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Identification of system parameters for end milling force simulation with tool and workpiece compliance

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Identification of system parameters for end milling force simulation with tool and workpiece compliance

Abstract
A Smart Machining System being developed at UNH has the potential to produce high quality machined parts in minimum time. Integral to the success of this system is the ability to accurately simulate cutting forces. In this current work, a time-domain milling simulation is developed with a tool-workpiece compliance model to predict dynamic cutting forces. The simulation computes milling forces, tool deflections, and workpiece vibration (surface waviness).

The accuracy of the simulation depends on finding reliable system parameters. In this work, an end milling parameter identification method is developed using linear predictive coding (LPC) and Extended Kalman Filtering. The milling simulation model is validated by comparison of simulation and experimental forces for a variety of end milling cuts. In-cut and out-of-cut damping is shown to be significantly different, and must be considered in the simulation model. This milling force simulation is shown to predict chatter reasonably well under controlled cuts.

Keywords
Engineering, Mechanical

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IDENTIFICATION OF SYSTEM PARAMETERS FOR END MILLING FORCE SIMULATION WITH TOOL AND WORKPIECE COMPLIANCE

BY

MIN HYONG KOH
B.S., Inha University, 2009

THESIS

Submitted to the University of New Hampshire in Partial Fulfillment of the Requirements for the Degree of

Master of Science in Mechanical Engineering

December, 2012
This thesis has been examined and approved.

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10 Dec 2012
Date
DEDICATION

To my family
ACKNOWLEDGEMENTS

I would like to thank the members of my committee, in particular Dr. Barry Fussell and Dr. Robert Jerard.

I would also like to thank Saman Nouri, Andrew Harmon, Yong Zhao, and Anthony Morin who work with me in the Design and Manufacturing Laboratory at the University of New Hampshire.
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Identification of System Parameters for End Milling Force Simulation with Tool and Workpiece Compliance

by

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University of New Hampshire, December, 2012

Degree Adviser: Barry Fussell

A Smart Machining System being developed at UNH has the potential to produce high quality machined parts in minimum time. Integral to the success of this system is the ability to accurately simulate cutting forces. In this current work, a time-domain milling simulation is developed with a tool-workpiece compliance model to predict dynamic cutting forces. The simulation computes milling forces, tool deflections, and workpiece vibration (surface waviness).

The accuracy of the simulation depends on finding reliable system parameters. In this work, an end milling parameter identification method is developed using linear predictive coding (LPC) and Extended Kalman Filtering. The milling simulation model is validated by comparison of simulation and experimental forces for a variety of end milling cuts. In-cut and out-of-cut damping is shown to be significantly different, and must be considered in the simulation model. This milling force simulation is shown to predict chatter reasonably well under controlled cuts.
CHAPTER I

INTRODUCTION

1.1 Introduction

Computer Numerical Control (CNC) machining has been developed to obtain high material remove rate without significant part error. In order to achieve this end, the effect of cutting forces and dynamic tool deflection must be considered. One obstacle to achieving this goal is the presence of chatter, which can lead to significant part error. Chatter is caused by significant dynamic deflections of the tool system. This research focuses on achieving a better understanding of the dynamic structures' deflection, as it affects chatter, by the development of a cutting force simulation model which includes tool-workpiece compliance.

The stability lobe diagram (SLD) was developed in order to determine the stability of the cutting process [1, 2]. However, the SLD has two major limitations: 1) it only provides a "global" idea of the stability, since it computes the cutting stability at a certain cutting point instead of an entire cutting region and 2) it does not consider variability in the model parameters such as stiffness, damping, and natural frequency. In order to consider the model parameters changes and achieve "local" stability, a time-domain milling simulation has been developed for calculation of regenerative force and dynamic deflections with the given cutting conditions and tool geometry [1-3]. Since the
workpiece system is often much stiffer than the tool system, general milling simulation considers only tool dynamic deflections, using constant parameters.

The *regenerative* milling forces, tool deflections, and workpiece vibration (surface waviness) can determine the milling stability and chatter development. Therefore, in this research the tool-workpiece compliance model is included in the simulation. The fact that the process damping ratio (in-cut) is significantly different from the dynamic damping ratio (out-of-cut) due to tool/workpiece engagement, has been considered in the development of the simulation.

Simulation of the *regenerative* cutting forces, tool/workpiece vibration, and surface waviness requires the identification of both the tool and workpiece system parameters, i.e. damping ratio, natural frequency, and stiffness. Force measurements are used to obtain these system parameters. For the workpiece, a bed dynamometer (Kistler), which is mounted under the workpiece, is used to measure cutting forces. These forces are affected by the workpiece and dynamometer behavior during the cutting. A Smart Tool is used to obtain force measurements acting on the cutting tool. This tool has strain gauges on the tool holder shank and transmits force data wirelessly to a PC [4, 5]. The measured force from this sensor is affected by the dynamics of the tool and tool holder.

The dynamic parameters of the tool and workpiece systems are identified from both force sensors and are used for the simulation. In this research, from the measured force vibrations such as the out-of-cut and in-cut vibrations, linear predictive coding (LPC), which predicts the pole-zero location [6], is used for identification of dynamic damping ratio and natural frequency (out-of-cut) and process damping (in-cut).
Additionally, the dynamic stiffness of the tool and workpiece systems is approximated from their static stiffness, determined when the CNC machine is stationary.

The simulation model developed in this research is validated for a variety of cutting conditions using the experimentally obtained system parameters. Validation is determined by comparing simulated force measurements to the experimentally measured forces in terms of the maximum peak force and force frequency content. In addition, the possibility of chatter prediction for a given maximum axial depth of cut has been evaluated.

The out-of-cut profile, i.e. tool not engaged with the workpiece, is the vibration signal. The in-cut force profile includes the cutting force and the in-cut vibration of the sensor. In order to estimate the in-cut parameters, the in-cut vibration must be separated from in-cut profiles by de-trending, i.e. subtracting the vibration induced forces, from the measured cutting force. An Extended Kalman Filter, which includes a harmonic force model and sensor dynamic model, is developed in order to estimate the actual cutting force. This Extended Kalman Filter is self-tuning using the harmonic components of the measured cutting force.

This work has resulted in several significant innovations for end milling research.

- In milling simulation, the tool and workpiece compliance model allows chatter prediction as well as cutting force estimation. This compliance model provides better understanding of both the tool and workpiece system vibrations.
- The Extended Kalman Filter, which is designed as a combination of the harmonic force model and sensor dynamic model, reduces the sensor dynamic effects and
estimates "actual" cutting force. The Extended Kalman Filter is self-tuned by harmonic computation of the measured cutting force.

- Through the use of the LPC and the Extended Kalman Filter, the tool and workpiece system parameters are identified from the force measurement.

1.2 Thesis Overview

Chapter 2 describes the basic theories of a time-domain milling force simulation, including the tool-workpiece compliance model. The simulation is based on regenerative chip thickness, linear milling force model, systems dynamics, and tool geometry. The system dynamics includes tool-workpiece compliance and process damping effects. In addition, the methodology for calculating cutting forces and tool/workpiece deflections is explained.

Chapter 3 discusses the method used to identify tool and workpiece system parameters (dynamic stiffness, dynamic natural frequency, dynamic damping ratio: $k_{dyn}$, $\omega_{n,dyn}$, $\zeta_{dyn}$, and process damping ratio: $\zeta_{process}$). These parameters are required in order to calculate the regenerative chip thickness and the dynamic deflection of the structures. Using linear predictive coding (LPC), $\omega_{n,dyn}$ and $\zeta_{dyn}$ are defined from the out-of-cut vibration when the tool rotates but does not engage in cutting. $\zeta_{process}$ is also defined by the LPC method based on the estimated cutting force from the in-cut profile. The dynamic stiffness of the tool and workpiece ($k_{dyn}$) is estimated using static parameters $k_{sta}$ and $\omega_{n,sta}$, that are obtained when the CNC machine is stationary.

In Chapter 4, the simulation results for the force profile, maximum peak force and force frequency content, using the systems parameters obtained by Chapter 3 methods,
are compared with experimental data for specific cutting conditions. Since the system parameters vary due to the stochastic nature of cutting, simulation sensitivity based on variation of the tool system parameters is evaluated. The simulation is used to predict the maximum axial depth of cutting as the milling stability remains.

Chapter 5 introduces the Extended Kalman Filter as a way to estimate the actual cutting force from the measured force. The Extended Kalman Filter includes a harmonic force model and a model of the sensor dynamics. Due to the limitation of the sensor's bandwidth, the force sensors data does accurately reflect the actual cutting forces. The Extended Kalman Filter estimates the actual cutting forces without phase delay and without the use of tedious Kalman tuning.

Chapter 6 summarizes the conclusion of this research in terms of practical application and discusses future works. Several methods to improve milling simulation and the Extended Kalman Filter are offered and real-time system identification is proposed. In addition, chatter frequency detection is discussed. Chatter frequency detection is necessary in order to optimize material remove rate and keep the cutting process stable.
CHAPTER II

TIME-DOMAIN MILLING SIMULATION WITH TOOL AND WORKPIECE COMPLIANCE

2.1 Introduction

The phase delay between the current and previous tool path, i.e. the regenerative tool path, is the main reason for chatter, an unstable cutting condition which can cause extremely large cutting forces. The large cutting force can potentially damage the tool, workpiece, and even the CNC (Computer Numerical Control) machine itself. This can lead to dramatic decreases in productivity and quality of products [2]. Because of these reasons, chatter should be avoided when machining.

Stability Lobe Diagrams (SLD), based on the frequency response function [1], have been developed to select spindle speeds and axial depths of cuts that avoid chatter. However, typical SLD assumes that the milling process is a time invariant system with no workpiece deflection. Because of this limitation, an SLD just gives a general idea for milling stability.

In this chapter, a time-domain milling simulation with tool-workpiece compliance is created for accurate estimation of cutting force and milling stability for a given set of cutting conditions. The simulation computes the cutting force based on a linear cutting force model, dynamic models of the tool and workpiece vibrations, process damping
effects, and given cutting conditions such as spindle speed, feedrate, tool geometry, and cut geometry. The *regenerative* chip thickness is determined from the current and previous tool/workpiece deflections and forms an important component of force simulation.

### 2.2 Regenerative Chip Thickness and Chatter

The *regenerative* chip thickness is the difference between the previous tool deflection and the current tool deflection. This is shown as feedback paths in Figure 2.1. The system dynamics block includes mass, damping, and compliance of the tool and workpiece systems. The regenerative chip thickness is added to the nominal chip thickness, \(h_0\) in Figure 2.1, to form the instantaneous chip thickness, which is used to determine the instantaneous cutting force. During the milling process, the cutting force deflects the tool and workpiece because they are flexible structures. The tool path waviness is affected by these deflections. Figure 2.1 shows all the various components that comprise the milling system.

![Figure 2.1: Milling Force Model based on Regenerative Chip Thickness](image-url)

*Figure 2.1: Milling Force Model based on Regenerative Chip Thickness*
Considering the time delay between current and previous tool deflections, the instantaneous chip thickness $h(t)$ is calculated by:

$$h(t) = h_0 - n(t) + n(t - \tau)$$  \hspace{1cm} (2.1)

where $h_0$ is the nominal chip thickness, $n(t)$ is the current tool deflection, $n(t - \tau)$ is previous tool deflection, and $\tau$ is time delay between the current and previous cutter teeth.

The nominal chip thickness is defined by:

$$h_0 = f_t \cdot \sin \phi$$  \hspace{1cm} (2.2)

where $f_t$ is the feed per tooth and $\phi$ is the angular position of the tooth which is engaged in milling. The feed per tooth ($f_t$) is given by:

$$f_t = \frac{F_R}{\Omega \cdot N_T}$$  \hspace{1cm} (2.3)

where $F_R$ is the feedrate, $\Omega$ is the spindle speed, and $N_T$ is number of teeth.

![Figure 2.2: Phase Delay and Chatter [7]](image)

Figure 2.2 shows the relation between phase delay and the variation of chip thickness. With zero phase delay, the cutting process is rigidly stable since chip variation $(n(t - \tau) - n(t))$ is negligible and the nominal chip thickness, a sine function of the tooth position in the cut, only affects the cutting force. On the other hand, the other two
figures show significant chip thickness variation with the phase shift. This significant variation of chip thickness can cause unstable cutting, resulting in chatter. The worst stability case is obtained when the phase delay is 270° (-90°) instead of 180° [1, 7].

2.3 Linear Milling Force Model

From the mechanics of milling, the linear milling force model described by Altintas is used to compute the cutting force based on the undeformed chip thickness [3]. This linear milling force model includes radial ($F_r$) and tangential ($F_t$) components, and is defined by:

$$F_r(t) = F_{rc} + F_{re} = K_{rc} \cdot h(t) \cdot a + K_{re} \cdot a$$

(2.4)

$$F_t(t) = F_{tc} + F_{te} = K_{tc} \cdot h(t) \cdot a + K_{te} \cdot a$$

(2.5)

, where $a$ is axial depth of cut, $h(t)$ is the instantaneous chip thickness including the regenerative term, $K_{rc}$ and $K_{tc}$ are cutting coefficients contributed by shearing action in the radial and tangential directions, and $K_{re}$ and $K_{te}$ are edge constants resulting from rubbing and ploughing. This linear force model assumes the cutting coefficients are constant values for a given tool-workpiece material pair and a sharp tool. As the tool wears, the edge coefficients ($K_{re}$ and $K_{te}$) increase in magnitude [8].

There are nonlinear force models of the end milling process as well [3]:

$$F_r(t) = K_R \cdot h(t)^{1-q} \cdot a$$

(2.6)

$$F_t(t) = K_T \cdot h(t)^{1-p} \cdot a$$

(2.7)

, where $K_R$ and $K_T$ are cutting coefficients for the nonlinear milling force model, and $p$ and $q$ are cutting force coefficients determined by cutting tests at a variety of chip thicknesses. This nonlinear model is more accurate than the linear for very small chip
thicknesses; however, it does not provide much advantage for normal cuts, where the chip thickness is considered medium too large. Either model can provide very good force estimation as long as the coefficients have been recently calibrated [4, 8].

2.4 System Dynamics with Compliant Tool and Workpiece

Deflection of the tool and workpiece has a significant effect on the regenerative chip thickness and is a major consideration for chatter simulation. An existing chatter simulation program [1] is adapted in this research to include a simple second-order model for the tool and workpiece systems. This includes damping (b), mass (m), and spring (k), as shown in Figure 2.3. Higher order models may be used; however, this makes system identification more complicated and possibly less accurate, leading to less reliable results. In this thesis, compliant second-order models are created for the tool and workpiece systems. Higher order models are left as future work.

![Compliance Model of the Tool and Workpiece Systems](image)

Figure 2.3: Compliance Model of the Tool and Workpiece Systems
Figure 2.3 shows the compliance models of the tool and workpiece systems. Based on this model, mathematical equations resulting from Newton's law for both the tool and workpiece systems are given by:

\[ F_x = m_{x,tool} \cdot \dot{x}_{tool} + b_{x,tool} \cdot \ddot{x}_{tool} + k_{x,tool} \cdot x_{tool} \]  
\[ (2.8) \]

\[ -F_x = m_{x,workpiece} \cdot \ddot{x}_{workpiece} + b_{x,workpiece} \cdot \dot{x}_{workpiece} + k_{x,workpiece} \cdot x_{workpiece} \]  
\[ (2.9) \]

\[ F_y = m_{y,tool} \cdot \dot{y}_{tool} + b_{y,tool} \cdot \ddot{y}_{tool} + k_{y,tool} \cdot y_{tool} \]  
\[ (2.10) \]

\[ -F_y = m_{y,workpiece} \cdot \ddot{y}_{workpiece} + b_{y,workpiece} \cdot \dot{y}_{workpiece} + k_{y,workpiece} \cdot y_{workpiece} \]  
\[ (2.11) \]

where \( b_{x,tool}, b_{y,tool}, b_{x,workpiece} \) and \( b_{y,workpiece} \) are damping coefficients, \( x_{tool}, y_{tool}, x_{workpiece} \) and \( y_{workpiece} \) are deflections, \( F_x \) and \( F_y \) are cutting forces, \( k_{x,tool}, k_{y,tool}, k_{x,workpiece} \) and \( k_{y,workpiece} \) are the spring stiffness values, and \( m_{tool} \) and \( m_{workpiece} \) are the effective masses. The total deflection between the tool and workpiece is given as:

\[ x_{total} = x_{tool} - x_{workpiece} \]  
\[ (2.12) \]

\[ y_{total} = y_{tool} - y_{workpiece} \]  
\[ (2.13) \]

From the total deflection, the instantaneous chip thickness is calculated by:

\[ h(t) = f_t \cdot \sin \phi - \begin{bmatrix} -\sin \phi & -\cos \phi \end{bmatrix} \begin{bmatrix} x_{total}(t) \\ y_{total}(t) \end{bmatrix} \]
\[ + \begin{bmatrix} -\sin \phi & -\cos \phi \end{bmatrix} \begin{bmatrix} x_{total}(t - \tau) \\ y_{total}(t - \tau) \end{bmatrix} \]  
\[ (2.14) \]

From the instantaneous chip thickness and the linear force model (Equations 2.4 and 2.5), the cutting force acting on a tooth \( F_t \) and \( F_r \) can be represented in X and Y coordinates.

\[ \begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} -\cos \phi & -\sin \phi \\ \sin \phi & -\cos \phi \end{bmatrix} \begin{bmatrix} F_t \\ F_r \end{bmatrix} \]  
\[ (2.15) \]
2.5 Process Damping Effects

The system dynamics of the milling simulation also includes the process damping effects as well as the tool-workpiece compliance model. When the differential equations (Equations 2.8-2.11) are solved to calculate the tool/workpiece deflection, different system parameters for both the tool and workpiece systems, i.e. in-cut and out-of-cut parameters, are used according to the process damping effects. The in-cut and out-of cut are distinguished by the tool/workpiece engagement. The in-cut properties differ from out-of-cut because of the complicated engagement between the tool and workpiece systems. As a result, the differential equations must be solved using system parameters that reflect the spindle speed of the tool and the states of the cut, i.e. cutter engagement.

During the cut, the tool dynamics, i.e. k, ζ, and ωn, have been shown to be a function of spindle speed and cutting conditions [10]. With a given spindle speed, the cutting process damping can be modeled by following the equation:

\[ m\ddot{u} + (b + B \frac{h}{V} \cos(\alpha)^2)\dot{u} + ku = F \cos(\beta - \alpha) \]

(2.16)

where B is the process damping coefficient, h is the chip width, V is the cutting speed, u is the displacement, \( \dot{u} \) is the tool system velocity, \( \ddot{u} \) is the tool system acceleration, \( \beta \) is the force angle that corresponds to the surface normal, and \( \alpha \) is the angle between the displacement and the surface normal [10]. In this research, the process damping effects are applied to both tool and workpiece systems. Instead of directly using Equation 2.16, the dynamic damping ratio \( \zeta_{\text{dyn}} = \frac{b \cdot \omega_n}{2 \cdot k} \), where \( \alpha = 0 \) and the process damping ratio \( \zeta_{\text{process}} = \frac{(b + \frac{bh}{V} \cos(\alpha)^2) \cdot \omega_n}{2 \cdot k} \) are used for the system dynamics according to the state of
the cut. The dynamic damping ratio refers to the out-of-cut damping ratio of the tool or workpiece systems at a given spindle speed. The process damping ratio refers to the in-cut damping ratio at a given spindle speed. The somewhat constrained end of the tool leads to a slightly higher damping in both the tool and workpiece systems. The natural frequency is the same for both the in-cut and out-of-cut since it is derived from the stiffness and the effective mass (Equation 2.16).

2.6 Tool Geometry of a Helical Cutter

Peripheral milling is typically performed using a helical tool so that the chip thickness gradually increases as the tool rotates. The cutting force model shown in Equations 2.4 and 2.5 are for a straight tooth cutter. To calculate the force on a helical tooth cutter, the cutter must be discretized (sliced) along the axial direction. By assuming that the tooth is straight for each slice, the force equations can be used on each slice and then summed to get the total force on the tooth. The lag angle between the various slices is important for determining the chip thickness of the cut.

![Figure 2.4: Discretized Axial Depth of Cut with Helix Angle](image-url)
Figure 2.4 shows a simple geometric description of the discretized helical tooth with a lag angle. In this case, the lag angle is defined as the delay angle between the bottom and top of the tooth. From this geometry, the relation between the helix angle ($\theta_{\text{helix}}$) and the lag angle ($\theta_{\text{lag}}$) is defined by:

$$\tan \theta_{\text{helix}} = \frac{R \cdot \theta_{\text{lag}}}{a}$$  \hspace{1cm} (2.17)

where $R$ is the tool radius and $a$ is the axial depth of cut. Equation 2.17 can be defined with a discretized lag angle ($\Delta \theta_{\text{lag}}$) and a discretized axial depth of cut ($\Delta a$). One method to discretize along the axial direction is the use a discretized angle ($\Delta \phi$) for tracking the angular position of each segment bottom in the angular summation loop. The discretized axial depth of cut is determined by the discretized lag angle using Equation 2.17. After discretization along the axial direction, the cutting force on each tooth segment engaged with the workpiece is summed to get the total force acting on the tool.

2.7 Simulation Description

The simulation outputs the chip thickness, the cutting force, and the deflection of the tool and workpiece systems for a given set of cutting conditions and tool revolutions. The simulation consists of three major parts: angular summation loop, axial summation loop, and deflection calculation from the system dynamics. Figure 2.5 shows the flow chart for the simulation.
Figure 2.5: Flow Chart of Simulation Program
2.7.1 Input Variables

In order to run the simulation, a minimum number of input variables are required. The input variables include:

- Simulation parameter: number of tool revolutions
- Tool geometry: number of teeth \( N_t \), tool diameter, helix angle
- Cutting conditions: spindle speed, feedrate, axial depth of cut, radial depth of cut, cutting type (1: down milling, 2: up milling)
- Cutting coefficients: \( K_{rc}, K_{re}, K_{tc}, K_{te} \)
- System dynamics:
  - static stiffness of the tool system \( (k_{x\_tool\_sta}, k_{y\_tool\_sta}) \)
  - static natural frequency of the tool system \( (\omega_{n\_tool\_sta}) \)
  - static stiffness of the workpiece system \( (k_{x\_workpiece}, k_{y\_workpiece}) \)
  - dynamic natural frequency and dynamic damping ratio of the tool system
    \( (\omega_{n\_x\_tool\_dyn}, \zeta_{x\_tool\_dyn}, \omega_{n\_y\_tool\_dyn}, \zeta_{y\_tool\_dyn}) \)
  - dynamic natural frequency and dynamic damping ratio of the workpiece system
    \( (\omega_{n\_x\_workpiece\_dyn}, \zeta_{x\_workpiece\_dyn}, \omega_{n\_y\_workpiece\_dyn}, \zeta_{y\_workpiece\_dyn}) \)
  - process damping ratio \( (\zeta_{x\_process}, \zeta_{y\_process}) \)

In order to solve the differential equations, i.e. Equations 2.8 - 2.11, system dynamics including stiffness \( (k) \), effective mass \( (m = \frac{k}{\omega_n^2}) \) and damping \( (b = \frac{2\zeta k}{\omega_n}) \) of both the tool and workpiece must be defined under dynamic conditions when the tool rotates. Natural frequency \( (\omega_n) \), damping ratio \( (\zeta) \) and stiffness are calculated from these parameters and used as simulation inputs. In this simulation, the dynamic parameters
refer to out-of-cut parameters of the tool and workpiece systems at a given spindle speed. The process parameters refer to in-cut parameters at a given spindle speed.

Experimental data from the Smart Tool and the Kistler force transducer are used to find dynamic system parameters of the tool and workpiece systems, i.e. $\omega_{n, dyn}$ and $\zeta_{dyn}$ during out-of-cut, and $\zeta_{process}$ during in-cut conditions. However, the dynamic stiffness of both systems, i.e. $k_{x, tool, dyn}$, $k_{y, tool, dyn}$, $k_{x, workpiece, dyn}$, and $k_{y, workpiece, dyn}$, remains as unknowns. These unknowns are approximated from the static characteristics of the tool and workpiece systems. Static characteristics of the systems are obtained from a simple tap test ($\omega_{n, sta}$) and linear calibration of tool deflection vs applied force ($k_{tool, sta}$) when the CNC machine is stationary. In this approximation, we make two different assumptions: 1) the dynamic stiffness of the workpiece system is the same as the static workpiece stiffness for small material removal rates and 2) the effective mass of tool system remains constant.

The stiffness of the workpiece is assumed as a constant value ($k_{workpiece, dyn} = k_{workpiece, sta}$) during the cutting process for small material removal rates or small variations of area moment of inertia.

![Figure 2.6: Geometry of Workpiece Material (Al6061, w=6[in], l=8[in], h=2[in])](image-url)
From Figure 2.6, the stiffness of material in X and Y directions are defined:

\[ k_x = \frac{3 \cdot E \cdot I_x}{h^3}, \quad k_y = \frac{3 \cdot E \cdot I_y}{h^3} \quad (2.18) \]

\[ I_x = \frac{1}{3} \cdot w \cdot h \cdot (w^2 + h^2), \quad I_y = \frac{1}{3} \cdot l \cdot h \cdot (l^2 + h^2) \quad (2.29) \]

where \( E \) is Young's modulus, \( I_x \) and \( I_y \) are the area moment of inertia. From Equation 2.18 and 2.19, the area moment of inertia and stiffness are increased or decreased since the geometry of the workpiece changes as material is removed. In order to accurately track the workpiece stiffness, a tap test [3] and measurement of the mass of the workpiece system, including material, connecting bolts, and Kistler dynamometer, should be performed before the cutting. However, in this research, the stiffness of the workpiece is assumed constant as very little material is removed during test cuts. The variation of workpiece's stiffness and natural frequency as function of time during the cut is left as a future work.

The stiffness of the tool system under dynamic conditions (\( k_{x_{\text{tool_dyn}}} \) and \( k_{y_{\text{tool_dyn}}} \)) is considered variable and can be estimated from the ratio of the dynamic and static natural frequencies if the effective mass is considered constant. From these ratios and the relationship among the natural frequency, mass, and stiffness, the stiffness of tool in dynamic conditions is obtained by:

\[ k_{x_{\text{tool_dyn}}} = k_{x_{\text{tool_sta}}} \left( \frac{\omega_{n_{x_{\text{tool_dyn}}}}}{\omega_{n_{x_{\text{tool_sta}}}}} \right)^2, \quad k_{y_{\text{tool_dyn}}} = k_{y_{\text{tool_sta}}} \left( \frac{\omega_{n_{y_{\text{tool_dyn}}}}}{\omega_{n_{y_{\text{tool_sta}}}}} \right)^2 \quad (2.20) \]

Specific methods and results about the identification of the tool and workpiece dynamics will be discussed in Chapter 3 in detail.
2.7.2 Additional Input Variables

From the input variables given in Section 2.7.1, additional variables are calculated in order to compute the cutting force and the tool and workpiece deflection. Figure 2.7 describes the simple tool geometry of a 4 tooth cutter in up milling. From the cutting type (up or down milling), tool diameter ($2R$), and radial depth ($r$), the start angle ($\theta_{\text{start}}$) and end angle ($\theta_{\text{end}}$) are calculated as shown in Table 2.1.

![Figure 2.7: Tooth's Angular Position in Up Milling](image)

Table 2.1: Start Angle and End Angle Based on Type of Cut

<table>
<thead>
<tr>
<th></th>
<th>Up Milling</th>
<th>Down Milling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start Angle ($\theta_{\text{start}}$)</td>
<td>$0$ [deg]</td>
<td>$180 - \cos^{-1}\left(\frac{R-r}{R}\right)$ [deg]</td>
</tr>
<tr>
<td>End Angle ($\theta_{\text{end}}$)</td>
<td>$\cos^{-1}\left(\frac{R-r}{R}\right)$ [deg]</td>
<td>$180$ [deg]</td>
</tr>
</tbody>
</table>
Since the bottom of the helical tooth is the reference angular position, the actual end angle is the end angle plus the lag angle \( \theta_{\text{actual end}} = \theta_{\text{end}} + \theta_{\text{lag}} \). Machining occurs in the cutting region between the start and actual end angles. Axial slices with tooth segments within this cutting region are engaged in machining.

For each angular position, deflections are determined from the cutting force and system dynamics of the tool and workpiece systems. The differential equations of the systems, Equations 2.8 - 2.11, are solved by Euler's numerical integration method, using a time step \( (\Delta t) \) based on the maximum natural frequency of the tool or workpiece systems. Numerical calculation requires the use of a smaller time step than the critical time step \( (t_{\text{cri}}) \) [11] defined by:

\[
\Delta t_{\text{cri}} = \frac{t_n}{\pi}
\]  

(2.21)

, where \( t_n \) is the natural period of system. When a larger time step than the critical is used, the calculation result is inaccurate because of round-off error, especially the higher order derivative terms. A typical time step is 3.42e-5 [s], approximately 30000 [Hz], that is almost 20 times faster than natural frequency of the structures.

There are two common methods for solving differential equations: Euler's and Runge-Kutta methods. Euler's method is easier to implement but the Runge-Kutta method is more accurate with the same step size. If the time step size is small enough, e.g. 20 times faster than the structure natural period, Euler's method is sufficient to accurately solve the differential equations instead of using the more complicated Runge-Kutta method. Figure 2.8 shows similar Euler's integration method results with different step sizes (peak force: 78.34 [N] with X=20, 79.88 [N] with X=100, and 80.41 [N] with
This demonstrates that a time step 20 times faster than the structure natural period is adequate to achieve accurate results. If the Runge-Kutta integration method is used with the same step size, the integration process becomes more complicated and slower.

From the selected simulation time step, the angular summation step size ($\Delta\phi$) and axial summation step size ($\Delta a$) are determined:

$$
\Delta\phi[\text{deg}] = \frac{360[\text{deg}] \cdot \Omega[\text{rpm}] \cdot \Delta t[s]}{60[\text{s}]} \quad (2.22)
$$

$$
\Delta a[\text{in or m}] = \frac{R[\text{in or m}] \cdot \Delta\phi[\text{rad}]}{\tan(\theta_{\text{helix}})[\text{rad}]} \quad (2.23)
$$
In addition, the lag angle is calculated by Equation 2.16 and the feed per tooth \( f_r \) is computed by Equation 2.3. The simulation begins with zero initial tool and workpiece position, and zero initial deflection of the tool and workpiece.

2.7.3 Angular Summation Loop, Axial Summation Loop, and System Dynamics

The outer loop of Figure 2.5 is the angular summation loop including the calculation of the system dynamics. The inner loop of Figure 2.5 is the axial summation loop.

The angular summation loop calculates cutting forces and deflections for every reference angular position \( \theta \). The bottom of the first tooth's angular position in Figure 2.7 is tracked as the reference angular position. If a multiple tooth cutter is used for milling, the position of each tooth is calculated based on the first tooth reference angular position. For a given reference angular position, the applied forces to all teeth on the cutting region are summed as the cutting force.

The axial summation loop is necessary to handle cutters with helical teeth. Along the axial direction, the tool is sliced resulting in tooth segments. When the angular position of a tooth segment is in the cutting region, the cutting force \( \Delta F_r \) and \( \Delta F_t \) acting on the segments are calculated. The forces on each tooth segment of each axial disc are summed to find the cutting force acting on tool.

With the cutting forces evaluated, the deflection of the tool and workpiece is calculated by numerical integration of Equations 2.8 - 2.11. Various parameters are used in these equations based on the state of the cut. When the teeth enter the cutting region, between the start and actual end angles, the process damping ratios \( \zeta_{\text{x,process}} \) and \( \zeta_{\text{y,process}} \)
are applied. Out of the cutting region, the dynamic damping ratios ($\zeta_{x\_tool\_dyn}$, $\zeta_{y\_tool\_dyn}$, $\zeta_{x\_workpiece\_dyn}$, and $\zeta_{y\_workpiece\_dyn}$) are applied. The same natural frequency is applied to both cutting regions.

2.7.4 Output Variables

For the given input variables, the simulation outputs the chip thickness, the cutting force, and the deflection of the tool and workpiece system for a given number of tool revolutions. Specifically, the output variables include the following:

- Chip thickness
- Cutting forces
  - cutting forces in X, Y, radial, and tangential, directions
    (ForceX, ForceY, ForceR, ForceT)
  - net forces (ForceN)
- Deflections
  - tool deflections in X, Y, radial, and tangential directions
    (xTool, yTool, rTool, tTool)
  - workpiece deflections in X, and Y directions (xWorkpiece, yWorkpiece)
- Force measurements
  - force measurements in X and Y directions from the Kistler dynamometer
    (FxKistler, FyKistler)
  - force measurements in radial and tangential directions from the Smart tool
    (FrSmart, FtSmart)
Because of the dynamic effect of the sensors, i.e. the Smart Tool and Kistler dynamometer, the force measurements are not typically the same as the cutting forces. The strain gauges from the Smart Tool and the piezoelectric force transducer from the Kistler dynamometer measure the displacement of the tool and workpiece systems which provides an accurate force measurement when the frequency content of the cutting forces is within the bandwidth of the sensors. More detail about the difference between the force measurement and the cutting force will be discussed in Chapter 4.

2.7.5 Simulation Description for the Smart Tool (One-Tooth Cutter)

In order to compare simulation results to experimental data from the Smart Tool, the simulated cutting forces and tool deflections are decomposed into radial and tangential directions. The simulation generally calculates the cutting force and the structure deflection in the X and Y directions which are defined as "global coordinates" based on the CNC machine. When a multiple teeth cutter is used, cutting forces cannot be decomposed in radial and tangential directions because each tooth has its own "local coordinate". However, since the Smart Tool uses a one tooth cutter, the cutting force and the structure deflection can be transferred into radial and tangential directions. These results are used to validate the force measurement from the Smart Tool.

The 10th version of the Smart Tool measures radial and tangential cutting forces on a single tooth cutter in "local coordinates". When a helical tooth cutter is used, the "local coordinate" moves on the helical tooth. Simulated cutting forces are calculated in the radial and tangential directions for the one tooth cutter. The segment forces are first transformed to the X and Y directions, and then summed to find the resultant X and Y
forces. Then, cutting forces in the X and Y directions are transformed to radial and tangential directions. The transform matrix is defined by:

\[
\begin{bmatrix}
F_x \\
F_r
\end{bmatrix} =
\begin{bmatrix}
-\cos(\phi_{\text{half}}) & \sin(\phi_{\text{half}}) \\
-\sin(\phi_{\text{half}}) & -\cos(\phi_{\text{half}})
\end{bmatrix} \cdot
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix}
\]  

(2.24)

\[
\phi_{\text{half}} = \frac{\phi + (\phi - \theta_{\text{lag}})}{2}
\]  

(2.25)

, where \(\phi_{\text{half}}\) is the average angle between bottom and top of the tooth based on a given axial depth. Tool deflections in the radial and tangential directions are calculated with the same coordinates using the modified natural frequency and damping ratio in the radial and tangential directions obtained from the experimental data without any coordinate transfer. In order to calculate total X and Y deflections by Equation 2.12 and 2.13, tool deflection in the radial and tangential directions should be transformed.

\[
\begin{bmatrix}
X_{\text{tool}} \\
Y_{\text{tool}}
\end{bmatrix} =
\begin{bmatrix}
-\cos(\phi_{\text{half}}) & -\sin(\phi_{\text{half}}) \\
\sin(\phi_{\text{half}}) & -\cos(\phi_{\text{half}})
\end{bmatrix} \cdot
\begin{bmatrix}
t_{\text{tool}} \\
r_{\text{tool}}
\end{bmatrix}
\]  

(2.26)

### 2.8 Summary

In Chapter 2, the basic background information for chatter time-domain simulation with a compliant tool and workpiece is explained. The *regenerative* chip thickness calculation is explained along with the force model, tool and workpiece compliance modeling, cutting process damping, and tool geometry. The simulation program, consisting of angular and axial summation loops and integration of the dynamic equations of the tool and workpiece systems, is described in detail.
CHAPTER III
SYSTEM PARAMETER IDENTIFICATION

3.1 Introduction

The milling simulation program calculates the regenerative chip thickness from the cutting force and the compliance of the tool and workpiece systems. This requires knowledge of the parameters of both the tool and workpiece systems including stiffness (k), damping ratio (ζ), and natural frequency (ω_n). The tool system includes the tool and spindle and the workpiece system includes the Kistler dynamometer, material (Aluminum 6061), mounting plate, and connecting bolts.

Tool and workpiece deflections are solved using a specific set of system parameters that depend on the state of the cut. When the tool is rotating but not cutting, i.e. out-of-cut, dynamic system parameters (ω_n_dyn, ζ_dyn, k_dyn) are used. When the tool rotates and engages the workpiece, in-cut vibrations are characterized with ω_n_dyn, ζ_process, and k_dyn. These parameters are identified experimentally and are used for milling simulation. The milling force profile with a one-tooth cutter can typically be separated into these two significant vibration modes based on the boundary conditions.

Chapter 3 discusses the identification of these system parameters under different operating conditions. The multi-component force dynamometer (9257B, KISTLER), and the Smart Tool (10th ver., Design and Manufacturing Laboratory, University of New
Hampshire) are used as measurement devices. As seen in Figure 3.1, the Smart Tool uses strain gauges to measure cutting forces in the radial and tangential directions acting on a helical cutting tooth. As a result of gauge placement, the Smart Tool is limited to a single tooth at a certain axial depth of cut. The Kistler force dynamometer measures forces in three directions, X, Y and Z using piezoelectric sensors. Since cutting forces in the Z direction are not a significant factor during milling, only X and Y forces are considered. The Kistler force dynamometer is used to identify the parameters of the workpiece system. The Smart Tool and a piezo accelerometer (353B03, PCB) are used to identify the tool system parameters. The resulting system parameters are used for the milling simulation results described in Chapter 4.

![Figure 3.1: Force Sensors; Smart Tool and Kistler Dynamometer [4]](image)

3.2 Relation between Experimental Oscillation and \( \omega_n \) and \( \zeta \) of Systems

The identification of the tool and workpiece system parameters during cutting is complicated because the milling force comprises the regenerative chip thickness and sensor dynamics with compliance. In this work, we simplify the milling force model with an assumption that the characteristics of the system dynamics are embedded in the experimental oscillations. With this assumption, system parameters are identified from the force vibration.
It is straightforward to measure the vibrations from the Smart Tool and the Kistler force dynamometers during a stable cutting process. These oscillations are related to tool and workpiece systems' parameters such as $\omega_n$ and $\zeta$. The overall milling process transfer function is shown in Figure 3.2. $K_S$ is the linear milling force model, $G$ is the transfer function of the tool (or potentially the workpiece) system, $k_c$ is the tool system structural compliance, $k_m$ is the relationship between structure deflection and force measurements, and $T$ is the delay term based on tooth pass or runout frequency.

$$G \text{ (compliance)} = \frac{k_c}{\frac{S^2}{\omega_n^2} + \frac{2 \cdot \zeta}{\omega_n} \cdot S + 1}$$

From the simplest block diagram in Figure 3.2, the characteristic equation is defined:

$$1 + (1 - e^{-TS}) \cdot K_S \cdot G = 0 \quad (3.1)$$

Equation 3.1 is expanded as:

$$S^2 + 2 \cdot \zeta \cdot \omega_n \cdot S + \omega_n^2 \cdot (1 - e^{-TS}) \cdot K_S \cdot k_c = 0 \quad (3.2)$$

Because of the time delay term, Equation 3.2 cannot be simplified as a typical second-order transfer function ($S^2 + 2 \cdot \zeta \cdot \omega_n \cdot S + \omega_n^2 = 0$, where $\omega_n$ and $\zeta$ are natural frequency and
damping ratio of the force measurement oscillations). Pade's 1st-order approximation of
the delay term [12] \((e^{-T \cdot S} \approx \frac{T_{S+1}}{T_{S+1}})\), where \(T\) is time delay) can be used in Equation 3.2 to
find a rational characteristic equation:

\[ S^2 + 2 \cdot \zeta \cdot \omega_n \cdot S + \omega_n^2 + \omega_n^2 \cdot K_S \cdot k_c \cdot \left(1 - \frac{-T \cdot S + 2}{T \cdot S + 2}\right) = 0 \]  (3.3)

By MATLAB, Equation 3.3 is solved with following solutions:

\[ S_1 = \frac{3\sqrt{A} + B - \frac{C}{3\sqrt{A} + B} - D}{3\sqrt{A} + B} \]  (3.4)

\[ S_2 = -\frac{3\sqrt{A} + B}{2} + \frac{C}{2 \cdot 3\sqrt{A} + B} - D + \frac{\sqrt{3} \cdot \left(\frac{3\sqrt{A} + B}{3\sqrt{A} + B} \frac{C}{\sqrt{A} + B}\right)}{2} \cdot i \]  (3.5)

\[ S_3 = -\frac{3\sqrt{A} + B}{2} + \frac{C}{2 \cdot 3\sqrt{A} + B} - D - \frac{\sqrt{3} \cdot \left(\frac{3\sqrt{A} + B}{3\sqrt{A} + B} \frac{C}{\sqrt{A} + B}\right)}{2} \cdot i \]  (3.6)

where \(A, B, C,\) and \(D\) are expressed by \(\zeta, \omega_n, K_S, k_c,\) and \(\tau\) (see details in Appendix B).

Even though the solutions of Equation 3.3 are defined, the solutions \(S_2\) and \(S_3\) are not
easy to represent as a function of \(\zeta\) and \(\omega_n\) \((-\zeta \cdot \omega_n \pm \omega_n \cdot \sqrt{1 - \zeta^2} \cdot i)\). Instead of using
these solutions, we simplify Equation 3.3 to a standard second-order equation with an
assumption that \(\lim_{T \to 0} \left(1 - \frac{-T \cdot S + 2}{T \cdot S + 2}\right) = 0\), when fast spindle speed cutting is considered.

Therefore, as a good approximation, parameters from experimentally measured
oscillations can be used to determine the tool system parameters (or potentially the
workpiece system parameters).
3.3 Parameter Identification Methods

Both the tool and workpiece systems are modeled as second-order systems with a damping ratio ($\zeta$), natural frequency ($\omega_n$) and stiffness ($k$). In order to identify system characteristics including $\omega_n$ and $\zeta$, the decaying oscillation of an impulse or step response is typically analyzed by two different methods: 1) logarithmic decrement from second-order free vibration and 2) Linear Predictive Coding.

3.3.1 Logarithmic Decrement

One classic method that identifies the parameters of a second-order system is the logarithmic decrement method [11]. This method can be applied to a system that is in free vibration resulting from an input or initial condition. The damped natural frequency ($\omega_d$) is determined from the period of oscillation and a damping ratio can be determined using the logarithmic decrement method (Equation 3.9). The decaying oscillation from a tap test is considered as a free vibration motion with no external force. The free vibration motion is described by:

$$x(t) = x_0 \cdot e^{-\zeta \omega_n t} \cdot \cos \omega_d t$$ (3.7)

where $x_0$ is the initial displacement. Since the first cycle of the transient response is affected by the impulse, 'tap', the transient response after the second cycle is analyzed. From that, the $\omega_d$ is calculated by:

$$\omega_d[Hz] = \frac{N_C}{T}$$ (3.8)

where $N_C$ is the number of oscillation cycles and $T$ is the period of these cycles. The parameter $\zeta$ is calculated by the logarithmic decrement:
\[
\zeta = \frac{1}{n-1} \cdot \left( \ln \frac{x_1}{x_n} \right) \sqrt{4\pi^2 + \left[ \frac{1}{n-1} \left( \ln \frac{x_1}{x_n} \right) \right]^2}
\]

, where \( n \) is \( n \)th cycle, \( x_1 \) is magnitude of first cycle and \( x_n \) is magnitude of \( n \)th cycle.

From the \( \omega_d \) and \( \zeta \), \( \omega_n \) is calculated:

\[
\omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}}
\]

### 3.3.2 Linear Predictive Coding (LPC)

"Linear Predictive Coding" (LPC) can be used to predict the pole-zero locations of a discrete-time signal assuming a white noise input [6]. It can also be used to find the natural frequency and damping ratio of a system by identifying the pole-zero position associated with free vibration, i.e. decaying oscillations.

The linear predictor model, autoregressive process (AR), is represented by:

\[
\hat{x}(m) = \sum_{k=1}^{P} a_k x(m - k)
\]

, where \( a_k \) are the predictor coefficients, \( m \) is the discrete time index, \( P \) is the number of past samples that defines the order of the system, \( x(m) \) is the signal and \( \hat{x}(m) \) is the prediction of \( x(m) \). The prediction error is defined as the difference between the sampled signal and the prediction:

\[
e(m) = x(m) - \hat{x}(m) = x(m) - \sum_{k=1}^{P} a_k x(m - k)
\]

The best coefficients, \( a_k \), are obtained by minimizing the mean square error (MSE):

\[
MSE = [e^T(m) e(m)]
\]
This leads to:

$$a = R_{xx}^{-1} r_{xx} \quad (3.14)$$

where $R_{xx}$ is the autocorrelation of the input vector, $r_{xx}$ is the autocorrelation of vector $x$ and $a^T$ is the predictor coefficient vector. See Harmon [4] for more details on the LPC procedures. The order of the linear predictive model ($P$) is determined by the number of the dominant peaks ($P/2$) from the power spectrum of the output signal. Recall that one dominant peak corresponds to a complex pole pair.

Both the logarithmic decrement method and the LPC method are common methods for analyzing second-order free vibration to identify $\omega_n$ and $\zeta$ of a system. However, if the transient signal is composed of multiple frequencies, the computed $\omega_n$ and $\zeta$ by the logarithmic decrement method are not precise values. On the other hand, the LPC can estimate multiple natural frequencies and damping ratios from the oscillations of a higher order system. Most of the transient signals from the tap test and the resulting system vibrations are higher than second-order, therefore, the LPC gives a more precise $\omega_n$ and $\zeta$ of the system fundamental. The order of the linear predictor model is determined from the shape of the transient signal power spectrum. Each distinct peak in the power spectrum density (PSD) requires 2 poles for accurate modeling. Even though a higher linear predictor model can be used, the system characteristics of only the first mode is considered because the tool and workpiece systems are modeled as second-order in the simulation program.
3.4 System Parameter Identification under Static Conditions

The system parameters under static conditions are determined from force measurements taken when the tool is not rotating. The system parameter identification under static conditions is used to estimate of the system stiffness under dynamic condition, i.e. when the tool rotates (recall from Section 2.7.1).

The following tool and workpiece systems parameters are needed to model static conditions based on the simulation inputs:

- static stiffness of the workpiece system \( k_{x\_workpiece\_sta}, k_{y\_workpiece\_sta} \)
- static stiffness of the tool system \( k_{x\_tool\_sta}, k_{y\_tool\_sta} \)
- natural frequency of the tool system under static condition \( \omega_{n\_x\_tool\_sta}, \omega_{n\_y\_tool\_sta} \)

3.4.1 Workpiece System Identification under Static Conditions

The X and Y stiffness of the workpiece system \( k_{workpiece\_sta} \) is identified from the static natural frequency of the workpiece system and the given technical parameters of the Kistler force dynamometer (stiffness: \( k_{kistler} \), natural frequency: \( \omega_{n\_kistler} \)). Since the force dynamometer is a part of the workpiece system, technical parameters of the force dynamometer cannot represent the static workpiece system parameters.

To obtain the natural frequency and the damping ratio of the workpiece system, tap tests are performed in both the X and Y directions (see Figure 3.3 and 3.4). Since the decaying oscillations indicate a higher order system, the data is analyzed by LPC instead of the logarithmic decrement. It is important to select an LPC order that is large enough to model every significant peak in the PSD. The PSD is estimated by two different methods: 1) standard periodogram and 2) Welch's method. The PSD by standard
periodogram is wavy compared to the smooth curve from Welch's method, but results by the two methods are similar. See "Advanced Digital Signal Processing and Noise Reduction" [6] for more details on the difference of the two PSD estimated methods.

In order to choose the minimum order of the LPC, the PSD from Welch's method is used as a basic criterion. If the selected LPC order is too small, distinct peaks in the data can be averaged and accurate first mode information can be lost. For this case, 8th and 10th order LPC models are chosen for the decaying oscillation in the X and Y directions. As see in Table 3.1 and 3.2, the 8th and 10th order LPC model detects four and five modes from the tap test data. The "avg" indicates average and "stdv" represent standard deviation among the 15 samples.

![Decaying Oscillation](image1)

![Welch Power Spectral Density Estimate](image2)

**Figure 3.3: Impulse Response of the Workpiece System, X direction**

**Table 3.1: LPC Derived Vibration Modes of the Workpiece system, X direction**

<table>
<thead>
<tr>
<th>( \omega_{n, \text{workpiece}, \text{std}} [\text{Hz}] )</th>
<th>1st Mode</th>
<th>2nd Mode</th>
<th>3rd Mode</th>
<th>4th Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>avg</td>
<td>997</td>
<td>2455</td>
<td>4529</td>
<td>6555</td>
</tr>
<tr>
<td>stdv</td>
<td>3.85</td>
<td>8.61</td>
<td>3.54</td>
<td>5.18</td>
</tr>
</tbody>
</table>
Figure 3.4: Impulse Response of Workpiece System, Y direction

Table 3.2: LPC Derived Vibration Modes of Workpiece System in Y direction

<table>
<thead>
<tr>
<th>ω_{n,y\text{-workpiece,sta}} [Hz]</th>
<th>1st Mode</th>
<th>2nd Mode</th>
<th>3rd Mode</th>
<th>4th Mode</th>
<th>5th Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>avg</td>
<td>1172</td>
<td>2019</td>
<td>3676</td>
<td>5228</td>
<td>6790</td>
</tr>
<tr>
<td>stdv</td>
<td>6.65</td>
<td>24.35</td>
<td>58.77</td>
<td>32.12</td>
<td>19.20</td>
</tr>
</tbody>
</table>

From these results, the ω_n and ζ in the X and Y directions of the workpiece system under static conditions is defined, as shown in Table 3.3.

Table 3.3: Parameters of the Workpiece System under Static Conditions by LPC

<table>
<thead>
<tr>
<th>X</th>
<th>ω_{n,x\text{-workpiece,sta}} [Hz]</th>
<th>ζ_{x\text{-workpiece,sta}}</th>
<th>Y</th>
<th>ω_{n,y\text{-workpiece,sta}} [Hz]</th>
<th>ζ_{y\text{-workpiece,sta}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>avg</td>
<td>997</td>
<td>0.0643</td>
<td>avg</td>
<td>1172</td>
<td>0.0560</td>
</tr>
<tr>
<td>stdv</td>
<td>3.85</td>
<td>0.0028</td>
<td>stdv</td>
<td>6.65</td>
<td>0.0071</td>
</tr>
</tbody>
</table>

Using the obtained static natural frequency of the workpiece, the static stiffness of the workpiece system is computed from the natural frequency expression:

\[ k_{x\text{-workpiece}} = m_{\text{eff\text{-workpiece}}} \cdot \omega_{n,x\text{-workpiece,sta}}^2 \]  \hspace{1cm} (3.15)

\[ k_{y\text{-workpiece}} = m_{\text{eff\text{-workpiece}}} \cdot \omega_{n,y\text{-workpiece,sta}}^2 \]  \hspace{1cm} (3.16)

, where the effective mass of the workpiece system \((m_{\text{eff\text{-workpiece}}})\) is unknown. For simplification, the \(m_{\text{eff\text{-workpiece}}}\) is assumed as the sum of the effective Kistler dynamometer mass \((m_{\text{eff\text{-kistler}}})\) and the total mass \((4.457 \text{ [kg]})\) of the mounting plate, connecting bolts, and material. The \(m_{\text{eff\text{-kistler}}}\) is obtained as 4.788 [kg] in both the X and
Y directions by the definition of natural frequency and technical parameters of the Kistler dynamometer, as given in Table 3.4 [13]. Then, an effective mass of the workpiece system is obtained (9.245 [kg]).

Table 3.4: Kistler Dynamometer Technical Data [13]

<table>
<thead>
<tr>
<th></th>
<th>Stiffness</th>
<th>(\omega_n) (mounted on flanges)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X &amp; Y</td>
<td>1e9 [N/m]</td>
<td>2300 [Hz]</td>
</tr>
</tbody>
</table>

From Equation 3.15 and 3.16, using the static natural frequency and the effective mass of the workpiece system, the stiffness of the workpiece system is calculated as \(k_{x\_workpiece\_sta} = 3.628e8\ [N/m] \pm 1.55\%\) and \(k_{y\_workpiece\_sta} = 5.017e8\ [N/m] \pm 2.28\%\) (with 15 samples taken).

As mentioned in Section 2.7.1, this stiffness corresponds to a certain mass and natural frequency of workpiece system which may changes during milling. In order to obtain an accurate values of \(k_{workpiece\_sta}\), the mass of the workpiece system and \(\omega_{n\_workpiece\_sta}\) should be determined before cutting.

### 3.4.2 Tool System Identification under Static Conditions

The stiffness of the tool system in the X and Y directions, \(k_{x\_tool\_sta}\) and \(k_{y\_tool\_sta}\), are obtained by linear calibration using an applied force on the tool system and calculating tool deflection. The workpiece system, mounted to the table, is used to push against the tool system. The table movement is known and the applied force is measured by the Kistler dynamometer, as seen in Figure 3.5. Since the stiffness of the workpiece system is known, the tool deflection is calculated from the known table movement and the calculated workpiece deflection, as described below.
From Figure 3.5, the following equations are derived.

\[ \Delta x_{\text{workpiece}} = \frac{-\Delta F_x}{k_{x,\text{workpiece, sta}}} \quad (3.17) \]

\[ \Delta x_{\text{tool}} = \Delta x_{\text{workpiece}} + \Delta x_{\text{table}} \quad (3.18) \]

\[ k_{x,\text{tool, sta}} = \frac{\Delta F_x}{\Delta x_{\text{tool}}} \quad (3.19) \]

The delta-force (-\( \Delta F_x \) and -\( \Delta F_y \)) are measured using the Kistler dynamometer, and the delta-displacement of table (\( \Delta x_{\text{table}} = \Delta y_{\text{table}} = 1.27e-5 \) [m]) is controlled using the CNC machine. Using Equations 3.17, 3.18 and 3.19, the stiffness of the tool system under static conditions is identified with average (avg) and standard error of the mean (SEM = \( \frac{\text{stdv}}{\sqrt{\text{number of sample}}} \)), shown in Table 3.5. Raw data of the linear calibration is provided in the Appendix C.

Table 3.5: Stiffness of Tool System under Static Conditions, 15 Samples

<table>
<thead>
<tr>
<th></th>
<th>( k_{x,\text{tool, sta}} ) [N/m]</th>
<th>( k_{y,\text{tool, sta}} ) [N/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>avg</td>
<td>3.064e6</td>
<td>2.082e6</td>
</tr>
<tr>
<td>SEM</td>
<td>0.064e6</td>
<td>0.130e6</td>
</tr>
</tbody>
</table>
Since the tool holder has symmetric geometry, and the effect of the spindle structure is considered negligible, the tool system model can be simplified such that $k_{x,\text{tool\_sta}}$ is the same as $k_{y,\text{tool\_sta}}$ based on symmetric geometry. Based on the Table 3.5, Figure 3.6 shows the error range of $k_{x,\text{tool\_sta}}$ and $k_{y,\text{tool\_sta}}$. The error range is determined by ±1.96% of SEM which indicates a 95% confidence interval. Since the average of $k_{x,\text{tool\_sta}}$ and $k_{y,\text{tool\_sta}}$ is inside the error range of X and Y, the tool system can be simplified with the assumption of symmetry in X and Y.

![Figure 3.6: Error Range of $k_{\text{tool\_sta}}$ Based on SEM](image)

The static natural frequency of the tool system is obtained by analysis of decaying oscillations from a tap test, similar to the workpiece system. Initially, the radial and tangential gauges of the Smart Tool are aligned with the Y and X directions of the CNC machine, and the impact hammer is used to tap in these directions ((1) and (7) in Figure 3.7).
Figure 3.7: Tap Tests with Different Rotation Angles

Based on the PSD of \( F_y \) in Figure 3.8, a 10th order LPC is used for identifying the first vibration mode. For the PSD of \( F_x \) in Figure 3.9, the second peak is not distinct as in \( F_y \). Still, a 10th order LPC is used to find five modes. Table 3.6 and 3.7 show the results of the avg and stdv of 15 tap tests.

Figure 3.8: Impulse Response of the Tool system, \( F_y \) to Radial gauge, Case (1)

Table 3.6: Vibration Modes of the Tool system, Y direction, Case (1)

<table>
<thead>
<tr>
<th>( \omega_{n_y, tool_sta} ) [Hz]</th>
<th>1st Mode</th>
<th>2nd Mode</th>
<th>3rd Mode</th>
<th>4th Mode</th>
<th>5th Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>avg</td>
<td>635</td>
<td>1382</td>
<td>2534</td>
<td>3618</td>
<td>4656</td>
</tr>
<tr>
<td>stdv</td>
<td>1.61</td>
<td>26.82</td>
<td>49.52</td>
<td>67.72</td>
<td>143.81</td>
</tr>
</tbody>
</table>
Table 3.7: Vibration Modes of Tool System, X direction, Case (7)

<table>
<thead>
<tr>
<th>$\omega_{n, x_{tool, sta}}$ [Hz]</th>
<th>1st Mode</th>
<th>2nd Mode</th>
<th>3rd Mode</th>
<th>4th Mode</th>
<th>5th Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>avg</td>
<td>641</td>
<td>1368</td>
<td>2437</td>
<td>3540</td>
<td>4639</td>
</tr>
<tr>
<td>stdv</td>
<td>3.41</td>
<td>38.66</td>
<td>66.22</td>
<td>72.13</td>
<td>68.99</td>
</tr>
</tbody>
</table>

Table 3.8 presents $\omega_n$ and $\zeta$ of the first mode vibration in the X and Y directions by using a 10th order LPC with average and standard error of the mean from 15 tap tests.

Table 3.8: First Vibration Mode of Tool System in X (Case 7) and Y (Case 1)

<table>
<thead>
<tr>
<th>X</th>
<th>$\omega_{n, x_{tool, sta}}$ [Hz]</th>
<th>$\zeta_{x_{tool, sta}}$</th>
<th>Y</th>
<th>$\omega_{n, y_{tool, sta}}$ [Hz]</th>
<th>$\zeta_{y_{tool, sta}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>avg</td>
<td>641</td>
<td>0.036</td>
<td>avg</td>
<td>635</td>
<td>0.029</td>
</tr>
<tr>
<td>SEM</td>
<td>0.88</td>
<td>0.0006</td>
<td>SEM</td>
<td>0.42</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

The tool system model can be simplified by assuming symmetric geometry. Under this assumption, the characteristic ($\omega_n$, $\zeta$, and $k$) of the tool system in the X and Y directions are equal to the mean value of $\omega_n$ and $\zeta$ of the tool system in the radial and tangential directions. Mathematical proof of this is found in Appendix D. This simplifies the use of the Smart Tool, since it measures cutting forces in the radial and tangential directions. These force measurements have to be converted to the X and Y directions in order to identify tool system parameters. This conversion requires the correct tooth angular position. However, this position is not trivial to determine because of the sensor
phase delay between the cutting force and the measured force. With the assumption that the tool system is modeled as "symmetric", tool system parameters in the X and Y directions can be directly estimated from the radial and tangential directions without conversion.

As shown in Figure 3.10, $\omega_{n,\text{tool,sta}}$ and $\zeta_{\text{tool,sta}}$ of tool system in the X and Y directions are inside the error range which is determined by $\text{avg} \pm 1.96 \times \text{SEM}$ based on Table 3.8. Therefore, a symmetric model is possible.

![Figure 3.10: Error Range of Tool System's Natural Frequency and Damping ratio](image)

**Figure 3.10: Error Range of Tool System's Natural Frequency and Damping ratio**

In order to support the symmetric tool system model, a tap test is performed with the tool at different rotation angles. The decaying oscillations from the radial and
tangential gauges of the Smart Tool in cases (1), (2), (3), (4), and (5) characterize the tool system parameters \((\omega_n \text{ and } \zeta)\) in the Y direction. Tool system parameters in the X direction are defined from the decaying vibrations in cases (6) and (7) from the Smart Tool in the radial and tangential gauges.

Table 3.9: Parameters \((\omega_n, \zeta)\) of Tool System with Different Rotation Angle

<table>
<thead>
<tr>
<th>Case</th>
<th>Gauge</th>
<th>(\omega_{n, \text{tool_sta}} [\text{Hz}]) avg SEM</th>
<th>(\zeta_{\text{tool_sta}}) avg SEM</th>
<th>2nd mode: (\omega_{n, \text{tool_sta}} [\text{Hz}]) Avg SEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>R</td>
<td>635 0.72 0.029 0.00067</td>
<td></td>
<td>1383 6.92</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td>R</td>
<td>632 0.77 0.035 0.00081</td>
<td></td>
<td>1376 13.23</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>644 2.84 0.041 0.00139</td>
<td></td>
<td>1359 17.94</td>
</tr>
<tr>
<td>(3)</td>
<td>R</td>
<td>618 1.27 0.042 0.00085</td>
<td></td>
<td>1389 6.12</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>643 1.72 0.043 0.00125</td>
<td></td>
<td>1353 35.52</td>
</tr>
<tr>
<td>(4)</td>
<td>R</td>
<td>632 6.52 0.022 0.00125</td>
<td></td>
<td>1399 15.89</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>645 2.91 0.041 0.0004</td>
<td></td>
<td>1380 29.50</td>
</tr>
<tr>
<td>(5)</td>
<td>R</td>
<td>645 0.63 0.041 0.00063</td>
<td></td>
<td>1354 17.15</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6)</td>
<td>R</td>
<td>644 0.71 0.029 0.00076</td>
<td></td>
<td>1274 15.01</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7)</td>
<td>R</td>
<td>641 1.52 0.037 0.00103</td>
<td></td>
<td>1368 9.98</td>
</tr>
</tbody>
</table>

Table 3.9 shows the results with average and SEM of the 15 tap tests corresponding to different tool rotation angles. Figure 3.11 shows the average with error range, determined by ±1.96 times of SEM, contains nearly all of the \(\omega_n\) and \(\zeta\) values in the X and Y directions. Inside of these range, an average \(\omega_n\) and \(\zeta\) can be defined as the tool system parameters \((\omega_{n, \text{tool_sta}} = 639.6 [\text{Hz}] \text{ and } \zeta_{\text{tool_sta}} = 0.0355)\).

Some of the tap test results do not correspond to the average value within the error range. There are several reasons for this: 1) the Smart Tool ver.10 cannot measure both radial and tangential at the same time, i.e. two separate tests must be performed, 2) the epoxy, which is used for mounting a strain gauge can affect the vibrations, 3) the tool
cannot be tapped at the same position every time, 4) the LPC has calculation errors and the minimum order of the LPC is not enough to separate two close peaks, and 5) the Tool has different vibration characteristics every time it is mounted in the CNC machine.

Figure 3.11: Error Range Based on Table 3.9
To confirm that the Smart Tool accurately measures vibrations, tap tests are performed with a piezo accelerometer with a magnetic mounting. Cases (1), (5), (6) and (7) are tested with 15 trials.

Table 3.10: Parameters ($\omega_n$, $\zeta$) of Tool System using a Piezo Accelerometer

<table>
<thead>
<tr>
<th>force</th>
<th>case</th>
<th>$\omega_n$, tool, sta [Hz]</th>
<th>$\zeta$, tool, sta</th>
<th>2nd mode: $\omega_n$, tool, sta [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_y$</td>
<td>(1)</td>
<td>avg 624</td>
<td>0.0344</td>
<td>1303</td>
</tr>
<tr>
<td></td>
<td></td>
<td>stdv 1.42</td>
<td>0.0033</td>
<td>27.41</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>avg 630</td>
<td>0.0342</td>
<td>1311</td>
</tr>
<tr>
<td></td>
<td></td>
<td>stdv 0.74</td>
<td>0.0028</td>
<td>28.07</td>
</tr>
<tr>
<td>$F_x$</td>
<td>(6)</td>
<td>avg 631</td>
<td>0.0359</td>
<td>1029</td>
</tr>
<tr>
<td></td>
<td></td>
<td>stdv 2.60</td>
<td>0.0039</td>
<td>72.71</td>
</tr>
<tr>
<td></td>
<td>(7)</td>
<td>avg 625</td>
<td>0.0341</td>
<td>1034</td>
</tr>
<tr>
<td></td>
<td></td>
<td>stdv 2.49</td>
<td>0.0044</td>
<td>69.37</td>
</tr>
</tbody>
</table>

Figure 3.12: Error Range Based on Table 3.10
As seen in Table 3.9 and 3.10, the results from the accelerometer are slightly different from the Smart Tool measurements, especially $\omega_n$. With the accelerometer's measurements, the tool system has similar vibration in the X and Y directions. From the results of the accelerometer, it is questionable why some vibration results (see in Table 3.10 and Figure 3.12) are out of the SEM range. The reason for this may be that during the tap test, the tool is taken out of the CNC machine in order to maintain an accurate rotation angle.

A number of tap tests are performed to explore the effect of tool mounting (locked into the spindle) on the tool system natural frequency. The 15 tap tests are performed for 15 trials in the X direction and 6 trials in the Y direction.

![Graphs showing natural frequency and damping ratio results.](image)

**Figure 3.13:** Statistical Plot of 15 trials in Y and 6 trials in X and Y to Explore the Effect of Tool Mounting
As seen in Figure 3.13, the average value of $\omega_n$ and $\zeta$ in some trials is below the error range. That means that the chuck affects the tool’s vibration. This result explains very well why the radial gauge measurement of case (3) in Figure 3.11 is below the error range. In addition, comparing the results in the X and Y directions, the vibration is similar in both X and Y directions, supporting the claim of tool symmetry.

From the tap test results of the tool system, it is concluded that the tool system has symmetric geometry. However, the vibration characteristics slightly change whenever the tool is remounted. Therefore, the on-line identification of the tool system parameters is important for accurate force simulation.

### 3.5 System Parameters Identification under Dynamic Condition

The tool and workpiece system parameters ($k_{dyn}$, $\omega_{n,dyn}$, $\zeta_{dyn}$, $\zeta_{process}$) under dynamic conditions, i.e. when the tool rotates, are used for solving Equations 2.8 - 2.11. From the Kistler force dynamometer and Smart Tool experimental stable cutting data, $\omega_{n,dyn}$, $\zeta_{dyn}$, and $\zeta_{process}$ are defined through use of the LPC. The dynamic condition is separated into two different types, i.e. out-of-cut parameters and in-cut parameters. Out-of-cut is when the tool rotates but does not engage the workpiece and in-cut is when the tool rotates and engages in cutting. According to the simulation inputs (see in Section 2.7.1), the parameters associated with each type include:

- **Out-of-Cut Parameters:**
  - dynamic natural frequency and damping ratio of the tool system
    
    $(\omega_{n,r\_tool\_dyn}, \zeta_{r\_tool\_dyn}, \omega_{n,t\_tool\_dyn}, \zeta_{t\_tool\_dyn})$
- dynamic natural frequency and damping ratio of the workpiece system
  \( (\omega_{n_x, workpiece, \text{dyn}}, \zeta_{x, workpiece, \text{dyn}}, \omega_{n_y, workpiece, \text{dyn}}, \zeta_{y, workpiece, \text{dyn}}) \)

- In-Cut Parameters:
  - process damping ratio \( (\zeta_{x, \text{process}}, \zeta_{y, \text{process}}) \)

Since the milling simulation calculates cutting forces and structure deflections in the X and Y directions, the tool system parameters in both X and Y direction must be obtained from experimentally determined radial and tangential directions. As in the static case, since the tool geometry is symmetric, the RMS value of the tool system parameters in the radial and tangential directions are used for the X and Y direction without force transformation. Based on the cutting process damping model, the natural frequency is the same for both the in-cut and out-of-cut cases (recall from Section 2.5).

The in-cut process damping ratio can be obtained from either the Smart Tool or Kistler data, since process damping occurs between the tool and workpiece systems. The estimated process damping ratio should be similar using either the Smart Tool or Kistler data. However, the LPC cannot be directly applied to the measured force profile for obtaining in-cut parameters because the in-cut profile includes structure vibration and cutting force harmonics of the tooth passing frequency and runout frequency [4]. The LPC can be applied to the measured cutting force by first subtracting (de-trending) the "actual" cutting force from the measured force. The resulting force profile can be used to estimate the process damping ratio. The estimated cutting force is obtained by the Extended Kalman Filtering, which is described in much more detail in Chapter 5.
Recall from Section 2.7.1, the dynamic stiffness of the tool system is approximated from the relationship between the static natural frequency and the dynamic natural frequency because of constant effective tool mass. In addition, the stiffness of the workpiece system remains constant for either static or dynamic conditions within small material remove rate.

Details for obtaining tool and workpiece system parameters are discussed in the next two sections using a sample experimental data set with the following cutting conditions, \( \Omega: 2600 \text{ [rpm]}, \) half immersion, \( a: 0.125 \text{ [in]}, \) \( h_{av}: 0.001 \text{[in]}, \) feedrate \( (F_R) = 4.084 \text{ [in]}, \) material: Al6061, \( N_T: 1, \) cutter: Sandvik Coromill 390 (d: 0.75 [in]), mill insert: R390-11 T3 08E-NL H13A, and \( \theta_{helix}: 17.869 \text{[deg]}. \)

3.5.1 Example Out-of-Cut Parameters

A single tooth cutter is used for the milling experiments, and the out-of-cut free vibration can be observed from the force profile of the measurement data from the Kistler and Smart Tool. Since the cutter is out of the workpiece, it is concluded that the cutting force oscillations are a result of tool and workpiece deflections instead of the actual cutting force. As seen in Figure 3.14, the out-of-cut vibration is separated from original measured force data by eliminating the cutting force during the tool engagement.

LPC is applied to each out-of-cut vibration of the separated signal in order to obtain a dominant pole which yields the natural frequency and damping ratio. Thirty cycles are separated and analyzed offline to provide statistical data. The example MATLAB code of signal separation and LPC application is shown in Appendix E. This
code estimates $\omega_n$ and $\zeta$ from the de-trended measured force without user input and as such could be used for real-time process application.

![Original and Separated Plots]

**Figure 3.14: Out-of-Cut Profile Separation, Kistler in X (top) and Y (bottom)**

The order of the LPC is determined by the PSD of the separated measured force. The PSD is calculated two different ways, 1) whole separated signal (30 cycles using periodogram and Welch) and 2) averaged decaying signal from 30 cycles using periodogram. The reason for two different methods of PSD is to obtain more signal characteristics. Usually, the LPC order is selected from the second method because it only considers the decaying portion of the signals. The PSD from the averaged cycles method is seen in Figure 3.15. The 14th order LPC is used for both X and Y directions to
estimate the first dominant peak as well as second peak near 2000 [Hz]. The same method is applied to the radial and tangential data from the Smart Tool for indentifying $\omega_{n,\text{dyn}}$ and $\zeta_{\text{dyn}}$. Table 3.11 shows the results for the tool and workpiece system under dynamic conditions for the given cutting conditions.

![Power Spectral Density](image)

*Figure 3.15: PSD from each cycle, Kistler in X and Y*

Table 3.11: Workpiece and Tool System Parameters from Out-of-Cut, 30 Cycles

<table>
<thead>
<tr>
<th>Workpiece System</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\zeta_{\text{x,workpiece,dyn}}$</td>
<td>$\omega_{\text{n,x,workpiece,dyn}}$</td>
<td>$\text{2nd} , \omega_{\text{n,x,workpiece,dyn}}$</td>
</tr>
<tr>
<td><strong>X</strong></td>
<td>avg</td>
<td>0.075</td>
<td>1211</td>
</tr>
<tr>
<td></td>
<td>stdv</td>
<td>0.0254</td>
<td>14.45</td>
</tr>
<tr>
<td></td>
<td>$\zeta_{\text{y,workpiece,dyn}}$</td>
<td>$\omega_{\text{n,y,workpiece,dyn}}$</td>
<td>$\text{2nd} , \omega_{\text{n,y,workpiece,dyn}}$</td>
</tr>
<tr>
<td><strong>Y</strong></td>
<td>avg</td>
<td>0.076</td>
<td>1462</td>
</tr>
<tr>
<td></td>
<td>stdv</td>
<td>0.0258</td>
<td>13.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tool System</th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\zeta_{\text{t,tool,dyn}}$</td>
<td>$\omega_{\text{n,t,tool,dyn}}$</td>
<td>$\text{2nd} , \omega_{\text{n,t,tool,dyn}}$</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td>avg</td>
<td>0.093</td>
<td>652</td>
</tr>
<tr>
<td></td>
<td>stdv</td>
<td>0.0029</td>
<td>5.19</td>
</tr>
<tr>
<td></td>
<td>$\zeta_{\text{r,tool,dyn}}$</td>
<td>$\omega_{\text{n,r,tool,dyn}}$</td>
<td>$\text{2nd} , \omega_{\text{n,r,tool,dyn}}$</td>
</tr>
<tr>
<td><strong>R</strong></td>
<td>avg</td>
<td>0.036</td>
<td>626</td>
</tr>
<tr>
<td></td>
<td>stdv</td>
<td>0.0031</td>
<td>2.29</td>
</tr>
</tbody>
</table>
3.5.2 Example In-Cut Parameters: Process Damping Effect

Similar to the out-of-cut case, in-cut profiles are separated from the original data and 30 cycles are each analyzed. Figure 3.16 shows the separated in-cut profiles from the Kistler measurements in the X and Y directions. The separated in-cut profiles include both cutting forces and structural vibration. The cutting forces used to de-trend the measured force are estimated by an Extended Kalman Filter, which will be discussed in Chapter 5.

Figure 3.16: In-Cut Profile Separation, Kistler in X (top) and Y (bottom)

Figure 3.17 shows 3 signals: experimental data, estimated cutting forces, and de-trended signals. The de-trended signals are periodic with magnitudes and phase shifts that vary from profile to profile. Since experimental data is influenced by the structure deflections, experimental data is not time aligned cycle to cycle even though data results
are from a stable cut. This prevents any averaging of the profiles. So, the LPC is applied to the de-trended signals of each cycle. From the PSD comparison between out-of-cut and in-cut both X and Y directions (see bottom plot in Figure 3.18 and 3.19), two more peaks around 700 and 5200 [Hz] are observed from the PSD of the in-cut profile. Therefore, the LPC order of the in-cut profile (18th) is chosen to be four more than that of the out-of-cut profile (14th). Since the de-trended signals vary from cycle to cycle, the process damping ratio from in-cut profiles have large standard deviations. In addition, the unknown peak around 22 [kHz] are continuously observed in all experimental data: X and Y of the Kistler, radial and tangential of the Smart Tool. The explanation of these peaks is left as future work.

Figure 3.17: De-Trended In-Cut Profile, Kistler in X (top) and Y (bottom)
Table 3.12 shows estimated process damping ratios obtained from the sample experimental data set. Figure 3.18, 3.19, 3.20, and 3.21 show the out-of-cut profile, in-cut profile, and their PSD from the entire signal and from each cycle of the Kistler and Smart Tool data.

Table 3.12: Tool and Workpiece System Parameters under Dynamic Condition

<table>
<thead>
<tr>
<th>Workpiece System</th>
<th>Out-of-Cut</th>
<th>In-Cut</th>
<th>Tool System</th>
<th>Out-of-Cut</th>
<th>In-Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>Y</td>
<td>R</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\zeta_{x,\text{workpiece_dyn}})</td>
<td>(\omega_{n_x,\text{workpiece_dyn}})</td>
<td>(2nd \omega_{n_x,\text{workpiece_dyn}})</td>
<td>(\zeta_{r,\text{tool_dyn}})</td>
<td>(\omega_{n_r,\text{tool_dyn}})</td>
</tr>
<tr>
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<td>avg</td>
<td>stdv</td>
<td>avg</td>
<td>stdv</td>
<td>avg</td>
</tr>
<tr>
<td></td>
<td>0.075</td>
<td>0.0254</td>
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<td>0.0765</td>
<td>0.0994</td>
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<tr>
<td></td>
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<td>1247</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>avg</td>
<td>stdv</td>
<td>avg</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.076</td>
<td>0.0258</td>
<td>0.188</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>1462</td>
<td>13.60</td>
<td>1545</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>avg</td>
<td>stdv</td>
<td>avg</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.163</td>
<td>0.0765</td>
<td>0.188</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1247</td>
<td>245.15</td>
<td>1545</td>
</tr>
<tr>
<td></td>
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<td>avg</td>
<td>stdv</td>
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<td>0.0254</td>
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<tr>
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<td></td>
<td></td>
<td>1247</td>
<td>245.15</td>
<td>641</td>
</tr>
</tbody>
</table>

53
Figure 3.18: Out-of-Cut and In-Cut Parameters identification, Workpiece in X
Figure 3.19: Out-of-Cut and In-Cut Parameters identification, Workpiece in Y
Figure 3.20: Out-of-Cut and In-Cut Parameters identification, Tool in Radial
Figure 3.21: Out-of-Cut and In-Cut Parameters identification, Tool in Tangential
Table 3.12 contains the damping ratio and natural frequency from the out-of-cut and in-cut measurements. Parameters in Table 3.12 can be used for milling simulation under the same experimental cutting conditions that the parameters were estimated, i.e. $\Omega$: 2600 [rpm], half immersion, $a=0.125$ [in], and $h_{avg} = 0.001$[in]. In addition, the mean value of the process damping ratio from the Kistler in X and Y directions (0.176) are similar to the mean value of the process damping ratio from the Smart Tool in the tangential and radial directions (0.172). From this, it confirms that the process damping ratio can be obtained using either the Kistler or Smart Tool data for this particular cut.

### 3.6 Asymmetric Tool System

This work assumes that the tool system is close to symmetric and the tool system parameters are similar in each direction, X, Y, radial, and tangential. As a result, the system parameters in the tangential and radial directions are used in both the X and Y directions. However, if a tool system is asymmetric, the measured forces from the Smart Tool should be converted to the X and Y directions. This conversion is not an easy process, since very accurate knowledge of the tooth rotational position is required. Because of the phase delay between structural deflections and cutting forces, the actual tooth angle is difficult to determine. Conversion of the Smart Tool measurement to the X and Y directions is left as future work.

### 3.7 Summary

In order to calculate the regenerative chip thickness by simulation, tool and workpiece deflections must be considered. To do this, the systems parameters ($\omega_{n_{dyn}}, \zeta_{dyn}, \zeta_{process}, k_{dyn}$) under dynamic condition are defined for the tool and workpiece systems.
Using an LPC with proper order, \( \omega_{d,dyn} \), \( \zeta_{dyn} \), and \( \zeta_{process} \) of each system under dynamic condition, i.e. the tool rotating, are defined from out-of-cut profiles and de-trended in-cut profiles. The unknown stiffness \( k_{dyn} \) of the systems can be estimated by using the system parameters \( (\omega_{n,sta}, k_{sta}) \) under static condition. Since the system parameters do not remain constant, e.g. when a tool holder is remounted in the spindle, identification parameters should be performed every time before cutting in order to have a correct model for simulation.
CHAPTER IV

COMPARISON OF SIMULATION RESULTS TO EXPERIMENTAL DATA

4.1 Introduction

In this chapter, a time-domain milling simulation is validated with experimental data from the Kistler and Smart Tool sensors. Specifically, the simulation is compared to experimental results using force profile, maximum peak force and force frequency content. The maximum peak force is used for process planning to maintain part quality and prevent tool damage. The frequency content is used to detect the onset of chatter. Since statistical values of the system parameters are used for simulation, the sensitivity of the simulation results to variations in system parameters is investigated. In addition, the capability of the simulation as a tool for predicting chatter will be discussed.

4.2 Force Sensor Models

The Kistler and the Smart Tool both measure force through the displacements of the sensor structure. Since the displacement is affected by system dynamics, the relationship between the actual cutting forces and the sensor data should be defined. The Kistler dynamometer uses a piezoelectric force transducer and the Smart Tool uses a strain gauge for measuring cutting forces. The force measurement of these two sensors is based indirectly on displacements of the tool and the workpiece. Any displacement changes resulting from the structural dynamics leads to errors in the sensor force measurement. Sensor models can help us understand these dynamic effects. Both the
Smart Tool and the Kistler Dynamometer can be modeled as a simple second-order dynamic models with mass (m), spring (k), and damper (b). The input to the sensor models is the cutting force between the tool and the workpiece and the output from the sensor models is the force measurement.

The Kistler Dynamometer uses a piezoelectric force transducer that can be modeled as a coupled mechanical and electrical system (Figure 4.1). From the mechanical model of the force transducer, the relationship between applied forces (F_C) and cover plate displacement (u) is derived as shown in Equation 4.1, where k_{kistler} is the stiffness, ω_n is natural frequency (ω_n = \sqrt{\frac{k}{m}}), and ζ is damping ratio (ζ = \frac{b}{2\sqrt{m\cdot k}}):

\[ F_C = k \cdot \left( \frac{1}{\omega_n^2} \cdot \ddot{u} + \frac{2 \cdot \zeta \cdot \omega_n}{\omega_n} \cdot \dot{u} + u \right) \]  \tag{4.1}

\[ C \cdot \frac{dE}{dt} + \frac{E}{R} = i = \frac{dq}{dt} = k_q \cdot \dot{u} \]  \tag{4.2}

The electric model of the piezoelectric force transducer can be analyzed resulting in Equation 4.2, where C is the total capacitance including crystal, cable, and scope, R is the total resistance, E is the voltage change, i is electric current, q is electric charge, and k_q is piezoelectric coefficient between the displacements of crystal and the resultant electric field. Equations 4.1 and 4.2 can be combined with the instrumentation sensitivity.
(k_{sensitivity}) to provide a model relating actual cutting force (F_C) to the measured force measurement (F_m):

\[
\frac{F_m}{F_C} = \frac{k_q \cdot k_{sensitivity} \cdot R \cdot \dot{u}}{\left(\frac{1}{\omega_n^2} \cdot \dot{u}^2 + \frac{2 \cdot \zeta \cdot \dot{u}}{\omega_n} + u\right) \cdot (C \cdot R \cdot \dot{u} + u) \cdot k}
\]  

(4.3)

\[
F_C(S) = \frac{1}{S^2 + \frac{2 \cdot \zeta \cdot S}{\omega_n} + 1}
\]

\[
u(S) = \frac{k_q \cdot R \cdot S}{C \cdot R \cdot S + 1}
\]

\[
E(S) = \text{constant}
\]

\[
k_{sensitivity} \quad F_m(S)
\]

**Figure 4.2: Piezoelectric Force Transducer Model with Laplace Transform**

Based on Equation 4.3, shown in block diagram form in Figure 4.2, the force measurement (F_m) is proportional to the crystal velocity (\(\dot{u}\)). However, the electric model of the piezoelectric force transducer can be approximated as a constant since the time constant of the crystal, when connected to the amplifier, is in the order of several minutes. As a result, the bandwidth is only limited by the mechanical natural frequency. Therefore, the relation between measurement (F_m) and displacement (u) can be approximated by:

\[
F_m = k_{\text{Kistler}} \cdot u
\]  

(4.4)

The Smart Tool sensor element is four strain gauges arranged in a Wheatstone bridge. The mechanical part of the Smart Tool is modeled the same as the piezoelectric force transducer (Equation 4.1). Figure 4.3 shows the electrical and mechanical model for the tool.
The relationship between displacement \( u \) and voltage change \( E \) is given by:

\[
E = \frac{E_{ex} \cdot \Delta R}{2 \cdot R_0} \cdot u
\]  

(4.5)

, where \( E_{ex} \) is the circuit input voltage, \( \Delta R \) is variable resistance per unit length, and \( R_0 \) is a constant resistance. Since \( E_{ex}, \Delta R, \) and \( R_0 \) are constant values, the force measurement \( (F_m) \) is proportional to displacement. Hence, the relation between displacement and force measurement in the strain gauge is:

\[
F_m = k_{\text{sensitivity}} \cdot \frac{E_{ex} \cdot \Delta R}{2 \cdot R_0} \cdot u = k_{\text{smart}} \cdot u
\]  

(4.6)

As seen in the general sensor model of Figure 4.4, the relationship between applied force \( (F_C) \) and force measurement \( (F_M) \) is given by \( k_{\text{sensor}} \) (\( k_{\text{istler}} \) or \( k_{\text{smart}} \)) that relates the force measurement \( (F_M) \) and displacement \( (u) \) for both sensors. Using this model, the simulated force measurements can be compared with experimental data. Noise is not considered in this model.

\[
\begin{align*}
F_C(S) & \quad \frac{1}{k} \quad u(S) \quad k_{\text{sensor}} \quad F_m(S) \\
S^2 & \quad \frac{2 \cdot \zeta \cdot S}{\omega_n} + 1
\end{align*}
\]

Figure 4.4: Applied Forces to Measurements

### 4.3 Cutting Conditions and Cutting Coefficients

Cutting conditions and cutting coefficients for a given set of tool and workpiece systems must be defined in order to simulate cutting forces. Tool and workpiece systems are defined with the system parameters obtained in Chapter 3 (see in Table 4.1).
Simulation and experimental data are compared for the following cutting conditions:

- Spindle Speed: 2600 [rpm]
- Cutting type: up-milling
- Radial depth of cut: half immersion
- Axial depth of cut: a=0.125 [in]
- Average chip thickness: $h_{avg} = 0.001$ [in]
- Material: Al6061
- Number of teeth: $N_T = 1$
- Cutter: Sandvik Coromill 390 (d: 0.75 [in])
- Mill insert: R390-11 T3 08E-NL H13A

<table>
<thead>
<tr>
<th>Workpiece System</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Static</strong></td>
</tr>
<tr>
<td><strong>dynamic</strong></td>
</tr>
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<td></td>
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<tr>
<td><strong>Process Damping</strong></td>
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<table>
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</thead>
<tbody>
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</tr>
<tr>
<td><strong>dynamic</strong></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Most cutting conditions can be assigned values, however the helix angle should be measured for the insert cutter. Figure 4.5 shows the how to get the helix angle from measurements. The helix angle for this cutter and insert is:

$$\theta_{\text{helix \ [deg]}} = \tan^{-1} \left( \frac{3.54 \text{ [mm]}}{10.98 \text{ [mm]}} \right) \times \frac{180 \text{ [deg]}}{\pi \text{ [rad]}} = 17.9 \text{ [deg]}$$
Based on the linear cutting force model shown in Equations 2.4 and 2.5, cutting coefficients are calculated using the average force method (see [9] for details). The obtained cutting coefficients are:

\[ K_{tc} = 688.0 \text{e}6 \text{ [N/m}^2\text{]}, \ K_{te} = 17.2\text{e}3 \text{ [N/m]}, \ K_{rc} = 229.4\text{e}6 \text{ [N/m}^2\text{]} \text{ and } K_{re} = 10.5\text{e}3 \text{ [N/m]} \]

### 4.4 Comparison with Stable Cutting Data

To validate the simulation model containing the tool-workpiece compliance, stable experimental cut data is compared with the simulated force measurements using the same cutting conditions as in the experiments. The model parameters used in the simulation are those derived from the experimental cuts as described in Chapter 3. Cutting force profiles, maximum peak forces, and frequency content of the cutting forces are compared.

Figures 4.6, 4.7, and 4.8 show the simulation results. Figures 4.6 and 4.7 show the comparison of the Kistler and Smart Tool data to the simulated cutting force and simulated force measurement. Figure 4.8 shows the comparison of the resultant force from the Kistler data. These figures show that the simulated force measurements are quite similar in pattern to the experimentally measured force data.
Figure 4.6: Simulated Results and Experimental Data, Kistler
Figure 4.7: Simulated Results and Experimental Data, Smart Tool
In the out-of-cut profile of Figures 4.6 and 4.7, both the simulated and experimental force measurements of the Smart Tool show *beating*, which occurs when two interacting structures have similar natural frequencies. For the in-cut profile, vibrations can be observed for both the simulation and the experiment, even though the cutting process is stable. This in-cut vibration remained the same cycle to cycle from the simulation results, but it varies somewhat for the actual cutting because of the stochastic nature of cutting. In general, the vibration grows during the in-cut portion of the actual cut and shrinks in the simulation. More sophisticated modeling is required to simulate this effect.

Table 4.2 gives the maximum peak force for both the simulation and the experiment along with the standard error. The maximum peak force from the simulation
is directly affected by the cutting coefficients values, as well as by the vibration model and the parameters of those models. The simulated results are slightly lower than the experimental data for this particular cutting conditions and system parameters, with an average error around 9%.

Table 4.2: Comparison of Maximum Peak Forces

<table>
<thead>
<tr>
<th>Direction</th>
<th>X</th>
<th>Y</th>
<th>Resultant Force</th>
<th>Radial</th>
<th>Tangential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation [N]</td>
<td>115.9</td>
<td>137.9</td>
<td>148</td>
<td>65.1</td>
<td>133.8</td>
</tr>
<tr>
<td>Experiment [N]</td>
<td>127.9</td>
<td>139.7</td>
<td>168.1</td>
<td>69.7</td>
<td>157.2</td>
</tr>
<tr>
<td>Error [%]</td>
<td>9.38</td>
<td>1.29</td>
<td>11.96</td>
<td>6.60</td>
<td>14.89</td>
</tr>
</tbody>
</table>

Figure 4.9: Maximum Peak Forces between Simulation and Experimental Data

The frequency content from both the simulation and experimental data from the Kistler and Smart Tool is shown in Figures 4.10 and 4.11. Accurate frequency content from the simulation is important in chatter prediction. Since the tool and workpiece systems are modeled as a second-order system, the frequency content of the simulation is similar to the frequency content of the experimental results until 2100 Hz, where the second mode of vibration occurs. Among the harmonics of the tooth passing frequency, enlarged peaks near $\omega_n$ of the structures can be observed in both the simulation results and experimental data (see in Figure 4.10 and 4.11).
Figure 4.10: FFT Comparison of Kistler and Simulated Data
Figure 4.11: FFT Comparison of Smart Tool and Simulated Data
Table 4.3: Experimental Data Sets for Validation of Simulation

<table>
<thead>
<tr>
<th>Axial Depth of Cut</th>
<th>Radial Immersion</th>
<th>Spindle Speed [RPM]</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.175 [mm] = 0.125 [in]</td>
<td>Half</td>
<td>600</td>
<td>stable</td>
</tr>
<tr>
<td></td>
<td>Slot</td>
<td>2600</td>
<td>stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3600</td>
<td>stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3600</td>
<td>stable</td>
</tr>
</tbody>
</table>

Previously, experimental data from a half immersion cut at 2600 rpm was used to validate the simulation model. In order to further validate the simulation, different experimental data sets are compared with simulation results. Table 4.3 shows the additional four cutting conditions that are used for the validation of the simulation. The same cutter and inserts are used in all four cuts. System parameters for selected cutting conditions, and cutting coefficients, are obtained experimentally (see for detail in Appendix F). Table 4.4 and Figure 4.12 show the maximum peak force comparison of the experimental and simulated data.

Table 4.4: Comparison of Experimental and Simulated Maximum Peak Forces

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simulation</td>
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<td>190.7</td>
<td>77.2</td>
<td>175.8</td>
</tr>
<tr>
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<td>148.7</td>
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<td>84.2</td>
<td>178</td>
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<tr>
<td>Error [%]</td>
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<td>20.4</td>
<td>1.97</td>
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<td>1.2</td>
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<tr>
<td>Simulation</td>
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<tr>
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<tr>
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<tr>
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</tr>
<tr>
<td>Error [%]</td>
<td>8.5</td>
<td>31.5</td>
<td>17.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Figure 4.12: Comparison of Experimental and Simulated Maximum Peak Forces

Figure 4.13, 4.14, 4.15, and 4.16 show resultant forces for both experiment and simulation. Since the Smart Tool cannot measure cutting force in the radial and tangential directions at the same time, the resultant forces are experimentally obtained from the Kistler dynamometer. For each case, the force profile and frequency content for both experimental data and simulation results are provided in Appendix F.
Figure 4.13: Simulation and Experiment, Resultant Force, Half Immersion, 600 [rpm]

Figure 4.14: Simulation and Experiment, Resultant Force, Half Immersion, 3600 [rpm]
Figure 4.15: Simulation and Experiment, Resultant Force, Slot, 600 [rpm]

Figure 4.16: Simulation and Experiment, Resultant Force, Slot, 3600 [rpm]
4.5 Simulation Sensitivity to Tool System Parameter Variation

The milling simulation as implemented uses invariant system parameters. Unlike the simulation, actual system parameters vary due to the stochastic nature of cutting and the complicated cutting geometry that is simplified in the model. In this section, we investigate how the simulation results are affected by variation of the system parameters. One cut is investigated, half immersion, 2600 [rpm].

Previously, the system parameters under dynamic condition were defined statistically from 30 cycles of the stable experimental cutting data through use of LPC. Figure 4.17 shows the pole position variation using LPC, which associates the parameters \( \omega_{n,dyn} \) and \( \zeta_{dyn} \). Both \( \omega_{n,dyn} \) and \( \zeta_{dyn} \) clearly change from cycle to cycle. The distance from zero to the pole's position indicates the natural frequency in Figure 4.17. Simulation results are investigated using min, max, and average statistical system parameter values (Table 4.5). Since the workpiece system has much greater stiffness than the tool system, only the variation of tool system parameters are examined by simulation. Table 4.6 shows the average and the standard error of the mean (SEM) of the tool's \( \omega_{n,dyn}, \zeta_{dyn}, \) and \( \zeta_{process} \) from experimental data both in radial and tangential directions. Based on the average and SEM, possible maximum and minimum system parameters are calculated with a 95 % confidence interval (avg±1.96·SEM). The simulation results are examined with possible combinations of maximum, minimum, and average tool system parameters (see Table 4.6).
Table 4.5: Possible Combinations (8 cases) of Tool System Parameters

<table>
<thead>
<tr>
<th>case</th>
<th>combination</th>
<th>case</th>
<th>combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\omega_{n, \text{dyn}}, \zeta_{\text{dyn}}$ avg $\zeta_{\text{process}}$ max</td>
<td>2</td>
<td>$\omega_{n, \text{dyn}}, \zeta_{\text{dyn}}$ avg $\zeta_{\text{process}}$ min</td>
</tr>
<tr>
<td>3</td>
<td>$\omega_{n, \text{dyn}}, \zeta_{\text{dyn}}$ max $\zeta_{\text{process}}$ avg</td>
<td>4</td>
<td>$\omega_{n, \text{dyn}}, \zeta_{\text{dyn}}$ min $\zeta_{\text{process}}$ avg</td>
</tr>
<tr>
<td>5</td>
<td>$\omega_{n, \text{dyn}}, \zeta_{\text{dyn}}$ max $\zeta_{\text{process}}$ max</td>
<td>6</td>
<td>$\omega_{n, \text{dyn}}, \zeta_{\text{dyn}}$ max $\zeta_{\text{process}}$ min</td>
</tr>
<tr>
<td>7</td>
<td>$\omega_{n, \text{dyn}}, \zeta_{\text{dyn}}$ min $\zeta_{\text{process}}$ max</td>
<td>8</td>
<td>$\omega_{n, \text{dyn}}, \zeta_{\text{dyn}}$ min $\zeta_{\text{process}}$ min</td>
</tr>
</tbody>
</table>

The simulation program is run for the eight conditions given in Table 4.5 and 4.6.

Figure 4.18 to 4.25 show the simulated cutting force and simulated measured cutting
force for the eight combinations min, avg, and max parameter values. Except for the case with minimum $\omega_{n, dyn}$, $\zeta_{dyn}$ and minimum $\zeta_{process}$ (see in Figure 4.25), peak force simulation results are not sensitive to the statistical maximum or minimum of the tool system parameters, as seen in from Figure 4.18 to 4.24. The variation of $\omega_{n, dyn}$ of the tool system does not significantly affect the simulation results. However, it is clearly observed that the oscillation magnitudes are changing depending on $\zeta_{dyn}$ and $\zeta_{process}$. The oscillation magnitudes decrease as $\zeta_{dyn}$ or $\zeta_{process}$ of the tool system increase. Figure 4.25 shows unstable cutting when using the minimum of $\omega_{n, dyn}$, $\zeta_{dyn}$ and $\zeta_{process}$ ($\omega_{n, dyn}$: 620.06 [Hz], $\zeta_{dyn}$: 0.031, and $\zeta_{process}$: 0.1126). For this particular case, it suggests that $\zeta_{dyn}$ and $\zeta_{process}$ help determine the stability of the cut, but it is unknown which has a greater effect. In general, one can state that the force simulation program can accurately predict cutting forces when supplied with good parameter values.

![Cutting Force Simulation](image1)

![Cutting Force Simulation](image2)

Figure 4.18: Simulation with Average of $\omega_{n, tool, dyn}$ and $\zeta_{tool, dyn}$ with Maximum $\zeta_{process}$
Figure 4.19: Simulation with Average \( \omega_{n, tool\_dyn} \) and \( \zeta_{tool\_dyn} \) with Minimum \( \zeta_{\text{process}} \)

Figure 4.20: Simulation with Maximum \( \omega_{n, tool\_dyn} \) and \( \zeta_{tool\_dyn} \) with Average \( \zeta_{\text{process}} \)
Figure 4.21: Simulation with Minimum \( \omega_{t_{ool_{dyne}}} \) and \( \zeta_{t_{ool_{dyne}}} \) with Average \( \zeta_{process} \)

Figure 4.22: Simulation with Maximum \( \omega_{t_{ool_{dyne}}} \) and \( \zeta_{t_{ool_{dyne}}} \) and Maximum \( \zeta_{process} \)
Figure 4.23: Simulation with Maximum $\omega_{n,dyn}$ and $\zeta_{tool,dyn}$ and Minimum $\zeta_{process}$

Figure 4.24: Simulation with Minimum $\omega_{n,dyn}$ and $\zeta_{tool,dyn}$ and Maximum $\zeta_{process}$
Figure 4.25: Simulation with Minimum \( \omega_{n \text{, tool \_ dyn}} \) and \( \zeta_{\text{tool \_ dyn}} \) and Minimum \( \zeta_{\text{process}} \)

4.6 Simulation: Prediction of Chatter

With well-defined system parameters, the capability of the simulation to predict chatter should be investigated. The simulation estimates cutting forces for a given cutting condition with system parameters derived from a stable experimental cut with the same cut conditions except for the axial depth of cut. As the axial depth of cut is increased, simulation results are examined to determine when instability or chatter occurs. Experimentally, at 2600 [rpm], half immersion chatter occurs when the axial depth of cut is 0.175 [in] (see in Figure 4.26). The simulation is run using cutting coefficient values and system parameters from previous stable cuts at 2600 [rpm], half immersion (see in Section 4.3).
Figure 4.26: Experimental Instability (Chatter), 0.175 [in] Axial Depth of Cut

Figure 4.27: Simulated Chatter, 0.189 [in] Axial Depth of Cut

From the simulation results as seen in Figure 4.27, instability occurs at an axial depth of cut 0.189 [in]. Experimentally, when $a=0.175$ [in], the milling process is unstable and chatter builds up. The maximum axial depth of the cut is hard to find experimentally because of the potential for machine and tool damage. Figure 4.28 shows how chatter builds up in the simulation. Due to the large deflections of the structures, the cutter does not stay in continuous contact with the workpiece inside the cutting region ($\theta_{\text{start}} \sim \theta_{\text{actual end}}$). The remaining material results in large cutting forces for the next cycle. A major benefit of a milling simulation is that one can examine how the cutting force stability is affected by different cutting conditions.
Table 4.7 shows more stable and unstable experimental data sets. At an axial depth of 0.125 [in] both slot cuts (600 [rpm] and 3600 [rpm]) are stable. Simulations for these cutting conditions (Table 4.3) also showed stable cuts. The axial depth is increased in the simulations to produce chatter. This occurs at 0.183 [in] and 0.198 [in], slightly deeper than the depths for experimental chatter. Figure 4.29 and 4.30 show the simulated...
and experimental chatter for the 600 [rpm] and 3600 [rpm] cut. The simulated results show the capability of the simulation program for chatter prediction.

Table 4.7: Experimental Data Sets (Stable Cut and Chatter)

<table>
<thead>
<tr>
<th>Axial Depth of Cut</th>
<th>Radial Immersion</th>
<th>Spindle Speed [RPM]</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.175 [mm] = 0.125 [in]</td>
<td>Slot</td>
<td>600</td>
<td>stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3600</td>
<td>stable</td>
</tr>
<tr>
<td>4.445 [mm] = 0.175 [in]</td>
<td>Slot</td>
<td>600</td>
<td>unstable</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3600</td>
<td>unstable</td>
</tr>
</tbody>
</table>

Figure 4.29: Experimental and Simulated Chatter, Slot, 600 [rpm]
4.5 Summary

In Chapter 4, a time-domain milling simulation program with tool-workpiece compliance is validated for a given set of cutting conditions. The system parameters used in the simulation are derived from the experimental cut at the same cutting conditions. To validate the simulation, maximum peak force, force profile, and force frequency content are compared to experimental data. Based on these criteria, it is concluded that the simulation can estimate cutting forces for a given set of cutting conditions if the system parameters are well defined. In addition, the simulation can also predict chatter with systems parameters obtained from stable experimental cutting data. Since Linear Predictive Coding (LPC) is used for system identification, system parameters are defined only from stable cutting experiments.
CHAPTER V

COMPENSATION OF SENSOR DYNAMICS USING AN EXTENDED KALMAN FILTER WITH SELF-TUNING

In order to monitor the cutting force during milling with a CNC machine, force transducers such as the Kistler dynamometer (piezoelectric sensor) and the Smart Tool (strain gauge) are used. Due to sensor dynamics, there are differences between the actual cutting force and the measured force, characterized by phase delay and additional vibrations in the cutting profile. In Chapter 5, actual cutting forces are estimated from the measured sensor data using an Extended Kalman Filter. The estimated cutting force is then used for system identification. Since cutting forces are composed of harmonics of the tooth passing frequency and runout frequency, both a sensor dynamic model and harmonic cutting force model are included in an Extended Kalman Filter model to improve filter performance. The harmonic computations of the cutting force model are used to self-tune the Extended Kalman Filter.

5.1 Introduction

For the Smart Machining System developed at UNH, two different force transducers are used to monitor cutting forces. Both force transducers have their sensors located away from the actual cut. The Kistler has piezoelectric crystals inside a load cell and the Smart Tool has strain gauges located on the upper part of the tool holder. Both sensors respond to deflection away from the cut and can be modeled as second-order
mechanical systems with a damping ratio, natural frequency and stiffness. The dynamics associated with each sensor can affect force readings near, or above their natural frequencies. In addition, excessive vibrations in the measured force profile are seen in the tooth out-of-cut regions, where the actual cutting force is zero. In 2004, Simon S. Park and Yusuf Altintas [13] showed that a Kalman filter with a pseudo integrator can compensate for spindle dynamic effects on strain gauges located on the spindle. Their method is sensitive to coefficients of the spindle dynamics such as natural frequency, damping ratio and sensor stiffness, and the input covariance matrix of the noise. The covariance matrix must be practically tuned for each case in order to apply the Kalman filter. In this work, an Extended Kalman Filter is used to reduce the effect of system dynamics and improve the performance of the Kalman filter. This Extended Kalman Filter model includes a harmonic force model and a sensor dynamic model reducing the need for Kalman tuning, and making it less sensitive to system parameter variation.

In order to estimate actual cutting forces from the force measurements, the inverse system model, as see in Figure 5.1, is used, based on a second-order system model of the sensor as shown in Figure 4.4.

\[
\frac{F_m(S)}{k_{sensor}} + \frac{u(S)}{\omega_n^2} + \frac{2 \cdot \zeta \cdot S}{\omega_n} + 1 \cdot \frac{1}{k} = F_C(S)
\]

*Figure 5.1: Ideal Conversion from Force Measurements to Cutting Forces, assuming a Second-order Dynamic Model for the Force Sensor*

The actual measurement data includes the force measurement and noise (see Figure 5.2). The noise causes significant error when converting the measured force to cutting force. If the noise is integrated, its effect on the measured force is reduced. However, if it is
differentiated, as it is necessary for this conversion, the noise results in significant errors in computation of the cutting force. Therefore, for accuracy, the noise should be completely filtered before calculation of the cutting force. Using a type of filter such as a low pass, the noise can be reduced, but some noise will remain causing significant error when differentiated twice. For this reason, the Kalman filter is used for estimation of cutting forces. The Kalman filter reduces the effect of noise and estimates all states such as displacement and velocity associated with a second-order state space model.

\[
\begin{align*}
F_c(S) & \xrightarrow{\frac{1}{k}} u(S) \xrightarrow{\frac{S^2}{\omega_n^2} + \frac{2 \cdot \zeta \cdot S}{\omega_n} + 1} F_m(S) \xrightarrow{\text{Noise}} u(S) \xrightarrow{\frac{S^2}{\omega_n^2} + \frac{2 \cdot \zeta \cdot S}{\omega_n} + 1} F_c(S)
\end{align*}
\]

Experimental Data from Force Transducer

**Figure 5.2: Conversion from Sensor Measurements with Noise to Applied Cutting Forces**

### 5.2 General Kalman Technique

The basic concept of the Kalman filter is recursively minimizing the mean square error between the actual states and estimated states, using state space methods. The Kalman filter has the ability to filter noise and estimate all of the system's states from one noisy state measurement. For example, velocity can be estimated from the position measurement.

The *recursive* operation is one of the key features of Kalman filtering. *Recursive* means that current desired value (\(\bar{M}_n\)) is estimated by a weighted sum of the previous measurements (\(\left(\frac{n-1}{n}\right) \cdot \bar{M}_{n-1}\)) and current measurement (\(z_n\)) without storing all previous measurements [15]. A simple *recursive* example is shown by:

\[
\bar{M}_n = \left(\frac{n-1}{n}\right) \cdot \bar{M}_{n-1} + \left(\frac{1}{n}\right) \cdot z_n, \quad n = 1, 2, \ldots
\]  

(5.1)
Since a weighted sum of the previous measurements and current measurement is discarded after the current desired valued is estimated, storage memory problems are not a concern.

The Kalman filter uses a state space approach with the state model (Equation 5.2) and observation model (Equation 5.3) defined as:

\[
\dot{X} = A \cdot X + B \cdot w \tag{5.2}
\]
\[
y = H \cdot X + N \tag{5.3}
\]

where \(A\) is the state transition matrix, \(B\) is the input transition matrix, \(H\) is the observation matrix, \(N\) is the additive noise process, \(X\) is the state vector, \(y\) is the noisy measurement vector, and \(w\) is the uncorrelated input vector which is assumed as a white noise with zero average value.

The Kalman filter is optimized using the least square error \(e_y = y - \hat{y}\) between current measurement \(y\) and recursive estimated measurement \(\hat{y}\) by the Kalman gain \(G\) which is derived from Equation 5.4 and 5.5 [15].

\[
\dot{X} = A \cdot \hat{X} + G \cdot (y - \hat{y}) \tag{5.4}
\]
\[
\hat{y} = H \cdot \hat{X} \tag{5.5}
\]

This estimated state \(\hat{X}\) is recursively calculated from the current measurement \(y\) and the previous state of \(\hat{X}\) using a recursive discrete formula [15]. After the state space model (Equation 5.2 and 5.3) is defined, the covariance matrix (Q) of input (w) and covariance matrix (R) of noise (N) are needed in order to calculate the Kalman gain. The method to find Q and R is discussed in the next section. The Kalman gain is calculated by minimizing the covariance \(v_e\) of the signal estimation error \(e = X - \hat{X}\) until the signal
estimation error becomes a constant value at steady state (ideally, $e_{k+1} = 0$ or $\dot{e} = 0$).

Figure 5.3 shows the Kalman gain calculation based on the covariance. $T$ is the sampling time.

```matlab
for i=1:step
  G = Ve*H'*1/R;
  Ve = Ve+T*((A-G*H)*Ve+Ve*(A-G*H)'+B*Q*B'+G*R*G');
end
```

*Figure 5.3: Numerical Calculation of the Discrete Kalman Gain [15]*

### 5.2.1 Kalman Filter with a Pseudo Integrator

In order to filter signal noise, a Kalman filter is applied to the second-order sensor model (see Figure 4.4). The state space model is defined as:

$$
\dot{X} = \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = 
\begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\cdot\zeta\cdot\omega_n \end{bmatrix}
\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + 
\begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix}
\frac{k}{k} F_c = A \cdot X + B \cdot w
$$

(5.6)

$$
y = F_m = \begin{bmatrix} k_{\text{sensor}} & 0 \end{bmatrix}
\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + N = H \cdot X + N
$$

(5.7)

The state space model is shown in Figure 5.4, with $X_1$ the displacement of the sensor ($x$) and the $X_2$ the velocity of the sensor ($\dot{x}$).
In this application, the Kalman filter minimizes the effect of noise and estimates the force measurement ($\hat{F}_m$) by estimating the displacement and velocity states ($\hat{X}_1$ and $\hat{X}_2$). However, the actual cutting forces cannot be estimated because an additional state $X_3$, related to acceleration of the sensor, is needed to provide an estimate of the cutting force. This is accomplished by adding a pseudo integrator to the state model as seen in Figure 5.5.

![Figure 5.5: Kalman Filter with a Pseudo Integrator for Estimation of Cutting Force](image)

The state space model for the Kalman filter with a pseudo integrator is defined as:

$$
\dot{\hat{X}} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = 
\begin{bmatrix} 0 & 1 & 0 \\ -2\cdot\zeta\cdot\omega_n & -\omega_n^2 & \frac{\omega_n^2}{k} \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot F_c = A \cdot \hat{X} + B \cdot w 
$$

(5.8)

$$
y = F_m = [k_{sensor} \quad 0 \quad 0] \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + N = H \cdot \hat{X} + N
$$

(5.9)

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The Kalman filter is applied to the above state space model (Equation 5.8 and 5.9) in order to estimate cutting forces from noisy measurements. Previous research by Park and Altintas [14] proves this application is reliable with calibrated system parameters and tuned covariance matrix \((Q)\) of inputs \((w_i)\). However, two key features limit their results. First, the spindle dynamics is characterized by system parameters under static condition, even though system parameters change during cutting. Second, tuning the Kalman filter, i.e. choosing the values of the covariance of the Kalman filter inputs \((w_i)\), is accomplished by empirical trials. The input covariance matrix \((Q)\) is related to the uncertainty of the sensor dynamics and the actual cutting forces that are unknown. Generally, there is no way to directly measure the input covariance matrix of the Kalman filter that is associated with \(w_1, w_2,\) and \(w_3\). Therefore, the covariance matrix \((Q)\) of the unknown inputs must be tuned instead of systematic determination. Even if the covariance matrix \((Q)\) of the unknown input \((w_i)\) is tuned, it can only be applied in certain cases when the system parameters are constant. If the parameters change, e.g. damping, the \(Q\) matrix must be tuned again.

### 5.3 Extended Kalman Filter Model

In order to avoid tedious Kalman tuning, i.e. finding the covariance matrix \((Q)\) of input \((w_i)\), an Extended Kalman Filter model that includes a harmonic cutting force model and a sensor dynamic model is considered. Theoretically, the milling force contains harmonics of the tooth passing frequency and runout frequency if the tool contains multiple teeth. An Extended Kalman Filter model is developed from a combination of the harmonic cutting force model and second-order sensor dynamic model as seen in Figure 5.6.
5.3.1 Sensor Dynamic Model

As mentioned before, the force transducer dynamics of both the Smart Tool and the Kistler dynamometer are modeled as a second-order system with mass, spring, and damper, as seen in Figure 5.7.

In order to improve model accuracy, a higher order system could be considered. However, if a higher order model is adapted, system coefficients must be accurately identified to
maintain overall model accuracy. For simple modeling and identification, a Kalman filter is developed for the second-order model as:

\[ F_C = \frac{1}{\omega_n^2} \cdot \ddot{F}_m + \frac{2 \cdot \zeta}{\omega_n} \cdot \dot{F}_m + F_m = \frac{m}{k} \cdot \ddot{F}_m + \frac{b}{k} \cdot \dot{F}_m + F_m \]  

(5.10)

The identification of system parameters is discussed in Chapter 3. Since system parameters, \( \omega_n \) and \( \zeta \), change depending on boundary conditions, one has to examine the effect of parameter variation on filter performance.

5.3.2 Harmonic Cutting Force Model

Instead of a pseudo integrator, as described in section 5.2.1, a harmonic force model is utilized as part of the Extended Kalman Filter. The harmonic force model provides some information of the unknown input (\( F_C \)) based on the harmonic force corresponding to the tooth passing frequency (\( \omega_{rt} \)) or runout frequency (\( \omega_{runout} \)) when considering a multiple tooth cutter. The tooth pass frequency and runout frequency are given in Equation 5.11, where \( N_T \) is the number of teeth.

\[ \omega_{runout} = \frac{\text{Spindle Speed [rpm]}}{60} = \frac{\omega_{rt}}{N_T} [\text{Hz}] \]  

(5.11)

The cutting force at the tooth passing frequency can be written as a Fourier series as shown in Equation 5.12:

\[ F_C(t) = A_0 + \sum_{i=1}^{\infty} (A_i \cos(i \cdot \omega_{rt} \cdot t) + B_i \sin(i \cdot \omega_{rt} \cdot t)) \]  

(5.12)

, where \( A_0 \), \( A_i \), and \( B_i \) are the Fourier coefficients. Assuming that the cutting force is an even function (\( f(x) = f(-x) \)), the coefficients (\( A_0 \), \( A_i \), and \( B_i \)) are defined in Equation 5.13 [16]:

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From Equations 5.12 and 5.13, the harmonic cutting force model is defined as:

\[ F_C(t) = A_0 + A_1 \cos(\omega_r t) + A_2 \cos(2 \omega_r t) + A_3 \cos(3 \omega_r t) + \ldots \] (5.14)

where \( A_0 \) is defined as the average value of \( F_C \), and \( A_i \) is the coefficient of the Fourier terms, which is the magnitude of the FFT (Fast Fourier Transform) at each harmonic of the tooth passing frequency.

In order to represent \( F_C \) (Equation 5.14) as a state space model, a model of each harmonic is defined. Equation 5.15 is the differential equation for a harmonic motion with known frequency (\( \omega \)).

\[ \ddot{y} + \omega^2 \cdot y = 0 \] (5.15)

Equation 5.16 is the homogeneous solution of Equation 5.15.

\[ y(t) = y(0) \cdot \cos(\omega \cdot t) + \frac{\dot{y}(0)}{\omega} \cdot \sin(\omega \cdot t) \] (5.16)

In addition, when \( \dot{y}(0) = 0 \), the general homogeneous solution becomes \( A_i \cos(i \cdot \omega_r \cdot t) \), which is a component of the \( F_C \) Fourier series (Equation 5.14). Now, \( y_1 = A_1 \cos(\omega_r \cdot t) \) can be represented in state space as:

\[
\begin{bmatrix}
\dot{X}_1 \\
\dot{X}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-\omega_r^2 & 0
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix}
+ \begin{bmatrix}
0 \\
1
\end{bmatrix}\cdot w = A \cdot X + B \cdot w
\] (5.17)

\[ y_1 = \begin{bmatrix} A_1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\
X_2
\end{bmatrix} \] (5.18)

where \( w \) is a white noise with zero mean, \( X_1 \) is \( \frac{y_1}{A_1} \), \( X_2 \) is \( \dot{X}_1 \), and \( y_1 \) is \( F_{C_1} \). Equations 5.17 and 5.18 represent the state space model of the first cutting force harmonic.
Each harmonic component of the cutting force can be combined to form the harmonic cutting force model that is part of the Extended Kalman Filter (Figure 5.8).

![Diagram of Harmonic Cutting Force Model](image)

**Figure 5.8: Parallel Structure of the Harmonic Cutting Force Model**

### 5.3.3 Kalman Tuning by Harmonic Computation

As mentioned before, a well-known drawback of the Kalman filter is "tuning the covariant matrix (Q) of input (w)". In this work, the value of the input covariant matrix is chosen by harmonic computation.

For the harmonic cutting force model in state space (Equation 5.17), the variance of $\mathbf{X}$ and $w$ are related by:

$$
\text{Var}(\dot{\mathbf{X}}) = \mathbf{A} \cdot \text{Var}(\mathbf{X}) + \text{Var}(\mathbf{X}) \cdot \mathbf{A}^T + \mathbf{B} \cdot \text{Var}(w) + \text{Var}(w) \cdot \mathbf{B}^T \tag{5.19}
$$

With an assumption that the covariance matrix of $\dot{\mathbf{X}}$ (Var($\dot{\mathbf{X}}$)) becomes a zero matrix at steady state, the relationship between the variance of $\mathbf{X}$ and the variance of $w$ becomes:
\[ \text{Var}(w) = (\omega_t^2) \cdot \text{Var}(x) \]  

(5.20)

From Equation 5.18, the relationship between the variance of \( F_{C_t} \) and the variance of \( x \) is defined as:

\[ \text{Var}(x) = \frac{\text{Var}(F_{C_t})}{A_1^2} \]  

(5.21)

Using Equation 5.20 and 5.21, the covariance of input \( (w) \) is defined as:

\[ \text{Var}(w) = \frac{(\omega_t^2) \cdot \text{Var}(F_{C_t})}{A_1^2} \]  

(5.22)

From Equation 5.22, the covariant matrix \( (Q) \) of inputs \( (w_n) \) is given by:

\[ Q_{(i+2)(i+2)} = \text{Var}(w_{i+2}) = \frac{(i \cdot \omega_t)^2 \cdot \text{Var}(F_{C_i})}{A_i^2}, \quad i = 1, 2, 3, \ldots, n \]  

(5.23)

As shown in Equation 5.23, the input covariant matrix \( (Q) \) is computed by the \textit{Fourier} coefficient \( (A_i) \) of the cutting force and the cutting force variance \( (\text{Var}(F_{C_i})) \).

The \textit{Fourier} coefficient \( (A_i) \) can be determined from an FFT of the cutting force. The magnitude of each tooth passing harmonics is equal to \( A_i \), where \( i \) represents the harmonic number. The cutting force variance \( (\text{Var}(F_{C_i})) \) is calculated from the power spectral density (PSD) of the cutting force. The variance of \( F_{C_i} \) is the area under the PSD curve [15] at each harmonic of the tooth passing frequency. The area is calculated using rectangular integration [see MATLAB code for detail in Appendix G].

In order to calculate the covariance matrix \( (Q) \) in Equation 5.23, the cutting force has to be defined. However, only the force measurement \( (F_m) \) with noise is available. These force measurements are analyzed to find the \textit{Fourier} coefficients \( (A_i) \) of the cutting force and the cutting force variance \( (\text{Var}(F_{C_i})) \). Through use of the milling simulation,
the relationship between cutting forces ($F_C$) and force measurements ($F_m$) are investigated to see if the FFT and PSD are similar for force content below the natural frequency of the sensor. If so, we can assume that the *Fourier* coefficients ($A_i$) of the cutting force and the cutting force variance ($\text{Var}(F_C)$) can be approximated from the measured data.

The force simulation program has been validated with a number of experimental cuts as described in Chapter 4. One specific cut for the Smart Tool (Radial) can been seen in Figure 4.7. The FFT and PSD of the simulated cutting force and simulated force measurements of this cut are computed and compared (see in Figure 5.9).

As seen in Figure 5.9, the frequency content of the simulated force measurements tend to have the same pattern as the content of the simulated cutting forces below the natural frequency of the sensor (Smart Tool: T, R $\approx$ 630 Hz, Kistler dynamometer: X $\approx$ 1200 Hz, Y $\approx$ 1400 Hz). Near the natural frequency of the sensors, the magnitude of the tooth passing harmonics increases from *resonance*. Based on these observations, one can assume that the *Fourier* coefficients ($A_i$) of the cutting force can be approximated by force measurements below the natural frequency of sensor. In addition, the PSD in Figure 5.8 shows that energy above the system's natural frequency is small enough to be ignored. Therefore, the cutting force variance ($\text{Var}(F_C)$) can be approximated by the covariance matrix of the force measurements below the sensor's natural frequency.
Figure 5.9: FFT and PSD of Simulated Cutting Forces and Force Measurements, Smart Tool in Radial
From Figure 5.9, the magnitude of the FFT and PSD from the simulated force measurements at each tooth pass frequency harmonic is larger than those magnitude from the simulated cutting force, even if they tend to be same pattern. Using the FFT and PSD of the force measurement instead of the cutting force makes $A_i$ and $\text{Var}(F_{C_i})$ larger. As a result, we cannot be sure whether the input covariance matrix increases or decreases (see in Equation 5.22). However, because $\text{Var}(F_{C_i})$ is generally larger than $A_i^2$ (see in Figure 5.10), the variance of $w_i$ is more affected by $\text{Var}(F_{C_i})$.

![Figure 5.10: Comparison of the variance of cutting force ($\text{Var}(F_{C_i})$) and the Fourier Coefficients Squared ($A_i^2$)](image)

Therefore, the input variance is increased when the FFT and PSD from the force measurement are used instead of the cutting force. For example, the covariance of inputs from the simulated force measurement is larger than the input covariance of the simulated cutting force (see in Table 5.1). Since larger values of the Q matrix represents higher model uncertainty, the input variance matrix from the force measurement can be used for the Extended Kalman Filter.
Table 5.1: Increasing the Input Covariance Matrix (Q), Model Uncertainty

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Var(w₃)</th>
<th>Var(w₄)</th>
<th>Var(w₅)</th>
<th>Var(w₆)</th>
</tr>
</thead>
<tbody>
<tr>
<td>fₘ</td>
<td>2.5878E+06</td>
<td>1.2455E+07</td>
<td>2.4331E+07</td>
<td>3.9328E+07</td>
</tr>
<tr>
<td>f_c</td>
<td>2.5876E+06</td>
<td>1.2454E+07</td>
<td>2.4329E+07</td>
<td>3.9325E+07</td>
</tr>
<tr>
<td></td>
<td>Var(w₇)</td>
<td>Var(w₈)</td>
<td>Var(w₉)</td>
<td>Var(w₁₀)</td>
</tr>
<tr>
<td>fₘ</td>
<td>6.3160E+07</td>
<td>1.0576E+08</td>
<td>1.3853E+08</td>
<td>1.5935E+08</td>
</tr>
<tr>
<td>f_c</td>
<td>6.3152E+07</td>
<td>1.0575E+08</td>
<td>1.3852E+08</td>
<td>1.5934E+08</td>
</tr>
<tr>
<td></td>
<td>Var(w₁₁)</td>
<td>Var(w₁₂)</td>
<td>Var(w₁₃)</td>
<td>Var(w₁₄)</td>
</tr>
<tr>
<td>fₘ</td>
<td>2.0079E+08</td>
<td>2.7894E+08</td>
<td>3.6121E+08</td>
<td>1.1241E+08</td>
</tr>
<tr>
<td>f_c</td>
<td>2.0078E+08</td>
<td>2.7892E+08</td>
<td>3.6119E+08</td>
<td>1.1240E+08</td>
</tr>
<tr>
<td></td>
<td>Var(w₁₅)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fₘ</td>
<td>1.2789E+08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f_c</td>
<td>1.2788E+08</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The order of the harmonic force model can be decided by how many tooth passing harmonics are incorporated for a given spindle speed. For example, with given spindle speed of 2600 [rpm], for the Smart Tool we use \( n = 13 \), while for the Kistler dynamometer we use \( n = 21 \) on X and \( n = 29 \) on Y. This keeps the considered harmonics below the system's natural frequency.

In conclusion, we claim that the Fourier series coefficient \( (A_i) \) of the cutting force and the cutting force variance \( (\text{Var}(F_{C_i})) \) can be approximated from force measurements below the system's natural frequency in order to tune the covariance matrix \( (Q) \) of unknown inputs \( (w_i) \). Unfortunately, the covariance of \( w_1 \) and \( w_2 \), which associate sensor dynamic model uncertainty, is tuned by empirical trials instead of systematic tuning. However, as soon as accurate system parameters are determined for the Extended Kalman Filter, the same values of the covariance of \( w_1 \) and \( w_2 \) with other experimental data sets can be used.
5.4 The Extended Kalman Filter Parameterization

In order to compute the Kalman gains and apply the Extended Kalman Filter, the state space model of the Extended Kalman Filter, the input covariance matrix (Q), and the noise covariance matrix (R) must be defined.

In this work, the Extended Kalman Filter is modeled with a continuous system model, which includes the harmonic force model and the sensor dynamic model, and a discrete measurement and filter model. The system model (Equation 5.24), measurement model (Equation 5.25), and the Kalman filter prediction model (Equations 5.26 and 5.27) are given by:

\[ \dot{X} = A_c \cdot X + B_c \cdot w \]  \hspace{1cm} (5.24)

\[ Z(k) = H \cdot X(k) + N(k) \]  \hspace{1cm} (5.25)

\[ \hat{X}(k + 1) = A_d \cdot \hat{X}(k) + G_d \cdot \left( Z(k) - H \cdot \hat{X}(k) \right) \]  \hspace{1cm} (5.26)

\[ \hat{Z}(k) = H \cdot \hat{X}(k) \]  \hspace{1cm} (5.27)

where \( A_c \) is the state transition matrix, \( A_d \) is the discretized state transition matrix, \( B_c \) is the input transition matrix, \( G_d \) is the discrete Kalman gain matrix, \( H \) is the observation matrix relating the noisy measurement (Z) to the state vector (X), N is the noise, \( \hat{X} \) is the estimated state vector, \( w \) is the input covariance vector, and \( \hat{Z} \) is the estimated measurements. The system model and the measurement model are shown in from the block diagram form in Figure 5.11.
The state transition matrix ($A_C$) is shown in Equation 5.28, and includes both the harmonic force model and the sensor dynamic model.

$$A_{C_{2n+2,2n+2}} = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{k}{m} & \frac{b}{m} & -\frac{A_1}{m} & 0 & -\frac{A_2}{m} & 0 & -\frac{A_n}{m} & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -\omega_{ft}^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -(2\omega_{ft})^2 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & -(n\omega_{ft})^2 & 0 & 0 
\end{bmatrix}$$ (5.28)
System parameters under dynamic conditions (\(k_{\text{dyn}}, \omega_{n_{\text{dyn}}}, \text{and } \zeta_{\text{dyn}}\)) are used for the sensor dynamic parameters including \(b, k, \text{and } m\) (see in Chapter 3). The tooth pass frequency (\(\omega_t\)) is calculated from the number of teeth and spindle speed using Equation 5.11. The Fourier coefficients (\(A_i\)) are obtained by the FFT of the experimentally measured force data. The input transition matrix (\(B_C\)) is expressed in Equation 5.29.

\[
B_{C_{2.n+2,n}} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \tag{5.29}
\]

The observation matrix (\(H\)) is defined in Equation 5.30, where \(k\) is sensor stiffness.

\[
H_{1.2.n+2} = [k \ 0 \ 0 \ 0 \ 0 \ 0 \ \ldots \ 0 \ 0] \tag{5.30}
\]

The state vector (\(X\)) and the input covariance vector (\(w\)) can be observed in Figure 5.11. The discretized state transition matrix (\(A_D\)) is obtained using the MATLAB built-in function "c2d" with zero-order hold.

The Extended Kalman Filter model does not require Kalman tuning for selected data sets in order to compensate for sensor dynamics. The input covariance matrix (\(Q\), except \(Q(1,1)\) and \(Q(2,2)\)), is calculated by PSD and FFT of the experimental data. Based on Equation 5.23, the variance of \(w_3\) to \(w_n\) can be approximated from the force measurement covariance and the Fourier coefficient of the force measurements. By empirical trials, both \(Q(1,1)\) and \(Q(2,2)\), which are related to the uncertainty of sensor dynamic parameters, are tuned to 0.01. As mentioned before, these values of the \(Q(1,1)\)
and Q(2,2) are general, and can be used for all cuts. The input covariance matrix can be expressed as:

\[
Q_{n+2,n+2} = \begin{bmatrix}
\text{Var}(w_1) & 0 & 0 & 0 & 0 \\
0 & \text{Var}(w_2) & 0 & 0 & \cdots & 0 \\
0 & 0 & \text{Var}(w_3) & 0 & 0 \\
0 & 0 & 0 & \text{Var}(w_4) & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \text{Var}(w_{n+2})
\end{bmatrix}
\]  

(5.31)

The noise covariance matrix (R) is determined from the variance of the force measurement noise when the CNC machine is stationary. Before the cutting process starts, noise on the force sensor signal can be measured. The noise (N) is experimentally confirmed as white noise since these are no dominant frequency components. The noise variance matrix (R) is computed as the variance of the measured noise using one of MATLAB built-in functions "var". In this work, the noise variance matrix becomes just one scalar value, since only one noise source is considered in the Extended Kalman Filter model.

After the state space model, the input covariance matrix (Q), and the noise covariance matrix (R) are defined, the Kalman gain (Gd) is calculated by minimizing the covariance of the signal prediction error (e = X -  \bar{X}). The Kalman gain is computed by the MATLAB built-in function "kalmd".

In addition, initial values of each state (X) are defined from the harmonic force model and initial values of the velocity and the displacement (see in Figure 5.10). X_1(0) is the initial displacement (\(\frac{F_m(0)}{k}\)) and X_2(0) is the initial velocity (\(\frac{(F_m(1)-F_m(0))}{k}f_s\)), where f_s is the sampling frequency). Since the force model is a harmonic function (Equation
5.14), $X_{2+i}(0)$ is defined as one because of $\cos 0 = 1$ and $X_{2+i}(0)$ is defined as zero because of $\sin 0 = 0$, where $i = 1, 2, 3, \ldots n$. These initial values are used for the discrete state space model of the Kalman filter in MATLAB SIMULINK.

After calculating the Kalman gain, the Kalman filter is applied to experimental data. Figure 5.12 shows the MATLAB SIMULINK block diagram for estimation of the cutting force from the measured cutting force.

\[
A = Ad - G \cdot H \\
B = G \\
C = H_{hat}
\]

Figure 5.12: Application of the Extended Kalman Filter

This application reduces the effect of signal noise and provides estimations of each state. The estimated cutting force is obtained from the estimated states by:

\[
\hat{F}_c = \hat{H} \cdot \hat{X}
\]

\[
\hat{H} = \begin{bmatrix} 0 & 0 & A_1 & 0 & A_2 & 0 & \cdots & A_n & 0 \end{bmatrix}
\]

The estimated observation matrix $\hat{H}$ is composed of the Fourier coefficients as seen in Figure 5.10. Typical Kalman technique assumptions require the inputs ($w_i$), white noise sources, to have zero averages. Therefore, the average cutting force ($F_c$) is biased to zero. After the cutting force is estimated, this offset value is added back.

The MATLAB code consists of a main program with function calls for the Extended Kalman Filter and can be found in Appendix G. The function code for the Extended Kalman Filter auto-calculates the input covariance matrix ($Q$) from the measured force. The inputs to the function code are the discrete measured cutting data as
a function of time, the sensor noise, the number of teeth, and the sensor system parameters including stiffness, natural frequency, and damping ratio. Because Kalman tuning is not required, this code automatically estimates the cutting force from the given input as long as it is in the proper format, as shown in Appendix G.

5.6 Estimate Cutting Force Results

The results of the Extended Kalman Filter are validated by comparing the estimated cutting force to the measured cutting force and the simulated cutting force. The Extended Kalman Filter is applied to the experimental data from both the Smart Tool and the Kistler dynamometer. The given cutting conditions are half immersion up-milling, with 2600 [rpm] spindle speed, 0.0254[mm] average chip thickness, 19.05 [mm] tool diameter with one tooth, with a sampling frequency of 10240 [Hz] for the Smart Tool and 5200 [Hz] for the Kistler dynamometer, which satisfies the Nyquist sampling theorem. The estimated cutting force (\( \hat{F}_c \)) is calculated with the Extended Kalman Filter for both the Smart Tool (radial and tangential directions) and the Kistler dynamometer (X and Y directions). The cutting force simulation computes the simulated cutting force (\( f_c \)) and the simulated force measurement (\( f_m \)) in X, Y, radial, and tangential directions.

In order to validate the Extended Kalman Filter model, four data sets (experimental data, estimated cutting force, simulated cutting force, and simulated force measurement) in each direction (X, Y, radial, and tangential) of each sensor (Smart Tool and Kistler dynamometer) are compared in Figure 5.13 through 5.24.
Figure 5.13: Comparison of Estimated Cutting Force from the Extended Kalman Filter to Measured and Simulated Force, Kistler, X-Direction
Figure 5.14: FFT of Estimated Cutting Force, measured force, and Simulated Force, Kistler, X-Direction
Figure 5.15: PSD of Estimated Cutting Force, measured force, and Simulated Force, Kistler, X-Direction
Figure 5.16: Comparison of Estimated Cutting Force from the Extended Kalman Filter to Measured and Simulated Force, Kistler, Y-Direction
Figure 5.17: FFT of Estimated Cutting Force, measured force, and Simulated Force, Kistler, Y-Direction
Figure 5.18: PSD of Estimated Cutting Force, measured force, and Simulated Force, Kistler, Y-Direction
Figure 5.19: Comparison of Estimated Cutting Force from the Extended Kalman Filter to Measured and Simulated Force, Smart Tool, Radial-Direction
Figure 5.20: FFT of Estimated Cutting Force, measured force, and Simulated Force, Smart Tool, Radial-Direction
Figure 5.21: PSD of Estimated Cutting Force, measured force, and Simulated Force, Smart Tool, Radial-direction
Figure 5.22: Comparison of Estimated Cutting Force from the Extended Kalman Filter to Measured and Simulated Force, Smart Tool, Tangential-Direction
Figure 5.23: FFT of Estimated Cutting Force, measured force, and Simulated Force, Smart Tool, Tangential-Direction
Figure 5.24: PSD of Estimated Cutting Force, measured force, and Simulated Force, Smart Tool, Tangential-Direction
Since simulated measured forces do not perfectly match the experimental data, direct comparison of the estimated cutting force and the simulated cutting force is not meaningful for validation of the Extended Kalman Filter. However, the trend between the experimental data and estimated cutting forces by Kalman filter clearly shows that vibrations from in-cut and out-of-cut profiles are decreased without a phase delay (see in Figures 5.13, 5.16, 5.19, and 5.22), even if some vibration components still remain. The FFT and PSD of the two forces show a significant reduction of tooth passing harmonics at or above the natural frequency of the sensor in Figures 5.14, 5.15, 5.17, 5.18, 5.20, 5.21, 5.23 and 5.24. Similar trends of the harmonic distortion are observed between the simulated forces.

The Extended Kalman Filter is successful in reducing the sensor vibration in the measured cutting force. An additional model of the cutting force harmonics is required; however all the calculations are automated in MATLAB with the measured cutting force as the only time based input. In particular, since the input covariance matrix (Q) is calculated from the experimentally derived PSD and FFT, the Extended Kalman Filter does not require tedious Kalman tuning.

5.7 Extended Kalman Filter Performance

Since the sensor system parameters change during a cut, e.g. in-cut and out-of-cut, and the Extended Kalman Filter uses a second dynamic model with constant parameters, the performance of the Extended Kalman Filter with different system parameters should be investigated. To investigate this, the Smart Tool, radial direction, is examined using experimental data from a 2600 [rpm], half immersion cut. Three different sets of system
parameters are used for this examination as seen in Table 5.2. The Extended Kalman Filter results for each of these set of constants is discussed below.

Table 5.2: Different System Parameter Sets, Smart Tool, Radial

<table>
<thead>
<tr>
<th>Condition</th>
<th>k [N/m]</th>
<th>$\omega_n$ [Hz]</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static</td>
<td>2.82e6</td>
<td>632.22</td>
<td>0.03227</td>
</tr>
<tr>
<td>Dynamic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In-Cut</td>
<td>2.78e6</td>
<td>627.24</td>
<td>0.14236</td>
</tr>
<tr>
<td>Out-of-Cut</td>
<td>2.77e6</td>
<td>626.03</td>
<td>0.03641</td>
</tr>
</tbody>
</table>

Figure 5.25: Extended Kalman Filter Performance in Estimating Cutting Force with Different Sets of Constant System Parameters

As seen in Figure 5.25, there are no significant differences in the results from the Extended Kalman Filter when using different system parameters sets. If parameters sets are significantly different from the nominal, the Extended Kalman Filter does not work in reducing vibrations. Therefore, the performance of the Extended Kalman Filter is seen as reasonable if near nominal system parameters are used. Since static system parameters

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are close to the dynamic parameters, the Extended Kalman Filter can accurately compensate for variation in the system under dynamic conditions, without on-line identification of the system dynamic parameters.

In addition, as seen in Figure 5.26, the Extended Kalman Filter performs better than a low pass filter in terms of filtering. The low pass filter has significant phase delay while the Kalman filter has no delay. With no phase delay, the estimated cutting force can be used to calibrate the cutting force model using the instantaneous method [9].

![Extended Kalman Filter VS Low Pass Filter](image)

Figure 5.26: Filtering with Extended Kalman Filter and Low Pass Filter

However, there is a problem in using the Extended Kalman Filter for on-line application. The Kalman filter requires a number of force revolutions for stabilization. For 2600 rpm, time for 10 revolution (0.23 [s]) is required for stabilization. In order to apply the Extended Kalman Filter on-line, a "moving window" may be needed in order to
reduce processing time. If force data is not needed instantly, e.g. force model calibration, the current delay time is acceptable.

5.8 Summary

An Extended Kalman Filter is applied to the measured force data to be compensate for sensor dynamics. The Extended Kalman Filter model includes a harmonic cutting force model and a second-order sensor dynamic model. Instead of using a trial and error tuning method to obtain the input covariance matrix (Q), it is approximated from a harmonic cutting force model derived experimentally from measured force data. The Extended Kalman Filter is implemented in MATLAB code. It is fully automated, requiring a minimum of user input, and very general in use. Several case studies show that the filter helps reduce the vibrations associated with the force sensor dynamics. The filter was also known to be robust in the presence of sensor model parameter variation, indicating that constant static model parameters are sufficient to obtain good results, avoiding on-line identification of the dynamic parameters.
CHAPTER VI

CONCLUSION AND FUTURE WORK

6.1 Conclusion

This research on CNC machining has been developed in order to achieve high productivity and excellent quality machined parts. To do this, the cutting conditions are optimized while maintaining cutting stability. The milling force simulation model with tool-workpiece compliance has been developed in order to predict dynamic cutting forces and determine cutting stability for a given set of cutting conditions. The main components of the simulation are a linear cutting force model, regenerative cutting force component, the tool geometry, and a description of the system dynamics including tool deflection, workpiece vibration (surface waviness), and process damping effects. The process damping model is simplified in that the damping ratio becomes slightly higher when the tool and workpiece engage, assuming that the natural frequency and the stiffness remain the same.

To accurately estimate cutting forces and predict chatter (instability) through use of a simulation, parameter identification of the tool and workpiece systems is essential. Since the system parameters vary due to the stochastic nature of cutting, dynamic system parameters should be considered in the simulation instead of static system parameters. In this thesis, a dynamic system parameter identification method was developed using the cutting force measurements from a bed dynamometer (Kistler) and the Smart Tool, which
is used to measure cutting forces acting on the cutting tool. The force measurements are distinguished as either "in-cut" or "out-of-cut" based on the tool/workpiece engagement. Similarly, the system parameters consist of dynamic system parameters of "out-of cut" and process damping parameters of "in-cut".

Two major parts of the identification method are the linear predictive coding (LPC) and the Extended Kalman Filter. The dynamic natural frequency and damping ratio are identified through use of the LPC on the out-of-cut force profile. However, since the in-cut profile includes the cutting force and the vibration, both the Extended Kalman Filter (EKF), used to estimate the "actual" cutting force, and the LPC are used for identifying the process damping ratio of the in-cut vibration. The estimated cutting force is subtracted from the in-cut measured force to obtain the de-trended in-cut profile. The LPC is used on the detrended profile to find the process damping.

In order to accurately estimate the natural frequency and damping ratio, the LPC order must be chosen based on the power spectrum density (PSD) of the separated vibration components. The EKF was designed containing the harmonic force model and the sensor dynamic model. This Extended Kalman Filter is self-tuned by the harmonic computation, which is the input covariance matrix (Q) determined from the PSD and FFT (Fast Fourier Transform) of the force measurements. Additionally, the dynamic stiffness of the tool and workpiece systems is estimated from the static stiffness and natural frequency of the tool system and the static workpiece system stiffness.

A milling force simulation with tool-workpiece compliance model was validated with a variety of experimental cuts. Model accuracy was assessed in terms of the
maximum peak force, the force profile, and the force frequency components. We confirm that a simulation can estimate the cutting force with well-identified system parameters. In addition, the simulation is investigated to see if it can predict chatter for a given axial depth of cut. Since chatter is affected by both the tool deflection and the workpiece vibration (surface waviness), it is uncertain that chatter can be predicted accurately considering only the tool system dynamic behavior, even if workpiece deflections are much smaller than tool deflections. Therefore, in terms of chatter prediction through use of simulation, the tool-workpiece compliance model must be considered. Figure 6.1 shows the simulated cutting forces without the tool-workpiece compliance.

![Simulation Results without Tool-Workpiece Compliance](image)

**Figure 6.1: Simulation Results without Tool-Workpiece Compliance**

Comparing these results, experimental data (see in Figure 4.7) and the simulated force measurement with tool-workpiece compliance (see in Figure 4.6), it clearly shows that simulation results with tool-workpiece compliance is much closer to the measured forces. In addition, the simulation without tool-workpiece compliance predicts instability at a
maximum axial depth of cut near 0.7 [in]. The actual axial depth of cut which produces chatter is 0.175 [in], while the simulation with compliance was 0.184 [in] (see in Section 4.6). These results support that the tool-workpiece compliance model must be considered in order to determine the conditions which lead to chatter.

The milling force simulation with tool-workpiece compliance can be used to optimize cutting conditions with the following procedures:

1) Identification of static system parameters ($k_{sta\_tool}$, $\omega_{n\_sta\_tool}$, $k_{sta\_workpiece}$)
2) Stable experimental data collection from the Smart Tool and bed dynamometer with selected spindle speed and small depth of cut.
3) Cutting coefficients calibration
4) Dynamic system parameter identification from the force measurements ($\omega_{n\_dya}$, $\zeta_{dyn}$, $\zeta_{process}$)
5) Running the simulation program and investigating the maximum axial depth of cut ($a_{lim}$) for optimization cutting conditions

However, this procedure still has issues. The system parameters vary due to many reasons, i.e. tool mounting effects, feedrate, type of cut, spindle speed, large material remove, etc. This variation of the system parameters makes it difficult to optimize cutting condition through the simulation.

**6.2 Future Work**

In this section, future work is suggested for improving the milling simulation, the system identification, the Extended Kalman Filter, and the chatter frequency detection.

**6.2.1 Milling Simulation Improvements**
A more sophisticated simulation model is required to account for the following effects:

1) The unexplained vibration modes near 2000 and 3500 [Hz] appear in the force measurements from both the Smart Tool and bed dynamometer (see in Section 3.5).

2) The simulated in-cut vibration shrinks but the experimental in-cut vibration grows as seen in the comparison of the simulation results and experimental data.

3) The chip thickness is reduced when the cutter enters and exits the cutting region.

4) The tool and workpiece systems parameters change during the cut, even if the cutting is stable.

5) The process damping ratio changes inside the cutting region due to the angle between the surface waviness and the flank face of the tool.

6.2.2 System Identification

Accurate system parameter identification is essential for accurate simulation. The simulation cannot work properly without well-identified system parameters. The LPC used for the system identification works well. However, in order to apply the LPC, experimental data should be separated into in-cut and out-of-cut vibrations. Due to the stochastic nature of the cut, the signal separation procedure requires more work in order to generally apply the LPC in real time. In addition, the dynamic stiffness identification method of the tool and workpiece systems must be developed for general use. Especially, the dynamic workpiece system parameters change a large amount with high material remove rates.
6.2.3 Developing Extended Kalman Filter

The Extended Kalman Filter has been developed in order to compensate for sensor dynamic effects and estimate cutting forces from the force measurements. As previously mentioned, the Extended Kalman Filter does not require Kalman tuning. However, the Kalman filter needs a time constant for stabilization (see in Section 5.7). Therefore, the real-time implementation of the Extended Kalman Filter needs more work, i.e. using a "moving window" which defines the minimum time constant to stabilize the filter itself. In addition, a more complicated sensor dynamic model is required as a part of the Extended Kalman Filter.

6.2.4 Chatter Frequency Detection

Cutting conditions can be optimized if the chatter frequency is known and therefore the optimum spindle speed chosen for stable operation, i.e. no chatter. Chatter frequency can be detected from the force measurement frequency contents using a comb filter. The frequency contents include harmonics of the tooth passing and runout frequency and the natural frequency of the structures. The chatter frequency remains after these frequency components are eliminated by the comb filter. Some concepts for the chatter frequency detection methods are described in Appendix H.

In addition, future work is needed to track the position of the cutter tooth for conversion of the Smart Tool force measurement in radial and tangential direction to X and Y direction. Also, the cutting force interpretation in radial and tangential directions with multiple teeth cutter remains as future work.
LIST OF REFERENCES


APPENDIX A

TIME-DOMAIN MILLING SIMULATION WITH TOOL AND WORKPIECE COMPLIANCE

MATLAB Program: Milling Simulation

% Based on p.6_2_3_1.m, T. Schmitz, University of Florida, April 1, 2008
% Changed by Min hyong Koh, University of New Hampshire

% Linear Force Model: F_t = Ktc*h*a+Kte*a
% F_r = Krc*h*a+Kre*a
% Regenerative Chip Thickness
% Tool and workpiece compliance model.
% Process damping effect
% Helix angle, Runout
% Calculation ratio

% The Tool system is considered symmetric geometry
% When the tool engages the workpiece, the process damping ratio is considered
% Calculation ratio is determined by the maximum frequency of structures
% It is 20 times faster than the maximum natural frequency of structures
% The tool & workpiece deflections calculate in X & Y direction

clc
close all
clear all

%% Simulation Parameters
rev = 50; % revolution for progress.

%% Tool Geometry
NT = 1;
ToolDiameter = 0.75*0.0254; %[m]
HelixAngle = 17.869 ; %[deg]

%% Cutting Conditions
SpindleSpeed = 2600; %[rpm]
FeedRate = 4.084*0.0254 ; %[m/min]
AxialDepth = 0.195* 0.0254; %[m] 0.184 maximum_axial depth
RadialDepth = ToolDiameter *(2/4); %[m] ex)1/4 quarter 1/2 half
CuttingType = 2; % CuttingType 1->Down Milling 2->Up Milling
% Slot Cutting can be either 1 or 2.
RO = 0*1e-6; %[m] tooth-to-tooth runout for 4 tooth cutter
%% Cutting Coefficients
Kt = 688.0240e6; %[N/m^2]
Kr = 229.4289e6; %[N/m^2]
Kte = 17.2290e3; %[N/m]
Kre = 10.5200e3; %[N/m]

%% Static Systems Parameters
% from Tap Test & linear calibration

% Tool System
StaK_T1 = 3.093e6; %[N/m]
StaK_T2 = 2.874e6; %[N/m]
StaKxT = sqrt((StaK_T1^2+StaK_T2^2)/2); %[N/m]
StaKyT = StaKxT; %[N/m]

StaWnxT = 639.4218; %[Hz]
StaWnyT = StaWnxT; %[Hz]

% Workpiece System
KxW = 3.628e8; %[N/m]
KyW = 5.0179e8; %[N/m]

%% Dynamic Systems Parameters
% from experimental data

% Tool System
DynWntT = 652.811; %[Hz]
DynWnrT = 626.03; %[Hz]
DynZetatT = 0.093534;
DynZetarT = 0.036408;
DynWnxT = sqrt((DynWntT^2+DynWnrT^2)/2);
DynWnyT = DynWnxT;
DynZetaxT = sqrt((DynZetatT^2+DynZetarT^2)/2);
DynZetayT = DynZetaxT;

% Workpiece System
DynWnxW = 1211.414; %[Hz]
DynZetaxW = 0.0752825;
DynWnyW = 1462.313; %[Hz]
DynZetayW = 0.076201;

%% Process Damping
% from Kiehler experimental data within the cutting region
zetaxPro = 0.163727;%
zetayPro = 0.188937;%

% Inputs are done.

%% Calculation System Parameters as variables
% Assumptions:
% 1) Effective Mass of the tool system does not change.
% 2) Stiffness of workpiece does not change with small MRR.

% Tool System
ratiox = (DynWnxT/StaWnyT)^2;
ratioy = (DynWnyT/StaWnyT)^2;
DynKxT = StaKxT * ratiox; %[N/m]
DynKyT = StaKyT * ratioy; %[N/m]
MxT = StaKxT / (StaWnxT*2*pi)^2; % [kg]
MyT = StaKyT / (StaWnyT*2*pi)^2; % [kg]

DynCxt = 2 * DynZetaxT * sqrt( DynKxT * MxT ); % [N-s/m]
DynCyT = 2 * DynZetayT * sqrt( DynKyT * MyT ); % [N-s/m]

zetaPro = sqrt(((zetaxPro^2+zetaayPro^2)/2);
ProCxt = 2 * zetaPro * sqrt( DynKxT * MxT ); % [N-s/m]
ProCyT = 2 * zetaPro * sqrt( DynKyT * MyT ); % [N-s/m]

% Workpiece System
DynKxW = KxW / (DynWnxW*2*pi)^2; %[kg]
DynKyW = KyW / (DynWnyW*2*pi)^2; %[kg]

DynCwT = 2 * DynZetawxW * sqrt( KxW * DynMxW ); % [N-s/m]
DynCyW = 2 * DynZetawyW * sqrt( KyW * DynMyW ); % [N-s/m]

ProCwT = 2 * zetaPro * sqrt( KxW * DynMxW ); % [N-s/m]
ProCyW = 2 * zetaPro * sqrt( KyW * DynMyW ); % [N-s/m]

% Variables
ToolRadius = ToolDiameter/2; %[m]
LagAngle = asin((AxialDepth*tan(HelixAngle*pi/180))/ToolRadius)*180/pi;
FeedperTooth = FeedRate/(SpindleSpeed * NT); %[m/teeth]

% Calculation ratio based on solving differential equations
W = max([DynWnxT DynWnyT DynWnxW DynWnyW]);
ratio = 20; % 20 times faster than maximum natural frequency of structures
fs = ratio * W ; % sampling frequency[Hz]
RevStep = fs/(SpindleSpeed/60);
% The force sensors both the Smart Tool and Kistler measure the displacement of
% the tool and workpiece systems instead of the cutting force.

temp = floor(RevStep/NT);
RevStep = temp * NT ;
steps = rev*RevStep;
fs = RevStep*(SpindleSpeed/60); %updated fs
dt = 1/fs;
time = ((1:steps)-1)*dt; %[s]
dphi = 360/RevStep; % Angular increment [deg]

% Calculation Start & End Angle depends on cutting type and radial depth
% For slot cutting, it does not matter whether type 1 or 2.
% first quadrant: +X & +Y, clockwise rotation
if CuttingType == 1
    StartAngle = 180 - acos( (ToolRadius-RadialDepth)/ToolRadius )*180/pi;
    EndAngle = 180 ;
elseif CuttingType == 2
    StartAngle = 0;
    EndAngle = acos( (ToolRadius-RadialDepth)/ToolRadius )*180/pi;
else
    disp('---Wrong Cutting Type---')
    return
end

ActualEndAngle = EndAngle + LagAngle;
% Discretized along the Axial direction
if HelixAngle == 0
    da = AxialDepth;
else
    % discretized axial depth [m]
    da = ToolDiameter*(dphi*pi/180)/(2*tan(HelixAngle*pi/180));
end
% number of steps along tool axis
AxialStep = floor(AxialDepth/da);

% average chip thickness [in]
havg = -FeedperTooth*(cos(EndAngle*pi/180) - cos(StartAngle*pi/180))/
    ((EndAngle*pi/180 - StartAngle*pi/180)*0.0254);

%% Initialize vectors
teeth = zeros(1,NT);
for cnt = 1:NT:
    teeth(cnt) = (cnt-1)*RevStep/NT + ceil(RevStep/4);
end

phi = zeros(1,RevStep);
for cnt = 1:RevStep
    phi(cnt) = (cnt - 1)*dphi;
end
% no negative phi value
surf = zeros(AxialStep,RevStep); % surface for previous tooth
ChipThickness = zeros(1,steps);
ForceX = zeros(1,steps);
ForceY = zeros(1,steps);
ForceT = zeros(1,steps);
ForceR = zeros(1,steps);
FxSmart = zeros(1,steps);
FySmart = zeros(1,steps);
FxKistler = zeros(1,steps);
FyKistler = zeros(1,steps);
FtSmart = zeros(1,steps);
FrSmart = zeros(1,steps);
xpos = zeros(1,steps);
ypos = zeros(1,steps);
tpos = zeros(1,steps);
rpos = zeros(1,steps);

% Euler integration initial conditions
xTool = zeros(1,steps);
yTool = zeros(1,steps);
xWorkpiece = zeros(1,steps);
yWorkpiece = zeros(1,steps);
dxTool = zeros(1,steps);
dyTool = zeros(1,steps);
dxWorkpiece = zeros(1,steps);
dyWorkpiece = zeros(1,steps);
ddxTool = zeros(1,steps);
ddyTool = zeros(1,steps);
ddxWorkpiece = zeros(1,steps);
ddyWorkpiece = zeros(1,steps);
x = xWorkpiece - xTool;
y = yWorkpiece - yTool;

%% Simulation Progress
handle = waitbar(0, 'simulation in progress');
for cnt1 = 1:steps
    waitbar(cnt1/steps, handle)
    format long

    % track to each teeth position
    for cnt2 = 1:NT
        teeth(cnt2) = teeth(cnt2) + 1;
        if teeth(cnt2) > RevStep
            teeth(cnt2) = 1;
        end
    end

    Fx = 0;
    Fy = 0;
    Ft = 0;
    Fr = 0;

    for cnt3 = 1:NT
        for cnt4 = 1:AxialStep
            cntphi = teeth(cnt3) - (cnt4-1); % based on bottom of teeth
            if cntphi < 1 % helix has wrapped through phi = 0 [deg]
                cntphi = cntphi + RevStep;
            end
            phia = phi(cntphi); % angle for given axial disk [deg]
            if (StartAngle <= phia) && (phia <= ActualEndAngle)
                n = -x(cnt1) * sin(phia*pi/180) ...
                -y(cnt1) * cos(phia*pi/180); % [m]
                h = FeedperTooth*sin(phia*pi/180) + ...
                surf(cnt4, cntphi) - n + RO(cnt3); % [m]
                if h < 0
                    dFt = 0;
                    dFr = 0;
                    surf(cnt4, cntphi) = surf(cnt4, cntphi) + ...
                    FeedperTooth*sin(phia*pi/180);
                else
                    dFt = Kt*h*da + Kte*da;
                    dFr = Kr*h*da + Kre*da;
                    surf(cnt4, cntphi) = n - RO(cnt3);
                end
            else
                h = 0;
                dFt = 0;
                dFr = 0;
            end
        end
        Fx = Fx - dFt*cos(phia*pi/180) - dFr*sin(phia*pi/180);
        Fy = Fy + dFt*sin(phia*pi/180) - dFr*cos(phia*pi/180);
        end
    end

    ChipThickness(cnt1) = h;
    ForceX(cnt1) = Fx;
    ForceY(cnt1) = Fy;
    offsetPhia = phia*dphi*round(AxialStep/2); % rearrange to Smart Tool's % gauge.position
\[
\begin{align*}
\text{ForceT}(cntl) &= \text{ForceX}(cntl) \cdot -\cos(\text{offsetPhi} \cdot \pi/180) + \ldots \\
\text{ForceY}(cntl) &= \text{ForceX}(cntl) \cdot \sin(\text{offsetPhi} \cdot \pi/180); \\
\text{ForceR}(cntl) &= \text{ForceX}(cntl) \cdot -\sin(\text{offsetPhi} \cdot \pi/180) + \ldots \\
\text{ForceY}(cntl) &= \text{ForceX}(cntl) \cdot -\cos(\text{offsetPhi} \cdot \pi/180); \\
\end{align*}
\]

% system dynamic with tool-workpiece compliance and process damping
if (StartAngle <= phi) && (phi <= ActualEndAngle)
\[
\begin{align*}
\text{ddxTool}(cntl) &= ( \text{Fx} - \text{ProCxT} \cdot \text{dxTool}(cntl) ) \\
&\quad - \frac{\text{DynKxT} \cdot \text{xTool}(cntl)}{\text{MxT}}; \\
\text{dxTool}(cntl+1) &= \text{dxTool}(cntl) + \text{ddxTool}(cntl) \cdot \text{dt}; \\
\text{xTool}(cntl+1) &= \text{xTool}(cntl) + \text{dxTool}(cntl+1) \cdot \text{dt}; \\
\text{ddyTool}(cntl) &= ( \text{Fy} - \text{ProCyT} \cdot \text{dyTool}(cntl) ) \\
&\quad - \frac{\text{DynKyT} \cdot \text{yTool}(cntl)}{\text{MyT}}; \\
\text{dyTool}(cntl+1) &= \text{dyTool}(cntl) + \text{ddyTool}(cntl) \cdot \text{dt}; \\
\text{yTool}(cntl+1) &= \text{yTool}(cntl) + \text{dyTool}(cntl+1) \cdot \text{dt}; \\
\end{align*}
\]
\[
\begin{align*}
\text{ddxWorkpiece}(cntl) &= ( -\text{Fx} - \text{ProCxW} \cdot \text{dxWorkpiece}(cntl) ) \\
&\quad - \frac{\text{KxW} \cdot \text{xWorkpiece}(cntl)}{\text{DynMxW}}; \\
\text{dxWorkpiece}(cntl+1) &= \text{dxWorkpiece}(cntl) + \text{ddxWorkpiece}(cntl) \cdot \text{dt}; \\
\text{xWorkpiece}(cntl+1) &= \text{xWorkpiece}(cntl) + \text{dxWorkpiece}(cntl+1) \cdot \text{dt}; \\
\text{ddyWorkpiece}(cntl) &= ( -\text{Fy} - \text{ProCyW} \cdot \text{dyWorkpiece}(cntl) ) \\
&\quad - \frac{\text{KyW} \cdot \text{yWorkpiece}(cntl)}{\text{DynMyW}}; \\
\text{dyWorkpiece}(cntl+1) &= \text{dyWorkpiece}(cntl) + \text{ddyWorkpiece}(cntl) \cdot \text{dt}; \\
\text{yWorkpiece}(cntl+1) &= \text{yWorkpiece}(cntl) + \text{dyWorkpiece}(cntl+1) \cdot \text{dt}; \\
\end{align*}
\]
else
\[
\begin{align*}
\text{ddxTool}(cntl) &= ( \text{Fx} - \text{DynCxT} \cdot \text{dxTool}(cntl) ) \\
&\quad - \frac{\text{DynKxT} \cdot \text{xTool}(cntl)}{\text{MxT}}; \\
\text{dxTool}(cntl+1) &= \text{dxTool}(cntl) + \text{ddxTool}(cntl) \cdot \text{dt}; \\
\text{xTool}(cntl+1) &= \text{xTool}(cntl) + \text{dxTool}(cntl+1) \cdot \text{dt}; \\
\text{ddyTool}(cntl) &= ( \text{Fy} - \text{DynCyT} \cdot \text{dyTool}(cntl) ) \\
&\quad - \frac{\text{DynKyT} \cdot \text{yTool}(cntl)}{\text{MyT}}; \\
\text{dyTool}(cntl+1) &= \text{dyTool}(cntl) + \text{ddyTool}(cntl) \cdot \text{dt}; \\
\text{yTool}(cntl+1) &= \text{yTool}(cntl) + \text{dyTool}(cntl+1) \cdot \text{dt}; \\
\end{align*}
\]
\[
\begin{align*}
\text{ddxWorkpiece}(cntl) &= ( -\text{Fx} - \text{DynCxW} \cdot \text{dxWorkpiece}(cntl) ) \\
&\quad - \frac{\text{KxW} \cdot \text{xWorkpiece}(cntl)}{\text{DynMxW}}; \\
\text{dxWorkpiece}(cntl+1) &= \text{dxWorkpiece}(cntl) + \text{ddxWorkpiece}(cntl) \cdot \text{dt}; \\
\text{xWorkpiece}(cntl+1) &= \text{xWorkpiece}(cntl) + \text{dxWorkpiece}(cntl+1) \cdot \text{dt}; \\
\text{ddyWorkpiece}(cntl) &= ( -\text{Fy} - \text{DynCyW} \cdot \text{dyWorkpiece}(cntl) ) \\
&\quad - \frac{\text{KyW} \cdot \text{yWorkpiece}(cntl)}{\text{DynMyW}}; \\
\text{dyWorkpiece}(cntl+1) &= \text{dyWorkpiece}(cntl) + \text{ddyWorkpiece}(cntl) \cdot \text{dt}; \\
\text{yWorkpiece}(cntl+1) &= \text{yWorkpiece}(cntl) + \text{dyWorkpiece}(cntl+1) \cdot \text{dt}; \\
\end{align*}
\]
end
\[
\begin{align*}
\text{x}(cntl+1) &= \text{xTool}(cntl+1) - \text{xWorkpiece}(cntl+1); \\
\text{y}(cntl+1) &= \text{yTool}(cntl+1) - \text{yWorkpiece}(cntl+1); \\
\end{align*}
\]
\[
\begin{align*}
\text{xpos}(cntl) &= \text{x}(cntl); \\
\text{ypos}(cntl) &= \text{y}(cntl); \\
\text{tpos}(cntl) &= \text{xpos}(cntl) \cdot -\cos(\text{offsetPhi} \cdot \pi/180) \\
&\quad + \text{ypos}(cntl) \cdot \sin(\text{offsetPhi} \cdot \pi/180); \\
\text{rpos}(cntl) &= \text{xpos}(cntl) \cdot -\sin(\text{offsetPhi} \cdot \pi/180) \\
&\quad + \text{ypos}(cntl) \cdot -\cos(\text{offsetPhi} \cdot \pi/180); \\
\end{align*}
\]
% Force Measurements of the Smart Tool
\[
\begin{align*}
\text{FxSmart}(cntl) &= \text{xTool}(cntl) \cdot \text{DynKxT}; \\
\text{FySmart}(cntl) &= \text{yTool}(cntl) \cdot \text{DynKyT}; \\
\end{align*}
\]
\[ \text{FtSmart(cntl)} = \text{FxSmart(cntl)} \cdot \cos(\text{offsetPhia} \cdot \pi/180) + \text{FxSmart(cntl)} \cdot \sin(\text{offsetPhia} \cdot \pi/180); \]
\[ \text{FrSmart(cntl)} = \text{FxSmart(cntl)} \cdot -\sin(\text{offsetPhia} \cdot \pi/180) + \text{FxSmart(cntl)} \cdot -\cos(\text{offsetPhia} \cdot \pi/180); \]

\% Force Measurements of the Kistler
FxKistler(cntl) = \text{KxW} \cdot -x\text{Workpiece(cntl)};
FyKistler(cntl) = \text{KyW} \cdot -y\text{Workpiece(cntl)};

end

\% resultant forces (net forces)
\text{ForceN} = (\text{ForceX}^2 + \text{ForceY}^2)^{0.5}; \% [\text{N}]
\text{ForceNKistler} = \sqrt{\text{FxKistler}^2 + \text{FyKistler}^2}; \%[\text{N}]
\text{ForceNSmart} = \sqrt{\text{FtSmart}^2 + \text{FrSmart}^2}; \%[\text{N}]

\text{close(handle);} \% close progress bar

\% ForceX, ForceY, ForceT, ForceR: Cutting Forces
\% FxKistler, FyKistler: Force measurements of the Kistler
\% FtSmart, FrSmart: Force measurements of the Smart Tool
\% Force: resultant cutting forces (net forces)
\% ForceNKistler: net force measurements of the Kistler
\% ForceNSmart: net force measurements of the Smart Tool
APPENDIX B

SOLUTION OF CHARACTERISTIC EQUATION

Using Model of System:

\[ h_{avg} \rightarrow \frac{K_s \cdot G}{1 + (1 - e^{-T \cdot S}) \cdot K_s \cdot G} \]

\[ x, y \rightarrow k_m \rightarrow F_m \]

\[ \frac{F_m}{h_{avg}} = \frac{K_s \cdot G \cdot k_m}{1 + (1 - e^{-T \cdot S}) \cdot K_s \cdot G} \]

Characteristic Equation:

\[ 1 + (1 - e^{-T \cdot S}) \cdot K_s \cdot G = 0 \]

\[ \left( \frac{S^2}{\omega_n^2} + \frac{2 \cdot \zeta \cdot S + 1}{\omega_n} \right) + (1 - e^{-T \cdot S}) \cdot K_s \cdot k_c = 0 \]

\[ S^2 + 2 \cdot \zeta \cdot \omega_n \cdot S + \omega_n^2 + \omega_n^2 \cdot (1 - e^{-T \cdot S}) \cdot K_s \cdot k_c = 0 \]

Using Pade' 1st-order approximation (\( e^{-T \cdot S} \approx \frac{T \cdot S + 1}{T \cdot S + 1} \))

\[ S^2 + 2 \cdot \zeta \cdot \omega_n \cdot S + \omega_n^2 + \omega_n^2 \cdot K_s \cdot k_c \cdot \left( 1 - \frac{T \cdot S + 2}{T \cdot S + 2} \right) = 0 \]

Solutions:

\[ S_1 = \sqrt[3]{\sqrt{A + B} - \frac{C}{\sqrt[3]{A + B}}} - D \]

\[ S_2 = \frac{3 \sqrt[3]{A + B} + C}{2 \cdot \sqrt[3]{A + B}} - D + \frac{\sqrt{3} \cdot \left( \sqrt[3]{A + B} + \frac{C}{\sqrt[3]{A + B}} \right)}{2} \cdot i \]

\[ S_3 = -\frac{3 \sqrt[3]{A + B} + C}{2 \cdot \sqrt[3]{A + B}} - D - \frac{\sqrt{3} \cdot \left( \sqrt[3]{A + B} + \frac{C}{\sqrt[3]{A + B}} \right)}{2} \cdot i \]

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where A, B, C, and D are defined:

\[
A = \frac{\omega_n^6}{27} + \frac{\omega_n^6 \cdot \zeta^2}{27} + \frac{8 \cdot \omega_n^5 \cdot \zeta}{27 \cdot T} + \frac{8 \cdot \omega_n^5 \cdot \zeta^3}{27 \cdot T^3} + \frac{8 \cdot \omega_n^4 \cdot \zeta^2}{27 \cdot T^2} + \frac{8 \cdot \omega_n^4 \cdot \zeta^4}{27 \cdot T^4} + \frac{16 \cdot \omega_n^4 \cdot \zeta^4}{27 \cdot T^6} + \ldots
\]

\[
+ \frac{32 \cdot \omega_n^3 \cdot \zeta}{27 \cdot T^3} + \frac{32 \cdot \omega_n^3 \cdot \zeta^3}{27 \cdot T^6} + \frac{16 \cdot \omega_n^2}{27 \cdot T^4} + \frac{16 \cdot \omega_n^2 \cdot \zeta^2}{27 \cdot T^4} + \frac{8 \cdot K_S^2 \cdot k_c^2 \cdot \omega_n^6}{27} + \ldots
\]

\[
+ \frac{4 \cdot K_S^2 \cdot k_c^2 \cdot \omega_n^6}{9} - \frac{4 \cdot K_S^2 \cdot k_c^2 \cdot \omega_n^6 \cdot \zeta^2}{27 \cdot T} - \frac{4 \cdot K_S \cdot k_c \cdot \omega_n^6}{27 \cdot T^2} + \frac{4 \cdot K_S \cdot k_c \cdot \omega_n^5 \cdot \zeta}{27 \cdot T} + \ldots
\]

\[
- \frac{16 \cdot K_S \cdot k_c \cdot \omega_n^5 \cdot \zeta^3}{27 \cdot T} - \frac{40 \cdot K_S \cdot k_c \cdot \omega_n^4}{27 \cdot T^2} - \frac{64 \cdot K_S \cdot k_c \cdot \omega_n^4 \cdot \zeta^2}{27 \cdot T^2} + \ldots
\]

\[
- \frac{16 \cdot K_S \cdot k_c \cdot \omega_n^4 \cdot \zeta^3}{27 \cdot T^3}
\]

\[
B = \frac{\omega_n^3 \cdot \zeta}{3} - \frac{8 \cdot \omega_n^2}{27 \cdot T^3} - \frac{2 \cdot \omega_n^2}{3 \cdot T} - \frac{8 \cdot \omega_n^3 \cdot \zeta^3}{27 \cdot T^3} + \frac{4 \cdot \omega_n^2 \cdot \zeta^2}{9 \cdot T} + \frac{4 \cdot \omega_n \cdot \zeta}{9 \cdot T^2} + \ldots
\]

\[
+ \frac{2 \cdot K_S \cdot k_c \cdot \omega_n^3 \cdot \zeta}{3 \cdot T} + \frac{2 \cdot K_S \cdot k_c \cdot \omega_n^2}{3 \cdot T}
\]

\[
C = \sqrt[3]{\frac{-\omega_n^2}{9 \cdot T} - \frac{2 \cdot \omega_n^2 \cdot \zeta^2}{9} + \frac{2 \cdot K_S \cdot k_c \cdot \omega_n^2}{3} + \frac{4 \cdot \omega_n \cdot \zeta}{9 \cdot T}}
\]

\[
D = \frac{2 \cdot \omega_n \cdot \zeta \cdot T + 2}{3 \cdot T}
\]

Q) Can \( \omega_n \) and \( \zeta \) be generalized from \( S_2 \) and \( S_3 \) with following format?

\[-\omega_n \zeta \pm i \cdot \omega_n \cdot \sqrt{1 - \zeta^2} \]

A) The exact solution is remained as future work. In this work, we assume that

\[
\lim_{T \to 0} (\omega_n^2 \cdot K_S \cdot k_c \cdot (1 - \frac{T \cdot S^2}{T \cdot S^2 + 2})) = 0 \text{ considering the high spindle speed cutting.}
\]

Therefore, as a good approximation, parameters from experimentally measured oscillations can be used to determine the tool system parameters.
APPENDIX C

STATIC STIFFNESS OF THE TOOL SYSTEM

Raw Calibration Data

$$\begin{array}{|c|c|}
\hline
F_x [N] & F_y [N] \\
\hline
2.256775E+01 & 1.841954E+00 \\
5.974452E+01 & 3.332520E+01 \\
1.009623E+02 & 6.745338E+01 \\
1.399231E+02 & 1.088206E+02 \\
1.766052E+02 & 1.435216E+02 \\
2.154888E+02 & 1.791509E+02 \\
\hline
\end{array}$$

Table C.1: Linear Calibration of Tool System

$$\begin{array}{|c|c|c|c|}
\hline
\Delta F_x [N] & \Delta x_{workpiece} [m] & \Delta x_{tool} [m] & k_{x\_workpiece\_sta} [N/m] \\
\hline
3.717677E+01 & -1.040225E-07 & 1.259598E-05 & 2.951479E+06 \\
4.121780E+01 & -1.153296E-07 & 1.258467E-05 & 3.275239E+06 \\
3.896077E+01 & -1.090143E-07 & 1.259099E-05 & 3.094339E+06 \\
3.668213E+01 & -1.026385E-07 & 1.259736E-05 & 2.911890E+06 \\
3.888356E+01 & -1.087982E-07 & 1.259120E-05 & 3.088153E+06 \\
\hline
\end{array}$$

$k_{x\_workpiece\_sta} = 3.093 [N/m]$

Table C.2: Stiffness of Tool System on X

$$\begin{array}{|c|c|c|c|}
\hline
\Delta F_y [N] & \Delta y_{workpiece} [m] & \Delta y_{tool} [m] & k_{y\_workpiece\_sta} [N/m] \\
\hline
3.148324E+01 & -6.345086E-08 & 1.263655E-05 & 2.491443E+06 \\
3.412819E+01 & -6.878145E-08 & 1.263122E-05 & 2.701929E+06 \\
4.136721E+01 & -8.37087E-08 & 1.261663E-05 & 3.278785E+06 \\
3.470103E+01 & -6.993594E-08 & 1.263006E-05 & 2.747949E+06 \\
3.562927E+01 & -7.180671E-08 & 1.262819E-05 & 2.821407E+06 \\
\hline
\end{array}$$

$k_{y\_tool\_sta} = 2.874e 6 [N/m]$

Table C.3: Stiffness of Tool System on Y
Problem Statement:

Can the tool system parameters \((k, \omega_n, \alpha, \text{and } \zeta)\) in \(X\) and \(Y\) directions be obtained from the tool system parameters in radial and tangential directions without the force conversion?

Assumption:

The tool system has symmetric geometry \((k=k_x=k_y, \omega_n=\omega_{n,x}=\omega_{n,y}, \zeta=\zeta_x=\zeta_y)\).

Given: \(F = k \cdot \left( \frac{1}{\omega_n^2} \cdot \hat{n} + \frac{2 \zeta}{\omega_n} \cdot \hat{n} + n \right)\)

\[ F_n = \sqrt{F_x^2 + F_y^2} = \sqrt{F_t^2 + F_r^2} \quad n = \sqrt{x^2 + y^2} = \sqrt{t^2 + r^2} \]

With transient matrix

\[
\begin{bmatrix}
[F_x] \\
[F_y] \\
[F_r]
\end{bmatrix} = \begin{bmatrix}
-\cos \theta & -\sin \theta \\
\sin \theta & -\cos \theta
\end{bmatrix} \cdot \begin{bmatrix}
[F_x] \\
[F_y] \\
[F_r]
\end{bmatrix}
\]

\[
\begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix}
-\cos \theta & -\sin \theta \\
\sin \theta & -\cos \theta
\end{bmatrix} \cdot \begin{bmatrix}
t \\
r
\end{bmatrix}
\]

\[ x = -t \cdot \cos \theta - r \cdot \sin \theta \quad (1) \]
\[ y = t \cdot \sin \theta - r \cdot \cos \theta \quad (2) \]
Solution

1) $\dot{x}$ and $\dot{y}$ are derived

\[
\begin{align*}
\dot{x} &= -t \cdot \cos \theta + t \cdot \dot{\theta} \cdot \sin \theta - r \cdot \sin \theta - r \cdot \dot{\theta} \cdot \cos \theta \\
\dot{y} &= t \cdot \sin \theta + t \cdot \dot{\theta} \cdot \cos \theta - r \cdot \cos \theta + r \cdot \dot{\theta} \cdot \sin \theta
\end{align*}
\]

Since $\dot{\theta} \approx 0$,

\[
\begin{align*}
\dot{x} &= -t \cdot \cos \theta - r \cdot \sin \theta \\
\dot{y} &= t \cdot \sin \theta - r \cdot \cos \theta
\end{align*}
\]

(3) (4)

2) $\ddot{x}$, and $\ddot{y}$ should be derived

\[
\begin{align*}
\ddot{x} &= -\ddot{t} \cdot \cos \theta + \dot{t} \cdot \dot{\theta} \cdot \sin \theta - \dot{r} \cdot \sin \theta - \dot{r} \cdot \dot{\theta} \cdot \cos \theta \\
\ddot{y} &= \ddot{t} \cdot \sin \theta + \dot{t} \cdot \dot{\theta} \cdot \cos \theta - \dot{r} \cdot \cos \theta + \dot{r} \cdot \dot{\theta} \cdot \sin \theta
\end{align*}
\]

Since $\dot{\theta} \approx 0$,

\[
\begin{align*}
\ddot{x} &= -\ddot{t} \cdot \cos \theta - \ddot{r} \cdot \sin \theta \\
\ddot{y} &= \ddot{t} \cdot \sin \theta - \ddot{r} \cdot \cos \theta
\end{align*}
\]

(5) (6)

3) $F_x$ and $F_y$ can be expressed in terms of $t$ and $r$ with Equation 1, 2, 3, 4, 5 and 6.

\[
\begin{align*}
k \cdot \left( \frac{1}{\omega_n^2} \cdot \ddot{x} + \frac{2 \cdot \zeta}{\omega_n} \cdot \dot{x} + x \right) \\
&= k \cdot \left( \frac{1}{\omega_n^2} \cdot (-\dddot{t} \cdot \cos \theta - \dot{r} \cdot \sin \theta) + \frac{2 \cdot \zeta}{\omega_n} \cdot (-\dot{t} \cdot \cos \theta - \ddot{r} \cdot \sin \theta) + (-t \cdot \cos \theta - r \cdot \sin \theta) \right) \\
&= -k_t \cdot \left( \frac{1}{\omega_{n,t}^2} \cdot \dddot{t} + \frac{2 \cdot \zeta_t}{\omega_{n,t}} \cdot \dot{t} + t \right) \cdot \cos \theta - k_r \cdot \left( \frac{1}{\omega_{n,r}^2} \cdot \dddot{r} + \frac{2 \cdot \zeta_r}{\omega_{n,r}} \cdot \dot{r} + r \right) \cdot \sin \theta
\end{align*}
\]

\[
\begin{align*}
k \cdot \left( \frac{1}{\omega_n^2} \cdot \ddot{y} + \frac{2 \cdot \zeta}{\omega_n} \cdot \dot{y} + y \right) \\
&= k \cdot \left( \frac{1}{\omega_n^2} \cdot (\dddot{t} \cdot \sin \theta - \dddot{r} \cdot \cos \theta) + \frac{2 \cdot \zeta_t}{\omega_n} \cdot (\dot{t} \cdot \sin \theta - \dot{r} \cdot \cos \theta) + (t \cdot \sin \theta - r \cdot \cos \theta) \right) \\
&= k_r \cdot \left( \frac{1}{\omega_{n,t}^2} \cdot \dddot{t} + \frac{2 \cdot \zeta_t}{\omega_{n,t}} \cdot \dot{t} + t \right) \cdot \sin \theta - k_r \cdot \left( \frac{1}{\omega_{n,r}^2} \cdot \dddot{r} + \frac{2 \cdot \zeta_r}{\omega_{n,r}} \cdot \dot{r} + r \right) \cdot \cos \theta
\end{align*}
\]
\[
(k - k_t) \cdot t \cdot \cos \theta + \left( \frac{2 \cdot \zeta - 2 \cdot \zeta_t}{\omega_n - \omega_{nt}} \right) \cdot \dot{t} \cdot \cos \theta + \left( \frac{1}{\omega_{nt}^2} - \frac{1}{\omega_{nt}^2} \right) \cdot \dot{t} \cdot \cos \theta \\
+(-k + k_r) \cdot r \cdot \sin \theta + \left( \frac{-2 \zeta + 2 \zeta_t}{\omega_n - \omega_{nt}} \right) \cdot \dot{r} \cdot \sin \theta + \left( \frac{1}{\omega_{nt}^2} + \frac{1}{\omega_{nt}^2} \right) \cdot \dot{r} \cdot \sin \theta = 0 \quad (7)
\]

\[
(-k + k_r) \cdot t \cdot \sin \theta + \left( \frac{-2 \zeta + 2 \zeta_t}{\omega_n - \omega_{nt}} \right) \cdot \dot{t} \cdot \sin \theta + \left( \frac{1}{\omega_{nt}^2} + \frac{1}{\omega_{nt}^2} \right) \cdot \dot{t} \cdot \sin \theta \\
+(k - k_r) \cdot r \cdot \cos \theta + \left( \frac{2 \cdot \zeta - 2 \cdot \zeta_t}{\omega_n - \omega_{nt}} \right) \cdot \dot{r} \cdot \cos \theta + \left( \frac{1}{\omega_{nt}^2} - \frac{1}{\omega_{nt}^2} \right) \cdot \dot{r} \cdot \cos \theta = 0 \quad (8)
\]

The solution of Equation 7 and 8 which satisfies within all \( t, r \) and \( \theta \) are

\[ k = k_t = k_r \]

\[ \omega_n = \omega_{nt} = \omega_{nr} \]

\[ \zeta = \zeta_t = \zeta_r \]

Therefore, the tool system parameters in X and Y directions should be defined the average value of the tool system parameters in radial and tangential directions with an assumption that the tool system has symmetric geometry.
APPENDIX E

EXAMAPLE MATLAB CODE OF SIGNAL SEPARATION AND APPLIED LINEAR PREDICTIVE CODING (LPC)

Example MATLAB Code for Separation In-Cut and Out-of-Cut and Applied LPC

Here is example code. Due to the oscillation, separation of signal is not applied whole cases. In addition, the proper order of LPC should be chose by user from PSD. There is not general use for every signal and every system.

```matlab
% % with the Extended Kalman Filter for the Smart Tool % automation after separation force

clear all
close all
clc

global Ad Gd H Hhat IC; % % require for the Kalman filter application

% % load data file
NT = 1;
load 'Smart/half_26.r.mat'

% % measurements [bits] to force
Force = [s(i:end),s(1:i-1)];

BitsperLbf = 1.64; %[bits / lbf]
dt = strain_time(2)-strain_time(1);
fs = 1/dt; %[Hz]
Force = Force/BitsperLbf; % [bits]->[lbf]
Force = Force * 4.44822; % [lbf]->[N]

% % offset Range & force separation by user inspection % Separation: noise and signal
offsetRange = 2.5e4; % input by user
Start = 184250;
End = 259092;

% % Compensate offset
offset = mean(Force(1,1:offsetRange));
Force = Force - offset;

noise = Force(1,1:offsetRange);
time = strain_time(1,Start:End) - strain_time(1,Start);
force = Force(1,Start:End)
```

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%% spindle speed
[frequency magnitude]=FD(force,fs);
[pks locs]=findpeaks(magnitude,'sortstr','descend');
FreTooth = frequency(min(locs(1:5))); %tooth passing frequency[Hz]
SpindleSpeed = 60*FreTooth;
RevStep = round(fs/(SpindleSpeed/60));

%% Identification out-of-cut
cycle = 30;
initial = zeros(1,cycle);
initial(1) = 50158; % user input
decay(:,1)= force(1,initial(1):initial(1)+round(RevStep*3/4*0.95));
% the length of decay depends on the radial depth

%% Out-of-Cut Signal Separation
for i=1:(cycle-1)
    cnt = find(force(1,initial(i)+RevStep-5:initial(i)+RevStep+5) == ...
                max(force(1,initial(i)+RevStep-5:initial(i)+RevStep+5)));
    initial(i+1) = initial(i)+RevStep-5+cnt-1;
    decay(:,i+1) = force(1,initial(i+1):initial(i+1)+round(RevStep*3/4*0.95));
end

%% ref_R: original; sep_R: separated
ref_R = force(1,initial(1):initial(30)+round(RevStep*3/4*0.95));
sep_R = zeros(1,length(ref_R));
[pks locs] = findpeaks(-decay(:,1));
templ = length(decay(locs:end,1));
sep_R(1,locs(1):locs(1)+ templ-1) = decay(locs:end,1);
clear pks locs templ;

%% analyze after 2nd peak, free vibration
for i=1:(cycle-1)
    [pks locs] = findpeaks(-decay(:,i+1));
templ = length(decay(locs:end,(i+1)));
temp2 = initial(i+1)-initial(1);
    sep_R(1,temp2+locs(1):temp2+locs(1)+templ-1) = decay(locs:end,i+1);
clear pks locs templ temp2
end

%% Confirm the order of LPC
h = spectrum.periodogram('hann'); %spectrum object (periodogram)
psd_est_lside=psd(h,sep_R,'spectrume_type','onesided','Fs',fs);
fre_Psd = psd_est_lside.Frequencies;
Psd = pow2db(psd_est_lside.data);
Hs=spectrum.welch('hann');
% plot
figure(1)
plot(fre_Psd/1000, Psd,'r--')
hold on
grid on
xlim([0 max(fre_Psd/1000)])
psd(Hs,sep_R,'Fs',fs)
title('PSD Estimate, Out-of-Cut')
legend('Periodogram','Welch')
hold off

clear psd_est_lside fre_Psd Psd

%% Linear Prediction Coding (Out-of-Cut)
order = 10;
for i=1:cycle
    [pks locs] = findpeaks(-decay(:,i));
    S = decay(locs:end,1);
[Ak Ve] = lpc(S.order); % Ak:coefficients, Ve:error variance
Zpoles = roots(Ak);
sysd = tf(1,Ak,1/fs);
syosc = d2c(sysd,'soh');
POLES = pole(syosc);

cnt = 0;
for j=1:length(POLES)
    if imag(POLES(j)) ~= 0
        cnt = cnt+1;
        pairPoles(cnt,1) = POLES(j,1);
    end
end
dominantP = pairPoles(real(pairPoles) == max(real(pairPoles)));
wd(i,1) = imag((dominantP(1)))/(2*pi);
wn(i,1) = (sqrt(real(dominantP(1))^2+imag(dominantP(1))^2))/(2*pi);
zeta(i,1) = (sqrt(real(dominantP(1))^2-wd(i,1)^2))/wn(i,1);
for j=1:(length(POLES)/2)
    wn_mode(i,j) = (sqrt(real(POLES( (length(POLES)-2*(j-1)),1 )))^2+...)
    imag(POLES( (length(POLES)-2*(j-1)),1 )))^2))/(2*pi);
end
clear pairPoles

dominantP = pairPoles(real(pairPoles) == max(real(pairPoles)));
wd(i,1) = imag((dominantP(1)))/(2*pi);
wn(i,1) = (sqrt(real(dominantP(1))^2+imag(dominantP(1))^2))/(2*pi);
zeta(i,1) = (sqrt(real(dominantP(1))^2-wd(i,1)^2))/wn(i,1);
for j=1:(length(POLES)/2)
    wn_mode(i,j) = (sqrt(real(POLES( (length(POLES)-2*(j-1)),1 )))^2+...)
    imag(POLES( (length(POLES)-2*(j-1)),1 )))^2))/(2*pi);
end
clear pairPoles

%% statistics of Out-of-Cut parameters
% the root mean square and 95% of confidence interval
mean_wn = sqrt(sum(wn.^2)/cycle);
error_wn = 1.96*std(wn);
mean_zeta = sqrt(sum(zeta.^2)/cycle);
error_zeta = 1.96*std(zeta);
for i=1:(order/2)
    error_mode(1,i) = std(wn_mode(:,i));
    mean_mode(1,i) = sqrt(sum(wn_mode(:,i).^2)/cycle);
end

%% average the decay after 2nd vibration, size should be checked
for i=1:cycle
    [pks locs]=findpeaks(-decay(:,i));
    temp_signal(:,i) = decay(locs(1):locs(1)+157,i);
    clear pks locs
end
for i=1:length(temp_signal)
    signal(i,1) = mean(temp_signal(i,:));
end

clear temp_signal

%% plot
figure(2);
plot(dt*(0:length(signal)-1),signal)
title('Out-of-cut Profile')
xlabel('Time [s]')
ylabel('Force [N]')
xlim([0 max(dt*(0:length(signal)-1))])

%% PSD from averaged decay
psd_est_lside=psd(h,signal,'spectrumtype','onesided','Fs',fs);
fre_Psd = psd_est_lside.Frequencies;
Psd = pow2db(psd_est_lside.data);
figure(3)
plot(fre_Psd/1000, Psd)
hold on
grid on
xlim([0 max(fre_Psd/1000)])

%% parameters static [input]
k_sta = 2.9855e+06;
wn_sta = 639.4218;

k = k_sta*mean_wn^2/wn_sta^2;

%% kalman filter: estimated actual cutting forces
Fm = [time' force'];
[Fchat, Fmhat] = estimated(Fm,noise,NT,k,mean_wn,mean_zeta);

%% plot
figure(4)
plot(time,force,'b:');
hold on
plot(Fchat.time,Fchat.signals.values,'r--')
title({'Estimate Cutting Force by Kalman Filter'; 'Smart Tool on Radial'})
legend({'Experimental Data', 'Estimated Cutting Force'})
xlabel('time [s]')
ylabel('Force [N]')

%% separation of in-cut vibration: separated in-cut profile and
% de-trended cutting force
for i=1:cycle
    decay2(:,i) = force(1,(initial(i)-round(RevStep*1/4*1.05)):initial(i)-1));
    pseudo(:,i) = Fchat.signals.values((initial(i)- ... 
       round(RevStep*1/4*1.05)):initial(i)-1),1);
    incut(:,i)= decay2(:,i)-pseudo(:,i);
end

%%
ref_R_2 = force(1,(initial(1)-round(RevStep*1/4*1.05)):initial(30)-1));
sep_R_2 = zeros(1,length(ref_R_2));
sep_R_p = zeros(1,length(ref_R_2));
sep_R_i = zeros(1,length(ref_R_2));

%%
for i=1:cycle
    temp = length(decay2);
    sep_R_2(1,initial(i)-(initial(1)-1):initial(i)-initial(1)+temp) ... 
        = decay2(:,i);
    sep_R_p(1,initial(i)-(initial(1)-1):initial(i)-initial(1)+temp) ... 
        = pseudo(:,i);
    sep_R_i(1,initial(i)-(initial(1)-1):initial(i)-initial(1)+temp) ... 
        = incut(:,i);
end

clear temp

%% confirm the order of LPC
psd_est_lside=psd(h,sep_R_i,'spectrumtype','onesided','Fs',fs);
fre_Psd = psd_est_lside.Frequencies;
Psd = pow2db(psd_est_lside.data); 
Hs=spectrum.welch('hann');
figure(5)
plot(fre_Psd/1000, Psd,'r--')
hold on
grid on
xlim([0 max(fre_Psd/1000)])
psd(Hs,sep_R_i,'Fs',fs)
title('PSD Estimate, In-Cut')
legend('Periodogram','Welch')
hold off

clear psd_est_iside fre_Psd Psd

%% Linear Prediction Coding (In-cut)
order = 10;
for i=1:cycle
    S = incut(:,i);
    [Ak Ve] = lpc(S,order); % Ak: coefficients, Ve: error variance
    Zpoles = roots(Ak);
    sysd = tf(1,Ak,1/fs);
    sysc = d2c(sysd,'zoh');
    POLES = pole(sysc);
    cnt = 0;
    for j=1:length(POLES)
        if imag(POLES(j)) ~= 0
            cnt = cnt+1;
            pairPoles(cnt,1) = POLES(j,1);
        end
    end
    dominantP = pairPoles(real(pairPoles) == max(real(pairPoles)));
    wd_2(i,1) = imag((dominantP(1))/2*pi);
    wn_2(i,1) = (sqrt(real(dominantP(1))^2+imag(dominantP(1))^2))/(2*pi);
    zeta_2(i,1) = sqrt((wn_2(i,1)^2-wd_2(i,1)^2))/wn_2(i,1);
    for j=1:(length(pairPoles)/2)
        temp_wn_mode_2(i,j) = (sqrt(real(pairPoles( (length(pairPoles)-2*(j-1)),1 )))^2+imag(pairPoles( (length(pairPoles)-2*(j-1)),1 ))^2))/2*pi);
    end
end
%
%% statistics of In-Cut parameters
% the root mean square and 95% of confidence interval
mean_zeta_2 = sqrt(sum(zeta_2.^2)/cycle);
error_zeta_2 = 1.96*std(zeta_2);
mean_wn_2 = sqrt(sum(wn_2.^2)/cycle);
error_wn_2 = 1.96*std(wn_2);
for i=1:(order/2)
    error_mode_2(1,i) = std(wn_mode_2(:,i));
    mean_mode_2(1,i) = sqrt(sum(wn_mode_2(:,i).^2)/cycle);
end
%
%% average signal
for i=1:length(decay_2)
    sig(i,1) = mean(decay_2(i,:));
    sig_1(i,1) = mean(pseudo(i,:));
    sig_2(i,1) = mean(incut(i,:));
end
%
figure(6);
plot(dt*(0:length(sig)-1),sig)
hold on
plot(dt*(0:length(sig)-1),sig_1,'b:');
plot(dt*(0:length(sig)-1),sig_2,'b-');

150
hold off

title('In-Cut Profile ')
xlabel('Time [s]')
ylabel('Force [N]')
xlim([0 max(dt*(0:length(sig)-1)])
legend('Experiment Data','Psuedo Force','Vibration')

%% PSD

psd_est_1sided = psd(h,sig_2,'spectrumtype','onesided','Fs',fs);
fre_Psd = psd_est_1sided.Frequencies;
Psd = pow2db(psd_est_1sided.data);
figure(3)
plot(fre_Psd/1000,Psdf,'r')
hold off
grid on
xlim([0 max(fre_Psd/1000)])

% title('Power Spectral Density')
legend('Out-of-cut','In-Cut')
ylabel('Power/frequency (dB/Hz)')
xlabel('Frequency (kHz)')
APPENDIX F

IDENTIFIED SYSTEM PARAMETERS AND VALIDATION OF SIMULATION RESULTS WITH EXPERIMENTAL DATA

Comparison of the Simulation Results to Stable Experimental Cuts

Table F.1: Cases for Comparison, \( h_{avg} = 0.001 \text{ [in]} \)

<table>
<thead>
<tr>
<th>Axial Depth of Cut</th>
<th>Radial Immersion</th>
<th>Spindle Speed [RPM]</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.175 [mm] = 0.125[in]</td>
<td>Half</td>
<td>600</td>
<td>stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2600</td>
<td>stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3600</td>
<td>stable</td>
</tr>
<tr>
<td></td>
<td>Slot</td>
<td>600</td>
<td>stable</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3600</td>
<td>stable</td>
</tr>
</tbody>
</table>

The case with half immersion at 2600 [RPM] is already discussed in Chapter 4 in terms of validation with stable experimental cut and chatter prediction.

Here is cutting coefficients (cutter and material pair) for other experimental cuts.

\[
\begin{align*}
K_{rc} & = 322e6 \text{ [N/m}^2]\] \\
K_{re} & = 10.7e3 \text{ [N/m]} \\
K_{tc} & = 856e6 \text{ [N/m}^2]\] \\
K_{te} & = 21.2e3 \text{ [N/m]} 
\end{align*}
\]

Figures F.1, F.2, F.3, and F.4 show the simulation results and the experimental data for variety cuts based on Table F.1. Same as previous case, the validation is performed in terms of the maximum peak force, the force profile, and the force measurements frequency contents.
Table F.3: System Parameters, Half Immersion, 600 [RPM]

<table>
<thead>
<tr>
<th></th>
<th>Workpiece System</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Out-of-cut</td>
<td>In-Cut</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\zeta_{x,workpiece_dyn}$</td>
<td>$\omega_{n_x,workpiece_dyn}$ [Hz]</td>
<td>$\zeta_{x,process}$</td>
</tr>
<tr>
<td>avg</td>
<td>0.248</td>
<td>913</td>
<td>0.25696</td>
</tr>
<tr>
<td>stdv</td>
<td>0.0514</td>
<td>54.57</td>
<td>0.12874</td>
</tr>
<tr>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Out-of-cut</td>
<td>In-Cut</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\zeta_{y,workpiece_dyn}$</td>
<td>$\omega_{n_y,workpiece_dyn}$ [Hz]</td>
<td>$\zeta_{y,process}$</td>
</tr>
<tr>
<td>avg</td>
<td>0.204</td>
<td>1024</td>
<td>0.23361</td>
</tr>
<tr>
<td>stdv</td>
<td>0.0920</td>
<td>72.18</td>
<td>0.11892</td>
</tr>
<tr>
<td>Tool System</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Out-of-cut</td>
<td>In-Cut</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\zeta_{r,tool_dyn}$</td>
<td>$\omega_{n_r,tool_dyn}$ [Hz]</td>
<td>$\zeta_{r,process}$</td>
</tr>
<tr>
<td>avg</td>
<td>0.065</td>
<td>645</td>
<td>0.19456</td>
</tr>
<tr>
<td>stdv</td>
<td>0.0151</td>
<td>4.41</td>
<td>0.07537</td>
</tr>
<tr>
<td>T</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Out-of-cut</td>
<td>In-Cut</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\zeta_{r,tool_dyn}$</td>
<td>$\omega_{n_r,tool_dyn}$ [Hz]</td>
<td>$\zeta_{r,process}$</td>
</tr>
<tr>
<td>avg</td>
<td>0.073</td>
<td>646</td>
<td>0.26700</td>
</tr>
<tr>
<td>stdv</td>
<td>0.0106</td>
<td>5.10</td>
<td>0.10531</td>
</tr>
</tbody>
</table>

Table F.4: System Parameters, Half Immersion, 3600 [RPM]

<table>
<thead>
<tr>
<th></th>
<th>Workpiece System</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Out-of-cut</td>
<td>In-Cut</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\zeta_{x,workpiece_dyn}$</td>
<td>$\omega_{n_x,workpiece_dyn}$ [Hz]</td>
<td>$\zeta_{x,process}$</td>
</tr>
<tr>
<td>avg</td>
<td>0.047</td>
<td>866</td>
<td>0.11641</td>
</tr>
<tr>
<td>stdv</td>
<td>0.0023</td>
<td>5.91</td>
<td>0.00432</td>
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<tr>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Out-of-cut</td>
<td>In-Cut</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\zeta_{y,workpiece_dyn}$</td>
<td>$\omega_{n_y,workpiece_dyn}$ [Hz]</td>
<td>$\zeta_{y,process}$</td>
</tr>
<tr>
<td>avg</td>
<td>0.101</td>
<td>1054</td>
<td>0.18891</td>
</tr>
<tr>
<td>stdv</td>
<td>0.0182</td>
<td>29.54</td>
<td>0.07904</td>
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<td>Tool System</td>
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<tr>
<td>R</td>
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<td></td>
<td>Out-of-cut</td>
<td>In-Cut</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\zeta_{r,tool_dyn}$</td>
<td>$\omega_{n_r,tool_dyn}$ [Hz]</td>
<td>$\zeta_{r,process}$</td>
</tr>
<tr>
<td>avg</td>
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<td>605</td>
<td>0.07016</td>
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<tr>
<td>stdv</td>
<td>0.0069</td>
<td>4.81</td>
<td>0.01593</td>
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<tr>
<td></td>
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<td>In-Cut</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\zeta_{r,tool_dyn}$</td>
<td>$\omega_{n_r,tool_dyn}$ [Hz]</td>
<td>$\zeta_{r,process}$</td>
</tr>
<tr>
<td>avg</td>
<td>0.113</td>
<td>598</td>
<td>0.18752</td>
</tr>
<tr>
<td>stdv</td>
<td>0.0059</td>
<td>13.17</td>
<td>0.02397</td>
</tr>
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</table>
Table F.5: System Parameters, Slot, 600 [RPM]

<table>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>Workpiece System</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>X</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Out-of-cut</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta_{x_workpiece_dyn}$</td>
<td>$\omega_{n_x_workpiece_dyn}$ [Hz]</td>
<td>$\zeta_{x_process}$</td>
</tr>
<tr>
<td>avg</td>
<td>0.073</td>
<td>895</td>
</tr>
<tr>
<td>stdv</td>
<td>0.0252</td>
<td>19.49</td>
</tr>
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<td><strong>Y</strong></td>
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<tr>
<td>Out-of-cut</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta_{y_workpiece_dyn}$</td>
<td>$\omega_{n_y_workpiece_dyn}$ [Hz]</td>
<td>$\zeta_{y_process}$</td>
</tr>
<tr>
<td>avg</td>
<td>0.123</td>
<td>1040</td>
</tr>
<tr>
<td>stdv</td>
<td>0.0794</td>
<td>25.61</td>
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<tr>
<td><strong>Tool System</strong></td>
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<td></td>
</tr>
<tr>
<td><strong>X &amp; Y</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Out-of-cut</td>
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<td></td>
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<tr>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta_{t_tool_dyn}$</td>
<td>$\omega_{n_t_tool_dyn}$ [Hz]</td>
<td>$\zeta_{t_process}$</td>
</tr>
<tr>
<td>avg</td>
<td>0.055</td>
<td>648</td>
</tr>
<tr>
<td>stdv</td>
<td>0.0091</td>
<td>5.11</td>
</tr>
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</table>

Table F.6: System Parameters, Slot, 3600 [RPM]

<table>
<thead>
<tr>
<th></th>
<th>Out-of-cut</th>
<th>In-Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Workpiece System</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>X</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Out-of-cut</td>
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<td></td>
</tr>
<tr>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta_{x_workpiece_dyn}$</td>
<td>$\omega_{n_x_workpiece_dyn}$ [Hz]</td>
<td>$\zeta_{x_process}$</td>
</tr>
<tr>
<td>avg</td>
<td>0.138</td>
<td>891</td>
</tr>
<tr>
<td>stdv</td>
<td>0.0535</td>
<td>27.72</td>
</tr>
<tr>
<td><strong>Y</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Out-of-cut</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta_{y_workpiece_dyn}$</td>
<td>$\omega_{n_y_workpiece_dyn}$ [Hz]</td>
<td>$\zeta_{y_process}$</td>
</tr>
<tr>
<td>avg</td>
<td>0.123</td>
<td>1098</td>
</tr>
<tr>
<td>stdv</td>
<td>0.0537</td>
<td>53.80</td>
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<tr>
<td><strong>Tool System</strong></td>
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<tr>
<td><strong>X &amp; Y</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Out-of-cut</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta_{t_tool_dyn}$</td>
<td>$\omega_{n_t_tool_dyn}$ [Hz]</td>
<td>$\zeta_{t_process}$</td>
</tr>
<tr>
<td>avg</td>
<td>0.091</td>
<td>602</td>
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<tr>
<td>stdv</td>
<td>0.062</td>
<td>11.56</td>
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</tbody>
</table>
Figure F.1: Force Profile of Simulation and Experiment, Half Immersion, 600[RPM]

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Figure F.1 (continued): FFT of Simulation and Experiment, Half Immersion, 600 [RPM]
Figure F.2: Force Profile of Simulation and Experiment, Half Immersion, 3600[RPM]
Figure F.2 (continued): FFT of Simulation and Experiment, Half Immersion, 3600[RPM]
Figure F.3: Force Profile & FFT of Simulation and Experiment, Slot, 600[RPM]
Figure F.4: Force Profile & FFT of Simulation and Experiment, Slot, 3600[RPM]
APPENDIX G

EXTENDED KALMAN FILTER

MATLAB Program (Function): Extended Kalman Filter

```matlab
%% Extended Kalman Filter
% This Kalman filter includes the harmonic force model and second order sensor
% dynamics.
% In order to execute this function, Ad, Gd, h, Hhat, IC have to be defined
% as global variables (define in main program)
%
% inputs: Fm: experimental data [n x 2] [time data]
% noise: noise matrix
% NT: number of Teeth
% k: dynamic stiffness
% wn: dynamic natural frequency
% zeta: dynamic damping ratio
% output: estimated applied force (Fchat)

function [Fchat] = estimated(Fm, noise, NT, k, wn, zeta)

global Ad Gd H Hhat IC;

time = Fm(:,1);
force = Fm(:,2);'

dt = (time(2)-time(1));
fs = 1/dt;

% system parameters
m = k / (wn^2*pi)^2;  % [kg];
c = 2*zeta * sqrt( k * m ); % [N-s/m];

% FFT(Frequency Response Function) of the Force Measurements
[frequency magnitude] = FD(force, fs);
% FD: function for FFT using hanning window

% Runout Frequency [Hz]
[pks locs]=findpeaks(magnitude,'sortstr','descend');
FreRunout = frequency(min(locs(1:5))); % tooth passing frequency
FreTooth = FreRunout * NT; % runout frequency
clear pks locs

% The order of Harmonic force model depends on natural frequency
% harmonic force model
limit = wn/FreRunout;
n = floor(0.9*limit);
% The number of harmonics is used for the Extended Kalman Filter
```

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%% Generate Coefficients of Fourier series \( \{ A_n, \text{Co\_info\_Position}(2) \} \)
indexS = zeros(1,n+1);
indexE = zeros(1,n+1);
DataIndex = zeros(1,n);
Co\_info\(1,n\) = struct('DataIndex',zeros(1),'Position',zeros(1,2));

indexS(1) = 1;
indexE(1) = 1;
for i=1:n
  if i==1
    while frequency(indexS(i)) < FreRunout
      indexS(i) = indexS(i)+1;
    end
    while frequency(indexE(i)) < FreRunout
      indexE(i) = indexE(i)+1;
    end
  else
    while frequency(indexS(i)) < floor(Co\_info\(1,1\).Position(1)*i)
      indexS(i) = indexS(i)+1;
    end
    while frequency(indexE(i)) < ceil(Co\_info\(1,1\).Position(1)*i)
      indexE(i) = indexE(i)+1;
    end
  end
  DataIndex(i) = find(magnitude == ...
                      max(magnitude(1,indexS(i)-1:indexE(i)+1)));
  Co\_info\(1,i\) = struct('DataIndex',DataIndex(i),...
              'Position',[frequency(DataIndex(i)),magnitude(DataIndex(i))]);
indexS(i+1) = indexS(i);
indexE(i+1) = indexE(i);
end

%% PSD from the force measurements
h = spectrum.periodogram('hann'); %spectrum object (periodogram)
psd_est\_lside = psd(h,force,'spectrumtype','onesided','Fs',fs);
frePsd = psd_est\_lside.Frequencies;
Psd = psd_est\_lside.data;

%% Area of PSD = variance
% find Peaks' Position from PSD: height
peak(1,n) = struct('DataIndex',zeros(1),'Position',zeros(1,2));
for i=1:n
  DataIndex(i) = Co\_info\(1,i\).DataIndex;
  peak(1,i) = struct('DataIndex',DataIndex(i),...
                   'Position',[frePsd(DataIndex(i)),Psd(DataIndex(i))]);
end

% define band of Peaks: width
band(1,n*2) = struct('DataIndex',zeros(1),'Position',zeros(1,2));
for i=1:n
  temp1 = peak(1,i).DataIndex;
  while Psd(temp1) > Psd(temp1-1)
    temp1 = temp1 - 1;
  end
  band(1,(2*i-1)) = struct('DataIndex',temp1,'Position',...
                            [frePsd(temp1),Psd(temp1)]);
  temp2 = peak(1,i).DataIndex;
  while Psd(temp2) > Psd(temp2+1)
    temp2 = temp2 +1;
end
end

\( \text{band}(1, (2*i)) = \text{struct}('DataIndex', temp2, 'Position', ... \{\text{frePsd(temp2)}, \text{Psd(temp2)}\}); \)

end

\% \% A.c.Bc,H: State Space Model of Kalman filter
\Ac = \text{zeros}((2*n+2), (2*n+2));
\Bc = \text{zeros}((2*n+2), n+2);
\H = \text{zeros}(1, (2*n+2));
\Hhat = \text{zeros}(1, (2*n+2));
\IC = \text{zeros}((2*n+2), 1); \% initial values
for i=1:n
\Ac(1,2) = 1;
\Ac(2,1) = -k/m;
\Ac(2,2) = -c/m;
\Ac((2*i+1), (2*i+2)) = 1;
\Ac((2*i+2), (2*i+1)) = (\text{Co}_\text{info}(1, i).\text{Position}(2)/m);
\Ac((2*i+2), (2*i+1)) = -(\text{Co}_\text{info}(1, i).\text{Position}(1)*2*\pi)^2); \% [Hz] to [rad/s]
\Bc(1,1) = 1;
\Bc(2,2) = 1;
\Bc((2*i+2), 2+i) = 1;
\H(1,1) = k;
\Hhat(1, (2*i+1)) = \text{Co}_\text{info}(1, i).\text{Position}(2);
\IC((2*i+1), 1) = 1;
end
\IC(1,1) = \text{force}(1)/k;
\IC(2,1) = ((\text{force}(2)-\text{force}(1))/dt)/k;

\% R: variance of noise (MATLAB built-in function: var)
\R = \text{var}([\text{noise}]);

\% Q matrix is the covariance of input
\Q = \text{zeros}(n+2, n+2);
for i=1:n
\Q(1,1) = 0.01;
\Q(2,2) = 0.01; \% uncertainty of sensor dynamic model
\numpoint = (\text{band}(1, 2*i).\text{DataIndex}(1) - \text{band}(1, 2*i-1).\text{DataIndex}(1)) + 1;
\temp3 = \text{band}(1, 2*i-1).\text{DataIndex}(1);
for j=1:\numpoint
\points(1,j) = \text{struct}('DataIndex', temp3, 'Position', ... \{\text{frePsd(temp3)}, \text{Psd(temp3)}\});
\temp3 = \temp3+1;
end
\width = \text{zeros}(1, \numpoint-1);
\height = \text{zeros}(1, \numpoint-1);
\area = \text{zeros}(1, \numpoint-1);
\% numerical integration (convert unit: [Hz] to [rad/s])
for k=1:(\numpoint-1)
\width(k) = \points(1, k+1).\text{Position}(1) - \points(1, k).\text{Position}(1);
\height(k) = (\points(1, k+1).\text{Position}(2) + \points(1, k).\text{Position}(2))*2;
\area(k) = (2*\pi*\width(k))*\height(k);
end

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variance = zeros(1,n);
variance(1,:)= sum(area);

% Equation 5.23
Q(i+i, i+) = variance(1,i) * ...
    ( (Co_info(1).Position(1)*2*pi)/Co_info(1).Position(2) )^2;
end

%% Compute Discrete Kalman Gain (MATLAB built-in function: kalmd)
sys = ss(Ac, Bc,H,0);
[KEST, Gd, P, M, Z] = kalmd (sys,Q,R,dt);

%% Continuous System to Digital filter (MATLAB built-in function: c2d)
sy sd = c2d(sys, dt, 'zoh');
Ad = sy sd(1,1).a;
Bd = zeros(2*n+2,n+2);
for i = 1: n
    Bd(:, i) = sy sd(1, i).b;
end

%% Applied Kalman Filter
period = max(time);
sim('ApplicationKalman', period); % load and run simulink(.mdl)
offset = mean(Fchat.signals.values) - mean(force); % offset
Fchat.signals.values = Fchat.signals.values - offset;
end

ApplicationKalman.mdl

Discrete Fourier transform function using Hanning Window

function [frequency magnitude] = FD( signal, fs )
    nfft = 2*nextpow2(length(signal)); % length of signal
    frequency = fs/2*linspace(0,1,nfft/2+1); %[Hz]
    w = hanning(length(signal));
    bf = var( signal-mean(signal) ) / var( w'*(signal-mean(signal)) );
    mag = bf * fft((signal-mean(signal))*w',nfft)./nfft;
    re = real(mag);
    im = imag(mag);
    magnitude = sqrt(re.^2+im.^2);
    magnitude = magnitude(1:nfft/2+1);
end
Example of Using Function Code in Main Program

% main program
clear all
close all
clc

global Ad Gd H Hhat IC;

%% (Input)
NT = 1; % number of teeth
load Smart/half_26_t.mat; % load experimental data

%% unit convert
Force = [s(i:end),s(i-1)];
BitsperLbf = 1.64; % [bits/lbf]
Force = Force/BitsperLbf; % [bits] -> [lbf]
Force = Force * 4.44822; % [lbf] -> [N]

%% offset Range & force separation by user inspection
% offset Range
offsetRange = ##; % noise separation {input}
offset = mean(Force(1,1:offsetRange));
Force = Force - offset;
noise = Force(1,1:offsetRange);
% Separation: check the start & end position.
Start = ##; % {input}
End = ##; % {input}
force = Force(1,Start:End); % force separation
time = strain_time(1,Start:End) - strain_time(1,Start);

%% system parameters {input}
k_sta = ##; % [N/s]
wn_sta = ##; % [Hz]
wn_dyn = ##; % [Hz]
zeta_dyn = ##;

% k_dyn = k_sta * mean_wn^2/wn_sta^2;
Fm = {time' force'};

%% function code call
[Fchat] = estimated(Fm,noise,NT,k_dyn,wn_dyn,zeta_dyn);
The Performance of Extended Kalman Filter in Other Cases

Table G.1: Case Study for Extended Kalman Filter

<table>
<thead>
<tr>
<th>Radial Immersion</th>
<th>Spindle Speed [RPM]</th>
<th>Profiles according to direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half</td>
<td>600</td>
<td>X, Y, Radial, and Tangential</td>
</tr>
<tr>
<td></td>
<td>2600</td>
<td>X, Y, Radial, and Tangential</td>
</tr>
<tr>
<td></td>
<td>3600</td>
<td>X, Y, Radial, and Tangential</td>
</tr>
<tr>
<td>Slot</td>
<td>600</td>
<td>X,Y</td>
</tr>
<tr>
<td></td>
<td>3600</td>
<td>X,Y</td>
</tr>
</tbody>
</table>

The compensated results by the Extended Kalman Filter from experimental data in each direction, X, Y, radial, and tangential with half immersion and 2600 rpm are shown in Chapter 5. Based on Table G.1, the Extended Kalman Filter applies other cases and compensate sensor dynamic effect. The MATLAB function code of Extended Kalman Filter is used for all cases with identified dynamic system parameters ($k_d$, $\omega_n$, and $\zeta$). The identified dynamic system parameters for each case are presented in Appendix F. Without any Kalman tuning process, the covariance matrix (Q) of inputs ($w_n$) is automatically calculated by MATLAB code. The estimated actual cutting force for each case is shown following figures:
Estimated Cutting Force by Extended Kalman Filter
Kistler in X, Half Immersion, 600[rpm]

Estimated Cutting Force by Extended Kalman Filter
Kistler in Y, Half Immersion, 600[rpm]
Estimated Cutting Force by Extended Kalman Filter
Smart Tool in Radial, Half Immersion, 600[rpm]

Estimated Cutting Force by Extended Kalman Filter
Smart Tool in Tangential, Half Immersion, 600[rpm]
Estimated Cutting Force by Extended Kalman Filter
Kistler in X, Half Immersion, 3600[rpm]

![Graph of Estimated Cutting Force vs. time, showing force [N] on the y-axis and time [s] on the x-axis. The graph includes data points for experimental data and estimated cutting force.](image)

Estimated Cutting Force by Extended Kalman Filter
Kistler in Y, Half Immersion, 3600[rpm]

![Graph of Estimated Cutting Force vs. time, showing force [N] on the y-axis and time [s] on the x-axis. The graph includes data points for experimental data and estimated cutting force.](image)
Estimated Cutting Force by Extended Kalman Filter
Smart Tool in Tangential, Half Immersion, 3600[rpm]

Estimated Cutting Force by Extended Kalman Filter
Smart Tool in Radial, Half Immersion, 3600[rpm]
Estimated Cutting Force by Extended Kalman Filter
Kistler in X, Slot, 600[rpm]

Estimated Cutting Force by Extended Kalman Filter
Kistler in Y, Slot, 600[rpm]
Estimated Cutting Force by Extended Kalman Filter
Kistler in X, Slot, 3600[rpm]

- - - - - - Experimental Data
- - - - - - - - - - - - Estimated Cutting Force

force [N]


time [s]

Estimated Cutting Force by Extended Kalman Filter
Kistler in Y, Slot, 3600[rpm]

- - - - - - Experimental Data
- - - - - - - - - - - - Estimated Cutting Force

force [N]


time [s]
APPENDIX H

CHATTER FREQUENCY DETECTION

Concept

The frequency content of the force measurement includes harmonics of tooth passing and runout frequency, natural frequency of structures, and chatter frequency \( (f_c) \). Using a comb filter, harmonics of the tooth passing and runout frequency, and natural frequency of structures are eliminated from the frequency contents of the force measurement. The remaining dominant frequency content is defined as the chatter frequency.

Observation from Simulated Chatter

In order to design chatter detection algorithm based on the comb filter, the relation between stable cutting and chatter is observed by simulation results and FFT's. The stable and unstable simulation results with selected cutting conditions, i.e. 3600 rpm, half immersion, \( h_{\text{avg}} = 0.001 \) [in], are investigated in terms of frequency content. With given cutting conditions, the tool and workpiece system parameters are defined as shown in Table F.4. Figure H.1 shows the different force profiles of stable cutting and chatter with simulated cutting forces and simulated force measurements. From these simulated results, chatter frequency can be detected. The FFT of the simulated cutting forces between stable cut and chatter (see in Figure H.2) shows that the magnitude of the chatter frequency increases as the instability enlarges.
Figure H.1: Stable Cut and Chatter, Simulated Cutting Force and Force Measurement
Figure H.2: FFT of Simulated Cutting Force both Stable and Chatter
Figure H.2 (continued): FFT of Simulated Cutting Force both Stable and Chatter
Figure H.3: FFT of Simulated Force Measurement both Stable and Chatter
In the unstable cases, there are two significant peaks (650.1 and 829.7 [Hz]) in FFT of the simulated cutting forces in X (see in Figure H.2). Since a peak at 650.1 [Hz] is affected by the tool system, chatter frequency is defined as 829.7[Hz]. Chatter frequencies in other directions are defined 770 [Hz] by significant peak that is not associated harmonics of tooth passing frequency (60 [Hz]). These chatter frequencies are not observed in stable cuts. In order to detect the chatter frequency from experiments, the FFT of simulated force measurements is examined. As seen in Figure H.3, chatter frequency can be detected from the stable cutting data. Table H.1 shows detected chatter frequency from different simulation results. There are some differences of detected chatter frequency from simulated cutting force and simulated force measurements. The systems vibration can explain these differences.

Table H.1: Detection of Chatter Frequency [Hz]

<table>
<thead>
<tr>
<th>Simulation</th>
<th>X</th>
<th>Y</th>
<th>Radial</th>
<th>Tange</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting Force (chatter)</td>
<td>829.7</td>
<td>769.4</td>
<td>769.4</td>
<td>769.4</td>
</tr>
<tr>
<td>Force Measurement (stable)</td>
<td>829.7</td>
<td>769.4</td>
<td>709</td>
<td>709</td>
</tr>
<tr>
<td>Force Measurement (chatter)</td>
<td>824.3</td>
<td>764</td>
<td>705</td>
<td>705</td>
</tr>
</tbody>
</table>

Chatter Frequency Detection from Experimental Cutting Data

Using the experimental data, chatter frequency is detected after eliminating harmonics of tooth passing frequency and the systems vibration modes. From the frequency contents of experimental data, $f_{tooth\, pass}$ is defined as 59.9 [Hz] from stable data and 59.7 [Hz] from chatter. Figure H.4 shows the force profile and the FFT of both stable data and chatter in X and Y direction with given cutting conditions, i.e., 3600 rpm, half immersion, $h_{ave}$=0.001[in]. As seen in Figure H.4, the force profile from stable data includes both stable cut and chatter.
Figure H.4: Stable Cut and Chatter from Force Measurements of the Kistler
Table H.2 shows detected chatter frequencies that are close to the detected chatter frequencies from simulation results.

<table>
<thead>
<tr>
<th>Experiments</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stable</td>
<td>829.4</td>
<td>709.6</td>
</tr>
<tr>
<td>Chatter</td>
<td>817.2</td>
<td>817.2</td>
</tr>
</tbody>
</table>

However, chatter frequency detection with the comb filter gives inaccurate results due to systems parameters variation. Filtered systems vibration is not straightforward because systems vibrations change by each revolution and the frequency contents of the tool and workpiece system compliance is not just the addition of the two frequencies. As the FFT of the stable experimental data in the Y directions shows, the frequency content has the tool system vibration (649.6 [Hz]) and the workpiece system vibration (1069 [Hz]). If the stable experimental data in Y directions is used to chatter frequency detection, chatter frequency is 709.6 [Hz]. However, since the experimental chatter shows the chatter frequency is near 817.2 [Hz], the frequency contents at 709.6 [Hz] is not chatter frequency and is somewhat affected by systems vibration. The frequency content of stable experimental data also contains possible chatter frequency 819.8 [Hz]. The accurate chatter frequency detection is required understanding the frequency contents changing unexpectedly due to the system vibration.