A DETERMINATION OF CONVECTIVE ELECTRIC FIELDS THROUGH IN-SITU MEASUREMENTS OF THE IONOSPHERIC F-REGION THERMAL ION SPECTRA

BARRY GENE MORGAN

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A DETERMINATION OF CONVECTIVE ELECTRIC FIELDS
THROUGH IN SITU MEASUREMENTS OF THE IONOSPHERIC
F-REGION THERMAL ION SPECTRA

by

BARRY G. MORGAN
B.S., University of New Hampshire, 1971

A DOCTORAL DISSERTATION

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The Requirements for the Degree of

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ABSTRACT

A DETERMINATION OF CONVECTIVE ELECTRIC FIELDS THROUGH IN SITU MEASUREMENTS OF THE IONOSPHERIC F-REGION THERMAL ION SPECTRA

by

BARRY G. MORGAN

A detector package designed to measure the bulk flow of heavy ions was developed for rocket studies of the lower ionosphere. From direct measurements of the particles' thermal energy spectra, estimates of the rocket payload potential, ion density, ion temperature, and bulk plasma flow can be obtained. By interpreting the bulk flow velocity transverse to the local magnetic field as convective drift it is possible to infer the strengths of convective electric fields. The technique of data analysis involves least-squares fitting a shifted, drifting Maxwellian distribution to the differential energy spectra measured by detectors looking parallel and perpendicular to the rocket's spin axis.

The first flight of the detector package was onboard Electron Echo III which was launched eastward from Poker Flats Research Range on April 17, 1974. Electric field results were found to compare favorably with independent measurements obtained by both the electron echo technique and the auroral backscatter radar at Chatanika, Alaska. Plasma
properties estimated by the present method gave an ion temperature of 1500°K and a bulk flow parallel to \( \vec{B} \) of about 21 m/sec upward. A payload potential of 1.2 volts was also measured. Convective electric fields inferred from the bulk velocity vector had values of 6 mv/m east and 25 mv/m north.
CHAPTER I

INTRODUCTION

Since the launch of the Pioneer I satellite in 1958, many space probes have investigated the earth's surrounding environment. The data which has been gathered identifies the continual presence of an earth-centered cavity embedded in the interplanetary medium. The region within the cavity is known as the magnetosphere; it contains the geomagnetic field $\mathbf{B}$ and delimits the immediate plasma environment of the earth. The cavity boundary itself is termed the magnetopause. Its location as well as the shape of the enclosed geomagnetic field has been extensively mapped by satellite payloads. The information which they have provided presents a rather striking picture of the magnetosphere. (See Figure 1.) Magnetic field lines originating in the polar regions are drawn back into a long tail on the evening side of the earth. This tail extends well beyond the orbit of the moon and may be as long as 1000 earth radii ($R_e$). Reversal of field direction across the magnetospheric equator results in a magnetic neutral sheet extending back along the length of the tail. On the dayside, field lines are compressed from a pure dipole configuration; they penetrate to a distance of roughly $10 R_e$. Just beyond the magnetopause lies a turbulent region designated the magnetosheath. It extends to a shock front marking the transition of solar wind flow from
Figure 1  Current view of the magnetosphere in the noon-midnight meridian plane. [After N.F. Ness, Rev. Geophys., 7, 97, 1969.]
supersonic to subsonic speeds. Beyond the shock wave, conditions are characteristic of streaming solar plasma; particles continue to move at speeds of 300-500 km/sec. The presence of the magnetized earth is not discernible.

While the picture just presented portrays an average magnetospheric configuration, it must be emphasized that the magnetosphere is a truly dynamic structure. The time dependent mass and energy input which it receives from the solar wind causes a continuous evolution of particles and fields within the cavity. Among the resultant physical effects are impressive auroral displays which occur intermittently within a narrow latitude range known as the auroral oval. Other simultaneous events include increased ionization of the lower ionosphere, x-ray bursts, VLF emissions, micropulsations, and geomagnetic fluctuations. Satellite data show these disturbances to be associated with violent distortions of the distant magnetosphere. Collectively these explosive processes are termed magnetospheric substorms. They have sharp onsets and typically last from one to three hours. Their origins are directly connected to events taking place on the surface of the sun.

Historically, the apparent relationship between terrestrial storms and solar activity was first noted in the middle of the last century. In 1858, Broun pointed out that the recurrence of geomagnetic storms appeared to follow the sunspot cycle discovered fourteen years earlier by Schwabe (1844). The report in 1859 of intense magnetic and auroral
disturbances on the day following a large solar flare led to further study of that possible interconnection. Direct causality between the two events was suggested by Hale in 1892. When Maunder (1915) and Chree (1912), at the turn of the century, established beyond doubt the validity of Broun's earlier finding, it became evident that any attempt to understand storm occurrences would have to address the relationship between the earth's environment and solar processes.

Modern magnetospheric physics began with an attempt by Chapman and Ferraro (1932) to understand the relationship between solar activity and geomagnetic storms. They concluded that boundary currents induced in the highly conductive solar wind prevent its penetration into the geomagnetic field. As a result, the earth's dayside field is compressed, magnetic neutral points form at high latitudes, and the solar flow is diverted around the magnetospheric cavity. While direct particle entry could occur at the dayside neutral points, no solar plasma was allowed to penetrate to the nighttime ionosphere. Despite further effort, the two scientists were unable to disclose any mechanism for generating magnetic and auroral disturbances in the late evening-early morning time sector.

In 1961 two methods were proposed for effecting energy transfer within the magnetosphere. Both invoked the hydrodynamic concept of moving magnetic field lines and frozen-in flux. (See Appendix A.) One of the suggestions, offered by Axford and Hines (1961), held that high latitude magnetic field lines
were dragged downstream by a viscous interaction with the solar wind. The establishment of a circulation pattern prevented complete erosion of the front of the magnetosphere. As additional flux joined to the outside of the tail region, field lines in the interior returned towards the earth. (See Figure 2.) The particles which they carried were responsible for generating both magnetic and auroral activity. In this view, magnetospheric substorms occurred whenever a gusty solar wind enhanced the circulation process.

The second proposal, advanced by Dungey (1961), emphasized the orientation of the interplanetary magnetic field lines carried by the solar wind to the front of the magnetosphere. Northward field components combined with the earth's magnetic field to form a structure similar to that envisioned by Axford and Hines. On the other hand, the possibility existed for southward directed interplanetary field lines to connect with the earth's dayside dipole thus forming an x-type neutral point. Newly merged field lines, along which solar and magnetospheric plasma could coexist, were then blown over the polar caps creating a long stressed tail. Through a process of reconnection and earthward return, magnetic flux lost from the dayside cavity was eventually replaced. (Refer to Figure 3.) The attendant transfer of energy from the magnetotail to the nightside auroral zone provided the trigger for polar substorms.

While these two theories predict different geomagnetic topologies, they both rely on a simultaneous circulation of
Figure 2 Axford-Hines model for substorm generation. Plasma and field lines near the outer boundary of the magnetosphere are dragged downstream by a viscous interaction with the solar wind. Return flow at lower latitudes maintains an overall circulation pattern. Particles convected back towards the earth are energized as they penetrate further and further into the geomagnetic field. [After W.I. Axford and C.O. Hines, Can. J. Phys., 39, 1438, 1961.]
Figure 3  Open magnetospheric model of J.W. Dungey. Southward directed interplanetary field lines carried by the solar wind to the front of the magnetosphere can connect to geomagnetic field lines. As the newly merged field lines are blown over the earth's polar caps they create a long stressed tail. Magnetic flux lost from the dayside cavity in this manner is replaced by a process of tail-field reconnection and subsequent earthward return.
field and plasma to effect energy transport within the magnetosphere. (See Figure 4.) Such integral motion is referred to as convection and is a direct consequence of the high conductivity of the plasma. (See Appendix A.) Particles attached to a magnetic field line at a particular time migrate with the convecting force tube. The convective velocity, $v_C$, which is given by

$$v_C = \frac{\hat{E} \times \hat{B}}{B^2}$$

is driven by a dawn-to-dusk electric field $\hat{E}$ which exists across the tail and over the polar caps. Although experimental difficulties have prevented direct detection of the cross-tail field, the effect of its presence has been noted. Satellite observations of earthward plasma flow have been reported by Frank (1967), Freeman and McGuire (1967), and Vasyliunas (1968). Over the polar caps, in situ measurements find the westward electric field to be a permanent feature of the magnetosphere. [Cauffman and Gurnett (1971), Frank and Gurnett (1971), Heppner (1972)]. In response to its existence, high latitude dayside magnetic flux convects to the earth's tail where a return flow is established at lower latitudes. Maintenance of this overall circulation pattern is now considered vital to the development of substorm activity. However, the current interest in electric fields is not limited to the generation of convective flow.

One of the more puzzling aspects of magnetospheric substorms is how the electrons which excite auroral displays
Figure 4 Successive positions of a convecting geomagnetic field line. High latitude dayside magnetic flux moves over the polar caps to form a long tail. Return flow at lower latitudes maintains the circulation pattern and supplies substorm energy to the nighttime ionosphere.
reach energies of several keV. Since the apparent source of these particles is the low energy solar wind, some method of local acceleration must energize the plasma. The exact mechanism is still unknown but could very well be rooted in the existence of electric fields. Naturally, the relationship between electric fields observed in the polar ionosphere and distant magnetotail is a key issue. Magnetic field lines which thread the two regions guide the motions of precipitating charged particles. As long as these lines of force are equipotential, electric fields in one part of the magnetosphere simply map to other locations. Relative electric field strengths would depend only on the geometry of the connecting magnetic field. On the other hand, if considerable potential drops do appear along magnetic field lines, electrons could be accelerated to the observed auroral energies. Measurements of field-aligned potential differences do exist (e.g. Mozer and Fahleson, 1971; Kelley et al., 1971) but they occur infrequently and seem to be statistically unimportant. A more likely source of acceleration by electric fields involves the dawn-to-dusk electric field itself. Particles convecting in from the magnetotail undergo gradient and curvature drifts (see Appendix B); protons move westward and electrons eastward. Those electrons drifting sufficiently far across the tail's total potential drop of about 40 kv could gain enough energy to excite visual aurora. Only additional studies will determine if this acceleration scheme actually influences magnetospheric disturbances.
Yet another manifestation of substorm activity also ties in with electric fields. Magnetometers flown aboard sounding rockets have identified intense ionospheric currents flowing at altitudes near 90 km. These currents, known as auroral electrojets, cause the large geomagnetic fluctuations seen by ground based auroral zone stations. Pedersen currents driven by the dawn-to-dusk electric field and Hall currents driven by north-south electric fields which evolve during typical substorms both contribute to electrojet strength. Continued debate still has not settled which current predominates but it seems clear that a final resolution will have a very basic impact on substorm development theory.

Although the importance of electric fields to magnetospheric processes has only recently been realized, the subsequent growth of interest has been very rapid. Much data has been gathered and substorm understanding has improved. However, a lot of work remains to be done. Correlated rocket, satellite, balloon, and ground studies should be intercompared and analyzed. To this end, a variety of electric field measurement techniques have been developed; one of them is the subject of this research. Before proceeding with a description of this experiment and its results, it seems in order to review some of the other currently used techniques. The terms parallel and perpendicular will be used to refer to vector components along and perpendicular to the local magnetic field, respectively.
CHAPTER II

SUMMARY OF SOME CURRENTLY USED IONOSPHERIC ELECTRIC FIELD MEASUREMENT TECHNIQUES

Each one of the many methods which now exists for measuring DC electric fields can be classified as either a potential difference or bulk flow technique. These may be grouped as follows:

2.1 Potential difference measurements with double probes

2.2 Bulk flow techniques

2.2.1 Remote measurements
   a) Barium cloud releases
   b) Whistler propagation
   c) Thomson backscatter radar results
   d) Artificial electron beam motions

2.2.2 In situ measurements
   a) Split Langmuir probe analyses
   b) Spacecraft wake locations
   c) Lunar shadowing
   d) Particle occlusion
   e) Thermal ion distribution

In broadest terms, the potential difference approach measures the voltage drop between pairs of separated conductors placed in the surrounding plasma. A determination of the electric field component along the line connecting the two probes is then made by dividing the potential difference between them by their spatial separation. Specification of
a complete vector quantity requires evaluation of three independent field components. On the other hand, bulk flow measurements use various schemes to ascertain an average or bulk plasma motion. By interpreting perpendicular velocities as the convective flow mentioned in the introduction it is possible to deduce perpendicular electric fields. However, parallel electric fields cannot be so obtained. Such fields are related to parallel flow by an extremely variable and poorly known parallel conductivity. The resulting limitation by no means reduces the value of bulk flow measurements. The capability of determining convective fields which play such an important role in substorm processes ensures the further use of this technique. In the sections that follow, brief descriptions of individual measurements methods will be given.

2.1 POTENTIAL DIFFERENCE MEASUREMENTS WITH DOUBLE PROBES

Most of the data currently available on electric fields has been obtained by potential difference measurements between pairs of conducting probes. While this technique has been employed in the magnetosphere it seems better suited for use in the upper atmosphere-lower ionosphere where typical electric fields are relatively strong. To understand the method first requires consideration of the interaction between plasma and individual probes.

Any conductor immersed in the earth's plasma is continually bombarded by both electrons and heavy ions. Since
the lighter electrons are more mobile they dominate the particle flux to the object. As a result the conductor gains some excess negative charge until it reaches a floating potential at which no further net current is collected. This potential, which in the ionosphere is normally about one volt negative with respect to the nearby plasma, decays to that of the ambient surroundings in a distance on the order of a Debye length (see Appendix C.) Particles in this intervening Debye sheath redistribute themselves to totally shield the more remote plasma from the effects of the charged conductor. Those distant electrons which eventually do strike such a probe must first pass through this sheath region. As they do so, their potential increases from the local ambient value to the floating potential of the conductor. When they further penetrate the surface of the probe their potential suddenly drops to that of the electron sea inside the metal. The amount of decrease is simply the work function of the conductor.

Connecting two of these probes together with a differential voltmeter allows determination of local electric fields. Since these fields cause the plasma potential at the two conductors to differ a net particle current flows on to one probe, through the voltmeter, and finally off the other probe. Although it is tempting to attribute the measured potential drop solely to the existence of external electric fields, such an interpretation is oversimplified.

Plasma anisotropies arising from the presence of magnetic fields are likely to cause the two conductors to
attain different floating potentials. The attendant DC offsets in the output of the voltmeter may be minimized by using spherically or cylindrically symmetric probes mounted symmetrically on the spacecraft. However, differences in floating potentials may still remain.

Probe surface contaminants as common as the ubiquitous fingerprint can change the relative particle collecting capabilities of the two probes and hence also lead to DC signal offsets. Similar results occur when the conductors are constructed of slightly nonuniform material for then their work functions differ. The constant potential differences which these sources of error cause may be avoided by spinning the entire payload and analyzing only the rotation periodic signal. However, new pitfalls are created by the spinning process and must also be overcome or minimized. For example, any probe rotating through the rarefied wake immediately behind a moving spacecraft is subjected to a different plasma environment than that of its companion. Perturbations in the output signal of the voltmeter are thus to be expected but may be reduced by mounting the probes on booms sufficiently long to assure that the spatially confined wake is never encountered. While considerations of signal size and wake avoidance argue for ever greater probe separations the practical problems of probe deployment and mechanical support necessitate the use of short booms. Unfortunately this limits measurement accuracy. For an electric field $\mathbf{E}'$ as seen by the spacecraft moving at velocity $\mathbf{v}$, the measured
signal is $E' \cdot \hat{d}$ where $d$ is the interprobe spacing. The actual earth referenced field $E$, which is the quantity desired, must be obtained by subtracting the vector quantity $\hat{v} \times \hat{B}$ from the field $E'$. That is

$$E = E' - \hat{v} \times \hat{B}$$

Because of less than perfect knowledge of vehicle orientation and trajectory, the largest individual error in the evaluation of $E$ is introduced in this operation. Nevertheless, intercomparison of various techniques has verified the overall quality of dual probe measurements.

2.2 BULK FLOW TECHNIQUES

2.2.1 Remote Measurements.

a) Barium Cloud Releases

Some of the more valuable measurements of ionospheric electric fields have been made by observing the motions of artificial Barium clouds ejected from rockets or satellites. Ultraviolet radiation incident on the released gas produces Barium ions which resonantly absorb and reradiate visible wavelength sunlight. At twilight, when the background atmosphere is not sunlit, this resonance radiation may be seen and its motion tracked by ground based optical stations. Perpendicular electric fields can then be determined from the influence which they exert on the clouds' trajectory.
Although the actual ion motion is quite complicated, the drift of small clouds released at altitudes above 180 km obeys the relation $v_d = (\mathbf{E} \times \mathbf{B})/B^2$ remarkably well. At lower altitudes, frequent collisions between ions and atmospheric neutrals produces a component drift along the electric field direction. Neutral wind velocities also impart bodily movement to the cloud as a whole. Problems with large releases occur because the highly conducting Barium plasma polarizes to ensure divergence free currents. Electric fields near the cloud then differ from the ambient fields.

Since these difficulties are fairly easy to avoid, the technique retains its value; the one unavoidable shortcoming is that the experiment can only be carried out at twilight.

b) Whistler Propagation

When impulsive radiation caused by a lightning stroke travels through the ionosphere, naturally occurring VLF waves are produced. The lower frequencies lie in the audio range and produce a tone of decreasing pitch when heard on radio receivers. For this reason the waves are known as whistlers.

Since the mode in which they propagate is marked by an index of refraction greater than unity, whistlers are not reflected in the ionosphere. In fact, the waves are guided by the enhanced ionization ducts surrounding geomagnetic field lines and continue on into the magnetosphere. Component frequencies in the original impulse separate out and travel at
different speeds. When they arrive at the conjugate point of a given field line they may be recorded by ground stations. Those whistlers that get reflected at the earth's surface return doubly dispersed to the point of origin. There, if the original pulse was received without dispersion, the travel times of the component frequencies can be determined. From this information the azimuthal component of magneto-spheric electric fields can be evaluated. The method is based on the fact that the changing path length of a convecting field line causes corresponding changes in the delay times of the guided waves.

Detailed calculations assuming the duct ionization to be in diffusive equilibrium show that the frequency of minimum time delay is

$$\omega_{\text{min}} = 0.42 \omega_e$$

where $\omega_e$ is the electron gyrofrequency at the top of the given field line. Since this latter quantity provides an estimate of equatorial field strength, a determination of $\omega_{\text{min}}$ can be used to obtain the radial position of the duct of interest. Monitoring this position as a function of time thus gives a measure of both bulk plasma flow and convective electric fields.

While this technique, which was pioneered by Carpenter and Stone (1967), can evaluate electric fields over long time periods, it suffers from two major disadvantages. Its inability to measure longitudinal drifts makes it impossible to
use the method to obtain north-south fields. And more importantly, when whistler activity is absent absolutely no electric fields can be determined by the method.

c) Thomson Backscattered Radar Results

Radio waves incident on the ionosphere from below are totally reflected when they reach the lowest altitude at which their frequency is equal to the plasma frequency, \( \omega_p \), given by

\[
\omega_p = \left( \frac{ne^2}{m\epsilon_0} \right)^{3/2}
\]

where \( n \), \( e \), and \( m \) are the electron density, charge, and mass, respectively and \( \epsilon_0 \) is the permittivity of free space. Through observations of the travel times of man made signals it is thus possible to accurately determine the distribution of density with altitude. The problem is that the technique cannot be used to probe those regions of the ionosphere above the level where electron density maximizes. Waves whose frequencies are high enough to penetrate this peak layer do not get reflected back to the earth's surface; rather, they are scattered by density fluctuations which arise from the random thermal motions of the electrons in the medium. Movements of such local irregularities have been successfully measured by Thomson scatter radars and interpreted as convective drifts driven by perpendicular electric fields.

Very briefly, Thomson scatter techniques rely on the incoherent scatter of waves falling on free electrons, Because
these electrons are arbitrarily distributed in space the waves which they scatter combine with random phases. At a receiver, signal powers then add and the total power received is directly proportional to the average density of free electrons in the scattering volume. By measuring this returned power it is thus possible to determine the density of charged particles in a given altitude range. However, since the individual scattering cross section is so small, about $10^{-28}$ m$^2$, the power returned from the ionosphere is quite weak. Large receiving antennas and long integration times are needed to obtain measurable signals. The method of experimentation is known as "incoherent" or "Thomson" scattering.

Recently Thomson scatter techniques have been applied to measurements of particle motion within the ionosphere. These studies indicate that the scattered wave spectrum is affected by the temperatures and flow velocities of both ions and electrons.

Doppler broadening of the emitted pulses is readily observed and was originally thought to be due to the random thermal velocities of free electrons. The surprise was the smaller than predicted frequency spread. However this was quickly explained by the fact that electrostatic restoring forces strongly couple electrons and heavy ions so that the scattering electrons are not truly free. More detailed analysis showed that, since probing wavelengths were typically much longer than the Debye length, it was the ion thermal velocities which controlled the Doppler spread. Electron
motion served only to change the shape of the spectrum. These results made possible the measurement of both ion and electron temperatures.

Similarly, the occurrence of Doppler shifts in the emitted frequency allows determinations to be made of bulk plasma flows. For electrons and ions moving together toward the radar with velocity $v_d$, the measured frequency shift, $\Delta f_d$, is given by

$$\Delta f_d = \frac{2v_d}{\lambda} \text{ Hz}$$

where $\lambda$ is the probing wavelength. Although it is not easy to locate the center frequency of a shifted spectrum, present uncertainties in ionospheric drift velocities are on the order of 40 m/sec. This translates into convective field uncertainties of about 2 millivolts/meter.

While Thomson scatter measurements seem to be fairly sensitive, the technique as a whole suffers from two shortcomings. The need to change viewing angles in order to determine three independent velocity components results in large spatial separations of the data. Field vectors at single points are then not determined. The second problem is caused by the necessity of using range gates to collect pulses scattered from an altitude range. Quantities measured at a given height are really averaged over the 10 km altitude interval selected by typical gates. Spatial details on scales smaller than this are thus lost forever.
d) Artificial Electron Beam Motions

The Minnesota group under the direction of J.R. Winckler has recently undertaken a series of experiments designed to study the mechanisms by which electrons precipitate from the geomagnetic field to produce auroral x-rays. To date, a total of four sounding rockets have been used to inject controlled beams of energetic 40 keV electrons into the earth's quiet-time magnetosphere. These electrons bounce (see Appendix D) between their conjugate mirror points near the rocket and in the southern hemisphere as they gradient and curvature drift across magnetic field lines. In the presence of perpendicular electric fields they also experience an $E \times B$ drift motion.

On three of the four "Electron Echo" flights, on board detectors succeeded in intercepting the electron beam after it had made one or more bounces from the southern hemisphere. When the predicted beam displacement due to gradient and curvature drifts in model magnetic fields was compared to the known rocket position, discrepancies were sometimes found to exist. Since the particular field models used predicted the particle bounce times quite accurately, it was assumed that these discrepancies were not due to model limitations but rather to $E \times B$ drift motions. Detection of electron echoes could thus be used to obtain an estimate of convective electric fields under the assumption that magnetic field lines were equipotential.
While electric field measurements were originally an unplanned bonus of the Echo program, the problems inherent in the technique are easy to see. Certainly, the experiment's applicability is limited to auroral quiet times because identification of a beam echo in the midst of auroral precipitation would be quite difficult if not impossible. Aside from this, the cost and complexity of the experiment absolutely prohibit its widespread use for measurements more easily obtained by other less expensive, more reliable methods. The one advantage of the ECHO technique is its ability to measure the integrated effect of electric fields too small to be detected by these other approaches.

2.2.2 In Situ Measurements.

a) Split Langmuir Probe Analyses

The split Langmuir probe developed by Bering (1974) consists of two conducting hemi-cylinders separated by a strip of insulating material. From measurements of the particle current collected by either sensor as a function of applied voltage, electron temperature and density can be obtained using conventional probe theory. The added feature of this detector is its ability to determine electric fields.

When both plates are held at a large negative potential collected particle current is dominated by ions in the surrounding plasma. If these are moving relative to the detector with the common velocity \( \vec{v}_D = (\vec{E}' \times \vec{B})/B^2 \) then a difference current, \( \Delta I \), flows between the two hemi-cylinders.
Its magnitude is given by

\[ \Delta I = n \, e \, f \left| \int d\vec{A} \cdot \frac{\hat{E}' \times \hat{B}}{B^2} \right| \]

where \( e \) and \( n \) are electron charge and density, \( d\vec{A} \) is an area element of either sensor and \( f \) is a factor which accounts for the focussing of ions onto the plates. Determination of \( \hat{E}' \) thus requires evaluation of \( \Delta I \) for three different probe orientations. This may be accomplished by a set of probes or by one probe placed on a spinning, coning payload. The only difference between the two approaches is in the time resolution attainable.

Uncertainties in \( n \) and \( f \) cause much of the inaccuracies inherent in the electric field evaluations obtained by this technique. But they are not the only pitfalls of the method. Difficulties are also caused by the spurious difference current arising from probe wakes. These in fact are generally considered to be the major source of discrepancy between split probe and other simultaneous but independent measurements.

b) Spacecraft Wake Locations

Spacecraft velocities are normally comparable to or larger than thermal ion velocities. As a result, ion wakes are created as moving vehicles sweep out the plasma behind them. Since these wakes lie along the direction of relative rocket-plasma motion, the direction of bulk flow in the
rocket's rest frame can be determined simply by detecting the wake orientation. This is accomplished by locating either the reduced ion densities or large negative potentials within the rarefied plasma.

Although such a position measurement can indeed identify the direction of relative plasma flow, it does not provide knowledge of the flow speed. This precludes determination of the vector flow and any electric fields which drive it. In practice, however, the problem is overcome by assuming an electric field direction in an earth-fixed reference frame. Calculations of field strengths are then straightforward, but the results are meaningful only insofar as the assumed directions are reasonably accurate. Naturally this technique is subject to gross uncertainties.

Attempts to improve the method by determining both field direction and magnitude have been largely unsuccessful. These efforts involved making two wake position measurements over a time interval during which the electric field was assumed constant but the vehicle velocity changed. None of the analyses gave results in agreement with independent measurements on the same flight. The approach thus seems destined to remain only an adjunct of other techniques.

c) Lunar Shadowing

When the moon is present in the earth's magnetotail, it casts shadows in the energetic electron fluxes streaming earthward. Lin (1968) and Anderson (1970) have used the
displaced edges of such shadows to determine magnetospheric electric fields. The method involves the use of particle detectors on a lunar orbiting satellite.

Quite simply, electrons moving up the tail travel on trajectories guided by magnetic field lines. Those particles which miss the moon eventually mirror in the earth's field and return. As they do so they also $\vec{E} \times \vec{B}$ drift under the action of perpendicular electric fields. The total convective displacement which they undergo during their bounce motion causes some of the electrons to impinge upon the moon's otherwise shadowed surface. It thus appears that the shadow border does not lie along the moon's limb but rather is displaced a constant distance in the $\vec{E} \times \vec{B}$ direction.

Edge identification is made by satellite detectors which measure electron fluxes, energies, and pitch angle distributions. From this information and knowledge of satellite position it is possible to compute electric field magnitudes by comparing the known shadow displacement with that predicted by integrating the convective velocity $E_x/B$ over the moon-mirror-moon travel time. This calculation requires the use of a field model for $\vec{B}$ and is applicable only to those particles which mirror sufficiently near the moon to avoid significant gradient and curvature drifts.

Naturally this approach is not without its problems. The most obvious of these are the cost and complexity of satellite programs as well as the lack of data when the moon is outside the magnetotail. But beyond these is the fact...
that the method can be used to evaluate only those field components perpendicular to the satellite's orbit plane.

d) Particle Occlusion

In the rest frame of a spacecraft ramming through the lower ionosphere the counterstreaming plasma represents a directed particle flow. The path of motion, whose orientation is determined by both convective drift and vehicle ram, can be ascertained from ion flux measurements made around the vehicle. Difference currents collected by coupled detectors provide sufficient information to allow determination of convective electric fields. The method can be applied to either satellite or rocket studies.

One satellite instrument designed for such measurements was developed by Hanson et al. [1972] at the University of Texas at Dallas. Its sensor head consists of a four-segment collector recessed behind negative suppressor grids whose function is to prevent entry of thermal electrons. Ion currents collected by coupled sensors are fed through a difference amplifier which may be switched between various segment pairs. When the ions enter perpendicular to the sensor face each of the collector segments receives the same ion current. This is not the case when the mean entry velocity is other than perpendicular for then the particles do not have equal access to all collectors. Ion drift components perpendicular to the satellite's ram vector can thus be derived from measured current ratios and knowledge of vehicle orientation.
Difficulties with the approach include an inability to evaluate plasma flow speeds along the satellite trajectory. The information gap which results can be filled only by measurements obtained with other techniques. Flow accuracies are all limited by vehicle attitude information to about ± 100 msec⁻¹. Also plaguing the experiment is the lack of spatial resolution caused by vehicle ram speeds of seven to eight km sec⁻¹. In the few seconds it takes to measure ion current ratios the spacecraft moves at least ten kilometers. Resultant data coverage precludes determination of local plasma behavior.

On rocket flights the experimental concept is similar to that for satellites but the equipment design is different. Carlson [1971], who has now made measurements on several rockets, flies coupled retarding potential analyzers mounted to look in opposite directions. The units are operated in a feedback configuration in which the applied retarding voltage is continuously adjusted to maintain a constant flux to one detector. The current difference collected by the two analyzers then becomes a measure of the \( \frac{(\mathbf{E} \times \mathbf{B})}{B^2} \) drift velocity.

Although spatial detail on these rocket flights is improved over the satellite data, time resolution presents something of a problem. Unless the magnetic field-bulk plasma flow plane is assumed, measurements must be made at various vehicle attitudes. This requires either multiple detectors or coning payloads with known spatial orientations.
e) Thermal Ion Distributions

One technique which has only recently been used to determine convective fields utilizes detailed measurements of thermal ion fluxes to evaluate bulk plasma flow. The method works because the average velocity of ionospheric particles may be comparable to the thermal speed of heavy ions. This results in significant changes to the measured ion distributions.

The first such experiment of this type, designed by D. Green and B. Whalen (1974) at the Division of Physics, National Research Council of Canada, was flown on a sounding rocket into the expansive phase of an auroral substorm on January 15, 1972. A thermal ion sensor mounted perpendicular to the rocket's spin axis sampled ion distribution functions in the energy range from 0 to 5 eV. Coning of the payload provided a limited range of viewing directions.

Estimates of the spin plane component of the mean plasma velocity were derived by calculating the first moment of the velocity distribution measured during a spin scan. Individual flow results obtained over a significant portion of the vehicle's precession period were then combined in a least squares analysis to evaluate the complete bulk plasma flow vector. Transformation of this quantity back to an earth-fixed reference frame led to a measurement of electric fields.
Additional information gleaned from this experiment include a determination of ion density, ion temperature, and negative rocket potential. Most easily obtained is a measure of the potential itself. Since no ions are detected below the accelerating voltage of the spacecraft the low energy cutoff of the ion spectrum readily identifies the vehicle potential. As noted before, this is usually about one volt negative with respect to the ambient plasma.

An experimental determination of ion density and temperature required the assumption of a specific distribution function. The authors related the quantities obtained by calculating the zeroth and second moments of a Maxwellian distribution to the same quantities evaluated from their data. They found reasonable particle densities but unusually high temperatures which were not obviously correlated with auroral activity.

Time resolutions of about 30 seconds prevented the association of shorter duration data fluctuations with specific auroral forms. No significance was attached to any trends persisting for less than this time period. The experiment, though it has sparked a lot of controversy since its results were published, appears to have established the viability of such a technique. Analysis of inherent problems will have to await further experimentation and data break down.
CHAPTER III

DETERMINATION OF IONOSPHERIC CONVECTIVE ELECTRIC FIELDS
FROM MEASUREMENTS OF THERMAL O\(^{+}\) ION SPECTRA

The research effort which forms the basis of this thesis is a further development of the convective field evaluation technique pioneered by the Canadians. Measurements of thermal ion spectra which are modified by the existence of a net plasma transport are used to ascertain the plasma's convective drift. Ionospheric electric fields are then calculated from the relation given in equation 1. The new approach, though similar in concept to the aforementioned experiment, differs in its manner of sampling and subsequently analyzing the collected data.

As noted before the crucial factor in the success of this type of experiment is the comparability of ion thermal and bulk flow speeds. It is shown in Appendix B that for typical ionospheric fields \( \mathbf{E} \) and \( \mathbf{B} \) the ambient plasma executes a bulk drift, \( \mathbf{v}_d \), that is well approximated by \( \mathbf{v}_d = (\mathbf{E} \times \mathbf{B})/B^2 \). To spacecraft detectors the surrounding plasma also appears to counterstream with the vehicle ram speed, \( v_r \). There results a net ion-vehicle velocity, \( \mathbf{v}_o \), given by

\[
\mathbf{v}_o = \frac{\mathbf{E} \times \mathbf{B}}{B^2} - \mathbf{v}_r + \mathbf{v}_n
\]

where \( \mathbf{v}_n \) represents the particle velocity along \( \mathbf{B} \).
If measured fluxes are to be used to evaluate such relative transport then the differential particle flux of a plasma moving at the bulk velocity $\mathbf{v}_c$ must be considerably modified from that of a plasma executing no bulk motion. This implies that the average particle speed, $v_0$, must NOT be much different than the total speed, $v$; otherwise the effects of a net plasma transport on the shape of measured energy spectra will not be discernible.

Since vehicle ram speeds are normally on the order of 1 km/sec and typical electric and magnetic field values in the lower ionosphere are 20 mV/m and 0.5 gauss, respectively, the relative drift speed between particles and detectors is about one to two kilometers/sec. Although this speed is much smaller than that of even 1eV electrons it does compare to the thermal speed of heavy ionospheric ions. For example, a typical temperature for the singly charged oxygen ions which usually predominate at rocket altitudes is 1000°K. This translates into a thermal speed of about 1 km/sec. As a result, measurements of thermal oxygen spectra are ideal for bulk flow analyses. Detected fluxes, significantly modified by a bulk transport velocity that is comparable to the total speed of sampled ions, provide the means of evaluating the net plasma drift. After correction for vehicle ram motion, according to equation 7, a straightforward measurement of convective electric fields remains.

To obtain the information necessary for determining ionospheric electric fields, an instrument package was designed.
which contained two cylindrical plate electrostatic analyzers. These units, dubbed EFMs for electric field meters, are essentially electrostatic lenses which use a radial electric field to select incoming charged particles within a given range of some mean energy. Potentials applied to concentric electrodes specify the value of this energy according to the relation

\[
\text{Energy} = \frac{eU_0}{2 \ln(R_o/R_i)}
\]

where \(e\) is the magnitude of the electronic charge and \(U_0\) is the voltage drop across the inner and outer deflection plates whose radii of curvature are \(R_i\) and \(R_o\), respectively. Instrument resolution for small plate separations is given by

\[
\frac{\Delta \text{Energy}}{\text{Energy}} = \frac{R_o - R_i}{\frac{1}{2} (R_o + R_i)}
\]

For the particular EFMs first flown into the ionospheric plasma these equalities reduced to

\[
E \text{ (electron volts)} = 5.24 U_0 \text{(volts)}, \text{ and}
\]

\[
\frac{\Delta E \text{ Energy}}{\text{Energy}} \approx 10\%
\]

In adapting the detector package to make the appropriate flux measurements it was decided that one analyzer (EFM 1) would look parallel, the other (EFM 2) perpendicular, to the spin axis of the payload on which they were included. Both units were to sample positive thermal ions in the
energy range from zero to five electron volts. The data which they collected would provide a limited sampling of the three dimensional ion distribution function once each spin period. From this information the bulk plasma flow vector could be evaluated.

Data analysis is handled by least squares fitting the particle fluxes predicted from an assumed Maxwell-Boltzmann distribution to those particle fluxes actually measured. The computer program which does all the work is a fortran routine known as General Least Squares With Statistics. Its algorithm, described in detail in Appendix E, interpolates between a Taylor series and Gradient approach to least squares analysis. Parameters determined in the fitting process include ion density, ion temperature, vehicle potential, and bulk flow velocity. Convective electric fields are deduced from this latter quantity.

For a payload which undergoes a significant precession as well as spin motion either detector alone could be used to evaluate the unknown quantities. However, data accumulated over extended time intervals would be required. With both detectors providing essentially simultaneous information, a least squares fit could be performed several times per precession period. Time resolutions would thus be markedly enhanced.

The first flight of this particular instrument was made on the third rocket in the ELECTRON ECHO program series being supervised by Dr. J.R. Winckler at the University of
Minnesota. The rocket, whose chief purpose was to study the motion of energetic electrons which it artificially injected into the earth's magnetosphere, was launched from Poker Flats Research Range outside Fairbanks, Alaska at 10 hrs 23 min 30.2 sec UT on April 17, 1974. Vehicle trajectory was directed nearly eastward so that the rocket could follow along the drift path of its injected electron beam. (Refer back to section 2.2.1 d on determining convective electric fields from observations of artificial electron beam motions.) A quiet auroral display some distance to the north was the only activity present.

Choice of the Poker Flats Research Range as the launch site for ECHO III was based largely on the availability there of an auroral backscatter radar station operated by Stanford Research Institute at Chatanika, Alaska. (See Figure 5 for relative locations of the radar and launch sites.) Since the rocket could not be launched on a trajectory which would allow it to adequately keep up with the gradient and curvature drift speeds of the injected energetic electrons it was necessary to await the development of a northward convective field which would give the particles an opposing bulk velocity component slowing them down. Interception of the bouncing beam would then be possible. The problem of monitoring the ionospheric electric field for satisfactory flight conditions was thus given to the personnel at the radar site. They maintained a quick look, real time analysis of changes in the electric field configuration by measuring the Doppler shift
Figure 5  Map showing relative location of Poker Flats launch range and Chatanika radar. Superimposed on the map is the horizontal projection of ECHO III's trajectory. [Map from experimenter's information packet on Poker Flats Research Facility.]
caused by the drift motion of ionospheric plasma. The information which they provided was not only a prerequisite for ECHO III's success; it was also a valuable adjunct to the onboard electric field experiment. Radar measurements made along the rocket's flight path both before and during the actual flight also provided information on particle temperatures and density profiles. A quick look at pre-launch data revealed the existence of a significant northward electric field of around 25 mv/m and F region ion densities near $2.5 \times 10^5$ /cc.

Advantages to flying the unit on ECHO series rockets are basically twofold. First ECHO's goal of intercepting and identifying a bouncing, drifting electron beam originally injected from the rocket precludes launch into any auroral activity; sorting the artificially produced electrons from an intense auroral background would be impossible. The quiet auroral surroundings of an ECHO launch thus provide an undisturbed plasma environment in which to make the prescribed ion flux measurements. Whatever electric fields exist are then more likely than not to be quasi-static as are also ion temperatures and densities. Dynamic variations in any of these quantities are not apt to be sources of concern. The second benefit afforded by an ECHO payload is its ability to make an independent determination of electric fields; the method was originally discussed in the previous chapter. Though its results may be somewhat questionable because of the assumption of equipotential field lines and the imprecision
inherent in magnetic field models chosen for data reduction, the value of intercomparisons cannot be denied. Gross problems in either approach could be quickly spotted.

Besides reaping the benefits of its association with ECHO III, this electric field experiment also became a boon to the personnel analyzing the electron gun data. From beam interceptions and knowledge of convective electric fields they were able to test and evaluate various magnetic field models for future reference. An unexpected bonus of the experiment also developed. The electrostatic analyzers immediately responded to the intense gun firings and measured particle spectra displaced in energy. Information on payload neutralization during beam injections was thus obtained by observing the ion spectra as they returned to their normal ambient forms. In this way the ECHO and electric field experiments truly complemented one another.

Details of the particular experiment package flown on ECHO III and the technique used for data reduction follow in the next two sections.
CHAPTER IV

EFM PACKAGE DESCRIPTION

The electrostatic analyzers used to measure the differential energy spectrum of ionospheric thermal ions were small units milled from solid aluminum blocks. (See Figure 6.) Protruding fingers left in the detectors' sidewalls served two purposes. Ostensibly they provided the means of positioning and holding the insulating Delrin pads which supported the units' cylindrical deflection plates. (Refer to the exploded analyzer drawing shown in Figure 7.) In addition they also acted as baffles for any particles which managed to enter the analyzer's interior without passing between the curved electrodes. Stray ions thus did not contribute to measured fluxes.

To maintain well defined analyzer look directions both entrance and exit apertures of the detectors were collimated. Small holes drilled front and rear had diameters of 0.059 and 0.078 inches, respectively. The resulting full acceptance cone of each device was then about 2°. No particles entering the front collimator more than 1° off the vertical were counted.

Before the electrostatic analyzers were assembled a uniform coating of colloidal graphite was applied to all interior surfaces including the insulating Delrin pads. This reduced the problem of contact potentials and allowed the
Figure 6 Mechanical drawing of EFM analyzer housing. Cylindrical deflecting plates which mount in the unit's front section select ions entering aperture G according to their energy. Those particles with the proper energy exit hole F and are counted by a solid state detector known as a channel electron multiplier (channeltron).
Figure 7  Exploded view of EFM detector. Shown are the analyzer housing, the cover plate, insulating Delrin pads, deflecting electrodes, and channel electron multiplier block.
bleed-off of static charges which could otherwise have built up on the insulators. Various current paths simultaneously created between the two electrodes and chassis ground were still sufficiently resistive as to draw negligible power from the one volt sweep supply used to bias the units' electrostatic deflection plates.

The ion sensors used to count selected particles were model 4025 channel electron multipliers (CEMs) supplied by Galileo Electro-Optics Corporation. These tiny devices (refer to Figure 8) are high gain ($\sim 10^6$) continuous dynode multipliers which put out a short duration electron current pulse for each particle detected. The units were mounted in Kel-F blocks and vacuum potted in place with Dow Corning's Sylgard 184 potting gel. The 5 mm CEM conical entrance apertures were biased 1500 volts negative with respect to the ambient plasma in order to post-accelerate analyzed ions and boost detector efficiency. A tail potential of +1500 volts provided the field needed to draw secondary electrons to the collector. Output pulses were fed through a resistor across which was connected a pulse transformer as shown in Figure 9. The signal developed on the transformer secondary was thus a voltage spike typically 50 to 100 nanoseconds wide. It was delivered to a fast preamp capable of handling megahertz counting rates. The diagram of the amplifier electronics is shown in Figure 10. In the circuit's first stage, output signals from the CEM were amplified and then sent to a discriminator which filtered out background noise. Real pulses
SHAPE: HELIX
SUBLTENDED ARC: 840°±10°
CONE INSIDE DIAMETER: 5MM NOM.
BACKGROUND COUNT: <.5/SEC.
@ 3000 V
RESISTANCE: 1×10⁹±30%

Figure 8 Model 4025 Channel electron multiplier supplied by Galileo Electro-Optics Corporation.
Figure 9 Schematic of the channeltron assembly. The sensor and illustrated electronics are mounted in Kel-F blocks and vacuum potted in place with Dow Corning's Sylgard 184.
Figure 10 Fast pulse amplifier and signal discriminator.
were further amplified in the third and final stage of the preamp electronics.

Bias voltages for the preamp and channel electron multiplier were furnished by the low voltage-high voltage power supply shown in Figure 11. Its input power was derived from the 28 volt battery packs included in the payload. To prevent arcing of the high voltage CEM biases at the intermediate plasma pressures encountered at low altitudes a control circuit was designed to delay the turn on of the high voltage section of the power supply. This turn-on circuit, which was triggered by a preset mechanical timer armed by the increased g-force of launch, is shown in Figure 12. It delivered low voltage input power to the high voltage supply approximately 90 seconds after liftoff when the rocket had attained an altitude of about 140 kilometers. The supply then continued to deliver the necessary bias voltages until loss of telemetry signal was encountered during vehicle reentry.

Deflection voltages applied to the analyzer electrodes were provided by the sweep logic diagrammed in Figure 13. A thirty-two step linear ramp, synced to timing pulses supplied by NASA electronics, was recycled every 25.6 milliseconds of flight time. During this period the potential on the detectors' outer plate was varied from 0. to +0.5 volts while the potential on the inner electrode was simultaneously changed from 0. to −0.5 volts. Because the analyzers' energy selection to plate voltage ratio was about 5, this resulted in
Figure 11  Schematic of low voltage-high voltage power supply. Preamp and channeltron biases were furnished by this circuit.
Figure 12  High voltage turn-on circuit. This unit, triggered by a preset mechanical timer armed by the increased g-force of launch, delivered low voltage (28 volts) input power to the high voltage supply approximately 90 seconds after lift-off.
Figure 13  Circuit logic for ±0.5 volt EFM electrode sweep.
the electrostatic sampling of thermal ion spectra in the energy range from 0 to 5 eV.

Analyzer calibration curves obtained in the lab were used to convert accumulated counts to absolute particle fluxes expressed in terms of particles/cm²/sec/ster/kev. The energy dependent conversion factor relating counts and flux is a function of analyzer design and is known as the detector's energy-geometry factor. In brief, it expresses the product of the unit's effective collecting area, acceptance solid angle, and energy resolution for a given integration time. Details of the laboratory scheme for obtaining experimental calibrations are given in Appendix F.

The package in which the two EFMs and associated electronics were housed was a small assembly fashioned from aluminum. (It is sketched in Figure 14.) An enclosed box in the lower half contained all circuits and one analyzer which looked out the side. The other analyzer was mounted on an extra cover plate secured to four struts atop the box. Mechanical drawings detailing the structure are included in Figures 15, 16 and 17. As can be seen from them, nice plane conducting surfaces surrounded both detectors.

When it was completed, the entire EFM assembly was given an exterior coating of colloidal graphite and affixed to the top of the ECHO III payload pictured in Figure 18. Mating with the rocket motor was accomplished at the launch site where last minute ground tests showed all experiments to be working properly.
Figure 14 Sketch of EFM package flown on ECHO III. The two small squares with enclosed dots show how the analyzers were mounted. All EFM circuits were housed in the lower half of the assembly.
Figure 15 ECHO III EFM package top deck and support struts. The rocket's nose cone eject spring rested on this deck.
Figure 16  ECHO III EFM package lower box. All EFM electronics and the spin (perpendicular) detector were mounted in this assembly.
Figure 17 Mechanical drawing of assembled EFM package flown atop the ECHO III payload.
Figure 18  Picture of assembled ECHO III payload. The EFM package is shown mounted on top; Minnesota's high energy electron gun is contained in the large can comprising the bottom \( \frac{1}{3} \) of the payload.
Rocket attitude data was provided by a gyroscope and a dual axis (pitch-yaw) magnetometer. A horizon detector included as a back-up aspect sensor failed before launch but its loss was inconsequential. The measured magnetic field and gyro angular momentum vectors were sufficient to determine detector aspect as a function of flight time.

To relay accumulated information to ground receivers a pulse code modulated/FM-FM telemetry system was installed on the rocket. It had the capability of continuously transmitting a paragraph of data every 25.6 milliseconds. Each paragraph contained 32 frames of data and each frame contained sixteen 9-bit words, the first two of which were used as sync codes. (See Figure 19.) Since an entire channel was devoted to each of the two EFMs, one 32 point spectrum from 0 to 5 eV was obtained for each analyzer during every paragraph readout. This meant that counts collected at a particular energy step were integrated over a period of 0.8 milliseconds before being transferred to the output buffer of the telemetry system. All bits in a given frame were encoded in digital form and then transmitted sequentially to ground receivers. If a particular bit read "0" one FM frequency was used to relay a pulse signal and if it read "1" another FM frequency was used. Back on the ground the transmitted information was serially recorded on video tape. A digital decoder was then used to reconstruct the data and write it on magnetic tapes which NASA sent to the experimenters. Computer words, for purposes of convenience, were grouped into data
Figure 19 ECHO III telemetry paragraph. Channels 8 and 9 were reserved exclusively for the two EFMs. One readout was obtained from each detector every 0.8 milliseconds.
records consisting of five paragraphs worth of information. Telemetry was maintained from 30 seconds before launch to loss of signal about nine minutes later.
CHAPTER V

EFM DATA HANDLING

In analyzing the collected thermal ion data it was necessary to relate experimentally measured particle fluxes to the three dimensional plasma distribution function, $f(\theta, \phi)$. This is accomplished by considering a well-collimated detector placed within the plasma medium. (Refer to Figure 20.)

The fraction of particles hitting the detector from an incremental volume element $d^3r$ a distance $r$ away is

$$\frac{dA}{4\pi r^2} \frac{f(\theta, \phi)}{\langle f \rangle_{\theta, \phi}} \cos \theta$$

where $dA$ is the detector collection area, $\theta$ is the angle between the normal $\hat{n}$ to the detector face and the position vector $\hat{r}$ from the detector to the volume element $d^3r$ and $\langle f \rangle_{\theta, \phi}$ is the anisotropic distribution function averaged over $4\pi$ steradians. In the same notation, the number of particles per unit volume with speeds between $v$ and $v + dv$ is

$$4\pi v^2 \langle f \rangle_{\theta, \phi} dv$$

Hence of all those particles in the volume element $d^3r$, the number with speeds between $v$ and $v + dv$ which strike the detector in the time interval $dt$ is

$$\frac{dA}{4\pi r^2} \frac{f(\theta, \phi)}{\langle f \rangle_{\theta, \phi}} \cos \theta \ 4\pi v^2 \langle f \rangle_{\theta, \phi} dv \ 4\pi v^2 \langle f \rangle_{\theta, \phi} dv \ r^2 v d\Omega d\Omega dt$$

$$= f(\theta, \phi) v^3 \cos \theta \ dA dv d\Omega dt$$
Figure 20  Collimated detector with collecting area $dA$ and unit normal $\hat{n}$ immersed in a plasma environment.
Since the actual detectors used for the thermal ion measurements accept only those particles entering the front collimator within \( \pm 1^\circ \) of the vertical it is reasonable to replace \( \cos \theta \) by 1. This substitution gives:

\[
\text{# particles in } d^3r \text{ with speeds in } dv \text{ hitting the detector in time } dt = f(v^3) \, dA \, dv \, d\Omega \, dt
\]

Writing \( E = \frac{1}{2}mv^2 \) allows the further substitution \( v \, dv = \frac{dE}{m} \). Thus the number of particles in the volume element \( d^3r \) with energies between \( E \) and \( E + dE \) which strike the detector in the time interval \( dt \) is

\[
(f(2E/m)(dE/m)) \, dA \, d\Omega \, dt = (2fE/m^2) \, dA \, dE \, d\Omega \, dt
\]

The differential particle flux, \( j \), which is defined as the number of particles striking the detector/cm²/sec/ster/keV then becomes

\[
j(E) = \frac{2E}{m^2} f
\]

Details of the data analysis thus depend not only on the choice of a particular distribution function but also on the actual ion mass composition. Since the thermal ion detectors designed to evaluate electric fields are not capable of sorting the various ion species, a mass composition must be assumed.

Rocket-borne ion mass spectrometer measurements made by Holmes, Johnson, and Young on a daytime ionsosphere show that above \( \sim 150 \) km, the main atmospheric constituents are \( O^+ \) ions. Similar measurements made by the same group on the
nighttime ionosphere reveal that $\text{NO}^+$ predominates between 120 and 200 kilometers while above 200 kilometers, $\text{O}^+$ is again most common. (See Figure 21.) The usual preponderance of $\text{O}^+$ ions at the 150 to 300 kilometer altitudes where ECHO III thermal ion data was collected led to the assumption that measured energy spectra were those of singly charged oxygen ions. Difficulties in extrapolating typical mass profiles to the particular conditions encountered during flight made it virtually impossible to assume a more accurate mass composition for the region of interest.

In order to proceed with the method of data analysis, the particle distribution function was taken to be a streaming Maxwell-Boltzmann. Predicted ion fluxes thus reduce to

$$j(E) = \frac{2E}{m^2} N \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{1}{2}m(\vec{v} - \vec{v}_o)^2 / kT}$$

where $N = \text{# ions/cc}$, $T = \text{particle temperature}$, and $m$ is the mass of oxygen ions. The bulk flow velocity, $\vec{v}_o$, is related to the plasma drift and vehicle ram as indicated in equation 7. Total particle velocities, $\vec{v}$, are determined from both analyzer energy selection and detector look direction, $\hat{n}$, as obtained from vehicle aspect.

Because electrons are much more mobile than heavy ions a rocket which is launched into the lower ionosphere will typically charge to a small negative potential of about one volt. The situation is identical to that of the conducting plasma probes discussed in section 2.1. Ions approaching the payload are accelerated to the rocket body by the electric
Figure 21 Typical ion compositions of the daytime and nighttime ionosphere. [After J.C. Holmes, C.Y. Johnson, and J.M. Young, "Ionospheric Chemistry," in Space Research V, 756, edited by R. Muller, John Wiley, New York, 1965.]
field which exists within roughly one Debye length of the vehicle's surface. It is important to point out here that the acceleration process does not introduce any appreciable change in the direction of net particle flow. Ions pass through the sheath region too quickly to undergo significant deflections. A better understanding of the situation can be obtained by considering the simple case of a planar conductor surrounded by a Debye sheath of thickness d. Figure 22 defines the geometry of the problem. Particles entering the sheath region at an angle \( \alpha \) with respect to the surface normal \( \hat{n} \) follow a curved trajectory to the charged conductor. During their transit of the Debye sheath they undergo a total angular deflection, \( \beta \), as indicated in the diagram. This quantity may be related to the entry angle, \( \alpha \), through simple kinematics.

An ion which accelerates to the conducting surface attains a final velocity

\[
\mathbf{v}_f = \sqrt{v^2 \cos^2 \alpha + (2q\Delta U/m)} \quad x - v \sin \alpha \hat{y}
\]

where \( m \) is the mass of the particle, \( v \) its sheath entry velocity, and \( \Delta U \) the floating potential of the conductor. The angle of deflection, \( \beta \), is calculated from the inner product of \( \mathbf{v} \) and \( \mathbf{v}_f \); i.e.

\[
\cos \beta = \frac{\mathbf{v} \cdot \mathbf{v}_f}{||\mathbf{v}|| \cdot ||\mathbf{v}_f||}
\]

Performing the above indicated operations gives the result
Figure 22 Deflection geometry of particles penetrating the Debye sheath surrounding a charged conductor.
\[ \beta = \cos^{-1} \left( \frac{\cos \alpha \sqrt{\cos^2 \alpha + \frac{q \Delta U}{\frac{1}{2} mv^2} + \sin^2 \alpha}}{\sqrt{1 + \frac{q \Delta U}{\frac{1}{2} mv^2}}} \right) \]

Values of \( \beta \) vs \( \alpha \) are tabulated below for typical parameters \( \Delta U = 1 \) volt and \( \frac{1}{2} mv^2 = \frac{3}{2} kT = 0.13 \) eV (\( T = 1000^\circ K \)).

**TABLE 1**

**ANGULAR DEFLECTION OF PARTICLES ACCELERATED ACROSS A DEBYE SHEATH**

<table>
<thead>
<tr>
<th>( \alpha^\circ )</th>
<th>( \beta^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>.5</td>
</tr>
<tr>
<td>2</td>
<td>.9</td>
</tr>
<tr>
<td>3</td>
<td>1.4</td>
</tr>
<tr>
<td>4</td>
<td>1.8</td>
</tr>
<tr>
<td>5</td>
<td>2.3</td>
</tr>
<tr>
<td>6</td>
<td>2.7</td>
</tr>
<tr>
<td>7</td>
<td>3.2</td>
</tr>
<tr>
<td>8</td>
<td>3.6</td>
</tr>
<tr>
<td>9</td>
<td>4.1</td>
</tr>
<tr>
<td>10</td>
<td>4.6</td>
</tr>
</tbody>
</table>

For small values of \( \alpha \), \( \beta \) is about \( \frac{1}{2} \alpha \). Since the EFMs are collimated to within 1° of the vertical, the only particles which can enter the detectors are those for which \( \alpha - \beta \gtrsim 1^\circ \). It is easy to see from the above tabulation that to be collected, particles must penetrate the sheath region within
2° or 3° of the surface normal; they can then be deflected through only 1° or 2° at most. The result is that no skewing of the bulk flow direction occurs because of sheath field acceleration of detected particles.

One other concern which arises out of the existence of a sheath field surrounding ionospheric spacecraft is the possible creation of extraneous $\vec{E} \times \vec{B}$ drifts. However, the apprehension is totally unfounded. Debye distances in the lower ionosphere are only about 1 cm; since this distance is so much smaller than the gyroradius of even 1 eV ions sheath fields can be felt by gyrating particles only within a very localized region of their gyro-orbits. No additional $\vec{E} \times \vec{B}$ drift can thus be imparted to the ions because such convection requires the existence of large scale electric fields. A more complete discussion of this phenomenon is given at the end of Appendix B.

The one effect of an accelerating sheath field is simply to change measured particle fluxes. Liouville's theorem (see Appendix G) expresses the amount of the shift from the requirement that the quantity $j(E)/E$ be conserved. This implies

$$\frac{j_1(E_1)}{E_1} = \frac{j_2(E_2)}{E_2}$$

If $U$ is used to denote the potential drop through which bombarding ions fall then $E_2 = E_1 + eU$ where $E_1$ is the particle energy at the outside edge of the sheath region, $e$ is the magnitude of the electronic charge, and $E_2$ is the energy of
an ion just as it enters the detector. Thus

\[ j_2(E_2) = \frac{E_1 + eU}{E_1} j_1(E_1) \]

Combining this result with equation 11 gives

\[ j(E_1 + eU) = \frac{2N}{m^2} \left( \frac{m}{2\pi kT} \right)^{3/2} (E_1 + eU) e^{-\left[ \frac{1}{2} \frac{m}{2} (\vec{\nu} - \nu_0)^2}{kT} \right]} \]

where \( E_1 \) and \( \nu \) are related by \( E_1 = \frac{1}{2} mv^2 \).

Since the particle energy selected by the analyzer is \( E = E_1 + eU \) it is convenient to rewrite the flux, \( j \), in the new form

\[ j(E) = \frac{2NE}{m^2} \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-\left[ \frac{1}{2} \frac{m}{2} (\vec{\nu} - \nu_0)^2}{kT} \right]} \]

where now \( \vec{\nu} = -\sqrt{2(E-eU)}/m \hat{n} \). This expression defines the theoretical shape of the measured differential energy spectra of thermal ions. Typical \( O^+ \) flux curves, constructed for an ion temperature of \( 1000^\circ K \) and a vehicle potential of one volt, appear in Figure 23. They show significant differences in the particle fluxes predicted for a plasma which remains stationary with respect to its detector and one which moves relative to the detector at a speed of 1 km/sec. When the ion sensor looks into the transporting plasma it measures increased fluxes while reduced flux levels are seen in the opposite direction. This type of angular dependence in the observed energy spectra can best be visualized by considering the information contained in Figure 24. Graphed here are contours of constant flux for the rocket and plasma properties
Figure 23 Ion fluxes predicted from a Maxwell-Boltzmann plasma of density $10^6$ particles/cc and temperature 1000°K. Detectors on a motionless spacecraft floating at a potential of 1 volt negative with respect to the nearby plasma would record the fluxes given by the curve labelled $v_0 = 0$. If the same spacecraft were moving relative to the ions at a speed of 1 km/sec then onboard detectors looking directly into the plasma onrush would measure the fluxes corresponding to the curve labelled $v_0 = +1000$ m/sec; those detectors looking 180° away from the bulk plasma flow would see the fluxes of the curve labelled $v_0 = -1000$ m/sec.
Figure 23

\[ J = \frac{2N}{m^2} \left( \frac{m}{2\pi kT} \right)^{3/2} \left( \frac{E+U}{kT} \right) e^{-\frac{1}{2}m\left(\mathbf{v} \cdot \mathbf{v}_0\right)^2} \]

- \( V_0 = +1000 \text{ m/sec} \)
- \( V_0 = 0 \)
- \( V_0 = -1000 \text{ m/sec} \)

\( T = 1000^\circ \text{K} \)
\( U = 1 \text{ Volt} \)
\( N = 10^6/\text{cc} \)
listed in the upper left-hand corner of the page. Particle velocities parallel and perpendicular to \( \mathbf{B} \) form the two axes of the graph. The plotted curves are those appropriate to the plane containing the local magnetic field line and the relative ion-vehicle velocity vector here taken to be along the horizontal. Note the following:

1) The centroid or peak of the constant flux curves has been shifted off to the right because of the relative transport which exists between spacecraft and detected ions. Such asymmetries are indicative of net motion between detectors and ambient plasma.

2) Particle fluxes are zero for velocities less than about 3600 m/sec corresponding to the energy ions gained in dropping through the specified 1.1 volt potential surrounding the rocket.

What a given detector placed in this particular plasma would see depends upon how its look direction cuts across the various contours. Maximum count rates are obtained when an EFM views directly into the transporting plasma for then the number of particles striking its collecting face in the time interval \( dt \) is as large as it ever gets. As the detector's view shifts away from this peak count direction measured fluxes decrease symmetrically about the relative ion-vehicle bulk velocity vector. Minimum count rates occur when the analyzer looks 180° away from the plasma onrush.
Figure 24 Contours of constant flux for a detector sampling an ion plasma having the listed properties. Velocities parallel and perpendicular to \( \mathbf{B} \) are the graph's ordinates and abscissae, respectively. The various curves were constructed for a rocket floating potential of -1.1 volts and a net ion vehicle velocity (assumed to be perpendicular to \( \mathbf{B} \)) of 1600 m/sec. A detector sampling ions in the plane containing the magnetic field and net ion transport vectors would record the count rate appropriate for its viewing direction.
N = 1.5 \times 10^5/\text{cc} \\
T = 900^\circ \text{K} \\
U = -1.1 \text{ volts} \\
relative ion-vehicle velocity = 1600 \text{m/sec}
Obviously the ability of a thermal ion sensor to identify the location of a flux peak depends on the amount of data coverage which it obtains. Unambiguous determinations of bulk plasma flow require numerous flux measurements made around the flow direction. The goal of the electric field experiment on ECHO III was to obtain sufficient amounts of data from which to evaluate $\vec{v}_0$ and hence $\vec{E}$. Information provided by the rocket flight included EFM look directions, energy selection, and particle counts integrated over 0.8 milliseconds. Data processing was done on an IBM 360 Model 50 computer. Particle energies, detector aspect, and ion fluxes from successive spectra were input to the General Least Squares With Statistics fitting program which compared measured and predicted ion fluxes. Best fit parameters determined in the analysis include particle density, ion temperature, vehicle potential, and plasma transport velocity. From knowledge of rocket ram motion (see Appendix H) an estimate of convective electric fields was obtained through application of equation 7.
CHAPTER VI

RESULTS FROM THE FLIGHT OF ECHO III

Raw EFM data telemetered back to ground receivers from the ECHO III experiment package has the general appearance shown in Figure 25. Displayed here is a 1.4 second sample of the count rate spectra collected by the two detectors at apogee. The top curve in the figure is labelled EFM 1; it presents data from the analyzer which was mounted to look along the rocket's spin axis. The bottom curve, labelled EFM 2, shows the corresponding data gathered by the detector whose view lay in the vehicle's spin plane. Small arrows under the individual curves identify the beginning of each 32 point sweep.

There are several noteworthy features in the raw data which deserve mention. Immediately obvious, of course, is the spin periodic nature of the spectra measured by EFM 2. Maximum count rates were recorded by the detector whenever its narrow field of view passed through the plane defined by the rocket's spin axis and the net ion-vehicle velocity. As the rotation of the payload shifted the analyzer's view away from this peak count direction the number of particles collected at a given energy step decreased. Flux levels remained above the threshold of the detector only within an angular

---

¹Naturally the number of particles collected by EFM 2 is determined by the convolution of energy step and spin phase. The referenced statement is true for a given particle energy.
Figure 25 Raw EFM data telemetered back from ECHO III at apogee. Each count rate spike corresponds to the particle spectrum obtained at the given time. Small arrows under both graphs mark the beginning of each 0-5 eV energy sweep.
range of ~90° either side of the count rate maximum. The limited dynamical range of the analyzer precluded measurement of particle spectra at larger displacements.

A second readily apparent observation made from the EFM data concerns the relative flux levels between EFM 1 and EFM 2. At apogee, and indeed for most of the flight, the parallel detector recorded higher fluxes than those measured by the perpendicular unit. This was interpreted to mean that, over the appropriate time interval, EFM 1 was more closely aligned along the plasma net transport vector than was EFM 2. The inescapable conclusion, since the rocket lay only a few degrees off the magnetic field line, was that a large plasma flow existed parallel to $\mathbf{B}$. At this point no consideration was given to the possibility that either or both detectors had malfunctioned.

One of the more surprising features in the EFM data is the apparent spin modulation of EFM 1. The results are evident even in Figure 25. Although it was not at first clear what could cause such an effect the answer now appears straightforward. The two detectors were originally designed to measure thermal ion fluxes corresponding to particle densities of a few times $10^6$/cc. However, Chatanika radar data collected over the five day period before launch indicated that ion densities encountered during flight would very likely not exceed $3 \times 10^5$ particles/cc. This meant that the analyzers would register very low count rates at best. To compensate for the problem, the detector entrance apertures
were drilled out from 0.0292 inches to 0.059 inches, accounting for a 4-fold increase in analyzer geometry factors. Since no machine shop was available at the launch site, the work was done by hand and it seems quite likely that the new holes were not drilled exactly perpendicular to analyzer entrance faces. This would account for a spin modulation in the EFM 1 data, however small. More will be said about the effect later.

The final point which should be made regarding the raw EFM spectra of Figure 25 centers around the data dropout at point P. This loss of data was not caused by a temporary telemetry failure but rather by a firing of the onboard electron gun. On such occasions the entire payload is driven slightly positive for a very short time. Thermal O⁺ ions are then retarded by the vehicle potential and may be totally stopped. The enhanced electron current collected by the rocket during this time period eventually swings the spacecraft voltage to a larger negative value than usual before equilibrium is reestablished. The whole process takes only about 30 or 40 milliseconds. Figure 26 lends experimental verification to the above discussion. In this illustration is shown a blowup of the typical EFM response to an 8 msec gun pulse. The event occurs shortly before apogee, at T = 245.173 seconds. The solid curve centered near 1.5 eV is the normally measured ion spectrum. It is unaffected by the gun firing whose beginning and duration is identified by the horizontal barred line at the bottom of the figure. Upon completion of
Figure 26 Distortion of ion spectra following an electron gun firing. The solid curve centered at 1.5 eV is the normally measured ion spectrum. A gun firing covering the time interval indicated by the horizontal barred line causes a distorted spectrum to appear at 4.5 eV. Within the next 10 milliseconds a new spectrum (dotted line) recorded at 1.7 eV shows that static equilibrium is already nearly reestablished.
the pulse a displaced spectrum centered at 4.5 eV appears. At this time the entire spacecraft is charged to a potential of about -3.9 volts. Within the next 10 miliseconds a new spectrum (dotted curve) measured by the analyzer shows that static equilibrium is already nearly reestablished. Hundreds of such gun firing-detector response sequences obtained during the flight all display the same general time development. Disturbed flux measurements appear routinely throughout the EFM data. To insure that no gun contaminated spectra were used in the evaluation of convective electric fields all EFM data within four telemetry paragraphs of the initiation of a gun pulse were ignored. Analyzed data then truly represent ambient plasma properties.

The initial analysis of thermal ion spectra was accomplished by least squares fitting Maxwell-Boltzmann particle fluxes to the data taken from both detectors over two successive spin periods. EFM 1 measurements were numerically averaged to remove the spin dependence of that unit's ion count rate. The coordinate system in which the selected particle data was referenced is shown in Figure 27. Angles $\theta$ and $\phi$ define the elevation and azimuth of the bulk plasma flow vector with respect to the rocket's spin axis, $\hat{n}$, and the look direction of EFM 2 at the beginning of the data input. Detected particles have the total velocity vector $\vec{v}$ as shown in the diagram. For EFM 1 the angle $\alpha$ was assumed to be $0^\circ$ and for EFM 2, $90^\circ$; the phase angle $\psi_0$ was defined to be $0^\circ$ for EFM 2 at time $t = 0$. At the bottom of the figure, the
Figure 27  Coordinate system used for spin period analyses of ECHO III EFM data. The rocket spin axis, $\hat{\Omega}$, forms the z axis of the system; the x and y axes are determined by the look directions of EFM 2 at the start of a data interval and $\frac{1}{2}$ spin period later, respectively. Angles $\alpha$ and $\psi$ denote the viewing angles in this spin system of an arbitrarily mounted detector. Predicted fluxes satisfy the expression at the bottom of the figure. The elevation, $\theta$, and azimuth, $\phi$, of the negatively directed bulk flow vector, $\vec{V}_0$, are among the parameters calculated from a least squares fit of the function $J(E)$ to the actual data.
\[ J(E) = \frac{2N}{m^2} \left[ \frac{-m}{2\pi kT} \right] \frac{3}{8} E_0^{\frac{1}{2}} \frac{(E-U+E_0)^{\frac{1}{2}}(E-U)}{(E-U)^{\frac{1}{2}}(E-U+E_0)} \left( \sin \theta \cos \phi \right)^{\frac{1}{2}} \cos \theta \] \]

\[ E_0 = \frac{1}{2} mV_0^2, \quad U = \text{payload potential} \]

Parameters: \( N, T, U, E_0, \theta, \phi \)

Figure 27
theoretical expression showing the angular dependence of expected ion fluxes is included. Parameter notation is unambiguous.

Spectral fits determined from a typical spin period analysis of EFM data are presented in Figure 28. It is clear from the drawing that the predicted "average" count rate of EFM 1 does a fair job of tracking the actual data. However the results of EFM 2 are harder to judge. Although the fit does reproduce the gross spin phase dependence of the fluxes measured by the analyzer, there seems to be considerably more noise in the spectra of this detector than in those of EFM 1. Excessive scatter of unselected high energy particles in the analyzer's interior may be responsible for the count rate irregularities of the unit. The same phenomenon can also explain the nonzero ion fluxes recorded by both detectors at energies below the predicted 1.1 eV cutoff.

Individual spin period analyses like that described above were run on the EFM fluxes measured over a 100 second interval centered about apogee. The results of the data reduction are shown in Figures 29 and 30. Graphed here as a function of flight time are the six parameters determined in the least squares analysis of the thermal ion spectra. Typical values for payload potential, ion density, and particle temperature are 1.1 volts, $1.4 \times 10^5$/cc, and 800°K respectively. The vector plasma flow, when transformed to geomagnetic coordinates and corrected for rocket ram motion (see Appendix H), implied the existence of east-west electric
Figure 28 Results of a typical spin period fit to the data of both EFM 1 and EFM 2. Since the initial studies did not allow an off axis look direction for EFM 1 the analysis routine simply fit to an average spectrum from the detector.
Figure 29 Results of individual spin period fits covering a 100 second interval centered about apogee.

I. Ion density and temperature and spacecraft potential.
Figure 30 Results of individual spin period fits covering a 100 second interval centered about apogee.

II. North-south and east-west convective electric fields and flow parallel to $\mathbf{B}$. 
fields which averaged to zero. A 35 mV/m northward convective field was also seen. Steady flow down the field line ranged between 600 and 1300 m/sec.

Two particularly disturbing aspects of these results were immediately apparent. First, the analyzers measured a large flow parallel to \( \hat{B} \) which the Chatanika radar station did not see; reconciling this difference was not easy. Second­ly, all fitted parameters were modulated at the 43 second pre­cession period of the payload. In an effort to understand and remove this artificial periodicity in the results, more least squares fits of the particle data were attempted. Various combinations of parameters were held fixed and a constant relative calibration between the detectors was introduced in the expression for theoretical fluxes. Nothing quashed the sinusoidal variation of the outputted parameters. Even changing the ion distribution function to a two temper­ature Bi-Maxwellian and referencing the EFM flux measurements to a geomagnetic coordinate system had no effect on the final results. Indeed, the fitting program refused to allow dif­ferent particle temperatures parallel and perpendicular to \( \hat{B} \); ion count rates were adequately represented by the fluxes predicted from a simple Maxwell-Boltzmann plasma.

Allowing a skew look direction for EFM 1 marked the next logical step in the refinement of the bulk plasma flow analysis. To take into account the unknown viewing angles of the upward looking detector, only minor modifications of the spin fit program were necessary. The addition of the two new
parameters, $\alpha$ and $\psi$, while increasing the computing time required for individual fits, did improve the reproducibility of EFM 1 spectra. For purposes of comparison Figure 31 details the newly revised skew fit to the same data displayed in Figure 28. No drastic differences occur in the flux levels predicted for EFM 2 although the qualitative improvement in the fit to EFM 1 spectra cannot be denied. A 3° detector offset calculated by the fitting program was all that was needed to achieve this result. Credence was thus lent to the theory that the spin modulation of EFM 1 was due to a slightly skewed analyzer viewing direction.

After re-analyzing with the new oblique-look approach all previously examined thermal ion data it was found that the calculated plasma parameters heretofore determined continued to oscillate at the vehicle precession frequency. The only logical explanation for the stubborn existence of this effect had to be an inherent inconsistency in the data of the two analyzers. To find such an incompatibility, individual energy spectra like those shown in Figure 28 were studied. More curves detailing only actual ion count rates are displayed in Figure 32. A quick look at either set of data immediately reveals how dirty the spectra of EFM 2 are in comparison to those of EFM 1. Mounting concern over whether or not the perpendicular unit had functioned properly prompted consideration of the information presented in Figure 33. The graph in this drawing displays the 1 second-averaged peak count rate of the two detectors as a function of flight time. The general
Figure 31  Results of a revised spin period fit to the data of both EFM 1 and EFM 2. The parallel detector (EFM 1) was allowed to have a skew-look direction which was determined by the analysis routine to be within 3° of the rocket's spin axis.
Figure 32  Typical sequence of spectra from both detectors. The data shown in the plots covers the time interval from 261.344 to 261.574 seconds after launch.
time profile of each curve is determined by both altitude dependent particle densities and the proximity of the analyzer's view to the changing net ion-vehicle transport. Clearly indicated in the data of EFM 1 is the 43 second precession period of the rocket payload. No equivalent oscillation is seen in the counts collected by EFM 2 until well after apogee. Such a result might be explained by a bulk plasma flow along the bisector of the cone described by the precessing peak count direction of EFM 2. However, the change in rocket ram velocity during the time span considered does not allow the perpendicular detector to continuously and symmetrically revolve about the relative ion flow. There thus seems to be no obvious physical reason for the lack of precession in much of the EFM 2 data. An equally puzzling observation concerns the maximum level to which the analyzer's varying count rate returns between $T = 300$ seconds and $T = 450$ seconds. Even if the spacecraft ram velocity had continually shifted the net plasma transport towards EFM 2 during this time period it seems unlikely that the resulting increased flux to the detector would have been exactly offset by a corresponding decrease in ionospheric particle densities. Yet the data, assuming static plasma conditions, would indicate such an interpretation.

Up to this point the evidence suggesting problems with EFM 2 had been somewhat circumstantial. It was the constant flux contours constructed from the data of the two detectors that illustrated the perpendicular unit's erratic nature. The separate curves, analagous to those predicted
Figure 33 One-second averaged peak count rate of EFM 1 and EFM 2 as a function of flight time.
in Figure 24, were produced in a two step process. Particle fluxes measured by each analyzer were first normalized to the ion densities encountered at payload apogee; for this purpose the ionospheric density profile was taken to be that obtained by the auroral backscatter radar at Chatanika, Alaska. During the upleg of the rocket flight the radar was set at an elevation angle of 67°; its preset range gates selected signals scattered from several altitudes. As the rocket passed over apogee, the radar elevation was dropped to 45° and more measurements were taken at different heights. Gaps in the data coverage at one elevation were filled in with the results from the other setting. For ease of analysis, vehicle altitude information was combined with the height dependent ion densities measured by radar to give the symmetric density-time profile shown in Figure 34. The solid curve fitted through the data is actually a composite of four separate piecewise continuous functions. Together they allow determination of the particle densities encountered by the rocket as a smooth function of flight time.

The second step in adapting the EFM measurements to the type of data display desired involved transforming the vector velocities of analyzed ions to a coordinate system with one axis along the net plasma-vehicle transport, $\mathbf{v}_0$. For a first look at the data, the radar determination of bulk particle flow was assumed to be correct. Ion velocity components both parallel and perpendicular to $\mathbf{v}_0$ were combined with the corresponding detector count rates to provide the
Figure 34 Ion densities encountered during flight as a function of flight time. The altitude profile was measured by Chatanika backscatter radar.
input for the mapping program described in Appendix I. The routine, known as PSUMAP, used the information to produce the two plots of Figures 35 and 36. Solid lines have been added to the maps to clearly delineate contour levels. Actual data is located at the site of arabic numerals; where two or more conflicting measurements overlap blank spots are left on the page. Interpolated results are indicated by the various symbols associated with each contour level; the legend on each figure coordinates the symbols and numerals.

Understanding of the two maps hinges on a discussion which relates their appearance to the theoretical constant flux contours of Figure 24. In particular, it must be pointed out that the labels \( \mathbf{v}_n \) and \( \mathbf{v}_\perp \) of the present graphs refer not to the ion velocity components parallel and perpendicular to \( \mathbf{B} \) but rather to those along and at right angles to the net plasma transport, \( \mathbf{v}_0 \). As long as the bulk ion flow determined by radar is reasonably accurate the data displayed in these plots should thus peak along the parallel map axis. From an examination of the separate graphs it can be seen that EFM 1 fluxes do indeed exhibit the general trend shown in Figure 24. The occurrence of zero count rates for velocities less than about 3300 m/sec is due to the fact that analyzed particles were pre-accelerated by the \( \approx 1 \) volt potential of the payload. Additionally, maximum detector fluxes lie along the \( \mathbf{v}_n \) axis as expected. There are seen to be no surprises in the data from this unit; now, however, consider the contour curves constructed from EFM 2 flux measurements. Maximum count rates
Figure 35  Constant flux contours generated from all EFM 1 data assuming the direction of bulk plasma flow to be that determined by Chatanika radar. Particle velocities parallel and perpendicular to the net ion transport form the two axes of the graph.
Figure 36 Constant flux contours generated from all EFM 2 data assuming the direction of bulk plasma flow to be that determined by Chatanika radar. The construction of the map is the same as that of Figure 35.
occupy a position off the parallel map axis and individual curves appear jagged. Flux plateaus near the center of the graph correspond to the precession free data mentioned in connection with Figure 33. The blank spots which appear throughout the plot indicate how often the analyzer recorded self-conflicting data. Based on this and the various other irregularities that have been pointed out in the data of the unit a decision was made to exclude EFM 2 measurements from further ECHO III ion studies. This meant that the experiment results would have to be evaluated with the particle spectra of EFM 1 alone.

From the contour map of Figure 35 it is clear that EFM 1 provided enough data coverage over the entire flight to allow at least one accurate determination of bulk plasma flow. Whether or not better time resolutions could be achieved required an evaluation of how long it took to collect sufficient information for believable least squares fits. To resolve this problem, an analysis was attempted on the data from one 43 second vehicle precession period. Since the range of altitudes encountered by the rocket during this time interval was so large, it was decided that the radar density profile would be used in the data reduction; the fitting program would be allowed to determine a constant normalization factor. No altitude dependence for particle temperatures was assumed. Anisotropic flux measurements were referenced to the local geomagnetic coordinate system. Out of the analysis effort came the fit shown in Figure 37. Sample spectra equally
spaced over the 24 second subinterval shown in the drawing are compared with the count rate curves predicted by the fortran fitting routine, GLSWS. The results are seen to be in good agreement; however, a PSUMAP contour plot generated from both the measured EFM fluxes and the calculated bulk ion flow vector shows that the amount of data coverage obtained in 43 seconds is, in general, inadequate for a realistic determination of the plasma parameters. The map, reproduced in Figure 38 clearly illustrates the paucity of actual data; measured fluxes all lie within the small circled region of the graph. The extent in velocity space of information collected from other precession intervals is similarly small but the absolute location of the data coverage evolves with time as the spacecraft ram vector slowly shifts the direction of net ion transport. This implies that even a few sequential analyses of EFM 1 spectra are not feasible. Any conclusions drawn from the experiment thus have to be based on a single simultaneous evaluation of the data from the entire flight. To this end, the fortran fitting routine was modified to accept larger amounts of input. EFM 1 spectra were averaged over successive spin periods and detector viewing angles were measured in the geomagnetic coordinate system. Particle data near electron gun firings were scrupulously avoided. As before, ion densities were specified by the radar determined density profile and the fitting program was allowed to select its own normalization factor. The four minutes worth of useful information accumulated by EFM 1 was finally boiled down
Figure 37 Results of a precession period fit to EFM 1 data only. Sample spectra equally spaced over the 24 second subinterval shown are compared with the curves predicted from a least squares fit to the data.
Figure 38  Constant flux contours generated from EFM 1 data from a 43 second interval centered about apogee. All EFM 1 flux measurements from this time period are contained in the small outlined region on the map. Data coverage from single precession periods is not sufficiently large to allow least squares evaluations of bulk ion flow.
Flux Contours for EFM I.

Bulk Plasma Flow taken to be in direction determined by a Least Squares fit to EFM I Data collected during a 43 second Precession Period centered about Apogee.

Contour Level Symbol
1 L
2 +
3 -
4 =
5 x
6 0
7 e
8

Figure 38
to the several thousand data points numerically analyzed on the IBM system 360 Model 50 computer. Results of the analysis were at first surprising. Predicted ion densities were an order of magnitude larger than what radar measurements showed and velocities parallel to \( \hat{B} \) were huge. What had happened was revealed by the statistical information output by the fitting program. The density normalization parameter was highly correlated with all other variables, especially ion flow along the magnetic field. Such a problem had not been encountered in the attempted spin period fits of the data from both detectors. Relative count rates had uniquely located the bulk flow direction with respect to each analyzer. By now ignoring the data from one unit, some parameter independence was being sacrificed. In particular, the fitting routine found it could achieve good spectral fits by balancing changes in particle density with simultaneous changes in vector ion flow. Physically, this may be understood by considering a hypothetical particle detector connected through some sort of feedback mechanism to a well collimated ion source; the goal of the experiment is to maintain a constant flux to the detector regardless of its viewing direction. As the unit is turned to expose more or less effective collecting area to the beam, its count rates tend to change; when they do, the feedback mechanism appropriately adjusts the source intensity to recover the original flux levels. In this present situation, the feedback mechanism is the fitting program. It allows EFM 1 to maintain a given count rate while looking in a
supposedly disadvantageous direction by increasing the apparent density of surrounding particles. The exact effect discussed here is shown in the PSUMAP of Figure 39. This graph assumes the direction of bulk ion motion to be that determined in the analysis of the ion data from the entire flight. Detector fluxes corresponding to measured vector velocities have been mapped to coordinate positions parallel and perpendicular to the resulting net ion transport. Peak fluxes are predicted along the plots' vertical ($v_{\|}$) axis but no actual data is seen to lie there. PSUMAP says the detector looked away from on-rushing plasma but recorded high count rates nevertheless. This pictorial representation of the foregoing discussion shows that the evaluated flow direction was extrapolated from "distant" measurements; as such it should not be believed. To properly analyze the thermal ion data the correlation between density and field-aligned flow must be overcome. This is most easily done by simply fixing the density normalization parameter to give absolute densities seen by radar. A quoted 5% uncertainty in the profile can be combined in quadrature with the 15% variability of EFM analyzer calibrations to provide estimates of fitted parameter uncertainties. Extreme values taken from separate data analyses inputting measured densities ± 17% would thus give the range of results for each parameter. No other changes in the analysis program are necessary.

With these last modifications of the data reduction scheme put into effect, a final computer run was made on the ECHO III thermal ion data. The results are tabulated here
Figure 39  Constant flux contours generated from all EFM 1 data assuming the direction of bulk plasma flow to be that determined by the unspecified density analysis of measured EFM 1 fluxes.
along with the corresponding independent measurements made by both radar and the electron echo technique:

TABLE 2

PARAMETRIC RESULTS FROM THE ANALYSIS
OF ECHO III ION DATA

<table>
<thead>
<tr>
<th>QUANTITY</th>
<th>EFM RESULTS</th>
<th>RADAR RESULTS</th>
<th>ECHO RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ion Temperature</td>
<td>1507 ± 94°K</td>
<td>~1,000°K</td>
<td></td>
</tr>
<tr>
<td>Rocket Potential</td>
<td>1.227 ± 0.001 volts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E$ north</td>
<td>25 ± 4 mv/m</td>
<td>~ 23 mv/m</td>
<td>~30 mv/m</td>
</tr>
<tr>
<td>$E$ east</td>
<td>6 ± 4 mv/m</td>
<td>(0)?</td>
<td>~7 mv/m</td>
</tr>
<tr>
<td>$V_{</td>
<td></td>
<td>}B$</td>
<td>21 ± 85 m/sec Up</td>
</tr>
</tbody>
</table>

While the EFM observed temperatures is about 50% higher than that seen by radar, its measurement of vector ion flow compares well with both of the independent determinations. The overall quality of fitted results can be seen in Figure 40. Shown here are the actual and theoretical peak sampling rates of EFM 1 for the whole flight. With a few minor exceptions the two curves track very well; differences may be attributed

1During ECHO III's flight the Chatanika radar antenna was set to look geomagnetically eastward along the rocket's trajectory. Without running a 360° azimuthal scan it was impossible for the radar people to provide information on North-South ion motions (East-West convective electric fields). Only preflight data exists to suggest $E$ east was zero.

2These results depend on the magnetic field model used to analyze the electron gun echo pulses; the values listed are median results for a series of different field models.
Figure 40 Comparison of EFM l's actual peak count rates with those predicted from a least squares fit to all its data. The parameters determined in the analysis are listed in the upper right-hand corner of the graph.

$T = 1507 \, ^\circ \text{K}$
$V = 1.227 \, \text{volts}$
$E_{\text{north}} = 25 \, \text{mv/m}$
$E_{\text{east}} = 6 \, \text{mv/m}$
$\vec{v}_L = 21 \, \text{m/sec Up}$

- Data
- Fit
to slight variations in plasma conditions over the four minute time interval studied. The sensitivity of expected results to changes in convective electric fields is illustrated in Figures 41 and 42. The predicted count rate curves of each plot are generated from the set of parameters listed thereon. Note that only the northward electric field value has been altered in either case; one set of theoretical data points corresponds to $E_{\text{north}} = 10 \text{ mV/m}$ and the other to $E_{\text{north}} = 40 \text{ mV/m}$. Both curves show gross distortions from measured count rate profiles and hence lend credence to the analyzed results.

New PSUMAP plots produced from the calculated flow parameters are shown in Figures 43 (EFM 1) and 44 (EFM 2). A comparison between these and the two graphs originally constructed from radar evaluated plasma velocities (Figures 35 and 36) reveals no significant contour changes. EFM 1 data conform nicely to model predictions but EFM 2 results do not. It seems apparent that the spin detector somehow malfunctioned during flight.

To hopefully learn more about the perpendicular unit's problem(s), parameter values estimated from the least squares fit to EFM 1 data were used to reconstruct the ion sampling rates expected for EFM 2. It was found that collected counts were at last a factor of 5 below predicted levels. Peak fluxes obtained during the rocket's ascent failed to show the precession modulation occurring in the theoretical data; after apogee, analyzer count rates remained independent of altitude while predicted sampling rates decreased with diminishing
Figure 41 Sensitivity check 1 to the fit results of Figure 40. Particle temperature, vehicle potential, ion flow along $\mathbf{B}$ and the eastward convective field strength remain unchanged. The predicted curves are, however, generated for the northward field value of 10 mv/m.
Figure 42  Sensitivity check 2 to the fit results of Figure 40. The only parameter varied is the northward convective field strength; the new curve is generated for a value of 40 mv/m.

\( T = 1507^\circ K \)
\( V = 1.227 \) volts
\( E_{\text{north}} = 40 \) mv/m
\( E_{\text{east}} = 6 \) mv/m
\( V_{\text{Up}} = 21 \) m/sec Up

○ Data
● Prediction
Figure 43 Constant flux contours generated from all EFM 1 data assuming the direction of bulk plasma flow to be that determined by the specified density analysis of the parallel detector's measured count rates.
Figure 44  Constant flux contours generated from all EFM 2 data assuming the direction of bulk plasma flow to be that determined by the specified density analysis of EFM 1 spectra.
particle densities. Very little correspondence was found between expected and measured values. The only suggestion coming from these results was that particles which should have entered the analyzer may have gyrated around the canted rocket body and collided with it before being detected. However, a careful trajectory trace, backward in time, of incoming ions revealed that only particles from in front of the rocket could have been counted; those passing behind the spacecraft were convected away before their gyro-motion could return them to the payload. No other ideas have been forthcoming on a way to understand or interpret EFM 2 data. It appears now that the results are totally anomalous.

In concluding the analysis of ECHO III ion data it is necessary to recalculate a "skew-look" direction for EFM 1 using the plasma parameters computed for the whole flight. The result, an equivalent offset angle of 1° to 3°, is evidence of the self-consistency of the data reduction scheme.
CHAPTER VII

CONCLUSIONS

For typical ionospheric electric fields plasma convection contributes significantly to the total motion of heavy thermal ions. Consequently the velocity distribution of such particles is anisotropic. Measurements of the ambient ion spectra can thus be used to recover information on bulk plasma flow and causative convective fields. A least squares analysis of the data also provides estimates of particle density and temperature as well as detector floating potential.

To date, experiment packages designed to collect thermal ion data in the earth's lower ionosphere have been flown on four separate rocket payloads. The results of the first flight, into quiet geomagnetic conditions, are generally encouraging. Despite the malfunction of one low energy detector an analysis of the various plasma parameters has been completed. Time resolutions, however, were severely compromised and the ability to independently evaluate ion densities was lost. Nevertheless, electric field values inferred from the fitted bulk flow velocity compare favorably with the field strengths determined by both backscatter radar and electron echo techniques. The estimated particle temperature is slightly higher than that reported by radar and the value deduced for the floating potential of the spacecraft is typical of ionospheric probes.
An unexpected bonus of the low energy ion detectors on ECHO III is the information provided on payload neutralization following intense electron gun firings. Subsequent distortions of measured energy spectra are seen to persist for about 30 milliseconds after beam injections. The ambient plasma obviously acts to quickly restore equilibrium conditions.

While the qualified success of this initial thermal ion experiment ensures that more such studies will be attempted, it is not yet possible to properly evaluate the technique. Of the four EFM packages already flown, only the first has given useful data. Two others, modified to fit smaller rockets, were launched into active aurora on March 18 and March 23, 1975. Because of an oversight, high voltage channeltron bias terminals were left exposed to the surrounding plasma. It appears that particle currents collected at these points were sufficiently large to force a reduction in detector voltages. As bias dependent channeltron gains consequently decreased, output signals fell below the discriminator level of the pulse amplifier and no counts were recorded. The fourth EFM package, installed on the ELECTRON ECHO IV payload was boosted into the lower ionosphere on January 31, 1976. Primary ground station antennae failed at launch and only small hand-held receivers were available as back-up units. Data quality from this flight is extremely poor; excessive noise levels make any analysis attempt very difficult if not impossible.
For the future, thermal ion flux measurements should concentrate on using multiple detectors to provide good data coverage over short time periods. Flights should be made into auroral events as well as quiet conditions. Wherever possible, analyzed results should be compared with those determined by independent methods. In particular, an experimental verification of the analyzers' ability to uniquely determine local ion densities will be important in assessing the capabilities and limitations of the approach.

An analyzer design modification now being incorporated into units to be flown in the spring of 1977 offers a way of improving the data reduction scheme. To the front end of individual detectors is being added a mass analyzer set to select $O^+$ ions. While the mass resolution will not be good enough to exclude passage of $N^+$ ions it will be good enough to block entry of heavy $NO^+$ and light $H^+$ ions. The need to assume an ion mass composition for purposes of data analysis will thus be obviated.

When the results of these recommended programs have been studied and intercompared it will be possible to thoroughly evaluate the continued viability of using ambient ion data to deduce convective electric fields. I predict that the present method will be found to be accurate, simple, and reliable.
BIBLIOGRAPHY


APPENDIX A

FROZEN-IN FLUX

For a plasma moving at velocity \( \vec{v} \) through a region of space in which both electric (\( \vec{E} \)) and magnetic (\( \vec{B} \)) fields exist, Ohm's law takes the form

\[
\vec{J} = \sigma (\vec{E} + \vec{v} \times \vec{B})
\]

where \( \vec{J} \) is current density and \( \sigma \), the plasma conductivity, is assumed to be a scalar quantity. If displacement currents are neglected, the electromagnetic fields are related by

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}
\]

\[
\nabla \times \vec{B} = \mu_0 \vec{J}
\]

where \( \mu_0 \) is the permeability of free space. These three results can be combined to yield the wave equation

\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times \nabla \times \vec{B} + \frac{1}{\sigma \mu_0} \nabla^2 \vec{B}
\]

expressing the time evolution of the magnetic field. The two terms on the right represent, respectively, field line transport and diffusion. For a stationary fluid (\( \vec{v} = 0 \)), A.4 reduces to the diffusion equation

\[
\frac{\partial \vec{B}}{\partial t} = \frac{1}{\sigma \mu_0} \nabla^2 \vec{B}
\]
which has the solution

\[ \vec{B} = \vec{B}_0 e^{-t/\tau} \]

where \( \tau \), the diffusion time, is related to the characteristic length \( L \) over which \( \vec{B} \) varies by

\[ \tau = \sigma \mu_0 L^2 \quad \text{A.6} \]

For a one centimeter radius copper sphere, \( \tau \) is about one second while for a typical magnetic field in the sun it is about \( 10^{10} \) years. Such extremely long diffusion times are characteristic of highly conducting cosmic plasmas; therefore, for times much shorter than the diffusion time, or equivalently for very large plasma conductivities \( (\sigma \rightarrow \infty) \) the temporal behavior of the magnetic field is controlled by flux transport. That is

\[ \frac{\partial \vec{B}}{\partial t} = \nabla \times \vec{V} \times \vec{B} \quad \text{A.7} \]

Equation A.7 can be shown to be "equivalent to the statement that the magnetic flux through any loop moving with the local fluid velocity is constant in time."\(^1\) To obtain this result consider a closed loop \( C \) moving with velocity \( \vec{V} \); it is bounded by the open surface \( S \) with unit normal \( \hat{n} \) as shown in Figure A-1. The total time rate of change of magnetic flux, \( \phi \), through the circuit is given by

\[ \frac{d}{dt} \int_{S} \vec{B} \cdot \hat{n} \, da \]

\(^1\)Jackson, J.D., *Classical Electrodynamics*, p. 313.
Figure A-1 Closed plasma loop moving through a magnetic field. The loop $C$ is bounded by the open surface $S$ with unit normal $\hat{n}$. 
where $da$ is the element of loop area. Contributions to the above integral arise not only from real time changes in the magnetic induction but also from changes in the boundary location of the moving contour. The flux variation due to a time dependent $\hat{B}$ is given simply by

$$\int \frac{\partial \hat{B}}{\partial t} \cdot \hat{n} da$$

Simultaneous changes in $\phi$ due to the motion of the circuit occur because the translating contour may envelop more or less flux. Since the magnetic field is divergence free there can be no net flux passing through the closed surface bounding the volume swept out by the circuit element. (See Figure A-2.) Any flux change through the loop itself must be accounted for by a corresponding change through the volume's sidewall. If $d\vec{l}$ is the element of length of the contour then $\vec{v}dt \times d\vec{l}$ is the incremental surface area swept out in the time $dt$. The flux through this area is

$$\oint_c \hat{B} \cdot \vec{v} dt \times d\vec{l}$$

which from Stoke's theorem can be transformed to

$$dt \int_S \vec{v} \times (\hat{B} \times \vec{v}) \cdot \hat{n} da$$

Thus

$$\frac{d}{dt} \int_S \hat{B} \cdot \hat{n} da = \int_S \frac{\partial \hat{B}}{\partial t} \cdot \hat{n} da + \int_S \vec{v} \times (\hat{B} \times \vec{v}) \cdot \hat{n} da \quad \text{A.8}$$

Substituting A.7 into A.8 yields the result
Figure A-2 Volume swept out in time $dt$ by a closed plasma loop moving at velocity $\vec{v}$. 
which is interpreted as showing that lines of force are frozen into the conducting plasma and are transported along with it; hence, there is no flux change through the moving circuit.

Since the plasma conductivity is so high (effectively infinite), Ohm's law reduces to

$$
\dot{\mathbf{E}} + \mathbf{v} \times \mathbf{B} = 0
$$

A.10

The velocity \( \mathbf{v} \) of magnetic field lines, defined to be perpendicular to \( \mathbf{B} \), is thus seen to be

$$
\mathbf{v} = \frac{\mathbf{E} \times \mathbf{B}}{\mathbf{B}^2}
$$

A.11

This is the convective velocity mentioned in the introduction.
APPENDIX B

GUIDING CENTER MOTION

Trajectories of charged particles in electromagnetic fields are quite complicated and must usually be obtained by numerically integrating the equation of motion. Analytic solutions are possible only in special cases generally because of some symmetry in the problem. An appropriate example is that of a uniform static magnetic field about which the particle gyrates in a helix.

If the particle is in a nearly uniform magnetic field, its path is approximately helical. By expanding the fields in first order Taylor series and treating deviations from pure helical motion as perturbations, it is possible to follow the particle over many gyration periods without resorting to numerical integrations. This is the essence of the "guiding center" or "adiabatic" approximation. Use of the term "guiding center" comes about because particles in a slowly varying field move roughly in a circle whose center advances across as well as along lines of force.

The nonrelativistic equation of motion of a charged particle is

\[ m\ddot{\mathbf{r}} = m\mathbf{G} + e\mathbf{E}(\mathbf{r}) + e\mathbf{r} \times \mathbf{B}(\mathbf{r}) \]  

where \( \mathbf{r} \) is the instantaneous position of the particle, \( m \) and \( e \) are its mass and charge, \( \mathbf{E} \) and \( \mathbf{B} \) are electric and magnetic
fields, and $\hat{G}$ represents all other forces per unit mass acting on the particle. As a first step in finding the trajectory of the guiding center, let $\dot{r} = \dot{R} + \dot{\rho}$ where $\dot{R}$ is the position of the guiding center and $\dot{\rho}$ is the vector from the guiding center to the particle. See Figure B-1.

A coordinate system is established by defining three orthonormal vectors. Let $\hat{e}_i = \hat{B}/|\hat{B}|$; choose $\hat{e}_2$ such that $\hat{e}_2 \cdot \hat{e}_1 = 0$; the third vector is determined by the relation $\hat{e}_3 = \hat{e}_1 \times \hat{e}_2$. Referenced with respect to the guiding center, a particle moving on a circle of radius $\rho$ is then located by the gyrovector $\rho = \rho (\hat{e}_2 \sin \theta + \hat{e}_3 \cos \theta)$ where $\theta = \int \omega \, dt$ and $\omega$ is the gyrofrequency, $eb/m$.

If it is assumed that the gyroradius $\rho$ is much smaller than some characteristic length $L$ over which the fields change, $\hat{E}$ and $\hat{B}$ may be expanded in Taylor series about $\dot{R}$ to give

$$\hat{E}(\dot{r}) = \hat{E}(\dot{R}) + \rho \cdot \nabla \hat{E}(\dot{R}) + O(\rho/L)^2$$

$$\hat{B}(\dot{r}) = \hat{B}(\dot{R}) + \rho \cdot \nabla \hat{B}(\dot{R}) + O(\rho/L)^2$$

The notation $O(\rho/L)^2$ has been used to indicate that second and higher order terms in the expansion parameter $\rho/L$ are negligible.

Substituting the first order field expansions (B.2) together with $\dot{r} = \dot{R} + \dot{\rho}$ into equation (B.1) gives

$$\ddot{R} + \ddot{\rho} = \hat{G} + e/m(\hat{E}(\dot{R}) + \rho \cdot \nabla \hat{E}(\dot{R}) + (\dot{R} + \dot{\rho}) \times [\hat{B}(\dot{R}) + \dot{\rho} \cdot \nabla \hat{B}(\dot{R})])$$

$$= \hat{G} + e/m(\hat{E}(\dot{R}) + \rho \cdot \nabla \hat{E}(\dot{R}) + \dot{R} \times \hat{B}(\dot{R}) + \dot{R} \times \rho \cdot \nabla \hat{B}(\dot{R}) + \dot{\rho} \times \hat{B}(\dot{R}) + \dot{\rho} \times \rho \cdot \nabla \hat{B}(\dot{R})]$$
Figure B-1  Guiding center coordinate definitions. The particle at point P gyrates about a "guiding center" which moves across as well as along lines of magnetic force.
Although it may at first appear that the term $\mathbf{\hat{p} \times \hat{p} \cdot \mathbf{V B}(\hat{R})}$ is of second order in $\rho/L$, a closer look shows it has the dimensions $(\omega \rho) \rho B/L \sim \frac{eB}{m} \frac{mv_1}{eB} \frac{mv_1}{eB} \frac{B}{L} = \rho/L v_1 B$ and is thus of the first order in the expansion parameter $\rho/L (<<1)$.

Time averaging equation (B.3) over a gyration period allows concentration on the motion of the guiding center.

It is immediately obvious that $\langle \dot{\rho} \rangle = \langle \dot{\rho} \rangle = \langle \dot{\rho} \rangle = 0$. A simple calculation shows

$$< \mathbf{\hat{p} \times \hat{p} \cdot \mathbf{V B}(\hat{R})} > = <[\omega \rho (\hat{e}_2 \cos \theta - \hat{e}_3 \sin \theta) + (\dot{\rho} \hat{e}_2) \sin \theta + (\dot{\rho} \hat{e}_3) \cos \theta] \times [\rho (\hat{e}_2 \sin \theta + \hat{e}_3 \cos \theta) \cdot (\hat{e}_1 \hat{e}_1 \cdot \mathbf{V} + \hat{e}_2 \hat{e}_2 \cdot \mathbf{V} + \hat{e}_3 \hat{e}_3 \cdot \mathbf{V})] B >$$

$$= \omega \rho^2 (\hat{e}_2 \cos \theta - \hat{e}_3 \sin \theta) \times (\sin \theta \hat{e}_2 \cdot \mathbf{V} + \cos \theta \hat{e}_3 \cdot \mathbf{V}) B > + O(\rho/L)^2$$

$$= \frac{1}{2} \omega \rho^2 [\hat{e}_2 \times (\hat{e}_3 \cdot \mathbf{V}) B \hat{e}_3 - \hat{e}_3 \times (\hat{e}_2 \cdot \mathbf{V}) B]$$

The result of time-averaging equation (B.3) is then

$$\ddot{\mathbf{R}} = \dot{\mathbf{G}} + \frac{e}{m} [\dot{\mathbf{E}}(\hat{R}) + \dot{\hat{R}} \times \dot{\mathbf{B}}(\hat{R})] + \frac{e \omega \rho^2}{2m} [\hat{e}_2 \times \hat{e}_3 \cdot \mathbf{V B}(\hat{R}) - \hat{e}_3 \times \hat{e}_2 \cdot \mathbf{V B}(\hat{R})] \quad B.4$$

Simplification of the last term can be accomplished as follows:

$$\hat{e}_2 \times (\hat{e}_3 \cdot \mathbf{V}) B = (\hat{e}_3 \times \hat{e}_1) \times (\hat{e}_3 \cdot \mathbf{V}) B = \hat{e}_1 [\hat{e}_3 \cdot (\hat{e}_3 \cdot \mathbf{V}) B] - \hat{e}_3 [\hat{e}_1 \cdot (\hat{e}_3 \cdot \mathbf{V}) B]$$

But

$$\hat{e}_1 \cdot (\hat{e}_3 \cdot \mathbf{V}) B = \hat{e}_1 \cdot (\hat{e}_3 \cdot \mathbf{V}) B \hat{e}_1 = \frac{1}{2} \mathbf{B} (\hat{e}_3 \cdot \mathbf{V}) \hat{e}_1^2 + \hat{e}_3 \cdot \mathbf{V B} = \hat{e}_3 \cdot \mathbf{V B}$$
because $\hat{e}_1^2 = 1$. Therefore

$$\hat{e}_2 \times \hat{e}_3 \cdot \nabla \vec{B} = \hat{e}_1 [\hat{e}_3 \cdot (\hat{e}_3 \cdot \nabla) \hat{B}] - \hat{e}_3 (\hat{e}_3 \cdot \nabla) \vec{B}$$

Similarly

$$\hat{e}_3 \times \hat{e}_2 \cdot \nabla \vec{B} = -\hat{e}_1 [\hat{e}_2 \cdot (\hat{e}_2 \cdot \nabla) \hat{B}] - \hat{e}_2 (\hat{e}_2 \cdot \nabla) \vec{B}$$

By expressing the operator $\nabla$ as $\nabla = \hat{e}_1 (\hat{e}_1 \cdot \nabla) + \hat{e}_2 (\hat{e}_2 \cdot \nabla) + \hat{e}_3 (\hat{e}_3 \cdot \nabla)$, and using the fact that $\nabla \cdot \vec{B} = 0$, one obtains

$$\hat{e}_2 \times \hat{e}_3 \cdot \nabla \vec{B} - \hat{e}_3 \times \hat{e}_2 \cdot \nabla \vec{B} = \hat{e}_1 [\hat{e}_2 \cdot \hat{e}_2 \cdot \nabla \vec{B} + \hat{e}_3 \cdot \hat{e}_3 \cdot \nabla \vec{B}] - [\hat{e}_2 \hat{e}_2 \cdot \nabla \vec{B} + \hat{e}_3 \hat{e}_3 \cdot \nabla \vec{B}]$$

$$+ \hat{e}_1 \hat{e}_1 \cdot (\hat{e}_1 \cdot \nabla) \hat{B} - \hat{e}_1 \hat{e}_1 \cdot (\hat{e}_1 \cdot \nabla) \hat{B}$$

$$= \hat{e}_1 \nabla \cdot \vec{B} - [\hat{e}_2 (\hat{e}_2 \cdot \nabla) \vec{B} + \hat{e}_3 (\hat{e}_3 \cdot \nabla) \vec{B}]$$

$$- \hat{e}_1 [\frac{1}{2} \vec{B} (\hat{e}_1 \cdot \nabla) \hat{e}_1 \cdot \hat{e}_1 + (\hat{e}_1 \cdot \nabla) \vec{B}]$$

$$= - [\hat{e}_1 (\hat{e}_1 \cdot \nabla) \vec{B} + \hat{e}_2 (\hat{e}_2 \cdot \nabla) \vec{B} + \hat{e}_3 (\hat{e}_3 \cdot \nabla) \vec{B}]$$

$$= - \nabla \vec{B}$$

Since the quantity $\frac{1}{2} e \omega \vec{p}^2$ is simply the charged particle's magnetic moment $\mu$, equation (B.4) has the new form

$$\frac{d}{dt} \vec{R} = \vec{G} + \frac{e}{m} [\hat{E}(\vec{R}) + \hat{R} \times \vec{B}(\vec{R})] - \frac{\mu}{m} \nabla \vec{B} \tag{B.5}$$

Crossing $\vec{B}$ into the above and solving for $\hat{R}_L$ gives

$$\hat{R}_L = \frac{\vec{E} \times \vec{B}}{B^2} + m \frac{\vec{G} \times \vec{B}}{B^2} + \frac{\mu}{e} \frac{\vec{B} \times \nabla \vec{B}}{B^2} + m \frac{\vec{B} \times \hat{R}}{B^2} \tag{B.6}$$

The calculation of $\hat{R}_L$ requires knowledge of the acceleration
of the guiding center. Since the term containing \( \dot{\hat{R}} \) is of first order in the expansion parameter \( \rho/L \), \( \hat{R} \) itself is needed only to zero order. Use of the notation \( \hat{\mathbf{U}}_{E} = \mathbf{E} \times \mathbf{B}/B^2 \) and \( v_{\|} = \hat{e}_1 \cdot \dot{\hat{R}} \) yields the acceleration

\[
\dot{\hat{R}} = \frac{d}{dt} \hat{R} = \frac{d}{dt} (v_{\|}, \hat{e}_1 + \hat{\mathbf{U}}_E) + O(\rho/L)
\]

\[
= \left[ \frac{\partial}{\partial t} + (v_{\|}, \hat{e}_1 + \hat{\mathbf{U}}_E) \cdot \nabla \right] (v_{\|}, \hat{e}_1 + \hat{\mathbf{U}}_E) + O(\rho/L)
\]

\[
= \hat{e}_1 \frac{\partial v_{\|}}{\partial t} + v_{\|} \frac{\partial \hat{e}_1}{\partial t} + \frac{\partial \hat{\mathbf{U}}_E}{\partial t} + v_{\|}^2 \hat{e}_1 \cdot \nabla \hat{e}_1
\]

\[
+ v_{\parallel} \hat{e}_1 \cdot \hat{\mathbf{U}}_E + v_{\parallel} \hat{\mathbf{U}}_E \cdot \nabla \hat{e}_1 + \hat{\mathbf{U}}_E \cdot \nabla \hat{\mathbf{U}}_E + O(\rho/L)
\]

where it has been assumed that the term \( \mathbf{E} \times \mathbf{B}/B^2 \) is of order greater than \( \rho/L \).

Putting this result back into equation (B.6) gives

\[
\dot{\hat{R}} = \hat{\mathbf{U}}_E + \frac{m}{e} \frac{\mathbf{G} \times \mathbf{B}}{B^2} + \frac{\mu}{e} \frac{\mathbf{B} \times \mathbf{V}_B}{B^2} + \frac{m}{e} v_{\|} \frac{\mathbf{B} \times (\partial \hat{e}_1/\partial t)}{B^2} + \frac{m}{e} \frac{\mathbf{B} \times (\partial \hat{\mathbf{U}}_E/\partial t)}{B^2}
\]

\[
+ v_{\|}^2 \frac{m}{e} \frac{\mathbf{B} \times \hat{e}_1 \cdot \nabla \hat{e}_1}{B^2} + v_{\|} \frac{m}{e} \frac{\mathbf{B} \times \hat{e}_1 \cdot \nabla \hat{\mathbf{U}}_E}{B^2}
\]

\[
+ v_{\parallel} \frac{m}{e} \frac{\mathbf{B} \times \hat{\mathbf{U}}_E \cdot \nabla \hat{e}_1}{B^2} + \frac{m}{e} \frac{\mathbf{B} \times \hat{\mathbf{U}}_E \cdot \nabla \hat{\mathbf{U}}_E}{B^2} + O(\rho/L)^2
\]

The guiding center is thus seen to drift across magnetic field lines with the velocity given by equation (B.8).

Three of the terms have been given special names; they provide an adequate description of particle drift in many
situations. The $\vec{E}$ cross $\vec{B}$ drift is denoted $\vec{U}_E$; it arises from the presence of crossed $\vec{E}$ and $\vec{B}$ fields. Magnetic field gradients appropriately cause the gradient drift $\frac{\mu}{e} \frac{\hat{\vec{B}} \times \nabla \vec{B}}{B^2}$ and curvature drift $v''_E \frac{m}{e} \frac{\hat{\vec{B}} \times \hat{\vec{e}}_1 \cdot \nabla \hat{\vec{e}}_1}{B^2}$ results from the motion of particles along curved lines of force.

Motion along $\vec{B}$ satisfies the equation:

$$\frac{dv''}{dt} = \frac{d}{dt} (\hat{\vec{R}} \cdot \hat{\vec{e}}_1) = \hat{\vec{R}} \cdot \hat{\vec{e}}_1 + \hat{\vec{R}} \cdot \frac{d\hat{\vec{e}}_1}{dt}$$

$$= \hat{\vec{R}} \cdot \hat{\vec{e}}_1 + (v''_E \hat{\vec{e}}_1 + \vec{U}_E) \cdot \frac{d\hat{\vec{e}}_1}{dt} + O(\sigma/L)$$

$$= \hat{\vec{R}} \cdot \hat{\vec{e}}_1 + \vec{U}_E \cdot \left[ \frac{d\hat{\vec{e}}_1}{dt} + (v''_E \hat{\vec{e}}_1 + \vec{U}_E) \cdot \nabla \hat{\vec{e}}_1 \right] + O(\sigma/L)$$

Or

$$\frac{dv''}{dt} = G'' + \frac{e}{m} v''_E \vec{E} - \frac{\mu}{m} \hat{\vec{e}}_1 \cdot \nabla \vec{B} + \vec{U}_E \cdot \left[ \frac{d\hat{\vec{e}}_1}{dt} + (v''_E \hat{\vec{e}}_1 + \vec{U}_E) \cdot \nabla \hat{\vec{e}}_1 \right] + O(\sigma/L) \quad B.9$$

Although equations (B.8) and (B.9) look rather formidable, in their application to ionospheric measurements they may be considerably simplified. For example, during quiet geophysical conditions, the upper ionosphere is in diffusive equilibrium along the magnetic field. Under such circumstances, any parallel motion which exists must be constant; i.e. $(dv''_E / dt) = 0$.

Also in the ionosphere, the guiding center drift velocity perpendicular to $\vec{B}$, given by equation (B.8), may frequently be approximated by the lead term $\vec{U}_E = \frac{\vec{E} \times \vec{B}}{B^2}$. Since
all other terms are of order \( \rho/L \), this result is valid whenever \( U_E = E_1/B \gg \rho/L \). Use of the ratio \( B/|V B| \) to estimate the characteristic length \( L \) transforms this inequality into the restriction

\[
E_1 > \frac{m v^2}{e} \frac{\nabla B}{B} \quad \text{B.10}
\]

For polar ionospheric thermal ions at a maximum temperature of 2000°K in a magnetic dipole field, inequality (B.10) becomes

\[
E_1 \gg 2.5 \times 10^{-4} \text{ millivolts/meter}
\]

That is, if the perpendicular electric field is of the order of 0.1 mv/m or larger, then to a very good approximation the guiding center drift velocity transverse to \( \vec{B} \) is

\[
\vec{v}_1 = \frac{\vec{E} \times \vec{B}}{B^2} \quad \text{B.11}
\]

and the bulk (average) ion velocity \( \vec{v}_0 \) simplifies to

\[
\vec{v}_0 = \frac{\vec{E} \times \vec{B}}{B^2} + \vec{v}_n \quad \text{constant} \quad \text{B.12}
\]

To gain a physical understanding of the \( \vec{E} \times \vec{B} \) drift first consider a region of space over which a uniform magnetic field exists. Particles moving through this region have velocity components both parallel and perpendicular to the magnetic field lines. That component parallel to \( \vec{B} \) is unchanged by the field's presence while motion perpendicular
to \( \hat{B} \) consists of gyrations about the field lines. The equilibrium condition on the perpendicular velocity component, \( \dot{v}_\perp \), is

\[
F_C = \frac{mv^2}{r_g} = qv_\perp B \tag{B.13}
\]

where \( r_g \) is the radius of gyration. Note that the centrifugal force, \( F_C \), is inversely proportional to the gyroradius.

Now superimpose upon this region of space a uniform electric field, \( \hat{E} \), which is perpendicular to \( \hat{B} \) and exists over a distance greater than the gyro-diameter of the particles of interest. See Figure B-2. Particle motion becomes more complicated. It might seem that a given particle would continue to orbit or gyrate about \( \hat{B} \) while accelerating along \( \hat{E} \) but this, in fact, is NOT the resultant motion. Recall that the centrifugal force on the particle is inversely proportional to its gyroradius and consult Figure B-3. On this diagram are shown the force vectors \( +q\hat{E} \) and \( \hat{F}_C \) at both the top and bottom of an undistorted gyro-orbit. Also shown are the resultant forces, \( \hat{F}_R \), at both locations. Since the net forces are different at top and bottom, so too are the particles' real gyroradii. At the bottom, because of the larger net force, the gyroradius is small, while at the top, the net force is smaller making the gyroradius larger. The result of the varying gyroradius is a particle drift with drift velocity given by equation B.11. The net motion appears as shown in Figure B-4.
Figure B-2 Crossed $\vec{E}$ and $\vec{B}$ fields.
Figure B-3 Net forces on a particle in crossed $\mathbf{E}$ and $\mathbf{B}$ fields. A particle which would normally gyrate about $\mathbf{B}$ executes a more complicated motion when a large scale electric field exists over a distance greater than its gyrodiameter. Although the force $q\mathbf{E}$ on the particle acts in one direction, the centripetal force, $F_c = q\mathbf{v} \times \mathbf{B}$, is always directed towards the center of the gyro-orbit; at the two extremes of the gyro-motion the resultant forces acting on the particle thus have different magnitudes.
Figure B-3
Figure B-4  Particle trajectory in crossed $\mathbf{E}$ and $\mathbf{B}$ fields. The difference in net force on a particle at the extremes of its gyro-orbit cause it to experience different radii of curvature. The result is a net particle drift given by $\mathbf{V}_{\text{drift}} = \mathbf{E} \times \mathbf{B}/B^2$. 
The foregoing remarks are valid only when the electric field exists over a distance larger than the particle's gyroradius. Smaller scale electric fields can contribute to the net centrifugal force on one side of a particle's gyro-orbit but not on the other. The result is that such fields accelerate particles along their vector direction. No additional drift motion arises due to these fields.
APPENDIX C

DEBYE LENGTH

Over a very small distance known as a Debye length the behavior of plasma particles can be described in terms of numerous two-body Coulomb collisions; however, on a larger scale the individual ions and electrons tend to act collectively. If a charge density perturbation occurs anywhere in the plasma, surrounding particles will redistribute themselves in an attempt to restore electrical neutrality. Coulomb fields are thus reduced to short-range interactions.

In order to relate the Debye scale length to known particle parameters, consider an electron plasma with a uniform background of positive charge. The electrons are in thermal equilibrium in an electrostatic potential $\phi$; their spatial distribution is governed by the Boltzmann factor $e^{-e\phi/kT}$ where $T$ is absolute temperature in degrees Kelvin, $e$ is the electronic charge and $k$ is Boltzmann's constant. In this notation the electron density, $n_e$, is expressed as

$$n_e = n_0 e^{-e\phi/kT} \quad \text{C.1}$$

where $n_0$, the equilibrium electron density, also represents the density of positive ions. The potential $\phi$ satisfies Poisson's equation:

$$\nabla^2 \phi = -\frac{n_0 e}{\varepsilon_0} (e^{-e\phi/kT} - 1) \quad \text{C.2}$$
which for $e\phi/kT$ small reduces to the linear relation

$$\nabla^2 \phi = \frac{n_0 e^2}{\varepsilon_0 kT} \phi$$

Equation C.3 has the spherically symmetric solution

$$\phi = \frac{\phi_0}{r} e^{-r/\lambda_D}$$

where $\lambda_D$, the plasma Debye length, is given by

$$\lambda_D = \sqrt{\frac{\varepsilon_0 kT}{n_0 e^2}}$$

It is thus seen that plasma potentials are effectively "screened" in a distance of a few Debye lengths. In the lower ionosphere where electron temperatures reach about 2000°K amid densities of roughly $2 \times 10^5$/cc, this Debye length is about 0.7 cm.

If the quantity $e\phi/kT$ is not small the potential $\phi$ must be found from a numerical integration of the nonlinear relation C.2. Application of any potentials exceeding those in the ambient plasma effectively results in thickened Debye sheaths. However, the Debye length is still defined as in Equation C.5.
APPENDIX D

MAGNETIC MIRRORING

In a magnetic field \( \hat{B} \), charged particles move in a helix along the lines of force. Their gyrations about \( \hat{B} \) give rise to a magnetic moment \( \mu \) given by

\[
\mu = \frac{1}{2} m v_\perp^2 \frac{B}{B}
\]

where \( m \) is the mass of the particle and \( v_\perp \) is the velocity transverse to \( \hat{B} \). For a slowly varying field the magnetic moment is nearly constant; its "adiabatic invariance" is especially useful for following the motions of magnetically trapped particles. A quantitative discussion of the phenomenon is given below.

Consider an axially symmetric field that is slowly converging in space as indicated in Figure D-1. The \( z \) direction is taken to be that of the flux line which passes through the guiding center of the particle of interest. In cylindrical coordinates the magnetic field is expressed as

\[
\hat{B} = B_x \hat{r} + B_z \hat{z}
\]

and the \( z \) component of the equation of motion becomes

\[
m \frac{dv_z}{dt} = - qV_\theta B_r
\]

This latter result may be rewritten
Figure D-1  Slowly converging, axially symmetric, magnetic field geometry.
\[ m \frac{dv''}{dt} = \pm qv_\perp B_r \] 

because \( \hat{v}_\perp = +v_\theta \hat{\theta} \) depending on the sign (\( \pm \)) of \( q \).

Since the divergence of \( \mathbf{B} \) is zero the two field components \( B_r \) and \( B_z \) are related by

\[ \frac{1}{r} \frac{\partial}{\partial r} (r B_r) + \frac{\partial B_z}{\partial z} = 0 \]  

D.5

If \( \frac{\partial B_z}{\partial z} \) is assumed constant across the particle's gyroradius \( \rho \) then D.5 can be integrated to give

\[ B_r = -\frac{1}{2} \rho \frac{\partial B_z}{\partial z} \]

which for a slowly converging field approximates to

\[ B_r = -\frac{1}{2} \rho \frac{\partial B}{\partial z} \]  

D.6

Combining D.6 with D.4 yields the equality

\[ m \frac{dv''}{dt} = -\frac{1}{2} q v_\perp \rho \frac{\partial B}{\partial z} \]  

D.7

The perpendicular component of the particle's equation of motion can now be used to show that the quantity \( \frac{1}{2} q v_\perp \rho \) is simply the magnitude of the magnetic moment \( \mu \). That is,

\[ - \frac{m v_\perp^2}{\rho} = - q v_\perp B \]

for \( \pm q \) implies

\[ m v_\perp = \pm q B \rho \]  

D.8
Substituting this equation back into D.7 gives

\[ m \frac{dv_{\perp}}{dt} = \pm \frac{1}{2} qv_{\perp} (\pm mv_{\perp} / qB) \frac{\partial B}{\partial z} \]

\[ = - \frac{1}{2} \frac{mv_{\perp}^2}{B} \frac{\partial B}{\partial z} \]

or finally

\[ m \frac{dv_{\perp}}{dt} = - \mu \frac{\partial B}{\partial z} \tag{D.9} \]

To show the invariance of \( \mu \) multiply both sides of equation D.9 by \( v_{\perp} \) \((= dz/dt)\) to get

\[ mv_{\perp} \frac{dv_{\perp}}{dt} = - \mu \frac{\partial B}{\partial z} \frac{d}{dt} \]

which simplifies to

\[ \frac{d}{dt} \left( \frac{1}{2} mv_{\perp}^2 \right) = - \mu \frac{d}{dt} \frac{B}{dt} \tag{D.10} \]

Since the particle's energy is conserved

\[ \frac{d}{dt} \left( \frac{1}{2} mv_{\perp}^2 \right) = - \frac{d}{dt} \left( \frac{1}{2} mv_{\perp}^2 \right) = - \frac{d}{dt} (\mu B) \]

The latter result is obtained by using equation D.1. Thus

\[ \frac{d}{dt} (\mu B) = \mu \frac{d}{dt} \frac{B}{dt} \tag{D.11} \]

It follows from equation D.11 that \( \mu \) is a constant of the motion. The implication is that particles moving in a magnetic field which converges at two points (see Figure D-2)
Figure D-2 Magnetic mirror geometry. Particles entering the regions of increased magnetic induction experience a force slowing their motion parallel to \( \vec{B} \). If the field is strong enough in the "mirror" region the particles are actually reflected back into the weak field area; they are said to be magnetically trapped.
can become trapped; they "mirror" or bounce between the high field regions. As a particle tries to enter an area of increasing magnetic induction, its perpendicular velocity must increase to keep $\mu$ constant. This increase in $v_\bot$ occurs at the expense of $v_\parallel$; hence, the particle motion along $\vec{B}$ is slowed. If the field becomes strong enough in the "mirror" region then particles are eventually stopped and reflected back into the weaker field area. It is thus possible to "trap" particles in a field geometry such as that of the earth. Of course, the force $F_\parallel$ given by equation D.9 actually causes particle mirroring.
APPENDIX E

LEAST SQUARES FITTING

General Least Squares With Statistics (GLSWS) is a fortran routine written to perform a least squares fit of both linear and nonlinear functions to a set of data. The algorithm developed for this purpose interpolates between two commonly used approaches each of which overcomes the practical difficulties of the other. The Taylor series method calculates corrections to the currently estimated parameter vector by expanding the fitting function to first order in a Taylor series. Because limitations on numerical procedures prevent the retention of higher order terms, correction vectors determined in this way may be sufficiently large to render the first order expansion invalid. If this happens, successive iterations will diverge. By contrast, convergence at every iteration can be guaranteed through the use of the Gradient or Steepest Descent method. Its difficulty lies in increasingly slower convergence as the fit draws nearer to the minimum of the residual surface. The present approach utilizes the best features of each of these methods. It may be developed with a simple modification to the linear Taylor series approximations.

If \( f(\hat{x}, \hat{b}) \) is the function chosen to describe a set \( \{y_1\} \) of \( n \) data points where \( \hat{x} = (x_1, x_2, \ldots, x_j) \) are \( j \) independent variables and \( \hat{b} = (b_1, b_2, \ldots, b_k) \) represents the \( k \)
parameters to be determined in the fit, then least squares modeling will adjust $\hat{b}$ to minimize the sum of square residuals

$$\phi = \sum_{i=1}^{n} [f_i(\hat{x},\hat{b}) - y_i]^2. \tag{1}$$

As previously mentioned, the neglect of higher order terms in the Taylor expansion of $f$ may lead to overly large corrections of the parameter vector. The result is divergence of successive iterates. One way to surmount this problem is to modify the Taylor technique of minimizing $\phi$ by requiring that the length of the correction vector be restricted. The linear approximation is then valid and the method of least squares can be applied to the sum of squares of both residuals and parameter vector increments.

Since the various $b_j$ may have different units, it is necessary to define dimensionless parameters $P_j$. The choice of scale $\xi_j^{-1} = \sqrt{\sum_{i=1}^{n} (\partial f_i/\partial b_j)^2}$ is one often used in least squares problems and permits the definitions $P_j = b_j/\xi_j$ for all $j$. Naturally, current estimates for the $b_j$ must be used as necessary in evaluating the derivatives $\partial f_i/\partial b_j$.

In terms of the dimensionless parameters, the constraint placed upon the length of the correction vector

\[ \sqrt{\sum_{i=1}^{n} (\xi_j^{-1})^2} \]

If the data points $y_i$ are to be weighted by some statistical uncertainty, $\sigma_i$, then the quantity $\phi$ to be minimized must be modified to $\phi = \sum_{i=1}^{n} [f_i(\hat{x},\hat{b}) - y_i/\sigma_i]^2$. For purposes of this project, all uncertainties $\sigma_i$ were taken to be equal.
$\delta \hat{p}$ (= $\hat{p}$ - $\hat{p}$ current estimate) is expressed as $\sum_{m=1}^{k} \delta p_{m}^{2}$ = constant.

Minimizing $\phi$ subject to this restriction is equivalent to treating the $P$'s as independent and using Lagrange's method of constant multipliers to minimize the auxiliary quantity $\phi'$ given by

$$\phi'(\hat{p}) = \sum_{i=1}^{n} [f_{i}(\hat{x}, \hat{p}) - y_{i}]^{2} + \lambda \sum_{m=1}^{k} \delta p_{m}^{2}; \lambda > 0 \quad \text{E.1}$$

Calculation of $\delta \hat{p}$ follows by replacing $f_{i}(\hat{x}, \hat{p})$ with its first order Taylor expansion and solving, as a function of $\lambda$, the following system of $k$ equations:

$$\frac{1}{2} \frac{\partial \phi'}{\partial p_{j}} = \sum_{i=1}^{n} \left[ f_{i} + \sum_{m=1}^{k} \frac{\partial f_{i}}{\partial b_{m}} \delta p_{m} - y_{i} \right] \frac{\partial f_{i}}{\partial b_{j}} + \lambda \delta p_{j} = 0 \quad \text{E.2}$$

for all $j$.

Rearranging terms gives

$$\sum_{i=1}^{n} (f_{i} - y_{i}) \frac{\partial f_{i}}{\partial b_{j}} + \sum_{i=1}^{n} \sum_{m=1}^{k} \frac{\partial f_{i}}{\partial b_{m}} \frac{\partial f_{i}}{\partial b_{j}} \delta p_{m} + \lambda \delta p_{j} = 0 \quad \text{E.3}$$

for all $j$.

In matrix notation, equation (E.3) has a particularly simple form.

$$[A(r) + \lambda(r) I] \delta p(r) = g(r) \quad \text{E.4}$$

where $I$ = identity matrix

$$A_{jm} = A_{mj} = \frac{\sum_{i=1}^{n} \frac{\partial f_{i}}{\partial b_{m}} \frac{\partial f_{i}}{\partial b_{j}}}{\sqrt{\sum_{i=1}^{n} \left( \frac{\partial f_{i}}{\partial b_{m}} \right)^{2} \sqrt{\sum_{i=1}^{n} \left( \frac{\partial f_{i}}{\partial b_{j}} \right)^{2}}}}$$
and the superscript \( r \) denotes the equality (E.4) at the \( r \)th iteration.

Note that for \( \lambda = 0 \), these equations degenerate to those of the normal Taylor approach. For \( \lambda \to \infty \), equations (E.4) reduce to

\[
\partial P^{(r)} = \lambda^{-1}(r) G^{(r)}
\]

or

\[
\partial P_j^{(r)} = \lambda^{-1}(r) \frac{n \sum_{i=1}^{n} (f_i - y_i)(\partial f_i / \partial b_j)}{\sqrt{n \sum_{i=1}^{n} (\partial f_i / \partial b_j)^2}}
\]

The expression in brackets is simply \( \frac{1}{2} \) of the \( j \)th component of the dimensionless gradient of \( \phi \). That is

\[
\partial P_j^{(r)} = \frac{1}{2} \lambda^{-1}(r) (-\nabla_P \phi)^{(r)}
\]

Therefore in the limit as \( \lambda \) becomes very large, the correction vector has a Gradient solution reduced by \( (2\lambda)^{-1} \).

The algorithm here developed is seen to be a combination of both the Gradient and linear Taylor methods. Interpolation between the two is based on the maximum region of applicability of the truncated Taylor series. Program strategy for converging to the minimum of \( \phi \) is as follows:

a) Let \( v \) be a constant greater than 1.
b) Denote the current estimate of the parameter vector by \( \hat{b}(r) \).

c) Let \( \lambda^{(r-1)} \) be the value of \( \lambda \) from the previous iteration. This requires initialization of \( \lambda \) at some convenient value, e.g. \( \lambda(0) = 10^{-2} \).

d) Compute \( \phi(\lambda^{(r)}) \equiv \phi(r) \)

e) Calculate \( \phi(\lambda^{(r-1)}) \equiv \phi[\hat{b}(r) + \lambda^{(r-1)}] \) and \( \phi(\lambda^{(r-1)}/\nu) \equiv \phi[\hat{b}(r) + \lambda^{(r-1)}/\nu] \).

f) If \( \phi(\lambda^{(r-1)}/\nu) \leq \phi(r) \), define \( \lambda^{(r)} = \lambda^{(r-1)}/\nu \)

If \( \phi(\lambda^{(r-1)}/\nu) > \phi(r) \), and \( \phi(\lambda^{(r-1)}) < \phi(r) \),

define \( \lambda^{(r)} = \lambda^{(r-1)} \)

If \( \phi(\lambda^{(r-1)}/\nu) > \phi(r) \), and \( \phi(\lambda^{(r-1)}) > \phi(r) \),

search for the smallest integer \( \omega \) such that \( \phi(\lambda^{(r-1)}/\nu) \leq \phi(r) \). Then define \( \lambda^{(r)} = \lambda^{(r-1)}/\nu^\omega \).

With the value of \( \lambda^{(r)} \) thus obtained, perform the next iteration and proceed in this manner until convergence is obtained. The criteria for convergence are

\[ |\lambda b_j^{(r)}| / (\tau + |b_j^{(r)}|) < \varepsilon \] for all \( j \), for a user determined value of \( \varepsilon \) (e.g. \( 10^{-4} \)) and some suitable choice of \( \tau \) (e.g. \( 10^{-3} \)).
After having satisfied the user's convergence criteria, GLSWS proceeds to output some statistical properties of the fit. Interesting information includes uncertainties and confidence levels associated with the evaluation of each parameter. Comments on these follow.

Consider a quantity \( z(x_1, x_2, \ldots, x_k) \) which is a function of \( k \) measured variables \( x_k \). If its most probable value \( \bar{z} \) is taken to be that determined by the mean
\[ z(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_k)^T \]
of \( N \) measurements then the standard deviation of \( z \) is written as
\[
\sigma_z = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} (z_i - \bar{z})^2
\]
The difference \( z_i - \bar{z} \) may be expanded as
\[
z_i - \bar{z} = (x_{i1} - \bar{x}_1) \frac{\partial z}{\partial x_1} + (x_{i2} - \bar{x}_2) \frac{\partial z}{\partial x_2} + \ldots + (x_{ik} - \bar{x}_k) \frac{\partial z}{\partial x_k}
\]
where each derivative is evaluated at \( x = \bar{x} \). The result of performing the indicated replacement is
\[
\sigma_z \approx \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \left\{ (x_{i1} - \bar{x}_1)^2 \left( \frac{\partial z}{\partial x_1} \right)^2 + (x_{i2} - \bar{x}_2)^2 \left( \frac{\partial z}{\partial x_2} \right)^2 + \ldots + (x_{ik} - \bar{x}_k)^2 \left( \frac{\partial z}{\partial x_k} \right)^2 \right\}^{1/2}
\]
If variations in \( x_m \) and \( x_j \) are uncorrelated, cross terms in the above expression will vanish as \( N \to \infty \). Equation (E.8) then becomes
\[
\sigma_z \approx \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \left\{ (x_{i1} - \bar{x}_1)^2 \left( \frac{\partial z}{\partial x_1} \right)^2 + (x_{i2} - \bar{x}_2)^2 \left( \frac{\partial z}{\partial x_2} \right)^2 + \ldots + (x_{ik} - \bar{x}_k)^2 \left( \frac{\partial z}{\partial x_k} \right)^2 \right\}^{1/2}
\]
Recognizing that $\sigma^2 x_j = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} (x_{ji} - \bar{x})^2$ allows the further simplification

$$
\sigma^2 z = \sqrt{\sum_{j=1}^{k} \sigma^2 x_j \left( \frac{\partial z}{\partial x_j} \right)^2}
$$

Because the solution obtained from the nonlinear fitting process was the result of a search along parameter space rather than a precise analytical solution, it is not reasonable to expect analytic expressions for the $\sigma^2_{P_j}$'s. However, Richard Arndt and Malcolm MacGregor\(^1\) have shown through a sophisticated mathematical proof that the analytic formula for calculating $\sigma^2_{P_j}$ for linear solutions provides a reasonable definition for nonlinear solutions as well. Derivation of this formula begins with the attempt to fit the function $f(\vec{X}) = \sum_{m=1}^{k} P_m X_m(\vec{x})$ to a set of $N$ data values $y_i$. For simplicity, the standard deviations of all measurements are assumed equal. Calculation of the parameter vector requires solution of the matrix equation

$$
\alpha P = \beta
$$

where

$$
\alpha_{jm} = \sum_{i=1}^{N} X_m X_j
$$

$$
P = (P_1, P_2, ..., P_k)^T
$$

$$
\beta_j = \sum_{i=1}^{N} y_i X_j
$$

\(^1\)Arndt, R.A. and M.H. MacGregor, Methods in Computational Physics, 6, 253, 1966.
Denoting the inverse of the matrix \( \alpha \) by \( \varepsilon \) leads to the solution

\[
P = \varepsilon \beta \text{ or}
\]

\[
P_{ij} = \sum_{m=1}^{k} \varepsilon_{jm} \beta_{mi} = \sum_{m=1}^{k} \varepsilon_{jm} \sum_{l=1}^{N} y_{l} x_{m} \]

(E.12)

Using the result of equation (E.10) to write

\[
\sigma_{Pj}^2 \approx \sum_{i=1}^{N} \sigma_i^2 (\partial P_j / \partial y_i)^2 = \sigma^2 \sum_{i=1}^{N} (\partial P_j / \partial y_i)^2
\]

and substituting \( \partial P_j / \partial y_i = \sum_{m=1}^{k} \varepsilon_{jm} x_m \) from E.12 gives

\[
\sigma_{Pj}^2 = \sigma^2 \sum_{m=1}^{k} \sum_{l=1}^{k} \varepsilon_{jm} \varepsilon_{jl} x_m x_l
\]

\[
= \sigma^2 \sum_{m=1}^{k} \sum_{l=1}^{k} \varepsilon_{jm} \varepsilon_{jl} \alpha_{ml}
\]

\[
\sigma_{Pj}^2 = \sigma^2 \varepsilon_{jj} \quad \text{E.13}
\]

This is the formula used to calculate \( \sigma_{Pj} \) for the linear and nonlinear fits performed by GLSWS. The curvature matrix at the minimum of \( \phi \) corresponding to the matrix \( \alpha \) above is just the matrix \( A \) defined in equation (E.4). Its inversion to the "error" matrix \( \varepsilon \) is straightforward. The determination of \( \sigma^2 \) for \( N \) measurements and \( k \) parameters is given by

\[
\sigma^2 \approx \frac{1}{N-k} \sum_{i=1}^{N} (y_i - f_i)^2 \quad \text{E.14}
\]
From the standard deviations of the dimensionless parameters $P_j$ are found the deviations in the true parameters $b_j$. The relationship between the two is

$$\sigma_{b_j} = \lambda_j \sigma_{P_j} \quad \text{E.15}$$

Assignment of actual individual uncertainties is based upon the establishment of parametric confidence levels. The Gaussian distribution seems to describe the scatter of random observations for most experiments where the Poisson distribution is not valid. From a parent population with mean $\mu$ and standard deviation $\sigma$, the probability a random measurement yields the value $x$ is

$$P_G(x,\sigma,\mu) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2} \quad \text{E.16}$$

In an arbitrary sampling, the most likely value to be obtained is thus seen to be the mean $\mu$. Similarly, each parameter determined in a least squares fit should provide a reasonable estimate of its parent mean value.

The following question naturally arises. "Within some uncertainty $U$, does the measured value of $x$ reliably approximate the mean?" Obviously, the answer depends upon how large that uncertainty is. For example, only with 0% confidence can it be said that a given parameter has exactly the value $x$. Likewise, it makes no sense to say with 100% confidence that the value determined lies somewhere in the range $-\infty$ to $\infty$. Neither quote is the least bit useful. Note,
however, that the probability any measurement $x$ falls in a range $2h$ centered about $\mu$ is given by

$$\frac{\text{Confidence level}}{100\%} = P_{\mu-h}^{\mu+h} = \frac{1}{\sigma \sqrt{2\pi}} \int_{\mu-h}^{\mu+h} e^{-\frac{1}{2} \left[ \frac{(x-\mu)}{\sigma} \right]^2} \, dx \quad \text{E.17}$$

The determination of $x$ may thus be quoted with any arbitrary confidence if it is assigned an uncertainty $h$.

GLSWS establishes only 0, 50, and 95 percent confidence levels. The associated uncertainties may be found by solving the integral equation (E.17) for each of these values. For example:

$$0.95 = \frac{1}{\sigma \sqrt{2\pi}} \int_{\mu-h}^{\mu+h} e^{-\frac{1}{2} \left[ \frac{(x-\mu)}{\sigma} \right]^2} \, dx = \frac{1}{\sqrt{2\pi}} \int_{-h/\sigma}^{h/\sigma} e^{-t^2/2} \, dt$$

$$= 2 \text{ erf} \left( \frac{h}{\sigma} \right) - 1$$

This has the solution $h_{.95} = 1.96\sigma$. Similarly $h_{.50} = 0.67\sigma$ and $h_0 = 0$.

To establish a confidence level for parameter $P_j$, compute the ratio $P_j/\sigma_{P_j}$. If $P_j/\sigma_{P_j} > 1.96$ then this parameter is assigned a 95% confidence level. For $0.67 \leq P_j/\sigma_{P_j} < 1.96$, a 50% reliability is assigned and if $P_j/\sigma_{P_j} < 0.67$ it is assumed that the value of $P_j$ can be quoted with no confidence.

Even with these aforementioned statistics, the differences between computed and observed functions probably remains the best single indicator of the goodness of fit.
In summary, it must be pointed out that GLSWS requires solution of equation (E.4) for at least two values of \( \lambda \) at each iteration (refer to the sequence of program strategy steps). It is, therefore, prudent to ascertain whether or not the greater number and complexity of calculations is warranted by the increased applicability of GLSWS.

Since the evaluation of the \( A \) matrix requires most of the numerical effort in a given iteration, the second linear equation solution represents only a small increase in the number of computations. Nevertheless the power of an iteration is increased dramatically. Table E-1 below\(^1\) presents a comparison between the Taylor, Gradient, and GLSWS methods of least squares fitting. The function modeled to actual data is \( y(x) = a_1 \exp[-\frac{1}{2}(x-a_2/a_3)^2] + a_4 + a_5 x + a_6 x^2 \). For each approach, the same initial estimate of parameters was made.

TABLE E-1
SEARCH PATHS FOR NON-LINEAR LEAST-SQUARES FITTING PROCEDURES

Fitting function is \( y(x)=a_1 \exp[-\frac{1}{2}(\frac{x-a_2}{a_3})^2]+a_4+a_5 x+a_6 x^2 \)

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<th>ROUTINE</th>
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<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
<th>( a_5 )</th>
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\(^1\)Bevington, P.R., *Data Reduction and Error Analysis for the Physical Sciences*, p. 216, (partial Table II-1).
**TABLE E-1—Continued**

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</table>

The Gradient method is seen to have closed in on the vicinity of the residual surface minimum in one iteration. However, subsequent progress towards converged values was markedly slower. The Taylor approach although taking longer to locate the neighborhood of final values proceeded from there to a converged condition more rapidly. In their applications to this problem, neither technique shows any clear superiority over the other. By way of comparison, GLSWS converges in fewer iterations. Not only is the vicinity of the terminal values reached quickly but final convergence from within this region is also rapid.

This practical example shows that GLSWS incorporates into one unifying algorithm the best features of the Taylor and Gradient methods. Individual limitations of either are avoided. The result is a powerful tool for statistical analysis of experimental data.
APPENDIX F

DETECTOR CALIBRATION

Since extensive data coverage is required for bulk flow analyses of convective electric fields it is most reasonable to perform the experiment with several detectors. Acceptable time resolutions can thus be obtained. However, accurate relative calibrations between the detectors are essential. This appendix describes a laboratory calibration scheme from which a particular unit's energy-geometry factor can be determined.

Consider an electrostatic analyzer set to look at particles in some finite energy range $\Delta E$; the detector's response curve varies continuously over the entire window width peaking at the mean energy $E_0$. Particle fluxes $j(E_0)$ are given in terms of measured count rates $C(E_0)$ by

$$j(E_0) = \frac{C(E_0)}{G(E_0) \Delta E(E_0)} \text{ particles cm}^{-2} \text{ster kev sec}$$

where $G(E_0)$ is the analyzer's average efficiency-geometry factor for the window of interest; it has the units cm$^2$ ster counts/particles. The indicated dependence of $\Delta E$ on $E_0$ is real and is determined by the analyzer design. For a detector with closely spaced deflecting plates the relationship between $\Delta E$ and $E_0$ is approximately linear.

To obtain the quantity $G(E_0)$ it is necessary to know the analyzer's response to particles whose energies fall
within the given window. If \( g(E) \epsilon(E) \) is used to denote the product of detector collecting area, acceptance solid angle, and counting efficiency for particles of energy \( E \) then the average efficiency-geometry factor \( G(E_0) \) may be calculated as

\[
G(E_0) = \frac{1}{AE} \int g(E) \epsilon(E) \, dE
\]

The problem of determining analyzer geometry factors thus reduces to that of finding the detector response curves for the energy windows considered. A laboratory scheme devised by Arnoldy et. al. (1973) uses a simulated omni-directional flux of monoenergetic particles to calibrate individual units. An electron gun and an automatic scanning platform known as a wabbler are the central pieces of equipment involved in the calibration process.

The electron gun is simply a hollow brass cylinder and enclosed molybdenum filament which is heated by a constant current supply. Three exit holes placed around the circumference of the can are drilled midway along the gun's length. A small negative potential is applied to the filament while the cylinder itself is held at ground. Since the length of the gun is roughly five times its diameter the electric field near the exit ports is approximately radial. Emitted electrons thus have an energy equal to the potential applied to the filament.

The automatic scanning platform is a movable base on which the detector to be calibrated is mounted; it uses a
series of cams and drive sprockets to produce motion in a four-dimensional space: \( \theta \) (elevation), \( \phi \) (azimuth), \( x \) (horizontal), and \( y \) (vertical). A schematic drawing of the wabbler is shown in Figure F-1. The total solid angle \( \Omega \) swept out during the platform's \( \theta-\phi \) scan is \( 2.2 \times 10^{-2} \) steradians; this is much larger than the acceptance cone of the EFM s. Similarly the area \( A \) determined by the wabbler's \( x-y \) motion is considerably greater than the detector's collecting area. In the time it takes the platform to describe the area \( A \), many individual scans are made in \( \theta-\phi \) space. Gear ratios are chosen to be nonintegral multiples of each other so that the platform cannot trace out identical paths on successive scans.

In order to calibrate a particular analyzer it is necessary to first mount the unit atop the automatic scanning platform; the assembly is then situated in front of the electron gun and the whole system placed under vacuum. A Faraday cup and electrometer are used to monitor the emission current. The experimental setup is shown in Figure F-2. As the wabbler moves relative to the fixed electron beam the analyzer is bombarded by an equivalent, omni-directional particle flux. The fraction of particles which the detector actually counts is determined by the ratio of its geometry factor to that of the automatic scanning platform. These remarks may be made more precise by defining the quantities of interest. For example, let \( N \) be the total number of electrons emitted from the gun per unit time; this quantity
Figure F-1 Schematic of the automatic scanning platform (wabbler). Drive sprockets and linkage chains are not shown for reasons of clarity. The platform, driven by a Slo-Syn 72 RPM motor, generates motion in $xy\theta\phi$ space.
Figure F-2 Experimental analyzer calibration setup. The alignment of electron gun, faraday cup, and wabbler platform is shown.
can be easily calculated from the electrometer's measurement of beam current, \( I_b \). The omni-directional particle flux, \( j_o \), is then expressed as

\[ j_o = \frac{NT}{A\Omega} \]

where \( T \) is the wabbler's full scan period (\( \sim 1000 \) seconds) and the product \( A\Omega \) is the geometry factor defined by the motion of the platform. Since both the analyzer and the Faraday cup look into the same size gun holes, the quantity \( g(E) \varepsilon(E) \) may be obtained by dividing the counts, \( C(E) \), collected by the analyzer by the flux \( j_o \). Successive scans made at different beam energies can thus be used to map out the response curve of the EFMs; these curves then define the energy window of the detector for the applied electrode deflection voltage. The combined analyzer energy resolution and efficiency-geometry factor for the window of interest is obtained by integrating the response curve over energy and dividing the result by the mean energy, \( E_0 \). The ratio

\[ \frac{\int g(E) \varepsilon(E) dE}{E_0} \equiv G \]

is found to be nearly constant for all detector energies. This simplifies the amount of work needed to completely calibrate a given analyzer. Calculation of differential energy fluxes from measurements of particle count rates are thus obtained by dividing those rates by the product of \( G \) and \( E_0 \). Answers are expressed in terms of particles/cm\(^2\)/ster/kev/sec. For further information on this calibration technique see the paper by Arnoldy et. al. (1973).
An actual efficiency-geometry curve for a flight EFM is reproduced in Figure F-3. The detector had an entrance aperture whose diameter was 0.0292 inches. Electrode voltages were symmetrically set at ±6 volts and the total drop held constant to within 10 millivolts. As can be easily seen from the figure, the analyzer selected a range of particles whose mean energy was 61 eV. This value agrees quite well with that predicted from equation 8. The energy resolution determined from the full width at half maximum of the curve is about 8%, also in agreement with theory. And when the response curve is numerically integrated over energy, the result is

\[ G_{0.0292}E_0 = 2.1 \times 10^{-5} \text{ cm}^2 \text{ ster kev} \]

Thus the analyzer's energy-geometry-efficiency factor is

\[ G_{0.0292} = 3.45 \times 10^{-7} \text{ cm}^2 \text{ ster kev/kev} \]

Although electron beams were used to reproduce detector response curves the results appear equally valid for heavy ion fluxes. The major reason relatively large and complex ion guns were not used in the calibration process was because of a lack of space in the available vacuum systems. However, this does not invalidate the detector calibrations. Since the electrostatic analyzers are essentially energy selectors they must select leV ions just as easily as they do leV electrons. Any difference between electron and ion efficiency-geometry factors is most likely due to a
Figure F-3  Actual calibration curve for EFM 1. The entrance aperture of the analyzer was 0.0292 inches in diameter. Electrode voltages were symmetrically set at ±6 volts and electron beam energy was continuously varied from 56eV to 67eV. The beam intensity was held constant at $6.79 \times 10^5 \text{e}^-/\text{sec}$. 
Analyzer Entrance Aperture is 0.0292 inches

\[
\frac{\int g(E)\varepsilon(E)\,dE}{E_0} \approx 3.45 \times 10^7 \text{ cm}^2 \text{ ster}
\]

\[
\frac{\delta E}{E_0} \approx 8\%
\]

Figure F-3
difference in channeltron counting efficiencies. This difference was minimized by appropriately post-accelerating analyzed particles. Electrons were accelerated to 500 eV by a +500 volt potential applied to the cone of the channel electron multiplier while detected ions were post-accelerated to 1500 eV in like manner. Channeltron efficiencies measured by various research groups reveal no significant differences in electron and ion counting capabilities at these respective particle energies. Hence the calculated energy-geometry-efficiency factors obtained from electron calibrations were assumed to be the absolute values for the EFM's on ECHO III.
APPENDIX G

LIOUVILLE'S THEOREM

A complete specification of the instantaneous state of a plasma requires information about the position and momentum of each constituent particle. Because these particles are so numerous, only a statistical treatment of the plasma is feasible.

Any collective behavior which exists may be developed from a distribution function \( f(\mathbf{r}, \mathbf{v}, t) \) so defined that

\[
f(\mathbf{r}, \mathbf{v}, t) \, d^3 \mathbf{r} \, d^3 \mathbf{v}
\]

represents the probable number of particles which, at time, \( t \), occupy an element \( d^3 \mathbf{r} \, d^3 \mathbf{v} \) of phase space centered about position \( \mathbf{r} \) and velocity \( \mathbf{v} \). The impossible problem of monitoring the entire system then reduces to that of determining the equation of motion of the distribution function.

Consider a collisionless plasma moving under the influence of an external force \( \mathbf{F} \). Particles of mass \( m \) whose coordinates at time \( t \) are \( (\mathbf{r}, \mathbf{v}) \) will have coordinates \( (\mathbf{r} + \mathbf{v} \delta t, \mathbf{v} + (\mathbf{F}/m) \delta t) \) after an infinitesimal time interval \( \delta t \). Statistically, then, all particles contained in the phase space volume element \( d^3 \mathbf{r} \, d^3 \mathbf{v} \) at \( (\mathbf{r}, \mathbf{v}) \) at time \( t \) will occupy a new element \( d^3 \mathbf{r}' \, d^3 \mathbf{v}' \) at \( (\mathbf{r} + \mathbf{v} \delta t, \mathbf{v} + (\mathbf{F}/m) \delta t) \) at time \( t + \delta t \). This may be expressed in mathematical notation as
\[ f(\vec{r} + \vec{v} \delta t, \vec{v} + (F/m) \delta t, t + \delta t) d^3r' d^3v' = f(\vec{r}, \vec{v}, t) \ d^3r d^3v \quad G.2 \]

For a force whose only velocity dependence is of the Lorentz force type \( q \vec{v} \times \vec{B} \), a simple relation exists between the elements \( d^3r' \ d^3v' \) and \( d^3r \ d^3v \). In fact, \( d^3r' d^3v' = d^3r d^3v \).

Suppose the element at time \( t \) is a six dimensional cube. To establish the above equality, it is sufficient to show that the area of any projection of this cube, say \( dx dv_x \), is invariant. See Figure G-1.

The projection at time \( t \) has the area \( \Delta x \Delta v_x \). To find the area of this projection at time \( t + \delta t \), it is necessary to know the coordinates of points 1 through 4 labelled in Figure G-1. These may be expressed as

\[
\begin{align*}
    x_1 &= x + v_x \delta t \\
    v_{x1} &= v_x + \frac{1}{m} F_x (x, v_x) \delta t \\
    x_2 &= x + \Delta x + v_x \delta t \\
    v_{x2} &= v_x + \frac{1}{m} F_x (x + \Delta x, v_x) \delta t \\
    x_3 &= x + \Delta x + (v_x + \Delta v_x) \delta t \\
    v_{x3} &= v_x + \Delta v_x + F_x (x + \Delta x, v_x + \Delta v_x) \delta t \\
    x_4 &= x + (v_x + \Delta v_x) \delta t \\
    v_{x4} &= v_x + \Delta v_x + F_x (x, v_x + \Delta v_x) \delta t
\end{align*}
\]

If the only velocity dependent forces acting are of the Lorentz type, then \( \partial F_x / \partial v_x = 0 \) and first order Taylor expansions of \( F_x \) about \( x \) reduce equations G.3 to

\[
\begin{align*}
    x_1 &= x + v_x \delta t \\
    v_{x1} &= v_x + \frac{1}{m} F_x (x) \delta t \\
\end{align*}
\]

G.4
Figure G-1  Time evolution in phase space of an element of plasma. Particles in the six dimensional volume $d^3r \, d^3v$ at time $t$ migrate to the new volume $d^3r' \, d^3v'$ at time $t + \Delta t$. Shown in the figure is the projection in the x$v_x$ plane of the plasma element at times $t$ and $t + \Delta t$.
Figure G-1
\[ x_2 = x + \Delta x + v_x \Delta t \quad v_{x_2} = v_x + \frac{1}{m} F_x(x) \Delta t + \frac{1}{m} \frac{\partial F_x(x)}{\partial x} \Delta x \Delta t \]

\[ x_3 = x + \Delta x + (v_x + \Delta v_x) \Delta t \quad v_{x_3} = v_x + \Delta v_x + \frac{1}{m} F_x(x) \Delta t + \frac{1}{m} \frac{\partial F_x(x)}{\partial x} \Delta v_x \Delta t \]

\[ x_4 = x + (v_x + \Delta v_x) \Delta t \quad v_{x_4} = v_x + \Delta v_x + \frac{1}{m} F_x(x) \Delta t \]

Although no two points have the same ordinate or abscissa, note that \( v_{x_4} - v_x = v_{x_1} - v_{x_2} \) and \( x_3 - x_4 = x_2 - x_1 \). This means that the projection of the six dimensional volume element at time \( t + \delta t \) is a parallelogram. Its area may be expressed as \( |\vec{A} \times \vec{B}| \) where \( \vec{A} = (x_2 - x_1) \hat{x} + (v_{x_2} - v_{x_1}) \hat{v}_x \) and \( \vec{B} = (x_4 - x_1) \hat{x} + (v_{x_4} - v_{x_1}) \hat{v}_x \). The result of performing the indicated operations to first order in \( \delta t \) is

\[
\text{Area (} t + \delta t \text{)} = \Delta x \Delta v_x = \text{Area (} t \text{)} \quad \text{G.5}
\]

thus establishing the constancy of the six dimensional volume \( d^3r \ d^3v \). Equation (G.1) then simplifies to

\[
f(\vec{r} + \vec{v} \Delta t, \vec{v} + (\vec{F}/m) \Delta t, t + \delta t) = f(\vec{r}, \vec{v}, t) \quad \text{G.6}
\]

Expansion of the left hand side to first order in \( \delta t \) gives the equation of motion for the distribution function:

\[
\left( \frac{\partial}{\partial t} + \vec{v} \cdot \Delta_x + \frac{\vec{F}}{m} \cdot \Delta_v \right) f(\vec{r}, \vec{v}, t) = 0 \quad \text{G.7}
\]

where \( \Delta_x \) and \( \Delta_v \) are, respectively, the gradient operators with respect to position and velocity. Since the operator \( \partial/\partial t + \vec{v} \cdot \Delta_x + F/m \cdot \Delta_v \) is simply the total time derivative
\[ \frac{df(\vec{r}, \vec{v}, t)}{dt} = 0 \]  

This result, known as Liouville's theorem, is to be interpreted as meaning that the density of particles in phase space is constant along a particle trajectory.

For experiments measuring the differential energy flux \( J \) of the particles in question, Liouville's theorem can be expressed in another useful form.

It has already been shown that \( J \) and \( f \) are related by the equation

\[ J(E) = \frac{2E}{mT} f(\vec{r}, \vec{v}) \]

where \( E = \frac{1}{2}mv^2 \) is the particle energy. Because \( f \) is constant along a particle trajectory, so also is \( J(E)/E \) for nonrelativistic particles. This is the result we set out to obtain.

In summary, for nonrelativistic particles of mass \( m \) moving under the influence of external forces whose only velocity dependence is of the Lorentz force type, the quantity \( J(E)/E \) in the absence of collisions is constant along a particle trajectory.
APPENDIX H

ECHO III RAM VELOCITY

Information on the trajectory of ECHO III was obtained from tracking radar operated by a NASA crew out of Wallops Island, Virginia. Rocket position and velocity data, made available to all experimenters, were tabulated in one second intervals from lift-off to impact. From these measurements, a smoothed ram vector was calculated as a function of flight time.

From 90 seconds to 440 seconds after launch, ECHO III was in free fall above 135 kilometers. Its horizontal ram velocity was determined by averaging the radar data collected during this time period and assuming the results to be constant. In geographic coordinates, the rocket ram had an eastward velocity component of 821.7 m/sec and a southward velocity component of 325.7 m/sec. The vertical ram motion was computed by differentiating with respect to time the parabolic curve which was least squares fit to the altitude data. A comparison between the vehicle's fitted trajectory and individual radar measurements is shown in Figure H-1. Results of the height calculations gave

\[ Z = -4.41668T^2 + 2353.261T - 40420. \]  

H.1

where \( Z \) is the rocket altitude in meters and \( T \) is flight time in seconds. The vertical ram speed is then
Figure H-1 Parabolic fit to ECHO III altitude data as a function of flight time. During the interval covered, the rocket was in free fall above 140 kilometers.
expressed in meters/second. Apogee, which occurred at a height of 273 kilometers, was reached at 266.4 seconds.

In summary, ECHO III spent most of its flight time in the ionospheric F region; its trajectory lay in a plane approximately 22° south of geographic east. The vehicle ram velocity, given in geographic coordinates East (E), North (N), and Up (Z), can be calculated as a function of time from the equation

$$\dot{V}_{\text{ram}} = 821.7E - 325.7N + (2353.3 - 8.833T)Z \text{ m/sec}$$
APPENDIX I

CONTOUR MAPPING WITH PSUMAP

PSUMAP (for Pennsylvania State University Map) is a fortran program using piecewise smooth functions to construct contour maps from irregularly spaced data. It is a modified version of a routine developed at the Laboratory for Computer Graphics and Spatial Analysis, Harvard University.

Briefly, data are located on a two-dimensional network of grid points. An interpolative scheme is then used to calculate contour values at grid positions not occupied by actual measurements.

A preliminary method of obtaining a contour value at some point \( P (x,y) \) uses the data locations \( (x_i,y_i) \) and values \( z_i \) of all \( N \) measurements. Weighting each data value by the inverse square of its distance to \( P \) and summing over all data points provides a weighted average contour level at position \( P \). That is

\[
z_i(x,y) = \frac{\sum_{i=1}^{N} \frac{1}{d_i^2} z_i}{\sum_{i=1}^{N} \frac{1}{d_i^2}}
\]

where \( d_i = d|P,P_i| = \sqrt{(x-x_i)^2 + (y-y_i)^2} \). As \( P \) draws arbitrarily close to the \( j \)th data point, the distance \( d_j \) approaches zero and the function \( z_i \) assumes the measured value \( z_j \) as desired.
While this method is simple, it suffers from several shortcomings. For example, increasingly larger amounts of data require proportionately longer computations. The resulting inefficiency is unnecessary. Since the contribution to $z$ from distant points is screened by the inverse square weighting, computational effort can be reduced by interpolating from nearby data only. In fact, a collection of the nearest four to ten points is sufficient to make a reasonable determination of the contour value at any location.

The details of a revised weighting factor lessening the numerical workload stem from the concept of an "initial search radius" $r$. With $A$ representing the area of the largest polygon enclosed by all $N$ measurements, $r$ is defined to be the radius of that circle containing an average of seven data points.

$$\pi r^2 = 7 \frac{A}{N} \quad \text{(1.2)}$$

Since initial search areas centered at various locations contain different numbers of data points, it may be necessary to expand or contract the radius $r$ to insure that all interpolations are performed using the four to ten nearest measurements.

The scheme used proceeds from the following definitions.

$C_p \equiv$ the collection of data points $D_i$ lying within the initial search area.

$n(C_p) \equiv$ the number of elements in $C_p$
\( r'(C^n_P) \equiv \text{the distance to the nearest data point outside the circle containing } n \text{ measurements} \)

For a reference location \( P \), new radii \( R \) are thus specified:

\[
R \equiv \begin{cases} 
  r'(C^n_P) & \text{if } n(C_p) \leq 4 \\
  r & \text{if } 4 < n(C_p) \leq 10 \\
  r'(C^{1.0}_p) & \text{if } n(C_p) > 10
\end{cases}
\]

From the above, new weighting factors \( S_i \) are constructed:

\[
S_i \equiv \begin{cases} 
  d_i^{-1} & \text{if } 0 < d_i \leq R/3 \\
  \frac{27}{4R} \left[ \frac{d_i}{R} - 1 \right]^2 & \text{if } R/3 < d_i \leq R \\
  0 & \text{if } d_i > R
\end{cases}
\]

This modification transforms the existing interpolation function given by equation (I.1) to

\[
z_{II}(x,y) \equiv \frac{\sum S_i^2 z_i}{\sum S_i^2} \quad \text{I.5}
\]

where the primed summation notation is used as a reminder that indicated sums are to be performed using only those data elements lying within a range \( R \) of point \( P \). Although much of the inefficiency of the original interpolation method has
been removed, no relevant properties of the surface $z$ have been changed.

Other problems also plague the simple inverse square weighting scheme.

1) Calculation of $z_{i}(x,y)$ uses distances to surrounding data points but ignores the corresponding directions. Two quite different arrangements of data could thus conceivably lead to identical interpolated values $z_{i}$ at $P(x,y)$.

2) The choice of functional form $z_{i}$ leads to zero directional derivatives at the site of all data elements. The constraint which this places on the contour surface is artificial and unwarranted.

3) Very near data points, distances $d_{i}$, are calculated from the differences of two nearly equal numbers. Computational error, based on the limits of machine precision, can become significant.

Each difficulty can be overcome by additional modifications to the function $z_{i}$.

Directional influence is taken into account with the introduction of a directional weighting term $t_{i}$ for each data point used in a given interpolation. If the angle formed by point $P$ and data elements $D_{i}$, $D_{j}$ is denoted by $\theta_{ij}$ then $t_{i}$ is expressed as

$$
t_{i} = \frac{\Sigma S_{j} (1 - \cos \theta_{ij})}{\Sigma S_{j}}
$$

I.6
The relative contribution to \( z \) from points lying in a given direction is ranked according to the distance weighting factor \( S_j \). Since \( \cos \theta \) can take on all values from -1 to +1, \( t_i \) itself ranges from 0 to +2.

When both distance and direction are considered in forming the contour surface, a revised weighting factor \( w_i \) given by

\[
w_i = S_i^2 (1 + t_i)
\]

leads to the new interpolation function

\[
z_{III}(x, y) = \frac{\sum w_i z_i}{\sum w_i}
\]

The directional derivatives calculated from contour function \( z_I \) are

\[
\frac{\partial z_I}{\partial x} = -2 \frac{\sum_{i=1}^{N} \sum_{j \neq i} d_i^{-2} d_j^{-2} (x-x_i) (z_i - z_j)}{\left( \sum_{i=1}^{N} d_i^{-2} \right)^2}
\]

\[
\frac{\partial z_I}{\partial y} = -2 \frac{\sum_{i=1}^{N} \sum_{j \neq i} d_i^{-2} d_j^{-2} (y-y_i) (z_i - z_j)}{\left( \sum_{i=1}^{N} d_i^{-2} \right)^2}
\]

At the site of data element \( D_i \) each derivative evaluates to
zero as previously mentioned. To overcome this undesirable trait, an increment \( \Delta z_i \) is added to the contour value at nearby data points. This increment, although small in comparison to \( z_i \), is so constructed that the directional derivatives at data grid locations are nonzero.

Constants \( A_i \) and \( B_i \), equal to the desired slopes at elements \( D_i \), are determined for every data point. To this end, search circles of radius \( R \) are established about each \( D_i \). All enclosed data except element \( D_i \) itself are used to obtain "weighted averages of divided difference of \( z \) about \( D_i \)."

That is,

\[
A_i = \frac{\sum'_{j \neq i} w_j \frac{(x_j - x_i)(z_i - z_j)}{(d_i, P_i', P_j')^2}}{\sum'_{j \neq i} w_j} \tag{I.10}
\]

\[
B_i = \frac{\sum'_{j \neq i} w_j \frac{(y_j - y_i)(z_i - z_j)}{(d_i, P_i', P_j')^2}}{\sum'_{j \neq i} w_j} \tag{I.11}
\]

The characteristic distance \( \varepsilon \) over which slope terms contribute significantly to the interpolation of \( z \) is limited to

\[
\varepsilon = b \left( \max \{ z_i \} - \min \{ z_i \} \right) / \left[ \max \{ A_i^2 + B_i^2 \} \right]^{\frac{1}{2}} \tag{I.11}
\]

Shepard, D., Proc. 23 Nat'l Conf. of the Ass'n for Computing Machinery, p. 520.
Choice of the factor $b$ is somewhat arbitrary but if slope effects are to be felt over some fraction $f$ of a contour interval then $b$ should roughly equal $f/n$ where $n$ is the number of contour levels specified for the map.

From the foregoing work, contour increments $\Delta z_i$, are defined as

$$\Delta z_i = \left[ A_i (x-x_i) + B_i (y-y_i) \right] \left[ \frac{\xi}{\xi+\delta_i} \right] \tag{I.12}$$

Since the partial derivatives of $\Delta z_i$ at the $i^{th}$ data point are simply the constants $A_i$ and $B_i$, the interpolation function which has the desired slopes but otherwise behaves like $z_{III}$ is

$$z_{IV}(x,y) = \frac{\sum w_i (z_i + \Delta z_i)}{\sum w_i} \tag{I.13}$$

With the attainment of this latest function, three of the four listed problems have been circumvented. Limited machine precision is the remaining annoyance. Rounding and truncation error can be responsible for large inaccuracies when interpolations are undertaken very near data points. Avoidance of this difficulty is achieved by establishing an $\varepsilon$-neighborhood of each measurement. Interpolations are then performed only outside this region. Within the neighborhood, the function $z$ is defined to be the average of the data values enclosed.

Although this plan creates a discontinuous surface, the discontinuity is given approximately by the product of
machine error and contour gradient magnitude. The subsequent loss of accuracy is of the same order as that produced by machine imprecision at some other location.

The final modification of $z_i$ results in the interpolation function $z$ actually used by PSUMAP.

$$z = \begin{cases} 
\frac{\sum' w_i (z_i + \Delta z_i)}{\sum' w_i} & \text{if } d_i \geq \epsilon \text{ for all } D_i \text{ used in the sum } \sum' \\
<z_i> & \text{if } d_i < \epsilon \text{ for some } D_i
\end{cases}$$

I.14

Contour maps produced by this algorithm are grey scales containing up to ten contour levels. While caution must be exercised in drawing conclusions about the surface behavior far from all data points, it must be recognized that the above approach can provide a valuable tool in the analysis of three-dimensional data.