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Thermally Driven Topology and the Topological Hall Effect in Chiral Magnets

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Abstract

Observations of topological spin textures in real materials provide promising platforms for physicists to study the fundamental physics in materials. In addition, the topological spin textures have potential applications in ultra-dense magnetic memories. A magnetic skyrmion is a nanoscale topological spin texture that can be manipulated by a small electrical current is promising for the next generation of ultra-dense memory technology. The topological Hall effect is used exclusively as a signature of the magnetic skyrmions in a material. However, this phenomenon might have originated from other mechanisms such as the addition of two anomalous Hall effects with different coercivity fields. On the other hand, the relationship between topological spin structures and the Hall effect has been investigated at low temperatures; however, much remained unknown for the emergent topology at high temperatures. In this thesis, I performed the Monte Carlo simulation of a two-dimensional chiral magnet and characterized the spins’ topology with the topological charge and scalar chirality. The Hall conductivity and longitudinal conductivity were calculated by applying the Kubo formula to the tight-binding model. Massive ensemble averages were taken to derive the thermal average of the observables. I discovered the correlation between thermally emergent topology and the topological Hall effect, and I present the scattering time dependence of the topological Hall effect in the strong and weak coupling regime. The results provide a new mechanism of the topological Hall effect induced by thermal fluctuation.

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I. BACKGROUND

A. Introduction to Skyrmion

Skyrmion was proposed as a theoretical model of fundamental particles by Professor Tony Skyrme in the 1960s. In 2009, the magnetic skyrmions were experimentally realized in a chiral magnet MnSi [1]. A magnetic skyrmion is a nanoscale vortex of spin texture characterized by a topological charge of $\pm 1$ (Fig. 1). Topological charge (TC) is an integer indexing the topology of a spin texture. In another word, TC indicates how many times the spin texture can wrap around a unit sphere. For a skyrmion, the spin texture can wrap around a unit sphere exactly one, and the sign of the TC depends on the spin directions. For example, if the spins in the center of a skyrmion are pointing up and the outer spins are pointing down then the skyrmion has a TC of 1. TC is equal to -1 for the opposite case. Skyrmion behaves as a particle that can be moved, created, and annihilated with a small electrical current. In addition, a skyrmion is a topological texture that is extremely robust against disorder. Therefore, magnetic skyrmions are promising for applications in low-energy memory and logic devices.

Figure 1. Hedgehog spin configurations on a unit sphere projected onto a two-dimensional plane via stereographic projection. The spin texture on a two-dimensional plane is a magnetic skyrmion [2].

Skyrmion is exciting because it has the potential to make a big impact on future technol-
ogy. Moreover, its discovery confirms the existence of quasi-magnetic monopole in nature.

Realizing skyrmion in real materials and probing them via experimental techniques allow
thoretical physicists to better understand more about the fundamental physics in materials.
This can lead to more exciting predictions. A mathematical description of a skyrmion is
well understood; however, its presence and behaviors in materials are subjects of ongoing
theoretical and experimental research. In this thesis, I studied the presence of thermally
emergent topology and its significance in transport measurements. The results allow us to
bridge the knowledge gap between the fundamental physics of topological spin texture and
the THE in transport physics.

B. Topological Hall Effect

To use skyrmions, we first need to find a material that can host skyrmions at room tem-
perature. Recent experiments claim the presence of skyrmions in common materials such as
the transition metal oxides at high temperatures using transport signatures. However, it is
ambiguous whether skyrmions can exist in this type of material and can exist at high tem-
peratures (near the critical temperature of the material). The most versatile and common
method that physicists use to observe the evidence of skyrmions is the topological Hall effect
(THE). The THE is a phenomenon in which the motion of electrons is deflected due to the
effective magnetic field that arises from skyrmions (Fig. 2). However, the THE is always
entwined with other Hall effects, such as the ordinary Hall effect and the anomalous Hall
effect (Fig. 2). The ordinary Hall effect arises from the Lorentz force when there is an
external magnetic field. The anomalous Hall effect arises from the intrinsic magnetization
of a material or the extrinsic properties such as impurity scatterings [3].

In transport experiments, the total Hall signal is denoted as $\rho_{xy}$, which is the sum of the
three Hall effects mentioned. Eq. (1) is the total Hall effect expressed in terms of the ordinary
($\rho_{xy}^0$), anomalous ($\rho_{xy}^A$), and topological ($\rho_{xy}^T$). In a typical transport experiment, $\rho_{xy}^0$ is propor-
tional to the external magnetic field ($B$) and $\rho_{xy}^A$ is proportional to the magnetization ($M$).
The coefficients for these two Hall effects can be found by fitting the transport data with $B$ and $M$. Then the topological Hall component can be found with Eq. (2).

$$
\rho_{xy} = \rho_{xy}^0 + \rho_{xy}^A + \rho_{xy}^T = R_0 B + \mu_0 R_s M + \rho_{xy}^T
$$

(1)
\[ \rho_{xy}^T = \rho_{xy} - R_0 B - \mu_0 R_s M \] (2)

However, separating the THE from other Hall effects is not a simple task. Some materials exhibit non-linear anomalous Hall effect due to inhomogeneous magnetizations [5-7]. For example, SrRuO\(_3\) is a transition metal oxide that exhibits a unique transport signature depending on the type of substrates. In particular, SrRuO\(_3\) on NdGaO\(_3\) and SrRuO\(_3\) on SrTiO\(_3\) have inhomogeneous magnetization regions that give rise to multi-channels anomalous Hall effect [7]. The combinations of the anomalous Hall effect give rise to a hump feature that looks similar to the THE (Fig. 3). However, the multi-channels AHE is not the only explanation for THE-like in these transition metal oxides. In particular, SrIrO\(_3\)/SrRuO\(_3\) exhibits a hump feature similar to SrRuO\(_3\)/SrTiO\(_3\) [8]. Magnetic force microscopy shows magnetic bubbles and domain walls in SrIrO\(_3\)/SrRuO\(_3\), which can be the evidence of skyrmions or topological spin textures. This means the THE may have originated from topological spin textures (e.g. skyrmions). In another case, the THE has been reported in SrRuO\(_3\) near the critical temperature, and the main mechanism for THE is possible the spin fluctuations [9]. The main objective of this project is to study the thermal fluctuation phase and its transport signatures. Since the thermal fluctuation phase gives rise to the thermally emergent
topology [10], we confirm that thermally emergent topology gives rise to the THE.

Figure 3. (a) The transport results at different temperatures in SrRuO$_3$. (b) The reconstruction of $R_{AH}$ data using two anomalous Hall data (green and purple). The sum of green plots and purple plots matches with $R_{AH}$ in (a) [5].

C. Thermal Emergent of Topological Charge

The main goal of this thesis is to study the transport signatures of spin fluctuations due to the thermal effect in a chiral magnet. The chiral magnet has non-centrosymmetric unit cells, meaning the inversion symmetry is broken which gives rise to the Dzyaloshinskii-Moriya interaction (DMI). The DMI is required for skyrmions formation in ferromagnets. In chiral magnets, skyrmions can form near the zero temperature and at some finite magnetic field [10]. However, the system also has a non-zero topology near the critical temperature and at a moderate magnetic field. We predict that this will induce an effective magnetic field that can contribute to the transport measurement. The mathematical definition of the effective magnetic field is similar to that of the topological charge,

$$B_{\text{eff}} = \frac{1}{2} \mathbf{S} \cdot (\nabla_x \mathbf{S} \times \nabla_y \mathbf{S}) \sim \mathcal{W} = \frac{1}{8\pi} \mathbf{S} \cdot (\nabla_x \mathbf{S} \times \nabla_y \mathbf{S}),$$

(3)

where $\mathcal{W}$ is the topological charge density [2]. However, Eq. (3) is only valid in the continuum limits, meaning the spins vary slowly. At high temperatures, the spins are randomly
distributed and the solid angle subtended by any three nearby spins is very large. Therefore, the correlation between TC and the effective magnetic field is not well understood at high temperatures.

This project relies on numerical simulations to better understand the correlation between spin topology and THE at high temperatures. The simulation work revolves around the thermal emergent of topological charge and its transport signature. Here, we also study the strong and weak coupling regimes to see if they match with the current theories. Scattering time is the key parameter that originated from inevitable impurity concentrations; therefore, we use the scattering time to identify the scattering mechanisms.

II. METHODS

A. Monte Carlo Simulation

In this section, I describe the Monte Carlo method, and the parameters that I used to simulate the spin lattice. The Monte Carlo method is a powerful statistical method for minimization problems and works very well with stochastic systems. At high temperatures, the spin lattice is driven by thermal fluctuation, which is a stochastic process. The role of the Monte Carlo method here is to minimize the energy and maximize the entropy of the spin lattice. Energy minimization and entropy maximization are governed by the Boltzmann distribution,

\[ P(s) = e^{-\beta E(s)}, \]

where \( E(s) \) is the energy and \( \beta = 1/k_B T \). Eq. 4 describes the probability of the system in a certain energy and temperature. The probability transition from energy \( E_i \) to \( E_j \) is \( P_i/P_j = e^{-\beta(E_j-E_i)} = e^{-\beta \Delta E} \). The Monte Carlo method compares this probability to a random number between zero and one to satisfy ergodicity (uniform random process). I used the Monte Carlo method to simulate the spin lattice near the critical temperature of the chiral magnet and then calculated the observables such as the topological charge, scalar chirality, and transport data.

The psuedocode for the Monte Carlo algorithm is the following,

1. Calculate the energy at site \( i \)
2. Change the spin at site \( i \) with a random spin on a unit sphere and then calculate the new
energy

if $\Delta E < 0$ then
  Accept the spin change
else
  if random $\#$ between 0 and 1 < $e^{-\beta \Delta E}$ then
    Accept the spin change
  end if
end if

Figure 4. Phase diagram of topological charge with $D = 0.3$. The colorbar represents the average topological charge per 1000 spins. The simulation is across the cyan dotted line in the phase diagram [10].

The energy of the system is given by the Hamiltonian,

$$H = \sum_{\langle ij \rangle} -JS_i \cdot S_j + D_{ij} \cdot (S_i \times S_j) - g\mu_B H' \sum_i S_i^z,$$  \(5\)

where the first summation is summing over the first nearest neighbors, $D_{ij}$ is the DMI vector, and $g\mu_B H' = B$ is the applied magnetic field. In this work, all fundamental constants are equal to unity and are unitless. The parameters in the spin Hamiltonian are ferromagnetic Heisenberg exchange $J = 1.0$, strength of the DMI vector $D = 0.3$, and applied magnetic field $B = 0.15$. These parameters create the thermal emergent of topological charge across the critical temperature (Fig. [4]). The Monte Carlo method was used to update the spin
lattice five thousand times at each temperature to reach equilibrium before calculating the observables.

1. Calculating the Observables

Once the spins have settled at a new temperature point, I calculated the observables such as the topological charge (TC) and scalar chirality. These quantities are used to characterize the spin lattice. A non-zero TC and scalar chirality indicate some topological features in the spin lattice. For skyrmions or ordered spins, a snapshot of the spins configuration will show clear evidence of topological spin texture exists or not because the ordered spins do not change their configuration after many Monte Carlo updates. However at high temperatures, the spins are randomly distributed from one Monte Carlo update to the next; therefore, it is not possible to look at a snapshot of a spin configuration and identify if there are any topological features. Massive ensemble averages of TC and scalar chirality are needed to characterize the topology of the system. The TC is a summation of all the solid angles, $\Omega$, divide by $4\pi$ (Fig. 5). TC can only be an integer value because it represents the winding number of spins on a unit sphere. The solid angle between three nearby spins can be calculated by the Berg formula [12],

$$\exp\left(\frac{i\Omega}{2}\right) = 1 + S_1 \cdot S_2 + S_2 \cdot S_3 + S_3 \cdot S_1 + i S_1 \cdot (S_2 \times S_3).$$

Scalar chirality is another quantity that describes the winding number of spins in the continuum limits (i.e. weak coupling regime and low temperatures). Scalar chirality is defined as the summation of all triple products of any three nearby spins,

$$\chi = \frac{1}{4\pi} \sum S_i \cdot (S_j \times S_k).$$

At low temperatures, the spins change slowly; hence TC and scalar chirality are the same and can be used to describe the same physics. At higher temperatures, TC and scalar chirality differ from one another, and their effects on transport measurement are unknown. I calculated TC and scalar chirality and studied their correlations with the THE in the strong and weak coupling regimes.
Figure 5. The three nearby spins $S_1, S_2,$ and $S_3$ form a solid angle $\Omega$. The summation of all solid angles divide by $4\pi$ is the topological charge [10].

B. Tight Binding Model

Figure 6. The energy dispersion of the tight-binding model with $J_H = 0$ and $t = 1$.

The tight binding model is a powerful tool in condensed matter physics use for obtaining the energy dispersion of a free, non-interacting system of fermions. The Hamiltonian for the conduction electrons is,

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j - J_H \sum_i c_i^\dagger \sigma \cdot S_i c_j,$$  \hspace{1cm} (7)
where the $t$ is the hopping amplitude, and the first term describes how the electron hops around its nearest neighbors. $J_H$ is the ferromagnetic Hund’s coupling, $\sigma$ is the vector of Pauli matrices, and the second term describes electrons and spins coupling. Under finite $J_H$, up and down spin bands split from each other, forming the conduction and valance bands respectively. $J_H$ is used to obtain the strong and weak coupling regime, where $J_H \leq 4t$ is in the weak coupling regime. More precisely, the weak coupling regime is when the conduction and valance band touch or overlap.

The dispersion relation of the tight-binding model consider only the first term in Eq. 7 is given by $E(k) = -t \sum_a \cos(k \cdot a)$, where $k$ is the momentum vector, $a$ is the lattice vector, and $t = 1$. Fig. 6 is the plot of the dispersion relation for $t = 1$ and $J_H = 0$. When $J_H > 4t$, the conduction and valence band separate with a bandgap of $(J_H - 4t)/2$. By plotting the dispersion relation, we can visualize the problem and adjust the Fermi energy to get a non-zero value when calculating the Hall conductivity and longitudinal conductivity.

C. Kubo Formula

The Hall conductivity and longitudinal conductivity were calculated by the Kubo formula [13];

\[
\sigma_{xy} = \frac{2\pi}{L} \mathcal{R}\{ \sum_{m \neq n} \frac{f_n - f_m}{(E_m - E_n)(E_m - E_n + i\eta)} \mathcal{I}(\langle m|J_x|n\rangle\langle n|J_y|m\rangle)\}
\]
\[
\sigma_{xx} = \frac{2\pi}{L} \mathcal{R}\{ \eta \sum_{m \neq n} \frac{f_n - f_m}{(E_m - E_n)^2(E_m - E_n + i\eta)} |\langle m|J_x|n\rangle|^2\},
\]

where $f_n$ is the Fermi-Dirac distribution $(\exp(\beta(E - \mu)) + 1)^{-1}$. I shifted the chemical potential (Fermi energy) so that the Hall conductivity gives a non-zero value. If the chemical potential is in the bandgap or at $J_H$ exactly, then there is no conductivity. $L$ is the total number of spins, $E_n$ is the eigenenergy, and $|n\rangle$ is the eigenstate of the Hamiltonian. $J_x$ and $J_y$ are current operators $\partial_{k_x}H$ and $\partial_{k_y}H$, respectively. $\mathcal{R}$ and $\mathcal{I}$ mean taking the real and complex part, respectively. The parameter of interest here is $\eta$ which is the inverse of scattering time, $\tau = \eta^{-1}$. In this thesis, we are interested in studying the scattering time dependence of the TEP. Scattering time is an important quantity and is discussed in section II C 2
1. **Cutoff Energy in the Kubo Formula**

The Hall conductivity in Eq. (8) can be simplified by multiplying $(\Delta E - i\eta)/(\Delta E - i\eta)$, where $\Delta E = E_m - E_n$. This simplifies the denominator of the Hall conductivity to $(\Delta E)^2 + \eta^2$. Here, I chose a cutoff energy to reduce the computation time. The cutoff energy is chosen such that it will have minimal effect on the final results. For the strong coupling regime, the bands are separated by a bandgap, and the calculation is only important near the Fermi energy. Therefore, summing over the entire energy spectrum is unnecessary. Another reason for using the energy cutoff is the Lorentz distribution, $((\Delta E)^2 + \eta^2)^{-1}$, decays very quickly for large $\Delta E$ and $\eta << 1$. These reasons justify my decision for using the cutoff energy in the strong coupling regime. The cutoff energy that I chose here is $\pm 3$ around the Fermi energy.

The bands overlap in the weak coupling regime. Therefore, I chose larger cutoff energy to account for the bands overlap. I noticed that there is no significant difference between the cutoff energy of $E_f \pm 4$ and the full energy spectrum. Therefore, I used $E_{\text{cutoff}} = E_f \pm 4$ for the weak coupling regime to minimize the computational time.

2. **Scattering Time**

Scattering time is an important quantity that describes the transport properties of charge carriers. The Hall conductivity and longitudinal conductivity dependence of scattering time in the Drude model are $\sigma_{xy} \propto \tau^2$ and $\sigma_{xx} \propto \tau$, respectively. The Hall resistivity (also known as THE) is defined as $\rho_{xy} = \sigma_{xy}/(\sigma_{xx}^2 + \sigma_{xy}^2)$. In the thermal fluctuating phase, the contribution from the Hall conductivity is very small compared to the longitudinal conductivity; therefore, $\rho_{xy} \approx \sigma_{xy}/\sigma_{xx}^2 = B/ne$, where $B$ is the effective magnetic field from the spin texture, and $n$ is the electron density. Here we assumed the thermally emergent topology gives rise to an effective magnetic field similar to the skyrmion. However, this is not the case as discussed in the results section. Also, the Drude model only works for $\Omega_c\tau << 1$, where $\Omega_c$ is the cyclotron frequency. The correction to the Drude model is presented in the results section.

Scattering time $(\eta^{-1})$ in Eq. (8) is a key parameter originated from the impurity concentrations. The Hall resistivity dependence of scattering time is complex because it may depend
on multiple scattering mechanisms. One of the scattering mechanisms is from the real space (i.e. effective magnetic field from non-zero topological charge), and the other mechanism is from momentum space. In the strong coupling regime, the real space scattering mechanism dominates the momentum space scattering. Hence, the THE in the strong coupling regime is typically attributed to the presence of skyrmions. However, in the weak coupling regime, the contributions from real space and momentum space may be more complicated. The resistivities from multiple independent scattering mechanisms can add via Matthiessen’s rule [14]. Using scattering time, we will be able to better understand the mechanisms of the THE in the thermal fluctuation phase in the strong and weak coupling regime.

D. Computation Details

The boundary condition for the simulation is the periodic boundary condition, and the size of the lattice is 16x16. In the thermal fluctuation phase, lattice sizes of 24x24, 48x48, and 64x64 have the same topological charge as the 16x16 [10]. This is clear evidence that the thermal fluctuation phase is immune to the finite size effect as long as the lattice size is larger than 16x16. I used a lattice size of 16x16 to reduce the computational time because the number of operations for diagonalizing a matrix scales like $N^3$, where $N$ is the matrix rank.

The codes for all simulations are written in Python and Cython. Cython is a C/C++ extension of Python that allows the program to run at a much faster speed. The simulations for this work were mostly done on plasma - a supercomputer cluster at the University of New Hampshire. At least ten thousand average were taken at each temperature to obtain the Hall conductivity and the longitudinal conductivity. The TC and scalar chirality were obtained with 500 thousand average at each temperature and saved in a separate file because the computations were not as intensive as calculating the conductivity. Source code can be found on my GitHub repository https://github.com/tan02dao/Thesis.
The figure shows TC (solid red), scalar chirality (dashed red), and Hall resistivity (blue plots) in the strong coupling regime. $J_H$ is the Hund’s coupling, $t$ is the hopping amplitude, and $\mu$ is the Fermi energy. $Q_T$ is the topological charge per 1000 spins. The scalar chirality is scaled by $\max(|TC|)/\max(|\chi|) = 3.374$. Hall resistivity at different scattering times with error bars.
Fig. 7 shows the results of TC, scalar chirality, and Hall resistivity at different scattering times. Scalar chirality and TC both have 500 thousand averages, and their errors are negligible because the error decreases proportional to $\sqrt{N_{\text{average}}}$, where $N_{\text{average}}$ is the number of averages. Errors for the Hall resistivity are much larger at high temperatures because the spins fluctuate randomly, and only 20 thousand averages were taken due to the intensive computations with the Kubo formula. The peak of the TC curve is around $T = 0.9K/J$, which is consistent with the peaks of the Hall resistivity at various scattering times. The scalar chirality’s peak is around $T = 0.8K/J$, which does not correlate to the Hall conductivity plots in the strong coupling regime. The result shows a strong correlation between TC and Hall resistivity, and the correlation coefficient is above 0.95 (95% correlation).

Here we observed the magnitude of $\rho_{xy}$ increases as the scattering time $(1/\eta)$ increases. This is unexpected because we expected $\rho_{xy}$ to be constant based on the Drude model. To investigate the scattering time dependence, we chose a temperature ($T = 0.9K/J$) and calculated $\rho_{xy}$ at various scattering times. An in-depth analysis of scattering time dependence of the THE is discussed in section III C.

B. Weak Coupling

In the weak coupling regime where $J_H \leq 4t$, the geometric picture of the Berry phase is no longer valid [15]. The relationship between the effective magnetic field and the spin texture can be non-local. In the weak coupling regime, the peak of the Hall resistivity has shifted to $T = 0.8K/J$, which is consistent with the peak of the scalar chirality. The result shows a strong correlation between scalar chirality and Hall resistivity (Fig. 8). In the weak coupling regime, the Hall conductivity is computed by the perturbation of the coupling [15]. The analytical calculations show that the Hall conductivity depends on the mixed-product of three non-local spins. As an approximation, the nearest neighbors coupling should give the largest weight. The mixed product of the three nearest neighbors is the scalar chirality. Therefore, the Hall resistivity computed here is consistent with the weak coupling theory because it correlates with the scalar chirality instead of TC. Moreover, the magnitude of the Hall resistivity decreased by a factor of 4.5 compared to the Hall resistivity in the strong coupling regime. This is consistent with our hypothesis that the Hall resistivity is controlled by the scalar chirality because the magnitude of the scalar chirality is smaller (by a factor
of 3.374) than TC.

Figure 8. Hall resistivity in the weak coupling regime. Where $J_H$ is the Hund’s coupling, $t$ is the hopping amplitude, and $\mu$ is the Fermi energy. $Q_T$ is the topological charge per 1000 spins. Hall resistivity at different scattering times with error bars.

Here we observed the opposite trend in the scattering time dependence of the Hall resistivity compared to the strong coupling regime. The Hall resistivity decreases as the scattering time increases. Similar to the strong coupling study, we chose a temperature ($T = 0.8K/J$) and study its scattering time dependence in section III C.

C. Scattering Time Dependence of the THE

1. Skyrmions Phase

I obtained the Hall resistivity as a function of scattering time for both the skyrmion and thermal fluctuation phase and analyzed them. The calculations with the skyrmions (in the strong coupling regime) are used as the control group because their effect on transport
measurement is well understood. We used a larger DM interaction and $B$ field to control the skyrmions size and density. We hope to use the skyrmions to better understand the transport signatures in the thermal fluctuation. In the strong coupling regime, the Hall conductivity and longitudinal conductivity agree with the transport equations. The transport equations here are the expansion of the Drude model derived using Boltzmann transport equations \[16\].

Figure 9. Conductivity and resistivity are plotted as a function of $\tau$. $D = 0.9$ and $B = 0.45$ were used in the spin Hamiltonian to increase the skyrmion density. 9(a - b) The conductivity and resistivity in the strong coupling regime. 9(c - d) The conductivity and resistivity in the weak coupling regime.

The conductivity tensor in two dimensions is,

$$
\sigma_{ij} = \frac{n e^2 \tau}{m} \left[ \frac{1}{1 + (\Omega_c \tau)^2} \pm \frac{\Omega_c \tau}{1 + (\Omega_c \tau)^2} \right] 
$$

where $\Omega_c = e B_{\text{eff}} / m^*$ is the cyclotron frequency. $m^*$ is the effective mass of an electron, and $B_{\text{eff}}$ arises from skyrmions. The diagonal terms are the longitudinal conductivity, and the off-diagonal terms are the transverse conductivity also known as the Hall conductivity.
The skyrmion phase shows expected behaviors of $\sigma_{xy}$ and $\sigma_{xx}$. $\sigma_{xy} \propto \tau^2$ for small $\tau$, and approaches a constant for large $\tau$ (Fig. 9a,b). $\sigma_{xx}$ is linear for small $\tau$ and approaches zero for large $\tau$ as expected. The Hall resistivity is constant at large $\tau$ as expected.

At high $\tau$, we can express $\sigma_{xy}$ (Eq. 10) in terms of $J_H$, $t$, and the solid angles ($\Omega$). Notice that Eq. 10 is only valid for a topological spin texture where the spins vary slowly. If we consider a limiting case where all spins are polarized then the solid angle is zero, which gives infinite $\sigma_{xy}$. Eq. 10 is the analytical expression for the Hall conductivity with $\epsilon$ measured from the bottom of the conduction or valance band to the Fermi energy. $e^2/h$ is the unit for quantized conductance.

$$\sigma_{xy} = \frac{e^2}{h} \frac{\epsilon t \Omega}{t} = \frac{e^2}{h} \frac{|J_H - 4t - E_f|}{t \Omega} \tag{10}$$

I calculated $\sigma_{xy}$ at $\tau = \infty$ using Eq. 10. The analytical results are consistent with the simulation results at various Fermi energies (see supplementary information). These results show that skyrmions give rise to the THE with $\Omega_c$ proportional to the effective magnetic field.

The transport results of skyrmions in the weak coupling regime exhibit similar behaviors to the strong coupling regime when the Fermi energy is not very close to $J_H$ (Fig. 9c,d). Skyrmions form a dense lattice and produce a local effective magnetic field throughout the whole lattice. Therefore, when the Fermi energy is sufficiently large, the scattering mechanism here is still local and obeys the local effective magnetic field.  

When the Fermi energy is close to $J_H$ value, where $J_H \ll 1$, the transport results of skyrmions in the weak coupling regime exhibit similar behavior to the weak coupling regime of the thermal fluctuation phase (Fig. 10). This may suggest that there are some correlations, and we may be able to use the skyrmion model to gain a better understanding of the weak coupling regime in the thermal fluctuation phase. Studying ordered phases such as the skyrmion phase is a lot easier than studying the thermal fluctuation phase. The similarities between the ordered phase and fluctuation phase in the weak coupling regime may help us better understand the weak coupling theory.

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2. Thermal Fluctuation Phase

I repeated the calculations for the strong and weak coupling regime of the thermal fluctuation phase to see if they agree with Eq. 9. The results in the strong coupling regime are shown in Fig. 10. $\sigma_{xy}$ is constant for large $\tau$, and $\sigma_{xx}$ has a local maximum around $\tau = 20$. Looking closely at the small $\tau$ limits, we can see that $\sigma_{xy} \propto \tau^2$ and $\sigma_{xx} \propto \tau$. These features of the conductivities are similar to the skyrmion phase. In the spins fluctuation phase $\sigma_{xx} >> \sigma_{xy}$, which makes $\rho_{xy} = (B_{\text{eff}}/ne)(1 + (\omega_c\tau)^2)$. In Fig. 10 the Hall conductivity curve falls between quadratic and linear $\tau$. A log-log plot shows nonlinear behavior of $\rho_{xy}$, which indicates exponential dependence of $\tau$ (Fig. 11). Further study is needed to describe this behavior.

![Plot of Conductivity and Resistivity](image)

Figure 10. Conductivity and resistivity plotted as a function of $\tau$. $D = 0.3$ and $B = 0.15$ were used in the spin Hamiltonian to obtain the thermal fluctuation phase. (a - b) The conductivity and resistivity in the strong coupling regime. (c - d) The conductivity and resistivity in the weak coupling regime. The Fermi energies are 10 and 0.1 in the strong and weak coupling case, respectively.

Qualitatively, the $\sigma$ curves have the same behavior as in the skyrmion phase. This may suggest that the THE arises from the emergent topological charge in the thermal fluctuation...
phase. However, we do not have an analytical model to relate the Hall conductivity to the observables. Though, the qualitative results show a promising sign that the THE may have originated from the thermal emergent of topological charge. Further study to identify the mechanism (real space or momentum space) that dominates the transport signatures will give valuable insight into the fundamental understanding of the spin topology and transport physics.

In the weak coupling regime $\sigma_{xy}$ has a maximum at around $\tau = 5$ and continue to decrease for larger $\tau$. $\rho_{xy}$ also decreases for increasing $\tau$. At the small $\tau$ limits, $\sigma_{xy}$ retains its quadratic behavior. Currently, there are no models that can explain the behaviors in the weak coupling regime of the thermal fluctuation phase. However, we also observed similar behavior in the weak coupling regime of the skyrmion phase. Here, we propose to use the skyrmion model to understand the weak coupling regime, and then we can generalize the skyrmion model to explain the thermal fluctuation phase. We predict that calculating the contribution from the conduction band, valance band, and cross term in the Kubo formula separately will allow us to understand the contribution from real space and momentum space. Understanding the transport mechanisms in the weak coupling regime will fill in the knowledge gap of scattering physics. This in turn can help theoretical physicists predict and explain complex phenomena in materials.
IV. CONCLUSION

I presented the correlation between thermally emergent topology and the THE in the strong and weak coupling regime and then showed the scattering time dependence of the THE for the skyrmion phase and thermal fluctuation phase. The results show strong correlations between thermally emergent topology and the THE. This suggests thermal fluctuation near the critical temperature is responsible for the THE. Moreover, the magnitude of the THE in the thermal fluctuation phase is about 30% of the THE in the skyrmions phase. This means the THE induced by thermal fluctuation is a significant phenomenon in transport physics.

Our results are consistent with experimental results as well. The THE observed in SrRuO$_3$ near the critical temperature originates from thermal fluctuation [9]. Here, we conclude that the THE can not be used as a signature of skyrmions at the high-temperature limits. More experimental evidence is needed to claim the presence of skyrmions. Methods such as magnetic force microscopy and small-angle neutron scattering are needed to observe domain walls and magnetic modulations, which are the characterizations of skyrmions [18, 19]. Combining these methods with transport measurements is needed to unambiguously identify skyrmions. Accurately identifying skyrmions in materials is crucial for applications in electronics and spintronics, as well as providing a material platform for further exploration of skyrmion physics.

On a more theoretical side, we observed the THE crossover from the weak to the strong coupling regime. This means the THE in the weak and strong coupling regimes are controlled by the scalar chirality and TC, respectively. Qualitative analysis of the scattering time dependence of the THE in the strong and weak coupling regime show that they agree with the skyrmion phase. Here we used the skyrmion phase as a control group to study the complex behaviors in the thermal fluctuation phase. The results will allow us to identify the dominant scattering mechanism in the thermal fluctuation phase.
V. ACKNOWLEDGEMENTS

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Figure 12. Conductivity and resistivity of the skyrmion phase in the strong coupling regime plotted as a function of $\tau$ and Fermi energy. $D = 0.9$ and $B = 0.45$ were used to increase the skyrmion density, with $Q_T = 12$ topological charge per 1000 spins. $\epsilon$ is the energy measured from the bottom of the conduction band to the Fermi energy. (a - b) $\epsilon = 0.2$, (c - d) $\epsilon = 2$, (e-f) $\epsilon = 3$, and (g-h) $\epsilon = 3.5$. 

SUPPLEMENTARY INFORMATION

Transport Results of Skyrmions in the Strong Coupling Regime
<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>Analytical $\sigma_{xy}$</th>
<th>Comp. $\sigma_{xy}$</th>
<th>Analytical $\rho_{xy}$</th>
<th>Comp. $\rho_{xy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1.35</td>
<td>0.5</td>
<td>0.800</td>
<td>0.8</td>
</tr>
<tr>
<td>2</td>
<td>13.5</td>
<td>12.5</td>
<td>0.080</td>
<td>0.065</td>
</tr>
<tr>
<td>3</td>
<td>20.4</td>
<td>20</td>
<td>0.053</td>
<td>0.04</td>
</tr>
<tr>
<td>3.5</td>
<td>23.7</td>
<td>25</td>
<td>0.045</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table I. Table comparing analytical and computational results of $\sigma_{xy}$ and $\rho_{xy}$.

Table I compares $\sigma_{xy}$ and $\rho_{xy}$ between the analytical (Eq. 10) and the computational (Fig. 12) results. $\Omega = 0.147$ is the average solid angle of the skyrmion lattice used in Eq. 10. $\sigma_{xy}$ decreases as $\epsilon$ decreases for both analytical and computational results. $\rho_{xy}$ increases as $\epsilon$ decreases, and the results agree very well at small $\epsilon$. Since I used the average solid angle, the small differences in the results are expected because some areas in the skyrmion lattice have zero solid angles. Therefore, the effective magnetic field is not uniformly distributed (Fig. 13). Overall the analytical results have the same trend as the computational results, and the values are within an order of magnitude. This shows the accuracy of the computational model.

Figure 13. (a) The skyrmion lattice. The colorbar represents the z-direction of the spins. (b) The solid angles distribution and the colorbar shows the magnitude of the solid angles. All calculations in the skyrmion phase used this skyrmion configuration.