Empirical modeling of the quiet time nightside magnetosphere

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Empirical modeling of plasma pressure and magnetic field for the quiet time nightside magnetosphere is investigated. Two models are constructed for this study. One model, referred to here as T89R, is basically the magnetic field model of Tsyganenko (1989) but is modified by the addition of an inner eastward ring current at a radial distance of ~3 RE as suggested by observation. The other is a combination of the T89R model and the long version of the magnetic field model of Tsyganenko (1987) such that the former dominates the magnetic field in the inner magnetosphere, whereas the latter prevails in the distant tail. The distribution of plasma pressure, which is required to balance the magnetic force for each of these two field models, is computed along the tail axis in the midnight meridian. The occurrence of pressure anisotropy in the inner magnetospheric region is also taken into account by determining an empirical fit to the observed plasma pressure anisotropy. This effort is the first attempt to obtain the plasma pressure distribution in force equilibrium with magnetic stresses from an empirical field model with the inclusion of pressure anisotropy. The inclusion of pressure anisotropy alters the plasma pressure by as much as a factor of ~3 in the inner magnetosphere. The deduced plasma pressure profile along the tail axis is found to be in good agreement with the observed quiet time plasma pressure for geocentric distances between ~2 and ~35 RE.

INTRODUCTION

Although the magnetosphere has been studied and surveyed extensively for over 3 decades since the advent of spacecraft, there are still outstanding tasks pertaining to the quiet time magnetosphere. One of these is a quiet time magnetospheric model with specification of the magnetic field configuration and the associated equilibrium plasma pressure distribution. In the quiet time Earth's magnetosphere, the inertial force \( \rho dv/dt \) is usually insignificant, and therefore force balance should exist between magnetospheric plasma pressure and electromagnetic forces. An explicit configuration resembling the magnetosphere and satisfying force balance has yet to be devised.

Equilibrium of the magnetosphere has been studied theoretically in the two limiting situations of the inner magnetosphere, where the Earth's dipole field dominates, and the magnetotail, where the field arises mainly from the theta-shape tail current system. The inclusion of the transition region between these two simplified situations has proved difficult [Voigt and Wolf, 1988]. In the inner magnetosphere, Schopke [1972] studied equilibrium under the assumption of isotropic plasma pressure in a perturbed dipole field. This type of investigation was recently extended by Zavriyev and Hasegawa [1989] and Cheng [1992] with the more realistic consideration of anisotropy in plasma pressure as observed [e.g., Lui and Hamilton, 1992]. The magnetotail equilibrium has been studied analytically [Harris, 1962; Kan, 1973] as well as examined by numerical simulation [Toichi, 1972; Birn, 1989; Hesse and Birn, 1992]. Unfortunately, no global three-dimensional equilibrium solutions are known. Present global MHD simulations do not stabilize enough to provide them. Two-dimensional numerical solution, when convection is included, appears to evolve in ways that may preclude a steady state equilibrium [Erickson and Wolf, 1980; Erickson, 1984; Hau et al., 1989], although this may be merely an artifact of the two-dimensional limitation [Kivelson and Spence, 1988].

An alternative approach to obtaining equilibrium configurations is to use data. For example, the magnetic field models of Tsyganenko [1987, 1989], which give averaged magnetic field values satisfying the static Maxwell's equations, have been used extensively in relating magnetospheric regions to the ionospheric level and vice versa [Elphinstone et al., 1990; Stasiewicz, 1991; Elphinstone and Hearn, 1992; Pulkkinen et al., 1992]. However, early models of this type were badly out of force balance [Walker and Southwood, 1982]. Particularly pertinent questions related to any empirical magnetic field model are therefore the following:

1. What is the plasma pressure distribution in force equilibrium with the empirical field model?
2. How is this plasma pressure distribution compared with actual measurements?
It is quite evident that the availability of plasma pressure distribution in force balance with the magnetic forces in an empirical field model will further enhance its utilization.

Spence et al. [1987, 1989] and Kan et al. [1992] have inverted the earlier magnetic field models [Tsyganenko and Usmanov, 1982; Tsyganenko, 1987] to obtain the equilibrium plasma pressure along the tail axis at midnight by assuming isotropic or nearly isotropic plasma. The goal of this paper is to extend these previous studies in deriving empirical magnetic field models for the quiet time magnetosphere, with the associated plasma pressure in force equilibrium with the model magnetic field and in good agreement with observations. Two features are incorporated, namely, the presence of an eastward inner ring current and the occurrence of anisotropic plasma pressure. These two features are found to be essential in arriving at a realistic plasma pressure profile. This study treats the simple situation in which the dipole tilt angle is zero, making the configuration symmetric and easy to handle without significantly altering the results.

**The Modified Magnetic Field Models**

The magnetic field models of Tsyganenko [1987, 1989], hereafter referred to as T87L (long version) and T89, have a number of features that lend themselves to easy applications. The magnetic field in these models is given in an analytical form, with most terms bearing physical insights on the various current systems contributing to the total magnetic field at a given location. The coefficients of these terms are obtained through a least-squares fit to actual measurements and are derived separately for different levels of the $Kp$ index to allow for the variability of the magnetic field with geomagnetic activity. Since we consider here the quiet time magnetosphere, the activity level chosen for this study corresponds to the $Kp = 0$ level. Modified versions of these field models form the basis from which magnetic field models are constructed here.

The ring current is dominated by the gradient of plasma pressure. At the inner edge of the ring current, the plasma pressure decreases rapidly inward, producing an eastward flowing ring current in the inner magnetosphere [Lui and Hamilton, 1992]. This ring current is featured in neither the T87L nor the T89 model. Although this eastward flowing ring current may not alter the magnetic field to a great extent, as will be demonstrated later, its absence tremendously affects the plasma pressure distribution required to maintain equilibrium with the magnetic forces. Without such an eastward flowing current, the plasma pressure in force equilibrium with the magnetic field will have to increase monotonically inward, as indicated by the earlier work of Spence et al. [1989] and Kan et al. [1992], and is contrary to the observed plasma pressure distribution. Even though the required volume current density for the eastward ring current is small, the $j \times B$ force is significant, since the field strength is high (about inversely proportional to the third power of the geocentric distance) in the inner magnetosphere.

To include the inner ring current in the model, we construct its vector potential using a procedure similar to the one employed in the T89 model to represent the outer ring current, i.e.,

$$A_{IRC} = C_{IRC} \rho S_{IRC}^3,$$  

where $C_{IRC}$ is a coefficient controlling the intensity of the inner ring current and

$$S_{IRC} = \sqrt{\rho^2 + (a_{IRC} + \xi_{IRC})^2},$$  

$$\xi_{IRC} = \sqrt{x^2 + (D_o + \gamma_{IRC} h_{IRC})^2},$$  

$$h_{IRC} = 0.5 \left[1 + x(x^2 + L_{IRC}^2)^{-1/2}\right].$$

The location of the point in space is given in cylindrical coordinates by $(\rho, \phi, z)$ and in Cartesian coordinates by $(x, y, z)$. The physical significance of the nonlinear parameters is the same as given for the outer ring current by Tsyganenko [1989]. The radial scale length $a_{IRC}$ relates to the geocentric distance for the inner ring current. The half thickness of the inner ring current region is represented by $D_o$. The variation of this thickness with local time, i.e., the difference between dayside and nightside, is controlled by the function $h_{IRC}$ with its scale length $L_{IRC}$. We find that the flow reversal of the ring current from westward to eastward inside $L = 3.5$ can be reproduced reasonably well by adopting the following values for the above parameters:

- $C_{IRC} = 760 \text{ nT}$, $a_{IRC} = 1.5 \text{ RE}$, $D_o = 1.8 \text{ RE}$, $\gamma_{IRC} = -0.2723$
- $L_{IRC} = 1.5 \text{ RE}$

To keep the total magnetic moment of the ring currents the same as the original ring current in the T89 model, we have modified the outer ring current strength by

$$C_{ORC} = C_{IRC} - C_{IRC} \times \left(a_{IRC}/a_{ORC}\right)^2,$$

where the subscripts RRC and ORC denote, respectively, the revised and original outer ring current parameters in T89. It turns out that the modification is extremely slight (only $-0.2\%$). This model with the inner eastward ring current and a correspondingly modified outer ring current strength is identified hereafter as T89R.

Since the T89 and T89R field models give rather weak magnetic fields in the magnetotail [Stern and Tsyganenko, 1992; Rostoker and Skone, 1993; Pereida et al., 1993], we have also explored the T87L field model. To take advantage of the fact that the T89R model is better than the T87L model in the inner magnetospheric region, but the reverse is true for the distant tail [Tsyganenko, 1989; Donovan et al., 1992], we combine the two field models by constructing a hybrid field model (designated hereafter as TH) with a transition parameter $\epsilon$ such that

$$B_{z,TH} = B_{z,87L} + (1-\epsilon)B_{z,89R},$$  

$$\epsilon = 0.5 - 0.5 \sin \left(\left[2(x - x_L)/(x_U - x_L) - 1\right]\pi/2\right),$$

where $B_z$ is the $z$ component of the magnetic field along the midnight meridian and subscripts 87L and 89R denote parameters from the T87L and T89R models, respectively. The parameters $x_L$ and $x_U$ are the lower and upper bounds of $x$ for the transition. The transition parameter $\epsilon$ is 0 at $x = x_L$ and 1 at $x = x_U$, with a smooth derivative $d\epsilon/dx$ at the end points. The boundary points adopted here are $x_L$
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We note that the region of interest in this paper is the equatorial plane along the tail axis in which the magnetic field has only the $z$ component (for a dipole tilt angle of $0^\circ$). The divergence of the magnetic field along the tail axis therefore reduces to $\partial B_z / \partial z$. It is clear then that the linear combination of the magnetic fields from the T87L and T89R models as expressed in (6) is also divergence free along the tail axis. On the other hand, if this hybrid model were to be extended beyond the tail axis, there would be a small nonzero divergence if the other field components were not modified accordingly.

Figure 1a shows the profiles of magnetic field values in logarithmic scale from the four magnetic field models, i.e., T87L, T89, T89R, and TH, at the equatorial region along the midnight meridian. Note that the four magnetic field profiles are quite different between $x = -8 R_E$ and $-30 R_E$. The highest magnetic field values come from T87L, followed by TH, T89, and T89R. Although the inclusion of the inner ring current only slightly weakens the magnetic field, this effect is manifested quite dramatically in the extremely low field region of the T89 model, where the field magnitude goes below 1 nT.

The volume current densities from these four models at the equatorial region along the midnight meridian are examined in Figure 1b. The T87L model typically gives the lowest current densities among these models. Tailward of $x = -10 R_E$, the T89 and T89R models give almost the same values, whereas the TH model shows noticeably lower current densities between $x = -10$ and $-30 R_E$. The lower values from the TH model, in comparison with the T89 and T87L models, are expected from the influence of the T87L model. Earthward of $x = -10 R_E$, both the T89 and T87L models show a persistent westward current. In contrast, the T89R and TH models show a reversal of westward ring current to an eastward ring current at $-3.5 R_E$. The presence of this current reversal is modeled to represent the reversal noted from the CCE observations [Lu and Hamilton, 1992]. In comparison, we find the observed current densities appear to be higher than the model values at the current density peak but smaller than the model values at distances further downstream. In addition, the observed current densities are more variable, reflecting the filamentary nature of current much like the magnetotail current [McComas et al., 1986].

The observed anisotropy of plasma pressure taken from Lu and Hamilton [1992] is shown in Figure 2 together with an empirical fit. The anisotropy is seen to be large in the inner region and decreases rather systematically at distances further downstream. The empirical fit to the anisotropy is a fifth-order polynomial function of $x^{-1}$, i.e.,

$$P_{\perp} / P_{\parallel}^{-1} = a_0 + a_1 x^{-1} + a_2 x^{-2} + a_3 x^{-3}$$

$$+ a_4 x^{-4} + a_5 x^{-5},$$

with coefficients $a_0 = -0.410554$, $a_1 = -0.94369$, $a_2 = -86.9877$, $a_3 = -504.066$, $a_4 = -1110.73$, and $a_5 = -847.912$. The range of validity for this fit is $-2.5 R_E > x > -15 R_E$ such that the plasma pressure is isotropic at $x = -15 R_E$. That the pressure should be isotropic downstream of $x = -15 R_E$ is indicated by the studies of Stiles et al. [1978] and Baumjohann and Paschmann [1989].

Given a magnetic field model, it is straightforward to compute the magnetic force $j \times B$ and determine the pressure force required to balance it. The radial profile of perpendicular plasma pressure can be obtained by integrating

$$\frac{\partial P_{\perp}}{\partial r} = [j \times B + (P_{\perp} - P_{\parallel})(b \cdot V)b]_r,$$

where the subscript $r$ indicates the radial component of the vector quantity, $b$ is the unit magnetic field vector $B$, $j$ is the volume current density, and $P_{\perp}$ and $P_{\parallel}$ are the plasma pressure components perpendicular and parallel to the observed magnetic field direction, respectively.

Figure 3a shows the result from this computation for the T89R model with the plasma pressure at $x = -34 R_E$ taken to be 0.074 nPa. This initial value of plasma pressure is obtained by computing the plasma pressure re-
The range of pressure values from Explorer 45 data indicated by the error bars actually spans the observed pressure during geomagnetically quiet to disturbed conditions, and thus the lower limit should be used for comparison with the model calculation. As can be seen from Figure 2b, the model pressure profile is in general above the observed pressure profile, especially in the outer region. This finding is understandable and is to be expected because the observed pressure is not necessarily obtained right at the equatorial plane and thus is expected to underrepresent the actual observed pressure at the equatorial plane. Inside 4 RE, the pressure determined from Explorer 45 is noticeably lower than that from CCE values and the model calculation. This difference may be due to the upper energy threshold being only 872 keV for Explorer 45 measurements, whereas that of CCE extends to >4 MeV. It is possible that a significant contribution to plasma pressure inside 4 RE comes from particles above the 872-keV level. We have evaluated this possibility using several midnight passes of CCE during quiet conditions. We found the ions with energy of >872 keV contributing typically ~50-65% to the plasma pressure at geocentric distances inside 4 RE. Since the disparity in the determination of pressure between Explorer 45 and CCE is about a factor of 6, we conclude that the different energy passbands can account for a significant portion, but not the entirety, of the observed pressure difference.

Overall, we find very good agreement exists between the observed and computed profiles of plasma pressure on the basis of the T89R model. Another notable feature of the model pressure profile is its relative constancy between the downstream distances of 16 and 25 RE. This finding indicates the small magnitude of the J × B force in this region. Reexamining Figure 1 reveals that the main reason for the decrease in the J × B force in the T89R model is the small value of the magnetic field in that region (Figure 1a), since the volume current density is still substantial at those distances (Figure 1b). The more abrupt change in the radial profile of plasma pressure occurs in the downstream distances between x = -12 RE and x = -15 RE, as noted earlier by Spence et al. [1989]. The J × B force is therefore relatively large at the transition region between the dipolelike and taillike field configurations. It is interesting to note that this region may correspond to the hinging point of the tail, where the tangential stress acted on the magnetotail by the solar wind at the tail magnetopause is balanced by the attractive force between the Earth's dipole and the tail current system [Siscoe and Cummings, 1969].

A similar plasma pressure profile can be obtained from the TH model, as shown in Figures 4a and 4b. The plasma pressure based on the formula from Spence and Kivelson [1993],

\[
P(nPa) = 89e^{-0.59|x|} + 8.9|x|^{-1.53}
\]  

is also shown for comparison; the downstream distance x is in Earth radii. The assumption of isotropic pressure gives an underestimate of plasma pressure by as much as a factor of ~3 in the inner magnetosphere as before. The most significant difference between the two anisotropic pressure profiles in Figures 3 and 4 lies in the downstream...
plasma pressure is inverted from the TH magnetic field model.

**Fig. 4.** (a) Similar to Figure 3a except that the perpendicular plasma pressure is inverted from the TH magnetic field model instead. (b) Similar to Figure 3b except that the perpendicular plasma pressure is inverted from the TH magnetic field model instead.

The plasma pressure shows a continuous change in the region of \( x = -10 \) to \(-20 R_E \). Whereas the T89R profile shows a relatively constant pressure in this region, the other profile shows a continuous pressure increase toward the Earth. This continuous rise, however, only contributes rather insignificantly to the plasma pressure earthward of \( x = -8 R_E \). The continuous change in plasma pressure reflects a more gradual transition between the dipolelike and taillike field configurations as well as a more \( x \)-dependent pressure for the near-Earth tail region in this model than in the T89R model. The empirical formula by Spence and Kivelson [1993] also compares reasonably well with the model profile. The formula gives a good fit to the plasma pressure within the range of \( x = -2.5 \) to \(-15 R_E \) but overestimates it inside this region and underestimates it further tailward.

**SUMMARY AND DISCUSSION**

We have investigated two empirical models for the specification of magnetic field and the plasma pressure for the quiet time nightside magnetosphere. The derived magnetic field models are extensions of the previous magnetic field models of Tsyganenko [1987, 1989] by the addition of an inner eastward ring current. Furthermore, in constructing the equilibrium plasma pressure along the tail axis in the midnight meridian, we have taken into account the presence of plasma pressure anisotropy in the inner magnetosphere earthward of \( x = -15 R_E \). An empirical fit to the observed pressure anisotropy in the inner magnetosphere (between 2.5 and 15 \( R_E \)) is also provided. These two new features profoundly affect the deduced plasma pressure profile. The first feature leads to an earthward reduction of the plasma pressure in the innermost part of the magnetosphere (geocentric distances of \(< -3 R_E \)) as observed. The second feature leads to an appropriate evaluation of the plasma pressure gradient. Without the consideration of plasma pressure anisotropy, the deduced plasma pressure may be underestimated by as much as a factor of \(-3 \).

The derived perpendicular plasma pressure is found to be in good agreement with the observed values from ISEE 2, CCE, ATS 5, and Explorer 45. The T89R and TH models therefore provide quite realistic representations along the tail axis in the midnight meridian of not only the quiet time magnetic field but also the associated equilibrium plasma pressure distributions needed to provide the force balance with the magnetic forces in these models. The range of validity for the T89R and TH models in representing the magnetic field, the volume current density, and the anisotropic plasma pressure considered here is \(-2.5 R_E > x > -35 R_E \). The present result also supports the earlier findings from Spence et al. [1989] that the gradient of plasma pressure shows a relatively large change in the transition region between dipolelike and taillike field configurations.

A region called the inner edge of the plasma sheet has been introduced in the early studies of the energetic particle environment in the magnetosphere [Vasyliunas, 1968; Frank, 1971]. It is a region characterized by an exponential decrease of electron energy density with decreasing radial distance and is generally located at geocentric distances of \(-6 \) to \( 8 R_E \) in the nightside. Many researchers implicitly assume that the inner edge of the plasma sheet is associated with an earthward decrease of the total plasma pressure, and theories on the close-up of the large-scale region 2 field-aligned current system [Iijima and Potemra, 1978] have been built upon this assumption (see, for example, the review by Mauk and Zanetti [1987]). As pointed out by Mauk and Zanetti [1987], the observed plasma pressure shows a persistent earthward increase rather than a decrease in the geocentric distances usually ascribed to the inner edge of the plasma sheet. In terms of total plasma pressure, there is no edge at those distances. The plasma commonly identified as constituting the plasma sheet in the outer magnetosphere (tailward of the geocentric distance of \(-10 R_E \)) gradually becomes the hot ring current particle population. The absence of a large-scale plasma pressure decrease in the so-called inner edge of the plasma sheet indicates the necessity to modify the conventional mechanism for closing the large-scale region 2 field-aligned current. An alternative means for the region 2 field-aligned current closure in an earthward increasing plasma pressure condition has been proposed by Lui and Hamilton [1992]. They have suggested, based on the work of Sato and Iijima [1979], that the correct sense of field-aligned current will be generated if the \( \mathbf{VP}_\perp \) vector is inclined toward the midnight meridian more than the \( \mathbf{VB} \) vector in both the premidnight and the postmidnight sectors, since the correct sense of field-aligned current is determined by the sign of the triple product of \( \mathbf{B}, \mathbf{VP}_\perp \), and \( \mathbf{VB} \).

At least two aspects of this study can be improved in the future. One is to extend the calculation to other local times besides midnight, and the second is to extend it in the \( z \) direction to cover the region away from the equato-
Generation of the Region II Field-Aligned Current

\[ \mathbf{V}_B \times \mathbf{B} < 0 \]

Post-midnight

\[ \mathbf{V}_P \times \mathbf{B} \]

(FAC from the ionosphere)

Pre-midnight

\[ \mathbf{V}_P \times \mathbf{B} > 0 \]

(FAC into the ionosphere)

Fig. 5. A schematic diagram to illustrate the relative orientation of the vectors \( \mathbf{V}_B \) and \( \mathbf{V}_P \) to give the correct sense of the field-aligned current system for region 2.

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