Characterization of the Wave Bottom Boundary Layer over Movable Rippled Beds

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Characterization of the Wave Bottom Boundary Layer over Movable Rippled Beds

BY

Sylvia Rodríguez-Abudo
Bachelor of Science, University of Puerto Rico at Mayagüez, 2007

THESIS

Submitted to the University of New Hampshire in Partial Fulfillment of the Requirements for the degree of

Master of Science in Ocean Engineering

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A Norma, Javier e Ingrid.
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LIST OF SYMBOLS

time average operation

phase average operation

spatial average operation

points located at the interface between the bed and the fluid

constant for bedload transport model, 10

wave orbital excursion

horizontal area occupied by the fluid

wave asymmetry

x-coordinate of points A and B

acceleration obtained from \( c_b \)

acceleration of the bedforms

correction constant employed for bed finding

wave celerity

ripple migration rate

median grain diameter

any flow quantity

force necessary to move the ripple formations

grain roughness friction factor

residual force in the momentum balance per unit volume

bedform-induced component of skin friction
\( g \)  gravitational acceleration

\( H \)  wave height

\( h \)  water depth

\( KC \)  Keulegan-Carpenter number

\( k \)  wave number

\( L_f \)  length of slab occupied by the fluid, \( f(z, \tilde{t}) \)

\( L_0 \)  total length of slab

\( n \)  sediment concentration

\( p \)  instantaneous pressure, \( f(x, z, t) \)

\( \bar{p} \)  time-averaged pressure, \( f(x, z) \)

\( (\bar{p})_b \)  time-averaged bedform-induced pressure, \( f(x, z) \)

\( \tilde{p} \)  phase-averaged pressure, \( f(x, z, \tilde{t}) \)

\( (\tilde{p})_b \)  phase-averaged bedform-induced pressure, \( f(x, z, \tilde{t}) \)

\( p' \)  temporal fluctuations in pressure, \( f(x, z, t) \)

\( Q \)  sediment transport rate

\( Q_0 \)  sediment transport rate at \( z = 0 \)

\( Re \)  Reynolds number

\( Re_s \)  Reynolds number of sediment particle

\( S \)  Sleath parameter

\( Sk \)  wave skewness

\( Stk \)  Stokes number

\( s \)  specific gravity of the sediment

\( T \)  wave period
$t$  time

$t_f$  characteristic time scale of the fluid, $\omega^{-1}$

$t_s$  particle relaxation time

$\tilde{t}$  phase

$u$  horizontal component of velocity

$u_s$  shear velocity

$u_\infty$  horizontal component of the free-stream velocity

$u_0$  horizontal velocity amplitude

$u_i$  total instantaneous velocity component in the $i$th direction, $f(x, z, t)$

$\bar{u}_i$  time-averaged velocity in the $i$th direction, $f(x, z)$

$(\bar{u}_i)_b$  bedform-induced time-averaged velocity in the $i$th direction, $f(x, z, \tilde{t})$

$\bar{u}_{off}$  quasi-steady average of horizontal velocity in the offshore direction

$\bar{u}_{on}$  quasi-steady average of horizontal velocity in the onshore direction

$\bar{u}_{stokes}$  Stokes drift

$\tilde{u}_i$  phase-averaged velocity in the $i$th direction, $f(x, z, \tilde{t})$

$\tilde{u}_{i,\infty}$  phase-averaged free-stream velocity in the $i$th direction, $f(\tilde{t})$

$(\tilde{u}_i)_b$  bedform-induced phase-averaged velocity in the $i$th direction, $f(x, z, \tilde{t})$

$u'_i$  temporal fluctuation of velocity in the $i$th direction, $f(x, z, t)$

$u_{piv}$  horizontal component of velocity as calculated by the PIV, $f(x, z, t)$

$u_{piv, filt}$  filtered horizontal PIV velocities, $f(x, z, t)$

$V_s$  volume of sediment

$w$  vertical component of velocity

$w_\infty$  vertical component of the free-stream velocity
\( w_0 \) particle settling velocity

\( \bar{w}_{cont} \) mean vertical velocity as obtained from the continuity equation

\( \bar{w}_{PIV} \) mean vertical PIV velocity

\( x \) horizontal coordinate parallel to wave direction

\( z \) vertical coordinate

\( z_0 \) elevation of zero-intercept in the logarithmic velocity profile

\( z_b \) bed height

\( z_c \) centroid of mean pixel image

\( z_{hc} \) elevation of the highest ripple crest

\( z_{initial} \) initial estimate of bed height

\( \gamma^2 \) coherence squared

\( \Delta z_{mov} \) thickness of the sediment moving layer

\( \delta \) boundary layer thickness

\( \delta^* \) displacement thickness

\( \delta_{99} \) boundary layer thickness obtained with \( u \geq 0.99u_\infty \)

\( \delta_{GM} \) boundary layer thickness as suggested by Grant and Madsen (1979)

\( \delta_K \) boundary layer thickness as suggested by Kajiura (1968)

\( \zeta \) proportionality function, \( L_f/L_0, f(z,\ell) \)

\( \eta_b \) ripple height

\( \theta_{2.5} \) grain roughness Shields parameter

\( \theta_{2.5, u_*} \) grain roughness Shields parameter using \( u_* \)

\( \theta_c \) critical Shields parameter for incipient motion

\( \theta_r \) Shields parameter as suggested by Du Toit and Sleath (1981)
$\theta_i$  Phase-dependent Shields parameter

$\theta_{u_*}$  Shields parameter obtained from $u_*$

$\theta_v$  Shields parameter obtained from $\tau_v$

$\kappa$  Von Karman coefficient, 0.4

$\lambda$  wavelength

$\lambda_b$  ripple wavelength

$\mu$  dynamic viscosity of water

$\nu$  kinematic viscosity of water

$\rho$  water density

$\rho_p$  density of the particle

$\tau_{MPM}$  shear stress from bedload transport model

$\tau_v$  viscous shear stress

$\tau_{xz}$  shear stress, $f(z, t)$

$\phi$  phase separation

$\omega$  wave radian frequency
ABSTRACT

Characterization of the Wave Bottom Boundary Layer over Movable Rippled Beds

by

Sylvia Rodríguez-Abudo
University of New Hampshire, May, 2011

Laboratory-scale observations of a two-dimensional near-bed velocity field were collected using a submersible PIV system. A quasi-steady approach was employed to characterize the observed wave-bottom boundary layer thickness and shear stress. Friction velocities, corresponding to $Re_*=15 - 40$, were obtained from a nonlinear least-squares regression of the velocity field averaged over individual half-waves. The regressions show zero-velocity elevations located roughly 1 cm under the ripple crests. The resulting Shields parameter is $O(1)$ greater than the reported threshold for sheet flow, even though bedforms persist. An unsteady analysis using the Double-Averaged Navier-Stokes equations show turbulent and wave-induced stresses dominating the total shear stress near the bed. Bedform-induced stresses are significant up to two ripple heights above the crests, while viscous stresses are negligible. The phase and magnitude of the total shear stress agrees with estimates inferred from a simple bedload transport model. Momentum imbalance below ripple crests is consistent with bedform acceleration.
CHAPTER 1

INTRODUCTION

Recently, investigations of sediment transport on beaches have become increasingly important. The United States Census Bureau reports that over 52% of the US population resides in coastal counties, occupying only one quarter of the total US land area. Beaches provide recreation, storm protection and a multi-million dollar industry (e.g. real estate, tourism, etc.) to these coastal communities. By the late 1990s, the federal government was spending an average of $100 million annually in shore protection projects (NOAA-CSC, 2009). Understanding and predicting the transport of sediment as it relates to storms, climate change and sea level rise is therefore vital for coastal management.

The initiation of sediment transport occurs in the wave bottom boundary layer as sediment particles become suspended after a threshold in the shear stress imposed by waves and currents is exceeded. Wave bottom boundary layers are constantly oscillating due to the nature of the free-stream flow, with thicknesses scaling with the wave radian frequency. For free surface gravity waves with periods of less than 10 s, the boundary layer thickness is generally less than 1 cm, although this estimate depends on sediment roughness and seabed geometry. In laboratory observations of the wave bottom boundary layer over flat fixed rough beds, the near-bed velocity leads the free-stream by $\sim 25^\circ$ (Jonsson and Carlsen, 1976; O’Donoghue et al.,
2008); phase shifts of less than ≈10° have been observed in the field over movable sand beds (Trowbridge and Agrawal, 1995; Foster et al., 2000).

In addition to entraining sediment, the wave bottom boundary layer is a region where significant energy dissipation occurs due to fluid-sediment interactions. Energy dissipation is induced by friction forces exerted by the fluid on the movable sediment bed, and transfer of momentum by coherent structures within the boundary layer. Bottom roughness and friction factors are often used to parameterize these processes in large-scale coastal circulation models (Warner et al., 2008). Accurate characterization of bottom roughness is therefore essential for sediment transport predictions, and has been the motivation for several modeling and observational efforts (Grant and Madsen, 1982; Wilson, 1989; Drake et al., 1992; Wiberg and Nelson, 1992; Newgard and Hay, 2007).

Quantifying bottom roughness is further complicated by the presence of large-scale, often two-dimensional (2D), bedforms known as ripples. These ripples are movable sedimentary features that respond to the changing hydrodynamic conditions. Commonly aligned with the wave crests, ripples induce vortex formation and ejection, which feeds back to the hydrodynamics and further complicates the momentum transfer in the wave bottom boundary layer. Orbital ripples scale with the wave orbital excursion and are commonly present in laboratory conditions, while anorbital ripples scale with the sediment grain diameter and are more commonly present in nature. For a further review on ripple geometry in wave dominated environments see Wiberg and Harris (1994).

Ripples impart an additional pressure gradient upon the oscillating flow, such
that when integrated over a ripple length results in form drag. Although most sediment transport is believed to be caused by skin friction, the name collectively given to viscous stresses acting on sediment grains plus pressure drag imparted by each individual grain (Maddux et al., 2003), understanding and predicting form drag is significantly important, since it affects the ability of flow shear to impact sediment grains through skin friction.

Ripples tend to migrate in the presence asymmetric wave forcing, as observed by Boyd et al. (1988), Traykovski et al. (1999) and Doucette (2002). Ripple migration is related to sediment transport rates by a statement of conservation of sediment mass, the Exner equation (Exner, 1920). In its simplest form, when bedforms travel with constant geometry, the sediment flux is directly proportional to the bedform migration rate (Nielsen, 1992). Deciphering the effects of wave forcing on ripple migration and associated sediment transport could allow for estimates of beach erosion or accretion.

Laboratory efforts investigating ripples under oscillatory flow date back to Darwin (1883), Ayrton (1910) and Bagnold (1946), each of which observed the presence of coherent vortices induced by the rippled bed and their effects on ripple evolution. Recently, more detailed observations of the dynamics within the wave bottom boundary layer have been obtained in oscillatory flow tunnels (Jonsson and Carlsen, 1976; Zala Flores and Sleath, 1998; Ahmed and Sato, 2001; O’Donoghue and Clubb, 2001; Sand Jespersen et al., 2004; Ourmieres and Chaplin, 2004; Doucette and O’Donoghue, 2006; O’Donoghue et al., 2006; Van der Werf et al., 2007; Carstensen et al., 2010). Oscillatory flow tunnels have a closed top and are therefore unable to
reproduce vertical orbital motions. Wave orbital motions are more realistically simulated in wave flumes, as examined in the studies by Earnshaw and Created (1998) and Fredsøe et al. (1999) over smooth fixed ripples; and Faraci and Foti (2001) and Coleman et al. (2008) over movable rippled sediment beds. At a slightly larger scale, Rankin and Hires (2000) obtained values for wave friction factors from shear plate observations which agree with the theory presented by Grant and Madsen (1979). Laboratory experiments under full scale conditions include Davies and Thorne (2005), Nichols and Foster (2007, 2009) over rippled beds, and Dohmen-Janssen and Hanes (2005) under sheet flow conditions over flat beds.

Early estimates of shear stress and bed roughness during field conditions relied on observations of flow velocity outside the wave bottom boundary layer (Drake and Cacchione, 1986; Drake et al., 1992). While Myrhaug et al. (1992) published evidence of a ‘seabed boundary layer’, Trowbridge and Agrawal (1995) were the first to systematically recognize the existence of a wave bottom boundary layer in the coastal ocean by observing a reduction in velocity magnitude and a phase lead near the seabed. Field scale observations of fluid-sediment interactions in the coastal ocean have primarily been limited to one-dimensional velocity profiles (Osborne and Vincent, 1996; Traykovski et al., 1999; Foster et al., 2000; Crawford and Hay, 2001; Doucette, 2002; Smyth et al., 2002; Thorne and Hanes, 2002; Chang and Hanes, 2004). Doron et al. (2000) and Nimmo Smith et al. (2005) were able to resolve the turbulence structure in the wave bottom boundary layer with 2D observations of the velocity field, although their measurements were limited to a minimum of 10 cm above the bed.
Due to the turbulent nature of the flow, the unsteadiness of the free-stream, and the spatial non-uniformity imposed by the rippled bed, parameterizations of the wave-bottom boundary layer are often approached with empirical and quasi-empirical formulations (for a review see Nielsen, 1992; and Fredsøe and Deigaard, 1992). A consensus is yet to be reached regarding the most appropriate approach to assess hydrodynamic quantities such as boundary layer thickness, friction velocity, shear stress, and friction coefficient. The purpose of the present study is to further explore, using improved measurement techniques, ripple dynamics and the hydrodynamic parameters describing wave bottom boundary layers. The hypothesis of this investigation is that fluid stresses are responsible for motion and migration of bedforms, and therefore appropriate characterization of the stress mechanisms within the wave bottom boundary layer is vital for modeling and prediction of sediment transport induced by bedform mobility.

In this thesis, the wave bottom boundary layer will be characterized by two different approaches. The first approach will utilize quasi-steady physics by time-averaging the flow field according to free-stream flow direction (onshore-offshore), similar to Cox et al. (1996). Friction velocities are estimated by fitting a logarithmic curve to the velocity profile in the bottom boundary layer, yielding estimates of boundary layer thickness and non-dimensional shear stress. The second approach will be based on the unsteady physics characterizing the flow in the wave bottom boundary layer. The equations of motion will be double-averaged (in time and space) following Gimenez-Curto and Corniero Lera (1996), resulting in an explicit expression for the shear stress. By double-averaging the flow, it is possible to assess
the relative contribution of each individual term in the phase-dependent momentum balance.

This thesis is organized as follows: Chapter 2 describes the experimental methods used to collect information of the wave bottom boundary layer over a movable rippled sand bed; Chapter 3 presents a characterization of the free-stream hydrodynamics, along with a quasi-steady analysis of the wave bottom boundary layer; Chapter 4 focuses on the unsteady analysis, including a brief introduction, a theoretical development of the governing equations and a results section; and Chapter 5 provides the relevant conclusions of this study.
CHAPTER 2

OBSERVATIONS

The observational data for this study was collected in a small-scale (1:15) wave flume at the Fluid Mechanics Laboratory at Delft University of Technology, Netherlands. The flume is 42 m long, 0.8 m wide, and 1 m high. A 1:20 rigid slope starting 32 m from the wave generator was covered with sediment, which extended to a few meters from the wave generation area (Figure 2-1). The wave generator is capable of producing regular waves as well as irregular bichromatic and Jonswap spectral waves with peak periods ranging from 1 to 3 s. The active reflection compensation system reduced the effect of reflected waves (its mechanisms are described in Van den Boomgaard, 2003). The present effort will focus on regular-sinusoidal waves, 5 cm in height and 2 s in wave period as observed 29 m from the wave generator in 31 cm water depth. Additional wave characteristics are summarized in Table 2.1, and are further discussed in Chapter 3 with the free-stream hydrodynamics.

As indicated by Henriquez et al. (2008), correct physical modeling of the nearshore sediment transport in a laboratory setting requires significant scaling of the sediment particles. In order to preserve the ratios of the Reynolds number, shear stress and particle settling, Henriquez et al. (2008) suggested a sediment specific gravity of 1.2, and mean grain diameter of 0.54 mm, for the Delft wave flume dimensions and specifications of the wave maker, according to the Froude scale.
These sediment characteristics produced a grain roughness Shields parameter of 0.26, Reynolds number of 3500, and ripples of approximately 1 cm in height and 5 - 7 cm in wavelength, for the prescribed wave conditions (Table 2.1). Since the evolution of a flat bed to a rippled one is out of the scope of this project, the bed was allowed to achieve equilibrium before initiating data collection.

A Dantec Particle Image Velocimetry (PIV) system was used to obtain optical observations of the two-dimensional flow field and bed geometry. The free-stream velocity was measured with an Acoustic Doppler Velocimeter (ADV), time-synchronized with the PIV. The PIV system consisted of a 120 mJ ND YAG laser vertically located 27 cm above the bed illuminating a vertical (x-z) slice of the water column. A 1 megapixel camera (1016 pixels x 1008 pixels) located outside the wave flume obtained image pairs over a 11 cm x 11 cm sampling window at 12 Hz for 60 s, as shown in Figure 2-2. The time between image pair members was selected as 10 ms to prevent particles from moving more than one third of a correlation window (21 pixels, 2.3 mm). Seeding material included sediment, micro-bubbles, and organic matter.

Velocity vectors were calculated using, MatPIV 1.6.1, developed by Sveen (2004). Correlations were calculated with two passes with interrogation windows of size 64
Table 2.1: Summary of Experimental Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave height, $H$</td>
<td>5 cm</td>
</tr>
<tr>
<td>Wave period, $T$</td>
<td>2 s</td>
</tr>
<tr>
<td>Wavelength, $\lambda$</td>
<td>3.3 m</td>
</tr>
<tr>
<td>Horizontal velocity amplitude, $u_0$</td>
<td>12 cm s$^{-1}$</td>
</tr>
<tr>
<td>Wave orbital excursion, $A_0 = u_0 T/2\pi$</td>
<td>3.8 cm</td>
</tr>
<tr>
<td>Water depth, $h$</td>
<td>31 cm</td>
</tr>
<tr>
<td>Wave asymmetry, $A_s$</td>
<td>0.58</td>
</tr>
<tr>
<td>Wave skewness, $Sk$</td>
<td>0.06</td>
</tr>
<tr>
<td>Kinematic viscosity of water, $\nu$</td>
<td>$1.307 \times 10^{-6}$ m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>Sediment median grain diameter, $d_{50}$</td>
<td>0.54 mm</td>
</tr>
<tr>
<td>Sediment specific gravity, $s = \rho_p/\rho$</td>
<td>1.2</td>
</tr>
<tr>
<td>Sediment settling velocity, $w_0$</td>
<td>1.8 cm s$^{-1}$</td>
</tr>
<tr>
<td>Ripple height, $\eta_b$</td>
<td>1 cm</td>
</tr>
<tr>
<td>Ripple wavelength, $\lambda_b$</td>
<td>5 cm</td>
</tr>
<tr>
<td>Migration rate, $c_b$</td>
<td>0.005 cm s$^{-1}$</td>
</tr>
<tr>
<td>Reynolds number, $Re = u_0^2 T/2\pi \nu$</td>
<td>3500</td>
</tr>
<tr>
<td>Particle Reynolds number, $Re_s = w_0 d_{50}/\nu$</td>
<td>7</td>
</tr>
<tr>
<td>Stokes number, $Stk = \pi s d_{50}^2/9 T \nu$</td>
<td>0.06</td>
</tr>
<tr>
<td>Wave friction factor, $f_{2.5} = e^{[5.213(2.5 d_{50}/A_0)^{0.194} - 5.977]}$</td>
<td>0.0387</td>
</tr>
<tr>
<td>Grain roughness Shields parameter, $\theta_{2.5} = f_{2.5} u_0^3/2(s - 1) g d_{50}$</td>
<td>0.26</td>
</tr>
<tr>
<td>Sleath parameter, $S = (du_\infty/dt)/(s - 1)g$</td>
<td>0.25 (at max. flow)</td>
</tr>
<tr>
<td>Keulegan-Carpenter number, $KC = u_\infty T/\eta_b$</td>
<td>24 (at max. flow)</td>
</tr>
</tbody>
</table>

pixels in the horizontal and 32 pixels in the vertical, with 50% overlap. The spatial resolution of the resulting velocity field is 3.48 mm in the horizontal and 1.74 mm in the vertical. Particle displacements were calculated at the subpixel level using a three-point Gaussian estimator (Raffel et al., 2007). Sample snapshots of instantaneous PIV images and corresponding velocity vectors are shown in Figure 2-3. The lower and upper 2 cm of the images and vector maps were discarded in this figure to better show the dynamics near the bed while preserving the correct axes scales. The image intensity shows the high energetics near the bed, with sediment plumes being shed above ripple crests. Although noisy, the velocity vectors correspond to the expected behavior of the flow field within the bottom boundary layer, showing
at times, signatures of vortical motion induced by the presence of bedforms. The free-stream velocity vectors were validated with point measurements provided by the ADV, as proposed by Nichols and Foster (2007). A cross-spectral analysis between the two approaches will be presented further in this chapter. The velocity measurements obtained within the rippled bed will not be used in this study, as their uncertainty is high due to lack of illumination in this region, and out-of-plane motions contaminating the signal.

The time-averaged ratio between the highest PIV correlation peak and the second highest and its standard deviation over the 60-s realization at each interrogation window after the second pass, is presented in Figure 2-4. This quantity provides
Figure 2-3: (a) Free-stream horizontal velocity. (b - i) Instantaneous image intensity with corresponding velocity vectors (red) as calculated by PIV. Each snapshot corresponds to the free-stream velocity indicated in (a). Scale vectors in the top right corner represent 10 cm s\(^{-1}\). Onshore flow is directed to the left. The lower and upper 2 cm of the images and vector maps have been discarded for plotting purposes. Velocity vectors below the mean bed elevation have been omitted.
information regarding the strength of the peak compared to its surroundings. Two correlation peaks within the same correlation window indicates that the PIV algorithm was able to resolve two different particle displacements. For example, an interrogation window within a flat plate boundary layer will have particles displacing at higher speeds at the top edge than at the lower edge, thus yielding more than one correlation peaks. The ratio between the highest correlation peak to the second highest is therefore a measure of regions with high velocity gradients and strong shear, as expected in the wave bottom boundary layer. Figure 2-4 shows large velocity gradients as far as 2 cm above the bed (at an elevation of roughly 5 cm). This may suggest that the region of strong shear extends as high as two ripple heights, and is cross-shore uniform. The lower correlation values found at the left side of the image (onshore direction) could be attributed to several things including camera-laser misalignment, focusing, and lower scatterers.

Figure 2-5a depicts the spectra of horizontal PIV velocities \( u_{piv} \) at a vertical slice located at \( x = 5.2 \) cm, as shown in Figure 2-4. Noise resulting from unrealistic velocity estimates associated with the PIV technique, was reduced with a three standard deviation filter. The total number of spurious vectors detected corresponded to less than 3.5% of the observations, for both \( u \) and \( w \) (Appendix A). After detection, outliers were replaced with the local ensemble average. The spectra of the filtered velocities \( u_{piv, filt} \) clearly shows a peak at the incident band (0.5 Hz), that extends deep inside bed (Figure 2-5b). The spectra also demonstrates that the filter improved the resolution of the higher harmonics (1 and 1.5 Hz) present in the skewed waves of this study. The skewness and asymmetry of the waves will be fur-
Figure 2-4: (a) Time average and (b) standard deviation of the ratio between the highest PIV correlation peak to the second highest, after the second pass as calculated by MatPIV 1.6.1. The time-averaged bed profile is shown in black. The black dashed line identifies the location of the spectral plots shown in Figure 2-5

ther explored in Chapter 3. Additionally, Figures 2-5b show a significant reduction of energy at the noise floor for free-stream velocity observations, corresponding to the smoother velocity ensembles shown in Figure 2-5d.

Ensemble-averaged velocities before and after filtering are shown in Figures 2-5c and 2-5d for the same vertical slice located at $x = 5.2$ cm. The smoothed velocity data shows the near-bed negative velocity peaking before the free-stream, and is indicative of a phase lead in this region. This feature has been previously observed in laboratory and field scale efforts (e.g. Foster et al., 2000), and will be further discussed in Chapter 3. A strong velocity signal is apparent at elevations within the bed for short periods of time (roughly around 100° and 270°). Since,
in order to obtain velocity estimates, the PIV technique resolves the displacement of particles, there is reason to believe that the bed slips and/or liquifies during maximum free-stream velocities. This is not surprising, as Henriquez et al (2008) saw bed liquefaction in their study with the same sediment properties and similar regular wave forcing.

A cross-spectral analysis between the free-stream horizontal velocity as measured by the ADV and PIV is presented in Figure 2-6. Coherence exceeding the 95% significance level and small phase separation at the incident wave frequency.
Figure 2-6: (a) Spectral density, (b) coherence, and (c) phase separation between free-stream horizontal velocities as measured by the PIV (solid line) and ADV (dashed line). Significance levels were calculated with 4 degrees of freedom.

(0.5 Hz), validate the PIV measurements in the free-stream. Both systems have significant amounts of energy at the higher harmonics and are consistent with a slightly skewed and asymmetric wave form (Table 2.1).

Previous studies employing PIV have been able to resolve the bed position with significant confidence (Nichols and Foster, 2007, 2009). However, they rely on time averages of image intensity over entire data realizations. Moreover, bed reflections associated with the laser sheet present serious difficulties. In this study, we will follow the method employed by (Nichols and Foster, 2007) to compute the phase-dependent bed height, \( z_b(x, \tilde{t}) \). However, since it is difficult to obtain a
Figure 2-7: (a) Phase-maximum pixel image and (b) phase-averaged pixel image at $\tilde{t} = 235^\circ$ showing: $z_{initial}$ (red), $z_c$ (blue) and $z_b$ (green). (c) Vertical profiles of phase-maximum (red) and phase-averaged (blue) pixel intensity at $x = 4.37$ cm with diamonds corresponding to the values in (a) and (b) as shown by the black dashed line. Images have been zoomed into the near-bed region and their vertical scale has been exaggerated by a factor of 3.

A reliable estimate from phase-averaged pixel images due to coherent sediment plumes obscuring the bed (Figure 2-7b), phase-maximum pixel images will be incorporated in the calculation.

An initial estimate of the bed position, $z_{initial}(x, \tilde{t})$, is taken as the lowest point with maximum pixel intensity in the phase-maximum pixel image (shown in red in Figure 2-7a). The actual bed position, $z_b(x, \tilde{t})$, is assumed to be the initial estimate plus a constant given by

$$C(\tilde{t}) = \langle z_c(x, \tilde{t}) - z_{initial}(x, \tilde{t}) \rangle,$$  \hspace{1cm} (2.1)  

where the angle brackets represent spatial average in the $x$ direction and $z_c$ is the centroid of the phase-averaged pixel image as suggested by Nichols and Foster (2007) and shown in blue in Figure 2-7b. The final bed height (shown in green in Figure 2-7b) is given by
Finally, to smooth out any high frequency signal present in the bed estimates, a running average of size 30 pixels (3.2 mm) was applied to \( z_b \). These bed elevation estimates will be used to infer the mobility of the bed in Chapters 3 and 4. Additionally, from this point on, velocity measurements and related estimates will be disregarded (masked) if they are located below the corresponding bed elevation estimate.
CHAPTER 3

QUASI-STEADY WAVE BOTTOM BOUNDARY LAYER ANALYSIS

3.1 Free-stream Hydrodynamics

In this study, a rippled sediment bed was subjected to regular, intermediate water waves with 2 s wave period, 5 cm wave height and 3.3 m wavelength. A 6 s sample time series of the free-stream horizontal \( u_\infty \) and vertical \( w_\infty \) velocities is shown in Figure 3-1. Onshore-directed flow shows maximum \( u_\infty \) values 2 cm s\(^{-1}\) higher than its offshore counterpart, with half-wave period 0.12 s shorter. Maximum \( w_\infty \) values are twice as high in the upward direction, although roughly one order of magnitude lower than \( u_\infty \). ADV and PIV velocity estimates are consistent, with \( R^2 = 0.98 \) for \( u_\infty \), and \( R^2 = 0.82 \) for \( w_\infty \). Low correlation values in the vertical velocity are attributed to uncertainty in the PIV measurements associated with sub-pixel resolution, which is inversely proportional to the average particle displacement (Raffel et al., 2007). Due to the nature of the flow below intermediate water waves, particles move further in the horizontal direction than they do in the vertical, and therefore the uncertainty in the vertical velocity measurements is higher. The uncertainty values due to sub-pixel resolution in the free-stream are
Figure 3-1: Sample time series of (a) $u_\infty$ and (b) $w_\infty$ as measured by the PIV (solid line) and ADV (dashed line). Positive velocities are directed offshore and upward.

1.30% and 10.4%, and were calculated using the root-mean-squared deviation of the horizontal and vertical velocity estimates, respectively.

Cross-spectral analysis between $u_\infty$ and $w_\infty$ shows coherence in the first three harmonics (Figure 3-2). The phase separation at the incident band is 90°, as expected. The second harmonic has a phase separation of 75°, whereas the third is 102° out of phase. These nonlinear effects are responsible for the peaked forward-leaning wave crest and the flat trough (note that the opposite will be true in the $u_\infty$ time series for the prescribed coordinate system, Figure 3-1), and are quantified with the wave skewness ($Sk$) and asymmetry ($As$). In this study $Sk = 0.06$ and
Figure 3-2: (a) Spectral density, (b) coherence, and (c) phase separation between $u_\infty$ (solid line) and $w_\infty$ (dashed line). Significance levels were calculated with 4 degrees of freedom.

As $= 0.58$, and were estimated using the free-stream velocity skewness and temporal asymmetry, respectively, following Gonzalez-Rodriguez and Madsen (2007). This method is more suited for this data set than that of Elgar and Guza (1985), since it does not require surface elevation data.

Horizontal velocity spectra for four different positions along a ripple wavelength are shown in Figure 3-3. At all positions, the first two harmonics are evident across the water column, except near the bed, where they reach the noise floor. Interestingly, the incident wave band is resolved within the bed. This could suggest a slip condition at the bed surface, although contamination of the signal by the flow
Figure 3-3: (b - e) Horizontal velocity spectra for four vertical slices located along a ripple wavelength shown in (a). The black dashed line denotes time averaged bed height. 95% significance levels were calculated with 2 degrees of freedom, corresponding to raw spectral densities.
between the camera and the laser sheet is also possible. The noise floor is lower going from left to right in the image, as also shown in Figure 2-4. No additional spectral differences are found along the ripple.

The boundary layer phase shift was computed with cross-spectral analysis. Space- and time-dependent horizontal velocities, $u(x, z, t)$, were compared against the free-stream velocity, $u_\infty(x, t)$. The coherence squared ($\gamma^2$) and phase separation ($\phi$) at the incident band are shown in Figure 3-4. In the trough, the boundary layer leads the free-stream by $7 \text{ - } 15^\circ$. These values are not inconsistent with previous observations of the wave bottom boundary layer over flat beds, which showed roughly a $10^\circ$ phase lead (Trowbridge and Agrawal, 1995; Foster et al., 2000). Interestingly, a phase lag of $10^\circ$ is observed $1.5 \text{ cm}$ above the main ripple trough, likely induced by vortices being shed above the crests during flow reversals.
Figure 3-5: Mean image intensity over the 60-s PIV realization with (a) root-mean-squared velocity field and (b) mean velocity field (red vectors). Scale vectors represent $8 \text{ cm s}^{-1}$ (a) and $1 \text{ cm s}^{-1}$ (b). Data under the mean bed elevation over the 60-s realization (green solid line) have been masked. Onshore flow is directed to the left.

Basic statistics of the velocity field are shown in Figure 3-5. The root-mean-squared (RMS) velocity field shows sharp gradients above ripple crests and milder gradients above the troughs. As expected, the mean velocity field averages to zero at the free-stream, but shows a strong vertical component with magnitude $\sim 1.5 \text{ cm s}^{-1}$ above ripple troughs. This is consistent with the expected settling velocity of the particles ($1.8 \text{ cm s}^{-1}$) following Cheng (2009), although sediment accretion is not observed in this area (see Section 3.2). Figure 3-5b also shows a net near-bed offshore flow (boundary layer streaming) potentially induced by wave asymmetry. Several authors have observed onshore-directed streaming, but only a few have reported this phenomena in the offshore direction (Ribberink and Al-Salem, 1995; Scandura, 2010). Both velocity fields in Figure 3-5 show noisy velocity profiles at the left side of the image which are consistent with the lower correlation area shown.
in Figure 2-4, and therefore care must be taken when interpreting velocity vectors in this area.

3.2 Bedform Migration

Wave-dependent bed elevation is shown in Figure 3-6a. As shown by the standard deviation (Figure 3-6b), bedforms do not migrate significantly over the realization. Cross-correlation analysis between the estimates of bed elevation for the first and last wave periods yields a migration rate of $5 \times 10^{-3} \text{ cm s}^{-1}$ in the onshore direction. This is one order of magnitude lower than the migration rates reported by Nichols and Foster (2007) during their full scale experiment, but consistent with observations by Crawford and Hay (2001) of linear transition ripples subjected to similar wave forcing.

3.3 Quasi-steady Analysis

Quasi-steady approximations of various hydrodynamic- and sediment transport-related characteristics are performed for regular monochromatic waves. The horizontal velocity field was time-averaged over half-wave periods, which were detected by finding the zero-crossings in the horizontal free-stream velocity (Figure 3-7a). Half-wave periods in each flow direction over an entire realization were time-averaged, resulting in a mean velocity field for each flow direction (Figure 3-7f and g).

Vertical gradients in the horizontal velocity are larger on the upstream side of the ripple as shown by the small separation between consecutive velocity profiles.
for both flow directions in Figure 3-8. This is consistent with flow acceleration as it constricts up the ripple face. Conversely, the flow decelerates on the downstream side of the ripple, forming a wake-like area that remains present until it is affected by the presence of the next ripple (Nelson and Smith, 1989; Wiberg and Nelson, 1992; Li, 1994; Coleman et al., 2008). This suggests the presence of a recirculating area after the ripple crest, in a manner analogous to a backward facing step (Nichols and Foster, 2007). Noisy velocity profiles in the left side of the image are attributed to low PIV correlation (Figure 2-4).

Although the shape of the velocity profiles is affected by vortex shedding, Fig-
Figure 3-7: Methodology for producing quasi-steady mean velocity profiles: (a) Half-wave periods were detected by finding the zero-crossings in the free-stream horizontal velocity time series. (b - e) Time averages were produced for each half-wave period indicated by superscripts (1) - (4). (f - g) Half-wave averages were subsequently averaged over the entire 60-s realization (only 4 s are shown) to produce mean onshore- (blue) and offshore- (red) directed velocity profiles. Black dashed lines represent the elevation of the highest ripple crest at each half-wave period (b - e), and the mean elevation of the highest ripple crest over the entire realization (f - g).
Figure 3-8: Mean horizontal velocity profiles for (a) offshore- and (b) onshore-directed flow. Profiles have been offset to the right by 1 cm s\(^{-1}\). Data under the mean bed elevation over the 60-s realization (black dots) have been masked.

Figure 3-8 suggests that the horizontal velocity decays logarithmically. In fully rough turbulent flow with steady and uniform forcing, the velocity profile has been characterized with

\[
u = \frac{u_*}{\kappa} \ln \left( \frac{z}{z_0} \right) \tag{3.1}\]

where \(u_*\) is the shear velocity, \(\kappa = 0.4\) is the Von Karman coefficient, and \(z_0\) is the hypothetical vertical position where \(u = 0\) (Davidson, 2004). Consistent with Cox et al. (1996) and Fredsøe et al. (1999), a least-squares regression to the observations (Figure 3-9) allows for estimates of \(u_*(x)\) and \(z_0(x)\). The limits for regression analysis were defined as the lower limit located at the first cell above the sediment bed, and the upper limit located at the lowest elevation where \(u \geq 0.99u_\infty\). The resulting estimates of \(u_*\) correspond to boundary Reynolds numbers \((Re_* = u_*d_{50}/\nu)\) between 15 and 40, and zero-velocity elevations located, at times, 1 cm below the bed. This technique allows for direct estimates of boundary layer thickness and non-dimensional shear stress.
3.3.1 Boundary Layer Thickness

Defining the wave-bottom boundary layer thickness on a rippled bed is not a trivial task. In turbulent boundary layers over flat rough beds, the most common approach is to use $\delta_{99}$, which is found by locating the position where the horizontal velocity is 99% of the free-stream. This definition can be problematic for boundary layers with overshoots. *Grant and Madsen* (1979) suggested a less arbitrary definition:

$$\delta_{GM} = \frac{2\kappa}{\omega} u_*$$  \hspace{1cm} (3.2)

where $\kappa = 0.4$ is the Von Karman coefficient, $\omega = \frac{2\pi}{T}$ is the wave radian frequency and $u_*$ is obtained from (3.1). This approach is frequently used in literature since it is a direct relationship between $\delta$ and the dynamics near the bed characterized by $u_*$ (*Trowbridge and Agrawal*, 1995; *Zou and Hay*, 2003; *Frank*, 2008). In this study, the estimates of $\delta_{GM}$ will be spatially dependent, as $u_*$ is a function of $x$. 

Figure 3-9: Mean bed elevation over the 60-s realization (beige) between the center ripple trough and the rightmost ripple crest with superimposed mean horizontal velocity profiles plotted every other in a logarithmic scale for (a) offshore- and (b) onshore-directed flow. The least-squares fit is plotted in black. Data under the bed have been masked.
Kajiura (1968) provided an empirical formulation using the wave friction factor to obtain the boundary layer thickness from the free-stream hydrodynamics,

\[ \delta_K = \frac{1}{2} f_{2.5} A_0 \]  \hspace{1cm} (3.3)

where \( A_0 = u_0/\omega \) is the wave orbital excursion with velocity amplitude \( u_0 \), and \( f_{2.5} \) is the wave friction factor as approximated by Swart (1974),

\[ f_{2.5} = \exp \left( 5.213 \left[ \frac{2.5 d_{50}}{A_0} \right]^{0.194} - 5.977 \right). \]  \hspace{1cm} (3.4)

Since the velocity at the free-stream is assumed uniform and the formulation for \( f_{2.5} \) does not consider ripple roughness, \( \delta_K \) is independent of \( x \), and therefore any ripple effects on the boundary layer thickness are disregarded, contrary to what is shown in Figure 3-9.

Another common approach relies on finding the distance the bed will be upwardly displaced in a frictionless flow so as to maintain the same mass flux (Kundu and Cohen, 2004). This is called the displacement thickness and is defined as

\[ \delta^*(x) = \int_{z_b}^{\infty} \left( 1 - \frac{u(x, z)}{u_\infty(x)} \right) dz. \]  \hspace{1cm} (3.5)

Figure 3-10 shows \( \delta^* \) for mean onshore and offshore-directed flows. Consistent with laminar boundary layers, \( \delta^* \) gets thicker as it responds to an adverse pressure gradient on the downstream side of the ripple. This increased thickness is also consistent with the expanded logarithmic layers shown on the downstream side of the ripples in Figure 3-9. However, the increased mixing in a turbulent flow
demands thicker boundary layers, justifying the use of $u_*$ in the definition by Grant and Madsen (1979). Therefore the displacement thickness method results in values of $\delta^*$ that are inversely proportional to $\delta_{GM}$ (Figure 3-11b). Furthermore, the displacement thickness approach becomes problematic when velocity overshoots ($u > u_\infty$), resulting in negative values of $\delta^*$, as seen in Figure 3-10a for $x < 1$ cm, and at the second ripple crest ($x = 7.5$), implying perhaps a slip condition.

The results obtained using the above-mentioned methods for calculating the boundary layer thickness are summarized in Figure 3-11a. Predictions for $\delta_K$ are comparable to the diameter of the sediment grains ($d_{50} = 0.54$ mm), which is unrealistic for the high energetics observed in this study. The formulation of $\delta_K$
Figure 3-11: (a) Boundary layer thickness for onshore- (red plus signs) and offshore- (black open circles) directed flow, as calculated with the different methods provided in Section 3.3.1. Offshore and onshore estimates of (b) $\delta^*$ and (c) $\delta_{GM}$ as a function of $\delta_{99}$ with least-squares regression line and correlation coefficient showing their relationship. The red line represents a 1:1 relationship.

does not account for bedform induced roughness, which in this case is significantly larger than the grain roughness, consequently making it more suitable for flat beds. The highest variability is found in the offshore estimates of $\delta^*$, and results from flow accelerating above ripple crests. Although problematic at regions of velocity overshoot, $\delta^*$ agrees with intuition (smaller above ripple crests and high velocities) and follows the same trend as $\delta_{99}$, although differing by a factor of 5 (Figure 3-11b). The values of $\delta_{GM}$ show the least variability in Figure 3-11a, but are inconsistent with $\delta_{99}$ and observations of the logarithmic layer (Figure 3-9). Generally, $\delta^*$, $\delta_{GM}$ and $\delta_{99}$ have similar order of magnitude, with correlations shown in Figure 3-11b and c.
3.3.2 Sediment Mobility

The thickness of the sediment moving layer can be estimated as $\Delta z_{mov}(x) = z_b(x) - z_0(x)$, where $z_b$ is the bed height, and $z_0$ is obtained from (3.1). Figure 3-12a shows $\Delta z_{mov}(x)$ for the onshore- and offshore-directed flows. Consistent with visual observations along the tank side wall during the experiment, $\Delta z_{mov}(x)$ can be as large as one ripple height ($\eta_b = 1$ cm). This differs significantly from the common definition of $z_0$ to be located at a distance equal to $d_{50}/30$ (Nikuradse, 1933). Despite this relatively large mobile layer, ripple formations persist and remain mostly stationary over the entire realization (Figure 3-6). One possible explanation for this is the low specific gravity of the sediment grains ($s = 1.2$) compared to regular sand grains ($s = 2.65$), and suggested by the scaling provided by Henriquez et al. (2008).

In steady flow, the non-dimensional number governing the ratio between the disturbing and stabilizing forces was suggested by Shields (1936),

$$\theta_{u*} = \frac{u_*^2}{(s - 1)gd_{50}}$$  \hspace{1cm} (3.6)

where $u_*$ is obtained from the least-squares fit performed on (3.1), $s$ is the specific gravity of the sediment, $g$ is the gravitational acceleration, and $d_{50}$ is the median grain diameter. This is the densimetric non-dimensionalisation utilized by Henriquez et al. (2008) in the scale analysis of his physical model. The critical Shields parameter for incipient motion, $\theta_c$, is taken from the Madsen-Grant diagram in Nielsen (1992) as 0.05. For $\theta_{u*} > 0.8$ the bed is said to be under sheet flow conditions, where a thickness of 10 - 60 grain diameters mobilize, and the bed
Figure 3-12: (a) Thickness of the sediment moving layer, $\Delta z_{\text{mov}}$, for each horizontal position. (b) Shields parameter, $\theta_u$, at each horizontal position. The dashed line indicates the threshold for sheet flow. (c) Mean bed elevation for each wave period starting with darker lines at the beginning of the realization and ending with brighter lines at the end of the realization.

The values of $\theta_u$ exceed sheet flow conditions for almost the entire bed profile, even though the bedforms persist. Carstens et al. (1969) and Lofquist (1986) (taken from Nielsen, 1992) reported same order of magnitude values of $\theta_u$ during their laboratory experiments over rippled beds.

Some additional observations regarding Figure 3-12 follow: (1) Higher onshore Shields parameter at the first crest is consistent with the direction of bedform migration, suggesting that the stress induced by the onshore-directed flow is respon-
sible for mobilizing the bedforms; (2) the balance between the onshore and offshore Shields parameter causes bedform flattening \((x = 8 - 10.5 \, \text{cm})\); and (3) the Shields parameter and the thickness of the moving layer have an inverse relationship. This can be easily visualized in Figure 3-13, where steeper slopes associated with smaller magnitudes of \(u_x\) lead to lower \(z\)-intercepts and therefore thicker moving layers. In this data set, high shear stresses and small moving layers are generally found at the ripple crests, whereas milder stress and thicker moving layers are more easily found at the ripple troughs.

Following Nielsen (1992), Crawford and Hay (2001), Nichols and Foster (2009) and others, the grain roughness Shields parameter, \(\theta_{25}\), is defined with

\[
\theta_{25} = \frac{f_{25}}{2} \frac{u_0^2}{(s - 1)gd_{50}},
\]

where \(f_{25}\) is the wave friction factor approximated with (3.4), and \(u_0\) is the hori-
Figure 3-14: Values of the Shields parameter for onshore- (red plus signs) and offshore- (black open circles) directed flow, as calculated with the different methods provided in Section 3.3.2. Other experimental values of $\theta_{2.5}$ on flat beds include: Zala Flores and Sleath (1998) with sand (black open diamond) and acrylic granules (black open square), and Ribberink and Al-Salem (1995) with sand (black crosses). The black line represents the threshold for sheet flow ($\theta = 0.8$).

Horizontal velocity amplitude in the free-stream. Assuming horizontally uniform flow outside the boundary layer, $u_0$ was taken as the spatially averaged free-stream velocity for each flow direction. Using (3.7) yields results closer to those reported in literature (Hanes et al., 2001; Sumer et al., 2003; Zou and Hay, 2003; Nichols and Foster, 2009), even though this formulations does not account for ripple roughness. The estimates of $\theta_{2.5}$ are lower than $\theta_u$ by almost two orders of magnitude (Figure 3-14), implying that the log-layer approach could be overestimating the friction velocities and consequently the shear stresses.

A modified space-dependent grain roughness Shield parameter, $\theta_{2.5,u_*}$, can be obtained by substituting $u_*^2(x)$ for $u_0^2$ in (3.7). This could give a different estimate of the effective stress exerted on the bed by means of the friction velocity, $u_*$. Although velocity is squared in this formulation, $\theta_{2.5,u_*}$ maintains similar order of magnitude
after the change (Figure 3-14), showing little sensitivity to the modification.

According to Du Toit and Sleath (1981), the presence of a vortex ripple can significantly enhance the Shields parameter at a ripple crest. They observed an amplification factor proportional to the ripple steepness given by

$$\theta_r = \frac{\theta_{2.5}}{(1 - \pi \eta_b / \lambda_b)^2},$$

(3.8)

where $\eta_b$ is the ripple height and $\lambda_b$ is the ripple wavelength. This approach results in a $\theta_r$ at the threshold for sheet flow (Figure 3-14), but can not fully account for the discrepancy between $\theta_{2.5}$ and $\theta_{u*}$. $\theta_r$ is almost one order of magnitude lower than $\theta_{u*}$, once again suggesting a possible overestimation using the log-law approach.

In Newtonian fluids, the viscous shear stress $\tau_v$ is given by

$$\tau_v = \mu \frac{\partial u}{\partial z},$$

(3.9)

were $\mu$ is the dynamic viscosity of the fluid. The viscous shear stress in the lower 2 cm of the water column is depicted in Figure 3-10 for both flow directions. The magnitude of $\tau_v$ is higher during the onshore-directed flow and comprises a bigger area. This is consistent with the direction of bedform migration (Figure 3-6a). Figure 3-10b suggests that high shear is generated at the stoss (upstream) side of the ripple, where the pressure gradient is favorable, and then shed into the water column, with little or no bed shear stress at the lee (downstream) side. This is less evident in Figure 3-10a, where the shear stress maxima is confined to a smaller layer. In this case, the shear stress becomes zero before it switches sign, which is consistent with the observed boundary layer streaming. The observations of shear
being shed into the water column after the ripple crests (more noticeable for the onshore case) are consistent with flow separation and vortex shedding, as observed by Nichols and Foster (2007).

An additional estimate of the Shields parameter, neglecting turbulent stresses, can now be obtained from \( \tau_v \):

\[
\theta_v = \frac{\tau_v}{(s - 1) \rho g d_{50}},
\]

where \( \tau_v \) is the viscous shear stress as defined in (3.9) at \( z = z_b \), and \( \rho \) is the density of water. The values of \( \theta_v \) are \( O(10^{-2}) \) (Figure 3-14), suggesting that viscous shear is not the only mechanism inducing stress on a rippled bed, as also noted by Smith and McLean (1977), Chriss and Caldwell (1982), Grant and Madsen (1982), Li (1994), among others. A more in depth analysis of the different mechanisms imparting stress on the bed will be presented in Chapter 4.

The different values for \( \theta \) using the approaches presented in this section, are summarized in Figure 3-14. They differ by as much as four orders of magnitude, with the lowest being \( \theta_v \), which only takes into account the viscous shear. Values for \( \theta_{2.5} \) agree with some definitions presented in the literature, however it is not a direct measurement of the non-dimensional shear stress, as it is calculated with free-stream hydrodynamic quantities and only considers grain-induced roughness. Using the shear velocity obtained from fitting a logarithmic curve to the velocity profile, spatially varying values of the Shields parameter were estimated (\( \theta_{u_+} \)). These are higher than the threshold for sheet flow (\( \theta > 0.8 \)), and are not inconsistent with the thick moving layers reported herein (2.5 - 10 mm, 4 - 18 \( d_{50} \)). These observations
agree with experimental values reported in the literature for flat bed conditions 
(Ribberink and Al-Salem, 1995; Zala Flores and Sleath, 1998), even though bedforms persist. Using $\theta_r$, a modified version of $\theta_{2.5}$ that accounts for the existence of ripples, does not significantly alter the results.

A characterization of the wave bottom boundary layer has been presented in this chapter along with the free-stream hydrodynamics describing the flow above it. A quasi-steady approach was employed to obtain averages of the offshore- and onshore-directed flow fields. By fitting a logarithmic curve to the velocity profiles, estimates of boundary layer thickness and non-dimensional shear stress were obtained. Several measures of the boundary layer thickness were explored, yielding inconsistent results that differ by orders of magnitude and were, at times, inversely proportional. This confirms the complexity of including bedform effects in boundary layer thickness formulations. Similarly, different methods for calculating the Shields parameter resulted in values differing by orders of magnitude for one specific data set. The direct estimates of the Shields parameter using the quasi-steady approach are much higher than the threshold for sheet flow and are consistent with a thick mobile layer, even though ripples were present. The inconsistencies in this analysis demand a more thorough look into the time- and space-dependent behavior of the flow field and the equations governing it. This will take place in Chapter 4, where an unsteady analysis will be performed to further characterize the wave bottom boundary layer.
CHAPTER 4

UNSTEADY WAVE BOTTOM BOUNDARY LAYER

ANALYSIS

4.1 Introduction

Shear stress plays a crucial role in bottom boundary layer dynamics, sediment suspension, and the transformation of waves across the shelf. Sediment suspension and transport generally occurs when shear stress dominates the stabilizing forces exerted on the particles (Shields, 1936). The development of the boundary layer profiles is affected by the shear stress behavior. Moreover, drag coefficients and friction factors formulated to parameterize the bed, are significantly important for predicting sediment transport and wave energy dissipation in coastal environments (Nielsen, 1992). Therefore, it is of great importance to accurately quantify the mechanisms responsible for imparting shear stress on the bed.

Over flat beds, the total friction at the bed is composed of the viscous stresses acting on the sediment grains plus the pressure drag imparted by each individual grain, collectively named skin friction (Maddux et al., 2003). In the presence of roughness elements, with length scales much larger than the sediment grain diameter, a second stress mechanism emerges, usually called form stress, resulting from flow separation above these boundary features. The region where flow separation
occurs is characterized by mean velocity profiles with gradients differing from those near the bed, as seen by Smith and McLean (1977), Paola (1983), Wiberg and Nelson (1992), Li (1994), among others. These efforts support the hypothesis that total shear can be linearly partitioned into a skin friction component and a form stress component, as proposed by Einstein (1951). Note that the instruments used in the previous studies were unable to resolve the viscous sublayer.

In Chapter 3, several quasi-empirical formulations for estimating shear stress based on quasi-steady physics were examined, all of which produced results that differed by orders of magnitude, depending on their fundamental definition. In this chapter, the equations governing fluid motion over a sediment bed are double-averaged (i.e. averaged in time and space), yielding a new expression for shear stress partitioning.

Smith and McLean (1977) first introduced spatial averaging when analyzing mean velocity profiles in the Columbia River. A more formal approach was undertaken by Wilson and Shaw (1977) when studying wind flow over plant canopies. Gimenez-Curto and Corniero Lera (1996) extended the formulation for horizontally oscillating flow over a variable averaging domain. Thereafter, several authors have modified the double averaging technique to study different flow environments (Nikora et al., 2001; Hsu et al., 2002; Maddux et al., 2003; Lowe et al., 2005). Recently, Coleman et al. (2008) expanded the formulation to include progressive waves, and implemented it with PIV observations of the two-dimensional velocity field over a rippled bed. In this study, we extend the efforts of Coleman et al. (2008) to yield a momentum balance between the acceleration deficit, form-induced drag.
and shear stress gradient, as a function of wave phase.

4.2 DANS Equations: Theoretical Development

In this section, the continuity and momentum equations describing flow in the wave bottom boundary layer over a rippled sediment bed are averaged in time and space resulting in the Double-Averaged Navier-Stokes (DANS) equations. Flow parameters are decomposed in time and space within a time-varying fluid domain, resulting in expressions that admit forcing from waves and currents, form-induced pressure gradients, and ripple migration.

In this effort, velocity and pressure are decomposed into mean (current), phase (wave), and fluctuating (turbulent) components defined by

\[
\begin{align*}
    u_i & \equiv \bar{u}_i + \tilde{u}_i + u'_i \quad (4.1a) \\
    p & \equiv \bar{p} + \tilde{p} + p' \quad (4.1b)
\end{align*}
\]

where overbar (\(\bar{\cdot}\)) represents the mean, over tilde (\(\tilde{\cdot}\)) denotes phase average with zero mean, and prime (\('\)) represents departure from the latter. Time and phase averages are calculated following Nielsen (1992):

\[
\begin{align*}
    \bar{u}_i &= \frac{1}{N} \sum_{n=1}^{N} u_i(t_n) \quad (4.2) \\
    \tilde{u}_i(\hat{t}) &= \frac{1}{M} \sum_{m=0}^{M} u_i(t + mT) - \bar{u}_i \quad (4.3)
\end{align*}
\]
where \( N \) is the total number of samples, \( M \) is the total number of waves, and \( T \) is the wave period.

Substituting (4.1) in the Navier-Stokes equations for incompressible flow in Cartesian tensor notation, and taking the phase average yields

\[
\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial (\bar{u}_i \bar{u}_j)}{\partial x_j} + \frac{\partial (\bar{u}_i' \bar{u}_j')}{\partial x_j} + \frac{\partial (\bar{u}_i \bar{u}_j')}{\partial x_j} = \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j},
\]

(4.4)

where tildes have been omitted from terms that are already a function of phase \( \bar{t} \), and fluctuating quantities are assumed zero when phase-averaged over several wave periods. The equations presented in (4.4) are also known as the Reynolds-Averaged Navier-Stokes (RANS) equations for flows with a periodic signal.

A second averaging operation for flow over bedforms is introduced following Gimenez-Curto and Corniero Lera (1996):

\[
\langle F \rangle = \frac{1}{A_f} \int_{A_f} F \, dx_1 \, dx_2
\]

(4.5)

where angle brackets denote spatial average, \( F \) represents any flow quantity averaged over a horizontal plane with area \( A_f \) defined in the \( x_1 \) (cross-shore) and \( x_2 \) (alongshore) directions. For bedforms uniform in the alongshore direction, (4.5) reduces to

\[
\langle F \rangle = \frac{1}{L_f} \int_{L_f} F \, dx_1,
\]

(4.6)
where $L_f$ is the length occupied by the fluid in a very thin slab of length $L_0$, as shown in Figure 4-1. For the simplified case of a single bedform, $L_f$ represents the horizontal distance between points $A(a, x_3)$ and $B(b, x_3)$, such that $L_f(x_3) = b - a$. This formulation requires points $A$ and $B$ to be at the boundary, therefore below ripple crests, $L_f$ is a function of $x_3$. Please note, this formulation is only valid if the variability of the wave forcing occurs over a significantly larger scale than the averaging scale, i.e. $\lambda \gg L_f$, where $\lambda$ is the wavelength.

Figure 4-1: Schematic of the spatial averaging domain. For each vertical location, $x_3$, the average operation is performed over the portion of the slab occupied by the fluid, $L_f$. The maximum length of the slab is $L_0$, which in this case is equal to the distance between consecutive ripple crests, $\lambda_b$. Points $A(a, x_3)$ and $B(b, x_3)$ are located at the bed and are different at each vertical position. Above ripple crests, $L_f(x_3) = L_0$.

Taking the spatial average of (4.4) yields
\[ \left\langle \frac{\partial \bar{u}_1}{\partial t} \right\rangle + \left\langle \frac{\partial (\bar{u}_i \bar{u}_j)}{\partial x_j} \right\rangle + \left\langle \frac{\partial (u'_i u'_j)}{\partial x_j} \right\rangle
\]
\[ + \left\langle \frac{\partial (\bar{u}_i u'_j)}{\partial x_j} \right\rangle + \left\langle \frac{\partial (\bar{u}_i u'_j)}{\partial x_j} \right\rangle \]
\[ = -\frac{1}{\rho} \left\langle \frac{\partial \bar{p}}{\partial x_i} \right\rangle + \nu \left\langle \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} \right\rangle. \]

The spatial average defined in (4.6) can be used to evaluate each term in (4.7). In the case of the cross-shore derivatives, \( \partial / \partial x_1 \), the spatial average becomes

\[ \left\langle \frac{\partial F}{\partial x_1} \right\rangle = \frac{1}{L_f} \int_{L_f}^{L_f} \frac{\partial F}{\partial x_1} \, dx_1 \]
\[ = \frac{1}{L_f} \int_{a}^{b} \frac{\partial F}{\partial x_1} \, dx_1 \]
\[ = \frac{1}{L_f} F|_a^b. \]

where \( L_f = b - a \). The no-slip boundary condition requires all components of \( u_i \) to be zero at \( a \) and \( b \). While \( F \) can be any flow quantity in (4.7), for the case of velocity, (4.8) reduces to

\[ \left\langle \frac{\partial F}{\partial x_1} \right\rangle = 0. \]

However, pressure is nonzero at \( a \) and \( b \) and will be considered later.

Uniformity in the alongshore direction requires that \( \partial / \partial x_2 = 0 \). Using (4.6) to evaluate the vertical derivatives yields

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\[
\left\langle \frac{\partial F}{\partial x_3} \right\rangle = \frac{1}{L_f} \int_{L_f} \frac{\partial F}{\partial x_3} \, dx_1,
\]

(4.10)

with limits of integration being functions of the differentiating variable \((L_f = f(x_3))\). Leibniz Integral Rule given by,

\[
\int_{\Phi(\alpha)}^{\Psi(\alpha)} \frac{\partial f(\alpha, \beta)}{\partial \alpha} \, d\beta = \frac{\partial}{\partial \alpha} \int_{\Phi(\alpha)}^{\Psi(\alpha)} f(\alpha, \beta) \, d\beta
\]

\[
- f(\Psi(\alpha), \alpha) \frac{d\Psi}{d\alpha} + f(\Phi(\alpha), \alpha) \frac{d\Phi}{d\alpha};
\]

yields

\[
\left\langle \frac{\partial F}{\partial x_3} \right\rangle = \frac{1}{L_f} \left[ \frac{\partial}{\partial x_3} \int_a^b F \, dx_1 - F(b, x_3) \frac{db}{dx_3} + F(a, x_3) \frac{da}{dx_3} \right],
\]

(4.12)

where \(F\) is any velocity product in (4.7). The no-slip, no-penetration boundary conditions require \(F\) to be zero everywhere on the bed \((F(a, x_3) = F(b, x_3) = 0)\).

It follows that

\[
\left\langle \frac{\partial F}{\partial x_3} \right\rangle = \frac{1}{L_f} \frac{\partial}{\partial x_3} \int_{L_f} F \, dx_1
\]

\[
= \frac{1}{L_f} \frac{\partial}{\partial x_3} \left[ \frac{L_f}{L_f} \int_{L_f} F \, dx_1 \right]
\]

\[
= \frac{1}{L_f} \frac{\partial L_f}{\partial x_3} \langle F \rangle.
\]

(4.13)

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Similarly, the temporal derivative becomes:

\[
\left\langle \frac{\partial \bar{u}_i}{\partial t} \right\rangle = \frac{1}{L_f} \frac{\partial L_f \langle \bar{u}_i \rangle}{\partial t},
\]

where bedform migration is assumed uniform along the bed profile, thus causing the last two terms in (4.11) to cancel.

Substituting (4.9), (4.13) and (4.14) into (4.7) and multiplying by \( L_f \) results in

\[
\frac{\partial L_f \langle \bar{u}_i \rangle}{\partial t} + \frac{\partial L_f \langle \bar{u}_i \bar{u}_j \rangle}{\partial x_j} + \frac{\partial L_f \langle \bar{u}' \bar{u}' \rangle}{\partial x_j} \\
+ \frac{\partial L_f \langle \bar{u}_i \bar{u}_j \rangle}{\partial x_j} + \frac{\partial L_f \langle \bar{u}_i \bar{u}_j \rangle}{\partial x_j} \\
= -\frac{L_f}{\rho} \left\langle \frac{\partial \bar{p}}{\partial x_i} \right\rangle + \nu \frac{\partial^2 L_f \langle \bar{u}_i \rangle}{\partial x_j \partial x_j}.
\]

In a manner analogous to a Reynolds decomposition, the mean and phase-averaged velocity and pressure can be decomposed into a spatial average plus a bedform-induced disturbance, such that:

\[
\bar{u}_i \equiv \langle \bar{u}_i \rangle + (\bar{u}_i)_b, \quad (4.16a)
\]

\[
\bar{p} \equiv \langle \bar{p} \rangle + (\bar{p})_b, \quad (4.16b)
\]

and
where angle brackets denote the spatial average given by (4.6), and the subscript \( b \) denotes the bedform disturbance. The inclusion of parentheses in the bedform component emphasizes the order of operation, where temporal averages precede spatial averages. Please note, spatial decomposition of the turbulent terms is not performed. Also, decomposition of \( \bar{p} \) is shown even though it will not be necessary in this analysis as this term goes to zero when phase-averaged using the pressure equivalent to (4.3). The above definitions require the boundary disturbances to spatially average to zero, i.e. \( \langle (u_1)_b \rangle = \langle (\bar{p})_b \rangle = \langle (\tilde{u}_t)_b \rangle = \langle (\tilde{p})_b \rangle = 0 \).

Substituting (4.16) into the first term of (4.15), yields the spatial decomposition of the phase-averaged component:

\[
\langle \tilde{u}_1 \tilde{u}_j \rangle = \left[ \langle \tilde{u}_1 \rangle + (\tilde{u}_t)_b \right] \left[ \langle \tilde{u}_j \rangle + (\tilde{u}_j)_b \right] \\
= \langle \tilde{u}_1 \rangle \langle \tilde{u}_j \rangle + \langle \tilde{u}_1 \rangle (\tilde{u}_j)_b + (\tilde{u}_t)_b \langle \tilde{u}_j \rangle + (\tilde{u}_t)_b (\tilde{u}_j)_b \\
= \langle \tilde{u}_1 \rangle \langle \tilde{u}_j \rangle + \langle \tilde{u}_1 \rangle (\tilde{u}_j)_b + \langle (\tilde{u}_t)_b \rangle \langle \tilde{u}_j \rangle + \langle (\tilde{u}_1)_b \rangle (\tilde{u}_j)_b \\
= \langle \tilde{u}_1 \rangle \langle \tilde{u}_j \rangle + \langle (\tilde{u}_1) \rangle (\tilde{u}_j)_b, \tag{4.18}
\]

where \( \langle \tilde{u}_1 \rangle \) and \( \langle \tilde{u}_j \rangle \) do not depend on \( x_1 \) and were therefore pulled out of the spatial
average. Similarly,

\[ \langle \tilde{u}_i \tilde{u}_j \rangle = \langle \tilde{u}_i \rangle \langle \tilde{u}_j \rangle + \langle (\tilde{u}_i)_b (\tilde{u}_j)_b \rangle, \quad (4.19) \]

\[ \langle \tilde{u}_i \tilde{u}_j \rangle = \langle \tilde{u}_i \rangle \langle \tilde{u}_j \rangle + \langle (\tilde{u}_i)_b (\tilde{u}_j)_b \rangle. \quad (4.20) \]

The periodic component of the cross-shore pressure gradient is now evaluated with

\[
- \frac{L_f}{\rho} \left( \frac{\partial \tilde{p}}{\partial x_1} \right) = - \frac{L_f}{\rho} \left( \frac{\partial \langle \tilde{p} \rangle}{\partial x_1} + \frac{\partial \langle \tilde{p} \rangle_b}{\partial x_1} \right) \\
= - \frac{1}{\rho} \int_{L_f} \frac{\partial \langle \tilde{p} \rangle}{\partial x_1} \ dx_1 - \frac{1}{\rho} \int_{L_f} \frac{\partial \langle \tilde{p} \rangle_b}{\partial x_1} \ dx_1. \quad (4.21) \]

For linear free-surface gravity waves, the variability of \( \tilde{p} \) occurs over a significantly larger scale than \( L_f \), as \( \lambda \gg L_f \). The first gradient on the right-hand-side of (4.21) is nonzero and can be approximated with the inviscid solution of the momentum equation in the free-stream, given, in the cross-shore direction, by

\[ \frac{\partial \langle \tilde{u}_\infty \rangle}{\partial t} = - \frac{1}{\rho} \frac{\partial \langle \tilde{p} \rangle}{\partial x_1} \quad (4.22) \]

where velocity and pressure have been decomposed as in (4.17), with zero boundary disturbance, and the pressure gradient assumed vertically uniform. While not included in the derivation, this formulation does not preclude higher order wave solutions from being considered. To first order, (4.21) becomes
Substituting (4.18 - 4.20) and (4.23) into (4.15) yields the unsteady Double-Averaged Navier-Stokes (DANS) equations for combined wave and current flow over a rippled bed:

\[
\begin{eqnarray*}
- \frac{L_f}{\rho} \left\langle \frac{\partial \bar{p}}{\partial x_1} \right\rangle &=& \frac{\partial L_f}{\partial t} \left\langle \bar{u}_\infty \right\rangle - \frac{(\bar{p})_b^b}{\rho} \frac{\partial}{\partial t} \frac{\partial}{\partial x_1} \\
&=& \frac{\partial L_f}{\partial t} \left\langle \bar{u}_\infty \right\rangle - \frac{(\bar{p})_b^b}{\rho}, \quad (4.23)
\end{eqnarray*}
\]

This equation is a simplified version of those presented by previous authors (Gimenez-Curto and Corniero Lera, 1996; Coleman et al., 2008), and was achieved by assuming that the boundary irregularities are uniform in the alongshore direction.

In the case of flows that are uniform in the alongshore direction, the horizontal component of the DANS equations in Cartesian coordinates becomes

\[
\begin{eqnarray*}
\frac{\partial L_f}{\partial t} \left\langle \bar{u}_\infty - \bar{u}_\infty \right\rangle + \frac{\partial L_f}{\partial x_j} \left\langle \bar{u}_i \right\rangle \left\langle \bar{u}_j \right\rangle + \frac{\partial L_f}{\partial x_j} \left\langle \bar{u}_i \right\rangle \left\langle \bar{u}_j \right\rangle + \frac{\partial L_f}{\partial x_j} \left\langle \bar{u}_i \right\rangle \left\langle \bar{u}_j \right\rangle \\
+ \frac{\partial L_f}{\partial x_j} \left\langle \left( \bar{u}_i \right) b \left( \bar{u}_j \right) b \right\rangle + \frac{\partial L_f}{\partial x_j} \left\langle \left( \bar{u}_i \right) b \left( \bar{u}_j \right) b \right\rangle \\
+ \frac{\partial L_f}{\partial x_j} \left\langle \left( \bar{u}_i \right) b \left( \bar{u}_j \right) b \right\rangle + \frac{\partial L_f}{\partial x_j} \left\langle \left( \bar{u}_i \right) b \left( \bar{u}_j \right) b \right\rangle \\
= - \frac{(\bar{p})_b^b}{\rho} + \nu \frac{\partial^2 L_f}{\partial x_j \partial x_j} \left\langle \bar{u}_i \right\rangle \quad (4.24)
\end{eqnarray*}
\]

\[
\begin{eqnarray*}
\frac{\partial L_f}{\partial t} \left\langle \bar{u} - \bar{u}_\infty \right\rangle + \frac{\partial L_f}{\partial z} \left\langle \bar{w} \right\rangle + \frac{\partial L_f}{\partial z} \left\langle \bar{w} \right\rangle + \frac{\partial L_f}{\partial z} \left\langle \bar{w} \right\rangle \\
+ \frac{\partial L_f}{\partial z} \left\langle \left( \bar{u} \right) b \left( \bar{w} \right) b \right\rangle + \frac{\partial L_f}{\partial z} \left\langle \left( \bar{u} \right) b \left( \bar{w} \right) b \right\rangle \\
+ \frac{\partial L_f}{\partial z} \left\langle \left( \bar{u} \right) b \left( \bar{w} \right) b \right\rangle + \frac{\partial L_f}{\partial z} \left\langle \left( \bar{u} \right) b \left( \bar{w} \right) b \right\rangle \\
= - \frac{(\bar{p})_b^b}{\rho} + \nu \frac{\partial^2 L_f}{\partial z^2} \left\langle \bar{u} \right\rangle \quad (4.25)
\end{eqnarray*}
\]

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Multiplying by $\rho/L_0$ and noting that

$$\nu \frac{\partial^2 L_f \langle \ddot{u} \rangle}{\partial z^2} = \nu \frac{\partial}{\partial z} \left( L_f \frac{\partial \langle \ddot{u} \rangle}{\partial z} \right) + \nu \langle \ddot{u} \rangle \frac{\partial^2 L_f}{\partial z^2} + \nu \frac{\partial L_f}{\partial z} \frac{\partial \langle \ddot{u} \rangle}{\partial z},$$  \hspace{1cm} (4.26)

yields

$$\rho \frac{\partial \zeta \langle \ddot{u} - \ddot{u}_\infty \rangle}{\partial t} = - \frac{(\ddot{p})_b}{L_0} \frac{\partial \zeta \tau_{xz}}{\partial z} + f_{sk},$$  \hspace{1cm} (4.27)

where $\zeta \equiv L_f/L_0$ is a weighting function that accounts for the reduced averaging areas below ripple crests, $\tau_{xz}$ is the shear stress given by

$$\tau_{xz} = \mu \frac{\partial \langle \ddot{u} \rangle}{\partial z} - \rho \langle \ddot{u} \rangle \langle \ddot{w} \rangle - \rho \langle \ddot{u} \rangle \langle \ddot{w} \rangle - \rho \langle \ddot{u} \rangle \langle \ddot{w} \rangle - \rho \langle \ddot{u} \rangle \langle \ddot{w} \rangle - \rho \langle \ddot{u} \rangle \langle \ddot{w} \rangle - \rho \langle \ddot{u} \rangle \langle \ddot{w} \rangle - \rho \langle \ddot{u} \rangle \langle \ddot{w} \rangle - \rho \langle \ddot{u} \rangle \langle \ddot{w} \rangle \hspace{1cm} (4.28)$$

and $f_{sk}$ is the bedform-induced component of skin friction given by

$$f_{sk} = \mu \langle \ddot{u} \rangle \frac{\partial^2 \zeta}{\partial z^2} + \mu \frac{\partial \zeta}{\partial z} \frac{\partial \langle \ddot{u} \rangle}{\partial z}. \hspace{1cm} (4.29)$$

Equation 4.27 shows that the acceleration deficit in the wave bottom boundary layer is balanced by the shear stress gradient plus the form and viscous drag per unit fluid volume. The stress terms in (4.28) represent: the viscous stress (first term); the wave stress (second term); the convective transfer of momentum (third and fourth terms, Coleman et al. (2008)); the form-induced stresses (fifth, sixth and seventh terms); and the turbulent stress (eighth term).

The expressions presented in (4.27) and (4.28) are the DANS equations for the
wave bottom boundary layer over a movable rippled bed, for the case of waves and
ripples oriented parallel to the shore. Note that these expressions allow for oblique
currents as long as they are alongshore uniform. These expressions are of significant
importance for the following reasons: 1) they provide a momentum balance relating
the acceleration deficit to the force exerted by the ripples and sediment particles
on the fluid, and the momentum transfer gradients; 2) they partition the stress
allowing for the assessment of the contribution of individual terms; and 3) they
allow for a sensible reduction of the two-dimensional time-varying observations.
The following section presents the results of an experimental investigation using
this technique to better understand the mechanisms governing the flow in the wave
bottom boundary layer.

4.3 Results

A 60-s realization of the two-dimensional velocity field over a rippled bed sub­
jected to a wave forcing was collected using PIV at a sampling rate of 12 Hz. The
monochromatic waves were 5 cm high, 3.3 m long, and 2 s in wave period, allowing
for 30 waves ensembles. Please refer to Chapter 2 for a more thorough description
of the experimental methods.

Phase-dependent bed elevation was estimated using the methodology described
in Chapter 2 with phase-averaged PIV images, which were obtained with the
method analogous to (4.3). Figure 4-2 shows selected image ensembles with corre­
sponding bed elevation estimates. A general description of the near-bed sediment
motion follows:
Figure 4-2: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b - i) Image ensembles for eight different phases indicated in (a). The green lines show bed elevation estimates. Onshore flow is directed to the left.
• At a phase of 0°, right before flow reversal, sediment plumes are observed in the onshore (left) side of the ripples. Free-stream vertical velocity is at a negative maximum (Figure 4-2b).

• At a phase of 45°, the sediment plumes are above the ripple crests, and the bed dilates (Figure 4-2c).

• At a phase of 105°, the free-stream horizontal velocity is at an offshore maximum and the sediment plumes are all located above ripple troughs (Figure 4-2d).

• At a phase of 150°, the sediment plumes have reached the crest of the next ripple (Figure 4-2e).

• Right before the next flow reversal, at a phase of 195°, the first sediment plumes have now moved an entire ripple length and are located above the next offshore ripple trough. They appear weaker as settling has occurred due to horizontal deceleration. A new high sediment concentration area appears offshore of the ripple crests (Figure 4-2f).

• At a phase of 240°, the new sediment plumes start moving onshore above the ripple crests (Figure 4-2g).

• By a phase of 285°, the sediment plumes have passed the ripple crest and are located above ripple troughs. They appear larger in size and lower in concentration. The free-stream horizontal velocity is at an onshore maximum (Figure 4-2h).
• At a phase of 330°, the sediment plumes have passed the next onshore ripple crest. A new and smaller concentration area forms onshore of the ripple crest. The bed flattens coincident with increasing negative vertical velocities (Figure 4-2i).

It is worth noting that the above description is solely based on image intensity and does not replace in situ measurements of sediment concentration. Nevertheless, the general motion of the sediment plumes agree with findings by Van der Werf et al. (2007), who also observed sediment plumes moving one ripple length within a half-wave period. The complete phase series of image ensembles and associated bed height estimates is included in Appendix B.

A time series of horizontal free-stream acceleration is shown in Figure 4-3a. Although some would argue that bed dilation occurs when the overlying wave acceleration is large (Sleath, 1999), for this specific case, the bed dilates back to its original geometry after it has been flattened by large negative wave vertical velocities (Figure 4-2i and b). Dilation does not occur during high acceleration near the second flow reversal (Figure 4-2f).

Figure 4-3 shows the general flow pattern for eight different wave phases, which can be described as follows:

• At a phase of 0°, right before on-offshore flow reversal, a small circulation area is observed on the onshore (left) side of the ripples (Figure 4-3b).

• At a phase of 45°, near-bed velocities are higher than the free-stream velocities, especially above ripple crests and offshore slopes (Figure 4-3c). This is consistent with the boundary layer phase lead shown in Figure 3-4.
Figure 4-3: (a) Free-stream phase-averaged horizontal velocity (black) and acceleration (blue). (b - i) Negative phase-averaged image intensity with phase-averaged velocity field (red) for eight different phases indicated in (a). Scale vectors in the top right corner represent 10 cm s$^{-1}$. Onshore flow is directed to the left.
• At a phase of $105^\circ$, the free-stream horizontal velocity is at an offshore maximum. This is consistent over the entire observation window, with the exception of the offshore ripple slopes near the bed, where velocities have started to decelerate due to an adverse pressure gradient imposed by the ripple (Figure 4-3d).

• At a phase of $150^\circ$, a boundary layer develops on the onshore slope of the ripples. Velocity overshoots above the offshore ripple slopes (Figure 4-3e).

• Right before off-onshore flow reversal, at a phase of $195^\circ$, the free-stream acceleration is close to a negative maximum and a strong vortex is observed above the center ripple trough (Figure 4-3f).

• At a phase of $240^\circ$, flow accelerates near the crests due to ripple constriction (Figure 4-3g).

• By a phase of $285^\circ$, the free-stream horizontal velocity is at an onshore maximum. Flow near the onshore slope starts to decelerate due to an adverse pressure gradient imposed by the ripple (Figure 4-3h).

• By a phase of $330^\circ$, the ripples have flattened considerably causing the flow above the onshore slopes to accelerate back, therefore delaying separation. The free-stream acceleration is at a positive maximum (Figure 4-3i).

It is clear from Figure 4-3 that the vortex formed at $\tilde{t} = 195^\circ$ is much stronger than the one present at $\tilde{t} = 0^\circ$. In the study by Van der Werf et al. (2007) with pure horizontally oscillating flow (no free-stream vertical velocity), stronger vortices
Figure 4-4: (a) Bed elevation as a function of wave phase (right vertical axis). Each bed profile is vertically offset by 3 mm. (b) Bedform migration rate between consecutive ensembles.

forming near on-offshore flow reversal were observed ($\hat{t} = 0^\circ$ in this study). We attribute the discrepancy to the fact that, in the present study, ripple slopes ($\eta_b/\lambda_b$), coincident with on-offshore flow reversal, are more gentle than their counterparts due to large downward velocities, thus creating a weaker vortex. Additionally, vertical velocities are not present in studies with oscillatory water tunnels, like that of Van der Werf et al. (2007). Nichols and Foster (2007) also observed stronger vortices, parameterized by the swirling strength (Zhou et al., 1999), near off-onshore flow reversal. Their full-scale experiment included wave groups and asymmetric ripples. For a complete series of flow maps at each wave phase please refer to Appendix C.

Phase-dependent bed elevation allows for a time-varying domain between ripple
crests. The bedform stacks in Figure 4-4a (offset by 3 mm) show how the bed elevation varies in \( x, z, \) and \( \tilde{t} \). This framework will allow the velocity quantities in Section 4.3.1 to be spatially averaged over a time- and spatially-varying domain. Furthermore, intra-wave bedform migration rates can be obtained from cross-correlation analysis of phase-dependent bedform elevation. Figure 4-4b depicts the migration rate between two consecutive bed elevation estimates. In general, these observations show bedforms migrating in the same direction as the free-stream velocity. The net migration is 0.03 cm s\(^{-1}\) directed onshore, and is found by correlating the first and last bed elevation estimates. This is consistent with migration rates reported by Nichols and Foster (2007) during their full scale experiment, although six times higher than those reported in Chapter 3. Because total migration rates are not the focus of this study, the discrepancy between these two values will be attributed to uncertainty in the methods of analysis.

4.3.1 Velocity Decomposition

In order to assess the effect of boundary disturbances (ripples) on the velocity field, mean and phase-averaged velocities are decomposed following (4.16) and (4.17). The resulting flow fields are composed of a spatial average (denoted by angle brackets, \( \langle \rangle \)), plus a boundary disturbance (denoted by the subscript \( b \)). The averaging operation is performed following (4.6), which in Cartesian coordinates becomes

\[
\langle F \rangle \equiv \frac{1}{L_f} \int_{L_f} F \, dx, \tag{4.30}
\]

where \( F \) can be \( \bar{u}, \bar{w}, \hat{u} \) or \( \hat{w} \), \( x \) is directed cross-shore, and \( L_f = L_0 = 10.5 \text{ cm} \).
Figure 4-5: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Mean horizontal and (e) vertical velocity fields decomposed into (c, f) a spatial average plus a (d, g) bedform disturbance. (h) Mean and (i) bedform-induced velocity fields over mean pixel intensity. Scale vectors in the top left corner represent 2 cm s\(^{-1}\). Negative flow is directed onshore and left.

above ripple crests. Please note that, in order to improve confidence in the results, the averaging domain was chosen to include two ripple wavelengths. This is only possible because of the symmetry of this data set, and will not be feasible otherwise.

As expressed in Section 4.2, below ripple crests, averaging is only performed within the area occupied by the fluid.
The spatial decomposition of mean velocities is shown in Figure 4-5. Spatially averaged mean horizontal velocity is slightly positive (less than 1 cm s$^{-1}$), and is attributed to recirculation in the wave flume. As explained by Nielsen (1992), zero water transport in the shoreward direction of a wave flume produces a mean steady offshore current, known as Stokes drift, with magnitude

$$
\bar{u}_{stokes} = \frac{gH^2}{8ch}
$$

where $g$ is the gravitational acceleration, $H$ is the wave height, $c = \lambda/T$ is the wave celerity, $\lambda$ is the wavelength, $T$ is the wave period, and $h$ is the water depth. For this data set $H = 5$ cm, $\lambda = 3.3$ m, $T = 2$ s, and $h = 0.31$ cm, resulting in $\bar{u}_{stokes} = 0.6$ cm s$^{-1}$. The bedform-induced mean horizontal velocity field is generally small, and shows small negative velocities at the offshore slope of the ripples and positive velocities at the onshore slopes.

The mean vertical velocity field is shown in Figure 4-5e. A strong signal of negative velocities above ripple troughs is present, and may be attributed to the settling of the particles above areas of high pressure. The expected settling velocity ($w_0$) is 1.8 cm s$^{-1}$ as calculated following Cheng (2009), and about twice as large as $\langle \bar{w} \rangle$. Nevertheless, the bedform-induced mean vertical velocity field follows an expected trend with upward velocities at the ripple crests and slopes, and downward velocities at the troughs.

The mean vector map over the mean pixel intensity is shown in Figure 4-5h, with corresponding bedform-induced mean vector map in Figure 4-5i. As expected, there is no mean contribution by the bedforms to the mean velocity field above
Figure 4-6 (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal and (e) vertical velocity fields at $\hat{t} = 0^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance (h) Phase-averaged and (i) bedform-induced velocity fields over phase-averaged image intensity Scale vectors in the top left corner represent 2 cm s$^{-1}$ Negative flow is directed onshore and left.

one ripple height from the crests In analogy to potential flow theory, the bedform-induced vector map shows ripple crests behaving like point sources and troughs behaving like sinks.

Figures 4-6 to 4-13 show the spatial decomposition of the phase-averaged velocities for eight selected phases Panels (b) and (e) show the horizontal and vertical
Figure 4-7: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at $\tilde{t} = 45^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform-induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent 2 cm s$^{-1}$. Negative flow is directed onshore and left.

phase-averaged velocity fields, respectively. Panels (c) and (f) show their spatial average, following (4.30). Panels (d) and (g) show bedform-induced horizontal and vertical velocity fields, respectively. And panels (h) and (i) show the phase-averaged vector map, corresponding to Figure 4-3, and bedform-induced vector map, respectively, over the phase-averaged pixel intensity. The decomposition of
the phase-averaged velocity field will be described as follows:

- At a phase of 0°, right before off-onshore zero crossing, \( \bar{u}_\infty \) is very small and a circulation area is evident in \( \bar{\omega}_b \) (Figure 4-6).

- At a phase of 45°, near-bed velocities are leading the free-stream. The pres-
Figure 4-9 (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal and (c) vertical velocity fields at $\bar{\phi} = 150^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance (h) Phase-averaged and (i) bedform-induced velocity fields over phase-averaged image intensity Scale vectors in the top left corner represent 10 cm s$^{-1}$ Negative flow is directed onshore and left

ence of bedforms starts causing positive $\langle \bar{u} \rangle_b$ above ripple crests. Similarly, the bed forms cause vertical velocities to point upward as they move up the ripple slope, and downwards as they pass the crests (Figure 4-7)

- At a phase of 105°, the free-stream horizontal velocity is at an offshore maxi-
Figure 4-10  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at $\theta = 195^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform-induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent 4 cm s$^{-1}$. Negative flow is directed onshore and left.

The pressure gradient imposed by the ripples is now entirely apparent, as $(\bar{u})_b$ and $(\bar{w})_b$ are both positive when flow moves up the ripples, and negative after flow passes the ripple crests (Figure 4-8).

- At a phase of 150°, the free-stream flow starts decelerating. Low momentum
Figure 4-11  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity  (b) Horizontal and (e) vertical velocity fields at $\bar{\ell} = 240^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance  (h) Phase-averaged and (i) bedform-induced velocity fields over phase-averaged image intensity Scale vectors in the top left corner represent 10 cm s$^{-1}$ Negative flow is directed onshore and left.

flow at the onshore slope of the ripples responds faster to the wave pressure gradient, therefore $\langle \bar{u} \rangle_b$ is negative in this region. A wake-like area characterized by positive $\langle \bar{u} \rangle_b$ and negative $\langle \bar{w} \rangle_b$ forms after ripple crests (Figure 4-9)
Figure 4-12 (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal and (e) vertical velocity fields at $\bar{t} = 285^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance (h) Phase-averaged and (i) bedform-induced velocity fields over phase-averaged image intensity Scale vectors in the top left corner represent 12 cm s$^{-1}$ Negative flow is directed onshore and left

- At a phase of $195^\circ$, right before off-shore flow reversal, a strong vortex appears at the center ripple trough. Its signature appears in both, $(\bar{u})_b$ and $(\bar{w})_b$. As shown by vector maps (h - i), its existence is almost solely due to the presence of bedforms (Figure 4-10)
Figure 4-13: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (c) vertical velocity fields at \( \theta = 330^\circ \) decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform-induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent 7 cm s\(^{-1}\). Negative flow is directed onshore and left.

- At a phase of 240\(^\circ\), \((\bar{u})_b\) is slightly negative above crests due to ripples constraining the flow. A boundary layer develops on the offshore slope of the ripple. \((\bar{w})_b\) is positive as flow moves up the ripple and negative as it moves down (Figure 4-11).
• At a phase of 285°, the free-stream horizontal velocity is at an onshore maximum. The pressure gradient imposed by the bedforms causes \((\bar{u})_b\) to be hindered after flow passes the ripple crests. \((\bar{u})_b\) behaves as in \(t = 240°\). A noisy area appears at the left edge of the domain (Figure 4-12).

• At a phase of 330°, the ripples have flattened considerably. A new circulation area emerges and its signature is present in both, \((\bar{u})_b\) and \((\bar{w})_b\). The vector maps show that its existence is mostly due to the presence of the bedforms (Figure 4-13).

The spatial decomposition shown in Figures 4-6 to 4-13 is revealing for several reasons: (1) Excluding the edges of the domain, whose noisy signal is attributed to image distortion associated with the PIV technique (Raffel et al., 2007), bedforms do not affect the flow field more than one ripple height above the crests. This agrees, within the same order of magnitude, with measurements of \(\delta_{99}\) and \(\delta_{GM}\) (see Section 3.3.1). (2) Vortices with scales similar to the ripple height are solely due to the presence of bedforms (Figures 4-10i and 4-6i). And (3) steeper bedforms cause stronger and bigger vortices, consistent with observations by Nichols and Foster (2009). The complete series of phase-averaged spatially decomposed velocity fields can be found in Appendix D.

So far, mean and phase-averaged velocities, and their spatial decomposition, have been presented. We now proceed to examine the turbulent velocities, which are found experimentally by solving (4.1a) for \(u'_t\). Figures 4-14 to 4-21 present four phase-averaged turbulent quantities: horizontal turbulent energy \((\bar{w}^2)\), vertical turbulent energy \((\bar{w}^2)\), Reynolds stress \((\overline{uw'})\), and turbulent kinetic energy
Figure 4-14: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{t} = 0^\circ$. Negative flow is directed onshore and left.

$(\overline{u'^2} + \overline{w'^2})$. Note that, the phase average definition requires subtraction of the mean, and therefore these turbulent quantities can have negative values.

Horizontal turbulent energy is more pronounced during high wave velocities. For both flow directions, $\overline{u'^2}$ is higher at the downstream slope of the ripple, suggesting that ripples act as a backward facing step for large wave forcings, independently of flow direction. Nichols and Foster (2007) observed similar correspondence with flow past a backward facing step, but only for one flow direction due to asymmetry of the bedforms. Also, these observations show that when $|\bar{u}_\infty| < 6 \text{ cm s}^{-1}$, there is no production of horizontal turbulent energy.
Figure 4-15  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at \( \ell = 45^\circ \) Negative flow is directed onshore and left

A vertical turbulent energy signal \( \overline{w'^2} \) appears above the ripple crests and onshore slopes at \( \ell = 0^\circ \). It moves with the direction of the flow until it reaches the offshore ripple trough by maximum offshore flow. At the ripple trough, the signal loses some strength. At flow reversal, it starts moving onshore as it becomes bigger and stronger. By maximum onshore flow, it has reached the onshore ripple slope. It remains at the onshore slope as it loses strength and decreases in size. Note in Figures 4-14 to 4-21, the color scale of \( \overline{u'^2} \) and \( \overline{w'^2} \) differs by one order of magnitude.

Reynolds stresses \( \overline{u'w'} \) are highly negative above offshore ripple slopes during phases of high offshore flow. Similarly when flow is large and directed onshore, \( \overline{u'w'} \)
Figure 4-16 (a) Free stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $t = 105^\circ$ Negative flow is directed onshore and left is highly positive above onshore ripple slopes. These areas of high $u'w'$ eventually reach the ripple trough and reduce their intensity before flow reversal. Note that when the magnitude of $u'w'$ is at a minimum, near flow reversal, noticeable large scale coherent structures are observed (Figure 4-10). This evidences the fact that turbulence can indeed be partitioned from bedform-induced motion.

Turbulent kinetic energy, TKE, follows almost the exact same trend as $\tilde{u}^2$. This is expected since $\tilde{u}^2$ is one order of magnitude lower than $\tilde{u}^2$. TKE is larger during high free-stream velocities and significantly diminishes by the time of flow reversal, as also observed by Hino et al. (1983) and Sleath (1987). Similar to observations...
by Foster et al. (2006a) over a flat bed in the coastal wave bottom boundary layer, the intensity of TKE is higher under wave crest than troughs, suggesting a dependance on acceleration. For the complete series of phase-averaged turbulent quantities, please refer to Appendix E.

4.3.2 Shear Stress

Time- and space-decomposition of the velocity field allows for individual estimates of the momentum flux terms responsible for imparting shear stress on the sediment bed (see expression 4.28). These terms are shown for eight different wave phases.
Figure 4-18: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $t = 195^\circ$. Negative flow is directed onshore and left.

in Figure 4-22, and can be divided in four major categories: wave- and current-induced momentum transfer terms (black), bedform-induced momentum transfer terms (red), turbulent (or Reynolds) stresses (blue), and viscous stresses (green). The complete series of phase-averaged stress terms can be found in Appendix F.

Figure 4-22 shows that the stress due to viscosity $(-\rho \frac{\partial \langle \vec{u} \rangle}{\partial z})$ is significantly lower than most of the other terms. Reynolds stresses $(-\rho \langle u'w' \rangle)$ are large during high free-stream velocities, and maximum at the wave crest. After off-onshore flow reversal, the portion of $-\rho \langle u'w' \rangle$ located below ripple crests switches sign before the portion above. $-\rho \langle u'w' \rangle$ is generally minor about two ripple heights above
Figure 4-19 (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $t = 240^\circ$. Negative flow is directed onshore and left the crest. These results are significantly different from those of Coleman et al (2008), who found the phase- and spatially averaged values of Reynolds stresses to be negligible at all heights and wave phases.

The bedform-induced terms $(-\rho \langle (\bar{u})_b(\bar{w})_b\rangle, -\rho \langle (\bar{u})(\bar{w})_b\rangle, \text{ and } -\rho \langle (\bar{u})_b(\bar{w})_b\rangle)$, and the term correlating the mean horizontal velocity with the wave vertical velocity $(-\rho (\bar{u})(\bar{w}))$, are significant below ripple crests, although small if compared to $-\rho (\bar{u})(\bar{w}), -\rho (\bar{u})(\bar{w})$ and $-\rho (\bar{u}'\bar{w}')$. Consistent with observations by Coleman et al (2008), the bedform-induced terms are negligible above one ripple height from the crest.
Wave-induced stresses, resulting from correlations between horizontal and vertical wave motions \(-\rho\langle \tilde{u}\rangle\langle \tilde{w}\rangle\), are generally high up in the water column, except near both flow reversals. These arise from wave orbital motions and agree with the trends observed by Coleman et al. (2008). Near the bed, \(-\rho\langle \tilde{u}\rangle\langle \tilde{w}\rangle\) is more pronounced during onshore-directed flow.

Perhaps the most significant momentum transfer term near the bed results from correlations between the wave horizontal velocity and the mean vertical velocity, \(-\rho\langle \tilde{u}\rangle\langle \bar{w}\rangle\). This term dominates at the trough and within two ripple heights above the crest, and is small away from the bed, as \langle \bar{w}\rangle becomes negligible.
Figure 4-21  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at \( \bar{t} = 330^\circ \). Negative flow is directed onshore and left.

It is, however, recognized that sediment settling velocity can potentially bias the estimates of \( \bar{w} \), and consequently \( -\rho(\bar{u})(\bar{w}) \) and \( -\rho((\bar{u})_b(\bar{w})_b) \). The Stokes number \( (Stk) \) was calculated to assess if the trajectory of the sediment particles was severely influenced by gravity. When \( Stk \gg 1 \), gravity dominates the inertial forces, impeding the particle from following the fluid’s trajectory, alternatively when \( Stk \ll 1 \) the particles tend to follow the streamlines closely. Crowe (2006) suggested

\[
Stk = \frac{t_s}{t_f}, \tag{4.32}
\]
where $t_f$ is the characteristic time scale of the fluid, taken conservatively as the inverse of the wave radian frequency, $\omega^{-1}$ (Kundu and Cohen, 2004). $t_s$ is the particle relaxation time as given by Raffel et al. (2007),

$$t_s = \frac{d_{50}^2 \rho_p}{18 \mu},$$

(4.33)

where $d_{50}$ is the median grain diameter, $\rho_p$ is the particle density, and $\mu$ is the dynamic viscosity of water. The resulting $Stk = 0.06$, and therefore inertial forces should dominate the particles’ trajectories.

Nevertheless, two alternative expressions for $\bar{w}$ will be explored. The first, makes use of the continuity equation for incompressible fluids to derive a new estimate of $\bar{w}$:

$$\bar{w}(z) = \int_z^\infty \frac{\partial \bar{u}}{\partial x} dz,$$

(4.34)

where $\bar{w}(\infty) = 0$ have been assumed. The shear stress utilizing this approach is depicted in Figure 4-23b. Evidently, in the near-bed region, this approach reduces the stress signal significantly (up to 50%), especially for phases higher than 220°. The disadvantage of using this approach is that an artificial signal is embedded in $\bar{w}$ due to the noise imposed by the differential quantity $\partial \bar{u}/\partial x$.

A second approach to deal with the potential bias imposed by the particle settling velocity is to make $\bar{w} = 0$. The rationale to use this method comes from the mean vertical velocity signal being significantly lower than the horizontal wave signal, $\bar{u}$. However, this assumption fails near the bed where large scale vortex formation and ejection occur. The shear stress as calculated with $\bar{w} = 0$ is presented
Figure 4.22: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b - i) Terms conforming expression (4.28) for each vertical position at eight different wave phases indicated in (a). The lower bound of the domain is located at the ripple trough, and the horizontal dashed line represents the vertical position of the highest crest. Negative flow is directed onshore and left.
in Figure 4-23c, which shows that this method yields results that are very similar to those using the continuity equation, with the highest mismatch occurring at roughly 250°.

In this analysis we will continue to use $\bar{w}$ as calculated by the PIV owing to the following reasons: (1) the use of $\bar{w} = 0$ is somewhat unrealistic near the bed due to vortical motion; (2) $\bar{w}_{cont}$, potentially affected by $\partial \bar{u}/\partial x$, yields results that are not significantly different from those using $\bar{w}_{PIV}$ (50% at the most); (3) no net deposition of sediment at the ripple troughs was observed by the end of the realization (Figure 3-6), suggesting that $\bar{w}_{PIV}$ is a measure of fluid and not sediment velocity; and (4) the Stokes number reveals that sediment particles should follow the fluid’s trajectory closely. The author also recognizes the uncertainty associated with image areas where one velocity component is much smaller than its counterpart (for a review on PIV uncertainty please refer to Raffel et al., 2007). Therefore, care was taken when interpreting $\bar{w}$ and the terms containing it.

The total stress in Figure 4-23a represents the total shear present in the water column for each different wave phase. A corresponding phase dependent Shields parameter $\theta_t$ can now be obtained by taking the ratio of the shear stress to the immerse weight of the grains:

$$\theta_t = \frac{\tau_{xx}}{(s - 1) \rho g d_{s0}},$$

(4.35)

where $\tau_{xx}$ is the total stress given by (4.28), $s$ is the specific gravity of the grains, $g$ is the gravitational acceleration, $\rho$ is the density of water, and $d_{s0}$ is the median grain diameter. At the ripple ripple crest level, $\theta_t$ reaches a peak value of roughly 0.65,
Figure 4-23: Total shear stress as calculated using $\bar{w}$ from (a) PIV, (b) integration of the continuity equation (4.34), and (c) $\bar{w} = 0$, for all wave phases (colorbar). The horizontal dashed line represents the vertical position of the highest crest.

which is less that the threshold for sheet flow ($\theta = 0.8$). $\theta_i$ is more appropriate at describing the near-bed stress than the quasi-steady estimates of the Shields parameter, $\theta_{u_+}$, presented in Chapter 3, as it situates the bed in the bedform regime.

The crest-amplified Shields parameter, $\theta_r$, suggested by Du Toit and Sleath (1981), is roughly 32% higher that $\theta_i$, and the closest to $\theta_i$ of the estimates presented on Chapter 3.

Shear stress has been extensively linked to the initiation of motion and subsequent sediment transport (Nielsen, 1992). An independent estimate of shear stress can be obtained from a simple bedload transport model proposed by (Meyer-Peter and Muller, 1948):

$$\tau_{MPM} = \rho \left[ \frac{Q(s - 1)g}{A_{MPM}} \right]^{2/3},$$  \hspace{1cm} (4.36)
where $A_{MPM}$ is a constant with value 10, and $Q$ is the sediment transport rate given by

$$Q(x, \tilde{t}) = Q_0 + nc_{b}z_{hc}(\tilde{t}), \quad (4.37)$$

where $Q_0$ is the sediment transport at $z = 0$ and assumed zero, $n = 0.7$ is the sediment concentration, $c$ is the bedform celerity, and $z_{hc}$ is the elevation of the highest ripple crest. The two estimates of shear stress are presented in Figure 4-24. The magnitude of the stresses agree remarkably, even at phases of local minimum and maximum ($100^\circ$ and $280^\circ$, respectively). Additionally, they are noticeably in phase.

In summary, partitioning of the phase- and spatially averaged stress terms suggests that: (1) viscous stresses are negligible; (2) the presence of bedforms affects the momentum flux only within a certain region in the water column; (3) Reynolds
stresses are significant; and (4) the total stress near the bed is dominated by wave-induced motion. When expressed as the Shields parameter, the total shear stress obtained from the DANS equations agrees within the same order of magnitude with bulk estimates formulated for ripple crests, \( \theta_r \), and its value is consistent with the presence of bedforms. The total shear stress derived from the DANS equations agrees remarkably with the stress inferred from a commonly used bedload transport model suggested by *Meyer-Peter and Muller* (1948).

### 4.3.3 Momentum Balance

Three of the four terms conforming the momentum balance presented in (4.27) can now be evaluated individually as a function of their vertical position and wave phase. The acceleration deficit, shear stress gradient, and bedform-induced skin friction terms are shown in Figure 4-25. The only unknown in (4.27) is the form drag term. Although possible, (4.27) will not be solved for the form drag term in this study due to the uncertainty associated with the other terms.

As expected, a negative acceleration deficit dominates within wave phases with positive acceleration (see Figure 4-3 for a time series of acceleration). Similarly, a positive acceleration deficit exists for wave phases with negative acceleration. The latter is stronger than its counterpart due to wave steepness. Approximately 50° before both flow reversals, an area of slight acceleration deficit develops above ripple crests \((\tilde{t} = 140°, z = 4 - 5 \text{ cm} \text{ and } \tilde{t} = 300°, z = 4 - 5 \text{ cm})\), and vanishes by the time of flow reversal. This is a signature of the jet-like area that develops due to ripples constricting the flow.
Figure 4-25. (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b - d) Terms conforming the momentum balance in (4.27) at each vertical position and wave phase: (b) acceleration deficit, (c) shear stress gradient, (d) bedform-induced skin friction. The solid line represents the vertical position of the highest crest at each wave phase. Negative flow is directed onshore and left.

Positive shear stress gradients are present below ripple crests during offshore-directed flow, and are ejected into the water column after flow reversal. Similarly, negative stress gradients are present during onshore-directed flow and climb up in the water column after the second flow reversal.

The acceleration deficit shows some agreement with the shear stress gradient in the near-bed region for wave phases higher than 200° (during onshore-directed
flow), although there is some vertical and temporal offset. Between 150° and 200°, a similarly sized signal of momentum, located at elevations between 4 and 5 cm, is present in both, the acceleration deficit and shear stress gradient. They are similar in magnitude but different in sign. This difference could be attributed to form drag due to vortex shedding during flow reversal. However, below ripple crests they seem to agree within the same order of magnitude.

The bedform-induced skin friction, resulting from rearrangement of the viscous term in (4.25), is almost two orders of magnitude lower than the other terms in the momentum balance, suggesting, not surprisingly that momentum transfer and form-induced pressure differences are responsible for most of the flow shear in this environment. As expected, bedform-induced skin friction acts only at heights below the ripple crest.

Similar to Figure 4-25, the integral form of (4.27) given by

$$\rho \int_{z}^{\infty} \frac{\partial \zeta \langle \bar{u} - \bar{u}_\infty \rangle}{\partial t} \, dz = - \frac{1}{L_0} \int_{z}^{\infty} (\bar{p})_b \big|_{\alpha}^\beta \, dz + \zeta \tau_{xz} \big|_{z}^{\infty} + \int_{z}^{\infty} f_{sk} \, dz, \quad (4.38)$$

and shown in Figure 4-26, shows the integrated acceleration deficit agreeing satisfactorily with the shear stress below ripple crests for the second half-wave period (steeper part of the wave). This may suggest less form drag effects due to bed liquefaction. When the bed liquifies and moves as a block, it is said to be under plug-flow conditions. The parameterization suggested by Sleath (1999),
Figure 4.26 (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b - d) Terms conforming momentum balance in integral form presented in (4.38) at each vertical position and wave phase (b) depth integrated acceleration deficit, (c) shear stress, (d) depth integrated bedform-induced skin friction The solid line represents the vertical position of the highest crest at each wave phase Negative flow is directed onshore and left

\[ S = \frac{\partial u_\infty}{\partial t} (s - 1) g \]  

(4.39)

identifies the limit for plug flow formation at 0.29. Plug flow is induced by pressure gradients and has only been observed in flat beds For example, in their field study,
Foster et al. (2006b) reported a lower critical value of $S = 0.1$. Significant pressure gradients associated with the steeper part of asymmetric waves could suggest plug flow conditions in the second half-wave period of this data set ($S = 0.25$). The persistence of bedforms is intriguing given the strong forcing regime, but could explain, by means of vortex shedding, the imbalance of momentum above ripple crests. Finally, it is worth noting that bedforms become flatter during the second half-wave period (Figures 4-2 and B-3), diminishing thereby the form drag.

Using the acceleration deficit in the momentum balance eliminates the two strongest signals produced by the unsteady term and the pressure gradient induced by the waves. In order to further explore the bulk balance of momentum, we have decomposed the acceleration deficit such that the expression presented in (4.27) becomes

$$\rho \frac{\partial \zeta \langle \tilde{u} \rangle}{\partial t} = \rho \zeta \frac{\partial \langle \tilde{u}_\infty \rangle}{\partial t} + \frac{\partial \zeta \tau_{xz}}{\partial z} + \text{residual}, \quad (4.40)$$

where the form-induced skin friction (previously shown to be negligible), form drag and any other physical process that we are unable to resolve, have been grouped in the residual.

The balance presented in (4.40) is shown in Figure (4-27). Clearly, there is a momentum balance above 5 cm. The right-hand-side of the equation (Figure 4-27e) shows a phase lead within two ripple heights above the crest, potentially induced by vortex shedding. The residual is mostly balanced by the shear stress gradient, except below ripple crests at phases of flow strengthening (50 - 100° and 210 - 270°), where it gets larger (-125 N m$^{-3}$ and 250 N m$^{-3}$, respectively). These phases are
Figure 4-27  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity  (b - d) Terms conforming momentum balance in (4.40) at each vertical position and wave phase  The solid line represents the vertical position of the highest crest at each wave phase  Negative flow is directed onshore and left
consistent with high ripple migration (Figure 4-4b). One can hypothesize that, below ripple crests, the residual represents the force necessary to move the ripple formations \( F_b \), and missing in the momentum balance. As a rough approximation, one can assume that the acceleration of the bedforms \( a_b \) is given by the residual force per unit volume \( f_r \) divided by the particle’s density \( \rho_p \), or

\[
F_b = m_s a_b
\]
\[
\frac{F_b}{V_s} = \frac{m_s}{V_s} a_b
\]
\[ (4.41) \]
\[
f_r = \rho_s a_b,
\]

where \( V_s \) is the volume of sediment, and \( f_r \) is taken from Figure 4-27f. For phases within 50 - 100°, the estimates of \( a_b \) using (4.41) agree remarkably with independent estimates of acceleration, \( a_{cb} \), using the bedform migration rate presented in Figure 4-4b \( a_b = -10.4 \text{ cm s}^{-2} \) and \( a_{cb} = -8.64 \text{ cm s}^{-2} \). These estimates are not as good for the second stage of flow strengthening (210 - 270°), potentially due to a local minimum in \( a_{cb} \) found near 270°. However, they are similar in magnitude for phases between 250 - 270° \( a_b = 20.8 \text{ cm s}^{-2} \) and \( a_{cb} = 9.47 \text{ cm s}^{-2} \).

The effort presented in Figures 4-25 - 4-27 is remarkable for the following reasons: (1) it shows, to the author’s knowledge, the first momentum balance performed in the wave bottom boundary layer; (2) an agreement between the two sides of (4.40) at heights above 5 cm reveals that form drag could act as far as two ripple heights above the crests; (3) better agreement at phases with flatter ripples suggests a strong dependence on form drag and possible plug flow conditions.
during the steeper part of the wave; (4) an imbalance of momentum below ripple crests can potentially represent the forces generating ripple migration; and (5) total skin friction (bedform-induced skin friction plus viscous diffusion, as described by (4.26)), commonly believed to be the principal mechanism for sediment transport (Nielsen, 1992), is small and confined to regions below ripple crests. This last statement suggests that, although small, the total skin friction is responsible for sediment pick-up. Once mobilized, the fate of the sediment particles is decided by the momentum transfer terms and the particle's ability to stay in suspension.
CHAPTER 5

CONCLUSIONS

In this laboratory study, a rippled sediment bed was subjected to regular, monochromatic wave forcing. The skewed and asymmetric intermediate water waves, had orbital velocities significantly larger in the horizontal direction. Observations of the two-dimensional near-bed velocity field and time-dependent bed elevation were collected using a submersible particle image velocimetry (PIV) system.

The wave bottom boundary layer thickness and the non-dimensional shear stress were evaluated with a quasi-steady approach. Friction velocities were obtained from a nonlinear least-squares regression of the averaged mean velocity field over individual half-wave periods. Friction velocities were used to estimate the boundary layer thickness as suggested by Grant and Madsen (1979). The results show boundary layer thicknesses that shrink above ripple troughs and expand above crests, and are inconsistent with $\delta_{99}$ and visual inspection of the velocity profiles over a rippled bed. The boundary layer thickness was also estimated using the displacement thickness method ($\delta^*$), which shows agreement with $\delta_{99}$, although they still differ by a factor of 5.

The ratio of shear stress to the immersed weight of the grains (Shields parameter) is one order of magnitude larger than the reported threshold for sheet flow, even though the ripple formations suggest a value of less that 1. Moreover, and not
surprisingly, this value is, at times, two orders of magnitude larger than the grain roughness Shields parameter ($\theta_{2.5} \sim 0.1$) frequently used in sediment transport formulations. According to Du Toit and Sleath (1981), the presence of a vortex ripple can significantly enhance the Shields parameter at a ripple crest. The amplification factor is a measure of shear enhancement due to ripple steepness. In these observations, the ripple steepness and amplification factor was found to be 0.2 and 7, respectively, and could not fully account for the discrepancy.

Higher onshore-directed Shields parameters over the ripple crest agree with the direction of bedform migration, whereas a balance between the onshore/offshore values produces bedform flattening. The nonlinear regression also shows sediment moving layers with thicknesses of roughly one ripple height. These agree with visual observations of the fluidizing bed, but were not consistent with the common definition of $z_0$ to be located at a distance equal to $d_{50}/30$ (Nikuradse, 1933).

In an effort to further explore the time- and space-dependent behavior of the wave bottom boundary layer flow over a rippled bed, an unsteady analysis of the momentum field was performed. The analysis consisted of double-averaging the momentum equations, to yield a spatially averaged representation of the momentum balance. This technique, originally proposed by Gimenez-Curto and Corniero Lera (1996), uses ensemble and subsequent spatial averaging to yield the Double Averaged Navier-Stokes (DANS) equations. The resulting formulation yields a balance between the acceleration deficit and various pressure- and shear-induced drag terms as a function of wave phase. It also provides an explicit expression for stress partitioning, allowing for evaluation of the relative contribution of each component to
the total stress.

An unsteady analysis of the phase-averaged PIV images provides qualitative information regarding sediment entrainment and bed geometry. Sediment plumes travel one ripple wavelength in half a wave period, consistent with previous observations by Van der Werf et al. (2007). This suggests that proper calibration of sediment concentration using image intensity could provide a way for inferring sediment transport. Phase-dependent estimates of bed elevation show a highly mobile bed and suggest bedform flattening as a response to high accelerations and negative vertical velocities.

Intra-wave analysis of the velocity field shows a strong influence of the ripple formations on the flow field. Coherent motions develop at the downstream side of the ripples near flow reversal. The off-onshore coherent vortex is the strongest, and is coincident with high accelerations and wave steepness over steeper bedforms. These vortices are also consistent with sediment resuspension and may provide phase-dependent information of the availability and fate of sediment in the water column.

Assessment of the phase- and spatially averaged shear stress and its components reveals that turbulence and wave-induced motions dominate the total shear stress near the bed. Bedform-induced stresses comprise roughly 16% of the stress variance and suggest that the flow is only affected by the presence of bedforms within two ripple heights above the crest. The magnitude and phase of the total shear stress agrees remarkably well with estimates of shear stress from a simple bedload transport model (Meyer-Peter and Muller, 1948). Additionally, when expressed
as the Shields parameter, the total shear stress agrees within the same order of magnitude with the ripple amplified Shields parameter suggested by *Du Toit and Sleath* (1981).

Showing the first momentum balance performed in the wave bottom boundary layer, this study reveals that form drag plays an important role near the bed and is significant as far as two ripple heights above the crests. Plug flow conditions are likely to occur during the steeper part of intermediate water waves with high skewness and asymmetry, due to the large pressure gradients, and is consistent with observations of bedform flattening. An imbalance of momentum below ripple crests provides with an idea of the forces causing ripple migration. Total skin friction, composed of bedform-induced skin friction plus viscous diffusion, is negligible when compared with other terms in the overall momentum balance.
Appendices
Figure A-1: (a) Free-stream phase-averaged horizontal (solid line) and vertical (dashed line) velocity. (b - c) Horizontally averaged percent vectors kept as a function of wave phase and elevation.
APPENDIX B

Phase-averaged pixel intensity
Figure B-1  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity  
(b - i) Image ensembles for eight different phases indicated in Figure 4-2a  
The green lines show bed elevation estimates  Onshore flow is directed to the left
Figure B-2  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b - i) Image ensembles for eight different phases indicated in Figure 4-2a. The green lines show bed elevation estimates. Onshore flow is directed to the left.
Figure B-3: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b-i) Image ensembles for eight different phases indicated in Figure 4-2a. The green lines show bed elevation estimates. Onshore flow is directed to the left.
APPENDIX C

Phase-averaged velocity field
Figure C-1  (a) Free-stream phase-averaged horizontal velocity (black) and acceleration (blue)  (b - i) Negative phase-averaged image intensity with phase-averaged velocity field (red) for eight different phases indicated in Figure 4-3a. Scale vectors in the top right corner represent 10 cm s⁻¹. Onshore flow is directed to the left.
Figure C-2: (a) Free-stream phase-averaged horizontal velocity (black) and acceleration (blue). (b - i) Negative phase-averaged image intensity with phase-averaged velocity field (red) for eight different phases indicated in Figure 4-3a. Scale vectors in the top right corner represent 10 cm s\(^{-1}\). Onshore flow is directed to the left.
Figure C-3  (a) Free-stream phase-averaged horizontal velocity (black) and acceleration (blue)  
(b - i) Negative phase-averaged image intensity with phase-averaged velocity field (red) for eight different 
phases indicated in Figure 4-3a. Scale vectors in the top right corner represent 10 cm s$^{-1}$  
Onshore flow is directed to the left.
APPENDIX D

Phase-averaged velocity decomposition
Figure D-1 (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal and (e) vertical velocity fields at $\theta = 0^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent 2 cm s$^{-1}$. Negative flow is directed onshore and left.
Figure D-2: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at $\bar{\theta} = 15^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent 2 cm s$^{-1}$. Negative flow is directed onshore and left.
Figure D-3: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at $\hat{\theta} = 30^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent 2 cm s$^{-1}$. Negative flow is directed onshore and left.
Figure D-4  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity  (b) Horizontal and (e) vertical velocity fields at $\bar{t} = 45^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance  (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity  Scale vectors in the top left corner represent 2 cm s$^{-1}$ Negative flow is directed onshore and left
Figure D-5. (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at $\theta = 60^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent 7 cm s$^{-1}$. Negative flow is directed onshore and left.
Figure D-6  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity  (b) Horizontal and (e) vertical velocity fields at $\gamma = 75^\circ$ decomposed into  (c, f) a spatial average plus (d, g) a bedform disturbance  (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity Scale vectors in the top left corner represent 7 cm s$^{-1}$ Negative flow is directed onshore and left.
Figure D-7  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal and (e) vertical velocity fields at $\dot{t} = 90^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance  (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity  Scale vectors in the top left corner represent 7 cm s$^{-1}$  Negative flow is directed onshore and left
Figure D-8  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity  (b) Horizontal and (e) vertical velocity fields at $\tilde{t} = 105^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance  (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity Scale vectors in the top left corner represent 5 cm s$^{-1}$  Negative flow is directed onshore and left
Figure D-9: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at $\phi = 120^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent 10 cm s$^{-1}$. Negative flow is directed onshore and left.
Figure D-10  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal and (e) vertical velocity fields at $\bar{\theta} = 135^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity Scale vectors in the top left corner represent 10 cm s$^{-1}$ Negative flow is directed onshore and left
Figure D-11: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at $\bar{\phi} = 150^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent 10 cm s$^{-1}$. Negative flow is directed onshore and left.
Figure D-12 (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at $\theta = 165^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent 10 cm s$^{-1}$. Negative flow is directed onshore and left.
Figure D-13: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at $\bar{\ell} = 180^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent 5 cm s$^{-1}$. Negative flow is directed onshore and left.
Figure D-14  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity  (b) Horizontal and (e) vertical velocity fields at $\bar{t} = 195^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance  (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity Scale vectors in the top left corner represent $4 \text{ cm s}^{-1}$. Negative flow is directed onshore and left.
Figure D-15: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at $\tilde{\ell} = 210^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent 4 cm s$^{-1}$. Negative flow is directed onshore and left.
Figure D-16: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at $t = 225^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent $7 \text{ cm s}^{-1}$. Negative flow is directed onshore and left.
Figure D-17  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity  (b) Horizontal and (e) vertical velocity fields at $\theta = 240^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance  (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity  Scale vectors in the top left corner represent 10 cm s$^{-1}$  Negative flow is directed onshore and left
Figure D-18: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at $\theta = 255^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent 12 cm s$^{-1}$. Negative flow is directed onshore and left.
Figure D-19 (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at $\tilde{t} = 270^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity Scale vectors in the top left corner represent 15 cm s$^{-1}$. Negative flow is directed onshore and left.
Figure D-20: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at $\hat{t} = 285^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent 12 cm s$^{-1}$. Negative flow is directed onshore and left.


Figure D-21: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at $t = 300^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent $12$ cm s$^{-1}$. Negative flow is directed onshore and left.
Figure D-22  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity  (b) Horizontal and (e) vertical velocity fields at $\theta = 315^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance  (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity Scale vectors in the top left corner represent 10 cm s$^{-1}$  Negative flow is directed onshore and left
Figure D-23 (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal and (e) vertical velocity fields at \( t = 330^\circ \) decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance. (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity. Scale vectors in the top left corner represent 7 cm s\(^{-1}\). Negative flow is directed onshore and left.
Figure D-24  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity  (b) Horizontal and (e) vertical velocity fields at $\bar{t} = 345^\circ$ decomposed into (c, f) a spatial average plus (d, g) a bedform disturbance  (h) Phase-averaged and (i) bedform induced velocity fields over phase-averaged image intensity Scale vectors in the top left corner represent 4 cm s$^{-1}$. Negative flow is directed onshore and left
APPENDIX E

Phase-averaged turbulent quantities
Figure E-1: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{t} = 0^\circ$. Negative flow is directed onshore and left.
Figure E-2: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $t = 15^\circ$. Negative flow is directed onshore and left.
Figure E-3 (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\theta = 30^\circ$. Negative flow is directed onshore and left.
Figure E-4: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{\ell} = 45^\circ$. Negative flow is directed onshore and left.
Figure E-5: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\bar{t} = 60^\circ$. Negative flow is directed onshore and left.
Figure E-6: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\bar{t} = 75^\circ$. Negative flow is directed onshore and left.
Figure E-7: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\bar{t} = 90^\circ$. Negative flow is directed onshore and left.
Figure E-8  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity  (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{t} = 105^\circ$. Negative flow is directed onshore and left.
Figure E-9: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{t} = 120^\circ$. Negative flow is directed onshore and left.
Figure E-10 (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{t} = 135^\circ$ Negative flow is directed onshore and left.
Figure E-11  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity  (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{t} = 150^\circ$  Negative flow is directed onshore and left.
Figure E-12: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{t} = 165^\circ$. Negative flow is directed onshore and left.
Figure E-13  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{t} = 180^\circ$ Negative flow is directed onshore and left
Figure E-14: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{t} = 195^\circ$. Negative flow is directed onshore and left.
Figure E-15  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity  (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{t} = 210^\circ$  Negative flow is directed onshore and left
Figure E-16 (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{t} = 225^\circ$ Negative flow is directed onshore and left
Figure E-17  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity  (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{t} = 240^\circ$  Negative flow is directed onshore and left.
Figure E-18  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{t} = 255^\circ$. Negative flow is directed onshore and left.
Figure E-19: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\bar{\theta} = 270^\circ$. Negative flow is directed onshore and left.
Figure E-20  (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{\tau} = 285^\circ$ Negative flow is directed onshore and left
Figure E-21 (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{t} = 300^\circ$ Negative flow is directed onshore and left
Figure E-22 (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\tilde{t} = 315^\circ$ Negative flow is directed onshore and left
Figure E-23: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\bar{t} = 330^\circ$. Negative flow is directed onshore and left.
Figure E-24 (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity, (b) Horizontal turbulent energy, (c) vertical turbulent energy, (d) Reynolds stress, and (e) turbulent kinetic energy phase-averaged at $\bar{\theta} = 345^\circ$. Negative flow is directed onshore and left.
APPENDIX F

Phase-averaged stress partitioning
Figure F-1: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b - i) Terms conforming expression (4.28) for each vertical position at eight different wave phases indicated in (a). The lower bound of the domain is located at the ripple trough, and the horizontal dashed line represents the vertical position of the highest crest. Negative flow is directed onshore and left.
Figure F-2: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b - i) Terms conforming expression (4.28) for each vertical position at eight different wave phases indicated in (a). The lower bound of the domain is located at the ripple trough, and the horizontal dashed line represents the vertical position of the highest crest. Negative flow is directed onshore and left.
Figure F-3: (a) Free-stream phase-averaged horizontal (black) and vertical (red) velocity. (b - i) Terms conforming expression (4.28) for each vertical position at eight different wave phases indicated in (a). The lower bound of the domain is located at the ripple trough, and the horizontal dashed line represents the vertical position of the highest crest. Negative flow is directed onshore and left.


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