SCALING PROBABILITY OF REINFORCEMENT WITH A SIGNAL DETECTION PROCEDURE

STEPHEN GEORGE WHITTAKER

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STEPHEN GEORGE WHITTAKER
SCALING PROBABILITY OF REINFORCEMENT WITH A SIGNAL DETECTION PROCEDURE

by

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B.S. Hobart College, 1972
M.A. University of New Hampshire, 1975

A THESIS

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ABSTRACT

Using a two alternative signal detection procedure, subjective probability of reinforcement was scaled as a function of objective probability. Rats were trained in a discrete trial procedure to press a right hand lever if an overhead light produced 1.20 ft·ca illumination (correct detection) and the left lever if the light produced less illumination (correct rejection). A .02 cc dipper of water was programmed to occur probabilistically following correct responses. If a right lever press occurred with the lesser illumination (false alarm) or a left lever press occurred with greater illumination (miss), a 3 second time-out ensued and reinforcement was never delivered. The probability of reinforcement for correct detections and a complementary probability of reinforcement for correct rejections were varied in successive 5000 trial conditions while the rats were discriminating 1.20 ft·ca and 1.05 ft·ca illuminations; and again while the rats were discriminating 1.20 ft·ca and .66 ft·ca illuminations. The overall probability of reinforcement for correct responses was always .5: The probability of reinforcement for correct rejections relative to the probability of reinforcement for correct detections ranged from .1/.9 to .9/.1.

The resultant relations between an index of response bias and the ratio of reinforcement probabilities for correct rejections over correct detections was plotted for the two
discrimination conditions. The bias index was a criterion likelihood ratio estimated from response frequencies using a signal detection analysis. The bias indices were power functions of the ratio of reinforcement probabilities. The exponents departed from 1.0, indicating that animals did not maximize the average total number of possible reinforcers in a session (maximize expected value). By assuming that animals adopted a strategy which maximized subjective expected utility, the bias index was used to define the subjective equivalent to an objective probability of reinforcement. Analysis of the results of this and other experiments indicate that the bias index exhibited properties of an interval scale of amount and probability of reinforcement. Thus, subjective probability, as defined by the bias index, has interval scale properties.

Subjective probability was a power function of objective probability under either discrimination condition. The exponents ranged from .49 to 1.57 and for some animals tended to be smaller under less discriminable conditions. Reanalysis of the results of other detection experiments in which stimulus probability was varied revealed a similar pattern of results. Thus, there was an interaction between the bias index and discriminability of the stimuli contradicting the assumption of signal detection analysis that animals adopt an optimum criterion. The interaction was neither large nor reliable across animals, suggesting that it could be eliminated by procedural refinement.
In the final two conditions the probability of the more intense stimulus was reduced to .4 and finally the probability of reinforcement for correctly detecting that stimulus was reduced to .4. Response bias was better predicted using subjective probability than objective probability. The findings of this and other experiments suggest that a signal detection procedure can be used to define subjective probability as a unique interval level scale, and encourage use of this procedure for operationally defining a utility scale.
SECTION I
INTRODUCTION

1.1 Purpose of the Research

During the 18th century, Court mathematicians who advised gamblers among the court nobility were puzzled by a phenomenon which became known as the "St. Petersburg Paradox." An experienced gambler often adopted a strategy that did not maximize average long run payoff or the expected monetary value of a gamble. For example, a gambler would typically choose a wager which would pay off $100 with a probability of .9 over a wager which would pay off $1000 with a probability of .1, where in each case the loss of the wager led to a payoff of $0. Suppose we define a random variable, w, which denotes the monetary winnings (or if negative, the losses) on a gamble. The expected monetary value, E(w), of a gamble with two possible outcomes can be defined as follows:

\[ E(w) = w_1p(w_1) + w_2p(w_2) \quad (1.1.1) \]

where \( w \) are the different outcomes that may occur and \( p(w) \) are the probability of those occurrences. In our gambling example above, the expected monetary value of the $1000 bet is $100 where for the $100 bet the expected monetary value is only $90. The gambler typically chose the $100 bet even though the $1000 bet had higher expected monetary value. Bernoulli attempted to explain this paradox.
by suggesting that a gambler acted to maximize his expected utility rather than the expected value of a gamble.

Utility is a psychological continuum functionally related to but not necessarily equal to the monetary value. For example, Bernoulli suggested that utility was a logarithmic function of monetary value. Such a mathematical transformation would account for the behavior of a gambler in the above example. The $1000 wager was less than ten times the utility of the $100 wager. Later, Savage (1954) suggested that the subjective probability estimate of an outcome may not equal the objective probability. Objective probability is the limiting relative frequency of an outcome over an infinite number of gambles; subjective probability is a psychological continuum which is a transformation of objective probability. If subjective probability does not equal objective probability, then both the utility of an outcome and its subjective probability must be defined before choice behavior can be predicted. If subjective probability is consistent with the axioms of objective probability and the assumptions underlying expected utility theory are correct then equation 1.1.1 can be rewritten in terms of subjective continua (Savage, 1954).

\[ E'[u(w_j)] = u(w_1)p'(w_1) + u(w_2)p'(w_2) \]  

where \( u \) and \( p' \) are algebraic transformations of the value of an outcome and the objective probability of an outcome, respectively.
Assuming that organisms chose alternatives in such a way as to maximize expected utility, Von Neumann and Morgenstern (1947) developed a mathematical theory of utility which has been incorporated into descriptive models of choice behavior (Shimp, 1969; Tversky, 1967), psychophysical detection and discrimination performance (Green & Swets, 1966) and motivation (Rachlin, Green, Battalio, & Kagel, 1976).

The purpose of this research was to determine if a signal detection analysis, which is itself based on decision theory, can be used to scale subjective probability and utility. In the first of two experiments dealing with this topic (Whittaker, in prep), the method was used to attempt to scale delay of reinforcement into a utility dimension. The method is used here to scale probability of an outcome.
1.2 Other Methods Used to Scale Utility or Probability

Various lines of research have directly or indirectly led to the development of utility scaling procedures. In one of the earlier such experiments (Mosteller & Nogee, 1951), a subject was presented with gambles, constructed for possible hands of poker dice, that he could accept or reject. He won \( w_1 \) cents if he beat the hand or would lose a nickel (i.e., \( w_2 = -5 \)) if he did not. By presenting the subjects with a variety of gambles, differing in payoff, \( w_1 \), and payoff probability, \( p(w_1) \), the expected utility of the gamble,

\[
E[u(w)] = p(w_1)w_1 + p(w_2)w_2
\]

(1.2.1)

was assumed to be zero at the indifference point where the subject accepted or rejected the gamble an equal number of times. By arbitrarily setting \( u(-5) = -1 \) and \( u(0) = 0 \), the authors then defined the utility of the payoff, \( w_1 \), in terms of the objective probabilities which produced indifference. By assumption,

\[
p(w_1)u(w_1) + p(w_2)w_2 = -1 = 0
\]

(1.2.2)

then

\[
u(w_1) = \frac{p(w_2)}{p(w_1)}
\]

(1.2.3)
The problem with this procedure is that two independent variables, outcome utility and the subjective probability of an outcome, contribute to the subjective expected utility of a gamble. Mosteller & Nogee assumed that subjective probability equalled objective probability, an assumption which has been questioned by Savage (1954). Probability must be scaled with an independent procedure, before utility of outcomes can be scaled in terms of probability. Other attempts to scale utility or subjective probability dealt with this problem.

Tversky (1967) used a choice procedure which offered a rather elegant solution. Subjects chose the minimum selling price of a gamble. Increases in the probability of winning and payoff of a gamble increased its selling price. Thus,

\[ E'[u(w)] = u(\$) = p'(w_1)u(w_1) \]  

(1.2.4)

where \( u(\$) \) is the utility of the selling price, \( p'(w_1) \) the subjective probability of the outcome of the gamble and \( u(w_1) \) the utility of the outcome. Since outcome and minimum selling price are functions of money, they should have the same utility function. Tversky assumed a power function so that

\[ \theta \log \$ = \theta \log w_1 + p'(w_1) \]  

(1.2.5)

and thus

\[ \log \$ = \log w_1 + \frac{1}{\theta} \log p'(w_1). \]  

(1.2.6)
Using a two way analysis of variance, Tversky showed that $w_1$ and $p(w_1)$ had additive effects on $\log p$ so subjective probability, $s(w_1)^{1/6}$, could be estimated as follows (the actual computing formula differs in detail):

$$p(w_1)^{1/6} = \$/w_1 \quad (1.2.7)$$

The subjective probability derived for individuals was a power function of objective probability with an exponent close to 1.0; very high probabilities were usually underestimated and very low probabilities usually overestimated.

Galanter (1962) used a direct estimation method developed by S. S. Stevens (1959) where human subjects directly estimated some physical quantity by assigning a number to it. Galanter had subjects assign numbers which represented "twice the happiness" produced by different amounts of money. Since the subjects arbitrarily chose different ranges of numbers to assign, data were normalized and then averaged across subjects and functionally related to the objective amount of money. He then had the same subjects directly estimate the likelihood of different statements such as "the likelihood it will not rain in Philadelphia in April" and scaled these events using three other related procedures with similar estimates of subject probability obtained for each statement. In order to verify these estimates, in a second experiment he used a choice procedure similar to Mosteller and Nogee's where subjects chose between hypothetical gambles such as:
G1: you win $1000 if it does not rain in Philadelphia in April

G2: you win $10 if the next person you meet had an appendectomy

Since the utility of each payoff and the subjective probability of each statement had been independently assessed, the subjects' choices could be predicted, and were in reasonable agreement with the data. Utility was found to be a power function of money; however, since the objective probability of the statements were not defined, the subjective probability function could not be specified. Another experiment (Beach & Phillips, 1967) had subjects estimate the objective probabilities of light illuminations and found that objective probabilities approximately matched subjective probabilities.

The experiments described above represent various procedures which could be used for measuring utility and subjective probability. The signal detection analysis is a direct scaling procedure in the sense that subjective expected utilities are inferred directly from the degree of preference in a choice situation. Because the analytical methods are derived from concepts of utility theory, they imply consistency with results of a choice procedure such as used by Mosteller & Nogee (1951) and Tversky (1967).
1.3 Signal Detection Paradigm

In a signal detection procedure, a subject must discriminate two stimulus conditions; differences in light intensity, for example. Typically one stimulus is presented per discrete trial and one of two responses can be made. One response (R₁) is correct in the presence of one stimulus (S₁) while the other response (R₂) is correct in the presence of the other stimulus (S₂). The two responses are usually defined on separate levers or keys in animal experiments. The responses can be defined as follows:

<table>
<thead>
<tr>
<th></th>
<th>R₁</th>
<th>R₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>(signal)</td>
<td>S₁</td>
<td>correct detections</td>
</tr>
<tr>
<td>(no signal)</td>
<td>S₂</td>
<td>false alarms</td>
</tr>
<tr>
<td></td>
<td>miss</td>
<td>correct rejections</td>
</tr>
</tbody>
</table>

Each of the responses has a corresponding outcome:

<table>
<thead>
<tr>
<th></th>
<th>R₁</th>
<th>R₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₁</td>
<td>w₁₁</td>
<td>w₁₂</td>
</tr>
<tr>
<td>S₂</td>
<td>w₂₁</td>
<td>w₂₂</td>
</tr>
</tbody>
</table>

w₁₁ and w₂₂ are favorable outcomes, where w₁₂ and w₂₁ are negative or null outcomes.

The dependent variables are the conditional probabilities estimated by the relative frequency of a response under stimulus condition S₁ or S₂: the probability of R₁ given S₁, denoted p(R₁|S₁) and the conditional probability of R₁ given S₂, denoted p(R₁|S₂). The remaining conditional probabilities of responding p(R₂|S₁) and p(R₂|S₂) are complements of p(R₁|S₁) and p(R₁|S₂) respectively. The latter two conditional response
probabilities are conventionally used in the signal detection analysis.

In this paradigm the physical stimuli ($S_1$ & $S_2$) and outcomes are independent determiners of particular probabilities of correct detections and false alarms. If $S_1$ and $S_2$ are not clearly discriminable, increasing payoff for correct detections ($w_{11}$) relative to correct rejections ($w_{22}$) would likely produce an increase in both $p(R_1|S_1)$ and $p(R_1|S_2)$. If the difference between $S_1$ and $S_2$ is held constant and the values of $w_{11}$ and $w_{22}$ are varied, the resulting relationship between $p(R_1|S_1)$ and $p(R_1|S_2)$ is called a receiver operating characteristic (ROC) curve. Specification of this function permits one to derive an index of sensitivity which is independent of biasing variables (e.g., outcomes): similar isosensitivity functions have been found in experiments with human (e.g., Galanter & Holman, 1967; Grier, 1971; Green & Swets, 1966; Swets, 1973; Swets, Markowitz & Franzem, 1969) and non-human subjects (Hobson, 1970; Hume, 1974; Hume & Irwin, 1974; Nevin, Olsen, Mandell & Yarensky, 1975; Schusterlman & Barrett, 1975; Stubbs, 1976). A bias-independent index of sensitivity has been used to scale the effect of stimulus dimensions such as sound intensity and wavelength of light on the behavior of human (e.g., Green & Swets, 1966) and non-human subjects (Wright, 1972) with remarkable agreement with other scaling methods.

The relation between $p(R_1|S_1)$ and $p(R_1|S_2)$ when outcomes are held constant and stimulus conditions are varied is called an isobias curve. Specification of the forms of the
function permits one to derive an index of response bias which may be independent of stimulus conditions but functionally related to outcomes. The form of the isobias functions has been less extensively studied (Terman & Terman, 1972; Hume & Irwin, 1974; Hume, 1974) and the independence of the bias index from stimulus conditions remains questionable. The relationship between this bias index and a specification of outcomes such as reinforcement probability, could provide an alternative method for scaling the utility of an outcome and thus warrants further investigation. The analytical methods presented in Green & Swets (1966) in their theory of signal detectability, were developed from some basic assumptions of utility theory.
1.4 Signal Detection and Utility Theory

In the theory of signal detectability, a particular stimulus condition may produce various sensory events: the sensory events associated with these stimuli are characterized as a single variable, $x$, which usually has a different value each time a stimulus condition is sampled. A unique distribution of $x$ is associated with each stimulus condition.

Choose $R_2$, $\beta=7$ Choose $R_1$, $\beta=1$

These distributions suggest that although a certain sensory event could occur during each stimulus condition, the probability of this sensory event differs if the stimulus conditions are not the same. For example, the animal could be looking away from the light source when the bright light ($S_1$) was presented with sensory event $x_1$ produced (see fig. 1.4.1).
Since $x_1$ is more likely if the dim light ($S_2$) occurs, one might say that the animal "saw" a dim light even though a bright light was presented, and would most likely report that the dim light occurred (miss). Since a particular value of $x$ can occur if either stimulus is presented, the subject bases his decision on a ratio of the likelihoods that some information, $x$, occurs given a stimulus.

$$P_{\text{obt}} = \frac{f_{S_1}(x)}{f_{S_2}(x)}$$

(1.4.1)

In order to make a decision, the subjects set a stable criterion, $P_{\text{obt}}$, which equals a particular likelihood ratio and corresponds to a particular value of $x$ (see the vertical lines in Fig. 1.4.1). If the likelihood ratio obtained on a trial exceeds the criterion, he chooses $R_1$; if it does not, then he chooses $R_2$. The criterion likelihood ratio indicated by the solid vertical line in Figure 1.4.1 equals 1 because the ordinate heights, $f_{S_1}(x)$ and $f_{S_2}(x)$, of each distribution are equal. With the other criterion the likelihood ratio would be smaller than 1. So in the above example the animal would be more likely to choose $R_1$ if either $S_1$ or $S_2$ occurred.

The criterion the animal adopted over a session of a few hundred trials can be estimated from $p(\text{correct detections})$ and $p(\text{false alarms})$. The area of distribution $S_1$ to the right of the criterion corresponds to $p(\text{correct detection})$; the probability $x$ greater than the criterion would be produced if $S_1$ occurred. The area of distribution $S_2$ to the right of the
criterion corresponds to the p(false alarms), the probability that x greater than the criterion would be produced if S₂ occurred. By assuming these distributions to be normal and of equal variance, the z equivalent to x can be found in a table of z scores for the unit normal distribution. The formula for the unit normal distribution can be used to calculate \( f_{S_1}(z) \) or \( f_{S_2}(z) \) (Hays and Winkler, 1971, p. 215). If both \( S_1 \) and \( S_2 \) distributions also have equal variance then the likelihood ratio based on z scores is equivalent to the likelihood ratio in equation 1.4.1 (Green & Swets, 1966, ch. 3; McNicol, 1972). The assumption of normal distribution of equal variance has been supported by experiments with pigeons (Hobson, 1970; Stubbs, 1976), porpoises, sea lions (Schusterman & Varrett, 1975), and rats (Hume, 1974; Hume & Irwin, 1974; Nevin, Olsen, Mandell & Yarensky, 1975; Terman & Terman, 1972) as well as humans (Green & Swets, 1966). The stimuli which the animals discriminated included: number of responses required for reinforcement, duration, presence of tone, noise intensity and light intensity.

The criterion, \( B \)-obtained, is assumed to depend on the utilities of certain outcomes. If a correct detection produced an outcome of greater utility than a correct rejection, then the subject would adopt a criterion less than 1. Such a criterion would increase \( p(\text{correct detections}) \) and decrease \( p(\text{correct rejections}) \). An experiment by Hume (1974) nicely illustrates this point.

Rats were trained to discriminate two noise intensities...
which were presented on separate trials. Pressing the right lever in the presence of the louder noise (correct detection), or the left lever in the presence of the less intense noise (correct rejection) produced reinforcing brain stimulation. Incorrect responses, right lever presses with a less intense noise (false alarms) or left lever presses with a louder noise (miss) were followed by a 5 second time out. After the animals were trained, Hume varied the number of brain stimulations for correct detections and correct rejections. For example, in one condition, 3 stimulations were produced by a correct detection and only 1 was produced by a correct rejection. The animals adopted a criterion less than 1 (e.g., $\beta = 1/7$); this increased the $p$(correct detection) and $p$(false alarms) with a net decrease in number of correct responses in a session. However, since more brain stimulations were associated with correct detections, the number of brain stimulations obtained over a session was increased by the adoption of this criterion.

Let us assume for the moment that subjective and objective probabilities are equal and that the objective value of an outcome equals its subjective utility. If one further assumes the animal chooses according to a strategy which maximized expected value (and expected utility) then for a given set of outcomes and a priori $p(S_1)$, there is a unique criterion which the animal will adopt. The relationship between the criterion and expected value can be derived as follows:

$$E(w_2|x) = w_{22} p(S_2|x) + w_{12} p(S_1|x)$$

$$E(w_1|x) = w_{11} p(S_2|x) + w_{21} p(S_1|x)$$

(1.4.2)
where \( p(w_{22}) \) and \( p(w_{21}) \) equals \( p(S_2|x) \), and \( p(w_{11}) \) and \( p(w_{12}) \) equals \( p(S_1|x) \).

Expected value of alternative \( R_2 \), \( E(w_2|x) \) is the sum of the products of the outcomes and their probability of occurrence given \( x \). Likewise, the expected value of \( R_1 \), \( E(w_1|x) \) can be defined given the same information, \( x \), obtained on a particular trial. The animal who maximizes expected value will choose \( R_2 \) if

\[
E(w_2|x) \geq E(w_1|x) \tag{1.4.3}
\]

Otherwise, he should choose \( R_1 \). Substituting the expressions for the expected values, we have

\[
w_{22}p(S_2|x) + w_{12}p(S_1|x) \geq w_{11}p(S_1|x) + w_{21}p(S_2|x) \tag{1.4.4}
\]

and rearranging terms, we have

\[
\frac{w_{22} - w_{21}}{w_{11} - w_{12}} \geq \frac{p(S_1|x)}{p(S_2|x)}. \tag{1.4.5}
\]

In equation 1.4.5 \( p(S_1|x) \) and \( p(S_2|x) \) are unknown; however, using Bayes' theorem we find:

\[
\frac{p(S_1|x)}{p(S_2|x)} = \frac{p(x|S_1)}{p(x|S_2)} \cdot \frac{p(S_1)}{p(S_2)} \tag{1.4.6}
\]
substituting 1.4.6 into 1.4.5 we can specify the relation between the criterion, computed from response probabilities and the outcomes and $p(S_1)$

$$\frac{p(x|S_1)}{p(x|S_2)} \leq \frac{p(S_2)}{p(S_1)} \cdot \frac{w_{22} - w_{21}}{w_{11} - w_{12}}$$  \hspace{2cm} (1.4.7)$$

Equation 1.4.7 implies that when $p(x|S_1)/p(x|S_2)$ exceeds a certain criterion ratio the animal will choose $R_1$. That criterion is $B$-obtained estimated from response probabilities (see equation 1.4.1). Assuming the animals maximize expected value, the likelihood ratio, $B$-obtained, should equal the ratio of a priori stimulus probabilities times the ratio of outcome values,

$$\frac{f(x|S_1)}{f(x|S_2)} = \frac{p(S_2)}{p(S_1)} \cdot \frac{w_{22} - w_{21}}{w_{11} - w_{12}}$$  \hspace{2cm} (1.4.8)$$

Given a certain set of outcomes and an a priori probability of stimulus presentation, the criterion, $B$-obtained, should remain constant as $S_1$ and $S_2$ become more or less discriminable.

Increasing the difference between $S_1$ and $S_2$ (e.g., difference in light intensity) would change the probability of correct detections relative to false alarms; with no false alarms if the stimuli were perfectly discriminable and an equal number of false alarms and correct detections if they were indiscriminable. Increasing the discriminability of $S_1$ and $S_2$ is represented in figure 1.4.1 as a greater difference between the mean of the two distributions. The relative
probability of correct detections and false alarms will change, however, if the animal's choice behavior is consistent with equation 1.4.8 and the hypothesis that it maximizes expected value, $B$-obtained should not change. Furthermore, an animal's criterion should be predicted under different stimulus conditions from the a priori probability of a stimulus occurrence and a measure of the value of the outcomes.

The application of utility theory to the signal detection situation leads to the predictions that an animal's criterion, $B$-obtained, should be determined by the a priori probability of a stimulus occurrence and a measure of the relative values of the outcomes according to equation 1.4.8. Moreover, the value of $B$-obtained and the relation between $B$-obtained and $B$-optimum should not be affected by changes in the stimulus condition because there is no parameter representing discriminability in equation 1.4.8. These two predictions can be examined with the results of other signal detection experiments where the relation between $B$-obtained and $B$-optimum has been studied.
1.5 Studies of Bias in Signal Detection Experiments

Galanter and Holman (1967) trained humans to discriminate differences in noise intensity using a two alternative signal detection procedure. Subjects won cash for correctly detecting a difference or reporting, correctly, no difference. Subjects lost money for incorrect choices. Thus, $w_{22}$, $w_{21}$, $w_{12}$, and $w_{11}$ were varied so that there were 5 combinations of costs and payoffs with a resulting response bias for each set of outcomes. Because $p(S_2) = p(S_1) = .5$, equation 1.4.8 becomes:

$$B_{obt} = \frac{w_{22} - w_{21}}{w_{11} - w_{12}}$$  \hspace{1cm} (1.5.2)

The results could not be described by equation 1.5.2 suggesting that the subjects did not maximize the expected values of the outcomes (i.e., amount of money). Green (1960) has argued that internal level measurement of $B$-obtained may not be meaningful because rather large changes in $B$-obtained result in rather small changes in obtained payoff over a session suggesting that quantitative predictions based on equation 1.5.2 are not meaningful. Another interpretation is that subjects maximize subjective expected utility rather than expected value. Equation 1.5.2 can thus be modified with a function transforming the payoff ratio to equal $B$-obtained which defines the payoff ratio of equivalent utilities. If maximization of subjective expected utility is assumed;
By replotting these data and the data from another similar experiment with humans (Green & Swets, 1966, pp. 83-93), the following formula was found to closely describe the relation between $B$-obtained and the payoff ratio,

$$B_{\text{obt}} = f\left(\frac{w_{22} - w_{21}}{w_{11} - w_{12}}\right)$$

(1.5.3)

where $\theta$ had a value less than 1.

Visual inspection of figure 1 indicates a rank order correlation of 1.0 between $B$-obtained and $B$-optimum; moreover there is some convergence on an approximately linear function on log-log coordinates with a slope, $\theta$, less than 1. The consistency of this function across experiments suggests that differences between $B$-obtained which correspond to differences in $B$-optimum represent a feature of the experiment, namely, the monetary value of the outcomes. Identification of $B$-obtained units with experimental condition (e.g., outcomes) is an essential criterion for an interval scale (Coombs, Dawes & Tversky, 1970, ch. 2). Further support for the view that $B$-obtained is meaningful as an interval scale would be a demonstration of additivity of values of $B$-obtained separately related to different outcome conditions.

Hobson (1970) varied relative probability and amount of reinforcement for correct detections and correct re-
FIGURE 1

The relation between $B$-obtained and $B$-optimum plotted on log-log coordinates. Values of $B$-obtained were determined by reanalysis of response probabilities reported graphically by Galanter & Holman (1967) and in a table by Tanner et al. (1956, in Green & Swets, 1966).
jections in an experiment where pigeons were trained to discriminate response number. In her experiment, the animals initially pecked a center key a requisite number of times in order to turn on a pair of side keys. Two response requirements were programmed to occur randomly; if the higher ratio (number of responses per consequence) had been programmed a single peck to one of the side keys was reinforced, and if the lower ratio had been programmed a single peck to the other side key was reinforced. The pair of side keys was one of three colors which occurred irregularly on each trial. The key colors signalled a different relative probability of reinforcement for correct discriminations so that three different biasing conditions were in effect during a particular session. In subsequent conditions different relative amounts of reinforcement were programmed to occur in the presence of the different key colors for one bird. Hobson found that in the presence of each key color the animal developed a response bias commensurate with the programmed consequences; e.g., a stronger preference for the right key if a greater probability or amount of reinforcement had been programmed for correct right key responses. Because the probabilities of stimulus presentation were equal and the consequences of errors, $w_{21}$ and $w_{12}$, were null, equation 1.5.3 could be further simplified so that $P_{obt}$ equalled a transformation of $w_{22}/w_{11}$. However, both probability of reinforcement and amount of reinforcement were varied so that the outcomes were actually gambles with subjective expected utilities, $E'[u(w)]$. 
Substituting for $w_{22}$ and $w_{11}$ equation 1.5.3 becomes

$$B_{obt} = f \left( \frac{p(w_{22})}{p(w_{11})} \right) = \frac{E'[u(w_{22})]}{E'[u(w_{11})]}$$

(1.5.5)

where $E'[u(w_{22})]$ and $E'[u(w_{11})]$ are subjective expected utilities of correct rejections and correct detections and equal the product of the utility of a reinforcer times its subjective probability (outcome if reinforcement is not delivered is assumed null).

When amount of reinforcer was varied, the probability terms cancelled and the ratio of utilities, $f(w_{22}/w_{11})$, could be estimated by $B$-obtained. Likewise, the ratio of subjective probability of reinforcement, $f[p(w_{22})/p(w_{11})]$ could be estimated when the amounts of reinforcement were equal. For each condition three values of $B$-obtained were obtained - one for each key color. In this way a total of six values of $B$-obtained were estimated for six pairs of relative probability and amount of reinforcement. The functions relating the log ratio of reinforcement probability to log $B$-obtained and the log ratio of amounts to $B$-obtained are plotted in the top of Figure 2. The linear regression equations have nearly identical slopes; $\theta_p = .82$ for probability and $\theta_A = .84$ for amount and identical intercepts, $a$, of .01. The data are too variable to determine if the functions are best described as linear; however, a linear function is assumed and used to transform amount into utility and probability into subjective probability:
\[ \log B'_A = \theta_A \log \left( \frac{w_{22}}{w_{11}} \right) + a \]
\[ \log B'_p = \theta_p \log \frac{p(w_{22})}{p(w_{11})} + a \]

(1.5.6)

where \( B'_A \) and \( B'_p \) are estimates of \( \beta \)-optimum based on, respectively, amount and probability of reinforcement. In the final condition of this series, Hobson varied both relative probability and amount of reinforcement. Using values of \( B'_A \) and \( B'_p \) estimated above \( \beta \)-obtained should be predicted by equation 1.5.5. If utility of an outcome and probability of an outcome combine multiplicatively (equ. 1.5.5) then their logarithms will combine additively,

\[ \log \beta_{\text{obt}} = \log B'_A + \log B'_p \]

(1.5.7)

The results are plotted in the bottom graph in Figure 2 and suggest that the predictions based on equation 1.5.2 were accurate. Probability and amount can apparently be rescaled onto an expected utility dimension based on the values of \( \beta \)-obtained. Moreover, these two dimensions combine additively to accurately predict response bias implying that expected utility may indeed be meaningful as an interval scale.

Thus, the results of experiments by Galanter & Holman (1967); Green & Swets, (1966); and Hobson (1970) rather convincingly suggest that \( \beta \)-obtained is an interval scale. In addition further support is provided by Galanter & Holman (1967) and Green & Swets (1966) who reported orderly and similar relations between \( \log \beta \)-obtained and \( \log \beta \)-optimum
FIGURE 2

The top panel presents the relation between log B-obtained and log B-optimum. Circles are values of B-obtained produced by different amounts of reinforcement with B-optimum the ratio of reinforcement amount for correct rejections over amount for correct detections. Triangles are values of B-obtained produced by different probabilities of reinforcement with B-optimum the ratio of probability of reinforcement for correct rejections over the probability of reinforcement for correct detections. The dashed line is fit to the circles by regression analysis. The solid line is fit to the triangles. The bottom panel values of B-obtained predicted from B'-optimum based on the combined effects of utility and subjective probability (see text). The solid line indicates matching between B-obtained and B'-optimum. Data are from Hobson (1970).
when stimulus probability was varied. With animals Hume & Irwin (1974) and Terman & Terman (1972) obtained orderly relations between \( \beta \)-obtained and \( p(S_2)/p(S_1) \); however, while the relation produced by Terman & Terman's (1972) procedure could be closely described by a power function, the data Hume & Irwin (1974) report systematically depart from a power function. The relations between \( \beta \)-obtained and \( \beta \)-optimum produced by individual animals in Hume & Irwin's experiment were S-shaped on log-log coordinates. In their experiment animals had rather brief, 500 trial, exposures to a particular \( p(S_1) \) condition followed by a different bias condition. Terman & Terman's animals had about 2000 trials of exposure to one \( p(S_1) \) condition. It is likely that Hume & Irwin's animals did not have sufficient exposure to a particular bias condition for stable performance. In Terman & Terman's experiment the discriminability of \( S_1 \) and \( S_2 \) was systematically decreased every 350 trials while a particular \( p(S_1) \) was in effect so it is possible that their animals were not responding at an asymptotic level. It remains unclear what function best describes the relation between \( \beta \)-obtained and \( \beta \)-optimum when probability of a stimulus is varied.

A particular function, which reliably describes the relationship between \( \beta \)-obtained and \( \beta \)-optimum when probability is varied, could be considered a transformation function for converting objective probability into subjective probability. The primary purpose of this research was to define the trans-
formation function for probability. This was done by re-analysing the results of other experiments in the Hume & Irwin (1974) study in which \( p(S_1) \) was varied according to different schedules (the bias functions were not reported) and an experiment was conducted in which the probability of an outcome was varied.

In the present experiment animals were given approximately 5000 trials of exposure to a particular outcome probability. Successive conditions with different outcome probabilities were programmed with one stimulus condition in effect, permitting the definition of a relation between \( \beta \)-obtained and outcome probability under a particular stimulus condition.

Equation 1.4.8 implies that \( \beta \)-obtained is related to the ratio of stimulus probabilities, \( p(S_2)/p(S_1) \), times a ratio of outcome values with no variable representing outcome probability. Using theorem 5.1 of expected utility theory (Luce & Raiffa, 1959) one can substitute the expected value of an outcome for an outcome so equation 1.4.8 becomes,

\[
\beta_{\text{obt}} = \frac{p(S_2)}{p(S_1)} \cdot \frac{E(w_{22}) + E(w_{21})}{E(w_{11}) + E(w_{12})} \quad (1.5.9)
\]

which says that the two alternatives available to an animal under a particular stimulus condition represent two gambles rather than two certain outcomes. The expected values of correct detections and correct rejections were the probability of the outcome times the value of the outcome, \( p(w)w \), where the outcome was always a .02 cc dipper of water and \( w \) was
assumed null if reinforcement did not occur. Because \( w_{21} \) and \( w_{12} \), the consequence of incorrect responses, were assumed null, the expected values of those alternatives were eliminated from equation 1.5.9. The ratio of stimulus probabilities was eliminated because in this experiment the programmed probability of either stimulus which the animals discriminated was .5 and, thus, \( p(S_2)/p(S_1) = 1 \). That left

\[
P_{\text{obt}} = \frac{E(w_{22})}{E(w_{11})} = \frac{p(w_{22})w_{22}}{p(w_{11})w_{11}}
\]

which reduced to

\[
P_{\text{obt}} = \frac{p(w_{22})}{p(w_{11})}
\]  

(1.5.10)

because \( w_{22} \) and \( w_{11} \) were identical .02 cc dippers of water. The right side of equation 1.5.10 is \( P \)-optimum. Equation 1.5.10 would hold only if an animal's subjective probability estimates matched objective outcome probability. Departures from a matching relation would be characterized as a relation between \( P \)-obtained and \( P \)-optimum, \( f[p(w_{22})/p(w_{11})] \), where \( f \) was the transformation function which converted the ratio of objective probabilities into the corresponding ratio of subjective probabilities. The transformation function was defined by \( P \)-obtained which represents the effect a particular set of outcome probabilities had on response bias; that is, subjective probability is being directly indicated by
a continuous measure of behavior \( B \)-obtained. If the relations between \( B \)-obtained and \( B \)-optimum measured for individual animals converged onto a particular function then \( B \)-obtained could be considered an interval level measure of outcome probabilities. If, in addition, this same function characterized the effects of varying stimulus probability in other experiments (Hume & Irwin, 1974; Terman & Terman, 1972) then a general transformation function for converting objective probability into subjective probability would be indicated.

In this experiment the scale of subjective probability was derived by varying relative probability of an outcome. Probability of a stimulus occurrence and probability of an outcome may be qualitatively different continua. Such a difference would be indicated if bias functions obtained in experiments where \( p(S_1) \) was varied did not have the same form as bias functions obtained in this experiment. Another indication of a qualitative difference would be a failure to predict the effects of \( p(S_2)/p(S_1) \) on behavior using a scale of subjective probability derived by varying probability of an outcome. In one condition of this experiment a particular value of \( p(S_1) \) was chosen and the bias effect predicted using the subjective probability scales derived by varying outcome possibilities. If varying the probability of either a stimulus or an outcome resulted in the same function relating \( B \)-obtained and \( B \)-optimum, then one should predict \( B \)-obtained when both are varied. The combinatorial rule when more than one variable affecting response bias changes is
\[ \beta_{obt} = \frac{p'(S_2)}{p'(S_1)} \cdot \frac{E'[u(w_{22})] - E'[u(w_{21})]}{E'[u(w_{11})] - E'[u(w_{12})]} \]  \hspace{1cm} (1.5.11) 

where \( p'(S) \) is the subjective probability of an outcome and \( E[u(w)] \) is the expected utility of the various outcomes indicated by the subscripts. That is, separate definition of the expected utilities of various outcomes and the subjective probability of a stimulus presentation should enable \( \beta \)-obtained to be predicted. Equation 1.5.11 is an analogue to equation 1.4.8 in terms of subjective expected utility. Assuming \( E[u(w_{12})] \) and \( E[u(w_{21})] = 0 \), equation 1.5.11 becomes, in logarithmic terms,

\[ \log \beta_{obt} = \log \frac{p'(S_2)}{p'(S_1)} + \log \frac{E'[u(w_{22})]}{E'[u(w_{11})]} \]  \hspace{1cm} (1.5.12) 

and if \( w_{22} \) and \( w_{11} \) are equal as they were in this experiment then equation 1.5.12 suggests that the bias effects of varying both \( p(S_2)/p(S_1) \) and \( p(w_{22})/p(w_{11}) \) will be additive,

\[ \log \beta_{obt} = \log \frac{p'(S_2)}{p'(S_1)} + \log \frac{p'(w_{22})}{p'(w_{11})} \]  \hspace{1cm} (1.5.13) 

In the final condition of this experiment both \( p(S_2) \) and \( p(w_{22}) \) were reduced to .4 and the corresponding subjective probabilities, \( p'(S_2) \) and \( p'(w_{22}) \), and their complements were computed using the subjective probability scales derived when outcome probability was varied. If \( \beta \)-obtained is indeed and interval scale then the effects of varying these two biasing variables should be additive.
Defining an interval level subjective probability scale which corresponds to the continuum of objective probability should enable one to quantitatively predict an individual's choice behavior in other situations. In the present study, these other situations were different stimulus conditions where a subject discriminated with a different level of accuracy than when the subjective probability function was defined. The relations between $\beta$-obtained and $\beta$-optimum should remain invariant when stimulus conditions are changed if the estimates of subjective probability for individuals are consistent across situations.
SECTION II

METHOD

2.1 Subjects

Seven albino rats, R10-R16, were used at various stages in this experiment. Only R12, R14, and R16 were run under all conditions, R13 and R15 died, and R10 and R11 were started after the experiment was underway. The animals were housed in a continuously lighted room. They had ad lib access to food and were maintained on a 15 - 20 cc daily water ration in addition to the water obtained during the session.

2.2 Apparatus

In a conventional rat chamber, 23.5 by 20.5 by 19.5 cm, two levers were mounted symmetrically on the front wall, 10 cm apart, with a .02 cc water dispenser centered between them. A single lever was mounted centrally at the rear of the chamber. A dim houselight provided general illumination throughout the session except when it was briefly turned off following incorrect responses (discussed below). Visual signals were produced by lighting a symmetrical array of bulbs in a matrix of 25 bulbs, mounted centrally above the chamber. The light was diffused through two layers of milk plastic, uniformly illuminating the chamber. Illumination was varied by lighting a different number of bulbs and was measured as illumination of the ceiling which was transluminated by the light source. The chamber and its associated equipment were enclosed in a sound-attenuating box which was located in the animal colony where
humidity and temperature were regulated. The electromechanical programming and recording equipment was located in a separate room.

2.3 Procedure

The rats R12-R16 were trained with water reinforcement to press the right bar in the presence of a 1.20 ft.-Ca illumination and to press the left bar while the light intensity was .37 ft.-Ca. They were trained to press the centrally located rear lever after all animals were reliably pressing the right lever when the brighter light was on and the left lever in the presence of the dimmer light. The animals were then put on a close approximation to the final procedure.

A single rear-lever press produced either S₁, the brighter light, (1.2 ft.-Ca) or S₂, the dimmer light (.37 ft.-Ca). If S₁ occurred then a single press on the right front lever (R₁), a correct detection, was reinforced with water; if S₂ came on the a single press on the left front lever (R₂), a correct rejection, produced water. Incorrect responses, a left lever press (R₂) when S₁ was on, a miss, or a right lever press (R₁) when S₂ occurred, a false alarm, never produced water. After the animals had learned this sequence, the probability of reinforcement was reduced to .5 for correct responses and after incorrect responses a 3 second time-out occurred. During the time-out the houselight was off and responses had no consequences. The time-out was added to disrupt stereotypic response patterns where an animal would rapidly alternate between the rear lever and a preferred front lever - collect-
ing reinforcement whenever a response happened to be correct. There was no intertrial interval, thus following correct responses the animal could immediately press the rear bar to initiate the next trial.

The intensity of $S_2$ was gradually increased to 1.05 ft.-Ca, and after performance stabilized by visual inspection, a series of biasing conditions was imposed. Approximately 70 training sessions were completed before the first condition. Sessions were, at first, 500 trials, however, following the first two bias conditions the session length was shortened to 300 trials because some animals stopped responding after 300 - 400 trials. During each biasing condition there was a different probability of reinforcement for correct detections and correct rejections. The reinforcement probabilities were always complementary so that overall the probability of reinforcement for correct responses remained .5. In the first bias condition the probability of reinforcement for correct detections was .9 and correct rejections produced reinforcement with a probability of .1. In subsequent bias conditions the probability of reinforcement for correct detections, $p(w_{11})$, was .1, .4, .6, and .5. The probability of reinforcement for correct rejections, $p(w_{22})$ was $1 - p(w_{11})$. The stimuli were made more discriminable by changing $S_2$ to .66. A series of 8 bias conditions was programmed with $p(w_{11})$ equal to .5, .9, .1, .6, .4, .3, .7, .5 and $p(w_{22})$ equal to the respective complements. Following this series of conditions the probability of $S_1$ was decreased to .4 and in the final condition $p(w_{11})$ was decreased from .5 to .4 in order to assess the combined
effects of these two biasing variables. The sequence of conditions is listed in Table 1 along with the animals who completed the particular conditions. R10 and R11 were trained using a procedure similar to the other animals and introduced into the experiment following the change in stimulus conditions after receiving approximately 40 training sessions. The number of sessions and trials per condition are also listed in Table 1. For the first 2 bias conditions longer and fewer sessions were run with all subsequent conditions 15 sessions of 301 trials each. The number of trials per condition exceeded the number used in other experiments where response bias was varied (Hume, 1974; Hume & Irwin, 1974; Terman & Terman, 1972) and was sufficient to produce performance which did not systematically increase or decrease (i.e., was stable) by the last 1000 trials for nearly all conditions in a previous experiment (Whittaker, in prep).
TABLE 1

The sequence of conditions is listed in the order of their presentation. Columns are, respectively, condition number, the illumination of the stimuli (S₁ and S₂) discriminated by the animals, programmed probability of reinforcement for correct detections (reinforcement probability for correct rejections is the complement), programmed probability of S₁ presentation (probability of S₂ presentation is the complement), number of trials per daily session, and number of sessions per condition. In the last 7 columns the animals tested under each condition are listed.
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<th>Condition</th>
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<th>S2 (ft-Ca)</th>
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<th>p(S1)</th>
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*Summary data not reported, sessions less than half completed.*
SECTION III
RESULTS

3.1 The Effects of Relative Reinforcement Probability on Response Probability

Changing the relative probability of reinforcement for correct detections usually brought about an abrupt shift in the probabilities of correct detections and false alarms. Increases in reinforcement probability for correct detections resulted in higher probabilities of reporting the bright light, $R_1$, in the presence of both stimulus conditions; when the probability of reinforcement for correct detections was reduced the probability of correct detections and false alarms decreased. When difference in luminance between $S_1$ and $S_2$ increased, the probability of correct detections and false alarms diverged. Changes in both discrimination accuracy and response bias can be seen in some session-by-session plots of response probabilities of randomly chosen animals in Figure 3. The top two panels present performance under less discriminable conditions. The middle and bottom panels are from conditions where the luminance difference between $S_1$ and $S_2$ was greater. The changes in behavior accompanying stimulus change can be clearly seen with a larger separation of the probabilities of correct detections and false alarms in the lower four panels. Modifying the relative probability of reinforcement drove both the probabilities of correct detections and false alarms to a different level - usually within 5 sessions.
FIGURE 3

The probabilities of correct detections \( (p(C.D.)) \) and false alarms \( (p(F.A.)) \) are plotted for successive daily sessions. Different panels include data for different animals. Different bias conditions are delineated with vertical dashed lines with the probability of reinforcement for correct detections which was in effect centered above the sessions. The top two panels include data from the less discriminable stimulus conditions. The bottom four panels include data from more discriminable stimulus conditions.
A subsequent systematic drift in response probabilities in some animals appears more as changes in accuracy than bias. For R11 (.9), R12 (.6), and R14 (.7) false alarm probabilities diminished with no discernible change in the probability of correct detections. Following a shift in reinforcement probability to .3, the probability of correct detection by R16 increased through that condition with no systematic change in false alarm probability. R12, however, shows a decrement in the probability of R1 in both stimulus conditions during the latter half of the .4 condition—a shift in bias. These animals typify the performance of other animals in the accommodation of their performance to new stimulus and reinforcement conditions. Usually session to session variation in performance by the last 3 sessions of a condition was either not systematic or suggestive of a gradual drift in response probability.

The pooled data from the last 3 sessions were used to characterize the effects of the relative probability of reinforcement for that condition. In all but the first 3 conditions of the experiment these response probabilities are based on a total of 900 trials, approximately 450 S1 presentations and 450 S2 presentations over 3 sessions. In the first 3 conditions there were 500 rather than 300 trials per session. To insure that measured performance in the 500 trial sessions was comparable to 300 trial sessions, the last 200 trials were excluded. For condition 3 a total of 900 trials was pooled, but for conditions 1 and 2 only 600 trials were used to compute the summary statistics because the first 100 trials of
these conditions were not counted. Decisions concerning the
data to be used were made prior to data analysis.

By examining summary response probabilities listed
in Table 2 one can see lower accuracy in conditions 1 through
5, when the luminance difference was smaller, than in the re­
maining conditions. Lower accuracy is indicated by less
difference between the probability of correct detections and
false alarms. The response probabilities also reveal a shift
in preference with both the probability of correct detections
and false alarms positively correlated with the relative
probability of reinforcement for correct detections.

Separation of changes in accuracy and bias by direct
examination of response probabilities is at best difficult
and can only be qualitative because an increase in preference
for one alternative will necessarily decrease the difference
between the probability of correct detections and false alarms
as they both converge on 1 or 0. Signal detection analysis is
preferred because it permits the separation of changes in bias
and discrimination accuracy into logically independent indices
computed from the probability of correct detections and false
alarms.
TABLE 2

Condition number, programmed probability of reinforcement for correct detections and summary statistics; probability of correct detections, probability of false alarms, probability of $R_1$, $d'$, and $B$-obtained listed for individual animals; conditions 1-13. If both probabilities of correct detections and false alarms were greater than .99 or less than .01, $d'$ and $B_{obt}$ were not computed because these statistics could not be accurately estimated by response probabilities estimated on the basis of 450 trials.
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3.2 Signal Detection Analysis

The performance of any detection device, human, animal or electronic operating on the basis of the theory of signal detectability outlined in the introduction, can be completely described by what is called the "receiver operating characteristic curve" or R.O.C. curve. The R.O.C. curve is a functional relationship between the probabilities of correct detections and false alarms. Because misses and correct rejections are respective complements of correct detections and false alarms, all the information about the receiver's performance is contained in the correct detection and false alarm rate. For any fixed set of experimental conditions the theory says there will be a \( \beta \), a criterion beyond which the observer will report \( S_1 \) (the brighter light). The observer's correct detection and false alarm probability will be the areas under the \( S_1 \) and \( S_2 \) distributions respectively, above the point defined by \( \beta \).

The assumption that the distributions are normal permits one to convert response probabilities into \( z \) scores using a table for the unit normal distribution. The \( z \) score equivalents to the probability of a correct detection are plotted as a function of the \( z \)-transformation of the probability of false alarms. An R.O.C. curve plotted on \( z \)-transformed coordinates more clearly illustrates changes in accuracy and shifts in bias than R.O.C. plots on linear-linear coordinates. The R.O.C. curves which characterize the performance of the observers in this experiment, in Figure 4, are on probability-probability coordinates which directly transform response probability into
a z equivalent. (Green & Swets, 1966, discuss this issue in depth).

The major diagonal which extends from the lower left to the upper right corners of the R.O.C. plots in Figure 4 is the expected R.O.C. curve if the receiver responds randomly with respect to the stimuli. Along this line the probability of correct detections equals the probability of false alarms suggesting that the distributions $S_1$ and $S_2$ are identical. A point on this curve merely reflects the observer's bias for $R_1$. If $R_1$ occurred randomly 40% of the time, then both $p(R_1|S_1)$ and $p(R_1|S_2)$ would equal .4. Along the major diagonal the likelihood ratio, $\Lambda$-obtained, would always be 1 because the ordinate heights of the hypothetical $S_1$ and $S_2$ distributions are identical. Curves plotted above this diagonal also describe both accuracy and response bias. In Figure 4, the distance of the curves from the major diagonal is constant at all points because they have identical unit slopes. This distance represents accuracy and the value of the parameter $d'$. When $d'$ is computed from the probability of correct detections and false alarms directly, it is the distance of the point representing these probabilities from the major diagonal. Points along each curve correspond to different response biases and in all cases, except the major diagonal, different likelihood ratios. The likelihood ratios can also be computed directly from the probabilities of correct detections and false alarms for each point. The likelihood ratio, called $\Lambda$-obtained, is the criterion chosen by the observer for a given payoff and stimulus condition.
FIGURE 4

R.O.C. plots of the probability of correct detections as a function of probability of false alarms for each animal. Points represent different bias conditions with the probability of reinforcement for correct detections indicated adjacent to each point. Open squares are data from more discriminable stimulus conditions with illuminance (ft.Ca.) values of $S_1$ and $S_2$ indicated in the legend. Closed squares are data from less discriminable stimulus conditions. The major diagonal extends from the lower left to the upper right corners of each plot. The other solid lines represent theoretical R.O.C. curves for median values of $d'$ across all bias conditions within a stimulus condition.
In this experiment $d'$ was computed by first converting
the probability of correct detections and false alarms into
the corresponding $z$ scores for a unit normal distribution, and
then computing the difference between the two $z$ scores. The
values of $d'$ for each relative reinforcement condition did
not change systematically for individual animals under the
less discriminable stimulus condition. This can be seen by
examining values of $d'$ in Table 2 or graphically as the dis­
tance of the closed points from the major diagonal. The R.O.C.
curve was estimated from the median $d'$ across reinforcement
conditions and is based on the assumption that the underlying
distributions of $S_1$ and $S_2$ are normal and of equal variance.
If the actual distributions were not normal, the R.O.C. curve
would not be linear. If the distributions did not have equal
variance, then the R.O.C. curve would not have a unit slope
(Green & Swets, 1966). Under the less discriminable conditions
the points do not depart systematically from predicted R.O.C.
curves. Under the conditions where the animals were discrim­
inating a greater difference in $S_1$ and $S_2$ the obtained prob­
abilities of correct detections and false alarms depart sys­
tematically from the predicted R.O.C. curve. The curve was
drawn assuming Gaussian distributions of equal variance and
a median $d'$ across relative reinforcement conditions 6 to 13.
The data under this stimulus condition are too variable to
evaluate the linearity prediction; however, a departure from
a unit slope is evident. Points resulting from the .7 and .9
relative reinforcement conditions tend to be closer to the
major diagonal (lower $d'$ in Table 2) than their complement
conditions, .3 and .1. This suggests that the distribution assumption of equal variance is incorrect. In addition, the assumption that the underlying distributions of \( S_1 \) and \( S_2 \) are normal is not clearly supported. Both assumptions are important for the computation of the bias index \( B_{obtained} \); fortunately, additional data from a nearly identical experiment are available.

In Figure 5, data are replotted from an experiment by Nevin, Olsen, Mandell, and Yarensky, (1975) which used rats trained to discriminate light intensity differences. The study was conducted in the same chamber used in this experiment with differences in light intensity controlled with an identical apparatus. Two rats were discriminating with accuracy similar to performance of rats in the more discriminable stimulus condition; however, their data are much more orderly. The data converge on a linear function when they are plotted on probability-probability coordinates supporting the assumptions that the underlying distributions of \( S_1 \) and \( S_2 \) are normal.

Although available evidence supports linearity of the R.O.C. curve, the discrepancy between the slopes obtained under more discriminable conditions (Figure 4) and the slopes obtained in the Nevin, et al. (1975) study leaves the assumption of equal variances questionable. This raises problems in computing an estimate of \( B_{obtained} \).

The formula for \( B_{obtained} \), \( B_{obt} \), is

\[
B_{obt} = \frac{f_{S1}(x)}{f_{S2}(x)} \quad (3.2.1)
\]
FIGURE 5

R.O.C. plots of the probability of correct detection as a function of probability of a false alarm for two animals discriminating the same stimuli. The major diagonal extends from the lower left to the upper right corners. The other diagonals are theoretical R.O.C. curves.

Data are from experiment #1 reported by Nevin et al., (1975).
Nevin et al. (1975)
$\beta$-obtained is estimated by locating the $z$ score corresponding to the probability of correct detections in a table for the unit normal distribution; the formula for the unit normal distribution (Hays & Winkler, 1971, p. 215) is used to compute the ordinate height, $f_{S1}(z)$. Likewise, $f_{S2}(z)$ is computed from the probability of false alarms

$$
\beta_{obt} = \frac{f_{S1}(z)}{f_{S2}(z)} \quad (3.2.2)
$$

If the variances of the $S_1$ and $S_2$ distributions are equal, then the $z$ scores represent equivalent units on $x$ and equation 3.2.2 produces the same estimate of $\beta$-obtained as equation 3.2.1. If the variances differ, however, then equations 3.2.1 and 3.2.2 are not equivalent (McNicol, 1972).

When $S_1$ and $S_2$ distributions do not have equal variance then two nonequivalent estimates of the likelihood ratio are available, equation 3.2.1 or 3.2.2. If equation 3.2.1 is used then the relation between $\beta$-obtained and $\beta$-optimum (equation 1.4.6) is

$$
\frac{f_{S1}(x)}{f_{S2}(x)} = \frac{p(S_2)}{p(S_1)} \cdot \frac{w_{22} + w_{12}}{w_{11} + w_{21}} \quad (3.2.4)
$$

If equation 3.2.2 is used, then equation 3.2.4 would be rewritten as

$$
\frac{f_{S1}(z)}{f_{S2}(z)} = \frac{p(S_2)}{p(S_1)} \cdot \frac{w_{22} + w_{12}}{w_{11} + w_{21}} \quad (3.2.5)
$$
An estimate of $\beta$-obtained based on values of $x$ (eqn. 3.2.2) requires an additional parameter, $m$, which describes the relative variance of the two distributions. Normal distributions of $z$ scores always have a variance of 1, so an additional parameter is not necessary and different variances of the $S_1$ and $S_2$ distributions can be ignored. As there are no available data to guide the selection of one of the two procedures for estimating the likelihood ratio, equation 3.2.2 was chosen because it is the more parsimonious of the two.
3.3 The Effects of Relative Reinforcement Probability on B-obtained

Expected utility theory predicts a unique relation between B-obtained and relative reinforcement probability. This relation, developed in the introduction as equation 1.5.5, is:

\[ B_{obt} = f \left( \frac{p(w_{22})w_{22}}{p(w_{11})w_{11}} \right) = \frac{E[u(w_{22})]}{E[u(w_{11})]} \]  \hspace{1cm} (3.3.1)

where \( E[u(w_{22})] \) and \( E[u(w_{11})] \) are the subjective expected utilities of, respectively, correct rejections and correct detections. The rule for transforming the ratio of the expected values of a correct rejection, \( p(w_{22})w_{22} \), over the expected value of correct detections, \( p(w_{11})w_{11} \), is \( f \). All rats received an identical .02 cc dipper of water for correct rejections, and correct detections. Accordingly the payoffs for correct rejections, \( w_{22} \), and correct detections, \( w_{11} \), can be considered equal so the ratio of expected values of correct discriminations become equivalent to the ratio of the probabilities of reinforcement, \( p(w_{22}) \) and \( p(w_{11}) \) which were programmed. The probability of reinforcement for correct rejections over the complementary probability of reinforcement for correct detections is called B-optimum because matching B-obtained and the ratio of reinforcement probabilities would produce the maximum number of reinforcements in a session. If matching occurred, then \( f \) would be linear with a unit slope and expected utility
would equal expected value. The relations between log $B$-obtained and log $B$-optimum are plotted for individual animals in Figure 6. Values of $B$-obtained derived from performance under different stimulus conditions are distinguished on the graphs of animals who were tested under both conditions. In cases where there were more than 3 points for a particular stimulus condition, a regression analysis was performed to determine the best fit linear function relating log $B$-obtained to log $B$-optimum under a particular stimulus condition. Regression lines for different stimulus conditions are also distinguished in Figure 6 and the parameters are listed in Table 3.

The data in Figure 6 are well described by linear functions. The correlation coefficients, with the exception of R16, are all above .94 (see Table 3) and departures from the regression lines are not systematic. A linear function on log-log coordinates implies that $B$-obtained is a power function of $B$-optimum.

The slopes of the regression lines differ from 1 for data obtained under both stimulus conditions for all animals, suggesting that animals did not maximize objective value. The objective probabilities of reinforcement could be rescaled as subjective probability according to a power function to produce matching between $B$-obtained and $B$-optimum. The animals could then be considered to have maximized expected utility. Such a transformation could take the form

$$ p'(w) = \left[ p(w) \right]^\theta \quad (3.3.2) $$
FIGURE 6

Log $B$-obtained as function of log $B$-optimum. Solid lines are fit to triangles by regression analysis; dashed lines are fit to circles. Triangles represent data generated under more discriminable stimulus conditions (illuminance values of $S_1$ and $S_2$ are specified in the legend); circles represent data obtained under less discriminable stimulus conditions. Two points are not plotted; for R10 the value of $B$-obtained was 0 when $B$-optimum = -.96 and for R12 the value of $B$-obtained under less discriminable stimulus conditions was infinity (division by zero) when $B$-optimum = .96.
where $p'(w)$ is subjective probability and $\theta$ the slope of the regression line listed in Table 3. This follows because if

$$\log \beta_{obt} = \theta \log \beta_{opt} + a$$

then

$$\beta_{obt} = a \beta_{opt}^\theta$$

and because

$$\beta_{opt} = \frac{p(w_{22})}{p(w_{11})}$$

then

$$\beta_{obt} = a \left(\frac{p(w_{22})}{p(w_{11})}\right)^\theta \hspace{1cm} (3.3.3)$$

This argument states that the exponent of the power function relating $\beta$-obtained to $\beta$-optimum also is the exponent of a power function which converts objective probability into subjective probability. The y-intercept in Table 3, $a$, indicates a general preference for $R_1$ or $R_2$ and could be considered the animal's subjective indifference point. It represents the predicted value of $\beta$-obtained if $\beta$-optimum equals 1 ($\log \beta$-optimum equals 0); that is, when the programmed outcomes of correct detections and correct rejections are equal.

Rescaling objective probability of reinforcement into subjective probability should permit $\beta$-obtained to be predicted under different stimulus conditions. The relations between $\beta$-obtained and $\beta$-optimum should not systematically change when stimulus conditions change. In Figure 6, the regression lines
<table>
<thead>
<tr>
<th>Animal Discrimination accuracy (median d')</th>
<th>intercept slope S.E. of slope</th>
<th>correlation S.E. of coefficient Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.8</td>
<td>-.125</td>
</tr>
<tr>
<td>11</td>
<td>2.2</td>
<td>.226</td>
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<td>2.0</td>
<td>.330</td>
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<td>13</td>
<td>.76</td>
<td>.050</td>
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<td>14</td>
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<td>.086</td>
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<td>16</td>
<td>.49</td>
<td>.196</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>.170</td>
</tr>
</tbody>
</table>

*Minimum slope since one point could not be defined on log coordinate (B=0)
through values of $p$-obtained resulting under more discriminable stimulus conditions generally have steeper slopes than the regression lines obtained under less discriminable conditions. (See also regression coefficients in Table 3). However, R16 and R12 are exceptions. A regression line is plotted for R16 which has a steeper slope under the less discriminable conditions. The regression line for the less discriminable conditions is not plotted for R12 because the animal exclusively preferred R2 when $p(w_{11}) = .1 \log \beta\text{-optimum} = .954$. Such an extreme preference results in $p$-obtained equal to infinity. Although the point could not be defined it is certainly suggestive of a steeper slope under less discriminable conditions. Although there are two animals who have $B$-$B$ functions which are steeper under less discriminable conditions, R13, R14, and R15 generated data which converged on linear functions with shallower slopes under less discriminable conditions. Their data were less variable (i.e., lower standard error of slope in Table 3) than data generated by R16 and thus are more reliable estimates of the slopes of regression lines. Shallower slopes under less discriminable conditions are thus indicated although not reliably.

Examination of the relations between $\log p$-obtained and $\log B$-optimum plotted in Figure 6 leads to two conclusions. Firstly, a power function converts objective probability of reinforcement into subjective probability. Secondly, estimates of subjective probability, obtained under one stimulus condition, do not reliably predict $B$-obtained under a very different stimulus condition.
3.4 Predicting $\beta$-obtained from Estimates of $p'(S_1)$ and $p'(w_{ll})$

If subjective probability of payoff and stimulus presentation are equivalent transformations of objective probability then the scale of subjective probability developed by varying relative reinforcement could be used to predict the effects of varying $p(S_1)$ on $\beta$-obtained. In condition 14 the probability of reinforcement for correct detections and correct rejections was equal and the stimulus conditions were not changed. The probability of occurrence of the bright light ($S_1$) was reduced to .4. Equation 1.4.8 predicts that this should result in a bias toward reporting the dim light. This prediction can be expressed (see also equation 1.5.1) quantitatively as,

$$P_{obt} = \frac{p(S_2)}{p(S_1)} = \frac{.4}{.6}$$

(3.4.1)

or if objective and subjective probabilities are not equal and maximization of expected utility is assumed, the analogous formula would be,

$$P_{obt} = a \frac{p'(S_2)}{p'(S_1)} = a \left(\frac{.4}{.6}\right)^6$$

(3.4.2)

The purpose of condition 14 was to determine whether subjective probability, $p'(S_2)$ and $p'(S_1)$ or objective probability would more accurately predict $\beta$-obtained. Subjective
probability was estimated by a power transformation of the objective probability with the exponent, $\theta$, estimated in the previous conditions 6-13 where probability of reinforcement was varied under the more discriminable stimulus conditions. The animal's general response bias for $R_1$ over $R_2$, expressed as $a$, is also estimated from the previous conditions (see Section 2.4) for each animal. The results of this condition are listed in Table 4. $\beta$-obtained did not vary systematically from $\beta$-optimum which was based on subjective probability (Equation 2.5.1); however, when $\beta$-optimum was based on the ratio of objective stimulus probabilities then $\beta$-obtained was underestimated for all animals. The difference between obtained and predicted was averaged across animals. When objective probabilities were used the mean difference was .194 and the standard error was .29; however, when scaled probabilities were used the mean difference reduced to .072 with a lower standard error of .166. In this case an estimate of subjective probability based on objective probability of reinforcement predicts the effects of probability of a stimulus occurrence on response bias with more accuracy and reliability than if objective probabilities were used.

The purpose of condition 15 was to determine if $p'(S_2)/p'(S_1)$ and $r'(w_{22})/p'(w_{11})$ have additive effects on $\beta$-obtained. Additivity is predicted by Equation 3.4.3 and the assumption that subjective probability is an interval scale.

In condition 15 the probability of the bright light remained at .4 and the probability of reinforcement for correct detections decreased to .4. As a consequence, all
TABLE 4

Summary statistics for individual animals; conditions 14 and 15. Columns are, respectively, animal number, programmed probability of reinforcement for correct detections (reinforcement probability for correct rejections is the complement), programmed probability of $S_1$ presentation (probability of $S_2$ presentation is the complement), probability of correct detections, probability of false alarms, predicted $\beta_{obt}$ on the basis of objective probabilities of reinforcement and stimulus presentation, predicted $\beta_{obt}$ on the basis of (scaled) subjective probabilities of reinforcement and stimulus presentation, $\beta$-obtained, and the differences between obtained and predicted.
<table>
<thead>
<tr>
<th>Animal</th>
<th>$\pi(W_{ap})$</th>
<th>$\pi(S_t)$</th>
<th>$\pi(C.D.)$</th>
<th>$\pi(F.A.)$</th>
<th>Predicted $\log P_{opt}$</th>
<th>Obtained $\log P_{obt}$</th>
<th>Difference $\log P_{obt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>unsealed</td>
<td>scaled</td>
<td>unsealed</td>
<td>scaled</td>
<td>unsealed</td>
<td>scaled</td>
<td>unsealed</td>
</tr>
<tr>
<td>10</td>
<td>.5</td>
<td>.4</td>
<td>.723</td>
<td>.062</td>
<td>.176</td>
<td>.151</td>
<td>.244</td>
</tr>
<tr>
<td></td>
<td>.4</td>
<td>.4</td>
<td>.643</td>
<td>.087</td>
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<td>.371</td>
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<td>12</td>
<td>.5</td>
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<td>.043</td>
<td>.176</td>
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<td>.607</td>
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<tr>
<td></td>
<td>.710</td>
<td>.011</td>
<td>.528</td>
<td>.011</td>
<td>.352</td>
<td>.731</td>
<td>1.07</td>
</tr>
<tr>
<td>14</td>
<td>.5</td>
<td>.4</td>
<td>.927</td>
<td>.022</td>
<td>.176</td>
<td>.187</td>
<td>.422</td>
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<tr>
<td></td>
<td>.4</td>
<td>.4</td>
<td>.919</td>
<td>.021</td>
<td>.352</td>
<td>.384</td>
<td>.473</td>
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<td>16</td>
<td>.5</td>
<td>.4</td>
<td>.674</td>
<td>.141</td>
<td>.176</td>
<td>.321</td>
<td>.207</td>
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<tr>
<td></td>
<td>.4</td>
<td>.4</td>
<td>.699</td>
<td>.053</td>
<td>.352</td>
<td>.473</td>
<td>.508</td>
</tr>
</tbody>
</table>
animals developed a stronger preference for \( R_2 \) indicated by a higher \( \log \beta \)-obtained which could be predicted as (see Equation 1.4.6):

\[
P_{\text{obt}} = \frac{p(S_2)}{p(S_1)} \cdot \frac{E(w_{22})}{E(w_{11})} \tag{3.4.3}
\]

where \( E(w) = p(w)w \) (outcome if reinforcement is not delivered is assumed null). Because \( w_{22} = w_{11} \) Equation 3.4.3 becomes:

\[
\log P_{\text{obt}} = \log \frac{p(S_2)}{p(S_1)} + \log \frac{p(w_{22})}{p(w_{11})} \tag{3.4.4}
\]

The prediction of \( \beta \)-obtained based on scaled probabilities is computed as follows:

\[
\log P_{\text{obt}} = a + \theta \log \frac{p(S_2)}{p(S_1)} + \theta \log \frac{p(w_{22})}{p(w_{11})} \tag{3.4.5}
\]

The slope, \( \theta \), and the y-intercept, \( a \), are parameters estimated for each animal in Table 3. In Table 4 the \( \beta \)-obtained exceed the predicted based on either objective or subjective probabilities; however, the differences between predictions based on Equation 3.4.5 are smaller for 3 of the 4 animals when subjective probabilities were used. For R10, R14, and R16 the estimates based on Equation 3.4.5 are within the standard error of estimates for these animals listed in the last column of Table 3. These standard error of estimates are based on the regression analysis of the relation between
B-obtained and B-optimum when probability of reinforcement was varied (conditions 6-13) and indicate expected variability of data around estimates based on transformation of objective probabilities into subjective probability.

It appears that predictions of the bias effects when the probability of a stimulus is .4 are more accurate and reliable when a subjective scale of probability is used than if an objective scale is used, even though the scale was derived by varying probability of reinforcement. The bias effects when both \( p(S_1) \) and \( p(w_2) \) were set to .4 did not deviate significantly from predictions based on the additivity hypothesis and scaled probabilities (i.e., Equation 3.4.5)
SECTION IV
DISCUSSION

4.1 The Interaction of $B$-obtained and Stimulus Conditions

All animals exhibited a stronger preference for an alternative if it was associated with a higher probability of reinforcement. Inspection of the probabilities of correct detections and false alarms or the unconditional probability of reporting the bright light ($R_1$) in Table 2 reveals this ordering of preferences as a positive correlation between the probability of $R_1$ and the relative probability of reinforcement for correct detections, $p(R_1/S_1)$. A higher probability of reinforcement for correct detections relative to correct rejections generally resulted in a higher probability of correct detections and false alarms under either discrimination condition. Although examination of response probabilities would reveal a rank ordering of preference, the quantitative effects of relative probability of reinforcement on preference (or bias) measured directly using response probabilities are obscured in a detection situation by changes in discrimination accuracy. A signal detection analysis was performed to disentangle changes in bias from changes in discrimination accuracy. Quantitative effects of changing the relative probability of reinforcement on behavior were examined using an index of response bias which was purportedly independent of changes in discrimination accuracy (Dusoir, 1975; Green & Swets, 1966).

The relation between $B$-obtained and $B$-optimum,
\( p(w_{22})/p(w_{11}) \), apparently was not independent of stimulus conditions. The slopes of the bias function tended to be more shallow under less discriminable stimulus conditions. By themselves, the differences in slopes obtained in this experiment were not large enough or consistent enough to conclusively suggest non-independence of \( B \)-obtained and discrimination performance; however, these data fit into a pattern of results from three other experiments in which a bias condition was systematically varied under different stimulus conditions.

In an auditory detection experiment, Terman & Terman (1972) trained rats to discriminate two tone intensities. The animals were reinforced with electrical brain stimulation for pressing a lever if the higher intensity tone, \( S_1 \), was presented, and for not pressing the lever if the lower intensity stimulus, \( S_2 \), was presented. Probability of the louder tone was varied from session to session in an irregular order. During each daily session the difference in stimulus intensity was decreased in a descending series of 8 intensity differences. A particular stimulus condition was in effect for 350 trials. Probability of the louder stimulus, \( p(S_1) \) was varied from session to session in irregular order, using the following values: 0.1, 0.3, 0.5, 0.7, 0.9 and for one animal, 0.4. The series of probability values was repeated 4 times. Because R.O.C. curves of Terman & Terman's data on probability-probability coordinates are well approximated by linear functions, using data provided by Terman, \( B \)-obtained was calculated from z-scores (see equation 3.2.2) and used as
an index of response bias. The relation between $\log B$-obtained and $p(S_2)/p(S_1)$, called $B$-optimum, is plotted in Figure 7 for the two animals used in Terman & Terman's experiment. A different bias function was plotted for each of three different stimulus conditions. As the two tones became less discriminable, the slopes of the $B$-$B$ functions systematically decreased for each animal. The apparent interaction between bias functions and stimulus conditions could have represented an order effect. Stimuli became less discriminable in successive conditions within the same session. Perhaps animals tend to decrease response bias toward the end of a session regardless of stimulus conditions.

In an experiment by Hume & Irwin (1974) order and discriminability factors were not confounded. Hume & Irwin used a two alternative signal detection procedure similar to the one used in this experiment to train rats to discriminate different noise intensities. (Their procedure is discussed in the introduction). An irregular order of stimulus probabilities was presented, with a series of 6 discrimination conditions presented in a descending, then ascending sequence of intensity differences. Only 2 stimulus conditions were presented per day in blocks of 500 trials.

Resultant R.O.C. curves from the Hume & Irwin experiments were symmetrical. By inspection they could be closely approximated by linear functions if probability-probability coordinates were used. Thus, $B$-obtained was computed according to equation 3.2.2 (assuming an underlying normal distribution).
FIGURE 7

Log B-obtained is plotted as a function of log B-optimum for 2 animals. Functions generated under different stimulus conditions are distinguished. The legend lists the difference between S₁ and S₂; the standard, S₁, was always 100 db. Data are from Terman & Terman (1972).
<table>
<thead>
<tr>
<th>Animal</th>
<th>Intensity Difference $S_1 - S_2$</th>
<th>Intercept</th>
<th>Slope S.E.</th>
<th>Correlation S.E. of Coefficient</th>
<th>S.E. of Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>R15A</td>
<td>-3db</td>
<td>-0.036</td>
<td>0.454</td>
<td>0.014</td>
<td>0.999</td>
</tr>
<tr>
<td></td>
<td>-5db</td>
<td>-0.043</td>
<td>0.608</td>
<td>0.013</td>
<td>0.999</td>
</tr>
<tr>
<td></td>
<td>-10db</td>
<td>-0.177</td>
<td>0.876</td>
<td>0.046</td>
<td>0.996</td>
</tr>
<tr>
<td>R7A</td>
<td>-3db</td>
<td>-0.009</td>
<td>0.213</td>
<td>0.054</td>
<td>0.890</td>
</tr>
<tr>
<td></td>
<td>-5db</td>
<td>-0.054</td>
<td>0.312</td>
<td>0.058</td>
<td>0.938</td>
</tr>
<tr>
<td></td>
<td>-10db</td>
<td>-0.107</td>
<td>0.735</td>
<td>0.074</td>
<td>0.980</td>
</tr>
</tbody>
</table>
Data were taken from a plot of the probabilities of correct detections and false alarms (Fig. 2, Hume & Irwin 1974) and were accurate to approximately ± .02. No attempt was made to reanalyze data which involved response probabilities greater than .98 or less than .02.

Values of $\beta$-obtained could be determined for all bias conditions experienced by 3 animals in the Hume & Irwin experiment. The relation between $\log \beta$-obtained and $\log \frac{p(S_2)}{p(S_1)}$, called $\log \beta$-optimum, is plotted for these animals in Figure 8. Separate functions were plotted for different stimulus conditions. For one animal the slope of the $\beta$-$\beta$ function clearly decreased when the discriminability of the stimuli decreased. The slopes of the bias functions obtained under different stimulus conditions did not differ for the other two animals. There is some indication, therefore, that bias functions have more shallow slopes under less discriminable conditions although the difference in slope is not as consistent as the interaction evident in Terman & Terman's (1972) experiment. The greater slope difference in Terman & Terman's experiment might have been due to an order effect, fewer trials per condition, or both.

A third experiment suggested that functions relating $\beta$-obtained to an outcome variable were not independent of stimulus conditions. In an earlier study (Whittaker, in prep), the author used a two alternative signal detection procedure similar to the one used in the present experiment. Delay of reinforcement for correct detections was varied and $\beta$-optimum
FIGURE 8

Log $B$-obtained is plotted as a function of log $B$-optimum for 3 animals. Functions generated under different stimulus conditions are distinguished. The legend lists the difference between $S_1$ and $S_2$; the standard, $S_1$, was always 69 db. Data are from Hume & Irwin (1974).
was computed as \( \log \frac{D_{22}}{D_{11}} \) where \( D_{11} \) and \( D_{22} \) were the delay of reinforcement for correct detections and correct rejections, respectively. The bias index, \( \beta \)-obtained, was computed according to procedures used in this experiment (equation 2.2.2). For one animal the relation between \( \beta \)-obtained and \( \beta \)-optimum had a more shallow slope under less discriminable conditions. For the other animals whose discrimination accuracy changed when stimulus conditions changed, the slopes did not significantly differ.

These experiments are currently the only signal detection experiments with animals having a sufficient number of bias conditions to define functions relating \( \beta \)-obtained to \( \beta \)-optimum under more than one stimulus condition. In all of these experiments, for at least one subject the slope of the bias functions clearly depended on stimulus conditions. When it occurs, the interaction between discrimination performance and response bias is nearly always in the same direction. Thus, the signal detection procedure did not result in estimates of \( \beta \)-obtained which were consistently independent of stimulus conditions. One reason for this interaction might have been an incorrect estimate of \( \beta \)-obtained.

The likelihood ratios, \( \beta \)-obtained, in Figures 6, 7, and 8, were estimated as \( \frac{f_{S_1}(z)}{f_{S_2}(z)} \), assuming underlying normal distributions of \( S_1 \) and \( S_2 \) of equal variance. Another estimate of the likelihood ratio, \( \frac{f_{S_1}(x)}{f_{S_2}(x)} \), could have been used (McNicol, 1972). If the underlying distributions are normal and of equal variance both estimates of \( \beta \)-obtained
are equivalent. In experiments by Hume & Irwin (1974) and Terman & Terman (1972) the R.O.C. curves were consistent with assumptions that \( S_1 \) and \( S_2 \) distributions are normal and of equal variance because R.O.C. curves obtained had unit slopes on probability-probability coordinates. The estimates of \( \beta \)-obtained were equivalent; however, in Figures 7 and 8 an interaction between stimulus conditions and \( \beta \)-obtained was evident. Therefore, the differences in slope in Hume & Irwin's (1974) and Terman & Terman's experiment were not due to incorrect estimates of \( \beta \)-obtained. In this experiment the equal variance assumption could not have been validated under the more discriminable condition. However, the general similarity of the results in this experiment with the results of other experiments suggests that equality of variance is not crucial. The slope differences, therefore, were not due to an incorrect method for estimating the likelihood ratio, \( \beta \)-obtained.

When it occurred, this interaction between discrimination performance and response bias was consistent in the same direction; however, the interaction tended to be idiosyncratically exhibited by individuals rather than reliably across animals. If the interaction had been reliable, an alternative detection model, not based on expected utility theory (e.g., Bush, Luce & Rose, 1964; Thomas & Legge, 1970) might have enabled an index of bias to be defined which was independent of stimulus conditions.

These alternate detection models predict that subjects will match response probabilities to relative probability of
stimulus presentations, a result not found in detection experiments by Hume (1974); Hume & Irwin (1974); and Terman & Terman, (1972). Before the appropriateness of alternative models can be considered, experimental conditions which affect the extent and reliability of the interaction between B-obtained and stimulus conditions must be discovered.
4.2 A Power Function Transforms Objective Probability into Subjective Probability

Although the slopes of the functions which relate log \( \beta \)-obtained to log \( \beta \)-optimum appeared to depend on the discriminability of \( S_1 \) and \( S_2 \), the form of the function in this experiment was independent of stimulus conditions. In Figure 6 the observed relation between log \( \beta \)-obtained and log \( \beta \)-optimum can be accurately described as linear; departures from linearity appear unsystematic and unrelated to the stimulus condition.

The functions have slopes which differ from 1.0; that is, \( \beta \)-obtained did not match \( \beta \)-optimum. The departures from matching indicated that the animals had not maximized the total number of available reinforcers in a session; in other words, they had not maximized expected value. By assuming that animals maximize subjective expected utility (rather than expected value), the objective outcome probabilities were transformed into subjective probabilities. The form of the relation between log \( \beta \)-optimum and log \( \beta \)-obtained determined the equation for converting objective into subjective probability.

As described more fully above, a linear relation between log \( \beta \)-obtained and log \( \beta \)-optimum when probability of an outcome is varied, can be expressed as:

\[
\log \beta_{\text{obt}} = \theta [\log p(w_{22}) - \log p(w_{11})] + a \tag{4.2.1}
\]
implying that

$$P_{\text{obt}} = a \left( \frac{p(w_{22})^\theta}{p(w_{11})^\theta} \right).$$  

(4.2.2)

If \( p'(w) = p(w)^\theta \) then \( \beta \)-obtained based on subjective probability can be predicted;

$$P_{\text{obt}} = a \left( \frac{p'(w_{22})}{p'(w_{11})} \right).$$  

(4.2.3)

Subjective probability of an outcome, \( p'(w) \), is thus a power transformation of objective probability of an outcome. The slope of the relation between \( \log \beta \)-obtained and \( \log \beta \)-optimum is a direct estimate of the exponent, \( \theta \). The \( y \)-intercept, \( a \), in equations 4.2.2 and 4.2.3 is an index of general preference for one alternative over the other and it should be independent of outcome probability; \( a \) also represents an animal's indifference point; that is, \( a = \log \beta \)-obtained when the expected utilities of \( R_1 \) and \( R_2 \) are equal. In this experiment expected utilities of each alternative equal the probabilities of the outcomes because \( w_{11} = w_{22} \). In equation 4.2.1, \( a \) would equal \( \beta \)-obtained when \( p(w_{22}) = p(w_{11}) \).

By assuming that animals maximize expected utility, objective probability can be rescaled into a subjective probability using the index of response bias, \( \beta \)-obtained. A signal detection procedure can thus be used to define a subjective probability scale.
Analysis of the results of experiments reveals that a linear function also describes the relation between log $\beta$-obtained and log $\beta$-optimum in most detection experiments where stimulus probability, $p(S_2)/p(S_1)$, was varied. Hume & Irwin reported the results of one experiment in which the stimulus probability was changed every 500 trials. The resultant relations between log $\beta$-obtained and log $\beta$-optimum (Figure 6, Hume & Irwin) for all 4 animals were, by inspection, clearly S-shaped indicating a systematic departure from linearity. However, the animals may not have had a sufficient number of trials to accommodate to the new bias condition. In another experiment in this series Hume & Irwin exposed animals to a $p(S_2)/p(S_1)$ condition for about 5000 trials; every 500 trials they changed the difference in noise intensities which the animals were discriminating. The results of 3 animals run under these conditions are plotted in Figure 8. One animal's behavior exhibited an S-shaped relation between log $\beta$-obtained and log $\beta$-optimum; however, a linear function would better describe the bias functions obtained for the other two animals. Terman & Terman (1972) also systematically varied stimulus probability in a signal detection experiment with rats. For the two animals run the relation between log $\beta$-obtained and log $\beta$-optimum, $p(S_2)/p(S_1)$, could be closely described as linear (Figure 7). Thus, if animals are given sufficient exposure to a bias condition to insure close approximation to asymptotic behavior, the observed relation between log $\beta$-obtained and log $\beta$-optimum can be accurately described as linear.
Green & Swets (1966) and Galanter & Holman (1967) also varied probability of stimulus presentation in signal detection experiments with humans. They reported approximately linear relations between $\log \beta$-obtained and $\log \beta$-optimum with slopes less than 1.0. Thus, when measured using signal detection procedures, subjective probability is a power function of objective probability. The form of the function does not depend on whether outcome probability or stimulus probability were varied.

The exponents of the transformation function varied from individual to individual within and across experiments and usually were less than one. This suggests that generally subjects underestimated programmed probabilities greater than .5 and overestimated probabilities less than .5; however, some animals significantly overestimated programmed probabilities (see R1O in Figure 6 and A-IV in Figure 8). In another experiment, not a signal detection procedure, Tversky (1967) also found that human subjects usually underestimated values of objective probability greater than .5 and overestimated lower probabilities with a few exceptions. Subjective probability does not match objective probability. Rather, subjective probability can be generally described as a power function of objective probability; however, the exponent is an individual difference which must be specified in order to predict choice behavior in other situations.
4.3 Subjective Probability As a Unique Continuum

Although a scale of subjective probability can be defined, it is not necessarily meaningful. A scale is meaningful, in an empirical sense, to the extent that numerals are uniquely representative of certain experimental conditions (Suppes & Zinnes, 1963). If $\beta$-obtained and subjective probability were meaningful scales of objective probability at an interval level, then successive intervals between scale values would represent a particular series of intervals on the objective probability scale. If $\beta$-obtained were not meaningful beyond the ordinal level then only the rank ordering of $\beta$-obtained rather than a particular function form relating $\beta$-obtained to objective probability could be stated. The reproducibility of a particular form of the bias functions across diverse experiments convincingly suggest that $\beta$-obtained is an interval scale.

Additivity is another test for an interval scale. If two intervals, $x-y$ and $y-z$ could be separately defined and $x$, $y$ and $z$ are numerals on an interval scale, then $(x-y) + (y-z) = x-z$ (Coombs et al., 1970). In Hobson's (1970) experiment, outcome probability and amount were scaled in terms of $\beta$-obtained (see introduction) and thus a change in $\beta$-obtained (from the animal's indifference point $a$) were separately defined for a particular outcome probability and for a particular amount. The additive effect of two intervals were accurately predicted in a successive condition for three separate combinations of probability and amount. In condition 15 of the present ex-
experiment the additive effects of a stimulus probability, $p(S_2)/p(S_1)$, of .67 and an outcome probability, $p(w_2)/p(w_1)$, of .67 were predicted. Although there was more variation between predicted and obtained values of $\beta$-obtained in this experiment than in Hobson's experiment, the systematic departures were small. Indeed the average value of $\beta$-obtained (across animals) in condition 15 where both stimulus and outcome probabilities were .67 was exactly twice (200%) the distance from the average indifference point, $a$, than $\beta$-obtained in condition 14 when only the stimulus probability was .67.

Existing evidence supports the contention that $\beta$-obtained and subjective probability are interval scales; however, if the scale is meaningful then a particular interval, for example, .30 - .40, must correspond to a unique interval on the objective probability continuum, for example .25 - .33. If the scale parameters are related to variables other than outcome probability, more than one rule for converting objective probability into subjective probability would exist. If $\theta$, the parameter in the transformation rule, had a higher value under a more discriminable stimulus condition then, continuing the above example, the subjective probability interval of .30 - .40 might correspond to the objective probability interval of .33 - .45. Thus there would be more than one interval on the objective probability scale for one interval on the subjective probability scale. The subjective probability scale would be ambiguous or, in the strict sense, as outlined by Suppes & Zinnes (1963) not meaningful. Accordingly, it is important that the subjective probability scale remain orthogonal to other variables.
Two variables were systematically studied to test for possible interactions; stimulus variables and the type of uncertain event. Stimulus conditions did appear to affect the estimate of the subjective probability scale for some individuals in this and other experiments. Further refinement of the signal detection procedures to eliminate this interaction is a necessary next step in the development of signal detection procedures for scaling utility and subjective probability.

The subjective probability function might also be dependent on the type of uncertain event, in which case, a subjective probability scale of a stimulus would differ from the subjective probability of an outcome. When he introduced the notion of objective probability into modern utility theory, Savage (1954) defined subjective probability as a continuum orthogonal to utility. Thus, the effects of varying the probability of an event should not interact with variations in utility. The independence of subjective probability from variations in utility implies that the same subjective probability scale should be obtained in a detection situation regardless of the value of an outcome. Instead of water reinforcement, the animals could have been reinforced in the present experiment with food pellets or brain stimulation with no effect on the derived subjective probability scales. Likewise, variations in stimulus probability and outcome probability should result in the same rule for transforming objective into subjective probability; otherwise, the subjective probability scale would not be meaningful across variations in the type of uncertain event. The application of expected utility theory
(including subjective probability) to signal detection thus implied that subjective probabilities, \( p'(w) \) and \( p'(S) \) are equivalent and that \( p'(w) \), defined by varying outcomes, could also be the scale of subjective probability of stimulus events, \( p'(S) \). Comparisons between the results of the present experiment and other experiments in which stimulus probability were varied showed no consistent difference between subjective probability scales derived by varying stimulus probability and outcome probability (see section 2.2). In addition, \( \beta \)-obtained was predicted with reasonable accuracy in condition 14 when the probability of a bright light, \( p(S_1) \), was set to .4. The ratio of stimulus probabilities was converted into subjective probability according to a power function. Parameters of this transformation function were determined for individuals by varying outcome probabilities in previous conditions. Thus a scale of subjective probability derived by varying outcomes was used with moderate success to transform objective stimulus probabilities into subjective probability. Condition 14, however, was not a very sensitive test. A more powerful test would involve, for an individual animal, first deriving a scale of subjective probability by varying outcome probability, and secondly, deriving a scale of subjective probability by varying stimulus probability. The resultant functions for converting objective into subjective probabilities should be the same.

Although additional experiments are needed, existing evidence, from this experiment and other experiments provides a firm empirical foundation for Savage's (1954) assumption that subjective probability is a separately definable continuum.
4.4 Summary

The present experiment was intended to determine if signal detection procedures could be used to scale subjective probability. A method was derived from expected utility theory to define a subjective probability continuum using $\beta$-obtained, the index of response bias. As a measure of the effects of biasing variables, $\beta$-obtained exhibited properties of an interval scale and thus subjective probability, as defined by $\beta$-obtained, would have properties of an interval scale. Subjective probability of either a stimulus event or outcome is a power function of the objective (i.e., programmed) probability with exponents that vary across individuals and have exponents which are, for some, greater than 1 and for others less than 1. Comparisons across stimulus conditions within the present experiment and across other signal detection experiments differing along many dimensions indicate that although further refinement of the signal detection procedure is needed, the detection procedure can produce a separately definable subjective probability scale. These findings encourage further use of detection procedures for scaling outcome dimensions such as reinforcement delay or amount in terms of $\beta$-obtained in order to map these outcome variables onto a utility dimension.
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