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Two Answers are Better than One

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What powerful mathematical tool do you use everyday? If you have ever considered how much food you can eat and still leave room for dessert, or made a rough calculation of how long it will take to travel to school, you are using *estimates* to help guide and inform your decisions.

Estimating means *approximating* your answer rather than making a precise calculation. Making an estimate begins with a specific question, like how long will you need to save in order to buy something that costs about $100. You earn about $20 per week from allowance, babysitting or other chores, and can usually save about half of that, then it will take you about 10 weeks to save the total amount:

$100 / $10 per week = 10 weeks

As you save, you may have to revise your estimate based on your actual earnings and savings each week.

Suppose your class is having a party at school, and you want to serve pizza, chips, juice and soda pop, and ice cream. About how much money will you need to raise to buy enough food for 100 people? You need to estimate how much food you will need, and the costs per item of the food. One pizza and one bottle of juice or soda pop will feed about 4 people, so you need 100 people/4 people per pizza or soda pop ~ 20-30 pizzas and 20-30 soda pops. A bag of chips and a half-gallon of ice cream are each enough for about 10 people, so you need ten of each of these. Suppose each pizza costs $7, a bag of chips is $2, a half-gallon of ice cream costs $4, and drinks costs $1 per 2 liter bottle. Then you need about:

30 x $7 + 10 x $2 + 10 x $4 + 30 x $1 = $300.00

As another check on your answer, express it in a different way: $300.00 for the party for 100 people is $3.00 per person, which seems pretty reasonable. Also, if you only ordered 25 pizzas, you would have some money left (how much?) for cups, paper plates, and napkins.

Scientists use estimates as a check on more precise measurements; estimation is an easy way to quantify early hypotheses that can be tested or checked later in formal analysis. This is where two answers are better than one. An initial estimated answer can be used to help interpret a second calculated answer, or identify problems with a methodology when the two don’t match. Estimating is also useful when you have a problem with several components. You can estimate the importance of each component and concentrate your research and thinking on the most important part of the problem.

When working with complex problems, especially those involving large numbers, it is useful write your numbers with only one significant figure, that is to say, your number will have only one numeral that is not a zero. For example, if you know the length of something is 2350 meters you would round that to 2000 meters (the '2' is the only significant figure). Now suppose you want to estimate one-seventh of that length to one significant figure, your answer will be 2000 ÷ 7 ~ 300 meters, though the precise answer would be 2350 ÷ 7 = 335.71428571... meters. Your answer of 300 meters conveys the same level of precision (1 significant figure) as your initial estimate of 2000 meters.

Using significant figures requires that you round each number to the nearest ten, hundred, thousand, etc. For example, both 8 and 12 are approximated by 10, and 75 is approximately 80; a year has approximately 50 weeks. There are specific rules for rounding numbers. However, you should realize that if you were to round every number down, your estimate would be too low, and if you were to round every number up, your estimate would be too high. So try to make sure that you round some things up and some things down; this way your final answer will be a better approximation.

Here are two more examples:

1. **You want to paint your room.** The room is rectangular, about 10 feet long by 10 feet wide by 8 feet high. There are two doors, 3 feet by 6 feet, and two windows, 2 ft by 3 ft. Your mom says, ‘Okay, you can paint the room, but you buy the paint and supplies.’ How much will it cost? Some useful information: paint is sold in gallons (1 gallon of paint covers about 450 square feet, and costs about $20) and quarts (1 quart covers about 100 square feet, and costs about $7).

The area of the walls and ceiling is about:

$$4 \text{ walls x (10 ft x 8 ft per wall) + 1 ceiling x (10 ft x 10 ft)} \sim 400 \text{ ft}^2$$

The area of the doors is about:

$$2 \text{ doors x (3 ft x 6 ft per door)} \sim 40 \text{ ft}^2$$

The area of the window trim is determined by multiplying the trim width by the distance around the window (perimeter) and is about:

$$2 \text{ windows x (0.5 ft wide x 10 ft perimeter)} \sim 10 \text{ ft}^2$$

So 1 gallon of paint should be enough for a single coat of paint on the walls and ceiling, and 1 quart of paint should be enough for two coats of paint on the doors and trim. The total cost would be about $30 - $50, depending on whether the walls needed one or two coats of paint. You might also need some money ($10 - $20) for paint brushes and/or paint rollers, masking tape, and drop cloths.

2. **How much water flows out of the Mississippi River in a day?** To estimate this, we have to estimate several characteristics of the river. Assume that near its mouth the river is about 1 kilometer (km) wide and perhaps 10 meters (or 0.01 km) deep. We could estimate that the rate of flow was 8 km per hour (~200 km/day); this is equivalent to a very fast walk (5 mph) and seems reasonable. Then to estimate the flow we would multiply the width by the depth by the rate of flow:

$$1 \text{ km x } 1/100 \text{ km x } 200 \text{ km/day} \approx 2 \text{ km}^3\text{/day}$$

Our estimate shows that the Mississippi River discharges about 2 cubic kilometers per day. But is it right? How would you check? We could check with another estimate. Almost all the water that flows out of the Mississippi River fell as rain or snow (some was pumped from groundwater aquifers, but we can ignore this). So the amount of rain and snow that falls on the area that is drained by the Mississippi River (called the drainage basin) should be about this amount. (Actually, it should be about a factor 2 bigger, because typically more than half of the rain that falls evaporates back into the atmosphere before getting to the river.) We can estimate this by multiplying three numbers: the area of the drainage basin, the depth of water in a typical rain or snow storm, and the typical storm frequency.

What is the area of the Mississippi River drainage basin? To estimate this, picture a map of the USA in your mind, or look at an atlas. The country (except Hawaii and Alaska) is roughly rectangular and is about 2000 km (north-south) by 5000 km (east-west), giving an area of about 10,000,000 square kilometers. The Mississippi drainage basin is *roughly* 1/4 of the country, or about 3,000,000 km^2. It rains or snows *about* once a week, and here in New Hampshire we get about 1 meter of precipitation (rain plus snow) per year. If we assume the New Hampshire annual total is typical for the Mississippi basin (some areas will be wetter, some drier; what is the annual precipitation in your area?) then a typical storm might produce 2 cm (= 1 m divided by 50 storms) or 0.00002 km of water. So the average daily precipitation on the Mississippi River drainage basin is *about:*

$$3{,}000{,}000 \text{ km}^2 \times 0.00002 \text{ km water/storm} \times (1 \text{ storm/7 days}) \approx 8 \text{ km}^3\text{/day}$$

Given all the estimating involved, these two results (2 and 8 km^3/day) are close enough to be considered consistent with each other. Neither is precisely right, but both are probably reasonable estimates.