} \left( \frac{3e}{\gamma 3c} \right)^{1/2} \frac{1+\gamma}{\nu} \frac{1-\gamma}{\nu} \]

Equation 2.6 describes emission that is highly polarized. Ginzburg and Syrovat-skii include the polarization effect by performing an integration which averages the emissivity over
all angles of inclination to the magnetic field. They thus obtain an additional multiplying factor which depends on the electron spectral index.

\[ \frac{1}{2} \int_0^\pi (\sin \theta)^2 \sin \theta \, d\theta \]  \hspace{1cm} 2.7

Unfortunately, the previous integration disturbs the overall conservation of energy; therefore, the final equations relating the intensity of radio noise to the electron spectrum presented by Ginzburg and Syrovat-skii (specifically, their eqns 4.33, 4.34, and Table 6) are only approximate. We carefully abstract the exact equation rather than simply copying Ginzburg and Syrovat-skii's text. From eqn 2.6 we see that the synchrotron radiation from electrons with a power law energy distribution of index, \(-\gamma\), has a power law frequency spectrum of index \(-\frac{(\gamma-1)}{2}\).

\[ I \propto v^{-\alpha}; \quad \frac{dI}{dE} = E^{-\gamma}; \quad \alpha = \frac{\gamma-1}{2} \]  \hspace{1cm} 2.8

The intensity of synchrotron emission along a particular line of sight to a distance \(L\) is

\[ I(v) = \int_0^L \varepsilon(v) \, dL \]  \hspace{1cm} 2.9

\[ \frac{\text{watts}}{m^2\text{-ster-Hz}} \]

and if the emissivity is uniform and isotropic

\[ I(v) = \varepsilon(v) \cdot L \]  \hspace{1cm} 2.10
II.2 Intensity Of The Diffuse Radio-Frequency Background

Through equation 2.6, then, one can relate the observed non-thermal radio spectrum to the spectrum of radiating electrons. Ideally one would like to determine the emissivity just outside the solar modulating region for comparison with the electron spectrum observed at earth. This is not possible, but several alternatives may be used to deduce the non-thermal emissivity. It should be pointed out that as one looks in directions within ±45° of the galactic center along the disk of the galaxy, the radio emission pattern is a complicated function of position reflecting passage of the line of sight through spiral arms and other features of the galaxy. The emissivity is probably a complicated function of position so that it is difficult to deduce a local emissivity from measurements of I(νu) in these directions. In the polar directions it appears that ~50% of the emission comes from beyond our own spiral arm so measurements of the intensity in these directions also are not representative of the emissivity spectrum near the sun. As one examines the radio distribution in the galactic disk but at directions >±45° from the center including the anti center one finds that although the intensity still changes significantly, the spectral shape is almost independent of position (Cane, 1977). In the anti-center direction a detailed radio spectrum is available and at the same time the interpretation of the line of sight is less complicated than
most directions. There is evidence that in this direction, for example, most of the emission comes from within 1-2 Kpc of the sun, e.g. mainly our own spiral arm (Parrish, 1972). Thus while there may be some uncertainty in the path length, L, to use in intensity calculations, the spectrum in this direction should be representative of the emissivity spectrum near the earth apart from a normalization constant. Hence, we shall use this as our base spectrum. In Figure 2.2 we present the anti-center spectrum derived from a large number of measurements at different frequencies. We have examined and inter-compared the original data using new, revised radio maps (Cane, 1977) and have made some changes in the originally quoted intensities as indicated in Table 2.1. We have also adjusted (convolved) the measured intensities to correspond to an angular resolution of 5x5 square degrees where necessary using published radio maps at nearby frequencies.

A smooth curve is drawn through the data (a dashed line is drawn above 1400 MHz where the data is extrapolated) and from this smooth curve a spectral index is deduced as shown in the bottom part of the Figure. Note that this radio spectral index changes slowly with energy—{}from a value of 0.9±0.1 above ~1000 MHz, to 0.55 at 100 MHz and 0.25 at 10 MHz as absorption effects set in and the spectrum begins to turn over. This effect is most clearly seen in the abrupt turn-over of the spectral index. For a 5 microgauss interstellar magnetic field, the corresponding electron
Figure 2.2 The intensity of the diffuse radio frequency background is shown. The anti-center spectral data are also tabulated in Table II.1 where the observers are listed by number. The spectrum observed in the direction of the North and South galactic pole is shown for comparison. Also shown, below the intensities, are the indices of the two spectra. The anti-center spectrum has been extrapolated below ~10 MHz. Below this frequency, the radio frequency spectrum becomes uncertain.
Figure 2.2
TABLE 2.1

NON-THERMAL RADIO INTENSITIES IN THE ANTI-CENTER DIRECTION (l=180°, b=0°)

<table>
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<th>MHz</th>
<th>W/m²-ster-Hz</th>
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<td>11.7 x 10^-21</td>
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<td>10.4 x 10^-21</td>
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<td>Milogradov, et al. (1973)</td>
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<td>55.</td>
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<td>Yates (1967)</td>
</tr>
<tr>
<td>150</td>
<td>3.5 x 10^-21</td>
<td>Landecker, et al. (1970)</td>
</tr>
<tr>
<td>178</td>
<td>3.2 x 10^-21</td>
<td>Turtle, et al. (1962)</td>
</tr>
<tr>
<td>234</td>
<td>2.60 x 10^-21</td>
<td>Davies, et al. (1962)</td>
</tr>
<tr>
<td>408</td>
<td>1.8 x 10^-21</td>
<td>Halsam (1970)</td>
</tr>
<tr>
<td>610</td>
<td>1.2 x 10^-21</td>
<td>Howell (1970)</td>
</tr>
<tr>
<td>1400</td>
<td>0.62 x 10^-21</td>
<td>Mathewson, et al. (1972)</td>
</tr>
<tr>
<td>2000</td>
<td>0.46 x 10^-21</td>
<td>Hirabayashi (1974)</td>
</tr>
<tr>
<td>3000</td>
<td>0.31 x 10^-21</td>
<td>Hirabayashi (1974)</td>
</tr>
<tr>
<td>4100</td>
<td>0.22 x 10^-21</td>
<td>Hirabayashi (1974)</td>
</tr>
<tr>
<td>10000</td>
<td>0.10 x 10^-21</td>
<td>Hirabayashi (1974)</td>
</tr>
</tbody>
</table>
energies and spectral indices at the above frequencies are $2.8 \pm 0.2$ above 3.3 GeV, 2.3 at 1.0 GeV and 1.8 at 0.31 GeV.

In Figure 2.2 we also show the average North and South polar spectra (Cane, 1977). These spectra could be used as an alternative but less good representation of the local spectrum since probably only ~30% of the emission in these directions comes from our own spiral arm. Note that this spectrum is distinctly steeper than the anti-center spectrum, but it still slowly flattens at lower frequencies changing from an index $\sim 0.6$ at 100 MHz to 0.4 at 10 MHz. The onset of free-free absorption is again seen to occur abruptly in the change of spectral index at ~5 MHz. Bridle (1967) has also noted that the spectral index at intermediate frequencies steepens with increasing distance from the galactic plane. Whether this steepening is associated with energy losses of the electrons contained for $\sim 10^8$ years or is merely due to lower magnetic fields at large distances from the galactic plane is not yet understood. However, the fact that the spectrum steepens in the polar direction along with the fact that the radio spectrum in the disk is uniform with location suggests that the disk radio spectrum near the anti-center direction is the best representation of the galactic electron spectrum in the spiral arm near the sun.
One should also note that the minimum radio intensity in the galactic plane (b=0°) is not in the anti-center direction (l=180°) but at l=225°-240°. In this direction the intensity is ~0.85 of that in the anticenter direction. Also in the polar directions the intensity at frequencies >100 MHz is ~0.60 of the anti-center spectrum. The polar spectra are somewhat steeper at lower frequencies, as noted above, and at 10 MHz the intensities are ~0.75 of the anti-center intensity. The absolute minimum of non-thermal radio emission in the sky occurs at l=225°-240° and b=±40° and the intensity here is ~0.3 of the anti-center intensity—the spectrum is not well defined and is consistent with either the anti-center or polar spectra.

II.3 Cosmic Ray Electron Spectra

The cosmic ray electron spectrum has been measured in a systematic fashion by 3 groups over the period since 1965 (Webber and Rockstroh, 1971; Webber, 1973; Fulks, 1975; Winkler and Bedijn, 1976). These measurements are over the energy range from a few hundred MeV to ~10 GeV, a range which is compatible with the frequency range over which the non-thermal radio spectra are available. The agreement between the separate measurements of these groups is generally good except for some uncertainties at low energies in the 1965-1966 period, and some discrepancies at higher energies in the 1972-1973 period. The inconsistencies in 1965-1966 data are particularly disturbing since this was at
sunspot minimum when the electron intensities near the earth were expected to be nearest to those in interstellar space and also because this 1965-1966 data has not yet been completely reproduced in the latest sunspot minimum period of 1973-1976. The general agreement of data at these lower energies, however, is in contrast to a much more uncertain situation that exists regarding electron intensities at ~10 GeV and above about which we will have more to say later.

In presenting the electron data we shall mainly use the self-consistent set of data from the University of Minnesota, University of New Hampshire telescope, using data from the other groups only where there is some discrepancy between results, or where the above data base is lacking. Figure 2.3 shows the spectra for the 1965-1969 period, the first part of the solar cycle when the modulation was increasing and the general intensity level was decreasing. During this period the electron intensities at low energies track quite closely with the high energy total cosmic intensity as reflected by the Mount Washington neutron monitor rates. Data are also presented from L'Heureux (1967) for 1965 and Bleeker, et al., (1970) for 1966 to illustrate the level of intercomparability in this period where the differences among groups are largest.

Figure 2.4 shows the data from 1969 to 1974. This was a period during which the high energy cosmic ray intensity as indicated by neutron monitors returned to a level
Figure 2.3 The electron spectra measured at earth are shown for the years 1965-1969. The diamonds indicate the measurements of the Webber Group. The 1965 measurement of L'Heureux, the 1966 measurement of Bleeker et al., and the 1968 measurement of the Chicago Group are also shown. There is general experimental agreement in the electron spectrum at these energies. The Mount Washington neutron monitor counting rate, when each measurement was made, is shown in the legend.
Figure 2.3
Figure 2.4 The electron spectra measured at earth are shown for the years 1969 to 1973. The diamonds again indicate the Webber measurements. Chicago data of the years 1970, 1972, and 1973 supplement our measurement. Above 4 GeV, the data of Buffington et al. (1975) are shown.
Figure 2.4

Electrons/m²·ster·sec·MeV

Primary Mt.
Electrons Wash.

1965
1966

1972-73 Buffington et al. (1975)

1972 2415
1971 2343
1969 2089

1970 2111 Fulks (1975)
1972 2404 Fulks (1975)
1973 2387 Caldwell et al. (1973)
approximating that existing in 1965-1966. The lower energy electrons, however, do not track this behavior in a simple manner. The lowest electron intensity was observed in 1970 at which time the high energy cosmic rays had already started to increase. The electron intensity increases further in 1971, 1972, and 1973, however, it is still below the level observed in 1965-66. Electron data from the time period 1975-77 when the neutron monitor rates were at a slightly higher level than in 1972-74 and which might indicate a further recovery of the low energy electrons to 1965-66 levels is not yet available.

Some interesting differences exist in the data above ~2 GeV for the years 1972-73 with the data from Fulks (1975) being notably higher than the Webber and Rockstroh (1973) data. In fact an examination of the Fulks data for 1968 and 1972-73 shows that the 1972-73 intensities are significantly higher than the earlier ones above ~5 GeV where the modulation effects would be expected to be small. This is unlikely to be a real effect and points up the fact that all three experiments are basically designed to measure electron intensities and time variations below a few GeV and do not completely discriminate against proton interactions which provide a significant and increasing background as one goes to higher energies. For this reason we also show in Figure 2.4 intensities above ~5 GeV measured in 1972 from the most sophisticated system yet flown for background rejection and analysis (Buffington, et al., 1975). This data suggests
that some of the fluxes determined from these low energy telescopes could be a factor of 1.5-2.0 high at energies above a few GeV.

This problem is not unrelated to the long standing controversy regarding the high energy electron intensity above ~10 GeV. We have collected data relevant to this controversy as well as the measurements by the above 3 groups at lower energies in Figure 2.5. In this Figure the differential fluxes are multiplied by $E^3$, a procedure which (1) visually enhances any flux differences at a particular energy, and (2) enables one to determine more easily the high energy spectral index and normalization. This Figure illustrates the generally good agreement in the low energy data up to ~3-4 GeV and the discrepancy of the Fulks (1975) data for 1968 and 1972-73 above ~5 GeV. It also clearly shows the high energy situation wherein the data points above ~10 GeV, all from quite elaborate and carefully conducted experiments, have a range of intensities covering at least a factor of 3. Even the latest experiment (Mueller, et al., 1977) does not convincingly solve this discrepancy, since those who measure the lowest intensities argue with a certain conviction, that if proton induced showers are indeed an important background, then lower intensities are more acceptable than higher ones simply because it is not possible to understand how events can be missed. Without going further into this intriguing controversy, it is evident that from the point of view of
Figure 2.5 A compilation of high energy electron measurements multiplied by $E^3$ is shown. Thus, the ordinate displays the numeric value of the constant, $N$, to be defined in equation 4.2. The normalization point is indicated at 4 GeV.
Figure 2.5

1972-73 Data
Mueller et al., 1973
Mueller et al., 1977
Freier et al., 1977
Fulks 1975
Burger & Swanenberg 1973
Webber et al. 1973
Buffington et al. 1975
Silverberg 1976
Anand et al. 1973
Meegan & Earl 1975
the high energy data any spectrum $dj/dE=(100\text{ to }300)\ E^{-3}$
can be argued to fit the data. As one goes to lower
energies the restrictions become greater, however, because
of the improved agreement in the data. The best agreement
is found at $\sim4$ GeV where the data suggests an interstellar
intensity of $2.8\pm0.4\times10^{-3}$ Peters, corresponding to a
spectrum of $dj/dE=160\pm40\ E^{-3}$. It is immaterial that
this spectrum does not fit all of the high energy data since
the radic spectrum is not well enough known at the
corresponding frequencies to provide a constraint on the
data. In other words, as we shall discuss further later,
from the point of view of normalizing the interstellar
electron spectrum and the radio spectrum we can bypass the
uncertainties in the electron spectrum at or above 10 GeV,
by considering a lower energy where the electron spectrum is
better known, but the solar modulation effects are still
small enough so as to be unimportant.
CHAPTER III

A BRIEF SURVEY OF SOLAR MODULATION THEORY

Cosmic-ray particle intensities observed at the orbit of earth are reduced from their galactic values by an interaction with the extension of the solar magnetic field carried in the solar-wind plasma. The amount of this reduction in intensity depends on both the particle energy and particle momentum and varies over the solar cycle. Because of the anti-correlation with solar activity, this phenomenon has been named the solar modulation.

The early attempts to explain the phenomenon of solar modulation achieved only limited success. It was imagined that some physical process, say diffusion, must be important, and the solution over some range of parameters was compared to observed fluxes in order to study that process. To emphasize the lack of rigor in the modelling process, Freier and Waddington, (1965), tabulated the various models, comparing them to observations. The most unlikely model, the absolute value electrostatic field model of Ehmert (1960), best represented the observed spectra.

Of the early models there is one worth discussing because it illustrates simply the main features of the current Fokker-Planck model. Parker (1963) proposed that the modulation was caused by convection of particles out of
the solar cavity by the solar wind. Let $V$ be the solar wind velocity and $U$ represent the particle number density per unit energy. Then, according to Parker, the outward convective current is given by $UV$. In this picture, the solar wind scatters the particles and sweeps them away from the sun, and in so doing creates a positive gradient. The cosmic rays are scattered by the magnetic field irregularities that are frozen into the solar wind, and because there is a gradient in number density, a diffusive current of $K \frac{dU}{dr}$ flows inward towards the sun. $K$ is the diffusion coefficient resulting from the magnetic-particle scattering; in addition to the obvious velocity dependence, $K$ is a function of the particle's magnetic rigidity and possibly a function of solar radius. Assuming that there are no sources or sinks within the solar cavity, Parker writes down the balance between the convective current and the diffusive current to obtain a differential equation which may be solved for the modulated spectra observed at earth.

$$UV = K \frac{dU}{dr}$$

3.1

Integration yields the following solution:

$$U(r_E, T, t) = U_o(r_B, T) \exp \left[ -\int_{r_E}^{r_B} \frac{V(r', t)}{K(r', T, t)} \, dr' \right]$$

3.2
1. \(U\) - Particle number density per unit energy
2. \(U_0\) - The galactic number density per unit energy
3. \(V\) - The solar wind velocity
4. \(K\) - The diffusion coefficient
5. \(r_E\) - 1 astronomical unit
6. \(r_B\) - Radius of the boundary of the solar cavity
7. \(T\) - Particle kinetic energy
8. \(t\) - Time of solar cycle when flow, \(U\), is measured

We define separability of the diffusion coefficient to mean it can be written in the following form:

\[
K(r,T,t) = \beta K_1(P,t)K_2(r,t)
\]  

where \(P\) is the particle rigidity.

Under assumption that \(K\) separates, the solution can be factored to give:

\[
U(r_E,T,t) = U_0(r_B,T) \exp \left[ -\frac{n(t)}{\beta K_2(P,t)} \right]
\]

where

\[
n(t) = \int_{r_E}^{r_B} \frac{V(r',t)}{K_1(r',t)} \, \text{d}r'
\]

The elegance of Parker's convection-diffusion model resides in the comparison of experiment to theory. Cosmic-ray particle spectra are usually reported in units of differential directional intensity, \(\frac{\text{d}j}{\text{d}E}\), and the conversion of intensity to density is given by \(U = 4\pi v/c \frac{\text{d}j}{\text{d}E}\). The differential directional intensities measured at two different times, \(\frac{\text{d}j}{\text{d}E}(r_E,T,t_1)\) and \(\frac{\text{d}j}{\text{d}E}(r_E,T,t_2)\), may
be compared by considering the modulation parameter, $M$, defined as:

$$M \equiv \beta \ln \frac{\frac{\partial U(r_B,T,t_1)}{\partial E}}{\frac{\partial U(r_B,T,t_2)}{\partial E}} = \beta \ln \frac{U(r_B,T,t_1)}{U(r_B,T,t_2)} \quad 3.6$$

If $K$ separates and the form of $K_2(P,t)$ is the same at $t_1$ and $t_2$, that is, $K_2(P,t_1) = K_2(P,t_2) = K_2(P)$, then the modulation parameter is given by $M = (n(t_1) - n(t_2))/K_2(P)$. The differential, $n(t_1) - n(t_2)$, is interpreted as a change in the depth of modulation. By plotting $M$, the diffusion coefficient responsible for the modulation can be directly seen, apart from a constant. If the actual total depth of modulation at a given time was known, the interstellar particle density $U_0(r_B,T)$ could be calculated.

The solar modulation at any one time is determined mainly by the diffusion coefficient. The boundary of the solar cavity may be thought of as the solar radius at which the diffusion coefficient becomes infinite. The diffusion coefficient should account for the modulated spectra of the electrons, the protons, and the alpha particles. These three cosmic-ray species are each considered because they differ in their charge-to-mass ratio.

The ease with which data may be fit to this model is deceptive—if one compares measured spectra from only two time periods, the agreement is excellent. As data from
other periods are included, the interstellar spectra do not change and the free parameters left are the depth of modulation and the functional form of the diffusion coefficient. If one fits the diffusion coefficient to one species at a particular time, then that diffusion coefficient ought to modulate the other two species correctly. As more and more data are added to the model, the number of degrees of freedom diminishes and the goodness-of-fit becomes impoverished. A phenomenological test of this model would require sufficient data to exhaust the freedom in the model; a true test of the model would be the derivation of its equations from first principles.

The diffusion-convection model presented above does not include the effect of betatron deceleration of particles by the diminishing magnetic field in the geometrically expanding solar wind. It is difficult to think of the differential equation responsible for diffusion, convection, and adiabatic deceleration as simply being the sum of three terms, one for each effect. Rather, both the physical laws and the transformation between the solar wind frame and the sun's frame become complicated; hence, the derivation of the transport equation is approached from other vantage points.

The conservation of particles and currents is described with differential equations, and because small terms in a differential equation are not necessarily small in their
effect, it is necessary to include all the physical processes simultaneously. Parker (1965) first derived a Fokker-Planck equation in particle differential number density, $U$, which describes the known physical interactions occurring in a spherically symmetric solar wind.

$$\frac{3U}{3t} + \frac{1}{r^2} \frac{3}{3r} r^2 \left[ U V - K \frac{3U}{3r} \right] = \frac{1}{3} \left[ \frac{1}{r^2} \frac{3}{3r} r^2 V \right] \frac{3}{3T} \alpha TU \quad 3.7$$

where
- $U$ is particle density per unit energy
- $V$ is solar wind velocity
- $K$ is diffusion coefficient
- $T$ is kinetic energy per nucleon
- $\alpha$ is $(\gamma + 1)/\gamma$ in terms of the Lorentz factor
- $t$ is time
- $r$ is distance from the sun

Where the differential operator in $r$ on the right hand side of the equation affects only the solar-wind velocity, $V$. The energy loss term, the right hand side, arises from betatron deceleration in the rest frame of the solar wind. Parker argues that the next higher order energy loss term, Fermi acceleration, is inconsequential and need not be included.
Gleeson and Axford (1967, 1968) in a series of related papers modelled the scattering process directly. They supposed that a forceless Boltzmann equation governed the particle motion in the cosmic-ray gas and that the collision term arose from elastic scattering of particle by field with an isotropic cross section in the rest frame of the solar wind. Gleeson and Axford presented and correctly described the transformations between solar wind and sun reference frames. The first-order Taylor expansion of their "Boltzmann" operator yielded Parker's Fokker-Planck equation.

Implicit in the Boltzmann formulation is the expression for the radial particle current

\[ S_r = C U V - K \frac{\partial U}{\partial r^2} \quad 3.8 \]

where

\[ C = 1 - \frac{1}{3U} \frac{\partial}{\partial T} a T U. \quad 3.9 \]

C has been called the Compton-Getting coefficient and is said to arise from the transformation of the convection current, UV, into the reference frame of the sun. Gleeson and Axford proved that at high energies the current of the Fokker-Planck equation approached zero and thus the equation

\[ C U V - K \frac{\partial U}{\partial r} = 0 \quad 3.10 \]

was a valid high energy approximation to the full transport equation.
Under the assumption that \( K \) separates, equation 3.10 also separates and may be integrated by the method of characteristics. The solution is

\[
\frac{\partial j(r,E,t)}{\partial E} = \frac{\partial j(r_B,E + \phi)}{E^2 - E_0^2 (E + \phi)^2 - E_0^2}
\]

where

\[
\phi = \frac{1}{3} \int_{r_B}^{r_E} \frac{V(r',t)}{K_1(r',t)} \, dr'
\]

\[
\rho = \int_E^{r_E} \frac{K_2(P',t)}{E \left( E'^2 - E_0^2 \right)^{1/2}} \, dE'
\]

\[
\phi = \chi(\rho + \phi, Z, t) - \chi(\rho, Z, t)
\]

\( \chi \) is the inverse function, \( \rho^{-1}(E) \) and \( E \) is the total particle energy.

Gleeson and Axford first applied equation 3.10 to the case of a diffusion coefficient where \( K2(P,t) = P \). Then PHI = \( |Z| \) e \( \phi \) and the solution, equation 3.11, is equivalent to an electrostatic repulsion from the potential PHI\((r,t)\). Thus equation 3.10 is called the "force-field" approximation. If the galactic spectrum is known, then the diffusion
coefficient responsible for a modulated spectrum can be calculated; Urch and Gleeson (1973) present the construction which is valid within the "force-field" approximation.

Most observations have been of spectra at energies too low for a valid "force-field" approximation to be applied. Fisk, Forman, and Axford (1973) have studied various simplifications to the Fokker-Planck equation, 3.7, for application at low energies. Because the Fokker-Planck equation is parabolic in energy, it is difficult to envision a valid low energy approximation. Therefore, the full transport equation must be solved in order to study the measured fluxes at low energy.

The numerical solution of equation 3.7 requires specification of the diffusion coefficient and boundary conditions on $U$. Since the $d/dt$ term is taken to be vanishingly small and the steady state is assumed, the time dependence of the solution $U$, resulting in what are known as solar-modulation effects, can arise only from temporal changes in the functional form of the diffusion coefficient or changes in the radial extent of the modulation region. The operator (3.7) is solved over a rectangle in the radius-energy plane, and the boundary conditions must be specified along three edges. We allow kinetic energy to vary from 1 MeV to 10 GeV per nucleon and solar radius to vary from the sun, $r=0$, to an imagined boundary at the edge
of the solar cavity, \( r_B \). At this boundary, \( r_B \), the density \( U(r_B, T) \) is set equal to \( 4 \pi \frac{1}{v} \frac{dj}{dE} \) where \( \frac{dj}{dE} \) is the galactic particle intensity. At the sun, a differential boundary condition is desired that the particle current vanish so the sun is neither a source nor a sink of particles. Fisk, who initially solved (3.7) numerically, first suggested that if \( r^2 U \) approaches 0 as \( r \) approaches 0, the radial current, \( S \), would be bounded and small (Fisk, 1969). Later he suggested the weaker condition \( r < 5 \) \( U \) approaches 0 which we use. At the third and last boundary, the particle density \( U(r, 10 \text{ GeV}) \) must be specified; that is, the solution of the operator over radius is needed at a high energy. Along this edge, the "force-field" approximation to 3.7 is solved for the particle densities (Gleeson and Axford, 1967). Finally, the Crank-Nicholson algorithm is applied to numerically calculate successive solutions \( U(r, T) \) over \( r \), as each successive \( T \) is diminished by \(-1\%\). Thus, the modulated spectrum \( U(1 \text{ AU}, T) \) is obtained. Carnahan, Luther, and Wilkes (1969) present an excellent derivation of the algebraic equations for the Crank-Nicholson method and we have followed their sample program.

The Fokker-Planck equation describes the diffusion of cosmic rays in the interplanetary magnetic field, but the relationship between the diffusion coefficient and the magnetic field is not stated. Roelof (1966) and Jokipii (1966) derived relationships between the diffusion coefficient and the power spectrum of the interplanetary
magnetic field. Their theories employed the concept of resonant scattering which asserted that a particle was most likely to be scattered by a magnetic field structure whose wave length corresponded to the particle's gyro radius. Resonant scattering predicted that if the power spectrum of the interplanetary field had a spectral index, $-\gamma$, in frequency, then the diffusion coefficient has a spectral index, $2-\gamma$, in rigidity. Verification of these theories has proved quite difficult. It was discovered that significant portions of the power of the interplanetary field occurred in the discontinuities, rotational and tangential, and it was never clear how the power in the discontinuities should be included in the theory of resonant scattering. Klimas and Sandri (1971), and subsequently several workers, have criticized the resonant theory as inadequate to explain the cosmic ray diffusion. The resonant theory is based on what is called the quasi-linear approximation, that is, that scattering is a "small" perturbation to the particle's orbit. In a general sense, the validity of the quasi-linear approximation has been questioned; and specifically, the resonant theory does not allow particles at 90 degree pitch angles to be scattered, which they certainly must be or else the isotropy of the cosmic rays would vanish. More advanced theories that include the higher-order correlation terms and certain other non-linear effects have recently been presented in an attempt to explain precisely how the magnetic field scatters
the cosmic rays (Jones, et al., 1976).

Additionally, the overall power level in the interplanetary magnetic field was envisioned to vary with solar cycle and thereby account for the diffusion coefficient's variation. Quiet time measurements near the earth of the magnetic field power showed little or no variation with solar cycle (Quenby, et al., 1973). The reason why the diffusion coefficient varies with solar cycle is currently unknown. The resonant theory of scattering does not adequately explain the diffusion phenomenon.

The particle spectra of electrons, protons, and alpha particles observed at earth constitute the empirical evidence for the solar modulation. In addition, the radial gradient of protons and alphas has recently been measured (McDonald, et al., 1977). The radial gradient is formally defined as

\[ G = \frac{1}{U} \frac{3U}{\delta r} \]

The integral of the radial gradient at a fixed energy from the earth to the boundary is the ratio at that energy of the interstellar density to the density at earth. The nucleonic gradient measurements place two constraints on the Fokker-Planck model. The radial dependence of the diffusion coefficient must be chosen so that the small gradients (a few % per AU) observed at earth are duplicated by the model. Secondly, the gradient's variation in energy and solar radius is indicative of the interstellar spectrum. Because
of convection and energy loss, the interstellar spectrum below roughly 200 MeV/nuc is obscured at earth. Whether the interstellar spectrum rises at low energies can be seen qualitatively from the gradient. If the maximum in the gradient with energy shifts to lower energy with increasing solar radius, a rising interstellar spectrum is indicated.
CHAPTER IV

DERIVATION OF THE LOCAL INTERSTELLAR ELECTRON SPECTRUM

The existence of cosmic-ray electrons was postulated by Ginzburg (1953) who suggested that the diffuse radio background was synchrotron emission from cosmic electrons. The presence of a galactic magnetic field had just been confirmed by studies of light reflected and polarized by magnetic dust grains aligned with the galactic magnetic field. This early concept of Ginzburg is now accepted and the directional variation of the radio emission provides a map of the spatial distribution of electrons in the galaxy. The radio frequency background consists of a component from the disk and a roughly spherical component thought to originate from a halo about the galaxy. The best supportive evidence for a radio halo about our galaxy is the occasional occurrence of a halo about other nearby galaxies, and in particular, about Andromeda which is of similar spiral structure. If the galaxy has a halo, the matter density and magnetic field strengths are essentially unknown and the star field consists only of a few globular clusters.

The structure of the galaxy must be considered in evaluating the radio frequency emission because the intensity seen in any direction is the line of sight average of the volume emissivity of the synchrotron radiation which depends on both the magnetic field and the electron
spectrum. When looking out of the galactic plane, the synchrotron radiation from both disk and halo is observed. It is likely that the magnetic field strengths in the halo are less than those in the disk and it is also likely that the electron spectra of the two regions differ as noted earlier. We shall therefore restrict our study of the radio intensity to the galactic plane and in particular to the galactic anti-center direction. At low frequencies, the synchrotron radiation is absorbed in the interstellar medium by the process of free-free absorption. In the direction of the galactic center, free-free absorption is significant below ~30 MHz, but in the anti-center direction, the absorption affects the spectrum only below ~10 MHz. In the galactic-center direction, there is on the average more absorber between the sun and the regions where the synchrotron emission is occurring than in the anti-center direction.

We shall follow the current literature and first illustrate the derivation of a local interstellar electron intensity from the radio frequency emission spectrum in the anti-center direction. As the electron intensity has been traditionally calculated, there has been, not only disagreement between the supposed and the measured local emissivities, but also a large uncertainty in the resulting electron spectrum itself. We shall then improve upon this derivation by requiring consistency with observation for both the high energy electrons and the local emissivity. We
shall show that our new calculation agrees with the estimate of the interstellar electron spectrum deduced from the positron ratio.

IV.1 Previous Estimates Of The Local Interstellar Electron Spectrum

The emissivity formula, eqn 2.6, can be related to the measured intensity by considering the galactic geometry. Consider the r-f spectrum from 10 MHz to 10 GHz observed in the anti-center direction. The sun is thought to be located on the inner edge of the local spiral arm, called Orion. So in the anti-center direction one is looking through the Orion arm and the next major arm, Perseus, which lies in that direction. Beyond Perseus it is thought that there might be a rather patchy arm, and then comes the faint outer arm of the galaxy. It is usually assumed that the magnetic field and electron distribution are uniform to the "edge" of the galactic disk which is typically placed 5 Kpc away—just beyond Perseus. The intensity seen is then the integral of equation 2.6 along the line of sight, which is computed with the following formulas:

\[
I(\text{MKS}) = a(\gamma) \frac{\gamma+1}{2} K \frac{L}{(\text{cm})} B \frac{12}{(G)} \left[ \frac{6.26 \times 10^{18}}{v(\text{Hz})} \right] \frac{\gamma-1}{2};
\]

\[
a(\gamma) = 9.33 \times 10^{-24} \frac{\gamma-3}{2} \frac{\gamma+7/3}{\gamma+1} \left( \frac{3\gamma-1}{12} \right)^{3/2} \left( \frac{3\gamma+7}{12} \right)
\]
\[ \frac{\partial N}{\partial E} \left( \frac{e^{-}}{m^2 \text{ster-sec-GeV}} \right) = N E^{-\gamma} \text{ (GeV)}; \]

\[ N(E) \left( \frac{e^{-}}{\text{cm}^3 \text{erg}} \right) = K E^{-\gamma} \text{ (ergs)}; \]

\[ K = \left[ \frac{4\pi}{3} 10^{-14} \left( 1.60184 \times 10^{-3} \right) \right] N \]

\[ v_m \text{ (MHz)} = 0.299 [16.1 B_\perp (\mu G) E^2 \text{ (GeV)}] \]

Cummings (thesis, 1973) has derived the local interstellar electron spectrum from the radio frequency emission in the anti-center direction, using in essence the above argument. (See also Daniel and Stephens (1975) for an essentially similar argument.) His model is complex and includes free-free absorption and the Bazin effect, both important at low frequencies. Above 10 MHz however, these effects are negligible and we shall discuss his model at the higher frequencies where it greatly simplifies.

The radio frequency data he uses are described by a power law segment in frequency with spectral index \(-0.4\) over the frequency range of 10 MHz to 150 MHz; above 150 MHz the curve is approximated by a power law of index \(-0.7\) or \(-0.8\). Cummings chooses reasonable galactic parameters with which to calculate the electron spectrum from the formula expressed by equations 4.1-4.3. Pertaining to frequencies above 10 MHz, the two determining parameters are the perpendicular
component of the B field (range 3-5 microgauss, nominally 5) and the line of sight distance in the anti-center direction L (range 2-6 Kpcs, nominally 4). His nominal electron spectrum, shown in Figure 4.1, is identical to the one derived by Goldstein, Ramaty, and Fisk, 1970. Limits on his spectrum are established by varying the galactic parameters. He himself notes "that at high energies there is roughly a factor of four between the bracketing lower and upper spectra."

Cummings also compares his nominal galactic electron spectrum to one deduced from the measured positron ratio. In order to obtain agreement he finds it necessary to propagate the cosmic ray nuclei through 10 gm/cm² of interstellar material. The fragmentation studies have shown, however, that the cosmic ray path length is ~5 gm/cm². Thus, his nominal electron spectrum is a factor of two higher than what a proper estimate of the electron spectrum from the positron ratio should be.

In formal appearance the synchrotron formulas relate the observed radio intensity to the electrons; however, the emissivity is the item of direct physical significance. Later in this thesis we report results on the local emissivity as deduced from measurements toward nearby H II regions. A comparison of this local emissivity to the emissivity inherent in Cummings' calculation is shown in Figure 4.2 (See also Webber, 1977.). There appears to be at
Figure 4.1 Cummings' estimate of the interstellar electron spectrum.
Figure 4.1

Electrons / m²-ster-sec-MeV

10²

10¹

10⁰

10⁻¹

10⁻²

10⁻³

Kinetic Energy (GeV)

1 9 6 5 - 6 6
Electrons at Earth

Dashed Lines are Cummings' Limits

Cummings' Nominal Interstellar Electron Spectrum
Figure 4.2 H II emissivity measurements are contrasted with the emissivity deduced by Cummings in his calculation of the interstellar electron spectrum.
Observe Electrons in 1965-66

Electron Energy (GeV) for B=5μG

Frequency (MHz)

Figure 4.2
least a factor of two between the average of the observed emissivities in the direction of the H II regions and that emissivity which corresponds to Cummings' nominal electron spectrum.

The background emissivity has often been compared to the measured electron spectrum under the assumption that the B field is between 3 and 5 microgauss. Several researchers have noted the discrepancy that there are insufficient electrons to account for the observed emissivity when this comparison is made, and two possible solutions have been offered. Cowsik and Mitteldorf (1974) have suggested that electron density fluctuations in the galaxy may correlate with magnetic field fluctuations producing a calculated emissivity more in accord with the observed emissivities. Freier, Gilman, and Waddington, (1977) have suggested that if gas clouds which comprise ~2% of the line of sight contain strong magnetic fields, say 70 microgauss, the quadratic B-field dependence of synchrotron emissivity is sufficient to account for the synchrotron emission with a mean galactic field of about 2 microgauss.

IV.2 Our Calculation Of The Interstellar Electron Spectrum

In order to resolve the disparity between the observed and the calculated emissivities, we shall normalize the path length, L, and the electron intensity, N, to the physical measurements which they represent. We shall determine an
integration path length in the anti-center direction which is consistent with the emissivities toward the further H II regions which lie in the anti-center direction. The calculated electron spectrum will then be made to agree assymtotically at high energies with the electron data that was discussed earlier in Chapter II.

Once L and N are specified, B is determined. The value of B is larger than what one would expect the average galactic field to be based on the above arguments. At this time we realize that there is a distribution of magnetic field strengths within the gas in the galaxy. The synchrotron formulas in Chapter II are modified to describe the radio frequency radiation of electrons in a region of varying B. With this interpretation, the local interstellar electron spectrum will be calculated from the anti-center radio frequency spectrum.

IV.2.A Effect of the B-field Distribution

In the traditional formulation it has been tacitly assumed that the B field is spatially uniform. Instead, the galactic B-field magnitudes will in all likelihood be distributed corresponding to portions of the spiral arms which have strong fields and to portions of the arms where the fields are weaker. On the average, the cosmic-ray electron ensemble will see the distribution of magnetic field strengths. The galactic B field is frozen into the
interstellar gas and, hence, the B-field strength should fluctuate with the gas density. Since distributions occurring in nature are frequently observed to obey the lognormal distribution (Aitchison and Brown, 1957), we suggest that the distribution of interstellar B-field strengths is likely to be lognormal. Although the lognormal distribution can approach a delta function, for finite variance the distribution is skewed by the presence of a tail.

How is it that we may suppose a B-field distribution in the galaxy when there are not direct measurements in support thereof? The interplanetary magnetic field is an accessible astrophysical phenomenon which most closely resembles the galactic field. Both fields originate within turbulent dynamos and both are frozen into their respective plasmas. The interplanetary magnetic field strength has been measured by Mariani, et al., (1975) who reported the distributions of daily average field strength. In general appearance, the distributions are skewed even though the averaging process tended to deskew the distribution of the instantaneous field strength. Figure 4.3 shows the distribution of daily field averages for 1972 (Mariani, et al., 1975). To emphasize the magnitude of the root-mean-square value of the field data, we also show, in the Figure, a lognormal distribution whose rms value is twice its most probable and one and one-half times its average. The random component of the interstellar field is thought to be comparable to the average
Figure 4.3 The distribution of interplanetary magnetic field strengths observed by Mariani in 1972 is shown by the solid line. The dashed line is a lognormal distribution characteristic of the interstellar field needed to account for the high synchrotron emissivities.
Figure 4.3

Interplanetary Field (1972)
Interstellar Field $B_{rms} = 2 \times B_{mp}$
interstellar field (Parker, 1971). It is well known that the skewedness of the field distribution increases as the magnitude of its random component increases. Since the random component of the galactic field is proportionately greater than the random component of the interplanetary field, it is reasonable to assume that the galactic field is at least as skewed as is the interplanetary field.

The effect of a galactic B-field distribution on the synchrotron emission is very interesting; it allows for an increase in the radiated synchrotron energy without requiring unrealistically large average B-field values. We shall now argue that the predominant effects can be evaluated with the above formulas using a root-mean-square $B$, $B_{rms}$, and an effective $B$, $B_{eff}$, from the B-field distribution.

Consider the simple case of a galaxy composed of two equally sized regions, randomly intermixed, of field strengths $B_0$ and $2B_0$ respectively. An isotropic, power law electron spectrum fills the model galaxy and emits synchrotron radiation. Firstly, the spectral shapes of the synchrotron emission from the two regions are the same, but $4/5$ of the intensity originates in the region of stronger B-field, $2B_0$. The distribution of intensity with frequency is given by $F(x) + F(x/2)/2$, where $F$ is the frequency dependent portion of $P(nu)$. Secondly, the energy of the electrons which produced the radiation at a given frequency
is different in the two regions and we see that the critical frequency also depends on the B-field distribution. Since each individual particle integrates the perpendicular component of B as it travels, a B-field distribution, \( p(B) \), requires the power formulas to be evaluated at \( B_{\text{rms}} \). Equation 4.1 then becomes

\[
I(\text{MKS}) = a(\gamma) K L (\text{cm}) \frac{\gamma+1}{2} B_{\text{rms}}^{\gamma-1} \left( \frac{6.26 \times 10^{18}}{\nu(\text{Hz})} \right) \]

Now the power radiated is distributed in frequency by the MacDonald functions which are to be weighted as the power itself is weighted—that is, according to \( B^2 p(B) \). A single electron of energy, \( E \), will emit maximum radiation at the maximum of the convolution of the MacDonald function and \( B^2 p(B) \). Let \( B_{\text{eff}} \) be the value where \( B^2 p(B) \) attains its maximum, then the frequency of maximum emission is

\[
\nu_m (\text{MHz}) = 4.81 B_{\text{eff}} (\mu\text{G}) E^2 (\text{GeV})
\]

For that particular lognormal distribution shown above, the effective \( B \) equals the root-mean-square \( B \). \( B_{\text{eff}} \) and \( B_{\text{rms}} \) are in general different values; for example, the class of gamma-distributions has an \( B_{\text{eff}} \) slightly greater than \( B_{\text{rms}} \).

**IV.2.B The Local Emissivity As Deduced From H II Region Studies And The Integration Path Length L**
The local emissivity can be measured to nearby H II regions under the assumption that (1) the H II region is opaque to radio frequency radiation, (2) it fills the beam of the radio telescope, and (3) its distance is known (Shain, 1958). We have tabulated the intensity measurements in the direction of the H II regions and analysed them to deduce estimates of the local emissivity (Table 4.1) (See also Cane, 1977). These data have been plotted as a function of frequency in Figure 4.2. At first glance, there is a dispersion in the emissivity measurements by a factor of 5 (8.0 x 10^{-4} MKS to 4.0 x 10^{-4} MKS) at 10 MHz, for example; however, this can be partially interpreted in terms of the spiral structure of the galaxy. Supposedly, the emission from the inter-arm regions will be less than that from the arms so that a path length directly into a spiral arm will have a greater emissivity than one into an inter-arm region. Measurements toward the galactic center are generally higher than those toward the galactic anti-center. This behavior is more easily seen in Figure 4.4 which shows the emissivity as a function of galactic longitude superimposed on the spiral arm structure.

In general, the emissivities deduced from the H II region studies are a factor of ~2 higher than those implied by Cummings' (1973) work (See Figure 4.2). This would suggest that the best value for the path length in the anti-center direction is ~2 Kpc instead of the value of 4 Kpc used by Cummings. A more direct determination of the
Figure 4.4 The emissivities measured toward the nearby H II regions in the disk are shown as a function of galactic longitude. Observe the variation in emissivity which indicates the spiral structure of the galaxy.
Figure 4.4
### TABLE 4.1

EMISSIVITIES OF THE H II REGIONS

<table>
<thead>
<tr>
<th>Source</th>
<th>Kpc</th>
<th>Direction (b,l)</th>
<th>Resolution</th>
<th>MHz</th>
<th>10^{-4.0}</th>
<th>MKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gum Nebula</td>
<td>0.35</td>
<td>263°,-5°</td>
<td>∼2.8°</td>
<td>2.1</td>
<td>3.0</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>∼2.5°</td>
<td>4.7</td>
<td>2.2</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>∼2.0°</td>
<td>9.0</td>
<td>2.2</td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>∼1.5</td>
<td>19.7</td>
<td>1.5</td>
<td>(4)</td>
</tr>
<tr>
<td>NGC-1499</td>
<td>0.44</td>
<td>160.5°,-12.5°</td>
<td>∼2.0°</td>
<td>10.0</td>
<td>2.7</td>
<td>(5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>∼2.0°</td>
<td>13.1</td>
<td>2.7</td>
<td>(6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>∼1.8°</td>
<td>50.0</td>
<td>1.4</td>
<td>(7)</td>
</tr>
<tr>
<td>IC-405</td>
<td>0.65</td>
<td>172.5°,-1.5°</td>
<td>∼2.0°</td>
<td>10.0</td>
<td>2.5</td>
<td>(5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>∼2.0°</td>
<td>13.1</td>
<td>1.6</td>
<td>(6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>∼1.6°</td>
<td>50.0</td>
<td>0.45</td>
<td>(7)</td>
</tr>
<tr>
<td>RCW-113</td>
<td>1.30</td>
<td>345°,+2.0°</td>
<td>∼2.0°</td>
<td>8.3</td>
<td>4.1</td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>∼1.5°</td>
<td>16.5</td>
<td>3.9</td>
<td>(3)</td>
</tr>
<tr>
<td>IC-1805</td>
<td>2.20</td>
<td>134.5°,+1.0°</td>
<td>∼2.0°</td>
<td>10.0</td>
<td>0.9</td>
<td>(5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>∼2.5°</td>
<td>22.6</td>
<td>0.8</td>
<td>(8)</td>
</tr>
<tr>
<td>NGC-7822</td>
<td>0.85</td>
<td>118.5°,+5.0°</td>
<td>∼2.0°</td>
<td>10.0</td>
<td>2.5</td>
<td>(5)</td>
</tr>
<tr>
<td>L-Ori-neb</td>
<td>0.50</td>
<td>196.8°,-12.0°</td>
<td>∼2.0°</td>
<td>10.0</td>
<td>2.9</td>
<td>(5)</td>
</tr>
<tr>
<td>NGC-2264</td>
<td>0.71</td>
<td>203.0°,+2.5°</td>
<td>∼2.0°</td>
<td>10.0</td>
<td>2.7</td>
<td>(5)</td>
</tr>
<tr>
<td>Barnard Loop</td>
<td>0.55</td>
<td>208.0°,-20.0°</td>
<td>∼2.0°</td>
<td>10.0</td>
<td>2.6</td>
<td>(5)</td>
</tr>
<tr>
<td>Carina Neb</td>
<td>2.50</td>
<td>287°,-1.5°</td>
<td>∼1.5°</td>
<td>13.0</td>
<td>0.9</td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>∼0.8°</td>
<td>29.9</td>
<td>0.7</td>
<td>(9)</td>
</tr>
<tr>
<td>RCW-110</td>
<td>1.25</td>
<td>341.8°,-0.8°</td>
<td>∼1.5°</td>
<td>19.7</td>
<td>1.9</td>
<td>(4)</td>
</tr>
<tr>
<td>RCW-132</td>
<td>1.40</td>
<td>355.4°,0.2°</td>
<td>∼1.5°</td>
<td>19.7</td>
<td>1.4</td>
<td>(4)</td>
</tr>
<tr>
<td>RCW-146</td>
<td>1.30</td>
<td>6.6°,-1.5°</td>
<td>∼1.5°</td>
<td>19.7</td>
<td>2.8</td>
<td>(4)</td>
</tr>
<tr>
<td>RCW-108</td>
<td>1.30</td>
<td>337°,-1.5°</td>
<td>∼0.8°</td>
<td>29.9</td>
<td>3.4</td>
<td>(10)</td>
</tr>
<tr>
<td>W-51</td>
<td>6.10</td>
<td>49.2°,-0.35°</td>
<td>1.8°</td>
<td>50.0</td>
<td>1.4</td>
<td>(7)</td>
</tr>
<tr>
<td>NGC-2175</td>
<td>2.0</td>
<td>190°,0.43°</td>
<td>1.8°</td>
<td>50.0</td>
<td>0.7</td>
<td>(7)</td>
</tr>
<tr>
<td>NGC-2244</td>
<td>1.5</td>
<td>206°,-2°</td>
<td>1.8°</td>
<td>50.0</td>
<td>1.1</td>
<td>(7)</td>
</tr>
</tbody>
</table>

(1) Beuermann, 1972  
(2) Ellis and Hamilton, 1966  
(3) Cane, 1975, 1977  
(4) Shain, 1959  
(5) Caswell, 1976  
(6) Andrew, 1969  
(7) Parrish, 1972  
(8) Roger, 1969  
(9) Jones, 1973  
(10) Jones and Finlay, 1974
value of \( L \) may be made using the following argument. The radio frequency background intensity is the integral of the volume emissivity over an unknown path length. This path length can be estimated by requiring an emissivity similar to the H II regions (>1 Kpc distant) in the anti-center direction. Let us consider an example. NGC 2175 is an H II region 2 Kpc distant in the anti-center direction \((l=190^\circ, b=0.43^\circ)\) and Parrish has measured its emissivity at 50 MHz to be \(0.7 \times 10^{-40} \) MKS. From Figure 2.2 we see that the intensity of the background radiation at 50 MHz is \(6.2 \times 10^{-21} \) MKS. The emissivity in the anti-center direction would be the same as that to NGC 2175 if the anti-center path length were 2.9 Kpc. The average of the anti-center path lengths deduced in this manner is 2.5 Kpc and their nominal range is the interval 2-3 Kpc.

IV.2.C Our Construction Of The Interstellar Spectrum

The galactic radio frequency spectrum does not obey a single power law in frequency, but rather, at 10 MHz it has a spectral index of about \(-.35\) which gradually steepens to \(-.95\) at 10 GHz (See Figure 2.2.). It is not reasonable to approximate the emission data with a single power law, or perhaps, with even two power law segments. Ginzburg and Syrovat-skii have shown by construction that a power law segment on \([\nu_1,\nu_2]\) in radio emission frequency results from a power law electron distribution over a somewhat greater energy interval than the expected interval \([E_1,E_2]\)
where each $n_u$ is 0.299 times the critical frequency associated with energy $E_i$. The conversion formula at the interval midpoint is accurate, however. This suggests that an electron spectrum can be constructed pointwise from the radio frequency emission. This construction approximates the solution of an integral equation. The error of the construction is estimated to be of order 2-3%.

The interstellar electron spectrum is constructed from the radio frequency data as follows: First the electron spectrum is normalized to the measured spectrum at high energies where the solar modulation is small. We normalize the interstellar electron spectrum at 4 GeV to a differential directional intensity of $2.8 \pm 0.4 \times 10^{-3}$ Peters (See Figure 2.5.). The electron spectrum measured at earth has a power law exponent of $-3$ in the energy region above ~4 GeV. It is seen in Figure 2.5 that this corresponds to a normalization constant of $160 \pm 40$ for an $E^{-3}$ spectrum—consistent with the high energy data. The spectrum measured at earth at solar minimum is expected to differ from (to be less than) the interstellar spectrum as a result of the residual solar modulation. Above a few GeV the solar modulation at sunspot minimum is believed to be small, less than 10%, and the interstellar spectrum should be of roughly the same slope and of slightly higher intensity than that measured at earth. This normalization is therefore taken to be about 10% higher than Buffington's measurement which we consider to be the most
reliable in this energy range. In effect, we view the radio frequency spectrum as continuing at higher frequencies with a spectral index of \(-0.95\) which corresponds to an electron spectral index of \(-2.9\) which is consistent with the measured index of \(-3.0 \pm 0.2\).

The normalization is completed by requiring the path length, \(L\), to be in the range 2-3 kpc which is consistent with the emissivities deduced from the \(\text{H II}\) region studies. This overall normalization then specifies the final parameter, the \(B\) field. An rms field between 7.1 and 10.5 microgauss is needed to normalize this electron spectrum to the radio frequency emission. It is emphasized again, that this is the rms perpendicular component of the \(B\)-field distribution; the corresponding value of the most probable \(B\) is 4-5 microgauss, from the argument in a preceding section.

The interstellar electron spectrum is constructed point by point from the galactic radio spectrum using equations 4.4 and 4.5. Spectral-index consistency attests to the validity of this approach. The interstellar electron spectrum deduced in this manner is shown in Figure 4.5 with the label, \(N=160, L=2.5\). This spectrum is similar to that derived by Daniel and Stephens (1975) using a simple normalization procedure.
Figure 4.5 Our estimate of the local interstellar electron spectrum is shown as the curve labelled $160,2.5$. The enclosed area is the bound set by our normalization procedure.
Electrons/m$^2$ ster-sec MeV

$B_{\perp \text{max}} = 10.47 \mu G$ 140-2Kpc

$B_{\perp \text{min}} = 7.10 \mu G$ 200-3Kpc

Figure 4.5
The central question that this normalization has answered is the discrepancy between the intensity of the radio emission that is observed and the intensity of the radio emission that is calculated from the measured electron spectrum assuming a 3-5 microgauss mean magnetic field in the galaxy. In the important energy range of 5-15 GeV, the extension of our normalized electron spectrum is in reasonable agreement with many of the experiments. However, a few experimenters have reported higher fluxes. Although the high electron fluxes are more in agreement with the surfeit of radio emission that is observed, we have presented an alternative picture that has not only removed the above discrepancy but also facilitated our understanding that the low measurements are correct.

We wish to point out that whatever the reasoning, it is important in constructing interstellar spectra from the radio frequency background to ensure that the constructed spectrum agrees with one's measured electron spectrum at energies where solar modulation is negligible.

IV.3 The Interstellar Electron Spectrum Deduced From The Positron Ratio

An alternative method of deducing the interstellar electron spectrum utilizes the measured positron ratio. Several workers have taken this approach (See Daugherty, et al., 1975, and Hartman and Pellerin, 1976.). We believe that in view of the uncertain positron measurements and
uncertainties in the calculated positron intensity, this approach is basically less accurate than the one we have used; however, good form requires us to reproduce these arguments here.

Cosmic ray electrons originate from two possible mechanisms: direct acceleration and nuclear interaction. Electrons may be accelerated along with the nuclei and these are called primary electrons. Since it is likely that the source material for the cosmic rays includes only negative electrons [any positrons would annihilate], the primary electrons are thought to be exclusively negatrons. As the cosmic ray nuclei traverse the interstellar medium, they interact strongly with the interstellar gas. The interaction debris includes nuclear fragments and created pions. The production of light nuclei by fragmentation indicates that the cosmic rays have traversed about 6 $\text{gm/cm}^2$ of interstellar material which numbers 90% hydrogen and 10% helium. The gamma ray emissivity observed along the disk provides evidence that these interactions are occurring. The neutral pi mesons decay immediately to gamma rays which escape from the galaxy before undergoing pair production. The charged pi mesons, however, are trapped in the galactic magnetic field and eventually decay to electrons, both negatrons and positrons. These electrons are called secondary electrons. The production rate of pi$^+$ mesons is greater than the production rate for pi$^-$ mesons, and thus a majority (about 70%) of the secondary electrons
are positrons.

Our knowledge, a priori, of the primary electrons is very limited. Analyses of simple acceleration mechanisms predict in general much smaller electron-to-proton ratios (about 1:100000) than the one which is empirically observed (1:50) (Ramaty, 1974). In effect, we have no explanation for the abundance of directly accelerated electrons. On the other hand, the galactic flux of secondary electrons can be calculated because the cosmic-ray nuclei path length and the interaction cross sections are thought to be known. Measurement of the positron ratio, therefore, indirectly provides information about the primary electron fluxes. Since all positrons are secondary, we have:

\[
\frac{e^+}{e^+ + e^-} = \frac{\text{secondary positrons}}{\text{interstellar electrons}} \quad 4.6
\]

Thus, the measured positron ratio can be set equal to the calculated secondary positron spectrum over the galactic electron spectrum.

Calculation of the secondary electron and positron production spectrum has been performed by Perola, Scarsi, and Sironi (1967) and confirmed by Ramaty (1972). The secondary negatron and positron densities per gram of interstellar material traversed by the nuclei are shown in Figure 4.6. The lines are Ramaty's calculations which are based on more complete cross sections than Peroni, Scarsi, and
Figure 4.6 The secondary positron production spectrum calculated by Peroni, Scarsi, and Sironi is shown by the circles. The curves are the later calculation by Ramaty.
Electrons/gm-sec-\gamma

Figure 4.6
Sironi's calculations which are shown by the circles. There is of late complete agreement between the two calculations.

The lifetime of the cosmic-ray nuclei is of order ten million years and consequently, the local secondary electrons must be roughly that age. The effect of ageing of the secondary electron spectrum is taken into account by modelling the galactic propagation of cosmic rays. There are two major conclusions from the propagation models: (1) Synchrotron emission steepens the aged spectrum above a certain energy (Syrovat-skii, 1958) and (2) transport of energetic electrons several Kpc in the disk of the galaxy is not allowed by current propagation models (Beedle, 1970). The transport is only important for primary electrons, since the secondary electrons are locally produced. As a result the galactic center is excluded as a possible source of the primary electron component. The effect of synchrotron energy loss on the secondary electrons is negligible up to at least 1 GeV and probably is unimportant over the entire energy range where the positron ratio has been accurately measured. Thus, galactic propagation does not much effect the production spectrum of the pi-mu-e decays.

We now wish to utilize the above arguments along with the calculated positron spectrum to deduce an interstellar electron spectrum using equation 4.6.
The positron ratio presents a difficult measurement; its history is so varied that no consensus among the measurements has yet evolved—especially at low energies. The published measurements of the positron ratio are shown in Figure 4.7. Below 100 MeV the experimental points are uncertain and we shall confine our attention to higher energies. We assume that the only source of galactic positrons (above 1 MeV) is the decay of secondary pions. A band is drawn through the positron ratio data points and is representative of the limits which we have assigned to the measurement.

The effect of solar modulation on the positron ratio can be thought of as distinct from the modulation of the positrons themselves. The solar modulation may severely attenuate the electron component in intensity; however, both the positrons and the sum of positrons and negatrons are diminished in the same fractional amount. Therefore, the positron ratio is not directly affected by the intensity reduction produced by the modulation. As particles enter the solar cavity, they lose energy, and both positrons and negatrons will lose the same amount. The solar modulation, therefore, shifts the galactic positron ratio down in energy as it is being modulated. The amount of this energy shift, called the energy loss, is determined by the diffusion coefficient which varies with solar cycle. Above 100 MeV, the positron ratio depends weakly on energy, and, hence, the effect of the modulation over this energy range on the
Figure 4.7 The positron ratio, the secondary galactic positron spectrum, and the inferred interstellar electron spectrum are shown. A banded region indicates our estimate of the positron ratio uncertainty. The secondary positrons have been calculated using Ramaty's production spectrum. The shaded interstellar electron spectrum includes the positron uncertainty and the range of path lengths from 5 to 7 grams of interstellar gas. The dotted line shows our interstellar electron spectrum determined from the radio frequency background.
Particles/m²-ster-sec-MeV

Interstellar Electrons

Radio Normalization

Calculated $e^+$ for $6 \pm 1 \text{ g/cm}^2$

$e^+$ for $6 \pm 1 \text{ g/cm}^2$

Energy (GeV)

Data Uncertain

Ratio

Figure 4.7

Beuerman, et al. (1970)
Daugherty, et al. (1975)
Fanselow, et al. (1969)
Hartman + Pellerin (1975)
Buffington, et al. (1975)
The positron ratio is small compared with the error band in Figure 4.7. The accuracy of the positron ratio measurement does not at this time warrant the inclusion of the small modulation effect.

An alternate procedure of analysis would be the direct modelling of the solar modulation of the positron component (See Beuermann, et al., 1970.). This procedure is based on the secondary interstellar positron spectrum, and is algebraically equivalent to the method of analysis presented above. Therefore, since the effects of the modulation are small, our simpler analysis produces the same result as does Beuermann's procedure.

The positron production spectrum obtained by propagating the cosmic-ray nuclei through 6 gm/cm² is shown in Figure 4.7. The cosmic-ray pathlength can be reasonably bounded between 5 and 7 gm/cm² as a result of complex arguments explaining the observed abundances of the various secondary, cosmic-ray nuclei. The general trend, however, shows evidence that the path length of the cosmic-ray nuclei is energy dependent. The current uncertainty in the path length as a function of energy further diminishes the quantitative validity of the positron-ratio analysis. We have chosen a path length of 6±1 gm/cm² and do not include the energy dependence.
The interstellar electron spectrum calculated using equation 4.6 is shown in Figure 4.7. The interstellar electron spectrum is shown as a band which includes both the uncertainty in the positron ratio and the uncertainty in the calculated positron spectrum. The "average" interstellar electron spectrum deduced from the radio frequency data is shown in the Figure as a dotted line. Although the spectra agree rather well, we again point out that the agreement is probably fortuitous because of the path-length approximations and uncertain data associated with the positron calculation.
CHAPTER V

DETAILS OF THE SOLAR MODULATION PROBLEM

We shall present solutions to the Fokker-Planck equation that represent the modulated particle spectra observed at the orbit of the earth. The representative functional dependencies of the diffusion coefficient have been obtained from solar particle propagation studies of observed particle diffusion. Using diffusion coefficients modelled from those obtained from the solar particle studies, the modulated spectra of electrons have been calculated. The modulation parameters determined from the electron data are then used to modulate the protons and helium nuclei, keeping the interstellar spectra of these nuclei a free parameter.

V.1 Selection Of The Diffusion Coefficient

Whereas studies of the galactic modulation of protons and helium nuclei yield little information on the functional dependences of the diffusion coefficient, such is not the case with solar particle propagation. Solar particle events are transient phenomena whose time scale of hours is quite distinct from the steady-state character of galactic modulation. The time structure of the event depends directly on the diffusion rate and hence is sensitive to the functional form of the diffusion coefficient. Zwickl
(thesis, 1976) has studied solar flare protons (at 3 and 30 MeV) observed by Pioneers 10 and 11 in order to determine their mean free path from 1 to 5 AU. Solar flares whose hydrogen alpha emission could be uniquely identified were selected according to solar longitude. Zwickl then catalogued 5 parameters that characterized the time profiles of the flare events as observed at the spacecrafts:

1. Tmax  
   time from H-alpha to maximum flux at the spacecraft
2. J(Tmax)  
   flux ratio of the spacecraft to the earth
3. Zeta(Tmax)  
   anisotropy at Tmax
4. delta t 5  
   duration of the flare protons
5. Tau  
   decay constant early into the decay phase

These parameters were statistically compared with the corresponding parameters calculated from the time dependent Fokker-Planck equation in order to fit a diffusion coefficient of the form \( K1 + K2 \cdot r^{**BETA} \). Zwickl found that the diffusion coefficient was constant with radius, \( BETA = 0.3 \), from 1 to 5 AU. He reports a diffusion coefficient of \( 1.2 \times 10^{21} \text{ cm}^2/\text{sec} \) for 3.4-5.2 MeV protons and of \( 2.6 \times 10^{21} \text{ cm}^2/\text{sec} \) for 24-30 MeV protons. Because of the velocity dependence, the mean free path is smaller at 24-30 MeV than at 3.4-5.2 MeV. The decrease in mean free path with increasing rigidity is atypical and has not been predicted by any of the scattering theories. Zwickl and Webber (1977) have derived the mean free path from both solar proton and electron events for the rigidity
range 1-3000 MV by analysing Tmax alone. An analysis based on a single parameter will tend to underestimate the mean free path but will, nevertheless, provide a relative comparison of mean free path with rigidity. Figure 5.1 shows the mean free path as a function of rigidity under the assumption that the radial dependence is constant. All points except the two Zwickl thesis points (marked Z) are from the analysis based on Tmax. The Zwickl points are based on all 5 parameters and are, therefore, more accurate. We interpret the set of analyses based on one parameter as defining the functional dependence with rigidity and the Zwickl points as suggesting the normalization and verifying the negative slope at 100 MV.

Following the suggestion by Gleeson and Urch, we shall refer to the normalization of the diffusion coefficient in terms of the "force-field" energy-loss parameter, phi, calculated at a 1 GV rigidity. Phi is also called the depth of modulation. The formula defining phi has been presented in equation 3.12, and is shown below.

\[ \phi(MV) = \frac{1}{3} \int_{r_{E}}^{r_{B}} \frac{V(r',t)}{K_{1}(r',t)} \, dr' \]

5.1

Note that phi differs by a factor of 3 from n(t), which also has been referred to as the depth of modulation in the earlier diffusion-convection model. Phi is chosen because it has the physical dimension of momentum and specifies the energy loss at energies where the diffusion coefficient
Figure 5.1 The Zwickl and Webber estimate of the mean free path is shown with rigidity. Zwickl's thesis points, marked Z, are better estimates than the other points are. The other points are indicative of the functional dependence, while the Zwickl points indicate the overall normalization.
Figure 5.1

- Zwickl and Webber (1977)
- Zwickl Thesis (1976)
- Lanzerotti et.al. (1975)
- Hamilton (1976)
- Datlowe (1971)
- Reinhard and Wibberentz (1974)
- McKibben et.al. (1975)

Note: Above data modified by Zwickl and Webber
varies linearly with momentum. Phi indicates the "size" of the diffusion coefficient. Phi may be used to compare not only those diffusion coefficients representing different time periods, but also those of different workers for the same time period.

The interstellar electron spectra derived in Chapter IV along with the measured electron spectra at earth are used to estimate diffusion coefficients in each of their respective years. In an attempt to represent accurately the data of Zwickl and Webber the diffusion coefficient was represented by the following functional form:

\[ K = \beta K_1(r) K_2(P) \]  

\[ K_1(r) = \frac{V(r_B - r_E)}{3\phi} \]  

\[ K_2(P) = \begin{cases} 
P & P_b < P \\
P_b & P_c < P < P_b \\
\frac{K_{c-b}}{P_{b-c}} & P_m < P < P_c \\
P_{1-P} & P < P_m 
\end{cases} \]  

\[ P_1 = \frac{\phi}{.75 + \frac{\phi}{6000}}; \quad P_b = \frac{\phi}{.75 + \frac{\phi}{1200}} \]  

\[ P_m = \frac{P_{b-c}}{P_1}^{P_{c-b}}; \quad P_c = \frac{P_{b-c}}{6} \]
Excellent agreement to the measured electron spectra was possible only because our function incorporates the negative slope of the diffusion coefficient as observed by Zwickl and Webber. The diffusion coefficient is characterized by and expressed as a function of the depth of modulation; it is shown in Figure 5.2.

V.2 The Electron Modulation

Our measured electron data points and the modulated spectra are shown for each year separately in Figures 5.3 through 5.8. The electron data are indicated by bands that include both the uncertainty of the data points and the systematic error between measurements. The solid and dashed lines represent the modulation needed to reproduce the data starting from the upper limit and lower limit interstellar electron spectra given in Figure 4.5. The values of phi required for these fits are shown in Table 5.1. The agreement to the electron data indicates that the curve-fitting has been well-performed.

There is an alternative to the process of estimating the diffusion coefficient by curve-fitting the modulated and observed electron spectra in the way we have done above. Gleeson and Urch (1973) have presented a direct construction. In general, any set of electron data can be matched to an interstellar spectrum by the appropriate diffusion coefficients. Initial estimates for the diffusion
Figure 5.2 The diffusion coefficient that we use is shown as a function of rigidity for certain values of phi, the depth of modulation.
Figure 5.2
Figure 5.3 The electron data of 1965-66 are indicated by the banded region. The band includes our estimates of the statistical and systematic uncertainty in the data. The solid line shows the modulated spectrum calculated from the upper-limit interstellar electron spectrum. The dashed line shows the modulated spectrum calculated from the lower-limit interstellar electron spectrum.
Figure 5.3

Electrons/m² ster-sec-MeV

Energy (MeV)

1965-66

High, 200 MV

Low, 150 MV
Figure 5.4 The electron data of 1968 are indicated by the banded region. The band includes our estimates of the statistical and systematic uncertainty in the data. The solid line shows the modulated spectrum calculated from the upper-limit interstellar electron spectrum. The dashed line shows the modulated spectrum calculated from the lower-limit interstellar electron spectrum.
Figure 5.4
Figure 5.5 The electron data of 1969 are indicated by the banded region. The band includes our estimates of the statistical and systematic uncertainty in the data. The solid line shows the modulated spectrum calculated from the upper-limit interstellar electron spectrum. The dashed line shows the modulated spectrum calculated from the lower-limit interstellar electron spectrum.
1969

High 700 MV
Low 590 MV

Figure 5.5
Figure 5.6 The electron data of 1970 are indicated by the banded region. The band includes our estimates of the statistical and systematic uncertainty in the data. The solid line shows the modulated spectrum calculated from the upper-limit interstellar electron spectrum. The dashed line shows the modulated spectrum calculated from the lower-limit interstellar electron spectrum.
Figure 5.6
Figure 5.7 The electron data of 1971 are indicated by the banded region. The band includes our estimates of the statistical and systematic uncertainty in the data. The solid line shows the modulated spectrum calculated from the upper-limit interstellar electron spectrum. The dashed line shows the modulated spectrum calculated from the lower-limit interstellar electron spectrum.
Figure 5.7
Figure 5.8 The electron data of 1972-73-74 are indicated by the banded region. The band includes our estimates of the statistical and systematic uncertainty in the data. The solid line shows the modulated spectrum calculated from the upper-limit interstellar electron spectrum. The dashed line shows the modulated spectrum calculated from the lower-limit interstellar electron spectrum.
Figure 5.8

1972-73-74

- High 450 MV
- Low 330 MV

Electrons/m^2 ster-sec-MeV

Energy (MeV)

$10^{-1}$ $10^{-2}$ $10^{-3}$

$10$ $100$ $1000$ $10,000$ $10,000$
TABLE 5.1

DEPTH OF MODULATION

<table>
<thead>
<tr>
<th>Year</th>
<th>Date</th>
<th>j(&gt;60 Mt. Wash MeV)</th>
<th>phi(NV) high j</th>
<th>low j</th>
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<tr>
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<td>July 28</td>
<td>1304 2451(2436)</td>
<td>180±40</td>
<td>130±40</td>
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<td>1162 2369(2393)</td>
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<td>150±25</td>
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<tr>
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<td>July 31</td>
<td>890 2210(2231)</td>
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<td>400±35</td>
</tr>
<tr>
<td>1969</td>
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<td>683 2089</td>
<td>700±50</td>
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<tr>
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<td></td>
<td>654 2101</td>
<td>880±60</td>
<td>750±50</td>
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<td>949 2343</td>
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<td>320±40</td>
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<tr>
<td>1974c</td>
<td></td>
<td>1042 2304</td>
<td>450±35</td>
<td>350±35</td>
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</table>

ELECTRONS

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<th>phi(NV) high j</th>
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<td>642 2070</td>
<td>(760)</td>
<td>(650)</td>
</tr>
<tr>
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<td>(460)</td>
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<td>1974c</td>
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<td>320?</td>
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PROTONS

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<th>phi(NV) high j</th>
<th>low j</th>
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<tr>
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<td>1168 2361</td>
<td>(460)</td>
<td>(360)</td>
</tr>
<tr>
<td>1974ac</td>
<td>July-Sept</td>
<td>1070 2318</td>
<td>(450)</td>
<td>(350)</td>
</tr>
<tr>
<td>1974b</td>
<td>July 21-Aug 3</td>
<td>1094 2340</td>
<td>(440)</td>
<td>(340)</td>
</tr>
<tr>
<td>1975c</td>
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<td>420?</td>
<td>320?</td>
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HELIUM NUCLEI

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<th>phi(NV) high j</th>
<th>low j</th>
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<td>Jan</td>
<td>642 2070</td>
<td>(760)</td>
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<tr>
<td>1970</td>
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<tr>
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<td>1070 2318</td>
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<tr>
<td>1975c</td>
<td>June-Sept</td>
<td>1240 2405</td>
<td>420?</td>
<td>320?</td>
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</table>
coefficients can be calculated by inverting the "force-field" differential equation to yield a differential equation to be solved for the diffusion coefficient. Then, by studying the numerical solution to the general transport equation, the diffusion coefficient can be "fine-tuned" to reproduce modulated electron spectra to any desired degree of accuracy. Although the diffusion coefficients generated by this construction are mathematically precise, the uncertainty in the electron measurement itself is amplified as the diffusion coefficient meaninglessly tracks the curve (and its slope) that must be drawn through the electron data points. We prefer our method whereby the functional dependence is determined from physical considerations and then is adjusted to reproduce best the measured data points.

The accuracy of the diffusion coefficients is based on knowledge of their correct functional form, an accurate interstellar electron spectrum, and the actual electron spectra observed at the earth. These factors also contribute to the accuracy of the values derived for phi. These values are shown in Figure 5.9. In this figure the values of phi required for the upper- and lower-limit interstellar electron spectra are shown along with errors inherent in the fit of the modulated spectra to the electron data.

V.3 The Modulation Of The Proton And Helium Nuclei
Figure 5.9 The depth of modulation, phi, is shown for solar cycle 20. Phi has been calculated from the measured electron spectrum and our inferred interstellar electron spectrum. The interstellar electron spectrum upper-limit and lower-limit estimates are indicated by the central element of each data point. The bars show the combined statistical and systematic uncertainty in the measured electron spectra.
Figure 5.9
Using the values of phi derived from the electrons and the appropriate rigidity dependence of the diffusion coefficient (See Figure 5.2.), we then proceed to calculate numerically the modulated spectra for protons and helium nuclei. The comparison of these calculated spectra with the measured ones is carried out in Figures 5.10 through 5.15. For the data base we have used mainly University of New Hampshire results on protons and helium nuclei (Webber and Lezniak, 1973; Lockwood, et al., 1973; Webber and Schofield, 1975; and Yushak, 1977) supplemented by University of Chicago data (Garcia-Munoz, et al., 1975,1977). Smoothed lines are drawn through the original data points and points are extracted from these lines at a fixed set of energies as shown in the Figures. Table 5.1 lists the dates of the nuclei measurements and the appropriately scaled values for phi from the electron modulation when the nuclei and electron measurements refer to different times.

The only remaining free parameters in this analysis are the interstellar proton and helium spectra. The upper and lower limits on these spectra are based on the upper and lower limits of phi derived from the 1965-1966 electron spectra. In other words, the interstellar proton and helium spectra are chosen so that by using the values of phi obtained from the electron modulation, the observed 1965-1966 proton and helium spectra are reproduced.
Figure 5.10 The measured proton spectra in the years 1965, 1968, and 1969 are shown along with our calculated, modulated spectra. The solid curve shows the modulated spectrum consistent with the depth of modulation determined from the electron measurements at the same time and the upper limit to the interstellar electron spectrum. The dashed line shows the modulated spectrum corresponding to the lower limit to the interstellar electron spectrum. The required interstellar proton spectrum is shown as the shaded band. For comparison, we also show the interstellar spectrum used by Garcia-Munoz, et al. (1975).
Figure 5.10
Figure 5.11 The measured proton spectra of the years 1970, 1971, and 1972 are shown along with our calculated, modulated spectra. The required interstellar proton spectrum is shown as the shaded band.
Protons/m²-ster-sec-MeV

Figure 5.11
Figure 5.12 The measured proton spectra of the years 1972, 1973, and 1975 are shown along with a calculated, modulated spectrum based on the electron modulation during this period. The interstellar spectrum of the previous figures is included. The dotted line (a) shows the effect in the calculated, modulated spectrum if the diffusion coefficient were to break to a $P^n$ dependence at 1 GV. The dotted and dashed line (b) shows the effect of placing the boundary for the calculation at 50 AU.
Figure 5.12

Energy (GeV)

Protons/m² ster sec MeV

1975
1974
1972

(a) \( P_R = 1 \) GV

(b)
Figure 5.13 The measured helium nuclei spectra of the years 1965, 1968, and 1969 are shown along with our calculated, modulated spectra. The required interstellar helium spectrum is shown as the shaded band. For comparison, we also show the interstellar spectrum used by Garcia-Munoz, et al. (1975).
Figure 5.13

Helium Nuclei/m$^2$ster-sec-MeV/nuc

Energy (GeV/nuc)
Figure 5.14 The measured helium nuclei spectra of the years 1970, 1971, and 1972 are shown along with our calculated, modulated spectra. The required interstellar helium nuclei spectrum is shown as the shaded band.
Figure 5.14

Helium Nuclei

Energy (GeV/nuc)

Helium Nuclei/m^2·ster·sec·MeV/nuc

1972
1971
1970

360
460
560

700
820

0.1
0.1
1
10

0.01
0.01

Figure 5.14
Figure 5.15 The measured helium nuclei spectra of the years 1972, 1973, and 1975 are shown along with a calculated, modulated spectrum based on the electron modulation during this period. The interstellar spectrum of the previous figures is included. The dotted line (a) shows the effect in the calculated, modulated spectrum if the diffusion coefficient were to break to a $p^2$ dependence at 1 GV. The dotted and dashed line (b) shows additionally the effect of placing the boundary for the calculation at 50 AU.
Figure 5.15
The modulation of the nuclei is seen to be quite well predicted above \( \sim 50 \text{ MeV/nuc} \) for the entire period 1965-1970 for either combination of interstellar spectra and \( \phi \), as set by the electron modulation. In 1971, however, the observed and predicted nuclei spectra agree at lower energies but diverge by a large amount at higher energies. The disagreement is even more severe in 1972 at both high and low energies. The predicted nuclei spectra are much less than those which are actually observed, or, in effect, the electron modulation is much less than that of the nuclei. This effect persists throughout the 1972-75 period and, in fact, becomes even more pronounced for both the low energy protons and helium nuclei.

V.4 Discussion Of The Modulation Reluctance Of The Electrons

We now wish to examine this interesting behavior more closely, and to do this we use the concept of differential modulation. This approach was used with some success by Lezniak and Webber (1971) to examine the modulation of protons, helium nuclei, and electrons during the 1965-68 time period and to show that the data were consistent with formulations of the cosmic-ray modulation equations that included the energy loss term. The differential modulation of a given species at a particular energy or rigidity is defined as \( dj = \ln(j_1/j_2) \), where \( j_1 \) and \( j_2 \) are the differential intensities measured at two times. We select
four time periods for this study: period 1, \( j_1 = 1965 \) and \( j_2 = 1968 \); period 2, \( j_1 = 1968 \) and \( j_2 = 1969 \); period 3, \( j_1 = 1971 \) and \( j_2 = 1969 \); and period 4, \( j_1 = 1972 \) and \( j_2 = 1971 \). The fractional modulation observed for protons in these four periods is shown in figure 5.16. The predicted fractional modulation for a \( \Delta \phi = 300 \) MV is also shown in the Figure. It is seen that the observations and predictions agree well for this value of \( \Delta \phi \), that is, the data show that the proton intensity changes were very similar for all of the four periods. Figure 5.17 shows the same data for helium nuclei. Here the scatter is somewhat larger, but again the data are consistent with theory, and the fractional modulation is similar for all four periods. The electron data are shown in Figure 5.18. Only the data for the first time period agree with the same value of \( \Delta \phi \) that applies to the nuclei. The fractional modulation of electrons in period two is smaller and implies a smaller \( \Delta \phi \approx 200 \) MV (See Table 5.1.). The electron modulation in period three is even smaller \( (\Delta \phi \approx 110 \) MV, see Table 5.1.). In period four the fractional electron modulation is larger \( (\Delta \phi = 180 \) MV, see Table 5.1.), but is still well below that observed in period 1. Since our predictions for the nuclei are based on the observed electron modulation, it is clear that the amount of the modulation of the nuclei will be underestimated.
Figure 5.16 The differential modulation of protons is shown for four time periods: Period 1 is 65/68; Period 2 is 68/69; Period 3 is 71/69; Period 4 is 72/71. The prediction for a difference in phi of 300 MV is indicated by the dashed line. The error in the data points is typically \( \pm 10\% \).
Figure 5.16

PROTONS

Prediction
$\Delta \phi = 300$ MV

$\ln\left(\frac{j_2}{j_1}\right)$

Period 1
Period 2
Period 3
Period 4

Rigidity (GV)

Figure 5.16
Figure 5.17 The differential modulation of helium nuclei is shown for four time periods: Period 1 is 65/68; Period 2 is 68/69; Period 3 is 71/69; Period 4 is 72/71. The prediction for a difference in phi of 300 MV is indicated by the dashed line. The helium spectrum includes an anomalous component at low rigidities. The error in the data points is typically ±20%. 
Figure 5.17

**Prediction**

\[ \Delta \phi = 300 \text{ MV} \]
Figure 5.18 The differential modulation of electrons is shown for four time periods: Period 1 is 65/68; Period 2 is 68/69; Period 3 is 71/69; Period 4 is 72/71. The prediction of the Fokker-Planck equation for a difference in phi of 300 MV is shown. The failure of the electron modulation to track the dashed line is indicative of the phenomena of modulation reluctance. The error in the data points is typically ±25%. 
ELECTRONS

Prediction
$\Delta \phi = 300$ MV

Figure 5.18
The solution of the Fokker-Planck equation makes certain specific predictions regarding the ratio of the fractional modulation of any two cosmic-ray species \( R(dj(1)/dj(2)) \) at a given rigidity. Numerical solutions lead to the values of \( R \), as shown in Figure 5.19. The value of \( R(dj(\text{He})/dj(p)) \) is \( \approx 1 \) at all rigidities for most values of \( \phi \). Only when \( \phi \) becomes small and the anomalous He component becomes important does this ratio deviate significantly from 1 at low rigidities. Likewise, the expected value of \( R(dj(e)/dj(p)) \) does not deviate greatly from 1 above a rigidity \( \approx 0.2 \) GV for all values of \( \phi \).

The observed values of \( R(dj(\text{He})/dj(p)) \) for the four time intervals are shown in Figure 5.20. These values are all consistent with 1 except the lowest energy value for period four, which is affected by the anomalous helium component. The observed values of \( R(dj(e)/dj(p)) \) are shown in Figure 5.21. As would be expected from the earlier Figure, only the data from period one are consistent with the theoretical predictions. Agreement between observations and predictions becomes progressively worse in the later periods when the electron fractional modulation is always much less than that of the protons.

It is well known that there has been a non-tracking, or hysteresis, effect observed between nuclei of different energies during the last cycle, (Lockwood, et al., 1972; Van Hollebeke, et al., 1972). This effect, however, began
Figure 5.19 The theoretical predictions for the ratio of fractional modulations between two species is shown. The dotted lines show the ratios of the diffusion convection model, while the shaded area indicates the range of prediction of the Fokker-Planck equation.
Typical $P(\frac{He}{p})$ for low $\phi$ with Anomalous He

Figure 5.19
Figure 5.20 The ratio of the differential modulation of the helium nuclei to the differential modulation of the protons is shown for the four time periods: Period 1 is 65/68; Period 2 is 68/69; Period 3 is 71/69; Period 4 is 72/71.
Figure 5.20

\[ R\left(\frac{\Delta j_{He}}{\Delta j_{p}}\right) \]

Rigidity (GV)
Figure 5.21 The ratio of the differential modulation of the electrons to the differential modulation of the protons is shown for the four time periods: Period 1 is 65/68; Period 2 is 68/69; Period 3 is 71/69; Period 4 is 72/71.
Figure 5.21

\[ R \left( \frac{\Delta j_e}{\Delta j_p} \right) \]

Rigidity (GV)
in 1969 and the hysteresis loop was closed in 1972 (Lockwood, et al., 1973), although a smaller secondary loop appeared in 1974 (Garcia-Munoz, et al., 1977). The behavior of the electron-nuclei modulation we have just discussed may be related to the nuclei hysteresis but appears to follow a different pattern.

The electron-nuclei differences began in 1968-69 and persisted throughout the remainder of the cycle. In effect, when compared with the nuclei, the electrons failed to recover their 1965-66 level. As we shall see later, however, such a consideration is too narrow.

How is it then that some workers are able to get generally excellent agreement between the observed and predicted proton and helium spectra at higher energies during the years 1972-75? (Garcia-Munoz, et al., 1975, 1977). First of all, these workers use essentially the same electron spectra in 1972-74 that we use, but they use the interstellar electron spectrum derived by Cummings (1973). As a result, they require somewhat larger values of phi than we obtain to reproduce the observed electron spectra in the 1972-74 period. This, in turn, means that to reproduce the observed nuclei spectra in these years, they require larger interstellar proton and helium nuclei spectra. Secondly, the predicted spectra fit well in these years basically because they are used as a reference. If Garcia-Munoz, et al., were to attempt to fit the nuclei
spectra between 1965-70 using the observed electron spectrum for these years and maintaining the same interstellar nuclei spectra as derived from the 1972-74 data, they would find a gross disagreement between prediction and observation. In fact, this problem is already evident in the work of Fulks (1975) and Caldwell and Meyer (1975), who make a brief comment about it.

The basic reason that the electron and nuclei modulation do not agree when compared over a whole solar cycle is that the differential electron modulation during the years 1968-72 is much too small relative to the nuclei. We name this new effect the modulation reluctance of the cosmic ray electrons. We believe that this is a fundamental observation and shall discuss its possible explanation along with its impact on the theory of modulation in a later chapter.

V.5 The Effect Of The Diffusion Coefficient On The Various Species

In view of the above problems connected with the relative electron and nuclei modulation, it is useful to examine more closely the significance of the various terms in the Fokker-Planck equation. The model of the solar modulation is a differential equation that comprises the physical effects of diffusion, convection, and betatron deceleration. To examine the influence of these three
physical processes, the terms of the Fokker-Planck equation,

\[ \frac{\partial}{\partial t} \left| \begin{array}{l} \frac{\partial U}{\partial r} \\ \frac{2V}{3r} \frac{\partial}{\partial T} UT \end{array} \right| \]

may be compared in value as a function of kinetic energy. The relative contribution of each effect to the modulated number density, U, is proportional to the relative magnitude of its term in the differential equation. Fractional convection, for instance, is defined as the ratio of the convective term to the sum of the absolute values of all three terms in the Fokker-Planck equation. We view the differential equation as a balance of the physical processes at each energy.

Figure 5.22 shows the fractional convection, diffusion, and energy loss as they vary with energy for solar maximum electrons (1969). By and large, the electron modulation is a balance between convection and diffusion. The energy-loss term is small and partially competes with diffusion in the energy region of modulation parameter maximum. It is easy to imagine that the electron data could be accurately represented in Parker's early convection-diffusion model (1963) with only slightly altered diffusion coefficients. The diffusion coefficient, which is a free parameter in the model, influences (by directly balancing convection) the modulated electron spectrum at all energies. Furthermore, each electron spectrum has its own diffusion coefficient. This accounts for the facility with which the electron
Figure 5.22 The relative terms in the Fokker-Planck equation are shown for the electron solution in 1969 at solar maximum. The diffusion coefficient maintains influence over the entire energy range.
Figure 5.22

Fokker-Planck Terms for Electrons (1969)

- Convection
- Energy Loss
- Diffusion

Relative Percent

Kinetic Energy (MeV/nuc)
fluxes may be modelled.

The nuclei, however, exhibit a different phenomenon. The differential fractions are plotted for the proton and helium nuclei solutions at solar maximum in Figures 5.23 and 5.24. Convection is balanced at low energies by betatron deceleration and at high energies by diffusion. From the viewpoint of curve-fitting, the one free parameter in the differential equation, the diffusion coefficient, has little impact on the low-energy portion of the proton and helium nuclei spectra. In fact, the high-energy portion of the interstellar proton or helium nuclei spectrum determines both the modulated spectrum at one energy-loss lower energy and also the spectrum in the $j$ proportional to $T$ region. Thus the model itself, rather than its free parameters, governs the low-energy protons and helium nuclei.
Figure 5.23 The relative terms in the Fokker-Planck equation are shown for the proton solution in 1969 at solar maximum. The diffusion coefficient maintains influence only at the high energies. At the low energies, the convective term is balanced by the energy loss term.
Figure 5.23

Fokker-Planck Terms for Protons (1969)

- Convection
- Diffusion
- Energy Loss

Relative Percent

Kinetic Energy (MeV/nuc)
Figure 5.24 The relative terms in the Fokker-Planck equation are shown for the helium nuclei at solar maximum. The diffusion coefficient maintains influence only at the high energies. At the low energies, the convective term is balanced by the energy loss term.
Figure 5.24

Fokker-Planck Terms for Helium Nuclei (1969)

- Convection
- Diffusion
- Energy Loss

Relative Percent

Kinetic Energy (MeV/nuc)
It is not clear at this point whether the different magnitudes of the electron and nuclei modulation reported in Chapter V are related to the hysteresis effect discussed extensively in the literature. Nevertheless, because of certain similarities between the two effects, it is necessary that the hysteresis effect be discussed here in order that we may understand possible differences between it and the effect we have discovered.

The non-tracking, or hysteresis, between high and low energy nuclei of a given nuclear species has been discussed by many workers (e.g. Van Hollebeke, et al., 1972; Lockwood, et al., 1972; Rygg, et al., 1974). Basically, the effect is that after the minimum intensity of nuclei of all energies was reached in 1969, the higher energies recovered much more rapidly. This was evident in 1970 and 1971. By 1972, however, the low energy nuclei had recovered and the intensities were on the same regression curve as the 1965-69 data (Van Hollebeke, et al., 1973; Lockwood and Webber, 1973) thus essentially completing the hysteresis loop for nuclei. A smaller hysteresis loop was observed for nuclei during the 1974 reduction of intensity (Garcia-Munoz, et al., 1977). The earlier attempts to explain the
hysteresis effect for nuclei were mainly in terms of a changing rigidity dependence for the diffusion coefficient, $K$ (e.g. Van Hollebeke, et al., 1972). A more recent attempt, solving the time dependent transport equation and recognizing the hysteresis as a rigidity dependent time lag, has also met with considerable success in fitting the data (O'Gallagher and Maslyar, 1975).

It has been recognized that studies of both the electron and the nuclei modulations are essential to the understanding of the problem. Electron and nuclei modulation effects during the 1965-69 period could generally be fit into a self-consistent modulation picture (e.g. Lezniak and Webber, 1971; Schmidt, 1972; Urch and Gleeson, 1973) with no evidence of differences in the modulation amplitude for electrons and nuclei other than those predicted by the Fokker-Planck equation. These studies were too early in the solar cycle to observe the hysteresis effects. The work of Burger and Swanenberg (1973) and later of Webber (1973) and Fulks and Meyer (1973) showed clearly a hysteresis for electrons similar to that for nuclei for the 1970-71 period. When these electron data were compared with the nuclei intensities measured at 60 MeV/nuc by Van Hollebeke, et al., in the same time period, it was evident that this hysteresis for all species was mainly a rigidity dependent effect. The modulation amplitudes observed for electrons and nuclei at 60 MeV/nuc (only) were still reasonably consistent with the expected variation based on
the Fokker-Planck models. Two studies using 1971 and later data, where the effects we observe become more obvious for both electrons and nuclei, have been reported. The Chicago data have been reported by Fulks (1975) and by Caldwell and Meyer (1975). Fulks obtains diffusion coefficients from the 1969-1972 electron data not greatly dissimilar to those found in this thesis, although because he uses Cummings' (1973) interstellar electron spectrum he gets considerably different values of phi. He then uses these quantities to fit the proton and helium data for 1968-1972. The fit is quite poor at both high and low energies and is limited by the lack of proton data above 100 MeV for several years. Fulks comments briefly on this lack of agreement on p. 1710.

Further studies of the Chicago group by Caldwell and Meyer (1975) using the 1973-74 electron data show even larger discrepancies for the predicted and observed nuclei spectra (See their Figure 3.), particularly the proton spectra. Caldwell and Meyer note: "Our success in applying the modulation parameters that are derived from the electron measurements to protons and alpha particles seems to have deteriorated in recent years."

The work of Winkler and Bedijn (1976) on this subject is very interesting. They mainly use the data on electrons from the Dutch group during the years 1968-1972. These data are basically identical to those used in this thesis. Winkler and Bedijn then predict quite accurately the modulation for protons and helium nuclei in the 1968 to 1969
period as we have been able to do, and have been able to accurately predict the 60 MeV/nuc proton and helium nuclei intensities in 1968-71 measured by Van Hollebeke, et al., 1972. This work, like that of Burger and Swanenburg (1973), clearly shows the similarity in the electron and nuclei hysteresis effects when compared as a function of rigidity. Winkler and Bedijn (1976) do not appear to have attempted to fit the Van Hollebeke, et al., 1972 proton and helium data; in any case, they seem to have mis-plotted it. Van Hollebeke, et al., (1973) observe a factor of 10 increase in 60 MeV protons between the lowest intensity in 1969-70 and in 1972, and a factor of 6 increase in 60 MeV/nuc helium nuclei (See also our data in Figures 5.10 through 5.15.). Winkler and Bedijn plot these increases as factors of ~5 and 4 respectively (Their Figure 6.). Thus they appear to have missed this effect! It should be noted that using either Winkler and Bedijn's data or those presented in this thesis, the electrons at the same rigidities increased only a factor of 4 and 3 respectively. This difference in the magnitude of the electron and nuclei modulation is precisely the effect we are trying to point out!

This gross difference in modulation amplitude, already discussed in Chapter V, can be seen in several other ways. Consider a comparison between the 1968 and 1972 electron spectra. All observers agree that the 1972 intensities are higher than those in 1968 by only 10-20%. Winkler and Bedijn themselves note that "the diffusion coefficients for
1972 and 1968 are equal within \( \pm 10\% \). Yet the 1972 protons are higher than those in 1968 by a factor of \(~3\) at 100 MeV and by a factor \(\geq 2\) at 400 MeV. For the He nuclei, the increases are a factor of 2.3 and 1.8 respectively at the same energies. It is difficult to relate these effects, which, as we have noted, become even more pronounced in 1973 and 1974, to the ordinary, rather-well established hysteresis, which for nuclei basically ended in 1972. Instead, it appears that they represent a new phenomenon not recognized in earlier work. This phenomenon is quite simply stated: The electron and nuclei fractional modulation amplitudes are inconsistent when interpreted within the framework of the Fokker-planck equation. In what follows, we shall examine possible explanations and some implications of these results.

Specifically, we consider: (1) The possibility that the 1965-66 electron data are wrong and what effect this might have. (2) The possibility that a different rigidity dependence of \( K \) at higher energies or a different location of the boundary than we have chosen could be responsible for the inconsistency. (3) The difference in magnitude of the electron and nuclei modulation could somehow be related to the hysteresis effect observed between nuclei of different energies and could be explained in terms of time-dependent modulation models. (4) The effects we observe could be explained by effects related to the perpendicular diffusion of cosmic rays across field lines. (5) The energy loss term
in the Fokker-Planck equation is incorrect; the equation itself is not applicable to the problem. This is not an exhaustive list, but will serve to delineate some of the problems connected with an explanation of this effect.

(1) A natural inclination is to suspect the 1965-66 electron data. Let us therefore suppose that the fluxes measured in these years are really too high and the correct fluxes are, in fact, in agreement with those measured in the 1972-1974 period. How will this change our arguments? In the first place, it will change the phi we have derived for sunspot minimum conditions from ~150-200 MV to a value ~350-450 MV consistent with that derived during 1972-74. As a result, the interstellar proton and helium spectra will increase, since a larger value of phi is required to demodulate them. These spectra will now be close to those used by Garcia-Munoz, et al. (1975), for example. Furthermore, the electron modulation during the decreasing part of the cycle (1965-69) will now be roughly consistent with that during the recovery (1970-75), and no peculiar modulation beyond the normal hysteresis effect will be needed. The fact will yet remain, however, that the electron modulation is smaller than it should be relative to that for protons and helium nuclei. This conclusion is based on the changes taking place from 1968 through 1974; decreasing the 1965-66 intensities will only make the electron modulation observed in this earlier period more in agreement with that during the later periods.
Can we really so boldly throw out the 1965-66 data? Even though the differences between intensities measured by the three groups are largest at this time, all three independent observations show considerably higher electron fluxes at low energies than were seen in 1972-74.

(2) The accuracy of the electron data is such that the diffusion coefficients are not well defined above ~1-2 GV. In our calculations, we have taken a simple \( K(P) \sim P \) dependence at the higher energies for all the time periods. It is well known that changes in the rigidity dependence of \( K \) can be invoked to explain the hysteresis effects between low and high energy nuclei. Any changes we make must, therefore, be consistent with neutron monitor observations. The most extreme assumption we can make and still be even passably consistent with the neutron monitor data is to consider \( K(P) \sim P^2 \) above ~3 GV during 1972-74. This does not affect the proton modulation and only slightly affects the helium modulation. In Figures 5.12 and 5.15 we show the effect if the change to \( P^2 \) is made at 1 GV. This does bring the predicted helium nuclei modulation into better agreement with observations at higher energies, but the effect on the protons is much less, and this form of the diffusion coefficient extending to high energies is totally inconsistent with the modulation observed by neutron monitors.
In general, in our calculations we have kept the modulation boundary fixed at 10 AU to conserve computer time. This was in recognition of the fact that the boundary distance generally acts as a scaling factor on the diffusion coefficient only for the same value of phi and has little effect on the intensities calculated for a given phi. Garrard (1973) has shown, however, that increasing the boundary distance does make small changes in the proton intensities calculated at low energies. Therefore, we have performed calculations for various boundary distances out to 50 AU. The effect of rB on the electron modulation is negligible and in Figures 5.12 and 5.15 we show the relatively small effects on the calculated proton and helium spectra at low energy.

We therefore conclude that changes in the rigidity dependence of K or in the location of rB can account for only a small fraction of the observed discrepancies between the amplitudes of the electron and nuclei modulation.

(3) We have already pointed out that the electron modulation appears to exhibit a hysteresis effect similar to that for nuclei. The data of Burger and Swanenburg (1973) show this quite clearly. The effect we are discussing here, namely the discrepancy between the amplitudes of the electron and nuclei modulations, persists throughout the 1972-74 period, even after the nuclei hysteresis loop has closed. Nevertheless, if the 1965-66 electron data are
accepted as accurate, then it is correct to say that the
electron intensities in 1972-74 have not recovered to their
previous level. In a sense, then, this is a
"hysteresis-like" effect, although presumably a different
one. Time-dependent models have had considerable success in
explaining the normal hysteresis effect for nuclei
(O'Gallagher and Maslyar, 1975). In these models the "delay
time" is specified by K, rB, and the solar wind velocity, V.
These are the same parameters that are involved in the
solution of the time-independent Fokker-Planck equation, so
it is not possible to introduce any new hysteresis effects
for electrons by this method.

(4) All of the Fokker-Planck transport models neglect
the effect of diffusion perpendicular to the magnetic field
lines on the basis that these terms either are small or are
implicitly included. Nevertheless, such terms could be
significant in non-spherically symmetric models where polar
diffusion across field lines is alleged to be important
(e.g., Fisk, 1976), or even in spherically symmetric models
where the field lines are wound up tightly in the outer part
of the solar system. The main process of cross-field
diffusion appears to be due to random walking field lines
(Jokipii and Parker, 1969). This process yields a diffusion
coefficient that is roughly $-1.2 \times 10^{21}$ cm$^2$/sec at
all energies. Another process that can lead to cross-field
diffusion is gradient and curvature drifts. The term for
perpendicular diffusion contains a factor of $\frac{Z^2}{(\gamma m)}$
which is significantly larger for electrons than for nuclei. In each of these cases, if perpendicular diffusion were significant, the electrons would have easier access to the inner solar system than nuclei, thus leading to relatively smaller modulation effects, as are observed.

(5) As we discussed in the last chapter, the energy loss term in the Fokker-Planck equation is much more important at low energies for nuclei than for electrons. If this term were incorrect or inapplicable, then it is possible that the electron-nuclei modulation differences could be explained. The simple diffusion-convection model discussed in Chapter III is essentially the Fokker-Planck equation without the energy loss term. It is interesting to examine what this theory predicts in terms of the relative modulation of electrons and nuclei. Using equations 3.4-3.6, it is easy to show that \( R(dj(e)/dj(p)) \sim (\text{proton velocity})/(\text{electron velocity}) \) and \( R(dj(\text{He})/dj(p)) \sim (\text{proton velocity})/(\text{Helium velocity}) \), e.g., if the relative modulation is plotted as a function of rigidity, the modulation of the different species is split according to their velocity, \( v \), at that rigidity. This behavior is shown in Figure 5.19. As far as the relative proton and helium modulation is concerned (Figures 5.16, 5.17 and 5.20), the data are consistent with an energy loss term being present. For the relative electron modulation, however, data after 1968 behave somewhat like those to be expected for the diffusion-convection model. This may be a clue that the
energy loss effects are not properly accounted for in the present Fokker-Planck equation, or that other terms, such as the drift terms, behave in such a way as to partially cancel the energy loss term.

Before concluding our discussion we should also note the work of Fisk (1976). In order to account for the modulation effects on the low energy anomalous oxygen component, assuming it had an interstellar origin, he suggested a modification to the strongly velocity dependent diffusion of conventional modulation theory. He suggested that to produce velocity independent diffusion, time variations of the magnetic field must be important, and proceeded to develop a model in which lower energy cosmic rays are trapped between magnetic mirrors. The escape of these particles is essentially independent of velocity and leads to a diffusive term which mainly depends on the distance between trapping regions. This model predicts a relatively smaller modulation for the nuclei as compared with that for electrons, contrary to the effect we have observed, and hence, does not seem applicable to our problem.
CHAPTER VII

UNIQUENESS OF THE DEPTH OF MODULATION

There are two possible goals to be attained from the study of the solar modulation: first, to ascertain that the equations describing the modulation do agree with observation and second, to determine the absolute depth of the modulation. To see that the equations are valid, mathematical tautology must be distinguished and separated from the physical predictions of the mathematical theory. The mathematical tautology which we refer to is the estimation of diffusion coefficients in order to calculate the modulated electron spectra. The physical predictions of the mathematical theory are then the modulated proton and helium spectra calculated by using these diffusion coefficients; here a test of the validity of the mathematics lies.

With regard to the absolute depth of modulation, within the framework of the Fokker-Planck equation, the total amount of modulation that relates the interstellar spectrum to the modulated spectrum has been determined by estimating the interstellar electron spectrum from the radio frequency emission and comparing this with the measured electron spectra at earth. Let us at this point recapitulate the error involved in our determination of phi.
This error arises from two uncertainties: (1) the interstellar spectrum and (2) the measured electron spectrum. We have discussed the errors on the interstellar spectrum in Chapter IV. These have been represented in Figure 4.5 which shows the interstellar spectrum and the upper- and lower-limit bounds, differing by ~50% at all energies. Observe that although there is a factor of 2 variance in the overall normalization procedure, the resulting electron flux variation is much smaller.

For the measured electron spectra we have taken fairly generous errors which include not only the quoted errors of the measurements, but also systematic differences observed between measurements. The resulting errors on phi are listed in Table 5.1 and are shown also in Figure 5.9. Thus the overall uncertainty of the absolute value of phi we deduce at any one level of modulation is ±20%.

VII.1 A Comment On The Interstellar Proton And Helium Spectra

At high energies (>10 GeV/nuc) the cosmic ray protons and helium nuclei obey a power law in energy with spectral index of 2.7±.1. At lower energies, however, the helium nuclei become relatively more abundant. The two spectra differ because of propagation through 6 grams of interstellar hydrogen and possibly because of complex acceleration mechanisms for cosmic ray injection, see
Lezniak and Webber (1974). The limits on the interstellar proton and helium spectra are shown above ~100 MeV/nuc in Figures 5.10-5.15. These limits are ascertained by requiring that the molulated proton and helium spectra be in agreement with the observations in the years 1965-69 for values of phi appropriate to the high and low interstellar electron spectra. It is clearly shown in Chapter V that the value of the diffusion coefficient has little influence on the solution in the region below a few 100 MeV/nuc for the nuclei. This is because most of the particles observed at these energies have suffered energy loss from a higher interstellar energy. Conversely, the interstellar spectra that we deduce become progressively less certain below a few 100 MeV/nuc.

We wish to point out again that if the 1972 electron spectrum is used along with Cummings' (1973) interstellar electron spectrum a significantly higher value of phi ensues. If this value of phi is then used to fit the 1972 nuclei spectra (in contrast to our fitting the 1965-69 nuclei spectra) much higher interstellar spectra are obtained for protons and helium nuclei. In Figures 5.10 and 5.13 we show the interstellar proton and helium spectra deduced in this manner by Garcia-Munoz, et al. (1975).
The major pursuits of this work have been (1) the explanation and elucidation of the relationship that exists among the cosmic ray electrons, the radio frequency background, and the magnetic fields in the galaxy, and (2) a study of the spherically symmetric Fokker-Planck equation in order to bound the extent of the modulation effect which in turn places limits on the estimates of the galactic proton and helium spectra.

The nature of the galactic magnetic field is the key to our understanding of the observed synchrotron radiation. Conservation of energy requires that the synchrotron formulas contain to good approximation the root-mean-square magnetic field component rather than the average field component. Of the three inter-related physical entities, the galactic magnetic field alone has not been accurately measured. At high energies, the electron spectrum measured at earth is the interstellar electron spectrum, and the radio emissivity is directly observable. Therefore, we have normalized the interstellar electron spectrum that we have calculated from the non-thermal background radiation to a consensus at high energy of the electron fluxes measured at earth. We have also studied the emissivities of the background radiation to nearby H II regions, and have
normalized the integration path length of the diffuse radio frequency background so that the background emissivity used to calculate the interstellar electron spectrum agrees with the measured emissivity toward the H II regions.

We further point out that past arguments, which have indirectly estimated the strength of the galactic magnetic field, have incorporated formulas consistent with, and have been prejudiced by, the assumption that the field strength was constant. Revision of the synchrotron formulas and normalization of the physical constants have enabled a consistent picture to emerge of the cosmic-ray electron spectrum producing the local emissivity.

Once the interstellar electron spectrum is accurately known the diffusion coefficients representative of the time periods for which extraterrestrial electron measurements exist have been calculated. Our diffusion coefficients embody the radial and rigidity dependencies observed from solar particle studies. In a sense, although the electron modulation theoretically determines the diffusion coefficients, we have constrained the diffusion coefficients to agree with the physically prescribed form based on the solar particle studies. The diffusion coefficients must then predict through the Fokker-Planck equation the modulation of the protons and helium nuclei. We find that when systematically reviewed over the last decade, both the particle fluxes and their modulation parameters are
inconsistent with the predictions of the spherically symmetric Fokker-Planck equation.

This inconsistency can be most readily seen as we follow in time the differential modulation of the electron fluxes. During the 1965-69 period, the modulation parameters of all three species followed a regression curve with depth of modulation, indicating that the fluxes at earth of the three species could be predicted at any one time by the Fokker-Planck equation containing a single diffusion coefficient. In the 1971 period, the proton and helium fluxes increased as solar activity continued its normal cycle, but the electron fluxes only slightly increased, based on a 1965-69 norm. By 1972, the proton and helium fluxes had increased to their 1965 values, but the electron fluxes had not. We have named this effect the modulation reluctance of the cosmic ray electrons.

Alternately, if one views the differential modulation, this effect manifests itself in an inconsistency between the modulation parameters of the protons and helium nuclei and the modulation parameters of the electrons.


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