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HYDRAULIC MOUNDING OF GROUNDWATER UNDER AXISYMMETRIC RECHARGE

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ABSTRACT

A comparison is made between several solutions of the equations describing groundwater flow as they apply to the local mounding of the groundwater that occurs under an isolated recharge system. Both time-dependent and time-independent solutions are considered. An additional, approximate approach is presented.

Mound heights calculated by the various methods are compared in eight separate cases, some involving hypothetical situations and some using field data obtained by others. The "worst case" conditions permitted by New Hampshire's regulations regarding subsurface disposal systems are examined. Nineteen reasons are discussed as to why mound heights calculated on the basis of idealizations might vary from field results. Because of these possible variations, it is difficult to conceive of field tests as the means of determining the accuracy of the mathematical solutions, or as the means of discriminating between them.

Because mound heights increase for soils of decreasing hydraulic conductivity, we cannot ignore the mounds which will develop in the soils at the "worst case" limits established by regulation dealing with rapid infiltration or subsurface disposal.

It is further concluded that what is needed is a solution which contains both time-dependence and accommodation of an asymmetric lateral control (down gradient water body).

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LIST OF SYMBOLS USED IN THIS PAPER

Symbol	Meaning	Units
D	Initial depth of saturation of the unconfined aquifer (water table height above impermeable material, ignoring capillary effects).	Feet
h	Saturated mound height (at radius r) above the underlying impermeable material as a result of local recharge, ignoring capillary effects.	Feet
R	Radius of circular recharge bed or field	Feet
h_R	h at r = R	Feet
H	h at r = 0	Feet
s	Saturated mound height above an initial water table, ignoring capillary effects (s = h - D)	Feet
S	s at r = 0 (S = H - D)	Feet
Q	Quantity of liquid recharged each day	Ft^3/Day
w	Recharge velocity ($w = Q/\pi R^2$)	Ft/Day
K	Saturated hydraulic conductivity	Ft/Day
V	Specific yield (drainable/fillable voids ratio)	---
t	Time	Days
L	Radius of influence or distance to the lateral control (parameter assumed for steady state solutions).	Feet
ℓ	Distance traveled through a porous medium under hydraulic head conditions.	Feet

INTRODUCTION

THE PROBLEM

The United State Environmental Protection Agency through its construction grants program has an important impact on the selection of sewage treatment and disposal works for municipalities. Particularly in the case of communities with a sewered population of 1000 or fewer, EPA is urging careful consideration of alternatives involving land disposal. Rapid infiltration (basins or leach fields), at first brush, often appears to be the most cost effective solution. However, most consulting engineers and soil scientists do not know how to evaluate the potential of a site to accept hydraulic loads of up to 100,000 gallons per day. If the site will not accept the load, effluent will spill over the ground and will be carried by surface runoff to streams and lakes. This paper is concerned with examining several approaches to the analysis of the hydraulic mounding of the groundwater which occurs under localized recharge.

In order for water to flow either on the surface or in the soil matrix, there must be a gravitational potential available to carry the water downhill (neglecting capillary effects). It is the groundwater mound which represents the mechanism by which water is carried away from the recharge site. The slower the hydraulic conductivity in the soil, the higher the mound must be to disperse a given amount of water recharged each day.

Since the mound is superimposed on the pre-existing groundwater free surface, the implication of a mound which far exceeds the available depth below the recharge bed is that the ground will become saturated below the bed and the recharge bed will not accept all the load. Spillage will occur. It should be noted, however, that surfacing of effluent can indicate either hydraulic failure (the mound is too big), or an infiltration surface failure (the water will not enter the ground fast enough, usually because the bed is too small).

It should also be mentioned that, if rapid infiltration is to achieve renovation of the recharged water, this may be defeated by a mound which rises close to the infiltration surface, decreasing the unsaturated depth to less than that required to achieve acceptable treatment. Adsorption of pollutants is far less efficient under saturated conditions, since the volume to surface ratio of liquid to soil particle surfaces is much less than with unsaturated conditions. In unsaturated flow only the smaller soil pores are active.

The problem, then, is to predict before construction how much unsaturated depth will remain after hydraulic mounding has been established. Only then can the rapid infiltration alternative be adequately compared with other means of disposal.

The issue of adsorption saturation and eventual breakthrough of pollutants is a separate issue and will not be treated here. For examples of work in this field of nutrient and pollutant flow from subsurface disposal systems the reader is referred to papers by Dudley and Stephenson (1973), Ellis and Childs (1973), Pickens and Lennox (1976), Reed, et al (1972), Reneau and Pettry (1976), Sawhney and Starr (1977), and Viraraghavan and Warnock (1976).

Groundwater flow can be submitted to rigorous analysis only in so far as solutions can be found of the fundamental differential equations which describe ideal behavior of liquids in porous media. Because of complexities of the problem and the generally intractable nature of the equations, simplifying assumptions have been required. With computers, iteration and relaxation techniques make it possible to closely approximate solutions. Usually, however, the consulting engineer designing wastewater facilities has neither the interest in complex computer analysis of groundwater mounds, nor personnel with sufficient mathematical skill to attempt solutions of the fundamental equations. Approximate methods may be all that he requires, because of the lack of adequate information about the proposed site.

THE ANALYSIS

The analytical approaches are usually of four types: assumption of an infinitely long source (two dimensional problem), assumption of an axisymmetric source (circular symmetry), assumption of a rectangular source (the square basin is a special case), and computer solution of finite element or finite difference nets for sources of arbitrary shape.

The two dimensional or linear source approach is here dismissed as being insufficiently appropriate for description of the average, isolated rectangular recharge field. However, the reader is referred to works by Amar (1975), Brock (1974), Cabrera and Marino (1976), Maasland (1959), Marino (1974a, 1974b, 1975), Nimr and Street (1972), Phillip and Forrester (1975), Sawhney and Parlange (1974), and Thomas, et al (1974).

The computer solutions are relevant and may be compared to analytical solutions, as we shall see. In the latter type a result can be obtained by substituting into an explicit solution of relatively minor complexity, while in computer solutions numerical techniques may be employed. These may involve perturbation theory, truncated infinite series and other techniques better handled by computers than by laborious hand computation. Examples not considered later in this paper include the use of Galerkin methods by Yoon and Yeh (1975), and use of the Hele-Shaw model by Tinsley and Regan (1968).

Computers can also be used to evaluate finite difference (FD) models and finite element models (FEM) in which a space network of points, or nodes, is used and what happens at one node in the grid is assumed to have an effect upon adjacent nodes. The computer is used to calculate all the interactions with iterative techniques. Examples of FEM applications to water movement in porous media are found in works by Cheng (1975), France (1974), and Reeves and Duguid (1975). Examples of applications of the FD approach may be found in works by Amerman (1976), Brock (1974), and Trescott, Pinder and Larson (1976).

It should be noted that it is characteristic of FD and FEM techniques that calculation parameters must be adjusted to fit an initial set of data before other results can be predicted. Unless initialization involves simple geometry, such as a flat plane-free surface, a known geometry must be used to adjust parameters.

However, numerical techniques for the solution of the fundamental equations may offer close to ideal solutions. Examples of this type will be considered later.

Readers interested in solutions for groundwater mounds involving perched water tables should consult Brock (1976) and Khan (1973).

ASSUMPTIONS

The basic time-independent equation of flow in unconfined porous media is the LaPlace equation, $\nabla^2\Phi = 0$, and it assumes that the liquid is incompressible. Generally, the solutions of LaPlace's equation, and the time-dependent form of this equation, have been sought for the simplest geometry. The usual assumptions involve an inert, non-expanding, homogeneous, isotropic medium characterized by a saturated hydraulic conductivity (coefficient of permeability), K, and a specific yield (ratio of drainable or fillable voids to total volume). Additional geometric assumptions are that the porous medium is underlain by a horizontal, impermeable base, and that there is some finite initial depth of saturation with an initially horizontal free surface.

It is assumed that capillary effects and lateral flow in the capillary and unsaturated zones can be neglected. It is assumed that no groundwater recharge or evapotranspiration takes place except for the constant and uniform application of water at the recharge basin or leach bed.

Darcy's law is assumed to hold. The temperature dependence of permeability (the viscosity of water is a relatively strong function of temperature) is handled by assuming a standard temperature, usually 20°C. Inertial effects are neglected. A specific yield is assumed which is appropriate for the particular soil being modeled, if no data are available.

The most common assumptions used to change the LaPlace equation into a manageable form are that the mound height is small compared to the initial saturated depth and Dupuit's assumption. These two are not the same. Dupuit assumed that the flow would be near enough to horizontal flow so that the slope of the free surface can be assumed to express the hydraulic gradient everywhere in a vertical section through the saturated zone. This can be shown to be an appropriate assumption for steady state flow solutions involving free surface slopes up to 10%. The assumption of relatively small mound height makes available the previously developed, analogous mathematical description of heat flow in thin metal plates.

Kozeny (see De Wiest, 1965) expressed Dupuit's assumption in the following form:

$$Q = -2\pi r h K \frac{dh}{dr} \tag{1}$$

where Q is the quantity of flow per unit time through a cylindrical surface of radius r, h is the saturated depth at radius r, K is the coefficient of saturated hydraulic conductivity, and $\frac{dh}{dr}$ is the slope of the free surface which is taken as the head-loss-over-distance in Darcy's equation. See Table 1 for designation of the variables used in this paper and by others.

If equation (2) is integrated from a radius of influence, or distance to the lateral control, L, inward to radius r, we then have, for a discharging well:

$$h^2 = D^2 - \frac{Q}{\pi K} \ln \frac{L}{r} , \tag{2}$$

and for the recharge well:

$$h^2 = D^2 + \frac{Q}{\pi K} \ln \frac{L}{r} , \tag{3}$$

where D is the initial saturated depth (see Appendix B).

Hantush (1962) and Polubarinova-Kochina (1952) have shown that equation (2) is an exact solution for the steady state discharging well with a finite radius of influence (neglect consideration of near-well problems such as a seepage face above the water surface in the well). The same treatment could be used to prove that equation (3) is exact. Since it is understood that both Dupuit's and Forchheimer's assumptions result in departures from the exact solution, the implication drawn from the proof that equation (2) or (3) is exact is that, in this example, the Dupuit departure must cancel the Forchheimer departure. To see that they are opposite, consider the upper portion of the flow from a recharge well as it traverses an incremental radial distance Δr as in Figure 1. In Darcy's law the head-loss per unit distance through the porous medium is expressed as $\Delta h/\Delta l$, where Δl is the distance traveled through the porous medium by the liquid, and Δh is the corresponding head-loss. From equation (1) we see that the available head-loss-over-distance is,

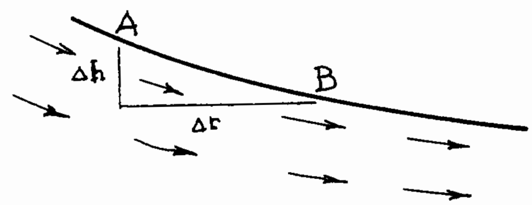


Figure 1. Flow Near a Free Surface

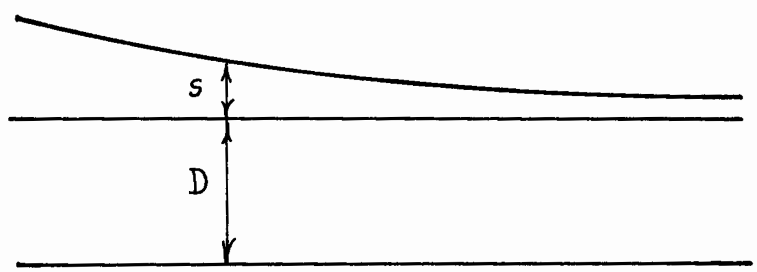


Figure 2. Depth of Flow

Table 1

Symbols Used in This Work and By Others

	Usual Units	This Paper	Hantush	Glover	Baumann	Hunt	Marino	Brock	Singh
Initial Depth of Saturation	feet	D	h_i	D	a_0	D	h_0	a	D
Total Mound Height Above Impermeables	feet	h	h	-	-	-	h	-	h
Mound Height Above Initial Water Table (h-D)	feet	s	-	h	$\bar{h}$	Ƶ	$h-h_0$	s	Ƶ-D
Mound Height at Center of Recharge System	feet	H,s	h_m	h_0	$\bar{\bar{H}}$	Ƶ$_c$	-	-	Ƶ
Hydraulic Conductivity	ft/day	K	K	K	K	K	K	K	K
Quantity of Water Recharged Each Day	ft^3/ day	Q	V	-	q_0	-	-	Q	-
Recharge Velocity ($Q/\pi R^2$)	ft/ day	w	ω	ω	Δq	ε	w	p_0	N
Radius of Recharge Bed	feet	R	R	a	R	R	a	R	R
Drainable/Fillable Voids Ratio (Specific Yield)	----	V	ε	V	-	σ	S	n_e	n_e
Time	days	t	t	t	-	t	t	t	t
Time Averaged Mound Height	feet	-	$\bar{b}$	-	-	-	$\bar{h}$	-	-
Radius of Influence or Distance to Lateral Control	feet	L	-	-	L	-	-	B	

essentially, $\Delta h/\Delta r$, but the distance traveled by the liquid is along the chord from A to B. This is considerably longer than Δr. The implication is that the slope must actually be steeper than as assumed by the Dupuit assumption, and the mound higher.

Now, if we consider the depth of flow to be D instead of D + s = h, as in Figure 2 (neglecting the effect of mounding as it contributes to transmissivity), the required dh/dr in our analysis will be greater than the dh/dr needed when mounding is taken into consideration, since in the latter case the depth of flow (and transmissivity) is greater. This means that the actual mound will be lower than that calculated by neglecting s in some terms of the non-linear differential equation to achieve linearization.

It has not been proved that these effects are equal, but it is apparent that they tend to cancel in the recharge case. However, it appears that the domain of validity of the DF approach should be greater than the domain of either assumption taken separately. Murray and Monkmeyer (1973) have examined carefully the domain of validity of the DF equation, generally, and conclude that, independent of the relative mounding, results for steady state recharge should be within 1% of the exact result, if the free surface does not have a slope in excess of 10%. They show that the DF assumptions are equivalent to the assumption that the vertical pressure distribution is hydrostatic.

In spite of great reservations about the Dupuit assumption by Muskat (1937) many tests have shown good agreement between well drawdown results and equation (2) or the time-dependent equivalent developed by Theis (1935).

Murray and Monkmeyer (1973) point out that DF time-dependent solutions more accurately describe a rising water table than a receding one. This can be understood by visualizing a wave of liquid moving outward from a source. The liquid may not be moving vertically, but the wave front may be very steep.

Khan (1973), in his analysis of perched mounds, found that the DF theory underestimated the maximum height of the free water surface.

Bouwer (1965) has indicated that equation (2) implies that, for infinite saturated depth an infinite Q may be obtained from a well with a finite draw-down. He has explored two-dimensional electric analogue results and concludes that, when the saturated depth equals or exceeds the width of the percolation zone, the results begin to deviate from the predicted values, because the lower zones do not contribute to the flow as much as is assumed in equation (2). The same reasoning should apply to equation (3). This would mean that the mound would be higher than predicted.

Suter (1956) has claimed that temperature measurements at a large recharge site demonstrated cold recharged water floating on top of the warmer groundwater, which is in keeping with laminar flow and the assumptions used herein.

The assumption of a radius of influence in equation (2) or (3) may appear to be arbitrary. The choice of this radius must be based upon experience. However, it can be shown (see the Discussion section) that equations (2) and (3) give results which are not as sensitive to the choice of the radius of influence as they are to the choice of other variables.

SOLUTIONS

Solutions to the problem, obtained by others, are discussed in Appendix A. An additional solution obtained by integrating equation (3) from $r = L$ to $r = R$ and then integrating from $r = R$ to $r = 0$ using the approach employed by Baumann (1965) and which has also been used by Bouma, et al (1973), for the linear recharge case. See Appendix B. For the center of the mound:

$$H^2 = D^2 + \frac{Q}{\pi K} \left(\ln \frac{L}{R} + \tfrac{1}{2}\right), \tag{4}$$

and for $R \leq r \leq L$:

$$h^2 = D^2 + \frac{Q}{\pi K} \left(\ln \frac{L}{r}\right). \tag{5}$$

Equation (4) involves the assumption that the vertical recharge inside a cylinder of radius r ($r \leq R$) is to be equated with the horizontal flow through the cylindrical surface in keeping with Dupuit's assumption. While this approach may appear suspect, the results should be considered satisfactory, if they differ from more accurate results by less than the dependent uncertainty due to uncertainty in the measured quantities. The contention made here is that the data available in the average problem confronted by the consulting engineer do not permit accurate prediction of mound heights but only approximate heights - for which equation (4) is adequate. However, it is not the intention here to under-value the development of accurate computer solutions for the ideal case in useful graphical form - a task not yet accomplished.

It should be noted that Baumann's result, equation (A-3) (Appendix A), can be rewritten (using $H - D = S$) in a form very similar to equation (4):

$$H^2 = D^2 + \frac{Q}{\pi K} \left\{\ln \frac{L}{R} + \tfrac{1}{2} - \frac{1}{L - R} \left[L \exp \left(\frac{L - R}{L}\right) - 2L + R\right]\right\} \tag{6}$$

Figure 3 presents a graphical representation of equation (4) after transformation to a non-dimensional form.

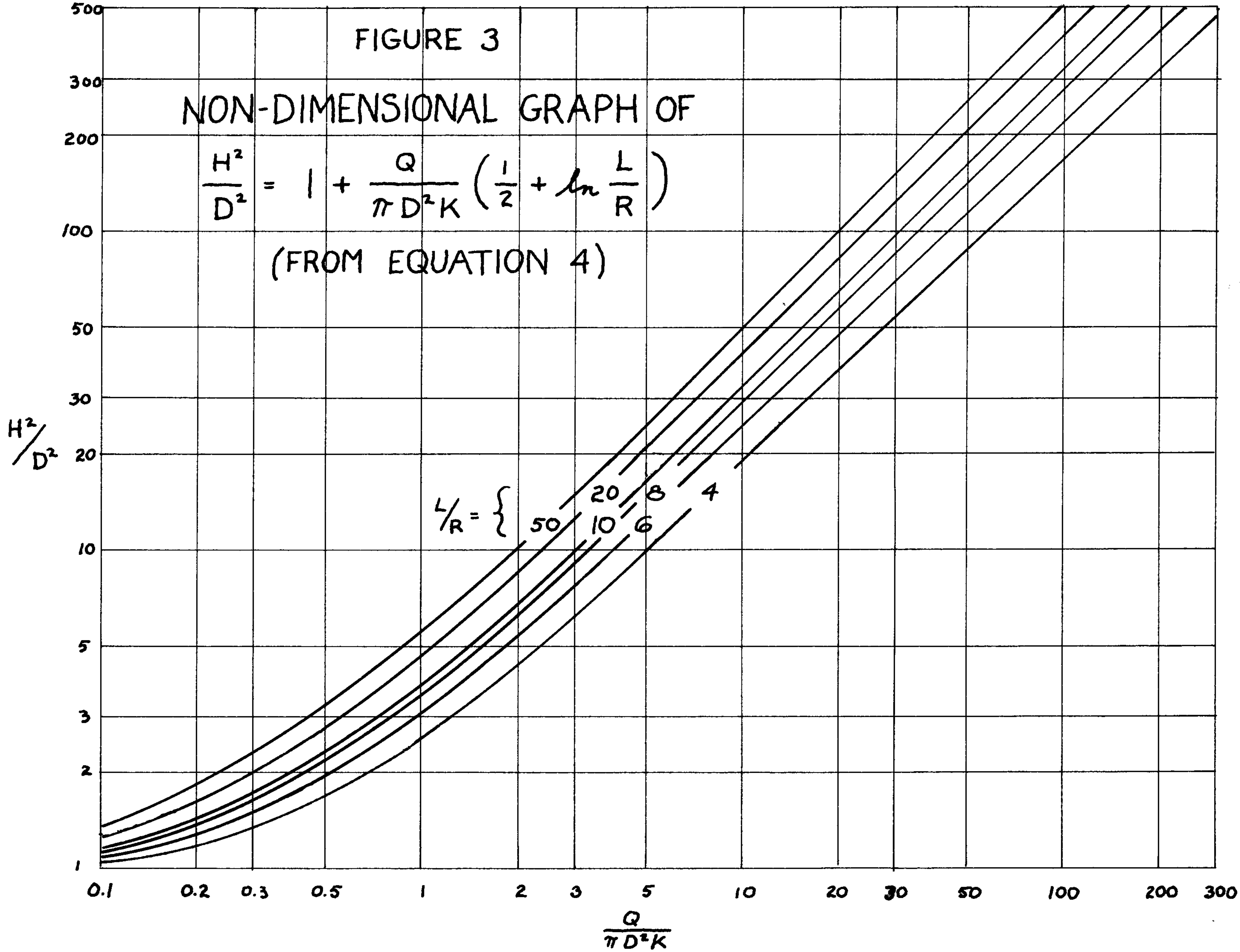


FIGURE 3
NON-DIMENSIONAL GRAPH OF
$$\frac{H^2}{D^2} = 1 + \frac{Q}{\pi D^2 K}\left(\frac{1}{2} + \ln \frac{L}{R}\right)$$
(FROM EQUATION 4)
$L/R = \{ 50, 20, 10, 8, 6, 4$
H^2/D^2
500
300
200
100
50
30
20
10
5
3
2
1
0.1
0.2
0.3
0.5
1
2
3
5
10
20
30
50
100
200
300
$\frac{Q}{\pi D^2 K}$

RESULTS

Several hypothetical cases have been examined and the mound heights have been calculated by use of the explicit equations developed by four different approaches. Additional results have been calculated for $r = R$ and $r = 2R$ so that some concept of the mound shape could be obtained. Where the graphical results of the computer solutions have permitted, mound heights have been compared to the results from the explicit equations. Some of these results were estimated by interpolation and extrapolation. The explicit equations, all involving the DF assumptions, have been applied to situations implying mound heights far in excess of the initial saturated depth. Even in the worst cases, the maximum slope of the free surface has been only about 10%.

The first two hypothetical examples were chosen to represent limiting cases within the current regulations of the New Hampshire Water Supply and Pollution Control Commission (Shepard, 1977), as they relate to private subsurface disposal systems. Other states have similar regulations derived from recommendations of the United States Public Health Service (1957). Since these regulations are phrased in terms of percolation test results, some means is needed to convert these results to corresponding values of hydraulic conductivity. The work by Bouma, et al (1973), includes the report of careful measurement of both hydraulic conductivity and percolation rate for a number of different soils. Analysis of their data by means of a best fit line on a log-log plot leads to the following equation:

$$\text{Standard Percolation Test (ft/day)} = 6.03\ K^{0.844}\text{(ft/day)} \qquad (16)$$

It was noted that two thirds of the data were within a factor of two of this equation.

Case I represents a limiting case for a single family residential lot with a leach field at grade. It involves the slowest percolation rate (60 minutes per inch), the greatest loading (4 bedrooms or 600 gallons per day (GPD)), and the minimum depth to impermeable material (8 feet) and water table (4 feet) implying an initial depth of saturation, $D = 4$ feet.

Case II represents the "worst case" for large, private, subsurface disposal systems which are limited in New Hampshire to a maximum flow of 10,000 GPD. A minimum of two leach fields are required. This means the maximum flow to any one leach field is limited to 5,000 GPD, the size considered here.

Case III represents the anticipated optimum soil condition for a large system. Soils with a faster hydraulic conductivity would not provide as much treatment to the effluent. A slower hydraulic conductivity would result in a higher mound.

Case IV compares the results obtained by Marino (1974) with other approaches.

Case V is a hypothetical case with parameters chosen to fit the graph presented by Singh (1976).

Case VI is based on parameters inferred for a recharge basin described by Suter (1956).

Case VII and Case VIII are based on data presented by Bianchi and Haskell (1975).

In Figure 4 and Figure 5, mounded water tables are presented as predicted by both time-dependent and time-independent equations. The height above the initial water table is plotted against radical distance measured from the center of the hypothetical leach field (axis of symmetry).

Guswa (1977) employed a calibrated, finite difference, three-dimensional, groundwater model to analyze the possible mounding of groundwater on Cape Cod which might result from moderate scale water import and recharge. Two of his examples were presented with sufficient assumptions to allow calculation of mound heights using equation (4). Guswa's results were roughly 10% higher than those developed by means of equation (4) for both examples and at various distances from the axes of symmetry.

Table 2

Height of Axisymmetric Recharge Mounds Above an Initial

Water Table (S), in Feet

		Analytic				Computer		Analytic	
		Time-dependent				Time-dep.		Time-independ.	
		Glover		Hantush		Hunt		Baumann	Allen
		t= 300 days	t= 3650 days	t= 300 days	t= 3650 days	t= 300 days	t= 3650 days	-	-
Case I Q=600 GPD L=300 ft. (assumed) K=0.27 ft./day (60 min/inch) D=4 ft. R=30.9 ft. V=0.15	r=0	15.8	30.3	9.3	14.1	(i) 20	(e) 45	10.8	12.6
	r=R	-	-	7.5	12.8	-	-	9.1	11.2
	r=2R	-	-	4.5	10.7	-	-	6.4	8.9
Case II Q=5000 GPD L=1000 ft. (assumed) K=0.27 ft./day (60 min/inch) D=4.ft. R=89.2 ft. V=0.15	r=0	44.5	150	24.5	41.3	(e) 45	(e) 180	38.6	44.1
	r=R	-	-	18.3	36.8	-	-	33.7	39.8
	r=2R	-	-	7.3	26.6	-	-	22.1	33.1
Case III Q=5000 GPD L=1000 ft. (assumed) K=15 ft./day (2 min/inch) D=4 ft. R=44.6 ft. V=0.15	r=0	5.2	7.4	3.8	4.9	(e) 4.8	-	3.6	4.2
	r=R	-	-	3.3	4.5	-	-	3.1	3.8
	r=2R	-	-	2.6	3.9	-	-	2.4	3.1

(i) implies interpolation, (e) implies extrapolation

Table 3

Height of Axisymmetric Recharge Mounds Above an Initial Water Table (S), in Feet

			Computer Solutions				Analytic Solutions			
			Time-dependent				Time-dep.		Time-ind.	
		Field Data	Marino	Singh	Hunt	Brock	Glover	Hantush	Baumann	Allen
Case IV Marion's Case Q=31,000 CF/D L=5000 ft. (assumed) K=10.2 ft./day D=84 ft. R=168 ft. V=0.15 t=30 days	r=0	-	10.5	-	(e) 12	-	10.5	10.0	16.8	20.3
	r=R	-	8.1	-	-	-	-	7.5	14.4	17.7
	r=2R	-	4.4	-	-	-	-	4.1	10.4	14.3
Case V Singh's graph Q=1211 GPD L=300 ft. (assumed) K=0.27 ft./day D=123.6 ft. R=30.9 ft. t=34.3 days V=0.15	r=0	-	-	3.4	(e) 5	-	1.5	1.5	1.7	2.1
	r=R	-	-	0.7	-	-	-	1.1	1.3	1.7
	r=2R	-	-	0.1	-	-	-	<0.1	0.8	1.2
Case VI Peoria pit Q=3 MGD L=10,000 ft. (assumed) K=1072 ft./day D=10 ft. (?) R=43.6 ft. t=210 days V=0.25(?)	r=0	? 7	-	-	-	-	32.6	16.6	16.9	18.4
	r=¼ mi.	5	-	-	-	-	-	5.6	5.7	8.4
	r=½ mi.	3	-	-	-	-	-	3.0	2.5	6.1

(e) implies extrapolation

Table 4

Height of Axisymmetric Recharge Mounds Above an Initial Water Table (S), in Feet

		Field Data	Computer Solutions				Analytic Solutions			
			Time-dependent				Time-dep.		Time-ind.	
			Marino	Singh	Hunt	Brock	Glover	Hantush	Baumann	Allen
Case VII Bianchi/Haskell Pond No. 1	r=0	5.6	-	-	(e) 4	-	4.9	4.5	7.0	8.3
Q=208,000 GPD, L=5000 ft. (assumed), K=104 ft./day, D=16 ft., R=166.4 ft., V=0.26 (?), t=28.1 days	r=R	3.9	-	-	-	-	-	3.5	6.1	7.4
	r=2R	2.5	-	-	-	-	-	2.0	4.7	6.1
Case VIII Bianchi/Haskell Pond No.2	r=0	6.6	* 7.5	-	(e) 6	** 3.6	3.7	3.7	7.2	8.6
Q=30446 CF/day, L=5000 ft. (assumed), K=26 ft./day, D=80 ft., R=166.4 ft., V=0.17 (?), t=8.9 days	r=R	4.4	* 5	-	-	3.5	-	2.7	6.1	7.6
	r=2R	2.1	* 2	-	-	2.7	-	0.7	4.6	6.1

(e) implies extrapolation

* Results obtained for: w=0.35 ft./day, D=84 ft., K=10.3 ft./day, V=0.15, R=168 ft.

** The result was 5.2 ft. when V was fitted to regions inside and outside r=R

FIGURE 4

MOUND SHAPE FOR CASE I

SCALE : 1" = 10'

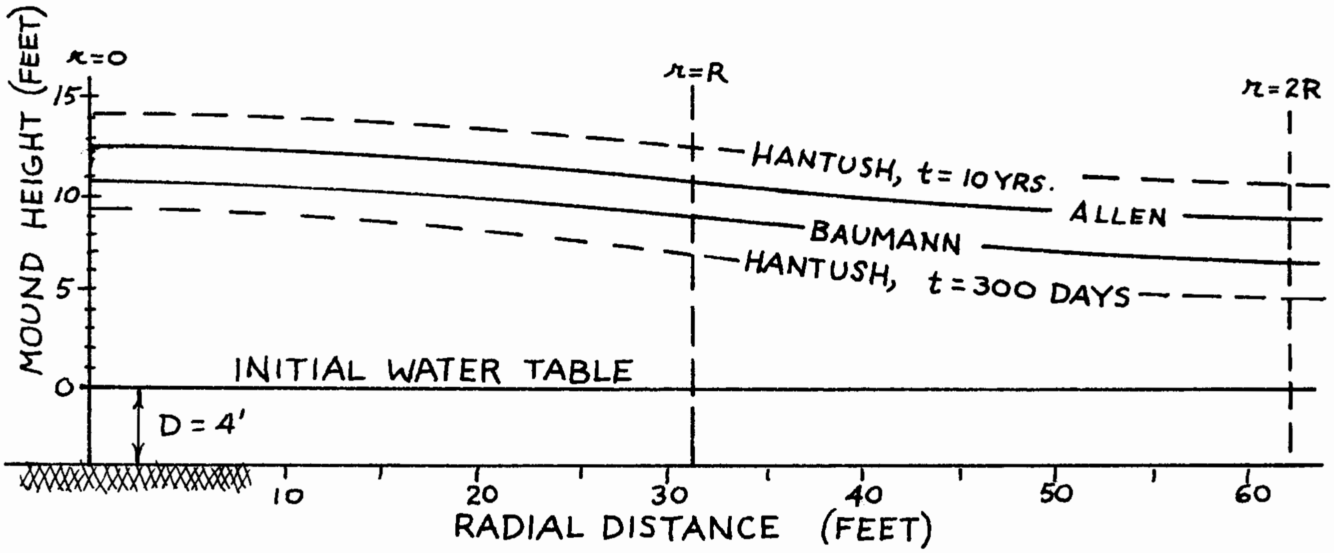
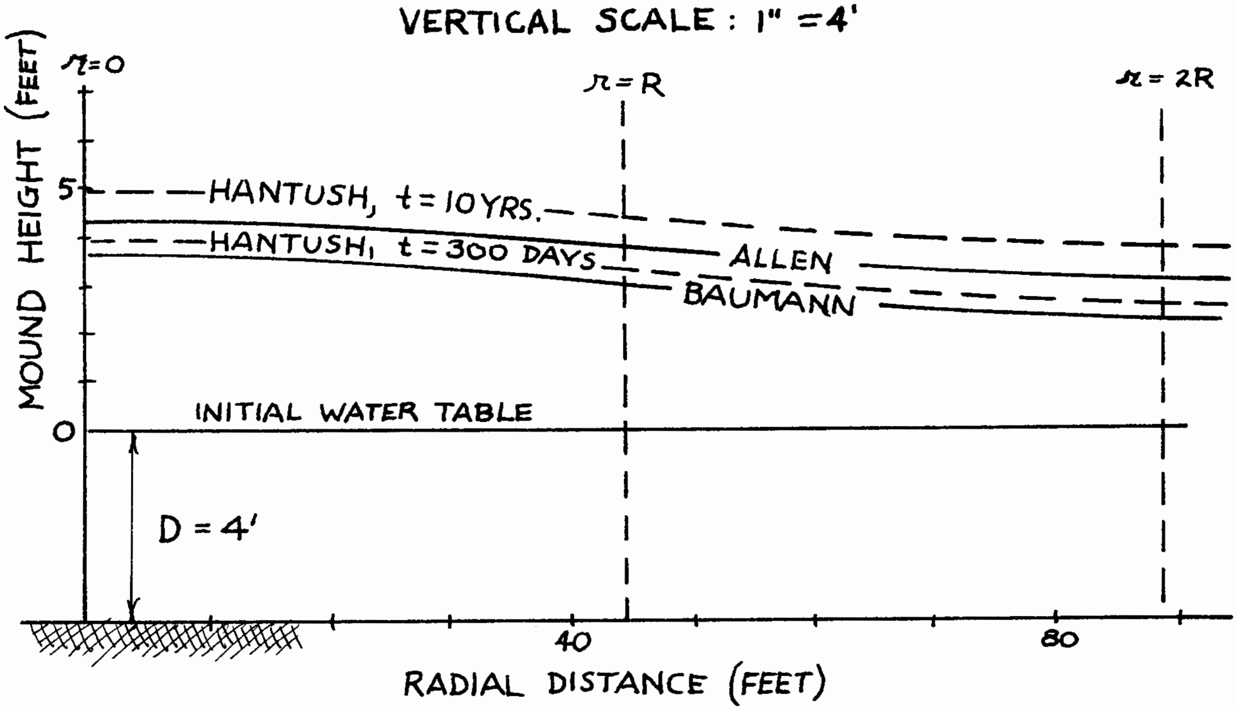


FIGURE 5

MOUND SHAPE FOR CASE III

HORIZONTAL SCALE : 1" = 16'

VERTICAL SCALE : 1" = 4'



DISCUSSION

All of the central mound heights for cases I and II, for t = 300 days, are within a factor of approximately two. Three hundred days was chosen as a time increment which might be associated with the period between wet seasons. However, when t = 3650 days (ten years), it becomes apparent, particularly in case II, that the lateral control, assumed for steady state solutions, must play an important role in keeping the mound heights from continuing to grow. In this example it appears difficult to reconcile the low mounds predicted by the Hantush solution with those obtained using solutions by Glover and Hunt. It should be noted that the Hantush approach relies on an assumed average saturated depth where the other solutions do not. The Hantush solution combines by means of quadrature the initial saturated depth and a term depending on the flow and hydraulic conductivity, just as in the solution by Baumann and as developed in this paper.

Mound heights for single family residential septic systems should not be ignored when marginal hydraulic conductivity is involved, as in case I, for which the mound height could reach ten to twenty feet, assuming a lateral control. With no lateral control and without wells to draw the water table down, the long-term mound height could be higher. Capillary effects reduce the unsaturated zone further. The likelihood of an unsaturated zone under the leach bed for the conditions of case I is nil. Because the unsaturated zone provides the potential for adsorption of pollutants essentially not present under saturated conditions, pollutants can travel long distances with the groundwater flow when no unsaturated zone is available.

In case III, and generally for the prediction of low mound heights, the results for t = 300 days and the time-independent results are all fairly close together. The steady state solutions appear to give mound heights which are lower than those which are time-dependent. This could be due to the assumption of a radius of influence which is too short. Choosing a larger value would result in a higher calculated mound height.

Since the mound heights developed in this example are almost all four feet or more, it should be noted that, even with rather rapid hydraulic conductivity assumed, the leach field would have to be over eight feet above the seasonal high water table to assure four feet of unsaturated soil.

In cases IV, V, VII, and VIII the time periods for the calculations were so short that steady state solutions should give results considerably higher than the time-dependent results. That this is not true in case V seems to indicate that solutions are sensitive to other factors, such as the very great initial saturated depth in this case.

In cases VI, VII, and VIII, field data are compared with calculated mound heights. In case VI the description of the recharge experiment is unclear as to the initial saturated depth and the resultant central mound height, however, the mound height at ¼ and ½ mile from the center of the recharge pit compares well with solutions by Baumann and Hantush. In cases VII and VIII, the specific yield was evaluated by three different methods both in the regions of the recharge pits and outside these areas. Great variability was reported between methods and between regions. These variations were not rationalized and lead to concern as to which single value might be used in the analytic solutions. However, Brock has obtained computer results using different values of specific yield for the two regions in case VIII. His best fit was obtained using the single value of specific yield assumed for case VIII.

Examination of the mound heights at $r = 0$, R, and 2R shows that for well established mounds (steady state or $t > 200$ days) the central mound height is roughly one fourth higher than the height at $r = R$. In the ideal case, then, the mound height is determined in large measure by conditions occurring outside of $r = R$. By comparison of the mound heights at the three locations we see that the slope of the free surface is not extreme. In the ideal case, we expect the point of inflection in the profile of the mound to occur at $r = R$, at which point the slope should be maximum. In case I and II, where mounding is far in excess of the initial saturated depth, there should be concern as to whether or not the mound slope exceeds 10%. Evaluation of dh/dr at $r = R$ in equation (4) gave the values of 10% and 10.1% for cases I and II, respectively.

NON-IDEAL CONSIDERATIONS

There are a number of reasons why, in practice, the results of recharge of groundwater might not turn out as calculated by even the best idealization.

1) The soil may be anisotropic, particularly with $K_{horizontal} > K_{vertical}$. Apparently, it is not unusual for the horizontal hydraulic conductivity to be as much as ten times the vertical hydraulic conductivity. This is true because soil is often constructed by processes which result in the deposition of horizontal layers. In such an event, the horizontal transmissivity is the arithmetic sum of the transmissivities of the various layers, but the vertical hydraulic conductivity depends in a geometric way on the hydraulic conductivities of the various layers. It is the horizontal hydraulic conductivity which is of most importance outside the recharge area.

2) The hydraulic conductivity of the soil, measured at a number of locations, may not give an average value which is truly representative of the overall site being utilized for recharge, particularly because of the difficulty of making point measurements of conductivity anywhere but on the ground surface. For discussion of variation of conductivity within soil units see Baker and Bouma (1976) and Winneberger (1967).

3) There may be some substantial flow laterally above the free surface in the saturated capillary zone. Luthin and Day (1955) have investigated this and found that the effect is important for soils with a substantial capillary zone such as clay soils. They mention that the saturated zone, even if at less than atmospheric pressure, should have flow which satisfies LaPlace's equation. Such flow should tend to increase the effective depth of flow, thereby reducing the free water mound height below the height which would be obtained by a calculation which ignores the capillary zone. The capillary rise of groundwater quoted by Lohman (1972) can be up to 40 centimeters for a fine sand. Additionally, there can be lateral flow in the unsaturated zone due to a horizontal gradient over a sloping free surface. Kroszynski and Dagan (1975) found the influence of the unsaturated zone to be negligible in most cases. There is a rich literature devoted to unsaturated flow. As an example, the reader is referred to work on radial unsaturated flow in the absence of gravity by Drake, et al (1969).

4) Evapotranspiration and rainfall may account for losses or gains of soil moisture. However, since we do not add the rainfall to the infiltration, rainfall should be considered the means by which the water table is maintained at a constant level of saturation, D. Evapotranspiration cannot be counted on to balance rainfall in the worst case condition occurring during the spring snowmelt. Clearly, transient events are capable of introducing large variations in soil moisture and the corresponding saturated depth. All calculations for the purpose of prediction of the maximum mound height should start from an assumed seasonal high water table. Evaporative losses are analyzed by Jayawardane (1976) who assumes that the moisture tension-unsaturated conductivity curve can be approximated by a line of constant slope.

5) The impermeable layer assumed to underlie the recharge site may not be impermeable. Water could be lost down through this layer, or water could be recharged to the overlying soil by a leaky underlying confined aquifer. Other sources or sinks of soil moisture, not shown in the ideal analysis, may occur in the field. Swales, springs, and wells may occur within the radius of influence.

6) Testing of the unconfined aquifer by means of the well draw-down test may not characterize the upper layers of soil in which the upper portion of a recharge mound would occur. Since it is likely that, for most New England soils (except for coarse sands and gravel deposits), the upper layers would be more permeable than the lower layers, the result of a mound calculation using the permeability of the lower zones will, generally, be on the safe side.

7) Soils undergoing chemical alteration, due to ions in the recharged water, may become altered in hydraulic conductivity. Clay soils are particularly prone to swelling and reduction in conductivity when salts are added.

8) The impermeable layer underlying the recharge site may not be horizontal, as assumed in the ideal analysis.

9) The topography of the impermeable layer can drastically change the potential of a site for subsurface disposal. A concave-up situation, whether it amounts to a so-called "bathtub" or a swale, is just not going to drain as well as the crest of a ridge, edge of a terrace, or other convex-up topography.

10) The water table without recharge might not actually be horizontal. As the average groundwater slope increases, the effect is to wash out the groundwater mound, carrying away the mound before it is established.

11) The recharged water may differ in temperature from the water involved in measuring the hydraulic conductivity, and the effective hydraulic conductivity depends on the viscosity of the water. Water recharged at a temperature of 97°F has only ½ the viscosity of water at 45°F, the year-round average temperature of groundwater in northern New England. However, Viraraghavan and Warnock (1977) have shown that their septic tank effluent had a temperature within a few degrees of the soil temperature, which varies seasonally. Effluent with a high temperature, as, perhaps, restaurant waste, would mean an enhanced effective hydraulic conductivity, and a mound lower than would be predicted using the assumption of a standard temperature.

12) Clay soils may exhibit non-Darcy behavior in that soil moisture in clay soils appears to have enhanced viscosity at low potential gradients. See Swartzendruber (1968), Childs and Tzimas (1971), and Basak (1977).

13) The assumption that unsaturated flow below the recharge bed will occur only in the vertical direction is not quite true. Some spreading of the unsaturated flow can be expected. This increases the effective size of the recharge bed by a minor amount and, thereby, reduces the height of the mound slightly.

14) Boundary effects, such as laminar flow next to the impermeable surface, are not accounted for in the ideal analysis.

15) Water is not quite the incompressible fluid assumed in the LaPlace equation, but the variation is not significant relative to many of the other considerations mentioned above.

16) For short time periods (probably for recharge events of less than 100 days) some error in time-dependent solutions may be associated with the vertical travel time lag for the unsaturated flow and for the associated initial reduction in effective conductivity due to trapped air.

17) Van Der Kamp (1976) has investigated groundwater waves resulting from start-up of a pumped well in the under-damped case. Similar wave propagation might occur from recharge, but is unlikely to produce significant

effects except, perhaps, initially when the mound is essentially insignificant.

18) When more than one leach field will be installed, the effects of each can be superposed as long as the resultant free surface slope does not exceed 10%. For instance, if four leach fields, each constructed and loaded as in case III, are arranged in a square array with separation of their centers equal to 3R, then the resultant mound height under each field will be 11 feet instead of the 4 feet, approximately, for each field alone. On other sites, well draw-down curves and other recharge or discharge effects may complicate the ideal case.

Superposition of the recharge mounds and well draw-down curves, averaged over a subdivision which uses shallow wells for water supply, should result in no net increase in the water table height. However, where water is imported, or is obtained from a non-communicating aquifer, a net increase in water table height is to be expected. Franks (1972) reports that a subdivision involving ½-acre lots and imported water resulted in an increase in the water table height which subsequently caused the subsurface disposal systems to fail.

19) The distance to the lateral control is not a simple measurement. The distance to the nearest wet area down-gradient from the recharge site is, probably, the best estimate to use. While the time-dependent solutions do not rely on a lateral control, this weakness suggests that these solutions are inappropriate for the long-term recharge case. If there is nothing closer, the oceans will provide a lateral control. The mound height will not rise to infinity. What is needed is a time-dependent solution which takes into account the lateral control of arbitrary shape.

A number of the foregoing considerations suggest that a method is needed which evaluates the overall performance of a proposed recharge site. The well draw-down test does evaluate all of the site except that portion above the initial water table. It can provide a much better estimate of the hydraulic conductivity, or what is better, the transmissivity, than can be obtained from localized conductivity tests performed on soil horizons near the ground surface. The well draw-down test should be considered an essential tool in site evaluation for significant recharge projects. Without use of this or a comparable test, the uncertainty in the conductivity should be considered to be substantially greater.

DEPENDENT UNCERTAINTY

The various analytical solutions for mound height can be submitted to uncertainty analysis which provides a measure of the fractional change in the mound height which takes place as a result of a fractional change in an independent variable. This relationship is determined by the functional relationship of the variables and is not a measure of variation relative to some known standard. To deal in terms of numbers requires that the functional relationship be evaluated for a particular set of parameters.

Generally, Q, t and R can be assumed to be known precisely, or they can be adequately controlled by careful execution of an experiment. However, the site conditions usually do not allow exact measurement of K, V, L and D.

In Table 5 dependent uncertainties in H are developed independently for the variables K, L, V and D by use of the assumption that the dependence of $\Delta H/H$ on, say, $\Delta K/K$ (designated $(\Delta H/H)_K$) is the same functional relationship as the dependence of $(\delta H/H)_K$ on $\delta K/K$ where δH and δK are infinitesimal quantities. The subscript here denotes that all the other independent variables are held constant.

The total uncertainty in H is found by combining by means of quadrature the dependent uncertainties due to all the independent variables after substituting fractional uncertainties for $\Delta K/K$, etc., as in equations (7) and (8).

$$\left(\frac{\Delta H}{H}\right)_T^2 = \left(\frac{\Delta H}{H}\right)_K^2 + \left(\frac{\Delta H}{H}\right)_D^2 + \left(\frac{\Delta H}{H}\right)_L^2 + \left(\frac{\Delta H}{H}\right)_V^2 \qquad (7)$$

If we assume a 10% fractional uncertainty in each of the independent variables (K, D, L and V), and evaluate the total fractional uncertainty in H for case I using equation (4) and values from Table 5, we have:

$$\left(\frac{\Delta H}{H}\right)_T^2 = \left(-0.48\,\frac{\Delta K}{K}\right)^2 + \left(.058\,\frac{\Delta D}{D}\right)^2 + \left(0.17\,\frac{\Delta L}{L}\right)^2 + \left(0 \cdot \frac{\Delta V}{V}\right)^2 \qquad (8)$$

$$\left(\frac{\Delta H}{H}\right)_T^2 = (-.48(.1))^2 + (.058(.1))^2 + (.17(.1))^2 \qquad (9)$$

$$\left(\frac{\Delta H}{H}\right)_T = 0.051$$

Table 5

Relative Values of Dependent Uncertainty in the Recharge

Mound Height (H)

			Glover	Hantush	Baumann	Allen
Case I (t=300 days)	K	$\frac{(\Delta H/H)_K}{\Delta K/K} \simeq$	-0.51	-0.32	-0.46	-0.48
	L	$\frac{(\Delta H/H)_L}{\Delta L/L} \simeq$	0	0	- .00018	0.17
	V	$\frac{(\Delta H/H)_V}{\Delta V/V} \simeq$	-0.28	-0.13	0	0
	D	$\frac{(\Delta H/H)_D}{\Delta D/D} \simeq$	-0.31	0.90	0.073	0.058
Case III (t=300 days)	K	$\frac{(\Delta H/H)_K}{\Delta K/K} \simeq$	-0.47	-0.37	-0.36	-0.38
	L	$\frac{(\Delta H/H)_L}{\Delta L/L} \simeq$	0	0	0.0013	0.11
	V	$\frac{(\Delta H/H)_V}{\Delta V/V} \simeq$	-0.10	-0.058	0	0
	D	$\frac{(\Delta H/H)_D}{\Delta D/D} \simeq$	-0.038	0.26	0.28	0.24

Note: 1) Subscripts designate variable in regard to which dependence is considered.

2) A negative sign indicates that an increase in H occurs when there is a decrease in the independent variable.

Note that, in Table 5 the uncertainty in H has the largest relative dependence on the conductivity. For the time-independent solutions, the dependence on L is relatively weak.

CONCLUSIONS

1) From this study it appears that it would be impossible to assure unsaturated conditions below leach beds situated on sites which have "worst case" conditions currently allowed under state regulations in New Hampshire. Not only are mound heights far in excess of the "four feet to seasonal high water table" rule used by a majority of state codes, but the low drainage potential of soils with a percolation rate of 60 minutes per inch is also accompanied by a very significant capillary potential which results in saturated conditions far above the free water surface observed in test pits. Even if the actual water usage were only 50% of the 75 gallons per capita-day assumed in case I, the mound height would still exceed the four feet of unsaturated soil assumed to be available.

2) From this study it can be concluded that, for well developed mounds in the idealized situation, the groundwater free surface slope is not excessive. In worst case conditions it was found to be on the order of 10%, the limit, as proposed by others, for application of solutions of the DF equation.

3) From this study it can be concluded that, for well developed mounds, the mounds extend horizontally to relatively large dimensions with flat slopes. The mounding under the recharge beds contributes only a small fraction of the total mound height. Apparently, the conditions outside the recharge areas, where quasi-horizontal flow occurs, are of primary importance in determining mound height.

4) Examination of the dependence of the viscosity of water on temperature, as part of this study of mound heights, showed that temperature might play an important role in subsurface disposal system performance through its effect on hydraulic conductivity. Within the likely seasonal temperature range for groundwater, the hydraulic conductivity could decrease by roughly 33%. This could result in a required 15% increase in mound height in the winter over that calculated for summer conditions.

5) If the mound heights found in this study are superposed for a square array of four fields, with centers separated by a distance equal

to three times the radius of each, the resultant mound height under each might be 2.8 times the mound height of a single field. In laying out large subsurface disposal systems with more than one field, the interaction is important.

6) This study has found in the literature no analysis of groundwater mounds which provided for both time-dependence and axisymmetric lateral control. This should not be difficult to provide. However, the average situation in the field involves an asymmetric lateral control. An analysis of this type, with time-dependence, should be sought by means of computer techniques, or, perhaps, by the method of images.

7) Of all the solutions examined here for the groundwater mound height in the ideal case, the work by Singh appears to give the solution most free from limiting assumptions. In case V, where a comparison could be made to other results, reasonable agreement was found. Comparison is really not possible between time-dependent and time-independent results.

8) The field data found, and the search for sources of field data made in this study, resulted in the conclusion that, not only is there a dearth of field studies, but the field studies may provide an inadequate basis for discriminating between various theories based on the predicted mound heights, because of uncertainties regarding site conditions.

9) The mound heights for large systems analyzed in this study are sufficiently impressive to bring attention to the need for adequate site evaluation before construction of such systems. The well draw-down test offers a means by which to evaluate the overall effective transmissivity and specific yield of a site. This information is not obtained from isolated surface tests of hydraulic conductivity employing the double tube permeameter, the percolation test, laboratory permeameter measurements of disturbed samples, or similar tests for hydraulic conductivity of small samples.

10) The equation for central mound height developed in this study was found to be of the same form as the other time-independent solution, by Baumann. However, the calculated mound heights were found to be consistently higher than those calculated by use of Baumann's equation. The difference in heights may well be within the uncertainty which should be anticipated for sites for which little data is available. The difference in results

predicted by the two approaches was not enough to change the basic conclusions to be drawn from use of one approach or the other. Certainly, the equation developed in this study offers far less intimidation to those who dislike complicated equations.

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APPENDIX A

THE SOLUTIONS

The solutions can be grouped into two broad categories, those which are expressed in explicit equations (analytic type) and those which are expressed in terms of routines requiring computer evaluation (computer solutions). The computer solutions break down into those involving finite difference or finite element methods to evaluate conditions at the nodes of a space grid superimposed on the porous medium, and a second category involving numerical solution of the fundamental equations by the computer using series approximations and other techniques.

The analytic solutions break down into time-dependent and time-independent equations. The time-independent equations require for steady-state solution that there be a control condition at some finite radius, at which there will be no resultant change in the free surface height. In practice, the control might be a stream or wet area.

In general, the time-dependent solutions do not assume a lateral control. The implication is that at infinite time the mound height will be infinite, and the mound will extend to infinity in a radial direction. In other words, they continue to grow.

All of the solutions can be broken down into those which employ the DF assumptions and those which do not. An additional consideration is whether or not linearization has been made use of.

The solutions found which involve axisymmetric recharge and which present results in either numerical, graphical, or explicit equation form are described below.

COMPUTER SOLUTIONS

HUNT (1971) has solved the LaPlace equation subject to linear boundary conditions. This approach is referred to as analysis of linear potential theory (not DF). The author used perturbation techniques, and feels that his techniques are only valid for a relatively short recharge event (t = 100 days). He shows comparison of his results to field data obtained by Bianchi and Haskell (1968) and concludes that the field data are in doubt.

MARINO (1975b) has applied the Douglas-Jones predictor-corrector methods to the second order, non-linear form of the DF partial differential equation. His results are presented for only one set of parameters (see Results). These results are compared with those of Hunt and Hantush (see below). It was found that Marino's results were always less than those of Hunt, and always greater than those of Hantush. The discrepancies increase as time increases.

BROCK (1974) has compared the linear DF results with non-linear DF results obtained by computer solution. Brock also calculates the error which results from assuming a square recharge basin is round, and he finds that the error does not exceed 3% with the greatest deviation occurring at the edge of the field. A graphical criterion is presented for determining when to use linear and when to use non-linear DF theory for the case of the square basin (see Figure A-1).

SINGH (1976) uses the finite difference technique and Gaussian elimination methods to solve LaPlace's equation with initialization involving a horizontal water table. "No simplifying assumptions or linearization of the phraetic surface conditions have been employed." His results are presented in graphical form, but only for a limited domain of the parameters.

It should be noted that none of the time-dependent computer results have been projected beyond about 100 days.

ANALYTICAL SOLUTIONS

GLOVER (1961) solved the time-dependent linear DF equation by means of Bessel functions for the center of the axisymmetric recharge basin. His equation calls for evaluation of the "well-function",

$$W(u) = \int_u^\infty \frac{e^{-x}}{x}\, dx \qquad \text{(A-1)}$$

using tables, and it is expressed as follows:

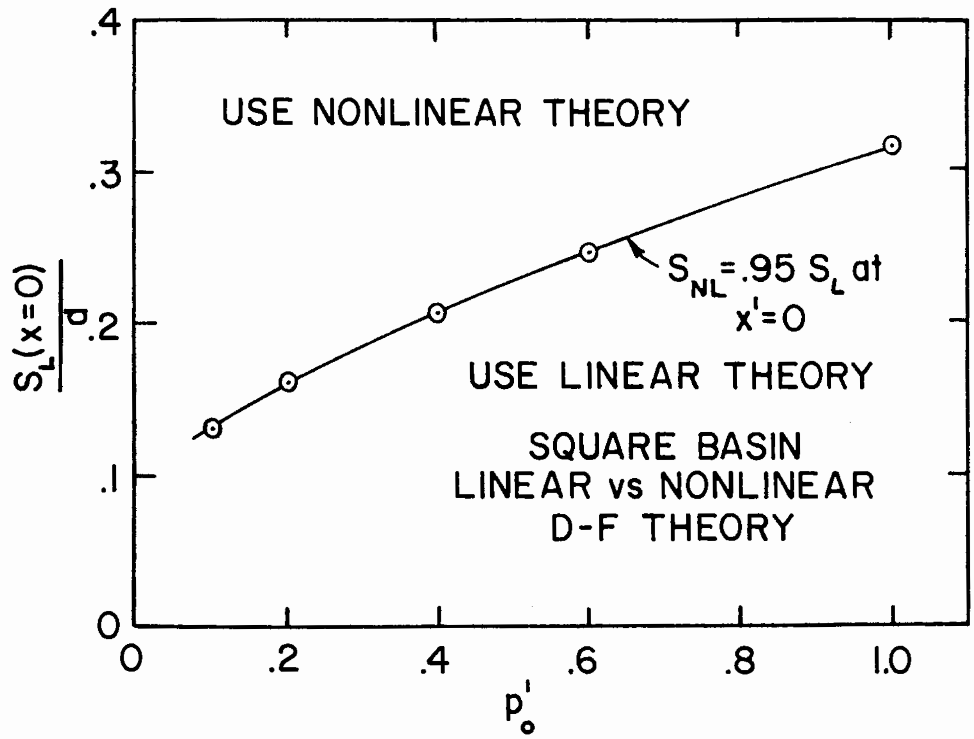


Figure A-1*

Square Basin: Linear vs. Nonlinear DF Theory

The following definitions apply to Figure A-1:

S_L = water table mound above initial depth (linear)

S_{NL} = water table mound above initial depth (non-linear)

x = 0, x' = 0 refer to distance from center of basin

a = initial saturated depth of unconfined aquifer

P_o' = $P_o L^2/Ka^2$

P_o = recharge rate

L = one half width of basin

K = hydraulic conductivity

*(Reproduced from Brock, 1974)

$$S = \frac{W}{V} t \; [1 - e^{-u} - u \; W(u)] \tag{A-2}$$

where $u = \frac{R^2}{4\alpha t}$ and $\alpha = \frac{KD}{V}$. However, Glover does provide a general solution for the case of a rectangular basin or leach field. In a later publication (Glover, 1965) comparison is made between DF theory and the field results which have been obtained. Glover concludes that DF theory is quite adequate when applied to appropriate situations.

BAUMANN (1965) has solved the steady state linear DF equation and obtained the following equations:

$$S = -D + (D^2 - \frac{Q}{\pi K} \{\ln \frac{R}{L} + \frac{1}{L-R} [L \exp(\frac{L-R}{L} - 2L + R] - \tfrac{1}{2}\})^{\frac{1}{2}} \tag{A-3}$$

but for r = R we have:

$$s = -D + (D^2 - \frac{Q}{\pi K} \{\ln \frac{R}{L} + \frac{1}{L-R} [L \exp \frac{L-R}{L} - 2L + R]\})^{\frac{1}{2}} \tag{A-4}$$

and, for r > R,

$$s = -D + (D^2 - \frac{Q}{\pi K} \{\ln \frac{R}{L} + \frac{1}{L-R} [(r + L - R) \exp(\frac{L-R}{L}) - 2L + R]\})^{\frac{1}{2}} \tag{A-5}$$

HANTUSH (1967) solved the time-dependent, linear DF equation by means of LaPlace and zero order Hankel transformations. His equations involve assumption of a weighted average saturated height ($\bar{h}$) and use of the well function:

$$H^2 = D^2 + \frac{Q}{2\pi K} [W(u) + (1 - e^{-u})/u], \tag{A-6}$$

where $u = \frac{R}{4\nu t}$ and $\nu = \frac{K\bar{h}}{V}$. For $0 < r < R$:

$$h^2 = D^2 + \frac{Q}{2\pi K} [W(u) - (\frac{r}{R})^2 e^{-u} + \frac{1}{u} (1 - e^{-u})] \tag{A-7}$$

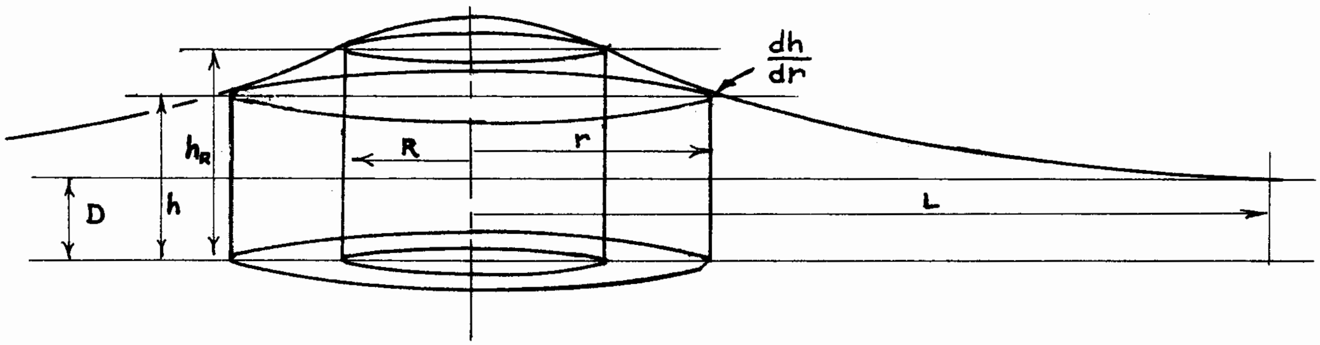
For $r > R$ and $t \geq 0.5 \frac{R^2}{\nu}$:

$$h^2 = D^2 + \frac{Q}{2\pi K} [W(u) + 0.5u \; e^{-u_1}] \tag{A-8}$$

where $u_1 = \frac{r^2}{4\nu t}$.

APPENDIX B

DEVELOPMENT OF EQUATIONS (3) and (4).



At the outer cylindrical surface and making the Dupuit assumption, the horizontal gradient is given by $-\frac{dh}{dr}$ and Darcy's Law says:

$$Q = KA\,\frac{dh}{d\ell} = -\,K2\pi rh\,\frac{dh}{dr}$$

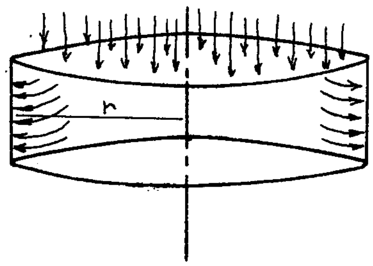
This implies: $\frac{Q}{2\pi K} \int_R^L \frac{dr}{r} = -\int_{h_R}^{D} h\,dh$

$$\frac{Q}{2\pi K}\,\ln r \,\Big|_R^L = \frac{h^2}{2}\,\Big|_D^{h_R}\ , \quad \frac{Q}{\pi K}\,\ln\frac{L}{R} = h_R^2 - D^2$$

$$h_R^2 = D^2 + \frac{Q}{\pi K}\,\ln\frac{L}{R}$$

This is a particular solution of the Forchheimer differential equation.

Now, inside the recharge area ($r < R$) we assume the conservation of flow such that $\frac{Q}{\pi R^2} \cdot \pi r^2$ is the quantity of fluid flowing through the horizontal recharge bed inside a circle of radius r.



While the flow may not be horizontal, the horizontal component is assumed to satisfy the following requirement:

$$\frac{Qr^2}{R^2} = -\,2\pi rh\,K\,\frac{dh}{dr}$$

This equation can be integrated from r = R to r = 0.

$$\frac{1}{2\pi K}\int_R^0 \frac{Qr}{R^2}\,dr = -\int_{h_R}^{H} h\,dh$$

$$\frac{Q}{2\pi KR^2}\left(\frac{r^2}{2}\right|_R^0 = \frac{h_R^2 - H^2}{2} = \frac{-Q}{2\pi KR^2}\left(\frac{R^2}{2}\right)$$

$$h_R^2 + \frac{Q}{2\pi K} = H^2 = D^2 + \frac{Q}{\pi K}\ln\frac{L}{R} + \frac{Q}{2\pi K}$$

$$\boxed{H^2 = D^2 + \frac{Q}{\pi K}\left(\ln\frac{L}{R} + 1/2\right)}$$