Spring 2019

Determination of Multi-Messenger Signals from Matter Outflows of Merger Systems

Ronny Nguyen

UNH Department of Physics

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Determination of Multi-Messenger Signals from Matter Outflows of Merger Systems

Ronny Nguyen
Advisors: Francois Foucart, Harald Kucharek

University of New Hampshire Department of Physics
UNH Institute for the Study of Earth, Oceans, and Space
May 22, 2019
Abstract

In 2017, LIGO detected gravitational waves from GW170817. This presented for the first time, gravitational waves originating from a neutron star - neutron star merger. Studies of neutron star mergers are significant because the multi-messenger signals in the form of gravitational waves and electromagnetic waves can inform us on the nuclear physics of neutron stars and the creation of heavy elements in the universe. Matter is ejected in the merging process and forms the outflow which provides a neutron-rich environment for rapid neutron capture (r-process) to occur leading to the nucleosynthesis of heavy elements. What we detect on Earth are kilonova emissions powered by the radioactive decays of these heavy, unstable elements. Therefore, simulations that accurately model the time evolution of matter outflows are needed to study neutron star mergers. In this study I have performed a simulation of a binary neutron star merger using the SpEC-Hydro code. The merger system consists of a $1.2M_\odot$ and $1.4M_\odot$ neutron star which are counter-rotating with respect to one another. With data from the simulation I used classical mechanics (accounting for relativistic corrections) to recreate the time evolution of particle trajectories in order to model the matter density distribution at any arbitrary time. This method is not limited to binary neutron star mergers and can be applied to any generic merger system. I will also be covering the work I have done in [1] regarding neutrino transport schemes. We used a Monte-Carlo (MC) algorithm to analytically solve Boltzmann’s equations of radiation transport. I analyzed the neutrinos emitted by a binary neutron star merger remnant and compared the neutrino distribution function of the MC scheme to a two-moment scheme.

* rn1007@wildcats.unh.edu
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I. BACKGROUND

A. Recent Developments in Multi-Messenger Astronomy

1. Gravitational Waves - A Brief Introduction

One of the core tenants of general relativity is that all objects in the universe possessing energy creates a curvature in spacetime. Gravitational waves are the ripples in curved spacetime caused by massive objects in motion. These ripples transport energy in the form of gravitational radiation and propagate at the speed of light. While any object with energy can produce gravitational waves, these signals are extremely weak and negligible for most systems. The strongest sources of gravitational waves that we are able to detect are from astronomical stellar events such as binary neutron star mergers, black hole mergers, and supernovae. Other events such as the big bang and early universe formation may also possibly be able to be studied with this new medium. Even with these stellar events as references in the search for ripples in spacetime, gravitational waves still prove difficult to detect with only a handful being published in the past 4 years. The difficulty of detecting gravitational waves comes from the weak interaction of the gravitational force, being a factor of $10^{-36}$ weaker than the electromagnetic force. However, by the same virtue of this weak interaction are we able to detect these signals because gravity is also a long range interaction that propagates across spacetime. When gravitational waves are produced by a stellar event, absorbing the radiant energy in the form of gravitational radiation is difficult. This is an advantageous property that gravitational waves have over other forms observational mediums such as electromagnetic waves. Therefore, gravitational waves allows us to observe objects in locations where electromagnetic signals would otherwise be absorbed or dispersed.

In summation, the nature of gravitational waves provides a window into the universe by allowing us a medium in which to study the formation of the early universe shortly after the big bang, the formation of heavy elements, and stellar evolution. Perhaps more importantly, gravitational waves provides a confirmation on the theory of general relativity which predicted the existence of these phenomena over a century ago.
2. 2015 *LIGO/Virgo Detection of GW150914*

On September 14, 2015, the LIGO and Virgo collaborations detected gravitational waves from a binary black hole merger designated GW150914. This constituted the first ever observation of gravitational waves and demonstrated the existence of binary black hole systems [2]. LIGO and Virgo confirmed this detection by matching the observed data from the black hole merger to gravitational wave templates created by simulations. Moving forward from the match made by simulations against the black hole merger data, LIGO and Virgo proceeded to plot the strain amplitude vs. time as well as the frequency vs. time (refer to Fig. 1 in [2] for visualization of graph). As the black hole binaries began spiraling inwards towards each other, the frequency of the signal begins to increase until the two objects merge together. This increase in frequency leads to what is commonly referred to as the “chirping” of the gravitational wave signal because the pitch increases along with frequency.

With real world data now available to the scientific community, numerical simulations of merger systems such as that performed by SpEC-Hydro, a relativistic hydrodynamics code, have taken on and have been an integral part in verifying gravitational wave detections. At the time of this writing, a total of 11 gravitational wave detections have been published from black hole mergers. Neutron star mergers have since also been adding on to these numbers. A variety of other theoretical sources such as neutron star - black hole mergers are currently undergoing studies but have so far not been detected yet.

3. 2017 *Joint Detection of GW170817 and GRB170817A*

On August 2017, the LIGO and Virgo collaboration detected a gravitational wave signal from GW170817 [3]. This detection was an important event for two reasons. First, it marked the first time gravitational waves were detected originating from a binary neutron star merger and second, the Fermi Gamma-ray Space Telescope reported a gamma-ray burst from the same source 1 second prior to the gravitational wave detection. The gamma-ray burst was later designated as GRB170817A. The joint detection from LIGO/Virgo and Fermi was the first time an electromagnetic counterpart to a gravitational wave source was observed [4]. Having an optical counterpart to a gravitational wave provides a verification of the detection
being real. Follow up studies to the LIGO/Virgo and Fermi detection confirmed that the electromagnetic counterpart detection was a kilonova which are optical and infrared signals from radioactive decay of unstable elements [5] [6]. The electromagnetic counterpart to GW170817 in particular was significant because it was an indication for the presence of matter and a confirmation of a neutron star source. The presence of matter is significant in determining the composition of the matter outflows and how the system evolves. The composition of matter will in turn change the composition of kilonova emissions that we detect on Earth. Kilonova emissions are created by the nuclear reactions that occur in the matter outflow. Rapid neutron capture by atomic nuclei results in the nucleosynthesis of heavy elements which then undergo radioactive decay into stable elements. It is the radioactive decay of these heavy elements in the outflow that powers kilonova emissions [7].

LIGO and Virgo inferred the masses of the neutron stars in the binary system to be between 1.17 – 1.60M\(_\odot\) with a total mass of 2.74\(^{+0.04}_{-0.01}\)M\(_\odot\). The masses of the neutron stars I used to for the SpEC simulation were 1.2M\(_\odot\) and 1.4M\(_\odot\) with the total mass of the system being 2.6M\(_\odot\) which is typical of neutron star mergers in the Milky Way. The model in this thesis differs from the GW170817 observation in that we have set both neutron stars to be counter-rotating whereas the neutron stars of GW170817 were not. This sets the stage for the analysis of possible future detections that have rotation in the system. Developing accurate models of neutron star merger system is essential because we use the modeled data and compare it to data from actual detections. The primary results of this thesis recreates the matter density distribution at any arbitrary time after merging which allows us to see how the system evolves over long period of time thus giving us an insight into the nuclear processes responsible for kilonova signals originating from neutron star mergers.

B. Neutron Star Merger Physics

1. Evolution and Outflow Processes

Binary neutron star mergers are phenomena where a neutron star is engaged in an inward spiral with another neutron star. As the orbits of the bodies get smaller, the orbital period decreases while tidal forces increases. The tidal forces that the neutron stars exert on each other start to stretch the shape of the objects – this can be observed in Fig. 3a and 3b
plotted from the data of our SpEC simulation. The frequency of the gravitational wave also increases rapidly as the orbits decrease. This is known as the “chirp” of a gravitational wave which is visualized in Fig. 1.

**FIG. 1:** The “chirping” of a generic gravitational wave. The frequency increases as the orbits of the binary merger objects gets smaller until they eventually merge together. All credit goes to A. Stuver of the LIGO collaboration for the publication this plot [2].

Using gravitational wave signals we can extrapolate the masses, spins and radii of the merger objects (albeit with large uncertainties for the spins and radii). With this information we can calculate the mass ratio of a merger system,

\[ q = \frac{m_2}{m_1} \]  

where \( m_1 \) and \( m_2 \) are the masses of the neutron stars and \( m_1 \geq m_2 \). The mass ratio of GW170817 was 0.7–1.0 with a total mass of the system being \( 2.74^{+0.04}_{-0.01}\text{M}_\odot \) [3] which is consistent with the population of neutron star binaries observed in the galaxy. We now have two ways in which we can identify the source objects of a gravitational wave – first by observing an electromagnetic counterpart (an indication of the presence of matter in the ejecta) and second by observing the mass ratio. Determining the size of the bodies in a merger system (particularly in neutron star mergers) is of interested to us because it gives us information on the composition and the size of the cores of the merger objects thus informing us on the nuclear physics that occur.
When the neutron stars collide two tidal tails form which eject matter. The matter can either be bounded and accrete which forms the disk around the post-merger object [8] [9] or become unbounded which forms the outflow of matter leaving the system. Inside the matter outflow, rapid neutron capture (r-process) occurs between neutrons, protons, and electrons resulting in the nucleosynthesis and creation of heavy elements. The heavy elements then undergo radioactive decay into stable elements which we observe on Earth. Entropy increases in the outflow while the temperature decreases due to the expansion of heated matter.

The ejected matter in the outflow significantly influences the optical emissions that are able to be detected. The mass of the ejecta determines the longevity of a kilonova emission – more mass being present results in brighter and longer emissions. The speed of the ejected matter will change the timescale of the emission as well. In terms of composition, more neutrons present results in emissions that are visible in the infrared spectrum while less neutrons results optical emissions.

After collision, the two neutron stars begin to merge into one singular object which is visualized in Fig. 3c and 3d. At this point in the evolution the cores of the neutron stars have completely merged while matter continues to be ejected and accreted around the post-merger object.

2. Neutrino Transport

Neutrinos play an important role in the composition of the matter outflows of neutron star mergers. First, nucleosynthesis is dependent on the number of neutrons and protons in the outflow – neutrinos are directly coupled to the number of these nucleons through absorption and emission. Protons and neutrons can capture neutrinos which changes the amount of each nucleon in the outflow. This becomes significant in the r-process that occurs; without a neutron-rich environment, neutron capture becomes less prevalent. As previously discussed, neutron capture results in the formation of heavy elements. Less neutrons leads to lower mass elements that are still unstable and undergo radioactive decay. Lower mass elements have a lower optical depth which allows photons to escape earlier in the merger evolution when matter in the outflow is hotter. Neutron-poor environments caused by neutrino absorption leads to kilonova emissions at higher temperatures and frequencies resulting in optical emissions instead of infrared emissions. Second, in addition to lower
thermal emissions due to lower rates of radioactive decay, neutrons take part in cooling of the post-remnant object by the following reactions,

\[ n + e^+ \rightarrow p + \bar{\nu} \]
\[ p + e^- \rightarrow n + \nu. \]  

(2)

Neutrino treatments such as that used by [1] and [10] utilizes a full general relativistic model of neutron star mergers with an approximate neutrino transport scheme. These treatments use a two-moment scheme which includes the energy density \( J \), momentum-density \( H \), and stress-energy tensor \( S \). These are known as the 0th, 1st, and 2nd moments respectively. They are explicitly written as momentum-space integrals as follows,

\[ J_{(\nu_i)} = \int d\nu \nu^3 \int d\Omega f_{(\nu_i)} \]
\[ H_{(\nu_i)}^\mu = \int d\nu \nu^3 \int d\Omega f_{(\nu_i)} l^\mu \]  
\[ S_{(\nu_i)}^{\mu\nu} = \int d\nu \nu^3 \int d\Omega f_{(\nu_i)} l^\mu l^\nu. \]  

(3)

Where \( f_{(\nu_i)} \) is the neutrino distribution function in momentum space, \( l^\mu \) and \( l^\nu \) are the direction of propagation of the neutrinos with respect to the 4-velocity of the fluid, and \( \nu \) is the neutrino energy in the fluid frame. The two-moment scheme is used to couple the evolution of neutrinos to the fluid. It should be noted that we are not given all the variables needed to close the system of equations and because of this we choose our own analytical closure called M1 in [1]. Further discussion of the closure along with my previous work is discussed in further detail in Sec. II B.

II. METHODS

A. Neutron Star Merger Simulation

In this study I simulated a counter-rotating binary neutron star merger consisting of 1.2\(M_\odot\) and 1.4\(M_\odot\) neutron stars. This simulation was created using SpEC-Hydro, a relativistic hydrodynamics code [11], [12]. SpEC creates a set numerical grid where the simulation takes place. We ran the simulation first on the Trillian computer cluster at the UNH Institute for the Study of Earth Oceans and Space. We later moved the simulation to the
Blue Waters computer cluster at the University of Illinois for further computational needs as the neutron star merger simulation evolved. The neutron stars begin at a distance of 50 km apart with initial orbital velocities of 0.12c. Each neutron star is also counter-rotating with respect to the direction of their orbits. The SpEC simulation allowed me to observe directly (with returned data) the evolution of the neutron star merger at different phases. I show the results of the simulation in Fig. 3 at different timestamps.

B. Neutrino Transport using a Monte-Carlo Algorithm

As part of this thesis I will discuss the work of a previous paper I have worked on [1]. This work pertained to neutrino transport using two different methods, the two-moment scheme with M1 closure and a newly developed MC algorithm (at the time of writing). The system we applied these methods to was another binary neutron star merger. I developed models for the analysis for neutrinos in the matter outflow for both methods. Specifically, I analyzed the energy distribution of neutrinos as well as their angular distributions. In this study I used the same method for analyzing the angular distribution of matter particles in my neutron star merger simulation.

In [1], \( J \) and \( H \) in Eq. 3 were evolved and we chose an analytical closure for \( S \) to close the system of equations which we call M1. We used an MC algorithm to solve for Boltzmann’s equations of radiation transport and compare it to the two-moment scheme with the M1 closure. As previously mentioned in Sec. 1B.2, the two-moment scheme is what couples neutrino evolution to the fluid so what closure we use to close the system of equations will impact the evolution of the neutron star merger.

C. Matter Density Distribution

SpEC outputs relevant data that I used to recreate the matter density distribution. It creates a numerical grid which is set where the simulation takes place. I want to analyze the evolution of the system beyond the grid boundary and so to do this I modeled the trajectory of particles using Newton’s law of gravitation and Kepler orbits conditions. It should be noted that by modeling the particles using classical mechanics that our velocities are in turn classical which does not match that of SpEC which is a fully relativistic code. This
would have implications with the initial conditions of the trajectories, \((x_0, y_0, z_0, v_{x0}, v_{y0}, v_{z0})\).

Therefore, I perform a linear transformation to ensure that energy is conserved between the classical and relativistic results. This returns the approximated relativistic velocities. With the relativistic velocities set, we use Newton’s law of gravitation which is integrated resulting in a 6D coupled differential equation. I solved this coupled system of six differential equations numerically using the approximated six initial conditions \((x_0, y_0, z_0, v_{x0,\text{gr}}, v_{y0,\text{gr}}, v_{z0,\text{gr}})\). This returned the trajectory of each particle as a function of time which I used to recreate the matter distribution for any arbitrary time in the future.

\section*{D. Particle Trajectories}

To recreate the trajectory of particles we use data sampled at the boundary of the SpEc numerical grid. This provided us with six initial conditions \((x_0, y_0, z_0, v_{x0}, v_{y0}, v_{z0})\) that allowed us to evolve the system. We modeled the particles leaving the grid using Newton’s law of gravitation,

\[
\vec{F}_{\text{grav}} = m\ddot{\vec{r}} = -\frac{GMm}{r^3}\vec{r}.
\]  \hspace{1cm} (4)

Where \(M\) is the summed mass of the neutron stars, \(m\) is the mass of the particle in the outflow, \(G\) is the Newtonian gravitational constant, and \(\vec{r}(x, y, z)\) is the variable we solved for. Integrating this equation results in a 6D coupled differential equation. We wanted to see the evolution of \(x, y, z, v_x, v_y, \) and \(v_z\) so we solved the system of differential equations for these variables numerically with Python. However, it must be noted that SpEC is a relativistic code but we modeled the particles trajectories using Newtonian mechanics. Therefore, if we want to accurately model the evolution of the neutron star merger system then we have to make a linear transformation to reconcile the discrepancies between our classical and relativistic results. To accomplish this, we looked at conservation of energy,

\[
\frac{(Av)^2}{2} - \frac{GM}{r} = -U_t - 1.
\]  \hspace{1cm} (5)

On the left-hand side is the classical model with kinetic and potential energy terms. On the right-hand side is the SpEC relativistic model where \(-U_t\) is the conserved specific energy for non-interacting particles in a time-dependent spacetime and \(-1\) is the subtracted
rest mass energy \((mc^2)\). \(A\) is the linear transformation term that is applied to the particle velocities. Solving for \(A\) gives

\[
A = \sqrt{2\left(\frac{-U_t + GM/r - 1}{v^2}\right)}.
\] (6)

We then applied the transformation term \(A\) to the velocities \(v\) of each modeled particle,

\[
(Av)^2 = (Av_x)^2 + (Av_y)^2 + (Av_z)^2 = v_{x,gr}^2 + v_{y,gr}^2 + v_{z,gr}^2.
\] (7)

The second line in Eq. 7 are the obtained relativistic velocities. With these relativistic velocities we now have the approximated initial conditions needed to numerically evolve the system; \((x_0, y_0, z_0, v_{x0}, v_{y0}, v_{z0}) \rightarrow (x_0, y_0, z_0, v_{x,gr}, v_{y,gr}, v_{z,gr})\).

We visualized the correction factor \(A\) in Fig. 2 and found that the majority of particles only needed a \(\sim 10\%\) correction. This can also be seen in Table I – as more particles were sampled, the correction factor decreased. This was because the first hundred thousand or so particles are close to the merger system which introduced large errors on the orders of 2–3. Later particles are farther away from the system and have a lower error. Some particle velocities could not be transformed into relativistic velocities due the fundamentally different metric in classical and relativistic mechanics. What this meant was that the classical energies in Eq. 5 could not be reconciled with the relativistic energies from SpEC. This resulted in the radicand in Eq. 6 to become negative and the value to be imaginary. We found that only \(\sim 0.0039\%\) out of the 3.5 million simulated particles could not be reconciled.

III. RESULTS

A. Evolving the Neutron Star Merger Simulation

Using SpEC, we recorded the matter density of the system as it was evolved. This allows us to graph the matter density and directly observe the evolution of the matter outflow at different stages in the simulation. Fig. 3 shows the evolution of the binary neutron star system at 12.65, 12.95, 13.39, and 14.06 ms. In Fig. 3a the tidal forces start to deform the shapes of both objects. We see that the neutron stars begin to merge into one singular shape in Fig. 3b while the system as a whole continues to rotate. After some time, the cores
TABLE I: Properties of the relativistic correction factor $A$ used to ensure that energy is conserved in our classical and relativistic simulations. We see that as more particles are sampled that the correction factor decreases. This is because earlier particles are much more closer to the merger itself which results in errors. Unsolvable $A$ values are due to the radicand of the square root in Eq. 6 becoming negative at certain values. This is due the different metrics in classical and relativistic mechanics.

<table>
<thead>
<tr>
<th>No. Particles</th>
<th>Average $A$</th>
<th>No. of Unsolvable $A$</th>
<th>Percentage of Unsolvable $A$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100,000</td>
<td>1.46</td>
<td>0</td>
<td>0.0000</td>
</tr>
<tr>
<td>1,000,000</td>
<td>1.31</td>
<td>7</td>
<td>0.0007</td>
</tr>
<tr>
<td>2,000,000</td>
<td>1.24</td>
<td>48</td>
<td>0.0024</td>
</tr>
<tr>
<td>3,533,246</td>
<td>1.19</td>
<td>138</td>
<td>0.0039</td>
</tr>
</tbody>
</table>

FIG. 2: Correction factor $A$ to transform our classical velocities to relativistic velocities. The peak near -1.0 indicates that the majority of particles roughly needed $\sim 10\%$ correction.

...of the neutron stars begin to merge together as seen in Fig. 3c and the overall shape of the system starts to resemble one object instead of two. It is important to notice the two tidal tails that form as a result of the inspiral motion of the neutron stars coming in. These tails...
eject more and more matter over time and form the accretion disk around the post-merger object which Fig. 3d shows. The ejected matter from the tidal tails also forms the matter outflow which we used to sample particles for our classical model in Sec. II D.

**B. Recreating the Matter Density Distribution**

In Sec. II D we recreated the trajectories of particles from classical mechanics while also making a correction to ensure that energy is conserved between the classical and relativistic results. We are now in a position to model the matter density distribution of particles at an arbitrary time in the future. Fig. 4 shows the evolution of particles at 1 ms and 37 ms after merging. Fig. 5 serves as a better visualization of the evolution of the system at various timestamps. Here the axes are set to give the reader a better sense of the distance the outflowing particles travel. We considered the bound conditions of particles using Eq. 5 as follows:

\[
\epsilon = \frac{(Av)^2}{2} - \frac{GM}{r}.
\]

If \( \epsilon < 0 \) the particle is bounded and if \( \epsilon > 0 \) the particle is unbounded. We are interested in the evolution of particles for a short time in the evolution. This is because unbound particle trajectories form a hyperbola and over long distances they will simply be a straight line. Bound particles will remain orbiting the post-merger object. We found that 82% of the 3.5 million particles were bound while 17% were unbound. Only 6 particles had \( \epsilon = 0 \) which are parabolic orbits. Unbound particles are more interesting to us because they are what power kilonovae.

**C. Angular Distributions**

1. **Matter Distribution**

The angular distribution of particles impacts the observable properties of kilonovae. Fig. 6 shows the angular distribution of particles using a spherical coordinate system. We use the following equation to calculate the angle at which particles leave the system with respect to the polar angle (z-axis).
\theta = \arctan\left(\sqrt{\frac{x^2 + y^2}{|z|^2}}\right) \tag{9}

We observe that the majority of particles leave at angles further away from the polar regions ($\theta > 30^\circ$). Around the polar region ($\theta < 30^\circ$), matter was not abundantly present. We found most of the matter leaving the merger system around the equatorial plane. This matter dominated region will be optically dense which will not allow for photons to pass through the matter outflow and leave the merger system in these directions. This is in agreement with the angular distribution of neutrinos in Fig. 7 (Fig. 8 in [1]) where it was observed that the neutrinos (not the matter particles) were mostly focused in the polar region. Although the M1 closure overestimates the abundance of neutrinos in the polar region, we observed that even in the MC plot that there is a greater amount of neutrinos in the this region than around the equatorial plane. The presence of neutrinos will result in the polar region being optically thin due to radioactive decay of less massive unstable elements which we discussed in Sec. 1B2. In summation, photons from kilonova emissions will be less likely to pass through optically dense regions near the equatorial plane due to absorption and dispersion from the matter particles. However, we note that the equatorial ejecta will only be optically thick on a timescale of days after merging but eventually becomes optically thin within a timescale of weeks [13]. When this happens, most of the energy of the kilonova will come from the equatorial ejecta.

2. **M1 and MC Neutrino Distributions**

We found with the M1 analytical closure that there was a large inaccuracy to actual neutron star merger models in the regions close to the polar axis ($\theta < 30^\circ$). This resulted to a $\sim 50\%$ overestimation of neutrinos in those regions. We surmise these regions to be more optically thin than leading to more photons passing through the matter outflow resulting in the kilonova to have a higher frequency emission which is the same process that we previously discussed when optically thick regions turn optically thin over time. Despite this, we found that the M1 closure performed well elsewhere. When we obtained the neutrino distribution with the MC algorithm we did not encounter the same overabundance of neutrinos in the polar regions as with the M1 closure which can be seen in Fig. 7.

With the error from the M1 closure in mind, we also state in [1] two ways in which the MC
scheme can improve upon current two-moment schemes. First, with a low resolution MC simulation we can forgo the need to use the M1 closure which resulted in large inaccuracies in the polar regions. This is done by using time-averaged moments as a closure for the two-moment scheme. From using time-averaged moments, we found that distribution of neutrinos was only weakly dependent on the time evolution. This allows our time-averaged MC simulations to be comparable to that of dynamically evolved systems. Second, the MC methods presented in [1] hints towards the viability of using an MC closure. Replacing the M1 closure with the MC closure in two-moment schemes can in principle reduce numerical errors by an order of magnitude for the current number MC packets that we can currently afford computationally.

IV. CONCLUSIONS AND DISCUSSION

In this thesis I have presented a methodology to recreate the matter density distribution of particles at any arbitrary time using classical mechanics and then making a linear transformation to reconcile with relativistic mechanics. Although the simulation I ran was a binary neutron star merger the method presented in this thesis can be applied to any generic binary merger system such as black hole mergers.

In making the linear transformation from classical and relativistic velocities I found that there were some particles that could not be reconciled to match that of our SpEC simulation. These irreconcilable particles comprised $\sim 1.7\%$ of the 3.5 million particles in the simulation. The irreconcilable particles were due to the fundamentally different metric used in classical and relativistic mechanics. In addition, abnormally large transformation factors, $A$, on the order of $2 - 3$ were because the first few hundred thousand particles were close to the merger system. As I sampled particles further away I found that the correction factor decreased with the majority only needing a $\sim 10\%$ correction. After performing the transformation from classical to relativistic mechanics, I obtained six initial conditions $(x_0, y_0, z_0, v_{x0,gr}, v_{y0,gr}, v_{z0,gr})$ which I used to solve for the particle motion numerically after integrating Newton’s law of gravitation. This gave the trajectories of the particles which allowed me to model the matter density distribution at any arbitrary time. The matter density distribution is plotted in Figs. 4 and 5.

I have also analyzed the angular distribution of particles leaving the system using the same
methods as in [1]. I found that particles were leaving at angles further away from the polar region ($\theta > 30^\circ$) and near the equatorial plane. This was where the accretion disk formed and where most of the matter outflow was present. The matter dominated regions results in r-process and where the creation of heavy elements takes place (heavier than elements created in neutron-poor environments). Radioactive decay of heavy elements results in an optically thick region around the equatorial plane where electromagnetic emissions from kilonovae is suppressed. However, this changes as the system continues to evolve. The equatorial ejecta is thick on a timescale of days after merging occurs. After a timescale of a week or so, the optically thick region becomes optically thin enough for kilonova emissions to penetrate through the ejecta. This has an affect of changing the composition of the kilonova – the late time emissions comes at lower temperatures due to cooling in the outflow.

Using returned data from the SpEC neutron star simulation, I created matter density plots. This allowed me to directly observe the evolution of the system at different timestamps in the simulation. From Fig. 3a and Fig. 3b I observed the deformation in the shape of the neutron stars due to tidal forces. The tidal tail that begins to form in Fig. 3c ejects matter which creates the matter outflow. The merging of the neutron star cores also occurs at this time and in Fig. 3c and 3d the post-merger object is formed while matter continues to to be ejected and accreted.

Finally, I have discussed the work I have done with neutrino transport in [1] and elaborated on the significance of the role of neutrinos in the evolution of neutron star mergers. We compared a newly developed MC simulation to a two-moment scheme that used the M1 closure. We saw that using the M1 closure resulted in an overestimation of neutrinos in the polar region ($\theta < 30^\circ$) by $\sim 50\%$ while performing well everywhere else. The overestimation of neutrinos in the polar regions changes the properties of the kilonova by reducing the amount of neutrons and protons through absorption. Consequentially, the polar regions become a neutron-poor environment. While the nucleosynthesis of heavy elements will still occur via r-process, these elements will have less mass compared to heavy elements in a neutron-rich environment. The radioactive decay of these less massive heavy elements results in an optically thin region that photons can pass through. This a similar (but not entirely the same) process that happens when the optically thick regions of the matter outflow become optically thin – the kilonova emission will occur from a higher temperature region have a higher frequency in the optical band rather than infrared. The MC simulation
on the other hand did not have the issue of neutrino overestimation in the polar regions. The comparison between the MC and two-moment scheme can be seen in Fig. 7.

Despite the errors of the M1 closure, we surmised that MC simulations can improve upon current two-moment schemes in two ways. First, the MC method provides a way to forgo the use of M1 closures by using low resolution MC simulations thus reducing the large inaccuracies in the polar regions. This was done by using time-averaged moments for the two-moment scheme. Therefore, our MC simulations can potentially be comparable to simulations that are dynamically evolved. Second, the MC method presents the possibility of using an MC closure to reduce numerical errors that plague the M1 closure by an order of magnitude.

V. ACKNOWLEDGMENTS

First and foremost I would like to express my gratitude to Dr. Foucart for his mentorship throughout this project and the opportunity to partake in research in computational astrophysics for the past 2 years. I would also like to thank the members of Dr. Foucart’s research group Alexander Chernoglazov, Alexander Knight, and Christian Krueger for being the most jovial people I have had the pleasure to work with and know. Acknowledgments also go to the UNH Institute for the Study of Earth Oceans and Space for the use of the Trillian computer cluster as well as the University of Illinois and NSF for the use and funding of the Blue Waters cluster to run the simulations.


FIG. 3: Density map of neutron star model at various stages of evolution after the beginning of merging together. (a) Shows the neutron stars before merging completely. (b) The cores of the neutron stars begin to coalesce into a single compact object. (c) At this point the cores have almost completely merged together. Matter from the tidal tail begins to whipped away from the system. (d) As the system continues to evolve, matter continues to be ejected which forms the matter outflow.
FIG. 4: Time evolution of particles 1 ms (LHS) after merging and 37 ms (RHS) after merging. Top images provides a 2D view of the system while the bottom images provides a 3D view. 82% of the 3.5 million simulated particles were bound with $\epsilon < 0$ while 17% were unbound with $\epsilon > 0$. We found only 6 particles with $\epsilon = 0$. After merging, the ejected particles continue to move at a fraction of $c$ which can be seen with the difference in order of magnitude for the axes scales for the LHS ($10^2$ km) and RHS ($10^3$ km) figures.
FIG. 5: Set axes visualization of the time evolution of particles 1, 8.5, 26, and 37 ms after merging.
FIG. 6: Angle distribution of particles leaving the merger system. Particles leave the system as angles further away from the polar angle (z-axis). This allows the emission of kilonova signals around the optically thin polar region with less matter that can potentially absorb or disperse photons.
FIG. 7: The left column shows the energy distribution of neutrinos leaving the SpEC numerical grid for 3 neutrino species: electron neutrinos $\nu_e$, electron antineutrinos $\bar{\nu}_e$, and heavy-lepton neutrinos $\nu_x$. The dashed vertical line shows the neutrino energy estimated by the M1 scheme, and the solid vertical line is the neutrino energy estimated by the MC scheme. The right column shows the angular distribution of neutrinos leaving the grid. The overestimation of neutrinos in the polar regions by the red M1 scheme is visually evident when compared to the grey MC scheme. For all of these plots the neutrino fluxes were integrated over a 50$\mu$s interval 14ms after merger.