The puzzling behavior of equity returns: The need to move beyond the consumption capital asset pricing model

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Abstract
This dissertation examines returns in equity markets and the ability of extant models to account for their behavior. I first consider the so-called "expectations hypothesis" (EH) and find that it fails empirically even when structural change is incorporated into the analysis. I then examine the consumption Capital Asset Pricing Model (CAPM), which has been the workhorse of financial economics for more than two decades. In its canonical version, researchers find that the model fails empirically along several dimensions. This failure has led them to explore several modifications, which they find perform better than the canonical model, although mostly in calibration exercises that ignore the time variation in the data. I show that one of the key features of this time variation is that the ex ante return on stocks in excess of the risk free rate undergoes extended periods of time in which it is largely positive and other periods of time in which it is largely negative. I also show that the canonical CAPM and all of its proposed modifications have essentially the same implication for such sign reversals. This result enables me to propose a simple test that confronts the entire class of consumption CAPM models with the time series data. I find that the canonical model and its modifications are unable to explain the pattern of sign reversals we observe in the data. This leads me to consider an alternative model that replaces expected utility theory with endogenous prospect theory and the Rational Expectations Hypothesis with an Imperfect Knowledge Economics representation of forecasting behavior. I find that my alternative model performs better than the consumption CAPM in accounting for the time variation of stock returns.

Keywords
Economics, General

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THE PUZZLING BEHAVIOR OF EQUITY RETURNS: THE NEED TO MOVE BEYOND THE CONSUMPTION CAPITAL ASSET PRICING MODEL

BY

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DISSERTATION

Submitted to the University of New Hampshire in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy in Economics

December, 2009
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ABSTRACT

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University of New Hampshire, December, 2009

This dissertation examines returns in equity markets and the ability of extant models to account for their behavior. I first consider the so-called “expectations hypothesis” (EH) and find that it fails empirically even when structural change is incorporated into the analysis. I then examine the consumption Capital Asset Pricing Model (CAPM), which has been the workhorse of financial economics for more than two decades. In its canonical version, researchers find that the model fails empirically along several dimensions. This failure has led them to explore several modifications, which they find perform better than the canonical model, although mostly in calibration exercises that ignore the time variation in the data. I show that one of the key features of this time variation is that the ex ante return on stocks in excess of the risk free rate undergoes extended periods of time in which it is largely positive and other periods of time in which it is largely negative. I also show that the canonical CAPM and all of its proposed modifications have essentially the same implication for such sign reversals. This result enables me to propose a
simple test that confronts the entire class of consumption CAPM models with the time series data. I find that the canonical model and its modifications are unable to explain the pattern of sign reversals we observe in the data. This leads me to consider an alternative model that replaces expected utility theory with endogenous prospect theory and the Rational Expectations Hypothesis with an Imperfect Knowledge Economics representation of forecasting behavior. I find that my alternative model performs better than the consumption CAPM in accounting for the time variation of stock returns.
INTRODUCTION

Financial markets, such as those for stocks and bonds, play a vital role in market economies: they help to channel savings from consumers, who have no productive use for their financial capital, to businesses, who would like to borrow capital to finance their investment projects. Well functioning financial markets promote efficient allocations and help to lay the foundation for future economic growth. A key question is whether financial markets perform their role well. To answer this question, researchers need models that can account for market outcomes.

In this dissertation, I examine the behavior of returns in equity markets and the ability of extant models to account for their behavior. My starting point is the so-called "expectations hypothesis" (EH), which assumes that investors are risk neutral and that their forecast errors are white noise. According to the pure EH, the \textit{ex post} return on stocks in excess of the return on risk free assets, such as short-term Treasury securities, should fluctuate randomly around zero.

Empirical studies generally find that EH is inconsistent with the data. However, most of the studies do not consider the problem of structural change, and when they do they impose break points a priori. This is a serious omission because in real-world markets, participants revise their forecasting strategies, leading to structural change in predictive regressions.
One of the contributions of this thesis is to re-examine EH using regression analysis that allows for temporal instability. I use two approaches to testing for structural change: the Bai and Perron (1998, 2003) procedure and a combination of the CUSUM test and a recursive Chow test. Both procedures enable me to test the stability of regressions without specifying break points a priori. I find much evidence of structural change. I also find that EH fails even when structural change is incorporated into the analysis.

The failure of EH has led economists to consider models that can account for the behavior of the excess return on stocks. For more than two decades now, the consumption Capital Asset Pricing Model (CAPM) has been the workhorse of financial economics. The canonical version uses a power utility function and expected utility theory to characterize the representative investor's preferences and the rational expectations hypothesis (REH) to represent her forecasting behavior. The model implies that the excess return on stocks is compensation to risk-averse investors for bearing consumption risk. The risk premium that emerges from the model depends on the degree of risk aversion and the covariance between the excess return and consumption growth.

By far, the most popular way to test the empirical validity of the canonical model has been to use historical averages of the excess return and the covariance. Researchers generally find that the average excess return on stocks is much too high given the covariation in the data and reasonable levels of risk aversion. This anomaly is the so-called the "equity premium puzzle."
There are also studies that examine whether the consumption CAPM can explain the time variation of the excess return. Here too, the model is found to be grossly inconsistent with the data.

There have been several responses in the literature to this failure. One has been to argue that the consumption CAPM is correct, but the time horizon over which consumption risk is typically measured needs to be lengthened. Several researchers have explored the implications of substituting the REH with a behaviorally motivated representation of forecasting behavior that presumes that individuals are irrational. But, by far, the most popular avenue of research has been to search for alternative specifications of preferences. Chief among these attempts are those that specify preferences to involve habit formation or that mix risk averse preference with the assumption of loss aversion.

Researchers report that these modifications to the consumption CAPM seem to do better empirically. Much of the empirical evidence is based on calibration exercises aimed at explaining the equity premium puzzle. Researchers generally find that the high equity premium found over 120 years of data can be largely accounted for by the model once it is modified with an alternative specification of preferences or forecasting behavior.

There are a few empirical studies that confront the habit-persistence models directly with time series data. Although this modification seems to match in calibration exercises, researchers find that it performs just as badly in explaining the time variation of the excess return as does the canonical
consumption CAPM. But, in general, researchers have not examined the ability of the recent modifications to account for the time variation in the data.

In this dissertation, I examine the key features of the \textit{ex ante} excess return on stocks that any empirically relevant model should explain. My sample runs from 1871:01 to 2008:12. I allow for the process driving the excess return on stocks to undergo structural change. I find two key features. One is that the \textit{ex ante} excess return varies widely over the sample. And, remarkably, this variation involves extended periods of time in which the excess return is largely positive and other periods of time in which it is largely negative.

There are a few empirical researchers that have noticed the tendency of excess returns to undergo sign reversals, but neither the canonical consumption CAPM nor any of its modifications have been confronted this feature of the data.

The main contribution of this dissertation is to fill this gap in the literature. I first show that the consumption CAPM is, in principle, consistent with sign reversals: such behavior would emerge if the conditional covariance between consumption growth and stock returns switches sign. Indeed, I find that the canonical model and all of its modifications has this implication for sign reversals. This result provides an easy way to confront the entire class of consumption CAPM models with the time variation of excess returns on stocks: I test whether these models can account for the pattern of sign reversals that we actually observe in the data.
To this end I make use of a test that Mark and Wu (1998) developed for modeling returns in the foreign exchange market. The results of my analysis are clear: the sign reversals in the data have no relationship to the switches in sign of the conditional covariance. The conclusion is that the canonical model and its modifications are grossly inconsistent with the time series data.

The failure of the entire class of consumption CAPM models leads me to consider an entirely new approach to modeling the excess return in equity markets. I adapt Frydman and Goldberg's (2007) model of the premium on currency to the stock market. This model replaces the assumptions of risk aversion and expected utility theory with endogenous prospect theory. It also replaces REH with an Imperfect Knowledge Economics representation of forecasting behavior. After sketching the model, I present some empirical evidence indicating that it does much better in accounting for the time series data.
CHAPTER I

THE REMARKABLE BEHAVIOR OF THE EX ANTE EXCESS RETURN ON STOCKS OVER BONDS

1.1 Introduction

Stock markets play a central role in our economy: they channel savings from households to firms, which use the financial resources to increase the productive capacity of the economy. This market process helps to lay the foundation for future economic growth. A key question is whether stock markets perform their function well. To answer this question, we need models that can account for the outcomes in these markets. The workhorse in the field is the consumption Capital Asset Pricing Model (CAPM). The canonical version of this model has encountered great difficulty in explaining the basic features of the data, especially concerning the excess return on stocks over risk free bonds.

The main purpose of this chapter is to examine the key features of this excess return, which any theory would need to explain. The starting point of the chapter is the so-called "expectations hypothesis," which implies that the excess return on stocks has a zero mean and is uncorrelated with its past and all other variables. Empirical studies have roundly rejected this prediction. For example, Fama and Schwert (1977) (hereafter FS) regress the future one-period stock return on the current risk free rate using a sample that runs from
January 1953 to December 1975. If the expectations hypothesis were correct, we would expect regression estimates to imply a zero intercept and a slope coefficient equal to one. However, FS and subsequent studies reject these hypotheses at very high significance levels.

One of the contributions of this chapter is that it examines whether the FS results are robust to the problem of structural change. Given that market participants must cope with ever-imperfect knowledge about the process driving stock returns,\(^1\) we would not expect the relationship between their forecasts of returns and any set of causal variables to remain stable over the entire sample period. This instability would then imply instability in the process driving excess returns.\(^2\) Indeed, Paye and Timmermann (2005) and Rapach and Wohar (2006) find evidence of structural change in regression models of stock returns.

To test for points of structural change, I employ recursive techniques that search the data for break points, rather than imposing them a priori. My regression analysis includes the Treasury bill rate, as well as other variables that have been shown to have predictive power. I use the Standard and Poor's (S&P) 500 stock price index to measure market prices. My data are monthly and run from January 1871 through December 2008.

---

\(^1\) No one has access to the exact causal mechanism underlying this process. They also change their understanding of the causal mechanism over time.

\(^2\) One would also not expect the social context in which excess returns are generated, including institutions and economic policies, to remain unchanged over time. Such change would also lead to temporal instability in the FS regression.
I find 14 break points over my sample, thereby implying 15 sub-periods of "statistical parameter constancy." The fitted values of my piece-wise-linear-regression analysis provide a measure of the *ex ante* excess return on stocks. I find that this excess return varies widely over a range between positive 20% and negative 15%. I also find that the FS results are robust to the problem of temporal instability: the expectations hypothesis is strongly rejected in most sub-periods.

Many researchers focus on the sample average of the excess return, which I find to be 4.67%. However, focusing on historical averages obscures not only the instability of the process driving excess returns, but a key feature of the time series data. Strikingly, I find extended time periods that are characterized by excess returns that are largely positive, while other time periods involve largely negative excess returns. Although other researchers have found such sign reversals, this key feature of the time path has received little attention in the literature.

In the next chapter, I explore the ability of the consumption CAPM to account for the behavior of excess returns, with particular emphasis on sign reversals. The remainder of chapter 1 is organized as follows. Section 2 defines the *ex post* and *ex ante* excess return and reviews the expectations hypothesis. In section 3, I review the existing evidence on the expectations hypothesis. Section 4 re-examines the evidence on excess returns and the expectations hypothesis by incorporating structural change into the analysis.
In section 5, I document the phenomenon of sign reversals in ex ante excess returns. Section 6 offers concluding remarks.

1.2 The Excess Return on Stocks and the Expectations

Hypothesis

The ex post return on a basket of stocks in excess of the risk free rate from time $t$ to $t+1$, which I denote by $er_{t+1}$, can be written as follows:

$$er_{t+1} = r_{t+1} - r^f_t$$

(1.1)

where the stock return, $r_{t+1} = \ln(P_{t+1} + D_{t+1}) - \ln(P_t)$, $P_t$ denotes the price index of the basket of stocks, $D_t$ denotes the end-of-period dividend on the stocks, and $r^f_t$ is the risk free rate that prevails between $t$ and $t + 1$. Researchers have documented that over the past century or two, the ex post return on stocks has, on average, been considerably higher than the average risk free rate. For example, Mehra and Prescott (2003) compute the mean of $er_{t+1}$ using the average annual real return on the US stock market and the return on a relatively risk free security from 1889 through 2000. They report an excess return of 6.92%.

---

3 Note that the dividend here is for the stock index. In order to calculate returns for the stock index, the dividend has to be an index number rather than a dollar value. The data in my empirical analysis is from Shiller's website. Shiller has transformed the dividend series into index.

In Table 1.1, I provide summary statistics on $er_{t+1}$, as well as on the underlying monthly data from 1881 through 2008. I use the S&P 500 stock price index and earnings series from Robert Shiller's website to compute the excess return.\footnote{See http://www.econ.yale.edu/~shiller/data.htm.} Shiller's data is based on the work of Cowles (1939), which uses a value-weighted portfolio for the price index. This index consists of 12 stocks in 1871 and 351 in 1938. From 1918, Shiller uses the S&P 500 industrial portfolio. The return on the risk free security is from Goyal and Welch (2008). Treasury certificates were first issued in 1920. GW uses the rate on short-term commercial paper for the period before 1920 and Treasury certificates from 1920-1930. From 1931 onward, they use the rate on 3-month Treasury bills.

Like Mehra and Prescott (2003), I find that the historical average nominal annual return on stocks is 8.28%, while the average nominal risk free rate over the period was 3.65%. As such, my data imply an average excess return on equities of 4.63%.

It is important to point out that focusing on the historical average obscures key features of the $er_{t+1}$ time series. In figure 1.1, I plot a 12-month moving average of this excess return, which I refer to as the trend excess return. The graph shows that the trend $er_{t+1}$ varies widely, from negative 112% to positive 97% over the period. Some periods, for example, the sub-period of 1929-1933, are characterized by a consistently negative trend excess return, whereas during other sub-periods, for example, the 1990s, the trend $er_{t+1}$ is consistently positive.
This behavior is suggestive that the *ex ante* (expected) excess return also undergoes such sign reversals.

1.2. 1. The *Ex Ante* Excess Return

Much of the focus in the literature is on *ex post* returns rather than on *ex ante* returns. We can decompose the *ex post* excess return into an expected excess return at time $t$ for $t+1$ and a forecasting error:

$$er_{t+1} = er_{t+1}^{\hat{}} + \varepsilon_{t+1}$$  \hspace{1cm} (1.2)

where $er_{t+1}^{\hat{}}$ is an aggregate of market participants forecasts of the excess return and $\varepsilon_{t+1}$ is a forecasting error.

The expectations hypothesis assumes that all market participants are risk neutral, implying that they will bid stock prices to the level at which $er_{t+1}^{\hat{}} = 0$. The idea is that if market participants are risk neutral, they care only about the expected returns across different assets. Let's consider two assets: stocks and bonds. If the return on stocks is larger than that on bonds, market participants will want to hold only stocks and in trying to do so, bid up their prices. Prices will rise until the expected returns on stocks and bonds are equal. This story assumes that market participants face no capital constraints and that they hold identical forecasts of returns at every point in time.\(^6\)

\(^6\) If one recognizes that market participants have heterogeneous expectations, then with no capital constraints, there would be no well-defined equilibrium price. To see this, suppose some market participants expect stock prices to go up, which we call “bulls”, while the rest of the participants expect prices to go down, which we call “bears”. As bulls expect prices to go up, they will want to hold more stocks. With no capital constraints, bulls will want to borrow an infinite amount of capital to place bets on a price rise, while bears will want to borrow an infinite amount
The expectations hypothesis also assumes that the forecast error is white noise, that is, mean zero and i.i.d. and thus uncorrelated with its past or any other variables. The literature commonly refers to this assumption as the so-called “Rational Expectations Hypothesis (REH).”

According to the expectations hypothesis, then, the ex post return is a mean zero, i.i.d. process:

\[ er_{t+1} = e_{t+1} \] (1.3)

that is, the expected return on stocks should, on average, be equal to the risk free rate. The hypothesis is thus a joint hypothesis of risk neutrality and REH.

Some economists appeal to a more general form of the expectations hypothesis, which recognizes that individuals are risk averse. According to this more general form, \( er_{t+1} \) is a constant that can differ from zero. This is because it is common to view stocks as risky compared to government treasury securities. After all, the future return on stocks is uncertain, whereas one can lock in a sure return on treasury securities as long as they are held to maturity. Consequently, if individuals are assumed to be risk averse, they should hold stocks only if they expect to earn a positive return—a premium—in excess of that on Treasury securities. This is the story that emerges from the traditional CAPM, which measures the riskiness of stocks in terms of the variance of their returns. Both forms of the expectations hypothesis imply that \( er_{t+1} \) is time invariant: it is zero.

---

7 In real-world markets, where individuals must cope with ever-imperfect knowledge, we would also expect that forecasting errors would not be systematically correlated with any information set in any fixed way.
under the assumption of risk neutrality and a constant with the assumption of risk aversion.

1.3 Tests of the Expectations Hypothesis

Testing the expectations hypothesis in its pure or general form is quite straightforward. We note that the finding of Mehra and Prescott (1985, 2003) and many others, that the historical average excess return over 100 years is significantly positive, immediately implies that, in its pure form, the expectations hypothesis is inconsistent with the data.

Tests of the general expectations hypothesis (and thus of its pure form too) generally involve regressing $e_{it+1}$ or $r_{it+1}$ on an information set.

1.3.1. The Fama-Schwert (FS) Regression

An important study is Fama and Schwert (1977), which uses OLS and regresses the ex post monthly stock return on the one-month Treasury bill rate:

$$r_{it+1} = \alpha + \beta r_{ft} + \epsilon_{it+1}$$  (1.4)

over a sample that runs from January 1953 to July 1971. Fama and Schwert's (1977) stock return data are based on a weighted portfolio of NYSE common stocks.

The expectations hypothesis in its pure form implies $\beta = 1$ and $\alpha = 0$, whereas in its general form, $\beta = 1$ and $\alpha \neq 0$. Both hypotheses imply that the error term has a conditional mean of zero.
FS report an estimate of the slope, $\hat{\beta}$, equal to -5, which they find is significantly different not only from unity, but also from zero. Other studies that run the FS regression include Fama (1981), Schwert (1981), Geske and Roll (1983), and Stulz (1986). All report results indicating a clear rejection of the expectations hypothesis. For example, Campbell (1985) replicates the FS regression, but instead of using the stock return as the dependent variable, he runs the equivalent regression:

$$er_{t+1} = \alpha' + \beta' r_{t} + \epsilon_{t+1}$$

where under the null, $\alpha' = \alpha = 0$ and $\beta' = \beta - 1 = 0$. Campbell (1985) uses return data based on the Center for Research in Security Prices (CRSP) stock price index and the 1-month T-bill rate over the sample period from February 1959 to September 1978. The study reports that $\hat{\beta}'$ is significantly less than zero. It also finds similar results for an extended sample period that runs until November 1983.

1.3.2. Additional Evidence of Predictability

Researchers have also tested the expectations hypothesis by exploring whether variables other than the interest rate have predictive power. Table1.2 lists major studies and the variables found to have predictive power. The general conclusion from this research is that a range of variables, including lagged excess returns
and price-earnings and price-dividend ratios help to predict future excess returns, thereby indicating a rejection of the EH hypothesis in both forms.\textsuperscript{8}

A number of studies examine the autocorrelation of stock returns, that is, whether returns depend on past realizations. As have discussed earlier, if the expectations hypothesis holds, then the stock return should not correlate with its past realizations. Fama and French (1988b) show that during the 1926-1985 period, long holding-period returns, i.e., 3-5 year, for New York Stock Exchange (NYSE) stocks based on CRSP data are significantly negatively serially correlated, implying that 25 to 40 percent of the variation of longer-horizon returns is predictable from past returns. Lo and MacKinlay (1988) find significant positive serial correlation for weekly and monthly holding-period returns. Using 1216 weekly observations from September 6, 1962, to December 26, 1985, for example, they compute the weekly first-order autocorrelation coefficient of the equal-weighted CRSP returns index to be 30%. Poterba and Summers (1989) examine market returns for the United States over the 1871-1986 period and for 17 other countries over the 1957-1985 period, as well as to returns on individual firms over the 1926-1985 period. They find consistent evidence that stock returns are positively serially correlated over short horizons, and negatively autocorrelated over long horizons.

The literature has given much attention to the price-dividend (P/D) and the price-earnings (P/E) ratios. This is because the present-value model, which is the

\textsuperscript{8} See Lewellen (2004) for a general discussion of return predictability and a review of the empirical literature. Goyal and Welch (2008) provide a comprehensive examination of the predictability of stock returns.
starting point of all asset pricing, implies that stock prices should equal the present discounted value of the expected future stream of dividends. Typical assumptions concerning the dividend process lead to the result that stock prices should fluctuate randomly around a constant P/D ratio. Although the P/D ratio is a natural selection to evaluate stock performance, it has its disadvantages. First, not all the companies pay dividends. Second, how much money being paid as dividends is determined by company dividend policy. As an alternative to the P/D ratio, the simplest and most widely used ratio to predict the market is the price–earnings ratio. The P/E ratio is an indicator of future stock performance as earnings predict a company's future profitability.

Shiller (1984), Keim and Stambaugh (1986), Fama and French (1988a), and Campbell and Shiller (1988) all find that the P/E and P/D ratios are predictors of future stock returns. For example, Campbell and Shiller (1988) propose a log-linear framework between stock prices, dividends and returns. They start with the definition of the log return on stocks,

\[ r_{t+1} = \ln(P_{t+1} + D_{t+1}) - \ln(P_t). \]

The log return is a nonlinear function of log prices \( p_t \) and \( p_{t+1} \) and log dividends \( d_{t+1} \), but it can be approximated around the mean log

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9 For example, see Campbell and Shiller (1988).

10 In this traditional framework, forecasting behavior is modeled using REH and there are no frictions in the market. Investors correctly process all information in forming expectations about future dividends. In the Gordon growth model, the dividend growth rate and discount rate are assumed to be constant, which leads to a constant P/D ratio. As a result, stock prices should fluctuate randomly around this “fundamental value”—the discounted sum of expected dividends.

11 In his review of the literature on the predictability of stock returns, Cochrane (2008) highlights the forecast ability of P/D and P/E ratios.
dividend-price ratio, \((d_t - p_t)\), using a first-order Taylor expansion. The resulting approximation is:

\[
    r_{t+1} \approx k + \rho p_{t+1} + (1 - \rho)d_{t+1} - p_t
\]

where \(k\) and \(\rho\) are the parameters of linearization and lowercase letters are logarithms of corresponding capital letters. Equation (1.6) is a linear difference equation for the log stock price. Solving forward, imposing the terminal condition that \(\lim_{j \to \infty} \rho^j p_{t+j} = 0\), taking expectations, and subtracting the current dividend, one gets

\[
    p_t - d_t = \frac{k}{1 - \rho} + E_t \sum_{j=0}^{\infty} \rho^j [\Delta d_{t+j} - r_{t+j}]
\]

(1.7)

This equation says that the log price-dividend ratio is high when dividends are expected to grow rapidly, or when stock returns are expected to be low. Intuitively, if the stock price is high today, then from the definition of the return and the terminal condition that the stock price is non-explosive, there must either be high dividends or low stock returns in the future. This popular framework gives a direct link between stock returns and the price-dividend ratio.

Campbell and Shiller (1988) estimate a Vector Autoregression (VAR) that entails the log P/D ratio and the dividend growth rate.\(^{12}\) The results show, contrary to the present-value model and the EH, that stock returns are predictable. The log P/D ratio is positively related to future stock returns, while

\(^{12}\) They employ two data sets: one involves the return based on the S & P 500 price index over the period 1871-1986 and the other the return based on the value-weighted NYSE index from CRSP over the period 1926-1986.
real dividend growth rate has a negative effect.\textsuperscript{13} The two variables are jointly significant at close to the 5 percent level.

Campbell (1991) estimates a VAR that entails the monthly excess return on stocks,\textsuperscript{14} $r_{t+1}$, the dividend-price ratio, D/P, and the relative bill rate, $r_{\text{rel}}$, which is the difference between the one-month T-bill rate and a one-year backward moving average:\textsuperscript{15}

$$\hat{r}_{t+1} = \alpha + \hat{\beta}_1 r_t + \hat{\beta}_2 (D/P)_t + \hat{\beta}_3 r_{\text{rel}}_t$$ (1.8)

Campbell defines the persistence coefficient as the ratio of the standard deviation of innovation in the expected present discounted value of future returns to the standard deviation of the innovation in the one-period-ahead expected return:

$$P_r = \frac{\sigma(\eta_{r_{t+1}})}{\sigma(\mu_{t+1})}$$ (1.9)

where $\sigma(x)$ denotes the standard deviation of $x$. Campbell defines $\eta_{r_{t+1}}$ to be the term which represents news about future returns:

$$\eta_{r_{t+1}} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j} ,$$ (1.10)

and $\mu_{t+1}$ is the innovation at time $t+1$ in the one-period-ahead expected return:

$$\mu_{t+1} = (E_{t+1} - E_t)r_{t+2} .$$ (1.11)

If the expected return follows an AR(1) process, then

\textsuperscript{13} See table 4 in Campbell and Shiller (1988), p 213.

\textsuperscript{14} He uses the return based on the value-weighted NYSE price index from CRSP over the period 1926-1988.

\textsuperscript{15} Researchers have noticed that the short-term rate itself may not be stationary. Subtracting its moving average is a crude way to detrend a nonstationary series.
Intuitively, if stock returns are serially correlated, the innovation or news in the one-period-ahead expected return will also affect the expected value of future returns. Campbell reports the persistence coefficient from the VAR model is about 5, which is equivalent to 0.8 for an AR (1) model. He also reports that the excess return is correlated with the P/D ratio and the relative T-bill rate.

Researchers have also examined the forecast ability of the aggregate dividend payout ratio, or dividend-earnings ratio (see Lamont, 1998), the consumption-wealth ratio (Lettau and Ludvigson, 2001), book to market ratio (see, Kothari and Shanken, 1997, and Pontiff and Schall, 1998), corporate bond returns (see, Fama and French, 1989 and Keim and Stambaugh, 1986) and other variables. These studies are based on different methodologies and sample periods, but they all report evidence of the predictability of future stock market returns.

Even though the predictor variables differ across the empirical studies, most find that predictability tends to be the strongest over long multi-year horizons. For example, by using earnings and dividend data on the S & P 500 stocks over the period of 1871-1987, Campbell and Shiller (1988a) show that the P/E ratio has strong predictability over 10-year horizons. The 10-year moving
average of earnings to price ratio can explain 38% of the variance. A moving
average of earnings is used because yearly earnings are quite noisy and they
could even be negative. I will also use this measure in my analysis. At the same
time, there is also evidence that the degree of predictability appears to have
diminished somewhat beginning in the mid-to-late 1990s.

1.4 Imperfect Knowledge and Structural Change

Like the FS regressions, the empirical work on the forecast ability of variables
other than the interest rate ignores the problem of structural change. Yet, in a
world of imperfect knowledge, no one knows the exact causal mechanism
underlying the market outcomes, and their understanding about this mechanism
also changes over time. They will modify their forecasts about future outcomes
and act upon it to invest in the market. As a result, we would not expect that the
process driving excess returns would be temporally stable.

Consider the definition of the expected excess return:

$$ e_r^{t+1} = \hat{r}_{t+1} - r^f_{t+1}, $$

For simplicity, assume that \( \hat{r}_{t+1} \) can be written as:

$$ \hat{r}_{t+1} = \beta x $$

where \( x \) is a vector that represents the variables that market participants use to
form their forecasts. Revisions of participants’ forecasting strategies is
represented by changes in \( \beta \).

\footnote{In Chapter 4, I will show that these results are consistent with the gap model in which the
P/E ratio plays a key role in the analysis.}
Now consider a linear projection of $\tilde{e}_{i,t+1}$ on $r_t'$, which can be expressed as follows:

$$E(\tilde{e}_{i,t+1} | r_t') = E(\tilde{r}_{i,t+1} | r_t') - r_{i,t+1}'$$

(1.16)

Since $E(\tilde{r}_{i,t+1} | r_t')$ depends on $\beta_i$, this FS-type regression will undergo structural change at points in time when market participants revise their forecasting strategies.

In real-world markets, no one has access to an exact model that adequately represents the causal process relating stock prices to fundamentals in all time periods, past and future. Market participants, who act on the basis of different preferences and constraints, will likewise adopt different strategies in forecasting the future. Because individuals' forecasts drive their trading decisions in the market, and individuals revise their forecasting strategies over time, one would not expect the relationship between the stock price or return and some set of causal variables to be time invariant. Not only might individuals alter the weights they attach to a set of causal variables in forming their forecasts, but the set of relevant causal variables might change from one time period to another. But, when market participants change their forecasting behavior, the relationship between the stock return and a set of causal variables also changes.

Not surprisingly, when researchers look for structural change, they usually find it. Some researchers consider parameter instability using techniques that impose the break points a priori. Campbell (1987), for example, investigates whether the term structure of interest rates can predict the stock return over a sample from 1959:04 to 1983:11. In September 1979, the Federal Reserve
adopted new operating procedures for implementing monetary policy. With that in mind, Campbell (1987) uses a Chow test to check for parameter stability across the pre- and post-1979 sub-periods, which strongly rejects this hypothesis.

Other researchers test for structural stability using techniques that do not specify the break points a prior. Rapach and Wohar (2006), for example, examine the stability of regression models of U.S. quarterly aggregate real stock returns over the postwar era (1953:2-2000:4). They consider models of the S&P 500 and CRSP equal-weighted real stock returns and eight financial variables that have displayed predictive ability in the extant literature (including the P/E ratio and the short term interest rate). The study uses different methods to test for structural stability, one of which is developed by Bai and Perron (1998, 2003, 2004). They find strong evidence of structural breaks in their regression models. Table 1.3 summarizes the results from the Bai and Perron tests for both bivariate and multivariate regressions using data on the S & P 500 index. For most of the regressions, they find only one break in the sample. Various results in the literature have pointed out that the critical aspect of testing for the presence of mean shifts is the low power of the test, especially in the case of multiple mean shifts.

Rapach and Wohar (2006) also show that the predictive ability of many financial variables varies considerably over time, indicating that failure to account for structural breaks in predictive regression models of S&P 500 returns can lead one to substantially overestimate or underestimate the predictive ability during certain periods. For example, the dividend-price ratio, in their bivariate regression
involving the dividend-price ratio, the slope coefficient is almost three times smaller as we move from the first regime, which ends in 1990:3, to the second regime, so that the predictive power of this variable is substantially reduced over the last decade of the full sample. For the price-earnings ratio, the slope coefficient is significant in each of the three regimes, while it is insignificant over the full sample. These results suggest that EH in both forms does not hold even when we consider structural breaks.

In the next section, I provide additional evidence of the temporal instability in the return regressions.

1.4.1 Additional Evidence of Temporal Instability

To add to the evidence that the failure of the expectations hypothesis is not due to temporal instability, I first examine the stability of regressions of the excess return on different information sets using recursive techniques that do not impose break points a priori. Once break points have been identified, I examine whether there is evidence of predictability within the separate subperiods of parameter stability.

My returns data is monthly and based on the S&P 500 price index and the risk free rate as discussed in section 1. I consider the stability of two regressions in my analysis. First, I run an FS-type regression, with only the short-term T-bill rate as the regressor. If the expectations hypothesis holds, then the estimate of the slope coefficient should be zero.
I also consider a multivariate regression, which in addition to the T-bill rate, includes a smoothed earnings-price ratio and lags of the excess return. Companies' earnings is one of the most important factors affecting stock prices, and the price-earnings ratio is often treated as a barometer of whether or not a stock is overvalued or undervalued. The smoothed earnings price ratio, proposed by Campbell and Shiller (1988b, 1998) uses a 10-year moving average of earnings. Campbell and Shiller argue that the ratio of smoothed earnings to price should have better forecasting power than the current earnings price ratio because aggregate corporate earnings display short-run cyclical noise; in particular, earnings typically drop to near zero during recession years such as 1934 and 1992 and this creates spikes in the current earnings price ratio that have nothing to do with stock market valuation.

The predictive regression model I estimate is an autoregressive distribution lag (ADL):

\[
er_{t-4} = \alpha + \sum_{i=1}^{2} \beta_i er_{t-i} + \sum_{i=0}^{2} \left( \beta_{2i} r_{t-i} + \beta_{3i} ep_{t-i} \right) + \varepsilon_t
\]

(1.17)

where \( er_{t-4} \) is the excess return on stocks over bonds and \( ep_t \) is the smoothed earnings-price ratio. The dynamic ADL specification helps to obtain white noise residuals and to deal with the problem of unit roots.\(^\text{18}\)

\(^{18}\) See Hendry and Juselius (2000, 2001) for a discussion of the ADL is able to deal with relationships involving unit-roots.
1.4.2. Tests for structural change

There is no one single way to check for structural breaks. A popular method is the structural change test of Bai and Perron (BP) (1998, 2004). BP’s methodology is designed explicitly for estimating and testing regression models with multiple breaks. But the test has low power in detecting the breaks. For example, Jones and Olken (2008) point out that the method is conservative in detecting breaks, capturing only major economic growth “accelerations and collapses”.

I also employ a combination of the CUSUM test of Brown, Durbin and Evans (1975) and a recursive Chow tests to determine the structural break points. Many studies have used the CUSUM test in testing for temporal instability.\(^{19}\) Like the BP tests, this procedure searches the data recursively for the possibility of one or more break points, rather than relying on tests that require the choice of break points a priori.

1.4.2.1. The Bai and Perron Procedure. Consider the following regression model with \( m \) breaks (\( m+1 \) regimes),

\[
er_t = z_t^j \beta^j + \epsilon_t, \quad t = T_{j-1} + 1, \ldots, T_j
\]

where \( j = 1, \ldots, m+1 \) and by convention, \( T_0 = 0 \) and \( T_{m+1} = T \). BP explicitly treat the break points (\( T_1, \ldots, T_m \)) as unknown. \( \beta^j \) is the corresponding vector of coefficients in the \( j \)th regime, and \( z_t \) are the explanatory variables. The equation

---

\(^{19}\) See, for example, Kramer, Ploberger and Alt (1988), Pagan and Schwert (1990), Hols and De Vries (1991), Han and Park (1989), Stern, Baum and Greene (1979), and Padovano and Galli (2001).
is estimated using least squares. For each \( m \)-partition \((T_1, \ldots, T_m)\), the least squares estimates of \( \beta^j \) are obtained by minimizing the sum of squared residuals,

\[
S_T(T_1, \ldots, T_m) = \sum_{i=1}^{m+1} \sum_{t=1}^{T_i} (e_{it} - z_i'\beta^j)^2.
\]  

(1.19)

Let \( \hat{\beta}((T_1, \ldots, T_m)) \) denote the regression coefficient estimates of a given \( m \)-partition. Substituting these estimates into the above equation, the estimated break points are obtained by

\[
(\hat{T}_1, \ldots, \hat{T}_m) = \text{arg min}_{T_1, \ldots, T_m} S_T(T_1, \ldots, T_m),
\]

where the breakpoint estimators are the global minimum of the sum of the squared residuals. Based on these breakpoint estimates, we can calculate the corresponding least squares regression parameters.

BP (1998) develop testing procedures to test the number of structural breaks in the regression. One involves a null hypothesis of no structural breaks against the alternative of a fixed number \( m \) of breaks by using a maximum \( F \) statistic, \( \text{Sup}F_T(m) \). They also develop so-called "double maximum" statistics, \( UD_{\text{max}} \) and \( WD_{\text{max}} \), for testing the null hypothesis of no structural breaks against the alternative hypothesis of an unknown number of breaks given an upper bound \( M \) where \( UD_{\text{max}} = \max_{1 \leq m \leq M} \text{Sup}F_T(m) \) and \( WD_{\text{max}} = \max_{1 \leq m \leq M} w_m \text{Sup}F_T(m) \). \( w_m \) is calculated as \( c(q, \alpha, 1)/c(q, \alpha, m) \), where \( c() \) is the asymptotic critical value of the \( \text{Sup}F \) test for a significance level \( \alpha \) and \( q \) denotes the degrees of freedom.
The difference between these two statistics is that they employ different sets of weights to the individual $SupF_T(m)$ statistics. $UD_{max}$ sets all weights to unity and $WD_{max}$ applies a set of weights such that the marginal $p$-values are equal across values of $m$. Finally, BP (1998) also develop a $SupF_T(l+1|l)$ statistic to test the null hypothesis of $l$ breaks against the alternative hypothesis of $l+1$ breaks.

How would the number of breaks be determined using the $SupF_T(l+1|l)$ statistics? BP (1998) start with the $SupF_T(1|0)$ statistic, which tests if there exists one break against 0 breaks. If this statistic is insignificant, we conclude that there are no structural breaks. If $SupF_T(1|0)$ is significant, we proceed to examine $SupF_T(2|1)$. If the $SupF_T(2|1)$ statistic is insignificant, we conclude that there is one structural break. If the $SupF_T(2|1)$ statistic is significant, we then proceed to examine the $SupF_T(3|2)$ statistic and continue in a similar manner.

BP (2004) show that this procedure performs well in a number of circumstances but its performance can be improved upon when multiple breaks are present. With multiple breaks, BP (2004) find in extensive Monte Carlo simulations that the double maximum statistics are much more powerful. They recommend the following strategy. First, check the double maximum statistics to determine if there are any structural breaks. If the double maximum statistics are significant, then use the $SupF_T(l+1|l)$ as we discussed above to decide the number of breaks.
This is the strategy I use in my applications. The results of my analysis for both models are reported in Table 1.4. I find 4 breaks in the FS regression and 6 breaks in the multivariate regression. Before I discuss these results, it is useful to present the results based on the CUSUM and sequential Chow tests.

1.4.2.2. The CUSUM and sequential Chow tests. In this procedure, I begin with the CUSUM test. This test uses an initialization period to estimate the ADL using OLS, and then rolls the regression forward through the sample one observation at a time, computing a residual for each recursion. The CUSUM test is based on the cumulated sum of standardized recursive residuals:

\[ W_r = \frac{1}{\hat{\sigma}_w} \sum_{j=k+1}^{r} w_j \]  

(1.20)

where \( k \) is the number of regressors, \( w \) is the recursive residual, \( r = k + 1, \ldots, T \), and \( \hat{\sigma}_w \) denotes the estimated standard deviation of \( w \).

Under the null hypothesis of no structural break, the cumulative sum of scaled residuals – the CUSUM – is expected to behave like a driftless random walk. A driftless random walk has no tendency to move in one direction or the other. However, if there is a structural break, the CUSUM will begin to grow persistently in one direction from the point of the break. The confidence bands of the CUSUM test are thus based on the crossing probabilities of a random walk. If a structural break occurs, then the CUSUM will eventually move beyond a threshold, either above or below the zero line, that a random walk would not be expected to cross. However, the CUSUM test cannot pinpoint exactly where the
break point is. In order to determine the location of the break points, I make use of sequential Chow test.

A Chow test is an F test used to determine whether the coefficients in a regression model are the same in separate subsamples. For example, my entire sample period runs from 1871 to 2008. If I knew that there was a break in 1929, I could divide my sample into two subsamples: one from 1871 to 1929, call it subsample 1, and the other from 1930 to 2008, call it subsample 2. Recall the FS regression

\[ r_{t+1} = \alpha + \beta r_t + \varepsilon_{t+1} \]

Running the regression for both subsample 1 and subsample 2, generates two sets of estimates for \( \alpha \) and \( \beta \), denote them as \( a_1 \) and \( b_1 \) and \( a_2 \) and \( b_2 \), respectively. The Chow test is based on the following F statistic:

\[ F = \frac{(RSSR - SSR_1 - SSR_2)/k}{(SSR_1 + SSR_2)/(n-2k)}, \tag{1.21} \]

where RSSR is the sum of squared residuals from a regression in which the parameters are assumed to be the same across subsamples; SSR_1 is the sum of squared residuals from a linear regression for sample 1, and SSR_2 is the sum of squared residuals from a linear regression for sample 2. For the sequential Chow tests, the F-statistic is computed at each point in time in the subsample implied by the CUSUM test. The break point is determined where the F statistic is the most significant.

Figure 1.2 plots the CUSUM for the excess return when based on the first 10 years of my sample (1881:01-1890:12) for the multiple-variable regression
model. The dotted lines in the figure are the confidence bands for the test based on a significance level of 10 percent.\textsuperscript{20} As can be seen, the CUSUM moves away from the zero line and crosses the upper band in April 1887, which indicates that a point of structural break occurred prior to this date. Recall that under the null hypothesis of no structural break, the CUSUM is expected to behave like a driftless random walk. But we can see that the CUSUM grows persistently upward and does not behave like a driftless random walk. The crossing of the confidence bound indicates that the null hypothesis of no structural change can be rejected.

To determine the location of the break point, I make use of sequential Chow tests. As discussed above, the F statistic is calculated at each point in time during the subsample. This statistic is most significant at 1884:04, indicating break point at that date.

To test for additional break points in the data, I re-run the CUSUM test from the first break point (1884:04) with another 10-year time period, i. e., 1884:04-1894:03. Using the sequential Chow test, I find another break point at 1893:05. I repeat this procedure until the end of the sample.

My structural change analysis delivers 14 break points over the sample from 1881:01 to 2008:12.\textsuperscript{21} The specific locations of the break points are reported

\textsuperscript{20} Structural break tests have low power. To reduce the type II error, I use the 10 percent significance level for my tests.

\textsuperscript{21} The full sample runs from 1871:01-2008:12. But, I used a 10-year moving average to calculate the smoothed earnings-price ratio. Consequently, the actual sample used in the estimation runs from 1881:01 to 2008:12.
in Table 1.5. The same procedure is also applied to the FS regression, which delivers 18 break points. These results are given in Table 1.6.

1.4.2.3. The Structural Change Results. Both the BP test and the CUSUM test identified points of structural change in the data. The results, however, depend on the test used. Although both tests use the 10% level, the number of breaks identified from the BP test is far less than with the CUSUM test. For the FS regression, the BP test delivers 4 break points while the CUSUM test produces 18. For the multivariate regression, the number of breaks from the BP and CUSUM tests is 6 and 14, respectively. Clearly, the BP test is weaker than the CUSUM test in identifying break points.

In this study, I rely on the results of the CUSUM test. There are several noteworthy aspects of these results. In the multivariate regression, some of the break points line up with major changes in the U.S. business cycle. Table 1.9 reproduces the business cycle expansions and contractions in the U.S. from the NBER. The break points at 1929:08 and 1933:04 coincide exactly with the peak and trough points of the Great Depression. Other breaks that are proximate to the turning points of the U.S. business cycle include 1893:5, 1907:03, 1937:05 and 1973:11 for peaks and 1919:03, 1927:11 and 1937:03 for troughs.

Break points during the post-war era also appear to be related to major economic events. The break point in November 1973 corresponds to a peak in US economy. But, it is also proximate to the oil embargo imposed by OPEC, which occurred in October of that year. This worsened an already poor inflation outlook in the US.
The break point of October 1987 is when the so-called "Black Monday" occurred. US stock markets suffered their largest one-day fall after WWII, when the Dow Jones Industrial Average Index of shares in leading American companies dropped 22%. In July 1997, the Asian financial crisis raised fears of worldwide economic meltdown due to financial contagion. And most recently, the break point in October 2008 captures the financial crisis in US.

1.4.3. Does EH Hold with Structural Change?

Previous tests have shown that the expectations hypothesis does not hold. In the literature, most researchers either do not consider structural change in their empirical analysis, or they specify it a priori. My structural change results show that the predictive regressions are not time invariant. The question, then, is: does EH hold after accounting for structural change?

The results from the FS regression for all the sub-periods are reported in Table 1.7. If EH holds, then the estimates of the slope coefficient should be zero. From the table, we can see that the results are mixed. For some periods, notably, from late 1940s to mid 1970s the coefficients are statistically different from zero, which implies that EH does not hold for those periods.

For the periods before the 20th century and after the late 1980s, the coefficients are not statistically different from zero. But these results may be due to model misspecification. For example, from the late 1980s to the late 1990s, the multivariate regressions show that lagged smoothed earnings-price ratios are highly significant. The multivariate regressions show that in 9 out of the 14
subperiods, there is at least one predictive variable that is significant. The regression results are reported in table 1.8. These results supplement previous studies in the literature showing that EH fails even when taking into account of structural change.

1.4.4 Accounting for the Failure of the Expectations Hypothesis

The literature has explored two major explanations for the failure of EH: the presence of a time varying risk premium and irrationality of market participants. To see how a time-varying premium and irrationality can account for the failure, recall that if EH holds, then the slope coefficient in the following FS regression

\[ r_{i+1} = \alpha + \beta r_i^f + \epsilon_{i+1}, \]  

(1.22)

should equal one. Following the analysis of Froot and Frankel (1989), I write the probability limit of the slope coefficient as

\[ \beta = \frac{\text{cov}(\epsilon_{i+1}, r_i^f) + \text{cov}(\tilde{r}_{i+1}, \epsilon_{i+1})}{\text{var}(r_i^f)} \]  

(1.23)

where \( \epsilon_{i+1} \) is market participants' forecasting error and \( \tilde{r}_{i+1} \) is the market expectation of the stock return. From the definition of the market premium,

\[ pr_{i+1} = \tilde{r}_{i+1} - r_i^f \]  

(1.24)

we can write \( \beta \) as equal to 1 (the null hypothesis) minus a term arising from a failure of rational expectations, minus another term arising from the premium:

\[ \beta = 1 - h_{re} - h_{pr}, \]  

(1.25)

where
\[
b_{r_{1+1}} = \frac{\text{cov}(e_{1+1}, r_{1+1})}{\text{var}(r_{1+1})} \quad \text{and} \quad b_{pr} = \frac{\text{var}(pr) + \text{cov}(r_{1+1}, r_{1+1})}{\text{var}(r_{1+1})}.
\]

We can see that if $\beta$ is not equal to 1, it is either because of a systematic forecasting error (irrationality of market participants) or the existence of a market premium that is correlated with the risk free rate.

The literature has explored these two explanations. But, by far, most researchers have searched for a valid model of the market premium.

In the next section, I examine the key features of \textit{ex ante} excess returns that any model of the premium should be able to explain.

1.5 Key Features of the \textit{Ex Ante} Excess Return

How do we measure the \textit{ex ante} excess return on stocks? One possibility would be to survey market participants about their forecasts of future returns. Unfortunately, survey data on stock returns over monthly horizons is available for only limited time periods.\textsuperscript{22} An alternative approach is to use the fitted values from predictive regressions. This is the approach I follow in this dissertation. I use the fitted values from the multivariate predictive regressions in each of the sub-periods, as a measure of the \textit{ex ante} excess return.

\textsuperscript{22} Money Market Services International (MMSI) used to provide a survey for monthly prices of S&P 500 and DJIA. But the survey was discontinued and the data was very noisy. I have not found any study using monthly survey data on stock prices. There are some studies using longer frequency survey data like Livingston survey. Livingston survey used to provide forecasts for 6-month-ahead and 12-month-ahead prices of S&P 500 index. The series was discontinued in 1990 when the Philadelphia Fed took it over. Studies using Livingston Survey data on stock index include Gultekin (1983) and Dokko and Edelstein (1989).
1.5.1. The Key Features of the Ex Ante Excess Return.

Figure 1.3 plots my measure of the monthly ex ante excess return for the entire sample period, 1881-2008, where I concatenate the fitted values from the 14 subperiods identified by the CUSUM test. The graph shows that the excess return varies widely, between positive 20% and negative 15%. It is difficult to reconcile the consumption CAPM with this large variation because the supposed fundamental, the covariation between consumption growth and stock returns just does not vary enough.

There is another key feature of the ex ante excess return: there are extended periods of time in which it is largely negative and other periods of time in which it is largely positive. Table 1.10 reports the means and variances of \( \hat{e}_{i+1} \) for the entire sample and the 14 sub-periods. The mean for the whole sample is 4.67% while the mean for the separate sub-periods varies widely and undergoes sign reversals. For the sub-periods 1881:01-1884:04, 1893:06-1895:04 and 1929:09-1933:04, the means of the ex ante excess return are negative! For example, from 1929:09 to 1933:04, the mean is negative 31%. However, during 1927:08-1929:08, the mean was a positive 33%, and more recently, from 1987:08-1997:07, the mean was a positive 10%.

Sign reversals can be seen in figure 1.4, which plots estimates of the ex ante excess return for each sub period. For some sub periods, \( \hat{e}_{i+1} \) are largely negative and for other periods it is largely positive. For example, the figure shows that \( \hat{e}_{i+1} \) was largely negative from 1929:08 until 1939:03 and from 1933:04 to 1937:02, it was largely positive. In the most recent subperiod, 1997:07-2008:10,
the excess return is largely negative from 2001:01 to 2004:01 and largely positive from 2004:01 to 2007:12.

Another notable feature of the excess return series is that the mean has a tendency to decline after World War II, which is consistent with the findings in the literature. This can be seen clearly from Figure 1.5.

1.6 Conclusion

According to the pure expectations hypothesis, the ex post excess return should fluctuate randomly around zero. Empirical studies have shown that EH fails. But most of the studies do not consider the problem of structural change. This is a serious omission because in real-world markets, market participants revise their forecasting strategies, leading to structural change in predictive regressions. To test for structural change, I used a combination of the CUSUM test and recursive Chow test. These procedures enabled me to test the stability of predictive regressions without specifying break points a priori. I found much evidence of structural change. I also found that EH fails even when structural change is incorporated into the analysis.

The data showed that the ex ante excess return on stocks varies widely over time. It also displays sign reversals: extended time periods in which it is largely positive followed by periods in which it is largely negative. Any empirically relevant model of the excess return should be able to explain these two key features.

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In the next chapter, I examine the ability of the workhorse in the field, the canonical CCAPM and its modifications, to account for the pattern of sign reversals we observe in the data.
2.1 Introduction

In chapter 1, I presented evidence showing that the time series behavior of the excess return on stocks over bonds is inconsistent with the expectations hypothesis (EH). Far from fluctuating randomly around zero, the average excess return was roughly 5% over the period from January 1871 through December 2008. Moreover, there were a number of variables, including past returns and the price-earnings ratio that had predictive power for stock returns. Strikingly, I also found that the expected excess return experienced alternating periods of time in which it was largely positive or largely negative.

As chapter 1 emphasized, EH is a joint hypothesis that assumes risk neutral investors and the rational expectations hypothesis (REH). The vast majority of researchers have attempted to account for the failure of EH by replacing the assumption of risk neutrality with the assumption that individuals are risk averse. The workhorse in the field for modeling the decision making of risk-averse investors is the consumption Capital Asset Pricing Model (CAPM). According to this model, the times series behavior of the excess return on stocks is due to a time-varying risk premium.
In this chapter, I review the empirical record on the ability of the canonical version of the consumption CAPM, as well as its recent modifications, to account for the behavior of the expected excess return.

The consumption CAPM implies that the riskiness of investing in an asset whose return is uncertain depends on the covariance of this return with the growth rate of consumption. The utility function in the model implies that consumers would like to smooth their consumption over their lifetimes relative to their income stream. Consequently, if an asset tended to pay off badly during times when income also tended to be low, it would lead to greater variability of consumption. In this case, a consumer would be willing to buy the asset only if she expected a return in excess of the risk free rate, that is, if it she expected a risk premium. On the other hand, if the asset tended to pay off well when income tended to be low, it would provide a hedge against decreases in consumption. In that case, the consumer would be willing to buy the asset at a discount, that is, if its expected rate of return was lower than the risk free rate.

The risk premium that emerges from this class of models also depends on investors' degree of risk aversion. A higher degree of risk aversion implies that investors will need a higher premium to hold risky assets. In its canonical version, researchers have assumed that the relevant horizon over which consumption risk (that is, the covariance between the excess return and consumption growth) should be measured is equal to the horizon over which excess returns are measured. Typically, this horizon is short-term, such as monthly or quarterly.
In testing the empirical validity of the consumption CAPM, most researchers focus on accounting for the historical average of the excess return. Mehra and Prescott (1985) and many others have found that the average excess return during the past 100 years is much too high to be explained by the consumption CAPM with reasonable levels of risk aversion. This failure is referred to as the “equity premium puzzle.” Studies that attempt to explain the time variation of the excess return have also found that the consumption CAPM is grossly inconsistent with the data.\textsuperscript{24}

Some researchers have responded to this failure by maintaining the validity of the consumption CAPM, but arguing that consumption risk should be measured over the medium term, for example, from one to three years.\textsuperscript{25} Others have investigated the implications of replacing REH with a behavioral representation of "irrational" forecasting behavior.\textsuperscript{26} But, by far, the most popular response has been to search for alternative specifications of preferences. Researchers have developed specifications that involve habit formation\textsuperscript{27} and that mix risk-averse preferences with the assumption of loss aversion (Barberis, Huang and Santos, 2001; hereafter, BHS).

\textsuperscript{24} See, for example, Hansen and Singleton (1983), Campbell and Mankiw (1989), Hall (1988), and Campbell (2003).

\textsuperscript{25} See, for example, Parker (2001, 2003), Bansal and Yaron (2004), Parker and Julliard (2005), Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2005).


\textsuperscript{27} Studies that develop a standard “internal habit” model include Constantinides (1990) and Sundaresan (1989), whereas for those that involve an “external habit” model, see Abel (1990, 1999) and Campbell and Cochrane (1999).
Researchers report that these modifications of the canonical model do better empirically. Much of the empirical evidence is based on calibration exercises aimed at explaining the equity premium puzzle.

However, a different story emerges when the modifications are confronted directly with the time variation in the data. Although habit-persistence models seem to match in calibration exercises, researchers find that when based on regression analysis, they perform just as badly as their canonical counterpart (Duffee, 2005). Parker's (2003) regression analysis suggests that the model with medium-term consumption risk does better in accounting for the time path of the excess return, however, Duffee (2005), which also considers medium-term risk, finds that this modification provides no help. As far as I know, the loss aversion and non-REH modifications to the canonical model have not yet been directly confronted with time series evidence.

Moreover, researchers have not explored the ability of the canonical model or any of its modifications to account for the tendency of the expected excess return on stocks to undergo sign reversals. Chapter 3 fills this gap in the literature by applying the test of Mark and Wu (1998).

The remainder of chapter 2 is organized as follows. Section 2 reviews the canonical consumption CAPM and the empirical record on it. Section 3 sketches the popular preference modifications to the canonical model and the empirical evidence on them. Section 4 examines the incorporation of irrationality into the model. Section 5 offers concluding remarks.
2.2. A Sketch of the Canonical Consumption CAPM

The canonical version of the consumption CAPM was developed by Rubinstein (1976) and Breeden (1979). The model assumes that asset returns are determined by the decisions of a representative investor. The basic problem facing the representative investor at time $t$ is to choose how much of her lifetime income to save and to consume and in what assets her savings should be held in the current and all future time periods. Because consumption has diminishing marginal utility in any period, the investor wants to smooth consumption over her infinite lifetime. The first order conditions emerging from the representative investor's maximization problem are used to determine how risky and non-risky assets should be priced.

More formally, the representative investor is assumed to maximize the expected present discounted value of utility flows from consumption,

$$E_t \left[ \sum_{i=0}^{\infty} \beta^i U(C_i) \right], \quad 0 < \beta < 1. \quad (2.1)$$

subject to a sequence of budget constraints, where $\beta$ is a discount factor, $U(\bullet)$ is a utility function, and $C_i$ is real consumption. The mathematical expectation, $E_t (\bullet)$, is conditioned on the information available to the representative investor at time $t$ and her forecasting strategy. Current and past values of consumption and asset returns are assumed to be included in the information set. Also, the investor has unrestricted access to financial markets and faces no borrowing or short-sales constraints.
Suppose that the investor has the choice of investing in a collection of \( N \) assets all with a holding period equal to one period.\(^{28}\) Let \( Q_{it} \) denote the quantity of asset \( i \) held at the end of time \( t \), \( P_{it} \) the price of asset \( i \) at time \( t \), \( X_{it} \) the time \( t \) payoff from holding a unit of an asset purchased at time \( t-1 \), and \( W_t \) real income at time \( t \). The consumption and investment plans must satisfy the sequence of budget constraints

\[
C_t + \sum_{i=1}^{N} P_{it} Q_{it} \leq \sum_{i=1}^{N} X_{it} Q_{it-1} + W_t.
\]  

(2.2)

The budget constraint says that the sum of the current consumption and investment should not be greater than the sum of current income and investment payoffs from previous periods.

The first order condition for maximization is,

\[
P_{it} U'(C_t) = \beta E_t [X_{it+1} U'(C_{t+1})].
\]  

(2.3)

This condition applies to all the assets that the investor holds. For example, suppose that the \( i \)th asset is a stock, which if held at time \( t \), pays a dividend per share of \( D_{it+1} \). For this asset, the one-period payoff would be \( X_{it+1} = (P_{it+1} + D_{it+1}) \) and (2.3) becomes

\[
P_{it} U'(C_t) = \beta E_t [(P_{it+1} + D_{it+1}) U'(C_{t+1})]
\]  

(2.4)

If we divide both sides of equation (2.4) by \( P_{it} \), we have

\(^{28}\) Hansen and Singleton (1982) consider \( N \) assets with different maturities. The simplification of the same holding period does not change the implications of the model.
\[ U'(C_i) = \beta E_t[(1 + R_{i,t+1})U'(C_{t+1})] \] (2.5)

where \(1 + R_{i,t+1} = (P_{i,t+1} + D_{i,t+1})/P_{i,t}\).

Alternatively, suppose the \(i\) th asset is a risk free asset, which if held at time \(t\), delivers a sure payoff of \(FV^i_{t+1}\) at time \(t+1\). In this case, the one-period payoff would be \(X_{i,t+1} = FV^i_{t+1}\) and (2.3) becomes

\[ P_{i,t}U'(C_i) = \beta E_t[FV^i_{t+1}U'(C_{t+1})] \] (2.6)

Again, if we divide both sides of the equation by \(P_{i,t}\), we have

\[ U'(C_i) = \beta E_t[(1 + R^f_{i,t+1})U'(C_{t+1})] \] (2.7)

where \((1 + R^f_{i,t+1}) = FV^f_{t+1} / P_{i,t}\).

These first-order conditions have a straightforward interpretation. The left-hand side gives the consumption lost if the investor gives up one real dollar of consumption at time \(t\) by investing it in asset \(i\) or \(j\). In equilibrium, this must equal the expected discounted marginal utility from selling asset \(i\) or \(j\) at time \(t+1\) and consuming the proceeds.

To see the implications of the model for determining the return on stocks relative to the risk free rate, assume we have just two assets, a stock and a risk free bond whose return at \(t+1\) is known at \(t\). If we divide the Euler equation for the stock, which is given in equation (2.5) by \(U'(C_i)\), we obtain:

\[ 1 = E_t\left[(1 + R_{i,t+1})\beta \frac{U'(C_{t+1})}{U'(C_i)}\right] = E_t[(1 + R_{i,t+1})M_{t+1}] \] (2.8)
where $M_{t+1} = \beta U'(C_{t+1})/U'(C_t)$ is the investor’s intertemporal marginal rate of substitution. $M_{t+1}$ is the rate at which the investor is ready to give up consumption at time $t$ in exchange for consumption at $t+1$ and is known as the stochastic discount factor (SDF hereafter). The right-hand side of equation (2.8) can in turn be written as:

$$E_t[(1 + R_{t+1})M_{t+1}] = Cov[R_{t+1}, M_{t+1}] + E_t[(1 + R_{t+1})E_t[M_{t+1}]]$$

(2.9)

Substituting this back into equation (2.8) and rearranging yields,

$$1 + E_t[R_{t+1}] = \frac{1 - Cov_t[R_{t+1}, M_{t+1}]}{E_t[M_{t+1}]}$$

(2.10)

Equation (2.10) implies that in equilibrium, the stock’s price will move so that the expected return on the stock increases as the covariance between its return and the SDF falls. If the stock pays off well when consumption is low and marginal utility is high—the covariance is positive—stocks provides a hedge against consumption risk. Therefore the investor will ask for a lower return to hold the stock. On the other hand, if the stock pays off badly when consumption is low and marginal utility is high—the covariance is negative—the stock then provides no hedge against the consumption risk. For the investor to hold the stock, she will ask for a higher return.

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29 We have used the definition of a covariance, $Cov(X, Y) = E(XY) - E(X)E(Y)$, which implies $E(XY) = Cov(X, Y) + E(X)E(Y)$.

30 To see this, note that a stock’s gross return from time $t$ to $t+1$ can be written as $1 + R_{t+1} = (P_{t+1} + D_{t+1})/P_t$. Mathematically, when the covariance decreases, the equilibrium gross return needs to increase, which implies a higher price.
Equation (2.10) holds for any asset. A risk free asset is one whose payoff is certain in the future. Thus, its return does not correlate with consumption. Consequently, the covariance between the risk free rate and the discount factor is zero. This gives the following equilibrium gross rate of return\(^3\) for the risk free asset:

\[
1 + R_{t+1}^f = \frac{1}{E[M_{t+1}]} \tag{2.11}
\]

This equation says that the gross rate of return for a risk free asset is equal to the reciprocal of the expectation of SDF. Equation (2.11) comes directly from the first order condition,

\[
U'(C_t) = \beta E_t[(1 + R_{t+1}^f)U'(C_{t+1})]
\]

The left-hand side gives the consumption lost if the investor gives up one real dollar of consumption at time \(t\) by investing it in the risk free asset. In equilibrium, this must equal the expected discounted marginal utility from selling the asset at time \(t + 1\) and consuming the proceeds.

If we now subtract equation (2.11) from equation (2.10), we obtain an expression for the expected "risk premium" implied by the consumption CAPM:

\[
E_t[R_{t+1}] - R_{t+1}^f = -\frac{\text{Cov}_t[R_{t+1}, M_{t+1}]}{E_t[M_{t+1}]} \tag{2.12}
\]

\(^3\) The gross rate of return for a risk free asset is \((1 + R_{t+1}^f) = FV_{t+1}^f / P_t\), where \(FV_{t+1}^f\) is its future payoff at time \(t + 1\). For example, consider a simple loan and assume that the principle is $100, and annual interest rate is 10%. Then the next year the payoff of the simple loan will be $110 and the gross rate of return for the simple loan will be 110%.
As \( 1 + R_{t+1}^f = \frac{1}{E[M_{t+1}]} \), we can rewrite the premium as:

\[
E_t[R_{t+1}] - R_{t+1}^f = -(1 + R_{t+1}^f)Cov_t[R_{t+1}, M_{t+1}]
\]

(2.13)

The risk premium depends on the product of the gross risk free rate and the covariance between an stock's return and the SDF.

In order to interpret equation (2.13), it is useful to assume a particular utility specification. The canonical version of the model\(^{32}\) assumes that utility is time separable:

\[
U(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma},
\]

(2.14)

where \( \gamma \) is a constant coefficient of relative risk aversion. This power utility function has two desirable features. First, it is scale-invariant, so that the risk premium does not change over time as aggregate wealth and the scale of the economy changes. Second, if the same utility function is attributed to all investors, then even with different endowments, individuals' utility functions can be aggregated into a single representative utility function.

Hansen and Singleton (1983) assume that the joint distribution of stock returns and consumption growth is lognormal and conditional on information

available in period $t$. With this assumption, the excess return on stocks can be written as:

$$E_t[r_{t+1} - r_{f,t+1}] = e_{t+1} = \gamma \text{cov}_t(\Delta \ln C_{t+1}, r_{t+1}).$$  \hspace{1cm} (2.15)

Because the risk free rate is non-stochastic, the expression for the excess return can also be written as:

$$e_{t+1} = \gamma \text{cov}_t(\Delta \ln C_{t+1}, er_{t+1}).$$  \hspace{1cm} (2.16)

The equation shows that the equilibrium ex ante premium that emerges from the canonical CAPM depends on the degree of risk aversion, $\gamma$, and the covariance between consumption growth and the excess return on stocks. The sign of the premium is determined by this covariance. If $\text{cov}_t(\Delta \ln C_{t+1}, er_{t+1})$ is positive, so that stocks tend to pay off badly when consumption is low, the asset will increase consumption risk and the representative investor would hold stocks only if she expects a positive risk premium. If, however, the covariance term is negative, the asset provides a hedge against consumption risk. In this case, the investor would be willing to buy stocks at a discount, that is, with a negative risk premium.

The empirical literature on the canonical consumption CAPM shows that the model fails along several dimensions.

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33 The expression in equation (2.15) follows much of the literature and assumes that we are computing the arithmetic average of the excess return on the left-hand side instead of the geometric average. If we were to use the geometric average, we would need to add $\frac{1}{2} \text{var}_t(r_{t+1})$ to the left-hand side of the equation.

34 Both of the expressions are used in the literature. For example, Mehra and Prescott (1985) use the covariance between the stock return and consumption growth whereas Parker (2001) uses the covariance between the excess return and consumption growth. I use the covariance between the excess return and consumption growth in my study.
2.3. Empirical Failure of the Canonical Model

2.3.1. The Equity Premium Puzzle

To estimate the model, one needs an estimate of consumption growth. The literature typically uses the growth rate of per capita aggregate consumption. Mehra and Prescott (1985) report that the historical average return on equities has exceeded the average return on risk free short-term Treasury securities.\footnote{Treasuries are risk free in the sense that investors will obtain a sure nominal return if they hold these securities to maturity. However, if real returns are considered, treasury securities are subject to inflation risk. But stocks are also subject to inflation risk. Thus, in the expression for the equity premium, which involves the difference between stock return and the Treasury bill rate, the inflation rates will cancel out. So the nominal equity premium is equal to the real equity premium.} Using a sample that runs from 1889 through 1978, they find that the average real yield on the Standard and Poor 500 Index was 7% per annum, while the average yield on short-term Treasury debt was less than 1% per annum. As such, they find an average equity premium over the 90 years equal to 6% per annum.

Mehra and Prescott (1985) examine whether the consumption CAPM of the preceding section can account for this premium. In their calibration exercise, they model equilibrium consumption growth rates so that the mean, variance, and serial correlation of the simulated series match what is observed for the U.S. economy over the 1889-1978 period. To this end, they assume that the consumption growth rate follows a Markov process. Their calibration exercise makes use of U.S. aggregate consumption data from the National Income and Product Accounts (NIPA) and measures consumption growth as the average growth rate of per capita aggregate consumption spending. They choose...
parameters in the Markov process to match the moments of the real consumption growth rate data. Table 2.1 provides the sample statistics from Mehra and Prescott (1985).

To pin down the degree of relative risk aversion, $\gamma$, Mehra and Prescott (1985) appeal to the literature. A number of studies show that this parameter lies between 0 and 10. To put these numbers in perspective, a risk aversion coefficient equal to 10 implies that an individual would be willing to choose a 19 percent sure decline in consumption over a gamble in which she would either win or lose 25% of her consumption. But, even when they use the maximum value of 10, Mehra and Prescott obtain a risk premium of only 1.41%.

The problem with the model is that the equity premium is tied to the growth rate of consumption, but consumption just does not co-vary with returns in the market! Indeed, relative to the variation of stock returns, consumption growth is a flat line. This means that stocks neither improve nor worsen an investor’s ability to lower consumption risk. Consequently, at plausible levels of risk aversion, the investor is roughly indifferent to holding stocks or the risk-free asset in her portfolio.

Of course, there exists a degree of risk aversion, $\gamma$, that is large enough to imply a risk premium equal to the 6 percent observed in the data. Mehra and Prescott find that a $\gamma=48$ delivers such a premium. However, a $\gamma=48$ is much too high to be even remotely plausible. The inability to account for the excess

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return on stocks with a plausible degree of risk aversion is referred to as the “equity premium puzzle.”

2.3.2 Tests of the Euler Equations

Researchers have confronted the consumption CAPM with the time variation in the data by estimating the Euler equations in (2.8) using Generalized Method of Moments (GMM). The basic idea underlying the estimation strategy is as follows. The Euler equations of the model imply a set of population orthogonality conditions:

$$E[(1 + R_{i,t+1}) \beta \frac{U'(C_{it+1})}{U'(C_i)} | I_t, -1] = 0.$$  (2.17)

where $\beta$ is the discount factor. This implies that, in equilibrium, the value of an asset, either a stock or bond, is its discounted future payoff weighted by the trade-off between future and present consumption, conditioned on the public information set at time $t$, $I_t$.

The left side of this expression can be thought of as an error term $u_{t+1}$ that should have a conditional mean 0, given the information set at time $t$, under REH. This defines a set of orthogonality conditions $E(u_{t+1} | Z_t) = 0$. If $Z_t$ is any subset of the variables in the current information set, these orthogonality conditions can be exploited to estimate the parameters of the model. Also, more orthogonality conditions are typically available for use in estimation than there are parameters to be estimated and, in this sense, the model is "overidentified."
overidentifying restrictions can be tested using a procedure called the Hansen's J test, which uses a chi-square statistic to examine how close sample versions of population orthogonality conditions are to zero.

Econometricians have observations on returns from time $t$ to $t+1$ for a subset of the assets implied by theory, and $C_t$. The parameters $\gamma$ and $\beta$ are unknown parameters to the econometricians. In Hansen and Singleton's setup, both the asset returns and consumption are assumed to be jointly determined in equilibrium and treated as endogenous. Hansen and Singleton (1982, 1984) then use 1, 2, 4 and 6 lags of $R_{t+1}$ and $\left(\frac{C_{t+1}}{C_t}\right)$ as instrumental variables. Using three sets of stock returns for the period February 1959 through December 1978, they test the Euler equations of the canonical consumption CAPM and report small values of $\gamma$ and a discount factor close to one. The standard errors on the risk aversion coefficients are very large, implying that the null of $\gamma=0$ cannot be rejected. A zero gamma suggests that the investor is not risk averse but risk

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37 In Hansen and Singleton's setup, investor can choose among N assets with different maturities. But for econometricians, they only observe a subset of all these assets.

38 The three sets of stock returns are: the equally-weighted average return on all stocks listed on the New York Stock Exchange (NYSE), the value-weighted average of returns on the NYSE (stocks are weighted by their market capitalization at the end of the previous period), and equally-weighted average returns on the stocks of three two-digit SEC industries. The industries chosen were chemicals, transportation and equipment, and other retail trade.
neutral. The estimates of $\gamma$ are also not stable across specifications. Moreover, their results of J-tests decisively reject the model.

Empirical studies have reported a sharp difference in the estimate of $\gamma$ depending whether or not conditional information is used. If unconditional moments are used, $\gamma$ tends to be large. Conversely, if conditional moments are used, $\gamma$ tends to be small. Hansen and Singleton's (1983) Table 5, reproduced here in Table 2.2, makes the story clear. In their 1983 paper, they use both unconditional and conditional information to estimate the preference parameters. They use the quarterly value-weighted average return of NYSE stocks and the treasury-bill rate from Ibbotson and Sinquefield (1979) over the period 1954:4-1978:4. Without instruments (lags), that is, using only unconditional moments, they report estimates of the coefficient of risk aversion that lie between 30 and 60 when the number of lags of consumption growth is zero. If instruments (lag terms) are used, a small estimate of $\gamma$ is obtained but again the chi-square statistic is huge, rejecting the orthogonality conditions of the model. Grossman, Melino and Shiller (1987) report similar results for the period of 1890-1981. Using six data sets, Grossman et al. report that when unconditional moments are

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39 Cochrane (2007) suggests that the problem is that Hansen and Singleton's instruments don't forecast either consumption growth or returns very well.

used, the estimated $\gamma$ is smaller than the estimate when conditional moments are used. The orthogonality conditions are also rejected. More recently, Lettau and Ludvigson (2009) use the quarterly value-weighted price index return from Center for Research in Stock Prices (CRSP), three-month Treasury-bill rate and six size and book-market sorted portfolio returns to estimate the Euler equations. Using only the unconditional moments, Lettau and Ludvigson report that $\beta$ is about 1.4 and $\gamma$ is implausibly high, between 87 to 90 for the period of 1951 through 2002.

The overall conclusion from tests based on the Euler equations is that the canonical CAPM is inconsistent with the time variation on the excess return on stocks. Other tests that confront the canonical model with the time variation in the data deliver the same conclusion.

2.3.3. Variance Bounds Tests

Another approach to confronting the consumption CAPM to the time variation in the data is provided by Hansen and Jagannathan (HJ, 1991). This approach exploits the implied properties of the stochastic discount factor, $M_{t+1}$. They show that the standard error of the SDF has a lower bound that is related to the Sharpe ratio. The Sharpe ratio is given by

$$\frac{\hat{R}_{t+1} - R'_{t+1}}{\sigma}$$  \hspace{1cm} (2.18)

\footnote{The data are available from Kenneth French's Dartmouth web site.}
Where $\hat{R}_{t+1}$ is expected return on stocks, $R_{t+1}^f$ is the return on risk free asset, and $\sigma$ is the standard deviation of the stock return. The Sharpe ratio is a reward-to-variability ratio that measures the excess return per unit of risk in the stock. Risk here is measured by the variance of the asset return. In finance, the fundamental idea of the selection of portfolios is based on the means and variances of their returns: we would select the portfolio that has the highest expected return given a certain risk; or we would select the portfolio that has the lowest risk given a certain expected return. The Sharpe ratio provides a simple way to compare the risk and return across different stocks or portfolios. It also serves as the lower bound on the standard error of SDF. To see this, consider equation (2.12),

$$\hat{R}_{t+1} - R_{t+1}^f = \frac{-\text{Cov}_t[R_{t+1}, M_{t+1}]}{E_t[M_{t+1}]} = -R_{t+1}^f \text{Cov}_t[R_{t+1}, M_{t+1}].$$

From the definition of the coefficient of correlation, $\rho_{xy} = \frac{\text{cov}(X, Y)}{\sigma_x \sigma_y}$, the covariance can be written as

$$\text{Cov}_t[R_{t+1}, M_{t+1}] = \sigma_t \sigma_m \rho_{im},$$

where $\sigma_t$ is the standard deviation of the asset return, $\sigma_m$ is the standard deviation of the stochastic discount factor, and $\rho_{im}$ is the correlation between the asset return and the stochastic discount factor. Since $|\rho_{im}| \leq 1$, we have $|\text{Cov}_t[R_{t+1}, M_{t+1}]| \leq \sigma_t \sigma_m$. Substituting this into the above equation, gives
\[
\frac{\hat{R}_{i,t+1} - R_{i,t+1}^f}{\sigma_i} \leq \frac{\sigma_m}{E_t[M_{t+1}]} \tag{2.19}
\]

where \(\frac{\hat{R}_{i,t+1} - R_{i,t+1}^f}{\sigma_i}\) is the Sharpe ratio. This inequality was first derived by Shiller (1982) and HJ derive a multi-asset version of it. It says that the Sharpe ratio is bounded by the volatility of the SDF, or alternatively, the Sharpe ratio is the lower bound of the volatility of the SDF.

We can reexamine the equity premium puzzle using this inequality. With a power utility function such as Mehra and Prescott use in their analysis, we have \(\sigma_m = \sigma\left(\frac{U_{i,t+1}'}{U_i'}\right) = \gamma \sigma\left(\frac{C_{t+1}}{C_i} \right)\). Substituting \(\sigma_m\) and \(E_t[M_{t+1}] = \frac{1}{1+R_{i,t+1}^f}\) into (2.19), gives

\[
\frac{\hat{R}_{i,t+1} - R_{i,t+1}^f}{\sigma_i} \leq \frac{\sigma_m}{E_t[M_{t+1}]} \approx \gamma \sigma\left(\frac{C_{t+1}}{C_i} \right) \tag{2.20}
\]

This inequality shows that to get a high Sharpe ratio, either the coefficient of risk aversion is high and/or consumption is volatile. In Mehra and Prescott’s analysis, the observed Sharpe ratio of stock market indices is about 1.09, while the volatility of consumption is about 3.37%. In order for the condition to hold, we have to assume unrealistically a high level of risk aversion of at least at 32.
2.3.4 Other tests

Mankiw and Shapiro (1986) examine whether the consumption CAPM provides an empirically more useful framework for cross-sectional stock returns than the traditional CAPM. The traditional CAPM relates the return of stock \( i \) to the risk-free rate \( R_f \) and the market return \( R_M \).

\[
R_{i,t+1} = R_f + [R_{t+1} - R_f] \beta_{Mi}
\]  

where \( \beta_{Mi} \) is the "market beta", a measure of systematic risk of stock \( i \).

As in the traditional model, the consumption CAPM relates the stock return to its systematic risk, but now measured by the covariance between its return and consumption growth. This covariance term is the "consumption beta."

Mankiw and Shapiro regress the average returns on the 464 surviving NYSE stocks over the period 1959-1982 on their market \( \beta \) and consumption betas, separately, as well as on both betas to explain the cross section of average returns. They report that the coefficient of market \( \beta \) is always far larger and far more significant than is the coefficient on the consumption beta. This result suggests that the market beta is a better measure of systematic risk for individual stocks and that the traditional CAPM provides a more empirically useful framework than the consumption CAPM for cross-sectional stock returns.

Cochrane (1996) also shows that the traditional CAPM substantially outperforms the canonical consumption-based model in pricing-size portfolios.\(^{42}\)

\(^{42}\) Cochrane uses the 10 portfolios of NYSE stocks sorted by market value (size) maintained by the Center for Research in Security Prices (CRSP).
sorted by market value maintained by CRSP. For example, CAPM's root mean square pricing error is 0.094 percent per quarter, while consumption CAPM's is 0.54 percent per quarter.

By contrast, Duffee (2005) tests the canonical consumption CAPM by relating the expected excess return directly to the conditional covariance. Appealing to equation (2.15), Duffee estimates the following regression:

\[ r_{t, t+1} - r_{t+1} + \frac{1}{2} \text{var}(r_{t, t+1}) = b_0 + b_1 \text{cov}(\Delta \ln C_{t+1}, r_{t, t+1}) + e_t. \]  

(2.22)

To do so, he first estimates the conditional covariance then regresses the excess return on the conditional covariance. Using monthly data on the excess return on CRSP stocks over the one-month T-bill from January 1959 to December 2001, Duffee reports that the contemporaneous monthly covariance is about $3.1 \times 10^{-5}$, which implies a coefficient of relative risk aversion of 160.

### 2.4. Do Modifications to the Canonical Model Perform Better?

The failure of the canonical model to explain the historical average of the excess return on stocks, as well as its time variation, has led economists to make modifications to the model. In this section, I review the four most popular modifications.
2.4.1 Medium-Term Risk and Limited Participation in the Stock Market

Parker (2001) contends that consumption risk is better measured over the medium term, which he defines as a period of one to three years. He argues that consumption responds with a lag to changes in wealth.\footnote{The slow adjustment of consumption to changes in wealth has been documented in the literature. See for example, Flavin (1981) and Hall and Mishkin (1982). Researchers also study the consumption response to the stock market. The results are rather mixed. See Parker (1999), Ludvigson and Steindel (1999), and Dynan and Maki (2001).}

Using quarterly data from 1959 to 2001, Parker shows that the covariance between the excess return on stocks and consumption growth at the longer horizons increases considerably. This, in turn, implies a lower estimate of the degree of risk aversion relative to the estimate based on contemporaneous consumption risk. However, despite this improvement, Parker’s estimate of $\gamma$ continues to be implausibly high. He still needs a $\gamma$ of roughly 40 to rationalize the high historical average excess return.

Beyond medium-term consumption risk, Parker also argues that the analysis should be confined to only those households that hold equity. To this end, he uses quarterly survey data from the Consumer Expenditure Survey (CEX) from 1980 to 1998. With these data, he estimates the coefficient of risk aversion to be between 10 and 20. Although much better, the estimate of $\gamma$ is still too high. Parker argues that during 1980-1998 period, the booming stock market leads to lower estimates of the covariance of consumption growth, which biases the estimation of $\gamma$. Once adjustment is made for this bias, the estimated...
coefficients\textsuperscript{44} of risk aversion lie between 4 and 8. Parker concludes that taking into account medium-term risk and limited participation in the stock market “leaves almost no equity premium puzzle.”

However, Duffee (2005) also considers medium-term risk and finds that this modification provides absolutely no help. He runs a regression similar to (2.22) and reports that the 4-month covariance is about $6.7 \times 10^{-5}$, which implies a coefficient of relative risk aversion of 75!\textsuperscript{45}

\subsection*{2.4.2 Habit Persistence}

There is much research exploring the implications of habit persistence or habit formation in the consumption CAPM. The idea is that people may get used to a certain standard of living (habits in consumption) after several good years, and a fall in consumption would hurt even though the level of the consumption itself is not bad at all. The central ingredient of the model is a slow-moving habit in consumption, added to the basic power utility function.

Habit persistence has been proposed in financial economics as a possible solution to the equity premium puzzle. Recall the equity premium puzzle is that, under the assumption of power utility, the observed excess return of stocks over the risk free asset is too high to be consistent with actual consumption behavior

\begin{footnotesize}
\begin{itemize}
\item \textsuperscript{44} Parker considers both conditional and unconditional estimates of the parameter with medium term consumption risk measured over 1-3 years.
\item \textsuperscript{45} But Duffee (2005) did not examine if limited participation in the stock market in addition to the medium-term risk would help.
\end{itemize}
\end{footnotesize}
unless households are assumed to be extremely risk averse. At the heart of the equity premium puzzle lies the low volatility of observed consumption growth.

To see why habit persistence has the potential to solve the equity premium puzzle, consider a habit persistence model by Campbell and Cochrane (1999). Following Abel (1990), Campbell and Cochrane propose an external, or "keep up with the Joneses" form of habit formation in which habits depend on the aggregate consumption rather than individual consumption. The external form of habit persistence simplifies the optimization problem of the consumer because the evolution of the habit is taken as exogenous by the individual.

Campbell and Cochrane replace the utility function $U(C_t)$ with $U(C_t - X_t)$ where $X_t$ denotes the level of habits:

$$U(C_t - X_t) = \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma}.$$  \hspace{1cm} (2.23)

The utility function is only defined when consumption exceeds habit.\footnote{In other habit persistence models, including those of Sundaresan (1989) Ferson and Constantinides (1991), Heaton (1995), and Chapman (1998), consumption can fall below habit.} They specify the functional form and parameters so that the risk-free rate is constant. They argue that the risk-free rate in the U.S. has limited variation. Keeping the risk free rate constant helps to show how the model can explain stock market behavior entirely by the variation in the risk premium. According to Campbell and
Cochrane, habits move slowly in response to consumption. A simple way to capture this is to assume that \( X_t \) follows an AR (1),

\[
X_t = \rho X_{t-1} + \lambda C_t. \tag{2.24}
\]

Just like consumers in the canonical consumption CAPM, who do not like variation in their consumption, \( C_t \), habit-forming consumers dislike variations in habit-adjusted consumption, \((C_t - X_t)\), rather than variations in consumption itself. A given percentage change in consumption produces a much larger percentage change in habit-adjusted consumption than in consumption itself. The idea is that small fluctuations in consumption growth can generate large variations in habit-adjusted consumption growth and hence explain sizable excess returns on risky assets even for moderate values of the degree of risk aversion.

In Campbell and Cochrane's model, they make use of the surplus consumption ratio, \( S_t \), to capture the relationship between consumption and habit. \( S_t \) is defined by

\[
S_t = \frac{C_t - X_t}{C_t}. \tag{2.25}
\]

By design, the habit is smaller than consumption. Therefore \( 0 < S_t < 1 \). The surplus consumption ratio is the fraction of consumption that exceeds habit. When consumption declines relative to the "habits" in a recession, that is, when

\(^{47}\)Campbell and Cochrane specify a non-linear version of (2.28). According to them, the non-linear specification is crucial for capturing the time variation on the stock returns and a constant risk-free rate.
When the consumption to habit ratio $S_t$ approaches zero, people will become more risk averse and demand a higher excess return in holding a risky asset like stocks. When consumption rises relative to the "habits" in an expansion, that is, when $S_t$ approaches one, people will become less risk averse and demand a lower excess return in holding a risky assets. Therefore the habit persistence model also provides variation in the coefficient of risk aversion.

To specify how the habit evolves over time in response to aggregate consumption, Campbell and Cochrane suggest an AR(1) model for the log surplus consumption ratio, $s_t = \log(S_t)$:

\[
s_{t+1} = (1 - \varphi) s_t + \varphi \xi_t + \lambda(s_t) \varepsilon_{c,t+1},
\]

where $\varepsilon_{c,t+1}$ is innovations in consumption growth. Consumption growth is modeled as an i. i. d process. This specification shows that today's habit is a complex nonlinear function of current and past consumption.

The expected excess return from Campbell and Cochrane's model can be written as:

\[
e_{r,t+1} = \eta_t \text{cov}(\Delta \ln C_{t+1}, e_{r,t+1})
\]

where $\eta_t = \gamma / S_t$, capturing the time variation in investor's degree of risk aversion.

To test this model, Campbell and Cochrane use post-war data on the value-weighted stock indexes from CRSP (1947-1995) and on the S&P 500 stock index (1871-1993), they show that the consumption CAPM with habit persistence can account for the risk free rate and the mean and standard deviation of stock
returns in calibration exercises. It can also generate a time varying risk premium. However, their estimation of the degree of risk aversion is high: about 80 at the steady state. It is above 100 when the surplus consumption ratio is low and is 60 when the surplus consumption ratio is at its maximum. Lettau and Ludvigson (2009) use post war CRSP data (1951-2002) to test the Euler equations of the habit persistence model. They also report an implausibly high estimate of $\gamma$ equal to 57.48.

Duffee (2005) tests of the habit persistence model estimates the following regression:

$$r_{t+1} - r_{t} = b_0 + (b_1 + b_2 \hat{s}_{t-1}) \sigma_y \rho_y \sigma_y \sigma_{C_t} + \epsilon_t$$

(2.28)

where $\hat{s}$ is a proxy for the surplus consumption. Duffee uses Wachter (2002)'s measure of surplus consumption as the proxy.\(^{48}\) The base model is the canonical model when $b_2 = 0$. His regression results show that the estimate of $b_2$ is not significantly different from zero, which means that the habit persistence does not provide help in explaining the excess return.

\(^{48}\) Wachter (2002) uses 10-year weighted moving average of consumption growth as the proxy of the surplus consumption ratio.
2.4.3 Loss Aversion/Narrow Framing

Another line of research that attempts to solve the equity premium puzzle incorporates loss aversion and narrow framing. An important research along this line is by Barberis, Huang and Santos (BHS, 2001). BHS modifies investors’ preferences by assuming that the representative agent derives direct utility not only from consumption (just like the canonical model) but also from changes in the value of her financial wealth. Getting utility directly from the outcome of one gamble (e.g. investment in a risky asset) is called narrow framing. It describes the phenomenon that when people are offered a new gamble, they sometimes evaluate it in isolation, not combining it with other risks they are facing to evaluate their overall wealth risk. People with narrow framing get utility directly from the outcome of the gamble itself, not from the gamble’s contribution to their consumption.

In BHS’s model, the representative investor chooses a consumption level $C_t$ and investment in the risky asset $S_t$ to maximize the following utility function:

$$E_0 \left[ \sum_{t=0}^{\infty} \left( \beta^t \frac{C_{t+1}^{1-\gamma} + b_t \beta^t v(X_{t+1}, S_t)}{1-\gamma} \right) \right]$$ (2.29)

Where $\beta$ is a subjective discount factor, $b_t$ is an exogenous scaling factor. The first term in this specification involving consumption is the same as that of

---

49 Barberis and Huang (2007) provide a review on this line of research. I will focus on the model of Barberis, Huang and Santos (2001) as it keeps the power utility function from the canonical consumption CAPM and adds the elements of loss aversion and narrow framing.
canonical consumption CAPM. The second term represents utility from fluctuations in the value of financial wealth.

BHS make use of narrow framing and assume that the representative investor also gets direct utility from changes in the value of financial wealth from time $t$ to $t+1$, which is denoted as $X_{t+1}$. Positive realizations of $X_{t+1}$ imply gains, while negative realizations imply losses. In addition to the investor’s aversion to consumption risk, BHS assume that the investor is much more sensitive to potential losses to her financial wealth than to gains of equal magnitude, a feature known as “loss aversion.”

BHS show that “loss aversion” and narrow framing help in accounting for the equity premium puzzle, but they are not enough to explain the entire puzzle. This leads them to assume that the investor’s degree of loss aversion varies with her prior investment performance. Gains from last period’s investment provide a cushion for the next period, and so the representative investor is assumed to become less loss averse. Conversely, prior losses are assumed to increase the investor’s degree of loss aversion. The idea that prior outcomes may affect subsequent risk-taking behavior is supported by the psychology literature. For example, Thaler and Johnson (1990) find that when faced with sequential gambles, people are more willing to take on risk if they made money on prior gambles, than if they lost. They interpret these findings as revealing that losses are less painful to people if they occur after prior gains, and more painful if they occur before prior gains.

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50 Loss aversion is one of the key elements of the “Prospect Theory” of Kahneman and Tversky (1979) and Tversky and Kahneman (1992).
follow prior losses. This result that risk aversion goes down after prior gains is called the “house money effect.”

To introduce a house money effect in the utility specification, BHS introduce a new variable to catch the loss aversion dynamics, denoted by $z_t$:

\[
E \left[ \sum_{t=0}^{\infty} \left( \beta^t \frac{C_{i+1}}{1 - \gamma} + b_t \beta^{t+1} v(X_{t+1}, S_t, z_t) \right) \right].
\]  

The changing degree of loss aversion generates more volatility and a higher excess return in the model. In their calibration exercises, BHS use CRSP data from 1926-1995 on NYSE stock and show that the change in the degree of loss aversion does lead to higher volatility in stock returns and a higher excess return. Without the house money effect, loss aversion alone generates a standard deviation of the stock return of 12 and an excess return of 2.88%; while with the house money effect the standard deviation of the stock return is above 20 and an excess return of 5.88%.

Kahneman and Tversky find in their experiments that the degree of loss aversion is roughly 2.25. However, to generate an average equity premium as high as that observed in the data, BHS must assume in their calibration exercises an average loss aversion of 3.5. This is still “not a small level of risk aversion” (BHS, p39). Consequently, BHS’s model with loss aversion leaves some of the equity puzzle unexplained.
2.4.4 Irrationality

The foregoing modifications to the consumption CAPM maintain REH in representing individuals' forecasting behavior. That is, market participants rely on one forecasting strategy, the one that is consistent with the economist's own aggregate model. Some researchers have explored replacing REH with a specification of irrational forecasting behavior. These studies assume that investors have distorted beliefs about the behavior of dividends or consumption growth.

In chapter 3, I consider a model of irrational forecasting behavior due to Cecchetti, Lam, and Mark (2000). Here I discuss Hansen, Sargent, and Tallarini (1999, HST hereafter), who use a permanent income framework to study consumption behavior and the price of risk. They propose a model in which a representative consumer with habit formation and facing a linear production technology and an exogenous endowment process, has a preference for "robustness." By robustness, HST mean that although decision makers share a common probabilistic specification of the income shocks with expected utility maximizers, they suspect specification errors and want decisions to be insensitive to them. This robustness preference allows for a concern about model misspecification, or pessimism, which distorts how expectations are formed, and thereby alter decisions. Because people are concerned with model misspecification, they make decisions based on the "worst scenario" even though they may possess a good model of the economy. HST also specify these
They show that increasing the preference for robustness stimulates a precautionary motive for savings and makes the SDF more volatile. The combination of habit persistence and robustness also leads to a higher premium.

Another attempt to explain the equity premium puzzle on the basis of irrationality is given by Cecchenetti, Lam and Mark (CLM, 2000). They propose a model in which individuals have "distorted" beliefs about endowment growth. Agents in the model have CRRA utility with a relative risk-aversion coefficient below 10 and a discount factor below 1. In their model, an individual's endowment can shift stochastically between a high- and a low-growth state. Individuals observe these two states but their beliefs about the transition probabilities between them systematically deviate from the true probabilities. Using the S&P 500 stock index data over the period 1871-1993, CLM's model can match the moments of the excess return in a calibration exercise. They also allow the degree of investors' irrationality to be stochastic and time varying, which helps explain the volatility of asset returns and the pattern of serial correlation and predictability exhibited in the data.

The excess return emerging from the model has the same implications as the canonical model:

\[ \hat{e}r_{t+1} = \gamma \text{cov}_{t}(\Delta \ln C_{t+1}, r_{t+1}). \]

However, the problem with this type of model is that it assumes that market participants make systematic forecasting errors endlessly. The learning
process is supposedly very slow. A hundred-year period is still not long enough for market participants to learn about their forecasting errors. This implies that market participants are not only "irrational", but grossly so, in that they systematically mis-predict stock returns and pass up the same exact profit opportunities for very long periods of time.

In this dissertation, I explore an alternative explanation. The problem with the consumption CAPM lies not with the assumption that individuals largely behave in rational ways, but that REH does not capture the behavior of rational individuals.

2.5. Conclusion

This chapter reviewed the empirical studies of the canonical consumption CAPM and its modifications. The main message from this research is that the canonical model has failed along a number of dimensions. The failure of the canonical model has led researchers to search for alternative specifications of preferences. Some researchers also consider irrationality. Although these modifications perform better in calibration exercises, the empirical studies pay little attention to accounting for the time variation of ex ante excess returns. No study examines the modifications' ability to account for the tendency of excess returns to undergo sign reversals.

I show in the next chapter that when the entire class of model is confronted with the pattern of sign reversals in the data, all of the models fail miserably.
These results suggest that there is something fundamentally wrong with the entire class of models and that we need to move beyond the consumption CAPM. In Chapter 4, I consider an alternative, the gap model of premium by Frydman and Goldberg (2007).
3.1 Introduction

Chapter 2 sketched the consumption CAPM and its main modifications and reviewed the empirical record on this class of models. The chapter highlighted that the canonical model has failed empirically along several dimensions, prompting researchers to search for alternative specifications of preferences or forecasting behavior. Much of the research examining the empirical performance of these modifications has been based on calibration exercises. The general conclusion from this research is that the modifications have led to better performance, particularly in accounting for the equity premium puzzle.

The general view among researchers in the field is that once modified, the consumption CAPM is a useful framework for understanding asset market outcomes. Constantinides (1990), for example, talks about the importance of modifying the model by introducing habit persistence:

The goal of this paper is to show that the equity premium puzzle is resolved in a rational expectations model, once we relax the time separability of preferences and allow for adjacent complementarily in consumption, a property known as habit persistence. Constantinides (1990; p520)
This positive conclusion stands in conflict with the few studies that have found alternative specifications of preferences provide no help in accounting for the time variation of excess returns.

In chapter 1, I showed that one of the key features of time series data on the excess return in equity markets is its tendency to undergo sign reversals, that is, to be largely positive for extended periods of time, followed by extended periods in which it is largely negative. None of the empirical studies of the canonical consumption CAPM or any of its modifications have examined the ability of this class of models to account for this feature of the data.

In the present chapter I fill this gap in the literature. I first show that the consumption CAPM is, in principle, consistent with sign reversals: such behavior would emerge if the conditional covariance between consumption growth and stock returns switches sign. Indeed, I find that the canonical model and all of its modifications has this implication for sign reversals. This result provides an easy way to confront the entire class of consumption CAPM models with the time variation of excess returns on stocks: I test whether these models can account for the pattern of sign reversals that we actually observe in the data.

To this end, I make use of a test that Mark and Wu (1998) developed for examining the ability of the consumption CAPM to explain sign reversals in currency markets. The results of my analysis are clear: the sign reversals in the data have no relationship to the switches in sign of the conditional covariance. The main conclusion is that if one wants to account for time series data on
excess returns in equity markets, one will need to jettison the entire class of consumption CAPM models. I begin this task in chapter 4 of this thesis.

The remainder of this chapter is organized as follows. Section 2 shows that the canonical consumption CAPM and its modifications have the same implications for sign reversals. In section 3, I sketch the Mark and Wu (1998) methodology and use it to test the entire class of models. Section 4 offers concluding remarks.

3.2 Sign Reversals in the Consumption CAPM

It is useful to reproduce equation (2.16) from chapter 2, which provides the causal relationship driving the expected excess return on stocks over bonds that is implied by the canonical consumption CAPM.\(^{51}\)

\[
e_{r_{p+1}} = \gamma \text{cov}_t (\Delta \ln C_{t+1}, e_{r_{p+1}}) \tag{3.1}
\]

where as before \(\gamma > 0\) is the representative investor's degree of relative risk aversion, \(e_{r_{p+1}}\) denotes her time-\(t\) expectation of the excess return on stocks over bonds held one period, and \(\text{cov}_t(\bullet)\) denotes a conditional covariance. The equation shows that the algebraic sign of this premium depends on the algebraic sign of the conditional covariance between consumption growth and excess return. In principle, then, the model can generate sign reversals.

The logic is as follows. As we have seen in chapter 2, the utility function in the model implies that consumers would like to smooth their consumption over

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\(^{51}\) As the literature largely ignores the variance term and use arithmetic mean of the stock return and excess return, the variance term is omitted here as well. See the discussion in chapter 2.
their lifetimes relative to their income stream. Consequently, if an asset tends to pay off badly during times when income also tended to be low, it would lead to greater variability of consumption. In this case, a consumer would be willing to buy the asset only if she expected a return in excess of the risk free rate, that is, if she expected a positive risk premium. On the other hand, if the asset tended to pay off well when income tended to be low, it would provide a hedge against decreases in consumption. In that case, the consumer would be willing to buy the asset at a discount, that is, if its expected rate of return was lower than the risk free rate or a negative premium.

The basic logic behind sign reversals provided by the canonical model does not change for the most part when alternative specifications of preferences and forecasting behavior are used. I now show this for each of the main modifications examined in the literature.

3.2.1 Medium-Term Consumption Risk

Parker (2001) explores the implications of medium-term (one to three years) consumption risk and limited participation for the equity premium in calibration exercises. The idea is to focus on the consumption of only those individuals who own stock. Moreover, the relevant covariance is between one-period returns and consumption growth over longer horizons because investors adjust their consumption with a time lag to the changes in their wealth. To measure consumption risk, Parker computes the following conditional covariance:

\[
\text{cov}_s \left( \Delta \ln C_{t+s}, \epsilon_r_{t+s+1} \right).
\] (3.2)

where \( s \) equals 12 to 36 months.
The expected excess return with medium term risk, then, is given by

\[ e_{r_{t+1}} = \gamma \text{cov}(\Delta \ln C_{t+2}, er_{t+1}) \]  

(3.3)

It is apparent from the expression that the sign of the expected excess return continues to be determined by the conditional covariance as with the canonical model, although the covariance is measured using consumption growth at longer horizons. As before, consumers want to smooth their consumption over their lifetimes because of diminishing marginal utility of consumption. If consumption tends to be low \( s \) periods after the stock tends to pay off well—that is, \( \text{cov}(\Delta \ln C_{t+2}, er_{t+1}) < 0 \)—stocks provide a good hedge against medium-term risk. In this case, investors would hold stocks at an expected discount (a negative premium). Conversely, if consumption tends to be high \( s \) periods after stocks tend to pay off well—that is, \( \text{cov}(\Delta \ln C_{t+2}, er_{t+1}) > 0 \)—stocks do not provide a hedge against consumption risk. They will therefore buy stocks only if they expect a positive premium from doing so.

### 3.2.2 Habit Persistence Model

Recall from chapter 2 that the idea of habit persistence is that individuals may get used to a certain standard of living (habits in consumption) after several good years, and a fall in consumption would hurt even though the level of the consumption itself may not be bad at all. To see the implication for sign reversals in the model, we again consider the popular framework by Campbell and Cochrane (1999). As I discussed in chapter 2, these authors propose an external, or “keep up with the Joneses,” form of habit formation in which habits...
depend on aggregate consumption, rather than on individual consumption. The external form of habit persistence simplifies the optimization problem of the consumer because the evolution of the habit is taken as exogenous by the individual. They replace the utility function \( U(C_t) \) with \( U(C_t - X_t) \), where \( X_t \) denotes the level of habits. The utility function takes the form:

\[
U(C_t - X_t) = \frac{(C_t - X_t)^{\gamma} - 1}{1 - \gamma}.
\] (3.4)

The utility function is only defined when consumption exceeds habit. They specify the functional form and parameters so that the risk-free rate is constant. This helps to show how the model can explain the behavior of stock returns entirely by the variation in risk premium. According to Campbell and Cochrane, habits move slowly in response to consumption. A simple way to capture this is to assume \( X_t \) follows an AR (1),

\[
X_t = \rho X_{t-1} + \lambda C_t.
\]

In Campbell and Cochrane's model, they make use of the surplus consumption ratio, \( S_t \), to capture the relationship between consumption and habit. \( S_t \) is defined by

\[
S_t = \frac{C_t - X_t}{C_t}.
\] (3.5)

By design, the habit is smaller than consumption. Therefore \( 0 < S_t < 1 \). The surplus consumption ratio is the fraction of consumption that exceeds habit. When consumption declines relative to the "habits" in a recession, that is, when \( S_t \) approaches zero, people will become more risk averse and demand a higher
excess return in holding a risky asset like stocks. When consumption rises relative to the "habits" in an expansion, that is, when \( S_t \) approaches one, people will become less risk averse and demand a lower excess return in holding a risky asset. Therefore, the habit persistence model also provides variation in the coefficient of risk aversion.

As shown in chapter 2, the expected excess return with habit formation can be written as

\[
er_{r_t+1} = \eta_t \text{cov}_t(\Delta \ln C_{t+1}, er_{r_t+1})
\]  

(3.6)

where \( \eta_t \equiv \gamma / S_t \), which captures the dynamics in the degree of risk aversion. Equation (3.6) shows that with \( S_t > 0 \), the consumption CAPM with habit persistence has the same implication for sign reversals as the canonical model: the sign of the premium changes only when the sign of the conditional covariance between the excess return and consumption growth changes. Although the habit persistence model implies a time-varying degree of risk aversion, this does not affect the model's implication for sign reversals of the premium.

### 3.2.3 Irrationality

Consider the model proposed by Ceccehetti, Lam and Mark (CLM, 2000). They propose a model in which individuals have "distorted" beliefs about endowment growth. Agents in the model have CRRA utility with a relative risk-aversion coefficient below 10 and a discount factor below 1. In their model, an individual's endowment can shift stochastically between a high- and a low-growth state.
Individuals observe these two states but their beliefs about the transition probabilities between them systematically deviate from the true probabilities.

The excess return in this model has the same expression as the canonical consumption CAPM

\[ e_{t+1}^* = \gamma \text{cov}_t (\Delta \ln C_{t+1}, e_{t+1}) \]

Just like the canonical model, the sign of the excess return is determined by the covariance.

3.2.4 Loss Aversion/Narrow Framing

Barberis, Huang and Santos (2001) (BHS henceforth) assume that the representative investor’s preferences involve both risk aversion and loss aversion. In chapter 2, I showed that the added assumption of loss aversion implies that the change in wealth (gains/losses) gets added to the investor’s utility function. Like with habit persistence, BHS assume that the investor’s degree of loss aversion varies over time. With loss aversion, the investor’s degree of loss aversion rises (falls) as she experiences losses (gains) on her portfolio.

Consider their model, where they represent investors’ decision problem with the following objective functional,

\[ E_0 \left[ \sum_{t=0}^{\infty} \left( \beta t \frac{C_t^{1-\gamma}}{1-\gamma} + b_t \beta^{t+1} v(X_{t+1}, S_t, z_t) \right) \right]. \quad (3.7) \]

The first term in each component of the equation is just the standard power utility function, which depends on the level of consumption. The second term adds or
subtracts from the investor's utility depending on whether she experiences a gain or a loss on her holding of the risky asset from time $t$ to $t+1$, denoted by $X_{t+1}$. The variable $S_t$ is the investor's risky asset holdings at time $t$. The variable $b_t$ serves as an exogenous scaling factor and BHS specifies it as $b_0 \overline{C_t}^{\gamma}$, where $\overline{C_t}$ is per capita consumption and $b_0$ controls for how much the investor cares about her financial wealth fluctuations. If she does not care about it at all, then $b_0$ is equal to zero and the utility function is the same power utility function as in the canonical model. The variable $z_t$ is defined as $Z_t / S_t$, where $Z_t$ is a historical benchmark level that the investor uses to see if "she is up or down" on her investment. If $z_t = 1$, then the investor has neither prior gains nor prior losses on her investments. If $z_t < 1$, she has prior gains and if $z_t > 1$, prior losses.  

The utility from gains or losses is a piecewise function:

$$v(X_{t+1}, S_t, z_t) = \begin{cases} X_{t+1}, & X_{t+1} \geq 0 \\ \lambda(z_t) X_{t+1}, & X_{t+1} < 0 \end{cases} \quad (3.8)$$

where $\lambda(z_t)$ measures the degree of loss aversion. BHS set

$$\lambda(z_t) = \lambda + k(z_t - 1)$$

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52 For example, suppose the investor's holding of stocks is currently $S_t = $100 and the historical benchmark she uses is $Z_t = $90. In this case, the change in the value of her stocks is $10 so she has a prior gain. As such, $z_t = Z_t / S_t = 90 / 100 = 0.9 < 1$. Conversely, if the investor's holding of stocks is currently $S_t = $110 and she uses the same benchmark, the change in the value of her stocks is negative $10. She thus experiences a prior loss and $z_t = Z_t / S_t = 110 / 100 = 1.1 > 1$. 

80
where \( k > 0 \). The larger \( z_t \) is, or equivalently, the larger the prior loss, the more painful subsequent losses will be. Consequently, investor will become more loss averse.

More specifically, to calculate subsequent gains or losses, we need to decide the reference level to compare with. BHS suggest using “the status quo scaled up by the risk free rate.” Suppose the investor invests in stocks. So a gain or loss, \( X_{t+1} \), from investing in stocks can be written as

\[
X_{t+1} = S_t R_{t+1} - S_t R_t^f.
\]

When the current value of the investment in the stochastic asset is no less than the historical value, that is, when \( z_t \leq 1 \), the utility function can be written as

\[
v(X_{t+1}, S_t, z_t) = \begin{cases} 
S_t R_{t+1} - S_t R_t^f & R_{t+1} \geq z_t R_t^f \\
S_t (z_t R_t^f - R_t^f) + \lambda (R_{t+1} - z_t R_t^f) & R_{t+1} < z_t R_t^f
\end{cases}
\]

For \( z_t > 1 \), or equivalently, when the current value of investment is less than the historical value, i.e., there are prior losses, the utility function becomes

\[
v(X_{t+1}, S_t, z_t) = \begin{cases} 
S_t R_{t+1} - S_t R_t^f & R_{t+1} \geq R_t^f \\
\lambda (S_t R_{t+1} - S_t R_t^f) & R_{t+1} < R_t^f
\end{cases}
\]

It may be helpful to illustrate these specifications through an example, which I reproduce from BHS (2001) in the Appendix.

BHS also specifies the dynamics of the historical benchmark. The only requirement they impose on the benchmark is that it responds sluggishly to changes in the value of the risky asset. When the stock price goes up by a lot, the benchmark will also move up, but by less. BHS suggests that one way to capture the dynamics is to specify
\[ z_{t+1} = \eta \left( z_t \frac{\bar{R}}{R_{t+1}} \right) + (1-\eta)(1) \]  

(3.11)

where \( \eta \) is between 0 and 1. BHS interprets \( \eta \) in terms of the investor's memory: how far back she can recall past gains or losses. When it is near zero, the investor has a short-term memory and only recalls the most recent prior outcomes. When it is close to one, she has long memory and the benchmark moves sluggishly. \( \bar{R} \) is the average return on stocks.

The equation for the premium that emerges from the BHS model is the following:

\[ \hat{\rho}_{t+1} = \gamma \text{cov}_t(\Delta \ln C_{t+1}, \epsilon r_{t+1}) + \log[1 - b_0 \beta v(R_{t+1}, z_t)] \]  

(3.12)

The equation shows that the expected excess return from holding stochastic assets stems from two components: 1) the representative investor’s aversion to consumption risk, like seen in the canonical consumption CAPM; and 2) the investor’s aversion to potential losses. But there is another difference from the canonical model. BHS assumes that the investor evaluates her financial gains or losses on an annual basis. Their reasoning follows the suggestion of Benartzi and Thaler (1995), who argue that a year is a natural evaluation period. Their reasons are as follows: 1) investors file taxes once a year and receive their most comprehensive mutual fund reports once a year; and 2) money managers’ performance is most carefully reviewed on an annual basis.

Consequently the implication for sign reversals with loss aversion is not as straightforward as with the other modifications to the canonical model. But, by subtracting the loss aversion term from both sides of the equation, gives,
To fix ideas, I call the left hand side of the equation,

\[ e^{\hat{I}_{t+1}} - \log(1-b_0\bar{\nu}(R_{t+1}, z_t)) = \gamma \text{cov}(\Delta \ln C_{t+1}, \epsilon_{t+1}) \]

(3.13)

To fix ideas, I call the left hand side of the equation, \( f[er_{t+1}, z_t] = E_t(\epsilon_{t+1}) - \log(1-b_0\bar{\nu}(R_{t+1}, z_t)) \), the loss-adjusted risk premium (LARP). Equation (3.13) shows that the sign of LARP depends on the sign of the covariance. Like in the canonical consumption CAPM, consumers want to smooth their consumption over their lifetimes because of diminishing marginal utility of consumption. If consumption tends to be low after the stock tends to pay off well—that is, \( \text{cov}(\Delta \ln C_t, \epsilon_{t+1}) < 0 \)—stocks provide a good hedge against consumption risk. In this case, investors would hold stocks at an expected discount (a negative LARP). Conversely, if consumption tends to be high after stocks tend to pay off well—that is, \( \text{cov}(\Delta \ln C_t, \epsilon_{t+1}) > 0 \)—stocks do not provide a hedge against consumption risk. They will therefore buy stocks only if they expect a positive LARP from doing so.

We can see that the entire class of consumption CAPM models, with a slight qualification for the loss aversion model, has the same implication for sign reversals. The expected excess return on stocks (the loss-adjusted excess return in the loss aversion model) has the same sign as the covariance between consumption growth and the excess return. In the next section, I use this insight to test this entire class of consumption CAPM models.

3.3 Can the Class of CAPM Models Account for Sign Reversals?

To test the ability of the class of CAPM models to account for the sign reversals in the data, I employ the test used by Mark and Wu (1998). This study examined
the ability of the consumption CAPM to explain the tendency of excess returns on the foreign exchange to undergo sign reversals. The application of their test to equity markets is straightforward.

### 3.3.1 The Mark and Wu (1998) Test

Mark and Wu's idea is to separate the paired observations on $er_{t+1}$ and $\Delta \ln C_{t+1}$ into two groups: 1) those observations that are preceded by a time-$t$ expectation of a positive excess return, that is, $er_{t+1} > 0$ and 2) those observations that are preceded by $er_{t+1} < 0$. Recall that the canonical model generates a positive premium only if $\text{cov}(\Delta \ln C_t, er_{t+1}) > 0$. Consequently, if this model is valid, we would expect that the set of paired observations on $er_{t+1}$ and $\Delta \ln C_{t+1}$ that are preceded by positive observations on $er_{t+1}$ would imply a positive $\text{cov}(\Delta \ln C_t, er_{t+1})$. Similarly, the set of paired observations on $er_{t+1}$ and $\Delta \ln C_{t+1}$ that are preceded by negative observations on $er_{t+1}$ should imply a negative $\text{cov}(\Delta \ln C_t, er_{t+1})$. Scatter plots and simple regressions are used to examine the covariation among the paired observations.

To apply this test to the loss aversion model, I separate the paired observations on $er_{t+1}$ and $\Delta \ln C_{t+1}$ into two groups: 1) those observations that are preceded by a positive time-$t$ expectation of LARP, that is, $LARP_{t+1} > 0$ and 2) those observations that are preceded by $LARP_{t+1} < 0$. If this model is valid, we would expect that the set of paired observations on $er_{t+1}$ and $\Delta \ln C_{t+1}$ that are
preceded by positive observations on $LARP_{t+1}$ would imply a positive $\text{cov}_t(\Delta \ln C_t, er_{t+1})$. Similarly, the set of paired observations on $er_{t+1}$ and $\Delta \ln C_{t+1}$ that are preceded by negative observations on $LARP_{t+1}$ should imply a negative $\text{cov}_t(\Delta \ln C_t, er_{t+1})$. Again, I use scatter plots and simple regressions to examine the covariation among the paired observations.

To sort the data, I use the estimates of $er_{t+1}$ from chapter 1. The estimates I use are based on the regressions involving the risk free rate and its lags, the smoothed earnings-price ratio and its lags, and lags of the excess return. Consumption data are taken from the Federal Reserve Economic Data base (FRED), which sum real consumption expenditures on nondurables and services. The sample period is from January 1959 to October 2008.

To calculate the loss aversion term, $\log[1-b_0\beta v(R_{t+1}, z_t)]$ in the BHS model, we need to specify the values for the parameters, $b_0$ and $\beta$, and find a way to calculate prior gains or losses. To that end, we need to decide the historical benchmark to compare with.

It would seem straightforward to calculate gains or losses using the stock return as specified in the dynamics in equation (3.11). BHS suggest using the average return as the historical benchmark. The question then is how far back should we take the average? Since the investor tends to have short memory, a year would be a reasonable horizon. But since $z_t$ is defined so that when it is less than one, we have prior gains, which excludes the possibility that $z_t$ might be less
than zero. Consequently, I use another way to calculate prior gains and losses. BHS also suggests using the stock price of a year ago as the historical benchmark. So the prior gain or loss can be calculated as $P_{t-12}/P_t$. If the ratio is greater than 1, there is a prior loss. Conversely, if the ratio is less than 1, there is a prior gain.

Consider the time frame the investor is facing, which is illustrated in the following chart. Time $t$ is in months. At time $t$ where the investor is standing, say December 2009, she has to do two things. First, she needs to calculate gains or losses from December 2008 to date. The gain or loss can be calculated as $P_{t-12}/P_t$. Second, she has to consider how much money to put into the stocks for the next period. This consideration is based upon: 1) her prior gains or losses and 2) her expected future return, $R_{t+1}$, and the potential gain or loss, $X_{t+1} = S_t R_{t+1} - S_t R_t$ from time $t$ to $t+1$. Notice here that although the investor evaluates her prior gains or losses on an annual basis, when she forms her expectation about the future, she is assumed to think about the shorter term, a month.

\[
\begin{array}{cccc}
  P_{t-24} & P_{t-12} & P_t & P_{t+1} \\
  R_{t-24} & R_{t-12} & R_t & R_{t+1} \\
\end{array}
\]

Time: \(t-24\) \(t-12\) \(t\) \(t+1\) (Jan. 2010)


\[53\] I tried to use 12-month average return of stocks as the benchmark. Not surprisingly, some periods have negative average returns.
If we set $S_t$ is $1$, then the gains or losses in equations (3.9) and (3.10) are linear combinations of stock returns and the risk free rate. Consequently, taking the natural log, the loss aversion premium term can be approximated by,

$$\ln[1 - b_0 \beta_1 (R_{t+1}, z_t)] \approx -b_0 \beta_1 (R_{t+1}, z_t).$$

(3.14)

I use this approximation to calculate the premium term associated with loss aversion. The calculation of the utility from gains and losses is summarized in Table 3.1. Parameter values used in the test and BHS's calibration are summarized in Table 3.2. For the first four parameters I use the same values as BHS use in their study. For parameters $k$ and $b_0$, BHS use a range of values whereas I only report one. 54

3.3.2 Results

3.3.2.1 The Canonical, Habit Persistence, and non-REH Models. I first examine the empirical performance of the canonical, habit persistence and non-REH models, which generate the same exact implications for sign reversals. Figures 3.1a and 3.1b provides scatter plots with consumption growth on the vertical axis and the excess return on equities on the horizontal axis. The first plot involves the paired observations preceded by a positive expected excess return, whereas the second plot involves the paired observations preceded by a negative expected excess return.

If the canonical, habit persistence, or the non-REH models were valid, we would expect to see that the paired observations show a positive relationship in

54 I also tried different values of $b_0$ and $k$ and the selection of the values of these parameters has no impact on the results of the Mark and Wu test.
figure 3.1a and a negative relationship in figure 3.1b. However, far from lining up on lines that slope downward and upward, respectively, the scatter plots show absolutely no relationship.

OLS regressions of the excess return on consumption growth, which are reported in the first rows of tables 3.3a and 3.3b, reveal a similar story. The $R^2$ of both regressions are very small. Moreover, the slope coefficient for the negative premium regression, which should be negative according to the models, is found to be significantly positive.

Clearly, the canonical, habit persistence, and non-REH models have no ability whatsoever to account for the sign reversals we actually observe in the data.

3.3.2.2 The Model with Loss Aversion. To test the loss aversion model, I follow the same procedure as discussed above, but use $LARP_{t+1}$ to sort the data. The sorted observations of consumption growth and the excess return are displayed in figure 3.2, where again consumption growth is on the vertical axis and the excess return is on the horizontal axis. According to the loss aversion model, paired observations of consumption growth and excess return that are preceded by a positive $LARP_{t+1}$ should be positively correlated. These are given in the A plots. Similarly, the paired observations preceded by a negative $LARP_{t+1}$ should be negatively correlated. These are given in the B plots.

Like with the other modifications, the graphs suggest that the two variables are uncorrelated. The OLS regressions of the ex post excess return on the growth rate of consumption, however, tell a more nuanced story. These results
are reported in the last lines of Table 3.3. Again, I find extremely small $R^2$'s. Both slope coefficients of consumption growth are positive and statistically significant. However, the slope coefficient for the paired observations that are preceded by a negative $LARP_{t|t+1}$ should be negative rather than positive. As such, the model with loss aversion is also unable to explain the sign reversals of $LARP_{t|t+1}$.

### 3.3.2.3 Medium Term Consumption Risk

To test the Parker’s specification of medium consumption risk, I follow the same procedure as discussed above while using a horizon of 12 to 36 months to compute the rate of consumption growth. That is, I use the paired observations of $(\Delta \ln C_{t+12}, er_{t+1})$ and $(\Delta \ln C_{t+36}, er_{t+1})$ to test the sign restrictions, where $t$ denotes months.

Figures 3.3a and 3.3b provide scatter plots with consumption growth over a 12-month horizon on the vertical axis and the excess return on equities on the horizontal axis. Similarly, Figures 3.4a and 3.4b provide scatter plots with consumption growth over a 36-month horizon on the vertical axis and the excess return on equities on the horizontal axis. The plots in a) involve the paired observations preceded by a positive expected excess return, whereas the plots in b) involves the paired observations preceded by a negative expected excess return.

If the medium risk model were valid, we would expect to see that the paired observations show a positive relationship in figures 3.3a and 3.4a, and a negative relationship in figures 3.3b and 3.4b. However, the scatter plots show absolutely no relationship.
OLS regressions of the excess return on consumption growth, which are reported in the second and third rows of tables 3.3a and 3.3b, reveal a similar story. The $R^2$ of both regressions are again very small. Only one slope coefficient for the 12-month risk model is found to be significantly positive. All the other slope coefficients are insignificantly different from zero. Clearly, the medium term risk model has no ability to account for the sign reversals we actually observe in the data.

3.4 Conclusion

In this chapter, I showed that the consumption CAPM and its recent modifications are, in principle, consistent with sign reversals. Such behavior stems from switches in the sign of the conditional covariance between consumption growth and excess returns. This insight enabled me to confront the entire class of consumption CAPM models with the pattern of sign reversals that we actually observe in the data.

My empirical results showed that the consumption CAPM and its modifications are unable to explain the sign reversals in the data. The inability of the entire class of consumption CAPM to explain market outcomes suggests that we need to move beyond this class of model.
CHAPTER IV

HISTORICAL BENCHMARKS AND THE EQUITY PREMIUM: IMPERFECT KNOWLEDGE AND ENDOGENOUS PROSPECT THEORY

4.1 Introduction

In previous chapters I showed that the canonical consumption CAPM and its modifications were unable to account for the time variation in the excess return of stocks. In this chapter, I consider an alternative model due to Frydman and Goldberg (2007, 2008; hereafter FG) that was developed to explain the behavior of excess returns in currency markets. After adapting this model to the stock market, I examine its ability to account for the time series data on excess returns. My results provide some evidence that this alternative model performs better than the consumption CAPM.

Following FG, I replace the usual specification of preference based on the assumptions of risk aversion and expected utility theory with preferences based on endogenous prospect theory (EPT). This alternative specification of preferences builds on Kahneman and Tversky (1979) and Tversky and Kahneman (1992) and assumes that an individual's degree of loss aversion
increases as the size of her open position increases in the market. With "endogenous loss aversion," a market participant will take a finite speculative position in stocks only if she expects a positive excess return—a premium—to compensate her for her extra sensitivity to potential losses. FG show that this premium, dubbed an "uncertainty premium," depends on an individual's forecast of the potential loss from speculating.

Modeling this forecast depends on whether an individual is a bull or a bear, that is, whether she holds a long or short position, respectively. To model the forecasts that underpin the uncertainty premium for bulls and bears, FG use Imperfect Knowledge Economics (IKE) to formalize an insight due to Keynes (1936) which emphasizes the importance of historical benchmark. Assets prices have a tendency to move persistently away from benchmark levels for protracted periods, but eventually these swings end and prices eventually undergo sustained movements back to benchmark levels. Keynes recognized that market participants are aware of this empirical regularity and use it in their attempts to assess the riskiness of their open positions.

FG suppose that bulls' and bears' forecasts of the potential losses from holding speculative positions depend on their assessments of the gap between the asset price and its perceived historical benchmark. In equilibrium, the premium on stock, ceteris paribus, co-varies positively with a measure of the

---

55 An individual is loss averse if her disutility from losses is greater than her utility from gains of the same magnitude.

56 A bull (a bear) is a market participant who speculates on the belief that the asset price will rise (fall). A long position in an asset that delivers a profit (loss) if price rises (falls). The converse is the case for a short position.
aggregate gap. The positive co-variation arises because if, for example, participants revise up their assessments of the gap, bulls increase and bears decrease their estimates of the riskiness of their positions and so the uncertainty premium required by bulls rises and the uncertainty premium required by the bears falls. Both of these movements lead to a rise in the aggregate premium.

I also show that the aggregate premium depends positively on the share of overall wealth that market participants hold in stocks. This extra term is analogous to FG's extra term, which in the context of the currency market, stems from the international financial position of the countries.

My empirical application makes use of two different data sets, the one used in chapter 1 and another due to Duffee (2005). The latter data set uses the ratio of stock market wealth to consumption as a proxy for the share of wealth held in stocks. In analyzing both data sets, I use the same methodology as in chapter 1 for incorporating temporal instability. My regression results provide support for the FG gap model.

The remainder of the chapter is structured as follows. Section 2 sketches the gap model of the premium for the stock market, leaving the full derivation of the model to an appendix. Section 3 discusses my empirical strategy and results. Section 4 offers concluding remarks.

4.2 A Gap Model of the Premium on Equities

In modeling individual decision making under uncertainty, economists typically assume that individuals are risk averse and that their preferences over gambles
can be represented by expected utility theory (EUT). Behavioral economists, however, have uncovered massive amounts of evidence showing that these standard assumptions are grossly inconsistent with how individuals actually make choices.\textsuperscript{57} This evidence has led Kahneman and Tversky (1979) and Tversky and Kahneman (1992) to develop an alternative approach to modeling preferences--dubbed “Prospect Theory”—that can account for the experimental evidence on individual decision making.

One of the key components of prospect theory is the assumption of loss aversion. An individual is loss averse if her disutility from losses is greater than her utility from gains of the same magnitude. We saw in chapter 2 that other studies have used the prospect theory and the assumption of loss aversion to model preferences.\textsuperscript{58} But, to model the speculative decision on whether and how much to gamble, these studies had to use a specification that mixed the assumptions of risk aversion and loss aversion.

To model an individual’s speculative decision solely on the basis of prospect theory, FG assume that an individual’s degree of loss aversion increases with her position size. They call their alternative specification of preferences, “endogenous prospect theory” (EPT). There are two key implications of EPT. First, an individual will be willing to hold a finite speculative position in stocks, but only if she expects a positive excess return—a premium—

\textsuperscript{57} Kahneman and Tversky (1979) is the classic study. See Barberis (2005) for a review article and references therein.

\textsuperscript{58} I am referring here to Barberis, Huang and Santos (2001). See Barberis and Huang (2007) for a review article and references therein.
from doing so. Second, this premium, which FG call an "uncertainty premium," depends on the individual's forecast of the potential losses that she might incur if the stock price moves against her.

In FG's model, this forecast depends on whether the individual is a bull or a bear, that is, whether she holds a long or short position in stocks. Like in previous chapters, a speculator's return on a long position in stocks in excess of return on bonds can be written as:

$$er_{t+1} = r_{t+1} - r_f$$  \hspace{1cm} (4.1)

The excess return on a short position of stock is then $$-er_{t+1}$$.

When realizations of $$er_{t+1}$$ are positive, bulls (bears), who hold long (short) positions and expect the stock price to rise (fall), gain (lose). Conversely, when the realizations of $$er_{t+1}$$ are negative, bears gain and bulls lose. An individual's forecast of the potential losses are conditional on her forecasting strategy and information set at time $$t$$. For a bull, we can represent this forecast, $$\hat{i}_{L_{t+1}}$$, by

$$\hat{i}_{L_{t+1}} = E_t[er_{t+1} < 0 | Z_{L_t}] < 0$$  \hspace{1cm} (4.2)

whereas the expected loss for bears, $$\hat{i}_{S_{t+1}}$$, is

$$\hat{i}_{S_{t+1}} = E_t[-er_{t+1} < 0 | Z_{S_t}] < 0$$  \hspace{1cm} (4.3)

The expression in equation (4.2) denotes the expected value of only the negative realizations of $$er_{t+1}$$, which implies losses for bulls. Similarly, the expression in equation (4.3) is the expected value of only the positive realizations of $$er_{t+1}$$, which implies losses for bears. $$Z_{L_t}$$ represents bulls' forecasting strategies and
information sets, and $Z_i^s$ represents bears'. The superscripts "L" and "S" denote long and short positions, respectively.

I show in the appendix that the uncertainty premium for the group of bulls and bears, denoted by $u\hat{p}_{t+1}^L$ and $u\hat{p}_{t+1}^S$, can be written as:

$$u\hat{p}_{t+1}^L = (1 - \lambda_1)\hat{L}_{t+1} > 0$$

(4.4)

$$u\hat{p}_{t+1}^S = (1 - \lambda_1)\hat{S}_{t+1} > 0$$

(4.5)

where $\lambda_1$ is a preference parameter that must be greater than unity for loss aversion. The logic here is straightforward. If an individual raises her forecast of the potential loss from holding a speculative position, she will raise the premium she expects to compensate her for her greater aversion to potential losses.

I also show in the appendix, that in equilibrium, the premium on stocks is determined by the uncertainty premium required by the bulls in relative to the bears, plus a term that depends on the value of stocks as a proportion of market wealth:

$$p_{t+1}^L = u\hat{p}_{t+1}^S + \lambda_1 \frac{S_t}{W_t^M}$$

(4.6)

where $p_{t+1}^L$ is the equilibrium premium, $S_t$ is the supply of stocks, $W_t^M$ is the total market wealth, and $\lambda_1 > 0$ is another preference parameter. The equilibrium condition in equation (4.6) shows that the market premium on stocks depends not only on an aggregate uncertainty premium, but also on the share of stocks in total market wealth.
To model the forecasts that underpin the uncertainty premium for bulls and bears, FG use IKE. They formalize an insight due to Keynes (1936) that what matters for assessing the riskiness of speculative positions is the divergence between an asset price and its perceived historical benchmark level.

Bulls and bears know that while stock prices can move persistently away from perceived benchmark levels, eventually such swings will end. But, when exactly a sustained movement back to benchmark levels will occur is unpredictable. Bulls of course expect that any upswing in price away from benchmark levels will continue for at least one more period. But, if the stock price does in fact rise further, they also become more concerned about an eventual reversal. FG assume that this would lead them to increase their forecasts of the potential capital losses from taking long positions and betting on a further rise in price. Bears, on the other hand, expect that the stock price will fall. Thus, a further rise in price away from perceived benchmark levels would give them a reason to become more confident in their forecasts of a movement back towards the benchmark. FG assume that this would lead them to decrease their forecasts of the potential losses.

FG formalize these assumptions as follows. They suppose that $\hat{L}_{t+1}$ and $\hat{S}_{t+1}$ depend on bulls' and bears' assessments of the gap between the stock price and their estimate of its benchmark level.\(^{59}\)

---

\(^{59}\) IKE's use of qualitative constraints of its representations of individuals' forecasting behavior enables an economist to recognize that individuals must cope with imperfect knowledge about the processes driving market outcomes without having to presume that they are irrational.
\[
\frac{\Delta \hat{L}_{t+1}^L}{\Delta \text{gap}_t^L} < 0 \text{ and } \frac{\Delta \hat{L}_{t+1}^S}{\Delta \text{gap}_t^S} > 0
\] (4.7)

where \(\Delta\) denotes a first difference operator and \(\text{gap}_t^L = \hat{p}_{t+1}^L - p_{t}^{\text{HB},L}\) and \(\text{gap}_t^S = \hat{p}_{t+1}^S - p_{t}^{\text{HB},S}\), and \(p_{t}^{\text{HB},L}\) and \(p_{t}^{\text{HB},S}\) denote bulls’ and bears’ assessments of the historical benchmark, respectively. The gap conditions in (4.7) imply that in equilibrium, the premium on stocks will, ceteris paribus, co-vary positively with the aggregate gap:

\[
\frac{\Delta \text{pr}_t}{\Delta \text{gap}_t} > 0
\] (4.8)

where \(\text{gap}_t = \frac{1}{2} (\text{gap}_t^L + \text{gap}_t^S)\).\(^{60}\) The positive co-variation arises because if, for example, participants revise up their assessments of the gap, bulls increase and bears decrease their estimates of the riskiness of their positions and so \(\hat{p}_{t+1}^L\) rises and \(\hat{p}_{t+1}^S\) falls. Both of these movements lead to a rise in the aggregate premium.

In real-world markets, participants alter their forecasting strategies over time, at least intermittently, as new information arises and the social context changes. Consequently, although the qualitative relationship between the market premium and the gap may be positive, we would not expect that the precise quantitative impact of \(\text{gap}_t\) on \(\text{pr}_t\) would remain unchanged at every point in

\(^{60}\) In deriving the positive relationship in (4.8), Frydman and Goldberg (2007, chapter 12) address distributional issues.
time. I address this problem by allowing for temporal instability in my regression analysis.

4.3. Empirical Methodology

Equation (4.6) shows that to test the implications of FG’s gap model, we need an aggregate measure of the benchmark level in the stock market. A natural candidate is the price-earnings (P/E) ratio. To calculate the historical benchmark for the stock price, I use a moving average of this ratio:

\[ P_t^{HB} = (P/E)_t^{MA} \times E_t, \]  
(4.9)

where \((P/E)_t^{MA} = \sum_{i=1}^{30-12} (P/E)_i\), is 30-year moving average of the P/E ratio, \(E_t\) is actual earnings at time \(t\), and time is measured in months.

Equation (4.6) also shows that we need a measure of the share of stocks in overall wealth. As far as I know, no monthly data exist on overall household wealth, let alone going back to the 1870s, which is when my sample begins. I address this problem in two ways.

First, I note that the variation in \(\frac{S_t}{W_t^{M}}\) is likely to be very small relative to the variation in excess returns. Indeed, the main problem with the traditional CAPM, which relates the market’s premium to asset supplies, is that asset supplies are a flat line compared to the time path of excess returns on stocks. I thus use my data from chapter 1 and omit \(\frac{S_t}{W_t^{M}}\) in my regressions. Ideally, I would like to regress a measure of the ex ante excess return on my gap variable, but monthly
survey data on participant's stock price forecasts is also very difficult to obtain. I thus regress the ex post excess return on stocks on my gap variable. This implies that the error in my regressions depends on market participants' forecasting errors.

I also make use of a data set from Duffee (2005), which runs from 1959:01 through 2002:05. Duffee (2005) argues that \( \frac{S}{W} \) can be proxied by the ratio of stock market value to consumption, which I denote by ME/C.\(^6\) Stock market wealth is measured by the month-end market capitalization of the CRSP value-weighted index, expressed in real per capita terms for comparability to the consumption data. Consumption is measured by Bureau of Economic Analysis estimates of monthly per capita expenditures on nondurables and services. Since the stock return in Duffee's data set is also ex post, the error term in my regressions also depends on market participants' forecasting errors.

### 4.3.1. Regression Methodology

To deal with the potential problem of unit roots, I follow the methodology outlined in Hendry and Juselius (2000) and estimate error-correction models. For my first data set, which omits \( \frac{S}{W} \), I first estimate an autoregressive distribution lag (ADL):

\[
er_{t-1} = \alpha + \sum_{i=1}^{2} b_{1i} e_{t-i} + \sum_{i=0}^{2} b_{2i} g_{t-i} + \varepsilon_t \tag{4.10}
\]

\(^6\) Duffee shows that S/W is a function of ME/C. When the stock market wealth in total wealth is relatively high, stock market wealth will be high relative to total consumption.
where $er_{t-1}$ is the excess return on stocks over bonds and $gap_t$ is the log difference between the stock price and its historical benchmark, $gap_t = \log(P_t) - \log(P_{t-1})$. The ADL can be written as in error-correction form,

$$
\Delta er_{t-1} = \alpha_0 + \alpha_1 \Delta er_{t-2,1} + \sum_{i=0}^{1} \alpha_{2i} \Delta gap_{t-i} + \alpha_3 ECT_{t-2} + \epsilon_t \tag{4.11}
$$

where the error correction term is given by

$$ECT_{t-2} = \beta_1 gap_{t-2} + \beta_0 - er_{t-1,2} \tag{4.12}$$

The static relationship in (4.14) can be solved as:

$$\Delta er_{t-1} = \gamma_0 + \gamma_1 \Delta gap_{t-1,2} - \gamma_3 ECT_{t-2} + \epsilon_t \tag{4.13}$$

This static relationship is typically referred to as the "long-run" relationship. In real-world markets, however, we would not expect the relationship between the equilibrium premium and the gap to settle down to any such relationship, as the influence of the gap on market participants' forecasts of potential losses is likely to depend on market conditions and thus vary over time. Consequently, if the gap model were consistent with the data, we would expect to find a short-run positive relationship between the monthly changes in $pr_t$ and $gap_t$, that is, $\gamma_1 > 0$. However, we would not expect a long-run relationship in levels, that is, $\beta_1 > 0$. For my second data set, which proxies $\frac{S}{W^M_t}$ with ME/C, I also run an ADL, which I write in error-correction form:

$$er_{t-1} = \alpha_0 + \alpha_1 \Delta er_{t-2,1} + \sum_{i=0}^{1} \alpha_{2i} \Delta gap_{t-i} + \sum_{i=0}^{1} \alpha_{3i} \Delta (ME / C)_{t-i} + \alpha_3 ECT_{t-2} + \epsilon_t \tag{4.14}$$

where the error correction term is given by
\[ ECT_{t-2} = \beta_1 gap_{t-2} + \beta_2 MEC_{t-2} + \beta_0 - er_{t-1|t-2} \]  

(4.15)

The static relationship in (4.1) can be solved:

\[ \Delta er_{t-1|t} = \gamma_0 + \gamma_1 \Delta gap_{t-1|t} + \gamma_2 \Delta (ME/C)_{t-1|t} - \gamma_3 ECT_{t-2} + \epsilon_t \]  

(4.16)

Like before, the gap effect implies that \( \gamma_1 > 0 \), i.e., in the short run, the excess return and the gap variable should be positively correlated. For the asset supply effect, we would expect \( \gamma_2 > 0 \).

### 4.3.2. Structural Change Results

As I discussed in chapter 1, one would not expect the underlying relationship between the excess return and the gap and asset supply variables to remain constant over time. I thus allow for structure breaks in my analysis. Since the CUSUM test performs better in identifying break points, I employ a combination of the CUSUM test and recursive Chow tests as discussed in chapter 1. In my analysis, I use an initialization period-10 years- to estimate the ADL using OLS and then roll the regression forward through the sample one observation at a time, computing a residual for each recursion. After the CUSUM test indicates the first point of structural change, I use the sequential Chow test to identify its most likely location. To test for additional break points in the data, I re-run the CUSUM and Chow tests.

The results of testing the temporal instability of the ADL specifications for the first and second data sets are reported in tables 4.1 and 4.2. For the larger sample, I find 20 break points, while for the smaller sample, I find five break points. Like before, many of the break points are proximate to major turning
points in the U.S. business cycle, for example, 1907:01 and 1937:04 for peaks and 1915:01, 1971:01 and 2001:10 for troughs. Break points are also found during the Great Depression, “Black Monday” in 1987, and the most recent financial crisis in 2008.

4.3.3. Regression Results

The regression results for my two samples are reported Tables 4.3 and 4.4. Table 4.3 reports the estimates of the error-correction model in equation (4.13), which includes only the gap variable. I find that the parameter estimates for this variable, $\gamma_1$, are significantly positive for all sub-periods. The $R^2$ of the regressions, which range between 0.24 and 0.68, are high compared to those obtained from regressions based on standard risk premium models. Table 4.3 also shows that the coefficient estimates of the error correction term are significant only for two sub periods, indicating little support for long-run relationship in levels. Also, the size of the estimates of $\gamma_1$ is much larger than for $\gamma_3$ for all sub-periods. As we would expect, the positive relationship between the equilibrium premium and the gap shows up largely through the short-run component of the model.

Table 4.4 reports the estimates of the error-correction model in equation (4.16), which includes both the gap and the stock supply variables. I find that the estimates of $\gamma_1$ are significantly positive for all sub-periods except the period from 1987:11 to 1996:07. The estimates of $\gamma_2$, the coefficient on ME/C, are significantly positive only for the sub-periods 1987:11 to 1996:07 and 1996:08 to
2001:10. Notice that during the period of 1987:11 to 1996:07, the coefficient on the gap variable is not significant. Moreover, during the sub period of 1996:08-2001:10, the most prominent bull market, both the coefficient estimates for the gap and ME/C are significantly positive. The $R^2$ of the regressions, which range between 0.35 and 0.63, are high compared to those obtained from regressions based on standard risk premium models. As with the results in table 4.3, those in table 4.4 show that the error correction term is largely not significant, with only one subperiod showing significance at 10% level. This suggests that the gap and asset-supply variables impact on the equilibrium premium mainly through the short-run component of the model.

Taken as a whole, the regression results in tables 4.3 and 4.4 reveal that the gap from a measure of the historical benchmark explains much of the short-run, month-to-month variation in the equilibrium premium. There is little evidence that there is a long-run relationship between these variables.

4.4. Conclusions

In this chapter, I adapted the FG model of the premium on foreign exchange to the stock market. The results of my empirical analysis provided some support that this alternative provides a better account of the time variation of the excess return on stocks than the class of consumption CAPM models. The empirical test showed that in the short run, the gap variable and the equilibrium premium co-vary positively. They also showed that the asset-supply effect is also significant during time periods in which the stock market value is high.
The empirical results for the FG model are promising, but more research needs to be done. In particular, its ability to account for sign reversals of the *ex ante* excess return needs to be examined. Equation (4.6) shows that the equilibrium premium in the FG model has two components— the uncertainty premium and the relative supply of stocks. As it stands, the FG model has no implication about which term might dominate.

One possibility would be to use a test similar to the Mark and Wu test. One could subtract the supply term from the equilibrium premium, and examine if the difference has the same sign as the uncertainty premium. Since the uncertainty premium is positively correlated with the gap variable, then the sign of the difference between the equilibrium premium and the supply term should be determined by the sign of the gap variable. I will leave this for future research.
Table 1.1 Summary Statistics
(1881:01-2008:12)

<table>
<thead>
<tr>
<th>variables</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>8.28</td>
<td>58.230</td>
</tr>
<tr>
<td>er</td>
<td>4.63</td>
<td>58.348</td>
</tr>
<tr>
<td>rf</td>
<td>3.65</td>
<td>2.52</td>
</tr>
<tr>
<td>se_p</td>
<td>-2.82</td>
<td>0.36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlations</th>
<th>Se_p</th>
<th>er</th>
<th>rf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Se_p</td>
<td>1.00</td>
<td>-0.067</td>
<td>-0.068</td>
</tr>
<tr>
<td>er</td>
<td></td>
<td>1.00</td>
<td>-0.069</td>
</tr>
<tr>
<td>rf</td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: r is return on S&P 500 index; er is excess return of stocks over bonds; se_p is the ratio of 10-year moving average of earnings to price; rf is risk free rate.
Table 1.2
Selected Studies with Predictive Variables

<table>
<thead>
<tr>
<th>Studies</th>
<th>Variables</th>
<th>Time periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fama and Schwert (1977)</td>
<td>T-bill</td>
<td>1953:01-1971:07</td>
</tr>
<tr>
<td>Fama and French (1988b)</td>
<td>Lagged return</td>
<td>1926-1985</td>
</tr>
<tr>
<td>Poterba and Summers (1989)</td>
<td></td>
<td>1871-1986</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1926-1986 for NYSE index</td>
</tr>
</tbody>
</table>
### Table 1.3
Rapah and Woah’s Results of Structural Change for S&P 500 Index

<table>
<thead>
<tr>
<th>Bivariate model</th>
<th>Break points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log(dividend-price ratio)</td>
<td>1990:3</td>
</tr>
<tr>
<td>Log (price-earnings ratio)</td>
<td>1972:4</td>
</tr>
<tr>
<td></td>
<td>1982:2</td>
</tr>
<tr>
<td>Log (Fed q)</td>
<td>1972:4</td>
</tr>
<tr>
<td></td>
<td>1990:3</td>
</tr>
<tr>
<td>Log (payout ratio)</td>
<td>1974:3</td>
</tr>
<tr>
<td>Cay</td>
<td>1976:1</td>
</tr>
<tr>
<td>Term spread</td>
<td>1974:3</td>
</tr>
<tr>
<td>Default spread</td>
<td>1967:3</td>
</tr>
<tr>
<td></td>
<td>1975:2</td>
</tr>
<tr>
<td>Short rate</td>
<td>1974:3</td>
</tr>
</tbody>
</table>

| Multivariable models:                    | Break points |
| Model selected by AIC                    |              |
| Log (dividend-price ratio)               |              |
| Log (price-earnings ratio)               |              |
| Log (payout ratio)                       |              |
| Cay                                      |              |
| Default spread                           |              |
| Short rate                               |              |
|                                          | 1988:4       |

| Model selected by SIC                    | Break points |
| Cay                                      |              |
| Default spread                           |              |
| Short rate                               |              |
|                                          | 1991:1       |
Table 1.4 Structural Changes (BP test)

<table>
<thead>
<tr>
<th>FS regression</th>
<th>Multivariate regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1916:10</td>
<td>1894:12</td>
</tr>
<tr>
<td>1932:04</td>
<td>1917:11</td>
</tr>
<tr>
<td>1945:12</td>
<td>1931:04</td>
</tr>
<tr>
<td>1974:08</td>
<td>1956:03</td>
</tr>
<tr>
<td></td>
<td>1972:09</td>
</tr>
<tr>
<td></td>
<td>1987:11</td>
</tr>
</tbody>
</table>
Table 1. 5 Structural Change of Multivariate Regression (CUSUM test)

<table>
<thead>
<tr>
<th>Break points</th>
<th>F statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1884-04</td>
<td>2.1344 [0.0456] *</td>
</tr>
<tr>
<td>1893-05</td>
<td>3.2937 [0.0005] **</td>
</tr>
<tr>
<td>1895-04</td>
<td>2.4659 [0.0255] *</td>
</tr>
<tr>
<td>1907-03</td>
<td>2.3589 [0.0435] *</td>
</tr>
<tr>
<td>1917-02</td>
<td>3.2319 [0.0432] *</td>
</tr>
<tr>
<td>1919-08</td>
<td>2.1837 [0.0390] *</td>
</tr>
<tr>
<td>1927-07</td>
<td>8.5111 [0.0000] **</td>
</tr>
<tr>
<td>1929-08</td>
<td>5.8039 [0.0001] **</td>
</tr>
<tr>
<td>1933-04</td>
<td>5.9562 [0.0197] *</td>
</tr>
<tr>
<td>1937-03</td>
<td>1.6845 [0.0370] *</td>
</tr>
<tr>
<td>1973-11</td>
<td>1.5381 [0.0208] *</td>
</tr>
<tr>
<td>1987-10</td>
<td>2.9810 [0.0013] **</td>
</tr>
<tr>
<td>1997-07</td>
<td>2.6106 [0.0394] *</td>
</tr>
<tr>
<td>2008-10</td>
<td>9.1085 [0.0000] **</td>
</tr>
</tbody>
</table>

Note: *denotes significance level at 10%, **denotes significance level at 5%.
Table 1.6 Structural Change of FS Regression (CUSUM test)

<table>
<thead>
<tr>
<th>Break points</th>
<th>F statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1884-05</td>
<td>10.583 [0.0024] **</td>
</tr>
<tr>
<td>1893-09</td>
<td>13.016 [0.0005] **</td>
</tr>
<tr>
<td>1901-06</td>
<td>6.5122 [0.0124] *</td>
</tr>
<tr>
<td>1929-10</td>
<td>40.574 [0.0000] **</td>
</tr>
<tr>
<td>1931-09</td>
<td>11.856 [0.0024] **</td>
</tr>
<tr>
<td>1938-03</td>
<td>6.7861 [0.0110] *</td>
</tr>
<tr>
<td>1940-05</td>
<td>7.2366 [0.0128] *</td>
</tr>
<tr>
<td>1948-11</td>
<td>6.8363 [0.0103] *</td>
</tr>
<tr>
<td>1955-11</td>
<td>4.5676 [0.0356] *</td>
</tr>
<tr>
<td>1962-05</td>
<td>8.1597 [0.0055] **</td>
</tr>
<tr>
<td>1974-10</td>
<td>20.170 [0.0000] **</td>
</tr>
<tr>
<td>1978-10</td>
<td>8.3963 [0.0057] **</td>
</tr>
<tr>
<td>1987-10</td>
<td>38.296 [0.0000] **</td>
</tr>
<tr>
<td>1988-06</td>
<td>11.272 [0.0153] *</td>
</tr>
<tr>
<td>1990-08</td>
<td>8.0593 [0.0091] **</td>
</tr>
<tr>
<td>1996-11</td>
<td>2.3741 [0.0005] **</td>
</tr>
<tr>
<td>1998-08</td>
<td>15.979 [0.0008] **</td>
</tr>
<tr>
<td>2008-10</td>
<td>26.541 [0.0000] **</td>
</tr>
</tbody>
</table>

Note: * denotes significance level at 10%, ** denotes significance level at 5%. 
Table 1.7 Regression Results for the FS Regression:

\[ r_{t+1} = \alpha + \beta r'_t + \varepsilon_{t+1} \]

<table>
<thead>
<tr>
<th>Period</th>
<th>( a )</th>
<th>( b )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1881:01-1884:05</td>
<td>-0.0091</td>
<td>0.7767</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>(0.0295)</td>
<td>(8.0402)</td>
<td></td>
</tr>
<tr>
<td>1884:06-1893:09</td>
<td>0.0194*</td>
<td>-5.6869*</td>
<td>0.0752</td>
</tr>
<tr>
<td></td>
<td>(0.0112)</td>
<td>(3.166)</td>
<td></td>
</tr>
<tr>
<td>1893:10-1901:06</td>
<td>-0.0012</td>
<td>3.4537</td>
<td>0.0074</td>
</tr>
<tr>
<td></td>
<td>(0.0118)</td>
<td>(4.1944)</td>
<td></td>
</tr>
<tr>
<td>1901:07-1929:10</td>
<td>0.0372</td>
<td>-10.3706</td>
<td>0.0426</td>
</tr>
<tr>
<td></td>
<td>(0.0087)</td>
<td>(2.6731)</td>
<td></td>
</tr>
<tr>
<td>1929:11-1931:09</td>
<td>-0.0711***</td>
<td>21.1297</td>
<td>0.0311</td>
</tr>
<tr>
<td></td>
<td>(0.0478)</td>
<td>(25.75)</td>
<td></td>
</tr>
<tr>
<td>1931:10-1938:03</td>
<td>0.0138</td>
<td>-34.7472</td>
<td>0.0224</td>
</tr>
<tr>
<td></td>
<td>(0.0149)</td>
<td>(26.3431)</td>
<td></td>
</tr>
<tr>
<td>1938:04-1940:05</td>
<td>-0.0215</td>
<td>772.631</td>
<td>0.0371</td>
</tr>
<tr>
<td></td>
<td>(0.0358)</td>
<td>(803.5508)</td>
<td></td>
</tr>
<tr>
<td>1940:06-1948:11</td>
<td>0.0108</td>
<td>-5.9366</td>
<td>0.0009</td>
</tr>
<tr>
<td></td>
<td>(0.0079)</td>
<td>(19.3173)</td>
<td></td>
</tr>
<tr>
<td>1948:12-1955:11</td>
<td>0.0456***</td>
<td>-24.2089**</td>
<td>0.0518</td>
</tr>
<tr>
<td></td>
<td>(0.0139)</td>
<td>(11.441)</td>
<td></td>
</tr>
<tr>
<td>1955:12-1962:05</td>
<td>0.0438***</td>
<td>17.6868***</td>
<td>0.0888</td>
</tr>
<tr>
<td></td>
<td>(0.0151)</td>
<td>(6.4988)</td>
<td></td>
</tr>
<tr>
<td>1962:06-1974:10</td>
<td>0.0348***</td>
<td>-8.4508***</td>
<td>0.0711</td>
</tr>
<tr>
<td></td>
<td>(0.0109)</td>
<td>(2.5201)</td>
<td></td>
</tr>
<tr>
<td>1974:11-1978:10</td>
<td>0.0056</td>
<td>-0.3969</td>
<td>0</td>
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<tr>
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<td>(0.0429)</td>
<td>(8.9095)</td>
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<tr>
<td>1978:11-1987:10</td>
<td>0.034**</td>
<td>-3.7043*</td>
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<td>(0.0163)</td>
<td>(2.0094)</td>
<td></td>
</tr>
<tr>
<td>1987:11-1988:06</td>
<td>0.3715</td>
<td>-74.467</td>
<td>0.0704</td>
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<tr>
<td></td>
<td>(0.539)</td>
<td>(110.4302)</td>
<td></td>
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<tr>
<td>1988:07-1990:08</td>
<td>-0.165</td>
<td>25.9579</td>
<td>0.0734</td>
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<tr>
<td></td>
<td>(0.1218)</td>
<td>(18.82206)</td>
<td></td>
</tr>
<tr>
<td>1990:08-1996:11</td>
<td>-0.0079</td>
<td>4.6526</td>
<td>0.0265</td>
</tr>
<tr>
<td></td>
<td>(0.0131)</td>
<td>(3.3015)</td>
<td></td>
</tr>
<tr>
<td>1996:12-1998:08</td>
<td>-0.9365</td>
<td>225.7721</td>
<td>0.0698</td>
</tr>
<tr>
<td></td>
<td>(0.7917)</td>
<td>(189.1462)</td>
<td></td>
</tr>
<tr>
<td>1998:09-2008:10</td>
<td>-0.0051</td>
<td>1.8239</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.0092)</td>
<td>(3.0138)</td>
<td></td>
</tr>
<tr>
<td>1881:01-2008:12</td>
<td>0.0077***</td>
<td>-1.228**</td>
<td>0.0030</td>
</tr>
<tr>
<td></td>
<td>(0.0022)</td>
<td>(0.5919)</td>
<td></td>
</tr>
</tbody>
</table>

Note: *denotes significance level at 10%, **denotes significance level at 5%, and ***denotes significance level at 1%. Standard errors are in parentheses.
Table 1.8 Regression Results for the Multivariate Regression: \( er_{-t} = \alpha + \sum_{i=1}^{2} \beta_i er_{-t-i} + \sum_{i=0}^{2} (\beta_3 r_{-i} + \beta_4 e_{-i}) + \varepsilon_{t} \)

<table>
<thead>
<tr>
<th></th>
<th>er(-1)</th>
<th>er(-2)</th>
<th>rf(-1)</th>
<th>rf(-2)</th>
<th>sep(-1)</th>
<th>sep(-2)</th>
<th>c</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1881:01-1884:04</td>
<td>31.1919***</td>
<td>0.0208</td>
<td>37.0638**</td>
<td>-12.2072</td>
<td>31.2093***</td>
<td>30.9246***</td>
<td>0.6448**</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>(10.6578)</td>
<td>(0.1679)</td>
<td>(15.3212)</td>
<td>(10.1984)</td>
<td>(10.678)</td>
<td>(10.5876)</td>
<td>(0.2694)</td>
<td></td>
</tr>
<tr>
<td>1884:05-1893:05</td>
<td>-0.3951</td>
<td>-0.1674</td>
<td>9.8872</td>
<td>18.8330**</td>
<td>-0.6759</td>
<td>0.7237</td>
<td>0.1564**</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>(1.6188)</td>
<td>(0.1028)</td>
<td>(8.0462)</td>
<td>(7.6293)</td>
<td>(1.6061)</td>
<td>(1.6101)</td>
<td>(0.0672)</td>
<td></td>
</tr>
<tr>
<td>1893:06-1895:04</td>
<td>-2.9278</td>
<td>0.0460</td>
<td>-18.1653</td>
<td>8.9190</td>
<td>-2.3497</td>
<td>3.1814</td>
<td>2.2191**</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>(18.8487)</td>
<td>(0.2240)</td>
<td>(22.9530)**</td>
<td>(13.6817)</td>
<td>(18.7967)</td>
<td>(18.7326)</td>
<td>(0.5960)</td>
<td></td>
</tr>
<tr>
<td>1895:05-1907:03</td>
<td>1.3800</td>
<td>0.0648</td>
<td>12.2000</td>
<td>-17.4330</td>
<td>1.1242</td>
<td>-1.0855</td>
<td>0.1289*</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>(1.0819)</td>
<td>(0.0899)</td>
<td>(7.2255)</td>
<td>(7.0299)</td>
<td>(1.0853)</td>
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<td>-2.0375</td>
<td>0.2183</td>
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<td>(8.5388)</td>
<td>(1.2979)</td>
<td>(1.2923)</td>
<td>(0.0700)</td>
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<td>6.6444</td>
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<td>-4.8990</td>
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<td>(0.2216)</td>
<td>(49.7044)</td>
<td>(44.9889)</td>
<td>(3.7783)</td>
<td>(3.7915)</td>
<td>(0.4096)</td>
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<tr>
<td>1919:09-1927:07</td>
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<td>-0.1865*</td>
<td>-17.4676</td>
<td>-9.2193</td>
<td>-3.1095*</td>
<td>3.1518*</td>
<td>0.2009***</td>
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<tr>
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<td>(0.1033)</td>
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<td>58.6448</td>
<td>0.2102</td>
<td>26.2479</td>
<td>35.36001</td>
<td>59.1751*</td>
<td>-58.5112*</td>
<td>1.8145*</td>
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<td></td>
<td>(33.8285)</td>
<td>(0.2348)</td>
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<td>(70.6384)</td>
<td>(33.8605)</td>
<td>(33.6400)</td>
<td>(0.7689)</td>
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<tr>
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<td>er(-2)</td>
<td>rf(-1)</td>
<td>rf(-2)</td>
<td>sep(-1)</td>
<td>sep(-2)</td>
<td>c</td>
<td>R²</td>
</tr>
<tr>
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<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>1929:09-</td>
<td>5.5677</td>
<td>-0.3018*</td>
<td>44.7228</td>
<td>-52.746</td>
<td>5.3413*</td>
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<td>0.0255</td>
<td>0.21</td>
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<td>(13.6628)</td>
<td>(0.1761)</td>
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<td>(68.407)</td>
<td>(13.0139)</td>
<td>(13.6476)</td>
<td>(0.1962)</td>
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<td>1933:05-</td>
<td>22.7350**</td>
<td>0.1174</td>
<td>144.2834</td>
<td>19.1104</td>
<td>22.7385**</td>
<td>(11.1428)</td>
<td>(9.3455)</td>
<td>(11.1635)</td>
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<tr>
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<td>(11.1428)</td>
<td>(0.118)</td>
<td>(122.7649)</td>
<td>(79.345)</td>
<td>(9.3455)</td>
<td>(11.1635)</td>
<td>(11.1719)</td>
<td>(0.0939)</td>
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<tr>
<td>1937:11</td>
<td>0.5613*</td>
<td>(0.0488)</td>
<td>(0.0488)</td>
<td>(0.0488)</td>
<td>(0.0488)</td>
<td>(0.0488)</td>
<td>(0.0488)</td>
<td></td>
</tr>
<tr>
<td>1973:12-</td>
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<td>-0.0063</td>
<td>-19.4339***</td>
<td>11.7378***</td>
<td>-4.3772</td>
<td>(12.9778)</td>
<td>(12.9778)</td>
<td>(0.0488)</td>
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<tr>
<td>1967:10</td>
<td>(0.5737)</td>
<td>(0.0118)</td>
<td>(13.0148)</td>
<td>(12.8778)</td>
<td>(12.8778)</td>
<td>(12.8778)</td>
<td>(12.8778)</td>
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<tr>
<td>1987:11-</td>
<td>7.9135***</td>
<td>-0.0444</td>
<td>-42.2990*</td>
<td>42.0064*</td>
<td>7.9272***</td>
<td>(6.2307)</td>
<td>(6.1113)</td>
<td>(6.2406)</td>
</tr>
<tr>
<td>2008:10</td>
<td>(2.6301)</td>
<td>(0.0797)</td>
<td>(21.4761)</td>
<td>(21.2476)</td>
<td>(2.6301)</td>
<td>(2.6064)</td>
<td>(2.6064)</td>
<td></td>
</tr>
</tbody>
</table>

Note: *denotes significance level at 10%, **denotes significance level at 5%, and ***denotes significance level at 1%. Standard errors are in parentheses.
<table>
<thead>
<tr>
<th>Peak</th>
<th>Trough</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 1857(II)</td>
<td>December 1854 (IV)</td>
</tr>
<tr>
<td>October 1860(III)</td>
<td>December 1858 (IV)</td>
</tr>
<tr>
<td>April 1865(I)</td>
<td>June 1861 (III)</td>
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<tr>
<td>June 1869(II)</td>
<td>December 1867 (I)</td>
</tr>
<tr>
<td>October 1873(III)</td>
<td>December 1870 (IV)</td>
</tr>
<tr>
<td>March 1882(I)</td>
<td>March 1879 (I)</td>
</tr>
<tr>
<td>March 1887(II)</td>
<td>May 1885 (II)</td>
</tr>
<tr>
<td>July 1890(III)</td>
<td>April 1888 (I)</td>
</tr>
<tr>
<td>January 1893(I)</td>
<td>May 1891 (II)</td>
</tr>
<tr>
<td>December 1895(IV)</td>
<td>June 1894 (II)</td>
</tr>
<tr>
<td>June 1899(III)</td>
<td>June 1897 (II)</td>
</tr>
<tr>
<td>September 1902(IV)</td>
<td>August 1904 (III)</td>
</tr>
<tr>
<td>May 1907(II)</td>
<td>June 1908 (II)</td>
</tr>
<tr>
<td>January 1910(I)</td>
<td>January 1912 (IV)</td>
</tr>
<tr>
<td>January 1913(I)</td>
<td>December 1914 (IV)</td>
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<tr>
<td>August 1918(III)</td>
<td>March 1919 (I)</td>
</tr>
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<td>January 1920(I)</td>
<td>July 1921 (III)</td>
</tr>
<tr>
<td>May 1923(II)</td>
<td>July 1924 (III)</td>
</tr>
<tr>
<td>October 1926(III)</td>
<td>November 1927 (IV)</td>
</tr>
<tr>
<td>August 1929(III)</td>
<td>March 1933 (I)</td>
</tr>
<tr>
<td>May 1937(II)</td>
<td>June 1938 (II)</td>
</tr>
<tr>
<td>February 1945(I)</td>
<td>October 1945 (IV)</td>
</tr>
<tr>
<td>November 1948(IV)</td>
<td>October 1949 (IV)</td>
</tr>
<tr>
<td>July 1953(II)</td>
<td>May 1954 (II)</td>
</tr>
<tr>
<td>August 1957(III)</td>
<td>April 1958 (II)</td>
</tr>
<tr>
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<td>December 1969(IV)</td>
<td>November 1970 (IV)</td>
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<tr>
<td>November 1973(IV)</td>
<td>March 1975 (I)</td>
</tr>
<tr>
<td>January 1980(I)</td>
<td>July 1980 (III)</td>
</tr>
<tr>
<td>July 1981(III)</td>
<td>November 1982 (IV)</td>
</tr>
<tr>
<td>July 1990(III)</td>
<td>March 1991(I)</td>
</tr>
<tr>
<td>March 2001(I)</td>
<td>November 2001 (IV)</td>
</tr>
</tbody>
</table>

Source: NBER
<table>
<thead>
<tr>
<th>time</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1881:01-1884:04</td>
<td>-5.184</td>
<td>14.3928</td>
</tr>
<tr>
<td>1884:05-1893:05</td>
<td>0.20262</td>
<td>14.1912</td>
</tr>
<tr>
<td>1893:06-1895:04</td>
<td>-3.60132</td>
<td>34.8516</td>
</tr>
<tr>
<td>1895:05-1907:03</td>
<td>5.86548</td>
<td>15.0516</td>
</tr>
<tr>
<td>1907:04-1917:02</td>
<td>2.44548</td>
<td>13.6044</td>
</tr>
<tr>
<td>1917:03-1919:08</td>
<td>2.56704</td>
<td>28.7688</td>
</tr>
<tr>
<td>1927:08-1929:08</td>
<td>33.5448</td>
<td>32.5656</td>
</tr>
<tr>
<td>1929:09-1933:04</td>
<td>-31.0356</td>
<td>78.3336</td>
</tr>
<tr>
<td>1933:05-1937:03</td>
<td>23.6328</td>
<td>30.7032</td>
</tr>
<tr>
<td>1997:08-2008:10</td>
<td>0.19812</td>
<td>8.48352</td>
</tr>
<tr>
<td>1881:01-2008:12</td>
<td>4.67328</td>
<td>22.2516</td>
</tr>
</tbody>
</table>
### Table 2.1

Sample Statistics for the U.S. Economy over the Period 1889-1978

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean risk-free rate</td>
<td>1.008</td>
</tr>
<tr>
<td>Mean return on equity</td>
<td>1.0698</td>
</tr>
<tr>
<td>Mean growth rate of consumption</td>
<td>1.018</td>
</tr>
<tr>
<td>Standard deviation of the growth rate of consumption$^1$</td>
<td>0.036</td>
</tr>
<tr>
<td>Mean equity premium</td>
<td>0.0618</td>
</tr>
</tbody>
</table>

Mean risk-free rate, mean return on equity and mean growth rate of consumption are gross rates.  

1. In Mehra and Prescott's setup, they show that the variance of the growth rate of consumption is equal to the covariance of the growth rate of consumption with return on equity.
Table 2.2

<table>
<thead>
<tr>
<th>Model</th>
<th>$\gamma^*$</th>
<th>$\beta^*$</th>
<th>Consumption data</th>
<th>Lags</th>
<th>Degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30.58</td>
<td>1.001</td>
<td>Nondurable</td>
<td>0</td>
<td>Just identified</td>
</tr>
<tr>
<td></td>
<td>(34.06)</td>
<td>(0.0462)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.205</td>
<td>0.999</td>
<td>Nondurable</td>
<td>4</td>
<td>170.25 (0.9999)</td>
</tr>
<tr>
<td>3</td>
<td>58.25</td>
<td>1.008</td>
<td>ND &amp; Services</td>
<td>0</td>
<td>Just identified</td>
</tr>
<tr>
<td></td>
<td>(66.57)</td>
<td>(0.0687)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.209</td>
<td>1.000</td>
<td>ND &amp; Services</td>
<td>4</td>
<td>366.22 (0.9999)</td>
</tr>
</tbody>
</table>

Source: Hansen and Singleton (1983), Table 5.
*Standard errors are in parentheses.
**Probability values in parentheses.
Table 3.1
Utility of Gains and Losses ($1 investment)

<table>
<thead>
<tr>
<th>$z, \leq 1$</th>
<th>$z, &gt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{t+1} \geq z_i R_i$</td>
<td>$R_{t+1} &lt; z_i R_i$</td>
</tr>
<tr>
<td>$= e \hat{\eta}_{t+1} \geq (z_i - 1) R_i$</td>
<td>$= e \hat{\eta}_{t+1} &lt; (z_i - 1) R_i$</td>
</tr>
<tr>
<td>$v = R_{t+1} - R_i = e \hat{\eta}_{t+1}$</td>
<td>$v = \lambda R_{t+1} + (z_i - \lambda z_i - 1) R_i$</td>
</tr>
<tr>
<td></td>
<td>$= \lambda e \hat{\eta}_{t+1} + (z_i - \lambda z_i - 1 + \lambda) R_i$</td>
</tr>
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</table>
Table 3.2
Parameter Values for the Test of Loss Aversion Model

<table>
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<th>Parameter</th>
<th>Zheng</th>
<th>BHS</th>
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<tr>
<td></td>
<td>0.01</td>
<td>0.91</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>2.25</td>
<td>2.25</td>
</tr>
<tr>
<td>$\gamma$</td>
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<td>0.1</td>
</tr>
<tr>
<td>$k$</td>
<td>3</td>
<td>3, 50, 100, 150</td>
</tr>
</tbody>
</table>
Table 3.3 Regressions of Excess Returns on the Growth Rate of Consumption According to Whether Observations are preceded by Positive or Negative Ex Ante Excess Return (or LARP)

A: Observations Preceded by positive Ex Ante Excess Return

<table>
<thead>
<tr>
<th>model</th>
<th>Observations preceded by positive premium</th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>constant</td>
<td>slope</td>
<td>R^2</td>
<td></td>
</tr>
<tr>
<td>Contemporaneous consumption risk</td>
<td>0.0071**</td>
<td>1.2518</td>
<td>0.01</td>
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</tr>
<tr>
<td>1-year medium risk</td>
<td>0.0161**</td>
<td>-0.1690</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>3-year medium risk</td>
<td>0.0151**</td>
<td>-0.0475</td>
<td>0.0008</td>
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</tr>
<tr>
<td>Loss Aversion</td>
<td>-0.0011</td>
<td>2.2981***</td>
<td>0.029</td>
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</tr>
</tbody>
</table>

B: Observations Preceded by negative Ex Ante Excess Return

<table>
<thead>
<tr>
<th>model</th>
<th>Observations preceded by negative premium</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>constant</td>
<td>slope</td>
<td>R^2</td>
<td></td>
</tr>
<tr>
<td>Contemporaneous consumption risk</td>
<td>-0.01207***</td>
<td>2.5316***</td>
<td>0.04</td>
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</tr>
<tr>
<td>1-year medium risk</td>
<td>-0.0215***</td>
<td>0.4770***</td>
<td>0.024</td>
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</tr>
<tr>
<td>3-year medium risk</td>
<td>-0.0185</td>
<td>0.1149</td>
<td>0.004</td>
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</tr>
<tr>
<td>Loss aversion</td>
<td>-0.0055</td>
<td>1.8340***</td>
<td>0.0258</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses. *** denotes significance at 1% level. ** denotes significance at 5% level. * denotes significance at 10% level. Standard errors are in parentheses.
Table 4.1 Structural Change of the Regression of the Gap Model (CUSUM test)

<table>
<thead>
<tr>
<th>Break points</th>
<th>F statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1905-01</td>
<td>33.724 [0.0000] **</td>
</tr>
<tr>
<td>1907-01</td>
<td>24.057 [0.0001] **</td>
</tr>
<tr>
<td>1915-01</td>
<td>118.94 [0.0000] **</td>
</tr>
<tr>
<td>1922-01</td>
<td>216.71 [0.0000] **</td>
</tr>
<tr>
<td>1929-10</td>
<td>15.290 [0.0002] **</td>
</tr>
<tr>
<td>1935-01</td>
<td>272.70 [0.0000] **</td>
</tr>
<tr>
<td>1937-04</td>
<td>9.7392 [0.0052] **</td>
</tr>
<tr>
<td>1938-10</td>
<td>31.930 [0.0001] **</td>
</tr>
<tr>
<td>1943-10</td>
<td>16.247 [0.0002] **</td>
</tr>
<tr>
<td>1946-07</td>
<td>5.7287 [0.0239] *</td>
</tr>
<tr>
<td>1956-01</td>
<td>4.6817 [0.0327] *</td>
</tr>
<tr>
<td>1959-07</td>
<td>15.856 [0.0003] **</td>
</tr>
<tr>
<td>1961-10</td>
<td>7.1610 [0.0141] *</td>
</tr>
<tr>
<td>1971-01</td>
<td>40.322 [0.0000] **</td>
</tr>
<tr>
<td>1976-01</td>
<td>33.520 [0.0000] **</td>
</tr>
<tr>
<td>1978-10</td>
<td>33.916 [0.0000] **</td>
</tr>
<tr>
<td>1987-07</td>
<td>47.736 [0.0000] **</td>
</tr>
<tr>
<td>1992-01</td>
<td>8.4551 [0.0055] **</td>
</tr>
<tr>
<td>2001-10</td>
<td>32.426 [0.0000] **</td>
</tr>
<tr>
<td>2008-10</td>
<td>315.27 [0.0000] **</td>
</tr>
</tbody>
</table>

Note:*denotes significance level at 10%, **denotes significance level at 5%.
Table 4.2 Structural Change of the Regression of the Gap/MEC Model 
(CUSUM test)

<table>
<thead>
<tr>
<th>Break points</th>
<th>F statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1976-01</td>
<td>35.564 [0.0000]  **</td>
</tr>
<tr>
<td>1978-10</td>
<td>41.564 [0.0000]  **</td>
</tr>
<tr>
<td>1987-10</td>
<td>33.118 [0.0000]  **</td>
</tr>
<tr>
<td>1996-07</td>
<td>10.775 [0.0014]  **</td>
</tr>
<tr>
<td>2001-10</td>
<td>35.399 [0.0000]  **</td>
</tr>
</tbody>
</table>

Note: *denotes significance level at 10%, **denotes significance level at 5%.
Table 4.3 Error Correction Model (Gap)

<table>
<thead>
<tr>
<th>Sample period</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$R^2$</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1901:5-1905:01</td>
<td>0.9131***</td>
<td>0.0009</td>
<td>0.44</td>
<td>2.08</td>
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<td>(0.1668)</td>
<td>(0.1731)</td>
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<tr>
<td>1905:02-1907:01</td>
<td>0.8982***</td>
<td>-0.1859</td>
<td>0.47</td>
<td>2.27</td>
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<tr>
<td></td>
<td>(0.2178)</td>
<td>(0.2253)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1907:02-1915:01</td>
<td>0.6962***</td>
<td>-0.2192**</td>
<td>0.37</td>
<td>2.41</td>
</tr>
<tr>
<td></td>
<td>(0.0985)</td>
<td>(0.1077)</td>
<td></td>
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</tr>
<tr>
<td>1915:02-1922:01</td>
<td>0.4336***</td>
<td>0.0164</td>
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<td>0.9364***</td>
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<td>(0.0992)</td>
<td>(0.1069)</td>
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<td>0.6597***</td>
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<td>0.3543***</td>
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<td>(0.0690)</td>
<td>(0.0043)</td>
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Note: * denotes significance level at 10%, ** denotes significance level at 5%, *** denotes significance level at 1%. Standard errors are in parentheses.
Table 4.4 Error Correction Model (Gap and ME/C)

<table>
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<tr>
<th>Sample period</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$\gamma_3$</th>
<th>$R^2$</th>
<th>DW</th>
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<td>1959:01-1976:01</td>
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<td>1976:02-1978:10</td>
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<td>(0.1677)</td>
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<td>0.5376**</td>
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<td>1987:11-1996:07</td>
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<td>(0.2050)</td>
<td>(0.0809)</td>
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</table>

Note: *denotes significance level at 10%, **denotes significance level at 5%, ***denotes significance level at 1%. Standard errors are in parentheses.
Figure 1.1 *Ex post* excess Return (1881:01-2008:12)
Figure 1.3 *Ex ante* Excess Return
(1881:03-2008:10)
Figure 1.4 *Ex ante* Excess Return

A: 1881:03-1884:03

![Graph A: Ex ante Excess Return (1881:03-1884:03)](image)

B: 1884:04-1893:04

![Graph B: Ex ante Excess Return (1884:04-1893:04)](image)
Figure 1.4 (continued)

C: 1893:05-1895:03

D: 1895:04-1907:02
Figure 1.4 (continued)

E: 1907:03-1917:01

F: 1917:02-1919:07
Figure 1.4 (continued)

G: 1919:08-1927:06

H: 1927:07-1929:07
Figure 1.4 (continued)

I: 1929:08-1933:03

J: 1933:04-1937:02
Figure 1.4 (continued)

K: 1937:03-1973:10

Figure 1.4 (continued)


N: 1997:07-2008:10
Figure 1.5 Means of *ex ante* Excess Return
(1881:01-2008:12)
Figure 3.1 Scatter Plots of Excess Return and the Growth Rate of Consumption (Contemporaneous Consumption Risk)

A: Observations Preceded by positive Ex Ante Excess Return

B: Observations Preceded by Negative Ex Ante Excess Return
Figure 3.2 Scatter Plots of Excess Return and the Growth Rate of Consumption (Loss aversion)

A: Observations Preceded by Positive LARP

B: Observations Preceded by Negative LARP
Figure 3.3 Scatter Plots of Excess Return and the Growth Rate of Consumption (1-year Medium Consumption Risk)

A: Observations Preceded by Positive Ex Ante Excess Return

B: Observations Preceded by Negative Ex Ante Excess Return
Figure 3.4 Scatter Plots of Excess Return and the Growth Rate of Consumption
(3-year Medium Consumption Risk)

A: Observations Preceded by Positive *Ex Ante* Excess Return

B: Observations Preceded by Negative *Ex Ante* Excess Return
References:


Appendices:

Appendix 3.1: Example from BHS (2001), p11

..An example may be helpful. Suppose that the current stock value is $S_t = $100, but the investor has recently accumulated some gains on his investments. A reasonable historical benchmark level is $z_t = $100, since the stock must have gone up in value recently. As discussed above, we can think of $90 as the value of the stock one year ago, which the investor still remembers. The difference, $S_t - z_t = $10 represents the cushion, or reserve of prior gains that the investor has built up. Suppose finally that the risk free rate is zero.

Imagine that over the next year, the value of the stock falls from $S_t = $100 down to $S_{R_{t+1}} = $80. In the case of $z_t = 1$, where the investor has no prior gains or losses, equations (3) and (4) show that we measure the pain of this loss as

$$(80 - 100)(\lambda) = -40$$

for a $\lambda$ of 2.

When the investor has some prior gains, this calculation probably overstates actual discomfort. We propose a more realistic measure of the pain caused: since the first $10 drop, from $S_t = $100 down to $z_t = $90, is completely cushioned by the $10 reserve of prior gains, we penalize it at a rate of only 1, rather than $\lambda$. The second part of the loss, from $z_t = $90 down to $S_{R_{t+1}} = $80 will be more painful since all prior gains have already been depleted, and we penalize it at the higher rate of $\lambda$. Using a $\lambda$ of 2 again, the overall disutility of the $20 loss is

$$(90 - 100)(1) + (80 - 90)(2) = (90 - 100)(1) + (80 - 90)(2) = -30,$$

or in general terms

$$(Z_t - S_t)(1) + (S_t R_{t+1} - z_t)\lambda = S_t(z_t - 1)(1) + S_t(R_{t+1} - z_t)(\lambda).$$
Appendix 4.1: A Gap Model of the Premium on Equities

A 4.1.1 The Speculative Decision Under Endogenous Prospect Theory

When modeling people's behavior of gambling, or speculation decisions, we generally assume that they are risk averse and their preferences over gambles can be represented by the expected utility theory if a number of axioms can be satisfied. However, there is a sizable literature documenting that expected utility theory cannot provide a coherent positive theory of risk averse behavior. In their gap model of premium, FG replace risk aversion and expected utility theory with endogenous prospect theory in modeling peoples' preference. I will follow FG and also use endogenous prospect theory in modeling people's behavior on the stock market.

4.1.1.1 The basic setup of the FG model of premium on the stock market

Assume a speculator holds her nonmonetary real wealth in either stocks or government securities. Government securities are relatively safe assets as people can get a sure return if holding them to maturity. They are to be referred to as bonds hereafter. At time t a speculator's wealth can be expressed as follows:

\[ W^i_t = S^i_t + B^i_t \quad i = 1, ..., N \tag{4.1} \]

where \( S \) = wealth held in stocks and \( B \) = wealth held in bonds. The nominal returns on stocks and bonds from time \( t \) to \( t + 1 \) are denoted by \( r^s_i \) and \( r^f_i \), respectively. We use log approximation to define a speculator's conditional forecast of the real return on stocks in excess of the real return on bonds:

\[ \tilde{r}^s_{i+1} = \tilde{r}^s_t - r^f_t = (\tilde{p}^d_{i+1} - p_t) - r^f_t \tag{4.2} \]

where \( p_t \) denotes log level of the time-\( t \) stock price and \( \tilde{p}^d_{i+1} \) is \( \ln(\tilde{p}_{i+1} + D_{i+1}) \) i.e. natural log sum of stock price and dividend at \( t + 1 \). Under prospect theory, loss-averse agents are more sensitive to losses than to gains of the same magnitude, therefore we assume that the carriers of value are gains and losses in wealth relative to some reference level.

Let \( a^i_t \) be the proportion of wealth that is held in stocks. The definition in (4.1) can be written as follows:

\[ W^i_t = a^i_t W''^i_t + (1 - a^i_t)W''^i_t \quad i = 1, ..., N \tag{4.3} \]

Given individual \( i \)'s portfolio composition, her wealth at time \( t + 1 \) depends on the real returns on holding stocks and bonds:

\[ W^i_{t+1} = S^i_t (1 + r^s_i - p_t) + B^i_t (1 + r^f_i - p_t) \quad i = 1, ..., N \tag{4.4} \]

where \( p_t \) is the non-stochastic rate of inflation.

From (4.3) and (4.4)\(^{62}\) we can rewrite agent \( i \)'s wealth at time \( t + 1 \) in terms of equity premium as:

\[^{62} \text{Since } S^i_t = a^i_t W''^i_t \text{ and } B^i_t = (1 - a^i_t)W''^i_t \text{, (4.4) can be rewritten as} \]
where the random variable $R_{t+1}$ is the excess return on stocks.

With endogenous prospect theory, we assume that people are more concerned about the change in their wealth rather than the level per se. To that end, we need to specify a reference level. Assume that the reference level of wealth for all wealth holders is the level of wealth they would be certain to receive if they held all of their wealth at time $t$ in bonds, i.e. $W_t(1 + r_t)$. With this assumption, the change in wealth for individual $i$, relative to her reference level, is

$$\Delta W_{t+1}^i = W_{t+1}^i - W_t^i(1 + r_t) = a_t W^i R_{t+1}$$  \hspace{1em} (4.6)

Whenever $\Delta W_{t+1}^i > 0$ ($\Delta W_{t+1}^i < 0$), an individual is said to experience a gain (a loss).

A positive realization of $R_{t+1}$, $r_{t+1}^+ > 0$, represents a gain for an individual who holds a long position (i.e., $a_t^+ > 0$) in stocks but a loss if she holds a short position (i.e., $a_t^- < 0$). A negative realization of $R_{t+1}$, $r_{t+1}^- < 0$, leads to the converse. We refer to agents with long and short positions as bulls and bears.

Kahneman and Tversky refer to each possible value of change in wealth, $\Delta W_{t+1}^i$, as a **prospect**. A speculator in the stock market has to decide the set of prospects she takes into account and how she weights these prospects in arriving at her decision. To model an agent’s speculative decision, an economist must specify this set of prospects and the weights a speculator associates with each one of these prospects.

Kahneman and Tversky (1992), followed by FG (2007), assume that an individual considers a finite set of prospects. I make the same assumption in my analysis. An individual attaches a value to each prospect and makes her decision on the basis of aggregate of these utility weighted prospects, known as prospective utility. Kahneman and Tversky assume that each individual uses a set of decision weights to aggregate her utility-weighted prospects. They also assume that these decision weights increase monotonically with the likelihood of each prospect.

Frydman and Goldberg (2007) extend prospect theory to endogenous prospect theory and propose the following specific functional form for the utility function for loss averse agents with imperfect knowledge,

\[
W_{t+1}^i = a_t W_t^i (1 + r_t^+ - p_t) + (1 - a_t^i) W_t^i (1 + r_t^- - p_t)
\]

\[
= a_t W_t^i (r_t^+ - r_t^-) + W_t^i (1 + r_t^-) + W_t^i (1 + r_t^+ - p_t)
\]

\[
= a_t W_t^i R_{t+1} + W_t^i (1 + r_t^- - p_t)
\]
\[ V(\Delta W) = \begin{cases} (W | a |)^{\Delta} | r^s | & \Delta W > 0 \\ -\lambda_1 (W | a |)^{\Delta} | r^l | - \frac{\lambda_2}{\Pi(\tilde{r})} (W | a |)^{\Delta+1} | r^l | & \Delta W < 0 \end{cases} \]

where \( \lambda_1 > 1 \) and \( \lambda_2 > 0 \) are preference parameters and constant. The gains and losses for bulls and bears can be written as

- For a bull: \( a > 0, \quad r^s = r^s_{t+1}, \quad r^l = r^l_{t+1} \)
- For a bear: \( a < 0, \quad r^s = -r^s_{t+1}, \quad r^l = -r^l_{t+1} \)

The degree of loss aversion implied by the utility function takes the following form:

\[ \Lambda = \lambda_1 + \lambda_2 W | a | \]

The utility function implies the following prospective utilities for holding long and short positions:

\[ PU_t^L = (a_t W_t)^{\Delta} [\hat{\Pi}(\hat{r}) - (1 - \lambda) \hat{\Pi}(\hat{r}^-)] + \lambda_2 (a_t W_t)^{\Delta+1} \hat{\Pi}(\hat{r}^-) \]

\[ PU_t^S = (-a_t W_t)^{\Delta} [\hat{\Pi}(\hat{r}) - (1 - \lambda) \hat{\Pi}(\hat{r}^-)] + \lambda_2 (a_t W_t)^{\Delta+1} \hat{\Pi}(\hat{r}^+) \]

where \( \hat{\Pi}(\cdot) \) are decision-weighted sums of prospects:

\[ \hat{\Pi}(\hat{r}) = \sum_k \hat{\pi}_{t|t+1,k} \hat{r}_{t|t+1,k} \]

\[ \hat{\Pi}(\hat{r}^-) = \sum_k \hat{\pi}_{t|t+1,k} \hat{r}^-_{t|t+1,k} \]

The decision problem facing each individual at time \( t \) is to choose the portfolio share, \( a_t^* \), that maximizes her prospective utility, given her assessments of the prospective potential gain and potential loss from a unit position in stock. We note that the prospective utilities, \( PU_t^L \) and \( PU_t^S \), are defined only for \( a_t^* > 0 \) and \( a_t^* < 0 \), respectively. Thus, an individual’s decision problem involves solving two constrained maximization problems, one for long positions using equation (4.10) and one for short positions using (4.11). An individual will then choose the long (short) solution if the long (short) solution delivers greater prospective utility. To this end, differentiating the prospect utility functions in (4.10) and (4.11) with respect to \( a_t^* \) and setting the results equal to zero yields the following solutions for the optimal portfolio share:

\[ a_t^L = \frac{\alpha}{\lambda (\alpha + 1) W} [\hat{\Pi}(\hat{r}) - (1 - \lambda) \hat{\Pi}(\hat{r}^-)] \]

and

\[ a_t^S = \frac{\alpha}{\lambda (\alpha + 1) W} [\hat{\Pi}(\hat{r}) - (1 - \lambda) \hat{\Pi}(\hat{r}^+)] \]
Following FG, I replace the decision-weighted sums with IKE representations of an individual's forecasts of the return and potential unit loss from holding speculative positions:

\[ a^L_i = \frac{\alpha}{W\lambda_3} [\hat{r}^L_{i|t+1} - (1 - \lambda_1)\hat{j}^L_{i|t+1}] \]  
\[ (4.16) \]

or

\[ -a^S_i = \frac{\alpha}{W\lambda_3} [\hat{r}^S_{i|t+1} - (1 - \lambda_1)\hat{j}^S_{i|t+1}] \]  
\[ (4.17) \]

where \( \lambda_1 = \frac{\lambda_2(1+\alpha)}{\alpha} \), \( \hat{r}^L_{i|t+1} \) (\( \hat{j}^L_{i|t+1} \)) is bull's (bear's) forecast of the return and \( \hat{r}^S_{i|t+1} \) (\( \hat{j}^S_{i|t+1} \)) is bull's (bear's) forecast of potential losses.

The solutions in (4.16) and (4.17) make it clear that with endogenous prospect theory, individuals limit the amount of capital they are willing to gamble when they perceive a profit opportunity. They will only take a position when expected return is greater than potential losses scaled by the degree of loss aversion. These solutions imply the following decision rules:

- stay out of market when \( \hat{r}^L_{i|t+1} \leq (1 - \lambda_1)\hat{j}^L_{i|t+1} \) and \( \hat{r}^S_{i|t+1} \leq (1 - \lambda_1)\hat{j}^S_{i|t+1} \);
- hold a long position in stock of size \( a^L_i \) when \( \hat{r}^L_{i|t+1} > (1 - \lambda_1)\hat{j}^L_{i|t+1} \)

or

- hold a short position in stock of size \( -a^S_i \) when \( \hat{r}^S_{i|t+1} > (1 - \lambda_1)\hat{j}^S_{i|t+1} \).

Equations (4.16) and (4.17) show that the minimum returns an individual requires for taking long or short position, denoted by \( u_{P^L_{i|t+1}} \) and \( u_{P^S_{i|t+1}} \), can be written as:

\[ u_{P^L_{i|t+1}} = (1 - \lambda_1)\hat{j}^L_{i|t+1} > 0 \]  
\[ (4.18) \]

\[ u_{P^S_{i|t+1}} = (1 - \lambda_1)\hat{j}^S_{i|t+1} > 0 \]  
\[ (4.19) \]

These minimum returns arise from an individual's uncertainty concerning the magnitude of the potential losses. FG refer to these premia as uncertainty premium.

The decision rule reveals three important implications of endogenous prospect theory for speculative behavior. In terms of stock market, first, an individual's assessment of the prospective potential loss and her degree of loss aversion may not be large enough so that although she may estimate \( \hat{r}^L_{i|t+1} > 0 \), she nonetheless decides to stay out of stock market.

Second, nonparticipation arises because speculators require a prospective return in excess of some minimum positive value in order to take open positions in stock.
Third, once an individual perceives a profit opportunity and enters the market under endogenous prospect theory, she takes a position of limited size. This limited position does not rise from the trade-off between the expected mean and variance of the portfolio return, as with risk aversion. Under endogenous prospect theory, individual limit their open position because their degree of loss aversion increases with the size of their positions.

These two implications – that endogenously loss-averse individuals limit the size of their speculative positions and that they require an uncertainty premium to compensate them for their greater sensitivity to losses – form the basis for a new equilibrium condition in the stock market.

**A 4.1.2 Equilibrium in the Stock Market**

**4.1.2.1 Momentary equilibrium in the stock market**

The momentary equilibrium condition in the stock market can be written as follows:

\[
\sum_i (S_i^{d_i} - S_i^i) = S_i^{d_i} - S_i = 0 \quad (4.20)
\]

where \( S_i^{d_i} \) denotes an individual's demand for stocks at time \( t \) (i.e., \( S_i^{d_i} = a_i^i W_i^i \)), \( S_i^i \) denotes her holdings of stocks entering time \( t \) (i.e., at time \( t - h \), where \( h \) denotes an infinitesimally small interval of time) and \( S_i^{d_i} \) and \( S_i \) denote the total demand for and supply of stocks at time \( t \).

We now substitute solutions in (4.16) and (4.17) into (4.20) to obtain the equilibrium condition under endogenous prospect theory. In doing so, we must account for the fact that, in general, individuals who decide to stay out of the market at any time \( t \) have nonzero assessments of \( r_{ij+t} \). This feature of the solutions under endogenous prospect theory leads to the following expression for equilibrium in the stock market:

\[
\sum_{i} \left[ \frac{1}{\lambda_3} \left( \hat{r}_{ij+t}^L - (1 - \lambda) \hat{r}_{ij+t}^M \right) W_i^{L,i} - S_i^i \right] + \sum_{i} \left[ \frac{1}{\lambda_3} \left( \hat{r}_{ij+t}^S - (1 - \lambda) \hat{r}_{ij+t}^M \right) W_i^{S,i} - S_i^S \right] + \sum_{i} 0 - S_i^{O,i} = 0 \quad (4.21)
\]

Multiply equation (4.21) by \( \lambda_3 \) and then divide it by \( W_i^M \), we get

\[
\sum_{i} \left[ \frac{1}{\lambda_3} \left( \hat{r}_{ij+t}^L - (1 - \lambda) \hat{r}_{ij+t}^M \right) W_i^{L,i} - S_i^{L,i} \right] + \sum_{i} \left[ \frac{1}{\lambda_3} \left( \hat{r}_{ij+t}^S - (1 - \lambda) \hat{r}_{ij+t}^M \right) W_i^{S,i} - S_i^{S,i} \right] + \sum_{i} \left( 0 - \lambda_3 \frac{S_i^{O,i}}{W_i^M} \right) = 0 \quad (4.22)
\]

From equation (4.22),
\[
\left( \frac{W_t^L}{W_t^M} \hat{r}_{t+1}^L - \frac{W_t^S}{W_t^M} \hat{r}_{t+1}^S \right) - (1 - \lambda_1) \left( \frac{W_t^L}{W_t^M} \hat{r}_{t+1}^L - \frac{W_t^S}{W_t^M} \hat{r}_{t+1}^S \right) = \lambda_3 \frac{S_t}{W_t^M}
\]

(4.23)

where \( N_t^L \) (\( N_t^S \)) denotes the number of individuals who take a long (short) position in stock at time \( t \). \( O_t \) denotes all individuals who hold no position at time \( t \), \( W_t^M \) denotes the total real wealth of all individuals who take an open position at time \( t \).

Let
\[
\hat{r}_{i,t+1} = \frac{W_t^L}{W_t^M} \hat{r}_{i,t+1}^L - \frac{W_t^S}{W_t^M} \hat{r}_{i,t+1}^S,
\]

\[
u^L_{i,t+1} = (1 - \lambda_1) \left( \frac{W_t^L}{W_t^M} \hat{r}_{i,t+1}^L - \frac{W_t^S}{W_t^M} \hat{r}_{i,t+1}^S \right),
\]

\[
u^S_{i,t+1} = \frac{W_t^L}{W_t^M} \hat{r}_{i,t+1}^L
\]

\[
u^S_{i,t+1} = \frac{W_t^S}{W_t^M} \hat{r}_{i,t+1}^S
\]

Equation (4.23) can be written as an equality between the aggregate forecast of the return on holding stock and the market premium:
\[
\hat{r}_{i,t+1} = p\hat{r}_{i,t+1}
\]

(4.24)

where
\[
p\hat{r}_{i,t+1} = \nu^L_{i,t+1} + \lambda_3 \frac{S_t}{W_t^M}
\]

(4.25)

In equilibrium, the premium on stocks therefore is determined by the uncertainty premium required by the bulls in relative to the bears, plus the term of supply of stocks as a proportion of market wealth.

This market equilibrium condition shows that the market premium depends upon the aggregate uncertainty premium and the proportion of supply of stocks in the market participants' total wealth. The market premium is negative if the uncertainty premium for the bears is greater than that of the bulls, and the absolute value of this negative aggregate uncertainty premium is greater than the supply of stocks. The sign of the expected market premium is determined by the sum of the uncertainty premium and the stock supply. In the next Chapter, I will test the implication of sign reversals of the premium.

4.1.2.2. Gap effect

To model the forecasts that underpin uncertainty premium for bulls and bears, FG use IKE to formalize an insight due to Keynes (1936) that what matters in
assessing the riskiness of speculative positions is the divergence between an asset price and its perceived historical benchmark level, a variable denoted as $gap$.

FG suppose that the potential losses for holding long and short positions, denoted as $\hat{\ell}_{t+1}$ and $\hat{s}_{t+1}$, depend on bulls' and bears' assessments of the gap and impose the following qualitative constraints on change in these representations:

$$\frac{\Delta \hat{\ell}_{t+1}}{\Delta gap^L_{t}} < 0 \text{ and } \frac{\Delta \hat{s}_{t+1}}{\Delta gap^S_{t}} > 0$$ (4.26)

where $\Delta$ denotes a first difference operator and $gap^L_t = \hat{p}^L_{t+1} - p^B_{t+1}$ and $gap^S_t = \hat{p}^S_{t+1} - p^B_{t+1}$ are defined in terms of bulls' and bears' forecasts of the stock price, $\hat{p}^L_{t+1}$ and $\hat{p}^S_{t+1}$, and their assessments of the historical benchmark, $p^B_{t+1}$ and $p^B_{t+1}$. The gap conditions in (4.26) imply that in equilibrium, the premium on stocks will, ceteris paribus, co-vary positively with the aggregate gap:

$$\frac{\Delta pr_{t}}{\Delta gap_{t}} > 0$$ (4.27)

where $gap_t = \frac{1}{2} (gap^L_t + gap^S_t)$. The positive co-variation arises because if, for example, participants revise up their assessments of the gap, bulls increase and bears decrease their estimates of the riskiness of their positions and so $u_p^L_{t+1}$ rises and $u_p^S_{t+1}$ falls. Both of these movements lead to a rise in the aggregate premium.

In real-world markets, participants alter their forecasting strategies over time, at least intermittently, as new information arises and the social context changes. Consequently, although the qualitative relationship between the market premium and the gap may be positive, we would not expect that the precise quantitative impact of $gap_t$ on $pr_t$ would remain unchanged at every point in time.

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63 IKE's use of qualitative constraints of its representations of individuals' forecasting behavior enables an economist to recognize that individuals must cope with imperfect knowledge about the processes driving market outcomes without having to presume that they are irrational.

64 In deriving the positive relationship in (4.27), Frydman and Goldberg (2007, chapter 12) address distributional issues.