RADIATIVE MEAN LIVES OF SOME EXCITED ELECTRONIC TERMS IN ATOMIC AND IONIC CARBON USING BEAM-FOIL EXCITATION

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AND IONIC CARBON USING BEAM-FOIL
EXCITATION

BY

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DEDICATION

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ABSTRACT

RADIATIVE MEAN LIVES OF SOME EXCITED ELECTRONIC TERMS IN ATOMIC AND IONIC CARBON USING BEAM-FOIL EXCITATION

by

DAVID JOHN PEGG

The radiative mean lives of twelve excited electronic terms of CI, CII, and CIII have been measured by the Beam-Foil method. A beam of $^{12}\text{C}^+$ ions (~350 keV) was passed through a thin $10\ \mu\text{g/cm}^2$ carbon foil which acted as the excitation medium. The beam emerges from the foil in various stages of ionization and various states of excitation. The decay of the light intensity of the multiplets produced when the excited electronic terms spontaneously decay is monitored photometrically as a function of distance downstream from the foil using a monochromator and a photomultiplier. External environmental influences on the excited system are negligible for the beam-foil source. Optical electric-dipole multiplet transitions were observed in the spectral region between $\lambda 750$ and $\lambda 5500$ Å. Some of the decay curves were affected by cascading and/or spectral multiplet blending, but many appear free of these effects. Where a transition is singular or dominant, it is possible to extract transition probabilities directly. Nine transition probabilities have been derived from the mean life results in this investigation.
SECTION I

INTRODUCTION

1.1 Purpose of the Investigation

Absolute transition probabilities (Einstein A-coefficients or oscillator strengths) for spontaneous radiative electric dipole transitions involving the emission of one photon are among the most important parameters characterizing atomic and atomic ion systems. Many problems in the fields of astrophysics, laser physics, laboratory plasma physics, and atomic structure studies, for example, depend greatly on the values of these atomic parameters.

1.2 Astrophysical Problems

The astrophysicist is faced with such problems as estimating the electron temperatures and abundances of various chemical elements in stellar atmospheres from the measured strengths (intensities) of the spectral lines emitted by the source. The intensity of such a spectral line is a function of environmental conditions such as temperature and pressure that govern the steady state populations as well as a fundamental atomic property called the radiative transition probability for spontaneous emission (the Einstein A-coefficient). A knowledge of these
transition probabilities thus allows one to calculate steady state populations and hence gain insight into the physical environmental conditions and the chemical composition present in the stellar atmospheres. Carbon is of particular importance in this respect since there remains many unsolved problems concerning emission lines from the hottest stars such as Wolf-Rayet (WC) stars and Of-stars which apparently contain this element in abundance. Swings (1968) indicates that there is a need for new work on spectral studies and transition probability measurements on the species CI – CIV with regard to this problem. Another outstanding astrophysical problem is concerned with line identifications in quasi-stellar objects (quasars). Lynds (1968) has compiled a list of lines of particular importance in the interpretation of quasar absorption spectra including many carbon lines in the visible and ultraviolet wavelength regions for which reliable radiative transition probability values are needed. The present investigation includes results on lines listed in this tabulation at λ1561 Å CI, λ1335 Å CII, and λ1176 Å CIII.

1.3 Atomic Structure Problems

Electronic energy level wave functions can now be calculated with sufficient accuracy (for the light atoms at least) to enable theoretical computations of radiative transition probabilities to be made. A set of tables of atomic transition probabilities for the first ten elements
has been compiled (using both theoretical and experimental results) by Wiese and his co-workers at the NBS (Wiese, 1966). Precise experimental measurements of radiative transition probabilities included in this tabulation afford good tests to the validity of the various assumptions and approximations (such as the Coulomb and self-consistent field approximations) used in some of the calculations. Of particular interest in this respect are transitions in which configuration interactions are involved. For example, transitions of the type \( 1s^22s^2m_2p^2n - 1s^22s^2m_2^{-1}2p^{n+1} \) in the lighter elements cannot be effectively tackled, theoretically, using a one-configuration model. Weiss (1967) has studied the effect of a superposition of several different configurations on the calculated transition probability values for some VUV transitions in CI and CII. This dissertation includes a measurement of the radiative transition probability for one such transition theoretically studied, i.e. \( 2s^22p^2^3P - 2s2p^3^3D^0 \) in CI at \( \lambda 1561 \) Å. Weiss' "superposition of configurations" value is seen to agree far better with the present experimental result than does a Hartree-Fock independent particle model (one-configurational) calculation.

Another interesting theoretical development in the past few years has been the effort to obtain a functional dependence for the values of transition probabilities on atomic numbers for the same electronic transition along an isoelectronic sequence. Wiese (1968) has stated the urgency
for precise experimental measurements to help ascertain this functional dependence. The present results include several radiative transition probabilities measurements that will be of use in this respect. These results are shown in Section IV. Section II contains more detailed descriptions of the following subjects; calculations of transition probabilities, configuration interaction problems and the f-dependence of transition probabilities within an isoelectronic sequence.

1.4 Introduction to Mean Life Measurements

Using Beam-Foil Excitation

One method of experimentally obtaining absolute radiative transition probabilities for atomic transitions is to measure the radiative mean life of the upper state involved in the transition. The radiative mean life is numerically equal to the inverse of the sum of the transition probabilities for all the possible radiative electric-dipole decay channels out of the state. Thus, if the branching ratio for a particular transition is unity, or if one transition in the sum is dominant (this is likely if the sum otherwise contains transitions of comparatively high frequency, since the transition probability is proportional to the frequency cubed) the lifetime measurements will lead directly to values for the pertinent transition probabilities. Measurements and calculations by other investigators can also be used effectively to estimate branching ratios.
that will in turn allow mean life measurements to lead to transition probability values.

The beam-foil excitation method has been used in the present investigation of the radiative mean lives of excited electronic states of atomic and ionic carbon. This technique was first introduced by Kay (1963) and developed by Bashkin (1968). Basically, this experimental method consists of passing a monoenergetic ion beam of the element under study through a thin foil which acts as an excitation region. Collisions between the incident beam ions and the atomic electrons of the foil material cause the beam to emerge from the foil in various stages of ionization and electronic excitation. A study of the subsequent spontaneous radiative decay of the excited electronic states as a function of the time after initial excitation allows one to directly measure the radiative mean lives of the states involved. In this particular investigation $^{12}$C$^+$ ions were passed through thin (10 $\mu$g/cm$^2$) carbon foils to create excited electronic states in atomic and ionic carbon (CI, CII, CIII). The resulting multiplet transitions were studied in the spectral range $\lambda$750 Å - $\lambda$5500 Å. The incident ion kinetic energies used were in the range 200 - 400 keV with most results being obtained at approximately 350 keV. At these energies the electronic states excited were in the neutral (CI) and single (CII), and doubly (CIII) ionized species of the elements (predominantly CII).
The excited beam-foil spectroscopic source has many advantages as a candidate for mean life studies. Some of the most important advantageous properties inherent in this source are:

(i) essentially no collisions (which could cause non-radiative decays) occur either between the beam particles themselves or between beam particles and residual gas atoms in the evacuated (P \( \sim \) 5 x 10\(^{-6}\) torr) target foil chamber since the particle densities involved are so small. The excited beam has a density of about 10\(^4\) particles/cm\(^3\) and the residual gas 10\(^{10}\) particles/cm\(^3\), thus the long mean free path of the particles (\( \sim \) 10\(^3\) meters) ensures that there are negligible interactions between particles in the beam and the residual gas atoms and molecules.

(ii) There is negligible absorption of any emitted radiation since, again, the source density is so tenous.

(iii) The probability of recombination of an ion in the beam with a free electron (leading to continuous radiation) is also small due to the low ion particle densities and low density of electrons in the beam.

(iv) Electric and magnetic fields generated by neighboring particles will be small, again due to the low particle densities in the beam and should not appreciably Stark-perturb the excited emitters.

(v) The times and places of excitation of all emitters are the same and occur, respectively, within a time
shorter than $10^{-13}$ sec and a distance of a few hundred angstroms (i.e. the foil thickness).

Thus, in summary, the precisely defined location of the excitation at the foil, the lack of subsequent interactions with the environment afterwards, and the constant velocity of the particles permit the investigator to relate the time evolution of the population of excited states of the ensemble of particles to the point downstream from the foil at which the radiation is detected. The radiation emitted by the source is also almost entirely due to pure spontaneous radiative decays of excited states. Non-radiative processes, which can also depopulate the excited states, will be negligible due to the low particle densities of the source.

Besides these decided advantages, the beam-foil method suffers from one major disadvantage which is also present in most other methods of mean life measurement. The main cause of error in these measurements is due to cascading or repopulation of the state under study by higher excited energy states. This effect must always be present, to some extent, if the state under study is not selectively excited. In some cases, however, where either the upper cascading states are not well populated or the transition probabilities for their decay channels to the state of interest are small, cascading effects do not become apparent in the decay curves. In many other cases where cascading
does occur, the decay constant of the cascading terms in the decay curve is widely different from that of the state under study, and the effect may be separated out and compensated for. Examples of both types of decay curves will be seen in the results section.

One disadvantage of the diffuse beam-foil source that may also become an experimental problem is due to the low level of light output. Thus, fast optical detection systems are demanded when complex atomic and ionic systems are studied since the spectra emitted by the source is then very rich in lines and high detection resolution is required to make sure blending of different multiplets does not occur. The blending of two multiplets originating from two different upper terms of different mean lives could show up in the decay curves, as does cascading, by an otherwise single exponential becoming a sum of more than one exponential. For carbon, this blending of different multiplets was not too great a problem (although it did occur) since the carbon emission spectrum produced in the low incident ion energy range used in this experiment was not very complicated, and very high resolution was only required in a few cases. At higher incident ion energies, where the spectra of high charge state ions begin to appear, and also for elements with larger numbers of valence electrons, the need for greater resolution becomes essential. Faster optical systems, more efficient collection of the emitted photons, and gratings with larger dispersion would all help to improve
the resolution of the system for the relatively weak beam-foil light source.
SELECTION II

THEORY

2.1 Theory of the Beam-Foil Excitation Method

To measure the radiative mean life of the upper level (term) of a transition in which a spectral line (multiplet) is emitted by the beam-foil source, one must record the number of photoelectric detector counts (proportional to the intensity of the line or the number of photons emitted by the segment of the excited beam that are collected by the detection system) accumulated during a fixed integration time interval for various different foil positions with the monochromator set at the wavelength of the spectral line under study. If cascading and multiplet blending can be neglected, the decay curve $I(x, \Delta x)$ is a simple exponential characterized by the decay length $v \tau_1$, where $v$ is the velocity of the beam and $\tau_1$ is the mean life of the upper level, $|i\rangle$. A semi-logarithmic plot of this decay will thus be linear and the parameter $\tau_1$ can be easily obtained from the logarithmic slope of the straight line. In some cases, however, cascading processes will be present, and the effect of this cascading will be to repopulate the upper level of the emission line along the length of the beam at a rate determined by the radiative mean life of the upper level of the cascading transition. The measured decay curve $I(x, \Delta x)$ will, therefore, no longer be a simple exponential, but a complex curve composed of a sum of terms, each characterized by a different
decay constant corresponding to the mean life under study, $\tau_i$, and the various "cascading" level mean lives. If the cascading decay constants are significantly different from the decay constant of the spectral line under study, the cascading processes can often be identified in the spatial decay curve and the data can be corrected to obtain the true mean life $\tau_i$. The same complex decay curve will result if multiplet blending is present.

A more detailed study of the beam-foil decay process will now be made in which an expression for the intensity $I(x, \Delta x)$ of the light emitted by a spectral line in the decay process as a function of distance downstream from the foil, $x$, is derived for the two cases where cascading is either present or absent.

First, consider the case where there is no cascading or multiplet blending present, and further consider an atom or atomic ion emerging from the foil ($x=0$) with a velocity, $v$, and in an excited electronic state $|1\rangle$. The excited state will decay both in time and in space since $x = vt$. If $N_i(0)$ atoms or atomic ions emerge from the foil at $x=0$ in the excited state $|1\rangle$ then the number per cm$^3$ $N_i(x)$ or $N_i(t)$, surviving at a distance, $x$, downstream from the foil is the solution of the simple differential rate equation governing the population of the state:

$$\frac{dN_i(t)}{dt} = -\sum_{j \neq i} N_i \omega_{ij} = N_i(t) / \tau_i$$

2.1.1
this solution, as a function of \( x \), is:

\[
N_i(x) = N_i(0) \exp \left( -\frac{x}{\nu_i(z)} \right)
\]

2.1.2

In the beam-foil experimental process, one counts the number of photons of a particular wavelength emitted in a certain time interval by a small segment of the excited beam. This small segment can be called the beam "window"; and its magnitude, which is measured experimentally, depends upon the length of beam "seen" by the monochromator diffraction grating. This is, of course, a function of the monochromator entrance slit width as well as geometrical distances. The diagram below shows the coordinate system adopted for this theoretical treatment.

![Diagram](image)

The observed decay curve \( I(x, \Delta x) \) is proportional to the number of photons emitted in the space interval from \( x - \Delta x/2 \) to \( x + \Delta x/2 \) for a certain instantaneous foil position \( x \).

\[
I(x, \Delta x) = k \int_{x-\Delta x/2}^{x+\Delta x/2} N_i(x') \, dx'
\]

2.1.3

where \( k = k(\lambda) \) is a constant for a particular wavelength which takes into account the cross-sectional area, \( S \), of the beam going through the foil and the overall efficiency of the detection apparatus for the radiation emitted in the
particular transition under study. This efficiency will be the product of such factors as the solid angle collection efficiency of the monochromator, the efficiency of the monochromator grating for the emitted radiation, the efficiency of the sodium salicylate wavelength conversion window for this radiation (if it is used), and the quantum and collection efficiencies of the photomultiplier tube. The solution to equation 2.1.3 may be written as:

$$ I(x, \omega) = k A_{ij} N_i(0) \exp\left(-\frac{x}{v_1}\tau_1\right) \left[2 \tau_1 \sinh\left(\frac{\Delta x}{v_1}\tau_1\right)\right] $$ 2.1.4

Thus, it is apparent that due to the finite "window" of the beam that the detection system views, the instantaneous intensity, $kA_{ij} N_i(0) \exp(-x/v_1)$ is multiplied by a "window factor" $2\tau_1 \sinh(\Delta x/v_1)$ which is really the fraction of the total number of decays which occur within the window. The size of this factor is a combined function of the velocity of the beam, the mean life under study, the monochromator entrance slit width, the effective area of the diffraction grating used in the monochromator and the beam-slits geometrical distance. The window size was experimentally determined by moving a small lamp attached to a foil holder in small steps across the face of the monochromator entrance slits and recording the number of time-integrated counts at each foil holder position. The full width at half maximum of the "foil-zero" curve gave the approximate size of the "window" for that particular diffraction grating, entrance slit width, and ion energy.
Now, consider the case where cascading effects are to be included in the theoretical treatment. One will now have additional terms present in the population rate equations. Consider, for example, the following decay scheme:

\[ |m\rangle \rightarrow |1\rangle \rightarrow |0\rangle \]

where the state \( |\rangle \) can depopulate to a number of lower energy states denoted collectively by \( |\rangle \) and is in turn repopulated from above by cascading from a number of higher energy states denoted collectively by \( |\rangle \). The population rate equation now becomes:

\[
\frac{dN_i(t)}{dt} = -\sum_{j \leq i} N_i(t) A_{ij} + \sum_{m > i} N_m(t) A_{mi} \tag{2.1.5}
\]

where \( A_{ij} \) is the transition probability for spontaneous electric-dipole transitions from the state \( |\rangle \) to the state \( |\rangle \) and \( \sum_{j} A_{ij} = (\tau_i)^{-1} \) (similarly for \( A_{mi} \)). The solution to equation 2.1.5 is:

\[
N_i(t) = N_i(0) \exp(-\frac{t}{\tau_i}) + \sum_{m > i} A_{mi} N_m(0) \frac{\exp(-\frac{t}{\tau_m}) - \exp(-\frac{t}{\tau_i})}{\tau_i - \tau_m} \tag{2.1.6}
\]

It will be necessary to add further terms to this sum if either the upper cascading states are themselves repopulated by transitions from still higher states, or if the passband of the detector includes wavelengths for additional transitions.
of different lifetimes. This latter process is usually called multiplet blending. The observed intensity decay curve \( I(x, \Delta x) \) is once again obtained by integrating the equation 2.1.6 over the variable \( x \) from \( x - \Delta x/2 \) to \( x + \Delta x/2 \), i.e., over the spatial beam segment. The result can be shown to be

\[
I(x, \Delta x) = k A_{ij} N_i(0) \exp\left(-\frac{x}{\alpha_{T_i}}\right) \left[ 2 \pi i \sinh \left( \frac{\Delta x}{\alpha_{T_i}} \right) \right] 
+ 2k A_{ij} \sum_{m \neq i} \left( \frac{\sigma_{m-1}}{\alpha_{T_m}} \right) N_{m,0} A_{m} \exp\left(-\frac{x}{\alpha_{T_m}}\right) \left[ \sinh \left( \frac{\Delta x}{\alpha_{T_m}} \right) \right]^{2.1.7}
\]

The second term of this expression is due to cascading and is summed over any upper state \( |m\rangle \) that can take part (i.e., obeying the electric-dipole selection rules) in a repopulation process to the state \( |i\rangle \). The effect of the finite beam "window" now appears in both terms. In general then, with cascading present, the shape of the decay curve depends on the size of the window parameter, \( \Delta x \). This is not the case when cascading is absent, in which case the shape of the decay curve is independent of \( \Delta x \), no matter how large the window factor \( \Delta x/vT \) is (as can be seen from equation 2.1.4).

### 2.2 Doppler Effects

The measured wavelengths of the observed beam-foil spectral lines (determined either by linear interpolation between beam and calibration lines or by measuring accurately from one well known beam-foil line) must be corrected for
Doppler effects that are present due to the high unidirectional velocity of the ion beam and the viewing window subtended by the acceptance angle of the monochromator.

The relativistic Doppler effect comes about due to the invariance of the phase of an electromagnetic wave under a Lorentz transformation. The measured wavelength, (in the moving frame) is related to the zero-velocity (rest frame) wavelength, \( \lambda_0 \), by the following equation:

\[
\lambda_0 = \lambda + \lambda \beta \cos \theta - \lambda \frac{1}{2} \beta^2 + \ldots 
\]

In this experiment, the observations on the beam are instantaneously made at \( \theta = \pi/2 \), but the "window" effect causes one to view a range of angles. The above equation can then be split into a second-order relativistic Doppler effect term \( \lambda(1- \frac{1}{2} \beta^2) \) which causes a shift in the spectral line, and the first order term, \( \lambda \beta \cos \theta \), which, when integrated over all the angles observed by the window, produces broadening of the spectral line. The carbon ion beam traveled at approximately \( 2 \times 10^8 \) cm/sec in this investigation. Thus, \( \beta = 6.7 \times 10^{-3} \) and the second order Doppler shift for \( \lambda 5000 \) Å (the worst case) becomes:

\[
\Delta \lambda = \lambda_0 - \lambda \approx 0.1 \text{ Å}
\]

This shift is negligible compared to the accuracy at which the wavelengths of the lines were determined. Doppler broadening of spectral lines, due to the finite acceptance angle of the monochromator, must also be taken into consideration.
since excessive spectral line widths can seriously affect the resolution of the emission spectrum. The beam velocity vector of a particle in the viewing "window" will have a component parallel to the line of sight and so the problem can be viewed as a classical longitudinal Doppler shift effect; i.e., the emitted radiation is shifted towards the blue part of the spectrum for viewing angles less than $\pi/2$ and shifted towards the red for viewing angles greater than $\pi/2$. The overall effect (when integrated over the range of angles in the viewing "window" of the beam) causes the spectral line to be broadening, i.e., shifted towards both lower and higher wavelengths. This Doppler correction can be written as:

$$\Delta \lambda = \lambda_0 \delta$$ \hspace{1cm} \text{2.2.3}$$

Where $\delta$ is the acceptance angle of the monochromator (in radians), $\lambda_0$ is the zero-velocity wavelength, and $\delta = \nu/c$. The acceptance angle was determined experimentally via a beam "window" determination using a small light bulb attached to the foil holder. The result was $\delta = 0.06$ radians for the worst case, where 2 mm entrance slits widths were employed. The calculated Doppler broadening correction factor, $\Delta \lambda$, for the present experimental arrangement was of the order of 2 Å for the worst case of $\lambda_5000$ Å and 2 mm entrance slits. When this value is combined in quadrature with the spectral-slit width factor, it is seen that the Doppler broadening result did not degrade the overall spectral resolution by very much since the spectral-slit widths employed were quite
large. For example, at $\lambda_0 = 4000 \AA$, Doppler broadening using a 336 keV $^{12}$C$^+$ ion beam was calculated to be approximately 1.6 $\AA$. The spectral-slit width is defined as the product of the reciprocal dispersion of the grating (33.3 $\AA$/mm for the 600 line/mm grating) and the entrance slit width (in this case, 300 $\mu$m = 0.3 mm). Thus, in this case the spectral-slit width is calculated to be 9.9 $\AA$. When combined in quadrature with the Doppler broadening factor of 1.6 $\AA$, the result for the full width at half maximum (fwhm) of a 4000 $\AA$ spectral line would be approximately 10 $\AA$. The experimental value (found from a spectral scan) was a little higher than this (~12 $\AA$) probably due to the finite time response of the ratemeter-chart recorder combination that was used to record the emission spectra in the spectral-scan experiment.

### 2.3 Theory of Atomic Structure

Descriptions of atomic structure theory can be found in many books, and this particular exposé is based on the text of Levinson and Nikitin (1962).

In order to determine the electronic wave functions (or eigenfunctions) and energy eigenvalues of an atomic or atomic ion system consisting of a nucleus of change $Ze$ surrounded by $N$ electrons ($N \leq Z$) one must solve the $N$-body Schrödinger time independent equation, $\hat{H} \left| \psi \right> = E \left| \psi \right>$, with the following Hamiltonian:

$$\hat{H} = \hat{T} + \hat{V} + \hat{Q} + \hat{W}$$
where \( \hat{T} = \sum_{i=1}^{N} \frac{\hat{P}_i^2}{2m} \) is the kinetic energy operator of the N electrons (\( \hat{P} \) is the electron linear momentum and \( m \) its mass).

\[ \hat{V} = -\sum_{i} \frac{Ze^2}{r_i} \] is the potential energy of the electrons in the field of the nucleus.

\[ \hat{Q} = \sum_{i<j} \frac{e^2}{r_{ij}} \] is a term describing the electrostatic interaction between the electrons;

\[ \hat{W} \] is a term that takes into account various other couplings such as spin-orbit and spin-spin. These interactions represent the energy of the electron spins in magnetic fields set up by the moving electrons and are small compared to other terms in \( \hat{H} \) (at least for L-S coupling). Hence, \( \hat{W} \) will be neglected at this point, but the spin-orbit part of it will be added later as a perturbation term. The exact solution of the Schrödinger equation for a many-electron system is unattainable in practice since the electrostatic interaction term, \( \hat{Q} \), will not allow the equation for each electron to be separated out. Various approximations can be made to help determine the wave functions and to classify the atomic states, levels, and terms (the energy eigenvalues) with the aid of approximate quantum numbers. Perturbation theory cannot be used directly since the electrostatic interaction between the electrons, \( \hat{Q} \), is of the same order of magnitude as the energy of the electrons in the nuclear
The central field approximation is, therefore, generally used in which one assumes that the greater part of the energy operator, \( \hat{Q} \), can be represented by a central field \( V^0(r) \), screening the nuclear field. The residual part of the electrostatic interaction, \( \hat{Q}' \), not represented by the central screening field, is considered as a small perturbation. Thus, in this approximation each electron moves, independently of the other electrons, in the central screening field produced by the other electrons and the nuclear field. These combined fields determine the state of the electrons. On the other hand, the field \( V^0 \) is set up by the electrons and therefore depends on their states. The determination of the field \( V^0 \) should, therefore, be carried out consistently with the determination of the wave functions of the electrons. This is best achieved by the Hartree-Fock method of the self-consistent field. To summarize then, the Hamiltonian for a many electron system in the zero central-field approximation is written as:

\[
\hat{H}^0 = \hat{T} + \hat{V} + \hat{Q}^0 = \hat{T} + U^0(r)
\]

Under this assumption, the electrons move independently, each in the effective central field \( U^0(r) = V - V^0(r) \). The remaining interactions:

\[
\hat{H}' = \hat{Q}' + \hat{W}
\]

are considered as a perturbation. By restricting the effective potential to spherical form, we assure the separation
of variables, and each electron can be ascribed a well-defined angular momentum. Each electron can thus be ascribed a set of one-electron quantum numbers:

\[ a^i = n^i, l^i, m_l^i, s^i, m_s^i \quad (i=1\ldots N) \]

and a wave function \( \Psi(a^i|x_i) \). In this approximation the total electron system can be described by a wave function which is a product of the one-electron functions of the type i.e., include a space and spin part.

\[ \Psi(a|x) = \phi(n_l m_s | r, \theta, \phi) \chi(s m_s | \sigma) \]

Thus, for the whole system,

\[ \Psi(A|x) = \Psi(a^1|x_1) \ldots \ldots \Psi(a^N|x_N). \]

This function must be antisymmetrized in order to satisfy the Pauli principle since electrons are fermions. The function \( \Psi(A|x) \) can be replaced by a Slater determinant consisting of the same one-electron functions

\[ \Psi(A|x) = \frac{1}{\sqrt{N!}} \left| \begin{array}{c} \Psi(a^1|x_1) \ldots \ldots \Psi(a^N|x_N) \\ \vdots \\ \Psi(a^N|x_N) \ldots \ldots \Psi(a^1|x_1) \end{array} \right| \]

Since in the central field approximation the electrons are assumed independent, the energy of the atomic state described by the determinant \( \Psi(A) \) is simply the sum of the one-electron
This energy is independent of all $m_l$ and $m_s$. i.e., degenerate in these quantum numbers and only depends, in this approximation, on the set of quantum numbers $n^1 \mathbf{L}^1 \ldots n^N \mathbf{L}^N$, which specify the configuration, $K$. Electrons with identical $n^1 L^1$ yield equal contributions to the energy and such electrons are called equivalent. A configuration is generally written with the equivalent electrons combined in shells, i.e., in the form $K = (n_1 L_1)^{N_1} \ldots (n_k L_k)^{N_k}$, where $N_r$ is the number of electrons in the $n_r L_r$ shell. Each state, $A$, possesses a definite parity, $\pi$, equal to the product of the parities of the individual one-electron states, thus:

$$\pi = (-1)^{\sum L_1}$$

2.3.9

The configuration thus entirely specifies the parity, so that $\pi$ is often omitted from a characterization of the configuration, $K$. Closed shells obviously do not influence the parity of the configuration.

The electrostatic electron-interaction term, which acts as a perturbation, can be thought of as being represented by interactions of various spin and angular momenta of the electrons. It turns out that the electrostatic interaction term, $\hat{Q}$, can be represented as the sum of two separate operators $\hat{F}$ and $\hat{G}$. $\hat{F}$ represents the Coulomb interaction between
the different charge distributions of densities \( \rho_1 = |\psi_1|^2 \) where \( \psi_1 \) is the spatial part of the one-electron wave function describing the \( i^{th} \) electron. Equivalently, the interaction can be represented as the potential energy of the charge density, \( \rho_1 \), in the field \( V^j \) set up by the charge density \( \rho_j \), summed over all possible electron pairs. The quantity, \( G \), cannot be interpreted classically. It is called the exchange interaction since it involves terms in which the electron coordinates \( x_i \) and \( x_j \) have exchanged their positions. The exchange term, \( \hat{G} \), vanishes for two electrons if the spins are oriented in opposite directions, but does not if the spins are parallel. The term can be thought of as due to an interaction of the electron spin momenta. The Coulomb term, \( \hat{F} \), is independent of the relative orientation of the spins, but can be thought of as an interaction between the orbital angular momenta, \( \hat{l}_1, \hat{l}_j \). Thus, the overall effect of the electrostatic interaction between the electrons of an \( N \)-electron system is to split the energy levels in the zero-order approximation, which were labeled by the configuration, into terms whose energy now depends upon the relative orientation of the angular and spin momenta taking part in the coupling to form the total spin, \( \hat{S} \), and orbital angular momentum, \( \hat{L} \), of the whole system. Thus, terms are usually labelled by their configuration and the \( L \) and \( S \) quantum numbers involved.

The vector model of the atom is based on the central field approximation, which makes it possible to ascribe to each electron a definite orbital and spin momentum and to
express the various interactions between the atomic electrons as interactions between the momenta \( l_1 \) and \( s_1 \). The unperturbed energy \( H^0 = T + V + Q^0 \) of the central field approximation does not contain interactions of momenta. Thus, in the zero approximation, all the states, \( A \), of the configuration, \( K \), differing from one another in the quantum numbers \( m_s^1 m_t^1 \), i.e., possessing different orientations of the momenta \( l_1 \) and \( s_1 \), have the same energy \( E^0(k) \). The interaction of the angular momenta is included in the perturbation energy \( H^t = Q^t + W \). The energy of interaction of two momenta depends upon the angle between them, and thus the influence of \( H^0 \) leads to a definite mutual orientation of all the one-electron momenta. States with different orientations of momenta have different coupling energies, \( E^1 \), of the momenta. The zero-approximation level \( E^0(K) \) is thus split into atomic levels of energy \( E = E^0 + E^1 \). In some cases all spin-orbit interactions, \( W \), may be assumed small in comparison with the electrostatic interactions, \( Q^t \). Thus, we may study the splitting of the \( E^0(K) \) level first under the influence of \( Q^t \); and then later each of the sublevels obtained are split under the influence of \( W \). When only \( Q^t \) is taken into consideration, there is no interaction between spin and orbital momenta of the electron; all the interactions are reduced to spin-spin or orbit-orbit. The total orbital momentum of the atom

\[
\hat{L} = \sum_{i} l_i
\]
and the total spin momentum of the atom

\[ \vec{S} = \sum_i \vec{s}_i \]  \hspace{1cm} 2.3.11

are therefore exact quantum numbers in this approximation.

In the coupling of all the spin momenta \((\vec{s}_1 \ldots \vec{s}_N)\vec{S}\) and all the orbital momenta \((\vec{l}_1 \ldots \vec{l}_N)\vec{L}\), the zero-approximation level, \(E^0(k)\), splits into the terms \(E(k\alpha SL)\) each of which is characterized by its value \(S\) and \(L\). The additional quantum number \(\alpha\) distinguishes between like terms. The states distinguished by a different orientation of the momenta \(\vec{S}\) and \(\vec{L}\), i.e., by different quantum numbers \(M_S\) and \(M_L\), belong to one term.

The interaction \(\hat{W}\) between \(\vec{l}_1\) and \(\vec{s}_1\) now becomes an interaction between \(\vec{L}\) and \(\vec{S}\), and in fact only the total angular momentum of the atom

\[ \vec{J} = \vec{L} + \vec{S} \]  \hspace{1cm} 2.3.12

is an exact quantum number. The coupling is denoted by \((SL)J\) due to the interaction \(\hat{W}\) and the term further splits into the levels \(E(k\alpha SLJ)\) characterized by the quantum number \(J\). Each level is further \((2J + 1)\)-fold degenerate (in the absence of external perturbations) since the momentum \(J\) can take on \(2J + 1\) different orientations if, for instance, a magnetic field is applied. Each of these states are characterized by the quantum number, \(M_J\). The coupling scheme just described is called L-S coupling (Russell-Saunders coupling or normal coupling). L-S coupling is significant to a problem if the electrostatic interactions predominate over spin-orbit interactions, and
this applies to most of the not too excited configurations of light atoms including, in this instance, carbon.

One may summarize the coupling scheme just described with an example. The following figure shows the splitting of an \( np^2 \) configuration (as for instance in the carbon atom) under L-S coupling:

\[
\begin{align*}
\begin{array}{c}
\text{\( \frac{1}{2} S \)} \\
- \rule{1.5cm}{0.15mm} - \rule{1.5cm}{0.15mm} - \rule{1.5cm}{0.15mm} \\
\text{\( \frac{1}{2} S_o \)}
\end{array} \\
\begin{array}{c}
\text{\( \frac{1}{2} D \)} \\
- \rule{1.5cm}{0.15mm} - \rule{1.5cm}{0.15mm} \\
\text{\( \frac{1}{2} D_2 \)}
\end{array} \\
\begin{array}{c}
\text{\( \frac{3}{2} P_2 \)}
\end{array} \\
\begin{array}{c}
\text{\( \frac{3}{2} P \)} \\
- \rule{1.5cm}{0.15mm} - \rule{1.5cm}{0.15mm} \\
\text{\( \frac{3}{2} P_1 \)} \\
\end{array} \\
\begin{array}{c}
\text{\( \frac{3}{2} P_0 \)}
\end{array}
\end{align*}
\]

configuration \hspace{1cm} terms \hspace{1cm} levels \hspace{1cm} states

Notice that the Pauli exclusion principle applies to this problem since the two electrons are equivalent. In L-S coupling, it can be shown that only states with \( L + S \) even are permitted since the wave functions of the other possible states are not overall antisymmetric as they must be to satisfy fermion statistics.

Wave functions in the one-configuration approximations may be constructed as linear combinations of the determinants pertaining to the given configuration:
The determinants $\Psi(A)$ describe the states with uncoupled one-electron momenta, and the functions $\Psi(k\alpha JM)$ describe the states in which these momenta are coupled to yield the total atomic momentum. In the case of L-S coupling, it is permissible to couple the orbital and spin momenta separately. We therefore have a state described by the function:

$$\Psi(k\alpha SLN_\delta M_\delta) = \sum_A \Psi(A) \langle A | k\alpha SLN_\delta M_\delta \rangle$$

The expansion coefficients are independent of the principal quantum number of the configuration and the summation extends only over those states $A$ for which

$$\sum_{i=1}^{N} m_{J}^i = M_\delta \quad ; \quad \sum_{i=1}^{N} m_{L}^i = M_\delta$$

Adopting the central-field approximation and this particular vector coupling scheme allows one to describe the atomic states with a set of quantum numbers such as the configuration, $K$, and the intermediate momenta $L, S$, etc. These quantum numbers are obtained in an approximate analysis of the Schrödinger equation, and, as such, are not exact. The only exact quantum numbers employed are the total momentum, $J$, its projection, $M_J$, and the parity, $\pi$.

The "exactness" of the approximate quantum numbers is determined with the aid of the energy matrix, whose elements
are:

\[ \langle \alpha \pi J M | \hat{H} | \alpha' \pi' J' M' \rangle \]  

2.3.16

Where \( \alpha \) stands for the set of all approximate quantum numbers it can be shown that the matrix elements only join states with the same exact quantum numbers \( J, \pi, \) and are independent of \( M. \) Matrix elements, however, with the same exact quantum number but with different approximate quantum numbers do not always vanish. The diagonal matrix elements represent the approximate energy levels

\[ E(\pi J) = \langle \alpha \pi J | \hat{H} | \alpha \pi J \rangle \]  

2.3.17

The energy matrix, when calculated in terms of the exact wave functions \( \Psi(v \pi JM) \) rather than the approximate wave functions \( \Psi(\alpha \pi JM) \), is diagonal in all the quantum numbers

\[ \langle v \pi JM | \hat{H} | v' \pi' J' M' \rangle = E \delta(v \pi JM) \delta(v') \delta(J' M') \]  

2.3.18

Its diagonal matrix elements

\[ E(v \pi J) = \langle v \pi JM | \hat{H} | v \pi JM \rangle \]  

2.3.19

give the exact energy levels. In contrast to the approximate additional quantum number \( \alpha \), the quantum number \( v \) is an exact additional quantum number. The approximate quantum number is obviously more "exact" the smaller the matrix elements \( \langle \alpha \pi J | \hat{H} | \alpha' \pi J \rangle \), which are nondiagonal in \( \alpha \). To measure the "exactness" of \( \alpha \),
these elements should be compared with the difference of the corresponding diagonal elements, i.e., \( E(\alpha\pi J) - E(\alpha'\pi J) \). If the nondiagonal elements are not sufficiently small, the energy matrix should be diagonalized. It is convenient to divide the energy matrix into squares corresponding to the various sets of the exact quantum numbers, \( \pi JM \). Then the non-diagonal squares will vanish. The diagonalization of such a matrix amounts to the diagonalization of the diagonal squares. Each square will consist of the elements \( \langle \alpha\pi J | \hat{H} | \alpha'\pi J \rangle \) with some \( \pi J \), but different \( \alpha \) and \( \alpha' \). The diagonalization in \( \alpha \) may thus be carried out separately for each \( \pi J \). The diagonalization process is actually a search for a representation of the exact wave functions as linear combinations of the functions \( \Psi(\alpha\pi JM) \) with different \( \alpha \)

\[
\Psi(\pi JM) = \sum_{\alpha} \Psi(\pi J) \langle \alpha \pi J | \pi J \rangle
\]

Only states with definite \( \pi JM \) enter the linear combination, since the diagonalization is carried out separately for each square. The coefficients should be chosen so that the square is diagonal in the \( \pi J \) scheme. In practice, however, this diagonalization is unfeasible. Generally one must be satisfied with making the matrix more diagonal; i.e., with replacing the quantum number, \( \alpha \) by another approximate quantum number, \( \beta \)

\[
\Psi(\beta \pi J) = \sum_{\alpha} \Psi(\alpha \pi J) \langle \alpha \pi J | \beta \pi J \rangle
\]
So that in the βJM scheme the nondiagonal elements are less than in the αJM scheme. The formation of the linear combinations described is equivalent to the mixing of states with different α but the same exact quantum numbers πJM. This implies that one can mix or superpose states described by different configurations, (but the same πJM quantum numbers) in order to improve the wave functions used in the one-configurational approximation. When this multi-configurational procedure is necessary, one says that configuration interaction is present in the problem. This technique becomes especially important in the calculation of some atomic transition probabilities in which either the initial, the final or both electronic wave functions are affected by configuration interaction. A specific example of this particular problem can be found in paragraph 2.5 of Section II, and experimental results for transitions prone to configuration interaction are discussed in the results section.

When the atomic structure problem is applied to atomic and ionic carbon, one can calculate approximate energy eigenvalues and hence draw term diagrams such as in Figures 1, 2, and 3. CI, CII, CIII contain six, five, and four electrons, respectively, with ground state configurations of \(1s^2 2s^2 2p^2\), \(1s^2 2s^2 2p\), and \(1s^2 2s^2\), respectively. CII and CIII are isoelectronic with BI and BeI, respectively. The various excited state configurations are formed when the "jumping" electron is excited into a higher energy shell. The radiative transitions shown in the partial term diagrams are electric-
Partial term diagram of CI

<table>
<thead>
<tr>
<th>2s² 2p ns</th>
<th>2s² 2p np</th>
<th>2s² 2p nd</th>
<th>2s² 2p³</th>
<th>2s 2p² ns</th>
</tr>
</thead>
<tbody>
<tr>
<td>1p⁰ 3p⁰</td>
<td>1s 1p 1D 3S 3p 3D</td>
<td>1p⁰ 1D⁰ 1p⁰ 3p⁰ 3D⁰ 3p⁰</td>
<td>1p⁰ 1D⁰ 3S⁰ 3p⁰ 3D⁰ 3S⁰</td>
<td>5p</td>
</tr>
</tbody>
</table>

![FIGURE 1](image)
Partial term diagram of CII

FIGURE 2
Partial term diagram of CIII

<table>
<thead>
<tr>
<th>2s ns</th>
<th>2s np</th>
<th>2s nd</th>
<th>2s ns</th>
<th>2s np</th>
<th>2s nd</th>
<th>2p 3s</th>
<th>2p²</th>
<th>2p 3p</th>
</tr>
</thead>
<tbody>
<tr>
<td>1s</td>
<td>1p</td>
<td>1p</td>
<td>3s</td>
<td>3p</td>
<td>3p</td>
<td>1p²</td>
<td>1s</td>
<td>1p</td>
</tr>
</tbody>
</table>

Energy (cm⁻¹) x 10⁴

- λ₂296.9
- λ₂236.9
- λ₄662.7
- λ₄263.1
- λ₄662.7
- λ₂313.9

FIGURE 3
dipole transitions, involving one jumping electron, which must obey the following selection rules:

\[ \Delta \pi = \text{odd} \leftrightarrow \text{even} \]
\[ \Delta n = 0, \pm 1, \pm 2, \text{etc.} \]
\[ \Delta S = 0 \]
\[ \Delta L = \pm 1 \]
\[ \Delta L = 0, \pm 1, \]

Where \( \pi \) is the parity of the state, \( S \) is the total spin momentum quantum number of the state, \( L \) is the orbital momentum quantum number of the state, \( n \) is the principal quantum number of the "jumping" electron and \( l \) is the orbital momentum quantum number of the "jumping" electron.

2.4 Quantum Mechanical Theory of Spontaneous Emission of Radiation

A quantum mechanical treatment of an isolated atomic system predicts that there will exist a set of discrete stationary states (the eigenfunctions of the Hamiltonian of the system) for an excited electron to reside in. These stationary states have no evolutionary time dependence and thus should not decay spontaneously. It is an experimental fact, however, that an excited atomic system does eventually spontaneously decay back to its ground state even if the atom is apparently isolated (i.e., not perturbed) in free space. The apparent contradiction lies in the fact that one can never treat an atomic system as completely isolated, but must study the combined quantized system of the atom and the elec-
tromagnetic radiation field generated by the charged particles of the atom itself. It is the interaction between these two quantum mechanical systems which causes the spontaneous decay of a "stationary" (or rather quasi-stationary) atomic state.

Many theoretical descriptions of this problem can be found in the literature, and this expose is based on work by Louisell (1964) and Nussbaum (1968). The most sophisticated approach to this problem could entail the use of a relativistic field theory in which there exists an interaction between a spin 1/2 (Dirac) field and a spin 1 (photon) field. This dual field approach is not strictly necessary, however, and the electrons may be treated as non-relativistic quantum mechanical particles obeying the Schrödinger equation. The free electromagnetic radiation field must still be treated as a quantized field. This field is found to be dynamically equivalent to an infinite set of uncoupled, one-dimensional, harmonic oscillator, one oscillator being associated with each value of the propagation vector of the radiation, $\vec{k}$, and polarization mode, $\lambda$. A stationary state of the free field is characterized uniquely by the photon occupation numbers $\{n_{\vec{k}\lambda}\}$; such a state has the total energy $\sum_{\vec{k}\lambda} \hbar \omega_k (n_{\vec{k}\lambda}^{\pm} +1/2)$. The lowest energy state is specified by the eigenvalues $n_{\vec{k}\lambda} = 0$ for every $\vec{k}$, $\lambda$. This state is called the vacuum state. A set of non-hermitian operators, $a_{\vec{k}\lambda}^\dagger (a_{\vec{k}\lambda}^\dagger)$ are introduced such that $a_{\vec{k}\lambda}^\dagger (a_{\vec{k}\lambda}^\dagger)$ creates (destroys) a single photon of momentum $\hbar \vec{k}$ and energy $\hbar c |\vec{k}|$ when operating on a state of the fields.
The total number operator \( N_{kx} = a^+_{kx} a_{kx} \) is defined such that its eigenvalue is the occupation number, \( n^x_{kx} \) of the state \( \{|n^x_{kx}\} \). The latter kets are states of the field containing \( n^x_{kx} \) photons in each of the modes \((k, \lambda)\). In terms of these definitions the Hamiltonian for a free electromagnetic field may be written as

\[
\hat{H}_{\text{Free Field}} = \sum_{kx} \hbar \omega_k a^+_{kx} a_{kx} 
\]

Consider now a system composed of an atom with a single bound electron and the radiation field to which it is coupled. (This simple system can be easily generalized to a more complex system later.) Such a system can be described by the following Hamiltonian (assuming, as can be done in this case, that electron spin can be neglected and the problem can be tackled non-relativistically).

\[
\hat{H} = \hat{H}_{\text{Free Atom}} + \hat{H}_{\text{Free Field}} + \hat{H}_{\text{Interaction}}
\]

where

\[
\hat{H}_{\text{Free Atom}} = \frac{\hat{p}^2}{2m} + V(r) \quad 2.4.3
\]

\[
\hat{H}_{\text{Free Field}} = \sum_{kx} \hbar \omega_k a^+_{kx} a_{kx} \quad 2.4.4
\]

\[
\hat{H}_{\text{Interaction}} = -e_y \sqrt{\frac{\hbar}{m}} \vec{A} \vec{F} + e_y \sqrt{\frac{\hbar}{m c}} \vec{A} \vec{\lambda} \quad 2.4.5
\]

and \( \vec{A} \) is the vector potential of the field. (The second term in the interaction Hamiltonian may be neglected for this particular problem since it involves the emission of two photons.)
Thus the unperturbed Hamiltonian for the problem is

\[ H_0 = H_{\text{Free Atom}} + H_{\text{Free Field}} \]

while the small perturbation is:

\[ H_I = -\frac{e}{m} \hat{A} \cdot \hat{p} \]

It can be assumed that the condition \( H_I \ll H_0 \) is valid and so perturbation theory is applicable. Let the state of the combined interacting system be denoted by \( |\Psi(r,t)\rangle \). The time evolution of this state is described by the time dependent Schrödinger equation.

\[
\text{i}\hbar \frac{\partial}{\partial t} |\Psi\rangle = (\hat{H}_0 + \hat{H}_I) |\Psi\rangle
\]

We may now expand \( |\Psi\rangle \) in terms of the eigenstates \( |\Psi_n(r,t)\rangle \) of \( \hat{H}_0 \), the Hamiltonian describing the combined uncoupled system. Thus

\[
|\Psi(r,t)\rangle = \sum_n C_n(t) |\Psi_n(r,t)\rangle
\]

\[
= \sum_n C_n(t) e^{-\frac{\text{i}}{\hbar} E_n t} |\Psi_n(r)\rangle
\]

where \( |\Psi_n(r)\rangle \) is an orthogonal eigenvector of \( \hat{H}_0 \). Insertion of equation 2.4.9 into equation 2.4.8, multiplying by \( \langle \Psi_m(r,t) | \) from the left, and integrating over all space leads to an expression for the time dependent projection, \( C_m(t) \), of
on the state \( |\Psi_m(\vec{r},t)\rangle \), or, in other words, the probability amplitude for finding the system in the state described by \( |\Psi_m(\vec{r},t)\rangle \). The time evolution of this projection is given by

\[
i\hbar \dot{C}_m(t) = \sum_n \langle \Psi_m(\vec{r},t) | \hat{H}_Z | \Psi_n(\vec{r},t) \rangle C_n(t).
\]

The eigenstates of a given uncoupled state of the system are simply products of a particular state of the free atom and a given state of the free radiation field. The states are conveniently designated by \( |i, \{n_{K,\lambda}\}\rangle \), where \( |i\rangle \) represents a free atom in its \( i \)th state and \( |\{n_{K,\lambda}\}\rangle \) represents a free field containing photons in an infinite number of different modes, with \( n_{K,\lambda} \) denoting the number of photons (in the \( k \)th mode) propagating in the direction \( \vec{k}/|\vec{k}| \), with energy \( \hbar c|\vec{k}| \) and circular polarization \( \lambda \). For notational convenience let us designate this composite state as \( |a\rangle = |i, \{n_{K,\lambda}\}\rangle \). Let us now consider the problem of specific interest, i.e., the spontaneous emission of one photon by an excited atom. The atom is placed in an excited state (in the vacuum electromagnetic field) initially. We desire to follow the development of coupled system in time, and, in particular, to calculate the amplitudes for the atom to be in its excited state, \( |i\rangle \), and ground state, \( |f\rangle \), respectively. The initial and final states of the combined interacting system may then be written for this special case as:

\[
|a\rangle = |i, 0\rangle \quad ; \quad |b\rangle = |f, \{n_{K,\lambda}\}\rangle
\]
Then equation 2.4.10 becomes

\[ i\hbar \dot{C}_b = \langle b | \hat{H}_z | a \rangle e^{i\omega_{ab} t} C_a \quad 2.4.12 \]

\[ i\hbar \dot{C}_a = \sum_b \langle b | \hat{H}_z | b \rangle e^{i\omega_{ab} t} C_b \quad 2.4.13 \]

where

\[ \omega_{ab} = (E_a - E_b) / \hbar = -\omega_{ba} \]

The matrix elements in equations 2.4.12 and 2.4.13 are time independent and so one may integrate directly (using the initial conditions \( C_a = 1, C_b(0) = 0 \)) equation 2.4.12 to give

\[ C_b(t) = \frac{1}{\hbar} \langle b | \hat{H}_z | a \rangle \int_0^t C_a(t') e^{i\omega_{ab} t'} dt' \quad 2.4.14 \]

If this equation is now substituted into equation 2.4.13 one gets

\[ \dot{C}_a(t) = -\frac{1}{\hbar} \sum_b |\langle a | \hat{H}_z | b \rangle|^2 \int_0^t C_a(t') e^{-i\omega_{ab}(t-t')} dt' \quad 2.4.15 \]

This equation is an integrodifferential equation for \( C_a(t) \) and may be solved approximately by means of Laplace transforms. Let us quote the approximate result (see Louisell, 1964) as

\[ C_a(t) = \exp (-\gamma t) \quad 2.4.16 \]

where \( \gamma \) is the complex quantity

\[ \gamma = \frac{1}{2} \Gamma + i \Delta \omega \quad 2.4.17 \]
The probability of observing the initial state is, therefore:

\[ |\langle a(t)\rangle|^2 = e^{\kappa \rho (-\Gamma t)} \quad 2.4.18 \]

Thus, the state will decay exponentially with a decay constant, \( \Gamma \). The mean life of the state against spontaneous decay will equal

\[ \tau = 1/\Gamma \quad 2.4.19 \]

Hence, \( \Gamma \) is equivalent to the total transition probability out of the initial state and may be defined as (see Louisell, 1964)

\[ \Gamma = \frac{2\pi}{\hbar^2} |\langle b|\hat{\mathcal{H}}_z|a\rangle|^2 \rho(\omega_b)|_{\omega = \omega_b} \quad 2.4.20 \]

where \( \rho(\omega_b) \) is the density of final states.

We may think of the interaction as shifting the energy of the previously stationary state \( |a\rangle = |i, o\rangle \), and consider the atom to evolve in time according to its new (complex) energy. Thus, we can write

\[ |a\rangle_t = e^{-i/\hbar \left[ \Delta E - i\hbar \frac{\omega}{2} \right] t} |a\rangle \quad 2.4.21 \]

The imaginary energy shift produced by the interaction of the atom with the field (of magnitude \( \hbar \Delta \omega / 2 \)) is responsible for the exponential decay of this state in time. The real energy shifts (\( \Delta E \)) caused by the interaction are responsible for such effects as the Lamb shift and the anomalous magnetic momenta of the electron. Both energy shifts are different for different states.
In order to solve the problem completely we must now solve equation 2.4.14 for \( C_b(t) \), i.e.:

\[
C_b(t) = \sqrt{\frac{1}{\hbar}} \langle b| \hat{H}_{\text{Ze}} |a \rangle \int_0^t C_a(t') e^{i \omega_{ba} t'} dt'
\]

2.4.14

If one now uses the expression below to sum over polarization directions and integrate over solid angles,

\[
C_b(\omega) = \sum \int d\Omega \; C_b(\hat{\Omega}, \lambda)
\]

2.4.22

and substitutes equation 2.4.16 into equation 2.4.14, one has

\[
C_{b(\omega)}(t) = \sqrt{\frac{1}{\hbar}} \int d\Omega \; \sum_k \langle b| \hat{H}_{\text{Ze}} |a \rangle \left[ \frac{1 - e^{-i (\omega_{ba} + \Delta \omega) t}}{\omega_{ba} + \Delta \omega} \right]
\]

2.4.23

where we have incorporated the real energy shift, \( \Delta \omega \), into the expression \( E_{ba} = \hbar \omega_{ba} \). We now ask for the distribution of emitted photon energies \( \hbar \omega |a| \) (or frequencies). To insure that the photon has actually been emitted one can evaluate equation 2.4.23 at \( t = \infty \).

\[
C_{b(\omega)}(\infty) = \sqrt{\frac{1}{\hbar}} \int d\Omega \; \sum_k \langle b| \hat{H}_{\text{Ze}} |a \rangle \frac{\omega_{ba} + \Delta \omega}{\omega_{ba} + \Delta \omega} \equiv C_b(\omega)
\]

2.4.24

and

\[
|C_{b(\omega)}|^2 = \sqrt{\frac{1}{\hbar}} \int d\Omega \; \sum_k |\langle b| \hat{H}_{\text{Ze}} |a \rangle|^2 \left[ \frac{\omega_{ba} + \Delta \omega}{\omega_{ba} + \Delta \omega} \right]^2
\]

2.4.25

Equation 2.4.25 represents the spectral distribution of spontaneously emitted radiation. This is associated with the exponential decay of an excited atom and is a distribution of...
possible energies that a photon emitted in such a decay can possess. Thus one sees that the spectral line shape, \( |C_b|^2 \), is Lorentzian and is peaked at \( \omega_{ba} = 0 \) and has a full width at half maximum equal to \( \Gamma \).

Thus, to summarize one can write the full expression for the state of the combined (atom plus field) coupled system as

\[
|\Phi\rangle_t = \sum_n C_n(t) |\Phi_n(t)\rangle e^{-i\omega_n t}
\]

2.4.26

and we have in the case of a two-state system

\[
|\Phi\rangle_t = C_a e^{-iE_a\hbar t} |a\rangle + \sum_b C_b e^{-iE_b\hbar t} |b\rangle
\]

2.4.27

\[
= e^{-iE_a\hbar t} e^{-i\hbar [E_a+AE]t} |a\rangle + \sum_b C_b e^{-iE_b\hbar t} |b\rangle
\]

The amplitude for the system (in the state \( |a\rangle = |1,0\rangle \) at \( t = 0 \)) to still be in the state \( |a\rangle \) at time \( t \) is

\[
\langle a |\Phi\rangle_t = e^{-iE_a\hbar t} e^{-i\hbar [E_a+AE]t}
\]

2.4.28

and the corresponding probability is

\[
|\langle a |\Phi\rangle_t|^2 = e^{-2\hbar t} = |C_a(t)|^2
\]

2.4.29

Thus

\[
\frac{d}{dt} |C_a(t)|^2 = -\Gamma e^{-\hbar t} dt
\]

2.4.30

and \(-d|C_a(t)|^2\) represents the probability for the atom to decay between \( t \) and \( t + dt \). Hence, the mean life of the \( |a\rangle \)
state may be expressed as:

\[ \tau = \int_0^\infty t \, f(t) \, dt = \frac{1}{p} \]

where \( F \) is the total transition probability. This result was previously quoted in equation 2.4.19.

2.5 Theoretical Calculations of Atomic Transition Probabilities

Transition probabilities for spontaneous electric-dipole transitions between electronic states of atoms and atomic ions described by L-S coupling can be calculated, in some cases, (for the lighter element at least) with sufficient accuracy that one can now compare theoretical values and experimental results with relative confidence. Among the theoretical approaches used, the self-consistent-field (Hartree-Fock) approximation and the Coulomb (Bates-Damgaard) approximation have provided the bulk of the data. This is the case for the values tabulated in the NBS monogram of atomic transition probabilities (Wiese, 1966) and with which many of the results of the present experiment are compared in the results section.

The transition probability or the Einstein A-coefficient for a spontaneous electric dipole transition between two non-degenerate atomic states \( |a\rangle \) and \( |b\rangle \) is given (as has been shown in equation 2.4.21) by

\[ A(a,b) = \frac{64 \pi^2}{3 \hbar c} \lambda_{ab}^2 |\langle a | F | b \rangle|^2 \]

2.5.1
where $\mathbf{P} = e \sum \mathbf{r}_i$ is the electric dipole operator summed over all the electrons, and $\lambda_{ab}$ is the wavelength of the radiation limited in the transition. If $|a\rangle$ and $|b\rangle$ are the magnetic substates of degenerate levels having angular momenta $J$ and $J'$, then one may write

$$A(JM, J'M') = \frac{C}{\lambda^3} |\langle J M \mathbf{P} | J'M' \rangle|^2$$

where $C = \frac{64 \pi^4}{3h^3}$ and $\gamma, \gamma'$ denote all other quantum numbers necessary for a complete definition of the states. The state $|J M \gamma\rangle$ may make transitions to several of the substates of the level $|J' \gamma'\rangle$. In this case the reciprocal of the radiative mean life of the state $|J M \gamma\rangle$ is the sum of the transition probabilities to all the substates of $|J' \gamma'\rangle$ with which it can combine, i.e.,

$$\sum_{J'} A(JM, J'M') = C \sum_{J'M'} |\langle J M \mathbf{P} | J'M' \rangle|^2 = \frac{1}{\tau(JM)}$$

The only non-zero matrix elements contributing to the sum are those for which $M' = M \pm 1$. It may be shown (Condon and Shortley, 1936) that $\sum_{M'} |\langle J M \mathbf{P} | J'M' \gamma'\rangle|^2$ is independent of $M$, which means that all the substates of $|J \gamma\rangle$ have the same lifetime. This is to be expected on physical grounds since, if it were not so, the relative populations of the $M_J$ states differing only in the value of $M$ would change with time, an initially isotropic situation would become anisotropic. This point is important since the beam-foil experimental arrangement
may not be isotropic, i.e. all the M states may not be necessarily equally populated. We may thus write \( \tau(J_\gamma) \) instead of \( \tau(JM_\gamma) \), and call this the lifetime of the level \( |J_\gamma> \). Since \( \sum_{M'} A(JM_\gamma, J'M'\gamma') \) is independent both of M and M', it may be written as \( A(J\gamma, J'\gamma') \). This quantity is now a property of the level \( |J_\gamma> \) and denotes the total transition probability from any substate of \( |J_\gamma> \) to all substates of \( |J'\gamma'> \).

If \( |J_\gamma >, |J'\gamma'> \) are members of two Russell-Sauncers terms \( |\gamma>, |\gamma'> \), having total orbital and spin quantum numbers \( L, S \) and \( L'S' \) respectively, then the level \( |J_\gamma> \) may make transitions to more than one level of the term \( |\gamma'> \). In this case, the total decay constant of the level \( |J_\gamma> \) is \( \sum_{J'} A(J_\gamma, J'\gamma') \) where the sum over \( J' \) is subject to the usual electric-dipole selection rule \( J' = J \) or \( J \pm 1 \). If two levels of the term \( |\gamma> \) have angular momenta \( J_1 \) and \( J_2 \), the sums of the intensities of all lines of the multiplet originating from these two levels are in the ratio

\[
N(J_\gamma) \sum_{J} A(J_\gamma, J'\gamma') = N(J_{\gamma}) \sum_{J'} A(J_\gamma, J'\gamma')
\]

where \( N(J_\gamma) \) is the population of the level \( |J_\gamma> \), assuming that the small differences in wavelengths of the lines of the multiplet are negligible. Now from an experimentally observed sum rule of Burger and Dorelgo (1924), for "natural excitation" (one in which the populations of the various levels conform to a Boltzmann distribution) the above ratio
is equal to the ratio of the statistical weights of the two levels, i.e., \((2J_1 + 1)/(2J_2 + 1)\). But in this case it is also true that

\[
N(J,J')/N(J,J'') = (2J_1 + 1)/(2J_2 + 1)
\]

so that

\[
\sum_J A(J_1, J_1', J_1'') = \sum_J A(J_2, J_2', J_2'')
\]

which implies that all levels of a Russell-Saunders term have the same lifetime. This is an important result since the present experimental apparatus does not have enough resolution to resolve component lines within a multiplet. Since \(\sum_J A(J_1, J_1', J_1'')\) is independent of \(J, J'\) we may write it as \(\sum_{J'} A(J, J', J'')\). It then denotes the total transition probability of any one state in the term \(|y\rangle\) to all states in the term \(|y'\rangle\). The mean life of the term \(|y\rangle\) which we denote by \(\tau(y)\) is therefore given by

\[
\frac{1}{\tau(y)} = A(y') = \sum_{J'N'} A(J_1, J_1', J_1'')
\]

and

\[
= C \sum_{J'M'} |\langle J_1 M_1 N_1 | J_1 M_1 N_1' \rangle|^2
\]

The quantity \(\sum_{J'M'} |\langle J_1 M_1 N_1 | J_1 M_1 N_1' \rangle|^2\) is often called the "strength" of the multiplet emitted in transitions between the terms \(|y\rangle\) and \(|y'\rangle\) and is denoted by \(S(y, y')\). Since the summations over \(J'\) and \(M'\) are independent of \(J\) and \(M\) and
there are $$(2L + 1)(2S + 1)$$ states of the type $$|JM\gamma>$$ in the term $$|\gamma>$$, the strength reduces to

$$S(\alpha, \beta') = (2L + 1)(2S + 1) \sum_{J, M} |<JM\beta |JM\beta'>|^2$$ ...

$$= (2L + 1)(2S + 1) \frac{A(\alpha, \beta')}{c}$$

Hence

$$\frac{1}{\tau(\gamma)} = A(\alpha, \beta') = c S(\alpha, \beta')(2L + 1)(2S + 1)$$

The expression for $$\tau$$ is written in this way because the strength $$S$$ is the quantity usually given in calculation of transition probabilities. If the term $$|\gamma>$$ can combine with several other terms, then we must write $$1/\tau(\gamma) = \sum_{\gamma'} A(\gamma, \gamma').$$

In order to evaluate the multiplet strengths (for L-S coupled systems) one needs to know the atomic wave functions of the initial and final states. These wave functions cannot be found exactly but approximate functions are usually constructed from linear combinations of Slater determinants (products of anti-symmetrized single-electron wave functions), which are solutions of Schrödinger equations with central potentials. This problem has been studied in more detail in Section 2.3.

The single-electron wave functions are each a product of a space part and a spin part, and the space part is in turn a product of a radial function and an angular function which is of spherical harmonic form. Consequently, the multiplet strength may be expressed as the product of an integral involving the radial parts only (the radial transition integral, $$\sigma$$) and factors resulting from integration over the angular
and spin parts (Condon and Shortley, 1935) i.e.,

\[ S = \sigma(m) \sigma(l) \sigma^2 \]  \hspace{1cm} 2.5.11

The angular and spin factors are readily evaluated for simple cases and have been tabulated, for example, in works by Allen (1963) and Shore and Menzel (1968). For the case involving two terms derived from the same parent term, it can be shown (Condon and Shortley, 1936) that the multiplet strength is given (in atomic units) by

\[ S(LS, L'S') = (2 \sigma^2)(L+1)(L+1)(iL+3) \sigma^2 \text{ for } L' = L+1, S' = S. \]  \hspace{1cm} 2.5.12

\[ = (2 \sigma^2)(L+1)(L-1) \sigma^2 \text{ for } L' = L-1, S' = S. \]  \hspace{1cm} 2.5.13

where \( \sigma^2 \) is given by the expression

\[ \sigma^2 = \frac{1}{(4L'_{-1})} \left[ \int_r^\infty R_i(r) R_f(r) r \, dr \right]^2 \]  \hspace{1cm} 2.5.14

and \( R_i \) and \( R_f \) are the radial parts of the initial and final wavefunctions of the jumping electron in atomic units, and \( L'_{-1} \) is the greater of its orbital quantum numbers in the initial and final states. Substituting numerical values for the atomic constants, one obtains for the case when \( \lambda \) is expressed in angstroms,

\[ A(LS, L'S') = (2.02 \times 10^{18}) \sigma^2 \lambda^3 \left\{ \begin{array}{ll}
(L+1)(L+1) & \text{for } L' = L+1, S' = S \\
(L)(L-1) & \text{for } L' = L-1, S' = S.
\end{array} \right. \]  \hspace{1cm} 2.5.15
The problem of calculating transition probabilities, under the assumption of L-S coupling at least (which is generally well fulfilled for the first ten elements) thus reduces to a problem of evaluating the radial transition integral, $\sigma^2$.

Bates and Damgaard (1949) showed that for most transitions the main contribution to the transition integral comes from a spatial region in which the deviation of the actual non-Coulomb potential of an atom or atomic ion from its asymptotic Coulomb form is so small that it may be replaced by the latter. Since for the Coulomb potential the solution to the radial Schrödinger equation, and therefore the transition integral, may be expressed in analytic form (Whittaker functions), it is possible to calculate $\sigma^2$ as a function of the observed term value and the azimuthal quantum number. Bates and Damgaard (1949) have thus compiled tables with numerical values of $\sigma^2$ for s-p, p-d, and d-f transitions.

The Coulomb approximation, as this method is sometimes called, is restricted to transitions between levels having the same parent term. It gives the best results if the degree of cancellation in the transition integral is small, i.e., is not too close to zero, and if the upper and lower levels of the excited jumping electron are in a shell which contains no other electrons. These conditions apply well to moderately and highly excited levels. But even if the lower level is in a shell which contains other electrons, the results often agree fairly well with those obtained by other methods. The Coulomb approximation has been used to obtain a calculated value for the transition probability for several transitions
studied in this experiment and the results can be seen in the results section. Wiese (1966) estimates that the uncertainties on the theoretical value generated by using the Coulomb approxima-
tion on transitions in atomic and ionic carbon are within 50% for moderately excited transitions including 3s-3p and within 25% for medium and highly excited transitions, i.e., transitions of the type 3p-3d, 3d-4f, etc.

Another method of calculation is based on the Hartree-
Fock self-consistent field (SCF) approximation. This method provides a set of wave functions for the atomic electrons which produces an approximately self-consistent electric field. The transition probabilities are then determined, via the transition integral, from the radial parts of these self-
consistent wave functions. It is assumed that the effective potential produced by all the electrons and the nucleus is central, i.e., the charge density distribution of the whole system is spherically symmetric. Exchange correlations between the electrons are at first neglected and the electrons are assumed to move independently in the central field experi-
encing only the averaged charge distribution of the other electrons and the nucleus. With these simplifications it is possible to calculate the motions of the individual electrons by first assuming a trial wave function for the others, and from the resulting wave functions the charge density distribu-
tion is calculated and compared with the initial one obtained with the trial functions. If self-consistency is not achieved, the new computed wave functions are used themselves as trial
wave functions, and the procedure is repeated. Initial and final charge distributions are identical, i.e., the electric field is self-consistent. This basic procedure can then be improved by including electron exchange effects.

Large uncertainties in SCF calculations as well as other theoretical treatments are to be expected for transitions in which configuration interaction becomes important. For the first ten elements these transitions are of the type $1s^22s^22p^n - 1s^22s^m1s^12p^{n+1}$. Only a few attempts have yet been made to take configuration interaction into account in calculations by superposing several configurations. Weiss (1967) has made calculations for some transitions in Cl including the $1s^22s^22p^2 3P - 1s^22p^3 3D^0$ transition which has been studied experimentally in the present experiment. Bolotin and co-workers (Bolotin, 1956) have attempted to take into account some of the configurational interaction in carbon, nitrogen and oxygen atoms and ions by including one interacting term for the lower state and none for the upper state. It appears that many more configurations should be superimposed. Very recently Westhaus and Sinanoğlu (1969) have studied transitions of the type $1s^22s^22p^n - 1s^22s2p^{n+1}$ in which configuration interactions should be important and have listed some new calculated values for transitions in carbon and carbon isoelectronic ions. It is of great help to the theoretician to have precise experimental transition probability values available for transitions involving configuration interaction in order to judge the success of his calculations. The 3%
precision measurement on the transition probability for the 2s^2 2p^2 3P \rightarrow 2p^3 3D^0 transition in Cl, made in the present experiment, will hopefully be of use in this direction.

Wiese (1968) is attempting to study systematic trends within the large amount of data on transition probabilities that is now being produced. Of particular interest in this respect is the search for a systematic trend in the values of transition probabilities studied as a function of nuclear charge. Perturbation theory predicts such a trend and precise experimental values of transition probabilities for a particular transition in different isoelectronic ions would help to ascertain the correct functional dependence of the transition probability, A(z), or absorption oscillator strength f(z), on the atomic number, z. The present investigation includes results for multiplet transitions that fit into this category since they are isoelectronic with other ions that have already been studied by beam-foil excitation or some other method and thus may be compared systematically within an isoelectronic sequence. The results can be seen in Section IV.

To summarize, one can say that the calculation of atomic transition probabilities to an accuracy of even 10% is a very difficult task except for the case of one-and two-electron systems. For more complex systems the Self Consistent Field and the Coulomb approximations have found widest applications. Difficulties arise especially when strong configuration interaction is present. Transitions involving a single jumping electron moving in and separated from a core
may be treated by the Coulomb approximation method with fairly good accuracy. For the general case it seems probable that at present the best results are to be achieved by the intelligent combination of results from different methods, both experimental and theoretical. This has been the policy of Wiese, et al (1966) in their NBS tabulation of atomic transition probabilities of the first ten elements.
SECTION III

EXPERIMENTAL METHOD

3.1 Introductory Description of
the Apparatus and Experimental Technique

A diagram of the beam-foil apparatus used in this experiment is shown in Figure 4. An overall description of the apparatus and the general experimental procedure will first be made followed by more detailed studies of the major components.

The University of New Hampshire 400 kV positive ion Van de Graaff electrostatic accelerator was used to accelerate singly charged carbon ions of mass twelve ($^{12}$C$^+$) to kinetic energies between 200 and 400 keV. The ion beam was then passed through a magnetic analyzing system which allowed only ions of a certain magnetic rigidity to pass into a 30° port beam tube at the end of which was situated the foil-excitation chamber. Typical analyzed beam currents were of the order of from 0.05 - 0.10 μA. Higher beam currents were not used since they were usually unattainable using the present ion source; and, in any case, they burned out the foils very quickly. There was no way of changing foils in the present chamber without breaking vacuum. Magnetic analysis was necessary since many other atomic and molecular ions such as N$^+$, O$^+$, CO$^+$, NO$^+$, N$^+_2$, O$_2^+$, and CO$_2^+$ were simultaneously produced in the ion source when a gaseous mixture (commercial grade)
EXPERIMENTAL APPARATUS

- Beam current
- Excited beam
- Monitor P.M.
- Carbon foil
- Entrance slit
- H.V. Signal
- P.M. Cooling chamber
- Exit slit
- Grating
- Beam aperture

(Not to scale)

To pumping system

FIGURE 4
of CO₂ and N₂ was put into it.

The foil-excitation chamber contained a moveable thin (10 μg/cm²) carbon foil placed perpendicular to the path of the beam. The particles of the ion beam, which undergo collisions with the atoms of the foil, emerge from the foil in various charge and electronic excitation states. The distribution of these charge states is, as expected, a function of incident ion energy. Thus, in this particular energy range (200 – 400 keV) it was found that neutral (Ci) and singly (CII) and doubly (CIII) ionized species of carbon were produced (the major species produced was CII). At still higher energies (not attainable with the present accelerator) it is possible to produce CIV, CV, and even completely stripped carbon.

A spectral scan of the beam-foil source was first made at a fixed spatial position close to the foil to see which multiplet transitions occurred in the excited beam. Typical carbon spectral scans are shown in the results section. The monochromator was then set in turn, to pass the particular wavelength of each multiplet, and a survey study of the spatial dependence of the intensity of the emitted radiation was made. The monochromator entrance and exit slits were set to optimize the intensity and still only allow one particular multiplet transition to be studied at a time; i.e., to try to avoid "blending" of two or more multiplets. If the spectral multiplet was found to be a good candidate for a mean-life study, the study was made as described in the next paragraph. To be a candidate for such a study, it was necessary that the
multiplet intensity (the signal) was above the photomultiplier noise at the distances behind the foil required to define the number of exponentials in the best fit. For example, it was necessary to take data out to at least one mean life of the cascading exponential in a two exponential fit.

The excited atoms and atomic ions of the emergent beam move at a constant velocity downstream from the foil; i.e., their point of initial excitation. Thus, the foil forms the time origin \( t=0 \) for a time evolution study of the excited specie. The atoms and atomic ions also spontaneously emit electric-dipole radiation as they travel away from the foil. The intensity of this radiation for a particular transition (or wavelength) is proportional to the population, at any instant, of the upper state involved in the transition. Thus a study of the time dependence of the intensity of radiation of any particular wavelength is equivalent to a study of the time dependence of the population of the upper state involved in the transition. A knowledge of the time dependence of the population of a state allows a direct measurement of the radiative mean life of the state since they are connected, functionally, by the expression:

\[
N_i(t) = N_i(0) \exp\left(-\frac{t}{\tau_i}\right)
\]

where \( N_i(t) \) is the population / cm\(^3\) of any state \( |i\rangle \) at any instant, \( t \), \( N_i(0) \) is the initial population of this state, and \( \tau_i \) is the mean life of the state.
The intensity time dependence is studied by allowing a monochromator (spectrometer and photoelectric detector) to view a small segment of the excited beam (the beam window) at different spatial positions behind the foil. This is accomplished experimentally by moving the carbon foil with a small electric stepping motor. The spatial coordinates can be converted to temporal ones since the beam travels at a constant velocity. The resulting intensity decay curve for any particular transition is fit to a sum of exponentials by a computer program. This program calculates the radiative mean life of the upper excited state involved in the transition.

3.2 The Van de Graaff Accelerator

A diagram of the Van de Graaff electrostatic accelerator is shown in Figure 5. This type of accelerator utilizes the force exerted on a charged particle (ion) in an electric field as a means of direct particle acceleration. The ions are produced in a radio-frequency (R.F.) source bottle situated at the high potential terminal of the accelerator (see Figure 6). The source bottle consists essentially of a glass bottle with a narrow exit canal made of tantalum. A mixture of CO₂ and N₂ gases are fed into the ion source via a thermomechanical mechanical leak. A small R.F. coil, wound around the source bottle, ionizes the gas present in the bottle and a small positive extraction
THE VAN DE GRAAFF ELECTROSTATIC ACCELERATOR

High voltage terminal
Charge extraction screen
Ion source
Accelerating column
Insulators
Equipotential planes
Power supply
Beam tube
Belt drive motor
Ion beam

FIGURE 5
BEAM OPTICS OF THE VAN DE GRAAFF ACCELERATOR

FIGURE 6
potential is added to an electrode at the end of the bottle in order to expel the positive ions produced. The CO$_2$/N$_2$ gas mixture produced $^{12}\text{C}^+$, $^{14}\text{N}^+$, $^{16}\text{O}^+$, $^{28}\text{CO}^+$, $^{28}\text{N}_2^+$, $^{30}\text{NO}^+$, $^{32}\text{O}_2^+$, and $^{44}\text{CO}_2^+$ ions. The ions leaving the ion source then enter the evacuated accelerating column which consists of a series of equipotential planes insulated from each other by ceramic insulators. The planes are connected to one another through a series of resistors called the column resistors. In this way the full potential of the high potential terminal is dropped slowly along the accelerating column. The current through the column resistors - the column current - can be used as an indication of the accelerating energy, but it is not a very accurate measurement since the column resistors change value after a period of time due to heating. An electrostatic generating voltmeter provides a more reliable method of determining the accelerating energy of the machine. This instrument is mounted in the pressure tank of the accelerator just above the high potential terminal. More will be said about this instrument in paragraph 3.5.

3.3 The Magnetic Analyzing System

The analyzing system was designed to deflect positive ions of a certain magnetic rigidity (in order to insure an isotopically pure beam) into a beam port making an angle of 30° (in the horizontal plane) with the straight-through port.
The figure of merit for such an analyzing system is called the mass-energy product. This is defined as $\text{ME}/^2$ where $M$ is the mass of the ion in atomic units, $E$ is the energy of the accelerated ion in MeV and $Z$ is the charge on the accelerated ion (usually unity). Equation A-7 of Appendix A shows how the mass-energy product is functionally related to such magnet parameters as radius of curvature of the ion passing through the magnetic field, and the value of the magnetic field itself, $B$. The equation shows that for a constant magnetic configuration the field required to deflect singly charged ions of mass $M$(a.m.u.) and $E$(MeV) through a constant angle is proportional to the square root of the product $MZ$. A fluxmeter (Sensitive Research Instrument Corporation), which had a guaranteed accuracy of 0.05% was used to calibrate the magnet's power supply in terms of field strengths. A calibration curve is shown in Figure 7. The field strength is proportional to the current supplied to the magnet's coils over a large range of field strengths. Thus, for a constant energy setting, the currents $i$, required to deflect ions of different masses $M$ are proportional to the square root of $M$. The mass spectrum of all the ions leaving the R.F. ion source bottle was studied by slowly increasing the current to the magnet in a continuous manner and monitoring the current registered by a Faraday Cup situated on the $30^\circ$ beam port. The power supply current
Calibration curve for the Diecraft magnet
(Pole gap = $\frac{3}{4}$ in)

FIGURE 7
required to deflect $C^+$ ions into the $30^\circ$ port foil chamber was found by this method. The power unit used to supply current to the magnet coils was made by Atomic Laboratories, Inc. (Model C). It is a solid-state regulator unit. The current regulator, when connected between the power supply and the magnet, maintains the current delivered to the coils, and hence the resulting magnetic field, constant to one part in $10^5$ over the range 0.4 to 9.0 amperes and can be read with an accuracy of 0.01 amperes. The magnet itself was made by the Diecraft Co. (Model 1). It is an electromagnet with circular tapered pole faces of diameter four inches. A continuous flow of cold water flowing at a rate of approximately 2-3 liters/min was used to cool the system. The field strength was guaranteed to stay constant to within 1% over a region less than two inches from the magnet's vertical axis.

3.4 The Energy Stabilization Unit

A corona stabilizer amplifier unit, made by High Voltage Engineering, Inc., was used to electronically stabilize the terminal voltage of the Van de Graaff positive ion accelerator. High voltage stability was maintained by regulating the corona current drain between the high voltage terminal and the grounded pressure tank wall. The magnitude of this corona current is controlled by corona points protruding from inside the tank wall towards the terminal. The
plate of a corona control tube (type 4-125A) is connected to the corona points. A variation in the conduction of this tube varies the voltage on its plate and thus, in turn, affects the potential between the terminal and the corona points. It is this potential which controls the amount of corona drain. If the plate voltage is lowered (raised) the corona drain increases (decreases) and the voltage on the terminal is consequently lowered (raised). The corona load is controlled by a corona stabilizer amplifier.

The beam magnetic analyzing system deflects the positive ion beam into a 30° port beam tube. A pair of vertical insulated slits are placed in this port beam tube between the magnet and the foil excitation chamber. The amount of deflection of the ion beam is dependent on the instantaneous value of the terminal voltage. When this voltage fluctuates from a set value, the beam will strike one slit edge more than the other and currents on the two slit edges will become unbalanced. This then forms a doubled-ended correction signal to the corona stabilizer amplifier. This amplifier possesses dual inputs and outputs. One channel is for the high-energy slit signal and the other is for the low energy slit signal. The slits are approximately 1/8" apart and are adjusted initially so that equal amounts of charge splash on each slit. This is the balanced configuration. If the system becomes unbalanced due to a small
fluctuation in the terminal voltage, the unbalanced difference signal will be used to correct the fluctuation by changing the amount of charge on the high voltage terminal via the corona drain current device.

3.5 The Energy Calibration of the Generating Voltmeter

The generating voltmeter that was used to determine the energy of the machine was designed and manufactured by High Voltage Engineering Corporation. It was designed to measure the potential at the high voltage terminal of a Van de Graaff accelerator and thus determines the energy that an ion will acquire as it is accelerated. The instrument is placed inside the pressure tank close to the high voltage terminal. The generating voltmeter consists basically of a rotor and a stator plate which is divided into eight insulated 45° sectors. As the rotor revolves, it alternately exposes the high voltage terminal to the stator and shields from it each sector. Essentially, triangular wave a.c. voltages are electrostatically induced between adjacent sectors at a frequency of about 230 cps. These induced voltages are proportional to the terminal voltages. The instrument has a nearly linear calibration curve and so only one calibration point is really necessary. The rectified voltages are fed to a microammeter readout unit which has an estimated accuracy of ±2%. It is expected that a more accurate readout unit (e.g. a digital voltmeter) will be purchased in the near future.
The $^{19}\text{F}(p,\gamma)^{16}\text{O}$ nuclear resonance was used to calibrate the generating voltmeter and thus the accelerator, in energy. This nuclear reaction is characterized by a sharp, well-defined increase in cross section at a well known proton energy (See Figure 8). The laboratory system energy for this resonance is 340.5 keV. A simple fluorine target was prepared by rubbing a piece of teflon material onto a copper backing. It was important to get as thin a target as possible so that the resonance would be sharp. A proton beam was directed onto the target and the incident beam energy was increased slowly through the resonance energy. This resonance point was signaled by a large increase in the yield of emitted gamma rays (~6 MeV gamma rays) that were monitored by a 2\textquotedbl NaI scintillation detector. The output pulses from the detector were fed into a preamp whose purpose was to drive a 40 foot line to an amplifier, single channel analyzer and a scaler. The number of counts for a preset time was recorded as a function of incident proton energy as read on the generating voltmeter output monitor microammeter scale. In this way one could calibrate the generating voltmeter readout directly in the terms of incident ion energy. The calibration was made at one energy and the generating voltmeter was assumed to be linear over the range 200–400 keV. The calibration has an estimated accuracy of approximately 2\%.
Calibration of generating voltmeter using the $^7\text{Li}(P,\alpha\gamma)^{16}\text{O}$ resonance.

FIGURE 8
3.6 The Foil Chamber

The excitation foil chamber (See Figure 9) consists of a vacuum tight, internally blackened, brass chamber fitted with an external mechanism for moving the carbon foil along the axis of the beam thus varying the distance between the foil and the monochromator entrance slits. The chamber also contains a window for viewing the luminous beam when desired (The excited beam is visible to the dark-adjusted eye.), four BNC outlets and a small RCA 8571 photomultiplier tube attached at a fixed distance from the foil. The RCA 8571 was used to monitor the total light output from the excited beam at the fixed position from the foil in a set time so that the monochromator photomultiplier output could be normalized to a value directly proportional to the number of particles passing through the foil. Thus, fluctuations in the beam current would not show up in the data, at a particular foil position, as increases or decreases in the normalized intensity of a spectral line. An unshielded Farady cup was placed at the end of the foil chamber in order to monitor the beam current after the foil. This reading could not be taken as an absolute measure of the incident ion beam current since the charge state composition of the beam changed on passage through the foil, but it was a good relative measured and enabled one to see if the beam current remained approximately
PLUNGER OF
RCA 8571 PHOTOMULTIPLIER FOIL MOVING MECHANISM

8571 SIGNAL TERMINAL
8571 H.V. TERMINAL

VIEWING WINDOW THAT MAY BE COVERED

ION BEAM

FOIL HOLDER AND CARBON FOIL
EXCITED ATOM BEAM VIEWED BY MONOCHROMATOR

FOIL CHAMBER

BEAM TUBE

MONOCHROMATOR SLITS

TO CONCAVE GRATING AND EMI 6256 S PHOTOMULTIPLIER

FIGURE 9
constant throughout a run. The beam current could not be used for normalization since it varied with foil position due to a combination of a small divergence of the beam after the foil caused by Rutherford scattering of the incident ions by the foil atoms and the chamber geometry.

The foil-moving mechanism consists of a "plunger" rod which can be moved to various depths of penetration inside the excitation chamber. A double O-ring seal is placed in position at the place where the plunger enters the chamber in order to keep a good vacuum-tight system. The foil holder and foil are attached to the internal end of the plunger. The plunger is kept from rotating by a collar that fits over the beam tube and guides the plunger's sliding action so that the carbon foil always remains aligned with the incident ion beam. The plunger's motion is also controlled by an accurate screw thread (twenty threads per inch) and an exact measure of the distance it is moved by this screw can be made using a micrometer depth gauge which is mounted on the beam tube. The gauge can be read to ± 0.002 inch. During the course of an experimental run the foil is moved remotely by a small electric stepping motor. The distance between successive foil position varied in the experiments from about 0.025 inch to about 2 inch depending upon the magnitude of the mean-life under study. Manual motion of the foil can be made if one desires the distance between successive foil position to be less than 0.025 inch.
A residual gas chamber pressure of from approximately $5 \times 10^{-6}$ to $10^{-5}$ torr was maintained in the foil chamber by two separate vacuum systems, one on the accelerator itself, about ten feet away and another close to the chamber at a distance of about two feet. The pressure in the chamber was monitored by a Phillips cold cathode ionization gauge. It was very important to keep the residual gas pressure both low and constant in value since the incident ions could excite the atoms and molecules in the residual gas (mostly oxygen and nitrogen). Light from decaying residual gas atoms was at all times monitored by the RCA 8571 normalization tube and sometimes (if the wavelength setting on the monochromator were correct) by the EMI 6256S photomultiplier. Separate runs without a foil (but with a foil holder) were made for all the spectral lines studied in order to determine, if any, the contribution to the photomultiplier outputs from the excited residual gas atom. Generally this contribution was found to be negligible for the EMI 6256S photomultiplier, but in a few cases it had to be taken into account in the overall data analysis. The RCA 8571 photomultiplier always had a definite contribution for the residual gas but as long as this contribution remained constant a correction to the RCA 8571 output was not necessary. This contribution was constant for constant ion currents and residual gas pressures. In later runs the background gas contribution was also found at the end of each run by moving the foil a long way upstream.
3.7 The Carbon Foils

The thin carbon foils that were used as the excitation medium in the foil chamber were obtained as thin carbon films deposited on microscope glass slides from the Yissum Research Development Company of Jerusalem, Israel. Foils of $(10 \pm 4) \mu g/cm^2$ thickness were used exclusively in this experiment. The foils were made by evaporating a known amount of carbon from a carbon arc source onto a known area of a glass slide for a known time (private communication.) The thickness was measured by the suppliers using a light-transmission technique. The method of selecting foils of the appropriate thickness is by visual comparison with foils of measured thickness. This is achieved by stripping part of the foil area from a slide and mounting it as a self-supporting foil. The thickness of the self-supporting foil is measured in a gauge based on the energy loss of alpha-particles. The foil remaining on the slide is then used as a standard for the light transmission technique.

In order to prepare the carbon foil targets one must take the foil from the glass slide and deposit it on a brass foil holder. The glass slide (with foil) is held at from $30^\circ - 45^\circ$ to the horizontal in a tank of clear water, and the level of water is slowly raised. In this way the carbon foil is made to float off the slide and to lie on the surface of the water. The foil is then picked up on the foil holder and the target is ready for use.

Target life is a sensitive function of the element being accelerated, the particle energy, and the particle beam
current. In general, the foil life increases as the particle energy is raised. Unfortunately, at the low ion energies used in the present experiment, the foil "lifetime" became a problem. The present foil chamber did not allow one to rotate a new foil into position (if one broke) without breaking vacuum. Thus, low beam currents of $^{12}\text{C}^+\text{ ions}$ were used throughout the experiment (at the expense of light output intensity) in order to conserve foils for longer periods of time. It was found that molecular ion beams such as $\text{CO}^+$ and $\text{CO}_2^+$ beams, which gave greater light output, destroyed foils very quickly when used. Target foils are also known to thicken as they are bombarded or just remain in the system for a period of time (Chupp, 1968). The thickening may be due to a deposition of carbon from the general hydrocarbon vapor which forms a component of the residual gas in the target chamber. Foils were not kept in the system for great periods of time.

The velocity of an excited specie after passing through the foil must be calculated from a knowledge of the velocity of the accelerated ion prior to its passage through the foil and the specific energy loss of the ion in the foil. The latter value is a function of the ion energy and of the foil material. Experimental specific energy loss curves for singly ionized carbon ions in carbon at the ion energies used in the present experiment can be found in at least two places in the literature. Both Northcliffe (1963) and Fastrup, et al (1966) show experimentally derived specific energy loss curves from which one can calculate the average energy that


will be lost by the incident ion. For example, a 336 keV $^{12}\text{C}^+$ ion loses approximately 36 keV of energy in passing through a 10 $\mu$g/cm$^2$ carbon foil. The energy loss figure is subtracted from the incident ion energy to ascertain the final kinetic energy, and hence, the velocity of the excited specie. It is this velocity which helps define the time scale for the intensity decay curves and the uncertainties associated with this velocity are reflected directly as uncertainties in the final mean life result. The procedure for finding the final error on the mean life result, due to the velocity uncertainty, is shown in greater detail in appendix B of this thesis.

3.8 The Monochromator

A McPherson (Model 235) 1/2-meter Vacuum Ultraviolet Monochromator with an EMI 6256S photomultiplier placed at the exit slits was used to study the intensity of radiation emitted by a small segment of the excited beam. The size of this viewing segment or window is determined partly by the effective area of the grating and partly by the width of the entrance slits. The geometrical arrangement is shown in Figure 10. The optical system used in the monochromator is a Seya-Namioka mounting with a concave grating of 1/2-meter radius of curvature. The whole instrument is evacuated to less than $10^{-6}$ torr by a NRC 4" diffusion pump, and its associated Welch mechanical pump (13CPM). A water cooled trap is provided to protect the grating and main chamber from long term oil contamination and to improve the ultimate vacuum.
FIGURE 10

Excited atomic beam and monochromator
Two different gratings were used to cover the spectral range $\lambda 750 \text{ Å} - \lambda 5500 \text{ Å}$. A 1200 lines/mm coated with magnesium fluoride, blazed at 1500 Å, was used in the ultraviolet region and a 600 lines/mm grating blazed at 3500 Å was used in the visible region. Both gratings have ruled areas of $30 \times 50 \text{ mm}$ and their first order reciprocal dispersions were 16.6 Å/mm and 33.3 Å/mm respectively. The gratings were obtained from the Bausch and Lomb Optical Company.

The monochromator has bi-lateral entrance and exit slits which are controlled from outside the chamber. The slits are adjustable from 10 to 2000 μ by the use of a small micrometer screw. A spectrum may be scanned at any of twelve different synchronous scanning speeds from 0.5 Å/min to 2000 Å/min. A manual scanning control is also provided for precise positioning of the grating for a particular wavelength. The wavelength was indicated on a mechanical register. This register was calibrated by studying both the beam-foil and discharge tube sources of hydrogen and helium in which well separated spectral lines occur at precisely known wavelengths. Any instrumental offset was taken into account when measuring the wavelength of experimentally observed lines in the carbon spectrum. Wavelength shifts of spectral lines due to the second-order (transverse) Doppler effect were not apparent as can be expected from a theoretical calculation (see Section 2.2). The full aperture ratio (speed) of the instrument is f/11.4 and the masked aperture ratio (for best resolution) is f/34.0. The full aperture ratio was used in the
ultraviolet region since line intensities from the source were weak and resolution in some cases was not as important as intensity. The masked aperture ratio was used in the visible region since more lines were present and good resolution was of primary importance.

A sodium salicylate window was placed at the exit slit of the monochromator when studying the ultraviolet spectral region. This window converted the lower wavelength (higher energy photons) ultraviolet radiation to visible (4,200 Å) radiation by a fluorescent process. The efficiency for this process is approximately 30 - 99% over this spectral range (Allison, 1964). The EMI 6256S photomultiplier which monitored the radiation transmitted by the window, has a quantum efficiency of approximately 13% (near maximum at 4200 Å).

The spectral-slit width of a monochromator is usually defined as the product of the reciprocal dispersion of the grating (in Å/mm) and the entrance slit width (in mm). The actual measured line width (fwhm of the spectral lines) found in the spectral scans will usually be slightly broader than the spectral slit width due to a combination of different possible factors:

(a) The spectral line may be Doppler broadened.
(b) The finite time constant of the rate meter and the strip chart recorder combination used in the spectral scan experiments tends to broaden the spectral lines by slightly distorting the trailing edges, especially at high scanning speeds.
If a scan was made at a low scanning speed (e.g. 100 Å/min), the spectral line widths were found to be only slightly greater than the calculated spectral slit widths. In this experiment the spectral line widths observed were not as small as might be preferred, thus limiting the resolution of the detection apparatus. The foil chamber did not contain a mechanism for externally rotating a new foil into the path of the beam when a foil broke without breaking vacuum. Thus, one had to run at low beam currents in order to preserve the foil during the run. Typical beam currents were of the order of from 0.05 µa - 0.10 µa (\(^{12}\text{C}^+\)). Since the number of ions passing through the foil is directly proportional to the beam current, the light output from the excited beam was smaller than what it would be for larger beam currents. The lack of a high level of light intensity demanded wider entrance slit settings which consequently increased the spectral line-widths. It can be said that since carbon has a relatively simple emission spectrum at these incident ion energies very high spectral resolution was not strictly necessary and blending of other spectral multiplets with the one under study was not often a problem. Every multiplet was treated individually as regards the choice of the entrance slit width. The smallest slit widths were used that were consistent with the requirement of having large enough line intensities at the late foil positions in order to be well
above tube backgrounds. If multiplet blending were a possibility, then in some cases multiplet intensity was sacrificed in order to achieve high resolution. In several cases multiplets that should have been present in a blend with the multiplet under study were found to be of negligible intensity. This was probably due to the upper term of the blending multiplet being only weakly populated in the excitation process and/or the transition probability involved in the relevant multiplet transition being small.

Figure 10 also shows that the monochromator grating "sees" a finite portion of the beam, usually called the beam "window", due to the acceptance angle subtended by the beam at the monochromator entrance slits. Thus, the amount of radiation collected at a particular instantaneous foil setting is actually integrated over this finite beam length and this fact is accounted for in the computer analysis. The value of the "window" must, therefore, be experimentally measured for each grating used, both masked and unmasked, and for each entrance slit width used. This was accomplished by using a small (Sylvania Mite-T-Lite, MT202) electric light bulb attached to the foil holder at a height equivalent to the beam height. The constant light source was then slowly scanned past the monochromator entrance slit and the EMI photomultiplier counts were taken for each foil holder position in the scan. The full width at half maximum of the resulting intensity v. position curve was taken as a measure of the beam
"Window". It was found that both the 600 line/mm and the 1200 lines/mm masked gratings gave the same window value, and that the window value was the same for all slit widths less than or equal to 1 mm but larger for a 2 mm slit width. This can probably be attributed to a diffraction effect. The masked gratings gave a symmetrical intensity v. position curve whereas unmasked gratings gave a curve asymmetrical on the downstream side of the foil holder.

3.9 The Photomultiplier Detectors and Associated Electronics

An EMI 6256S photomultiplier was used at the exit slit of the monochromator to study the intensity decay of spectral lines in both the visible and ultraviolet (in conjunction with a sodium salicylate window) regions of the carbon spectrum. The dark current of this photomultiplier at 1850 volts H.V. is quoted to be $0.2 \times 10^{-9}$ amperes. The tube was run at +1700 volts H.V. in this particular experiment. The background count rate at this H.V. setting was approximately 200 cts/min (uncooled). The tube was housed in an aluminum can which contained facilities for cooling with either LN$_2$ or dry ice, although cooling was not used in this experiment. The photocathode of the EMI 6256S, which is an S(Q) surface, is made of a bialkali material (SbCs being the basic constituent). The spectral response of this surface is shown in Figure 11. The tube was powered by a Power Designs Pacific, Inc., (Model 2k-10) high voltage power supply. The wiring of the base is shown in Figure 12.
SPECTRAL RESPONSE OF THE EMI 6256 S PHOTOMULTIPLIER

\((S(Q)\) photocathode surface\)

![Graph showing quantum efficiency in percent vs. wavelength in angstroms.

**Figure 11**
BASE DIAGRAM OF THE EMI 6256 S PHOTOMULTIPLIER

FIGURE 12
The normalization beam monitor photomultiplier tube was an RCA 8571 model. The tube showed dark current bursts from time to time and so it was decided to run this tube in a direct current-measuring mode since the dark current bursts were then rendered negligible compared with the overall current measured. The average dark current of this tube at -1000 volts H.V. was found experimentally to be approximately $1 \times 10^{-9}$ amperes. The tube was also powered by a Power Designs Pacific, Inc., (Model 2k-10) high voltage power supply and run at -1000 volts H.V. in this experiment. The wiring of the base is shown in Figure 13. The RCA 8571 has an S-4 photocathode surface and the spectral response curve for this surface is shown in Figure 14.

Figure 15 shows a block diagram of the electronics used to process the signals from the two photomultiplier tubes. The EMI 6256S was operated in a photon-counting mode in which small amplitude voltage pulses are generated across a load resistor (at the output) of the photomultiplier when photons impinge upon the photocathode. These output pulses were then passed through a preamplifier whose purpose it was to shape the pulses for entry into an amplifier. The amplifier both amplifies and reshapes the pulses for the next stage of processing. The pulses from the amplifier pass through a single channel analyzer whose discriminator is set to pass pulses of a certain amplitude only and reject all others.
Base diagram of the RCA 8571 photomultiplier

Keithley picoammeter
Signal out (DC)
Anode

Cathode

H.V. (-1000V)

FIGURE 13
SPECTRAL RESPONSE OF THE RCA 8571 PHOTOMULTIPLIER
(S-4 photocathode surface)

![Graph showing spectral response of the RCA 8571 photomultiplier. The x-axis represents wavelength in Angstroms (2000 to 8000), and the y-axis represents quantum efficiency in percent (0.1 to 40). The curve peaks around 6000 Angstroms, indicating the maximum quantum efficiency.]
Block diagram of electronics

FIGURE 15
In this way small amplitude pulses from noise events that were generated further down the dynode chain than the photocathode can be eliminated thus cutting down the counting rate without affecting the signal counts appreciably. This discrimination process improves the signal-to-noise ratio of the overall counting system. An integral spectrum (Figure 16) of the background count rate of the EMI 6256S photomultiplier was first taken in order to find the optimum setting for the discrimination level. The integral spectrum was relatively flat indicating that it was not really important where the discriminator bias was set for this tube. It was arbitrarily set at 100 (arb. units). A second function of the single channel analyzer was to shape the pulses passing the discriminator bias into pulses compatible with the input requirements of the scalers. The shaped pulses leaving the single-channel analyzer passed into a scaler which counted the number of pulses from the 6256S photomultiplier in a preset integration time. This time interval was present on an electronic timer which sent a gating pulse to the scalers stopping them when the required time interval was finished, thus eliminating human "time-lag" errors that would be generated in starting and stopping stop-watches.

The RCA 8571 beam monitor normalization photomultiplier was operated in a direct current measuring mode. Thus, small currents of the order of nanoamperes at the output circuit of
Integral spectrum of the EMI 6256S photomultiplier
the photomultiplier were passed into a Keithley (Model 610BR) picoammeter which indicated the instantaneous current from the photomultiplier. A voltage proportional to this current was generated at one output of the picoammeter and this signal was passed into a voltage-to-frequency converter circuit (See the General Electric Transistor Manual, General Electric Company, Syracuse, New York, 1964.). The output pulses from the circuit were counted on a scaler for a preset time interval.

3.10 The Data Taking Format

The counting rates from both photomultiplier tubes in this experiment were determined to be the sum of three distinct contributions.

(i) the background contribution of the photomultipliers, the major component of which came from the dark current noise pulses of the tubes

(ii) the residual gas background radiation contribution caused by collisions between the beam particles and the residual gas atoms and molecules in the foil chamber (primarily oxygen and nitrogen). This contribution effected the RCA 8571 monitor photomultiplier at all wavelengths but did not effect the EMI 6256S monochrometer photomultiplier as much, if at all, at some wavelengths since this part of the detection system has a much smaller bandpass associated
with it. The background gas contribution was, as expected, a function of residual gas pressure and incident ion beam current and possibly incident ion energy, since this effected the charge state composition of the excited beam.

(iii) the contribution from the excited beam-foil source caused by spontaneous electric dipole decays of the atom and atomic ions of the accelerated element. This is the true signal that must be extracted.

The data taking format was designed to extract the contribution (iii) from the sum of contributions (i), (ii), and (iii). A separate experiment was run in order to find the contribution (ii) at all wavelengths studied. In this experiment a foil holder (with no carbon foil) was placed in the foil-chamber. The only radiation registered by the detection photomultipliers now originated by decay of residual gas atoms that had been excited by inelastic collisions with the incident ion beam particles. It was found that this contribution to the EMI 6256S photomultiplier varied slightly as a function of wavelength but was nearly always of the order of from 0 - 100 counts/min at the residual gas pressures maintained during a normal experimental run (i.e., \( p \sim 5 \times 10^{-5} \) torr). Once the contribution (ii) had been found for a particular wavelength, entrance and exit slit widths, residual gas pressure and incident ion beam current, it was then necessary to determine only the tube background counting rate contributions, (i).
At any given foil position the "signals" (i.e., contributions (i), (ii), (iii)) from both tubes were taken for four consecutive separate minute counts. A pneumatic gate valve, situated in the 30° port beam tube, was then closed for another minute in order to determine background counting rates of the tubes (contribution (i)) with no source of radiation present. The foil was then moved to a new position, the pneumatic gate valve reopened, and the data taking procedure repeated. The average of the four "signal" rates and of the tube background rates, both before and after the four "signal" determinations, were calculated. The true beam-foil signals (contribution (iii)) were calculated using the known average background rates (contribution (i)) and the experimentally determined residual gas contributions (contribution (ii)). The true EMI 6256S monochrometer photomultiplier signals were normalized to the true RCA 8571 beam monitor photomultiplier signals to obtain the final true normalized EMI 6256S beam-foil source counting rate. The whole process was repeated at all foil positions. The averaging, subtracting and normalizing processes were all made using a computer program called CONVERT (See Appendix D). The actual procedure can be summarized by the following equations:

\[ N_{\text{norm, em}}^i = (N_{\text{em}}^i - n_{\text{em}}^i) \times \frac{k}{(N_{\text{aca}}^i - n_{\text{aca}}^i)} \]

\[ N_{\text{ave, norm, em}} = \frac{\sum_{i=1}^{p} N_{\text{norm, em}}^i}{p} \]
\[ N'_{\text{AVG, EMI}} = N_{\text{AVG, EMI}} - n_{\text{EMI, BKG GAS}} \times k/k'. \]

(Corrected for bkg gas contribution.)

where \( N^1_{\text{EMI}} \) and \( n^1_{\text{EMI}} \) are the "signal" (i.e., true signal plus background) and background counting rates for the EMI photomultiplier for each individual data taking 'run' at a particular foil position. There were usually four such 'runs' to be averaged at each foil position. Similar definitions apply to \( N^1_{\text{RCA}} \) and \( n^1_{\text{RCA}} \). The true foil signal, i.e., \( (N^1_{\text{EMI}} - n^1_{\text{EMI}}) \) for each run in a particular foil position set, \( K \) is the normalization constant chosen arbitrarily (usually taken to be close in value to the RCA "signal" counting rate). The averaging process is done by summing the normalized foil data at each foil position and dividing by the number of runs (\( P \)) in that particular foil position set. The background gas contribution to the EMI photomultiplier, i.e., \( n_{\text{EMI, BKG GAS}} \) is normalized and subtracted from the average EMI true signal rate to find the true rate corrected for background gas contributions. This final data set can then be plotted as a function of distance (or time) to form the intensity decay curve, and it also forms the input for the curve fitting program FRANTIC.
3.11 The Analysis of Raw Data

The output of the CONVERT program, i.e., the normalized monochrometer photomultiplier counting rate (for a particular transition present in the beam-foil source) as a function of foil position is used as the input to another computer program called FRANTIC (Rogers, 1962, see Appendix C.) whose function it is to fit (by the method of iterative least squares) this normalized intensity decay data to a sum of exponential terms and from the best fit calculate the initial intensities and decay constants of the exponential terms involved. In the transitions studied in this experiment the decay data was best fit by either the sum of two exponentials, one exponential plus a constant or simply one exponential. The program generated a goodness-of-fit parameter so that one could determine statistically the best fit to the data. Appendix C discusses more fully the computer program FRANTIC.
SECTION IV

RESULTS

4.1 The Emission Spectra of Beam-Foil Excited Carbon

Figures 17, 18, 19, and 20 show typical carbon emission line spectral scans made on the beam-foil source. In all cases the incident ion was $^{12}\text{C}^+$ and the carbon foil thickness was 10 µg/cm². The resolution was such that blends of lines originating from transitions between levels of the same terms were obtained. Such a blend is called a multiplet. The emission lines were all due to allowed electric dipole multiplet transitions in which only one "jumping" electron was involved. The low level light output from the source, which was due to the low beam currents (0.05 - 0.10 µa) used, combined with the relatively slow optical detection system (f/34.0 grating masked, f/11.4 grating unmasked) allowed only the strongest multiplets to be detected efficiently. The spectral resolution was relatively poor (due also to the low light output from the source and the optically slow detection system) and partial blending of different multiplets is apparent at some wavelengths.

In order to identify the emission lines or multiplets in the spectral scan, the monochromator wavelength scan register was calibrated in terms of known lines from both a helium and hydrogen beam-foil source and from a helium discharge tube. A good check was used to measure the distances of the lines from one well known line in the spectrum. These
Beam-foil excited U.V. spectrum of carbon

Incident ion: $^{12}$C$^+$

Ion energy: 356 KeV

Foil: 10 $\mu$g/cm$^2$ carbon

FIGURE 17
Beam-foil excited UV spectrum of carbon

Incident ion: $^{12}\text{C}^+$
Ion energy: 275 KeV
Foil: 10μg/cm$^2$ carbon
Beam-foil excited visible spectrum of carbon

Incident ion: $^{12}\text{C}^+$
Ion energy: 337 KeV
Foil: 10$\mu$g/cm$^2$ carbon

FIGURE 19
Beam-foil excited visible spectrum of carbon

Incident ion: $^{12}$C$^+$
Ion energy: 337 KeV
Foil: 10 μg/cm$^2$ carbon

Wavelength (Å)
wavelengths were then compared with both tabulated wavelengths (Wiese, 1966) and the wavelengths of lines seen by other authors (Poulizac, 1969), (Bergström, 1969) using a beam-foil carbon source. A computer program was also written in order to find all the possible allowed electric dipole one- and two-electron transitions for CI, CII, and CIII from all the possible energy levels tabulated by Moore (1949). In this manner all the observed lines were identified and assigned upper and lower term values as shown in Tables I and II.

The spectral range covered, \( \lambda 750 \, \text{Å} - \lambda 5500 \, \text{Å} \), is conveniently separated into the ranges \( \lambda 750 \, \text{Å} - \lambda 3000 \, \text{Å} \) and \( \lambda 3000 \, \text{Å} - \lambda 5500 \, \text{Å} \) and each will now be treated separately.

A. The UV Emission Spectrum of Carbon (\( \lambda 750 \, \text{Å} - \lambda 3000 \, \text{Å} \))

Figures 17 and 18 show the beam-foil excited UV multiplet spectrum of carbon. The incident ion energy in Figure 17 was 356 keV and in Figure 18 was 275 keV. The spectral-slit widths used in both the scans was approximately 17 Å, but the experimentally observed resolution is seen to be approximately 30 Å (fwhm). Some portions of this value must be attributed to the finite time constant of the rate-meter-chart recorder combination that was used to record the spectra. Most UV runs were made with an unmasked grating in order to increase the optical speed of the detection system with a resulting loss in resolution. A sodium salicylate wavelength conversion window was also used in all UV runs. It is interesting to notice the change in the ratio of the
### TABLE I

BEAM-FOIL EXCITED UV EMISSION MULTIPLETS OF CARBON

\[ E = 356 \text{ keV}; \text{ }^{12}\text{C}^+ \text{ IONS} \]

<table>
<thead>
<tr>
<th>WAVELENGTH (Å)</th>
<th>SPECIE</th>
<th>MULTIPLET ASSIGNMENT</th>
<th>LOWER TERM</th>
<th>UPPER TERM</th>
<th>REF.</th>
<th>M.S.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>904.1</td>
<td>CII</td>
<td>(2s) (2p) (P)</td>
<td>(2) (2) (0)</td>
<td>(2s2p) (P)</td>
<td>3a,3b</td>
<td>M</td>
</tr>
<tr>
<td>977.0</td>
<td>CIIC</td>
<td>(2s) (S)</td>
<td>(2) (1)</td>
<td>(2s(\ S)2p) (P)</td>
<td>1a,1b</td>
<td>M</td>
</tr>
<tr>
<td>1010.2</td>
<td>CII</td>
<td>(2s2p) (P)</td>
<td>(2) (4) (0)</td>
<td>(2p) (S)</td>
<td>4a,7b</td>
<td>W</td>
</tr>
<tr>
<td>1036.8</td>
<td>CII</td>
<td>(2s) (2p) (P)</td>
<td>(2) (2) (0)</td>
<td>(2s2p) (S)</td>
<td>2a,2b</td>
<td>W</td>
</tr>
<tr>
<td>1175.7</td>
<td>CIIC</td>
<td>(2s2p) (P)</td>
<td>(3) (0)</td>
<td>(2p) (P)</td>
<td>4a,4b</td>
<td>M</td>
</tr>
<tr>
<td>1247.4</td>
<td>CIIC</td>
<td>(2s2p) (P)</td>
<td>(1) (0)</td>
<td>(2p) (S)</td>
<td>6a,9b</td>
<td>W</td>
</tr>
<tr>
<td>1335.3</td>
<td>CII</td>
<td>(2s) (2p) (P)</td>
<td>(2) (2) (0)</td>
<td>(2s2p) (D)</td>
<td>1a,1b</td>
<td>S</td>
</tr>
<tr>
<td>1561.0</td>
<td>CI</td>
<td>(2s) (2p) (P)</td>
<td>(2) (3) (3) (0)</td>
<td>(2s2p) (D)</td>
<td>2a,3b</td>
<td>M</td>
</tr>
</tbody>
</table>

\(\text{a(Wiese, 1966); b(Moore, 1959)}\)

\(W = \text{Weak, M = Medium, S = Strong}\)

\(*\text{M.S. indicates multiplet strength (arb units)}*\)
intensity of the λ1561 Å CI line to the λ1176 Å CIII line when the incident ion energy changes from 275 keV to only 356 keV. The cross section for producing neutral emitters is larger at the lower energies than at the higher energy; whereas the reverse is true for the production of the doubly ionized specie. This fact was used as a test of identification in this particular case. Table I lists the UV multiplets, in increasing order of wavelength, that were emitted at incident ion energies of 356 keV, along with the upper and lower electronic term assignments for the transitions leading to the observed spectral multiplets. The references (a,b) included in Table I are the multiplet numbers as indicated in the tabulations of Wiese (1966) and Moore (1959), respectively. The multiplet strengths (intensities) are arbitrarily classified as weak, medium, and strong, and can be compared with multiplets close in wavelength but not over the whole wavelength range since the detection efficiency is, of course, a function of wavelength.

The mean lives of the upper terms leading to the following spectral multiplets were measured: λ904 Å CII, λ1176 Å CII, λ1335 Å CII, and λ1561 Å CI. Other UV emission multiplets, not shown in the figures, were studied at λ2297 Å CIII, and λ2511 Å CII. A parasitic multiplet is seen at approximately λ1216 Å. This may be due to spectral emission from the residual background chamber gas excited by the incident ions, probably the λ1217 Å OI multiplet, or from the λ1216 Å HI (Lyman α line) of hydrogen which is present in the beam due to probable outgassing of hydrocarbons in the foil when
TABLE II

BEAM-FOIL EXCITED VISIBLE EMISSION MULTIPLETS OF CARBON

$E = 337 \text{ keV}; \quad ^{12}\text{C}^+ \text{IONS}$

<table>
<thead>
<tr>
<th>WAVELENGTH (Å)</th>
<th>SPECIE</th>
<th>LOWER TERM</th>
<th>UPPER TERM</th>
<th>REF.</th>
<th>M.S.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>2296.9</td>
<td>CIII</td>
<td>2s2p$^1P^0$</td>
<td>2p$^{2}D$</td>
<td>5a,8b</td>
<td>W</td>
</tr>
<tr>
<td>2511.0</td>
<td>CII</td>
<td>2s2p$^{2}P$</td>
<td>2p$^{3}D^0$</td>
<td>7a,14b</td>
<td>W</td>
</tr>
<tr>
<td>3361.3</td>
<td>CII</td>
<td>2s$^23d^2D$</td>
<td>2s$^25p^2P^0$</td>
<td>27a,7b</td>
<td>W</td>
</tr>
<tr>
<td>3589.3</td>
<td>CII</td>
<td>2s2p3p$^4D$</td>
<td>2s2p4s$^4P^0$</td>
<td>18a,23b</td>
<td>W</td>
</tr>
<tr>
<td>3876.7</td>
<td>CII</td>
<td>2s2p3d$^4P^0$</td>
<td>2s2p4f$^4G$</td>
<td>22a,33b</td>
<td>S</td>
</tr>
<tr>
<td>3920.2</td>
<td>CII</td>
<td>2s$^23p^2P^0$</td>
<td>2s$^24s^2S$</td>
<td>17a,4b</td>
<td>M</td>
</tr>
<tr>
<td>4075.7</td>
<td>CII</td>
<td>2s2p3d$^4D^0$</td>
<td>2s2p4f$^4F$</td>
<td>23a,36b</td>
<td>S</td>
</tr>
<tr>
<td>4186.0</td>
<td>CIII</td>
<td>2s4f$^4F^0$</td>
<td>2s5g$^1G$</td>
<td>not listed</td>
<td>W</td>
</tr>
<tr>
<td>4267.2</td>
<td>CII</td>
<td>2s$^23d^2D$</td>
<td>2s$^24f^2F^0$</td>
<td>21a,6b</td>
<td>S</td>
</tr>
<tr>
<td>4314.0</td>
<td>CII</td>
<td>2s2p3p$^4P$</td>
<td>2s2p4s$^4P^0$</td>
<td>not listed</td>
<td>W</td>
</tr>
<tr>
<td>4374.0</td>
<td>CII</td>
<td>2s2p3d$^4P^0$</td>
<td>2s2p4f$^4D$</td>
<td>25a,45b</td>
<td>W</td>
</tr>
<tr>
<td>4411.4</td>
<td>CII</td>
<td>2s2p3d$^2D^0$</td>
<td>2s2p4f$^2F$</td>
<td>24a,39b</td>
<td>M</td>
</tr>
<tr>
<td>4619.1</td>
<td>CII</td>
<td>2s2p3d$^2F^0$</td>
<td>2s2p4f$^2G$</td>
<td>26a,50b</td>
<td>W</td>
</tr>
<tr>
<td>4648.8</td>
<td>CIII</td>
<td>2s3s$^3S$</td>
<td>2s3p$^3P^0$</td>
<td>12a,1b</td>
<td>S</td>
</tr>
<tr>
<td>4662.7</td>
<td>CIII</td>
<td>2p3s$^3P^0$</td>
<td>2p3p$^3P$</td>
<td>15a,5b</td>
<td>W</td>
</tr>
<tr>
<td>5041.7</td>
<td>CI</td>
<td>2p3s$^3P^0$</td>
<td>2p4p$^1D$</td>
<td>32a,4b</td>
<td>W</td>
</tr>
<tr>
<td>5052.1</td>
<td>CI</td>
<td>2p3s$^1P^0$</td>
<td>2p4p$^1D$</td>
<td>36a,12b</td>
<td>W</td>
</tr>
<tr>
<td>5141.8</td>
<td>CII</td>
<td>2s2p3s$^4P^0$</td>
<td>2s2p3p$^4D$</td>
<td>12a,14b</td>
<td>M</td>
</tr>
<tr>
<td>5263.1</td>
<td>CIII</td>
<td>2p3s$^3P^0$</td>
<td>2p3p$^3D$</td>
<td>14a,4b</td>
<td>W</td>
</tr>
</tbody>
</table>

a(\text{Wiese, 1966}); \quad b(\text{Moore, 1969})

W = Weak, M = Medium, S = Strong

*M.S. indicates multiplet strength (arb units)
the ion beam traverses it. A relatively strong oxygen multiplet has also been seen by this investigator at this wavelength in a beam-foil scan of oxygen in a separate experiment.

B. The Visible Emission Spectrum of Carbon (\(\lambda 3000 \text{ Å} - \lambda 5500 \text{ Å}\))

Figures 19 and 20 show the visible multiplet spectrum that was excited in this experiment. The incident ion energy in both cases was 337 keV. The spectral-slit widths used in Figure 19 and 20 were approximately 17 Å and 12 Å, respectively. The experimentally observed resolution is seen to be approximately 20 Å (fwhm) in the worst case, (i.e., Figure 19). All visible runs were made with a masked grating in order to improve the spectral resolution at the expense of optical detection speed since the visible spectrum is more abundant in lines.

Table II lists, in increasing order of wavelength, the visible multiplets that were emitted, using incident ion energies of 337 keV, along with the upper and lower electronic term assignments for the transitions producing the multiplets. The references (a,b) included in the table are the multiplet numbers as indicated in the tabulations of Wiese (1966) and Moore (1959), respectively. The multiplet strengths (intensities) are again arbitrarily classified as weak, medium, and strong.

The radiative mean lives of the upper terms leading to the following spectral multiplets were measured: \(\lambda 3360 \text{ Å} \) CII, \(\lambda 3877 \text{ Å} \) CII, \(\lambda 4076 \text{ Å} \) CII, \(\lambda 4267 \text{ Å} \) CII, \(\lambda 4649 \text{ Å} \) CIII, \(\lambda 5142 \text{ Å} \) CII. Several parasitic multiplets are seen in the
in the spectrum due primarily to spectral emission from residual background chamber gas atoms or molecules excited by collisions with ions of the incident beam. A $\lambda 4861$ Å line, corresponding in wavelength to the $H_\beta$ HI line, is seen in the spectrum and can probably be attributed to the outgassing process from the foil that has already been described.

4.2 The Radiative Mean Lives

Table III lists the radiative mean lives that were measured in the experiment. The table includes the upper excited electronic term of the transition studied as well as the wavelength of the emitted multiplet radiation that was monitored. An asterisk in the mean life results column of the table indicates whether the semi-logarithmic intensity decay curve was linear (L) (indicating the probable absence of cascading and/or blending) or non-linear (NL) (indicating possible cascading and/or blending). The final tabulated result is then an average of the individual runs. The uncertainty quoted on the results is due to a combination of the curve fitting error that signifies the statistical spread in the results and the uncertainty in velocity of the emitting specie. Another column shows other experimental mean-life results for the same upper term. References a(Heroux, 1969), c(Poulizac, 1969), d(Bergström, 1969) and e(Curnutte, 1968) are beam-foil measurements at approximate incident ion energies of 1 MeV, 1 MeV, 76 keV and 1 MeV, respectively. Reference b(Lawrence, 1966) is a pulsed-electron gun phase shift
<table>
<thead>
<tr>
<th>SPECIE</th>
<th>EXCITED UPPER TERM</th>
<th>WAVELENGTH (A)</th>
<th>RADIATIVE MEAN LIFE (nsec)</th>
<th>OTHER EXP. RESULTS</th>
<th>THEORY†</th>
</tr>
</thead>
<tbody>
<tr>
<td>CII 2s2p^2P</td>
<td>904.1</td>
<td>0.34 ± 0.04*</td>
<td>0.32 ± 0.03^a</td>
<td>0.24(E)</td>
<td></td>
</tr>
<tr>
<td>CIII 2p^3P</td>
<td>1175.7</td>
<td>0.79 ± 0.02*</td>
<td>0.80 ± 0.04^a</td>
<td>0.77(D)</td>
<td></td>
</tr>
<tr>
<td>CII 2s2p^2D</td>
<td>1335.3</td>
<td>3.7 ± 0.1</td>
<td>3.9 ± 0.4^a</td>
<td>1.7 (E)</td>
<td></td>
</tr>
<tr>
<td>CI 2s2p^3D^0</td>
<td>1561.0</td>
<td>7.3 ± 0.2</td>
<td>8.0 ± 0.8^b</td>
<td>5.5 (D)</td>
<td></td>
</tr>
<tr>
<td>CIII 2p^2D</td>
<td>2296.7</td>
<td>6.9 ± 0.2</td>
<td>7.1^c, 9.3 ± 0.5^d, 7.4 ± 0.6^e</td>
<td>2.8 (D)</td>
<td></td>
</tr>
<tr>
<td>CII 2p^3D^0</td>
<td>2511.0</td>
<td>2.0 ± 0.1</td>
<td>1.8^c, 2.1 ± 0.2^d</td>
<td>1.0 (E)</td>
<td></td>
</tr>
<tr>
<td>CII 2s^5p^2P^0</td>
<td>3361.3</td>
<td>5.1 ± 0.3*</td>
<td>5.2^c</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>CII 2s2p4f^4G</td>
<td>3876.7</td>
<td>3.2 ± 0.1*</td>
<td>3.5^c, 3.6 ± 0.2^d</td>
<td>3.8 (C)</td>
<td></td>
</tr>
<tr>
<td>CII 2s2p4f^4F</td>
<td>4075.7</td>
<td>2.9 ± 0.2*</td>
<td>3.5^c, 3.3 ± 0.2^d</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>CII 2s^24f^2F^0</td>
<td>4267.2</td>
<td>3.7 ± 0.3*</td>
<td>3.8^c, 3.6 ± 0.2^d</td>
<td>4.1 (C)</td>
<td></td>
</tr>
<tr>
<td>CIII 2s3p^3P^0</td>
<td>4648.8</td>
<td>12.5 ± 0.4</td>
<td>12.9^c, 14.8 ± 0.9^d</td>
<td>12.8 (C)</td>
<td></td>
</tr>
<tr>
<td>CII 2s2p3p^4D</td>
<td>5141.8</td>
<td>10.7 ± 0.3</td>
<td>11.4^c, 11.8 ± 0.6^d</td>
<td>11.6 (C)</td>
<td></td>
</tr>
</tbody>
</table>

*Indicates non-linear decay curve

^a (Heroux, 1969), ^b (Lawrence, 1966), ^c (Poulizac, 1969), ^d (Bergström, 1969), ^e (Curnutte, 1968)

†Theory derived by inverting theoretical transition probabilities of Table IV. Uncertainty parameter in parentheses.
method result. No errors were quoted on the results of reference c. It can be seen that relatively good agreement exists especially with the results of references a and c. The final column of Table III shows theoretical mean life results derived from taking the reciprocal of the NBS (Wiese, 1966) tabulated transition probabilities wherever applicable; i.e., where only one dominant decay channel is present. An exception to this case is for the $\lambda 1561 \AA$ result which is obtained from the calculation of Weiss (1967).

The following paragraphs individually treat each of the multiplets experimentally studied. As an aid to the repopulation and depopulation analysis, two computer programs were written in order to determine the possible cascading and decay channels. The program generated all the allowed one- and two-electron transitions (up to $n = 8$ with the selection rule $\Delta n = 0 \pm 1, \pm 2, \pm 3$) between the energy levels of CI, CII, and CIII as given by Moore (1949). These programs will be referred to, in what follows, as REPOP and DEPOP, respectively. Each paragraph contains the semi-logarithmic intensity decay curve figures for all the experimental runs at the particular wavelength studied. The results of the individual runs are also listed in the "rounded-off" form with the raw results from FRANTIC given in parentheses. The errors quoted on these results are just the statistical curve fitting errors. The goodness-of-fit parameter (G.O.F.) that is generated by FRANTIC, is also given for each individual result.
The errors quoted on the final averaged results are a combination of the statistical curve-fitting errors and the error due to the uncertainty in the specie velocity after the foil. It was thought to be unmeaningful to attempt a full statistical error analysis on the individual results for the cases where only a very small number of measurements were made at the same wavelength. It was decided, instead, to adopt (in most cases) as a measure of the statistical spread in the individual results, a deviation equal to the full range between the results divided, by the number of measurements made. This estimate was cited by Evans (1955) to apply reasonably well to data in which $n < 10$. In a few cases it was found that one of the individual values had a fractional curve-fitting error that was greater in magnitude than the aforementioned "range" type error estimate. If this was found to be the case, the greater of the two cited errors was used to signify the statistical spread in the results. In either case the final uncertainty on the averaged results was derived by summing, in quadrature, the fractional deviations due to the statistical spread and the velocity uncertainty. This velocity uncertainty was, in turn, found to be the combination of several smaller systematic errors. These include the following:

1) An uncertainty in the final beam energy due to the energy lost by the incident ion in traversing the foil which, in turn, includes uncertainties in the specific energy loss value, $dE/dx$ ($\sim 10\%$), and in the foil thickness ($\sim 40\%$).
ii) An uncertainty in the final beam energy due to the energy profile of the ion beam leaving the source bottle. This was estimated to be approximately 1.5% at the accelerating energies used in the experiment.

iii) An uncertainty in the energy calibration point of the generating voltmeter (and therefore in the energy of the accelerator itself). This value was estimated to be approximately 2%.

An appropriate combination of the above uncertainties leads to an overall value of \( \sim 5.4\% \) for the uncertainty in the final beam energy implying an uncertainty of \( \sim 2.7\% \) in the final velocity of the emitting specie. The uncertainty in the velocity will obviously be reflected as a proportional fractional error in the time coordinate, and thus also in the radiative mean life result which is the slope of the semi-logarithmic intensity (I v.t) decay curve. An actual example calculation of the velocity uncertainty can be found in Appendix B of this thesis.

4.3 Atomic Transition Probabilities

Atomic transition probabilities for allowed one-electron transitions between two terms, i.e., multiplets, can be calculated directly from the mean life results in some special cases. The total transition probability is equal to the sum of the transition probabilities for all the possible decay transitions out of the excited term. This total transition probability is also equal to the reciprocal
of the radiative mean life of the excited term that is decay­ing; i.e., \( \sum_{j<i} A_{ij} = (\tau_i)^{-1} \). Thus, a mean life measurement leads directly to an exact or approximate transition prob­ability value for a particular transition if all the other transition probabilities are known exactly, or approximately either experimentally or theoretically. The situation be­comes particularly simple for the case when only one tran­sition out of the term is possible or if one transition is much stronger than others. This situation prevails for nine of the transitions studied. To make this point more lucid, one can define a quantity called the branching ratio \( F(i|k) \) for a particular transition \((i \rightarrow k)\) as:

\[
F(i|k) = \frac{A_{ik}}{\sum_{j \neq k} A_{ij}}
\]

Thus, transitions for which the branching ratio is unity allow direct conversion of mean lives to transition prob­abilities. This is also true if the branching ratio is close to unity; i.e., if the particular transition is dominant over other possible transitions out of the term. It has been assumed in the analysis of the results that some transitions can be neglected when the following conditions apply:

1) If a transition which either repopulates or depop­ulates a term whose mean life is under study and is within the observable spectral range of the apparatus (\( \sim 750 - 5500 \) Å) but is not observed, then the corresponding transition prob­ability must either be so small or the upper term so weakly populated in the excitation process that it may be neglected.
11) If the pertinent repopulating or depopulating transitions lie in the infra-red, then since $A_{ij}$ is proportional to $(v_{ij})^3$ these transitions may be neglected compared to visible and ultraviolet transitions to and from the same term.

The computer program DEPOP lists all the possible one- and two-electron transitions out of a particular excited term. DEPOP proved to be an invaluable aid in the calculation of atomic transition probabilities. When only one decay channel is allowable the calculated transition probability will carry the same fractional uncertainty as the radiative mean life result. If more than one decay channel is open and the values of other listed transition probabilities are used in the calculation of a particular transition probability, then the uncertainty quoted on this value will depend upon the estimated uncertainties on the other transition probabilities used. The transition probability values are derived individually in the separate paragraphs on each tabulated transition, and the final results are tabulated in Table IV. Table IV lists all of the transition probabilities that are obtainable from the mean life measurements. The estimated accuracy parameter that is used in the table is the same as that used in the NBS tabulation (Wiese, 1966); i.e., AA(< 1%), A(< 3%), B(< 10%), C(< 25%), D(< 50%), and E(> 50%) with the addition of BB(< 5%). The present results are also compared with theoretical results for the same multiplets which are tabulated in the last column of Table IV. The theoretical values as derived from calculations...
<table>
<thead>
<tr>
<th>SPECIE</th>
<th>LOWER TERM</th>
<th>UPPER TERM</th>
<th>WAVELENGTH, (Å)</th>
<th>TRANSITION PROBABILITY (x 10^8 sec⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>PRESENT RESULT</td>
<td>AP⁺</td>
</tr>
<tr>
<td>CII</td>
<td>2s 2p 2p³ P₀</td>
<td>2s2p 2² P</td>
<td>904.1</td>
<td>29.4</td>
</tr>
<tr>
<td>CIII</td>
<td>2s2p 3² P³</td>
<td>2p 2³ P</td>
<td>1175.7</td>
<td>12.7</td>
</tr>
<tr>
<td>CII</td>
<td>2s 2p 2p² P</td>
<td>2s2p 2² D</td>
<td>1335.3</td>
<td>2.7</td>
</tr>
<tr>
<td>CI</td>
<td>2s² 2p² 3³ P</td>
<td>2s2p² 3³ P₀</td>
<td>1561.0</td>
<td>1.4</td>
</tr>
<tr>
<td>CIII</td>
<td>2s2p 1² P</td>
<td>2p 2¹ D</td>
<td>2296.9</td>
<td>1.4</td>
</tr>
<tr>
<td>CII</td>
<td>2s2p3d 4³ P</td>
<td>2s2p 4f 4² G</td>
<td>3876.7</td>
<td>3.13</td>
</tr>
<tr>
<td>CII</td>
<td>2s 3d 1² D</td>
<td>2s³ 4f ² P₀</td>
<td>4267.2</td>
<td>2.70</td>
</tr>
<tr>
<td>CI</td>
<td>2s3s 3³ S</td>
<td>2s3p ³ P₀</td>
<td>4648.8</td>
<td>0.80</td>
</tr>
<tr>
<td>CII</td>
<td>2s2p³ 4 P</td>
<td>2s2p³ 4² D</td>
<td>5141.8</td>
<td>0.93</td>
</tr>
</tbody>
</table>

*Accuracy parameters (AP): Estimated Uncertainty A(< 3%), BB(< 5%), B(< 10%), C(< 25%), D(< 50%), E(> 50%).

†Theoretical values taken from NBS (Wiese, 1966) tables with the exception of result for 11561 Å CI (Weiss, 1967).
(either Hartree-Fock or Coulomb approximation). It can be seen that the present results are of a higher precision than the best calculated results, but overall agreement is noted in most cases where the Coulomb approximation has been used. Wherever configuration interaction is involved in the transition poor agreement exists since the theoretical values for such transitions are still relatively inaccurate. The present experimental results should be a far better indicator of the true transition probability for such configuration interaction prone transitions.

4.4 Tabulated Transitions - CI

\[ \lambda_{1561.0} \, \text{Å} \, \text{CI (2s}\,2p\,^2\,P - 2s2p\,^3\,D) \]

The multiplet observed was a spectral blend of the lines produced in the transition \( 2s\,2p\,^2\,P\,^J - 2s2p\,^3\,D\,^J \), with \( J-J' \) of 2-3, 1-2, 0-1, 2-2, 1-1, 2-1. A blend of lines originating on different levels of the same term will not affect the shape of the decay curve since each level of a term has the same radiative mean life. The observed spectral blend occurred since the experimental apparatus was unable to resolve lines within a multiplet.

Figures 21 through 25 show the experimental decay curves for the \( \lambda_{1561.0} \, \text{Å} \) radiation that is emitted when the excited \( 2s2p\,^3\,D \) term decays to the \( 2s\,2p\,^2\,P \) term. A linear semi-logarithmic intensity decay curve is obtained in all cases indicating that both cascading and/or blending of other multiplets are absent. The results of the individual runs are:
INTENSITY DECAY CURVE FOR λ1561.0 Å RADIATION
(2s^2 2p^2 3P - 2s^2 2p^3 3D^0) CI

DATE = 11.22.68.
RUN = 1

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 21
The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 22
The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 23
INTENSITY DECAY CURVE FOR $\lambda 1561.0$ Å RADIATION
$(2s^22p^23p - 2s2p^33D^0)\text{Cl}$

$\tau(2s2p^33D^0) = 7.214 \pm 0.198 \text{ nsec.}$

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 24
INTENSITY DECAY CURVE FOR λ1561.0 Å RADIATION
(2s²²p²³P→2s²p³³D) CI

DATE= 112569.
RUN = 5

\[ \tau(2s²p³³D) = 7.371 \pm 0.060 \text{ nsec.} \]

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 25
The errors quoted on the above results are due to the statistical curve fitting error only. An average of the five individual runs was made and the final result is quoted as

$$\tau(2s2p^{33}D^0) = 7.3 \pm 0.2 \text{nsec} \ (2.7\%)$$

The error on this result is due to a combination of the statistical spread error and the uncertainty in the velocity of the emitting specie.

The computer program REPOP indicates that there exist no possible one-electron cascading channels into the 2s2p^{33}D^0 term. Blending of several other multiplets in CI, CII and CIII is possible but since the decay curve is always linear, it appears that there exists no appreciable blending (i.e. either the upper terms of the blending multiplet are weakly populated and/or the particular multiplet transition probability is small).

The program DEPOP lists only one possible one-electron transition decay channel out of the 2s2p^{33}D^0 term, i.e.,

$$2s^22p^2^3P \rightarrow 2s2p^{33}D \ (\lambda1561 \ \AA).$$

Thus, the transition
probability for this resonance multiplet is easily obtained, i.e.:

\[ A(2s^2 2p^2 3\mathrm{P} - 2s2p^3 3\mathrm{D}^0) = \frac{1}{\tau(2s2p^3 3\mathrm{D}^0)} \]

\[ = (1.37 \pm 0.04) \times 10^8 \text{sec}^{-1} \]

This value is assigned an accuracy parameter of A using the NBS (Wiese, 1966) notation. The latter tables list a value for this multiplet of \(1.5 \times 10^8 \text{sec}^{-1} \) [D]. This is the experimental emission-arc value of Boldt (1963). The present result is of much higher precision than the latter values. The transition involves configuration interaction and theoretically calculated values (such as Weiss, 1967) are still considerably accurate; nevertheless, a calculation involving the superposition of several configurations in both the initial and final electronic wave functions has been made by Weiss (1967) (accuracy parameter, D). This result, along with an experimental phase-shift result of Lawrence et al (1966) and the Boldt (1963) measurement is shown in Figure 26. In this figure the ordinate is plotted as an f-value (absorption oscillator strength) and the abscissa is the atomic number or charge on the nucleus. This plot is of interest when one considers the variation of f-value for the same transition within an isoelectronic sequence. The absorption f-value for a transition \((j \rightarrow i)\) is related to the transition probability for the emission transition \((i \rightarrow j)\) by

\[ f_{ij} = 1.499 \frac{g_i}{g_j} (\lambda_{ij})^2 A_{ij} \]
EXPERIMENTAL AND THEORETICAL \( f \)-VALUES WITHIN THE CARBON ISOELECTRONIC SEQUENCE

FIGURE 26
where $\lambda_{ij}$ is the wavelength of the transition in cm, and $g_i$, $g_j$ are the statistical weight factors for the terms; i.e., $g_i = (2S_i + 1)(2L_i + 1)$. The results for this same transition in isoelectronic NII and OII are plotted in the figure. These values are also beam-foil measurements, (Heroux, 1967) and (Bickel, 1967) respectively. In addition the plot shows theoretical $f$-values for this particular transition in both NII and OIII with a very small amount of configurational superposition included in the calculation (Bolotin, 1956) and with more recent theoretical values (Westhaus, 1969) involving a more sophisticated attempt to allow for the configurational interaction.

### 4.5 Tabulated Transition - CII

$\lambda_{904.1} \quad \text{Å} \quad \text{CII} \quad (2s^22p^2P^0 - 2s2p^2P)$

The multiplet observed was a blend of the spectral lines produced in the transition $2s^22p^2P^0_J - 2s2p^{22}P_J$, with $J-J'$ of 3/2-3/2, 1/2-1/2, 3/2-1/2, 1/2-3/2. The multiplet wavelength was tabulated (NBS Wiese, 1966) as $\lambda_{904.1} \quad \text{Å}$.  

Figures 27 and 28 show the experimental semi-logarithmic intensity decay for the $\lambda_{904.1} \quad \text{Å}$ radiation. A non-linear best fit to the decay curves was obtained in all cases. The computer best-fit was a two exponential fit indicating that cascading and/or blending was present producing the second exponential. The decay constant of the first exponential, however, is very short and is not affected greatly by the second exponential term. The results of the
INTENSITY DECAY CURVE FOR λ904.1 Å RADIATION
(2s²2p²P⁰-2s2p²²P)CII

DATE = 123068.
RUN = 2

The solid lines show computer best-fit. The uncertainty quoted on the results is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 27
The solid lines show computer best-fit. The uncertainty quoted on the results is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 28
individual runs are:

<table>
<thead>
<tr>
<th>Run Nos.</th>
<th>((2s^2p^{22}P) ) (nsec)</th>
<th>G.O.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1230692</td>
<td>0.33 ± 0.01(0.331 ± 0.009)</td>
<td>10.7</td>
</tr>
<tr>
<td>116702</td>
<td>0.34 ± 0.04(0.342 ± 0.039)</td>
<td>2.1</td>
</tr>
</tbody>
</table>

The errors quoted on the results are due to the statistical curve-fitting error only. An average of the individual runs was made and the final result is quoted as

\[
\tau(2s2p^{22}P) = 0.34 \pm 0.04 \text{ nsec (11.8\%)}
\]

The error quoted on this result is due to a combination of the statistical spread error and an uncertainty in the velocity of the emitting specie. The result agrees well with the beam-foil result of Heroux (1969).

REPOP indicates that there exist seven possible repopulating channels to the \(2s2p^{22}P\) term including the \(2p^{32}^D^0 - 2s2p^{22}P\) multiplet at \(\lambda2511.8\ \AA\) which is studied individually in the present investigation, and therefore known to be present in the source. There are also four possible VUV blending multiplets. The second exponential in the two exponential best-fit must be attributed to one or a combination of these cascading or blending terms, although a positive identification cannot be made in this case.

DEPOPOP indicates that there is only one decay channel (for one-electron transitions) out of the excited \(2s2p^{22}P\) term; i.e., \(2s^{2}2p^{2}P^0 - 2s2p^{22}P\) (\(\lambda904.1\ \AA\)). Thus, the transition probability for this multiplet is easily obtained, i.e.
\[ A(2s^2 2p^2 P^0 - 2s2p^{22} P) = \frac{1}{\tau}(2s2p^{22} P) = (29.4 \pm 3.5) \times 10^8 \text{ sec}^{-1} \]

This value is assigned an accuracy parameter of C. The tabulated NBS (Wiese, 1966) value is 42.0 \times 10^8 \text{ sec}^{-1}(E). The value tabulated is from a self-consistent field calculation of Weiss (1964) that does not include the important effects of configurational interaction, hence the accuracy parameter of E(>50%). The present result (precision \sim 10\%) is expected to be a better indicator of the true transition probability for this multiplet transition.

\[ \lambda 1335.3 \text{ Å CII} (2s^22p^2P^0 - 2s2p^{22} D) \]

The multiplet observed was a blend of lines produced in the transition \( 2s^22p^2P^0_J - 2s2p^{22}D^0_{J'} \), with J-J' of 3/2-3/2, 1/2-3/2, 3/2-3/2. The weighted tabulated (NBS Wiese, 1966) multiplet wavelength for this blended transition is \( \lambda 1335.3 \text{ Å} \).

Figures 29 through 32 show the experimental intensity decay curves for the \( \lambda 1335.3 \text{ Å} \) radiation. A linear semi-logarithmic plot is obtained in all cases indicating that probable blending of other multiplets and/or cascading do not affect the mean life results. Four separate runs were made on this line and an average was taken as the final result. The individual results and their goodness-of-fit parameters are:

<table>
<thead>
<tr>
<th>Run Nos.</th>
<th>( (2s2p^0 D) ) (nsec)</th>
<th>G.O.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1125692</td>
<td>3.8 \pm 0.1(3.804 \pm 0.019)</td>
<td>4.2</td>
</tr>
<tr>
<td>123691</td>
<td>3.7 \pm 0.1(3.658 \pm 0.022)</td>
<td>3.9</td>
</tr>
</tbody>
</table>
INTENSITY DECAY CURVE FOR \( \lambda 1335.3 \) Å RADIATION
\((2s^22p^2P^0 - 2s2p^2D)\) CII

DATE: 112569.
RUN: 2

\[ \tau(2s2p^2D) = 3.804 \pm 0.019 \text{ nsec.} \]

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 29
The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 30
The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 31
INTENSITY DECAY CURVE FOR λ1335.3 Å RADIATION
(2s^2 2p^2 P^0 - 2s2p^2 D) CII

DATE = 123069.
RUN = 3
Begin fit

\[ \tau(2s2p^2D) = 3.435 \pm 0.017 \text{ nsec.} \]

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 32
The errors quoted are due to the statistical curve-fitting error only. An average of the four runs was made and the final result is quoted as:

\[ \tau(2s2p^{22}D) = 3.7 \pm 0.1 \text{ nsec (3.6\%)} \]

The error quoted is due to a combination of the statistical spread error and the uncertainty in the velocity of the emitting specie. The result is in agreement with phase-shift value of Lawrence (1966).

The computer program REPOP indicates that there are nine possible one-electron repopulation channels into the 2s2p^{22}D term. Blending of other multiplets is also possible. Since the decay curves are always linear, one must assume that either the upper terms of the cascading and blending multiplets are weakly excited and/or the transition probabilities for the particular transitions are small.

DEPOP indicates that there exists only one possible decay channel out of the excited 2s2p^{22}D term, i.e. \(2s^22p^2p^0 - 2s2p^{22}D (\lambda 335.3 \text{ Å})\). Thus, the transition probability for this multiplet is easily derived, i.e.:

\[ A(2s^22p^2p^0 - 2s2p^{22}D) = \frac{1}{\tau(2s2p^{22}D)} \]

\[ = (2.7 \pm 0.1) \times 10^8 \text{ sec}^{-1} \]

This value is assigned an accuracy parameter of BB. The
NBS (Wiese, 1966) value for this transition is $6.0 \times 10^8$ sec$^{-1}$. This result is derived from a theoretical self-consistent field calculation of Weiss (1964). The transition involves configuration interaction and theoretical values thus contain large uncertainties ($E > 50\%$). It is expected that the present result (precision $\sim 3.5\%$) is a much better indicator of the true transition probability for this transition. The present transition probability result may be compared with two other beam-foil measurements (Heroux, 1967) and (Bickel, 1967) for the same transition in isoelectronic (Boron sequence) NII and OIII, respectively in order to study the functional dependence of the f-value for a particular transition on the nuclear charge within an isoelectronic sequence.

$$\lambda 2511.0 \, \text{Å CII} \ (2s2p^{22}P - 2p^{32}P^0)$$

The multiplet observed was a spectral blend of the lines produced in the transition $2s2p^{22}P_J - 2s^{32}P_0^J$, with $J$-$J'$ of $3/2$-$5/2$, $1/2$-$3/2$, $3/2$-$3/2$. The multiplet wavelength was tabulated (NBS Wiese, 1966) as $\lambda 2511.0 \, \text{Å}$.

Figures 33 and 34 show the experimental semi-logarithmic intensity decay curves for the emitted $\lambda 2511.0 \, \text{Å}$ radiation. A linear computer best-fit to the curves was obtained in all cases indicating that cascading and/or blending of other multiplets was negligible. The results of the individual runs are:
INTENSITY DECAY CURVE FOR λ2511.0 Å RADIATION
(2s2p22P-2p32D°)CII

DATE = 121669.
RUN = 1

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 33
INTENSITY DECAY CURVE FOR \( \lambda 2511.0 \) Å RADIATION

(\( 2s2p^2 2p^2 - 2p^3 3p^0 \)) CII

\[ \tau(2p^3 3p^0) = 2.063 \pm 0.063 \text{ nsec.} \]

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 34
Run Nos. \(\tau(2p^{32}P^0)\) nsec G.O.F.

<table>
<thead>
<tr>
<th>Run Nos.</th>
<th>(\tau(2p^{32}P^0))</th>
<th>G.O.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1218691</td>
<td>2.0 ± 0.1(1.963 ± 0.025)</td>
<td>12.7</td>
</tr>
<tr>
<td>117702</td>
<td>2.1 ± 0.1(2.063 ± 0.063)</td>
<td>8.7</td>
</tr>
</tbody>
</table>

The errors quoted on the above results are due to the statistical curve-fitting errors only. An average of the individual runs was made and the final result is quoted as:

\[
\tau(2p^{32}P^0) = 2.0 ± 0.1\text{ nsec (5\%)}
\]

The error quoted on this result is due to a combination of the statistical spread error and the uncertainty in the velocity of the emitting specie. The result is in good agreement with the experimental beam-foil results of both Poulizac (1969) and Bergström (1969).

REPOP indicates that there are no possible one-electron cascading transitions to the \(2p^{32}P^0\) term. There is also no multiplet blending possible. Thus the multiplet decay curve should be linear as it is.

DEPOP lists three decay channels out of the \(2p^{32}P^0\) term including one that is negligible compared to the others since \(\lambda = 2\nu\) and \(A_{ij} = a_i^3\). The other two depopulating multiplets are listed as \(2s2p^{22}D - 2p^{32}P^0\) at \(\lambda1323.9\) \(\text{Å}\) and \(2s2p^{22}P - 2p^{32}P^0\) at \(\lambda2511.0\) \(\text{Å}\). Both these multiplets are listed in the NBS Tables (Wiese, 1966) and so one can sum the listed transition probabilities to obtain a "theoretical" mean life value. It is not possible to obtain an experimental transition probability for the \(2s2p^{22}P - 2p^{32}P^0\) multiplet, however, since the theoretical tabulated value
for the $2s2p^{22}D - 2p^{32}P^0$ transition is not accurate enough to be subtracted from the inverted experimental mean life measurement (the final result becomes negative). The theoretical mean life is calculated using the NBS transition probabilities, to be

$$\tau_{\text{theory}} = \frac{1}{[A\ (2s2p^{22}P - 2p^{32}P^0) + A(2s2p^{22}D - 2p^{32}P^0)]^{-1}}$$

$$= (0.97 + 8.7) \times 10^8 \text{ sec}^{-1}$$

$$\sim 1.03 \text{ nsec} \ (E)$$

The tabulated transition probability for the $2s2p^{22}P - 2p^{32}P^0$ transition is from a self-consistent field calculation by Weiss (1964) in which configuration interaction has been neglected. The accuracy parameter for this transition is given as $E$. The tabulated value for the $2s2p^{22}D - 2p^{32}P^0$ transition is the experimental arc-method result by Maecker (1953). This also has an accuracy parameter of $E$. The present mean life result (precision $\sim 5\%$) is in poor agreement with the derived theoretical result which is known to be very inaccurate ($E > 50\%$).

$\lambda_{3361.3}^{\text{AII}} (2s^23d^2D^0 - 2s^25p^2P^0)$

The multiplet observed was a spectral blend of lines produced in the transition $2s^23d^2D^J - 2s^25p^2P^0_{J'}$ with $J-J'$ of 5/2-3/2, 3/2-1/2, 3/2-3/2. The multiplet wavelength was tabulated (NBS Wiese, 1966) as $\lambda_{3361.3}^{\text{A}}$.

Figures 35 and 36 show the experimental semi-
INTENSITY DECAY CURVE FOR λ3361.3 Å RADIATION
(2s^23d^2D - 2s^25p^2P^0)CII

DATE = 11870.
RUN = 1

Begin fit

τ(2s^25p^2P^0) = 5.377 ± 0.167 nsec.

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 35
INTENSITY DECAY CURVE FOR \( \lambda 3361.3 \) Å RADIATION

\((2s^23d^2D-2s^25p^2P^0)\) CII

\( \text{DATE} = 12/66 H \)
\( \text{RUN} = 3 \)

\( \tau(2s^25p^2P^0) = 4.748 \pm 0.065 \) nsec.

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 36
logarithmic intensity decay curves for the emitted $\lambda 3361.3 \text{Å}$ radiation. A non-linear computer best fit to the decay curves was obtained in all cases. The best fit was one exponential plus a constant indicating that either the cascading and/or blending component is long lived compared to the decay constant of the first exponential or that the experimental data was not taken out to large enough distances behind the foil to define the second exponential. This is thought to be the case since a slight increase in the result of the individual runs was obtained when data was taken out to a further distance from the foil. The average result is in agreement with the other beam-foil results of Poulizac (1969) and a reasonably large error is quoted to allow for the spread in the individual values.

The results of the individual runs are:

<table>
<thead>
<tr>
<th>Run Nos.</th>
<th>$\tau(2s^25p^2P^0)$ nsec</th>
<th>G.O.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1216693</td>
<td>4.7 ± 0.1(4.748 ± 0.065)</td>
<td>1.8</td>
</tr>
<tr>
<td>118701</td>
<td>5.4 ± 0.2(5.377 ± 0.167)</td>
<td>7.9</td>
</tr>
</tbody>
</table>

The errors quoted on the above results are due to the statistical curve fitting error only. An average of the individual runs was made and the final result is quoted as:

$$\tau(2s^25p^2P^0) = 5.1 \pm 0.3 \text{ nsec (5.9\%)}$$

The error quoted on this result is due to a combination of the statistical spread error and the uncertainty in the velocity of the emitting specie. This value is in good agreement with the beam-foil result of Poulizac (1969).
REPOP indicates that there is only one possible one-electron cascading channel to the $2s^25p^2P^0$ term, i.e., $2s^25p^2P^0 - 2s2p5p^2P$, at $\lambda 1900.8 \, \text{Å}$. A strong line at this wavelength is not observed in the spectral scan of the source and the other beam-foil result (Poullizac, 1969) produced a linear decay curve. (This experiment was run at an incident ion energy of 1 MeV.) There exist no possible blending multiplets within the system's resolution. It is thought that the non-linearity in the decay curve is produced by either some unlisted cascading transition or possibly a large background gas contribution (a constant) that was not correctly compensated for.

DEPOP lists eight possible one-electron decay channels out of the excited $2s^25p^2P^0$ term of which two will be negligible due to being of very high wavelength. It is thus impossible to quote a transition probability for the multiplet studied as the branching ratios of the other possible decay multiplets are not known to this investigator. It is hoped that the radiative mean life of the upper term will be of use to other investigators.

$\lambda 3876.7 \, \text{Å} \text{CII} \,(2s2p3d^4F - 2s2p4f^4G)$

The multiplet was a spectral blend of lines produced in the transition $2s2p3d^4F_J - 2s2p4f^4G_J$, with $J-J'$ of 9/2-11/2, 7/2-9/2, 5/2-7/2, 3/2-5/2, 9/2-9/2, 7/2-7/2, 5/2-5/2, 9/2-7/2, 7/2-5/2. The multiplet wavelength was tabulated (NBS Wiese, 1966) as $\lambda 3876.7 \, \text{Å}$. 
Figures 37 and 38 show the experimental semi-logarithmic intensity decay curves for the emitted 3876.7 Å radiation. The best fit was a two exponential fit in all cases indicating cascading and/or blending was present producing the second exponential. The results of individual runs are:

<table>
<thead>
<tr>
<th>Run Nos.</th>
<th>$\tau(2s2p4f^4G) \text{ nsec}$</th>
<th>G.O.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>125693</td>
<td>$3.2 \pm 0.1 (3.184 \pm 0.103)$</td>
<td>9.7</td>
</tr>
<tr>
<td>120701</td>
<td>$3.2 \pm 0.1 (3.193 \pm 0.111)$</td>
<td>5.3</td>
</tr>
</tbody>
</table>

The errors quoted on the above results are due to the statistical curve-fitting error only. An average of the individual values was made and the final result is quoted as:

$$\tau(2s2p4f^4G) = 3.2 \pm 0.1 \text{ nsec (3.1\%)}$$

The error quoted on this result is due to a combination of the statistical spread error and the uncertainty in the velocity of the emitting specie. The result agrees reasonably well with the beam-foil results of Poulizac (1969) and Bergström (1969).

REPOP indicates that there are no one-electron possible cascading channels to the 2s2p4f$^4$G terms. Blending of multiplets at $\lambda3889.1$ Å CII $2s2p3d^2P^0 - 2s2p5p^2P$ and $\lambda3888.2$ Å CIII $2s4d^3D - 2s4f^3F^0$ is possible, but the latter multiplet has a measured mean life of 1.5 nsec (Poulizac, 1969) and so could not explain the longer lived second exponential of the best fit to the present data. The second exponential may be due to a combination of the blended
INTENSITY DECAY CURVE FOR $\lambda$3876.7 Å RADIATION
$(2s2p3d^{4}F^{0}-2s2p4f^{4}G)_{\text{CII}}$

**DATE**: 12569.  
**RUN**: 3

$\tau(2s2p4f^{4}G) = 3.185 \pm 0.103$ nsec.  
$\tau_{c} = 23.804 \pm 2.878$ nsec.

The solid lines show computer best-fit. The uncertainty quoted on the results is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 37
INTENSITY DECAY CURVE FOR λ3876.7 Å RADIATION

(2s2p3d4F^0 → 2s2p4f^4G)CII

DATE = 12070.
RUN = 1

The solid lines show computer best-fit. The uncertainty quoted on the results is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 38
multiplet at $\lambda 3889.1$ Å and a cascading multiplet that was not listed by REPOP. A cascading term might be expected since both Poulizac (1969) and Bergström (1969) got non-linear fits even at higher spectral resolution.

DEPOP indicates that there is only one dominant decay channel out of the $2s2p4f^4G$ term; i.e., $2s2p3d^4F - 2s2p4f^4G$ ($\lambda 3876.7$ Å). Two other multiplets listed at $\lambda 5\mu$ and $\lambda 21\mu$ will be negligible since $A_{ij}a_1a_2^3$. Thus, the transition probability for the multiplet is easily obtained, i.e.:

$$A(2s2p3d^4F - 2s2p4f^4G) = \frac{1}{\tau(2s2p4f^4G)} = (3.1 \pm 0.1) \times 10^8 \text{ sec}^{-1}$$

This value is assigned an accuracy parameter of BB. The tabulated NBS (Wiese, 1966) value is $2.66 \times 10^8 \text{ sec}^{-1}(C)$. The value is obtained from a Coulomb approximation calculation which is well applicable to this type of transition; i.e. 3d-4f transition-medium to highly excited line. The present result (precision ~ 3%) is in reasonable agreement with the Coulomb approximation result, but it is expected that the measured mean life is a little low (compared with other beam-foil results) which would increase the transition probability value and hence get better agreement with the Coulomb approximation calculation.

$\lambda 4075.7$ Å CII ($2s2p3d^4D^0 - 2s2p4f^4F$)

The multiplet observed was a blend of spectral lines produced in the transition $2s2p3d^4D_j^0 - 2s2p4f^4F_j'$ with $J-J'$
of 7/2-9/2, 7/2-7/2, 7/2-5/2, 5/2-3/2, 5/2-5/2, 5/2-7/2, 3/3-3/2, 3/2-5/2, 1/2-3/2. The multiplet wavelength was calculated to be $\lambda_{4075.7}$ Å (Moore, 1949 and DEPOP Program.)

Figures 39 and 40 show the experimental semi-logarithmic intensity decay curves for the $\lambda_{4075.7}$ Å radiation. A non-linear best fit was obtained in all cases indicating cascading and/or blending are present. The results of individual runs are:

<table>
<thead>
<tr>
<th>Run Nos</th>
<th>$\tau(2s2p4f^4F)$ nsec</th>
<th>G.O.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>124691</td>
<td>3.1 ± 0.1 (3.054 ± 0.053)</td>
<td>2.5 (Forced Fit)</td>
</tr>
<tr>
<td>119703</td>
<td>2.8 ± 0.2 (2.817 ± 0.135)</td>
<td>3.2</td>
</tr>
</tbody>
</table>

The errors on the above results are due to the statistical curve fitting error only. An average of the individual runs was made and the final result is quoted as:

$$\tau(2s2p4f^4F) = 2.9 \pm 0.2 \text{ nsec (6.9%)}$$

The error in this result is due to a combination of the statistical spread error and the uncertainty in the velocity of the emitting specie. The result is somewhat lower than the beam-foil results of Poulizac (1969) and Bergström (1969).

REPOP indicates that there exist only one possible one-electron cascading channel, i.e. $2s2p4f^4F \to 2s2p6d^2D^0$ at $\lambda_{6520}$ Å. This cascading multiplet probably causes the non-linear nature of the decay curves. Blending of one other multiplet, i.e., $2s\,2p3s^2P^0 \to 2s\,2p5p\,D$ at $\lambda_{4065.3}$ Å in CI is also possible.
INTENSITY DECAY CURVE FOR λ4075.7 Å RADIATION
(2s2p3d^4D_0- 2s2p4f^4F)CII

DATE= 11970.
RUN = 3

The solid lines show computer best-fit. The uncertainty quoted on the results is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 39
INTENSITY DECAY CURVE FOR λ4075.7 Å RADIATION

\[(2s2p3d^4D^0 - 2s2p4f^4F)\text{CII}\]

\[\tau(2s2p4f^4F) = 3.054 \pm 0.053 \text{ nsec.}\]

\[\tau_c = 17.784 \pm 1.970 \text{ nsec.}\]

The solid lines show computer best-fit. The uncertainty quoted on the results is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 40
DEPOP lists two possible one-electron decay channels out of the 2s2p4f^4F (two others at \( \lambda \lambda 6.6 \mu, 10.3 \mu \) will be weak). The two multiplets listed are 2s2p3d^4F^0 - 2s2p4f^4F at \( \lambda 3949 \ \AA \) and 2s2p3d^4D^0 - 2s2p4f^4F at \( \lambda 4075.7 \ \AA \). An experimental transition probability for the 2s2p3d^4D^0 - 2s2p4f^4F multiplet could not be derived since a calculated value for the F(2s2p3d^4D^0 - 2s2p4f^4F), branching ratio, could not be found in the literature at the time of writing; although f-values for some lines of the multiplet are listed by Griem (1964). It was not checked to see if this set of lines was complete, i.e. all the lines of the multiplet listed. The transition involved is applicable to Coulomb approximation calculations with reasonable accuracy expected. It is hoped that the tabulated mean-life value will be of use to other investigators.

\[ \lambda 4267.2 \ \AA \ C II (2s^2 3d^2 D - 2s^2 4f^2 F^0) \]

The multiplet observed was a spectral blend of lines produced in the transition 2s^2 3d^2 D_J - 2s^2 4f^2 F^0_J, with J-J' of 5/2-7/2, 3/2-5/2, 5/2-5/2. The multiplet wavelength is tabulated (NBS, Wiese, 1966) as \( \lambda 4267.2 \ \AA \).

Figure 41 through 43 show the experimental semi-logarithmic intensity decay curves for the \( \lambda 4267.2 \ \AA \) radiation. A non-linear curve was obtained in all cases indicating that probably cascading and/or blending is present. The computer program FRANTIC gave a two exponential best-fit for a run in which data was taken out to large distances
The solid lines show computer best-fit. The uncertainty quoted on the results is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 41
The solid lines show computer best-fit. The uncertainty quoted on the results is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

**FIGURE 42**
The solid lines show computer best-fit. The uncertainty quoted on the results is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 43
behind the foil and a one exponential plus a constant fit for shorter runs. This clearly indicated that these latter runs should have been two exponential fits if enough data had been taken to define the second exponential. It was decided to salvage these runs by "forcing" a two exponential fit, i.e., by feeding the decay constant for the second exponential, \( \tau_2 \), from the two exponential fit run into FRANTIC for the one exponential plus a constant run. This method yielded results that were consistent in value with the result obtained for the first exponential in the two exponential fit and so the values were used in the averaging process. The results of individual runs were:

<table>
<thead>
<tr>
<th>Run Nos.</th>
<th>( \tau(2s^24f^2F^0) ) nsec</th>
<th>G.O.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1215693</td>
<td>3.4 ± 0.1(3.433 ± 0.072)</td>
<td>5.7 (Forced Fit)</td>
</tr>
<tr>
<td>125691</td>
<td>3.9 ± 0.2(3.891 ± 0.106)</td>
<td>4.3 (Forced Fit)</td>
</tr>
<tr>
<td>119702</td>
<td>3.8 ± 0.3(3.776 ± 0.262)</td>
<td>1.5</td>
</tr>
</tbody>
</table>

The errors quoted on the above are due to the statistical curve-fitting error only. An average of the individual runs was made and the final result is quoted as:

\[
\tau(2s^24f^2F^0) = 3.7 ± 0.3\text{ nsec} \quad (8.1\%)
\]

The error quoted on this result is due to a combination of the statistical spread error and the uncertainty in the velocity of the emitting specie. This result agrees well with the other beam-foil results of Poulizac (1969) and Bergström (1969).
REPOP indicates that there exists four possible one-electron cascading channels to the $2s^24f^2F^0$ term. Three of these multiplets lie in the near UV (1895 - 1920 Å) range, and one at $\lambda6628.7$ Å. The three UV multiplets were not seen strongly in the spectral scan of this wavelength region and so it is expected that the upper terms of these transitions were not strongly populated and/or the pertinent transition probabilities were small. The cascading multiplet (the second exponential in the computer fit) could be due to the $\lambda6628.7$ Å radiation. Blending of two multiplets, one in CI and one in CIII, is also possible; and so the non-linearity of the decay curve could be due to a combination of cascading and blending. It is thought that cascading must be present since both Poulizac (1969) and Bergström (1969) have obtained non-linear best fits even with better spectral resolution (i.e. less chance of multiplet blending).

DEPOP indicates that there is only one strong decay channel out of the $2s^24f^2F^0$ term, i.e. $2s^23d^2D - 2s^24f^2F^0$ ($\lambda4267.2$). There is another possible transition at $\lambda11.7\mu$ but this multiplet will have a negligibly small transition probability compared to other possible transitions since $A_{ij} = 1$. Thus the transition probability for the multiplet in question is easily obtained, i.e.

$$A(2s^23d^2D - 2s^24f^2F^0) = \frac{1}{\tau(2s^24f^2F^0)} = (2.7 \pm 0.2) \times 10^8 \text{sec}^{-1}$$
This value is assigned an accuracy parameter of B. The tabulated NBS (Wiese, 1966) value is \(2.46 \times 10^8 \text{ sec}^{-1}\). The value is computed using a Coulomb approximation calculation which is quite accurate for this type transition, i.e., the upper and lower terms of the excited "jumping" electron are in a shell which contains no other electrons. The present result (precision \(\sim 8\%\)) is in agreement with the Coulomb approximation value.

\[ \lambda_{5141.8} \text{Å CII} (2s2p3s^4P^0 \rightarrow 2s2p3p^4P) \]

The multiplet observed was a spectral blend of the lines produced in the transition \(2s2p3s^4P^0 \rightarrow 2s2p3p^4P_J\) with \(J-J'\) of 5/2-5/2, 3/2-3/2, 1/2-1/2, 5/2-3/2, 3/2-1/2, 3/2-5/2, 1/2-3/2. The multiplet wavelength was tabulated (NBS, Wiese, 1966) as \(\lambda_{5141.8} \text{Å}\).

Figures 44 and 45 show that experimental semi-logarithmic intensity decay curves for the \(\lambda_{5141.8} \text{Å}\) radiation. A linear computer best fit to the data was obtained in all cases indicating that cascading and/or blending is negligible for this transition. The results of the individual runs are:

<table>
<thead>
<tr>
<th>Run Nos.</th>
<th>(\tau(2s2p3d^4P)) nsec</th>
<th>G.O.F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1216692</td>
<td>10.8 (\pm) 0.3(10.807 (\pm) 0.096)</td>
<td>2.6</td>
</tr>
<tr>
<td>125692</td>
<td>10.6 (\pm) 0.3(10.630 (\pm) 0.060)</td>
<td>2.1</td>
</tr>
</tbody>
</table>

The errors quoted in the above results are due to the statistical curve-fitting error only. An average of the
INTENSITY DECAY CURVE FOR λ5141.8 Å RADIATION

(2s2p3s^4p^0 - 2s2p3p^4D)CII

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 44
The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 45
individual runs was made and the final result is quoted as:

\[(2s2p3^d^4P) = 10.7 \pm 0.3 \text{ nsec} \ (2.8\%)\]

The error quoted on this result is due to a combination of the statistical spread error and the uncertainty in the velocity of the emitting specie. The result is slightly lower than the beam-foil results of both Poullizac (1969) and Bergström (1969).

REPOP indicates that there exist eight possible one-electron cascading channels to the \(2s2p3^p^4P\) term. Apparently either the upper terms of the cascading transition are weakly populated and/or the pertinent transition probability is small since cascading effects are not observed in the decay curves. There should be no blending of other multiplets.

DEPOP indicates that there exists only one one-electron decay channel out of the excited \(2s2p3^p^4P\) term, i.e. \(2s2p3s^4P^0 - 2s2p3p^4P (\lambda 5141.8 \ Å)\). Thus, the transition probability for this multiplet is easily obtained,

\[A(2s2p3s^4P^0 - 2s2p3p^4P) = \frac{1}{\tau(2s2p3p^4P)}\]

\[= (0.93 \pm 0.03) \times 10^8 \text{ sec}^{-1}\]

This value is assigned an accuracy parameter of \(A\). The tabulated NBS (Wiese, 1966) value is \(0.86 \pm 10^8 \text{ sec}^{-1}\) (C). The value tabulated is from a Coulomb approximation calculation which is reasonably applicable to this type of
transition, i.e. a moderately excited transition of the 3d-3p type. The present result (precision ~ 3%) is in good agreement with the Coulomb approximation value, indicating that this approximation gives quite good results for a transition of this type.

4.6. Tabulated Transitions - CIII

\[ \lambda 1175.7 \text{ Å} \text{ CIII (2s2p}^3P^0 - 2p}^2P \]

The multiplet studied was a blend of the spectral lines produced in the transition \( 2s2p^3P_j - 2p^2P \) with \( J-J' \) of 2-2, 1-1, 2-1, 1-0, 1-2, 0-1. The tabulated multiplet wavelength (NBS, Wiese, 1966) was \( \lambda 1175.7 \text{ Å} \).

Figures 46 through 50 show the experimental semi-logarithmic intensity decay curves for the \( \lambda 1175.7 \text{ Å} \) radiation. A linear and a non-linear best fit to the decay curve was obtained depending to what distances behind the foil the data was taken. One exponential plus a constant was the best computer fit to the data in most cases. This indicates that the best fit would have been a two exponential fit, and that either the decay constant of the second exponential is much larger than that of the first (which itself is short in this case) or that the data was not taken out to great enough distances behind the foil in order to define the second exponential. In either case, the decay constant of the first exponential should not be greatly affected since it is so short. The results for the individual runs are:
INTENSITY DECAY CURVE FOR λ175.7 Å RADIATION

\( (2s^2p^3p^0 - 2p^3P) \) CIII

DATE = 112468.
RUN = 3

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant. A constant contribution of 73 cts is not shown in the figure.

FIGURE 46

\( \tau(2p^3P) = 0.796 \pm 0.010 \) nsec.
INTENSITY DECAY CURVE FOR λ1175.7 Å RADIATION
(2s2p^3P^0 - 2p^2^3P)CIII

DATE = 112568.
RUN = 4

\[ \tau(2p^2^3P) = 0.761 \pm 0.013 \text{ nsec.} \]

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 47
The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 48
INTENSITY DECAY CURVE FOR \( \lambda 1175.7 \) Å RADIATION

\((2s2p^3p^0 - 2p^23p)_{\text{CIII}}\)

*DATE* = 123069.
*RUN* = 1

 Begin fit

\( \tau(2p^23p) = 0.797 \pm 0.018 \) nsec.

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 49
The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

**FIGURE 50**
<table>
<thead>
<tr>
<th>Run Nos.</th>
<th>$\tau(2p^{23}P)$ nsec</th>
<th>G.O.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1124693</td>
<td>0.80 ± 0.01 (0.796 ± 0.010)</td>
<td>0.33</td>
</tr>
<tr>
<td>1125694</td>
<td>0.76 ± 0.01 (0.761 ± 0.013)</td>
<td>0.8</td>
</tr>
<tr>
<td>123692</td>
<td>0.82 ± 0.02 (0.815 ± 0.018)</td>
<td>3.2</td>
</tr>
<tr>
<td>1230691</td>
<td>0.80 ± 0.02 (0.797 ± 0.018)</td>
<td>8.7</td>
</tr>
<tr>
<td>116701</td>
<td>0.80 ± 0.02 (0.803 ± 0.020)</td>
<td>8.2</td>
</tr>
</tbody>
</table>

The errors quoted on the results are due to the statistical curve-fitting error only. An average of the individual runs was made and the final result is quoted as:

$$\tau(2p^{23}P) = 0.79 ± 0.02 \text{ nsec (2.9%)}$$

The error quoted on this result is due to a combination of the statistical spread error and the uncertainty in the velocity of the emitting specie. The result agrees very well with the beam-foil result of Heroux (1969).

REPOP indicates that there are nine possible VUV repopulation channels for the $2p^{23}P$ term. There are also six possible blending multiplets present. The non-linear character of the decay curve is very likely due to some of these cascading and/or blending multiplets, although positive identification cannot be made in this case.

DEPOP indicates that there is only one decay channel out of the $2p^{23}P$ term, i.e., $2s2p^3P^0 - 2p^{23}P$ ($\lambda 1175.7$ Å). Thus the transition probability for this multiplet is easily obtained,

$$A(2s2p^3P^0 - 2p^{23}P) = \frac{1}{\tau(2p^{23}P)}$$

$$= (12.7 ± 0.4) \times 10^8 \text{ sec}^{-1}$$
This value is assigned an accuracy parameter of $A$. The tabulated NBS (Wiese, 1966) value is $13 \times 10^8$ sec$^{-1}$(D). This value is from a self-consistent field calculation of Weiss (1964) that does not include the important effects of configurational interaction, hence the accuracy parameter of $D (< 50\%)$. The present result (precision $\sim 3\%$) is expected to be a better indicator of the true transition probability for this multiplet transition.

$$\lambda_{2296.9} \text{ Å CIII} (2s2p^1P^0 - 2p^2^1D)$$

The multiplet observed was mainly due to the lines produced in the transition $2s2p^1P^0_J - 2p^2^1D_J$, with $J-J'$, of 1-2. The listed (NBS Wiese, 1966) multiplet wavelength was $\lambda_{2296.9} \text{ Å}.$

Figures 51 and 52 show the experimental semi-logarithmic intensity decay curves for the $\lambda_{2296.9} \text{ Å}$ radiation. A linear plot is obtained in all cases indicating that cascading and/or blending of other multiplets are absent for this transition. The results of the individual runs are:

<table>
<thead>
<tr>
<th>Run Nos.</th>
<th>(2p$^2^1$D) nsec</th>
<th>G.O.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1218692</td>
<td>6.8 ± 0.1(6.849 ± 0.091)</td>
<td>8.2</td>
</tr>
<tr>
<td>117701</td>
<td>6.9 ± 0.1(6.946 ± 0.084)</td>
<td>4.3</td>
</tr>
</tbody>
</table>

The errors quoted on the above results are due to the statistical curve-fitting error only. An average of the individual runs was made and the final result is quoted as:

$$\tau(2p^2^1D) = 6.9 \pm 0.2 \text{ nsec} \ (2.9\%)$$
INTENSITY DECAY CURVE FOR  \( \lambda 2296.9 \) Å RADIATION
\((2s2p^1p^0 - 2p^2^1D)\) CIII

\[ \text{DATE} = 121869. \]
\[ \text{RUN} = 2 \]

\[ \tau(2p^1D) = 6.849 \pm 0.091 \text{ nsec.} \]

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 51
INTENSITY DECAY CURVE FOR λ2296.9 Å RADIATION
(2s2p^1P^0 - 2p^21D)CIII

DATE = 11770.
RUN = 1

τ (2p^2_1D) = 6.946 ± 0.084 nsec.

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 52
The error quoted on this result is due to a combination of the statistical spread error and the uncertainty in the velocity of the emitting species. The result is in agreement with the beam-foil measurements of Poulizac (1969) and Cumutte (1968) but not with the rather large value of Bergström (1969).

The program REPOP indicates that there are seven VUV transitions that could repopulate the 2p21D term in CIII. Blending of some four other multiplets is also possible. Apparently, either the upper terms of these contributing transitions are not greatly populated or the pertinent transition probabilities are small since no cascading and/or blending is observed in the decay curves.

The program DEPOP indicates that there exists only one decay channel out of the term 2p21D, i.e. 2s2p1P0 - 2p21D (λ2296.9 Å). Thus, the transition probability for this multiplet is easily obtained, i.e.:

\[ A(2s2p^1P_0 - 2p^{21}D) = \frac{1}{\tau(2p^{21}D)} \]

\[ = (1.4 \pm 0.1) \times 10^8 \text{ sec}^{-1} \]

This value is assigned an accuracy parameter of A. The NBS (Wiese, 1966) value for this multiplet is given as 3.6 \times 10^8 \text{ sec}^{-1}(D). This value is from a self-consistent field calculation by Weiss (1964) and does not include the important effects of configurational interaction, (hence the large uncertainty, D < 50%). The present result (precision \(\sim 3\%\)) is expected to be a much better indicator of
the true transition probability for this multiplet.

\[ \lambda 4648.8 \AA \ \text{CIII} (2s3s^3S - 2s3p^3P^0) \]

The multiplet observed was a spectral blend of lines produced in the transition 2s3s^3S_J - 2s3p^3P_J, with J-J' of 1-2, 1-1, 1-0. The multiplet wavelength was tabulated (NBS Wiese, 1966) as \( \lambda 4648.8 \AA \).

Figures 53 and 54 show the experimental semi-logarithmic intensity decay curves for the \( \lambda 4648.8 \AA \) radiation. A linear computer best-fit was obtained in all cases indicating that cascading and/or blending were negligible for this transition. The results of the individual runs are:

<table>
<thead>
<tr>
<th>Run Nos.</th>
<th>( \tau(2s3p^3P^0) ) nsec</th>
<th>G.O.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>125694</td>
<td>12.4 ± 0.4(12.385 ± 0.067)</td>
<td>2.7</td>
</tr>
<tr>
<td>1216691</td>
<td>12.6 ± 0.4(12.632 ± 0.104)</td>
<td>2.5</td>
</tr>
</tbody>
</table>

The errors quoted on the above results are due to the statistical curve-fitting error only. An average of the individual runs was made and the final result is quoted as:

\[ \tau(2s2p^3P^0) = 12.5 ± 0.4 \ \text{nsec} \ (3.2\%) \]

The error quoted on this result is due to a combination of the statistical spread error and the uncertainty in the velocity of the emitting specie. The result is in agreement (within the quoted uncertainties) with the beam-foil results of Poulizac (1969) and Bergström (1969), although the actual absolute value is somewhat lower than these two results.
INTENSITY DECAY CURVE FOR $\lambda 4648.8$ Å RADIATION

$(2s3s^3S-2s3p^3P^0)\text{CIII}$

Begin fit

$\tau(2s3p^3P^0) = 12.385 \pm 0.067$ nsec.

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 53
INTENSITY DECAY CURVE FOR λ4648.8 Å RADIATION
(2s3s^3S-2s3p^3P^0)CIII

DATE = 121669.
RUN = 1

The solid line shows computer best-fit. The uncertainty quoted on the result is due to the statistical curve-fitting error. Counting rate errors are indicated by flags, where significant.

FIGURE 54
REPOP indicates that there exist many possible one-electron cascading channels to the 2s2p3P0 term, but no cascading effects are seen in the decay curves. Thus, either upper terms of the cascading transition are weakly populated and/or the pertinent transition probabilities are small. Blending of two other multiplets is possible, but again it is not observed in this investigation.

DEPOP indicates that there is only one one-electron decay channel out of the 2s2p3P0 term, i.e. 2s3s3S – 2s3p3P0 (λ4648.8 Å). Thus, the transition probability for this multiplet is easily obtained, i.e.:

\[
A(2s3s^3S - 2s3p^3P^0) = \frac{1}{\tau(2s3p^3P^0)}
\]

\[
= \left(0.80 \pm 0.03\right) \times 10^8 \text{ sec}^{-1}
\]

This value is assigned an accuracy parameter of BB. The tabulated NBS (Wiese, 1966) value is 0.78 x 10^8 sec^{-1}(C). This value is from a Coulomb approximation calculation which is reasonably applicable to this type of transition, i.e., a 3s-3p moderately excited transition. The present result (precision ~3%) is in very good agreement with the Coulomb approximation value indicating that this approximation will give accurate results for this type of transition.

4.7 Curve-Fitting Criteria

It was found that the width of the observation window was sometimes appreciable compared to some of the decaying terms studied. Therefore, in these cases, it was
necessary to insure that the shape of the initial part of the decay curves, used in the curve-fitting process, was not affected by this window. To be certain that this did not happen the following criteria were used:

1) For a long lived term, curve fitting began at a distance downstream from the foil equal to one half the observation window profile width (h.w.h.m);

ii) For a short lived term, curve fitting began at a distance downstream from the foil where the intensity of the decay curve fell to a half maximum value. (At this point it is clear that the observation window will not be perturbing the decay curve.) The point on the decay curves at which curve-fitting began is designated by an arrow.
SECTION V.

CONCLUSIONS

5.1 Summary of Results

The beam-foil spectroscopic technique has been used to measure the radiative mean lives of twelve excited electronic terms in CI, CII and CIII. The results are listed in Table III. There exists relatively good agreement with other beam-foil measurements and with two phase-shift measurements.

Atomic transition probabilities are easily calculable from the radiative mean-life results when the branching ratio of the multiplet under study is equal to unity (or close to it). This was found to be the case for nine of the twelve multiplets studied in the present investigation. The transition probabilities are listed in Table IV. Good agreement exists between the recent experimental results and theoretical calculations applied to transitions for which the Coulomb approximation is applicable. Poor agreement is obtained for other transitions in which configuration interaction is an important factor. This is to be expected since present theoretical calculations involving the superposition of many configurations are not common or very extensive.
5.2 Analysis of Beam-Foil Technique

The experimental method used in this investigation, i.e. the beam-foil technique, is now well established as a method for determining radiative mean lives of atomic and ionic species to high precision. Many important atomic transition probabilities have been derived from these mean life results. The two primary sources of error in this technique are cascading and the uncertainty in the final velocity of the emitting species. The velocity error could be reduced by velocity analyzing the beam after the foil (as has been done by other experimenters), but this would not reduce the total error on the result by very much. Cascading cannot be eliminated unless the excited levels are excited selectively, which is not possible when using the foil as the excitation medium. A delayed photon coincidence study of cascade transitions in the beam-foil source after the foil would eliminate both quoted sources of error, but the technique would be limited to transitions emitting suitable wavelength photons and connecting well populated levels. The branching ratios for the transitions involved should also be large. The method has been attempted by this investigator using a beam-foil helium source with negative results due probably to poor branching ratios for the transitions studied. It might be feasible using a heavier ion beam-foil source such as carbon, but the experiment would only
be academic, really, since it would be limited to a small number of transitions only.

Instead of attempting to minimize the mentioned errors in the beam-foil method, it is thought by this investigator that the best way of improving the source and the method is to increase the source intensity by either collecting the radiation emitted by the source more efficiently, i.e., increase the collection efficiency, the speed of the monochromator, etc., and/or by producing more intense ion beams and foil materials that do not break so easily. The beam-foil spectra of the heavier elements could then be studied in more detail due to the improved resolution. For example, at present the beam-foil group at the California Institute of Technology (Whaling, 1969) is studying the element Iron with a resolution of approximately \(1^R\) (fwhm) and still they get some blending of spectral lines due to the extreme complexity of the iron spectra.
REFERENCES


Weiss, A. W., private communication to NBS (1964).


APPENDIX A

Mass-Energy Product of a Magnet

Consider a positively charged ion of mass \(m\) (grms) and charge \(Ze\) (c.g.s.) moving with a velocity \(\vec{v}\) (cm/sec) through a magnetic field \(\vec{B}\) (gauss) whose lines of force are perpendicular to the path of the particle. The ion is acted on by the Lorentz force which gives it a curved trajectory while in the magnetic field region. Thus the force equation becomes, for the case of \(\vec{v}\) perpendicular to \(\vec{B}\),

\[
\vec{f} = \frac{Ze}{c} \vec{v} \times \vec{B} = \frac{m}{p} \vec{v} \times \vec{B}
\]

but the kinetic energy (ergs) of the particle is

\[
E = \frac{1}{2} mu^2 \Rightarrow \vec{v} = \left(\frac{2E}{m}\right)^{\frac{1}{2}}
\]

Substitution of A-2 in A-1 gives

\[
B^2 = \frac{2E}{c^2} \left(\frac{m}{Ze}\right)^{\frac{1}{2}}
\]

\[
B_{p^2} = \frac{ze^2}{c^2} \left(\frac{mE}{Ze}\right)
\]

\[
\therefore \frac{meE}{Zc^2} = \frac{B_{p^2}e^2}{Zc^2} \text{ (c.g.s. units)}
\]
The term $ME/Z^2$ is often called the mass-energy product of the magnet. Converting now to the more usual units in which mass is expressed in a.m.u. and $E$ in MeV one gets

$$\frac{mE}{Z^2} = (4.8 \times 10^{-11}) B^2 \rho^2$$

It can also easily be shown that for a circular polefaces

$$\rho = R \frac{\cos \theta}{\tan \theta}$$

where $R$ is the effective radius of the polefaces of the magnet and $\theta$ is half-deflection angle. Thus equation A-6 may be rewritten as

$$\frac{mE}{Z^2} = (4.8 \times 10^{-11}) B^2 R^2 \frac{\cos^2 \theta}{\tan \theta}$$

In a separate experiment the radius of curvature $\rho$ was found by using a known particle and a known measured magnetic field. The maximum magnetic field that could be used was also found using a flip-coil gaussmeter. A knowledge of those two parameters allowed one to calculate the maximum mass-energy attainable with the magnet, now installed at the University of New Hampshire. The value was calculated
to be approximately 14 for the configuration with the smallest possible pole-gap (3/4") i.e., maximum field strength. This implied that singly charged ions of mass \( m \) (a.m.u.) and energy \( E \) (MeV) could be deflected into the 30° port by this magnet if the product \( mE \) was less than or equal to 14.

\[ \text{i.e. } mE \leq 14 \]

For example, at 350 keV it would be possible to analyze ions of mass up to \( m = 40 \) a.m.u., i.e. Argon. By running at lower energies one could raise this figure.
APPENDIX B

Determination of Final Beam Velocity

As a specific example of a velocity determination this appendix will calculate the final velocity of an incident 336 keV carbon ion beam after passage through a 10 µg/cm² carbon foil. At 336 keV, Northcliffe's curves (Northcliffe, 1963) give a specific energy loss,

\[
\frac{dE}{dx} = 3.64 \pm 0.40 \ (\sim 10\%) \text{ keV/}
\]

and the foil thickness is quoted as,

\[
\Delta x = 10 \pm 4 \ \mu\text{g/cm}^2 \ (\sim 40\%).
\]

The ion energy loss in the foil (combining fractional errors of a product in quadrature) is found to be

\[
\Delta E = (3.64 \pm 0.4) (10 \pm 4)
\]

\[= 36.4 \pm 14.6 \text{ keV}
\]

Thus the final beam energy will be 299.6 keV. The uncertainty on this value is found by combining the "energy loss in foil" uncertainty with further uncertainties due to the finite incident beam energy profile and in the calibration reading of the generating voltmeter. The beam profile error is taken to be the f.w.h.m. of the Gaussian-type
shaped incident beam energy profile and can be approximately
found experimentally by slowly sweeping the analyzing magnetic
field through its correct value and monitoring the beam cur-
rent registered by the Faraday Cup detector. This ($\Delta E$) pro-
file was found to be approximately $\pm$ 5 keV at 336 keV. The
uncertainty in reading the energy from the generating volt-
meter is mainly due to the 2% uncertainty in the energy
calibration point (obtained from a separate nuclear reson-
ance experiment) on the microammeter scale from which the
beam energy is read. Combining the two independent (the
beam profile and generating voltmeter calibration) energy
uncertainties in quadrature (fractional standard deviations),
one gets a combined uncertainty of $\pm$ 8.0 keV. Combining,
in quadrature, this value with the $\pm$ 14.6 keV uncertainty
due to the "energy loss in the foil", one gets an overall
uncertainty of $\pm$ 6.6 keV. This implies a 5.4% uncertainty
in the 299.6 keV final beam energy which in turn implies
a 2.7% uncertainty in the final beam velocity at this
energy. Thus the final velocity of a specie in the excited
beam after a 336 keV $^{12}$C$^+$ ion beam passes through a 10 $\pm$
4 $\mu$g/cm$^2$ carbon foil can then be calculated to be 2.19 $\pm$
0.06 cm/sec. This procedure must be repeated for all ion
energies used in order to define the time scale (and its
uncertainty) of the intensity decay curves. The fractional
uncertainty in the meanlife value due to the velocity un-
certainty will be equal in magnitude to the fractional
velocity uncertainty.
APPENDIX C

FRANTIC:

Program for Analysis of Exponential Decay Curves

This computer program was written for use with IBM Computers by Dr. P. C. Rogers of the M.I.T. Laboratory for Nuclear Science (Technical Report No. 76, June, 1962) (Rogers, 1962). The function of the program is to process raw counting data and fit to these data, by the least squares techniques, equations for multiple exponential growth or decay. Although written originally for radioactivity studies using a multichannel analyzer, the program is sufficiently general to be applied to the beam-foil data analysis problem.

The least-squares best fit of a calculated curve to actual data is defined as that fit in which the sum of the weighted squares of the residuals is a minimum. This sum, divided by the number of degrees of freedom, is known as the "variance of fit". In order for a linear least squares analysis to be applicable, there must exist a set of simultaneous equations which are linear in the parameters whose values are to be determined. If the equations to be used are non-linear in their parameters, then they must be linearized before the least squares method can be applied.
One method of linearization of non-linear equations consists of expanding each expression in a first-order Taylor series about the point defined by the previous estimates of the parameters. By neglecting all terms of the series beyond first order, there results a set of simultaneous equations which are linear in the first power of the \( \Delta \) terms (differences between the estimates of the parameters and the actual values) but not necessarily linear in the original parameters themselves. In order to determine the parameters of the non-linear equations, it is necessary only to evaluate the terms and correct the previous estimates for these differences. Each repetition is called an iteration and several iterations are usually required to meet a preset convergence criteria. This method is known as an iterative least-squares analysis. In order for convergence to occur, the value of each parameter and the variance of fit must not deviate from their values in the preceding iteration by more than one part in \( 10^6 \). When convergence occurs the results are printed out before the next analysis is begun. If convergence has not occurred by 25 iterations, the result at that time will be printed out.

In order to estimate the goodness of the fit, the program also generates calculated values of the weighted variance of fit and \( \chi^2 \). The variance of fit is the sum of the weighted squares of the residuals divided by the degrees of freedom, where each weighted residual is expressed in units
of its individual $\sigma$. This has been termed the goodness-of-fit parameter (G.O.F.) in this thesis.

In order to statistically find the best fit from a number of different attempted computer fits, i.e., one exponential, one exponential plus a constant, two exponentials, etc., one must apply an F-test in conjunction with the $\chi^2$ results. The ratio of two $\chi^2$ variables divided by their respective degrees of freedom ($m_1$, $m_2$) is called a Snedecor, $F(m_1, m_2)$ distribution. The number of degrees of freedom for a $\chi^2$ variable is given by $m = n - p - 1$, where $n$ is the number of experimental data points and $p$ is the number of fitted parameters. One can determine, at a certain confidence level, by use of $F$-distribution tables if there is a significant difference between two separate fits containing different numbers of parameters.

D I M E N S I O N T ( 1 0 0 ) , D T ( 1 0 0 ) , C ( 1 0 0 ) , R ( 1 0 0 ) , W ( 1 0 0 ) , A ( 1 0 0 ) , A C ( 1 0 0 )
1 * A E ( 5 , 1 0 0 ) , D A ( 1 0 0 ) , I X ( 1 0 ) , I S ( 6 ) , P C ( 1 0 ) , P G ( 1 0 ) , B M ( 1 0 )
2 B M O ( 1 0 ) , A M O ( 1 0 ) , A M O ( 1 0 ) , F M T ( 2 0 ) , X I D ( 3 )
D O U B L E P R E C I S I O N A N * B M * P I V O T * S N A P * D E T * A M A X * T A * A E * P A R T * X * D T * P C * T * Y
1 : E X P P F . D A . D P
C O M M O N D E T * A N * B M * T * D T * A E * P C * D A * K * A M O * B M O * F M T * X I D * N * J M A X * I C * I D * I S
2 W * A * A C * I X * P G
C O M M O N S L I G H T
1 F O R M A T ( 1 1 1 1 )
1 0 S L I G H T = 0
C A L L I N P U T
J C A L C = J M A X
J C A L C 2 = J M A X + J M A X
I T ( I C - 1 ) = 1 0 , 1 1 , 1 3
1 1 J C A L C = 0
1 2 C O N T I N U E
1 3 W R I T E ( 1 3 , 1 *)
C A L L L E S T I
W R I T E ( 1 3 , 1 *)
C A L L O U T P U T
I F ( J C A L C = J M A X ) 1 2 , 1 0 , 1 0
E N D

I N P U T 1 T O B E U S E D W I T H B E A M F O I L D A T A,

S U B R O U T I N E I N P U T
D I M E N S I O N T ( 1 0 0 ) , D T ( 1 0 0 ) , C ( 1 0 0 ) , R ( 1 0 0 ) , W ( 1 0 0 ) , A ( 1 0 0 ) , A C ( 1 0 0 )
1 * A E ( 5 , 1 0 0 ) , D A ( 1 0 0 ) , I X ( 1 0 ) , I S ( 6 ) , P C ( 1 0 ) , P G ( 1 0 ) , N M ( 1 0 )
2 B M O ( 1 0 ) , A M O ( 1 0 ) , A M O ( 1 0 ) , F M T ( 2 0 ) , X I D ( 3 )
D O U B L E P R E C I S I O N A N * B M * P I V O T * S N A P * D E T * A M A X * T A * A E * P A R T * X * D T * P C * T * Y
1 : E X P P F . D A . D P
C O M M O N D E T * A N * B M * T * D T * A E * P C * D A * K * A M O * B M O * F M T * X I D * N * J M A X * I C * I D * I S
2 W * A * A C * I X * P G
1 0 F O R M A T ( 2 0 A 4 )
1 1 F O R M A T ( 2 A 4 , A 2 , I 4 , 2 9 1 2 )
1 2 F O R M A T ( F 1 0 . 5 , 2 F 1 0 . 2 )
R E A D ) 1 * 1 0 , 1 2 0 # * F I X T ) 4 , 1 = 1 , 2 0 *
R E A D ) 1 * 1 0 1 , 1 4 * X I D ) 1 2 * X I D ) 3 , N , J M A X , I C , I D *
I ( I S ) = I ( I S ) + 1 , 6 , I ( X I ) = I ( X I ) + 1 , 1 0 *)
I F ( I I ) I = 1 1 5 , 1 1 4
1 1 4 F O R M A T ( I 2 M S , I C ) - 2 1 1 0 3 , 1 0 1 , 1 0 1
1 0 1 D O 1 0 2 I = 1 , J M A X
1 4 F O R M A T ( 2 F 1 0 . 4 )
1 0 2 R E A D ) 1 * 1 0 1 , 1 4 * P G ) 1 2 * I 4 * P G ) 1 2 * I 1 , 1 *
R E A D ) 1 * 1 3 , * D E L T
1 3 F O R M A T ( F 1 0 . 4 )
1 0 3 I F ( I C ) 1 1 0 , 1 1 0 , 1 0 4
1 0 4 D O 1 0 5 I = 1 , N
SUBROUTINE LL5FIT

DIMENSION T(100), DT(100), C(100), R(100), X(100), A(100), AC(100)
1, AL(5:100), DA(100), IX(100), IJ(10), PC(10), PG(10), BM(10).
25MC(10), AM(10:10), AMO(10:10), FMT(25), XID(3)
DIMENSION DP(10), P(10)
DOUBLE PRECISION AM, BM, PIVOT, SWAP, DET, AMAX, TA, AE, PC, T, Y
1, FXP, PC, T, Y
COMMON OLT, AM, 3M, T, QT, AE, PC, X3, Y, TAU, AMO, X, AC, PC
COMMON SLIGHT

C = 0.0
1 = 0, JCALC = 1, JCALC2 = 1
GO TO 510
510 FORMAT(/1X, 2A4, A2, 14X, 14, 14X, 14, 14X, 14, 14X, 14, 14X, 14,
1P8E15.7)

EXP(X) = 1.0 + X / 6.0 + X * 2 / 120.0 + X * 3 / 5040.0
1 = 0
IMA = 0
IML = 0
VAR = 0.0
DC 514 = 1, JCALC2 = 2
IF (IX(I)) = 1, 514 = 512, 511, 510
510 IMA = IMA + 1
511 IF (IX(I)) = 1, 512, 513, 512
512 IML = IML + 1
513 IX(I+1) = IX(I) + 100
DP(1) = 0.0
DP(I+1) = 0.0
PC(I+I) = PG(I)
514 PC(I+1) = PG(I)
DO 520 L = 1, N
DO 520 = 1, JCALC
J = 2
X = PC(J) / DT(L)
Y = PC(J) / T(L)
IF X = 1.0, 519, 518
518 AE(I, L) = EXP(-Y) * (1.0 - EXP(-X)) / X
GO TO 520
519 AE(I, L) = EXP(-Y-X/2.0) * EXPF((X/2.0)**2)
520 CONTINUE
K=JCALC-IMA
521 IT=1
523 CALL MATRIX
      CALL MTINV
      J=1
      DO 525 I=1,JCALC2
         IF (IT-5)526,526,521
         IF (IT)526.526.521
         IF (I*J)522,522,526
925 PC(I)=B(I,J)
         J=J+1
525 CONTINUE
526 IF(JCALC=1)527,527,525
527 SLIGHT=1
      GO TO 550
ITERATIVE SECTION
533 IT=IT+1
      CALL MATRIX
      CALL MTINV
      J=1
      IF (IT-5)536,536,537
536 H=0.7
      GO TO 536
537 H=0.98
538 DO 543 I=1,JCALC2
         IF (I*J)545,540,545
540 DP(I)=H*BM(J)
541 PC(I)=PC(I)+DP(I)
         IF (I*J)544,544,544
542 IF (PC(I)*I)543,543,544
543 DP(I)=DP(I)/2.0
      GO TO 541
544 J=J+1
545 CONTINUE
CALCULATION OF FIT
550 VAR2=0.0
      DO 563 L=1,IN
         AC(L)=0.0
      DO 562 I=1,JCALC
         J=2*I
         IF (I*J)562,559,562
559 X=PC(I)*DT(L)
      Y=PC(I)*T(L)
         IF (X-Y)560,560,566
560 AE(I,L)=EXP(-1)*((1.0-EXP(-X))/X
      GO TO 562
561 AE(I,L)=EXP(-Y)/(1.0-EXP(X/2.0))*(X/2.0)**2
562 AC(I,L)=AC(I,L)+PC(I)*AE(I,L)
      DA(L)=AC(L)-AC(L)
563 VAR2=VAR2+DA(L)**2*(L)
TEST FIT
564 DVAP=AE5((VAR2-VAR1)/VAR2)
      IF (IT)564,564,564
564 WRITE(13,501+XID1+XID2+XID3+1T+JCALC+VAR2*
DIMENSION T(100), DT(100), C(100), R(100), W(I, U), A(I), X(I, J)
1 + AL (5+100), G(100), I(10), I(15), PC(I), PG(I), BMI(I)
25+ AE(I, J), AM(I, J), ANM(I, J), EXPM, DA, DP
DIMENSION PART(10)
DOUBLE PRECISION AM, B, P, D, A, PC, T, Y,
1 EXP, X, Y, Z, TAU, J, A, AC, I, X, w, p, c, j, k
COMMON DT, AM, BM, T, DT, AE, PC, JA, K, AMO, G, FMT, XID, N, JMAX, t, I, ID, I5,
1 JCALC, JCALC2, I, TAU, TAU, E, S, X, NORM, VAR2, DVAR, SUM, DTAU, C2,
2, A, AC, IX, PG
EXP(X) = DEXP(X)
DO 600 1 = 1, K
DO 605 J = 1, N
600 AM(I, J) = 0.0
605 BM(I) = 0.0
DO 625 L = 1, N
J = 1
DO 613 J = 1, JCALC
JA = 2 * J - 1
IF (X(JA - 1)) 608, 607, 608
607 PART(J) = AE(J + 1)
J = J + 1
608 IF (X(JA)) 612, 609, 613
609 X = PC(JA) + DT(L)
IF (X = 0.0) 611, 611, 610
610 PART(J) = ((X, PC(JA) + T(1) + DT(L)) * EXP(-X) - 1.0 / PC(JA) - T(1) / X)
GO TO 612
611 PART(J) = T(L) - DT(L) / 2.0 + X / 2.0 + T(L) + DT(L) - (1.0 / PC(JA) + T(L) +
1 * DT(L)) * (X**2 / 6.0 - (X**3) / 24.0 + (X**2) * 2 / 120.0 + (X**3) / 720.0)
612 PART(J) = PART(J) * PC(JA - 1) * EXP(-PC(JA) * T(L))
J = J + 1
613 CONTINUE
DO 620 1 = 1, K
DO 620 J = 1, N
620
620 AN(i,j):=AN(i,j)+PART(i)*PART(j)*W(L)
IF(IT-1)621,621,623
621 DO 622 I=1,K
622 BH(i):=BH(i)+PART(i)*W(L)*A(L)
GO TO 625
623 DO 624 T=1,K
624 BH(i):=BH(i)+PART(i)*W(L)*DA(L)
625 CONTINUE
DO 627 I=1,K
DO 626 J=1,K
625 AMO(I,J)=AMO(I,J)
627 BH(0(I):=BH(1)
RETURN
END

MATINV

SUBROUTINE MATINV
DIMENSION A(10,10),B(10),PIVOT(10),IPIVOT(10),INDEX(10,2)
T(10),DET(10),AE(10),PC(10),DA(10)
DOUBLE PRECISION AM,AE,PIVOT,SWAP,DET,TMAX,TA,PC,T
1,EXPERT,DA,DP
COMMON LET,AM,BM,T,OT,AE,PC,DA,K
EQUIVALENCE (IROW,JROW),(ICOLUM,JCOLUMN),(AMAX,TA,SWAP)
70: DET=1.0
702 DO 703 J=1,K
703 IPIVOT(J)=0
704 DO 741 I=1,K
705 AMAX=0.0
706 DO 715 J=1,K
707 IF(IPIVOT(J)-1)708,715,708
708 DO 714 M=1,K
709 IF(IPIVOT(M)=1)710,714,753
710 IF(IPIVOT(M)-1)711,713,714
711 IROW=J
712 ICOLUMN=M
713 AMAX=AM(J,M)
714 CONTINUE
715 CONTINUE
716 IPIVOT(ICOLUMN)=IPIVOT(ICOLUMN)+1
717 IF(IPIVOT(ICOLUMN)=IROW)718,726,718
718 DET=-DET
719 DO 722 L=1,K
720 SWAP=AM(IROW,L)
721 AM(IROW,L)=AM(ICOLUMN,L)
722 AM(ICOLUMN,L)=SWAP
723 SWAP=BM(IROW)
724 BM(IROW)=BM(ICOLUMN)
725 UM(ICOLUMN)=SWAP
726 INDEX(I+1)=IROW
727 INDEX(I+2)=ICOLUMN
728 PIVOT(I)=AM(ICOLUMN,ICOLUMN)
729 DET=DET*PIVOT(I)
730 AM(ICOLUMN,ICOLUMN)=1.0
731 DO 732 L=1,K
732 AM(ICOLUMN,L)=AK(ICOLUMN,L)/PIVOT(I)
733 BM(ICOLUMN)=BM(ICOLUMN)/PIVOT(I)
734 DO 741 L=1,K
735 IF(L+ICOLUMN)736,741,736
736 TA=AM(L,ICOLUMN)
AM(L:ICOLUMN)=0.0
738 DO 739 L=1,K
739 AM(L:L)=AM(L:L)-AM(ICOLUMN:L)*TA
740 BM(L:L)=BM(L:L)-BM(ICOLUMN:L)*TA
741 CONTINUE
742 DO 752 I=1,K
743 L=K+1-I
744 IF(INDEX(L,1)-INDEX(L,2))745,752,745
745 JROW=INDEX(L,2)
746 JCOLUMN=INDEX(L,1)
747 DO 751 M=1,K
748 SWAP=AM(M,JROW)
749 AM(M,JROW)=AM(M,JCOLUMN)
750 AM(M,JCOLUMN)=SWAP
751 CONTINUE
752 CONTINUE
753 RETURN
END

SUBROUTINE OUTPUT
DIMENSION T(100), C(100), R(100), W(100), A(100), AC(100),
1, AC(100), DA(100), X(100), IS(100), PC(100), PG(100), BM(100),
25 MO(10), AM(10,10), AMO(10), FMT(100), XID(100),
D DIMENSION SP(10), XNORM(10), ENORIG(10), HL(10), EHL(10),
1, INST(100), NP(100), Y(100), IDEV(100),
2,  DOUGLE, PRECISION AM, BM, PIVOT, SWAP, DET, AMAX, TA, AL, PART, X, DT, PC, T, Y
1, EXP, DDA, DO
COMMON DET, AM, BM, T, DT, ENORIG, AMO, FMT, XID, XNORM, VAH2, DVAR, Y, DTAU, C
2, AC(100), IP, PG, X
EXP(X)=DEXP(X)
SORT(X)=DSORT(X)

601 FORMAT(/5X,2A4,A2,2SH = FRANTIC IDENTIFICATION, 19X,F10.5,25H = TA
1U (TIME TO COUNTING))
602 FORMAT(115,23H = NUMBER OF COMPONENTS, 21X,6PF10.5,46H = TAUO (DEA
1D TIME FACTOR IN MICRO TIME UNITS))
603 FORMAT(115,34H = NUMBER OF PARAMETERS HELD FIXED, 115,6PF10.5,47H
1 = ERROR IN DEAD TIME FACTOR (MICRO TIME UNITS))
604 FORMAT(115,24H = NUMBER OF DATA POINTS, 20X,F10.5,13H = BACKGROUND
1)
605 FORMAT(115,21H = DEGREES OF FREEDOM, 23X,F10.5,47H = UNCERTAINTY I
IN TIMING INTERVAL IN TIME UNITS)
606 FORMAT(115,13H = ITERATIONS, 31X,F10.5,20H = DATA SCALE FACTOR)
607 FORMAT(115,13H = UNIT WEIGHTS)
608 FORMAT(115,13H = STATISTICAL WEIGHTS)
609 FORMAT(115,13H = SPECIAL WEIGHTS)
610 FORMAT(1X,L14.5,28H = LEAST SQUARES DETERMINANT)
611 FORMAT(1H,50X,12H(LOUSY LUCK))
612 FORMAT(1X,F14.5,27H = WEIGHTED VARIANCE OF FIT)
613 FORMAT(1X,F14.5,13H = CHI SQUARE)
614 FORMAT(1H,55X,F10.5,23H = NORMALIZATION FACTOR)
615 FORMAT(///118H = SIGMA N (ORIGINAL) SIGMA MEAN LIFE SIGMA LAMUDA
1 SIGMA N (ZERO) SIGMA MEAN LIFE SIGMA LAMUDA
2///)
616 FORMAT(1X,F14.3,F15.3,F15.3,F15.3,F15.3,F15.3)
617 FORMAT(27H0 ORIGINAL ESTIMATES/)
194.

819 FORMAT (///6X.11HBEGINNING INTERVAL ORIGINAL CORRE-
ICTED CALCULATED INSTANTANEOUS WEIGHTS DELTA/
2X.4HTIME 8X.4HTIME 10X.GCOUNTS 11X.4HDATA 12X.4HRATE 12X.4HRATE
3X.7HX 10(3)*10X.4HRATE//)
820 FORMAT (I4.F11.3.F12.3.F15.1.F3F16.3.3PFI9.5.QFI9.3)
821 FORMAT (///4H I,39X.6HA(I,J),61X.4MB(I,J)//)
822 FORMAT (I4.1PE17.7/(1PE21.7;1M17.7))
823 FORMAT (1H..1PE19.7//)
824 FORMAT (4H I.35X.17HINVERSE OF A(I,J)//)
825 FORMAT (6PF15.5.32H = DELTA VARIANCE OF FIT X 10(6))
826 FORMAT (113.51H = NUMBER OF POINTS (*) DEVIATING MORE THAN 2 SIGMA
1)
827 FORMAT (1H..18X.1H*)
828 FORMAT (20X.20A4)
829 FORMAT (///40X.26HANALYSIS OF THE DEVIATIONS//)
830 FORMAT (5X.22I5/1X.114HSIGMA -XS -4.5 -4.0 -3.5 -3.0 -2.5 -2.0 -1
1.5 -1.0 -0.5 -0.0 0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.
25 XT)
831 IF (IDF=N-K)
832 DO 835 J=1,JCALC2
833 SP(I)=DSGRT(DABS(AM(J,J)*VAR2))
834 DO 836 J=J+1
835 GO TO 835
836 IF (PC(J)) 837,838,840
837 HL(I)=1.000000/PC(J)
838 IF (XLOG(Y(JP))=3.7D01 839,840,842
839 XNCRIG(1)=C(J-1)/PC(J)*EXP(X)
840 ENORIG(1)=ABS(XNORIG(1)*SORT(Y(1)**2+Y(2)**2))
841 CONTINUE
842 ENORIG(1)=1.0
843 CONTINUE
844 XNORIG(1)=1.0
845 CONTINUE
846 CONTINUE
847 CONTINUE
NP(L) = 0
A(INST(L)) = 0.0
DO 845 L = 1, JCALC + 2
845 A(INST(L)) = A(INST(L)) + PC(L) * EXP(-PC(L) * (T(L) + DT(L) / 2.0))
X = DA(L) * 2.
CHISQ = CHISQ + DT(L) / AC(L) * X
X = SQRT(X * W(L))
I = (2.0 * DSIGN) * 2.0 * X * DA(L) * 2.
IF(I) 850, 850, 851
850 I = 1
GO TO 853
851 IF(I-22) 853, 853, 852
852 I = 22
853 IDEV(1) = IDEV(1) + 1
IF(X-2.0) 847, 846, 846
846 NP2S = NP2S + 1
NP(L) = 1
847 CONTINUE
CHISQ = CHISQ / XNORM
WRITE(3, 828) IFMT(I) * (I.LT.20)
WRITE(3, 821) XID(I), XID(1), XID(2), XID(3), TAU
WRITE(3, 823) JCALC, TAU
WRITE(3, 824) XID(I), DT(I)
WRITE(3, 803) N
WRITE(3, 804) 1DF * EB
WRITE(3, 805) IT * 5
IF(ID-2) 850, 851, 853
859 WRITE(3, 906) IDF
GO TO 864
861 WRITE(3, 907) I
GO TO 864
863 WRITE(3, 908) I
864 WRITE(3, 914) XNORM
WRITE(3, 826) NP2S
WRITE(3, 809) DET.
IF(DET) 866, 867, 867
866 WRITE(3, 910)
867 WRITE(3, 925) DVAP
WRITE(3, 811) VAR2
WRITE(3, 812) CHISQ
IF(CHISQ/FLAT(1DF) = 16.0) 876, 875, 875
873 WRITE(3, 813)
875 WRITE(3, 915)
WRITE(3, 816) PC(L) * (1-I), SP(L) * (1-I), PC(L) * (1-I), SP(L) * (1-I)
I = XNORM(I) * CHNORM(1) * HL(I) * EHL(I) * I = 1 * JCALC
IF(ID-1) 890, 880, 881
391 WRITE(3, 917)
WRITE(3, 918) PC(L) * I * I = 1 * JCALC2
880 IF(IDC-1) 883, 883, 882
882 WRITE(3, 929)
WRITE(3, 930) IDEV(I) * I = 1 * 22
863 WRITE(3, 819)
DO 895 L = 1, N
WRITE(3, 820) L, T(L), DT(L), C(L), A(L), AC(L),
A(INST(L)) * W(L) * DA(L)
IF(NP(L) 885, 885, 884
884 WRITE(3, 827)
885 CONTINUE
IF(ISC(I) 887, 887, 884
887 WRITE(3, 921)
DO 890 I=1,K
WRITE(3,922*AM) I,J
890 WRITE(3,923*BN) I*
WRITE(3,924*)
DO 894 I=1,K
WRITE(3,922*AM) I,J,J=1,K*
994 WRITE(3,923*BN) I*
895 RETURN
END
APPENDIX D

The Convert Program

This computer program was written in order to prepare the raw experimental data for use as the input to FRANTIC. The program subtracts the backgrounds from both the photomultiplier tubes and normalizes the resultant EMI count rate to the resultant RCA count rate. The average of the four such EMI values at each foil position is determined and its standard deviation computed. This standard deviation is shown in the semi-logarithmic decay curves as an error flag on the individual data points (only when the flag is larger than the symbol used to mark the data point).

The listing of the CONVERT program is given in the following pages.
PROGRAM FOR DATA CONVERSION

THIS PROGRAM FIRST SUBTRACTS THE BACKGROUND CTS FROM BOTH
THE SIG AND MONITOR TUBES AND THEN NORMALIZES THE SIG TUBE TO
THE MONITOR TUBE. THE ERROR(SIG) IN THE AV SIGNAL AT EACH FOIL
POS IS DETERMINED FROM THE SUM SQR OF RESIDUALS IF MORE THAN
ONE SIGNAL IS TAKEN AT THAT FOIL POS. IF ONLY ONE SIG IS TAKEN A
CALC BASED ON ERROR=COUNT OF CTS IS USED AND IS CALLED ERR(I).
THE CORRELATION COEFFICIENT BETWEEN THE SIGNAL
AND MONITOR IS CALC. IF ONLY ONE SIG IS TAKEN AT ONE FOIL POS
CORR IS SET TO 0.0. THE BACKGROUND GAS RATE NORMALIZED TO THE
CHosen IP21 COUNT RATE (SNORM) IS SUBTRACTED FROM
THE AVG SIG CTING RATE AT EACH FOIL POS. THIS VALUE+ITS STANDARD
DEV APL PRINTED AND PUNCHED OUT ALONG WITH THE FOIL POS.

LOCALS OF FOIL POSITIONS

FOIL-FOIL THICKNESS
MAX=TOTAL NUM OF DATA PTS AT ALL FOIL POSNS. VEL=VELOCITY OF ION.
RUN=THE NUMBER DESIGNATING THE CHRONOLOGY OF THE DATA TAKEN ON
THE SAME DAY.

SNORM=IP21 CTS TO WHICH DATA IS NORMALIZED. BACKGR AND BACKGP
=62565 AND IP21 COUNTS RESPECTIVELY PER MIN WITH BEAM ON NO FOIL.
GENORM=THE IP21 COUNT RATE THAT WOULD HAVE IF A FOIL WERE IN
THE HOLDER WHEN TAKING BACKGROUND GAS COUNT RATE WITH NO FOIL.
THIS IS DETERMINED BY MATCHING THE BEAM CURRENT USED WHEN
MEASURING BACKG WITH THAT USED DURING A REGULAR RUN. GSNORM
AND SNORM WILL USUALLY BE ABOUT EQUAL.

BACKS=62565 DARK COUNTS BEFORE SIGNAL AT EACH FOIL POS.
BACK2=62565 DARK COUNTS AFTER SIGNAL AT EACH FOIL POS.
DE=FOIL POSN IN INCHES. T=SEC TIME WHEN GETTING BACKS AND BACKG.
A=62565 COUNTS. X=IP21 COUNTS. T=TIME WHEN CTING A AND W.
Y(I)=NORMALIZED 62565 DATA WITH BACKGROUND SUBTRACTED.
X(I)=TIME AFTER INITIAL EXCITATION TIMES 108.
Z(I)=IP21 COUNTS-BACKGROUND.
C(I)=62565 CTS-BACKGROUND UNNORMALIZED
AVE=AVE OF NORMALIZED 62565 CTS AT EACH FOIL POS
ERR(I)=STANDARD OF EACH Y(I). USING SIG=SQR OF CTS.
RES=SUM OF SQ OF RESIDUALS. SIG=STANDARD DEV OF Y(I) BY RESIDUALS.
CORR=CORRELATION COEFF FOR Y(I) AND Z(I).
XMAX=MAXIMUM VALUE OF X(I) ROUNDED UP TO NEAREST .1

DIMENSION Y(30), Z(30), X(30), ERR(30), G(30)

A SAMPLE SET OF DATA TAKEN AT 21 FOIL POS WITH 4 READINGS AT
EACH POS. TAKEN ON 5/6/68 WITH A 20 MICROGRAV. FOIL WOULD BE

<table>
<thead>
<tr>
<th>FOIL</th>
<th>COUNTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>64</td>
</tr>
<tr>
<td>3514</td>
<td>50000</td>
</tr>
<tr>
<td>623</td>
<td>624</td>
</tr>
<tr>
<td>8491</td>
<td>3496</td>
</tr>
<tr>
<td>11125</td>
<td>48042</td>
</tr>
<tr>
<td>12766</td>
<td>52961</td>
</tr>
<tr>
<td>12436</td>
<td>51190</td>
</tr>
<tr>
<td>624</td>
<td>626</td>
</tr>
<tr>
<td>8713</td>
<td>60898</td>
</tr>
<tr>
<td>7436</td>
<td>50805</td>
</tr>
</tbody>
</table>
The first two header cards are being read and printed out.

19 WRITE(6,16)
16 FORMAT(1HI)
  L=8
  READ(5,100)LFOIL,MAX,DATE
100 FORMAT(313,F6.0)
  IF(MAX)36,36,88
88 READ(5,101)VEL,SNORM,BACKG,GSNORM,FOIL
101 FORMAT(5F10.4)
  WRITE(6,200)
200 FORMAT(10H0,6X3HVEL,5X10HNORM CONST,2X16HEMI: TUBE GAS/IN.3X9FOIL
  1THMXX,3X3HRUN,4X4HDATL,5X8HDATA PTS,4X14FOIL POSITIONS,2X14HGAS "NO-
  IR" CONST)
  WRITE(6,201)VEL,SNORM,BACKG,FOIL,MAX,DATE,LFOIL,GSNORM
201 FORMAT(1H +2(F12,4,2X),F6.0,8X,F12,4,9X,12,2X,F6.0,4X,15,1UX,15,
  13XF12,4,7/)

Control card is read and calculation of backs and backp is carried out.

2000 READ(5,102)BACKS1,BACKS2,BACKP1,BACKP2,D,TIMBSK
102 FORMAT(6F10.4)
  FACTOR=TIMBSK
999 WRITE(6,202)
202 FORMAT(10H0,3X8HEMI,BKGD,5X9H1P21,BKGD,6X12HFOIL POS(IN),3X16H5KGD
  1CT TIME)

When changing background counting time calculation of backs and backp must be modified to obtain the correct answer. The ratio

BACKS=(BACKS1*(TIMBSK/FACTOR)+BACKS2)/2.
BACKP=(BACKP1*(TIMBSK/FACTOR)+BACKP2)/2.
FACTOR=TIMBSK
WRITE(6,203)BACKS,BACKP,D,TIMBSK
203 FORMAT(3(F12,4,2X),3X,F12,4,7/)
WRITE(6,204)
204 FORMAT(1H +4X3H1EMI,1UX4H1P21,11X4HTIME,12X4HY(1),10X,4HX(1),7X6HER-
  10(1),6X9H1P21-BKGD,5X8HEMI-BKGD)
IN THE LAST CARD READ IN AT EACH FOIL POS WILL BE THE SIGNAL AND MONITOR TUBE'S BACKGROUND CARD OF THE NEXT FOIL POS. THIS IS STORED IN THE 6 VARIABLES IN READ 102. THEREFORE THE LAST CARD OF A DATA SET MUST BE A DUMMY CARD WITH A 1.0 IN COLUMNS 31-40 IN ORDER FOR IT TO STOP TRYING TO READ MORE DATA IN. THIS QUANTITY IS CALLED CHECK IN THE PROGRAM AND WAS INCORPORATED TO ALLOW A VARYING NUMBER OF SIGNAL READINGS TO BE READ IN AT EACH FOIL POS (I.E. NOT JUST 4 AT EACH POSITION).

5 READ(5,103)A,N,T,CHECK,ADD1,ADD2
103 FORMAT(6F10.4)
   1IF(CHECK) 36,3,4
   3 X(I)=C/VEL
   4 T=AT
   5 CT=BACKS/TIMBKS
   6 Z(I)=W/CT-(BACKP/TIMBKS)
   7 CT=CT-BT
   8 CT=(CT-BT)*S/IZ(I)
   9 Y(I)=CT-(BACK2*S/IZ/IZSNORM)

THE STANDARD DEV OF EACH SIGNAL READING IS CALCULATED IT IS SIMPLY THE SQUARE OF THE SUM OF SQUARES OF THE RELATIVE ERRORS TIMES THE NORMALIZED VALUE OF THE SIGNAL MINUS BACKGROUND, ASSUMING NO ERROR IN THE BACKGROUND GAS CONTRIBUTION (BACKGS).

AERR=ET/ST/(2.0*TIMBKS)
CERR=ET/(2.0*TIMBKS**2)
THE FACTOR OF 2 IN DENOMINATOR IS THERE SINCE BACKS + BACKP ARE THE MEAN OF 2 VALUES, THEREFORE WANT ST. DEV. OF MEAN.

ER=SQRT((AERR/O(I)**2+CERR/Z(I)**2)*Y(I)**2)
ERR(I)=ER
SUM=SUM+Y(I)
SUMQ2=Q(I)*Z(I)+SUMQ2
SUMQ=Q(I)+SUMQ
SUMZ=Z(I)+SUMZ
SUMC2=Q(I)**2+SUMC2
SUMC=Q(I)+SUMC
SUMZ=Z(I)**2+SUMZ

WRITE(6,205)A,N,T,Y(I),X(I),ERR(I),Z(I),O(I)
205 FORMAT(6H,3(F10.2,3X),5X,F10.2,3X,F10.4,3X,F10.4,3X,F10.2,3X)
1F10.2)
1 I=1+1
GO TO 5
4 R=I-1
K=I-1
BACKS1=A
BACKP1=T
BACKP2=CHECK
D=ADD1
TIMBKS=ADD2
AVE=SUM/R
YAVE=ALOG(AVE)/2.30258

SIG1=0.0
DO 21 I=1,K
   1IF(R=I) 3001,3001,3002
   299 SIGI=SIGI+(ERR(I)/R)**2
   3001 SIG1=SIG1+(ERR(I)/R)**2
   3002 CORR=0.0
GO TO 21
RES = (Y(i) - AVE)^2 + RES
SIG = SQRT(RES / (R * (R - 1.0)))
CORR = 0.0
CONTINUE
WRITE (0, 206)
206 FORMAT (3H10, 4X, 3HAVE, 5X, 7HLOG AVE, 2X, 3HSIG, 12X4H, CORR)
WRITE (6, 207) AVE, YAVE, SIG, CORR
207 FORMAT (4X, 12HLOG AVE, SIG, CORR)
WRITE (7, 300) X(K), AVE, SIG
300 FORMAT (10.5, 2F10.2)
L = L + K
IF (MAX - L .GT. 19, 19, 999)
36 CALL EX
END
APPENDIX E

DEPOP and REPOP Programs

These two computer programs were used to determine all the one- and two-electron allowed transitions into and out of a particular electronic term. The selection rules that must be obeyed for these electric-dipole transitions are

\[ \Delta n = 0, \pm 1, \pm 2, \pm 3, \]
\[ \Delta S = 0, \]
\[ \Delta L = 0, \pm 1, \]
\[ \Delta \pi = \pm 1 \]
\[ \Delta \ell = \pm 1 \quad \text{one-electron transition} \]
\[ = \pm 2 \quad \text{two-electron transition} \]

The listing of the program is given in the following pages.
ENERGY LOSS
STEP EXEC FOR PCC
PORT SYSINPUT
DECP AND REPUP-PROGRAM DETERMINING THE DEPOLULATING AND
REPOPULATING TERM FOR A GIVEN TERM.
CARD 1=NC. DATA SETS. CII, CII, CIII ARE EACH DATA SETS.
   COL. 1-5 FORMAT 15
CARD 2=NO. OF AVERAGE ENERGY TERMS.
   COL. 1-5 FORMAT 15
CARD 3=NO. ENERGIES TO BE AVERAGED FOR A PARTICULAR TERM; THE
QUANTUM NUMBERS OF THE TERM N. L. J AND TOTAL PARITY.
   FORMAT 6/I
CARD 4=INDIVIDUAL ENERGIES TO BE AVERAGED.
   FORMAT 15/1
CARD 5=QUANTUM NUMBERS J FOR EACH OF AVERAGED ENERGIES.
   FORMAT 15/1
CARD 6=QUANTUM NUMBERS L FOR EACH OF THE AVERAGED ENERGY TERMS.
CARD 7=END OF DATA.
ONE CARD GIVING NO. OF ENERGY TERMS TO BE STUDIED
   FORMAT 15
ONE CARD FOR EACH ENERGY TERM TO BE STUDIED LABELING IT WITH ITS
POSITION IN THE LIST OF AVER. ENERGY TERMS IS THE FIRST ONE
WOULD BE LABELLED 1.
   FORMAT 15
SELECT J. PROMPT ENTER
DELTA NO. 1, 2, -1, 2, -2, 2, 3, -3.
DELTA 2=1=0
DELTA TOTAL L=O, -1, -1.
DELTA PARITY=O, -1, 1.

IMPLICIT REAL*8 (A-H,0-Z)
REAL INDEX
DIMENSION N(10), L(100), J(100), KKK(100), J(100), IP(100)
DIMENSION EX(5), IND(5), E(100), IPH(10), IFLU(5)
DIMENSION WAVE(100)
IR=1
IP=0
N=NUMBER OF PROBLEMS
READ(10,1)PROBS
DO 100 J=1, N, 1
   READ(10,1)J,
N=NUMBER OF ENERGY LEVELS
1 FORMAT(6/15)
READ IN ENERGY AND AVERAGE
DO 100 KKK=1, NEL
   READ(10,1)J, N, KKK, L(KKK), J(KKK), K(KKK), IP(KKK)
J=NUMBER OF J FOR SPECIFIC ENERGY LEVEL
   S=0
   L=0
   IS=2S+1
   K=K
   IP=PARITY O IF ODD, 1 IF EVEN
2 FORMAT(6/15)
   READ(10,1)INDEX(J), J=1,JN
3 FORMAT(6/15)
   INDEX=J VALUE FOR ENERGY LEVEL
   S=0
   DENOM=0.0
DO 10 J=1,JN
S=S+(2.*INDEX(J)+1.)*SE(J)
10 DENOM=DENOM+2.*INDEX(J)+1.
E(KKK)=S/DENOM
READ(IR,1)KPROB
DO 999 KK=1,KPROB
ICHECK=0
READ(IR,1)ISEL
ICOUNT=0
IF=1
ISEL1=ISEL-1
13 DO 15 I=IF,ISEL1
IF(ABS(N(I)-K(ISEL1))=3)14,14,15
14 IF(IS(I))=15(ISEL))=15,16,15
16 IF(IS(I))=1P(ISEL))=18,15,18
18 IF(ABS(N(I)-K(ISEL1))=1)19,19,15
19 ICOUNT=ICOUNT+1
IENH(I)=IENH(I)+1
IENH(I)=IENH(I)
COUNT=COUNT+1
CONTINUE
13 DO 15 I=1,NUM
33 IF=1+1
35 WAVE(I)=ABS(1(ISEL)-E(IELOW(I)))*1.D+0B
33 IF(NUM)=25,25,26
26 DO 40 I=1,NUM
39 IJF=1+1
40 J=JF+NUM
IF(WAVE(1)-WAVE(J))=39,40,40
39 AS=WAVE(1)
WAVE(1)=WAVE(J)
WAVE(J)=AS
ISE=IENH(I)
IENH(I)=IENH(J)
IENH(J)=ISE
ISE=IELOW(I)
IELOW(I)=IELOW(J)
IELOW(J)=IES
40 CONTINUE
25 WRITE(IW,9)
9 FORMAT(IH1)
IF(ICHECK)=27,28,27
28 WRITE(IW,11)
11 FORMAT(* LOWER ENERGIES,*)
20 WRITE(IW,4)
4 FORMAT(* NLSK,5X,ENERGY,5X,NLSK,5X,ENERGY,5X,LENGTH
1(ANGSTROMS),5X)
IF(ICHECK)=30,31,30
31 DO 50 I=1,NUM
50 WRITE(IW,5)N(ISEL),L(ISEL),IS(ISEL),K(ISEL),E(ISEL),N(ILOW(I)),
1L(ILOW(I)),IS(ILOW(I)),K(ILOW(I)),E(ILOW(I)),WAVE(I)
ICHECK=1
IF=ISEL+1
ISEL=ISEL+1
50 CONTINUE
GO TO 13
27 WRITE(IW,12)
12 FORMAT(* HIGHER ENERGIES,*)
GO TO 20
42 DO 43 I=1,NUM
43 WAVEL(I) = 1.0 * (E(IEHIGH(I)) - E(ISEL)) * 1.D+08
GO TO 45
30 DO 55 I = 1, NUM
55 WRITE(1M5) N(IEHIGH(I)), L(IEHIGH(I)), IS(IEHIGH(I)), K(IEHIGH(I)),
    1E(IEHIGH(I)), N(ISEL), L(ISEL), IS(ISEL), K(ISEL), E(ISEL), WAVEL(I)
5 FORMAT(1X,41Z, ('F10.2'), 2X, 'F10.2')
999 CONTINUE
1000 CONTINUE
CALL EXIT
END