INTERRELATIONSHIP OF ENERGETIC PARTICLES, PLASMA AND MAGNETIC FIELDS IN THE INNER HELIOSPHERE

JEROME THEODORE NOLTE

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INTERRELATIONSHIP OF ENERGETIC PARTICLES, PLASMA AND MAGNETIC FIELDS IN THE INNER HELIOSPHERE

Keywords
Engineering, Aerospace

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INTERRELATIONSHIP OF ENERGETIC PARTICLES, PLASMA
AND MAGNETIC FIELDS IN THE INNER HELIOSPHERE

by

JEROME T. NOLTE

B. A., Mankato State College, 1966
M. S., University of New Hampshire, 1970

A THESIS

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ABSTRACT

INTERRELATIONSHIP OF ENERGETIC PARTICLES, PLASMA
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by

JEROME T. NOLTE

This study of energetic solar particles, solar wind plasma and magnetic fields in the inner heliosphere divides naturally into two parts. One part is a study of the solar corona, and the other is an investigation of energetic particle propagation in the interplanetary medium, based on recent reports of measurements of highly anisotropic solar particle fluxes.

Coronal magnetic field structure is investigated through the use of solar and interplanetary magnetic polarity measurements, and observations of solar wind plasma and energetic particles during the first eight months of 1965. The means of investigation is a cross-correlation of chromospheric with high coronal magnetic polarity. The high coronal (at altitudes of 20-50 \( R_\odot \)) polarity is deduced from observations of the interplanetary magnetic field and solar wind plasma. The principal results are that low energy solar protons and fast solar wind are preferentially associated with two different kinds of
coronal magnetic field structure from the usual structure at this time period. Investigation of the individual particle events provides evidence in support of this conclusion. This apparent ordering of particle events by coronal magnetic structure is also consistent with the idea that these particles faithfully follow interplanetary field lines, which can be traced to their high coronal connection points.

Recent reports of highly anisotropic particle fluxes up to or beyond the time of maximum flux demonstrate that scattering was negligible in the interplanetary medium inside 1 AU at those times. I have therefore carried out a calculation of pitch angle distributions, assuming scatter-free propagation of energetic particles in the ideal spiral field. In this calculation, an exponentially decreasing injection, and a beginning of a scattering region between 1.5 and 3 AU are assumed. The pitch angle distributions are converted to idealized detector counting rates for comparison with spacecraft observations. Two events which are apparently relatively free from effects due to coronal structure are shown to be in semi-quantitative agreement with predictions of this simplified theory. The theory also provides a basis for interpretation of events which are highly anisotropic through the time of maximum flux.
CHAPTER I

INTRODUCTION

The principal mathematical theories of energetic solar particle propagation in the interplanetary medium which have been applied to flare-associated solar particle events over the last ten years are inconsistent with recently reported observations of events through the time of maximum particle intensity.

These theories are based on the assumption that particles undergo much scattering between the sun and 1 AU. Pitch-angle anisotropy measurements of strongly anisotropic fluxes often persisting up until and even past the time of maximum intensity of solar particle events demonstrate directly that there is insufficient scattering of energetic particles in the inner solar system to justify a description of propagation by a diffusion theory.

Many of these theoretical descriptions have assumed diffusion in longitude in the interplanetary medium. Multispacecraft observations of non-relativistic particles show that there is no measurable diffusion perpendicular to the interplanetary field.

In this thesis I present an alternative description, which is a continuation and extension of recent efforts by Roelof and Krimigis (1973) and Roelof (1973 and 1974). In this description, particles are organized near
the sun by coronal magnetic structure, and injected into the interplanetary medium over extended time periods (>1 day). The source of this extended injection may be either an extended acceleration (particularly in the case of protons of energy less than 1 MeV), or impulsive acceleration and storage. Then, as suggested by observational evidence, particles propagate with no appreciable scattering in the inner solar system.

This model is to a degree a return to the past. In the first attempt to describe a flare-associated solar particle increase with a diffusion theory, Meyer et al. (1956) found it necessary to assume no significant scattering occurred within 1 AU, and that the diffusing region was beyond the earth. The current description includes the interplanetary magnetic field, the existence of which had not been established in 1956.

Before proceeding with the discussion of the new description, it is appropriate to review the reasons why the older theories may no longer be considered to be adequate.

Most theoretical descriptions of energetic solar particle propagation in the last decade have been based on the assumption that these particles diffuse through the interplanetary medium. Following the early theories which used unspecified "scattering centers" (e.g., Parker, 1963; Krimigis, 1965), the theory of scattering from irregularities in the interplanetary magnetic field was developed by Jokipii (1966) and Roelof (1966).
Other effects such as anisotropic diffusion (Axford, 1965; Burlaga, 1967) and convection of the interplanetary field in the solar wind and adiabatic deceleration (Parker, 1967; Gleeson and Axford, 1967; Fisk and Axford, 1968) have been included in detailed calculations by Forman (1971) and Englade (1971) which provide reasonable good fits to the time histories of the omnidirectional fluxes of particles observed and to post-maximum anisotropy measurements in medium and high energy events.

To obtain these fits, it has been necessary to use small values for the diffusion coefficient, and also to assume an outer boundary to the diffusing region at ~3 AU. Additionally all such theories based on a diffusion approximation to a Fokker-Planck formulation of stochastic random walk assume a "nearly isotropic" pitch angle distribution, and are known to be invalid for anisotropies ~30% (see e.g. Forman, 1971, and Englade, 1971). Thus a different approach is required for a description of particle events when a large pitch-angle anisotropy persists from onset to the time of maximum.

An attempt to present a better description of the onsets of energetic particle events has been made by Fisk and Axford (1969), who derive a "telegraph" equation for early times after a flare. Their solutions do provide an initial high anisotropy, decaying fairly rapidly from 100% to values equal to those predicted by the diffusion approximation well before the time of maximum particle flux.
The observational evidence that these diffusion-based theories are inadequate is the simultaneous observation of both anisotropy and omnidirectional flux in a large number of events over a wide range in energy. At the lowest energy extreme of measured proton anisotropies, the detailed analysis by Roelof and Krimigis (1973) of the data from three solar rotations in 1967 (presented by Krimigis et al., 1971) leads them to conclude that:

300-keV proton anisotropies are large during all flare rises and during quasi-stationary events. The implication is inescapable: These particles undergo negligible scattering in the inner solar system, and the coronal injection process must function over long times (>1 day).

In another recent paper Innanen and Van Allen (1973) present an analysis of the time dependence of the anisotropy of 0.3 MeV protons during ten events between 1967 and 1970. They find that the field-aligned component of the anisotropy decreases only after a day or two following the flare to the value in the decay phase of most events. Their analysis shows that the high anisotropy (nearly 100%) often persists up to or even beyond the time of maximum flux. This observation cannot be explained by any diffusion-based theory; similarly, it is not accounted for by the "telegraph" equation calculation of Fisk and Axford (1969).

At intermediate energies, excellent work has been done by McCracken and coworkers, much of which is summarized by McCracken and Rao (1970). This review sets forth the basic description of solar particle event observations made up to that time.
The basic picture which McCracken et al. (1967 and 1968) find is an initial high field-aligned anisotropy, generally of the order of 20-50%, and a late-time "equilibrium anisotropy" of 5-15%. However, they do find some notable examples of persistent high field-aligned anisotropy (e.g. McCracken et al., 1968) at these energies also.

At the highest energies (>1 GeV) at which solar protons are observed, the observation of high anisotropy later than the time of maximum flux is also reported by Maurer et al. (1973). They find that, for the only four highly anisotropic (but otherwise dissimilar) ground level events which had occurred between 1960 and 1970, the highly anisotropic phase also lasts through the maximum of the event.

The inability of a diffusion theory to explain this high anisotropy at time of maximum flux can be demonstrated by a relatively simple calculation for the diffusion equation with constant coefficients. In this case the solution is given by (e.g. Parker, 1963)

\[ U = \frac{N_0}{2\pi^{1/2}([\kappa t]^{3/2})} \exp \left( -\frac{r^2}{4\kappa t} \right) \]

where \( U \) is particle density, \( \kappa = \lambda v/3 \) is the diffusion coefficient, \( N_0 \) is the total number of particles released and \( r \) and \( t \) are radius and time. The anisotropy \( \xi = 3S/Uv = (3/\nu U)\Delta U = 3r/2vt \), while the time of maximum flux \( t_{\text{max}} \) can be found by differentiation of the density to be
\[ t_{\text{max}} = \frac{r^2}{6\kappa} \]

Therefore,
\[ \xi_{\text{max}} = \frac{3r}{2vt_{\text{max}}} = \frac{9\kappa}{rV} = \frac{3\lambda}{r} \]

with \( \lambda \) the diffusion mean-free path, required to be smaller than \( r \) for the diffusion theory with a boundary to be valid. A more general restriction is that \( r << vt \), so at \( t_{\text{max}} \), \( r << vt_{\text{max}} = r^2/2\lambda \), so \( \lambda << r/2 \).

Thus a simple diffusion theory cannot tolerate a large (\( \xi \approx 1 \)) anisotropy at the time of maximum. More complicated theoretical descriptions (e.g. Englade, 1971) also predict much lower anisotropies near \( t_{\text{max}} \), closer to 10 or 20\%. Additionally, the solutions of the "telegraph" equation discussed by Fisk and Axford (1969) reduce approximately to the diffusion solutions before the time of maximum.

These results, coupled to the well-known inapplicability of diffusion theory at times of high anisotropy, demonstrate that diffusion theories are not the correct description of interplanetary propagation of energetic particles.

Further evidence that solar particles do not propagate diffusively is found in the measurements of a completely different species. Observations of low energy (\( \approx 40 \) keV) electron events have been reported by Lin and Anderson (1967) and Lin (1970). Many of these events were shown to be scatter-free, i.e., there is no scattering of these particles between the sun and 1 AU. Thus these low energy solar electron events cannot be described by a diffusion
Roelof and Krimigis (1973) list four points at which low energy solar charged particle observations differ from the predictions and assumptions of the theories which have been used to describe high energy solar cosmic ray propagation. Since these points provide the motivation for the present investigation, I shall briefly summarize them here; for the detailed evidence supporting these statements, the interested reader is referred to the work of Roelof and Krimigis, and the other references also listed below.

These four points, and references other than Roelof and Krimigis (1973) are:

Firstly, low energy solar charged particles are often associated with solar active regions rather than specific solar flares, and are almost continuously present in the interplanetary medium (Fan et al., 1968; Krimigis, 1969; McDonald and Desai, 1971; Pick, 1972).

Secondly, low energy solar charged particles are extensively redistributed in the corona, and often released into the interplanetary medium at locations far from the associated active region (Fan et al., 1968; Balogh et al., 1971; Keath et al., 1971; Innanen et al., 1973; Gold et al., 1973).

Thirdly, there is no measurable diffusion of low energy solar charged particles perpendicular to the interplanetary field (Krimigis and Van Allen, 1967; Fan et al., 1968; Lin et al., 1968; Krimigis, 1969; Anderson, 1969; Lin, 1970).
Finally, low energy solar charged particles can exhibit field-aligned anisotropies in quasi-stationary (corotating) events, and even during the zero-gradient decay phase of a flare-associated event (Krimigis et al., 1971; Roelof, 1973).

The first and last of these points taken together directly imply extended coronal injection, which Roelof and Krimigis also inferred (see quote above).

Based on a detailed analysis, Roelof and Krimigis (1973) concluded that low energy particle events in the summer of 1967 can be explained by three concepts (in addition to the required extended coronal injection, from either a continuing acceleration or a storage region):

1) Interplanetary propagation of these particles is a "collimated convection"; i.e. the particles on a given field line have their motion strongly collimated along it, and the transverse motion is only that of the field line itself.

2) The interplanetary particle fluxes may be traced back to their high coronal source longitudes using solar wind velocity data (Snyder and Neugebauer, 1966).

3) Coronal magnetic structure provides the fundamental ordering of particle profiles; these structures may be deduced from Hα filtergrams.

My work has consisted of further investigation of the validity and extent of applicability of these three concepts. The second, that solar wind velocity may be used to determine the high coronal connection longitudes of the
interplanetary field, has been discussed in two papers
(Nolte and Roelof, 1973a and 1973b). In these two papers
we have shown theoretically that the high coronal connection
points can be determined within \( \sim 10^\circ \) in latitude and longi­tude, using an "extrapolated quasi-radial hypervelocity"
(EQRH) approximation. These connection points are approxi­mately at the altitude of the magnetohydrodynamic (MHD)
critical points, estimated by Weber and Davis (1967) to be
at 20-50 \( R_\odot \), since the solar wind plasma takes on its inter­
planetary characteristics at the Alfvénic critical point
(Parker, 1969).

The EQRH approximation consists of determining the
connection point with the assumption that solar wind plasma
observed near 1 AU propagated radially at constant velocity
from the center of the sun. The approximation produces
accurate high coronal connection points only because inter­
planetary acceleration compensates for the extrapolation to
the center of the sun. The approximation does not determine
the interplanetary field configuration from one spacecraft's
data (except in special circumstances), but just the high
coronal connection points of the interplanetary field. Data
from several spacecraft can, however, be used to reconstruct
interplanetary field "snapshots" even during rapidly-evolving
solar wind configurations, by using the EQRH connection
points as labels for the field lines (Nolte and Roelof,
1973b). For references demonstrating observational verifi­
cation of the applicability of the EQRH approximation, the
interested reader is referred to Nolte and Roelof, 1973a.

In Chapter II of this thesis I analyze coronal and interplanetary magnetic field, plasma and energetic particle data from nine solar rotations (1489-1497) in 1965 (January-August). This time period provides two principal advantages for study. One is that there are energetic particle, solar wind plasma and interplanetary and coronal magnetic field data available (see Chapter II), which are necessary for a detailed study. The other advantage is that this time period was near the minimum in solar activity, so that it is usually possible to identify the solar flare or active region source of low energy protons unambiguously.

There are, unfortunately, no anisotropy data at this time. Additionally, multiple spacecraft low energy proton data are only available when the spacecraft (Mariner 4 and IMP 3) are too widely separated to measure interplanetary propagation effects directly.

I have therefore performed the entire analysis in terms of investigating the third concept above. That is, in Chapter II I demonstrate statistically that there is indeed a low coronal/chromospheric signature of the (presumably) high coronal magnetic structures which control the release of low energy solar protons into the interplanetary medium (and additionally of those structures which are associated with fast solar wind). I also show that the individual particle events support the statistical result. This explanation of the particle events in 1965 in terms of coronal
structure indirectly verifies the other two concepts (that low energy particles follow the interplanetary field lines, and the coronal connection points of these field lines can be determined) since it is highly improbable that the coronal structure at the inferred connection points would provide the ordering of the data by chance.

The other major chapter is a theoretical examination of scatter-free propagation of energetic particles in the ideal spiral field. This theory is obviously applicable to the onsets of a large number of solar particle events. In addition, Roelof (1973) has also shown for a particular simple proton event that the assumption of scatter-free propagation is adequate to describe the late-time decay.

It is therefore appropriate to begin a detailed theoretical investigation of scatter-free propagation in the interplanetary medium, to determine whether it is possible to explain all interplanetary propagation without the assumption of scattering. This calculation extends the original calculation of Roelof (1974) for a magnetic field diverging as $1/r^2$ with simplified boundary conditions. I have considered the most general boundary conditions consistent with the mathematical technique and developed an approximation that enables one to examine scatter-free propagation in an Archimedean spiral field, which is considerably closer to the field observed out to 5 AU than a radial $(1/r^2)$ field.

I have obtained numerical results for a simplified
model which assumes exponential decay of the coronal injection (except for one case where I have assumed constant injection) and an outer boundary to the scatter-free region (between 1.5 and 3 AU). In the mathematical description, the "outer boundary" does not mark a "thin" barrier, but rather indicates the inner edge of a scattering region that may be allowed to extend to infinity. Thus this boundary is the reverse of the boundaries required by diffusion theories for propagation in the inner solar system, such as Burlaga's (1967) "anisotropic diffusion with a boundary" (ADB) theory, or Forman's (1971) "anisotropic diffusion-convection with a boundary" (ADCB) theory.

Although this model is an oversimplification of the actual situation, it does provide quantitative predictions which can be compared with the observations cited above. The quantitative calculation done here is for protons of energy \( \sim 400 \text{ keV} \) which is comparable to those (\( >300 \text{ keV} \)) reported by Innanen and Van Allen (1973) and those discussed by Roelof and Krimigis (1973), and also to the energy (\( >500 \text{ keV} \)) of the particles observed by the University of Iowa detector on Mariner 4 (Krimigis, 1969), which are analyzed extensively in Chapter II.

The most prominent prediction of this simplified scatter-free theory is a high anisotropy persisting through the time of maximum flux for a decreasing injection. Thus this theory is the first which adequately describes the onsets of flare-associated low energy solar charged particle
events. Additionally, the simple theory does predict time histories of both flux and anisotropy which are remarkably similar to those observed events in which the effect of coronal structure is minimal, even though it is known that the coronal structure almost always dominates events at these energies (e.g., Roelof and Krimigis, 1973; and Chapter II of this thesis). These preliminary successes suggest that a continuing investigation of scatter-free propagation theory, preferably using multiple spacecraft particle flux and anisotropy measurements combined with solar wind plasma and magnetic field data, can provide an adequate theoretical basis for understanding low energy solar charged particle injection and propagation in the interplanetary medium and thus provide a better tool for the investigation of the magnetic structure of both the solar corona and the outer heliosphere.
CHAPTER II

CORONAL MAGNETIC FIELD STRUCTURES IN 1965

1. Introduction

The major portion of this chapter consists of an analysis of the latitude dependence of the cross-correlation of the chromospheric and interplanetary magnetic field polarities during the first eight months of 1965 (Carrington rotations 1489-1497). The technique used is similar to that of Roelof and McIntosh (1972), who used an extrapolated quasi-radial hyper-velocity (EQRH) approximation (Nolte and Roelof, 1973a) to map the interplanetary magnetic field polarity back to the sun and compared these mapped-back polarity measurements with the chromospheric magnetic polarity inferred from H$_a$ filtergrams (McIntosh, 1972).

This problem of the quantitative statistical determination of the relationship between the polarities of solar and interplanetary magnetic fields has received considerable attention, beginning with a study by Ness and Wilcox (1964) and also in subsequent papers by Wilcox and a number of coworkers. Such a relationship provides statistical information on the source and propagation of the solar wind (Wilcox, 1968) that may be compared with direct observations (Krieger et al., 1973), and also provides a framework for the discussion of the propagation of low energy solar protons since, as Roelof and Krimigis (1973) have shown, these particles follow interplanetary field lines with negligible perpendicular diffusion.
There are two major differences between this method and the techniques used by Ness and Wilcox (1964, 1967), Wilcox and Ness (1967), Schatten et al. (1969) and Scherrer et al. (1972). Firstly, the EQRH-approximation is used to correct for the variability of the solar wind velocity which would otherwise affect the comparison of interplanetary measurements near 1 AU with solar observations (since the transit time for solar plasma can easily vary from 3 to 6 days); and secondly, chromospheric polarities inferred from Hα filtergrams are used for solar data instead of direct measurements of the photospheric fields with a magnetograph. This method is described more fully in Section 2.

This Hα inference procedure is advantageous for the present investigation since the weak field polarity boundaries are directly visible as filaments and filament channels in Hα filtergrams, while the same fields are too weak for the boundaries to be delineated as precisely by the magnetograph. Roelof and McIntosh (1972) contrast the significant equatorial correlation ($\chi^2 > 15$, implying significance at the .01% level) found for July-October 1967 using the combined Hα/EQRH method with $10° \times 10°$ disc resolution with the lack of correlation which Scherrer et al. (1972) find at the appropriate 4-day lag for July-December 1967 using their smallest spatial smoothing aperture of radius $0.1 R_\odot (\approx 6°)$. This direct comparison of the Hα/EQRH technique with the magnetograph/solar wind-independent timelag method demonstrates that the Hα/EQRH method offers
an advantage for comparison with weak-field regions on a scale of \( \sim 10^\circ \) in solar longitude.

Another indication that the \( H_\alpha/EQRH \) technique is superior to the direct cross-correlation of solar magnetograph data with interplanetary field measurements for correlations on a scale of \( \sim 10^\circ \) is provided by the comparison of the results of this study of nine solar rotations from January to August 1965 (Carrington rotations 1489-1497) with those of Schatten et al. (1969) from nine solar rotations between June 1965 and February 1966. A direct cross-correlation of the observed photospheric fields with the interplanetary field observations (their Figure 9) yields essentially no significant correlation, while the \( H_\alpha/EQRH \) method applied in Section 3 of this chapter to a partially overlapping period shows significant correlation at all latitudes between N30 and S30 (See Figures 3 and 4).

The use of the EQRH approximation to remove interplanetary propagation effects also permits the interpretation of results directly in terms of coronal magnetic field structure. By "corona," using the same distinction as Nolte and Roelof (1973a), I mean that part of the solar atmosphere extending from the chromosphere out to the magnetohydrodynamic (MHD) critical points (estimated by Weber and Davis (1967) to be at altitudes of 20-50 \( R_\odot \)). Since Nolte and Roelof (1973a) have argued that the EQRH connection longitudes in the high corona should be accurate within \( \sim 10^\circ \), the observational relationship between
chromospheric and interplanetary fields provides a direct indication of whether or not there is continuity between the large-scale (~10° in longitude and latitude) field in the corona between the chromosphere and the MHD critical points, beyond which the solar wind takes on its interplanetary character.

The cross-correlation study which I am presenting consists of three main parts. In Section 3 the method of Roelof and McIntosh (1972) is applied to the time period from January-August 1965 (near solar minimum), and the latitude dependence of the correlation between interplanetary and chromospheric field polarities is determined. In the next two sections, using the same technique on selected subsets of the data, I demonstrate that both energetic solar protons and fast solar wind streams come preferentially from coronal magnetic field configurations different from those primarily responsible for the general latitudinal pattern in the correlation found in Section 3. These cross-correlation results may be interpreted in terms of generally "open" and "closed" coronal magnetic structures such as suggested theoretically by Pneuman (1973) and observationally by Krieger et al. (1973).

The final section of this chapter consists of a reexamination of the individual particle events, to show that they are consistent with the interpretation of the statistical results.
2. **Data and Analysis Method**

The time period covered by this study is January to August 1965. For this time period synoptic charts of chromospheric polarity, inferred from H$_\alpha$ filtergrams, are available (McIntosh and Nolte, 1974) as well as interplanetary field polarity measurements from Mariner 4 (Coleman et al., 1967) and unpublished solar wind velocity data from the same spacecraft, which have been supplied through the courtesy of A. J. Lazarus of Massachusetts Institute of Technology, and John Davis of American Science and Engineering, Inc.

The H$_\alpha$ synoptic charts used to indicate solar magnetic field polarity have been constructed using the method described by McIntosh (1972). Briefly, this technique consists of marking the locations of well-defined structures (filaments and filament channels in the weak field regions; plage corridors, fibril patterns and arch filaments in and around the strong fields of active regions) observed in H$_\alpha$ filtergrams on a synoptic chart. These well-defined locations provide the basis for inferring the chromospheric magnetic field polarity pattern.

As in Roelof and McIntosh (1972), I have divided the H$_\alpha$ synoptic charts into 10° bins in latitude and in longitude, and assigned a polarity to each bin: positive or negative if one polarity is dominant (>75% of the area),
otherwise mixed. In the correlation analysis, mixed polarities will be considered to be half positive and half negative. I assign an interplanetary polarity (also positive, negative or mixed) to each 10° in solar longitude by mapping back the interplanetary polarity observed at Mariner 4 (Coleman et al., 1967) to the high coronal connection points of the interplanetary field lines, using solar wind velocity data from Mariner 4 (J. Davis, private communication) in the EQRH approximation (essentially instantaneous ideal spirals: Nolte and Roelof, 1973a).

The correlation for each 10° latitude swath for all rotations analyzed is determined by constructing the 2 by 2 contingency table from the comparison of the interplanetary polarity, mapped back to the corona, and the $H_\alpha$ polarity in that latitude swath. In constructing the contingency table, a mixed polarity in either data set compared with a definite polarity in the other is considered a chance occurrence (half agreement, half disagreement), and mixed polarity in both sets is generally considered to be full agreement. Roelof and McIntosh (1972) find that the overly stringent condition of considering mixed polarity in only one set to be full disagreement gives the same pattern for the latitude dependence of the correlation, but a reduced statistical significance. I have therefore used only the chance occurrence interpretation of mixed polarities (which is perhaps more reasonable) to determine this pattern, except
in the following case. If both mixed polarity assignments are due to a definite change in polarity (in interplanetary polarities, a sector boundary; in Hα, a N-S oriented neutral line) the polarities are considered to be in full agreement only if both halves of the bin agree, and in full disagreement if the polarities in each half of the bin disagree. From each contingency table \( \chi^2 \) and \( \rho \) (the cross-correlation coefficient) are derived using the same methods as Roelof and McIntosh (1972). A sample table is shown as Table 1.

I have done the study twice, once using only those bins from the Hα charts whose polarity (+, -, or mixed) is defined by nearby Hα structures (within \( \pm 20^\circ \) of the bin); and a second time, using an "extrapolation" for the chromospheric polarity to regions \( \pm 20^\circ \) beyond the nearest Hα structure. This extrapolation consists of closing all neutral lines except in polar regions, based wherever possible on the assumption of continuity of magnetic structure from one solar rotation to the next.

A sample Hα map used for the definite polarity study is shown in Figure 1. Figure 2 shows the charts used in the estimated polarity study, with polarity indicated by the shading (gray is negative, white positive). The interplanetary polarities mapped back to the high corona using the EQRH solar wind technique are indicated at the bottom of these shaded maps. The source longitudes of energetic particles and fast solar wind, which are used to
Table 1

<table>
<thead>
<tr>
<th>Solar Polarity</th>
<th>+</th>
<th>-</th>
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<tbody>
<tr>
<td>+ n++</td>
<td>n+-</td>
<td>NS</td>
</tr>
<tr>
<td>- n-+</td>
<td>n--</td>
<td>NS</td>
</tr>
<tr>
<td>N_+^I</td>
<td>N_-^I</td>
<td>N</td>
</tr>
</tbody>
</table>

\[ \chi^2 = N \frac{(n_{++}n_{--} - n_{+-}n_{-+})^2}{N_+^I N_-^I S_+^I S_-^I} \]

\[ \rho = \sqrt{\frac{\chi^2}{N}} \]
Figure 1. A sample Hα synoptic chart, Carrington rotation 1492. Filaments are indicated by crosshatch, plage corridors and filament channels by solid lines and other (weak) structures or lines inferred from continuity from previous or subsequent rotations by dashed lines. Date of central meridian passage is indicated at the top, and Carrington longitude at the bottom of the chart. Polarities are inferred from sunspot groups, and by comparison with the Mt. Wilson magnetograph records (Howard et al., 1967).

Figure 2. The Hα synoptic charts with all neutral lines at latitudes below 60° closed. White areas are positive (out of the sun) polarity, grey negative. The interplanetary polarity, mapped back to the high coronal connection longitude using the EQRH-approximation (Nolte and Roelof, 1973a) is indicated at the bottom; again, white is positive, grey negative; the crosshatch represents mixed interplanetary polarity. Above the interplanetary polarity stripe, sources of fast solar wind (velocity >400 km/sec) are indicated by horizontal lines, and sources of enhanced 0.5 MeV proton flux by vertical lines.
Figure 2a
Figure 2b
Figure 2c
select subsets of the data in Sections 4 and 5, are also marked on these maps. Vertical lines between the interplanetary polarity strip and the Hα chart indicate the connection longitudes of large-scale interplanetary field lines populated with 0.5 MeV protons at Mariner 4. The source locations of fast solar wind are marked by horizontal lines.
3. **Comprehensive Cross-Correlation Study**

The results of the study using all data available during this entire period are shown in Figures 3 and 4. The latitude dependence of $\chi^2$ is shown in Figure 3, and that of $\rho$ in Figure 4. The general pattern of the latitude dependence of both $\chi^2$ and $\rho$ is the same in both studies (using the definite or estimated $H_\alpha$ polarities). In fact, the correlation coefficients from the two different studies are nearly equal in each latitude swath (Figure 4), demonstrating that the extrapolated closure of neutral lines has not distorted the statistics. There is no point for S50-60 in the definite $H_\alpha$ polarity study due to lack of definite neutral-line structure south of 50° latitude.

As shown by Chapman and Bartels (1940), once the cross-correlation has been calculated, the statistical significance may be determined by estimating the number of independent measurements from an "appropriate" length (or time) scale. A reasonable estimate of this appropriate length is 30°, since Wilcox and Ness (1967) find that the autocorrelation function of the photospheric fields decreases to zero within two days lag, or ≈25° in longitude. Thus the statistical significance can be estimated directly from Figure 3 simply by dividing each $\chi^2$ by 3. Thus the cross-correlation at N10-30 is significant at the 1% level (Pearson and Hartley, 1970). Clearly the hypothesis that interplanetary (high coronal) polarity is related by chance to chromospheric polarity on a scale of 10° must be rejected.
Figure 3. $\chi^2$ as a function of latitude for the comparison of interplanetary polarity (mapped back to the corona) and chromospheric polarity for nine solar rotations in 1965. For each 10° latitude swath on the sun, the contingency table (see Table 1 for an example) was constructed using first only definite, then also including the estimated Ha polarities of Figure 2. The larger $\chi^2$ at every latitude except S40-50 is due primarily to the larger number of points when the estimated polarities are also used. See text for significance.

Figure 4. The cross-correlation coefficient as a function of latitude for the same nine rotations as in Figure 3. As shown by Roelof and McIntosh, $\rho = \sqrt{\chi^2 / N}$, where $N$ is the number of data points. The correlation coefficients at each latitude are quite similar for the two studies (definite and estimated solar polarities). Therefore, either method may be used to determine the pattern of the latitude dependence of the correlation, since the estimated closure of neutral lines has not distorted the statistics.
Rotations 1489–97

- Definite Solar Polarity
- Estimated Solar Polarity

Figure 3
Rotations 1489–97

Figure 4
Schatten et al. (1969), using data from nine solar rotations from June 1965 through February 1966, were able to obtain a significant cross-correlation of mid-latitude solar fields calculated at a "source surface" 0.5 $R_\odot$ above the photosphere with the interplanetary field observed near the earth. Their results indicate that the pattern found in this study (best correlation of interplanetary polarity with solar polarity at latitudes removed from the equator) persisted throughout 1965.

A similar tendency for correlation of interplanetary with both northern and southern mid-latitude solar fields was deduced by Wilcox and Ness (1967) from a different line of reasoning. They compared the autocorrelation of latitude swaths of photospheric polarity with the autocorrelation of the interplanetary polarity for three different rotations (during Carrington rotations 1474-1477) but also near solar minimum. The patterns in the solar field autocorrelation at N10, N15, N20 and S20 are similar to the interplanetary pattern, i.e., they also find a good agreement between interplanetary and solar field autocorrelation at northern solar latitudes, and a weaker agreement between interplanetary and southern solar autocorrelations. Although the similarity of the statistical measures of southern and interplanetary fields in both studies may be the result of chance, the observation of this tendency for correlation in two different studies, using different techniques for different time periods, does suggest that the correlation is physically significant. It is of particular interest
to consider why the southern cross-correlation peak is observed, since solar activity was very weak in the southern hemisphere near solar minimum. One interpretation consistent with both this possible southern influence on the interplanetary polarity and the relative absence of strong-field solar active regions in the southern hemisphere at this time is that the large-scale mid-latitude chromospheric fields (both northern and southern) influence the equatorial interplanetary polarity, rather than the strong fields in mid-latitude solar active regions.

To test this interpretation I have examined the cross-correlation between interplanetary polarity mapped back to the corona and the polarities of solar active regions (as indicated by the occurrence of $H\alpha$ plage) at latitudes between $N10$ and $N30$. The correlation coefficient of 0.192 is smaller than the coefficients for both $N10-20$ ($\rho=0.299$) and $N20-30$ ($\rho=0.256$). Since the $H\alpha$ plage regions also occupy less than one-fourth of the longitudes at these latitudes, it is clear that the correlation previously found between mid-latitude solar polarity and interplanetary polarity is not due primarily to any agreement between active region and interplanetary polarity. This result also quantitatively supports the interpretation of Scherrer et al. (1972) that their best correlation (using a solar area $\sim 1/4$ disk) is due to large-scale regions rather than strong-field regions.
This conclusion suggests that the correlation at northern latitudes is not an indication of direct connection of the interplanetary field in the ecliptic to mid-latitude solar active regions. This suggestion is also supported by the results (presented in the next section) of the analysis of only those times when energetic protons were observed.

To further investigate the meaning of the southern cross-correlation peak, I have also cross-correlated the chromospheric polarities at N10-20 and S20-30 (the maxima in the latitudinal pattern of the cross-correlation with interplanetary polarity) at the same longitudes. The resulting $\chi^2$ is 10.2, demonstrating that the northern and southern chromospheric polarity data sets are not statistically independent. The interpretation of this result in terms of coronal magnetic field structure is discussed in Section 6.
4. Polarity Cross-Correlation during Times of Enhanced Energetic Particle Flux

The next part of this study is motivated by the realization that low energy ($\gtrsim 0.5$ MeV) solar protons are often not observable in the data from Mariner 4 for this same time period (Krimigis, 1969) even though these particles are often associated with centers of activity rather than specific solar flares (Fan et al., 1968; Krimigis, 1969; Krimigis et al., 1971; McDonald and Desai, 1971; Pick, 1972; Roelof and Krimigis, 1973). For instance, Fan et al. (1968) found that particle fluxes above detector threshold from a single solar active region could be observed near 1 AU over a spread of $\sim 180^\circ$ in heliocentric longitude.

Recently more detailed evidence for injection of low energy solar particles into the interplanetary medium at locations far removed from the active region accelerating source has been presented by Gold et al. (1973) and Innanen et al. (1973), who find, at two different times in 1967, that the energetic particles observed during an entire solar rotation were predominantly produced by a single active region and transported in the solar corona to the foot of the interplanetary field lines leading to the earth. Since these particles can be transported for large distances (at times completely around from the back side of the sun), and there are almost always active regions
visible on the sun even in 1965 (near solar minimum), 0.5 MeV protons might have been expected to be almost continually present in the interplanetary field. Therefore the absence of particles during much of this time period could indicate that they escape the corona preferentially from certain equatorial magnetic field configurations.

It is therefore reasonable to ask whether the polarity correlation differs in any significant way at times when low energy solar protons are observed from times when they are absent, since a different correlation pattern would imply a different "average" coronal magnetic field structure. To answer this question I have repeated the polarity correlation study, restricting it to times when 0.5 MeV solar protons were present in the interplanetary medium at flux levels >0.5 (cm²sec sr)⁻¹. These times totaled only one-sixth of the entire period.

The results of this study for χ² and ρ are shown in Figures 5 and 6. The change in the latitude dependence from the study using all the data from the same period is quite striking. The correlation now peaks strongly near the equator. The maximum χ² of 1.4 when the appropriate length scale is used is not very significant due to the reduced number of data points; however, the correlation coefficient (which is independent of the number of points) has increased from 0.18 to 0.28 and is now comparable to the maximum correlation coefficient (at N10-20) found in the study of this entire period.
Figure 5. Same as Figure 3, but restricted to times during the same nine solar rotations when fluxes of 0.5 MeV solar protons at Mariner 4 exceeded 0.5 (cm$^2$sec sr)$^{-1}$. The correlation as a function of latitude now peaks strongly near the solar equator.

Figure 6. Same as Figure 4, but again restricted to times when energetic particles were present in the interplanetary medium.
Figure 5

- Definite Solar Polarity
- Estimated Solar Polarity
Figure 6
More important than the absolute significance of the equatorial correlation found when particles were present is the change in the latitude dependence from the study of the entire time period. Not only has the equatorial correlation coefficient increased; the correlation at N10-20 and S20-30 (the maxima of the previous study) has almost completely disappeared! The maximum $\chi^2$ (0.067) at these mid-latitudes when energetic protons are present implies a probability of 79% that these chromospheric and interplanetary polarities are related by chance. This change is in striking contradiction to the interpretation that the correlation between solar mid-latitude polarity and interplanetary polarity in the ecliptic plane indicates direct connection of field lines from the mid-latitude solar regions to the equatorial interplanetary field. If 0.5 MeV solar protons are accelerated in the mid-latitude solar active regions, then the correlation at N10-30 when these particles were present should have been even better than the correlation in the general case.

To quantify the significance of the change in latitude dependence, I have calculated the frequencies of occurrence expected in the contingency table for each latitude, using the hypothesis that the subset of points obtained by restricting the study to times when particles were present is a representative sample of the general study. That is, each of the four frequencies ($n_{++}$, $n_{+-}$, $n_{-+}$ and $n_{--}$ in the sample Table 1) in the contingency table for each
latitude from the general study has been scaled by a factor: the number of points in restricted study divided by that number in the comprehensive study. The values of $\chi^2$ for each latitude from the comparison of these expected and the observed frequencies are shown in Figure 7. The change in the distribution of frequencies at each latitude is not too significant when the appropriate length scale is used. However, the sum of the $\chi^2$ from N40 to S40, a measure of the significance of the change in the entire pattern, is significant at the 2% level.

It is interesting to note that the significance of the change in the relative frequencies in the contingency tables is nearly independent of latitude. This independence is related to the primary cause of the different statistical properties of the correlation restricted to times when particles were present: during the general study, 60% of the measured interplanetary polarities were positive; during the particle events, 65% of the interplanetary polarities were negative. This observation, together with the significance of the change in pattern, estimated from the frequencies in all of the contingency tables, shows that the 0.5 MeV protons had a strong tendency to escape from a coronal field configuration different from the usual configuration in 1965. This observational result implies directly that the coronal magnetic field controlled the access of energetic solar protons into the interplanetary medium. Further discussion of the two kinds of field configurations is deferred until Section 6.
Figure 7. The latitudinal dependence of the significance of the change in polarity cross-correlation pattern from the comprehensive study to the study restricted to times when energetic protons were present. See text for significance.
Figure 7
One further aspect of both the study for the entire period and the study restricted to times when particles were observed is worth noting. In both studies, there is a correlation between high latitude fields, primarily N40-60, and the interplanetary polarity. Since the polarity regions at these latitudes usually extend uninterrupted for many tens of degrees in longitude, these high latitude field regions represent in a sense the large-scale solar field. The observed correlation therefore is not interpreted as indicative of the direct influence of these high latitude fields on the polarity of the interplanetary field in the ecliptic on the scale of ~10° appropriate for this study; rather, this correlation is another indication of the general correlation between very large-scale solar fields and the interplanetary polarity such as found by Scherrer et al. (1972) (averaging over ~1/4 of the solar disk), and by Severny et al. (1970), using daily solar average field measurements.
5. **Polarity Cross-Correlation during Times of Enhanced Solar Wind Velocity**

The results from the first two parts of this study show that there are (at least) two significantly different kinds of coronal magnetic field structures. These two kinds of structures could well be the "open" and "closed" structures suggested by Pneuman (1973), who also argues that solar wind should escape preferentially from open coronal magnetic field configurations. This argument is substantiated by the work of Noci (1973). He considers the energy budget in coronal "holes" and concludes that these magnetically open structures should be sources of strong solar wind. Krieger et al. (1973) provide observational support for this idea, finding that the source of a recurrent high speed solar wind stream is indeed the equatorial region of a coronal hole observed in an X-ray image of the sun. Further evidence for both the division of the corona into open and closed magnetic field regions, and the association of high speed solar wind with open regions is provided by Cuperman and Roelof (1973). They find that the dominant statistical relationship between solar wind velocity and coronal green-line emission is an anti-correlation at the appropriate lag (corrected for the interplanetary transit time of the solar wind). This anti-correlation is interpreted as a manifestation of the tendency
for fast solar wind to escape preferentially from open magnetic field structures, while enhanced green-line emission tends to be associated with the higher temperatures in closed magnetic configurations.

Since these studies show that strong solar wind may be associated with coronal magnetic field structures, I have also used a similar cross-correlation analysis during the same time period in 1965, but restricted to only those times when the solar wind velocity observed at Mariner 4 was greater than 400 km/sec. The latitude dependence of $\chi^2$ and $\rho$ for this study is shown in Figures 8 and 9. I have carried this study only to north and south 40° latitude, because the higher latitudes seem to reflect the large-scale field correlations (see Section 4).

The latitudinal pattern found here peaks near the solar equator, as is expected if high speed solar wind streams are associated with open (radial) coronal magnetic field configurations. The maximum $\chi^2$ (at N0-10) is again not very significant. However, the corresponding correlation coefficient (0.29) is comparable to the maximum coefficients found in the first two parts of this study.

A comparison of expected and observed frequencies in the contingency tables for this study yields a result (Figure 10) similar to that found in Section 4. The changes in the frequencies at each latitude are again not very significant, but the sum of these $\chi^2$, an indicator of the change in pattern, is significant at the 5% level.
Figure 8. Same as Figure 3, but restricted to times when solar wind velocity exceeded 400 km/sec. As in Figure 5, the correlation peaks strongly near the equator.

Figure 9. Same as Figure 4, but restricted to times when solar wind velocity exceeded 400 km/sec.
Figure 8
Figure 9
Figure 10. The latitudinal dependence of the significance of the change in polarity cross-correlation pattern from the comprehensive study to the study restricted to times of fast solar wind. See text for significance.
Figure 10
The interplanetary polarity during the fast solar wind streams was positive in 79% of the longitude bins. This dominance of positive polarity may not be significant, however, since the data is dominated by the recurrence of a single fast solar wind stream which originated near Carrington longitude 300° for six solar rotations (1490-1495). Over half of the longitude bins with solar wind faster than 400 km/sec were from this one recurrent stream.

This dominance of the data by one recurrent series, which makes the quantitative interpretation of statistical inferences somewhat uncertain, does, however, emphasize the principal result of this section: Fast solar wind does tend to come from a coronal magnetic field configuration different from the "average" configuration in 1965.

Before proceeding with the interpretation of this result, it is necessary to discuss a significant distortion of the time sequence of interplanetary data resulting from the application of the EQRH approximation to solar wind streams. Since in 1965 these streams represent only a small fraction (less than one-fifth) of the entire period, it is not necessary to correct for this distortion in either of the previous two sections (see below for a further discussion of particle events during solar wind streams).

This distortion is simply the rapid shift in connection longitude during the rising portion of the solar wind velocity time history and the slower than usual change during the decrease in velocity. During the rise, the
connection longitude often shifts by more than 10° in the three hours over which both magnetic field and solar wind velocity are averaged, while during the decrease, the connection point may move as little as 10° in several days (though this usually includes a decrease to velocities below 400 km/sec). The interplanetary polarity pattern is also much more likely to be distorted locally in the stream-stream interaction during the rise in velocity than in the rarefaction during the decrease. Thus the net effect of the application of the EQRH approximation to solar wind streams in the polarity cross-correlation is to emphasize the most uncertain interplanetary polarity measurements (during the velocity increase), while de-emphasizing the measurements at just those times when the EQRH approximation source locations are expected to be best, i.e., in the rarefaction following the peak of the high speed stream (Nolte and Roelof, 1973b).

To correct for this distortion, I have repeated the study of this section (restricted to times of fast solar wind), weighting each longitude bin by the length of time that the EQRH approximation connection longitude of the interplanetary field at Mariner 4 remained in that bin. The results of this weighted study are shown in Figures 11 and 12.

The pattern now peaks much more strongly near the solar equator, at a level of significance comparable to the maximum significance in the comprehensive study ($\chi^2 = 7.8$,
Figure 11. $\chi^2$ vs solar latitude for times of fast solar wind, but with each longitude bin weighted for the length of time the connection longitude remained in that bin. See text for significance.

Figure 12. Cross-correlation coefficient vs solar latitude for the same study as Figure 11.
Figure 11
Figure 12
Q(\chi^2) = .005). The maximum correlation coefficient (\rho = 0.53) is much larger than any in the previous studies reported here. This striking improvement in the equatorial correlation due to the weighting described above provides strong evidence that the interpretation of the unweighted study is correct: Fast solar wind exhibited a very strong tendency to come from a different kind of coronal magnetic field structure from the usual configuration during the first eight months of 1965. As with the energetic protons, this observational result immediately implies that the coronal magnetic field exerted a strong effect on the solar wind.

This result is easily interpreted in terms of open and closed magnetic field configurations (see the beginning of this section). Fast solar wind tends to come from open structures which extend nearly radially from the chromosphere out through the corona to the interplanetary medium, whereas the general study also includes closed structures. The highly significant change in the pattern of the latitude dependence from the general study to the time-weighted, fast solar wind study shows that the usual coronal field configuration at this time was closed. This inference of general coronal field configuration from interplanetary data will be compared to solar data in the form of H\alpha synoptic charts in the next section.

One final point requires some discussion: what is the relationship between enhanced solar wind velocity and 0.5 MeV solar proton increases? To answer this
question, I have studied those times when solar wind velocity was over 400 km/sec and 0.5 MeV protons were observed at Mariner 4. This data set consists of only eleven 10° bins in longitude, and is therefore inadequate to produce a statistically significant cross-correlation. However, it is worth noting that the occurrence of both enhanced solar wind and energetic protons in these eleven longitude bins could have been due to chance: since 54 out of 287 bins "contained" particles and 59 out of 287 had enhanced solar wind, a chance relationship between solar wind and particles would result in eleven bins with both. Furthermore, the solar protons are present in interplanetary field regions which are dominantly negative polarity, while the dominant polarity in the fast solar wind streams is positive. It therefore seems likely that fast solar wind and energetic solar protons escape preferentially from different coronal regions. This topic is discussed further in the next section.
6. **Interpretation of Cross-Correlation Results**

I have suggested above that the change in the latitudinal dependence of the cross-correlation of interplanetary with chromospheric polarity when the study is restricted to times when energetic particles and/or fast solar wind were observed is an indication of a different coronal magnetic field structure. I now wish to examine this hypothesis further.

The different coronal field structures have been identified here by their polarity signatures in the chromosphere. I have therefore used this polarity signature, defined by three chromospheric polarities, northern (N10-20), equatorial (0-S10) and southern (S20-30), to investigate the relationship of different coronal configurations to the interplanetary medium in the following manner.

The data from each of the four studies (comprehensive, and times of enhanced particle flux, fast solar wind, and fast solar wind with each longitude bin weighted by the time the connection point remained there) has also been divided into three subsets, based on the agreement, half-agreement (one polarity mixed) or disagreement of the interplanetary and equatorial polarities. Then, for each of these twelve subsets, and also for the four totals, the frequencies of occurrence of the four independent chromospheric polarity signatures (not considering a change in sign of all three
Then, using the hypothesis that the total from the comprehensive study is the set from which all subsets are randomly drawn, I have also calculated an expected frequency for each case. These expected and observed frequencies are shown in Table 2. Also shown are the values of $\chi^2$ calculated in each case where the expected frequencies are large enough for statistical significance.

The polarity structure in the two cases with significant deviations from the "normal" are shown schematically in Figure 13. The four arrow diagrams can be interpreted as follows.

In the first case, if the interplanetary and equatorial polarities disagree, it is significantly more likely than usual that both northern and southern polarities disagree with the equatorial polarity (and agree with the interplanetary polarity). On the other hand, it is less likely than normal that all three solar polarities agree, but disagree with the interplanetary polarity. Both of these situations seem reasonable. At those times when the interplanetary field is not extending radially out from the chromospheric fields near the equator, it must still connect somewhere nearby (within a few tens of degrees).

The other case is also quite reasonable. If there is fast solar wind and if the interplanetary and equatorial polarities agree, it is more probable than usual that all three solar polarities agree, and less likely that both
Using S 0-10 for Equatorial Solar Polarity

<table>
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$\chi^2=3.9$  
$\chi^2=2.6$  
$\chi^2=9$  
$\chi^2=5.3$

Table 2a
### Fast Solar Wind

#### Unweighted

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**Total:** 31

\[ \chi^2 = 9.8 \]

#### Weighted

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**Total:** 43

\[ \chi^2 = 9.8 \]

#### I & E agree

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**Total:** 17

\[ \chi^2 = 1.3 \]

#### I or E mixed

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**Total:** 11

\[ \chi^2 = 1.3 \]

#### I & E disagree

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<tr>
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**Total:** 11

\[ \chi^2 = 2.8 \]

#### Total

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**Total:** 59

\[ \chi^2 = 2.8 \]

#### Table 2b

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**Total:** 82.5

\[ \chi^2 = 2.8 \]
Figure 13. Comparison of chromospheric polarity signatures with interplanetary polarity for those structures which show significantly different statistical properties from normal. The top two diagrams illustrate that if the interplanetary polarity disagrees with the equatorial polarity, it is more likely than usual that both northern and southern polarities agree with the interplanetary field. The bottom diagrams demonstrate that when the interplanetary and equatorial polarities agree in a fast solar wind stream, it is more likely that the source is a large unipolar region (stretching from northern to southern mid-latitudes) than a small polarity cell.
More than chance

Comprehensive (I & E disagree)

Less than chance

Solar Wind (I & E agree)

N = 10°-20°N
E = 0°-10°S
S = 20°-30°S

ROTATIONS 1489 - 97
(Dec 1964 - Aug 1965)
northern and southern disagree with the equatorial polarity. That is, fast solar wind propagating directly out from an equatorial solar region is more likely to come from a large, presumably open unipolar field region than from a small polarity cell.

Two other aspects of this table are worth noting. Firstly, for the study when particles were present, there is no strong evidence of coronal structure different from the usual. However, as found in Section 4, the polarity is an important signature of the differing structure in this case, so this result is not too surprising. Secondly, there is no significant tendency for the mixed polarity subsets to be different from normal. The mixed polarities are primarily interplanetary, and this result implies that these interplanetary mixed polarities may be randomly associated with coronal field structures (as indicated by their chromospheric polarity signatures), i.e., the interplanetary mixed polarities are generated by interplanetary, not solar, processes.
7. **The Low Energy Solar Proton Increases**

As a final step in this analysis of solar and interplanetary data from near solar minimum I shall test whether the principal conclusion concerning energetic particle propagation from the statistical study, namely that low energy solar proton propagation was greatly influenced by the coronal magnetic field structure, is consistent with the observations of the individual events.

The overall picture is shown in Figure 2 (preceding page 27), to which I again direct the reader's attention. Note that all of the energetic particle enhancements except the flare-associated increase on May 25 (Carrington longitudes 170-185 on Rotation 1494) occur in three "recurrent" series. The January 8 and February 5 increases are both observed from a high coronal region including 180° to 210° Carrington longitude. On the three consecutive rotations 1493, 1494 and 1495 there are particles observed from a region near 90°. Finally, on rotations 1495, 1496 and 1497, particles are observed on field lines connected near 250°.

These series are not recurrent in the usual sense, as can be seen from the time history of the Mariner 4 proton observations, shown in Figure 14 (from Krimigis, 1969). The January and February events occurred on Bartels rotations 1799 and 1800. The February event is flare-associated, and therefore not a second observation
Figure 14. The time history of 0.5 MeV protons observed at Mariner 4 (from Krimigis, 1969). Here the detector counting rates are plotted by Bartels rotations.
Figure 14
of the same particle population. Similarly the series between days 10 and 15 on Bartels rotations 1803 to 1805 (the series from longitudes near Carrington 90°), is not a simple decay of a long-lived event. Both the first and second events of the last series (near day 25 of Bartels rotations 1804 to 1806) are flare-associated, and therefore not simply recurrent. However, the striking tendency for occurrence of particle events from the nearly same longitudes is an argument for a recurrence of a coronal region which preferentially permits the escape of energetic particles into the interplanetary medium.

The hypothesis that the coronal magnetic field configuration dominates the observed time history of energetic particle events is quite strongly supported by analysis of the January 8, 1965 event. This increase is a "precursor" of the largest event of the entire period, the February 5 flare associated event.

Figure 15 shows the three-hour averages of the counting rate of University of Iowa detector on Mariner 4 for protons between 0.50 and 11 MeV plotted vs. Carrington connection longitude. This is the particle data to be compared with the indicated interplanetary field polarity and the $H_\alpha$ synoptic chart for Carrington rotation 1489.

The negative polarity cell crossing the equator between 240° and 195° Carrington is in good agreement with the interplanetary polarity at the leading edge. The poorly marked eastern edge of this region seems to be almost
Figure 15. Three-hour averages of the University of Iowa detector $D_1$ counting rates from Mariner 4 for January 8-13, 1965, plotted at the EQRH-approximation connection longitudes of the interplanetary field lines through the spacecraft.
Counts/sec.

Mariner 4
Jan. 7-13, 1965

0.50 ≤ \( E_p \) ≤ 11 Mev

Inflight Source
Background

Counts/sec.

Connection Longitude

Figure 15
20° removed from the eastern boundary of the mapped back interplanetary sector boundary. This is an example of weak equatorial Hα features in a region where the relatively weak equatorial chromospheric field does not map faithfully up to the high corona.

The origin of this particle increase is uncertain. Neither Krimigis (1969) nor O'Gallagher and Simpson (1966) find a reliable flare association for this event. Krimigis suggests two possible visible sources for these particles: the 1+ flare in region 7626 (260, N20) at ~0830 January 6, or region 7630 (140, N20), which produced a number of small flares and radio bursts during its disk passage. The alternative explanation offered is acceleration in a flare on the invisible hemisphere, followed by eventual corotation to the field lines connected to the spacecraft.

Both the relatively slow onset of the increase on January 8 coincident with an interplanetary field reversal from positive to negative, and the sharp drop to background on January 12 and 13, coincident with the reversal of the interplanetary polarity back to positive, strongly suggest that the event is a quasi-stationary corotating particle increase. Thus, specific association with the January 6 flare may not be correct. However, if this association is correct, the observation of the increase, which is confined to a negative sector of the interplanetary field, is delayed until two days after the flare.

Next, suppose the particles were accelerated in 7630 (either continuous acceleration, or repeated, small impulsive
events). There is no observable increase when the field line through the satellite passes the longitude of the active region 7630, even though this region produces additional small flares on January 11, 13 and 14. Thus, if this region is the source of the observed particles, the increase is again confined to a negative interplanetary field sector 30° to 90° in longitude distant from the active region source.

The final alternative source of these particles is an active region or flare on the invisible hemisphere, followed by high coronal storage, and corotation of the "leaking" storage region (which must be located in the negative polarity region between 180° and 240° Carrington) to the foot of the field line through Mariner 4. The activity during the previous solar rotation suggests only one other potential active region source for the observed particles, region 7606, which returns as regions 7629b and 7631. This region is also considerably removed from the position of release of these particles into the interplanetary field; thus if a storage region for particles accelerated in a backside flare was releasing the observed protons, there must be a storage region releasing particles only into the negative polarity region of the corona.

Thus all three possibilities for the source of these particles imply control and ordering of the release of 0.5 MeV solar protons into the interplanetary medium by the coronal magnetic field.
Following a solar flare of importance 2 on February 5, 1965, at ~1800 UT, the largest solar particle event of the entire period occurred. This flare also had the highest comprehensive index (9) for all flares between January and September 1965 (Dodson and Hedeman, 1971). This event has been discussed extensively in the literature (see references above), and the flare association is well documented. However, the availability of solar wind plasma data from the separated spacecraft provides significant additional information which can be used to interpret the observations of this event. The discussion here will be restricted to the particle data from the University of Iowa detector on Mariner 4, which supplies nearly continuous low energy data throughout the event (Krimigis and Van Allen, 1967; Krimigis, 1969), and the University of Chicago detectors on Mariner 4 and IMP 2, which provide nearly continuous data from separated comparable detectors (O'Gallagher and Simpson, 1966; O'Gallagher, 1970). There is, however, a more recent recalibration of the threshold of the IMP 2 detector (Englade, 1971), so that these data are somewhat uncertain.

In Figure 16 the particle data from the University of Iowa Mariner 4 detector during the February 5 event is presented, using the same format as used for the January 8 event in Figure 15. The event onset occurs while Mariner 4 is connected within the recurring negative polarity sector which contained the (quasi-stationary) particle increase
Figure 16. Same as Figure 15, but for February 3-13, 1965.
Mariner 4
Feb. 3-13, 1965

0.50 ≤ E_p ≤ 11 Mev

Counts/sec.

Connection Longitude

Inflight Source

Background
of January 8-13. Thus the January event was in a sense the precursor of the February 5 event; however, the accelerating source of the protons in January must have been different from the source of the February 5 event since the flare region, McMath plage 7661 (160, N08), was not seen on the previous rotation.

It is interesting to note that the decay rate of the particle intensity increases sharply as the field line connection point shifts to the longitude of the flare (the first decrease on February 9). During the previous day, while the connection point changed only slightly (called a solar wind "dwell" by Gold et al., 1973) the counting rate decreased slowly. Although it is impossible to rule out completely a temporal change in the source of the particles (whether the "source" was continuing acceleration in region 7661 or a storage region in the high corona), this change in decay rate suggests a spatial (longitudinal) gradient of energetic solar protons in interplanetary space. If this is the case, late in the event fewer particles were observed coming from the flare site than from regions nearby.

Further support for the suggestion that the change in decay rate is a spatial, rather than temporal, effect is supplied by a comparison with the interplanetary magnetic field. The first sharp decrease on February 9 occurs in near coincidence with a change in interplanetary polarity from negative to mixed (as assigned by Coleman et al., 1967). Thus, if the decay was purely temporal, a sudden change in
either the acceleration or release rate of 0.5 MeV protons must have coincided with the change in field polarity.

Since such a coincidence is unlikely, I suggest that this decrease in counting rate is the result of an interplanetary longitudinal gradient of these particles.

If the source of the particles is either a continuing acceleration process, of duration of ~5 days, or a storage region located above (or near) the flaring active region, the low fluxes from the flare site imply preferential release of energetic particles from "selected" coronal regions. The strong fields in the active region itself may imply a closed configuration; thus, it is not impossible that low energy solar protons may be preferentially released into the interplanetary field near the active region, and not as effectively released directly from the region itself.

Data from the University of Chicago telescopes on IMP 2 and Mariner 4 for protons of energy greater than ~15 MeV (from O'Gallagher, 1970) are replotted vs connection longitude in Figure 17. I have multiplied the IMP 2 points by a factor of 1.5 to attempt to correct for the recalibration of the IMP 2 detector threshold referred to by Englade (1971). Since this correction is somewhat uncertain, an absolute comparison of the fluxes at the two spacecraft is not possible.

The plot does show, however, that there is definite solar wind structure at this time, which may affect the observed profiles. There are two solar wind "dwells," one
Figure 17. Data from the University of Chicago telescopes on Mariner 4 and IMP 2 from February 5-8, 1965, (O'Gallagher, 1970) plotted vs EQRH connection longitudes. The IMP 2 counting rates have been adjusted so that both data sets represent (approximately) flux of protons of energy greater than 15 MeV.
Figure 17
between 200° and 210°, the other near 180°. Between 1200 and 1800 UT on February 6 IMP 2 shifts from one stream to the other; the shift at Mariner 4 is on February 7, between 0300 and 1200 UT.

Any longitudinal structure which is present might be expected to show up as a change in the ratios of the counts in the two detectors while either is switching from the first solar wind stream to the second. In Figure 18 I have marked these times on a plot of the ratio of the counts at IMP 2 to the counts at Mariner 4 (from O'Gallagher, 1970). Due to the recalibration of the IMP 2 detector threshold, only changes in this ratio, and not the magnitude, are significant.

By the time Mariner 4 switches streams, the flux levels are too low to show a significant change in the ratio. However, the sharp change in the ratio as IMP 2 changes streams does provide evidence that there was a gradient of particle fluxes across magnetic field lines in the interplanetary medium at this time.

This inferred gradient just to the east of the first stream provides an explanation for the significantly lower fluxes of 0.5 MeV protons observed early on February 7 (near the maximum of the event) by the University of Iowa detector on Mariner 4 (Figure 16). This one three-hour average plotted at 200° Carrington longitude is 40% lower than the points on either side, consistent with a reduced access of particles to the interplanetary medium from the region between the two solar wind dwells.
Figure 18. The ratio of the counts at IMP 2 to the counts at Mariner 4 vs time (from O'Gallagher, 1970). The times when the connection longitudes of the two spacecraft were switching from one solar wind stream to another are also marked.
The University of Chicago

Ratio of Proton intensity of IMP II to that at Mariner IV for the 5 February 1965 Solar Proton Event.

Figure 18
By the time the spacecraft were connected to the longitude of the active region, the higher energy event had decayed below background. It is therefore not possible to use multiple spacecraft data to determine whether the decrease in the low energy proton counting rate at Mariner 4 on February 9 is indeed a spatial gradient, as suggested above.

In summary, there is evidence that the coronal magnetic fields, at least to some extent, controlled the release of flare-associated energetic protons into the interplanetary medium during February 1965. The data available are not adequate to determine accurately the degree to which the field structures influenced the observed interplanetary proton fluxes.

The next event with flux of 0.5 MeV protons greater than 0.5 (cm² sec sr)⁻¹ was a small increase, primarily on May 6 and 7 (from Carrington longitudes 65°-105° on rotation 1493) in the University of Iowa detector counting rate. Peak flux of protons $E_p > 0.5$ MeV was $0.8 \pm 0.2$ (cm² sec sr)⁻¹ (Krimigis, 1969). The interplanetary field remains negative during the entire increase. There are two possible active region acceleration sources for these protons--regions 7799 (75, N35) and 7794 (15, N30). Region 7799 was located at the longitude from which the particles were seen (see Figure 2), but 7794 was considerably more active. One of the five flares during this period, classified as major by Dodson and Hedeman (1971), took place in 7794 on May 1 at
1430 UT. Two other flares on May 2 also produced short wave fadeout (Solar Geophysical Data). Thus it is more likely that 7794 had accelerated particles.

The increase did not persist throughout the negative sector; however, the event terminates just prior to a definite increase in field strength. Also, the termination of the event is sharper than the onset (Figure 14). Thus it seems likely that the event was primarily spatial rather than temporal.

The identification of 7794 as the source of this quasi-stationary event, occurring within a single polarity interplanetary field region, provides evidence that the large-scale coronal magnetic field configuration controlled the release of these particles into the interplanetary magnetic field.

On the next solar rotation (Carrington 1494) there were two periods of enhanced 0.5 MeV proton fluxes (Figure 2). The first increase, on May 26-27, was associated with a "major" flare at 2240 on May 25 in region 7809 (200, N25). At the time of the flare, Mariner 4 was connected near the longitude of this region. This event was primarily an electron event (Van Allen and Krimigis, 1965).

A pair of events in the time period from June 1-8 presents an interesting comparison of the Iowa detector on Mariner 4 and the University of Chicago detectors on Mariner 4 and IMP 3. In Figure 19 I show the counting rates of these three detectors plotted vs connection longitude. The
Figure 19. The counting rates of the 0.5 MeV proton Iowa detector on Mariner 4 (top panel); the Chicago 1 MeV proton channel on Mariner 4 (middle panel); and the Chicago 1 MeV proton channel on IMP 3 for June 1-8, 1965, plotted vs EQRH connection longitudes.
Mariner 4
May 25 - June 4, 1965

Counts/Second

Mariner 4
June 1-7, 1965
Ep > 1 MeV

Counts/second

IMP 3
June 1-7, 1965
Ep > 1 MeV

Figure 19
first event, a gradual increase in the ≥0.5 MeV proton flux observed by the Iowa detector, began on June 1, coincident with the appearance on the disk of active region 7840 (20, S10) and 7842 (40, S10). The detector on Mariner 4 showed only a slight increase in the flux of protons £p≥1 MeV, while a similar detector on IMP 3 showed no increase above background. The second event was a flare-associated electron event (reported by several authors), associated with a "major" flare (index of 8, the second highest major flare index in the first eight months of 1965) in region 7842. The sensitivity of the Chicago detectors to 200 keV electrons is demonstrated by this event.

The coincidence between the birth of the region which later produces energetic particles, and the increase on June 1 in the proton flux, suggests that 7842 was also the source of the earlier increase. The lack of protons from the active region longitude at a time when energetic electrons were present may again be indicative of preferential release of protons at locations far from the accelerating source.

On Carrington rotation 1495 there were also two periods of enhanced proton fluxes, noted in Figure 2. The first period was a complex series of events, which began with a gradual increase in the 0.5 MeV proton counting rate on Mariner 4 on June 11, and included particles accelerated in two distinct flares, which propagated in an evolving solar wind/interplanetary field configuration. Particle
data from this time period are shown in Figure 20, in the same format at Figure 19.

The two flares were a "major" flare at 0330 on June 13, and a 1+ flare at 0745 on June 15, both in active region 7847 (260, N20). Krimigis (1969) notes a sudden change in the spectral characteristics near the end of June 15, associated with an increase in particle flux. The double peak in the Chicago data from Mariner 4 (June 13 and 15), not observable in the Iowa data, is again due to electron contamination following the June 13 flare. Electrons were observed by Van Allen and Krimigis (1965) and Lin and Anderson (1967).

The evolving solar wind configuration swept the near earth connection point into the region of enhanced particle fluxes after the second flare; the IMP 3 (and OGO 3) data therefore show a single major maximum. It is interesting to note that although a steady solar wind velocity of 400 km/sec would have put the IMP 3 interplanetary field connection point ~15° to the west of the Mariner 4 connection point, the observed solar wind velocities in this evolving configuration put IMP 3's connection point 20° to the east of Mariner 4's on day 167 (June 16)! It is clearly important to use solar wind velocity data from each spacecraft whenever spacecraft become widely separated.

During the other period of interest on rotation 1495, June 28 to July 6, the time history of the proton fluxes was also quite involved. Figure 21 shows the data from
Figure 20. Same as Figure 19, for June 11-20, 1965.

Figure 21. Same as Figure 19, for June 28-July 7, 1965.
Figure 20
Figure 21
The three detectors. The most likely source of these particles is region 7878 (10, N30), which produced a 2+ flare on June 28 at 1030 UT.

The Iowa data provides an indication that there may have been three distinct interplanetary regions with different particle accessibility. The boundaries of these regions, near 50° and 15°, corresponded nicely with equatorial Hα polarity boundaries. This suggests agreement with the hypothesis that low energy solar particle access to the interplanetary medium was controlled by a magnetic field structure with a well-defined chromospheric polarity signature (the conclusion from the statistical study).

The Chicago time-history data (Figure 22) demonstrates the effects of temporal changes in the solar wind on observations of energetic particles. At Mariner 4 the solar wind speed was relatively steady near the onset of the particle event, and Mariner 4 saw a gradual rise in intensity. Near earth there was a sudden change in solar wind velocity, and the region of enhanced fluxes swept rapidly over the IMP 3 detector. This variability in solar wind speed also explains the discrepancy between the observed and calculated delay between the maximum intensities at the two spacecraft (O'Gallagher and Simpson, 1966). This event is an example of a field-aligned particle event contained within an evolving solar wind/interplanetary field configuration such as described by Nolte and Roelof (1973b).

The only enhancement observed on rotation 1496 was a flare associated increase which began just prior to the
Figure 22. Time histories from the 1 MeV proton channels of the Chicago detectors on Mariner 4 and IMP 3 for June 28-July 4, 1965 (from O'Gallagher, 1970).
turn-off of the Mariner 4 detectors for encounter with Mars. The flare at 1100 on July 13 (in region 7886, at 270, N20) occurred when Mariner 4 was connected nearby. Neither Chicago detector observed a large increase.

The final period of increased fluxes occurred on August 3-9 at Mariner 4 (Carrington rotation 1497). These particles were probably related to the recurrence of region 7886 (270, N20). The Iowa detector observed a small increase on the third and fourth, and a larger increase coincident with an increase in the Chicago counting rate on August 7-9 (Figure 23). IMP 3 didn't detect an increase; however, an evolving solar wind configuration swept the interplanetary region containing the enhanced particle flux past the earth on day 219 (August 7), just when IMP 3 was in the magnetosphere. This interpretation can only be made using solar wind data from both spacecraft, since a steady solar wind of 400 km/sec would cause IMP 3 to observe the increase 3 1/2 days after Mariner 4, instead of 1 day before.

I summarize this reexamination of the individual particle events briefly. None of the events contradict the conclusion of the statistical study, that the coronal magnetic field (generally) controls the release of low energy solar protons into the interplanetary medium. Three of these events (January 8-13, May 6-7 and June 28-July 6) show strong evidence of this coronal control. The others each show some evidence of preferential release. The June 13-15 and August 7-9 periods also show large effects
Figure 23. The counting rates of the Iowa 0.5 MeV and the Chicago 1 MeV proton detectors on Mariner 4 plotted vs EQRH connection longitude for August 3-9, 1965. The IMP 3 counting rate remained near background.
Figure 23
on the observed time histories of particle fluxes due to evolving solar wind configurations.
CHAPTER III

THEORY OF SCATTER-FREE PROPAGATION
OF ENERGETIC PARTICLES

1. Introduction

It has been well-documented in the literature that current theories of energetic particle transport do not adequately explain the observations of particles of energy below a few MeV per nucleon (see references in Chapter I). Here I present a calculation of predicted idealized detector counting rates and anisotropy measurements, based on the assumption of no scattering of low energy solar particles in a region extending from the high solar corona to beyond the earth.

I shall show, using a simplified model for the injection of these particles at the sun, and the reflection at the outer boundary of the scatter-free region, that the scatter-free propagation theory produces a wide range of time histories of both flux and anisotropy near 1 AU, and that these theoretical predictions are in semi-quantitative agreement with events which have been reported in the literature. However, the coronal and solar wind structures which dominate the time histories of low energy events (see Chapter I) mask the effects of propagation along a single field line in the data from a single spacecraft. In order
to make a detailed comparison of theoretical predictions with data, it is therefore necessary to first separate the temporal and longitudinal effects by using multiple spacecraft particle flux and anisotropy data and field and plasma data. When it becomes possible to compare this theory with such detailed multi-faceted multiple spacecraft data, it will also be necessary to investigate the effects of a more realistic modeling of the injection and reflection of the energetic particles. For the present work it is sufficient to show that the most prominent aspect of the single spacecraft data, the time history of the anisotropy near the onset of the events, is described semi-quantitatively by this simplified scatter-free propagation theory.

The situation is described schematically in Figure 1. Particles are injected at an inner boundary $x_1$ (at the sun) into the scatter-free region, and propagate to the outer boundary $x_2$. Here they enter the scattering region, and may be reflected. The modeling of both injection and reflection is discussed in Section 3.
Figure 1. A schematic representation of the scatter-free propagation theory. Particles are injected at the sun (x₁) and travel through the scatter-free region to x₂, where they may be reflected or transmitted.
Figure 1

SUN

$X_1$  Scatter - free  Propagation Region

X

X$_2$  Scattering Region

In any region through which energetic charged particles propagate, if there is no scattering (i.e., randomization of the particles' velocities), the dynamical trajectories are deterministic. It is therefore appropriate to begin a scatter-free propagation theory from a mathematical expression of Liouville's theorem: Phase space density along a dynamical trajectory is a constant.

With Roelof (1973) I shall use the coordinates \((x, \phi, \theta)\) for position, with \(x\) the distance from the sun along an ideal spiral field line, and \(\phi\) and \(\theta\) the azimuthal and polar angles respectively. The momentum coordinates are chosen to be velocity \(v\), cosine of the pitch angle \(\mu\) and the angle \(\phi\) of rotation about the field line. Then for a dynamical trajectory with coordinates \((x, \phi, \theta; v, \mu, \phi; t)\) at time \(t\) and \((x', \phi', \theta'; v', \mu', \phi'; t')\) at time \(t'\), Liouville's theorem can be explicitly stated

\[
W(x; \phi, \theta; v, \mu, \phi; t) = W(x', \phi', \theta'; v', \mu', \phi'; t')
\]

(2-1)

In two papers, Roelof (1973 and 1974) has argued that the dependence on \(v, \phi\) and \(\theta\) (other than the motion of the field lines themselves) can be neglected in the discussion of solar particle events. Additionally the dependence on azimuth about the field line will be assumed
to be negligible, or if not, averaged over, so that the expression of Liouville's theorem which is applicable to this first discussion of scatter-free propagation in the ideal spiral field may be written

$$W(x, \mu, t) = W(x', \mu', t')$$  \hspace{1cm} (2-2)

if \((x, \mu, t)\) and \((x', \mu', t')\) are on the same dynamical trajectory.

The first adiabatic invariant \([1 - \mu^2(x)]/B(x) = \text{constant}\) determines a relationship between \(\mu\) and \(\mu'\)

$$\mu' = \frac{\mu}{|\mu|} \left[1 - \frac{B(x')}{B(x)} (1 - \mu^2)\right]^{1/2}$$  \hspace{1cm} (2-3)

The times \(t\) and \(t'\) are related by the transit time

$$\tau_{x, x'}(\mu) = t' - t, \text{ with } \tau_{ab}(\mu_a) \text{ determined from}$$

$$v_{\tau_{ab}}(\mu_a) = \left[\int_{x_a}^{x_b} \frac{dx'}{\mu(x')} \right] = \left[\frac{\mu_a}{|\mu_a|} \int_{x_a}^{x_b} \frac{dx'}{\sqrt{1 - \frac{B(x')}{B(x)}} (1 - \mu^2_a)} \right]$$

(2-4)

Since the interplanetary field diverges with distance from the sun, at any point \(x\) the pitch angle distribution may be separated into three parts (see Figure 2): Region a, which is given by

$$\sqrt{1 - \frac{B(x)}{B_1}} \leq \mu(x) \leq 1$$  \hspace{1cm} (2-5)

and particles are coming out from the sun; region b where

$$-1 \leq \mu(x) \leq -\sqrt{1 - \frac{B(x)}{B_1}}$$  \hspace{1cm} (2-6)
Figure 2. The three pitch angle regimes in the scatter-free region. Particles approaching a detector with pitch angles in region a are coming from the sun ($x_1$); those in region b will return to the sun; the reflected particles which either have mirrored or will mirror have pitch angles in region c.
which consists of all particles which will return to the sun; and region c, with

\[-\sqrt{1 - \frac{B(x)}{B_1}} < \mu(x) < \sqrt{1 - \frac{B(x)}{B_1}} \tag{2-7}\]

which contains those particles which mirror before reaching the inner boundary \(x_1\).

With \(\tau_1\) and \(\tau_2\) defined by

\[\tau_1 = \int_{x_1}^{x} \frac{dx'}{v_\mu(x')} \tag{2-8}\]

and

\[\tau_2 = \int_{x}^{x_2} \frac{dx'}{v_\mu(x')} \tag{2-9}\]

the pitch angle distribution at point \(x\), in regions a and b is given by

\[W(x, \mu, t) = W_1[\mu_1(x, \mu), t-\tau_1(x, \mu_1)] \tag{2-10}\]

\[= W_2[\mu_2(x, \mu), t+\tau_2(x, \mu_2)] \tag{2-11}\]

In region c, the distribution is

\[W(x, \mu, t) = W_2[\mu_2(x, \mu), t+\tau_2(x, \mu)] \tag{2-12}\]

\[= W_2 \left\{ \mu_2(x, \mu), t+\tau_2(x, \mu)-2\tau_m[\mu_2(x, \mu)] \right\} \tag{2-13}\]

where \(W_i(\mu_i, t) = W(x_i, \mu_i, t)\), and \(\tau_m(\mu)\) is the mirroring time for a particle of pitch angle \(\mu_2\) at \(x_2\). These equations specify \(W(x, \mu, t)\) completely if \(W_2(\mu_2, t')\) is known.
It is therefore necessary only to determine the pitch angle distribution $W_2(\mu_2, t)$ at the outer boundary $x_2$. The equation relating $W_1(\mu_1, t)$ and $W_2(\mu_2, t')$ in regions $a$ and $b$ is

$$W_1(\mu_1, t) = W_2[\mu_2(\mu_1), t + \tau_{12}(\mu_1)] \quad (2-14)$$

while in region $c$,

$$W_2(\mu_2, t) = W_2[-\mu_2, t - 2\tau_m(\mu_2)] \quad (2-15)$$

At $x_1$, for $0 \leq \mu_1 \leq 1$ (the outgoing component of $W_1(\mu_1, t) = W_1^+(\mu_1, t)$) the distribution may be expressed generally as a source (injected flux) term plus a reflection term

$$W_1^+(\mu_1, t) = \frac{J_1(\mu_1, t)}{v} + \int_{-1}^{0} d\mu' \int_{0}^{t} dt' R_1(\mu_1, \mu', t - t') W_1^- (\mu', t') \quad (2-16)$$

where $J_1(\mu_1, t)$ is the injected (outgoing) flux, $R_1(\mu_1, \mu', t - t')$ is the probability that a particle which approaches $x_1$ from the right in Figure 2 with pitch angle $\mu_1$ at time $t'$ will be reflected into pitch angle $\mu_1$ at time $t$, and $W_1^- (\mu', t') = W_1(\mu', t')$ for $-1 \leq \mu' \leq 0$.

Using the same general form and notation, the inward directed component of $W_2(\mu_2, t) = W_2^- (\mu_2, t)$ (i.e., $-1 \leq \mu_2 \leq 0$) at $x_2$ may be expressed as

$$W_2^-(\mu_2, t) = \frac{J_2(\mu_2, t)}{v} + \int_{0}^{1} d\mu' \int_{0}^{t} dt' R_2(\mu_2, \mu', t - t') W_2^+ (\mu', t') \quad (2-17)$$
I next apply the Laplace transform to equations 2-14 to 2-17, and solve the resulting system of equations in $w_1^+(\mu_1,s)$, $w_2^+(\mu_2,s)$ for $w_2^-(\mu_2,s)$. The transformed equations are

\begin{align*}
  w_1(\mu_1,s) &= e^{s\tau_{12}(\mu_1)}w_2[\mu_2(\mu_1),s] \\
  w_2(\mu_2,s) &= e^{-2s\tau_m(\mu_2)}w_2(-\mu_2,s) \\
  w_1^+(\mu_1,s) &= \frac{j_1(\mu_1,s)}{v} + \int_{-1}^{0} d\mu_1 r_1(\mu_1,\mu_1',s)w_1^-(\mu_1',s) \\
  w_2^-(\mu_2,s) &= \frac{j_2(\mu_1,s)}{v} + \int_{0}^{1} d\mu_2 r_2(\mu_2,\mu_2',s)w_2^+(\mu_2',s)
\end{align*}

Separating the integral over $d\mu_2'$ into integrals over region a and region c, and substituting from equation 2-18 in region a results in

\begin{align*}
  w_2^-(\mu_2,s) &= \frac{j_2(\mu_2,s)}{v} \\
  &+ \int_{-1}^{1} d\mu_2' r_2(\mu_2,\mu_2',s) e^{-s\tau_{12}[\mu_1(\mu_2')]} w_1^+[\mu_1(\mu_2'),s] \\
  &+ \int_{0}^{1} d\mu_2' r_2(\mu_2,\mu_2',s) w_2^+(\mu_2',s)
\end{align*}

Equation 2-19 may be used to rewrite the integral over region c in terms of $w_2^-(\mu_2',s)$.
\[ \int_0^{\mu_{12}} d\mu_2' r_2(\mu_2', \mu_2, s) w_2^+(\mu_2', s) \]

\[ = \int_0^{\mu_{12}} d\mu_2' r_2(\mu_2', \mu_2, s) e^{-2s\tau_m(\mu_2')} w_2(-\mu_2', s) \]  \hspace{1cm} (2-23)

\[ = \int_0^{\mu_{12}} d\mu_2' r_2(\mu_2', -\mu_2, s) e^{2s\tau_m(\mu_2')} w_2(\mu_2', s) \]  \hspace{1cm} (2-24)

For region a, substitution from equations 2-20 and 2-18 results in

\[ \int_{\mu_{12}}^1 d\mu_2' r_2(\mu_2', \mu_2, s) e^{-s\tau_{12}[\mu_1(\mu_2')] v} w_1[\mu_1(\mu_2'), s] \]

\[ = \int_{\mu_{12}}^1 d\mu_2' r_2(\mu_2', \mu_2, s) e^{-s\tau_{12}(\mu_2')} \left\{ \frac{j_1[\mu_1(\mu_2'), s]}{v} \right\} \]

\[ + \int_{-1}^0 d\mu_1' r_1[\mu_1(\mu_2'), \mu_1, s] w_1(-\mu_1', s) \]  \hspace{1cm} (2-25)

\[ = \int_{\mu_{12}}^1 d\mu_2' r_2(\mu_2', \mu_2, s) e^{-s\tau_{12}(\mu_2')} \left\{ \frac{j_1[\mu_1(\mu_2'), s]}{v} \right\} \]

\[ + \int_{-1}^0 d\mu_1' r_1[\mu_1(\mu_2'), \mu_1, s] e^{s\tau_{12}(\mu_1')} w_2[\mu_2(\mu_1'), s] \]  \hspace{1cm} (2-26)

so that equation 2-22 may be rewritten as
This is a completely general form for the function $w_2(u_2,s)$, in terms of flux injected at the two boundaries, the reflection properties at the boundaries, and the dependence of the magnetic field on distance, which enters through the delay times $\tau_{12}(\mu)$ and the mirroring time $\tau_m(u_2)$.

Since the Laplace transform is linear, each term in equation 2-27 may be individually interpreted. The first term ($j_2/v$) represents particles injected at the outer boundary. The next term represents particles injected at the inner boundary ($j_1/v$) which travelled to the outer boundary ($e^{-s\tau_{12}}$), and were reflected ($r_2$). The third term represents the incoming distribution in region $b(w_2^-)$ which travels to the sun ($e^{s\tau_{12}}$), is reflected ($r_1$), returns to the outer boundary ($e^{-s\tau_{12}}$) and is again reflected ($r_2$). The last term represents the part of the incoming distribution at the outer boundary ($w_2^-$) which mirrors and returns ($e^{2s\tau_m}$) and is reflected ($r_2$).
In this form, by appropriate choice of the injection and reflection functions (i.e., the boundary conditions) equation 2-27 may be used to describe any problem of the form of Figure 2 (a scatter-free region of propagation of energetic particles in a diverging magnetic field, with boundaries at \( x_1 \) and \( x_2 \)). The specialization to the impulsive onset solar particle problem is discussed next.
3. **Special Cases, Numerical Solution**

The general equation (2-27) can be reinverted only when the functions describing the injection and reflection at both boundaries are specified; if these functions are of a rather general form, clearly the inversion can be done only in an approximate numerical manner, since \( w_2^- (\mu, s) \) appears on both sides of the equation. There is, however, a rather weak restriction which can be applied, which permits an exact solution for \( w_2^- (\mu, s) \) in the Laplace transform space. This restriction is simply that both the reflection and injection are independent of pitch angle. Though this assumption may not be strictly accurate in the real situation, the simplification introduced in the mathematical treatment is sufficient to warrant investigation, to determine whether this scatter-free theory can produce a realistic pitch angle distribution as a function of time.

For solar particle events, the source at the outer boundary may also be set equal to zero, with little effect on the actual distribution. I shall also make one final assumption, that the reflection coefficient at the sun is zero, i.e., the sun absorbs any returning flux of particles. This assumption will be shown later to be of small consequence during the initial phases of an impulsive event.
The conditions, then, to be applied to equation 2-27, are: \( p_1(\mu_1, \mu_1', s) = 0; \ j_2(\mu_2, s) = 0; \ p_2(\mu_2, \mu_2', s) = p_2(s); \) and \( j_1(\mu_1, s) = vA_0 \bar{l}_1(s). \) Under these special conditions, the equation for \( w_2(\mu_2, s) \) becomes
\[
\begin{align*}
  w_2(\mu_2, s) &= p_2(s) \int^1_{\mu_12} \frac{-s \tau_{12}(\mu_2^{'})}{1} d\mu_2' e^{A_0 \bar{l}_1(s) + p_2(s) \int^0_{\mu_12} d\mu_2' e^{2s \tau_m(\mu_2^{'})} w_2(\mu_2^{'}, s)} \\
  &= p_2(s) \int^1_{\mu_12} \frac{-s \tau_{12}(\mu_2^{'})}{1-p_2(s) \int^0_{\mu_12} d\mu_2' e^{2s \tau_m(\mu_2^{'})}} d\mu_2' e^{A_0 \bar{l}_1(s) + p_2(s) \int^0_{\mu_12} d\mu_2' e^{2s \tau_m(\mu_2^{'})}} w_2(\mu_2^{'}, s)
\end{align*}
\] (3-1)

There is no dependence on \( \mu_2 \) on the right hand side of equation 3-1; therefore, \( w_2(\mu_2, s) = w_2(s) \) is not a function of \( \mu_2. \) This is the simplification alluded to above; now 3-1 may be immediately solved for \( w_2(s) \).
\[
\begin{align*}
w_2(\mu_2, s) &= \overline{w_2(s)} = \frac{A_0 \bar{l}_1(s) + p_2(s) \int^1_{\mu_12} d\mu_2' e^{A_0 \bar{l}_1(s) + p_2(s) \int^0_{\mu_12} d\mu_2' e^{2s \tau_m(\mu_2^{'})}} w_2(\mu_2^{'}, s)}{1-p_2(s) \int^0_{\mu_12} d\mu_2' e^{2s \tau_m(\mu_2^{'})}} \\
  &= \frac{A_0 \bar{l}_1(s) + p_2(s) \int^1_{\mu_12} d\mu_2' e^{A_0 \bar{l}_1(s) + p_2(s) \int^0_{\mu_12} d\mu_2' e^{2s \tau_m(\mu_2^{'})}} w_2(\mu_2^{'}, s)}{1-p_2(s) \int^0_{\mu_12} d\mu_2' e^{2s \tau_m(\mu_2^{'})}} \\
  &= \overline{w_2(s)}
\end{align*}
\] (3-2)

The next step is to use a model for the injected flux, and for the reflection function at the outer boundary. As a rather general choice for impulsive onset events, I have chosen to represent the injection function \( J_1(t) \) as \( J_1(t) = A_0 e^{-\beta t}; \) then \( \bar{l}_1(s) = \frac{1}{s + \beta}. \) A fairly general choice for the reflection function is \( R_2(t-t') = k_0 e^{-k(t-t')} \). Then, \( p_2(s) = \frac{k_0}{s+k}. \) These assumptions then permit a quantitative
evaluation of the pitch angle distribution near 1 AU, and
the dependence of these distributions on the parameters \( \beta \)
(the injection time constant), \( k \) (the reflection time
constant), \( k_0 \) (the "efficiency of reflection"), and the
location of the outer boundary of the scatter-free region.

The location of the inner boundary at \( x_1 \) enters
this equation only through \( \tau_{12}(\mu_1^2) \) and \( \mu_{12} \). Since for this
model \( x_1 \) is expected to be near the sun (definitely within
0.5 AU), \( \tau_{12}(\mu_1^2) \), which is linear in \( (x_2-x_1) \) for distances
larger than \( \sim 1 \) AU, is essentially independent of \( x_1 \). \( \mu_{12} \)
also depends only weakly on both \( x_1 \) and \( x_2 \) as may be seen
from Table 1 (where the boundaries are indicated by their
radial distance from the sun). Moreover, even this weak
dependence may be greatly reduced by an appropriate choice
for the normalization constant \( A_0 \), as will be seen below.
I have therefore chosen the inner boundary to be at 0.1 AU
radially from the sun (also 0.1 AU along the ideal spiral).

The principal restriction on the outer boundary is
that the travel time for particles from 1 AU to the boundary
and back must be consistent with observations of the onset
of return flux in actual events. The duration of the extreme
anisotropic phase of both relativistic particle events
(\( \gtrsim 1 \) hr; Maurer et al., 1973) and low energy solar proton
events (\( \lesssim 2 \) days; Innanen and Van Allen, 1973) suggests path
lengths of less than 10 AU as a maximum for most events.
Since the distance along an ideal spiral from a radius of
1 AU to 3 AU is \( \sim 4.5 \) AU, I shall use 3 AU as the outer limit
\begin{tabular}{|c|c|c|c|c|}
\hline
$R_1$ & .1 & .2 & .3 & .4 \\
\hline
1 & .99294 & .97187 & .93706 & .88877 \\
1.5 & .99601 & .98416 & .96485 & .93860 \\
2 & .99721 & .98898 & .97561 & .95758 \\
3 & .99825 & .99309 & .98474 & .97355 \\
\hline
\end{tabular}

Table 1
for the outer boundary. The lack of observation of a clear boundary in the Pioneer 10 particle, plasma and magnetic field data indicated in preliminary reports (e.g., Smith et al., 1973; Collard et al., 1973; Intriligator and Wolfe, 1973; Lentz et al., 1973; Teegarden et al., 1973; Van Allen, 1973) emphasizes that the boundary, assumed for the sake of mathematical simplicity to be fairly sharp, must actually be a smooth, gradual transition from the scatter-free region to a domain where the propagation is dominated by scattering if the scatter-free theory is the correct description of energetic particle propagation in the inner solar system. However, the use of a well-defined boundary reduces the mathematical complexity considerably, so that the scatter-free theory can be shown to produce realistic pitch angle distributions. The additional effort required to introduce more realistic boundary conditions may in this way be shown to be a promising approach toward the development of a more satisfactory theory of energetic particle propagation in the inner solar system than currently exists for the description of the onsets of impulsive events.

The meaning of the reflection coefficients can be clarified by a brief investigation. The probability that a particle has been reflected by time \( t \) after first reaching \( x_2 \), for the reflection function chosen here is

\[
p(t) = \int_0^t k_0 e^{-kt'} dt'
\]

(3-4)
The total probability that a particle will be reflected is therefore given by

\[
\lim_{t \to \infty} P(t) = k_0 \int_0^\infty e^{-kt'} \, dt' = \frac{k_0}{k} \leq 1 \quad (3-5)
\]

so that \( k_0 \) must be less than (or equal to) \( k \).

The next step is to evaluate the two integrals in equation 3-3 for the ideal spiral field in the ecliptic plane. It is therefore necessary to determine the dependence of field strength on distance along the ideal spiral, so that 2-4 may be used to find \( \tau_m \) and \( \tau_{12} \). This dependence of field strength can be determined from the geometry shown in Figure 3. With \( \Omega \) representing the solar sidereal angular velocity, and \( V \) the solar wind velocity (here assumed constant, i.e., the ideal spiral field), the angle \( \psi \) is determined by

\[
\tan \psi = \frac{\Omega r}{V} \quad (3-6)
\]

The spiral distance element \( dx \) is related to \( dr \) by

\[
dx = \sec x \, dr = \sqrt{1 + \left( \frac{\Omega r}{V} \right)^2} \, dr \quad (3-7)
\]

so that

\[
x = \frac{r}{2} \sqrt{1 + \left( \frac{\Omega r}{V} \right)^2} + \frac{V}{2\Omega} \ln \left[ \frac{\Omega r}{V} + \sqrt{\left( \frac{\Omega r}{V} \right)^2 + 1} \right] \quad (3-8)
\]

Conservation of magnetic flux requires that
Figure 3. Spiral geometry notation. Spiral field direction is $\hat{x}$, radial direction is $\hat{r}$, and the angle between is $\psi$. 
\[ \dot{B}(r) \cdot \hat{r} = B(r) \cos \psi = \dot{B}(r_0) \cdot \hat{r} = B(r_0) \cos \psi_0 \quad (3-9) \]

so that

\[ B(r) = B(r_0) \left( \frac{r_0^2}{r^2} \right) \frac{1}{1 + (\frac{\Omega r}{V})^2} \quad (3-10) \]

Although \( B(x) \) cannot be expressed analytically, the two equations 3-8 and 3-10 may be used to determine \( B(r) \) and \( x(r) \) for use in a numerical integration of equation 2-4. The results of this numerical integration for \( \tau_{12}(\mu_2) \) and \( \tau_m(\mu_2) \) for the outer boundary at 1.5, 2, and 3 AU are shown in Figures 4, 5, 6, and 7. Based on the results shown in Figure 7, I have used the approximation that \( \tau_{12}(\mu) = \text{constant} \), so that

\[ \int_{\mu_{12}}^{1} -s \tau_{12}(\mu'_2) \, d\mu'_2 \approx (1-\mu_{12}) e^{-sT_{12}} \quad (3-11) \]

Also, for the outer boundary at 2 or 3 AU, \( \tau_m(\mu_2) \) may be reasonably approximated by three straight lines, the second of slope zero (i.e., \( \tau_m \) is constant in that range of \( \mu_2 \)). Then, since

\[ \tau_m(\mu_{12}) = a_1 M_1 + a_2 M_2 - a_2 \mu_{12} \quad (3-12) \]

the integral over region c may be expressed approximately:
Figure 4. Mirroring path length $V_{t_m}$ in AU vs $\mu$ for outer boundary at 1.5 AU (solid line), and the approximation to this curve used in generating pitch angle distributions (dashed line).

Figure 5. Same as Figure 4, for outer boundary at 2 AU.

Figure 6. Same as Figure 4, for outer boundary at 3 AU.

Figure 7. Path length $V_{t12}$ from 0.1 AU to 1.5, 2.0 and 3.0 AU vs pitch angle $\nu_2$ at the outer boundary. Path length from inner to outer boundary is essentially independent of pitch angle.
Figure 6
\[
\int_{0}^{\mu_{12}} e^{-2s\tau_{m}(\mu_{2}')} \, d\mu_{2}' = \int_{0}^{M_{1}} e^{-2a_{1}s\mu_{2}'} \, d\mu_{2}' + \int_{M_{1}}^{M_{2}} e^{-2a_{1}s\mu_{2}'} \, d\mu_{2}'
\]
\[
-2s(a_{1}M_{1} + a_{2}M_{2}) \int_{0}^{\mu_{12}} e^{-2a_{2}s\mu_{2}'} \, d\mu_{2}'
\]
\[
+ e^{-2a_{2}s} \int_{M_{2}}^{\mu_{12}} e^{-2a_{1}s\mu_{2}'} \, d\mu_{2}'
\]
\[
(3-13)
\]
\[
= \frac{1}{2a_{1}s} \left( 1 - e^{-2a_{1}s} \right) + (M_{2} - M_{1}) e^{-2a_{1}s}
\]
\[
+ \frac{1}{2a_{2}s} \left[ e^{-2s(a_{1}M_{1} + a_{2}M_{2} - a_{2}\mu_{1})} - 2a_{1}s \right]
\]
\[
(3-14)
\]
\[
= \frac{1}{2a_{1}s} + \left( M_{2} - M_{1} \right) - \frac{1}{2a_{1}s} - \frac{1}{2a_{2}s} \right) e^{-2a_{1}s} + \frac{e^{-2s\tau_{m}(\mu_{12})}}{2a_{2}s}
\]
\[
(3-15)
\]
For an outer boundary at 1.5 AU, \( \tau_{m}(\mu_{2}) \) may be approximated by two straight lines. The same equations as 3-12 and 3-13 apply in this case also, with \( M_{2} = \mu_{12} \) so that
\[
\int_{0}^{\mu_{12}} e^{-2s\tau_{m}(\mu_{2}')} \, d\mu_{2}' = \frac{1}{2a_{1}s} + \left( M_{2} - M_{1} \right) - \frac{1}{2a_{1}s} - \frac{1}{2a_{2}s} \right) e^{-2a_{1}s}
\]
\[
(3-16)
\]
Then, using the assumed forms for \( \tau_{1}(s) \) and \( \rho_{2}(s) \), and equations 3-11 and 3-15 (in the more general case) in 3-3,
\[
\overline{w_{2}}(s) = \frac{\frac{1}{s + \beta}}{A_{o} \left( \frac{1}{s + \beta} \right) \left( \frac{k_{o}}{s + \beta} \right) \left( 1 - \mu_{12} \right) e^{-sT_{12}}}
\]
\[
1 - \left( \frac{k_{o}}{s + \beta} \right) \left[ \frac{1}{2a_{1}s} + \left( M_{2} - M_{1} \right) - \frac{1}{2a_{1}s} - \frac{1}{2a_{2}s} \right) e^{-a_{1}s} + \left( \frac{1}{2a_{2}s} \right) e^{-2s\tau_{m}(\mu_{12})}
\]
\[
(3-17)
\]
Defining \( C_{1} \) (which is a function of \( s \)) by
\[
C_{1} = M_{2} - M_{1} - \frac{1}{2a_{1}s} - \frac{1}{2a_{2}s}
\]
\[
(3-18)
\]
equation 3-17 may be written as

\[
\tilde{w}_2^2(s) = \frac{A_0 k_0 (1-\mu_{12})}{(s+\beta)(s+k)} e^{-sT_{12}}
\]

\[
\sum_{n=0}^{\infty} \left[ \left( \frac{k_0}{s+k} \right) \left( \frac{1}{2a_1 s} \right) e^{-2a_1 \mu_{12} s} \right] e^{-2s\tau_m(\mu_{12})} = 0
\] (3-19)

In this form, since the exponentials acts as time shifts, and \( \tilde{w}_2^2(t) = 0 \) for \( t<0 \), it is clear that there is a range in time,

\[
T_{12} \leq t \leq T_{12} + 2\tau_m(\mu_{12})
\] (3-20)

(since 3-12 shows that \( \tau_m(\mu_{12}) \leq a_{1,m} \)) for which the exponential terms in the infinite sum are all zero. For this range in time, these exponential terms in 3-17 may be ignored, so that

\[
\tilde{w}_2^2(s) = \frac{A_0 (s+\beta)(s+k)(1-\mu_{12})e^{-sT_{12}}}{1 - \left( \frac{k_0}{s+k} \right) \left( \frac{1}{2a_1 s} \right)}
\] (3-21)

\[
A_0 k_0 (1-\mu_{12})e^{-sT_{12}}
\]

\[
(s+\beta) [s(s+k)-k_0/2a_1]
\] (3-22)

\[
\frac{A_0 k_0 s(1-\mu_{12})e^{-sT_{12}}}{(s+\beta)(s+k/2+\sqrt{k^2+2k_0/a_1}/2)(s+k/2-\sqrt{k^2+2k_0/a_1}/2)}
\] (3-23)
There are only three simple poles of $w_2(s)$, located at

$$s = -\beta, \frac{-k\pm\sqrt{k^2+2k_0/a_1}}{2}$$  (3-24)

Therefore, the reinversion may be easily accomplished through the use of the Bromwitch integral, resulting in

$$\frac{-A_0k_0\beta(1-\mu_{12})e^{-\beta(t-T_{12})}}{\omega w_2(t) (\beta^2-\beta k-k_0/2a_1)}$$

$$+ \frac{A_0k_0(1-\mu_{12})(-k+\sqrt{k^2+2k_0/a_1})e^{-(t-T_{12})(k-\sqrt{k^2+2k_0/a_1})/2}}{(2\beta-k+\sqrt{k^2+2k_0/a_1})(\sqrt{k^2+2k_0/a_1})}$$

$$+ \frac{A_0k_0(1-\mu_{12})(k+\sqrt{k^2+2k_0/a_1})e^{-(t-T_{12})(k+\sqrt{k^2+2k_0/a_1})/2}}{(2\beta-k-\sqrt{k^2+2k_0/a_1})(\sqrt{k^2+2k_0/a_1})}$$  (3-25)

This isotropic distribution at the outer boundary is mapped into a pitch angle distribution near 1 AU by the transit time (which depends on $\mu$) from the outer boundary to 1 AU, for both regions b and c from Figure 2. Transit times as a function of pitch angle for the three choices of outer boundary, determined from a numerical integration of 2-4 using the ideal spiral field described by equations 3-8 and 3-10 are shown in Figures 8, 9 and 10. These transit times, together with equation 3-25 have been used to generate plots of the predicted pitch angle distribution near 1 AU at various values of the parameters $\beta$, $k$, $k_0$ and the three
Figure 8. Path length in AU from the outer boundary to 1 AU (top curve), and pitch angle cosine at the outer boundary (bottom curve) vs pitch angle cosine at 1 AU for outer boundary at 1.5 AU.

Figure 9. Same as Figure 8, for outer boundary at 2 AU.

Figure 10. Same as Figure 8, for outer boundary at 3 AU.
Figure 8
Figure 9
Figure 10
choices for the location of the outer boundary. A selected sample of these plots are shown as Figures 11-20, to illustrate the dependence of the back-scattered flux on these four parameters.

The normalization of the flux injected at the sun ($A_0$ in 3-25) has been chosen so that the total outgoing flux at 1 AU at the onset of the event, averaged over the outward hemisphere, is $2\pi$. That is,

$$\int_0^1 \frac{J_1[\mu(\mu_1),0]}{v} \, d\mu = 1$$

or

$$A_0 \int_{\mu_1,1AU}^1 d\mu = A_0 (1-\mu_1,1AU) = 1$$

This normalization then implies that the dependence of $W_2(t)$ on the location of the boundary at $x_1$ enters only (weakly) through the transit time $T_{12}$, and through the ratio $(1-\mu_{12})/(1-\mu_1,1AU)$. This ratio depends only slightly on $x_1$ for a fixed $x_2$ (see Table 2), so that a single choice for $x_1$ is certainly adequate to show the general nature of the pitch angle distributions predicted by the assumption of scatter-free propagation.

In Figures 11-20 only the back-scattered component of the total distribution is shown. The injected flux is present only in a narrow cone ($\mu \geq 0.99294$). The magnitude of the injected flux at a time $t$ after the onset of the
\[ \{1-\mu_{R_1,R_2}\}/\{1-\mu_{R_1,1 \text{ AU}}\} \]

<table>
<thead>
<tr>
<th>(R_1)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>.565</td>
<td>.563</td>
<td>.558</td>
</tr>
<tr>
<td>2</td>
<td>.395</td>
<td>.392</td>
<td>.388</td>
</tr>
<tr>
<td>3</td>
<td>.248</td>
<td>.246</td>
<td>.242</td>
</tr>
</tbody>
</table>

Table 2
event is \( J(t=0) e^{-\beta t} \). For comparison with the back-scattered component, \( W(t=0) = \frac{J(t=0)}{v} \) in the injection cone is simply \( A_0 \), and in the same units as Figures 11-20, is 142.4.

Since the back-scattered distribution in Figures 11-20 never exceeds 0.5, there is a definitive test for the extent to which the scatter-free theory is applicable to solar particle events. This theory predicts a very high outgoing flux in a narrow cone centered on the interplanetary field line. I shall show in the next section that the predictions of this theory produce profiles consistent with spacecraft data currently available; further refinement of the angular resolution of the detectors is necessary to determine whether scatter-free propagation is characteristic of normal or abnormal conditions in the inner solar system.

In Figure 11 I show the results of the calculation of the pitch angle distribution near 1 AU for an outer boundary at 1.5 AU (spiral distance 1.95 AU from the sun), with the injection time constant \( \beta=1 \), and the reflection parameters \( k \) and \( k_0 \) also 1. The five curves in the figure are for times 0.48, 0.64, 0.80, 0.96 and 1.12 days after the onset of the event.

This figure demonstrates a second major prediction of the scatter-free theory. The pitch angle distribution outside the injection cone region fills up gradually, over \( \sim 3/4 \) of a day, beginning with the particles returning directly along the field line. Testing of this prediction
Figure 11. Backscattered pitch angle distributions calculated for outer boundary at 1.5 AU, $\beta = k = k_0 = 1 d^{-1}$, at times 0.48 (A), 0.64 (B), 0.80 (C), 0.96 (D) and 1.12 (E) days after onset.
with currently available spacecraft data is discussed in
the next section.

Figure 12 shows the back-scattered profiles for the
same injection and reflection parameters as Figure 11
($\beta=k=k_\theta=1$), but for the outer boundary at 2 AU (spiral
distance 2.96 AU). The times for which these curves are
calculated are 0.96, 1.20, 1.44 and 1.68 days after onset.
It is interesting to note that the rate of filling up of
the distribution is nearly independent of the change in
outer boundary location. In Figure 11, this rate is
indicated by the time between curves A and D (0.48 days)
while similar distributions in Figure 12 are curves A and
C (also separated by 0.48 days).

The principal effects of the increase in distance to
the outer boundary are a longer delay before back-scattered
particles arrive at 1 AU, and a lower level of back-scattered
flux. That is, the distribution near earth responds more
quickly to a nearby boundary, and a greater percentage of
the particles return. The gradual filling of the pitch
angle distribution is, however, a more general prediction
of the scatter-free propagation theory.

Figure 13 demonstrates the effects of "hardening"
the outer boundary. For this figure, the boundary location
is again at 2 AU and $\beta=1$. The reflection parameters $k$
and $k_\theta$ have both been increased to 10, and distributions
were calculated for the same times as in Figure 12. As is
anticipated, there are larger gradients in the distribution
Figure 12. Backscattered pitch angle distributions calculated for outer boundary at 2 AU, $\beta = k = k_0 = 1d^{-1}$, at times 0.96 (A), 1.20 (B), 1.44 (C), and 1.68 (D) days after onset.

Figure 13. Same as Figure 12, except $k = k_0 = 10d^{-1}$. 
Figure 12
Figure 13
during the filling phase, since there is much less delay in the response of the outer boundary to the injected flux. Once again, however, the rate of filling is not greatly altered. The decrease in back-scattered flux from 1.44 to 1.68 days after onset (shown by curves C and D) is due to the decrease in the injected flux; this is evident here, but not in Figure 12, due to the quicker response of the hard outer boundary.

Figure 14 shows the return flux for the same injection and reflection parameters as Figure 13 (β = 1, k = k₀ = 10), but for an outer boundary at 3 AU (5.65 AU along an ideal spiral). The times are 2.28, 2.76, 3.24 and 3.72 days after onset. Once again I find less difference in the rate of filling than in the time until the first return flux is seen or in the final magnitude of the return flux. This result is anticipated on the basis of Figures 8, 9 and 10. There is much less difference in the relative shapes of the three curves for transit time from the three different outer boundaries to 1 AU as functions of μ than there is in the displacement of the entire curve to larger transit times for greater distance to the boundary. The rate of filling of the pitch angle distribution for the same values of the injection and reflection parameters depends only on the differences between transit times for different pitch angles, and Figures 8-10 show that these differences depend only weakly on location of the outer boundary.
Figure 14. Backscattered pitch angle distributions calculated for outer boundary at 3 AU, $\beta=1d^{-1}$, $k=k_0=10d^{-1}$, at times 2.28 (A), 2.76 (B), 3.24 (C) and 3.72 (D) days after onset.
Figure 14
The lower levels of returning flux for a more distant outer boundary is simply a geometrical effect. A larger percentage of the particles at the outer boundary mirror outside of 1 AU if the boundary is further away.

Figures 15 and 16, together with Figure 13, show the effects of a less "efficient" reflection ($k_0 < k$), with the other parameters held constant (boundary at 2 AU, $\beta = 1$, $k = 10$, and times 0.96, 1.20, 1.44 and 1.68 days after onset). For Figure 15, $k_0 = 8$; the calculation of Figure 16 used $k_0 = 5$. Just as might be expected, the only significant difference as $k_0$ decreases is a reduction in the magnitude of the back-scattered component.

The dependence of the distribution on the injection time constant $\beta$ is investigated in Figures 17-20, in conjunction with Figure 15. The other parameters are held constant at $R_2 = 2$ AU, $k = 10$, and $k_0 = 8$. In Figures 15, 17 and 18 A, B, C and D represent the same times as above (0.96, 1.20, 1.44 and 1.68 days), and E in Figure 17 is calculated for $t = 1.92$ days. Values of $\beta$ are 0, 1 and 8 for Figures 17, 15 and 18 respectively.

For a constant injection (Figure 17) the flux continues to rise through the first two days (at which point the "early time" term is no longer the only term in equation 3-15). A rapid turnoff of the injection results in a relative maximum progressing through the pitch angle distribution (Figure 18) followed by a decay to a rather low, nearly isotropic level within two days of the onset.
Figure 15. Backscattered pitch angle distributions calculated for outer boundary at 2 AU, $\beta=1d^{-1}$, $k=10d^{-1}$, $k_0=8d^{-1}$, at times 0.96 (A), 1.20 (B), 1.44 (C) and 1.68 (D) days after onset.

Figure 16. Same as Figure 15, except $k_0=5d^{-1}$.

Figure 17. Same as Figure 15, except $\beta=0$. Curve E is drawn for 1.92 days after onset.

Figure 18. Same as Figure 15, except $\beta=8$. 
Figure 15
Figure 16
Figure 17
Figure 18
The intermediate $\beta=1$ of Figure 15 produces an intermediate maximum flux level and shows the decay of the nearly isotropic back-scattered component after 1 1/2 days.

Figures 19 and 20 are the distributions calculated at 1.20 days after onset, for $R_2$, $k$ and $k_0$ as above (2 AU, 10 and 8), for changing values of $\beta$. In Figure 19, curves A, B, C and D are calculated for $\beta$ of 0, .1, .5 and 1.
The curves in Figure 20 were determined for $\beta=1, 2, 4$ and 8.
These two figures illustrate the gradual transition from the "standard" filling and continuing increase of the entire distribution for small $\beta$, through the filling and decay of a nearly isotropic back-scattered component for $\beta=1$, to the relative maximum sweeping through the distribution, and the rapid decay to low flux levels for large $\beta$.
Discussion of anisotropies has been intentionally postponed until the next section, where the response of a "real" detector to these calculated pitch angle distributions is considered.
Figure 19. Backscattered pitch angle distributions calculated for outer boundary at 2 AU, $k=10d^{-1}$, $k_0=8d^{-1}$, at 1.20 days after onset for $\beta$ of 0 (A), 0.1 (B), 0.5 (C) and 1 (D) inverse days.

Figure 20. Same as Figure 19, except here $\beta$ is 1 (A), 2 (B), 4 (C), 8 (D) inverse days.
Figure 20
4. **Response of a Spinning Detector**

The pitch angle distributions calculated in the previous section provide the basis for comparison of the scatter-free propagation theory with observations of solar particle events. However, the theory predicts a very narrow (≈7° half-width) cone containing a high flux of particles from the sun. Since this narrow cone may significantly affect the observed sectored counting rates in a moderately wide detector, I shall derive a fairly general scheme for translating a known (or assumed) pitch angle distribution into counting rates in arbitrary sectors for a spinning detector. Application of this scheme to typical pitch angle distributions from Section 3 for a rather simple detector geometry shows that the scatter-free propagation theory presented here does indeed predict anisotropy and time histories at early times in impulsive onset low energy solar proton events which are in good agreement with observations.

The determination of the number of counts a detector will make in a certain sector is essentially a triple integral. This triple integral may be most easily described as a double integral over the acceptance cone of the detector as it is pointing in each particular direction, followed by an integration over all pointing directions within the particular sector under consideration. This
way of looking at the detector response is not particularly
advantageous for calculation in the present case. Since
the calculated pitch angle distribution is highly aniso-
tropic, and is also known numerically only, the entire
triple integral must be done numerically; i.e., the double
integral over the acceptance cone depends on the direction
in which the detector is pointing.

I shall present a different scheme for breaking up
the triple integral mathematically. This scheme consists
of first integrating over the detector response to particles
coming from a particular direction, and following with a
double integral over all directions from which particles
can be counted in the sector of interest. This scheme is
advantageous because the integral over the detector response
is determined only by the detector geometry, and is inde­
dependent of the angle of rotation. This integral over the
detector response may therefore be done independently,
which then reduces the triple integral to a double integral.

The scheme is developed as follows. The flux of
particles of a particular velocity \( v \) toward the detector
from the direction \( (\theta, \phi) \) through an element of solid
angle \( d\Omega \) is \( vW[\mu(\theta, \phi)] \sin \theta d\theta d\phi \). (The coordinate system
is chosen so that the axis of rotation of the spacecraft
is at \( \theta=0 \).) The number of counts from the direction \( (\theta, \phi) \)
for one rotation of the detector can be written

\[
dN(\theta, \phi) = vW[\mu(\theta, \phi)]\sin\theta d\theta d\phi \int_{\phi}^{\phi'} \frac{E(\theta, \phi - \phi')}{\phi} \quad (4-1)
\]
where $\phi$ is the angular velocity of the detector, and $E(\theta, \phi-\phi')$ is the efficiency of the detector in counting particles approaching at an angle $\gamma = \cos^{-1}[\sin \theta \cos(\phi-\phi')]$ from the center of the detector, if the detector is symmetric about its center.

For a small solid state detector with a moderately large acceptance cone of half-width $\delta$ (on the order of 30°), $E(\theta, \phi-\phi')$ may be written approximately as $A \cos \gamma = A \sin \theta \cos(\phi-\phi')$, with $A$ proportional to the area of the detector. With $\ell(\theta)$ defined as the half-angle (in $\phi$) for which directions of polar angle $\theta$ are in the acceptance cone of the detector, the counts from the direction $(\theta, \phi)$ on one rotation may be written as

$$dN(\theta, \phi) = vW[u(\theta, \phi)] \sin \theta \, d\theta \, d\phi \int_{\phi-\ell(\theta)}^{\phi+\ell(\theta)} A \sin \theta \cos(\phi-\phi') \, d\phi' \tag{4-2}$$

For the particular case of an acceptance cone of half-width $\delta$,

$$\ell(\theta) = \cos^{-1}\left(\frac{\cos \delta}{\sin \theta}\right) \tag{4-3}$$

The simplification inherent in the present formulation of the problem is evident in equation 4-2; the integration over the detector response as a function of azimuthal angle may be immediately performed, yielding

$$dN(\theta, \phi) = \frac{2Av}{\phi} \sin^2 \theta \sin[\ell(\theta)]W[u(\theta, \phi)] \tag{4-4}$$
For the circular detector, it also follows that

$$\sin(\ell(\theta)) = \sqrt{1 - \cos^2 \delta / \sin^2 \theta} = \frac{\sqrt{\sin^2 \theta - \cos^2 \delta}}{\sin \theta}$$  \hspace{1cm} (4-5)$$

The integration over a particular sector in the present scheme must deal directly with the overlap in counting at the edge of the sector. This overlap at a particular polar angle $\theta$ near a sector terminating nominally at an angle $\phi = \zeta$ is in the region $\zeta - \ell(\theta) < \phi < \zeta + \ell(\theta)$.

At this point I introduce the simplifying assumption that the spacecraft spin axis is perpendicular to the magnetic field. Then $w(\theta, \phi)$ can be simply expressed as

$$w(\theta, \phi) = \sin \theta \cos \phi$$  \hspace{1cm} (4-6)$$

and $W(w)$ is symmetric in $\theta$ around $\theta = \pi/2$. The $\theta$-integration, which ranges from $\pi/2 - \delta$ to $\pi/2 + \delta$ may therefore be written as twice the integral from $\pi/2 - \delta$ to $\pi/2$.

With this restriction, elementary geometrical considerations show that the number of counts in a sector extending nominally from $\zeta_1$ to $\zeta_2$ for the circular, small solid-state detector may be written
\[ N_{\zeta_1, \zeta_2} = \frac{2A v}{\phi} \left\{ \int_{\pi/2-\delta}^{\pi/2} d\theta \sin^2 \theta \left[ \int_{\zeta_1 + \ell(\theta)}^{\zeta_1 + \ell(\theta)} d\phi W[\mu(\theta, \phi)] \right] \int_{\zeta_1 - \ell(\theta)}^{\phi + \ell(\theta)} d\phi' \cos(\phi' - \phi) \right. \\
+ \int_{\pi/2-\delta}^{\pi/2} d\theta \sin^2 \theta \left[ \int_{\zeta_2 + \ell(\theta)}^{\zeta_2 + \ell(\theta)} d\phi W[\mu(\theta, \phi)] \right] \int_{\zeta_2 - \ell(\theta)}^{\phi - \ell(\theta)} d\phi' \cos(\phi' - \phi) \right\} \\
+ \int_{\pi/2-\delta}^{\pi/2} d\theta \sin^2 \theta \left[ \int_{\zeta_2 + \ell(\theta)}^{\zeta_2 + \ell(\theta)} d\phi W[\mu(\theta, \phi)] \right] \int_{\phi - \ell(\theta)}^{\phi + \ell(\theta)} d\phi' \cos(\phi' - \phi) \right\} \\
(4-7) \]

which reduces to

\[ = \frac{2A v}{\phi} \int_{\pi/2-\delta}^{\pi/2} d\theta \sin^2 \theta \left\{ \sin[\ell(\theta)] \left[ \int_{\zeta_1 + \ell(\theta)}^{\zeta_1 + \ell(\theta)} d\phi W[\mu(\theta, \phi)] \right] \right. \\
+ \int_{\zeta_1 - \ell(\theta)}^{\zeta_1 + \ell(\theta)} d\phi \sin(\phi - \zeta_1) W[\mu(\theta, \phi)] \\
+ 2\sin[\ell(\theta)] \left[ \int_{\zeta_1 + \ell(\theta)}^{\zeta_1 + \ell(\theta)} d\phi W[\mu(\theta, \phi)] + \sin[\ell(\theta)] \right] \int_{\zeta_2 - \ell(\theta)}^{\zeta_2 + \ell(\theta)} d\phi W[\mu(\theta, \phi)] \\
+ \int_{\zeta_2 - \ell(\theta)}^{\zeta_2 + \ell(\theta)} d\phi W[\mu(\theta, \phi)] \right\} \\
(4-8) \]

when the \( \phi' \)-integrations are performed.
Since the pitch angle distributions which I wish to use are numerically derived, rather than analytic, I shall assume

\[ W[\mu(\theta,\phi)] = W[\mu(\theta_i,\phi_j)] = W_{ij} \]  \hspace{1cm} (4-9)

for a range in \( \theta \) and \( \phi \) given by

\[ \theta_i - \frac{\Delta \theta}{2} \leq \theta \leq \theta_i + \frac{\Delta \theta}{2} \]  \hspace{1cm} (4-10)

and

\[ \phi_j - \frac{\Delta \phi}{2} \leq \phi \leq \phi_j + \frac{\Delta \phi}{2} \]  \hspace{1cm} (4-11)

Then, rewriting 4-8 as a sum, and using 4-5 for \( \sin[\ell(\theta)] \),

\[
N_{\zeta_1,\zeta_2} \approx \frac{2AV}{\phi} \int_{\theta_i - \frac{\Delta \theta}{2}}^{\theta_i + \frac{\Delta \theta}{2}} d\theta \sin^2 \theta \left\{ \sum W_{ij} \int_{\phi_j - \frac{\Delta \phi}{2}}^{\phi_j + \frac{\Delta \phi}{2}} d\phi \left[ \frac{\sqrt{1 - \cos^2 \delta}}{\sin^2 \theta} + \sin(\phi - \phi_j) \right] \right\}
\]

\[ + 2\Delta \phi \sqrt{1 - \cos^2 \delta / \sin^2 \theta} \sum_{k} W_{ik} \]

\[ + \sum_{k} W_{il} \int_{\phi_i - \frac{\Delta \phi}{2}}^{\phi_i + \frac{\Delta \phi}{2}} d\phi \left[ \frac{\sqrt{1 - \cos^2 \delta}}{\sin^2 \theta} + \sin(\zeta_2 - \phi) \right] \]  \hspace{1cm} (4-12)
\[
\begin{align*}
&= \frac{2A_v}{\phi} \left\{ \sum_{j} W_{ij} \left[ \Delta \phi \int_{\theta_i+\Delta \theta/2}^{\theta_i+\Delta \theta/2} d\theta \sin^2 \theta - \cos^2 \theta \right] \right. \\
&\quad + [\cos(\phi_j - \zeta_1 - \Delta \phi/2) - \cos(\phi_j - \zeta_1 + \Delta \phi/2)] \int_{\theta_i+\Delta \theta/2}^{\theta_i+\Delta \theta/2} d\theta \sin^2 \theta \\
&\quad + 2\Delta \phi \sum_{k} W_{ik} \int_{\theta_i+\Delta \theta/2}^{\theta_i+\Delta \theta/2} d\theta \sin^2 \theta - \cos^2 \theta \\
&\quad + \left. \sum_{l} W_{il} \left[ \Delta \phi \int_{\theta_i+\Delta \theta/2}^{\theta_i+\Delta \theta/2} d\theta \sin^2 \theta - \cos^2 \theta \right] \right\} \\
&\quad + [\cos(\zeta_2 - \phi_k - \Delta \phi/2) - \cos(\zeta_2 - \phi_k + \Delta \phi/2)] \int_{\theta_i+\Delta \theta/2}^{\theta_i+\Delta \theta/2} d\theta \sin^2 \theta \\
&\quad + 2\Delta \phi \sum_{k} W_{ik} I_{1i}
\end{align*}
\]

where \( I_{1i}(\theta) \) and \( I_{2i}(\theta) \) are given by

\[
I_{1i} = \int_{\theta_i+\Delta \theta/2}^{\theta_i+\Delta \theta/2} d\theta \sin^2 \theta - \cos^2 \theta
\]

and

\[
I_{2i} = \int_{\theta_i+\Delta \theta/2}^{\theta_i+\Delta \theta/2} d\theta \sin^2 \theta = \int_{\theta_i+\Delta \theta/2}^{\theta_i+\Delta \theta/2} d\theta \left( \frac{1-\cos^2 \theta}{2} \right)
\]
\[ I_{1i}(\theta) = \frac{\Delta \theta}{2} + \frac{1}{4}[\sin(2\theta_i + \Delta \theta) - \sin(2\theta_i - \Delta \theta)] \] (4-17)

\[ I_{1i}(\theta) = \frac{\Delta \theta}{2} + \frac{1}{2} \sin \Delta \theta \cos 2\theta_i \] (4-18)

\[ I_{1i}(\theta) \text{ may also be determined explicitly:} \]

\[ I_{1i} = \int_{\theta_i - \Delta \theta/2}^{\theta_i + \Delta \theta/2} d\theta \sin \sqrt{(1 - \cos^2 \delta) - \cos^2 \theta} \] (4-19)

\[ = -\int \frac{\cos(\theta_i + \Delta \theta/2)}{\cos(\theta_i - \Delta \theta/2)} \frac{dx}{\sqrt{\sin^2 \delta - x^2}} \] (4-20)

\[ = \left[ \frac{x \sqrt{\sin^2 \delta - x^2}}{2} + \frac{\sin^2 \delta \sin^{-1}\left(\frac{x}{\sin \delta}\right)}{2} \right] \cos(\theta_i + \Delta \theta/2) \] (4-21)

\[ = 1/2 \left\{ \cos\left(\frac{\theta_i - \Delta \theta}{2}\right)\sqrt{\sin^2 \delta \cos^2\left(\frac{\theta_i - \Delta \theta}{2}\right) + \sin^2 \delta \sin^{-1}\left(\frac{\cos\left(\frac{\theta_i - \Delta \theta}{2}\right)}{\sin \delta}\right)} \right. \]

\[ - \cos\left(\frac{\theta_i + \Delta \theta}{2}\right)\sqrt{\sin^2 \delta - \cos^2\left(\frac{\theta_i - \Delta \theta}{2}\right) - \sin^2 \delta \sin^{-1}\left(\frac{\cos\left(\frac{\theta_i + \Delta \theta}{2}\right)}{\sin \delta}\right)} \right\} \] (4-22)
Then, using 4-23 and 4-18 for $I_{1i}$ and $I_{2i}$ in 4-14, and numerical pitch angle distributions, the expected sectored detector counting rates can be determined.

I have used a four-sector example, with the field line ($\mu=0$) between sectors 1 and 4 (i.e., sector 1 is centered on the sun) to demonstrate this method, applying it to the pitch angle distributions calculated above. A table of results of this calculation for the pitch angle distributions shown in Figures 11-18 is presented in the Appendix.

For easier comparison of the scatter-free theory predictions with previously reported data, I have also calculated the "experimental" anisotropy $A$ (more correctly, the amplitude of the first Fourier harmonic of the sectored counting rates), and a relative counting rate (normalized to 0.5 at the onset of the event). These two "experimental" parameters have been plotted in Figures 21 to 28 as functions of time. In every case except Figure 27, the direction of the anisotropy remains outward along the field line.

Comparison of Figures 21 and 22 (see Figures 11 and 12 for the pitch angle distributions) shows that the
Figure 21. Calculated ideal detector counting rate and anisotropy time histories for the pitch angle distributions of Figure 11 (boundary at 1.5 AU, $\beta=\kappa=k_0=1d^{-1}$). Here $t=0$ is the time of the flare. The detector has a 30° half-width cone of acceptance, and data are taken in four sectors, one centered on the sun.

Figure 22. Same as Figure 21, but for the distributions of Figure 12 ($R_2=2$AU, $\beta=\kappa=k_0=1d^{-1}$).
principal effect of an outer boundary closer to 1 AU is an earlier decay of the initial high anisotropy. Figures 22, 23, 25 and 26 show that there is no drastic effect due to changes in $k$ or $k_o$, at least during the early phases of the events, for which the present calculation is applicable. The major change, as might be expected from Figures 12, 13, 15 and 16 respectively, is that the anisotropy decays faster (initially) with increasing $k$, and more slowly with decreasing $k_o/k$.

Figure 24 shows a different story, however. For this case, with the boundary at 3 AU, and $\beta=1 \text{ d}^{-1}$, $k=k_o=10 \text{ d}^{-1}$, the injected flux has decayed to a low enough level so that the back-scattered component produces more counts than the injection component, while the present calculation is still valid. This leads to the "double-humped" time-history of the counting rate, and also to the sharp decrease in the anisotropy parameter $A$, followed by an increase, as the back-scattered pitch angle distribution fills up (as discussed above).

Figure 27 shows this double-humped event exaggerated by a rapid turn-off of the injection ($\beta=8 \text{d}^{-1}$) for an outer boundary at 2 AU (see also Figure 18). Here the injected flux is so low by 0.96 and 1.20 days after onset that the anisotropy changes direction, and is actually inward along the field line until the maximum in the distribution moves out of sectors 2 and 3 (past $\mu=0$, as discussed in Section 3).
Figure 23. Calculated ideal detector counting rate and anisotropy time histories for the pitch angle distributions of Figure 13 ($R_2=2\text{AU}$, $\beta=1d^{-1}$, $k=k_0=10d^{-1}$). Time starts at time of the flare, and a $30^\circ$ half-width conical detector counts in four sectors, one centered on the sun.

Figure 24. Same as Figure 23, but for the distributions of Figure 14 ($R_2=3\text{AU}$, $\beta=1d^{-1}$, $k=k_0=10d^{-1}$).

Figure 25. Same as Figure 23, but for the distributions of Figure 15 ($R_2=2\text{AU}$, $\beta=1d^{-1}$, $k=10d^{-1}$, $k_0=8d^{-1}$).

Figure 26. Same as Figure 23, but for the distributions of Figure 16 ($R_2=2\text{AU}$, $\beta=1d^{-1}$, $k=10d^{-1}$, $k_0=8d^{-1}$).

Figure 27. Same as Figure 23, but for the distributions of Figure 18 ($R_2=2\text{AU}$, $\beta=8d^{-1}$, $k=10d^{-1}$, $k_0=8d^{-1}$).
Figure 23

Flux (arbitrary units)

A

$I_1$

$t$ (days)

0.0

0.4

0.8

1.2

0.0

0.5
Figure 28 shows the profiles for constant injection (see Figure 17). As expected, the counting rate rises, and the anisotropy remains high. For this case I also demonstrate the possible effects due to choice of sectors, with Figure 29. For this figure I have used the same pitch angle distributions as for Figure 28, but have chosen the sectors so that the field line is in the center of sector 4. This results in a considerably higher observed "anisotropy" $A$ which remains above 1.2 throughout the time of validity of the calculation. Clearly, in all the other cases, too, the magnitude of the "observed" anisotropy is dependent on the choice of sectors (in the four-sector case).
Figure 28. Calculated ideal detector counting rate and anisotropy time histories for the pitch angle distributions of Figure 17 ($R_2=2$AU, $\beta=0$, $k=10d^{-1}$, $k_0=8d^{-1}$). Time starts with the flare, and a 30° half-width conical detector counts in four sectors, one centered on the sun.

Figure 29. Same as Figure 28, except one sector now starts at the sun.
Flux (arbitrary units)

- $t$ (days)
- Flux

Figure 28
Figure 29

FLUX (arbitrary units)

A

$t$ (days)

0  10  12  14  16  18  20  22  24

0  10  12  14  16  18  20  22  24

1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0  9.0  10.0
5. **Comparison of Scatter-Free Propagation Theory with Spacecraft Data**

Observations of low energy (0.3 MeV) protons (Innanen and Van Allen, 1973; Roelof and Krimigis, 1973), protons more energetic by a factor of $10^3$ (>1 GeV) (Maurer et al., 1973) and also of near-relativistic electrons (Lin and Anderson, 1967; Lin, 1970) have shown that often solar charged particles propagate from the sun to 1 AU with no observable effects due to scattering. The aspect of these events which demonstrate that scattering is negligible is the **persistent** high anisotropy, often near 100% at the time of maximum of the event. Here "observed anisotropy" $\xi$ is defined in the standard way in terms of the maximum flux $j_+$ and the flux in the opposite direction $j_-:

$$\xi = (j_+ - j_-)/(j_+ + j_-)$$

This persistence of the high anisotropy completely rules out any possibility of significant scattering in the interplanetary medium between the sun and 1 AU.

As shown in the figures in the previous section, this persistent high anisotropy is a basic feature predicted by the scatter-free propagation theory. This alone is sufficient verification that the simplified scatter-free theory presented here is a more nearly correct theoretical description of many flare-associated energetic particle events than any diffusion-based theory.
A quantitative comparison of the scatter-free theory with observations of low energy (~300 keV) solar protons is a more difficult problem. The difficulties are primarily caused by the fact that coronal structure usually dominates the time-histories of these low energy proton events (Lin et al., 1968; Roelof and Krimigis, 1973). Sharp longitudinal gradients in long-lived particle populations often can even "mimic" the fast-rising time history of an impulsive event. It should therefore be possible to obtain a better comparison if multiple spacecraft flux and anisotropy, magnetic field and solar wind plasma data, together with solar data sufficient to show the coronal magnetic structures, were available to separate out the interplanetary propagation effects for comparison with the theory.

Even though the assumptions of exponential decay of the injection and a sharply defined inner boundary to the outer scattering region are an over-simplification of the actual situation (though this assumption is somewhat relaxed by permitting delayed reflection), this simple model still warrants a quantitative comparison with the gross structure of observed events. I shall make a specific comparison of the theory to observations of two events reported by Innanen and Van Allen (1973). These two events (on January 24 and March 21, 1969) are ones in which coronal structure does not seem to be dominant, or during which specific effects apparently due to coronal structure can be (tentatively) identified from one spacecraft's data.
Before proceeding with this analysis, it is appropriate to discuss briefly some of the improvements, both theoretical and experimental, which can be expected in future work. A possible improvement in the modeling of the boundaries (involving considerably increased mathematical complexity in the reinversion of the Laplace transform) has been suggested by Roelof (1974). This improvement consists of treating the reflection at either the inner or the outer boundary of the scatter-free region as essentially a Green's function for the diffusing region beyond the boundary.

Better resolution of the injection time history requires improved experimental techniques. If detectors of sufficient angular resolution to obtain a time history of the flux in the narrow injection cone can be developed, they will measure directly the time history of the injection for each individual event when the effects of coronal structure have been removed. The first back-scattered particles can then be used to determine the location of the outer boundary, which is interpreted as the beginning of the region where scattering becomes significant. Then, using the observed injection flux, models of the reflection can be tested.

The first example is the event of March 21 and 22, 1969. This event looks very similar to the scatter-free calculation for an outer boundary at 2 AU, and constant injection of particles at the sun. Figure 30 shows the data from Innanen and Van Allen (1973) redrawn in the same form as Figures 21-29. As in those figures, time in figure 30 is
Figure 30. The event of March 21, 1969 (from Innanen and Van Allen, 1973) plotted in the same format as Figures 21-29. Figure 28 shows a similar high initial anisotropy, with decay to a lower level at the same time as the flux level increases.
March 21 and 22, 1969

Counts/second

A

0 0.2 0.4 0.6 0.8 1.0 1 1.2 1.6 2.0

t(days)

Figure 30
measured from the time of the flare. The increase shown here was associated with an importance 2N flare of 0141 UT on March 21, 16° E of central meridian (Innanen and Van Allen, 1973). This flare was in McMath plage region 9994, one of the largest active regions ever observed, which extended for more than 60° in solar longitude.

This figure compares well with Figure 28 for particles of approximately the same energy (~400 keV) as those measured by Innanen and Van Allen (>300 keV). Note that the initial anisotropy remains quite high for about one day after onset, though not quite at the maximum possible level. The observed anisotropy is expected often to be reduced from the maximum theoretical value due to a small amount of scattering, and to temporary (<1 hour) excursions $\leq 30°$ in the local magnetic field direction.

In addition to the initial persistent high anisotropy, there are other notable similarities between this event and the results of the calculation of scatter-free propagation for constant injection of particles at the sun and outer boundary of the scatter-free region at 1 AU. At 0.9 days after onset the flux begins to rise, just as in the calculation presented in Figure 28. At the same time the anisotropy begins to decrease, just as in the calculated profile.

Quantitatively, this particular choice of parameters in the model results in an overestimate of the anisotropy, (predicting a decrease of 30% compared to the observed 50% decrease) and underestimates the flux increase (the predicted
increase is 40% of the maximum, while the observed increase is 70% of the maximum flux). "Hardening" the outer boundary would increase the back-scattered component of the flux (compare Figures 13 and 15). Similarly an increase in the back-scattered component would be caused by the occurrence of some scattering between 1 and 2 AU. It seems necessary to assume that some scattering occurred within 2 AU; however, the high initial anisotropy and the semi-quantitative fit to a particular scatter-free propagation calculation indicate that scattering is not the dominant effect.

The similarity between these observations and the scatter-free predictions for constant injection at the sun would provide stronger verification of the validity of the assumption of almost negligible scattering in the inner heliosphere throughout the first two days of this event if comparable data were available from at least one other spacecraft. With data from one spacecraft only, it is not possible to determine beyond all possible doubt whether the changes beginning at day 1.5 in Figure 30 were due to field-aligned propagation or to a switch from one particle regime to another. However, for most such changes from one particle population to another, both the flux and anisotropy change abruptly (see e.g. Roelof and Krimigis, 1973). This event is therefore a good example of one which was more nearly scatter-free than diffusive, and not strongly distorted by coronal structure.

On the other hand, the event of January 24, 1969,
shows considerable evidence of coronal structure. Figure 31, taken from the work of Innanen and Van Allen, shows the observed flux and anisotropy parameters in the top two plots, and the field-aligned component of the anisotropy in the bottom plot. The importance 2B flare associated with this particle increase was at 0803 UT on January 24, 9° west of central meridian, and 20° north latitude in McMath plage region 9879.

During this event, the time history of the flux, on a time scale of about half a day, seems to show much more structure than that of the anisotropy. The anisotropy remains very high throughout the period of maximum flux, which is unusually flat, and terminates in an abrupt drop of an order of magnitude. The nearly simultaneous decrease in both flux and anisotropy on day 24 (January 25) may be due to either a rapid shift from one interplanetary particle regime to another (i.e. the change is caused by coronal and solar wind structure) or to a rapid turn-off of the coronal injection. To make a distinction, it would be necessary to have multiple spacecraft data.

The "notch" in the flux from about hour 20 on day 24 to hour 8 on day 25 appears to be an effect of coronal structure, that is, a different particle population injected onto these interplanetary field lines, since the anisotropy remains high and unchanged.

In addition to the initial persistent high anisotropy, there are a number of other striking similarities between
Figure 31. The 0.3 MeV proton event of January 24, 1969 (from Innanen and Van Allen, 1973). Top panel is the omnidirectional intensity; middle is the observed anisotropy; bottom is the field-aligned component of the observed anisotropy.
Event of January 24, 1969.

Figure 31
this event and the results of the scatter-free calculation presented in Figure 24, for an injection time constant $\beta$ of $1 \text{ d}^{-1}$, and outer boundary at 3 AU. Most notable is the temporary increase in the anisotropy, lasting about a day beginning at day 2.8 in Figure 24, and occurring on day 26, $\sim 2.8$ days after the flare.

On day 25, there was a 50% enhancement in flux superimposed on the decay. Similarly, beginning one-half day prior to the temporary increase in the anisotropy, there is a 50% enhancement in the calculated flux in Figure 24.

The injection function used to calculate Figure 24 (exponentially decreasing) was clearly not the same as the injection which occurred on January 24 and 25 (constant for $\sim 1$ day, then possibly a rapid turn-off) so that the early profiles do not agree. However, the profiles of both intensity and anisotropy quantitatively agree amazingly well later in the event. This may well be simply because the temporary increase in both flux and anisotropy is due to the first arrival and filling up of the pitch angle distribution of the back-scattered particles.

These two examples, together with the prediction of persistent high anisotropy, clearly demonstrate that the simplified scatter-free theory of this chapter is a promising approach for the development of a more satisfactory theory of energetic particle propagation in the inner solar system.
CHAPTER IV

SUMMARY AND CONCLUSIONS

In order to investigate the origin and acceleration of energetic solar charged particles, and their injection into the interplanetary medium, it is necessary to study the early time-histories of flare-associated events, principally the highly anisotropic phase often lasting until the time of maximum flux. This thesis has presented a two-pronged attack on the problem of establishing and refining a description of energetic particle propagation in the inner heliosphere. The discussion here emphasized the propagation of low energy (~0.5 MeV) protons; however, the model can be extended to interpret many high energy observations also.

The model used here was formulated by Roelof and Krimigis (1973), based primarily on a detailed analysis of coronal and multiple spacecraft particle, plasma and interplanetary magnetic field data from three solar rotations in the summer of 1967. The solar aspects of the model are a continuing injection of energetic particles into the interplanetary medium, which is organized by coronal magnetic field structures. The continuing injection of 300 keV protons, supplied either from a long-term acceleration process or from a storage region, must function for times longer than a day.
Interplanetary propagation in this model is described by collimated convection. Therefore, since the high coronal connection points of the interplanetary field can be determined within \( \sim 10^\circ \) using the EQRH approximation (Nolte and Roelof, 1973a), the coronal source locations of interplanetary particle fluxes may also be found.

In the first major part of this thesis I demonstrated that this model provides the framework for understanding 0.5 MeV proton observations in 1965 also (near solar minimum). This demonstration consisted of a statistical study, and also a detailed examination of the individual particle events.

The statistical study was comprised of three major parts. The first part was a comprehensive cross-correlation study of the latitude dependence of the relationship between the interplanetary and chromospheric magnetic polarities from January to August, 1965. In this study I found a best agreement between interplanetary and solar polarity at solar mid-latitudes (N10-30 and S20-30). I also showed that this agreement was not due to a direct connection of interplanetary field lines into the mid-latitude solar active regions.

The next part of the study was a similar cross-correlation analysis of the same time period, but restricted to only those times when 0.5 MeV protons were observed in the interplanetary medium. The results of this study are a striking demonstration that coronal field structure (at least on a statistical basis) controlled the access of
these particles into the interplanetary medium at this time. In contrast to what might be expected, the cross-correlation in this restricted study peaked strongly near the solar equator, and there is no significant correlation at the latitudes of the solar active regions.

If the interpretation that this change in pattern was due to differing coronal magnetic field structure at the times when particles were seen (implying that coronal structure controlled the energetic particles) is correct, recent ideas concerning the origin of fast solar wind streams (e.g., Krieger et al., 1973; Pneuman, 1973) suggest that there should also be an evident change of pattern when the study is restricted to times of fast solar wind. This prediction was verified in the third part of the statistical study, in which the cross-correlation peaked near the equator, at the highest value of the cross-correlation coefficient observed in the entire analysis. This result supports the conclusion above, that the change in the cross-correlation pattern is an indication of a different coronal magnetic field configuration.

A more detailed investigation of the different polarity signatures of the different subsets of the comprehensive study demonstrated that the interpretation of the statistical results in terms of coronal structure was quite reasonable. This investigation showed that if equatorial and interplanetary polarities disagreed, it was
quite likely that the equatorial polarity was a relatively small cell, and also that fast solar wind tended to come from large unipolar regions extending across the equator.

I have also presented a detailed examination of the individual particle events, to test whether they are consistent with the interpretation from the statistical study that the coronal magnetic fields controlled the access of these particles into the interplanetary medium. Several of the events demonstrate definite indication of effects due to such coronal influence.

The increase beginning at Mariner 4 was not associated with a particular solar flare. This event also occurred far from any possible solar active region accelerating source. The time-history observed therefore must have been dominated by the coronal ordering of the particle fluxes, either by means of a long term (longer than a few days) storage and gradual release, or by a preferential injection of particles onto certain interplanetary magnetic field lines.

There are two indications that coronal structure influenced the propagation of energetic particles during the flare-associated particle event of February 5, 1965, the largest increase of this time period. The first is the observation of lower fluxes of 0.5 MeV protons from the flare active region than from regions nearby. Additionally, a comparison of $\gamma$15 MeV proton fluxes at Mariner 4 and near earth shows some evidence of longitudinal structure associated with solar wind streams.
The increase of May 6 was apparently due to a quasi-stationary corotating particle population. However, this event was observed 50-90° removed in longitude from the most likely active region source. The most logical explanation is again that coronal structure controlled the release of these particles.

The June 28-July 6 particle increases at 0.5 MeV are divided into three interplanetary regimes. The boundaries of these interplanetary particle populations correspond very well with low coronal structures observed in Hα filtergrams. This correspondence implies that this complicated time-history of low energy proton fluxes was also dominated by coronal structure.

Comparison of observations at Mariner 4 and near earth during this period at a slightly higher energy (>1 MeV) also demonstrates the possible effects on observed time-histories due to solar wind structure. At Mariner 4 the Chicago detector sees a symmetric profile typical of a quasi-stationary corotating event; IMP 3 sees a much sharper rise, similar to a flare onset, but due to a sudden increase in solar wind velocity which shifts the coronal connection point rapidly to the east.

Another event which demonstrated the effects of solar wind structure occurred in early August. At this time IMP 3 (near earth) failed to observe a quasi-stationary increase seen by Mariner 4, but solar wind mapping showed that the spacecraft was in the magnetosphere just when the event should have been seen.
These events prevent the strongest individual support for the results of the statistical study. The other events also exhibit some indication of coronal control and lead to the conclusion that the model described above explains low energy solar proton events in 1965, as well as in 1967 as shown by Roelof and Krimigis (1973). This work is thus a major contribution to the growing observational evidence that these concepts provide a means for understanding low energy solar charged particle events.

In Chapter III I demonstrated the first propagation theory using the ideal Archimedean spiral field which accurately describes events which are anisotropic up to the time of maximum flux. This theory is an extension of the work of Roelof (1974), who used an $r^{-2}$ field and simple boundary conditions. I have extended the calculation to the most general case for which the mathematical technique is applicable, and have obtained a numerical solution in cases of exponential decrease of the injection and "diffuse reflection" for 400 keV protons. I have also converted the calculated pitch-angle distributions to an idealized detector response, for comparison with spacecraft observations.

The principal verification of scatter-free theory as more nearly valid than diffusion theory for interplanetary propagation is the prediction of high anisotropy up to the time of maximum flux. This alone is sufficient to
demonstrate the desirability of further investigation along these lines. Additionally, Roelof (1973) has shown that scatter-free theory fits the late-time decay of a particular simple event (August 5-7, 1967). Here I have also demonstrated that the scatter-free theory fits other events well past the initial onset, in one case semi-quantitatively (March 21, 1969), in another (January 24, 1969) very well at times more than 2.5 days after the flare, even with a somewhat different injection profile.

Further comparison, preferably with multiple spacecraft data, is necessary to determine the extent to which the assumption of negligible scattering in the inner solar system is valid. However, this extreme case of collimated convection provides the best description of many particle events up to the time of maximum flux that is currently available. This semi-quantitative agreement of the most extreme case with observations strongly supports the hypothesis that the less restrictive concept of collimated convection is the correct description of low energy solar charged particle propagation in the interplanetary medium.
BIBLIOGRAPHY


APPENDIX

Sected Counting Rates

**Figure 21**

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