Phase and Amplitude Interferometry Based Radio Frequency Direction Finder

Joseph Phillip Balsamo

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Phase and Amplitude Interferometry

Based RF Direction Finder

Faculty Advisor: Dr. Kent Chamberlin

Joseph Balsamo, Computer Engineer
University of New Hampshire
May 10, 2018
Abstract

Direction finding (DF) systems have been around for decades, preceding WWII. The main function of these systems is to calculate the direction of arrival of an electromagnetic wave. There are many real-world applications which utilize direction finders and direction-finding techniques, from recreational “fox hunts” to military geolocation systems. The following approach for implementing a direction finding system revolves around the phase and amplitude of a signal that is being radiated at an unlicensed frequency of 2.45Ghz by an RF source.

The system is comprised of an antenna array of 4 antennas which can be used receive the radiated signal. By comparing the amplitudes of the signal received by each antenna relative to each other, the quadrant from which the RF source is located in can be identified. By comparing the phase difference, 0° to +/- 180°, of the signal received by each antenna relative to each other, four possible directions can be calculated, one in each quadrant. Using the information discovered from comparing the phase and the amplitudes of the received signal at each antenna, the direction of the RF source can be found. The system runs the direction finding algorithm when the user commands it to from the graphical user interface (GUI), iterates it hundreds of times per second, and averages the found direction to reduce the effects of noise. The direction is then displayed on the GUI.
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Introduction

Direction finding (DF) systems have been around for decades, preceding WWII. The main function of these systems is to calculate the direction of arrival of an electromagnetic wave. There are many real-world applications which utilize direction finders and direction-finding techniques, from recreational “fox hunts” to military geolocation systems.

There are several basic properties of electromagnetic waves that can be used to help calculate a direction, also called the angle of arrival (AOA), of an incoming wave. The amplitude and phase of the incoming wave are the two characteristics used to determine AOA. The design implemented in this effort is to utilize a phase and amplitude based direction-finding technique for finding a stationary RF source radiating a continuous signal at the unlicensed frequency of 2.45 GHz.

AOA (Direction Line) Relative to the Detector’s Origin

Figure 1
Choosing Direction Finding System Approach

There were several other direction finding approaches considered for the implementation. The first approach was using a null-based technique. Each antenna has a specific radiation pattern which shows how well the antenna is able to receive a signal at different areas around it. For this approach, the antennas being used would be loop antennas which show the following radiation pattern:

![Loop Antenna Radiation Pattern](image)

As can be seen in the radiation pattern, and as the name of the approach might suggest, there are 2 null points in the radiation pattern. The DF that would be constructed would have to be able to rotate the loop antenna and search for those null points. Once these null points are found, it reveals 2 possible directions, the true direction to the RF source and the opposite direction to it. In order to isolate one direction, another loop at a different location would have to be used. Clearly having multiple devices which have to be a known, and considerable distance (multiple meters) away in order to calculate one direction would’ve been inefficient.

Another approach that was considered was using a Yagi-Uda antenna. This approach is similar to the null-based approach as it looks to find the high points of the antenna’s radiation pattern.
The benefit of using this type of antenna would’ve been that there wouldn’t have been a need for multiple devices since the high point in the radiation pattern is so prevalent. There is still the issue of having to rotate the antenna to find that high point and Yagi-Uda antennas can be very expensive, which eliminated the possibility of using this approach.

The last two approaches mentioned were based on signal strength, but the following approach, which was almost the approach that was adopted for this DF implementation, focuses on frequency. The doppler effect is the apparent increase or decrease in frequency as the source is moving closer or further away from the receiver.

A real-world example of this would be when an ambulance siren is approaching from the distance. When it’s moving closer, the pitch (frequency) gets higher whereas it’s the opposite when it moves away.
A method to creating this effect with the RF source’s signal would be to rotate an antenna in a circle and see where the frequency is the highest by frequency-demodulating the received signal. Instead of moving an antenna rapidly in a circle, this effect can be emulated by rapidly sampling one antenna at a time in a circular antenna array.

By sampling through the antennas at a rate around 1000Hz and frequency demodulating the signal, the rate of change of frequency can be seen. The demodulated signal of the live data can be compared to a 0° reference frequency-demodulated signal, which would have to previously be calibrated through testing, and the phase difference between these 2 signals would actually represent the direction of the RF source.

The only reason that this approach was not taken is because of the limited capabilities of commercially available receivers. There are software-defined radio (SDR) receivers that have the capability to receive and frequency-demodulate incoming signals in a specified frequency band, and for a price as low as $20. The only issue with these SDRs is that, in regard to saving data points, they are activated via shell script. Running a program that can automatically run shell script is not a challenge, but the timing of this shell script running is inconsistent. The only possible way to compare the received frequency-demodulated signal to the 0° reference is if the signal starts being read at a specific point in time. If the frequency-demodulated signal is off by a
Background Material of Chosen Direction Finding Approach

When an incoming electromagnetic wave is incident on an antenna, a copy of the transmitted signal will appear on the terminals of that antenna. This received signal will have a frequency, amplitude, and phase associated with it. This phase, which varies from 0° to 360°, is not an absolute value (unlike the amplitude and frequency), but is relative to a known, or unknown, reference. Typically, this reference is a local oscillator of the same frequency or another received signal. For the work described here, that reference is in relation to the received signal from another antenna allocated a known distance from the antenna.

The reason there is this phase difference between the two signals received by the antennas is because the incoming wave travels a different path for each antenna.

Examples of an Incoming Wave

Figure 6
There is a relationship between AOA and the phase difference between two antennas. This phase difference between 2 antennas of a known distance will generate 2 possible directions. There are 2 possible directions generated, opposed to 1, since there are 2 locations around the antenna array that can produce a given phase difference. It should be noted that the separation distance between antennas for each pair must be less than one wavelength to avoid aliasing errors.

By sampling the phase difference of 2 antenna pairs, there will be sufficient information to calculate the AOA. As an example, consider Figure 3. There are 2 ambiguous directions calculated in quadrants 2 and 3 and there are 2 overlapping directions in quadrant 1. We know the valid direction is in quadrant 1 because both antenna pairs were able to deduce a direction in that quadrant.

Example of the DF Deducing a Direction Using Phase

Figure 7
It’s important to note using the phase is more reliable than using the amplitude. The amplitude of an incoming wave decreases the further away the RF source is. If the wave’s source is too far away then it may be hard to accurately use the received signals. Phase remains unaffected from the distance of the RF source. By using the phase of the incoming wave to determine direction, there is not as much sensitivity to the strength of the incoming wave and there is no limitation on the range from the RF source.

Despite phase being more reliable to calculate the direction of an RF source, amplitude can still be helpful. By comparing the amplitudes received by each antenna, the quadrant of which the RF source lies in can be determined. It’s optimal to use 2 antenna pairs that are orthogonal to each other in order to get symmetrical quadrants, such as using a North-South pair and an East-West pair. For example, in figure 4 it shows the quadrant being chosen as quadrant 1. This is because the North antenna is receiving a stronger signal than the South antenna, meaning the RF source is in the northern quadrants, and the East antenna is receiving a stronger signal than the West antenna, meaning the RF source is in the Eastern quadrants. Quadrant 1 is the only quadrant that satisfies the conditions found.

Example of the DF Deducing a Direction Using Amplitude

Figure 8
System Design

I’ll be explaining the system that was created and its components.

Figure 9
The first observation to notice is that there are 2 pairs of azimuthally omni-directional antennas, which are capable of receiving an incoming wave from any direction in the azimuthal plane, used for the system. The antennas chosen for this system have to be able to cover a signal being transmitted from every position around the direction finder. For this reason, Dipole antennas were used.

These antennas pairs are \( \lambda/2 \) away from each other, which means the maximum phase shift that can occur between them is a 180°. This distance ends up being roughly 6.1224 cm since the RF source being used is transmitting at 2.45GHz.

The distance between the antennas within each antenna pair wasn’t chosen at random. The reason that this distance has to be at most \( \lambda/2 \) is to prevent any aliasing errors. If the distance between them is more than \( \lambda/2 \), say \( 1\lambda \), then a value of phase difference of 360° can’t be distinguished from a phase difference of 0°. Another reason \( \lambda/2 \) is chosen is because the phase detectors used can only measure a phase difference of +/- 180°. To prevent any aliasing from occurring, the maximum phase difference allowed should be within the limits of the phase detector, which is +/-180°.
The 2 antenna pairs, the North-South pair and the East-West pair, are placed on the vertical axis and the horizontal axis respectively. The reason that these antenna pairs are orthogonal to each other, as explained before, is to form 4 symmetrical-in-size quadrants that the direction calculated can be in. It also made the conditional logic used to adjust the direction to be relative to the chosen 0° point around the DF easier to implement.

The antennas were connected, via SMA cables, to the phase detecting amplitude comparator modules based on the Analog Devices AD8302 chip (AD8302 module). This module ideally outputs a voltage, 0V – 1.8V, which represents a phase shift of 0° to +/- 180° or a magnitude difference of -30dB to 30dB, depending on which pin is being read. The reference inputs are the North antenna, to which was compared with the South antenna, and the East antenna, to which was compared with the West antenna. These inputs have to be between 0dBm - -60dBm in order for the modules to function correctly.

AD8302 Phase Detector and Amplitude Comparator Outputs

Figure 11
It is important to note that the voltage outputs from the AD8302 modules are analog. Analog values have to be put through analog to digital converters (ADC) in order for Raspberry Pi 3s to be able to read them since Raspberry Pi 3s do not have a built in ADCs. The ADC that was used had to be able to distinguish 10mV (since $10\text{mV} = 1^\circ$) intervals at the minimum. The ADC used has 4-channels and uses 16-bits to represent a voltage input. It was able to be programmed to have an input range of 0V - 2.048V so it has intervals of $0.0625\text{mV}$ ($1/160^\circ$).

The Raspberry Pi 3 reads the live voltages from the ADC and uses that data to calculate a direction which is then displayed onto a graphical user interface (GUI) and will repeat the process of sampling the data and calculating the direction until ordered to stop (via the GUI).

The Raspberry Pi 3 supplies 5V to the AD8302 modules and 3.3V to the ADC as the modules’ power source. The system uses 50Ω terminated cables, antennas, and circuits since the AD8302 modules are 50Ω terminated. Making every component 50Ω terminated reduces transmission loss. The GUI displays onto a monitor connected to the Raspberry Pi 3 and is interacted with via a mouse also connected to the Raspberry Pi 3.

**Calculating Direction**

After researching Phase Interferometry, from reference 3, the following equation was found (derived using trigonometry) for determining the phase difference between an antenna pair:

$$\Delta \Phi = \frac{-2 \pi B \sin(\theta)}{\lambda}$$
This can be rearranged to solve as:

\[ \theta = \sin^{-1} \left( \frac{-\Delta \Phi \cdot \lambda}{2 \cdot \pi \cdot B} \right) \]

This equation was used to calculate the 2 possible directions based on the difference of phase between the two antennas within each antenna pair (\(\Delta \Phi\)), the wavelength of the frequency, and the length of the baseline (B) between the two antennas (\(\lambda/2\)).

An observation to make is that, unlike the amplitude comparator output of the AD8302 module, the phase output of the detection module is not linear. This means that for each output of the phase detector that’s read, it has 2 interpretations, one positive value and one negative value. For example, a phase detector output of 1.2V should both represent a -60° phase difference and a 60° phase difference. This means that for each value received from the phase detector output, both possible phase differences have to go through the equation above and 4 possible directions that the RF source can be coming from are calculated. This also means that both antenna pairs, relative to the 0° reference, reveal the same 4 possible directions, one per quadrant.

Direction Calculation Before Quadrant Determination

Figure 12
The next step is to determine which quadrant the RF source is located in. To do this, the amplitude comparator output was used. Since this is a linear output, there shouldn’t be any confusion when determining which antenna within each antenna pair is closer to the RF source. As was discussed before, and shown in figure 6, determining the quadrant is very straightforward. If the amplitude comparator output is read to be below 0.9V then the reference antenna is the closer antenna to the RF source. If not then the other antenna in the antenna pair is closer to the RF source. The only downside to relying on amplitude for direction calculation is that the range of the RF source is limited. If the RF source is too far away, there won’t be much of an amplitude difference between the 2 antennas within each antenna pair and the quadrant determination may be inaccurate and, although the phase detectors might calculate a correct value, the direction calculated could be off by as much as 180°.

**MATLAB Simulation**

A MATLAB simulation was developed to simulate the different phases that each antenna of the antenna array would receive from an RF source based on the RF source’s radius from the center of the array and angle from the baseline of the West-East pair of antennas. By creating this simulation, all possible RF source placement scenarios were modeled. After modeling the phases received at each of the antenna’s based on the RF source’s location, the MATLAB simulation calculated the direction using the algorithm previously developed. The success rate of this simulation was 100% as was to be expected.
Construction of the Direction Finder

The construction of the direction finder itself was fairly simple. Both the stand for the direction finder and the stand for the transmitter were 3D printed. The DF stand was 18cmx18cmx18cm while the transmitter stand was 10cmx10cmx18cm. The heights of the stands were the same since it would ensure the strongest signal would be received by the DF. There were holes in the top of the box, as can be seen in figure 9, that held the antennas in place.

The antennas were connected to the AD8302 modules (the purple chips) via SMA cables.

The ADC was connected to the Raspberry Pi 3 via the Pi’s I2C bus. The 4 inputs to the ADC were the magnitude and phase outputs of both of the AD8302 modules. The ADC was powered from the Raspberry Pi’s 3.3V power pin.

The other cables that were connected to the Raspberry Pi 3 were from the power source, the HDMI connector going to a monitor, the keyboard, and the mouse.
Algorithm Implementation

The algorithm and GUI developed was made using Python 3. This was chosen since the Raspberry Pi 3 already has the required packages installed to create and run a Python 3 program. There were also Python 3 libraries available for installation that easily allowed reading of the ADC.

Direction Finder Algorithm:

1. Read the live voltages from the ADC representing the phase detector and amplitude comparator outputs from both AD8302 modules
2. Translate the voltage outputs to their associated degree and decibel values
3. Use the decibel values to determine the sign of the degree values
   I. + degree value if the transmitter is closer to the North/East antennas
   II. – degree value if the transmitter is closer to the South/West antennas
4. Use the degree values to determine a direction relative to the baseline between the antennas within each antennas pair using the equation previously found on page 13
5. Offset the East-West antenna pair direction by -90° since the antenna pairs are orthogonal to each other
6. Use conditional logic to determine the quadrant of the direction and offset the found directions appropriately (since the phase detector outputs only go up to +/-180°)
   I. If direction is in the North-East:
      • East-West direction = -90 – Current East-West direction
   II. If direction is in the North-West:
      • East-West direction = 90 + Current East-West direction
• North-West direction = ( Current North-South direction + 360 ) % 360

III. If direction is in the South-East:

• East-West direction = 270 – Current East-West direction

• North-West direction = 180 – Current North-South direction

IV. If direction is in the South-West:

• East-West direction = 90 + Current East-West direction

• North-West direction = 180 – Current North-South direction

7. Take the average of the 2 directions (ideally they would be the same) and modulo them by 360° in case they are over 360°

The algorithm runs several hundred times a second and the average direction is displayed on the GUI using a unit circle. The average is used to eliminate the effects of possible noise. The more iterations of the algorithm that are performed, the more accurate the resulting direction should be. The user has the option to stop the processing, reset the average direction found, and resume sampling.

Direction Finder GUI

Figure 15
Complications with the Implementation

There were several complications with the implementation of the DF that altered its performance and changed the initial algorithm described above.

When unit testing the AD8302 modules, they did not work as described from the specifications page (figure 7). The reference voltage pin measured from the AD8302 modules was correct (1.8V), but the center voltage (0° and 0dB) found for the outputs was not 0.9V, but rather they were about 0.546875V for the North-South phase output, 0.453125V for the North-South amplitude comparator output, and 0.46875V for the East-West amplitude comparator output. The phase output for the East-West antenna pair was so inconsistent that it couldn’t even be used.

The center voltages being different from the specification sheet wasn’t really an issue since it could simply be accounted for when converting the voltages read from the ADC to their associated values, as long as the side bounds for the phase detectors (+/- 180°) were consistent and the voltage levels changed enough for the amplitude outputs when the transmitter got closer to one of the antennas. Unfortunately, this was not the case.

When the transmitter was nearing the direction of 270°, the amplitude differences detected didn’t seem to be significant enough for the AD8302 module to distinguish whether or not the transmitter was north or south relative to the center point of the DF, even though the radius was only 5 feet. This caused some quadrant inaccuracies.

When the transmitter was in the south of the DF, the side bound for the phase detector (-180°) was 0.425V whereas when it was in the north it was 0.3875V. To account for this major issue, whenever the transmitter was detected to be in the north, the side bound was set to be
0.425V. When it wasn’t, it was set to be 0.3875V. The equation for converting the voltage values to degree values therefore became:

\[
\text{Phase Difference (°) = } \left[ \frac{(\text{North-South Center Bound} - \text{Detected Phase})}{(\text{North-South Center Bound} - \text{North-South Side Bound})} \right] \times 180°;
\]

For the quadrant determination, the voltage was just looked at to see if it was above or below the center bound. If it was above, the transmitter was determined to be in the North/East. If it was below, the transmitter was determined to be in the South/West.

As might be obvious, the phase side bound chosen might be the wrong one when the direction approached 270°, causing some inaccuracies.

**Testing Procedure and Performance**

The testing procedure for the DF was the following:

1. Set up and run the transmitter approximately 5 feet away from the DF
2. Run the DF GUI
3. Hit “Start” to begin processing the sampled data
4. After 5 minutes, stop the processing and record the currently displayed direction
5. Rotate the DF by 5° about its center point
6. Reset the DF’s shown direction by hitting “Reset” on the GUI to reset the current results since the direction has now changed
7. Start sampling the data again (step 3)
8. Repeat steps 4 – 7 until a full rotation of the DF has occurred
The DF, allowing a $10^\circ$ tolerance, performed with an accuracy of 68.05%. This is fairly low, but certain areas of the unit circle had higher/lower accuracies than others. We can see this in figure 13 below. As expected, the DF performed worse when nearing $270^\circ$.

![DF Performance](image)

**Bill of Materials**

**Transmitter**

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<th>Group</th>
<th>Description</th>
<th>Price Per Unit</th>
<th>Number of Units</th>
<th>Total Price</th>
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**Figure 17**
Direction Finder

There were only a couple constraints with this project that led to some issues. One of these constraints was where to buy the AD8302 modules. The AD8302 evaluation board from Analog Devices was over $300 per unit. This is about 50 times the price the AD8302 modules from Bangood were. Seeing how these modules not performing as specified were the main reason for inaccuracies with the DF, this constraint was costly performance-wise.

Another constraint was the fact that the AD8302 modules were only able to detect phase differences of +/- 180 degrees and how its output was sinusoidal. If its output was linear and its

<table>
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Figure 18

Outside Constraints
range was 0 to 359 degrees then there would be no need to rely on the amplitude for quadrant determination and the system would only need to rely on phase outputs.

One constraint that was relevant was having a proper testing area. The best possible testing area would’ve been a very open area, ideally outside, with no objects. When testing the DF, because a slightly crowded lab was the only texting area that was available, signal distortion was prevalent, thereby affecting the performance.

**Final Remarks**

When implementing the DF, it was important that professional standards are met. Acknowledgement for outside resources used is very important. Stealing any equations or information is required or else it’s considered plagiarism.

There are many reasons why this was a good senior project. First, this project involves both hardware processing and software processing. Getting to develop an algorithm from the data received is very important to be able to do. As stated before, using the phase over other properties of an electromagnetic wave was a good initial approach, but having to adjust the approach based off of what pieces of hardware were available was an important lesson. Another reason this was a good senior project idea is because this device was testable, meaning there was be measured data collected with which was used to evaluate the device performance.

Creating a Phase-Based RF Direction Finder was not just a good senior project idea because they are used for many different types of applications in the real world, but because developing and implementing an actual system comprised of a variety of parts and processes, such as this design, and it was great hands-on experience.
Acknowledgements:

I would like to acknowledge Dr. Kent Chamberlin for advising and funding this project, Kathy Reynolds for helping ordering parts, James Abare for assisting in setting up lab space and 3D printing parts, and the Electrical and Computer Engineering Department at the University of New Hampshire for funding this project.

References:

