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A Comparison of Absolute and Relative Stand Density Measures Used in the Northeastern United States

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**A COMPARISON OF ABSOLUTE AND RELATIVE STAND DENSITY MEASURES
USED IN THE NORTHEASTERN UNITED STATES**

BY

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THESIS

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ABSTRACT

A COMPARISON OF ABSOLUTE AND RELATIVE STAND DENSITY MEASURES USED IN THE NORTHEASTERN UNITED STATES

by

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University of New Hampshire, December 2014

The quantification of observed stand density relative to a desired density, a measure known as relative density, is a critical component of many silvicultural treatments. Calculating stand density is more complicated when there are multiple species involved. I compared four regionally appropriate relative density measures to frequently used absolute density measures such as biomass, basal area, trees per area and stand density indices. I found absolute measures inferior to relative measures of stand density in that they lack an accepted reference point that allows for the comparison of one stand to the next in a meaningful and biologically accurate way. Focusing on the four relative density measures I explored the effects of species groups, specific gravity, plot size and definition of maximum density in mixed-species forests of the northeastern United States (New England and New York). The comparison of the relative density measures considered here resulted in conclusions similar to Curtis (1970) in that the choice among the measures is, in part, a matter of available information and convenience of computation. The cluster analysis implied measures form clusters based on the lumping viewpoint versus splitting viewpoint. Rater agreement analysis, used as a novel method of

comparing relative density models, suggests that Ducey and Knapp (2010) density estimates fall in between other density model estimates and thus if a single relative density model needs to be used in this region the Ducey and Knapp one seems most appropriate. Although on average FOXDEN2.1 (Desmaris, 2001) and Stout and Nyland (1986) provide higher density estimates than Ducey and Knapp (2010) and Woodall et al. (2006) when density estimates are assigned to categories, Ducey and Knapp (2010) and Woodall et al. (2006) consistently place more plots in higher density categories than FOXDEN2.1 (Desmaris, 2001) and Stout and Nyland (1986). The small FIA plot size used to estimate model coefficients may explain why Ducey and Knapp (2010) and Woodall et al. (2006) behave this way.

CHAPTER 1

INTRODUCTION

The term stand density, in general, refers to the amount of tree vegetation per unit land area and is often expressed in terms of number of trees, amount of basal area, or biomass (Smith et al., 1986). The search for the ideal stand density amount that maximizes total volume growth has continued since the beginning of forestry without a definite answer (Zeide, 2004). The relationship between maximum density and stand productivity is an important relationship to understand for maximizing economic gains and achieving desired forest management goals. The quantification of observed stand density relative to a desired density, a measure known as relative density (RD), is a critical component of many silvicultural treatments. RD is also key to ecological endeavors such as managing for fire risk (Raulier et al. 2013), developing owl habitat (Fiedler and Cully, Jr. 1995) and studying songbird response to harvesting (Costello et al. 2000). By its definition, relative density is not affected by any management objective. The same RD will indicate the same thing at the same levels at all local levels.

A key objective of the US Climate Change Science Program is to characterize the carbon sink in the United States. This requires an understanding of how to quantify "the role of terrestrial ecosystems in the global carbon cycle" (Birdsey and Heath, 1995) which can be accomplished through the use of relative density. RD has also been used to

understand the effects of stand and inter-specific stocking on carbon stocks (Woodall et al., 2011). Keyser and Zarnoch (2012) utilized relative stand density as a means for enhancing model predictions of aboveground live tree carbon (ATC). Of the factors they studied, Keyser and Zarnoch (2012) found that next to basal area (BA), RD is the next most important predictor of ATC levels.

Additionally, although silviculture guides like Leak et al.'s (1986) *Silvicultural Guide for Northern Hardwood Types in the Northeast* and Frank and Bjorkbom's (1973) *Silvicultural Guide for Spruce-Fir in the Northeast* rely on stocking guides and diameter distributions, resource managers are constantly exploring ways of including more environmental details in their assessments. Decision making and planning software programs like NED2, which includes silvicultural and wildlife models, have addressed this by including a greater number of data entry variables (Nute et al. 2004; Twery et al. 2005). While not all researchers have included relative density as a study variable (Keeton 2006; Kelty et al. 2003; Allison et al. 2003) many do. Long (1985) advocated for the use of size-density based indices in the development of density management regimes based on their independence of site quality and stand age. While Hillebrand et al. (1992) proposed using relative diameter to improve the correlation between relative stand density and growth, Reyes-Hernandez et al. (2013) focused on site quality and species composition effect on maximum size-density relationships in aspen and white spruce stands and Bennett and Maguire (1995) used relative density as a thinning target.

After a brief overview of the concepts of stand density and relative density, I examine the similarities and differences between four competing relative density measures and five absolute density measures and evaluate their overall usefulness.

1.1 Importance of Accurate Stand Density Measures

Forest ecosystem management must often make due with using only a few easily measured variables to develop plans and make decisions. Zeide (2004) notes that one of the reasons why we search for the right stand density measure is that the effect of density on growth is not separate from tree size and age. As trees get older, they tend to get bigger and as trees get bigger there tend to be fewer of them, thus a lower density is not necessarily evidence of an empty stand but perhaps an old one. Therefore if the density measure confuses an empty stand for an old stand and recommends inappropriate management it can lead to loss of time and resources.

1.2 Defining Stand Density

Stand density is different from stocking in that stand density is an amount independent of management objectives whereas stocking refers to a specific amount compared to a desired amount based on a specific management goal (Curtis, 1970). In other words, stand density is a measurement of a stand in square feet of basal area, number of trees or volume per acre whereas stocking is a relative term that describes how closely a stand meets management objectives (Husch et al., 1972). Stocking capacity is a loose term in that the amount can change depending on the management objective. If old growth is the goal then a stand at 70% maximum stocking might still be considered under-stocked. However, if a stand is being managed for early successional tree species, then a stand at 70% maximum would be considered seriously overstocked.

Daniel et al. (1979) suggested that stand density is, in fact, the second most important factor, after site quality, in determining the productivity of a site. Since foresters can more easily manipulate stand density over site quality in order to influence productivity, much research has been dedicated to how to accurately measure and calculate stand density. Unless stand density is controlled at the time a stand is established or during its development, it is almost sure to depart from optimum density at some stage of its life (Smith et al., 1986). In some parts of the country, the question of forest management is made simpler by the limited number of tree species. However, the more favorable climate of the eastern third of the United States, according to Barrett (1980), "supports one of the most complicated and variable aggregations of vegetation in the temperate regions of the world". Thus efforts to cope with and manage a more complicated system like the one in the northeast has resulted in a prolific number of methods for calculating stand density. Also, while system complexity is a factor, there are different histories and theories that have developed over the past century that result in fundamentally different approaches to this aspect of forest measurement.

1.3 Some Absolute Measures of Density

Absolute measures include number of trees, biomass, and basal area per unit area. Often the unit area is per hectare or per acre. Density as defined by "number of individuals per unit area is of limited usefulness, since trees increase in size more or less indefinitely and change in dimensions and ability to utilize available site resources in response to the influence of adjacent trees" (Curtis, 1970). A few researchers have used height-based equations to quantify stand density but since height is difficult to measure it

has never been a preferred technique (Wilson, 1946; Czarnowski, 1961). Shinozaki and Kira (1964a; 1964b) proposed using tree biomass or weight to calculate stand density. Later, Drew and Flewelling (1977) proposed using tree volume in place of weight since they are approximately proportional. Since the precursor to basal area is the measurement of diameter at breast height and this is almost always available, early foresters focused on using this measurement to estimate stand density.

1.4 Stand Density Management Diagrams

A common way of using stand density to manage a forest is through the use of density management diagrams. (Jack and Long, 1996; Long and Daniel, 1990). Stocking guides, like Gingrich (1967), are the precursors to stand density management diagrams (DMDs). The stand DMD is a simple biological model relating yield and density at any stage of stand development (Kershaw and Fischer, 1991). The diagram works well for even aged single species stands but is not as effective for uneven aged or multi-species stands and is thus not often used in the northeast United States. Uneven-aged forests, although offering a great deal of management options and flexibility, require more attention to silvicultural detail than even-aged systems (Long and Daniel 1990).

1.5 Reineke's Stand Density Index

Stand density indices (SDI) have been designed to produce a trees per unit area value relative to a reference diameter. Reineke was the first to examine this in 1933 as he sought to find "an adequate expression of density of stocking in even-aged forests". By

plotting the maximum number of trees per acre over average diameter in inches on logarithmic paper, Reineke developed the following relationship:

$$\log N = -1.605 \log D + k$$

where N is the number of trees per hectare, D is their quadratic mean diameter and k is a constant which would vary with species. The reference curve is established by finding the point on the graph where D is equal to 10 inches or 25.4 centimeters. Based on this curve, Reineke created the Stand Density Index or SDI, such that:

$$SDI = N \left(\frac{D}{25} \right)^{1.6}$$

Note that the reference diameter has been simplified to 25 (eg. Pretzsch and Biber 2005) and the allometric coefficient (-1.605) to -1.6 (eg. Curtis 1970).

Reineke's stand density rule is related to Yoda's rule, also known as the $-3/2$'s power rule, which describes the self-thinning line in plant populations; although they were developed independently of each other (Pretzsch and Biber, 2005; Drew and Flewelling, 1977). Drew and Flewelling (1977) explain that Yoda et al. (1963) examined the relationship between average weight, as opposed to average tree diameter, and density in even-aged plant populations. Curtis (1970) pointed out that one shortcoming of Reineke's density measure is that it is only applicable to relatively uniform, homogenous even-aged stands. Over the next several decades many tried comparing other variables or made different assumptions to develop new ways of quantifying stand density in systems more complicated than Reineke originally considered. Curtis (1970) notes that "a ratio of observed basal area to that of a normal stand of the same age and site, frequently used as an expression of relative density, is not directly interpretable as a comparison of areas" but that otherwise, "most common measures appear to be practically equivalent". His

review article focuses mainly on Reineke's SDI (1933), Chisman and Schumacher's Tree-Area Ratio (1940) and Krajicek et al.'s Crown Competition Factor (1961). More recently theorists have attempted to deal with the shortcoming in Reineke's stand density index (Dean and Baldwin, 1996; Zeide, 2005; Vanderschaaf and Burkhart, 2007).

1.6 Calculating Stand Density in Mixed Species Stands

Calculating stand density is more complicated when there are multiple species involved and many researchers have tackled the problem from just as many angles (Sterba and Monserud, 1993; Hasenauer et al., 1994; Woodall et al., 2005; Solomon and Zhang, 2002). Krajicek et al. (1961) identified crown competition factor as a way of calculating stand density. Chisman and Schumacher (1940) recognized that basal area is a number that does not convey frequency of trees according to size and that number of trees in a stand alone does not convey diameter distribution. In an effort to develop a number that better captured the full biological picture they found that by dividing stands into species groups they noted significant improvement in the fit of the tree-area ratio equation. Stout and Nyland (1986) built upon this idea and tried applying it to mixed species stands. There are no theoretical limits to the number of species that tree area ratio equations could be developed for but practically speaking it would be tedious work to find three coefficients per species in order to apply the tree-area ratio procedure (Stout et al. 1987). Baskerville (1992) noted that "when stands are grouped in a type for forest level forecasting but they do not have the same dynamic characteristics, the biological realism of the forecast is reduced". Age at establishment and species composition, which often only focus on a few major species, are only capturing stand appearance. Stout and

Nyland (1986) point out that variability in tolerance to crowding can result in basal areas that differ by as much as 50% among undisturbed stands of the same average diameter, but dissimilar species composition. This helps to explain why researchers continue to look for something as easy as BA to measure and use but which more accurately reflects the state of the stand.

Forecasts can be no better than the degree to which the initial conditions have been characterized and so including information that goes beyond age and basal area may be key to capturing a fuller picture of the forest as it is so that we can better predict the outcomes of the prescriptions used. Perhaps this can, in part, be accomplished through the use of relative density which not only gives the forecaster an idea of current quantity but also an idea about current quantity in relation to possible total quantity. According to Puettman et al. (2009), a major challenge for silviculture as a discipline is the development of an "overarching set of principles and strategies that could encompass the diversity of practices without sacrificing the heterogeneity that arouse from local ecological, economical, and social conditions". Many have sought to develop these strategies and have thus created a multitude of options. Given that not all relative stand density measures are equivalent I examine here how they compare to each other as related to density predications in the northeastern United States. It may be that the measures are redundant enough that each would serve the same purpose. Additionally I examine how these relative density measures compare to absolute density measures.

My primary objective is to compare four regionally appropriate relative density measures to five absolute density measure (biomass, biodensity, basal area, trees per area

and the Additive Stand Density Index). To accomplish this I seek to answer the following questions:

1. Generally, while absolute and relative density measures are sometimes treated as synonymous are the measures presented here similar enough to be redundant?
2. More specifically, do the relative density measures agree or disagree with each other at all relative densities?
3. Finally, how does the handling of variables such as species group, specific gravity and definition of maximum density affect density predictions?

CHAPTER 2

METHODS

2.1 Data Description

2.1.1 Forest Inventory and Analysis (FIA) Data

This study was based on measurements taken as part of the USDA Forest Service's Forest Inventory and Analysis (FIA) program. The data presented here were collected between 1983 and 2007. The study period does span some plot design changes as well as changes from periodic to annual surveys. For details on the FIA Program see Bechtold and Patterson (2005). A subset of the data from the more recent fixed area sampling design was examined and did not differ from the full dataset, which included fixed and variable radius plots. Therefore I determined that the changes did not affect our results. The tree tables for this study area, which includes Connecticut, Maine, Massachusetts, New Hampshire, New York, Rhode Island and Vermont, are available from the FIA website <http://www.fia.fs.fed.us/> [accessed on 18 Sept 2008, except VT accessed on 28 Nov 2007]. These states are dominated by northern hardwoods (Westveld, 1949) and are often grouped together because of their shared ecologies (Barrett, 1980, pg. 25). Table 1 shows the breakdown of the top 10 species represented in the data.

Table 1. Top 10 Species in Data

<i>Common Name</i>	<i>Number of Tree Records</i>	<i>Percent of Total</i>
red maple	116037	14.56%
American beech	43655	9.21%
northern red oak	25199	8.34%
sugar maple	61833	7.81%
paper birch	34377	5.12%
eastern hemlock	48933	5.07%
yellow birch	32828	4.84%
white ash	22292	4.79%
quaking aspen	16177	4.79%
black cherry	11739	3.55%
TOTAL	413070	68.08%

First, I combined the individual state files into one regional file containing all tree records. Since stand density does not include dead trees I removed all of the dead trees from the regional data. Additionally, for unknown reasons, some trees were missing an expansion factor and in these cases the tree record was excluded from the data. In sum, these anomalies accounted for 14% of the total (Table 2). On average, 86% of the trees from each state were included in the analysis.

Table 2. List of Percent Contribution to Total Data Set by State

<i>State</i>	<i>Number of trees included in analysis</i>	<i>Percent of Total</i>	<i>Number of plots</i>	<i>Percent of Total</i>
Connecticut	24950	90	448	0.9
Maine	334,371	84	7705	38.3
Massachusetts	46894	89	743	4.0
New Hampshire	92665	88	1205	8.6
New York	182783	89	4581	35.6
Rhode Island	9873	89	166	1.6
Vermont	68155	89	1018	11
TOTAL	759691	86	15866	100

The tree level FIA data used in this study included Inventory Year, State, County and Plot (which, when combined, uniquely identified a plot) along with a tree record number, a status code (where 1=live), diameter at breast height (DBH), trees per acre (TPA), a species identification code and finally, a species group code (USFS, 2008). The TPA value, also known as an expansion factor (EF), varies depending on whether the plot was fixed or variable radius and in the case of fixed, also depends on plot size. The FIA database includes its own density value in the STOCKING column. This was not included because it was only available for less than 60% of the plots and according to the FIA User Guide "stocking values are computed using several specific species equations that were developed from normal yield tables and stocking charts" which does not clearly explain how the values were calculated (USFS, 2008). Values were then converted from English units, DBH in inches and trees per acre, to metric units, DBH in centimeters and trees per hectare (TPH), as both are needed for this study.

2.1.2 Specific Gravity

Two of the models explored in this study use specific gravity as a variable that accounts for species contribution to stand density. In general the specific gravity of wood depends on the size of the cells, the thickness of the cell walls and the interrelationship between the number of cells of various size and with varying cell wall thickness (Panshin and de Zeeuw, 1970). I tabulated specific gravities for all tree species in the dataset using over a dozen sources (See Appendix A). In some cases, if specific gravity was not available, I estimated it based on the specific gravities of other species in the same genus.

For unknown conifers and unknown broadleaved trees I calculated the median

conifer specific gravity and the median broadleaved specific gravity respectively. The database included species with specific gravities ranging from a low of .35 (eastern white pine) to a high of .84 (osage-orange). Although specific gravity values can vary within a tree species and even within an individual tree, I selected specific gravity at a moisture content of 12% because I found the greatest number of known values and it is considered an average air-dry condition reached without artificial heating (Markwardt, 1930).

2.2 Data Analysis

Cluster analysis and analysis of correlations were used to compare all 9 stand density measures and rater agreement was used to compare the four relative stand density measures. Using these techniques I sought to evaluate the similarities and differences between all the measures, absolute and relative.

2.2.1 Summary Statistics

First I examined the summary statistics including range, mean, standard deviation (SD) and the coefficient of variation (CV). I excluded standard error because with such large sample sizes it is less relevant. Instead I have favored CV which can be used to compare the amount of variation in populations having different means (Sokal and Rohlf, 1973).

$$CV = SD * 100 / \text{mean}$$

CV is independent of the unit measurement and is expressed as a percentage. It is essentially the standard deviation expressed as a percentage of the mean.

2.2.2 Cluster Analysis

In order to see how potentially redundant some of the measures are I also examined a dendrogram using Ward's (1963) method of hierarchical clustering. I treated each stand density index or predictor as a case, treated each plot observation as a variable, and created clusters based on how often those predictions were similar. Hierarchical clustering required that I rank all the measures prior to analysis because the predictions are on different scales. To do this I sorted the plots based on one density measure at a time and assigned a number from 1 to 15866 (the total number of plots). I did this for each density measure such that I had a rank value for all measures that was independent of scale. This allowed me to include both the absolute and relative measures in the cluster analysis.

I choose Ward's (1963) method of clustering because I have no preconceived notion of how many clusters actually exist. I might have chosen K-means clustering due to the large number of records, but since I do not expect a large number of possible classification groups, Ward's method was most appropriate. With this method, the distance between two clusters is equal to the ANOVA sum of squares between the two clusters added up over all the stand density predictions. The clusters are created by minimizing the sum of squares within the cluster and maximizing the distance between clusters.

2.2.3 Correlations

Since the data, as would be expected, are not normally distributed I chose to use nonparametric methods to analyze the relationships between density measures. To confirm the non-normality of the data I conducted a visual inspection of the distributions

of stand density predictions (Appendix B) and also an examination of the KSL statistics (Appendix C). When the KSL statistic D is less than .01 the data are considered normally distributed. The hump at the low end of the distributions of the stand density predictions, shown in Appendix B, is likely due to several factors. Low density stands are more abundant in a managed system and also regular disturbance can lead to a larger number of low density plots. If sampling design favored the detection of low density plots then the data would be skewed to detect a greater number of low density plots.

Examination of a scatterplot matrix illustrating each pair-wise correlation between all the stand density measures considered in this study and a table comparing the Spearman's rank correlation coefficient rho (P) values provided insight into the relationships between measures. Using P values allowed me to easily include both the absolute and relative measures regardless of units of scale. The Spearman's rank correlation coefficient is a quantity arrived at by first assigning a rank to each row in a measurement variable column and then calculating the difference between ranks assigned by that measurement (Pagano and Gauvrea, 2000). I chose this method because it does not require that the data be normally distributed nor do the variables need to be on the same scale. Spearman rank correlation coefficients range from -1 to 1 where zero implies a lack of linear association between variables.

I defined a strong correlation as a Spearman's coefficient of .9500 or greater, a mild correlation between .9000 and .9499 and a weak correlation as a value of less than .9000. Using Spearman's P, values I compared the relative strengths of the pair-wise correlations among the nine stand density measures.

2.2.4 Rater Agreement

For the rater agreement phase of the analysis I focused on the relative measures. By creating relative density categories I could use rater agreement analysis to explore how much the density predictions based on the relative density models agree with each other. I go into greater detail about the rater agreement density categories shortly and explain the relative density models in great detail in the description of the data. Also, rater agreement does not require that the data be normally distributed. One set of categories I used to examine overall differences while the other set of categories was aimed at quantifying differences when the categories were based on management objectives.

To include all measurements, regardless of scale, I would have to create categories based on ranks and while that is theoretically easy to apply it does not make much practical sense. Those categories would only apply to this data set because its based on this particular sample size and population characteristics. Disregarding the absolute measures for this part of the analysis, I compare relative density measurements by creating categories based on equally sized density ranges and also relevant % RD thinning groups.

First, I calculated % RD from 0 to 100 for each plot. I then placed each plot into a category based on that predicted % RD. In the framework of rater agreement each RD model is treated as an individual observer or rater. Since each relative density measure has slight differences in underlying philosophy, each is coming at the question from a slightly different perspective and like different observers that perspective changes the

answer to the question from the observer to observer. Inter-observer agreement is evaluated using the Kappa statistic (Landis and Koch 1977, Cohen 1960).

Next, I created management relevant stand density categories. The Marquis et al. (1992) Allegheny Hardwoods silviculture guide recommends thinning at relative densities between 40 and 60% and so to that end I created the following four possible density categories:

Cat 1: RD less than 40% = Low density, don't thin

Cat 2: RD between 40 and 50% = Possible thin

Cat 3: RD between 51 and 60% = Recommended thin, 50-60%

Cat 4: RD greater than 60% = High density, requires further evaluation

I assigned the stand density prediction of each plot by each relative density measure into one of these categories and evaluated how often the predictions agreed. The estimate of the crude proportion of agreement between two observers is the sum of the observed proportions on the main diagonal of the corresponding 2-way table (Landis and Kohn, 1977). Figure 1 shows this diagonal shaded gray; these are the instances the observers agreed on the category assignments.

		Rater A			
Category		1	2	3	4
Rater B	1	Counted here if A=1 AND B=1 (observers agree)	Counted here if A=2 but B=1 (observers disagree)	Counted here if A=3 but B=1 (observers disagree)	Counted here if A=4 but B=1 (observers disagree)
	2	Counted here if A=1 but B=2 (observers disagree)	Counted here if A=2 AND B=2 (observers agree)	Counted here if A=3 but B=2 (observers disagree)	Counted here if A=4 but B=2 (observers disagree)
	3	Counted here if A=1 but B=3 (observers disagree)	Counted here if A=2 but B=3 (observers disagree)	Counted here if A=3 AND B=3 (observers agree)	Counted here if A=4 but B=3 (observers disagree)
	4	Counted here if A=1 but B=4 (observers disagree)	Counted here if A=2 but B=4 (observers disagree)	Counted here if A=3 but B=4 (observers disagree)	Counted here if A=4 AND B=4 (observers agree)

Figure 1. Illustration of How Rater Agreement is Quantified.

2.3 Stand Density Measure Descriptions

From the long list of possible stand density measures to include in this analysis I chose a range of absolute measures that vary in prevalence of use. Most common are biomass, basal area and trees per acre. Slightly less common are biodensity and the additive stand density index. Of the possible relative measures I chose two developed specifically for this region (Desmaris, 2001; Ducey and Knapp, 2010), one that was developed for use anywhere in the United States (Woodall, 2006) and one that was developed for nearby Pennsylvania (Stout and Nyland, 1986; Stout et al., 1987). A brief description of each of the absolute measures is followed by a more detailed description of each of the relative density models.

2.3.1 Basal Area and Trees per Acre (BA and TPH)

Basal area was calculated using FIA recorded diameter at breast height measurements and multiplied by the appropriate expansion factor (TPH). The expansion factor will change based on the plot design. The variable “Trees per acre” is set at a constant and comes from the subplot, microplot or macroplot radius on which the trees are sampled. For more information on plot design see also Curtis and Marshall (2005) and the FIA Fact Sheet Series (Brand, 2005; Burkman, 2005a and 2005b).

2.3.2 Biomass

Total biomass is a common characteristic used to describe and compare forest stands. Although there are thousands of biomass equations that are specific to a particular species, group of species or geographic region, I chose to use Jenkins et al.’s (2003) national-scale biomass estimators as it simplifies the analysis. As Tritton and Hornbeck (1982) point out, estimating tree biomass (weight) based on parameters that are easily measured in the field is a fundamental task in forestry. Although the emphasis on ease of field measurements is important, Jenkins et al. (2003) offers a means of easily calculating biomass from those field measurements without having to sift through a long list of biomass equation options. Jenkins et al. (2003) compiled over 2,500 equations and selected 707 equations for over 100 species from 104 sources. Although the data used to build the original equations under represent large trees (Jenkins et al. 2003) the FIA data used here contains a small (less than 10%) portion of trees of a diameter greater than 35 centimeters (~14 inches). Based on the 707 equations Jenkins et al. (2003) developed the following national-scale biomass estimator for tree species in the United States:

$$bm = \text{Exp}(\beta_0 + \beta_1 \ln dbh)$$

where bm is the total aboveground biomass (kg) for trees 2.5 cm dbh and larger; dbh is the diameter at breast height in centimeters; Exp refers to the exponential function; ln is the natural log base “e” (2.718282). The model coefficients β_0 and β_1 differ based on species group, of which Jenkins et al. (2003) identified 10; four hardwood, five softwood and one woodland. Each of the trees in the FIA data discussed earlier was assigned to one of the ten groups Jenkin et al.(2003) identified and this variable was used to calculate each tree's total aboveground biomass in kilograms.

2.3.3 Biodensity

Using the biomass values calculated as explained above, I also calculated a biodensity value for each tree. For the purpose of this analysis, biodensity was calculated based on Drew and Flewelling's formulation (1979). They began with

$$v = a * p^{-3/2}$$

where v = mean tree volume (or in this case biomass), a is a constant and p is the stand density (or in this case trees per acre). This equation only works for the limiting (maximum) condition, not the equation for the density of any particular stand. So, if we re-label p as p_{max} , to clarify that this is the maximum number of trees that can be packed in at a given weight we get:

$$v = a * p_{\text{max}}^{-1.5}$$

Solving for p_{max} as a function of v , and dividing both sides by the constant a and finally taking both sides to the $-2/3$ power we arrive at:

$$a^{2/3} * v^{2/3} = p_{\text{max}}$$

where we can calculate the maximum number of trees possible at a given weight by multiplying some constant by mean biomass. If we want to know what fraction of the maximum density of any given stand we essentially want to calculate T divided by p_max which gives us

$$T/p_{\max} = a^{2/3} * T * w^{2/3} = \text{constant} * T * w^{2/3}$$

This is a non-additive density equation using trees per hectare and the average value of a biomass that can be easily used to calculate stand level summaries but not as easy to separate the contributions of individual trees and to calculate a stand error. If we want to solve for the constant we need to know the maximum stand density. In the absence of this information we can set the constant equal to 1 and allow biodiversity to take on whatever range of values it may be. Setting the constant, a, to 1 we arrive at

$$p_{\max} = T * w^{2/3}$$

where w came from Jenkins biomass equations and T came from the unadjusted TPH values as reported by FIA.

2.3.4 Additive Stand Density Index (ASDI)

The last absolute measure used in this study is an additive form of Reineke's (1933) Stand Density Index (SDI) developed by Long and Daniel (1990). Long and Daniel (1990) reasoned that since the contribution of individual stand components to both total SDI and total site utilization is additive, Reineke's (1933) SDI could be modified and used to manage uneven aged stands and assess stocking levels. The equation they developed is now often referred to as the additive or summation method:

$$ASDI_i = \sum TPH_i \times \left(\frac{DBH_i}{25} \right)^{1.6}$$

where ASDI is the additive stand density index at the tree level, TPH_i is the expansion factor trees per hectare and DBH_i is the diameter at breast height of the i^{th} tree. The slope coefficient of 1.6 does vary by species, but was held constant for simplification. This absolute measure is in trees per hectare. Pretzsch and Biber (2005) found the slope coefficient varied by species with a low of -1.424 for common oak in Norway and a high of -1.789 for common beech. I argue that given the mixed species system that is examined here the average slope coefficient is sufficiently close to -1.6 and thus an acceptable simplification. Long and Daniel (1990) suggest that ASDI can be used for uneven-aged management because it accommodates the skewed diameter distribution, typical of uneven-aged forests, which is weighted towards smaller trees. ASDI is also better than basal area for comparing and managing forests (Long and Daniel, 1990).

2.3.5 Relative Density Model Descriptions

The expansion factor was used to calculate tree values on a per hectare basis for basal area, biomass, biodiversity, ASDI and four regionally appropriate relative density models described in the methods section. The expansion factor is determined by the plot radius for that tree which varies depending on plot size and sampling method. Of the four models examined three could be calculated at a tree level (Stout and Nyland, 1986; Desmaris, 2001; Ducey and Knapp, 2010). A fourth relative density measure (Woodall et al., 2006) is added to the data set at the plot level. It is not an additive measure and so could not be calculated on a tree by tree basis. For each plot I also added mean specific gravity values and weighted specific gravity values. The straight mean was calculated as:

$$\text{Mean SG} = \Sigma(\text{SG})/\text{Number of species}$$

Where as the weighted mean was calculated as:

$$\text{Weighted Mean SG} = \Sigma [\text{SG}_i * \text{TPH}] / \Sigma \text{TPH}$$

The two will be different when TPH varies from tree to tree. This could happen if the plot was part of periodic inventories when variable radius plots were used. This also happens as part of the fixed radius plot sampling design; trees less than 5" in diameter are measured on a smaller fixed radius plot and so those TPH values are different from those sampled on the full 0.04 acre plot. Ultimately, on the large scale, this difference was insubstantial. However, it might be a future point of interest to analyze the impact on a smaller scale.

Using the data outlined above, I compared four possible mixed species relative density measures (Table 3).

Table 3. List of Relative Density Equations

<i>Authors</i>	<i>Name</i>	<i>Model</i>
Desmaris (2001)	FOXDEN2.1: A Relative Density Spreadsheet	$RD_f = \sum_i \text{TPH}_i \times (b_{0i} + b_{1i} \text{DBH}_i + b_{2i} \text{DBH}_i^2)$
Stout and Nyland (1986)	Allegheny Hardwood Relative Density	$RD_s = \sum_i \text{TPH}_i \times (b_{0i} + b_{1i} \sum_j \text{DBH}_{ij} + b_{2i} \sum_j \text{DBH}_{ij}^2)$
Ducey and Knapp (2010)	Stand Density Index for Mixed Species Stands	$RD_d = \sum_i \text{TPH}_i \times (b_0 + b_1 \text{SG}_i) \left(\frac{\text{DBH}_i}{25} \right)^{1.6}$
Woodall et al. (2006)	Relative Density of Mixed Species Forests	$RD_w = \frac{\sum \text{TPH}_i \times \left(\frac{\text{DBH}_i}{25} \right)^{1.6}}{b_0 - b_1 (\text{SG}_m)}$

The four models that were chosen for analysis vary in levels of complexity.

Height measurements are not required for any of the models. FOXDEN2.1's form has

been simplified in the table above in the sense there are 16 species groups and each group has its own set of coefficients. The form above is giving the relative density contribution of one tree. You would multiply this RD_f by trees per acre to get a total RD for that species and then sum RD_f 's for all species on the plot to get a total RD. Stout and Nyland (1986) also utilizes different coefficients for different species groups and is handled similarly. These models are explained in detail below.

Relative Density Models - Using Species Groups. Two of the four models rely on assigning species into groups and then applying coefficients based on those group assignments. Both of these models also have their basis in the tree area ratio; they are FOXDEN2.1 developed by Ken Desmaris (2001) and the Relative Density Equation for the Allegheny Hardwoods by Stout and Nyland (1986).

Desmaris (2001) - RD_f .

FOXDEN2.1 (Desmaris, 2001) was developed for use in the Relative Density Spreadsheet developed by the State of New Hampshire Caroline A. Fox Research and Demonstration Forest. It uses similar algorithms found in NED, the Northeast Decision Model Stand Inventory Processor and Simulator (Simpson, 1995). FOXDEN2.1 breaks the species of the forest into 16 groups and each group has its own set of coefficients (Table 4) which are used in the following equation to estimate relative density:

$$RD_f = \sum_i TPH_i \times (b_{0i} + b_{1i}D_i + b_{2i}D_i^2)$$

Individual tree contributions are calculated using the RD_f equation above. Then, in order to get a per plot estimate, each RD_f is multiplied by the expansion factor, and then each

species total RD_f is summed to get the total plot RD_f . In some case RD_f results in a negative value, since it doesn't make sense that a tree would have a negative contribution to the plot density this was changed to a very small positive number (10^{-6}). FOXDEN2.1 (Desmaris, 2001) did not directly account for all species in the FIA data set. Of the 132 species represented in the data, 85 were not explicitly placed in a FOXDEN2.1 group however overall close to 85% of the nearly 760,000 trees are accounted for by the model.

Table 4. List of FOXDEN2.1 Model Coefficients

<i>Species Group</i>	<i>FIA Code(s)</i>	<i>B0</i>	<i>B1</i>	<i>B2</i>
white pine*	129	0.034975	0.00979025	0.002183375
red pine	125	-0.025418	0.014753	0.0016229
jack pine and all other unspecified softwoods	105; and anything >300 not specified elsewhere	-0.072197	0.034163	0.0010222
hemlock	261	-0.011528	-0.00085458	0.0026439
spruce-fir	12; 90-98	-0.019701	0.02164	0.00031039
white cedar	241	-0.0024055	0.0049422	0.0022667
Northern hardwoods (beech, birch, sugar maple)	318; 371; 531	0.0077041	0.0062613	0.003848
red maple and all other unspecified hardwoods	316; and anything <300 not specified elsewhere	-0.017979	0.021425	0.001711
paper birch	375	0.044283	-0.012946	0.0058748
CAPS (cherry, ash, yellow poplar)	541; 621; 762	0.027937	0.015452	0.000871
green ash, elm, cottonwood	544; 742; 972	0.032589	0.0074386	0.0038338
black walnut	602	0.030878	0.018058	0.0042321
hickories and other oaks	400-409, 800-837 (except 833)	0.002802	0.011881	0.003546
red oak	833	-0.0053402	0.0073765	0.004321
basswood	951	-0.0081504	0.0008167	0.0028048
aspen	743; 746	0.0041871	0.012551	0.0023796

*The original formula for white pine was based on an A-line = 80%; in order to convert values, coefficients were divided by 0.8.

Stout and Nyland (1986) - RD_s.

The Stout and Nyland (1986) model is a modification of the tree area ratio (TAR) (Chisman and Schumacher 1940) which they developed for use in the Allegheny hardwood region. Stout and Nyland (1986) include species composition, not a part of the original TAR, such that the modified model takes the following form:

$$100 = \sum_i (b_{0i}N_i + b_{1i} \sum_j D_{ij} + b_{2i} \sum_j D_{ij}^2)$$

where N_i is the number of trees per unit area of the i^{th} species, D_{ij} is the diameter of the j^{th} tree of the i^{th} species on the plot, and b_0 , b_1 , and b_2 are the model coefficients that they estimate. By breaking their dataset up into species groups, Stout and Nyland (1986) found values for the model coefficients for each of three species groups. Thus the Stout and Nyland (1986) relative density (RD_s) is:

$$RD_s = \sum_i (b_{0i}N_i + b_{1i} \sum_j D_{ij} + b_{2i} \sum_j D_{ij}^2)$$

where the estimates of coefficients are reproduced from Stout and Nyland (1986) in Table 5.

Table 5. List of Coefficient Estimates for Stout and Nyland's (1986) Model

<i>Species group</i>	<i>b₀</i>	<i>b₁</i>	<i>b₂</i>
Sugar maple- American beech	-0.03082	0.06272	0.04690
Red maple	-0.17979	0.21425	0.01711
Black cherry	0.27937	0.15452	0.00871

This density metric is designed to fall between 0 and 100, where 100 would indicate a full or maximally stocked plot. This density measure is based on Chisman and

Schumacher's (1940) Tree Area Ratio, which is reported in milacres. Stout and Nyland (1986), however, report values in $1/100^{\text{th}}$ of an acre, so RD_s values had to be divided by 10. Also, since this relative density measure did not necessarily apply to all the species in my dataset I used Marquis et al. (1992) as guidance and assigned all unaccounted for species to Group 2. This meant that 111 out of 132 species were placed in group 2. This accounted for nearly 45% of the data.

Relative Density Models - Using Specific Gravity. The following two models use specific gravity as a variable that accounts for species contribution to stand density. Specific gravity is a good index of mechanical properties (Forest Products Laboratory, 1999) and is negatively correlated with maximum SDI (Dean and Baldwin, 1996). For example, take two stands with the same mean basal area (BA) but composed of two different species, A and B. If A has a higher specific gravity than B, it would reason that trees in stand A can support more foliar biomass per tree. Thus it should take fewer of tree A to completely occupy a stand, implying that the maximum SDI for species with high specific gravity will be lower than that of species with low specific gravity (Ducey and Knapp, 2010). How this relationship is exploited by Ducey and Knapp (2010) and Woodall et al. (2006) to attempt to increase the accuracy of relative density measurements is explained below.

Ducey and Knapp (2010) - RD_d .

The Ducey and Knapp (2010) model was developed using the Forest Inventory and Analysis (FIA) database, maintained by the United States Department of Agriculture.

This relative density measure is additive and based on the relationship between specific gravity and stand density. Ducey and Knapp (2010) proposed that relative density is related to both specific gravity and the diameter distribution of the stand:

$$RD_d = \sum_i TPH_i \times (b_0 + b_1 SG_i) \left(\frac{DBH_i}{25} \right)^{1.6}$$

where b_0 and b_1 are model coefficients, SG_i is the specific gravity of the i^{th} species and DBH_i is the diameter at breast height of the i^{th} species. This gives the relative density contribution of the tree of the i^{th} diameter. Utilizing quantile regression and a specialized quantile selection technique, the relative density of mixed species stands in the northeastern United States was found to be:

$$RD_d = \sum_i TPH_i \times (0.00015 + 0.00218 * SG_i) \left(\frac{DBH_i}{25} \right)^{1.6}$$

where the model coefficient values are based on which quantile produced density values that most closely agreed with current single species standards as well as several well established standards for commonly occurring simple mixtures such as spruce and fir.

Woodall 99th (RD_{w99}) and Woodall Max (RD_{wMax}).

The Woodall et al. models are based on data from the National Resource Planning Act (RPA) database and were developed for predicting maximum stand density based on the mean specific gravity of the tree species on each plot (Woodall et al., 2005). Note the use of specific gravity differs from Woodall et al. (2005) in that each tree's contribution is calculated separately using its own specific gravity as opposed to Woodall et al. (2005) which used the mean specific gravity for the plot.

Woodall et al. (2005) proposes that the maximum SDI for each plot was estimated using the following model:

$$E(SDI_{max}) = 3546.7 - 3927.3(SG_m)$$

where $E(SDI_{max})$ is the statistical expected value of maximum stand density and SG_m is the mean specific gravity for each plot. Woodall et al. (2006) went on to show that relative density (RD_w) can then be estimated by dividing the additive form of SDI (ASDI) by $E(SDI_{max})$ for each plot:

$$RD_{wMAX} = \frac{\sum \left(\frac{DBH}{25} \right)^{1.6}}{3546.7 - 3927.3(SG_m)}$$

Woodall et al. (2006) found that the relative density at the 99th percentile had the greatest predictive power. Similar to the equation above:

$$E(SDI_{99}) = 2057.3 - 2098.6(SG_m)$$

So that the relative density equation becomes:

$$RD_{w99} = \frac{\sum \left(\frac{DBH}{25} \right)^{1.6}}{2057.3 - 2098.6(SG_m)}$$

The relative density at the 99th percentile has different coefficients and so I compared this density measure to the others as well to see if this did, in fact, make a difference.

A summary of the background information on each of the models can be found in Table 6. RD_s and RD_f are on a scale of 0 to 1 where as RD_d and the two RD_w models are on a 0 to 100 scale. RD_d and RD_w also allow for an unlimited number of species where RD_f has 16 groups and RD_s has 3. This difference results in RD_d and RD_w accounting for 100% of the species in this study where as RD_f accounts for approximately 85% and RD_s

directly accounts for approximately 55%. RD_s study plots were in Pennsylvania; RD_d and RD_f primarily focused on forests in New England and New York and the RD_w models included plots from the entire United States. They also each differ in how they defined maximum density and how plots were selected but are similar in what data are needed to use them.

Woodall et al. (2006) found the maximum similar to how Reineke (1933) did, by seeking out the visual upper limit. Where Reineke did this by eye, Woodall et al. (2006) used a computer to fit the highest percentiles and both used the upper limit to estimate model coefficients. Stout and Nyland (1986), on the other hand, prescreened plots they believed to be normally stocked and used that information to estimate coefficients that would allow estimation of relative density in comparison to the study plots. Finally, Ducey and Knapp (2010) used quantile regression and some regionally specific stocking guides to find the percentile that would produce results that would best match those stocking guides. Woodall et al. (2006) and Ducey and Knapp (2010) used the FIA database and tens of thousands of plots to estimate equation coefficients where as Stout and Nyland (1986) and the multiple equations in FOXDEN2.1 (Desmaris, 2001) are based on prescreened plots chosen to represent stands of a specific stocking amount.

Table 6. Summary of Background Information on Relative Density Models

	<i>RD_s</i>	<i>RD_f</i>	<i>RD_d</i>	<i>RD_{w99}</i>	<i>RD_{wMax}</i>
What region was this developed in?	Pennsylvania	New England	New England and New York	United States	United States
What is range of data?	0 to 100	0 to 100	0 to 1	0 to 1	0 to 1
Are trees treated individually or as part of a species group?	Group	Group	Individual	Individual weighted by plot mean specific gravity.	Individual weighted by plot mean specific gravity.
No. of species groups	3	16	Unlimited	Unlimited	Unlimited
% of tree species accounted for by model	55%	85%	~99%	~99%	~99%
How is maximum found?	Visual inspection of stands	Using stocking charts	Using quantile regression	Using 99th percentile	Using 100th percentile
How is maximum (A-line) defined?	At 100% (like Gingrich)	Variable depending on stocking chart	At 85% of max	At 99% of max	At 100% of max
What plot size was used?	0.10 to 10.37 hectare (or .25 to 25.6 acre)	Variable. Depended on stocking chart.	.04 acre	.04 acre	.04 acre
Sample size of original study?	201 plots	Unknown	15,866 FIA plots	119,235 FIA plots	119,235 FIA plots

CHAPTER 3

RESULTS

3.1 Summary Statistics

Table 7 shows why direct comparison of all the measures is difficult. They are on different scales and have different ranges. For this reason I first ranked each of the measures and performed a cluster analysis based on Ward's (1963) methods.

Table 7. Comparing Distribution and Error of Absolute and Relative Stand Density Predictions

<i>Measurement</i>	<i>Units</i>	<i>Min</i>	<i>Max</i>	<i>Mean</i>	<i>SD</i>	<i>CV</i>
TPH	(trees per hectare)	3.11342	20856.5	2088.1752	1949.4266	93.4
BA	(m ² /ha)	.07509	75.1365	22.017657	11.342132	51.5
Biomass	(kg/ha)	113.588	490998	115359.39	67068.03	58.1
Biodensity	(kg/ha)	124.115	68571.3	20583.925	9990.7392	48.5
ASDI	(trees per hectare)	3.81796	1771.44	502.86224	251.68907	50.0
%RD _d	%	.349	202	61.3	29.4	48.0
%RD _f	%	.000002*	353	77.6	39.2	50.5
%RD _s	%	.309	270	79.0	39.5	49.6
%RD _{w99}	%	.289	155	50.5	24.3	48.2
%RD _{wMax}	%	.176	98	32.5	15.9	48.6

*Some plots contained so few trees that FOXDEN2.1 densities were negative, I arbitrarily assigned a very small positive value resulting in this minimum value.

Based on this data set we see that all but two of the measures have a coefficient of variation of roughly 50% (See Table 7). The coefficient of variation of biomass is a bit higher at 58% but TPH is the most variable at 93%. This means that any estimate of TPH

may vary by nearly 100% or double the error of the other measures. The difference between TPH and the other density measures is reiterated by the dendrogram.

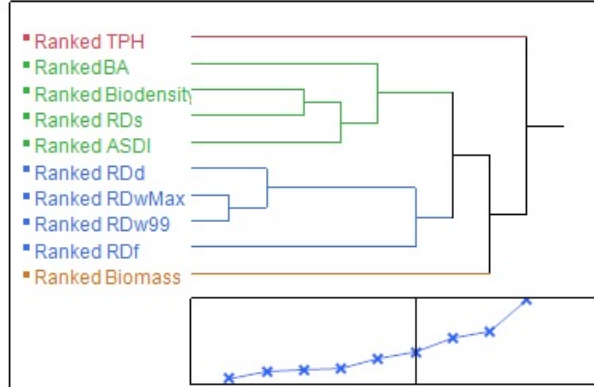
3.2 Cluster Analysis

As discussed earlier, the dendrogram is an example of an hierarchical algorithm where the distance between each stand density prediction is a measurement of dissimilarity. This distance, or numerical difference, is used to assign each of the stand density measures to a cluster. The dendrogram and distances listed in the table in Figure 2 show that there are essentially four clusters. TPH and Biomass each make up their own cluster. Another cluster is formed by the measures that account for 10 or less species groups. This includes ASDI and BA which do not directly account for species; Stout and Nyland's (1986) relative density measure (RD_s) which forces all species into one of only 3 species groups; and Biodensity, which based on Jenkin's et al. (2004) findings could be best estimated using only 10 species groups. The last cluster is made up of the density measures that take species most into consideration. FOXDEN2.1 (RD_f) includes 16 species groups and both of the RD_w measures and RD_d allow for an unlimited number of species groups so long as you have a specific gravity value for each species.

Hierarchical Clustering

Method = Ward

Dendrogram



Clustering History

Number of Clusters	Distance	Leader	Joiner
9	21.9555623	Ranked RDwMax	Ranked RDw99
8	44.5200541	Ranked RDd	Ranked RDwMax
7	49.7108698	Ranked Biodensity	Ranked RDs
6	53.1823744	Ranked Biodensity	Ranked ASDI
5	83.6004050	Ranked BA	Ranked Biodensity
4	103.5354437	Ranked RDd	Ranked RDf
3	145.7248818	Ranked BA	Ranked RDd
2	165.6999293	Ranked BA	Ranked Biomass
1	261.9724394	Ranked TPH	Ranked BA

Cluster 1: Ranked TPH
Cluster 2: BA, Biodensity, RDs, ASDI
Cluster 3: RDd, RDwMax, RDw99, RDf
Cluster 4: Biomass

Figure 2. Dendrogram Based on Ward's (1963) Method of Hierarchical Clustering.

3.3 Correlations

Examination of the scatterplot matrix (Figure 3) shows that some of the stand density measurements are highly correlated as is evident by a near straight line relationship (e.g. RD_{w99} and RD_{wMax}) while other are not correlated at all as is the case with TPH and all the other measurements. The cone shape of many of the diagrams in Figure 3 is also an indication that the data is not normally distributed, however, this is not an assumption made by this statistical method.

Using Spearman rho (P) values (Table 8), I compared the relative strengths of correlations between the remaining density measures. Although BA and Biomass are mildly correlated they are not identical and Biomass is only weakly correlated with the other density measures. By comparison BA is mildly correlated with five out of seven of the other measurements but shows no strong correlation with any of them.

Of the remaining 7 density measures, RD_d , RD_{w99} and RD_{wMax} are most strongly correlated with each other. Based on Spearman's P value of .9965, RD_{w99} and RD_{wMax} are nearly identical, which makes sense as they are only different by a single percentile. And, although RD_d and RD_{w99} treat stand variables differently, based on a Spearman's P of .9937 they, too, are nearly identical. Although TPH is very weakly correlated with all the other measures, the Spearman's P value for TPH and Biomass is so small as to be nearly zero indicating a lack of correlation between the two variables. Basal area is only ever mildly correlated with the other measures. $ASDI$, RD_s and RD_f are occasionally strongly correlated with another measure but in most cases they, too, are only mildly correlated with the other measures. The size and range of data is why the correlation values are so

large. This does not mean that all the stand density measures agree. This is better illustrated by the rater agreement analysis.

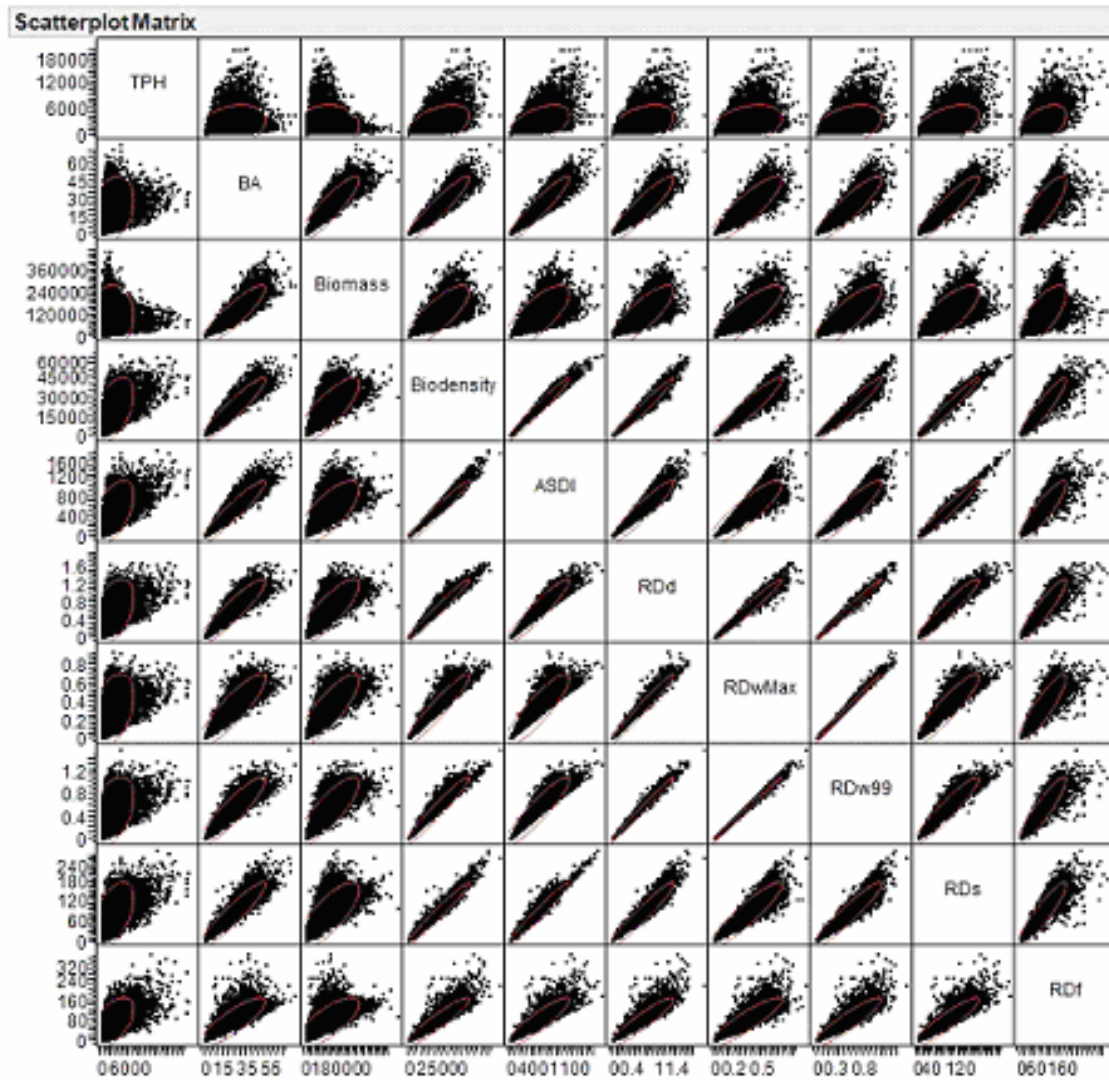


Figure 3. Scatterplot Matrix of Stand Density Measures Predictions.

Table 8. Spearman Rank Correlation Coefficients (rho)

	<i>TPH</i>	<i>Biomass</i>	<i>BA</i>	<i>ASDI</i>	<i>RD_s</i>	<i>Biodensity</i>	<i>RD_f</i>	<i>RD_d</i>	<i>RD_{w99}</i>	<i>RD_{wMax}</i>
TPH	X	.1455	.3730	.5884	.5677	.5539	.5512	.5158	.4952	.4681
Biomass		X	.9123	.7760	.8076	.8324	.7896	.8520	.8555	.8572
BA			X	.9493	.9373	.9458	.8773	.9248	.9085	.8832
ASDI				X	.9757	.9792	.9182	.9432	.9192	.8851
RD _s					X	.9810	.9376	.9601	.9457	.9214
Biodensity						X	.9509	.9840	.9687	.9468
RD _f							X	.9477	.9370	.9221
RD _d								X	.9937	.9836
RD _{w99}									X	.9965
RD _{wMax}										X

3.4 Rater Agreement

To further investigate the differences between the relative density measures I utilized the concept of rater agreement by treating each measure as an individual rater. I sought to compare how often they agreed with each other and at what densities. Based on the scatterplot matrix (Figure 3) and Spearman's P coefficients (Table 8) it appears that several of the relevant density measures may be redundant. However, the number of plots each measure places above 100% relative density, as a representation of overestimation, is the first clue that the predictions are not all the same at all density values (See Table 9).

Table 9. Comparison of Number of Plots Assigned to the Maximum Stand Density Category

<i>Model</i>	<i># of plots out of 15866 that are above 100%</i>	<i>Percent of Total</i>
RD _d	1327	8.36
RD _s	4890	30.82
RD _f	4344	27.38
RD _{w99}	238	1.5
RD _{wMax}	0	0

To explore this further I first divided the plots into 10% density categories with Category 1 containing plots with relative density 0 to 10%, category 2 containing plots 11 to 20% and so forth. As discussed earlier, inter-observer agreement is evaluated using the Kappa statistic which can be interpreted as percent agreement. The larger the Kappa value the higher the level of agreement between pair-wise comparisons between stand density predictions (Cohen, 1960). A low Kappa value means that the stand density measures assigned very few plots to the same relative density category. Obviously if I decreased the sensitivity of the categories I would see an increase in Kappa but for the purposes of comparison here 10% density ranges are sufficient to illustrate the differences in density measures from category to category.

The frequency plot and table in Figure 4 show visually how many of the plots have been assigned to each category by each relative density measure. The RD_{wMax} model is clearly very different from the rest, while RD_f and RD_s appear most similar. Since there were a large number of categories the kappa value spread is quite low ranging from about zero to 0.5. As is obvious from the frequency plot RD_f and RD_s assign plots to the same category the most often, or about half the time. RD_d has a kappa of 0.202, 0.184 and 0.223 with RD_{w99} , RD_s and RD_f respectively thus indicating that RD_d agrees with those three measures approximately an equal amount of the time or roughly one-fifth of the time. RD_{w99} and RD_{wMax} almost never assign plots to the same category as each other or RD_s and RD_f .

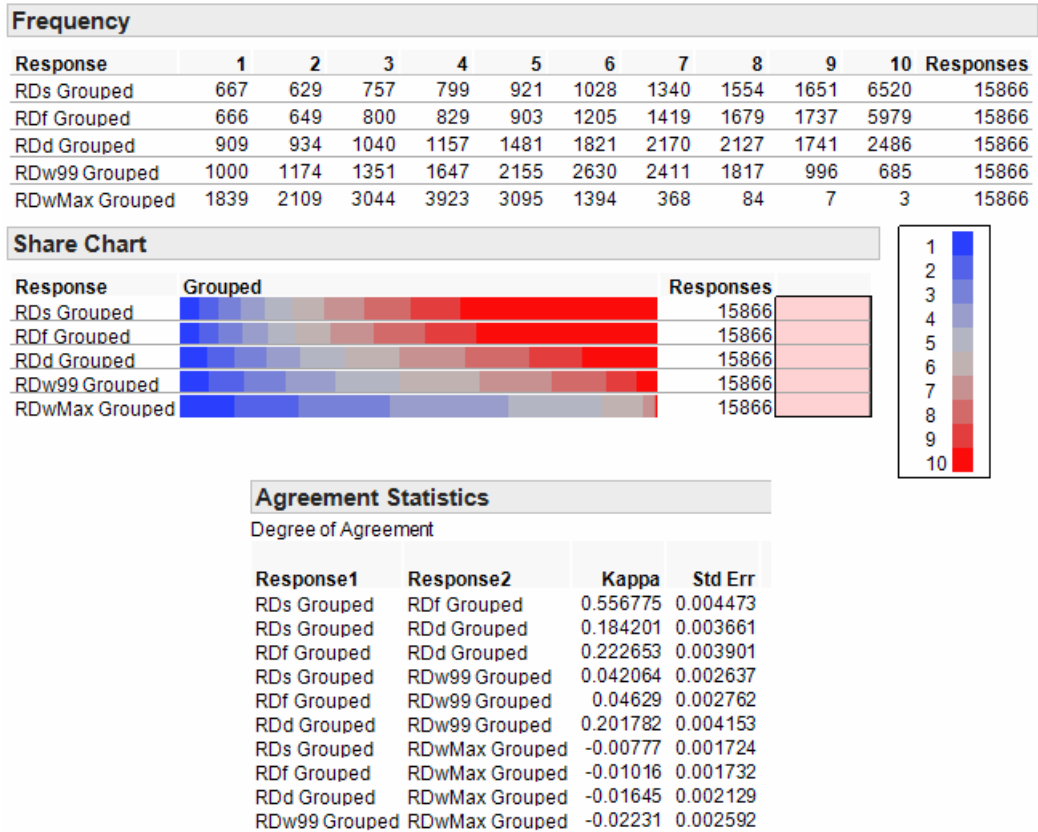


Figure 4. Rater Agreement Analysis Based on 10% Relative Density Increments.

This analysis shows a very different picture compared to straight correlations. The relative density measurements do not agree equally well at all densities and so correlations are high but rater agreement is low. The overlay plot of the 10% categories (Figure 5), again shows that RD_{wMax} is completely different while the other four are much more similar. Note that RD_s and RD_f , while mostly agreeing with RD_d up to the 80th percentile, differ sharply in the 90-100% category. It is unlikely then that the measures are assigning the same plots into the same category or the frequencies would be equivalent at all densities. Table 9 shows that RD_f and RD_s put between 4,000 and 5,000 plots at a relative density of over 100%. That works out to between 27 and 31% of the plots, compared to RD_d 's distribution of plots, which only places 8% in the 90-100% category. The overlay plot (Figure 5) illustrates how wide spread the distribution of density is depending which measure is taken into consideration.

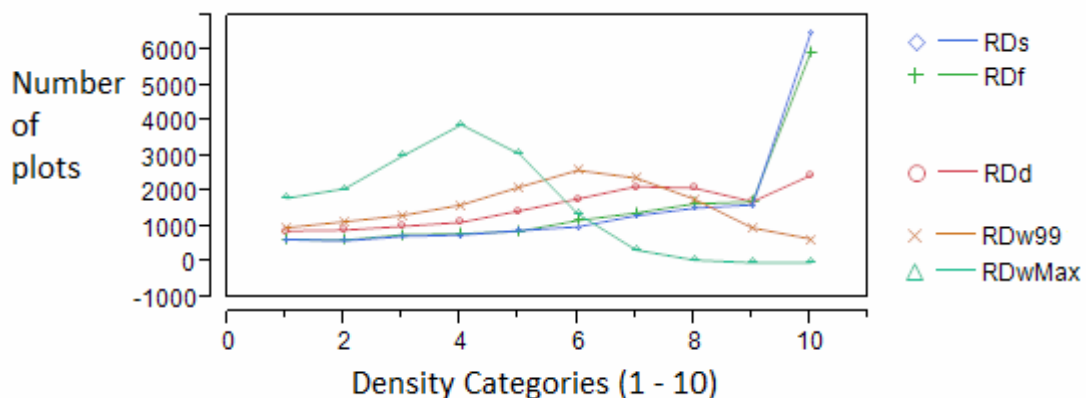


Figure 5. Overlay Plot of Relative Density Measures Based on Relative Density Divided into 10% Increment

The rater agreement analysis was repeated, as described in the Methods section, using the following management relevant categories as an example to illustrate the versatility of this statistical analysis technique:

Cat 1: RD less than 40% = Low density, don't thin

Cat 2: RD between 40 and 50% = Possible thin

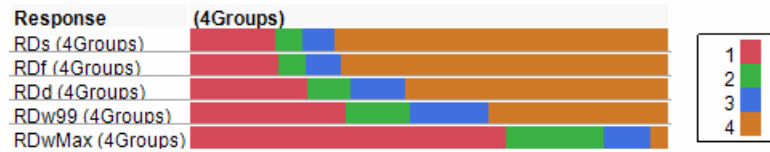
Cat 3: RD between 51 and 60% = Recommended thin, 50-60%

Cat 4: RD greater than 60% = High density, requires further evaluation

The change in the category assignment saw the maximum Kappa statistic increase from 0.55 in the previous analysis to 0.774, or from 55% agreement to just over 77% agreement. This is because I reduced the sensitivity of the categories which increased the overall amount of agreement. The frequency plot in Figure 6 shows that, as expected, RD_{wMax} is still assigning plots to completely different categories than the other 4 measures. RD_s and RD_f are still most often in agreement, assigning plots to the same category about 70% of the time. Once identifying plots ready for thinning became the characteristic by which density categories were assigned, RD_d actually agrees with the other three measures between 55 and 58% of the time.

Frequency					
Response	1	2	3	4	Responses
RDs (4Groups)	2852	921	1028	11065	15866
RDf (4Groups)	2944	903	1205	10814	15866
RDd (4Groups)	3916	1446	1799	8705	15866
RDw99 (4Groups)	5172	2154	2630	5910	15866
RDwMax (4Groups)	10501	3272	1555	538	15866

Share Chart



Agreement Statistics

Degree of Agreement			
Response1	Response2	Kappa	Std Err
RDs (4Groups)	RDf (4Groups)	0.774287	0.004741
RDs (4Groups)	RDd (4Groups)	0.552453	0.005097
RDf (4Groups)	RDd (4Groups)	0.580962	0.005054
RDs (4Groups)	RDw99 (4Groups)	0.329417	0.004403
RDf (4Groups)	RDw99 (4Groups)	0.341311	0.004436
RDd (4Groups)	RDw99 (4Groups)	0.478824	0.004615
RDs (4Groups)	RDwMax (4Groups)	0.062835	0.001705
RDf (4Groups)	RDwMax (4Groups)	0.06511	0.001747
RDd (4Groups)	RDwMax (4Groups)	0.087367	0.002218
RDw99 (4Groups)	RDwMax (4Groups)	0.119969	0.002868

Figure 6. Rater Agreement Analysis Based on Thinning Categories.

CHAPTER 4

DISCUSSION

My key goal was to compare absolute and relative measures of stand density commonly used in the northeastern United States. Absolute measures are inferior to relative measures in that they lack an accepted reference point that allows for the comparison of one stand to the next in a meaningful and biologically accurate way. The comparison of the relative density measures considered here resulted in conclusions similar to Curtis (1970) in that the choice among the measures is, in part, a matter of available information and convenience of computation. There are certainly fundamental differences in both the approach taken to find the maximum size-density line and also in the inclusion of supporting variables like specific gravity. Ducey and Knapp (2010) points out that there are arguably other factors besides specific gravity that relate to the competition dynamics in mixed-species forests and the analysis here supports that assertion. Rater agreement analysis highlighted the fact that handling of species and species groups as well as selection of location of maximum-size density line plays a role in relative density estimates. Woodall et al. (2006) utilized the 99th percentile compared to Ducey and Knapp (2010) which utilized the 85th percentile and that alone may account for the average 10.8% difference between the resulting density predictions. It is difficult to know an exact relative density for any stand but the comparison here shows that depending on the reason for using relative density many of the measures overlap, most

notably when broken in management relevant densities groups aimed at developing a thinning regime (See Figure 6).

From the cluster and correlation analyses, RD_d , RD_{w99} and RD_{wMax} are strongly correlated with each other, so strongly that based on this alone one might argue they are redundant. The Spearman rho values are so close to one because they are largely influenced by the size of the data set. That said, the distribution of results shows they are clearly not redundant. Rater agreement analysis shows that the correlation between the two measures does not mean that they actually rank plots the same (See Figure 5).

Chisman and Schumacher (1940) found that dividing study stands into species groups significantly improved the fit of tree area ratio equation so one would expect that the more an equation accounts for species the better. Grouping has practical disadvantages because groups must be decided upon. However, the cluster analysis and the rater agreement analysis support the notion that grouping at the very least changes the relative density predictions. The three models that most account for species group did tend to agree with each other. It could be argued that the dendrogram shows density measures grouping based on the lumping viewpoint versus the splitting viewpoint where the lumpers (RD_s , ASDI, BA and Biodensity) versus the splitters (FOXDEN2.1, RD_d and both RD_w 's). The overlay plot (Figure 5) illustrates that the relative density measures do not agree at all densities and, in fact, start to strongly disagree at higher density values.

For the most part the comparison done here ignores the variations in plot size within and between the studies that developed the relative density models under review. The plot size selection criterion for a research study are not the same as an inventory, and are influenced by such considerations as intended purpose, stand variability and cost

(Curtis and Marshall, 2005). Plot size varied from as small as 0.25 acre to as large as 25.6 acre in Stout and Nyland's (1986) study which led to the development of their relative density model. There were only 201 plots in the study and the article does not specify the distribution of size among the study plots but 0.25 acres or rather roughly 11,000 square feet is substantially larger than the FIA plots which were used in the Ducey and Knapp (2010) and the Woodall et al. (2006) relative density models. The FIA plots are broken up in to different sizes based on data collection goals and the plots on which tree data is collected are only 0.04 acres (Brand, 2005; Burkman, 2005a, 2005b). So while there are many more of them (over 15,000 used to develop RD_d and over 110,000 for the RD_w 's) it is possible that because of the small plots the estimates are biased and may result in values higher than realistically attainable on a stand basis (Curtis and Marshall, 2005). Although on average RD_f and RD_s provide higher average density estimates than RD_d and both RD_w 's (See Table 7) when we examine density categories, RD_d and both RD_w 's consistently place more plots in higher density categories than RD_f and RD_s . The small FIA plot size may explain why RD_d and both RD_w 's behave in this way.

Measures in this study were chosen based on their relevance to the study region, their prevalence of use and an understanding that they are all implicitly linked. That said, FOXDEN2.1 (2001) and Stout and Nyland (1986) may not work best for my study area (New England and New York) given that they unrealistically predict approximately 30% of the plots are above 100% (See Table 9). This may be a result of the study plot selection process used by Stout and Nyland (1986) in which the researchers hand selected normally stocked stands. Normal stocking is defined by Gingrich (1967) as "the mean stocking level of a large number of undisturbed stands". Ducey and Knapp (2010)

pointed out that when Stout and Nyland (1986) relative density measure is used to estimate the difference between RD in mixed stands of black cherry and sugar maple versus pure black cherry stands RD_s is approximately 40% greater than Woodall et al. (2006) and Ducey and Knapp (2010) estimate. If Stout and Nyland's (1986) underestimated maximum stand density by selecting plots actually containing a lower density of trees than maximum this could explain why RD_s is placing a much greater quantity of stands into the over maximum category. Both Stout and Nyland (1986) and FOXDEN2.1 (Desmaris, 2001) rely on the tree-area ratio (TAR) and perhaps a function of TAR is higher density estimates. This may also explain why those measures place a fewer number of plots in the lower density categories since the distribution of estimates at all densities would be affected by the believed state of the original study plots (See Figure 5). Arguably these differences could also be related to the definition of maximum density (A-line), however if that was the most compelling part of the equation then RD_s and RD_{wMax} would agree more often since both define maximum at 100% whereas RD_d defines maximum at 85% (Ducey and Knapp, 2010). Additionally, RD_s only directly accounts for approximately 55% of the species in the study region. Given that RD_s had the highest level of agreement with RD_f and RD_f accounts for 85% of the species this seems to indicate that the lack of direct accountability that occurs in RD_s alone does not explain the large number of plots estimated at over 100% relative density. This is perhaps because while Stout and Nyland's (1986) model requires lumping many species into one catch all group, 8 out of 10 of the top 10 species (Table 1) were included in their original study. Red oak was later added to Group 2 by Marquis et al. (1992) and aspen is not accounted for.

Based on the comparison conducted here future studies conducted in the northeastern United States would be best served by using the relative density measure developed by Ducey and Knapp (2010). RD_d and RD_{w99} result in similar predictions of relative density about 20% of time on a case by case basis and approximately 60% of the time when based on density categories. However, compared to the other measures, RD_d appears to agree with RD_{w99} and RD_f/RD_s an equal number of times (See Figure 5: RD_d is line in between RD_{w99} and RD_f/RD_s which essentially overlap). Compared to the other measures RD_d more evenly distributes the study plots into relative density categories, suggesting that the study area is made up of stands that are only slightly weighted towards 60% and greater relative density. RD_f and RD_s , as discussed earlier, place a large (30%) percent of plots at greater than 100% where as RD_{w99} has a noticeable hump at 60% relative density suggesting that the region is composed of fewer lower and higher density plots and more medium density plots. Based on Irland's (1999) estimate that 30% of the region's current forest area is located on abandoned fields and are often nearly 100 years old, and if relative density reaches maximum in that amount of time or possibly exceeds maximum due to underestimation of maximum, RD_s and RD_f may be completely accurate in their estimate of roughly 30% of the plots containing over 100% relative density. Additional research into land use history could make it possible to declare a "correct" relative density for a region based on the expected distribution of relative density measures. Patterns of agreement and disagreement between density measures compared to projections of expected density based on land use could also be conducted on a national scale but with greater effort.

Woodall et al. (2006) attempted a national scale relative density study using RD_{w99} (per conversation with the author it was confirmed that although the coefficients for the 100th percentile were quoted in the published paper, this was a typographical error and the coefficients for the 99th percentile were used). RD_d and RD_{w99} did not completely agree with each other so it appears clear that the use of the 85th versus the 99th percentile and/or the use of mean stand specific gravity (Woodall et al., 2005) versus individual species specific gravity (Ducey and Knapp, 2010) made a difference. Perhaps if Ducey and Knapp (2010) had selected the 99th percentile RD_d and RD_{w99} would have been in such close agreement as to suggest that the handling of specific gravity was not as important as the mere inclusion of it. Discovering this would aid in determining if the relative density measures estimated by Woodall et al. (2006) are reproducible by the Ducey and Knapp (2010) method in that if we isolated specific gravity as the only variable different between the two it could be determined if that difference is meaningful. This information could then be used to determine if the results in Woodall et al.'s (2011) study of tree carbon stocks in the eastern United States correctly estimates the influence of relative species composition on carbon sequestration. Woodall et al. (2011) calculates species relative density by utilizing a model that uses mean plot specific gravity and this may confound the actual species composition purity ratio. If RD_d was generated using coefficients from the 99th percentile and then those plot estimates were compared to RD_{w99} a lack of difference would imply that using mean stand specific gravity is a sufficient substitute for including each species specific gravity. Also, Woodall et al. (2011) cites the coefficients from the 100th percentile, and if they did use those

coefficients instead of the 99th, then they may be underestimating relative density across the board by allowing density outliers to drive the equation.

In addition to clarifying the influence of specific gravity on relative density, if multiple regional stocking guides could be compared to find the best mean prediction at a variety of relative density values then it would be possible to aid in decisions about which stocking guides are most useful for a particular area. Rater agreement can be a useful tool for conducting these comparisons as it allows for the creation of relevant categories and treats the models as observers of the same phenomenon. Ultimately, if a method could be developed that quantified RD independent of location or age there would be the potential to perform the historical analysis necessary to begin to resolve the debate over whether growth enhancement in the United States is due to global atmospheric change or land use change (Caspersen, et al., 2000; Houghton et al., 1999; Joos, et al., 2002). Relative density is a more robust method to absolute density for comparison and management of forests. Competing relative density measures can be compared and that comparison used to identify the most appropriate model to use for a particular region.

CONCLUSIONS

In summary, several statistical methods were used to compare absolute density measures and relative density models to evaluate the similarities and differences in their predictions. Based on this analysis, it is clear that all relative density measures are not the same but certain characteristics like species grouping or use of specific gravity definitely influence how much overlap there is between model density predictions. Also, relative density measures tend to converge the more inclusive of individual species composition.

Drew and Flewelling (1979) noted that "the relationship between density and yield will not be resolved until a general framework relating these variables has been developed and conceptualized in a manner that allows ideas and experimental evidence to be transferred from one experiment to another, from one region to another, and even from one species to another." They proposed using relative density as that unifying framework but much work is still needed to identify the appropriate relative density model and, in the absence of this, rater agreement could be used to determine if competing theories agree on predictions across a range of possible densities. By identifying which models agree despite their differences it could be feasible to hone in on the variables that are most predictive and also which models might work best for cross-region analysis.

Based on the key goals of the study, reiterated below, the major conclusions reached are:

Research Question 1: Generally, while absolute and relative density measures are sometimes treated as synonymous are the measures presented here similar enough to be redundant?

- The measures presented are not always similar enough to be redundant and the choice among them is, in part, a matter of available information and convenience of execution.
- Additionally, absolute measures were found inferior to relative measures in that they lack an accepted reference point that allows for comparison of one stand to the next in a meaningful and biologically accurate way.

Research Question 2: More specifically, do the relative density measures agree or disagree with each other at all relative densities?

- RD_d and RD_{w99} result in similar predictions of relative density about 20% of time on a case by case basis and approximately 60% of the time when based on density categories.
- RD_d agrees with RD_{w99} and RD_f/RD_s an equal number of times. Note that in Figure 5 the RD_d line is in between RD_{w99} , which is the line above, and RD_f/RD_s , which essentially overlap, and are the lines below.
- RD_f and RD_s agree most often, resulting in similar predictions 55% of the time.

Research Question 3: Finally, how does the handling of variables such as species group, specific gravity and definition of maximum density affect density predictions?

- Based on the analysis performed here it could be argued that density measures form clusters based on the lumping viewpoint versus the splitting viewpoint where the lumpers have few to no species groups (RD_s , ASDI, BA and Biodensity) and the splitters have a dozen or more (FOXDEN2.1, RD_d and both RD_w 's).
- Specific gravity clearly affects the outcome of the density prediction. If RD_d was generated using coefficients from the 99th percentile and then those plot estimates were compared to RD_{w99} a lack of difference would imply that using mean stand specific gravity is a sufficient substitute for including each species' specific gravity.

- Rater agreement analysis highlighted the fact that handling of species and species groups as well as selection of location of maximum-size density line plays a role in relative density estimates. If 85% was functionally equivalent to 99% then RD_d and RD_{w99} would likely have been in greater agreement.

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APPENDIX A

TABLE OF SPECIFIC GRAVITY BY SPECIES

Note: Specific gravity is based on weight when oven-dry and volume when at 12% moisture content.

Genus	Species	12% Specific Gravity	Source
<i>Abies</i>	<i>balsamea</i>	0.3500	<i>a</i>
<i>Abies</i>	spp.	0.3500	<i>l</i>
<i>Acer</i>	<i>negundo</i>	0.4440	<i>c</i>
<i>Acer</i>	<i>nigrum</i>	0.5700	<i>a</i>
<i>Acer</i>	<i>pensylvanicum</i>	0.4600	<i>i</i>
<i>Acer</i>	<i>platanoides</i>	0.6200	<i>h</i>
<i>Acer</i>	<i>rubrum</i>	0.5400	<i>a</i>
<i>Acer</i>	<i>saccharinum</i>	0.4700	<i>a</i>
<i>Acer</i>	<i>saccharum</i>	0.6300	<i>a</i>
<i>Acer</i>	<i>spicatum</i>	0.4600	<i>l</i>
<i>Acer</i>	spp.	0.5050	<i>l</i>
<i>Aesculus</i>	<i>glabra</i>	0.3800	<i>b</i>
<i>Aesculus</i>	spp.	0.3800	<i>l</i>
<i>Ailanthus</i>	<i>altissima</i>	0.5300	<i>e</i>
<i>Alnus</i>	<i>glutinosa</i>	0.5100	<i>h</i>
<i>Amelanchier</i>	<i>arborea</i>	0.6100	<i>k, m</i>
<i>Amelanchier</i>	spp.	0.6100	<i>l</i>
<i>Asimina</i>	<i>triloba</i>	0.3969	<i>f</i>
<i>Betula</i>	<i>alleghaniensis</i>	0.6200	<i>a</i>
<i>Betula</i>	<i>lenta</i>	0.6500	<i>a</i>
<i>Betula</i>	<i>nigra</i>	0.6200	<i>l</i>
<i>Betula</i>	<i>papyrifera</i>	0.5500	<i>a</i>
<i>Betula</i>	<i>populifolia</i>	0.4800	<i>a</i>
<i>Betula</i>	spp.	0.6200	<i>l</i>
<i>Carpinus</i>	<i>caroliniana</i>	0.7200	<i>b</i>
<i>Carya</i>	<i>alba</i>	0.7200	<i>a</i>
<i>Carya</i>	<i>cordiformis</i>	0.6600	<i>a</i>
<i>Carya</i>	<i>glabra</i>	0.7500	<i>a</i>

<i>Carya</i>	<i>laciniosa</i>	0.6900	<i>a</i>
<i>Carya</i>	<i>ovata</i>	0.7200	<i>a</i>
<i>Carya</i>	spp.	0.7200	<i>l</i>
<i>Castanea</i>	<i>dentata</i>	0.4300	<i>a</i>
<i>Catalpa</i>	<i>speciosa</i>	0.4000	<i>k, m</i>
<i>Catalpa</i>	spp.	0.4000	<i>l</i>
<i>Celtis</i>	<i>occidentalis</i>	0.5300	<i>a</i>
<i>Cercis</i>	<i>canadensis</i>	0.6300	<i>g</i>
<i>Chamaecyparis</i>	<i>thyoides</i>	0.3200	<i>a</i>
<i>Cornus</i>	<i>florida</i>	0.7500	<i>b</i>
<i>Crataegus</i>	spp.	0.6900	<i>j, m</i>
<i>Fagus</i>	<i>grandifolia</i>	0.6400	<i>a</i>
<i>Fraxinus</i>	<i>americana</i>	0.6000	<i>a</i>
<i>Fraxinus</i>	<i>nigra</i>	0.4900	<i>a</i>
<i>Fraxinus</i>	<i>pennsylvanica</i>	0.5600	<i>a</i>
<i>Fraxinus</i>	<i>quadrangulata</i>	0.5800	<i>a</i>
<i>Fraxinus</i>	spp.	0.5700	<i>l</i>
<i>Gleditsia</i>	<i>triacanthos</i>	0.6650	<i>j, m</i>
<i>Ilex</i>	<i>opaca</i>	0.6000	<i>b</i>
<i>Juglans</i>	<i>cinerea</i>	0.3800	<i>a</i>
<i>Juglans</i>	<i>nigra</i>	0.5500	<i>a</i>
<i>Juniperus</i>	spp.	0.4700	<i>l</i>
<i>Juniperus</i>	<i>virginiana</i>	0.4700	<i>a</i>
<i>Larix</i>	<i>laricina</i>	0.5300	<i>a</i>
<i>Larix</i>	spp.	0.5300	<i>l</i>
<i>Liquidambar</i>	<i>styraciflua</i>	0.5200	<i>a</i>
<i>Liriodendron</i>	<i>tulipifera</i>	0.4200	<i>a</i>
<i>Maclura</i>	<i>pomifera</i>	0.8400	<i>b</i>
<i>Magnolia</i>	<i>acuminata</i>	0.4800	<i>a</i>
<i>Magnolia</i>	<i>fraseri</i>	0.4300	<i>k, m</i>
<i>Malus</i>	<i>coronaria</i>	0.6800	<i>l</i>
<i>Malus</i>	<i>fusca</i>	0.6800	<i>l</i>
<i>Malus</i>	spp.	0.6800	<i>j, m</i>
<i>Morus</i>	<i>alba</i>	0.6500	<i>l</i>
<i>Morus</i>	<i>rubra</i>	0.6500	<i>b, m</i>
<i>Morus</i>	spp.	0.6500	<i>l</i>
<i>Nyssa</i>	<i>sylvatica</i>	0.5000	<i>a</i>
<i>Ostrya</i>	<i>virginiana</i>	0.7800	<i>b</i>
<i>Oxydendrum</i>	<i>arboreum</i>	0.5900	<i>b</i>
<i>Paulownia</i>	<i>tomentosa</i>	0.4000	<i>j, m</i>

<i>Picea</i>	<i>abies</i>	0.4300	<i>h</i>
<i>Picea</i>	<i>glauca</i>	0.4000	<i>a</i>
<i>Picea</i>	<i>mariana</i>	0.4600	<i>a</i>
<i>Picea</i>	<i>pungens</i>	0.4300	<i>l</i>
<i>Picea</i>	<i>rubens</i>	0.4000	<i>a</i>
<i>Picea</i>	spp.	0.4300	<i>l</i>
<i>Pinus</i>	<i>banksiana</i>	0.4300	<i>a</i>
<i>Pinus</i>	<i>nigra</i>	0.4650	<i>l</i>
<i>Pinus</i>	<i>pungens</i>	0.4700	<i>d</i>
<i>Pinus</i>	<i>resinosa</i>	0.4600	<i>a</i>
<i>Pinus</i>	<i>rigida</i>	0.5200	<i>a</i>
<i>Pinus</i>	spp.	0.4650	<i>l</i>
<i>Pinus</i>	<i>strobus</i>	0.3500	<i>a</i>
<i>Pinus</i>	<i>sylvestris</i>	0.4900	<i>h</i>
<i>Platanus</i>	<i>occidentalis</i>	0.4900	<i>a</i>
<i>Populus</i>	<i>balsamifera</i>	0.3400	<i>a</i>
<i>Populus</i>	<i>deltoides</i>	0.4000	<i>a</i>
<i>Populus</i>	<i>grandidentata</i>	0.3900	<i>a</i>
<i>Populus</i>	<i>heterophylla</i>	0.4000	<i>b</i>
<i>Populus</i>	spp.	0.3900	<i>l</i>
<i>Populus</i>	<i>tremuloides</i>	0.3800	<i>a</i>
<i>Prunus</i>	<i>americana</i>	0.5000	<i>l</i>
<i>Prunus</i>	<i>avium</i>	0.5000	<i>l</i>
<i>Prunus</i>	<i>pennsylvanica</i>	0.3800	<i>k, m</i>
<i>Prunus</i>	<i>persica</i>	0.5000	<i>l</i>
<i>Prunus</i>	<i>serotina</i>	0.5000	<i>a</i>
<i>Prunus</i>	spp.	0.5000	<i>l</i>
<i>Prunus</i>	<i>virginiana</i>	0.3800	<i>l</i>
<i>Pseudotsuga</i>	<i>menziesii</i>	0.4800	<i>a</i>
<i>Quercus</i>	<i>alba</i>	0.6800	<i>a</i>
<i>Quercus</i>	<i>bicolor</i>	0.7200	<i>a</i>
<i>Quercus</i>	<i>coccinea</i>	0.6700	<i>a</i>
<i>Quercus</i>	<i>ellipsoidal</i>	0.6300	<i>l</i>
<i>Quercus</i>	<i>ilicifolia</i>	0.6100	<i>l</i>
<i>Quercus</i>	<i>macrocarpa</i>	0.6400	<i>a</i>
<i>Quercus</i>	<i>michauxii</i>	0.6700	<i>a</i>
<i>Quercus</i>	<i>muehlenbergii</i>	0.6700	<i>l</i>
<i>Quercus</i>	<i>palustris</i>	0.6300	<i>a</i>
<i>Quercus</i>	<i>prinus</i>	0.6600	<i>a</i>
<i>Quercus</i>	<i>rubra</i>	0.6300	<i>a</i>

<i>Quercus</i>	spp.	0.6600	<i>l</i>
<i>Quercus</i>	<i>stellata</i>	0.6700	<i>a</i>
<i>Quercus</i>	<i>velutina</i>	0.6100	<i>a</i>
<i>Robinia</i>	<i>pseudoacacia</i>	0.6900	<i>a</i>
<i>Salix</i>	<i>alba</i>	0.3900	<i>l</i>
<i>Salix</i>	<i>amygdaloides</i>	0.3900	<i>l</i>
<i>Salix</i>	<i>bebbiana</i>	0.3900	<i>l</i>
<i>Salix</i>	<i>nigra</i>	0.3900	<i>a</i>
<i>Salix</i>	spp.	0.3900	<i>l</i>
<i>Sassafras</i>	<i>albidum</i>	0.4600	<i>a</i>
<i>Sorbus</i>	<i>americana</i>	0.4500	<i>j, m</i>
<i>Sorbus</i>	<i>aucuparia</i>	0.4500	<i>j, m</i>
<i>Taxodium</i>	<i>distichum</i>	0.4600	<i>a</i>
<i>Thuja</i>	<i>occidentalis</i>	0.3100	<i>a</i>
<i>Tilia</i>	<i>americana</i>	0.3700	<i>a</i>
<i>Tilia</i>	spp.	0.3700	<i>l</i>
<i>Tsuga</i>	<i>canadensis</i>	0.4000	<i>a</i>
<i>Tsuga</i>	spp.	0.4000	<i>l</i>
<i>Ulmus</i>	<i>alata</i>	0.6600	<i>2</i>
<i>Ulmus</i>	<i>americana</i>	0.5000	<i>a</i>
<i>Ulmus</i>	<i>rubra</i>	0.5300	<i>a</i>
<i>Ulmus</i>	spp.	0.5800	<i>l</i>
<i>Ulmus</i>	<i>thomasii</i>	0.6300	<i>a</i>
Unknown	broadleaf	0.5121	<i>l</i>
Unknown	conifer	0.4450	<i>l</i>
Unknown	unknown	0.5279	<i>l</i>

Footnotes

^a Forest Products Laboratory (1999)

^b Panshin and de Zeeuw (1970)

^c Maeglin and Ohmann (1973)

^d Burns and Honkala (1990)

^e Alden (1995)

^f Nugent and Boniface (2005)

^g Armstrong (2008)

^h Kollmann et al. (2008)

ⁱ Forest Products Laboratory (2008)

^j Jenkins et al. (2003)

^k Markwardt (1930)

^l Specific gravity based on closely related species.

^m Specific gravity converted from green volume to 12% dry.

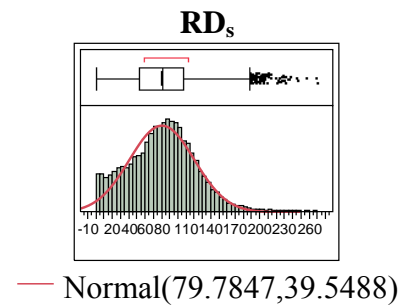
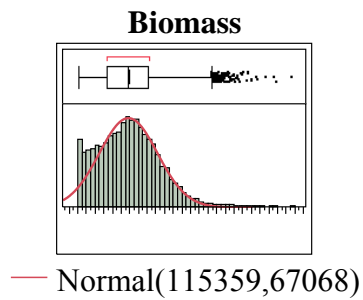
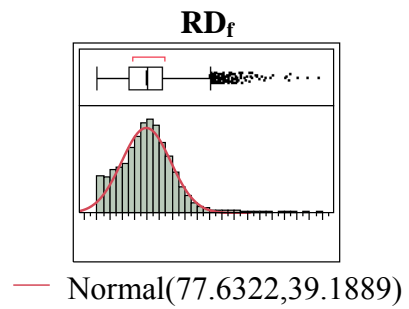
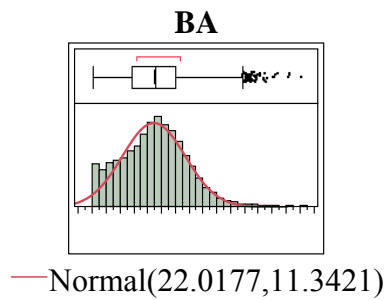
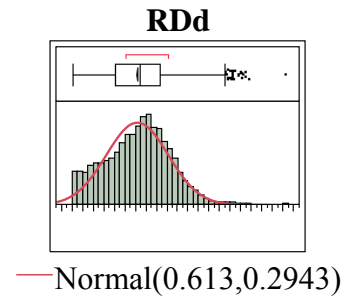
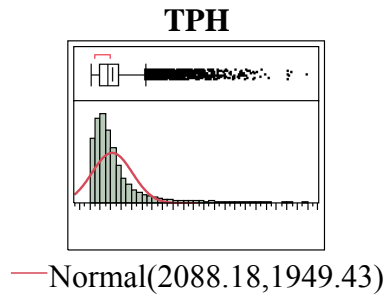
APPENDIX A

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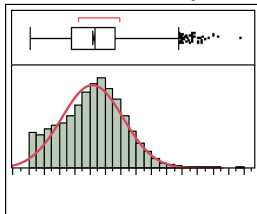
APPENDIX B

COMPARISON OF DISTRIBUTION OF STAND DENSITY MEASURE PREDICTIONS



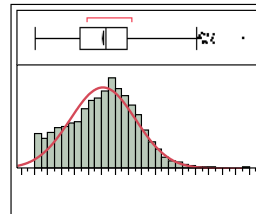
APPENDIX B CONTINUED

Biodensity



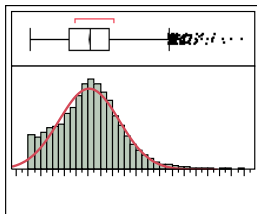
—Normal(20583.9,9990.74)

RD_{w99}



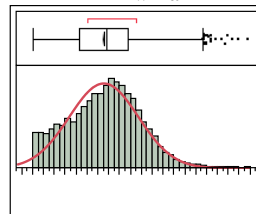
—Normal(0.50507,0.24338)

ASDI



—Normal(502.862,251.689)

RD_{wMax}



—Normal(0.32488,0.15873)

APPENDIX C

RESULTS OF GOODNESS OF FIT TEST USING KSL STATISTIC

Relative Density Measure	D (KSL Statistic)
ASDI	.023696
RD _f	.023865
RD _s	.027936
BA	.028822
Biodensity	.031726
RD _{wMax}	.031948
RD _{w99}	.035582
RD _d	.035765
Biomass	.042867
TPH	.146418