AURORAL PARTICLE ACCELERATION BY FIELD ALIGNED POTENTIALS

DAVID NORTON WALKER

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AURORAL PARTICLE ACCELERATION BY
FIELD ALIGNED POTENTIALS

by

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B.S., University of Maryland, 1965
M.S., University of New Hampshire, 1972

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ABSTRACT

AURORAL PARTICLE ACCELERATION BY FIELD ALIGNED POTENTIALS

by

David N. Walker

We attempted to ascertain whether certain auroral electron spectra could be explained by a parallel electric field acceleration mechanism. "Parallel" refers to the direction of the static magnetic field. The data which we used was taken from three separate auroral sounding rocket flights of Dr. R. L. Arnoldy.

We developed a simple scatter-free model of parallel electric field acceleration and compared the predictions of this model to data gathered by the detectors aboard the rocket flights. For the purpose of developing the model we assumed an initially Maxwellian plasma which we then allowed to fall through a potential drop. From this basic idea, we developed a number of models by varying the injection position, mirror effect parameters, number and form of accelerating potentials, etc. The basic conclusion drawn from these models is that in the simplest scatter-free case they are insufficient to describe the data adequately. The reasons for this are: (1) The model predicts discontinuous behavior ("cut-offs") which is not present to any degree in
the data and, (2) the data shows the existence of an isotropic component of the differential flux, in addition to a field aligned, or anisotropic, component. This behaviour is not predicted by a simple scatter-free single source model.

For the purpose of explaining the difference between the model and the data, multiple sources, scattering and fluctuations were studied. It was concluded that the data could be sufficiently represented with either a one source or a two source model if one allowed wave-particle scattering to occur.

As wave-particle scattering can arise from, for one, an electrostatic velocity space instability, the form of the parallel distribution function, \( F_{\parallel} \), was calculated by numerical methods from the computer fit to the data. We conclude from this form that the time segments which were investigated represent distributions which appear to be evolving toward monotonic decreasing distribution functions. Quasi-linear stability theory would predict these forms for \( F_{\parallel} \) for a distribution which was linearly unstable at an earlier time. The analysis indicates that the direction of the ambient magnetic field is the direction in which the instability seems likely to proceed (\( \mathbf{\hat{r}} \cdot \mathbf{\hat{B}} = 0 \)).
CHAPTER I

INTRODUCTION

The subject of this investigation is the acceleration of auroral particles by magnetic field aligned potential differences. Specifically, we wish to determine if it is reasonable to associate certain auroral electron spectra (See Figures (1) through (7)) with a simple non-interacting model of parallel electric field acceleration. As will be seen, it is ultimately necessary to introduce stability considerations and associated wave particle scattering phenomena into the interpretation of the data.

In beginning these ideas, the following areas should be clarified: (1) the data, (2) the relevant concepts underlying electric field formation, (3) the significance of non-interacting and, (4) the sense in which we treat the question of stability. Hopefully, the following few pages will serve this purpose.

It has become increasingly popular, particularly within the last decade, to invoke magnetic field aligned electric potentials as the explanation of the source of certain auroral particle energy (Swift (1965), Carlqvist and Bostrom (1970), Block (1972), Carlqvist (1972), Evans (1974)). Electrons thought to be responsible for aurorae range in energy from 0 to 10 kev as measured above the auroral region. Roughly, the differential energy flux spectrum,
Figures (1) through (7), can be characterized by three different electron populations. The spectrum of Figure (2) is a typical illustration of this division into separate particle groups. Notice that the fluxes in this figure are sorted with respect to pitch angle, the angle of the electron total velocity vector with respect to the magnetic field direction. In the low energy range (0-1 kev), there are isotropic fluxes of low temperature electrons. These particles are considered to be a combination of the ambient plasma at the detector location, along with backscattered primaries and secondaries produced by atmospheric scattering (Nagy and Banks (1974)). In addition to the low energy electrons, one observes another isotropic distribution which has maximum flux at approximately 5.6 kev. The temperature of this particle group is much higher than the lower energy one. These temperatures are more on the order of hundreds of ev to several kev. Finally the spectrum shows a highly anisotropic (field aligned) distribution whose peak flux occurs at approximately 4.8 kev. The temperature of these particles is low (40-100 ev). The electrons comprising the field aligned group have a relative velocity along B. They are said to be streaming at an energy of 4.8 kev ($\sim40x10^8$ cm/sec). One notices that the streaming energy of the field aligned peak is less than the peak energy of the nearby isotropic distribution (5.6 kev). This is the usual case when the two particle groups are observed together as in Figure (2) of Arnoldy (1974a,b). It is the existence of the field aligned, highly
variable, "mono-energetic" peak in the spectrum which initially prompted speculation regarding electric field acceleration as the source of the streaming energy. These electrons appear further to be related to the existence of parallel (or Birkeland) current systems (Arnoldy (1974)).

Since the magnetosphere and upper regions of the ionosphere closely approximate a collisionless plasma, it was difficult to understand how one could maintain, or in fact form, a potential difference along a direction in which the conductivity was essentially infinite. However, early experimental work on low pressure gas vapor discharges (Langmuir and Mott-Smith, Jr. (1924), Tonks (1937), Hull and Elder (1942)) showed that there existed a limitation on the current which could be carried by a low impedance plasma. Specifically, it was found that the plasma ceases to be conducting and can support a potential drop several orders of magnitude higher than the thermal energy of the system. This phenomenon is presently referred to as the formation of "anomalous resistivity" or "turbulent resistivity". Qualitatively, in the low impedance conducting situation, the lighter electrons are responsible for the current, their space charge being neutralized by a background of heavier ions. When the current flow is interrupted, the condition of charge neutrality no longer applies and high electric fields can then be supported. The potentials thus formed are intrinsically different from thermal potentials that can arise in differing regions of a plasma due to pressure (temperature) variations (Alfvén (1963)). The latter energies are more on
the order of the electron thermal energies. The potential regions described here are variously referred to as "double layers", "sheaths", or "space charge regions" in analogy to similar regions formed near the physical boundaries of confined plasmas. The regions are typically on the order of several Debye lengths in thickness and considerably less than the mean free path.

Broadly, the problem of producing parallel electric fields is treated on two levels. The first approach (macroscopic) is at root hydrodynamic. Notable among these models are those of Alfven and Carlqvist (1967), and later Carlqvist (1972) and Block (1972). The initial state is that of a cold beam-plasma system: an ion background and an electron current. The plasma is subjected to a density perturbation (decrease) and current conservation is required. The initial phase resembles a "double-double layer" in that there is no net potential drop across the region. This is so because the electric fields so established tend to deplete the region of electrons symmetrically from the disturbance point. It is shown that the unstable growth is limited by a critical density, $n_C$, in the evacuated area. For a density lower than $n_C$, a displacement current is required to conserve total current. This results in a net potential difference between opposite sides of the layer. By introducing non-zero electron and ion temperatures, the growth of the instability is no longer a certainty and depends critically upon the current density. Carlqvist (1972) has shown that the
current density necessary for the onset of this instability is given by,

$$\mathbf{J}_c = e \mathbf{n}_0 \left[ \frac{\gamma K (T_{e0} + T_{i0})}{m_e} \right]^{1/2}$$

where $n_0 = n_e = n_i$ is the steady state density, $\gamma$ is the adiabatic constant and $T_{e0}, T_{i0}$ are the initial electron and ion temperatures, respectively.

The second approach (microscopic) treats the problem through solution of the collisionless Boltzmann transport (or Vlasov) equation. This equation is,

$$\frac{\partial f_i}{\partial t} + \mathbf{V} \cdot \nabla f_i = 0$$

where,

$$f_i d\mathbf{r}_i d\mathbf{v}_i = \text{the number of particles with space coordinates between } \mathbf{r}_i \text{ and } \mathbf{r}_i + d\mathbf{r}_i \text{ and velocity between } \mathbf{v}_i \text{ and } \mathbf{v}_i + d\mathbf{v}_i \text{ at time } t.$$

By integrating this equation over all velocity space we obtain the equation of charge continuity (0th moment). Multiplying by $\rho$ and performing the same integration we arrive at the momentum transport equation (1st moment). These two integrations along with the thermodynamic relations,

$$p_i = n_i k T_i$$

constitute the basis for the macroscopic approach outlined above. For a cold plasma this approach is sufficient to predict 1st order instability. For finite electron and ion temperatures, particle dynamics become important and it is strictly no longer correct to neglect the form of the distribution function, $f_i$, on the growing disturbance field.
This is so because the particles now have a range of velocities which can interact differently with the electric field of a perturbation and are themselves responsible for its creation. In this case, one solves the Vlasov equation along with Maxwell's equations in a self-consistent manner (Hasegawa (1975) gives an excellent account of this procedure.). If one finally assumes that wave growth and distribution function changes occur on the same time scale, a linear approximation to the solution of these equations is not valid. The interaction between the two (referred to as wave-particle scattering) is intrinsically nonlinear. The complete solution of the nonlinear problem (i.e., a theory of "strongly turbulent" interactions) is not yet at hand. Some success in the theory of "weakly turbulent" interactions has been put forward in the past few years. The basis for a "weakly turbulent", or quasi-linear, approach is the assumption that time changes in the form of the original, unperturbed distribution function occur much more slowly than variations in the perturbed quantities. (In the linear treatment, no regard is given to changes in the unperturbed distribution function.) Complete introductory notes on these subjects are to be found in Krall and Trivelpiece (1973).

It should be pointed out here that the discussion of instabilities undertaken in Chapter V is not for the purpose of determining how an acceleration region would form. We assume everywhere in this paper that the electric field already exists and then go about finding whether it is rea-
sonable to attribute certain characteristics of our obser-
vations to velocity space instabilities generated by the
independent field. The term "non-interacting" arises from
these considerations.

From the brief outline above, one begins to have the
feeling that the extent to which auroral particle spectra
are related to the sheath formation is a sticky problem.
Put another way, are some of the particle spectral charac-
teristics related to a role they might have played in form-
ing the region or; do the spectra represent a non-interact-
ing acceleration of a particle population largely indepen-
dent of the distribution responsible for creation of the
potential? In either event considerable fluxes of field
aligned electrons are observed. As the first approach to
the problem, it seems reasonable to investigate whether the
observations are consistent, in the main, with electric
field acceleration. In doing this we ignore any effects
arising through self-consistent interaction with the accel-
eration region. This is justifiable in that if we are, in
some sense, to correlate a particular mechanism with spec-
tral events, we are not interested to a first approximation
in what particle population is energized, but only in the
fact that it is energized.
CHAPTER II
DATA PRESENTATION

2.1 Introduction of Spectral Plots and Contour Maps

The spectral data which we use in this investigation was obtained during three separate auroral sounding rocket flights (Arnoldy, et al (1974a)). A description of each flight along with the particulars of each rocket (detectors, etc.) is provided in the Appendix to this work.

Detector spectral measurements provide the differential energy flux \( j (\#/cm^2\cdot sec\cdot ster\cdot kev) \). If one is concerned with questions relating to the stability of the distribution, the distribution function \( f (\alpha j/E) \) is the relevant quantity. There are two ways in which we present the data. In the first, or original, form we plot the actual differential flux measurements, \( j \), versus energy (kev) sorted with respect to pitch angle. This is the customary form in which the data is presented. Another method of viewing the measurements of flux at a given energy and pitch angle is to plot these points in velocity space (\( V_{\parallel} \) vs. \( V_{\perp} \)). By preparing contours of constant flux \( j \) (or \( f \)) in this space, we can compare these curves with those predicted by our model. The velocity space contours are particularly well-suited for comparison to a model in that pitch angle dependence is more clearly discernible than in the usual spectral format. The contouring of the data is accomplished using the contour map.
generating scheme, PSUMAP, developed at Harvard University and modified at Pennsylvania State University (1969). The elements of this program are outlined in the Appendix.

Since the velocity space contours provide perhaps a better overall view of the data, we introduce each auroral sounding rocket flight through these plots while pointing out the salient characteristics of each. The $j$ versus $E$ spectra are also provided and may be compared when possible to the contour plots. Finally, we conclude this section with the results of the functional fit to the data. The fitting routine, GLSWS (General Least Squares With Statistics), was developed at the University of Maryland by Walter E. Daniels, Jr. (1965). This program is also covered in the Appendix.

The first of the present set of rocket flights being analysed here, Flight 18:91, suffered substantial time variations during the only period (135-145 sec) in which streaming was apparent. Nevertheless, it can be seen from Figure (1) that there is a clear field aligned peak at approximately 3 kev which was constant throughout the sample period. This peak appeared whenever small pitch angles were sampled. Figure (12) of Arnoldy, et al (1974b) shows sequential time segments of this interval. No other streaming was observed during this flight. Because of the time changes, particularly in the 2 second peak where both field aligned and isotropic distributions seem to merge in Figure (1), we could not prepare reliable contour diagrams of this flight.
(Note that Figure (1) is a time averaged, pitch angle sorted spectrum for the interval 135-145 seconds.)

The second flight considered here is Flight 18:109. For comparison, Figures (2) and (20) show one time segment during which field alignment was apparent. Figure (2) is the customary spectral representation and Figure (20) is the velocity space contour of the same time interval. We examine now the contours of Figure (20) in a bit more detail. Notice that the fluxes exceed $10^7.9$ e-/cm$^2$-sec-ster-kev at low velocities, drop to below $10^7.3$ at intermediate velocities (near $20.10^8$ cm/sec), and finally show a peak near $40.10^8$ cm/sec. (As pointed out in the List of Illustrations, the contours are plotted as the common logarithm of $j$ (or $f$).) We mention here that the entire distribution beyond $V_{\parallel} = 33.10^8$ cm/sec "is" the monoenergetic peak seen in Figure (2). The peak is most intense at small pitch angles ($j > 107.9$) but is in evidence up to the 70° pitch angle scan limit. The 800 data points in this scan are uniformly distributed throughout the region in which the contours of Figure (20) are shown.

Figures (3) and (21) were prepared from another time segment of the same flight. Comparing this time interval of the flight with the previous one above, one notices a difference in the pitch angle dependence of the monoenergetic peak. For the 121-125 sec scan, the peak flux is almost independent of pitch angle except for a small rise at low pitch angles where the peak becomes more intense and shifts to
slightly lower energy (Figure (20)). This is not the case for the 261-265 second scan of Figure (21). The peak flux in this case is smallest at an intermediate pitch angle (approximately 40°) and increases both near 0° and 70°.

The secondary peak at high pitch angles seen in Figure (21) is typical of most of the data segments analysed on Flight 18:109. This type of peak has also been observed by O'Brien and Reasoner (1971) and Venkatarangen, et al (1975). One notices that the peak energy of the higher pitch angle flux is invariably slightly higher than the field aligned peak. This leads some observers to interpret these two peaks as characteristic of two separate distributions, both of which are nearly monoenergetic. These considerations are covered in Chapter V.

The above examples are illustrative of essentially all spectra analysed; the field aligned flux is highly variable while the isotropic fluxes are regularly seen. For example, Figure (23) shows data from an interval during which there was no streaming. The monoenergetic peak observed is roughly isotropic extending to the lower pitch angle range. There appears to be no dependence of peak flux on pitch angle.

The pitch angle dependence of the two time segments 121-125 seconds and 261-265 seconds of Flight 18:109 is shown in Figures (4) and (5). In these plots, we see j versus pitch angle at three constant energies. The spectacular dependence of the field aligned distribution is evident in
the middle plots of both figures. The isotropic fluxes both show very slight angular dependence.

In the study of plasma instability, it was mentioned that one is more interested in the form of \( f \) rather than \( j \). In particular, it is possible that \( j \) might display "humps" (i.e., "gentle bumps") when plotted versus velocity whereas this behavior might be smoothed over if the plot is of \( f \). The "humps" in the parallel distribution function (See Chapter V) are critical in the determination of a possible velocity space instability. Figure (22) is a plot in velocity space for the same time interval as in Figure (20) which is a plot of \( j \). For comparison, \( f \) is given by,

\[
\frac{f(v)}{j} = \left( \frac{m^2}{2E} \right)
\]

or,

\[
\frac{f(v)}{j} = 1.616 \times 10^{-7} \left( \frac{j}{E} \right)
\]

where \( f \) has units of electrons-sec\(^3\)/km\(^6\) and \( j \) has units of electrons/cm\(^2\)-sec-ster-kev for \( E \) in kev. For this time segment, \( f \) has a peak at all pitch angles sampled albeit not as intense as those of \( j \) in Figure (20).

The final flight analysed is Flight 18:152. This flight data represents a significant improvement over that of the previous two in that three detectors covered the entire pitch angle range from 0° to 180°. Data for one other flight, 18:91, covered both up-going and down-going electrons. However, only two detectors were in use on Flight 18:91.

There were a number of considerable field aligned periods during this flight. Figure (24) shows contours of
j from an interval in which the down-going electron flux contours were similar to those observed on Flight 18:109, i.e., there is the basic structure of the anisotropic peak along with the isotropic monoenergetic peak. This period, 225-230 seconds, showed the simplest structure on this flight when streaming was present. Notice that there is no corresponding field aligned peak near the 180° pitch angle range. Apparently atmospheric scattering has served to eliminate any initial field alignment tendency. It is also worthwhile to mention that any particle which mirrored below the detector position would necessarily have a pitch angle in the range, 

\[ 90° < \alpha < 110° \]

at the detector location. Electrons with \( \alpha > 110° \) were therefore not mirrored but backscattered from the atmosphere. Elimination, or broadening, of the field aligned peak has been previously noted by Reasoner and Chappell (1973).

Figures (8a) through (8f) are contour maps of f for the time interval 220-247 seconds taken in segments of three detector duty cycles (One duty cycle is the time required to sample the same energy at the same pitch angle. Here this time is 1.5 seconds.). Figures (9) and (10) are the same for the interval 277-331 seconds. The continuous type display is informative in that we cover a period which begins with only the isotropic peak, shows the gradual emergence of the streaming peak and, finally, ends again with the isotropic peak alone. The dashed lines which enclose no contour lines represent gaps present in the data samples. We note that
Figure (24) is a plot of $j$ which corresponds to Figures (8b) and (8c) which are contours of $f$.

In Figure (8) the center of the streaming peak is approximately $40 \times 10^8$ cm/sec. In Figures (8d) through (8f) the peak gradually becomes more isotropic and less intense. Finally, by Figure (8f), the appearance of the contours is qualitatively similar to those of Figure (8a). In no case seen here or in any of the other segments is there evidence of a streaming behaviour near the $180^\circ$ pitch angle position.

The time segment 277-331 seconds is a period on Flight 18:152 when there appeared multiple peaking. Figures (6) and (7) show portions of this interval in pitch angle sorted spectral plots. An entire "calm" to "calm" period is covered in Figures (9) and (10). Approximately midway through the series, Figures (9d) through (10b) show a reasonably steady field aligned distribution at $V_\parallel \sim 30 \times 10^8$ cm/sec. We notice, beginning with Figure (9f), a gradual downward (in velocity space) expansion of the $10^{1.4}$-$10^{1.7}$ contours from the low energy contours near the origin of the system. There is again no noticeable expansion (or effect) of the same contours for pitch angles greater than $\frac{\pi}{2}$. The expansion ceases and the contours begin to approach isotropy again starting with Figure (10d). Figures (10a) through (10c) show the possibility of further field alignment tendencies in the expanding region. For example, in Figure (10b) we begin to see what might be interpreted as a tendency toward a separate anisotropic peak at $V_\parallel \sim 21 \times 10^8$ cm/sec for the $10^{1.4}$ contour.
Pitch angle sorted spectra of this time segment show multiple peaking at intermediate energies.
FLIGHT 18:109
120→125 sec.

Figure (2)
FLIGHT 18:109
260—265 sec.

- 0° - 10°
- 30° - 40°
- 50° - 60°

Figure (3)
Figure (4)
Figure (5)
Figure (6)
ELECTRONS (cm²·sec·ster·keV)⁻¹ × 10⁻⁸

Figure (7)

FLIGHT 18:152
325 - 335 sec
0 - 90°
Figure (8)
Figure (9)
Figure (10)
2.2 Functional Fit Forms

The fitting function routine is GLSWS. It was developed by Walter E. Daniels, Jr. (1965) at the University of Maryland. The fundamentals of this program are outlined in the Appendix. The functional fit in the final format includes the following fit forms to the observational spectra:

2.2a Low Energy Spectrum

The differential energy flux spectra of the low energy particles (E < 1 MeV) was fit quite well with the power law form,

\[ J \propto E^{-\gamma}, \quad \gamma \approx 1. \]

This differs from a number of theoretical calculations (See Nagy and Banks (1974) for the most recent) which are based on treatments of atmospheric backscatter and secondary production. These arguments lead to a power law whose form is more closely approximated by an \( E^{-2} \) dependence. Our observations are only valid to approximately 50 ev. Below this range, there is no data. There are very few investigations to date of the expected form of the distribution in the ultra low energy range (\(< 25 \) ev). The supposition is that this range is composed primarily of the Maxwellian ambient plasma background at the detector location. The ultra low energy electrons should display huge differential flux spectra near the origin based simply on a calculation of the electron density at the detector altitude (\( n_e \sim 10^6/cm^3 \)).
2.2b Field Aligned Peak

The anisotropic distribution was fit using two similar fitting functions. We describe both of these methods below and discuss briefly why they are interchangeable. Notice in all cases, the velocity contours generated from the data show the field aligned peak more elongated perpendicular to B than parallel to this direction. See Figure (21) as an example of this behaviour. We can incorporate this into the fit by: i) the mirror effect (See Chapter III) or, ii) the introduction of different parallel and perpendicular temperatures.

The first fit form used for the field aligned particles was the streaming Maxwell-Boltzmann distribution modified by the magnetic mirror effect. This form consists of a Maxwellian distribution, each particle of which has a non-relativistic streaming velocity, \( V_0 \), in the streaming direction. The differential flux spectrum is provided by,

\[
j = C_s E \exp \left( -\frac{1}{E_o} \left( E + E_{sm} - 2 \sqrt{E E_{sm} (1 - \beta \sin^2 \alpha)} \right) \right)
\]

where,

\[
C_s = \text{the normalized streaming constant}
\]

\[
E_o = k T_0 = \text{the characteristic temperature}
\]

\[
E_{sm} = m \nu_0^2 / 2 = \text{the streaming energy}
\]

\[
\alpha = \text{the particle pitch angle}
\]

\[
\beta = \frac{\beta_0}{\beta_i} = \text{the mirror ratio}
\]

The second method by which the field aligned particle peak was fit was by using different parallel and perpendicular temperatures instead of the ratio \( \frac{\beta_1}{\beta_i} \) to provide the
divergence from circular form for the contours in velocity space. The use of these two different temperatures is widespread today and although at first puzzling is easily explained when it is recalled that in the presence of a magnetic field, the velocities perpendicular to the direction of the field depend upon the field while those parallel to it do not (i.e., one expects an anisotropy in velocity).

The contours of a streaming Maxwell-Boltzmann distribution are those shown in Figure (27b). They are ideally perfect circles centered at the streaming velocity. The effect of a mirror geometry on a distribution of this sort as it enters a region of increasing magnetic field is to degenerate the circle into an ellipse-like figure, the "major-axis" being perpendicular to \( B \). This is shown in Figure (27c). The deviation from a perfect circle is a function of the mirror ratio. A distribution of this sort has different parallel and perpendicular temperatures from another viewpoint. We note, in passing, that when we consider different temperatures, the elongated figures are ellipses.

In light of the above illustration, we replace the mirror ratio of the preceding equation with the parameters \( \varepsilon_0 \|_0 \varepsilon_0 \|_1 \). The form of our fit now appears as,

\[
J = C \varepsilon e \left( - \varepsilon_1 \| \varepsilon_0 \| - \varepsilon_2 \| \varepsilon_0 \| \right)
\]

where, except for obvious notation, we have,

\[
\begin{align*}
C &= C \left( N, V_0 \|_1, V_0 \|_2 \right) \\
\varepsilon_1 &= \frac{1}{2} m \left( V_1 \| - V_0 \|_0 \right) \\
\varepsilon_2 &= \frac{1}{2} m \left( V_2 \| - V_0 \|_0 \right)
\end{align*}
\]
2.2c Isotropic Distribution

The monoenergetic isotropic peak was fit using a Gaussian distribution. This particular form was determined also by Nagy and Banks (1974) in a theoretical calculation of the expected form of this higher energy peak. The differential flux is given by,

\[ j = (C_1 + C_2 \alpha) E e^{-\frac{(E - E_0)^2}{E_0^2}} \]

where,

- \( C_1 \) = \( j \) intercept at \( \alpha = 0 \)
- \( C_2 = \frac{d\alpha}{d\alpha} \), the slope
- \( \alpha \) = the pitch angle
- \( E_p \) = the peak energy
- \( E_0 \) = the thermal spread of the Gaussian

2.2d Discussion and Figures

Figures (11) through (14) show a portion of the fit to two separate time segments of Flight 18:109 during which streaming occurred. The plots shown give log \( j \) versus log \( E \) in pitch angle sorted spectral format. In Figure (11) we concentrate on the field alignment. Here we show the pitch angle range 0-10 degrees for the fitting function and the actual data. Figure (12) is a plot of the same time segment, 121-125 seconds, but now it covers the pitch angle range 30-40 degrees in order to show the isotropic part of the spectrum more clearly. Figures (13) and (14) are plots of the same pitch angle ranges as above. The time segment covered in this case is 261-265 seconds.
18:109
T = 121-125
pa range: 0°-10°

Figure (11)
$\log_{10} j(e^-/cm^2\cdot sec\cdot ster\cdot keV)$

$\log_{10} E (keV)$

Figure (12)
18:109
T=261-265
Para range: 0°-10°

Figure *(13)
Log\textsubscript{10} j (e\textsuperscript{-} / cm\textsuperscript{2} \cdot sec \cdot ster \cdot keV)

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure14}
\caption{18:109}
\begin{align*}
T &= 261-265 \\
\text{pa range:} &= 30^\circ - 40^\circ
\end{align*}
\end{figure}
CHAPTER III

SIMPLE ELECTRIC FIELD MODEL

There are numerous approaches in the literature to the problem of a homogeneous plasma immersed in a uniform external electric field (e.g., Field and Fried (1964)). These models differ primarily in their treatment of the collisional (particle-particle, wave-particle, etc.) aspects of the Boltzmann equation. Following our general outline of viewing the problem through this equation, we present here the zeroth order solution for the time-independent acceleration of a one component ($e^-$) plasma by a localized electric field. As we assume the absence of collisions, Liouville's theorem applies and is equivalent to the Vlasov equation,

\[
\frac{df}{dt} = \frac{\partial f}{\partial t} + \nabla_x \cdot \mathbf{v}_f + \mathbf{a}_i \cdot \nabla_v f_i = 0
\]

The electric field is given by (See the drawing below),

\[
\bar{E} = -E_o \hat{\alpha} \delta (y - z_o)
\]

Since the "sheath" region is thought to be on the order of several Debye lengths, any variation of the external magnetic field in this region is negligible. In this sense, the
assumption of a delta function-like potential drop region is reasonable. If we consider extended field regions, this assumption is no longer valid. For steady state conditions \( \frac{\partial f}{\partial t} = 0 \) and one dimensional considerations, equation (3.1) becomes upon substitution of (3.2),

\[
35 \frac{\partial f}{\partial z} - \frac{1}{m} \epsilon \frac{\partial f}{\partial \nu} = 0
\]

or,

\[
\nu \frac{\partial f}{\partial z} + \frac{1}{m} \epsilon \nu (z - z_o) \frac{\partial f}{\partial \nu} = 0
\]

where,

\[
E = - \nabla \phi, \quad \phi = E_o H(z - z_o)
\]

This differential equation may be solved by separation of variables and the boundary condition that for \( z < z_o \) the distribution function is Maxwellian. The result is,

\[
f_z = \mathcal{Z}(z) \nu_v(z) \nu, \quad f_z(E) = \left( C_4 \frac{\epsilon e^{\nu_1 \phi}}{\nu_0} \right) e^{-\frac{E_z}{\nu_0}}
\]

where, \( \nu_0 = \frac{kT}{\epsilon} \) = the thermal energy of a particle with one degree of freedom.

In addition, the differential energy flux spectrum would appear as,

\[
J_z = f_z(E)E_z = E_z \left( C_4 \frac{\epsilon e^{\nu_1 \phi}}{\nu_0} \right) e^{-\frac{E_z}{\nu_0}} \equiv C_2 E_z e^{-\frac{E_z}{\nu_0}}
\]

where, \( J_z = e^{-/cm^2-sec-ster-kev} \)

Comparing \( J_z \) with \( J_1 \) where,

\[
J_1 = E z C_1 e^{-\frac{E_z}{\nu_0}}
\]
we notice the absence of any particles in the final distribution \( (j_2) \) with energy less than \(| e \phi | \), the energy which would be gained by an electron which entered the acceleration region with negligible parallel velocity.

The assumption of electric field localization provides a further view of this discontinuity. Since perpendicular energy, or the magnetic moment, is conserved,

\[(3.9) \quad E_1 \sin^2 \alpha_1 = E_2 \sin^2 \alpha_2 \]

then,

\[(3.10) \quad \sin^2 \alpha_2 = \frac{E_1}{E_2} \sin^2 \alpha_1 = \frac{E_2 - |e\phi|}{E_2} \sin^2 \alpha_1 \]

so that for a single particle of energy \( E_2 \), there is a maximum allowable pitch angle given by,

\[(3.11) \quad \alpha_{2 \text{ max}} = \sin^{-1} \left( \frac{E_2 - |e\phi|}{E_2} \right)^{\frac{1}{2}} \]

In summary, the model is seen to require of particles which are observed directly after acceleration:

1. Energy \( \geq |e\phi| \)
2. \( \alpha_2 \leq \sin^{-1} \left( \frac{E_2 - |e\phi|}{E_2} \right)^{\frac{1}{2}} \)

A particularly transparent way in which to view these results and eventually to compare them with observation was introduced in the preceding chapters: the use of velocity space contours of constant \( f \) (distribution function) or \( j \) (=\( fE \)). Figure (27) is an example of such a plot of a streaming \( (\vec{v}_0, \vec{z}_0) \) Maxwell-Boltzmann distribution. These contours, as seen in Figure (27b), are circular and centered
at the streaming velocity. Figure (15b), however, shows velocity space contours as seen directly below the modeled acceleration region; they show a sharp discontinuity or "cut-off". The observations are made at point $A'$ and the plasma source supplies a Maxwellian plasma of temperature 450 ev. (These figures are characteristic of the streaming peak observed between 261-265 seconds of Flight 18:109, Figure (3)) to point A.

As an addition to the description of the contours provided in the List of Illustrations, we provide the contour drawing below as an example of the model. The contour lines are individually labelled by the common logarithm of the flux $j$. In the example here the peak flux occurs at the minimum allowable velocity, $\sqrt{\frac{2e}{m}}$, in accordance with the example potential drop of 1 volt. $V_u$ refers to the velocity along the magnetic field direction. Finally one notices that the contours are circular and that there is a maximum allowable pitch angle, as illustrated, for any given energy.

This completes the fundamentals of the simple scatter free mode. What follows are modifications to this model due to magnetic field effects and different configurations of plasma source and potential regions in space.
3.1 The Effect of a Convergent Magnetic Field

A well-known effect of a non-uniform (convergent) static magnetic field on a charged particle in cyclotron resonance is the ability of the field to reverse the sense of the particle's translational velocity with respect to the primary magnetic field direction, i.e., to "mirror" it. To see this it is sufficient to consider a static magnetic field $B_{0z}$ and superimpose independent (for ease in handling) radial and longitudinal perturbations $B_r$, $B_z$. If the force on the particle is calculated, it is found to be always in a direction opposite to the gradient of $B$ along $z$. For one particle it is given by,

$$F_{\parallel} = -\mu \frac{dB}{dz}$$

(3.12)

where $\mu$ = the magnetic moment. As a consequence of this force, an electron traveling in the direction of $\nabla B$ will find its pitch angle (the angle between its velocity vector and the magnetic field direction) gradually increasing until it "mirrors", at which time this angle is $\frac{\pi}{2}$.

An alternate, and more exact, way of expressing the pitch angle effect is by noticing that in the absence of external energy sources, the magnetic moment of the particle is conserved. As the total energy cannot change in a static $B$ field, the incremental work done on the particle in traversing a distance $\delta z$ must be equal in magnitude and of opposite sign to the work done on the particle in the radial
direction,

\begin{equation}
\delta E_u = F_3 \delta z_3 = -\mu \frac{\partial B}{\partial z_3} \delta z_3 = -\delta E_\perp
\end{equation}

But,

\begin{equation}
E_\perp = \mu B_3
\end{equation}

or,

\begin{equation}
\delta E_\perp = \mu \frac{\partial B}{\partial z_3} \delta z_3 + B_3 \frac{\partial \mu}{\partial z_3} \delta z_3
\end{equation}

or,

\begin{equation}
\frac{\partial \mu}{\partial z_3} = 0
\end{equation}

so that,

\begin{equation}
\mu = \frac{E_\perp}{B} = \frac{1}{2}mv^2 \frac{\sin^2 \alpha_1}{B_1} = \frac{1}{2}mv^2 \sin^2 \alpha_2
\end{equation}

or,

\begin{equation}
\sin^2 \alpha_2 = \frac{B_2}{B_1} \sin^2 \alpha_1 \quad B_2 > B_1
\end{equation}

Therefore, the maximum allowable pitch angle at point C, Figure (15c) is given by,

\begin{equation}
\sin^2 \alpha_{2\text{max}}(E_2) = \frac{B_2}{B_1} \sin^2 \alpha_{1\text{max}} = \left(\frac{B_2}{B_1}\right) \left(\frac{E_2 - \frac{1}{2}eV_A}{E_2}\right)
\end{equation}

The cut-off contour is curved allowing for the fact that although the electrons observed must have the minimum energy, eV, the mirror effect allows particles to be observed with parallel velocities less than that which would be imparted in the absence of this effect.
3.2 Multiple Acceleration Regions

The suggestion that observed field aligned fluxes are due to multiple sheath-like regions has been put forward by a number of authors (Albert and Lindstrom (1970), Block (1972)). Retaining the assumption of localized acceleration regions, we derive here spectral characteristics to be expected from this suggestion. As above, we require a source which provides a Maxwellian plasma to the region above the potentials (point A, Figure (16a)).

If the distance between the two potential regions, (A'-B) of Figure (16a), is sufficiently small so as to allow neglect of magnetic field variations, it is clear that it would be impossible to distinguish (at B') this case from that of a single potential, $V_A+V_B$. In the event that this separation is significant, the contours at B' would appear as in Figure (16b). The curvature is again due to mirror effects, i.e.,

$$\sin^2 \alpha_{B'}^c (E_{B'})_{\text{max}} = \left( \frac{B_B}{B_A} \right) \left( \frac{E_{B'} - E(V_A+V_B)}{E_{B'}} \right)$$

and there is no particle flux for $V_{||} < \left( \frac{2eV_B}{m} \right)^{\frac{1}{2}}$. Observations at point C add a further mirror curvature to the cutoff contour provided by the potential $V_B$. Notice in addition that the "kink" in the contour has moved toward larger pitch angles. The angle at which the "kink" occurs is a function of where the observations are made (i.e., the distance between D and B'). As seen, if they are made dir-
ectly below the potential drop, it is not possible to observe any particle whose parallel velocity is less than \( eV_B \). This is not true between B' and D. In fact, if we require that,

\[
\frac{(B_B - B_A)}{V_A} = \frac{(B_D - B_B)}{V_B}
\]

so that \( \sin^2 \alpha' (E_0)_{\text{max}} \approx \frac{1}{2} \)

any parallel velocity can be observed and the situation below point D is indistinguishable from Figure (15).

The case where we allow potentials of opposite sign fails to contribute any significant change to the model. By an appropriate arrangement of positive potentials and mirror effect geometries, we can produce essentially the same results as would occur with negative potentials.

Further, extended regions of electric field can be approximated by a series of electrostatic potential drops. In this case observations made directly below the acceleration region would be expected to show various "kinks" and curvatures in the velocity space contours. Well below this region, as discussed above, this behaviour would no longer be observed and there would be no way to differentiate between a single potential region and an extended one.

In summary, extended or multiple potential regions, predict a sharp flux cutoff as in the case of the single region. The mirror effect can cause the cutoff contour to be a function of pitch angle and add curvature and "kinks" to the contours, but the basic discontinuity is still intact.
3.3 Particle Injection into the Acceleration Region

In all cases above, we have assumed the source of Maxwellian electrons to be far removed from the potential region(s). We now take an alternative viewpoint and require electron injection directly into the area. Such a view, for example, is held necessary by Whalen and McDiarmid (1972) in order to explain certain auroral spectral characteristics. Since the determination of particle sources is at least as important as the electric field formation itself, it seems reasonable to investigate this alternative. For example, the internal sources might arise from particle distributions that were present during formation of the region, or they might represent fluxes of secondaries due to atmospheric scattering.

Perhaps the most important contribution of the change in particle source is the predicted existence of fluxes below the cutoffs which are provided by the models with only a far removed plasma source. The differential flux observed at $A'$ (Figure (17a)) due to injection of $dn$ electrons at the potential $V'$ is given by,

\[ dJ(E', V') = C dn C E \exp \left( -\frac{(E - \frac{1}{2} kT)}{E_0} \right) \]

where again $E_0$ denotes the characteristic temperature. If we assume that a constant number of particles $dn$ is injected per unit change in $V$,,
(3.23) \[ \frac{d\eta}{d\nu} = \left( \frac{\eta}{\nu} \right) d\nu \]

where,

\( \eta \) = the total number of electrons injected

\( \nu \) = the total potential drop

then the total flux of particles with \( E_{\|} < \nu \) is,

(3.24) \[ j(E_{\|}, E_{\perp}) = C \frac{n \nu}{\nu} \int e^{-\frac{(E - (E_{\|}d\nu))}{E_{0}}} d\nu \]

\[ = C n \nu e^{-\frac{E}{E_{0}}} \left( e^{\frac{E_{\|}}{E_{0}}} - 1 \right) E_{\|} < \nu \]

Similarly, the total flux of particles for \( E_{\|} > \nu \) is,

(3.25) \[ j(E_{\|}, E_{\perp}) = C n \nu e^{-\frac{E}{E_{0}}} \left( e^{\frac{E_{\|}}{E_{0}}} - 1 \right) \]

where the upper integration limit of Equation (3.24) has been extended to \( \nu \), the full potential range. These contours are shown in Figure (17b).

The contours of Figure (17b) show no horizontal flux cutoff since all plasma was injected inside the potential region. If we allow, in addition, a source above the drop (point A of Figure (18a)) and assume that 90% of the observed flux at A' is injected from a source above this point, the contours of Figure (18b) result. Fluxes drop suddenly at the old horizontal cutoff, but not to zero as before.

Figures (17c) and (18c) show the effect of magnetic field convergence (mirror ratio=2.) for the two cases above when observed at point B. We note that the "kinks" in the contour of Figure (17c) follow the old cutoff line of Fig-
ure (15c). Similar behaviour is noticed in Figure (18c); however, in this instance, as with no mirror effect, the injection of electrons above the electric field retains the abrupt falloff in flux. Once more the flux drop is not to zero.
3.4 A Plasma Source Between Two Isolated Potentials

As noted in the Introduction and data sections of Chapter II, auroral spectra are often seen with two peaks in the higher energy range: one isotropic, the other anisotropic. In previous models we have produced at most a single peak in the predicted results (See Figures (15) through (18)). We are, in this regard, prompted to search for a mechanism which in some manner is capable of producing the double peaked structure so apparent in the data. One such model consists of providing a source between two isolated, delta function like potentials, as opposed to the continuous structure of the last section. We then observe either in the area between the potentials (point B of Figure (19a)) or below both potential regions (point B' of Figure (19a)).

With the above assumption, the flux at point B is the sum of contributions from a source above A and a source between A' and B. The fluxes are then given by the sum of,

\[
\begin{align*}
(3.26) \quad j(\varepsilon) &= \Delta n \cdot \mathcal{E} \cdot \mathcal{E} \cdot \frac{\varepsilon}{E_0} , \quad \varepsilon > \varepsilon_z \left(1 - \frac{\beta_B}{\beta_A} \right) +\left(\frac{\beta_B}{\beta_A}\right) |e| V_A \\
(3.27) \quad j(\varepsilon) &= (\Delta n - \Delta n) \cdot \mathcal{E} \cdot \mathcal{E} \cdot \frac{\varepsilon}{E_0} \quad \forall \varepsilon
\end{align*}
\]

where we assume that both sources are characterized by the same temperature.

The resulting flux contours are shown in Figure (19b) where we have allowed 90% of all electrons to be injected above point A and 10% to be provided by the source which
supplies electrons to the intermediate region. This figure includes no mirror effects for particles injected between A' and B because we have assumed that they are injected isotropically. In this instance, the mirror effect does not modify j. We have also assumed that the temperature of each distribution is 453 ev. Once more, it is observed that there is no flux cutoff entirely as in the case of the single source above point A.

Figure (19c) shows velocity space contour behaviour at B', below the second potential. There are two peaks to be observed in this case; there is a field aligned peak near the horizontal cutoff and there is a second peak, more closely isotropic, beyond the cutoff. Finally, Figure (19d) shows the contours at C for a mirror ratio of 1.2:1 (relative to B').
3.5 A Plasma Source Below the Potential Region

Evans (1974) has discussed the fact that up-going \( \langle \alpha \rangle \frac{\pi}{2} \) electrons with parallel energy less than that of the potential drop would simply be reflected from this site. Hence, to an observer below the potential, the contribution to the velocity space plots would be the introduction of electron fluxes at energies less than cutoff. Figure (19b) is an example of such a contribution, where we have provided the source below the electric field as described in the last section.

It should be pointed out that any of the models above could be altered to include sources below the acceleration site and hence to provide "fill-in" for the contours below the minimum velocity. The specific form of the contours would depend crucially on the plasma source, i.e., backscattered primaries, secondaries etc. A study of these low energy electrons (Nagy and Banks (1974)) is quite complicated and will not be considered further.
3.6 Summary

In conclusion, the simple time-independent, non-interacting electric field model predicts discontinuities in the flux spectra of accelerated Maxwellian plasmas. Specifically, these discontinuities assume the form of a flux cut-off or "kink" in the velocity space contours. Depending upon the form of acceleration model, one finds fluxes dropping abruptly to zero or to lower values. In the case of injection into the potential region, the contours of constant flux are seen to display a sharp discontinuity of slope in the space. The mirror effect can further bend and twist the basic contours, but the cutoff behaviour remains intact.

It is possible to envision an electrostatic acceleration region extending from the top of the ionosphere to the equator. In this case all electrons are injected directly into the potential region. A number of experimental studies (Albert and Lindstrom (1970), O'Brien and Reasoner (1971), Gurnett and Frank (1972), Berko and Hoffmann (1974)) and theoretical (Kindel and Kennel (1971), Block (1972)) studies suggest however, that the acceleration must take place at an altitude of not more than a few earth radii.

The model of Evans (1974) perhaps deserves the most attention in that the proposal in that paper was parallel electric field acceleration associated with auroral particle observations. In particular, it was pointed out that if the
acceleration were in some sense stationary (no mention is made in this paper of time or space variations) the low energy spectrum would be composed primarily of secondaries and backscattered primaries with insufficient energy to mount the potential barrier in the opposite direction (i.e., the up-going electrons). The fundamental model assumed in the paper is that of a Maxwellian distribution of temperature 800 ev and density 1.5 particles/cm$^3$ accelerated by the parallel electric field. A numerical model of the backscattered primaries and secondaries, based on a similar model of Nagy and Banks (1974), was employed to calculate this contribution to the final flux spectrum. The resultant flux is then the sum of the unscattered accelerated Maxwellian and the scattering contributions. Among other things the model predicts an increasing peak energy with increasing pitch angle as observed here on Flight 18:109 for example. There is provided a model fit of the 261-265 second data segment of this flight both at 0° and 45° pitch angle. Viewed in contour space this fit is quite good. Further comparisons of the model to data is limited to one other 0° pitch angle spectrum of Frank and Ackerson (1971).
Figure (15)

Figure (16)

Figure (17)
Figure (20)

Figure (21)
Figure (22)

Figure (23)
CHAPTER IV

COMPARISON OF OBSERVATION TO MODEL

The previous two chapters have presented observations and a model purported to represent the physical basis of those observations. The intent of this brief section is to underscore the differences between the two.

The most readily apparent difference between the data contours and the theoretical contours is the lack of any discontinuous cutoff in the data contours as a function of pitch angle. Contour plots of both flights, described in previous chapters fail to show any predilection toward maintaining "forbidden" regions as predicted by the model of Chapter II. For example, if this were the case, we should expect as an indication that contours on the low velocity side would show a tendency to be more closely spaced than those on the high velocity side. Figures (20) through (23) (Flight 18:109) and Figures (8) through (10) (Flight 18:152) fail to show this behaviour to any degree. The peaks in these cases are all relatively sharp but are not limited by the detector energy resolution (Arnoldy, et al. (1973)). In Figure (21) the most closely spaced contours represent the approximate limit of the detector resolution. Most all of the contours in this figure and nearly all of the contours of Figure (20) are spaced at least twice as widely as they would be if the actual electron flux dropped drastically and contour spacing was limited solely by finite detector resolution.
None of the contours of the model shows the isotropic peak at large pitch angles as seen in Figure (21). However it is possible to modify the theoretical pitch angle distribution by injecting a non-isotropic plasma source. It is also possible to produce some peaking near by trapping particles between a magnetic mirror point below the rocket and an assumed higher altitude region of potential drop (Evans (1974)). Only electrons with pitch angles between 70 and 110 degrees could be trapped by such a mechanism at the rocket altitudes studied here.

Notice that the only theoretical curves that contain clearly distinct field aligned and nearly monoenergetic isotropic peaks are those of Figure (19) that require a second source of low energy plasma between two potential drop regions. The model shown in Figure (19) predicts two peaks which, although not clearly isotropic and anisotropic as seen in the data, show increasing pitch angle with increasing peak energy.

In conclusion, if we are to retain the model of localized acceleration, further conditions must be placed on the simple configurations of Chapter III. Perhaps the most urgent of these conditions to consider are: (1) the existence of non-zero fluxes in the "forbidden regions" and, (2) the problem of two separate peaks at slightly different energy. The latter relates to a consideration of two separate sources of different temperature or, of appropriate scattering mechanisms acting on one distribution. These ideas and their implications are the subject of the next chapter.
CHAPTER V

SCATTERING AND FLUCTUATIONS

An explanation of the observed differences between the model and the observations requires a number of considerations. The first and perhaps most important topic is the question of whether the observed quantities represent a single distribution that has been accelerated and subsequently scattered or, whether what is seen is actually two distinct distributions: one isotropic, the other anisotropic. As mentioned in Chapter I, there are some convincing reasons for interpreting the data as being representative of two distributions. Figures (28a) through (28f) are contour plots of the time interval 113-137 sec on Flight 18:109. We choose to look at this interval because it is a time interval during which there was initially no streaming present, the streaming began to appear in Figure (28c), and by the end of the time segment (Figure (28f)) has receded once again. The darkened contours show most clearly the effect of the field aligned peak. The only noticeable change occurs at the low pitch angles as the contour lines become closely spaced in the presence of the field aligned peak. If this peak arises from the same distribution, it would be expected that there would be some effect on the contour levels of the steady isotropic peak. None is apparent from these results.

We will consider now the effect of scattering and electric field fluctuations. We expect that wave particle...
scattering is a possibility due to the form of the parallel
distribution function. The reasons for this are outlined in
the next section. Intense wave particle scattering is not
new to this area and is required in some models which produce
magnetic field aligned electric potentials by the nonlinear
process of the formation of anomalous resistivity (Kindel and
Kennel (1971)). Below we discuss the requirements which must
be placed on a scattering mechanism, or a fluctuating elec­
tric field, if it is to explain the discrepancies between
model and data. In the next section, we give some insight
into the theoretical reasons responsible for our considera­
tion of wave particle scattering. (This section is not meant
to be a complete investigation of the stability properties
of our distributions. Instead, it is included as an introduc­
tion to a highly complex and interesting subject. Indeed,
future investigations into these areas should include a more
detailed treatment of stability properties as the full under­
standing of these could weigh heavily in a final answer to
auroral particle acceleration.)
5.1 One Source Consideration

5.1a One Scattering Process or a Fluctuating Potential

In an attempt to isolate effects, we will begin by considering the differences between previous theoretical predictions and actual fluxes when there is no field aligned peak present. In the absence of this peak, our contours show: (1) No flux cutoff behaviour as predicted and, (2) A large flux of low energy electrons ($v < 15 \times 10^8$ cm/sec). The low energy electrons must be produced by a source below the detector position. This is discussed by Evans (1974). We do not consider these particles any further in this section. The sharp cutoffs in the flux contours could be eliminated either by providing a source of pitch angle scattering or by providing a fluctuating (in time or space) electric field region. (The rocket covers approximately 1 km in two seconds.)

Figure (25b) shows flux contours that result if we assume that some mechanism randomises or destroys the dependence of the contours of Figure (15b) on pitch angle. This scattering process will multiply the fluxes observed in Figure (15b) by some factor which is proportional to the solid angle within which all flux is contained (i.e., $\mathcal{N} = \frac{1 - \cos \alpha}{\frac{A}{A'}} = 1 - \left( \frac{eV}{E_A} \right)^\frac{1}{2}$).

Figure (25c) shows the effect of adding a fluctuating electric field. We required that the characteristic period
of the variations be shorter than the 4 second data accumulation interval in order to produce the observed smooth contours. Kintner and Hallinan (1975) have detected oscillations in the local perpendicular electric field on Flight 18:152 with a characteristic period of one second. In order to produce Figure (25c) we have assumed a linear variation in the potential from \( V_0 \) to \( V_0 + V_l \) where \( V_0 = 1137 \) volts and \( V_l = 420 \) volts. The contours in Figure (25c) correspond to a point just below the acceleration region (point A' in Figure (25a)). Figure (25d) is the result of taking observations at point B if the mirror ratio is assumed to be 2:1.

We conclude that it is possible to produce distributions which are qualitatively similar to the fluxes observed when only the monoenergetic isotropic peak is present (Figure (23)). Either of the two methods described above will serve this purpose. The study here has not included any fits using this sort of model. It seems likely, however, that almost any observed isotropic distribution could be fit by application of a suitable fluctuating electric field. We emphasize that there is no distinction here between time and space variations.

The processes considered in this section cannot simultaneously produce the field aligned and isotropic peak that are observed together. It appears necessary to introduce a second source, or a different scattering mechanism, in order to describe these contours. O'Brien and Reasoner (1971), Whalen and McDiarmid (1972) and Venkatarangen, et al (1975)
have also noted that the two distributions appear to have been produced by different processes. If all electrons observed were accelerated by a parallel electric field, they must arrive at the detectors by different mechanisms. This is the topic of the next subsection.

5.1b A Selective Scattering Process

We now discuss the possibility that all electrons were field aligned immediately after acceleration and some of these were then scattered to produce the isotropic distribution. In the next section two separate sources will be considered.

The models presented previously produced a field aligned peak. However, in no case did they produce distinct isotropic and anisotropic electrons from a single source. Also, there remains the problem of the discontinuous cutoff. If all electrons were field aligned after acceleration, then we must introduce a selective pitch angle scattering mechanism to redistribute the pitch angles of only a portion of the group. In the last section, the pitch angles of all particles were affected. The idea here is that the scattered electrons produce the isotropic peak while those not participating in the scattering produce the field aligned peak. One type of scattering process which would be selective would be one that scatters only electrons with velocities in a range appropriate to Doppler shift their cyclotron frequency to a scattering wave frequency. Another possibility is that the
scattering process is intermittent, or flickering, with a period much shorter than the 4 second interval required for one data sample. The field alignment then appears during those intervals when scattering is not taking place. We also must probably add either fluctuations, or a second scattering process, in order to eliminate the predicted sharp flux cutoff in the field aligned group. Finally, our observation that up-going electrons exhibit either a very broad energy peak, or no peak at all (See Reasoner and Chappell (1973)), strongly suggests that all scattering to produce the monoenergetic peaks occurs above the detector position.
5.2 Two Source Consideration

In presenting the basic model we found that it was possible to produce the anisotropic group along with the isotropic electrons by introducing a second source (Figure (19)) between two isolated potentials. This is also possible in the extended potential region case. Here we must add either an energy scattering mechanism or a fluctuating electric field in order to eliminate the sharp flux cutoff and to merge the two predicted distributions into a single one at low pitch angles. Therefore in this instance we need not require selective pitch angle scattering in order to explain the existence of both peaks. One difficulty that arises in using this mechanism is that both distributions peak at approximately the same energy but have widely differing pitch angle dependences. This feature is best seen in the data presentation of Arnoldy, et al (1974a) (Figures (5) through (8)). In our report, Figure (19) disagrees with actual data contour maps (Figures (20), (21) and (24)) in a number of respects. It is possible that with enough parameter variation this model could be made to fit the data more closely. However, an extensive investigation of these possibilities does not seem warranted at this time.
5.3 Projection of Measured Contours up the Field Line

Assuming that our detectors are below the acceleration region, as indicated by the least squares fitting routine, we should be able to project our measurements up the magnetic field line to determine the position of the region. This amounts to subtracting the mirror effect from our observations.

Figure (26a) is a magnification of the streaming region shown in Figure (22). Figure (26b) shows these same contours projected up the magnetic field line by conserving the magnetic moment of the electrons: \( \sin^2 \alpha / B = \text{constant} \). The projection here is based on the least squares fitting routine, GLSWS, described in the Appendix. Basically, the mirror ratio, \( B_2/B_1 \), is a parameter in this routine; determination of this ratio allows us to find the altitude at which the contours most closely approximate those of Figure (27b). In Figure (26b) the magnetic field strength is 0.098 times the field strength at the rocket. The contours in this figure are those that would be seen by an observer at an altitude of 7600 km on the \( L = 8.4 \) field line (if no scattering is allowed between the actual detector position and the 7600 km point). Figures (26c) and (26d) are similar comparisons for the time segment of Figure (21). In this instance the ratio of \( B_2 \) to \( B_1 \) is 0.18 which corresponds to an altitude of 5400 km. The location we deduce from these considerations is within the range predicted by Gurnett and
Frank (1972) for the generation of VLF hiss, a phenomenon possibly associated with the formation of electric field regions. Siren (1975) also concluded that hisslers are produced in this region.

Figure (27b), as mentioned earlier, is a plot of a streaming Maxellian distribution and (27c) incorporates the mirror effect. We recall these contours separately to show that although we can fit the data quite well with the streaming form, the form itself is significantly different from the form of the accelerated simple Maxwellian. The fitting routine uses either this form for the fitting function of the field aligned peak, or the form which uses different parallel and perpendicular temperatures. It is interesting to note that the characteristic thermal energy of the streaming peaks is on the order of tens of electron volts. From this observation we would conclude that the electrons do not originate within the Plasma Sheet where temperatures are more on the order of hundreds of electron volts. The temperatures seen are more typical of the Magnetosheath. The ionospheric plasma has lower characteristic energy than the field aligned particles show, but it could be the source if there is some heating associated with the acceleration process. The temperatures of the isotropic monoenergetic distribution are more on the scale of several hundred electron volts to the kev range. These particles have therefore either been selectively heated more than the field aligned ones, or they simply represent a separate source of different temperature.
5.4 Conservation Laws

The field aligned distribution with which we are concerned may be characterized by a number flux ($\Phi$), a momentum flux ($\lambda$), and an energy flux ($\Theta$) along the magnetic field direction. These quantities are defined through,

$$\Phi = \int_{\mathbb{R}_+} v_n f(\vec{v}) \, dv$$

$$\lambda = \int \left( m v_n \right) v_n f(\vec{v}) \, dv$$

$$\Theta = \int \left( \frac{1}{2} m v_n^2 \right) f(\vec{v}) \, dv$$

In general, any scattering process that converts a simple accelerated distribution (Figure (15a)) to a streaming distribution (Figure (27)) must redistribute the energy and momentum of the electrons but conserve total momentum or energy flux (we assume no particle collisions). We provide in Table 1 the results of the integrations indicated in Equations (5.1) through (5.3) for three forms of $f(v)$ of interest to us:

1. the Maxwellian distribution,
2. the unscattered accelerated distribution and
3. a streaming distribution. The streaming distribution is considered to have arisen from a scattering of the accelerated distribution as covered in the earlier parts of this Chapter. For reference these three distributions are collected here as,

$$f(\vec{v}) = \pi \left( \frac{m}{2 \pi E_0} \right)^{3/2} e^{-\frac{m v^2}{2 E_0}} \quad (\text{Maxwellian})$$
These results are listed in Table I for the purpose of determining if they place any restrictions on the acceleration and scattering processes that we consider. For example, we have normalized the entries of Table I so that the down-going Maxwellian electrons at point A of Figure (15a) carry the same number flux as the accelerated electrons at point A'. As seen in the derivations of Chapter III, the characteristic energy does not change in the acceleration process. However, it may also be seen from the Table that the accelerated electrons carry higher momentum and energy flux than before acceleration as expected. As the acceleration process must supply these differences, the energy is supplied by the agent that sets up the electric field region. Further, it is clear, that in order to produce continuous acceleration there must be a constant supply of energy. It is not necessary that the acceleration be taking place constantly (we only observe it rarely as compared to the isotropic peak); it might be intermittent or flickering. This sort of cycle is more easily associated with some sort of instability than the former continuous case. Possible mechanisms for these productions have been discussed by Kindel and Kennel (1971) and Block (1972). There are other ways in which the momentum flux, for example, could be conserved. Ions with an equal

(5.5) \[ f(\vec{v}) = \frac{m}{2\pi E_0}^{3/2} e^{-\frac{m\nu^2}{2E_0}} e^{\frac{eV}{E_0}} \]  

(5.6) \[ f(\vec{v}) = \frac{m}{2\pi E_0}^{3/2} e^{-\frac{m(\vec{v}-\vec{v}_0)^2}{2E_0}} \] (Streaming)
<table>
<thead>
<tr>
<th>Flux</th>
<th>Downward MB</th>
<th>Accelerated MB</th>
<th>Streaming MB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi$</td>
<td>$n \left( \frac{E_0}{2\pi m} \right)^{\frac{1}{2}}$</td>
<td>$n \left( \frac{E_0}{2\pi m} \right)^{\frac{1}{2}}$</td>
<td>$n E_o$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$\frac{n E_0}{2}$</td>
<td>$nE \left[ \left( \frac{V}{\pi E_0} \right)^{\frac{1}{2}} + \frac{e^{-V}}{2 \pi} \right] \left[ 1 - \text{erf} \left( \frac{V}{2E_0} \right) \right]$</td>
<td>$n \left( E_0 + mV_o^2 \right)$</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>$\frac{mn}{2\pi^2} \left( \frac{2\pi E_0}{m} \right)^{\frac{3}{2}}$</td>
<td>$\frac{nm}{2\pi^2} \left( \frac{\pi E_0}{m} \right)^{\frac{3}{2}} \left( 1 + \frac{V}{2E_0} \right)$</td>
<td>$nV_o \left( \frac{5\pi}{2} E_0 + \frac{mV_o^2}{2} \right)$</td>
</tr>
</tbody>
</table>

Conservation Laws.
momentum flux could be accelerated upward, or momentum could be carried away by plasma wave motion.
5.5 The Scattering Source-Stability

The previous section described qualitatively the requirements which must be placed on a scattering mechanism if it is to explain the discrepancy between the model and the data. This section is concerned with a possible source of this scattering.

Viewed from one perspective, the scattering process we consider is a "rearrangement" of the particle distribution function in velocity space. In order to produce significant distribution function changes, self consistency requires that we create internal fields in the absence of external ones. By self consistency we mean here that plasma particles, once perturbed, create fields which themselves will affect the particles which were responsible for their creation. It is this nonlinear interaction between distribution function change and wave growth that we refer to as wave particle scattering. To examine the possibility that this interaction can become significant, we first should determine whether wave growth is possible in the simplest case. A necessary step in this direction, for the class of electrostatic instabilities, is the determination of the form of the parallel distribution function ($f_{\parallel}$) in velocity space. The purpose of the following few pages is to clarify this statement.

Velocity space instabilities are distinguished from position space instabilities in that they may be said to arise from the form of the distribution function in velocity space.
space. In the electrostatic case the instability is associated with double or multi-humped distributions. The familiar two stream instability is an example of such a distribution. In the following, we will briefly outline the theory of the electrostatic plasma instability. Emphasis is placed on those facets of the linear theory necessary for understanding why the form of \( f_{uu} \) is important. For complete treatments of the topic see the original paper of Landau (1946) in addition to Davidson (1972) and Krall and Trivelpiece (1973). For discussions of the electrostatic instability applied to auroral phenomena, see Perkins (1968).

When treating the question of stability of a finite temperature plasma, the Vlasov theory of Plasma Stability is necessary. This theory consists of a self consistent solution of the collisionless Boltzmann transport (or Vlasov) equation and Maxwell's equations. The most general Vlasov-Maxwell system for \( N \) different particle species is,

\[
\begin{align*}
\frac{\partial f_i}{\partial t} + \nabla \cdot \mathbf{v}_i f_i + \frac{q_i}{m_i} \left( \mathbf{E} + \frac{\mathbf{v}_i \times \mathbf{B}}{c} \right) \cdot \nabla_v f_i &= 0 \\
\nabla \cdot \mathbf{E} &= 4\pi \rho &= 4\pi \sum_i q_i \int f_i \, d\mathbf{v} \\
\nabla \times \mathbf{B} &= \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \sum_i q_i \int \mathbf{v}_i f_i \, d\mathbf{v} \\
\nabla \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}
\end{align*}
\]

where we assume no external current or charge densities and,

\[ f_i \, d\mathbf{r}_i \, d\mathbf{v}_i \text{ the number of particles with space coordinates between } \mathbf{r}_i \text{ and } \mathbf{r}_i + d\mathbf{r}_i \text{ and velocity between } \mathbf{v}_i \text{ and } \mathbf{v}_i + d\mathbf{v}_i \text{ at time } t. \]
so that,

\[(5.11) \quad \rho_i = \int f_{i\lambda} \, d\vec{v} = \text{particle density}\]

\[(5.12) \quad \rho_i \vec{v}_i = \int \vec{v} f_{i\lambda} \, d\vec{v} = \langle \vec{v} \rangle\]

There are no averaging processes involved as in the fluid approach; further, there are no assumptions required as to an equation of state for the system. These considerations arise naturally in the process of solving the system above.

From the form of the Vlasov equation and the fact that \(\vec{E}\) and \(\vec{B}\) are expressed in terms of \(f\), Equation (5.1) is nonlinear through the third term. This substantiates the qualitative feeling presented in the introductory remarks. In order to linearize these equations a perturbation expansion is undertaken,

\[(5.13) \quad f_{i\lambda} = f_{i\lambda 0} + f_{i\lambda 1}\]

\[(5.14) \quad \vec{E} = \vec{E}_0 + \vec{E}_1\]

\[(5.15) \quad \vec{B} = \vec{B}_0 + \vec{B}_1\]

The resulting Vlasov-Maxwell system is,

\[(5.16) \quad \frac{d f_{i\lambda 1}}{dt} + \vec{v} \cdot \nabla f_{i\lambda 1} + \frac{q_i}{m_i} (\vec{E}_0 + \vec{v} \times \vec{B}_0) \cdot \nabla \vec{v} f_{i\lambda 0} = \frac{q_i}{m_i} (\vec{E}_1 + \vec{v} \times \vec{B}_1) \cdot \nabla \vec{v} f_{i\lambda 0}\]

\[(5.17) \quad \nabla \cdot \vec{E}_1 = 4 \pi \sum_\lambda q_b \int f_{i\lambda 1} \, d\vec{v}\]

\[(5.18) \quad \nabla \times \vec{B}_1 = \frac{1}{c} \frac{d\vec{E}_1}{dt} + \frac{4 \pi}{c} \sum_\lambda q_b \int \vec{v} f_{i\lambda} \, d\vec{v}\]
where we neglect 2nd order terms in comparison with 1st order ones. The assumption of small perturbations, although invalidating any serious attempt to describe the rearrangement of the distribution function, is sufficient to predict 1st order growth or decay of an instability. A specific prediction of the form of the changes which occur in \( f_i \) is nonlinear, as stated, and is beyond the scope of the present work. Investigations of this sort related to auroral phenomena have most recently been undertaken by Papadopolous and Coffey (1974).

Returning to the Vlasov system of equations, we make the electrostatic approximation,

\[
\nabla \times \vec{E}_i \sim 0, \quad B_i \sim 0
\]

The equations now reduce to,

\[
\frac{2f_{ii}}{\lambda} + \vec{v} \cdot \nabla f_{ii} + \frac{q_i}{m_i} \left[ \frac{\nabla \times \vec{B}_0 \cdot \nabla v_{ii} + \vec{E}_i \cdot \nabla f_{ii}}{c} \right] = 0
\]

\[
\nabla \cdot \vec{E}_i = \frac{q_i}{m_i} \sum_{\alpha} \int f_{i\alpha} \, d\vec{\nu}
\]

The method of solving these equations was introduced originally by Landau in his paper of 1946. The method consists of an integral transform technique: a Fourier transform in space and a Laplace transform in time. Once the transform is completed, the equation is solved for the transformed potential (or electric field). Neglecting the static magnetic field for the moment in the interest of clarity, the
solution for the transformed potential, $\phi(k, s)$, is given by,

$$
(5.23) \quad \phi(k, s) = \frac{4\pi}{k^2} \sum_i q_i \int \frac{q_i (k, \nu, \nu t = 0)}{s + i \nu} \, d\nu, \quad \text{Re} \, s \geq S_0
$$

where $q_i (k, \nu, \nu t = 0)$ is an initial condition on the distribution function which arises from the process of taking the Laplace transform of the partial time derivative of $f_i$.

"$s$" is the Laplace transform parameter and $S_0$ is chosen large enough to insure convergence of the transformation integral. $\hat{k}$ is the direction of wave propagation and the integrals are taken over the total velocity $\nu$. These integrals are considerably simplified if we choose our coordinate system such that one of the coordinate axes lies along the direction on the wave vector $\hat{k}$. We first define $F_{i0}$ as the integral over the other two coordinates,

$$
(5.24) \quad F_{i0}(u) = \int_{-\infty}^{\infty} f_{i0}(u) \, d\nu
$$

With this definition and with our axis along the direction $\hat{k}$, Equation (5.23) becomes,

$$
(5.25) \quad \phi(k, s) = \frac{-4\pi}{k^2} \sum_i q_i \int \frac{F_{i0}(u, k_1 \nu = 0)}{u - \frac{i S}{k_1}} \, d\nu, \quad \text{Re} \, s \geq S_0
$$

By making the substitution $S = -i \omega$ for the Laplace transform parameter $s$, we can write,

$$
(5.26) \quad \phi(k, \omega) = \frac{-4\pi}{k^2} \sum_i q_i \int \frac{F_{i0}(u, k_1 \nu = 0)}{u - \frac{\omega}{k^2}} \, d\nu
$$
In order to obtain a real world solution to the problem, one must perform the inverse transformation on \( \phi(k,\omega) \). It can be shown (after a number of non-trivial considerations, See Landau (1946)) that the steady state solution for the inverse Laplace transformed potential is such that,

\[
\lim_{t \to \infty} \phi(k, t) \propto e^{\omega_i t}
\]

where,

\[
\omega_i \propto \left( \frac{\partial \phi(k, \omega)}{\partial \omega} \right)
\]

where \( \omega_i \) is the pole of \( \phi(k, \omega) \) with the largest positive value of \( \text{Im} \omega \) (Note that if there is more than one pole of approximately the same order of magnitude, they are summed). Being a pole of \( \phi(k, \omega) \), \( \omega_i \) is also a root of the denominator of Equation (5.20). This denominator is referred to as the dielectric function and the complete solution of it yields the dispersion relation of the plasma. We mention that in performing the velocity integrals in Equation (5.26), one must specify the contours chosen in addition to the sign of the imaginary part of \( \omega \). The choice of the integration contour is that of Landau and the contours chosen are referred to (appropriately enough) as the "Landau Contours". (The particular choice of the contour is such that the velocity integrals above are performed in the complex plane and that their contours always enclose any pole of the integrand. This choice is required in order to analytically continue \( \phi(k, s) \) since, strictly, \( \phi(k, s) \) is not defined unless \( \text{Re}s \geq s_o \).
The correct choice of these contours leads, for example, to the prediction of stability of the Maxwell Boltzmann distribution, or in fact any monotonically decreasing distribution function.

From the above remarks, the form of $f_{\omega}(z_{\mu})$ begins to emerge as crucial to the question of stability. If $f_{\omega}$ is monotonically decreasing ($\frac{\partial f_{\omega}}{\partial \omega}$ ), from Equation (5.21) the distribution is stable in the long time limit (it is said to be Landau damped). On the other hand, a distribution with a double hump where $\frac{\partial f_{\omega}}{\partial \omega}$ can be unstable if there exists a solution (root) of the denominator in the right half complex $\omega$-plane. The question of linear stability of the plasma then reduces to the algebraic question of determining whether such a root exists. We mention in passing that one method of doing this relies upon application of the Penrose Criterion (1960) which is a simplification of the Nyquist Stability Criterion familiar to electrical engineers.

Figures (29) and (30) are the results of computing $\tilde{f}_{\omega} = \tilde{f} \mu$ numerically for our distribution function observations (i.e., actually performing the integrations indicated in Equation (5.24)). These plots were prepared using the fitting function which was described in Chapter II. From the form of these figures, it can be concluded that the distribution, for the time segments considered, represents a state of marginal stability. It seems likely, based on quasi-linear stability theory (See Davidson (1972)), that what we are seeing is a distribution that was unstable at some position.
further up the field line but has begun an approach toward a stable, or monotonically decreasing, distribution. The "fill-in" of this region before the streaming peak is what is predicted as the result of the instability based on this quasi-linear theory (See Davidson (1972) and Papodopolous and Coffey (1974)). We mention again that these plots were obtained from a numerical integration scheme which employed the integrand in the form of the fitting function. In both cases plotted, the region of marginal stability is just at the position of the streaming peak in that instance. The isotropic contribution to this peak is negligible. The plots each display two curves: one of the curves corresponds to electrostatic wave propagation along the direction of the magnetic field (\( \mathbf{E} \cdot \mathbf{B} \)), the other at a large angle (80°) with respect to this direction. All other cases (different angles with respect to \( \mathbf{B} \)) fall between the two. As this angle increases, the tendency toward double-humped behaviour is lessened. Finally, at large angles it is practically non-existent.
Figure (24)

Figure (25)
Figure (26)

Figure (27)
Figure (28)
Figure (29)
$T = 261 - 265$

- $\theta = 0^\circ$
- $\theta = 80^\circ$

Figure (30)
CHAPTER VI

CONCLUSIONS

We have attempted in this investigation to clarify the implications of simple parallel electric field accelerations of auroral electrons. We mentioned previously that this is widely regarded as the source of the field aligned energetic distributions. Our fundamental conclusion is that a scatterfree non-interacting model of this mechanism is insufficient to adequately explain the observations of at least three separate sounding rocket flights. Modifying the basic scheme allows the model to remain plausible. As concluding statements, we review the fundamental results which in sum, constitute these overall conclusions.
6.1 Field Aligned Monoenergetic Peaks

We have presented two possible mechanisms for the production of field aligned electrons. They are:

(i) **The single source.** In this model we inject electrons of characteristic temperature 10-100 ev above a potential drop region. The result of this, after scattering or some other energy broadening process, is a streaming Maxwellian plasma with a streaming energy on the order of several kev. The observed field aligned contours have higher perpendicular energy than parallel energy. This is probably due to mirror effect considerations but could arise from an intrinsic property of the scattering or acceleration mechanism.

(ii) **The two source model.** The second model proposed here required two sources. In this model, the electrons which produce the field aligned peak are injected by a source between two separated electric fields. For example, the source could be composed of ionospheric electrons which were present before formation of the lower electric field. The field aligned peak could disappear once the electrons between the two fields had been depleted. In order to follow this problem further requires a study of the formation and maintenance of the acceleration region. The fundamental difficulty with this mechanism is the observation of two peaks at nearly the same energy but with drastically different pitch angle distributions. Again a scattering or energy broadening process must be introduced to explain the absence of sharp cutoffs in the data contours.
6.2 Isotropic Electrons

Again the models proposed divide into the single source and multiple source models:

(i) **The single source.** This model requires selective pitch angle scattering to produce the isotropic peak. By selective is implied that not all electrons can be affected since the field aligned ones are observed to occur in the presence of the isotropic ones. It is possible that the process could represent an intermittent instability (or flickering instability) or could scatter only those electrons which satisfy a resonance condition. The fact that the isotropic electrons have slightly higher energies than the field aligned monoenergetic electrons could be a result of a resonance requirement in a scattering process.

(ii) **The two source model.** The two source model requires no selective pitch angle scattering process for the isotropic peak. It introduces a second intermittent peak in lieu of this. The isotropic monoenergetic peak arises from a source above the acceleration region. The isotropy is produced by the mirror effect.
6.3 Low Energy Electrons

Electrons that have energy less than those in the monoenergetic peak must be produced (or lose energy) below the potential region (Evans (1974)). There are some divergences between our observations and theoretical predictions based on backscattered and secondary electron production (Nagy and Banks et al. (1974)). Our observed power law spectrum goes most closely as $E^{-1}$; the predictions of the models go as $E^{-2}$. The fairly broad region of constant $j$ between the monoenergetic peak and the low energy peak differs also from the atmospheric scattering model. Papadopoulos and Coffey (1974) have discussed a wave-particle interaction which could be important in these considerations.
6.4 Stability

From the form of the parallel distribution function, \( f_{\parallel}(\pm F_{eq}) \), for the two flight times of Flight 18:109 considered, we are able to answer the question of instability with a maybe. The conclusion to be drawn from our numerical integration is that the distribution that we observe was unstable at some position further up the field line from the observation point. This was probably as a result of the electric field acceleration. The extent to which this is related to the field formation is unknown. Instabilities of this sort have been predicted earlier (Perkins (1968)) and from these and other considerations outlined it is felt that this is a fruitful area for further study.
APPENDIX A

Data Flights Description

The data upon which the investigation is based were obtained from spectral observations of three auroral sounding rocket flights. This section provides a cursory description of those flights together with any pertinent characteristics of each. The details of the individual flight spectra are covered in the body of the thesis. Each of the flights is well documented by Arnoldy et al. (1974a). The electron spectra (Figures (1) through (7)) were obtained from point sample flux measurements taken by a configuration of electrostatic analyzers which selected electrons of energies 0-15 kev in 50-52 equal increments. Each flight described below varies as to the time required for the detectors to sweep the entire energy spectrum. In addition, the payload spin rate differs on each flight. Although ion detectors were aboard one rocket, Flight 18:91, none measured ion fluxes above the detector threshold. Analyser characteristics and calibration are described by Choy et al. (1971) and Arnoldy (1973).

Flight 18:91

The rocket was launched on April 11, 1970 into a bright auroral band. Field alignment was noticeable during only one 10 second interval from 135-145 sec (Figure (1)). The detectors made one energy spectrum sweep every second and the payload spin period was 2.5 seconds. Therefore, the time
required between samples of approximately the same energy and pitch angle is 5 seconds. Considerable time variations were noted in the interval between 135-140 seconds and the 140-145 sec spectra. This is not reflected in the composite 10 second time averaged pitch angle sorted spectrum of Figure (1). The field aligned peak near 3 kev appears fairly constant over the entire 10 second interval. This is the only instance during which there was a pure field aligned monoenergetic peak.

**Flight 18:109**

The rocket was launched on 5 April 1972 into an active aurora. Spectacular streaming was apparent during two time segments of this flight. (See Figures (2) through (5)). Of interest here is that during these segments the basic structure of a field aligned peak at a lower energy than the accompanying less intense isotropic peak is apparent. The detectors on this flight employed 52 discrete energy channels which were swept 4 times per second. In order to repeat an identical pitch angle-energy measurement required 4 seconds. Therefore during one 4 second sweep a total of 800 flux measurements (16/channel) were made as compared to 250 (5/channel) on flt 18:90. The pitch angles scanned ranged from 0 to 70 degrees for down going electrons.
Flight 18:152

The flight launch date was 16 March 1973. Multiple peaks were apparent in the spectra during two time intervals. See Figure (6) and (7). Flight 18:152 employed three electron detectors which each made 2 complete energy scans per second. In addition, each detector on this flight scanned all pitch angles from 0 to 180 degrees (down going to upgoing electrons). The time required for the three detectors to repeat a given pitch angle-energy measurement was $1\frac{1}{2}$ seconds. During one $1\frac{1}{2}$ second sweep a total of 450 flux measurements (9/channel) were made. Along with the field aligned and isotropic peaks, at the earlier flight times, there are multiple peaked spectra occurring during the time segment 275-330 seconds.
APPENDIX B

The contour mapping program referred to extensively in the thesis is titled PSUMAP ("PSU" designates Pennsylvania State University). The routine is a version of the program SYMAP which was originally developed by the Laboratory for Computer Graphics and Spatial Analysis, Harvard University. The purpose of this section is to briefly describe this program in addition to the elements of the interpolation scheme employed in preparing the contour maps.

The list of options available to the user in this routine seems practically endless. For a detailed listing of the program along with these options see Introductory Manual for Synagraphic Computer Mapping listed in the reference section of this report. The minimum information which must be supplied by the user is the following:

(1) The data points \( \{ x_i, y_i, z_i \} \), where \( \{ x_i, y_i \} \) are position coordinates in the two dimensional plane and the \( z_i \) are the contour values associated with each data point (e.g., flux),

(2) The dimensions of the mapping area,

(3) The intervals between contour levels and finally,

(4) The maximum and minimum contour level to be included in the spacing considerations. Any contour value greater than the minimum is designated high or low and there is no differentiation made between the levels which are included in the regions. (If the user does not specify these levels, they are automatically assumed to be the highest and lowest value
found in the data. In this case the contours are spaced according to the difference between the highest and lowest contour divided by 10. Therefore, in addition to the present item, item 3 above is not necessary.)

Given the points \( \{x_i, y_i, z_i\} \) and their contour values \( z_i \), the most important consideration in producing the contour maps is that of assigning \( z \) values, or contour values, to points for which there is no data. This procedure is fundamentally what differentiates contour mapping schemes of this sort. The particular method used here was developed by Donald K. Shepard (1968). The basic idea is to begin by calculating a network of grid points in the \((x, y)\) plane. Once the grid is established, any point which lies within the grid, and which is not a data point, is found through linear interpolation. The formation of the regular grid array (i.e., establishing the value of each grid point) is the difficult task and the one which requires some method of establishing how these points are to be determined from the data set.

The basic scheme for calculating \( z \) at a point \( P(x, y) \) is to weight each data point \( z_i \) by the inverse square of the distance from \( z_i \) to \( P(x, y) \) and then to sum over all data points, or

\[
(B.1) \quad z(P(x, y)) = \frac{\sum_{i=1}^{N} \frac{z_i}{d_{i}^2}}{\sum_{i=1}^{N} \frac{1}{d_{i}^2}}, \quad d_{i} \neq 0
\]

where \( d_{i} = \sqrt{(x-x_i)^2 + (y-y_i)^2} \).
One notices from the form of $\mathcal{J}_j$ that the function passes continuously from $d_{j\neq 0}$ to $d_{j=0}$ and further that the function is bounded, i.e.,

$$
\lim_{\mathcal{P} \to (x', y')} \mathcal{J}_j = \mathcal{J}_j.
$$

Closer inspection of the method by which the grid points are to be determined shows some undesirable features. We shall mention these and indicate their solution. The interested reader is referred to the references previously cited in this section.

Since determination of $\mathcal{J}_j$ requires sums over all data points, it is easy to see that with large numbers of data points the grid point calculations become more time consuming than necessary. In any event, the inverse square weighting function rapidly damps contributions from all but the nearest few data points. Briefly, this problem was solved by requiring:

1. that the number of data points (n) chosen to calculate $\mathcal{J}_j$ be such that $4 \leq n \leq 10$ and,

2. that there be an initial search radius, $r$, which is a function of the data point density. "r" is defined such that on the average there are seven data points in a circle of this radius (i.e., $\pi r^2 = 7 \left( \frac{A}{N} \right)$).

Further problems which exist with the pure inverse square weighting form are related to:

1. Only magnitude, and not direction, is used in calculating $\mathcal{J}_j$. This leaves an ambiguity in the arrangement of data points about a particular point $P(x, y)$. 
(2) The directional derivatives one would calculate from \( \mathbf{3} \),

\[
\frac{\partial \mathbf{3}}{\partial x} = \sum_i \sum_j \frac{1}{d_i^4} \frac{1}{d_j^2} \left( \frac{x - x_i}{d_i^2} \right) \mathbf{3}_i \left( \frac{3_i - 3_j}{d_j^2} \right)
\]

have an arbitrary zero at any data point \((x_i, y_i)\). This places an unnecessary restriction on the contour surface.

(3) In the neighborhood of data points \((x_i, y_i)\), the computational error is large. This is so because the largest term comes from the difference of two nearly equal numbers.

In a capsule, (1) above was solved by introducing a directional weighting factor, (2) by adding increments to function values at nearby data points and, (3) by defining an \( \mathcal{E} \)-neighborhood of the data point as a limit; If several data points fall within this neighborhood their values are averaged.

This completes an outline of the contour map generating scheme from which our velocity space contours were developed. Further details are provided in the references cited.
APPENDIX C

The function fit program as referred to in the thesis body is GLSWS (General Least Squares With Statistics). It was developed by Walter E. Daniels, Jr. (1965). The program provides for a set of user supplied subroutines defining the function, reading the parameters, data, etc. The master control program, MAIN, is the driver program and an inspection of its flowchart, Figure (31), is sufficient to gain an insight into the fundamental operation sequence.

The heart of the program is the fitting routine, FIT, and subsequent to this the statistical analysis of the final parameters compatible with the convergence tolerance, STAT. Both routines are called by MAIN. Below we give a brief description of the fitting routine.

FIT Description

The FIT subroutine as described by GLSWS employs a general least squares fit which uses the Maximum Neighborhood method of Marquardt (1963). This method is also covered sufficiently for user purposes by Bevington (1969). Much of the following account is outlined in this book along with other references cited above.

Maximum Likelihood-Goodness of Fit

Let \( \{ y_i \}_{i=1}^N \) be the set of measurements made of a function \( y(x) \). We assume further that there exists a relationship between \( y \) and the independent variable \( x \). For
GLWS MAIN
FLOWCHART

COMMON
DIMENSIONS
INITIALIZE

1

M=-1

M=-1 Read More Data

READ HEADING

READ ANOTHER DATA SET ?

T

F

END PROGRAM ?

T

F

READ MAXIMUM ITERATIONS

READ CONVERGENCE TOLERANCE

READ PARAMETERS

COMPUTE \( F(x) = y \)

ERROR AND ITERATIONS LESS THAN MAX. ?

T

F

PRINT RESULTS

COMPUTE FIT STATISTICS

M=0

WRITE END OF JOB

CALL EXIT

Figure (31)
purposes of illustration we let this relationship be given by,

\[(C.1) \quad y = a_0 + b_0 x \]

where \(a_0, b_0\) are assumed true values of the two parameters \(a, b\).

We let these measurements \(\{y_i' | i = 1, \ldots, N\}\) be normally distributed and further assume the \(y_i'\) are more likely to be elements of the set \(\{y_i = a_0 + b_0 x_i\}\) than of any other distribution with different parameters (Principle of Maximum Likelihood).

If this is true the probability of measuring the set \(\{y_i' | i = 1, \ldots, N\}\) is greatest. This probability for normally distributed errors is given by,

\[(C.2) \quad P(a_0, b_0) = \prod_{i=1}^{N} \left( \frac{1}{\sqrt{2\pi\sigma_i^2}} \right) \exp \left( -\frac{1}{2} \sum_{i=1}^{N} \frac{(y_i - \eta_i(x_i))^2}{\sigma_i^2} \right) \]

Therefore, in general, the process of finding the maximum probability for normally distributed errors is the same as minimizing the least squares sum or,

\[(C.3) \quad \chi^2 = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \left( y_i - (a_i + b_i x_i) \right)^2 \]

where \(a, b\) are the initial parameter estimates.

**Linear Functional Form**

In the special case in which our functional fit form is linear, the process is particularly simple. We require,

\[(C.4) \quad \frac{\partial \chi^2}{\partial a_1} = \frac{\partial \chi^2}{\partial a_2} = \ldots = \frac{\partial \chi^2}{\partial a_V} = 0 \]
and solve for \( q_1, \ldots, q_r \)

This leads to a set of \( r \) simultaneous equations in \( r \) unknowns which can be solved by the method of undetermined coefficients. In the case of two parameters,

\[
\begin{align*}
(n+1)q_i + \left( \sum_{i=1}^{N} \chi_i^2 \right)q_{i+1} &= \sum_{i=1}^{N} y_i^2 \\
(n+1)q_{i+1} + \left( \sum_{i=1}^{N} \chi_i^2 \right)q_i &= \sum_{i=1}^{N} y_i^2 \chi_i
\end{align*}
\]

Non-Linear Functional Form

Notice now were we to choose, for example, a fit of the form

\[
y_i = f(x_i) = q_i e^{q_{i+1}x_i}
\]

the normal equations introduced above are non-linear in \( q_1, \ldots, q_r \).

The general problem is now viewed (Bevington (1969)) as that of finding the minimum of a function \( \chi^2 \) in parameter space which, for the case of two parameters, may be visualized as shown below.
One way to proceed from this point is to expand the function in a Taylor series in increments of the parameters, $\Delta q'_i$. This done, we use the method of linear least squares as above to solve for the $\Delta q'_i$s. Notice this method amounts to linearizing the function. Another method is to calculate the gradient of $\chi^2$ (or the direction of steepest descent) and then to adjust the parameters such that the calculation of the new $\chi^2$ follows this particular path. Both methods are outlined below along with the Marquardt (1963) method which is an optimum combination of the two.

**Taylor Expansion of the Fitting Function**

Using this method one expands $f$,

\[ f(x_1, q_1, a_2 + \Delta a_2, \ldots, q_r + \Delta a_r) = f(x_1, q_1, a_2, \ldots, q_r) + \sum_{j=1}^{r} \frac{\partial f(x, a)}{\partial a_j} \Delta a_j + \frac{1}{2} \sum_{j=1}^{r} \sum_{k=j+1}^{r} \frac{\partial^2 f(x, a)}{\partial a_j \partial a_k} \Delta a_j \Delta a_k. \]

Neglecting second order contributions for clarity,

\[ \chi^2 = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \left[ f(x_i, q) + \sum_{j=1}^{r} \frac{\partial f(x, a)}{\partial a_j} \Delta a_j - y_i \right]^2. \]

Differentiating with respect to $\Delta q_k$,

\[ \frac{\partial \chi^2}{\partial \Delta q_k} = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \left[ f(x_i, q) + \sum_{j=1}^{r} \frac{\partial f(x, a)}{\partial a_j} \Delta a_j - y_i \right] \frac{\partial f(x, a)}{\partial q_k} \partial q_k = 0 \]

or, using matrix notation,

\[ P_k = \sum_{j=1}^{r} \Delta a_j \Delta a'_j k. \]
where,

\[(C.12) \quad \beta_k = \frac{1}{\delta_i^2} \left( y_i - f(x_i, \alpha) \right) \frac{\partial f(\alpha)}{\partial \alpha_k} \]

and,

\[(C.13) \quad \chi_{jk} = \sum_{i=1}^{N} \frac{1}{\delta_i^2} \left[ \frac{\partial^2 f(\alpha)}{\partial \alpha_j \partial \alpha_k} \right] \]

Hence the process of solving for the increments to the parameters involves a matrix inversion,

\[(C.14) \quad \Delta \alpha = \beta \chi^{-1} \]

Notice that we can calculate the derivatives either from the fit or from an empirical determination.

Therefore, inherent to this method is the matrix \(\alpha\) which is symmetric and contains the off-diagonal terms that have arisen as a result of the expansion of \(f(\alpha + \Delta \alpha)\) in terms of \(\Delta \alpha\). One drawback to this method is that convergence is slow for initial bad estimates to the parameter values. Inclusion of the second order terms in the expansion can help the convergence process with good parameter guesses. This will slow individual loop computation time however.
The Gradient Method

In the gradient $\chi^2$ method one calculates the gradient of $\chi^2$ in parameter space and adjusts all parameters simultaneously. In this case the direction in which the search proceeds is along the path of steepest descent toward the minimum or,

\[ \nabla \chi^2 = \sum_j \left( \frac{\partial \chi^2}{\partial a_j} \right) \hat{a}_j \]  

To determine $\nabla \chi^2$ within one loop of stepsize $\Delta a_j$, a finite difference technique is employed, i.e.,

\[ (\nabla \chi^2)_j = \frac{\partial \chi^2}{\partial a_j} \approx \frac{\chi^2(a_j + \Delta a_j) - \chi^2(a_j)}{\Delta a_j} \]

where, $\Delta' a_j < \Delta a_j$

Roughly then, the approximation to the new increment, $\delta a_j$, is given by,

\[ \delta a_j = - (\nabla \chi^2)_j \Delta a_j \]

(Actually, since the parameters in general do not have the same dimensions, they are normalized with respect to the step size, $\Delta a_j$

\[ \hat{a}_j \equiv \left( \frac{a_j}{\Delta a_j} \right) \]

and a dimensionless gradient defined.)
The key point here is that we have $r$ separate equations for each increment $\delta a_j$ and have no cross terms (by definition of the gradient) as appeared in the Taylor expansion above.

This method is quite good for approaching the minimum from far away but begins to cause problems near the minimum of $\chi^2$. This is because in approaching the minimum, the gradient should go to zero. Hence the finite difference technique for calculating the gradient leads to round-off problems and, in general, slow convergence.

**The Marquardt (1963) Method-Gradient Expansion Algorithm**

The Marquardt Method combines the best features of the expansion (good convergence near the minimum) and the gradient (fast convergence toward the minimum for points far away). This is accomplished by introduction of a factor, $\lambda$, which controls interpolation between the two. The basic (normal) equations of the expansion scheme are rewritten as,

\[(C.19) \quad \Delta^2 z \overset{\lambda}{\sim} \Delta^2 a \Rightarrow \Delta^2 \chi \]

where,

\[(C.20) \quad \Delta_{jk} \overset{\lambda}{=} \begin{cases} \alpha_{jk}(1+\lambda) & j = k \\ \alpha_{jk} & j \neq k \end{cases} \]

Hence for large $\lambda$, the diagonal terms of the matrix dominate so that we arrive at $r$ separate equations for the gradient expansion,
(C.21) \[ \beta_j = \lambda \delta a_j \delta^*_j \]

with the \( \delta a_j \)'s now in the same direction as the steepest descent with the \( \delta a_j \)'s now in the same direction as the steepest descent and scaled in magnitude by the factor \( \lambda a_{jj} \). \( \chi^2 \) is scaled in magnitude by the factor \( \lambda a_{jj} \). Equations to which the off-diagonal terms contribute. For most intermediate points then, \( \lambda \) determines the optimum amount of each method to use in locating the minimum. (The actual method by which \( \lambda \) is calculated is found in the references listed above.)
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