ANALYTICAL, EXPERIMENTAL AND NUMERICAL STUDY ON THE MECHANICAL BEHAVIOR OF 3D PRINTED AUXETIC CHIRAL STRUCTURES

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ANALYTICAL, EXPERIMENTAL AND NUMERICAL STUDY ON
THE MECHANICAL BEHAVIOR OF 3D PRINTED
AUXETIC CHIRAL STRUCTURES

BY

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THESIS

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## Table of Contents

Chapter 1  Background and Introduction .................................................................1

1.1  Review on auxetic materials........................................................................1

1.2  The missing rib model..................................................................................2

1.3  Fabrication of auxetic materials ..................................................................4

1.4  Broad applications .......................................................................................5

Chapter 2  Analytical Mechanical Model of Auxetic Chiral Structures .............6

2.1  A rigid-rod-rotational-spring model ...............................................................6

2.2  Negative Poisson’s ratio and internal rotation ..............................................7

2.3  Analytical prediction of the effective stiffness .............................................14

2.4  Stability analysis .........................................................................................15

Chapter 3  Conceptual Design of New Chiral Structures ........................................19

3.1  Design and 3D printing of auxetic chiral structures ......................................19

3.2  Mechanical experiments on materials used in the 3D printer ......................21

3.3  Hyperelastic modeling of materials in the 3D printer ...................................25

3.4  Mechanical experiments on the 3D printed structures ..................................27

3.5  Finite element (FE) simulations of the experiments .....................................33

Chapter 4  FE Simulations on Auxetic Chiral Structures with Soft Hinges ........38

4.1  The effects of the stiffness of soft hinges ....................................................38

4.2  The effects of soft hinges on the active ribs ..............................................45
4.3 The effects of rib thickness ................................................................. 49
4.4 The auxetic effects under large deformation ...................................... 53

Chapter 5 FE Simulations of Auxetic Chiral Structures with Center Cores ........ 57
5.1 The effects of the stiffness of core materials ...................................... 57
5.2 The effects of core size ......................................................................... 64

Chapter 6 Conclusions and Future Work ................................................... 70
6.1 Conclusions ......................................................................................... 70
6.2 Future work ......................................................................................... 71

References ................................................................................................. 75

Appendix A: Hyperelastic Models for the Materials in the 3D Printer ............. 84
Appendix B: The Effect of Interphase in the 3D Printed Specimens ................ 89

Figure List ................................................................................................ 92
Table List ................................................................................................... 98
ABSTRACT

ANALYTICAL, EXPERIMENTAL AND NUMERICAL STUDY ON

THE MECHANICAL BEHAVIOR OF 3D PRINTED

AUXETIC CHIRAL STRUCTURES

by

Yunyao Jiang

University of New Hampshire, September, 2016

A unique deformation mechanism of auxetic chiral structure was investigated via integrated analytical, experimental and numerical approaches. This unique deformation mechanism is the auxetic effect due to internal rotation. In this thesis, first, a rigid-rod-rotational-spring model was developed to capture the quantitative relation between the Poisson’s ratio and internal rotation of the cells in the auxetic chiral structure. It was concluded that the auxetic effects will be amplified with elevated internal rotation. Then, based on this concept, two new auxetic chiral structures were designed: (1) auxetic chiral structure with soft hinges at the corners, and (2) auxetic chiral structure with hard cores in the center. The original and the two new chiral structures were fabricated via a multi-material 3D printer (Objet Connex260). Mechanical experiments under uni-axial compression were performed to prove the concept. It was proved that both new structures can effectively amplify the auxetic effects through increased internal rotation. In addition, to further explore the mechanical behaviour of the two new designs, thorough parametric study was performed via Finite Element (FE) simulations. Design guidelines for the auxetic chiral structure were provided.
The new designs of auxetic chiral structures enable tailorable negative Poisson’s ratio under large deformation. These new cellular structures have broad potential applications in designing industrial energy absorption foams, impact and tearing resistant materials, drug-delivery medical bandages, biomedical scaffolds, responsive/smart façade of architectures, and metamaterials for optical, electro-magnetic devices and components.
Chapter 1  Background and Introduction

1.1 Review on auxetic materials

The concept of materials with negative Poisson’s ratio i.e. auxetic materials first appeared in 1944 [1]. The term ‘auxetic’ was first used in [2]. The first artificial specimen of auxetic re-entrant structure was proposed in [3-5]. Since then, different auxetic models have been proposed and analyzed [6–16]. It is well known that auxetic materials have many engineering advantages due to their superior properties. For example, compared with the conventional materials, auxetic materials have increased indentation resistance [17–19], shear resistance, energy absorption capability [20], and variable permeability [21, 22]. They can be used as fasteners [23, 24], to achieve synclastic curvature [5, 25], and to develop shape memory materials [26]. In addition, auxetic materials have better acoustic and vibration properties over their conventional counterparts [27–30].

Artificial auxetic materials include cellular foams [5, 31], two-phase composites [32], and several molecular structures [33–35]. Some auxetic materials were created via innovative design [3-5], some were obtained via topology optimization [36, 37] and periodic tessellation [38]. There are also a few auxetic natural materials [1, 34, 39, 40] and biomaterials, such as cow teat skin [41] and cancellous bones [42]. Among all of them, auxetic cellular materials are one very important category due to their lightweight and excellent energy absorption capability for engineering applications.

Based on different deformation mechanisms, existing auxetic cellular structures can be
roughly classified into two categories, (1) structures with re-entrant angles, and (2) structures that rotate when deform. Models beyond these two categories can be found in [43-51]. Extended reviews of auxetic materials can be found in [52–54].

1.2 The missing rib model

For symmetric cellular materials such as honeycomb and re-entrant honeycomb, when the macroscopic strain reaches a critical value, instability will occur to break the symmetry; after the instability, the Poisson’s ratio goes to zero or positive, therefore, the auxetic effects disappear beyond the critical strain to instability. Compared with these conventional symmetric cellular materials, chiral structures are expected to be more stable under both external loads and internal pressure. Therefore, they are better candidates for preserving auxetic effects under large deformation.

The current study was motivated by the pioneering work on one of the earliest chiral structures, the ‘missing rib model’ [55, 56]. In 90s, when conventional polymeric foams with positive Poisson’s ratio were treated via the compression/heat-treatment process, they became auxetic. It was observed that during the process, some of the ribs in the foam broke, as was the reason for the auxetic effects after the treatment. An analytical 2D ‘missing rib model’ was proposed [55] to explain the auxetic effects due to rib missing. The unit cells of the intact model and the ‘missing rib model’ are shown in Fig.1.1. Fig.1.1a shows the unit cell of a intact diamond-shaped cellular structure, and Fig.1.1b shows that by selectively deleting some ribs (dash lines) from the intact model, a chiral structure is generated, which was then called ‘missing rib model’.
Figure 1.1 The unit cells of (a) a diamond-shaped intact model, and (b) the ‘missing rib model’.

In the analytical ‘missing rib model’ [55], the Poisson’s ratio of the classical chiral structure was derived as -1. However, finite element simulations of the missing rib model showed that the Poisson’s ratio of a real ‘missing rib’ structure is ∼0.4 [56, 57], as was confirmed by a recent uni-axial tension experiment on a missing rib structure fabricated in micro-scale [57]. This big difference between the analytical prediction and experiments is because of the unrealistic assumptions made in the analytical model: the middle cross in the ‘missing rib model’ is rigid, and the connections between other ribs are rotational free. To overcome this drawback of the original analytical model, a modified ‘missing rib model’ was proposed later [56], in which, a new parameter was introduced to relate the deformation of the center cross to the macroscopic strain. However, the new parameter lacks a clear physical meaning and is hard to be determined. The modified missing rib model was therefore mainly used for explanation rather than prediction. A predictive analytical model is needed to systematically explore the mechanical properties of this type of chiral structure.

In this thesis, an analytical rigid-rod-rotational-spring model is developed in Chapter 2 to fill this gap in literature. This model consider the structural resistance of both the center cross
and the corners and therefore can predict the real physical behavior of the missing rib type of chiral structures under both small and large deformation.

### 1.3 Fabrication of auxetic materials

For auxetic materials, existing manufacturing methods were specifically designed for a certain type of auxetic foam such as bio-medical scaffolds [58], auxetic composite laminates [59] and auxetic textiles made of helical yarns [60-62]. A lot of these manufacturing processes have many limitations. For example, the traditional manufacturing method of making auxetic foams is to convert conventional foam to an auxetic foam [5, 63-65] via compression and heating. This process usually contains several steps including compression in three directions, heating, cooling, and relaxation. Different types of polymeric foams require different heating time and temperature and a lot of foams are very sensitive to processing temperature and humidity. These manufacturing parameters can only be empirically obtained. These factors make the traditional manufacturing process to make auxetic foams very difficult and not very reliable. Also, through this process, the microstructures and the size of the cells are stochastic which will affect the overall mechanical properties.

The fast development in additive manufacturing enables easy and precise control of the geometry and material composition of complex structures [66], providing an opportunity to release those constraints and drawbacks in the traditional manufacturing process of auxetic materials. The proposed research will take this opportunity, to first expand the database of auxetic structures via innovative conceptual designs, and then fabricate the new designs via a multi-material 3D printer (Objet Connex 260). Finally, mechanical experiments and FE
simulations will be performed to further evaluate the auxetic chiral structures designed.

### 1.4 Broad applications

The auxetic chiral structures studied in this thesis have broad applications in the following areas: (1) lightweight auxetic materials for protection and energy absorption. For example, energy absorption foams in body and vehicle armors, packaging industry and aerospace industries; (2) bio-medical material and devices. For example, the auxetic chiral structures can be used to design medical scaffolds, bandages, drug reservoirs and Drug Eluting Stents (DES) [67-69]; (3) metamaterials with unique properties such as negative refractive index, and superior optical activities, also with superior wave-propagation properties, such as wave guides, and noise reducers [66, 70-76]; (4) smart responsive composites, which can be used to design next generation of actuators [77] and sensors, smart composites and responsive surfaces.
Chapter 2 Analytical Mechanical Model of Auxetic Chiral Structures

2.1 A rigid-rod-rotational-spring model

One of the unique deformation mechanisms of the chiral structure is the auxetic effect due to significant internal rotation. In this chapter, a rigid-rod-rotational-spring model is developed to study this unique deformation mechanism. In this model, the internal rotation is modelled via the deformation of the rotational springs. The schematics of the theoretical model for a general chiral cell are shown in Fig.2.1.

![Figure 2.1](image)

Figure 2.1. (a) The geometry of a general chiral cell and (b) the rigid-rod-rotational-spring model.

Fig.2.1a shows that the length of all ribs are the same $a$, which determines the scale of the cell. Based on different functions, the ribs are categorized into active ribs and passive ribs. By
definition, the active ribs are those located along the loading direction, such as ribs \( A-B-O-C-D \) in Fig.2.1a; and the passive ribs are those located along the direction orthogonal to the loading direction, such as ribs \( E-F-O-G-H \). As shown in Fig.2.1a, the configuration of the chiral cell is determined by two angles, the angle \( \alpha \) between the active ribs \( A-B \) and \( B-O \), and the angle \( \beta \) between the passive ribs \( E-F \) and \( F-O \). To model the constraints from the neighbouring cells, during deformation, points \( E \) and \( H \) are always on the same horizontal level, and points \( A \) and \( D \) are always along the same vertical line.

To model the relative rotation between the ribs and the rotational flexibility of the ribs, one rotational spring with rotational stiffness \( K_\theta \) is introduced to the center \( O \), as shown in Fig.2.1b, and two rotational springs with rotational stiffness \( K_\alpha \) are introduced to the corners \( B \) and \( C \) on the active ribs, and two rotational springs with rotational stiffness \( K_\beta \) are introduced to the corners \( F \) and \( G \) on the passive ribs. In this model. The ribs are assumed to be rigid, the bending effects of the ribs and the rotational flexibility of the cells were both represented by the deformation of the rotational springs. Thus, the chiral cell becomes a system of rotational springs and rigid rods.

### 2.2 Negative Poisson’s ratio and internal rotation

When the cell is subjected to uni-axial compression at \( A \) and \( D \), as shown in Fig.2.1a, the active ribs \( B-O-C \) will rotate clockwise with an angle \( d\varphi_a \), which will drive the rotation of the passive ribs \( F-O-G \) to rotate with an angle \( d\varphi_p \) in the same direction. The rotation of the ribs inside the cell leads to the overall negative Poisson’s ratio of the chiral cell. To quantify the internal rotation of the cell, the instantaneous internal rotational efficiency \( R_{p/a} \) is defined,
which is the incremental form of the ratio between the rotation angle of the passive ribs and that of the active ribs as shown in Eq. (2.1):

$$ R_{p/a} = \frac{d\varphi_p}{d\varphi_a} \quad (2.1) $$

The rotation efficiency $R_{p/a}$ can only vary from 0 to 1.

The instantaneous Poisson’s ratio under uniaxial loading along direction 2 is defined as

$$ \nu_{21} = -\frac{d\varepsilon_1}{d\varepsilon_2} \quad (2.2) $$

From the general geometry of the unit cell (Fig. 2.1), the cell length $L_1$ along direction 1 ($L_1 = EH$ in Fig. 2.1a) is related to the length of the rib segment $a$ and the angle $\beta$ as:

$$ L_1 = 4a \sin \frac{\beta}{2} \quad (2.3) $$

Similarly, the cell length $L_2$ along direction 2 ($L_1 = AD$ in Fig. 2.1a) is related to $a$ and $\alpha$ as:

$$ L_2 = 4a \sin \frac{\alpha}{2} \quad (2.4) $$

During the deformation, the change of cell length are related to the change of angles $\alpha$ and $\beta$ via the differential form of Eqs. (2.3) and (2.4) as the following:

$$ \frac{dL_1}{d\beta} = 2a \cos \frac{\beta}{2} \quad (2.5) $$

$$ \frac{dL_2}{d\alpha} = 2a \cos \frac{\alpha}{2} \quad (2.6) $$

According to Eq. (2.2), the instantaneous Poisson’s ratio can be expressed as,

$$ \nu_{21} = -\frac{\frac{dL_1}{L_1} \frac{L_2}{dL_2}} \quad (2.7) $$

and then based on Eqs. (2.5), (2.6), and (2.7), the instantaneous Poisson’s ratio is derived as:

$$ \nu_{21} = -\frac{\tan \frac{\alpha}{2} \frac{d\beta}{d\alpha}}{\tan \frac{\beta}{2} \frac{d\beta}{d\alpha}} \quad (2.8) $$

As shown in Fig. 2.1a, based on the kinematics, $d\beta$ and $d\alpha$ are related to $d\varphi_p$ and $d\varphi_a$ via:

$$ d\beta = -2d\varphi_p \quad (2.9) $$
\[ d\alpha = -2d\varphi_a. \]  

Therefore, according to Eqs. (2.1), (2.9) and (2.10), the instantaneous internal rotational efficiency \( R_{p/a} \) was derived to be related to the ratio between \( d\beta \) and \( d\alpha \) as:

\[ R_{p/a} = \frac{d\beta}{d\alpha}. \]  

By substituting Eq. (2.11) into Eq. (2.8), the Poisson’s ratio of the chiral cell is related to the rotational efficiency \( R_{p/a} \) and the angles \( \alpha \) and \( \beta \) as:

\[ \nu_{21} = \frac{\tan(\alpha/2)}{\tan(\beta/2)} R_{p/a}. \]  

Eq. (2.12) shows that for a certain configuration of the chiral cell, the larger \( R_{p/a} \) is, the smaller the negative Poisson’s ratio. Eq 2.12 is plotted in Fig.2.2.

![Figure 2.2](image_url)

**Figure 2.2.** The influences of geometry on the negative Poisson’s ratio and the rotation efficiency predicted by the rigid-rod-rotational-spring model.

Fig.2.2 shows that by varying \( \alpha \) and \( \beta \), the negative Poisson’s ratio can be tailored in a very large range \((0, -\infty)\). It can be seen that when \( \alpha \) increases and/or \( \beta \) decreases, the Poisson’s ratio \( \nu_{21} \) decreases; and when \( \alpha \) equals to \( \beta \), \( \nu_{21} = -R_{p/a} \).
Interestingly, from the equilibrium of the rotational-spring-rigid-rod system, the rotational efficiency $R_{p/a}$ can be directly related to the rotational stiffness ratio $R_s$ between the central spring and the corner springs on the passive ribs, where, $R_s$ is defined as

$$R_s = \frac{K_\theta}{K_\beta} ,$$

(2.13)

where $K_\theta$ is the rotational stiffness of the center spring and $K_\beta$ is the rotational stiffness of the corner springs on the passive ribs.

The free body diagrams of the passive ribs are shown in Fig.2.3. Since points $E$ and $H$ can move freely along the horizontal direction, no horizontal forces exist along the horizontal direction and only vertical forces $F_2$ exist at the two points.

![Free body diagram of passive ribs](image)

Figure 2.3 The free body diagrams of the active ribs in the unit cell.

The rotational moments generated by the rotational spring in the center and at the corners are $M_\theta$ and $M_\beta$, respectively. The constitutive equations of the rotational springs are:

$$M_\beta = K_\beta d\beta ,$$

(2.14)

$$M_\theta = K_\theta d\theta .$$

(2.15)

According to Fig.2.3, the moment equilibrium of the rib $E-F$ or $G-H$ yields

$$\alpha F_2 \sin \frac{\beta}{2} = K_\beta d\beta .$$

(2.16)
The moment equilibrium of the ribs $F-G$ shown in Fig. 2.3 yields

$$2aF_2 \sin \frac{\beta}{2} + 2K_\beta d\beta = K_\theta d\theta .$$  \hspace{1cm} (2.17)

Then, Eqs. (2.16) and (2.17) yields

$$4K_\beta d\beta = K_\theta d\theta .$$  \hspace{1cm} (2.18)

From the geometry shown in Fig. 2.1a, the angle $\theta$ between ribs $F-O$ and $C-O$ is related to the rotation angle of the active and passive ribs as

$$\theta = \varphi_p + \frac{\pi}{2} - \varphi_a .$$  \hspace{1cm} (2.19)

The angels $\varphi_a$ and $\varphi_p$ are related to the angles $\alpha$ and $\beta$ from the geometry shown in Fig. 2.1a as

$$\varphi_a = \frac{\pi - \alpha}{2} ,$$  \hspace{1cm} (2.20)

$$\varphi_p = \frac{\pi - \beta}{2} .$$  \hspace{1cm} (2.21)

Therefore, the angle $\theta$ can also be related to the angle $\alpha$ and $\beta$ as

$$\theta = \frac{\alpha}{2} + \frac{\pi}{2} - \frac{\beta}{2} .$$  \hspace{1cm} (2.22)

The differential form of Eq. (2.19) gives

$$d\theta = d\varphi_p - d\varphi_a .$$  \hspace{1cm} (2.23)

By substituting Eqs. (2.9) and (2.23) into Eq. (2.18), Eq. (2.24) is obtained:

$$-8K_\beta d\varphi_p = K_\theta (d\varphi_p - d\varphi_a) .$$  \hspace{1cm} (2.24)

Therefore, according to Eqs. (2.1), (2.13), and (2.24), the rotational efficiency $R_{p/a}$ can be directly related to the rotational stiffness ratio $R_s$ as,

$$R_{p/a} = \frac{R_s}{R_s + 8} .$$  \hspace{1cm} (2.25)

Eq. (2.25) is plotted in Fig. 2.4, which shows that the rotational efficiency $R_{p/a}$ is only
determined by $R_s$ and does not depend on the configuration of the cell and the rotational stiffness $K_{\alpha}$ of the springs on the active ribs.

Eq. (2.25) is plotted in Fig. 2.4. Fig. 2.4 shows that generally, when $R_s$ increases, the rotational efficiency $R_{p/a}$ will increase, and for a certain range of $R_s$ (~1-100), $R_{p/a}$ is very sensitive to $R_s$. Specifically, when $R_s$ increases from zero to infinity, the rotational efficiency $R_{p/a}$ will increase from zero to 1. When $R_s$ changes from ~1-100, the rotational efficiency $R_{p/a}$ will dramatically increase from ~0.11 to ~0.93, while when $R_s$ changes beyond this range, $R_s$ only slightly influences $R_{p/a}$.

![Figure 2.4](image)

Figure 2.4. The relation between the rotation efficiency and the rotational stiffness ratio of the center and corner springs.

According to the theoretical model, (Eqs.2.12 and 2.25), the instantaneous Poisson’s ratio of the chiral cell is determined by both the geometry of the cell and the rotational stiffness ratio between the center spring and that of the corner springs on the passive ribs via

$$\nu_{21} = \frac{\tan(\alpha/2)}{\tan(\beta/2)} \frac{R_s}{R_s + 8}.$$  \hspace{1cm} (2.26)
To explore the behavior of different cell geometry, three different cell configurations are selected, which are $\alpha = 60^\circ, \beta = 120^\circ$; $\alpha = 90^\circ, \beta = 90^\circ$; and $\alpha = 120^\circ, \beta = 60^\circ$; as shown in Fig.2.5.

Figure 2.5 Poisson’s ratio predicted via the rigid-rod-rotational-spring model with different geometries.

Fig.2.5 shows the prediction of the Poisson’s ratios for chiral cells with different geometries and different rotational stiffness ratios via Eq. (2.26). It can be seen that when the rotational stiffness ratio increases, the Poisson’s ratio will decrease, and when the stiffness ratio changes from zero to infinity, the Poisson’s ratio will change from zero to a certain value: -0.66 for geometry 1, -1 for geometry 2 and -3 for geometry 3 in Fig.2.5. It worth noting that the rotational stiffness ratio $R_s$ is nonlinearly related to the rotational efficiency $R_{p/\alpha}$ and the
Poisson’s ratio $\nu_{21}$. When $R_s$ is in the range of ~1 to 100, the $R_{p/a}$ and $\nu_{21}$ will be significantly influenced by $R_s$.

### 2.3 Analytical prediction of the effective stiffness

The effective stiffness of the cell along direction 2 (loading direction) $E_2$ can be derived from the energy method. The variational form of strain energy $U$ of the unit cell is equal to the deformation energy of the rotational springs,

$$\delta U = 2 \left[ \frac{1}{2} K_\alpha (\delta \alpha)^2 \right] + 2 \left[ \frac{1}{2} K_\beta (\delta \beta)^2 \right] + \frac{1}{2} K_\theta (\delta \theta)^2 .$$  \hspace{1cm} (2.27)

On the other hand, the virtual work $\delta W$ to deform the unit cell is:

$$\delta W = \frac{1}{2} L_1 L_2 E_2 \left( \frac{\delta \theta}{l_2} \right)^2 .$$  \hspace{1cm} (2.28)

By substituting Eq.(2.11), the second term in Eq.(2.27) which is $2 \left[ \frac{1}{2} K_\beta (\delta \beta)^2 \right]$ can be re-written in terms of $R_{p/a}$ and $\delta \alpha$ as,

$$2 \left[ \frac{1}{2} K_\beta (\delta \beta)^2 \right] = K_\beta \left( \frac{R_{p/a}}{a} \right)^2 (\delta \alpha)^2 .$$  \hspace{1cm} (2.29)

By combining Eqs. (2.25), (2.13), (2.23), (2.9), and (2.10), the third term in Eq.(2.27) which is $\frac{1}{2} K_\theta (\delta \theta)^2$ can also be re-written in terms of $R_{p/a}$ and $d\alpha$ as,

$$\frac{1}{2} K_\theta (\delta \theta)^2 = \left[ R_{p/a} - \left( \frac{R_{p/a}}{a} \right)^2 \right] K_\beta (\delta \alpha)^2 .$$  \hspace{1cm} (2.30)

Thus, by combining Eqs. (2.27), (2.29), and (2.30), the work done to deform a unit cell can be written into a function of $R_{p/a}$ and $d\alpha$ as

$$\delta W = \left[ K_\alpha + \left( \frac{R_{p/a}}{a} \right) K_\beta \right] (\delta \alpha)^2 .$$  \hspace{1cm} (2.31)

The principle of virtual works yields,

$$\delta U = \delta W .$$  \hspace{1cm} (2.32)

By combining Eqs. (2.3), (2.4), (2.6), (2.28), (2.31), and (2.32), the effective stiffness along the
loading direction 2 is derived as
\[ E_2 = \frac{1}{2a^2} \frac{\sin \alpha_2}{\sin \beta_2 \cos \alpha_2} \left( K_\alpha + \frac{R_p/a K_\beta}{K_\alpha + K_\beta} \right). \] (2.33)

Then, by substituting Eqs. (2.13) and (2.25) into Eq. (2.33), the effective stiffness along the loading direction 2 can also be derived as:
\[ E_2 = \frac{1}{2a^2} \frac{\sin \alpha_2}{\sin \beta_2 \cos \alpha_2} \left( K_\alpha + \frac{K_\beta K_\theta}{K_\theta + 8 K_\beta} \right). \] (2.34)

Form this equation, it can be seen that the effective stiffness along the loading direction 2 is not only related to the cell geometry \((a, \alpha, \beta)\) but also related to the rotational stiffness \((K_\alpha, K_\beta, K_\theta)\) of the rotational springs.

### 2.4 Stability analysis

Based on the rigid-rod-rotational-spring model, the micro-stability analysis was performed via the energy method. If the applied external force along the loading direction 2 is \(P\), the total potential energy \(\Pi\) of a unit cell is
\[ \Pi = U - 2P\Delta, \] (2.35)

where, \(U\) is the strain energy of the system, and \(\Delta\) is the displacement of the external force \(P\) along the loading direction 2, which can be derived from the kinematics as,
\[ 2\Delta = 4a \sin \frac{\alpha_0}{2} - 4a \sin \frac{\alpha}{2} = 4a \left( \sin \frac{\alpha_0}{2} - \sin \frac{\alpha}{2} \right), \] (2.36)

where, \(\alpha_0\) is the initial angle between active ribs \(A-B\) and \(B-O\), and \(\alpha\) is the current angle between the active ribs \(A-B\) and \(B-O\).

By substituting Eqs. (2.27) and (2.36) into Eq. (2.35), the total potential energy \(\Pi\) can be written as,
\[ \Pi = C(\alpha - \alpha_0)^2 - 4Pa \left( \sin \frac{\alpha_0}{2} - \sin \frac{\alpha}{2} \right) , \]  \hspace{1cm} (2.37) 

where \( C \) is determined by

\[ C = K_\alpha + R_{p/a} K_\beta . \]  \hspace{1cm} (2.38) 

The system has only one degree of freedom. Thus, the equilibrium path of the system can be obtained by taking

\[ \frac{d\Pi}{d\alpha} = 0 , \]  \hspace{1cm} (2.39) 

which yields

\[ P = \frac{c(\alpha_0 - \alpha)}{\alpha \cos \frac{\alpha_0}{2}} . \]  \hspace{1cm} (2.40) 

If the nondimensional load \( \bar{P} \) is defined as

\[ \bar{P} = \frac{pa}{C} , \]  \hspace{1cm} (2.41) 

by combining Eqs. (2.40) and (2.41), \( \bar{P} \) can be expressed as

\[ \bar{P} = \frac{\alpha_0 - \alpha}{\cos \frac{\alpha_0}{2}} . \]  \hspace{1cm} (2.42) 

The equilibrium paths of Eq. (2.42) are plotted in Fig.2.6. For three different values of \( \alpha_0 \), (10°, 90°, 170°), when \( \alpha < \alpha_0 \), the unit cell is under compression and when \( \alpha > \alpha_0 \), the unit cell is in tension. It can be seen from Fig.2.6 that for compression, \( \bar{P} \) is positive; and for tension, \( \bar{P} \) is negative. Fig.2.6 also shows that the chiral structure is getting harder to deform in tension as the slopes of the curves increase and getting softer to deform in compression as the slopes of the curves decrease.
The equilibrium paths of a unit cell with $\alpha_0 = 10^\circ, 90^\circ, \text{and } 170^\circ$.

To judge the stability along this equilibrium path as shown in Fig.2.6, the following criteria were used [78]:

$$\begin{align*}
\frac{\partial^2 \Pi}{\partial \alpha^2} > 0 & \quad \text{stable} \\
\frac{\partial^2 \Pi}{\partial \alpha^2} = 0 & \quad \text{no conclusion} \\
\frac{\partial^2 \Pi}{\partial \alpha^2} < 0 & \quad \text{unstable}
\end{align*}$$

which are equivalent to:

$$\begin{align*}
\cot \frac{\alpha}{2} > \frac{\alpha_0 - \alpha}{2} & \quad \text{stable} \\
\cot \frac{\alpha}{2} = \frac{\alpha_0 - \alpha}{2} & \quad \text{no conclusion} \\
\cot \frac{\alpha}{2} < \frac{\alpha_0 - \alpha}{2} & \quad \text{unstable}
\end{align*}$$

To further study the stability of a unit cell, $y_1 = \cot \frac{\alpha}{2}$ and $y_2 = \frac{\alpha_0 - \alpha}{2}$ are plotted in Fig.2.7 for $\alpha_0 = 180^\circ$. Under compression, when $\alpha > \alpha_0$, the unit cell is in tension. So the stability of the unit cell under both compression and tension can be investigated.
Fig. 2.7 shows that when in tension, \( y_2 < 0 \), then for all possible \( \alpha \) and \( \alpha_0 \), \( \frac{\partial^2 \Pi}{\partial \alpha^2} \) is always larger than zero, thus the chiral structure is always stable in tension.

Fig. 2.7 also shows that when in compression, when \( \alpha_0 = 180^\circ \), and \( \alpha \) changes from 0° to 180° which covers all possible angles of \( \alpha \), the value of \( y_1 \) is always larger than the value of \( y_2 \). As according to the criteria in Eq. (2.44), the unit cell of chiral structure is then always stable during compression. When \( \alpha_0 \) decreases, \( y_2 \) also decreases, therefore, for all values of \( \alpha_0 \), the value of \( y_1 \) is always larger than the value of \( y_2 \). This stability analysis showed that based on the rigid-rod-rotational spring model, the chiral cell is always stable during both compression and tension. However, the macro-instability of the structure with infinite number of cells needs further investigation in the future.
Chapter 3  Conceptual Design of New Chiral Structures

3.1 Design and 3D printing of auxetic chiral structures

According to Eqs. (2.12), (2.25), and (2.26), for a certain geometry, by increasing $R_s$, the internal rotational efficiency $R_{p/a}$ of the cell can be increased, and the negative Poisson’s ratio of the structure can be reduced. Thus, in order to amplify the auxetic effects (i.e. to reduce the negative Poisson’s ratio), two strategies were developed: (a) to decrease $K_\beta$ by introducing soft hinges at the corners on the passive ribs, and (b) to increase $K_\theta$ by adding hard cores in the center of the cell. Guided by this design concept developed from the rigid-rod-rotational-spring model, specimens of three periodic chiral structures were designed, as shown in Fig.3.1: (1) a basic chiral structure with uniform thickness and the same material in all ribs, (2) chiral structures with hard circular core in the center, and (3) chiral structures with soft material at the corners.
Based on the rigid-rod-rotational-spring model, structures 2 and 3 are expected to have amplified auxetic effects and elevated internal rotation than the basic structure 1. To quantitatively prove the concept, the three designs were fabricated via a multi-material 3D printer (Objet Connex 260), and mechanical experiments and finite element (FE) simulations were performed to quantify the Poisson’s ratio and the internal rotation.

The three specimens were first designed in the 3D CAD design software SOLIDWORKS. Then the .stl files of the 3D model were sent to the 3D printer. Single material printing was used to fabricate structures 1 and 2. For structure 3, different materials were assigned to different parts of the materials. The SOLIDWORKS models and the pictures of 3D printed specimens are shown in Fig.3.1. The dimensions of all three specimens are 75mm x 75mm x 20mm along directions 1, 2 and 3. The in-plane thickness of the ribs is 2mm for all three
For structure 2, the radius of the circular cores in the centre is 2.65mm and the length of the ribs from centre to corner is 5.3mm. Therefore, the ratio between the radius of the circular cores and the length of the ribs is 0.5.

In each printing job, the 3D printer has 12 material options, which include the stiffest material VeroWhite (~2GPa), and the softest material TangoPlus (~1MPa), and another 10 Digital Materials (DM) defined by mixing the two base materials with different ratios to obtain mechanical properties between ~1MPa to ~2GPa. The 3D printer can make very high quality bonding between dissimilar materials.

As shown in Fig.3.1, specimens 1 and 2 were printed as the same digital materials DM9760, which behaves like a rubbery material, while specimen 3 used the two base materials: the majority of the ribs were printed as the hardest material in the printer, VeroWhite (~2GPa), while the corners were printed as the softest rubbery material TangoPlus (~1MPa). Therefore, the only difference between specimens 1 and 2 is that specimen 1 does not have center core and specimen 2 has center core. The difference between specimens 1 and 3 is that specimen 1 has only one material, while specimen 3 has two different materials.

### 3.2 Mechanical experiments on materials used in the 3D printer

Mechanical experiments were first performed to test the stress-strain behaviour of the three materials used in the specimens ~24 hours after printing in room temperature. Both uni-axial tension and uni-axial compression experiments were performed on the two rubbery materials, TangoPlus and DM9760. Based on the ASTM standard D638-10 [79], the SolidWorks model and the dimensions of the dogbone specimens for TangoPlus and DM9760 are shown in Fig.3.2.
The thickness of the dogbone specimens is 2mm. Because TangoPlus is soft material and DM9760 is relatively rigid, the geometries and the dimensions of the two specimens are different.

![Dogbone Specimens](image)

Figure 3.2. The dogbone specimens for (a) TangoPlus and (b) DM9760 (unit: mm).

The dogbone specimens were tested on a material testing machine (ZwickiLine). Displacement-controlled experiments of uniaxial tension were performed with a strain rate of $10^{-3}$ per second. Digital Image Correlation (DIC) was used to get the stress-strain curves of the two materials. Each material was tested with three specimens. The experimental results for these specimens are shown in Figs.3.3 and 3.4.
Based on the ASTM standard D575-91[80], cylindrical specimens for uniaxial compression experiments were designed, as shown in Fig.3.5. The diameter of the cylinder is 28.6mm and the height of the cylinder is 12.5mm. The same design was used for both TangoPlus and DM9760 materials.
To perform uniaxial compression experiments, the cylindrical specimen was placed on a compression disk which is fixed on the Zwick machine. Another compressive disk was mounted on the machine and moved down with a displacement rate of 0.0125mm/s, which provided a strain rate of $10^{-3}$ per second to the specimen. To reduce friction, Teflon paper was added between the specimens and the disk.

The cylindrical specimens were also tested under a two-cycle loading condition. In the experiments, it was controlled that when the displacement of the upper disk reaches 6.25mm which is 50% of the height of the cylindrical specimen, the unloading occurs, the upper disk then moves upwards to its initial position with the same strain rate as the loading process which was $10^{-3}$ per second. There was no time break between the loading and unloading processes. The displacement-force curves are shown in Fig.3.6. Both TangoPlus and DM9760 show certain hysteresis. For TangoPlus, the load-displacement curves of the two cycles are identical,
indicating the material can fully recovered right after unloading under this strain rate. While for DM9760, certain plasticity/viscosity was observed after the first cycle, as shown in Fig.3.6b.

![Graphs showing force vs displacement for TangoPlus and DM9760](image)

**Figure 3.6** Two-cycle loading: (a) TangoPlus (b) DM9760.

### 3.3 Hyperelastic modeling of materials in the 3D printer

The hyperelastic Mooney-Rivlin model was used to model the TangoPlus and the digital material DM60 from the 3D printer. The same model was also used in the parametric study in Chapter 4. Other hyperelastic models also work for the two materials, although Monney-Rivlin fits a little better, as shown in Appendix A. Therefore the Mooney-Rivlin model is chosen. The strain energy density function $W$ of the Mooney-Rivlin model [81] is:

$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) + D_1(J - 1)^2,$$  \hspace{1cm} (3.1)

where $C_{01}$ and $C_{10}$ are fitting parameters from the experiments. $D_1$ is assumed to be zero due to the incompressibility of both materials. For TangoPlus (strain range ~0.5-0.2), $C_{01} = 0.0986$ MPa, $C_{10} = 0.0314$ MPa, and for the digital material (strain range ~0.1-0.1), $C_{01} =
0.4584 MPa, $C_{10} = 0$ MPa. The materials parameters were then input to ABAQUS for FE simulations. The fitting curves for TangoPlus and DM9760 are shown in Fig.3.7 and 3.8, respectively. The material parameters for both materials are shown in Table.3.1.

![Figure 3.7. Mooney-Rivlin model fitting for TangoPlus.](image1)

![Figure 3.8 Mooney-Rivlin model fitting for DM9760.](image2)
<table>
<thead>
<tr>
<th></th>
<th>$C_{01}$ (MPa)</th>
<th>$C_{10}$ (MPa)</th>
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<td>TangoPlus</td>
<td>0.0986</td>
<td>0.0314</td>
</tr>
<tr>
<td>DM9760</td>
<td>0.4584</td>
<td>0</td>
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Table 3.1 The material model used for FE simulation.

3.4 Mechanical experiments on the 3D printed structures

Mechanical experiments were performed for all three specimens under uni-axial compression on the Zwick material testing machine. The specimens were set between two compression disks. The bottom disk was fixed, and the top one was controlled to slowly move down to compress the specimens. Due to the chirality, when the vertical displacement is controlled, shearing stress will be generated at the boundary of the specimens. To prevent the shear-induced sliding between the specimens and the disks, sand paper was added between the disks and the top and bottom surfaces of the specimens. In this way, the overall shear deformation of the specimen was eliminated.

As shown in Fig.3.9, dark markers were added at different locations of the specimens. A high resolution camera was used to record the images of the specimens during deformation at each time instant. One image was taken every one second. The images in Fig.3.9 were chosen at three stages during the deformation (a) before the contact between ribs, (b) densification, and (c) recovered configuration after unloading and the red square with dash line represents the boundary of the undeformed configuration. All specimens showed auxetic effect that the horizontal dimension of all specimens decreased under the uniaxial compression along the
vertical direction. All the specimens are reconfigurable after unloading as shown in Fig.3.9.

Figure 3.9 Images of the deformed specimens at different levels of overall strain, (a) the basic chiral specimen, (b) chiral specimen with a center core, and (c) chiral specimen with soft hinges (initial area showed as red square).

Interestingly, due to the chiral geometry and the internal rotation of the cells, the compression and shear deformation are always coupled. For the displacement-controlled uni-axial compression experiments, the chiral structures are subjected to a shearing stress at the boundary. The shearing deformation was constrained by the sand paper, thus shear stress was provided by the sand paper and caused a non-uniform strain in the specimens. The central cell
is least influenced by the shear stress from the boundaries.

As shown in Fig.3.9c, due to the handness (counter-clockwise) of the specimens, the non-uniform strain first occurs at top-left and bottom-right corners of the specimens. The non-uniform strain distribution caused self-contact with the cell and local instability of the ribs. The self-contact in these regions complicated the loading condition of the centre cell. Therefore, we will focus on analysing the data before self-contact. After unloading, all three structures almost fully recovered to their initial shape after the large deformation (~40% for the specimens 1 and 2, and ~30% for specimen 3).

The force-displacement curves of these three specimens are shown in Figs.3.10. Each specimen was tested three times and there was about 30 minutes break between each test to let the specimens fully recover. It can be seen that the curves for three tests are almost the same for all three specimens, indicating little damage on the specimens after each experiment.
Figure 3.10 The load-displacement curves of (a) the basic chiral specimen 1, (b) the chiral specimen 2 with a center core and (c) chiral specimen 3 with soft hinges.

To measure the deformation of the chiral specimens, a high resolution camera was used to record the deformed configuration of the specimens at each time instant. As shown in Fig.3.11, by post-processing the images, the displacement history of each marker point was obtained.
Since the center of the specimen was least influenced by the boundary, the origin of the material coordinate was set in the centre of the specimen. To calculate the Poisson’s ratio of the chiral specimens, the displacements of the black marks at the centre of the four cells around the centre cell were tracked. Because the coordinate will slightly rotate during the deformation due to the shear stress from the boundary, as shown in Fig.3.11, the absolute distance of the horizontal and vertical length $L_1$, $L_2$ were output at each time step to calculate the Poisson’s ratio (all $\nu$ used in this chapter is $\nu_{21}$ as derived in Chapter 2) via

$$\nu = -\frac{L_1^i - L_1^0}{L_1^0} \frac{L_2^0}{L_2^i - L_2^0},$$

(3.2)

where, $L_1^0$ and $L_2^0$ are the initial length in the undeformed configuration, $L_1^i$ and $L_2^i$ are the
length in the deformed configuration, \( i \) is the time step number and \( i = 1, 2, 3, \ldots \), as shown in Fig. 3.11a.

To calculate the rotational efficiency of the chiral specimens, the displacements of the black marks (Fig. 3.11) at the four corner of the central cell were tracked. The material coordinate will slightly rotate during the deformation, as shown in Fig. 3.11. The rib angles \( \varphi_a \) and \( \varphi_p \) between the ribs and rotating material coordinate are output to calculate the rotational efficiency via:

\[
R_{p/a} = \frac{\varphi_p^i - \varphi_p^0}{\varphi_a^i - \varphi_a^0},
\]

where, \( \varphi_p^0 \) and \( \varphi_a^0 \) are the initial angles of the active and passive ribs in the material coordinate in the undeformed configuration, \( \varphi_p^i \) and \( \varphi_a^i \) are the angles of the active and passive ribs in the deformed configuration at time step \( i \).

The measured Poisson’s ratio and rotational efficiency of the three specimens are shown in Fig. 3.12. The error bar was calculated by the standard deviation of the measurements of the tests.

Figure 3.12 Experimental results of (a) the Poisson’s ratio vs. overall strain and (b) the
Fig. 3.12 shows that the initial Poisson’s ratio of the basic chiral structure is ~ -0.3. By adding hard cores and soft hinges, the initial Poisson’s ratio reduced to ~-0.5 and ~-1, respectively. The rotational efficiency of the three specimens is about 0.6, 0.8 and 1, respectively. It clearly shows that when the rotational efficiency increases, the auxetic effect will be amplified. When the overall compressive strain increases, the rotational efficiency for all three specimens and the Poisson’s ratio for basic chiral structure and chiral structure with a centre core are barely changed, while the Poisson’s ratio of the chiral structure with soft hinges will decrease. The decrease might be because the chiral structure with soft hinges is more sensitive of the influences from the boundaries.

3.5 Finite element (FE) simulations of the experiments

To further explore the deformation mechanisms of the specimens, FE simulations of the experiments on the three specimens were performed in ABAQUS/STANDARD V6.13. Four-node 2D plane stress elements (CPS4) were used. The geometry and the dimension of the FE models are the same as the SOLIDWORKS model used for 3D-printing. There is a modification of the FE model of chiral structure with soft hinges due to the interphase effect during multi-material 3D printing (more details can be found in Appendix B).

In the FE simulations, as shown in Fig. 3.13, two analytical rigid surfaces were set-up to represent the top and bottom compression disks. The FE models of the chiral specimen are in between.
Figure 3.13 FE model and setup of chiral structure.

The bottom rigid surface is fixed, and the top rigid surface moves downwards with a prescribed displacement. Hard contact was defined between the rigid surfaces and specimens. Friction coefficient with the value of 1 was used to model the sand paper. Frictionless self-contact was also defined within the FE model of the chiral specimen. Mooney-Rivlin hyperelastic models were used for the 3D printed materials. The material parameters were obtained from Section 3.3. For specimen 3, the deformations in the harder ribs are very small, so the VeroWhite material in the harder ribs of specimen 3 was modelled as linear elastic isotropic material with Young’s Moduli $E = 2\ \text{GPa}$ and Poisson’s ratio $\nu = 0.45$ [82]. Six elements were used across the in-plane thickness of the ribs and thirty one elements were used along the ribs.

As we know that for cellular materials, the influences of the boundary are largely determined by the number of cells in the specimen, to further investigate the boundary effects, FE models with different numbers of cells (5x5, and 7x7) were set-up. Also, to represent a
specimen with infinite numbers of cells, FE models with periodic boundary condition were set-up for all three designs. The results of all these FE models were compared.

The stress-strain curves (before self-contact and densification) from the FE simulations are compared with the experimental results in Fig.3.14.

![Stress-strain curve comparison](image)

**Figure 3.14** The stress-strain curves from the experiments (symbols with error bars) and FE simulations (solid lines) for the three specimens.

Fig. 3.14 shows that the original chiral structure and chiral structure with hard cores have very similar strain-stress behaviour which indicates that the cores barely deform during the deformation and the function of the core is just to elevate the internal rotation. The chiral structure with soft hinges has much higher strain-stress curve because the ribs are relative rigid so that the rotational stiffness of the center spring $K_\theta$ is larger. Although according to Eq. (2.34), the effective stiffness is not only determined by $K_\theta$, the competition results among several factors, the higher stress-strain curves of the specimen with soft hinges indicates that $K_\theta$ is the dominant factor for this case. The kink on the stress-strain curve (Fig.3.14) around
7% strain of the chiral structure with soft hinges was caused by the self-contact at the left top and right bottom corners as shown in Fig. 3.11b. For all three cases, the FE results are consistent with the experimental measurement.

In the FE simulations, the Poisson’s ratio and the rotational efficiency were obtained in a similar method as in the experiments, as was described in details in Section 3.4. The FE results of the Poisson’s ratio and the internal rotation efficiency are compared with experiments in Figs. 3.15a and 3.15b respectively.

Figure 3.15 (a) Poisson’s ratio vs. overall strain, and (b) the rotational efficiency vs. overall strain.

Fig. 3.15 shows that the FE simulations accurately captured the physical behaviour of the three specimens up to 10% strain. The FE results further proved that the chiral structure with a centre core and soft hinges has smaller Poisson’s ratio and larger rotational efficiency, as was conceptually developed from the rigid-rod-rotational-spring model. Among these three chiral structures, the chiral structure with soft hinges has the smallest Poisson’s ratio and largest rotational efficiency.

To evaluate the influences from the number of cells in the specimen and the boundary
effect, the FE results of Poisson’s ratio vs. rotational efficiency for FE models with different numbers of cells and boundary conditions were compared with experiments and analytical prediction (Eq. 2.12) in Fig. 3.16.

Figure 3.16 The comparison of experiments, FE results and analytical prediction with different cell sizes, boundary conditions: Poisson’s ratio vs. the rotational efficiency.

Fig. 3.16 shows that when the numbers of cells in the FE models of the specimens increases, the results become closer to the analytical prediction from Eq. 2.12. The error between the experiments/FE results and the theory prediction are 32.98% for experiments, 24.68% for FE 5x5 cells, 19.18% for FE 7x7 cells, and 6.55% for FE with PBCs. The results of the model with periodic boundary condition (infinite numbers of cells) are almost the same (6.55% error) as the analytical prediction. This indicates that the differences between experiments and the analytical prediction from the rigid-rod-rotational-spring model (Eq. 2.12) is mainly due to the boundary effects.
Chapter 4  FE Simulations on Auxetic Chiral Structures with Soft Hinges

In Chapter 3, the experimental results showed that the chiral structure with soft hinges can significantly amplify the auxetic effect. To further explore the mechanical behavior of the chiral structure with soft hinges, a parametric study was conducted via FE simulations. The design parameters that we are interested in include the stiffness ratio between the soft hinges and ribs, the location of the soft hinges, and the thickness of the ribs. The influences of each parameter on the strain-stress curves, the Poisson’s ratio (all \( \nu \) used in this chapter is \( \nu_{21} \) as derived in Chapter 2), and the rotational efficiency were quantified.

The simulations were performed under uniaxial compression along direction 2. A 2D FE model with 2x2 cells were set-up in ABAQUS/STANDARD V6.13. 2D plane stress elements (CPS4) were used for all simulations. To avoid the boundary effect and also to represent materials with infinite numbers of cells, periodic boundary conditions were used in all simulations. Shear-deformation was constrained for all simulations. Mooney-Rivlin hyperelastic model was used for all simulations.

4.1 The effects of the stiffness of soft hinges

In the FE models shown in Fig.4.1, the shear modulus \( \mu^2 \) of the soft hinges was kept the same (0.26 MPa) which is same as the material parameter used in Chapter 3.4 for TangoPlus material, while the shear modulus \( \mu^1 \) of the majority of the ribs varied from 0.26 MPa to 260 MPa. Thus the stiffness ratios \( \mu^1/\mu^2 \) varied from 1,10,100,400,700,1000. The shear moduli
of Mooney-Rivlin model was calculated via equation $\mu = 2(C_{01} + C_{10})$.

![Diagram showing quasi-isotropic and anisotropic geometries]

Figure 4.1. Unit cells of five different geometry of chiral structures with soft hinges.

For each stiffness ratio, five geometries were explored. The unit cells and the meshes of the five different models are also shown in Fig.4.1. Eight elements were used across the in-plane thickness of the ribs. By varying $\alpha$ and $\beta$, the three quasi-isotropic geometries (geometries 1, 2, 3 in Fig.4.1) were modelled with $\alpha = \beta = 80^\circ$, $90^\circ$, and $120^\circ$; and the two anisotropic geometries (geometries 4 and 5 in Fig.4.1) were modelled with $\alpha = 60^\circ$, $\beta = 120^\circ$ and $\alpha = 120^\circ$, $\beta = 60^\circ$.

The contours of the maximum principle strain of geometries 2, 4, 5 with different stiffness ratios are shown in Fig.4.2, in which, the red dash lines represent the undeformed configurations.
Figure 4.2 Strain contours of different geometries and stiffness ratios for chiral structures with soft hinges ($\varepsilon$ represents the maximum principle strain).

Fig.4.2 shows that for all geometries, the strain localized in the soft hinges and the deformation in the harder ribs is negligible. All of the models show that the auxetic effects are different for different geometry and different stiffness ratios.

For geometry 2 with $\alpha = \beta = 90^\circ$, with different stiffness ratio, the curve the horizontal strain $\varepsilon_1$ and vertical strain $\varepsilon_2$ (2 is the loading direction), and the curve of the rotation angles of the active ribs and passive ribs during the deformation are shown in Fig.4.3a and b, respectively, and the strain-stress curves are shown in Fig.4.3c.
Figure 4.3 FE results of chiral structure with soft hinges with $\mu^1/\mu^2 =$ 1, 10, 100, 400, 700, 1000: (a) the horizontal strain vs. vertical strain, (b) rotation angles of passive ribs vs. active ribs, and (c) strain vs. stress. ($\alpha = \beta = 90^\circ$).

For the horizontal strain $\varepsilon_1$ and vertical strain $\varepsilon_2$ as shown in Fig.4.3a, the negative slope of each curve represents the instantaneous Poisson’s ratio during the deformation. Fig.4.3a shows that the horizontal strain is almost linearly related to the vertical strain for all different
\( \mu^1/\mu^2 \), which indicates that the Poisson’s ratio during the deformation is close to a constant. When \( \mu^1/\mu^2 \) increases from 1 to 1000, the Poisson’s ratio decreases and eventually when \( \mu^1/\mu^2 \) increases beyond 100, the Poisson’s ratio approaches the asymptotic limit -1 for isotropic structures.

For the rotation angles of the active ribs and passive ribs shown in Fig.4.3b, they are also almost linearly related, indicating a constant rotating efficiency during the deformation. When the shear stiffness ratio \( \mu^1/\mu^2 \) increases, the slopes of these curves also increase, indicating the elevated internal rotation. When \( \mu^1/\mu^2 \) increases beyond 100, the rotational efficiency approaches the asymptotic limit 1 for isotropic structures.

Fig.4.3c shows that the stiffness ratio significantly influences the stress-strain behaviour of the structures. The stress along direction 2 were non-dimensionalized with the Young’s modulus of the material in the hard ribs. It can be seen that When \( \mu^1/\mu^2 \) increases, the non-dimensionalized stiffness decreases. For all values of \( \mu^1/\mu^2 \), the stress increases dramatically after self-contact. The self-contact occurs earlier when \( \mu^1/\mu^2 \) is larger. This is because that the bending of the ribs can delay self-contact, while when \( \mu^1/\mu^2 \) is large, ribs barely bend.

For all five different geometries, the rotational efficiency and the Poisson’s ratio are plotted as a function of shear stiffness ratio \( \mu^1/\mu^2 \) in Fig.4.4a and 4.4b, respectively.
Figure 4.4 FE results of the five chiral structures with soft hinges with $\mu^1/\mu^2 = 10, 100, 400, 700, 1000$: (a) rotation efficiency vs. stiffness ratio, and (b) Poisson’s ratio vs. stiffness ratio.

Fig.4.4a shows that the rotational efficiency $R_{p/a}$ is mainly determined by $\mu^1/\mu^2$, and slightly influenced by $\alpha$ and $\beta$. When the stiffness ratio $\mu^1/\mu^2$ increases, for all geometries, $R_{p/a}$ increases and eventually becomes asymptotic to the limitation value of 1. Roughly speaking, the stiffness ratio $\mu^1/\mu^2$ is proportional to the rotational stiffness ratio $R_s$ in the rigid-rod-rotational-spring model. When $\mu^1/\mu^2$ is smaller than 100, the $R_{p/a}$ is sensitive to $\mu^1/\mu^2$, but when $\mu^1/\mu^2$ increases beyond 100, $R_{p/a}$ only slightly increases towards 1. This trend is consistent with the trend predicted by rigid-rod-rotational-spring model (Fig.2.4). The relation between $R_{p/a}$ and $\mu^1/\mu^2$ was captured by the rotational-spring-rigid-rod model, while the influences from $\alpha$ and $\beta$ which was not included in the analytical
model. In real designs, when $\alpha$ and $\beta$ changes, the geometry of the soft hinges changes accordingly, as will slightly influence $R_s$.

As shown in Fig.4.4b, for the three quasi-isotropic structures, the Poisson’s ratios of them are very close to each other. When $\mu^1/\mu^2$ increases, for all three quasi-isotropic geometries, the Poisson’s ratio decreases and is asymptotic to -1; for geometry 4 with $\alpha = 60^\circ$, $\beta = 120^\circ$, the Poisson’s ratio is asymptotic to -0.33; for geometry 5 with $\alpha = 120^\circ$, $\beta = 60^\circ$, the Poisson’s ratio is asymptotic to -3. When $\mu^1/\mu^2$ is large, the Poisson’s ratio is not sensitive to $\mu^1/\mu^2$, while when $\mu^1/\mu^2$ is smaller than 100, when $\mu^1/\mu^2$ decreases, the Poisson’s ratio $\nu$ will increase.

For all five different geometries and all stiffness ratios, the FE simulation results of the Poisson’s ratio vs. rotational efficiency were compared with the analytical prediction (Eq. 2.12) in Fig.4.5, in which the FE results are symbols and the analytical predictions are solid lines.

Figure 4.5 Comparison of the FE results and the analytical predicton for various chiral structures with soft hinges: Poisson’s ratio vs. the rotational efficiency.

Fig. 4.5 shows that the Poisson’s ratio is linearly related to the rotational efficiency which
is consistent with the analytical prediction in Eq. (2.12). The theoretical results and the FE results match well (with average error of 4.64%) for all geometries.

4.2 The effects of soft hinges on the active ribs

According to Eqs. (2.25) and (2.26), the rotational efficiency \( R_p/\alpha \) and the Poisson’s ratio are only related to the stiffness ratio \( K_\theta/K_\beta \) between the rotational spring at center and that on the passive ribs but not influenced by the stiffness \( K_\alpha \) of the rotational spring on the active ribs. To verify this conclusion from the analytical prediction, FE simulations were performed by varying the shear modulus \( \mu_3 \) of the soft hinges on the active ribs as shown in Fig.4.6.

![Strain contours of the chiral structures with soft hinges with different stiffness ratios](image)

Figure 4.6 Strain contours of the chiral structures with soft hinges with different stiffness ratios \( \mu_1/\mu_3 \) (\( \varepsilon \) represents the maximum principle strain, \( \mu_1/\mu^2 = 1000 \)).

FE models with \( \alpha = \beta = 90^\circ \) were set up in ABAQUS/STANDARD V6.13. Same mesh size as in Chapter 4.1 was used in FE models. Mooney-Rivlin hyperelastic models with different shear moduli were used in the FE models. The parameter were determined in Chapter 3.3. For each FE model, the majority of the ribs were assigned with harder material with shear modulus \( \mu_1 = 260 \) MPa, and the soft hinges on the passive ribs were assigned shear modulus
of $\mu^2 = 0.26$ MPa. Thus, the stiffness ratio between these two materials was controlled as $\mu^1/\mu^2 = 1000$ for all models. While the shear moduli $\mu^3$ for hinges on active ribs varied from 0.26 MPa to 260 MPa, providing a stiffness ratio of $\mu^1/\mu^3 = 10,100,400,700, \text{and } 1000$.

The contours of the maximum principle strain of the FE models with different stiffness ratios $\mu^1/\mu^3$ are shown in Fig.4.6, in which the undeformed configurations are shown by red dash lines. It can be seen that for all structures, the strain localized in soft hinges and the deformation in the harder ribs is negligible. All of the models show the auxetic effects and the auxetic effects are quite similar for all three cases at the same overall strain.

The relation between the horizontal strain $\varepsilon_1$ and vertical strain $\varepsilon_2$ (2 is the loading direction) and the relation between the rotation angles of the active ribs and passive ribs during the deformation are shown in Fig.4.7a and b, respectively. The strain-stress curves are shown in Fig.4.7c.
Figure 4.7 FE results of the chiral structures with soft hinges with $\frac{\mu^1}{\mu^3} =$ 10, 100, 400, 700, 1000: (a) horizontal strain vs. vertical strain, (b) rotational angle on the passive ribs vs. active ribs, and (c) strain vs. stress. ($\frac{\mu^1}{\mu^2} = 1000$)

Figs.4.7a and b show that the results for all five cases are almost the same which indicates that the stiffness of the soft hinges on active ribs doesn’t have large influence on the Poisson’s ratio and rotational efficiency.
Fig. 4.7c, shows that the effective stiffness of the structure is significantly influenced by $\mu^1 / \mu^3$. This is consistent with the analytical predication of Eq. (2.34), in which the Young’s moduli $E_2$ is also a function of $K_\alpha$.

The FE results of the Poisson’s ratio and the rotational efficiency are compared as a function of shear stiffness ratio $\mu^1 / \mu^3$ in Fig. 4.8. It can be seen that both the Poisson’s ratio and rotational efficiency only slightly varies for different $\mu^1 / \mu^3$, especially when $\mu^1 / \mu^3$ is larger than ~100.

Figure 4.8 FE results of the chiral structures with soft hinges with $\mu^1 / \mu^3 = 10,100,400,700,1000$: (a) Rotation efficiency vs. stiffness ratio, and (b) Poisson’s ratio vs. stiffness ratio ($\mu^1 / \mu^2 = 1000$).

The FE simulation results of the Poisson’s ratio vs. rotational efficiency were compared with the analytical prediction (Eq. 2.12) in Fig. 4.9, in which the FE results are the symbols and the analytical predictions are solid lines. Fig. 4.9 again shows that $\mu^1 / \mu^3$ barely influence the Poisson’s ratio and the rotational efficiency, and the FE results are very consistent with the
analytical prediction (with average error of 2.08%). All the points from different value of \( \mu^1/\mu^3 \) are close to the case of \( \mu^1/\mu^2 = 1000 \) in Section 4.2.1.

![Graph showing comparison of FE results and analytical prediction](image)

Figure 4.9 Comparison of the FE results and the analytical prediction for chiral structures with soft hinges with different stiffness ratio \( \mu^1/\mu^3 \): Poisson’s ratio vs. rotation efficiency \((\mu^1/\mu^2 = 1000)\).

### 4.3 The effects of rib thickness

To investigate the influences of the rib thickness on the mechanical behaviours of the chiral structures, FE simulations of chiral structures with \( \alpha = \beta = 90^\circ \), shear stiffness ratio of \( \mu^1/\mu^2 = 1 \), and different thickness of \( t = 1, 1.5, 2, 2.5, 3 \) mm were performed. Thus, in the five FE models the ratios between the rib thickness and rib length are \( t/a = 0.19, 0.28, 0.38, 0.47, \) and 0.57. The unit cells and the meshes of the five FE models are shown in Fig.4.10. All models have the mesh size with the eight elements across the in-plane thickness of the ribs. Mooney-Rivlin hyper-elastic models with shear moduli \( \mu^1 = 0.26 \) MPa was used in all simulations which is the material property of the TangoPlus determined in Chapter 3.3.
The contours of the maximum principle strain of the FE models with different rib thickness are shown in Fig. 4.11. The undeformed configurations are shown in red dash lines. It can be seen that all structures show the auxetic effects and the auxetic effects are quite similar for all cases at the same overall strain.
The relation between the horizontal strain $\varepsilon_1$ and vertical strain $\varepsilon_2$ (2 is the loading direction) and the relation between the rotation angles of the active ribs and passive ribs during the deformation are shown in Fig.4.12a and b, respectively. The strain-stress curves are shown in Fig.4.12c.

Figure 4.12 FE results of the chiral structures with $t/a = 0.19, 0.28, 0.36, 0.47, 0.57$: (a) horizontal strain vs. vertical strain, (b) rotational angles on the passive ribs vs. those on the active ribs, and (c) strain vs. stress.
Figs.4.12a and 4.12b show that the results for all five cases are almost same which indicates that the rib thickness barely influences the Poisson’s ratio and rotational efficiency. Fig.4.12c show that the effective stiffness of the structure is significantly influenced by the rib thickness. In general, when the rib thickness increases, the chiral structure becomes stiffer and the self-contact occurs earlier.

The FE results of the Poisson’s ratio and the rotational efficiency are output as a function of the ratio between the rib thickness and the rib length \( t/a \) in Fig.4.13. It can be seen that both the Poisson’s ratio and rotational efficiency only slightly vary for different \( t/a \), especially when \( t/a \) is smaller than 0.4.

![FE results of the chiral structures with \( t/a = 0.19, 0.28, 0.36, 0.47, 0.57 \): (a)](image)

Figure 4.13 FE results of the chiral structures with \( t/a = 0.19, 0.28, 0.36, 0.47, 0.57 \): (a) Rotation efficiency vs. stiffness ratio, and (b) Poisson’s ratio vs. stiffness ratio.

The FE simulation results of the Poisson’s ratio vs. rotational efficiency were compared with the analytical prediction according to Eq. (2.12) in Fig.4.14, in which the FE results are
the symbols and the analytical predictions are solid lines.

Figure 4.14 Comparison of the FE results and the analytical predicition for chiral structures with rib thickness: Poisson’s ratio vs. rotation efficiency.

Fig. 4.14 again shows that $t/a$ barely influence the Poisson’s ratio and the rotational efficiency, and the FE results are very consistent with the analytical prediction (with the average error of 9.27%) (Eq. 2.12). All the points from different values of $t/a$ are close to the case of $\mu_1/\mu_2 = 1$ in section 4.2.1.

**4.4 The auxetic effects under large deformation**

For the auxetic chiral cells, if $\alpha_o$ and $\beta_o$ are the initial values of $\alpha$ and $\beta$ in the undeformed configuration, when the cell deforms, $\alpha$ and $\beta$ will change, the current $\alpha$ and $\beta$ in the deformed configuration become $\alpha = \alpha_o + d\alpha$, $\beta = \beta_o + R_{p/a} d\alpha$. From the kinematics, the overall strain $\varepsilon_1$ is related to angle $\alpha$ as

$$\varepsilon_1 = \ln \left( \frac{\sin \alpha}{\sin \frac{\alpha_o}{2}} \right).$$

(4.1)

According to Eq. (2.12), the instantaneous Poisson’s ratio will change with $\alpha$ and therefore
with $\varepsilon_1$ as well. The history of the instantaneous Poisson’s ratio was quantified by solving the incremental form of Eqs. (2.12) and (4.1) at each time step.

To verify the analytical prediction, FE simulations for chiral cells with soft hinges were performed under large deformation. The FE models included three different geometries: $\alpha_o = 60^\circ$, $\beta_o = 12$; $\alpha_o = 90^\circ$, $\beta_o = 90$; and $\alpha_o = 120^\circ$, $\beta_o = 60^\circ$. Mooney-Rivlin hyperelastic model was used for both the materials on the soft hinges and the ribs. The soft hinges were modelled with smaller shear moduli (0.2601 MPa). To get different rotational efficiency $R_{p/a}$ for each geometry, three different shear moduli (0.2601 MPa, 2.6010 MPa, and 26.0100 MPa) were used for the ribs.

In Figs.4.15a, 4.15b, and 4.15c, the theoretical predictions of the instantaneous Poisson’s ratio vs. the overall strain were compared with the results from the FE simulations for three geometries. It can be seen that the theoretical prediction (solid lines in Fig. 4.15) accurately captured the change of the instantaneous Poisson’s ratio during the deformation from the FE simulations (points in Fig 4.15).
Figure 4.15 The history of the instantaneous Poisson’s ratio for (a) $\alpha_o = 60^\circ$, $\beta_o = 120^\circ$, (b) $\alpha_o = 90^\circ$, $\beta_o = 90^\circ$, and (c) $\alpha_o = 120^\circ$, $\beta_o = 60^\circ$.

Fig. 4.15 shows that for all cases, the instantaneous Poisson’s ratio only slightly changed (<5%) during the deformation. For most of the cases, the theoretical predictions are consistent with the FE simulations which proved the accuracy of the analytical prediction even for large deformation. However, for the cases of $\alpha_o = 60^\circ$, $\beta_o = 120^\circ$ and $\alpha_o = 120^\circ$, $\beta_o = 60^\circ$, when $R_{p/a}$ is close to 1, relatively large difference between the analytical and numerical
predictions was observed, as shown in Figs.4.15a and 4.15c. This is because when \( \alpha \) and \( \beta \) change, the geometry of the hinges changes accordingly, and leads to different strain in the hinges under the same rib rotation. When the hinges deform, the size of the hinges will increase, which leads to the decrease of both \( \varepsilon_1 \) and \( \varepsilon_2 \), therefore, the Poisson’s ratio will be influenced. This effect of the change of hinge size during the deformation was not considered in the analytical model because the rotational springs were assumed to have zero dimension.

When \( \alpha_o = \beta_o = 90^\circ \), the deformation of the hinges on the active ribs and the passive ribs are almost proportional to \( d\varepsilon_2 \) and \( d\varepsilon_1 \), respectively. So that the Poisson’ ratio was barely influenced by the non-zero size of the hinges. However, when \( \alpha_o = 60^\circ \beta_o = 120^\circ \), the hinges on the active ribs has larger deformation than those on the passive ribs, leading to larger influence on \( \varepsilon_2 \) than \( \varepsilon_1 \), and therefore a smaller Poisson’s ratio than the analytical prediction was observed; when \( \alpha_o = 120^\circ \beta_o = 60^\circ \), the hinges on the passive ribs have larger deformation than those on the active ribs, leading to larger influence on \( \varepsilon_1 \) than \( \varepsilon_2 \), and therefore a larger Poisson’s ratio than the analytical prediction was observed.
Chapter 5  FE Simulations of Auxetic Chiral Structures with Center Cores

In Chapter 3, the experimental results showed that the chiral structure with a center core can also significantly amplify the auxetic effect. To further explore the mechanical behavior of the chiral structure with center cores, a parametric study was conducted via FE simulations. The design parameters that we are interested in include the stiffness ratio between the center core and the ribs and the size of the center core. The influences of each parameter on the strain-stress curves, the Poisson’s ratio (all $\nu$ used in this chapter is $\nu_{21}$ as derived in Chapter 2), and the rotational efficiency were quantified.

The simulations were performed under uniaxial compression along direction 2. A 2D FE model with 2x2 cells were set-up in ABAQUS/STANDARD V6.13. 2D plane stress elements (CPS4) were used for all simulations. To avoid the boundary effect and also to represent materials with infinity numbers of cells, periodic boundary conditions were used in all simulations. Shear-deformation was constrained for all simulations. Mooney-Rivlin hyperelastic model were used for all simulations.

5.1 The effects of the stiffness of core materials

In the FE models shown in Fig.5.1, the shear modulus $\mu_2$ of the ribs was kept the same (0.2601 MPa) which is the same as the material parameter used in Section 3.4 for TangoPlus material, while the shear modulus $\mu_1$ of the centre cores varied from 0.26 MPa to 260 MPa. Thus the stiffness ratios $\mu^1/\mu^2$ varied from 1,10,100,400,700,1000.
For each stiffness ratio, five geometries were explored. The unit cells and the meshes of the five different models are shown in Fig. 5.1. Eight elements were used across the in-plane thickness of the ribs. By varying $\alpha$ and $\beta$, the three quasi-isotropic geometries were modelled with $\alpha = \beta = 80^\circ$, $90^\circ$, and $120^\circ$; and the two anisotropic geometries were modelled with $\alpha = 60^\circ$, $\beta = 120^\circ$ and $\alpha = 120^\circ$, $\beta = 60^\circ$.

The contours of the maximum principle strain of geometry 2, 4, 5 with different stiffness ratios are shown in Fig. 5.2, in which the undeformed configurations are shown in red dash lines. It can be seen that for all structures, the major deformation mechanism of the cells is the bending of the ribs, and the cores barely deform. All of the models show the auxetic effects and for different geometry and stiffness ratio, the auxetic effects are different.
Figure 5.2 Strain contours of different geometries and stiffness ratios for chiral structures with a centre core (ε represents the maximum principle strain).

For geometry 2 with $\alpha = \beta = 90^\circ$, the relation between the horizontal strain $\varepsilon_1$ and vertical strain $\varepsilon_2$ (2 is the loading direction) and the relation between the rotation angles of the active ribs and passive ribs during the deformation are shown in Fig.5.3a and b. The strain-stress curves are shown in Fig.5.3c.
Figure 5.3 FE results of chiral structure with a centre core with $\frac{\mu^1}{\mu^2} =$ 1, 10, 100, 400, 700, 1000: (a) the horizontal strain vs. vertical strain, (b) rotation angles of passive ribs vs. active ribs, and (c) strain vs. stress. ($\alpha = \beta = 90^\circ$).

In Fig.5.3a, the negative slope of each curve represents the instantaneous Poisson’s ratio during the deformation. Fig.5.3a shows that the horizontal strain is almost linearly related to the vertical strain for all different $\frac{\mu^1}{\mu^2}$, which indicates that the Poisson’s ratio during the deformation is close to a constant. When $\frac{\mu^1}{\mu^2}$ increases from 1 to 1000, the Poisson’s ratio...
decreases a little bit, while when \( \mu_1/\mu_2 \) is larger than \(~100\), the Poisson’s ratios are very similar. Compared with the results for chiral structures with soft hinges, by changing the stiffness of the centre core, the amplification of the auxetic effects is limited.

Fig.5.3b show that the rotation angles of the active ribs and passive ribs are also almost linearly related, indicating a constant rotating efficiency during the deformation. When the stiffness ratio \( \mu_1/\mu_2 \) increases, the slopes of these curves increase a little bit, indicating the elevated internal rotation, while when \( \mu_1/\mu_2 \) is larger than \(~100\), the rotational efficiency is very similar which indicates a limitation in increasing the rotational efficiency.

In Fig.5.3c, the stress along direction 2 was non-dimensionalized with the Young’s modulus of the material in the hard ribs. Similar to the results of the strain and rotation angles, when \( \mu_1/\mu_2 \) increases, the slope of the strain-stress curves increases a little bit, and when \( \mu_1/\mu_2 \) is larger than \(~100\), the strain-stress curve are very similar. For all values of \( \mu_1/\mu_2 \), the stress increases dramatically after self-contact. When \( \mu_1/\mu_2 \) is larger, the self-contact occurs earlier. This is because that the bending of the ribs can delay the self-contact, while when \( \mu_1/\mu_2 \) is larger, ribs bend less.

For all five different geometries, the Poisson’s ratio and the rotational efficiency are output as a function of stiffness ratio \( \mu_1/\mu_2 \) in Fig.5.4.
Figure 5.4 FE results of the five chiral structures with a center core with $\mu^1/\mu^2 = 10, 100, 400, 700, 1000$: (a) Rotation efficiency vs. stiffness ratio, and (b) Poisson’s ratio vs. stiffness ratio.

Fig. 5.4a shows that the rotational efficiency $R_{p/a}$ is determined by both $\mu^1/\mu^2$ and $\alpha$ and $\beta$. When the stiffness ratio $\mu^1/\mu^2$ increases, for all geometries, $R_{p/a}$ increases and eventually becomes asymptotic to a limit. For the current core size and rib thickness, this limit of $R_{p/a}$ is $\sim 0.85$ for geometry 4 with $\alpha = 60^\circ$, $\beta = 120^\circ$, $\sim 0.66$ for the three quasi-isotropic geometries 1, 2, 3, and $\sim 0.55$ for the geometry 5 with $\alpha = 120^\circ$, $\beta = 60^\circ$. $R_{p/a}$ is sensitive to $\mu^1/\mu^2$ only when $\mu^1/\mu^2$ is smaller than 10. When $\mu^1/\mu^2$ is larger than 10, $R_{p/a}$ almost keeps unchanged.

As shown in Fig. 5.4b, for the three quasi-isotropic structures, the Poisson’s ratios of them
are very close to each other. When the stiffness ratio \( \mu^1 / \mu^2 \) increases, for all geometries, the Poisson’s ratio decreases and eventually becomes asymptotic to a limit. This limit is \(-0.2\) for geometry 4 with \( \alpha = 60^\circ, \beta = 120^\circ \), \(-0.66\) for the three quasi-isotropic geometries 1,2,3 and \(-1.45\) for geometry 5 with \( \alpha = 120^\circ, \beta = 60^\circ \). The Poisson’s ratio is sensitive to \( \mu^1 / \mu^2 \) only when \( \mu^1 / \mu^2 \) is smaller than 10. When \( \mu^1 / \mu^2 \) increases beyond 10, the Poisson’s ratio is almost unchanged.

For all five different geometries and different stiffness ratios, the FE results of the Poisson’s ratio vs. rotational efficiency are compared with the analytical prediction (Eq. 2.12) in Fig. 5.5, in which the FE results are the symbols and the analytical predictions are solid lines.

Figure 5.5 Comparison of the FE results and the analytical prediction for various chiral structures with a centre core: Poisson’s ratio vs. rotation efficiency.

Fig. 5.5 shows that the Poisson’s ratio is linearly related to the rotational efficiency which is consistent with the analytical prediction in Eq. (2.12) (with the average error of 10.50%). The theoretical results and the FE results match for all geometries. Again, it shows the limitation of tailoring Poisson’s ratio and rotational efficiency by only varying the stiffness of
the centre core.

5.2 The effects of core size

To investigate the influences of the core size, a chiral structure with $\alpha = \beta = 90^\circ$ and stiffness ratio between the centre core and the ribs $\mu^1/\mu^2 = 100$ was chosen. FE simulations with four different core sizes were performed. Same mesh sizes as in Chapter 5.1 were used in the FE models. The length of the ribs $a$ is 5.30 mm for all models but the radius of the centre core $a_c$ varied from 1.75 mm to 4.40 mm. Thus the ratio between the core radius and the rib’s length is $a_c/a = 0.33, 0.5, 0.66, \text{and } 0.83$. The unit cells of the four different FE models are shown in Fig.5.6.

![Unit cells of four different core sizes of chiral structures with a centre core.](image)

Figure 5.6. Unit cells of four different core sizes of chiral structures with a centre core.

The contours of the maximum principle strain with different core size are shown in Fig.5.7. The undeformed configurations are shown in red dash lines. It can be seen that for all structures, the major deformation mechanism of the cells is the bending of the ribs, and the cores barely deform. All of the models show the auxetic effects and for different core sizes, the auxetic effects are different.
Figure 5.7 Strain contours of different core sizes for chiral structures with a centre core ($\varepsilon$ represents the maximum principle strain).

The relation between the horizontal strain $\varepsilon_1$ and vertical strain $\varepsilon_2$ (2 is the loading direction) and the relation between the rotation angles of the active ribs and passive ribs during the deformation are shown in Fig.5.8a and b, respectively. The strain-stress curves are shown in Fig.5.8c.
Figure 5.8 FE results of chiral structure ($\alpha = \beta = 90^\circ$) with a centre core with $a_c/a =$ 0.33, 0.5, 0.66, 0.83: (a) the horizontal strain vs. vertical strain, (b) rotation angles of passive ribs vs. active ribs, and (c) strain vs. stress.

In Fig. 5.8a, the negative slope of each curve represents the instantaneous Poisson’s ratio during the deformation. Fig. 5.8a shows that the horizontal strain is almost linearly related to the vertical strain for all different $a_c/a$, which indicates that the Poisson’s ratio during the deformation is close to a constant. When $a_c/a$ increases, the Poisson’s ratio decreases.
Comparing to the results by varying the stiffness of the centre core, the Poisson’s ratio is more sensitive to the core size.

Fig.5.8b shows that the rotation angles of the active ribs and passive ribs are also almost linearly related, indicating a constant rotating efficiency during the deformation. When $a_c/a$ increases, the slopes of these curves also increase, indicating the elevated internal rotation. The results also show that the rotational efficiency is more sensitive to the core size than to the stiffness of the centre core.

In Fig.5.8c, the stress along direction 2 was non-dimensionalized with the Young’s modulus of the material in the hard ribs. It can be seen that the core size also influences the stress-strain behaviour of the structures. When $a_c/a$ increases, the non-dimensionalized stiffness will increase. Similar to the results of the Poisson’s ratio and the rotational efficiency, the non-dimensionalized stiffness of the chiral structure is also more sensitive to the core size than to the stiffness of the centre core.

The Poisson’s ratio and the rotational efficiency are output as a function of core size $a_c/a$ in Fig.5.9. It can be seen that when the core size increases, the rotational efficiency $R_{p/a}$ will dramatically increase, while the Poisson’s ratio will significantly decrease. Compared with increasing the core stiffness, enlarging the core size can more effectively amplifying the auxetic effects of the chiral structures with cores.
Figure 5.9 FE results of the chiral structures with a centre core with $a_c/a = 0.33, 0.5, 0.66, 0.83$: (a) the rotational efficiency vs. stiffness ratio, and (b) the Poisson’s ratio vs. stiffness ratio.

The FE simulation results of the Poisson’s ratio vs. rotational efficiency were compared with the analytical prediction (Eq. 2.12) in Fig.5.10, in which the FE results are symbols and the analytical predictions are solid lines.
Figure 5.10 Comparison of the FE results and the analytical prediction for chiral structures with soft hinges with $a_c/a = 0.33, 0.5, 0.66, 0.83$: Poisson’s ratio vs. rotation efficiency.

It can be seen that the Poisson’s ratio is linearly related to the rotational efficiency which is consistent with the analytical prediction in Eq. (2.12) (with average error f 6.36%). For all geometries, the theoretical results and the FE results are consistent. Fig.5.10 also shows that compared with the stiffness of the centre core, the core size more effectively influences the Poisson’s ratio and the rotational efficiency of the chiral structures with a centre core.
Chapter 6  Conclusions and Future Work

6.1 Conclusions

An analytical rigid-rod-rotational-spring model was developed to quantify the deformation mechanisms of auxetic chiral structures. The overall mechanical properties such as the Poisson’s ratio, Young’s modulus, and the internal rotation of the cells can be predicted via this model as functions of the cell geometry and material composition. The unique deformation mechanism of the rotation-induced auxetic effects was accurately captured by this model. The relations between the Poisson’s ratio, rotational efficiency and rotational stiffness were derived through equilibrium and kinematics. The Young’s modulus of the chiral structure was also derived and related to the cell geometry and rotational stiffness of the chiral structure by using the energy method. In addition, the micro-stability analysis was performed and it was proven that based on the rigid-rod-rotational-spring model, the auxetic chiral cells are always stable in-plane under both tension and compression.

According to the concept provided by the analytical model, new chiral structures were designed to enlarge the auxetic effect. The new chiral structures include a chiral structure with center cores, and a chiral structure with soft hinges at the corners. The original and new designs were then fabricated via a multi-material 3D printer (Objet Connex 260). To evaluate the new designs, the deformation mechanisms of the original and new auxetic chiral specimens were explored via mechanical experiments and FE simulations. The original chiral structure showed an initial Poisson’s ratio \(\sim 0.3\), the new chiral structure with a circular core was proved to have
a Poisson’s ratio of \( \sim 0.5 \), and the one with soft hinges showed a Poisson’s ratio \( \sim -1 \). The FE simulation results were consistent with the experiments.

Then, thorough parametric studies were performed for the two new chiral structures via FE simulations, which provide design guideline for the new auxetic chiral structures. For the auxetic chiral structures with soft hinges, the parameters explored included the stiffness ratio between rib material and hinge material, the geometry of the cells, the location of the soft hinges, and the thickness of the ribs. It was shown that the stiffness of the hinges on the passive ribs significantly influences the Poisson’s ratio, the rotational efficiency, and the stress-strain curves. However, the stiffness of the hinges on the active ribs only influences the stress-strain curves but not the Poisson’s ratio and the rotational efficiency. The thickness of the ribs also barely influences the Poisson’s ratio and rotational efficiency but has large influences on the strain-stress curves.

For the auxetic chiral structures with hard cores, the parameters explored included the stiffness and the size of the cores. It was shown that the Poisson’s ratio and the rotational efficiency are sensitive to both the core stiffness and the core size. However, when the stiffness of the core increases beyond a certain level, the Poisson’s ratio and the rotational efficiency are barely influenced by the core stiffness.

### 6.2 Future work

By taking the work in this thesis as the starting point, the future work can be extended in many different directions. Several major directions of the future work are summarized as the following:
(1) Based on the concept of the rigid-rod-rotational-spring model developed in this thesis, further new designs can be made to extend the current chiral structures into a new family of chiral structures. For example, the distribution of soft hinges and cores can be varied, the core shape can be tailored, and the handness of each cell can vary etc. By varying each geometric factor, a new series of chiral structures can be designed. Due to the new geometric design, new deformation mechanisms can be obtained. For example, by introducing auxetic cores in the center of the original chiral structures, new sequential cell-opening mechanisms can be obtained.

By introducing spatial variation of certain geometric parameters, functional graded auxetic chiral structures can be designed. The new designs are expected to have new properties and therefore new functions and applications.

(2) The unique deformation mechanism of the chiral structure with significant internal rotation makes it a good candidate to investigate the micro-polar theory in mechanics. A Micro-polar model will be used to quantify the constitutive relation of the new family of chiral structures. The advantage of this model is that each parameter in the micro-polar model for the chiral structures will have clear physical meaning, and can be calculated from the geometry and material combination. FE simulations and mechanical experiments on 3D printed chiral structures will be performed to verify the new models developed.

The coupling effect between chirality and auxeticity of this type of material provides a new challenge to the micro-polar theory. This study in this direction will advance the micro-polar theory. Also, if successful, the micro-polar model for the chiral structures will make it possible
to predict the mechanical response of large-scale structures or objects made of micro-structured chiral composites.

(3) Another unique deformation mechanism of the chiral structures is the coupling between normal strain and shear strain even when the external traction is uni-axial. This coupling effect is due to the chirality. A thorough integrated analytical, numerical, and experimental study will be performed to explore this coupling effect.

(4) Based on the 2D designs, three dimensional designs on the auxetic chiral structures will be developed. All 2D analytical, numerical and experimental study will be extended to a more general 3D version. Very limited 3D designs on auxetic materials exist and no 3D open-cell chiral structure exists. The study in this direction will significantly expand the data base for the designs on auxetic materials.

Also, the new 3D auxetic chiral structures can be used to solve the second planar problem in micro-polar theory, and can significantly advance the theory.

(5) The mechanical behavior of the auxetic chiral structures under bi-axial compression loading will be studied. A new fixture to perform bi-axial compression loads on a uni-axial material testing machine will be designed and fabricated. Both FE simulations and mechanical experiments on 3D printed chiral structures will be performed under bi-axial compressive loads.

(6) The micro and macro-stability of the auxetic chiral structures will be studied. Since the auxetic effects will disappear under macro-instability, Micro-instability/no-instability are desired in design. Criteria and design space will be developed for both micro and macro instability. The bi-axial loads are expected to help avoiding/delaying macro-instability.
(7) The applications of this family of auxetic chiral structures will be extended to photonic and photonic composites, and smart/adaptive composites. Wave-propagation properties of the auxetic chiral structures will be studied. Shape-memory polymers will be used to fabricate the auxetic chiral structures. The multi-functionality of the new adaptive composites will be explored.
References


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Appendix A: Hyperelastic Models for the Materials in the 3D Printer

In this appendix, several hyperelastic models were used to model the stress-strain behavior of the materials in the 3D printer. The materials include model material TangoPlus and the digital material DM9760. The hyperelastic models used include Neo-Hookean, Mooney-Rivilin, 3rd order Yeoh, 3rd order Ogden, and Arruda-Boyce hyperelastic models. A brief review of all these models is provided as the following:

The strain energy density function of the Neo-Hookean model is [81]:

\[ W = C_1 (I_1 - 3) + D_1 (J - 1)^2 , \]  
(A.1)

where \( C_1 \) is determined by fitting the experimental data and \( D_1 \) is assumed to be zero here, to represent the incompressibility of the materials. \( I_1 \) is the first invariant of the right Cauchy-Green deformation tensor and \( I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \), where \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) are the principal stretches along directions 1, 2, and 3. \( J \) is the determinant of Jacobian matrix and \( J = \lambda_1^2 \lambda_2^2 \lambda_3^2 \).

The strain energy density function of the Mooney-Rivlin model is [81]:

\[ W = C_{10} (I_1 - 3) + C_{01} (I_2 - 3) + D_1 (J - 1)^2 , \]  
(A.2)

where \( C_{01} \) and \( C_{10} \) are determined from the experiments. \( D_1 \) is assumed to be zero to represent an incompressible hyper-elastic model. \( I_2 \) is the second invariant of the right Cauchy-Green deformation tensor and \( I_2 = \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2 \).

The strain energy density function of the Yeoh model is [83]:

\[ W = \sum_{i=1}^{n} C_{i0} (I_1 - 3)^i + \sum_{k=1}^{n} C_{k1} (J - 1)^{2k} , \]  
(A.3)

where \( n \) is the order of the strain energy potential. \( C_{i0} \) are determined from experiments. For
incompressible materials, $C_{k1}$ are all zero.

The strain energy density function of the Ogden model is [84]:

$$W = \sum_{p=1}^{n} \frac{\mu_p}{\alpha_p} (\lambda_1^{\alpha_p} + \lambda_2^{\alpha_p} + \lambda_3^{\alpha_p} - 3),$$  \hspace{1cm} (A.4)

where $n$ is the order of the strain energy potential. For incompressible hyperelastic model, $\lambda_1\lambda_2\lambda_3 = 1$. $\mu_p$ and $\alpha_p$ are determined from experiments.

The strain energy density function of Arruda-Boyce model is [87]:

$$W = C_1 \sum_{i=1}^{5} \alpha_i \beta^{i-1} (I_1^i - 3) + D_1 \left( \frac{j^2 - 1}{2} - \ln j \right),$$  \hspace{1cm} (A.5)

where $\beta = \frac{1}{\lambda_m}$, $\alpha_1 = \frac{1}{2}$, $\alpha_2 = \frac{1}{20}$, $\alpha_3 = \frac{11}{1050}$, $\alpha_4 = \frac{19}{7000}$, and $\alpha_5 = \frac{519}{673750}$, $C_1$ and $\lambda_m$ are determined from experiments. For incompressible materials, $D_1$ is zero.

For TangoPlus and DM9760, both uni-axial tension and uni-axial compression experiments were performed. The best fitting results of each model are shown in Fig. A.1 and A.2. The parameters of each model are provided in Table A.1.

![Figure A.1 Experimental and fitting results for TangoPlus.](image)
Figure A.2 Experimental and fitting results for DM9760.
<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter 1 (MPa)</th>
<th>Parameter 2 (MPa)</th>
<th>Parameter 3 (MPa)</th>
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Table A.1 Parameters of each hyperelastic model for TangoPlus material.
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<th>Parameter 2 (MPa)</th>
<th>Parameter 3 (MPa)</th>
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<td>0.3525</td>
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</table>

Table A.2 Parameters of each hyperelastic model for DM9760.

In the FE simulations in the thesis, the Mooney-Rivlin model was chosen to model the TangoPlus and DM9760 in the auxetic chiral structures.
Appendix B: The Effect of Interphase in the 3D Printed Specimens

Due to the material jetting process of the 3D printer, an interphase exists at the boundary of two different materials in the 3D printed multi-material specimens. The interphase is a transition zone between the two materials, and the material property in this zone is between the two materials.

According to the tension experiments on the specimens of flat soft interfacial layer, when the interphase thickness is ~0.4 mm, the Young’s modulus is ~3 MPa. To simulate the mechanical behaviors of the auxetic chiral structures with soft hinges and evaluate the interphase effects, FE models with and without the interphase were set up, as shown in Fig. B.1.
Figure B.1 The FE models of the chiral structure with soft hinges (a) with interphase and (b) without interphase.

The experimental results and the strain-stress curves of these two models are compared in Fig.B.2, in which, the symbols with error bars are the experimental data, the blue dash line is the FE result from the model without interphase, and the black solid line is the FE result from
the model with interphase.

Figure B.2 FE and experimental results of the stress-strain curves (experimental data: symbols with error bar; FE model without interphase: blue dash line; FE model with interphase: black solid line).

Fig. B.2 shows that the FE model with interphase accurately captures the experimental data, while the FE model without interphase under predicts the experiments. It can be seen that the influence of the interphase on the stress-strain behavior is significant. More sophisticated methods to determine the thickness and mechanical properties of the interphase needs to be developed in the future.
Figure List

Figure 1.1 The unit cells of (a) a diamond-shaped intact model, and (b) the ‘missing rib model’ .................................................................3

Figure 2.1. (a) The geometry of a general chiral cell and (b) the rigid-rod-rotational-spring model..................................................................................................................6

Figure 2.2. The influences of geometry on the negative Poisson’s ratio and the rotation efficiency predicted by the rigid-rod-rotational-spring model.................................9

Figure 2.3 The free body diagrams of the active ribs in the unit cell. .................................10

Figure 2.4. The relation between the rotation efficiency and the rotational stiffness ratio of the center and corner springs.................................................................12

Figure 2.5 Poisson’s ratio predicted via the rigid-rod-rotational-spring model with different geometries .................................................................13

Figure 2.6 The equilibrium paths of a unit cell with $\alpha_0 = 10^\circ, 90^\circ, \text{and } 170^\circ$..........17

Figure 2.7 Instability criteria ........................................................................................................18

Figure 3.1 (a) SOLIDWORKS models for the three designs (different color represents different materials), (b) the 3D printed specimens with markers on .........................20

Figure 3.2. The dogbone specimens for (a) TangoPlus and (b) DM9760 (unit: mm).......22

Figure 3.3 Experimental data for the dogbone specimens of TangoPlus, (a) the load-displacement curves, and (b) the true stress-strain curves.................................23

Figure 3.4 Experimental data for the dogbone specimens of DM9760, (a) the load-displacement curves, and (b) the true stress-strain curves.................................23
Figure 3.5 The cylindrical specimen for uni-axial compression test of TangoPlus and DM9760.

Figure 3.6 Two-cycle loading: (a) TangoPlus (b) DM9760.

Figure 3.7. Mooney-Rivlin model fitting for TangoPlus.

Figure 3.8 Mooney-Rivlin model fitting for DM9760.

Figure 3.9 Images of the deformed specimens at different levels of overall strain, (a) the basic chiral specimen, (b) chiral specimen with a center core, and (c) chiral specimen with soft hinges (initial area showed as red square).

Figure 3.10 The load-displacement curves of (a) the basic chiral specimen 1, (b) the chiral specimen 2 with a center core and (c) chiral specimen 3 with soft hinges.

Figure 3.11 The measurement of internal rotation, (a) the initial undeformed configuration (left), and (b) the deformed configuration (right).

Figure 3.12 Experimental results of (a) the Poisson’s ratio vs. overall strain and (b) the rotational efficiency vs. overall strain.

Figure 3.13 FE model and setup of chiral structure.

Figure 3.14 The stress-strain curves from the experiments (symbols with error bars) and FE simulations (solid lines) for the three specimens.

Figure 3.15 (a) Poisson’s ratio vs. overall strain, and (b) the rotational efficiency vs. overall strain.

Figure 3.16 The comparison of experiments, FE results and analytical prediction with different cell sizes, boundary conditions: Poisson’s ratio vs. the rotational efficiency.
Figure 4.1. Unit cells of five different geometry of chiral structures with soft hinges.

Figure 4.2. Strain contours of different geometries and stiffness ratios for chiral structures with soft hinges (ε represents the maximum principle strain).

Figure 4.3. FE results of chiral structure with soft hinges with $\mu_1\mu_2 = 1, 10, 100, 400, 700, 1000$: (a) the horizontal strain vs. vertical strain, (b) rotation angles of passive ribs vs. active ribs, and (c) strain vs. stress ($\alpha = \beta = 90^\circ$).

Figure 4.4. FE results of the five chiral structures with soft hinges with $\mu_1\mu_2 = 10, 100, 400, 700, 1000$: (a) rotation efficiency vs. stiffness ratio, and (b) Poisson’s ratio vs. stiffness ratio.

Figure 4.5. Comparison of the FE results and the analytical prediction for various chiral structures with soft hinges: Poisson’s ratio vs. the rotational efficiency.

Figure 4.6. Strain contours of the chiral structures with soft hinges with different stiffness ratios $\mu_1\mu_3$ (ε represents the maximum principle strain, $\mu_1\mu_2 = 1000$).

Figure 4.7. FE results of the chiral structures with soft hinges with $\mu_1\mu_3 = 10, 100, 400, 700, 1000$: (a) horizontal strain vs. vertical strain, (b) rotational angle on the passive ribs vs. active ribs, and (c) strain vs. stress ($\mu_1\mu_2 = 1000$).

Figure 4.8. FE results of the chiral structures with soft hinges with $\mu_1\mu_3 = 10, 100, 400, 700, 1000$: (a) Rotation efficiency vs. stiffness ratio, and (b) Poisson’s ratio vs. stiffness ratio ($\mu_1\mu_2 = 1000$).

Figure 4.9. Comparison of the FE results and the analytical prediction for chiral structures.
with soft hinges with different stiffness ratio $\mu_1\mu_3$: Poisson’s ratio vs. rotation efficiency ($\mu_1\mu_2 = 1000$). ..............................................................49

Figure 4.10 Unit cells of five chiral structures with different rib thickness. ..................50

Figure 4.11 Strain contours of the chiral structures with different rib thickness ($\varepsilon$ represents the maximum principle strain). .................................................................50

Figure 4.12 FE results of the chiral structures with $t_a = 0.19, 0.28, 0.36, 0.47, 0.57$: (a) horizontal strain vs. vertical strain, (b) rotational angles on the passive ribs vs. those on the active ribs, and (c) strain vs. stress.................................................................51

Figure 4.13 FE results of the chiral structures with $t_a = 0.19, 0.28, 0.36, 0.47, 0.57$: (a) Rotation efficiency vs. stiffness ratio, and (b) Poisson’s ratio vs. stiffness ratio. ....52

Figure 4.14 Comparison of the FE results and the analytical prediction for chiral structures with rib thickness: Poisson’s ratio vs. rotation efficiency .........................53

Figure 4.15 The history of the instantaneous Poisson’s ratio for (a) $\alpha_0 = 60^\circ, \beta_0 = 120$, (b) $\alpha_0 = 90^\circ, \beta_0 = 90$, and (c) $\alpha_0 = 120^\circ, \beta_0 = 60^\circ$. .........................................................55

Figure 5.1 Unit cells of five different geometry of chiral structures with a centre core. 58

Figure 5.2 Strain contours of different geometries and stiffness ratios for chiral structures with a centre core ($\varepsilon$ represents the maximum principle strain).................................59

Figure 5.3 FE results of chiral structure with a centre core with $\mu_1\mu_2 = 1, 10, 100, 400, 700, 1000$: (a) the horizontal strain vs. vertical strain, (b) rotation angles of passive ribs vs. active ribs, and (c) strain vs. stress. ($\alpha = \beta = 90^\circ$). ......60

Figure 5.4 FE results of the five chiral structures with a center core with $\mu_1\mu_2 =$
10,100,400,700,1000: (a) Rotation efficiency vs. stiffness ratio, and (b) Poisson’s ratio vs. stiffness ratio.

Figure 5.5 Comparison of the FE results and the analytical prediction for various chiral structures with a centre core: Poisson’s ratio vs. rotation efficiency.

Figure 5.6. Unit cells of four different core sizes of chiral structures with a centre core.

Figure 5.7 Strain contours of different core sizes for chiral structures with a centre core ($\varepsilon$ represents the maximum principle strain).

Figure 5.8 FE results of chiral structure ($\alpha = \beta = 90^\circ$) with a centre core with $aca = 0.33, 0.5, 0.66, 0.83$: (a) the horizontal strain vs. vertical strain, (b) rotation angles of passive ribs vs. active ribs, and (c) strain vs. stress.

Figure 5.9 FE results of the chiral structures with a centre core with $aca = 0.33, 0.5, 0.66, 0.83$: (a) the rotational efficiency vs. stiffness ratio, and (b) the Poisson’s ratio vs. stiffness ratio.

Figure 5.10 Comparison of the FE results and the analytical prediction for chiral structures with soft hinges with $aca = 0.33, 0.5, 0.66, 0.83$: Poisson’s ratio vs. rotation efficiency.

Figure A.1 Experimental and fitting results for TangoPlus.

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Figure B.2 FE and experimental results of the stress-strain curves (experimental data: symbols with error bar; FE model without interphase: blue dash line; FE model with interphase: black solid line).
Table List

Table 3.1 The material model used for FE simulation..................................................27

Table A.1 Parameters of each hyperelastic model for TangoPlus material.................87

Table A.2 Parameters of each hyperelastic model for DM9760. .................................88