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JOEL EDWARD HENKEL

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BY
JOEL EDWARD HENKEL
M.S., University of New Hampshire, 1958

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CHAPTER ONE

INTRODUCTION

1.1. Introduction. Shortly after the introduction of stripping as a reaction mechanism (Butler 1950), (Butler 1951), (Bhatia 1952), investigators began to study (d,p) reactions on light nuclei at low energy in order to see if the observed angular distributions could be explained by stripping theory. Table 1.1 shows the reactions studied by different investigators. Stripping was indeed observed to play an important role in most cases, (Grosskreutz 1956), (Jengerius 1954), (Koudijs 1952), in competition with compound nucleus formation, (Paris 1954), (Booth 1957), (Juric 1955).

Later, modifications to the simple plane wave theory of Butler were made to take into account the various distortions to the plane wave assumption arising from the coulomb field and the residual nuclear interaction, other than stripping, (Tobocman 1954), (Tobocman 1955). Coulomb distortion would be expected to be particularly strong for bombarding energies well below the coulomb barrier, which is the case for the energy range $0.3 < E < 1.5$ MeV; covering the work cited in the literature survey of Table 1-1. Among others, Juric, (Juric 1956), has found that Tobocman theory, which takes into account these distortions, fits the observed data better than the plane wave theory.
Previous Work on (d,p) Reactions on Light Nuclei at Low Energy

The energies listed are the kinetic energies of the incident deuteron in the laboratory system.

*Li$^6$(d,p)Li$^7$ (Krone 1950), (Whaling 1950),
0.4 to 8 MeV 0.4 to 1.4 MeV
(Dunbar 1951), (Nickell 1954)
0.4 to 0.8 MeV 0.825 to 3.0 MeV

*Be$^9$(d,p)Be$^{10}$ (Resnick 1951), (De Jong 1952a,b),
0.3 to 0.88 MeV 0.3 to 0.8 MeV
(Juric 1955), (Juric 1956), (Smither 1959)
0.6 to 1.45 MeV 0.1 to 0.25 MeV

B$^{10}$(d,p)B$^{11}$ (Endt 1952), (Paris 1954)
0.31 MeV 0.18 to 0.58 MeV

*C$^{12}$(d,p)C$^{13}$ (Koudijs 1952), (Takemoto 1954),
0.37 MeV 0.52 to .84 MeV
(Juric 1956)
0.6 to 1.4 MeV

C$^{13}$(d,p)C$^{14}$ (Koudijs 1952), (Koudijs 1953)
0.37 MeV 0.282 to 0.637 MeV

*N$^{14}$(d,p)N$^{15}$ (Jongerius 1954), (Booth 1957)
0.4 to 0.6 MeV 0.595 to 0.99 MeV

*O$^{16}$(d,p)O$^{17}$ (Juric 1955), (Juric 1956),
0.6 to 1.45 MeV
(Grosskreutz 1956), (Richter 1958)
1.05 to 2.51 MeV 2.0 MeV

F$^{19}$(d,p)F$^{20}$ (Takemoto 1956), (Borecka 1963)
0.75 to 1.4 MeV 0.5 to 0.65 MeV

Table 1-1
More recently, it has been suggested, (Wilkinson 1959), that the original plane wave theory can be expected to hold for energies well below the coulomb barrier, provided that the Q value of the reaction is very small. There is still considerable discussion about the theoretical model to explain these low energy stripping reactions, (Glendenning 1963). Wilkinson's suggestion has been experimentally investigated by Sellschop, (Sellschop 1959), (Sellschop 1960), (Sellschop 1963a&b), who has observed the reactions: 
\[ \text{Li}_7(d,p)\text{Li}_8, Q = -0.192 \text{ MeV}; \text{C}^{12}(d,p)\text{C}^{13*}, Q = -0.368 \text{ MeV}; \]
and \[ \text{B}^{11}(d,p)\text{B}^{12}, Q = 0.195 \text{ MeV}. \]
In these very low Q value (< 0.2 MeV reactions, Wilkinson's suggestion has been verified. Now, the Q values for the majority of the reactions cited in Table 1-1 range from 1 to 8 MeV. It is of interest to test the upper limit of Q for Wilkinson's low Q value reactions by observing the extent to which plane wave theory holds for higher Q value reactions. The starred reactions listed in Table 1-1 were chosen for study.

The purposes of this thesis are:
1.) determine the relative contributions of compound nucleus and stripping mechanisms for the reactions, 
\[ \text{Li}_6(d,p)\text{Li}_7 Q = 4.027 \text{ MeV}, \text{Li}_6(d,p)\text{Li}_7* Q = 4.55, \text{Be}^9(d,p)\text{Be}^{10} Q = 4.585, \text{Be}^9(d,p)\text{Be}^{11*} Q = 1.217, \text{C}^{12}(d,p)\text{C}^{13} Q = 2.719, \]
\[ \text{N}^4(d,p)\text{N}^{15} Q = 8.615, \text{O}^{16}(d,p)\text{O}^{17} Q = 1.919, \text{and} \text{O}^{16}(d,p)\text{O}^{17*} Q = 1.048 \]
in the energy range 200 to 350 KeV, and
2.) test the validity of the plane wave Born approximation for these same reactions.
STRIPPING AND COMPOUND NUCLEUS PROCESSES

STRIPPING PROCESS

\[ \text{APPROACH} \rightarrow \text{STRIPPING} \rightarrow \text{FINAL STATE} \]

\[ \text{d} \quad \rightarrow \quad \text{Li}^6 \quad \rightarrow \quad \text{Li}^7 \]

COMPOUND NUCLEUS PROCESS

\[ \text{APPROACH} \rightarrow \text{COMPOUND NUCLEUS} \rightarrow \text{FINAL STATE} \]

\[ \text{d} \quad \rightarrow \quad \text{Li}^6 \quad \rightarrow \quad \text{Be}^8^* \quad \rightarrow \quad \text{Li}^7 \]

FIGURE I-1
This has been done by studying the angular distributions of the protons produced by the reactions.

The thesis is organized in the following way. The rest of this chapter is devoted to a qualitative picture of the nuclear reaction processes. Chapter 2 contains a discussion of why the plane wave Born approximation should be expected to apply for these reactions, followed by the development of a mathematical expression for the plane wave Born approximation angular distribution, called a Bhatia distribution, that can be compared with the experimental angular distribution. Chapter 3 contains a description of the experimental method. Chapters 4 and 5 contain the observed angular distributions and a comparison of these with other work at higher energies, respectively. Chapter 6 contains a discussion of how the results can be used to determine the relative contributions of compound nucleus and stripping mechanisms, and a discussion of the interpretation of the fit, or lack of fit, to the plane wave Born approximation.

1.2. Description of Stripping and Compound Nucleus Processes. Figure 1-1 shows schematically how a stripping and a compound nucleus process are visualized, with the Li\(^6\)(d,p)Li\(^7\) reaction as an example. These two reaction processes can be described as limits to an intermediate, more generally observed process, in the following way.

Stripping is a form of direct reaction, where the incoming particle interacts directly with one nucleon or group of nucleons in the target nucleus; the rest of the target nucleus plays no part in the reaction. In compound
nucleus formation the incoming particle amalgamates with the target to form an intermediate state. Here an incoming particle has initiated a sequence of reactions, which eventually involve all of the nucleons in the target nucleus. The transition from direct reaction to well developed compound nucleus formation can be thought of as a statistical process where the energy of the system tends toward equipartitism. The most important characteristic of this energy equalization is the loss of phase relation or coherence between the incoming particle and the system of interacting particles. This coherence decreases as the reaction proceeds, until, at equipartition or "thermal equilibrium", complete incoherence or random phase occurs. This can be stated in another way: the extreme compound nucleus has no memory of how it was formed. If the thermalization process is never completed, before the end of the reaction, an intermediate general process operates. Some coherence with the incoming particle will remain, while the target as a whole will be involved.

1.3. Angular Distributions Predicted by Stripping and Compound Nucleus Processes. If a reaction is an example of extreme compound nucleus formation, the absence of any coherence of the scattered particle with respect to the incoming particle implies that no direction is defined in the center of mass system. Hence, an isotropic angular distribution is predicted in the center of mass system. Since momentum must be conserved, the beam axis is defined in the laboratory system through the recoil particles. For beam
energies much less than the Q of the reaction, the recoils can be neglected, and angular distributions in the laboratory and center of mass systems are approximately the same.

When an extreme compound nucleus is not formed before the reaction is completed, some residual phase relation will occur. This intermediate process still involves the whole nucleus, so it is still a compound nucleus process. However, the angular distribution predicted will depend on the number of final states participating.

If only a single state participates, which will be the case for light nuclei at low excitation energies where the density of states is low, the following parity argument enables one to restrict the complexity of the angular distribution. Since parity is assumed to be conserved in strong interactions, the initial and final state functions will have definite parity, either even or odd. Now, the angular distribution is determined by the square of the final state wave function. Thus, it is necessarily even, which implies a symmetry about $\Theta = 90$ degrees, where $\Theta$ = the scattering angle, i.e. the angle between the incident beam and the scattered particle in the angular distribution.

If the final state of the residual nucleus is made up of two or more states of definite parity and angular momentum, interference terms are possible. Interference between two states of opposite parity will yield odd cross terms in the square of the final state wave function, and hence an asymmetry about 90 degrees in the angular distribution.
MOMENTUM VECTORS IN THE STRIPPING PROCESS

\[ \text{\( \vec{h}_K = \vec{h}_q \)} \]

\[ \Theta_{sc} \]

\[ \text{\( \vec{h}_K \)} \]

\[ \text{\( \vec{h}_d \)} \]

\[ \text{\( \vec{h}_p \)} \]

\[ \text{\( \vec{h}_n = \vec{h}_q \)} \]

\[ \text{FIGURE 1-2} \]

\( \vec{h}_q \) is the momentum transferred to the target. \( \vec{h}_K \) is the momentum contributed to \( \vec{h}_p \) by the internal motion of the deuteron.

HUBY DIAGRAM
The angular distribution predicted by a stripping process becomes apparent when one describes the stripping process. In Figure 1-1 the incoming deuteron is polarized by the coulomb field of the target nucleus, i.e. the proton of the deuteron is repelled while the neutron passes close to the target. When the neutron of the deuteron comes under the influence of the nuclear force of the target nucleus, it is stripped from the deuteron and captured by the target. The proton, remaining at a much greater distance, continues more or less in the direction of the original deuteron, the beam direction. Thus, a strong forward peaking in the angular distribution of the protons is predicted.

Figure 1-2 shows the momentum vectors involved in the stripping process, recognizing that momentum is conserved. Kinematics determine the magnitude of \( \hbar k_p \) as a function of the scattering angle, \( \Theta_{sc} \). Another parameter, the magnitude of \( \hbar k_n \), must be specified in order to complete the picture. This is determined by the structure of the residual nucleus, which is formed when the target captures the neutron. If a simple shell model for the residual nucleus is used, with single particle wave functions of definite angular momentum, then a simple argument determines \( \hbar k_n \). Let the angular momentum of the state of the captured neutron be \( \ell \hbar \), where \( \ell = 1, 2, 3, \text{ etc.} \), but \( \hbar k_n \times \bar{R} = \ell \hbar \), where \( \bar{R} \) is the effective radius for the capture of the neutron. Hence the magnitudes satisfy \( \hbar k_n R = \hbar \), and \( \hbar k_n = \hbar / R \). So \( \hbar k_n \) is determined by the value of the captured neutrons wave function. The
Huby diagram, (Huby 1953), for the case where $l = 2$ is shown in Figure 1-2. This semiclassical picture gives a simple explanation for the observed forward peaking in (d,p) angular distributions. It is seen that for a given $l$ value, there is a definite $\Theta_{sc}$ which satisfies the condition mentioned. A maximum in the cross-section corresponding to this angle might then be expected.

The momentum contributed to $\hbar k_p$ by the internal motion of the deuteron, shown in Figure 1-2, can be understood as follows. If there were no internal motion of the deuteron, the proton and neutron would be free particles, each with half the deuteron momentum, $1/2 \hbar k_d$. The internal momentum contribution of the proton momentum, $\hbar k$, is added to the free particle proton momentum to sum to the total proton momentum.
CHAPTER 2

THEORY

2.1. Introduction. This chapter treats the application of the Born approximation to the results of the present experiment. The chapter is organized in the following way. In Section 2.2, the Born approximation is expressed in terms of the differential cross-section for a scattering or reaction process. The method used to introduce the approximation is discussed. Section 2.3 discusses the validity of the Born approximation for direct reactions. Section 2.4 distinguishes two types of Born approximations, the plane wave Born approximation, hereafter called PWBA, and the distorted wave Born approximation, hereafter called DWBA. Section 2.5 discusses whether DWBA is always a better approximation than PWBA, and concludes that distortions should be minimum for special reactions called Oppenheimer-Phillips reactions. Section 2.6 discusses more explicit experimental situations where PWBA may apply, namely, low Q stripping reactions at low energy. Section 2.7 extends the discussion to reactions with larger Q. Section 2.8 describes the selection of definite reactions to experimentally test the validity of PWBA for stripping reactions.

The chapter ends with Sections 2.9 and 2.10, which develop an explicit expression for the PWBA to compare with experiment.
2.2. **The Differential Cross-section in Terms of the Born Approximation.** The differential cross-section for a scattering or reaction process is defined in terms of the scattering amplitude, \( f_k^{(+)}(\hat{\mathbf{r}}) \), by

\[
\frac{d\sigma}{d\Omega} = \left| f_k^{(+)}(\hat{\mathbf{r}}) \right|^2
\]

(1)

the scattering amplitude, \( f_k^{(+)}(\hat{\mathbf{r}}) \) can be defined by the following treatment, following Merzbacher (Merzbacher 1961). The wave equation describing the scattering or reaction process, in the C.M. system is

\[
\left( \mathbf{\nabla}^2 + \mathbf{k}^2 \right) \psi_k^{(+)}(\hat{\mathbf{r}}) = \frac{2\mu}{\mathbf{k}^2} U(\hat{\mathbf{r}}) \psi_k^{(+)}(\hat{\mathbf{r}}),
\]

(2)

where \( U(\hat{\mathbf{r}}) \) is the potential causing the scattering or reaction, \( \psi_k^{(+)}(\hat{\mathbf{r}}) \) are the asymptotic wave functions, \( (+) \) for outgoing waves, \( (-) \) for incoming waves. The wave functions \( \psi_k^{(+)}(\hat{\mathbf{r}}) \) satisfy the integral equation

\[
\psi_k^{(+)}(\hat{\mathbf{r}}) = \frac{i}{(2\pi)^3} \int \frac{\exp(\pm ik|\hat{\mathbf{r}} - \hat{\mathbf{r}}'|)}{|\hat{\mathbf{r}} - \hat{\mathbf{r}}'|} U(\hat{\mathbf{r}}') \psi_k^{(+)}(\hat{\mathbf{r}}') d\hat{\mathbf{r}}' \]

(3)

where \( \frac{\exp(\pm ik|\hat{\mathbf{r}} - \hat{\mathbf{r}}'|)}{|\hat{\mathbf{r}} - \hat{\mathbf{r}}'|} \) is the free particle Green's function,

\( \hat{\mathbf{r}}' \) is the position vector of the source point, where scattering takes place,

\( \hat{\mathbf{r}} \) is the position vector of the asymptotic wave functions,
\( \hat{r} \) is a unit vector along \( \vec{r} \),

\( k \) is the wave number of the scattered particle.

In order to simplify equation (3), approximations must be made. If the exponential is expanded in powers of \( \vec{r}' \),

\[ k |\vec{r} - \vec{r}'| = k (\vec{r} - 2 \vec{r} \cdot \vec{r}' + \vec{r}'^2)^{1/2} = k \vec{r} - k \vec{r} \cdot \vec{r}' + \frac{k (\vec{r} \times \vec{r}')^2}{2k} + \ldots \]  

(4)

Now the maximum value of \( \vec{r}' \), the source coordinate, is determined by the range of \( U(\vec{r}') \). If \( U(\vec{r}') \) is taken as a short range potential, the maximum value of \( \vec{r}' \) will be small. Thus, for large \( \vec{r} \), the quadratic term in the expansion can be neglected and the \( \vec{r}' \) in the denominator can be neglected.

So for large \( \vec{r} \),

\[ \Psi^{(\pm)}_k(\vec{r}) = \frac{1}{(2\pi)^{3/2} E} e^{i \frac{\vec{r} \cdot \vec{r}}{E}} \int \frac{e^{i \vec{k} \cdot \vec{r}'}}{\sqrt{4\pi \hbar}} U(\vec{r}') \Psi^{(\pm)}_k(\vec{r}') \, d^3 \vec{r}' \]  

(5)

\[ \psi^{(\pm)}_k(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \left[ e^{i \frac{\vec{r} \cdot \vec{r}}{E}} + \frac{e^{i \vec{k} \cdot \vec{r}}}{\sqrt{\hbar}} \right] \]  

(6)

\[ f^{(\pm)}_k(\vec{r}) = -\frac{(2\pi)^{3/2}}{4\pi} \int e^{i \vec{r} \cdot \vec{r}'} U(\vec{r}') \psi^{(\pm)}_k(\vec{r}') \, d^3 \vec{r}' \]  

(7)
In equations (6) and (7), \( f_k^{(+)}(\vec{r}) \) is the coefficient of the outgoing spherical wave, hence it is called the scattering amplitude. \( \Psi_k^{(+)}(\vec{r}') \) is the exact solution to equation (2). The Born approximation simplifies the integrand in equation (7) by an approximate form for \( \Psi_k^{(+)}(\vec{r}') \). The first Born approximation, or PWBA, uses

\[
\Psi_{\text{First Born},k}(\vec{r}') = e^{i\vec{k}.\vec{r}'},
\]

which ignores the effect of scattering on \( \Psi_k^{(+)}(\vec{r}') \). The second Born approximation uses the \( \Psi_{\text{Second Born},k}(\vec{r}') \) found by substituting \( \Psi_{\text{First Born},k}(\vec{r}') \) in the integral of equation (5) and integrating, which is a better approximation. Higher order Born approximations are obtained by iterating this procedure on equation (5).

2.3. Validity of the Born Approximation for Direct Reactions. The question of the validity of the Born approximation in direct reactions has been approached from two viewpoints. In the first, the coupling between reaction channels is discussed; in the second the relative variation of nuclear potentials is used as a criterion of validity. These two approaches will be considered in turn.

Preston (Preston 1962) divides the potential in equation (2) into components, one describes elastic scattering, another describes compound nucleus formation, another a direct reaction, and so on. The \( \Psi_{\text{Born},k}(\vec{r}') \) then are
solutions to equation (2) with potentials describing the various types of interaction. These solutions define channels associated with the various possible interactions, so that the approach is called a partition into channels. To the extent that equations (2) with different potentials can be separated or uncoupled, the solutions will be different; to the extent that the equations couple, the solutions will mix and overlap. This mixing is called coupling of channels.

Austern and Preston, (Austern 1963), (Preston 1962), mention two situations, in terms of the coupling between channels, where the Born approximation should be valid for direct reactions. First, if all the cross-sections, as given by equation (1), are small, then a given channel is only weakly coupled to other channels. If the particles are in one incoming channel, that is, described by a\( \Psi_{\text{Born}, k}^{(-)} (r') \), the wave function describing the interaction contains only a small admixture of wave functions from the other channels. Austern mentions that this should hold for very low energy and very high energy. This is the first situation where the Born approximation should be valid for direct reactions. The second situation occurs when the incoming channel is strongly coupled to a great many channels through the compound nucleus, then, when the reacting particles come together, a compound nucleus will certainly be formed. Hence, contributions to direct reactions must come when the deuteron is: 1.) far from the center of the target nucleus,
2.) near the nuclear surface, and 3.) on the tail of the target nucleus wave function. This is the second situation where the Born approximation should be valid. Austern comments that even under these circumstances it is not easy to defend the Born approximation, since the nuclear interactions for direct reactions are themselves strong. The defense depends on the short range of the interaction, which means that it makes little difference how the process is computed, the initial and final state functions will always be made up of independent particle wave functions. Due to the short range of the interaction, there will be no correlation of the reacting particles with any other particles in the nucleus. The wave functions will be determined by the overall size and shape of the nucleus, hence they will be made up of low momentum components. The Born approximation potential induces transitions between wave functions made up of low momentum components, so it should be valid for short range interactions.

The use of small cross-sections as a condition for validity of the Born approximation is also used by Daitch and Bhatia, (Daitch 1952), (Bhatia 1952). The second point of view, the use of the relative radial variation of nuclear potentials as a criterion for validity, is taken by Tobocman, (Tobocman 1957) in a comparison of the Born approximation with the impulse approximation. The condition is that the nucleus-proton potential be a slowly varying radial function. In the Born approximation, this potential variation should be small compared with the ratio of the binding energy of the
deuteron to the radius of the deuteron. In the impulse approximation, the nucleus-proton potential variation is compared with a neutron-proton potential variation. The nucleus-proton attractive potential varies by about 50 MeV per Fermi, \((10^{-13} \text{ cm})\), while the neutron-proton potential can be represented by one with a range of \(10^{-13} \text{ cm}\) and a depth of several thousand MeV. The condition of validity can be more easily satisfied by the impulse approximation than by the Born approximation. This comparison requires that the proton of the deuteron penetrate the nuclear surface in order for a non-vanishing nucleus-proton potential to exist, hence this applies to reactions above the coulomb barrier.

The above discussions can be summarized in two points. 1.) The Born approximation can be valid for direct reactions in some situations. 2.) One such situation is a very low bombarding energy.

2.4. Types of Born Approximation: PWBA and DWBA

The two Born approximations, PWBA and DWBA, are distinguished by the form of the wave function used in equation (7); PWBA uses a plane wave, given by equation (8), DWBA uses coulomb waves, which are exact solutions to equation (2) when the potential includes the coulomb field of the target nucleus. These coulomb waves differ from plane waves in that they are "distorted" by the coulomb potential from plane waves, hence the designation distorted wave Born approximation. The complete DWBA includes the effects of nuclear distortion by the use of optical potentials in equation (2) to describe
the nuclear scattering.

2.5. Question of Universality of DWBA. The question of the universality of DWBA is central to this thesis. If this approximation is indeed universal, then no experimental situation exists where PWBA is valid. We may ask, is there any set of experimental conditions where distortions may be expected to be at a minimum? Consider the effect of the coulomb barrier of a target nucleus on deuterons with extremely low bombarding energy. Coulomb and nuclear distortions will decrease as the impact parameter of the deuteron increases. Of course, at a sufficiently large impact parameter, no nuclear reaction will occur at all. However, for the largest impact parameter at which a nuclear reaction occurs, a special type of (d,p) reaction, called an Oppenheimer-Phillips reaction, (Oppenheimer 1935), will be favored because the deuteron is an extremely loose structure. In this case, a neutron from the deuteron passes near a target nucleus and is captured by it, while the proton of the deuteron remains far from the neutron and even farther from the target nucleus. This is an experimental situation where distortion of the proton trajectory due to nuclear interaction with the target nucleus may be completely neglected and, at the same time, the coulomb distortion should be at a minimum. PWBA may be expected to apply here.

2.6. PWBA and Low Q Stripping Reactions. Explicit details of an experimental situation where distortions may be a minimum has been given by Wilkinson, (Wilkinson 1958), who has suggested that examples of PWBA will occur as low energy
low Q value (d,p) reactions. Two reasons for the lack of distortion are given by Glendenning, (Glendenning 1963), in his discussion of the Wilkinson suggestion. First, for (d,p) reactions with low Q value, i.e. small $Q = B_n - B_d$, where $B_n$ is the binding energy of the neutron to the target and $B_d$ is the binding energy of the deuteron, the neutron is loosely bound to the target. Hence, its wave function extends some considerable distance from the main distribution of matter in the nucleus. The neutron can then be stripped at a large distance where the distortion of the free particle wave functions from plane waves is slight. Second, the outgoing momentum of the proton is very nearly one half of the momentum of the deuteron, for low Q value reactions. Hence the contribution to the outgoing momentum from the internal motion of the deuteron, $K$, can then be very small; see Figure 1-2. Therefore, the neutron and the proton of the deuteron can be very far apart at the instant of stripping.

2.7. PWBA and Larger Q Stripping Reactions. The lack of distortion for low Q stripping reactions has been associated with a proximity of this type of reaction to the "stripping pole" in the complex momentum plane; a treatment of the stripping pole has been made by Warburton, (Warburton 1960), and Amado, (Amado 1959). At the stripping pole, i.e. when $E_d = -2Q$, no distortion should exist. As distance from the stripping pole increases, distortions should increase. Variation of deuteron energy and Q value both affect distance to the stripping pole. Sellschop, (Sellschop 1963a), (Sellschop 1963b), relates "breaks" in observed angular distributions
of low Q stripping reactions to distance, in MeV, from the stripping pole. This distance is constant over a wide range of deuteron energy. Sellschop concludes that plane wave stripping will hold for low Q value reactions up to 1 MeV deuteron energy; above 1 MeV, it will hold at forward angles. Larger Q value reactions should be further from the stripping pole, so observation of angular distributions of larger Q value, low energy stripping reactions could test the relation of distortion to distance from the stripping pole.

2.8. Choice of Reactions to Test PWBA. It would be nice to observe reactions with Q values ranging from negative values to very large positive values. There is, however, a lower limit on the Q value of observable reactions. This lower limit is associated with the lowest detectable energy of the outgoing particle, which is determined by the Q value for reactions with low energy bombarding particles. The necessity to discriminate against elastically scattered deuterons places a minimum on the energy of the outgoing particle which is observable. This, in turn, puts a lower limit on the Q of these reactions. With deuteron energy available \( < 400 \) KeV, the lower limit on Q for observable reactions is about 1 MeV.

Only nuclear reactions with the lightest elements are observable with bombarding deuterons of energy \( < 400 \) KeV, due to the coulomb barrier of the target seen by the incoming charged particles. Even if complete penetration of the barrier is not required, e.g. in stripping reactions, the incident energy must be at least within an order of magnitude
of the target barrier energy for any observable reaction to
occur. A comparison of the available 400 KeV with the cou-
lomb barriers for light elements, e.g. 1.9 MeV for Li\textsuperscript{6} to
3.9 MeV for P\textsuperscript{19}, shows that observable reactions must be
restricted to those on elements lighter than \sim P\textsuperscript{19}. The
choice of observable reactions made on the basis of these
restrictions is shown in Figure 2.1.

2.9. Explicit Forms of PWBA. Explicit forms for
PWBA have been given by both Bhatia, (Bhatia 1952), and
Butler, (Butler 1950), (Butler 1951), where expressions are
developed for the differential cross-section for (d,p)
stripping reactions. These have been compared by Daitch,
(Daitch 1952) and Glendenning, (Glendenning 1963).

In the notation of Glendenning, the differential
cross-section for (d,p) reactions in the CM system is

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \frac{m_d^* m_p^*}{(2\pi k_d^*)^2} \frac{k_p}{k_d} \frac{z J_f + 1}{z J_i + 1} \sum_{j m} \left| \beta_{j m} \right|^2.$$  \hspace{1cm} (9)

Here \(m_d^*, m_p^*\) are the reduced masses,

\(k_d, k_p\) are wave numbers for the relative motion in
the initial and final states; see Figure 1.2.

\(J_i, J_f\) are the angular momenta of the initial and
final states,

\(J, l, m\) are the angular momentum quantum numbers of
the captured neutron,

\(\beta_{j m}\) are expansion coefficients for the final states.

They are given by
where \( \Psi_{J_f} \) is the wave function for the final state, 
\[
\Phi \left( J_{\lambda}, J_l \right) J^M_f \] is a wave function resulting from vector coupling the extra nucleon in the spin orbit state \( \phi_{l, J} \) to a target wave function with angular momentum \( J_c \).

\[
\Phi \left( J_{\lambda}, J_l \right) J^M_f = \sum_{m_c, m_l} \left( J_c m_c j m_l | J_l^M m_f \right) \Psi_{J_c}^{m_c} \phi_{l, j}^{m_l} \left( n_m, \sigma_m \right). \tag{11}
\]

\[
\left( J_c m_c j m_l | J_l^M m_f \right) \]

is a Clepsch-Gordon coefficient.

\( B^m_\ell \) are stripping amplitudes,
\[
B^m_\ell \left( \vec{k}_d, \vec{k}_d' \right) = i^{-\ell} \left( 2 \ell + 1 \right)^{-\frac{1}{2}} \int \psi^{\ell+} \left( \vec{k}_d', \vec{r}_d' \right) \phi_{l, j}^{m} \left( \vec{r}_d \right) \Psi_{J_c}^{m_c} \left( n_m, \sigma_m \right) \left( J^M_f \right) \times V_{m_d} \left( n_d \right) \psi \left( \vec{k}_d, \vec{r}_d \right) \phi \left( \vec{k}_d' \right) \right. \right. \left. \left. \right) d \vec{r}_m d \vec{k}_d' d \vec{k}_d'. \tag{12}
\]

Selection rules for the reactions are
\[
J_f = J_c + j = J_c + \ell \pm \ell_c
\]
\[
\pi_f = \pi_c \left( - \right)^\ell,
\tag{13}
\]
which express conservation of angular momentum and parity, respectively.

\( B^m_\ell \) can also be written
\[
B^m_\ell = i^{-\ell} \left( 2 \ell + 1 \right)^{-\frac{1}{2}} \int e^{-i \vec{k}_d \vec{r}_d} \Psi_{J_c}^{m_c} \left( n_m, \sigma_m \right) \left( J_l^M \right) \times \int u \left( n_d \right) \psi \left( \vec{k}_d, \vec{r}_d \right) \phi \left( \vec{k}_d' \right) d \vec{r}_m d \vec{k}_d'. \tag{14}
\]
where $\mathbf{\mathbf{ \overset { - } { q } = h \mathbf{ \overset { - } { q } } - M_1 \mathbf{ \overset { - } { q } }_1 \mathbf{ \overset { - } { q } }_p}$ is the momentum carried into the nucleus by the neutron,

$\mathbf{\mathbf{ \overset { - } { K = h \mathbf{ \overset { - } { K } } - \frac { i } { 2 } \mathbf{ \overset { - } { K } }_p \mathbf{ \overset { - } { K } }_d}}$ is the momentum transferred to the proton by its interaction with the neutron in the deuteron,

and $\mathbf{\mathbf{ \overset { - } { L }}}$ is the angular momentum transferred to the residual nucleus. These are shown in Figure 1.2.

The first integral in equation (14) is called the deuteron factor, which is related to the Fourier transform of the deuteron wave function.

$$G(K) = \int e^{-i \mathbf{K} \cdot \mathbf{r}} V_{np}(\mathbf{r}) \phi_d(\mathbf{r}) d\mathbf{r}$$  \hspace{1cm} (15)$$

while

$$\phi(K) = \int e^{-i \mathbf{K} \cdot \mathbf{r}} \phi_d(\mathbf{r}) d\mathbf{r}$$  \hspace{1cm} (16)$$

is the Fourier transform of $\phi_d(\mathbf{r})$. $G(K)$ gives the probability for finding a relative momentum $K$ in the internal motion of the deuteron. $G(K)$ is peaked for small $K$ and tails off as $K$ increases. $G(K)$ is angular dependent through the angular dependence of $K$. Small $K$ means the proton has about one half of the deuteron momentum, requiring $\mathbf{K}_p$ to point forward. Large $K$ requires $\mathbf{K}_p$ to make a large angle with the forward direction, as can be seen from Figure 1.2. The result is that $G(K)$ is peaked for forward angles. The second integral in equation (14) depends on the $\mathbf{R}$ value for the reaction. The integral can be simplified using the
plane wave expansion

\[ e^{i \hat{\mathbf{q}} \cdot \mathbf{r}} = 4\pi \sum_{L=0}^{\infty} i^L j_L(q, r) \sum_{\ell=0}^{\infty} Y_{\ell m}^m (\hat{\mathbf{q}}) Y_{\ell m}^\ell (\hat{\mathbf{r}}) \]  

(17)

and the orthogonality of the spherical harmonics, and becomes

\[
\int \psi_\ell (n_m) Y_{\ell m}^m (n_m) e^{i \hat{\mathbf{q}} \cdot \mathbf{r}} d^2 \mathbf{r} = 4\pi i^L Y_{\ell m}^\ell (\hat{\mathbf{q}}) \int \psi_\ell (n_m) j_\ell (q, r) n_m^2 d^2 \mathbf{r} \tag{18}
\]

The Bhatia and Butler theories differ in the evaluation of the integral

\[ \int \psi_\ell (n) j_\ell (q, r) n^2 d^2 \mathbf{r} \]

where the subscript is dropped, i.e. \( r = r_n \). The Bhatia theory uses a surface approximation and assumes \( \psi_\ell (r) \) to have a maximum near the nuclear surface, \( r = R_N \), and to be small elsewhere, while \( j_\ell (q, r) \) is a relatively slowly varying function of \( r \). Hence, by the Bhatia theory, the integral in equation (18) becomes

\[ \int \psi_\ell (n) j_\ell (q, r) n^2 d^2 \mathbf{r} \approx j_\ell (q, R_N) \]

(19)

The Butler theory uses a cutoff approximation, neglecting contribution to the integral for \( 0 < r < R_N \). Butler theory is seen to be a bit more realistic because it simulates the absorption of particles into the compound nucleus if they penetrate the nuclear surface. However, it has a more complex form. Using the Schrodinger equations for \( j_\ell (q, r) \) and \( \psi_\ell (r) \)
\[\left(- \frac{d^2}{dn^2} + \frac{\ell(\ell+1)}{n^2} - \alpha^2\right) n^2 j_{\ell}(q_n) = 0\]

\[\left(- \frac{d^2}{dn^2} + \frac{\ell(\ell+1)}{n^2} + \alpha^2\right) n u_{\ell}(n) = 0 \quad n \geq R_N\]  \hspace{1cm} (20)

where \[\frac{\hbar^2 \alpha^2}{2 M^*} = B_m\] is the binding energy of the deuteron,

\[M^*\] is the reduced mass of the neutron-target system, we find,

\[\left(q_n^2 + \alpha^2\right) \int_{R_N} j_{\ell}(q_n) u_{\ell}(n) n^2 dn =\]

\[- \int_{R_N} \frac{d}{dn} \left\{ \left[n u_{\ell}\right] \frac{d}{dn} \left(n j_{\ell}\right) - \left[n j_{\ell}\right] \frac{d}{dn} \left(n u_{\ell}\right) \right\} dn =\]

\[\left. R_N \ u_{\ell}(R_N) \left[ \frac{d}{dR_N} j_{\ell}(q R_N) - \frac{j_{\ell}(q R_N)}{\hbar \ell(i \alpha R_N)} \frac{d}{dR_N} \ h_{\ell}(i \alpha R_N) \right] \right\} \]

Equation (21) is more complex than the Bhatia expression equation (19). Since they are equivalent in that they are both plane wave approximations, we will use the simpler Bhatia form.

Equation (14) becomes, for the Bhatia theory,

\[B_m^{\text{Bhatia}} = 4\pi (2\ell+1)^{-1/2} G(K) \ Y_{\ell}^{m*}(q) j_{\ell}(q R_N) \]

(22)

Using

\[\sum_{m=-\ell}^{\ell} |Y_{\ell}^{m*}(q)|^2 = \frac{2\ell+1}{4\pi} \quad \text{AND} \quad \beta_{\ell} = \sum_{j} \beta_{j,\ell} \]

(23)
the expression for the differential cross-section for Bhatia theory becomes

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \frac{m_t^+ m_p^+ k_p}{(2\pi)^2} \sum_{l_2} \frac{2J_l+1}{2J_l+1} \sum_{l_4} B_l^2 \mathcal{G}(k) \left| j_{l_4}(qR_N) \right|^2$$  \hspace{1cm} (24)

1 is the kinematic factor
2 is the spectroscopic factor
3 is the deuteron factor
4 is the Bessel function factor
1, 3, and 4 contain the angular dependence. Therefore, the $\theta$ dependence of the cross-section in c.m. coordinates is given by:

$$\frac{d\sigma}{d\Omega}(\theta) \sim \frac{k_p}{2\pi} \sum_{l_2} \mathcal{G}(k) \left| j_{l_4}(qR_N) \right|^2$$  \hspace{1cm} (25)

2.10. Expression of PWBA to Compare with Experiment.

In order to compare the predictions of the Bhatia theory with experiment, an explicit form must be developed. The angular distributions observed were taken relative to the yield at a particular angle, hence only the angular dependence of the Bhatia differential cross-section, equation (25), is required.

In order to use the Bhatia theory, equation (25), the $l$ values for the reaction must be determined. A single $l$ value will occur when the captured neutron occupies only one single particle state. This was assumed in the present experiment because light nuclei at low excitation have
widely spaced levels of the residual nucleus, consequently, protons from reactions leading to different final states have markedly different energies of the order of an MeV, see Figure 2-1. Note the energy levels are not to scale. The $l$ value for each reaction is determined by the selection rules for angular momentum and parity, equations (13) and the $J^\pi$ values for well known nuclear levels. Figure 2-1 shows the $l$ values for the reactions observed.

To obtain a usable form for the deuteron factor, we proceed as follows. The deuteron factor in equation (15) depends on form of the wave function $\phi_d(r)$, and neutron-proton potential, $V_{np}$, used for the deuteron. These have been treated by Bhatia, Glendenning, and Hulthen, (Bhatia 1952), (Glendenning 1963), (Hulthen 1957). The Hulthen wave function for the deuteron is similar in form to the Yukawa wave function; the Hulthen form is usually used because it is easier to handle. The Hulthen wave function is given by

$$\phi_d(r) = \frac{\omega(n)}{\sqrt{n}} = e^{-\alpha_n} \left(1 - e^{-\mu_n r}\right)$$

without the normalization factor. With the normalization,

$$\omega(n) = \sqrt{\frac{2\alpha(\alpha+\mu)(2\alpha+\mu)}{\mu^2}} e^{-\alpha_n} \left(1 - e^{-\mu_n r}\right)$$

(26)

where $\mu = \frac{m_e}{\mu}$ and $m = \pi$ meson mass,

$$\frac{1}{\mu} = 1.414 \times 10^{-13} \text{ cm},$$

$$\alpha = \frac{B.E. \times 2M^2}{\hbar^2}$$

$$\frac{1}{\alpha} = 4.315 \times 10^{-13} \text{ cm}.$$
ENERGY LEVEL DIAGRAMS FOR OBSERVED REACTIONS

Li$^6$(d,p)Li$^7$

BE$^9$(d,p)BE$^{10}$

C$^{12}$(d,p)C$^{13}$

N$^{14}$(d,p)N$^{15}$

O$^{16}$(d,p)O$^{17}$

FIGURE 2−1
The Fourier transform of the Hulthen wave function is

\[ \phi(\kappa) = \sqrt{\frac{2}{\pi}} \left[ \frac{1}{\alpha^2 + \kappa^2} - \frac{1}{(\alpha + \mu)^2 + \kappa^2} \right] \]  

(27)

The second term of the Fourier transform, equation (27), contains the effect of the finite range of the nuclear force through \( \mu \). It is seen to be smaller than the first term by an order of magnitude. A good first approximation is to ignore the second term and use a zero range approximation for the nuclear force. Following the presentation of Bhatia, the deuteron factor is

\[ G(\kappa) = \frac{\chi((2\pi\alpha)^{1/2})}{\alpha^2 + \kappa^2} \]  

(28)

Thus,

\[ G(\kappa) = \frac{8\pi\alpha}{(\alpha^2 + \kappa^2)^2} \]  

(29)

The maximum of \( G(\kappa) \) for small \( \kappa \) is evident in this expression. The angular dependence of the differential cross-section, equation (23), becomes, if we insert equation (27) and use the single \( \ell \) value final state assumption,

\[ \frac{d\sigma}{d\Omega}(\theta) \sim \frac{k^2}{\kappa^2} \frac{1}{(\alpha^2 + \kappa^2)^2} \left| j_\ell(qR) \right|^2 \]  

(30)
A Fortran program was developed for computing on an IBM 1650 computer. The details of this program are given in Appendix One.

A factor $b$ can be introduced as a Bhatia amplitude, which can be adjusted to fit the observed angular distributions, giving

$$\frac{d\sigma}{d\Omega}(\Theta) = \mathcal{E} \frac{k_p}{k_e} \frac{1}{(\omega^2 + \kappa^2)^2} \left| J_{\ell} (\varphi_R) \right|^2$$

(31)
CHAPTER THREE

EXPERIMENTAL METHOD

3.1. **Description of Apparatus.** Figure 3-1 is a view of the Van de Graaff accelerator, the scattering chamber, the detector preamplifiers, the detector voltage bias circuitry, and the vacuum system for the beam tube. Figure 3-2 is a view of the control room. The left panel of the console contains the patch board for accelerator room cables; the central panel contains the accelerator controls; and the right panel contains routing pulse circuitry to enable the Radiation Counter Laboratory (RCL) pulse height analyzer to be used in a selective storage mode, to analyze two separate channels. The control room is separated from the accelerator room proper to provide protection against radiation when the machine is used as a source of $14\text{ MeV}$ neutrons.

The accelerator is a High Voltage Engineering Corporation (HVEC) model PN-400. It will deliver from 0.5 to 150 microamperes at 100 to 400 KeV. The terminal voltage is stabilized to the order of 1\% against drift by the addition of a servo loop, sensing the column resistor current and controlling the belt charge by means of a variable reactance in the charge power supply. This circuit was designed and built by Robert L. Dubois (Dubois 1963). The beam traversing a 7\' beam tube is directed into the scattering
ACCELERATOR AND SCATTERING CHAMBER

FIGURE 3-1
CONTROL ROOM

FIGURE 3-2
FIGURE 3—3
SCATTERING CHAMBER GEOMETRY

Beam cross section
Beam forming slit 3mm wide
Beam cross section 3mm x 12mm

MOLECHEM "N" Detector
MOLECHEM "B" Detector

Target
1-2 cm projected length
6.5 cm
10-15°

FIGURE 3-4
BLOCK DIAGRAM OF DETECTION SYSTEM

CHANNEL #1
DETECTOR #1
MOLECHEM N-50-40
TENNELEC 100A
CHARGE SENSITIVE
PREAMP
GAIN X 8
LOW IMPEDANCE OUTPUT

CHANNEL #2
DETECTOR #2
MOLECHEM B-50-40
HAMNER N 358A
CHARGE SENSITIVE
PREAMP
GAIN X 5
HIGH IMPEDANCE OUTPUT

DOUBLE EMITTER FOLLOWER CABLE DRIVER

~40'RG-58 CABLES TO CONTROL ROOM

COINCIDENCE MONITOR (SHOWED NO COUNTS FOR COUNTING RATES USED)

DOUBLE EMITTER FOLLOWER

RC-7 SET (128 CHANNELS)

PHA INPUT

RADIATION COUNTER LABORATORY MODEL 20617, 256 CHANNEL PULSE HEIGHT ANALYZER

INITIAL CHANNEL SET AT EXTERNAL U.L.D.
SET AT CHANNEL 128.

FIGURE 3-5
chamber without energy or composition analysis.

Figure 3-3 is a schematic view of the beam optics. Figure 3-4 shows the geometry of the scattering chamber. The beam impinges on thick targets mounted centrally in such a way that the beam hits the target at a glancing angle. Consequently, the detectors see the target from about 20 degrees or more to the beam direction. Two detectors are mounted on separately rotatable arms with vacuum-tight swivels.

The vacuum system on the beam tube maintains a pressure of about $10^{-4}$ mm. Hg. sufficiently low to prevent excessive beam scattering, even during relatively large current runs (about 10 microamperes), when target heating will evolve considerable gas. Vacuum valves on the beam tube allow the chamber to be isolated from the vacuum system for target replacement. The targets can be replaced and the chamber then evacuated in about one hour.

Figure 3-5 is a block diagram of the detection system. The detectors are Molechem surface barrier silicon solid state detectors, types N-50-40 and B-50-40. Their outputs are fed to Hamner model N 358A and Tennelec model 100A preamplifiers. The Tennelec output will drive the 40 foot cable between the accelerator and the control room directly, whereas the Hamner output requires a double emitter follower circuit. The voltage gains of the preamplifiers are about $X10$, providing pulses of about 0.1 v. for particles incident on the detector with a few MeV energy.

The voltage signals appearing at the control room are then
fed directly to a Radiation Counter Laboratory 256 channel pulse height analyzer, model 20617. This pulse height analyzer is operated in a selective storage mode. To accomplish this, the two detector outputs must be added by means of two double emitter follower circuits, the outputs of which are tied together before being fed to the analog input. One output, channel 2, is also fed to a Tracer Laboratory model RLA-6A coincidence-anticoincidence analyzer, operated as a routing pulse generator. The routing pulse is fed to the external initial channel input, RC-7 Set, of the pulse height analyzer. The upper level discriminator, ULD, is set at channel 128 to prevent overflow of channel 1 into channel 2. The system becomes two 128 channel analyzers, operating on a time sharing basis. No provision has been made for ignoring coincidence events, which would give false pulses due to pile up. The coincidence count has been monitored and found to be zero for the very low counting rates used. For larger counting rates, coincidences could be rejected by use of the delayed coincidence mode of the pulse height analyzer.

3.2. **Beam Characteristics.** The cross-section of the beam is limited by a slit mounted in the beam tube in front of the cavity of the scattering chamber. The slit width is about 3 mm. The beam diameter depends strongly on the focus voltage setting. When the beam strikes the target at grazing incidence, as shown in Figure 3-4, the projected length varies from 1 to 2 cm., depending on the exact angular setting of the target. Without the forming slit, the
variation in the length of the beam projected on the target would be enormous. With the slit, variations depend mostly on target setting, and so can be minimized. This length must be minimized in order that the detectors see approximately a point source for good geometry. This will be discussed further when the geometry of the detectors is treated.

The energy calibration of the beam was performed using the resonance at 340 KeV in the $^{19}\text{F}(d,\alpha\gamma)^{16}\text{O}$ reaction and observing the 6 to 7 Mev $\gamma$'s from the $^{16}\text{O}^*$ de-excitation. Teflon powder was rubbed on an aluminum backing to form the target. The observed width of the resonance was about 10 KeV. This was surprising for a thick target measurement, since all energies from the maximum to zero could be expected to contribute to the reaction, yielding a step function when counting rate is plotted against proton beam energy.

The energy spread and ion purity were not measured. Estimates for these, as given by HVEC, are the order of a few percent for the energy spread and greater than 50 percent for the deuteron atomic ion concentration. A thick target experiment does not require a monoenergetic beam. A relative yield experiment does not require knowledge of the beam current, but the current striking the target was always monitored.

3.3. Target Characteristics. The targets consisted of deposits on copper backing, except the metallic beryllium target and the mica used as an oxygen target. The Be
targets were self-supporting pieces of 20 mil beryllium sheet supplied by the Brush Beryllium Co., Cleveland, Ohio. Flakes of mica, thickness 0.001 in., were supplied by the Macallen Mica Products Co., Newmarket, N. H. The lithium target was made by evaporating separated lithium 6 metal, provided by the Isotope Division of ORNL, Oak Ridge, Tenn., onto the backing in vacuum. The nitrogen target was aluminum nitride, supplied by the Norton Research Laboratory, New Bond St., Worcester, Mass. The carbon targets were made by burning matches against the copper backing, forming soot deposit.

3.4. Detector Characteristics. Surface barrier silicon detectors were used, although this type of detector is still in the developmental stage. They are extremely versatile because they are compact, with good energy resolution, and are convenient. This convenience should be emphasized. Before solid state detectors were developed, low energy particle detection was accomplished by means of gas counters and/or nuclear emulsions. Emulsions were used for extremely low yield reactions where the collection of every particle was important, involving the enormous inconvenience of developing the emulsion and then counting tracks under a microscope. Now, the same reactions can be studied in a much more sophisticated way using solid state detector systems, where the data are immediately available.

The surface barrier silicon detectors had a nominal energy resolution of 40 KeV, which represented a noise figure for the detector only. In practice, the observed width of
DETECTOR CHARACTERISTICS

Range in Microns

Energy in MeV

RESISTIVITY, BIAS VOLTAGE AND DEPLETION LAYER RELATIONS

SILICON RESISTIVITY N-TYPE ohm-cm

DEPLETION LAYER microns

CAPACITANCE (pf per mm²)

DETECTOR BIAS volts

FIGURE 3-6
Am 241 alpha peak was of the order of 80 KeV.

It was observed that several detectors became noisy after operation for some time. No satisfactory explanation was ever found for this behavior. The detectors could no longer be used, since the energy resolution deteriorated to > 400 Kev, and the noise at low energies, or small voltage pulses, increased to about 1 MeV equivalent.

Several precautions were followed in the use of these detectors. Since the surface of the detectors was extremely sensitive to contamination, nothing could be allowed to touch the surface. The ambient pressure could not be changed with bias voltage applied. Therefore, the bias had to be turned off before the vacuum chamber could be evacuated or brought up to atmospheric pressure. Failure to observe this, even once, would result in a breakdown of the detector, ruining it. Electrical shielding of the detector lead inside the scattering chamber was required to eliminate pickup associated with the accelerator beam. The beam current had a 400 cycle component induced in the source bottle by the 400 cycle generator in the high tension terminal. This 400 cycle modulation was suppressed by a single ground shield on the detector lead.

The thickness of the depletion layer of the detector must be greater than the range of the particle to be counted. Then the particle will be stopped in the depletion depth, or sensitive region, of the detector. The depletion depth is a function of both the bias voltage and silicon resistivity. The monographs shown in Figure 3-6 illustrate this. For the
detectors used, the depletion depth at full bias voltage was about 6 MeV for protons. Hence, protons of higher energy had to be degraded to at least 6 MeV so that their energy could be entirely dissipated in the detector. For the 7 MeV protons from the \( ^{14}\text{N}(d,p)^{15}\text{N} \) reaction an 8 mil aluminum absorber reduced the protons energy to about 4 MeV.

3.5. Geometry. Figure 3-4 shows the geometric arrangement of beam, target, and detectors. From the figure it is evident that, with the detectors located 6.5 cm. from the target, the angular spread, due to the 1 - 2 cm. projected length of the target, is the arctangent of 2/6.5 or 17 degrees for the worst case at 90 degrees. Most of the data were taken with much better geometry. However, even this poor geometry would not affect the experimental results because the angular dependence of the yield for the reactions did not change appreciably over this angular range. As the two detectors had diameters of 0.2 cm. for their sensitive regions, the solid angles subtended were \( 2.4 \times 10^{-4} \). The angular opening was 0.031 radians or 1.8 degrees.

3.6. Target Loading Effect. As a run progresses, deuterium builds up on the target surface. This deuterium then reacts with the incoming deuterons to produce D-D reactions. These reactions can mask a proton group under study if the energy of the reaction products is the same as that of the proton group. In order to minimize the effect of target loading, fresh targets must be inserted at frequent intervals.
3.7. Extraction of Particle Groups. A thick target experiment does not seem suitable, superficially, as a method for studying angular distributions of resolved particle groups. Indeed, this is true for the energy range covered by most accelerators. However, the very peculiar set of effects that dominate reactions well below the coulomb barrier allow just such a thick target experiment to be feasible. The key to this type of experiment is the extremely rapid decrease of the reaction cross-section as the accelerator beam penetrates the target and loses energy. This cross-section curve follows the Gamow curve for the penetration of charged particles into the nucleus, an exponential form far from any single resonances, which is the case for our work. This means that the largest portion of the cross-section is contributed by the very thin surface layer of the target material. Hence, for reactions well below the coulomb barrier, where the exponential dependence of the cross-section dominates, a pseudo thin target situation exists.

The product groups are well defined in energy because of the following two effects, peculiar to these low bombarding energy reactions. First from the kinematics, it is seen that the energies of the outgoing particles are dominated by the Q values of the reaction, when the Q value is large compared to 0.35 MeV. The energies are thus independent of the bombarding energy. Second, the outgoing particles have a range much greater than the thickness of the very thin surface layer of target which they must
traverse to leave the target. Hence, they will lose a negligible fraction of their energy in escaping from the target. These two effects cause the product particle energy to depend only on the scattering angle through kinematics.

The energy width of the groups is spread somewhat by absorber foils mounted over the detectors. Plastic absorbers of 0.5 x 10^{-3} in. thickness are used to stop elastically scattering deuterons, which would otherwise swamp the detectors. These foils will stop about 0.7 MeV deuterons. When a very high energy proton group, such as that from N^{14}(d,p)N^{15} at 7 MeV, is being studied, a thicker degrading absorber is introduced. This further spreads the group in energy. The group can still be easily resolved, however. Resolution of groups is the only criterion that has been considered. No attempt has been made to interpret the energy widths of the groups.

3.8. Effective Deuteron Energy. For a thick target experiment, the effective beam energy is not coincident with the maximum beam energy, as it is for a thin target experiment, where the beam loses a negligible amount of energy in passing through the target. Since the beam is stopped in a thick target, all beam energies from maximum to zero are present. The effective energy for causing a particular reaction is the mean energy, weighted by the product of the excitation function for that reaction and the energy spectrum of the beam as it slows and stops in the target, over the energy range from zero to the maximum beam energy. This is expressed by:
EXCITATION FUNCTIONS FOR THE OBSERVED REACTIONS

- $\text{Li}^6(d,p)\text{Li}^7$ (WHALING, 1950)
- $\text{C}^{13}(d,p)\text{C}^{14}$ (CURLING, 1950)
- $\text{Li}^6(d,p)\text{Li}^7(^{3}H)$ (WHALING, 1950)
- $\text{N}^{14}(d,p)\text{N}^{15}$ (JONGERIUS, 1954)
- $\text{Be}^9(d,p)\text{Be}^{10}$ (DEJONG, 1952b; RESNICK, 1951)
- $\text{O}^{16}(d,p)\text{O}^{17}$ (HEYDENBURG, 1948; GROSSKREUTZ, 1956)
- $\text{Be}^9(d,p)\text{Be}^{10}(^{3}H)$ (RESNICK, 1951)

FIGURE 3-7
The exponential chosen increases one order of magnitude for every 100 KeV.

\[
E_{\text{Eff.}} = \frac{\int E \sigma(E) \, dE}{E_{\text{Max.}}} = \frac{\int \sigma(E) \, dE}{E_{\text{Max.}}}
\]
Since for (d,p) reactions, the excitation functions all follow similar Gamow curves, these can be used as a factor in the weighting function. The excitation functions for the reactions observed are shown in Figure 3-7. It is seen that no single resonances occur for the energy range of the experiment. Which would invalidate the effective energy argument. Figure 3-8 is a typical example of an effective energy computation. It uses an exponential form, increasing one order of magnitude for every 100 KeV. It is seen from Figure 3-9 that the excitation functions are all exponential for energy $\leq 400$ KeV; the excitation function for a target between C and N corresponds to the typical example. It is seen that the effective energy does not deviate considerably from the maximum or beam energy. For this reason, the energy given in the figures is the maximum energy.

3.9. **Identification of Particle Groups.** The pulse height spectra observed by deuteron bombardment of light nuclei have characteristic appearance. For an example, refer to the spectrum for the Li(d,α) reactions (Manalis 1964). A particular target nucleus has a spectrum with a distinctive, recognizable appearance, which can almost be used to identify the nuclide present. This recognition of
features of the spectrum helps in a preliminary identification of the particle groups. However, more quantitative methods must also be employed.

The particle can be identified by at least two methods. The first method is to observe the kinematical shift of the particle energy with scattering angle. For example, the predicted shifts for protons are much less than for alpha particles. The second method is by measuring the energy lost after the particles have passed through an absorber foil. The specific ionization or energy loss is much less for particles with $Z = 1$ than it is for $Z = 2$ particles. Thus, the energy loss for $Z = 2$ particles in passing through the absorber will be much greater. Protons and tritons can be distinguished by this method from helium three and alpha particles in the present experiment. The only deuterons present will be elastically scattered deuterons with much less energy than any of the reaction products observed. In fact, the absorber foil in front of the detectors completely eliminates the deuterons. These methods were used initially in this experiment to separate the types of particle present.

The primary characteristic for particle group identification is the group energy. This can be related to the observed pulse height in the following way. The energy of the protons emitted from $(d,p)$ reactions is determined by kinematical analysis, knowing the beam energy, masses of the reacting particles, and scattering angle. This analysis is discussed in appendix one. This kinematic energy is the
Figure 3-9

Specific Energy Loss for Particles

- Be
- Al
- Cu
- Au

Proton Energy (MeV)

- .2 .4 .6 1.0 1.4 2.0 4.0 6.0 8.0 10.0 14.0 20.0
- 40 60 80 100 140 200

Deut. Energy (MeV)

- .3 .6 .9 1.5 2.1 3.0 6.0 9.0 12.0 15.0 21.0 30.0

Trit. Energy (He^3 Energy (Mult. de E X 4))

- .4 .8 1.2 2.0 2.8 4.0 8.0 12.0 16.0 20.0 28.0 40.0

Alpha Energy, (Mult. de E X 4)

- 20 28 40
RESIDUAL ENERGY FOR VARIOUS PARTICLES AFTER PASSING THROUGH ABSORBER OVER DETECTORS

FIGURE 3-10
ENERGY CALIBRATION

CHANNEL 2
GAIN 5X1X#2
"B" DETECTOR

CHANNEL 1
GAIN 8X1X#5
"N" DETECTOR

FIGURE 3-11
energy of the particle as it impinges on the absorber covering the detectors, if it is assumed that no energy has been lost in escaping from the target, which is the case, as pointed out in the section describing the extraction of particle groups. The energy lost by the particle as it passes through the absorber covering the detectors must be determined. This energy loss is then subtracted from the kinematical energy resulting in the residual energy of the particle, or its energy as it enters the solid-state detector. The observed pulse height is proportional to this residual energy.

The procedure to identify particle groups is as follows. The kinematic energy is found from the tables, described in appendix one. The corresponding residual energy is determined from Figure 3-10. An association is then made between observed particle groups at definite pulse heights with predicted residual energy for specific reactions present. The association is verified by: 1.) consistency of the energy calibration for all groups, as illustrated in Figure 3-11; 2.) consistency of the identified groups with the proper kinematic energy shift with scattering angle. This verification procedure has given unambiguous results for all of the reaction groups studied.

The energy lost by the particles in passing through the absorber over the detectors which consisted of thin polyethylene, can be evaluated in the following way. The thickness of the absorber was estimated to be 1.1 mg./cm.$^2 \pm 20\%$. A measurement of the thickness with a micrometer
gave $0.35 \times 10^{-3}$ in. Assuming a density of 1.3 gm. cm.$^{-3}$, the thickness of 1.25 mg./cm.$^2 \pm 20\%$. Direct weighing of the absorber material gave a thickness of 1.35 mg./cm.$^2 \pm 20\%$. With the thickness of 1.3 mg./cm.$^2$, and knowing the rate of energy loss of the particles as given in Figure 3-9, the energy lost by particles in the absorber can be calculated. The results are shown in Figure 3-10.
CHAPTER 4

OBSERVED ANGULAR DISTRIBUTIONS

Figure 4-1 shows the relation of data to the experimental setup. The yield was extracted as follows. The proton group of interest was identified by methods described in Chapter 3. \( N_1 \) and \( N_2 \) are the total counts for the proton group in channels 1 and 2, corresponding to detectors 1 and 2, respectively. The yield is the ratio \( N_2/N_1 \), which is a relative quantity. The angular dependence of the yield is obtained by changing the angle of one detector and keeping the other at a fixed angle, for normalization. Although \( N_1 \) and \( N_2 \) are measured independently, no absolute cross-section was measured, for this requires knowledge of the effective number of atoms in the target. A thick target experiment cannot determine directly the effective number of atoms in the target.

The observed angular distributions are shown in Figures 4-2 through 4-16. The curves are smooth lines drawn through the experimental points.

The error bars indicate only counting statistics. Other sources of error are the following.

The target was set at a glancing angle to the beam, as shown in Figure 3-4, so that the target subtended an angle of the order of 10 degrees, as seen by the detectors.
This geometry effect resulted in a spread in the scattering angle of the order of 10 degrees. This angular resolution was adequate for the present experiment, since the angular distributions had a simple form.

Errors encountered in extraction of the yield involve background and overlapping particle groups. Groups that sat on a flat background were separated without difficulty. Overlapping groups prevented measurements in some cases, for example, Li$^6$(d,p)Li$^{7*}$ at 350 KeV below 80 degrees and C$^{12}$(d,p)C$^{13}$ at 200 KeV above 100 degrees.

The results are not sensitive to variations in deuteron energy, since fluctuations in yield due to beam variations would occur in both channels and be eliminated in the ratio.

It is not possible to give a precise determination of the error in the yield, since there is a large variation in the background and prominence of groups from run to run.
RELATION OF DATA TO EXPERIMENTAL SETUP

\[ \text{YIELD} = \frac{N_2}{N_1} \]

**FIGURE 4-1**
OBSERVED ANGULAR DISTRIBUTION

$\text{Li}^6(d,p)\text{Li}^7$

200 KeV

FIGURE 4-2
OBSERVED ANGULAR DISTRIBUTION
Li$^6$(d,p)Li$^7$
350 KeV

FIGURE 4-3
OBSERVED ANGULAR DISTRIBUTION

Li$(d,p)$Li$^7$

200 KeV

FIGURE 4-4
OBSERVED ANGULAR DISTRIBUTION

\[ \text{Li}^6(d,p)\text{Li}^7^* \]

350 KeV

FIGURE 4-5
OBSERVED ANGULAR DISTRIBUTION

Be$^9$(d,p)Be$^{10}$

200 KeV

FIGURE 4-6
OBSERVED ANGULAR DISTRIBUTION
\[ \text{Be}^9(d,p)\text{Be}^{10} \]
350 KeV

FIGURE 4-7
OBSERVED ANGULAR DISTRIBUTION

Be$^9$(d,p)Be$^{10*}$

200 KeV

FIGURE 4-8
OBSERVED ANGULAR DISTRIBUTION

Be$^9$(d, p)Be$^{10\ast}$

350 KeV

FIGURE 4-9
OBSERVED ANGULAR DISTRIBUTION
\[ {\text{Cl}}^{12}(d,p)\text{Cl}^{13} \]
200 KeV

FIGURE 4-10
OBSERVED ANGULAR DISTRIBUTION

$^{12}\text{C}(d,p)^{13}\text{C}$

350 KeV

FIGURE 4-11
OBSERVED ANGULAR DISTRIBUTION

$N^4(d,p)N^{15}$

200 KeV

YIELD, NORMALIZED TO 90°

FIGURE 4-12
OBSERVED ANGULAR DISTRIBUTION

$N^{14}(d,p)N^{15}$

250 KeV

FIGURE 4-13
OBSERVED ANGULAR DISTRIBUTION

$N^{14}(d,p)^{15}$

350 KeV
OBSERVED ANGULAR DISTRIBUTION

$^16\text{(d, p)}^17$

350 KeV

FIGURE 4-15
OBSERVED ANGULAR DISTRIBUTION

\[ {}^{16}\text{O}(d,p){}^{17}\text{O}^* \]

350 KeV

FIGURE 4-16

YIELD, RELATIVE TO 90°
CHAPTER 5

COMPARISON WITH OTHER WORK

Figures 5-1 through 5-8 show a comparison of the results of the present experiment with other work.
COMPARISON WITH OTHER WORK

\[ \text{Li}^6(d,p)\text{Li}^7 \]

(NICKELL 1954) — (DUNBAR 1951)

(WHALING 1950) — PRESENT EXPERIMENT

FIGURE 5-1

YIELD, NORMALIZED TO 90°

\( \theta_{\text{cm}} \)

2000 KeV
1000 KeV
800 KeV
600 KeV
400 KeV
350 KeV
200 KeV
FIGURE 5-2

COMPARISON WITH OTHER WORK

- Li$^6$(d,p)Li$^7$
- (Dunbar 1951)
- (Nickell 1954)
- (Whaling 1950)
- Present Experiment
COMPARISON WITH OTHER WORK

$^{9}\text{Be}^{(d,p)}^{10}\text{Be}$

- --- (JURIC 1955)
- ---(RESNICK 1951)
--- --- PRESENT EXPERIMENT

FIGURE 5-3
COMPARISON WITH OTHER WORK
BE⁹(d,p)BE¹⁰⁺

--- (Resnick 1951)

PRESENT EXPERIMENT

YIELD, NORMALIZED TO 90°

\[880 \text{ KeV}\]
\[600 \text{ KeV}\]
\[400 \text{ KeV}\]
\[350 \text{ KeV}\]
\[300 \text{ KeV}\]
\[200 \text{ KeV}\]

\[0\]
\[.5\]
\[1\]
\[1.5\]

\[0\]
\[40\]
\[80\]
\[120\]
\[160\]

\[\theta_{\text{CM}}\]

FIGURE 5-4
COMPARISON WITH OTHER WORK
$^{12}\text{C}^\text{(d,p)}^{13}\text{C}$

- (TAKEMOTO 1954)
- (KOUDIJS 1952)
- PRESENT EXPERIMENT

YIELD, NORMALIZED TO 90°

$\theta_{CM}$

FIGURE 5-5
COMPARISON WITH OTHER WORK
$^N\text{I}^4(\text{d}, \text{p})N^1\text{S}^5$

--- (JONGERIUS 1954)
--- PRESENT EXPERIMENT

FIGURE 5-6
COMPARISON WITH OTHER WORK
$^{16}\text{O}(d,p)^{17}\text{O}$

--- (JURIC'1955)
--- PRESENT EXPERIMENT

**Figure 5-7**
COMPARISON WITH OTHER WORK

$^{16}\text{O}(d,p)^{17}\text{O}$

--- (GROSSKREUTZ 1956)

--- PRESENT EXPERIMENT

**FIGURE 5-8**
CHAPTER 6

DISCUSSION OF RESULTS

6.1. Method Used to Compare Results with Bhatia Theory. The results of the present experiment are compared with Bhatia theory in Figures 6-1 through 6-8 where

\[ \frac{d\sigma}{d\Omega} \text{observed vs. } \Theta_{c.m.} \text{ at 200 and 350 KeV is fitted by} \]

\[ \frac{d\sigma}{d\Omega} \text{theoretical from Bhatia theory. No absolute } \frac{d\sigma}{d\Omega} \text{ was measured. The method used to compare results with Bhatia theory is the following.} \]

The parameters \( l \), \( R \), and \( b \) appearing in equation (31) were adjusted one at a time. \( l \), the orbital angular momentum transferred to the residual nucleus, was assigned from known \( J^\pi \) values of the nuclear levels of the initial and final states, as shown in Figure 2.1. \( \frac{d\sigma}{d\Omega} \) theoretical was computed for different values of \( R \), the stripping radius, and was found to be relatively insensitive to \( R \), i.e. \( \Delta \left( \frac{d\sigma}{d\Omega} \right) \) theoretical < \( \Delta R \) for \( E_d \ll 1 \text{ MeV} \) by the following argument.

1. \( \frac{d\sigma}{d\Omega} \) theoretical \( \propto |j_l (qR)|^2 \), in equation (31).
2. \( q \) is a function of \( \Theta_{c.m.} \) and \( E_d \); see Figure 1.2.
3. \( \Delta q (E_d \ll 1 \text{ MeV}) < q \), where \( \Delta q \) is the range of \( q \) over \( 0 \leq \Theta_{c.m.} \leq 180^\circ \).
4. \( \Delta (qR) (E_d \ll 1 \text{ MeV}) < qR \), for \( \Delta R \ll .5 \) \( R \).
5. \( \Delta j_l (qR) \) in equation (31) goes through only one or two nodes in the range \( 0 \leq \Theta_{c.m.} \leq 180^\circ \) for \( E_d \ll 1 \text{ MeV} \).
FIT TO PLANE WAVE THEORY

$^{6}\text{Li}(d,p)^{7}\text{Li}$

--- ZERO OF PLANE WAVE FIT

$L = 50 \text{ KeV}$
$R = 5.5$

$40 \quad 80 \quad 120 \quad 160$

YIELD, NORMALIZED TO 90$^\circ$

$\theta_{CM}$

FIGURE 6-1
FIT TO PLANE WAVE THEORY
\( \text{Li}^6(d, p)\text{Li}^7^* \)

--- ZERO OF PLANE WAVE FIT

\( L = 1 \) 350 KeV
\( R = 5.5 \)

\( L = 1 \) 200 KeV
\( R = 5.5 \)

YIELD, NORMALIZED TO 90°

\( \theta_{\text{CM}} \)

FIGURE 6-2
FIT TO PLANE WAVE THEORY
$^{8}\text{Be}(d, p)^{10}\text{Be}$

FIGURE 6-3
FIT TO PLANE WAVE THEORY
\[ \text{Be}^9(d,p)\text{Be}^{10*} \]

--- ZERO OF PLANE WAVE FIT

\[ L = 1 \quad 350 \text{ KeV} \]
\[ R = 5.8 \]

\[ L = 1 \quad 200 \text{ KeV} \]
\[ R = 5.8 \]

FIGURE 6-4
FIT TO PLANE WAVE THEORY

$^{12}\text{C}(d,p)^{13}\text{C}$

--- ZERO OF PLANE WAVE FIT

\[
\text{YIELD, NORMALIZED TO 90°}
\]

\[
\theta_{\text{CM}}
\]

**FIGURE 6-5**
FIT TO PLANE WAVE THEORY
$N^ {14}(d,p)N^ {15}$
--- --- ZERO OF PLANE WAVE FIT

\[ L=1 \ 350 \text{ KeV} \]
\[ R=5. F \]

\[ L=1 \ 200 \text{ KeV} \]
\[ R=5. F \]

YIELD, NORMALIZED TO 90°

\[ \theta_{CM} \]

FIGURE 6-6
FIT TO PLANE WAVE THEORY

$^{16}\text{O}(d, p)^{17}\text{O}$

--- ZERO OF PLANE WAVE FIT

$L = 2 \quad 350 \text{ KeV}$

$R = 5.1604$ cm

YIELD, NORMALIZED TO 90°

$\theta_{CM}$

FIGURE 6-7
FIT TO PLANE WAVE THEORY

\( \text{O}^{16}(d, p)\text{O}^{17} \)

--- ZERO OF PLANE WAVE FIT

--- ZERO OF PLANE WAVE FIT

FIGURE 6-8
6. Thus, \( \left( \frac{d\sigma}{d\alpha} \right)_{\text{theoretical}} \) goes through only \( \sim 1 \) minimum for \( \Delta R \lesssim 0.5 R \) and \( E_d \ll 1 \text{ MeV} \).

This argument shows 1.) \( \left( \frac{d\sigma}{d\alpha} \right)_{\text{theoretical}} \) has a simple form, i.e. goes through only \( \sim 1 \) minimum, and 2.) \( \left( \frac{d\sigma}{d\alpha} \right)_{\text{theoretical}} \) is insensitive to change in \( R \) for \( \Delta R \lesssim 0.5 R \). A definite value of \( R, R = 5F. \), was used in the comparison of theory with experiment.

Finally, the Bhatia amplitude, \( b \) in equation (31), was adjusted so that \( \left( \frac{d\sigma}{d\alpha} \right)_{\text{theoretical}} \) gave a smooth curve through the experimental points. The adjustment of \( b \) is shown in Figure 6-1 through 6-8 by the dashed line, which indicates the origin for \( \left( \frac{d\sigma}{d\alpha} \right)_{\text{theoretical}} \).

6.2. Comments on Agreement of Experimental Results with Bhatia Theory. The value of the stripping radius, \( R = 5F. \), used in the comparison of theory with experiment agrees, (Macfarlane 1960), with stripping radii given for these reactions. Various workers, using various stripping formulations, use a range of stripping radii, \( 4F. \lesssim R \lesssim 7F. \), for light nuclei; many use \( R = 5F. \).

The simple form of \( \left( \frac{d\sigma}{d\alpha} \right)_{\text{theoretical}} \) in the sense of Tobocman, (Gibbs 1961), means that it will be difficult to distinguish between PWBA and DWBA.

A detailed description of the agreement of experiment with theory can be made by defining three categories for extent of agreement: good agreement, poor agreement, and no agreement. These categories are defined in terms of a comparison of deviations of experiment from theory with
## COMPARISON OF RESULTS WITH BHATIA THEORY

<table>
<thead>
<tr>
<th>REACTION</th>
<th>ENERGY, KEV</th>
<th>STRIPPING C.N. = b/a</th>
<th>ANGULAR VARIATION, V</th>
<th>CATEGORY FOR EXTENT OF AGREEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li⁶(d,p)Li⁷</td>
<td>200</td>
<td>4.7/1.5 = 3.1</td>
<td>.6</td>
<td>Good</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>2.7/3.6 = .75</td>
<td>.4</td>
<td></td>
</tr>
<tr>
<td>Li⁶(d,p)Li⁷*</td>
<td>200</td>
<td>Only Stripping</td>
<td>.7</td>
<td>Good</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>4.6/1.6 = 2.9</td>
<td>.6</td>
<td></td>
</tr>
<tr>
<td>Be⁹(d,p)Be¹⁰</td>
<td></td>
<td>No Fit Obtained</td>
<td>.9</td>
<td>None</td>
</tr>
<tr>
<td>Be⁹(d,p)Be¹⁰*</td>
<td>200</td>
<td>5.2/.35 = 3.9</td>
<td>.2</td>
<td>Poor</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>3.8/2.5 = 1.5</td>
<td>.4</td>
<td></td>
</tr>
<tr>
<td>C¹²(d,p)C¹³</td>
<td>200</td>
<td>Only Stripping</td>
<td>.2</td>
<td>Poor</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>3.0/2.25 = 1.3</td>
<td>.4</td>
<td></td>
</tr>
<tr>
<td>N¹⁴(d,p)N¹⁵</td>
<td>200</td>
<td>Only Stripping</td>
<td>.6</td>
<td>Good</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>4.0/1.1 = 3.6</td>
<td>.7</td>
<td></td>
</tr>
<tr>
<td>O¹⁶(d,p)O¹⁷</td>
<td>350</td>
<td>3.0/9.1 = 3.3</td>
<td>.3</td>
<td>Poor</td>
</tr>
<tr>
<td>O¹⁶(d,p)O¹⁷*</td>
<td>350</td>
<td>2.0/2.0 = 1.0</td>
<td>.6</td>
<td>Poor</td>
</tr>
</tbody>
</table>

Table 6-1
the statistical error, given by the error bars in Figures 6-1 through 6-8 although other errors are present. The description of agreement described here ignores these other errors, since they are difficult to evaluate. Chapter 4 contains a brief description of these errors. Definitions for the categories are as follows. Good agreement was found for \( \text{Li}^6(d,p)\text{Li}^7 \), \( \text{Li}^6(d,p)\text{Li}^7^* \), and \( \text{N}^{14}(d,p)\text{N}^{15} \). For these reactions deviations of the observed points from \( \frac{d\sigma}{d\alpha} \) theoretical were comparable, i.e. \( \ll \) twice as large as, the statistical error over most of the angular range. There was poorer agreement for \( \Theta_{\text{c.m.}} \lesssim 100 \) degrees for \( \text{N}^{14}(d,p)\text{N}^{15} \) and one or two experimental points of \( \text{Li}^6(d,p)\text{Li}^7 \). Poor agreement was found for \( \text{C}^{12}(d,p)\text{C}^{13} \), \( \text{Be}^9(d,p)\text{Be}^{10^*} \), and \( \text{O}^{16}(d,p)\text{O}^{17} \), where deviations of the observed points from \( \frac{d\sigma}{d\alpha} \) theoretical were comparable, i.e. \( \ll \) twice as large as, the statistical error over only about half of the angular range. Structure in the experimental distributions was evident for these reactions that was not reproduced by \( \frac{d\sigma}{d\alpha} \) theoretical. No agreement was found for \( \text{Be}^9(d,p)\text{Be}^{10} \); it was not possible to fit \( \frac{d\sigma}{d\alpha} \) theoretical at all.

Table 6-1 shows parameters pertinent to a comparison of results with Bhatia theory. \( b/a \) is the ratio of the stripping amplitude to the compound nucleus amplitude. The Bhatia amplitude, \( b \), of equation (31) is taken as the stripping amplitude. In Figures 6-1 through 6-8, \( \frac{d\sigma}{d\alpha} \) theoretical is normalized at \( \Theta_{\text{c.m.}} = 90 \) degrees. Thus, \( b \) is the amplitude of \( \frac{d\sigma}{d\alpha} \) theoretical above the dashed line at 90 degrees. \( a \), the amplitude of the dashed line, or isotropic component,
LEVELS IN COMPOUND NUCLEI

\[ \text{Li}^6 + d \rightarrow \text{Be}^8^{+} \]

\[ \begin{array}{c}
23.85 \\
22.66 \\
21.5 \\
\hline
\end{array} \]

\[ \text{Be}^9 + d \rightarrow \text{B}^{11+} \]

\[ \begin{array}{c}
14.9 \\
16.6 \\
14.0 \\
\hline
\end{array} \]

\[ \text{B}^{12} + d \rightarrow \text{N}^{14+} \]

\[ \begin{array}{c}
10.66 \\
10.43 \\
9.49 \\
\hline
\end{array} \]

\[ \text{N}^{14} \]

\[ \begin{array}{c}
21.9 \\
20.7 \\
19.3 \\
\hline
\end{array} \]

\[ \begin{array}{c}
21.12 \\
20.713 \\
\hline
\end{array} \]

\[ \text{F}^{18} \]

\[ \begin{array}{c}
7.93 \\
7.7 \\
\hline
\end{array} \]

\[ \begin{array}{c}
7.1 \\
6.36 \\
\hline
\end{array} \]

TABLE 6-2
is taken as the compound nucleus amplitude. \( V \) is a measure of the structure of angular variation of the experimented points. It is defined by, \( V = \frac{\text{Maximum Yield} - \text{Minimum Yield}}{\text{Maximum Yield}} \).

6.3. Detailed Summary of Comparison of Results with Theory.

1. \( \text{Li}^6(d,p)\text{Li}^7 \)
   Agreement was within statistical errors over entire angular range.

2. \( \text{Be}^9(d,p)\text{Be}^{10} \)
   No fit was possible.

3. \( \text{Be}^9(d,p)\text{Be}^{10*} \)
   Agreement was poor. Experimental structure was not reflected in \( \frac{\text{d}N}{\text{d}n} \) theoretical.

4. \( \text{C}^{12}(d,p)\text{C}^{13} \)
   Agreement was good for \( \theta < 100 \) degrees. For \( \theta > 100 \) degrees, the experimental points exceeded \( \frac{\text{d}N}{\text{d}n} \) theoretical.

5. \( \text{N}^{14}(d,p)\text{N}^{15} \)
   Agreement was poor.

6. \( \text{O}^{16}(d,p)\text{O}^{17} \)
   Agreement was poor. Large backward peaking in experimental distribution.

7. \( \text{O}^{16}(d,p)\text{O}^{17*} \)
   Agreement was poor. Large backward peaking in experimental distribution.

6.4. Interpretation of Results: Relative Importance of Stripping versus Compound Nucleus Mechanism. There are two types of evidence for the importance of stripping versus C.N. in the present experiment: 1.) the asymmetry in the angular distributions, and 2.) the large value of \( b/a \), the stripping amplitude to C.N. amplitude ratio. These will be discussed in turn.
1. The observed distributions in Figures 6-1 - 6-8 are generally asymmetric about 90 degrees with forward peaking in some cases. These characteristics can be used as evidence for stripping by means of the following arguments. Firstly, a forward peaked angular distribution is characteristic of a stripping mechanism, as discussed in Section 1.3. Secondly, an asymmetric angular distribution is possible for a C.N. mechanism, only if interference occurs between members of a pair, or set of pairs, of levels, with opposite parity, as discussed in Section 1.3 and in (Wildenthal 1964) and (Ericson 1960). Pairs of closely spaced known levels at the proper excitation are not found, in general, for the compound nuclei of the observed reactions, as shown in Table 6-2. In the case of Be^9(d,p)Be^{10}, Smither, (Smither 1957), has invoked pairs of unknown levels to explain the asymmetry by a C.N. mechanism. It can be concluded that C.N. importance for the observed reactions requires the invocation of interference between members of pairs, or sets of pairs, of unknown levels for all reactions, hence C.N. importance seems unlikely.

2. The values of b/a in Figure 6-1 are generally >1. If b/a is taken as a measure of stripping, stripping is seen to dominate the observed reactions. A further trend can be seen in the behavior of b/a; it increases with decreasing energy. This is significant in terms of the greater coulomb repulsion as the energy in the entrance channel is decreased. Greater coulomb repulsion would increasingly favor stripping outside the nucleus over penetration to form a C.N.
This section can be summarized by the following remarks. Stripping is important for the observed reactions. A C.N. mechanism seems unlikely, requiring many pairs of unknown levels. Stripping increases as the energy in the entrance channel decreases, consistent with the effect of an increasing coulomb barrier.

6.5. Interpretation of Results: Mechanism to Explain Results and/or Agreement with Bhatia Theory. A survey of Figures 6-1 through 6-8 reveals marked variety in the observed angular distributions. On the face of it, it seems unlikely that a simple picture or mechanism could explain all the data. Indeed, an effort to construct such a mechanism has not given convincing results. However, a review of the attempt to invoke various semi-classical pictures will be given. By semi-classical, it is meant that that uncertainty principle has not been considered, for that principle would prohibit the picture of incoming particles of well-defined energies approaching a well-defined region of space, near the surface of the target nucleus and retaining thin well-defined energies. The semi-classical pictures are thus unrealistic. Nevertheless, it may be worthwhile to see what behavior these mechanisms would predict for the angular distributions.

In the discussion of the observed angular distributions that will follow, only reactions with marked features will be treated separately, i.e. $\text{Be}^9(d,p)\text{Be}^{10}$, $\text{N}^{14}(d,p)\text{N}^{15}$, and $\text{O}^{16}(d,p)\text{O}^{17}\times$. These reactions have $V \geq .6$, where $V$ is the structure or variation parameter of Table 6-1.
PARAMETERS FOR TEST OF PWBA

**REACTION**

$\text{Li}^6(d,p)\text{Li}^7$

<table>
<thead>
<tr>
<th>SHELL STRUCTURE</th>
<th>SURFACE</th>
<th>CHAR. OF</th>
<th>COUL.</th>
<th>COUL.</th>
</tr>
</thead>
<tbody>
<tr>
<td>RESIDUAL NUCLEUS</td>
<td>CHAR. OF</td>
<td>OBS. ANG.</td>
<td>EN. AT</td>
<td>EN. AT</td>
</tr>
<tr>
<td>$\odot$ PROTON 'O NEUTRON</td>
<td>RES. NUC.</td>
<td>Q, l</td>
<td>R = 5F</td>
<td>R/Q</td>
</tr>
</tbody>
</table>

Open 5.027 l Forward Peaked .86 MeV .17
Shell Diffuse

$\text{Li}^6(d,p)\text{Li}^7^*$

Open 4.549 l Forward Peaked .86 .19
Shell Diffuse

$\text{Be}^9(d,p)\text{Be}^{10}$

Closed 4.585 l Marked Backward 1.14 .25
Shell Sharp

Backward Peak

Table 6-3
<table>
<thead>
<tr>
<th>REACTION</th>
<th>SHELL STRUCTURE</th>
<th>SURFACE</th>
<th>CHAR. OF RES. NUC.</th>
<th>OBS. ANG.</th>
<th>Q</th>
<th>DIST.</th>
<th>R = 5F</th>
<th>R/Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be(^9)(d,p)Be(^{10*})</td>
<td>Open Shell Diffuse</td>
<td>1.217 3,1</td>
<td>Central Peak</td>
<td>1.14</td>
<td>.94</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(^{12})(d,p)C(^{13})</td>
<td>Open Shell Diffuse</td>
<td>2.719 1</td>
<td>Central Peak</td>
<td>1.72</td>
<td>.63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N(^{14})(d,p)N(^{15})</td>
<td>Closed Shell Sharp</td>
<td>8.6151 1</td>
<td>Marked Forward Peak</td>
<td>2.0</td>
<td>.23</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6-3
$^1_6(d,p)^{17}$

**Reaction:** $^1_6(d,p)^{17}$

**Shell Structure of Residual Nucleus:**

<table>
<thead>
<tr>
<th>Surface Char. Q</th>
<th>COUL. EN. AT R = 5F</th>
<th>COUL. EN. AT R/Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open 1.919 2</td>
<td>Central 2.3</td>
<td>1.2</td>
</tr>
<tr>
<td>Shell</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diffuse Peak</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6-3

$^1_6(d,p)^{17*}$

**Reaction:** $^1_6(d,p)^{17*}$

**Shell Structure of Residual Nucleus:**

<table>
<thead>
<tr>
<th>Surface Char. Q</th>
<th>COUL. EN. AT R = 5F</th>
<th>COUL. EN. AT R/Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open 1.048 0</td>
<td>Small 2.3</td>
<td>2.2</td>
</tr>
<tr>
<td>Shell</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diffuse Peak</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

- Marked Backward Peak
- Forward Peak
The remaining reactions have $V \lesssim 0.6$, hence are considered not markedly structured.

The semi-classical mechanisms to be considered in turn are: 1.) Coulomb Distortion, 2.) Deuteron Structure with Coulomb Distortion, and 3.) Shell Structure.

1. **Coulomb Distortion.** Qualitatively, coulomb distortion predicts backward peaked angular distributions, as found, for example when heavy target nuclei are bombarded by particles with energies below the coulomb barrier, (Stokes 1961), (Erskine 1962). Conversely, particles with energies above the coulomb barrier should have no backward peaking due to coulomb distortion. The entrance channel distortion and exit channel distortion can be considered separately. A quantitative measure of coulomb distortion in the entrance channel is the ratio, \( \frac{\text{coulomb energy of the deuteron at the stripping radius, } R}{\text{the kinetic energy of the deuteron}} \). This ratio increases by $\sim \times 2$ over the range of $Z$ of the target, as seen in Table 6-3. A rough measure of the coulomb distortion in the exit channel is the ratio, \( \frac{\text{coulomb energy of the proton at the stripping radius, } R}{\text{the kinetic energy of the proton}} \). This ratio is approximately, \( \frac{\text{the coulomb energy}}{\text{the Q for the reaction}} \). Exit channel distortion should reflect variations among the reactions more strongly than entrance channel distortion, so it will be used as a measure of overall coulomb distortion, and is tabulated in Table 6-3.

Strong backward peaking should occur when the ratio, \( \frac{\text{coulomb energy at R}}{\text{Q is large}} \) is large. Table 6-3 shows that
$^0_{16}(d,p)^{17}*$ has the largest ratio and a marked backward peak. Except for $^{9}_{Be}(d,p)^{10}_{Be}$ the reactions all have small ratios and no marked backward peak. This is reasonable from the following qualitative picture. When the outgoing proton has an energy greater than the coulomb energy, distortion in the exit channel is then minimized. It will be shown that shell effects may play a part in the $^{9}_{Be}(d,p)^{10}_{Be}$ reaction.

2. **Deuteron Structure with Coulomb Distortion.** The above argument required that $R$ = the position of the proton, and that the deuteron be considered as a point. Allowing deuteron structure would enable the position of the proton to be > $R$. The coulomb energy could then be diminished in the ratio, $(\text{coulomb energy of proton}) / Q$. A given Q value reaction would then have diminished coulomb distortion if deuteron structure were included. In order to test this reasoning, compare the backward peaking for reactions with roughly the same ratio, $(\text{coulomb energy with deuteron structure}) / Q$, but differing Q values. The reaction with the lower Q value should show less backward peaking if the argument is valid. Compare $^{9}_{Be}(d,p)^{10}_{Be}$ and $^{14}_{N}(d,p)^{15}_{N}$ in Table 6-3. $^{9}_{Be}(d,p)^{10}_{Be}$ has the lower Q value, hence it should have less backward peaking. This is not consistent with the observed distributions. The conclusion can be drawn that coulomb distortion and deuteron structure cannot yield a prediction for the angular distributions, consistent for all observed reactions.

3. **Shell Structure.** Residual nuclei, i.e. target plus captured neutron, that are closed shells have nuclear
surfaces with well-defined radii. Conversely, open shell residual nuclei have a diffuse surface; the last neutron wave function has a long tail. The following arguments conclude that the closed shell nuclei should show more distortion from Bhatia theory than the open shell nuclei. Two arguments can be made: 1.) Surface effect, and 2.) Q value in the shell structure.

1.) A closed shell has its wave function "pulled in". The result is a tight structure with small spatial extension, hence a small geometric cross-section. For a neutron to be captured to form a residual nucleus, it must be in an available final state of this residual nucleus at the moment of capture. Hence it must come in very close to the target nucleus, if the residual nucleus is a closed shell. The neutron will pull the proton in with it, more or less, depending upon the deuteron structure used. The conclusions drawn are: 1.) coulomb distortion is greater for closed shell nuclei, and 2.) there is a possibility of nuclear distortion if the proton reaches the nuclear surface.

2.) The Q value in the shell structure determines the binding energy of the last neutron, through \( Q = B_n - B_d \). Closed shell residual nuclei bind the captured neutron strongly, requiring a large Q value for the (d,p) reaction. If deuteron structure is considered, the large Q value, i.e. large momentum for the outgoing proton, requires the proton to be very close to the neutron when stripping occurs, since the proton must obtain its momentum from the internal motion
of the deuteron and large relative momentum components are associated with small neutron-proton separation. The conclusions drawn are similar to the ones from the surface argument: 1.) coulomb distortion is greater for large Q value reactions, associated with closed shell nuclei, and 2.) nuclear distortion is more probable for large Q value reactions with closed shell nuclei.

These shell structure predictions will be compared with the observed angular distributions in the following.

The diffuse surface residual nuclei Li$^7$ and C$^{13}$ have associated angular distributions that are forward peaked, and peaked about 90 degrees, as shown in Figures 6-1 and 6-5 respectively. The lack of backward peaking, showing minimum coulomb distortion, is consistent with the shell structure argument.

The closed shell residual nuclei Be$^{10}$ and N$^{15}$ have associated angular distributions that have strong backward peaking and strong forward peaking, respectively. The forward peaking in N$^{14}$(d,p)N$^{15}$ suggests that nuclear distortion of the proton trajectory dominates coulomb distortion. Nuclear attraction of the proton to the nucleus would enhance forward peaking. For nuclear distortion to be present, the proton must penetrate to the nuclear surface. Both shell structure arguments, the surface and Q value discussions, predict such a penetration for the proton, since N$^{15}$ is a closed neutron shell nucleus, and the Q = 8.615 is the largest for the group of observed reactions. Such a picture has been treated by Stokes, (Stokes 1961). The Be$^9$(d,p)Be$^{10}$
reaction having a closed shell $\text{Be}^{10}$ residual nucleus, and a $Q = 4.585$, uses the same shell structure arguments. The backward peaking implies strong coulomb distortion and minimal nuclear distortion to be predicted from the arguments. This is possible if we consider as significant the difference in the $Q$ values for the two reactions. The lower $Q$ of the $\text{Be}^9(d,p)\text{Be}^{10}$ may be insufficient to require the proton to penetrate fully to the nuclear surface. If it penetrates only partially, then coulomb distortion would dominate, yielding a backward peak. It can be concluded that shell structure arguments can qualitatively predict the observed angular distributions for $\text{Be}^9(d,p)\text{Be}^{10}$ and $\text{N}^{14}(d,p)\text{N}^{15}$.

To summarize this section, various semi-classical pictures, i.e., pictures ignoring the uncertainty principle, can be used to give a qualitative explanation of some of the features of the observed angular distributions. Coulomb and nuclear distortion, deuteron structure, and closed shell effects can be used in combination to predict the behavior of $\text{Be}^9(d,p)\text{Be}^{10}$ and $\text{N}^{14}(d,p)\text{N}^{15}$ angular distributions.

6.6. Comparison of Results of Present Experiment with Other Work. Figures 5-1 through 5-8 show $\frac{d\sigma}{d\Omega}$ observed versus $\theta_{c.m.}$ as a function of $E_d$, comparing the results of the present experiment with other work. Figure 3-7 shows the excitation functions for the observed reactions, obtained by various workers.

Inspection of Figures 5-1 through 5-8 and Figure 3-7 shows three trends in the results: 1.) the angular distributions are asymmetric about 90 degrees. 2.) the
asymmetry is constant over the energy range 200 to 1000 KeV.

3.) the excitation functions are smooth for \( E_d < 1 \) MeV.

6.7. Interpretation of Comparison of Results with Other Work. Comparison of results with other work yields three types of evidence that stripping dominates the observed reactions between 200 and 1000 KeV, corresponding to the three trends observed above.

1.) The asymmetry about 90 degrees has been cited as evidence in Section 6.4.

2.) The constancy of the asymmetry from 200 to 1000 KeV puts a further constraint on the interference between members of pairs or groups of pairs of levels in the compound nucleus, as discussed in Sections 1.3 and 6.4. The further constraint is that the cross terms or interference terms, which contain the asymmetry, be independent of energy from 200 to 1000 KeV. Since different pairs of levels would be expected to interfere at energies separated by many level widths, very detailed relations among pairs of interfering levels must exist in order to explain the constancy of the asymmetry.

3.) Smooth excitation functions can be used as evidence for stripping by means of the following argument. First, smooth excitation functions have been cited, (Wilkinson 1960), as characteristic of direct reactions. Second, smooth excitation functions for C.N. require a continuum of compound levels with no isolated resonances; found at high excitation, while only moderate excitation would occur for C.N. in the present experiment. The observed
excitation functions thus make C.N. appear unlikely and definitely rule out isolated levels for $E_d \leq 1000$ KeV.

This section can be summarized as follows. The asymmetry, its independence of energy, and the smooth excitation functions are evidence for the dominance for stripping in the observed reactions for $E_d \leq 1000$ KeV.

6.8. **Discussion of PWBA and DWBA.** The following short survey of the literature attempts to give some perspective to the problem of the application of DWBA to the present experiment.

Tobocman, (Tobocman 1959), has shown how distorting effects due to the coulomb field and nuclear interactions generally affect the angular distribution of $(d,p)$ reactions. Correction terms are progressively added to the predicted distributions, until, when all corrections have been applied, a very good fit is obtained. That the distorted wave Born approximation is universal in its application has been held by Lee et al., (Lee 1964), who makes the statement, "We now know that distortion by nuclear scattering and absorption is always important. In fact, under no circumstances met with experimentally are distortion efforts negligible, so a plane wave theory is never a good approximation." When applying this statement to $(d,p)$ stripping reactions, for distortions to be felt by the proton, the deuteron as a whole must penetrate the nuclear field of the target. This situation is clearly not the case well below the coulomb barrier, where the proton is prevented from
approaching the target nucleus. Here, nuclear distortion of the proton can be completely neglected.

Bethe, (Bethe 1937), gives a quantitative estimate of the relative penetration of the deuteron as a whole and the neutron alone into the target nucleus. For \( Z = 17 \), and \( E_d = 0.55 \) MeV, neutron penetration is 60% more probable than deuteron penetration. The difference is smaller for higher energy and larger for lower energy. For \( E_d = 1.1 \) MeV, the difference is only 20%, for \( E_d \geq 2.0 \) MeV, the difference is unnoticeable. An interesting observation of Bethe, (Bethe 1938), is the independence of the breakup distance of the deuteron to variation in \( E_d \) and the target nucleus radius. The breakup distance is the distance of the center of mass of the deuteron from the target. Even for \( E_d = 0 \), this distance remains constant at the place where the coulomb potential is slightly less than the binding energy of the deuteron. Notice that this still allows the proton and the neutron of the deuteron to be widely separated at the instant of stripping.

Recall the discussion about the low Q value \((d,p)\) reactions, where the proton could obtain its small outgoing momentum from the internal motion of the deuteron, even though it was widely separated from the neutron of the deuteron. Thus, distortions would be expected to be small and a PWBA should hold. Gibbs and Tobocman, (Gibbs 1961), have made a DWBA calculation in which the Q value of the reaction was varied. Large distortions were found at very low energies, where the PWBA should hold. Gibbs and Tobocman claim
the success of the plane wave theory is due to the simpli-
city of the shape of the angular distributions. However,
Glendenning, (Glendenning 1963), and Wilkinson, (Pullen
1961), have pointed out that the distorted wave calculations
employ a zero range force between the neutron and the proton
to reduce the integrations. Thus, deuteron structure
effects, which appear to be very important for this type of
reaction, are ignored completely in distorted wave calcula-
tions.

As discussed in Section 2.7, the PWBA seems justified
at very low Q value, at low energy. The question of the
upper limit to the Q value of (d,p) reactions where PWBA
applies was asked at the beginning of this thesis. From the
analysis of results it is seen that no simple answer can be
given to this question. Indeed, one conclusion that may be
drawn from this survey and analysis is that the subject is
not closed on the mechanism for (d,p) stripping reactions at
low energy.
CHAPTER 7

CONCLUSIONS

The conclusions that can be drawn from this thesis are summarized below.

1.) Stripping appears to be an important mechanism for \((d,p)\) reactions on light nuclei for energies less than 1 MeV.

2.) Stripping seems to become relatively more important as the bombarding energy decreases and the total cross-section decreases. This effect is consistent with the increased coulomb repulsion, which increasingly inhibits formation of a compound nucleus.

3.) The simplicity of the angular distributions prevents detailed determination of the stripping radius, \(R\), using PWBA.

4.) Only a rather special use of a combination of semi-classical pictures, involving coulomb distortion, deuteron structure, and shell structure, can predict, qualitatively, the variety of angular distribution observed for \((d,p)\) reactions in the energy range 200 to 350 KeV.

5.) No upper limit on the \(Q\) value of \((d,p)\) reactions where PWBA applies can be set by our analysis.
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<thead>
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<tr>
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APPENDIX ONE

DETAILS OF A FORTRAN PROGRAM TO COMPUTE THE BHATIA DIFFERENTIAL CROSS-SECTION CALLED SIGL CALC.
APPENDIX 1.1

FORTRAN PROGRAM PRINTOUT

Col. Num. 1 5 7

C SIGL FIVE CALC

8 READ 5,PM1,PM2,PM3,PM4,Q
KOUNT=0
READ11
PRINT11
PRINT 13
ED=.2
ETOT=Q+ED
X=(PM1+PM2)*(PM3+PM4)
Y=ED/ETOT
Z=1.+(PM1*Q)/(PM2*ETOT)
B=(PM1*PM3*Y)/X
D=(PM2*PM4*Z)/X
A=(PM1*PM4*Y)/X
C=(PM2*PM3*Z)/X
DIMENSION R(3)
3 READ 6,R(1),R(2),R(3)
DO 2 I=1,3
THET=0
1 RAD=(3.1416*THET)/180.
EP=ETOT*(B+D+2.*(SQRTF(A*C))*COSF(RAD))
PKP=.219**SQRTF(EP)
PKD=.3099*SQRTF(ED)
PROD=(PM2*PKP*PKD*COSF(RAD))/(PM2+1.)
QUE=SQRTF((PKD**2)+(PKP**2)-2.*PROD)
ALPH=.23
SQKB=PKP**2+((PKD**2)/4.)-PKP*PKD*COSF(RAD)
C=(ALPH**2+SQKB)**2
PIE=1./C
ARG=QUE*R(I)
XJO=(SINF(ARG))/ARG
XJ1=(SINF(ARG))/ARG**2-(COSF(ARG))/ARG
XJ2=(SINF(ARG))*((3./ARG**2)-(1./ARG))- (3./ARG**2)*COSF(ARG)
XJ3A=(SINF(ARG))*(15./(ARG**3))
XJ3B=6./ARG**2
XJ3C=(COSF(ARG))*(1./ARG)-15./ARG**3
XJ3=XJ3A-XJ3B+XJ3C
SQJ0=(ABSF(XJ0))**2
SQJ1=(ABSF(XJ1))**2
SQJ2=(ABSF(XJ2))**2
SQJ3=(ABSF(XJ3))**2
APPENDIX 1.1 CONT.

Col. Num. 1  5  7

C SIGL FIVE CALC

GAMA=(PKP/PKD)*PIE
SIG0=GAMA*SQJ0
SIG1=GAMA*SQJ1
SIG2=GAMA*SQJ2
SIG3=GAMA*SQJ3
PRINT 9,ED,R(I),THET,EP,QUE,PIE,GAMA,SIG0,
      SIG1,SIG2,SIG3
KOUNT=KOUNT+1
IF(KOUNT-57)20,21,21
21 PRINT 12
   KOUNT=0
20 CONTINUE
   THET=THET+10.
   IF(THET-190.)1,2,2
1 CONTINUE
   ED=ED+.05
   IF(ED-.45)3,4,4
4 GO TO 8
5 FORMAT(F6.4,F7.4,F6.4,F7.4,F5.3)
6 FORMAT(3F5.2)
9 FORMAT(1H ,F4.2,F6.2,F5.0,F6.2,F6.3,2F7.2,
       4F10.2)
10 FORMAT(1H ,F3.1,F7.0,F12.2,F11.2)
11 FORMAT(1H0,12H)
12 FORMAT(1H1,2X,2HED,2X,4HR(I),2X,4HTHET,2X,
      2HEP,5X,3HQUE,3X,3HPIE
14X,4HGAMA,6X,4HSIG0,6X,4HSIG1,6X,4HSIG2,6X,
      4HSIG3)
13 FORMAT(1H ,2X,2HED,2X,4HR(I),2X,4HTHET,2X,
      2HEP,5X,3HQUE,3X,3HPIE
14X,4HGAMA,6X,4HSIG0,6X,4HSIG1,6X,4HSIG2,6X,
      4HSIG3)
END
APPENDIX 1.2

SIGL FIVE CALC GLOSSARY

PM2(PM1,PM3)PM4 is the reaction in conventional notation.
PM(I) are particle masses in atomic mass units.
PM2 is the target mass.
PM4 is the residual particle mass.
Q is the Q value of the reaction in MeV.
KOUNT = 0
KOUNT = KOUNT+1 control data output onto
IF (KOUNT-57)20,21,21 consecutive pages.
ED is the bombarding deuteron energy in MeV.
EP is the emitted proton energy in MeV.
THET is the scattering angle in the center of
mass system in degrees.
RAD is THET in radians.
PKP is the wave number of the emitted proton
in units of $10^{13}$ cm$^{-1}$.
PKD is the wave number of the deuteron in units
of $10^{13}$ cm$^{-1}$.
QUE is the wave number of the momentum trans­
ferred to the target by means of the
neutron in units of $10^{13}$ cm$^{-1}$.
ALPH is $\alpha$, the wave number associated with the
binding energy of the deuteron in units
of $10^{13}$ cm$^{-1}$.
SQKB is the square of the wave number $K$, associated with the momentum component
of the proton contributed by the
internal motion of the deuteron.
PIE is $G^2(K)$, the deuteron factor.
ARG is the argument of the spherical Bessel
functions.
R(I) are interaction radii in fermis.
XJI are spherical Bessel functions, of order I.
SQJI are the absolute squares of the spherical
Bessel functions of order I.
GAMA is $\frac{K_p}{K_d} G^2(K)$.
SIGI is the Bhatia angular distribution for
angular momentum transfer I.
APPENDIX 1.3

NOTES ON THE USE OF SIGL FINE CALC.

Sample Input Cards for Li$^6$(d,p)Li$^7$

Input Data Card
No.  Col. Num. 1
1.  2.014706.01701.008107.01825.027
    PM1  PM2  PM3  PM4  Q
2.  LL16(DP)LL7
    l is the comment column, controlling the holorith field.
3.  05.0005.5006.00
    R(1)  R(2)  R(3)  for ED = .2
4.  04.5005.0005.50
    R(1)  R(2)  R(3)  for ED = .25
5.  
    for ED = .30
6.  
    for ED = .35
7.  
    for ED = .40
8.  Data cards for next reaction

Sample Output Printout for Li$^6$(d,p)Li$^7$

ED  R(I)  THET  EP  QUE  PIE  GAMA  SIG0  SIG1  SIG2  SIG3
.20  5.00  90.  4.53 .487  13.13  44.27  3.13  7.87  8.82 .85
APPENDIX TWO

VAN DE GRAAFF OPERATING PROCEDURE

1. Add liquid nitrogen to the vacuum system traps.
2. Check the insulating gas pressure on the Van de Graaff. It should be near 100 psi.
3. Check cooling water valves. Make sure all are on.
4. Make sure beam tube gate valve is open.
5. Turn on detector bias. Turn up slowly, monitoring the detector current while doing so. If the detector current rises sharply, back off; come up again slowly. Do not exceed specified maximum detector bias.
7. Turn on terminal voltage stabilizer.
8. Set focus voltage for corresponding terminal voltage.
9. Bring gas from 5 microamperes to about 12 microamperes. Let gas come to equilibrium.
10. Monitor beam current meter on most sensitive scale. Wait until bottle lights. About a half microampere current will appear.
11. Adjust focus voltage to maximize target current and minimize beam forming electrode current.
12. Bring beam current up to desired value. Overheating of the target will show as an increase in the vacuum current, indicating evolution of gas from the target. This will cause excessive scattering of the beam.
13. Monitor signal pulses from the preamps on the oscilloscope.
14. If signal behavior is proper, proceed with data recording.
15. At the end of the run, bring beam down to zero.
16. Turn off terminal voltage stabilizer and Van de Graaff.
TARGET REMOVAL PROCEDURE

1. Turn off detector bias.
2. Remove preamps from chamber cable connectors.
3. Close beam tube vacuum system gate valve.
5. Open roughing pump beam tube valve.
6. Open roughing pump valve, allowing the beam tube and scattering chamber to come up to atmospheric pressure. If humid air is present, bleed air through liquid nitrogen filled trap on roughing pump.
7. Reverse this procedure to evacuate scattering chamber. Rough down at least ten minutes before closing roughing pump beam tube valve and opening beam tube vacuum system gate valve. Be sure nitrogen trap on beam tube vacuum system is filled.