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Chosen Multiplication Algorithms and the Ability to Learn New Methods

Catherine A. Tarushka
University of New Hampshire - Main Campus

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Catherine Tarushka

Advisor: Dr. Sharon McCrone

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University of New Hampshire

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Introduction

When observing students attempt middle school mathematics, the question of multiplication competency tends to arise. When students struggle to answer multiplication questions which require an algorithm rather than memorization of multiplication facts, the algorithm that they choose can be telling about their deeper understanding of multiplication. For instance, if they use a calculator, it could be because they have opportunities to use calculators at every turn. Or, it could possibly mean that students have trouble with written or mental calculations. The use of any number of algorithms could also indicate either a strength or a weakness in number sense or multiplication capability.

According to Brocado, Serrazina, and Rocha, education researchers in Portugal, number sense is "Knowledge and facility with numbers, which include multiple representations of numbers, recognizing the relative and absolute magnitudes of numbers, composing and decomposing numbers and selecting and using benchmarks… Applying knowledge of and facility with numbers and operations to computational settings," (Brocardo 407). Therefore, number sense is the ability to visualize and understand numbers in different ways and to manipulate them using a variety of methods including decomposition in order to complete computations. Number sense, then, would contribute to a student's ability to perform mental multiplication. These findings led me to ask if there was any particular multiplication algorithm which most encouraged students to develop strong number sense or that would demonstrate if a student had strong number sense. Because one method of measuring number sense is to be able to use a variety of methods when solving problems, I decided to investigate whether and how well students could learn a new method. My reasoning was that if they were capable of performing multiplication when learning a new method, then they had strong number sense. So, I could see if there was a method which most consistently was used by those who were successful.

Opinions about the use of algorithms in the mathematics classroom differ widely. Fosnot and her colleagues found that some educators believe that "using algorithms… is antithetical to calculating with number sense" (Fosnot 102). This is because these educators see algorithms as concrete methods imposed on students such that they fail to develop number sense whatsoever. Others believe that "Algorithms are one means by which we can look into one another's' minds and see what thought processes we are using and why," (Morrow 38). In this view, algorithms are not hindrances to student learning but merely a way of communicating thoughts. No matter any mathematician's or educator's opinions, however, algorithms exist in classrooms and offer students methods for finding solutions to common mathematical problems. Therefore, the question of which algorithms to teach remains a dominant question for educators. If there was a multiplication algorithm which could contribute to a student's growth of number sense, then this algorithm would be important because it would indicate a best way to teach multiplication.

In order to determine if there is a particular method which contributes to number sense, I decided to test how students learn new methods. If students are capable of learning a new method, then that shows that they are well equipped for future learning and that they can use their number sense to inform their algorithmic thinking, rather than following an algorithm blindly. This is imperative because in today's age there are multitudes of algorithms for multiplication. If any particular method would give students an advantage toward meaningful understanding or multiplicative ability then that method is vital. This advantage might not be what the teacher would expect, since all students learn differently. Instead, an algorithm that is best for a students might be one that appeals to their sense of logic.

When looking at algorithms for multiplication, there are two mediums through which students typically carry out multiplication computations: mental multiplication and pencil and paper multiplication. Although calculator use is certainly another medium for this, it does not allow the student to perform the computation. I intended to investigate number sense and algorithm use by teaching an algorithm that is best for pencil and paper calculations. Thus, paper and pencil would be the primary way to assess students' number sense via their multiplication methods. By exploring the ways in which students completed multiplication problems with pencil and paper, and then teaching students a new algorithm, I could investigate if there was any connection between pencil and paper algorithms and number sense. I hypothesized that if a student was able to learn and use a new method, then that student probably had a stronger number sense than a student who was unable to make sense of or use a new method.

Although educators place heavy emphasis on paper and pencil calculations, Morrow and Kenney assert in "The Teaching and Learning of Algorithms in School Mathematics", that students use more mental calculations than pencil and paper methods, and on top of that, students use a variety of mental methods that are typically different from pencil and paper methods. When students are presented with similar problems, they are even inclined to use different methods for each (Morrow 44), indicating that students' mental mathematical computation skills are more heavily dependent upon their number sense. Because of this, understanding how students multiply mentally would greatly indicate their number sense. Therefore, I intended to find out how students thought about and computed multiplication problems mentally. I was curious to find out how they thought about mental multiplication and how they understood what they were doing in their heads. That way, I could view their processes and visualize their methods which to them, may seem to be just intuitive and disconnected from paper and pencil tasks. To that end, I wished to have them compare what they were doing in their heads to what they were doing using a pencil and paper. If students were able to see a connection between all of the ways in which they multiply, it would indicate that they have greater understanding of multiplication, a deeper number sense, and hence will be able to carry these skills successfully into higher mathematics.

Rationale

Number sense is of great importance in today's mathematical community. In an era of technology, where calculators abound, students can frequently be seen to be relying on calculators when trying to multiply problems with single digits, a task which should be within their grasp. As related in "The Teaching and Learning of Algorithms in School Mathematics", more calculator algorithms are being taught in this era and technology changes the importance of different algorithms. Calculator algorithms consist of the ways to use a calculator correctly, such as knowing when to use parentheses or being able to find the program in a graphing calculator which will best suffice for a problem. This reliance on technology can lead to blind acceptance of results, overuse of calculators, and helplessness when a particular algorithm fails (Morrow 15). Therefore, number sense, which contributes to the ability of students to analyze answers and find different methods to meet a goal when one method fails, is surely an important quality to possess in any mathematical classroom. That is why Brocardo, Serrazina, and Rocha state that "number sense has been considered one of the most important components of elementary mathematics curriculum" (Brocardo 407). Number sense can contribute to the ability of students to succeed in future mathematical environments, because it means that they can work out difficult calculations and then assess those answers. On top of that, competency with computation is dropping. According to Karen Fuson in her work "Developing Mathematical

Power in Whole Number Operations", research has "identified aspects of computation in which children's performance was disappointing" (Fuson 71). This drop in ability to multiply will only cause problems for students as they crawl through future math. I believe that the drop in students' computational competency noted by Fuson indicates problems with number sense, and so an algorithm which best fuels understanding and competency would be a valuable find (Fuson 71).

Some of the many algorithms for multiplication include: the Long or Standard Method, Napier's Method, the Lattice Method, the Box Method, and the Russian Peasant Method. In addition, many students create their own multiplication algorithm, thus providing them and their classmates with slightly different methods (Morrow 38). Therefore, the question of what methods are most conducive to learning new methods and thus developing number sense is an important one. It is also valuable to consider students' personal algorithm choices as these might indicate student thinking and perhaps a "best" way to think about multiplication. If there is a best way to think about multiplication, then all students should have the opportunity to know it.

The Box Method

One multiplication algorithm that I found interesting is the Box Method. I do not claim that this is a "best" method for students, but it is a method which shows each and every component involved in multiplying multi-digit numbers. Using the Box Method, one begins by constructing an array based upon the number of digits in each number being multiplied. For instance, when multiplying two two-digit numbers, a two by two array is required. The digits of one number are aligned with each row (reading top to bottom) and the digits of the other number are aligned with each column (left to right). Then one multiplies the digits in the following way:

The digit in the first row is multiplied by the digit in the first column and the result is placed in the upper left box of the array; the digit in the second row is multiplied by that in the first column and the result is placed in the lower left box of the array; the digit in the first row is multiplied by that in the second column and placed in the upper right box of the array; the digit in the second row is multiplied by that of the second column and placed in the lower right box of the array (See Figure 1). For a larger array, this pattern of column and row multiplication continues. Students must remember that for each digit, place value must be taken into account. For instance, in the number 35, the digit 3 would be aligned with one row, however, that 3 is in fact an 30, so when students multiply with that 3, they are really multiplying with 30 (See Figure 1). Therefore, this method encourages students to think about place value and the importance of it in multiplication.

The Box Method allows students to see how every digit of a number is multiplied by every digit of the other number. Thus, the methodology behind multiplication is in plain view. After every box in the array has been filled, students find the sum of the boxes and that number is the product of the original two numbers (See Figure 1). Another bonus of the Box Method is that it is more versatile in terms of multiplying larger numbers. So long as a person is willing to make an array large enough, he or she can multiply any values using the Box Method.

Figure 1 – Example of Box Method

24 * 35

	2	4
3	600	120
5	100	20

600

100

120

+20

840

<u>The Long Method</u>

The Long Method, which may also be known as the traditional or standard method is a common multiplication method taught in U.S. schools. It consists of arranging each number so that digits are lined up in columns according to place value. Then, the ones place of the second number, or multiplier, is multiplied by the ones place of the upper number or multiplicand (5* 4 in Figure 2). The ones place of their product is written below and if there is any number other than zero in the tens place of this initial product, that number is “carried”. “Carrying” refers to the spare tens place of a product which is added to the subsequent higher place value product (The 2 from 5*4 = 20 is carried in Figure 2). Then, the ones place of the multiplier is multiplied by the tens place of the multiplicand and any carried numbers are added (In Figure 2, 5*2 and add 2 that was carried). The student must continue to write down each of the products below in the adding area (120 is in the adding area in Figure 2). This process is repeated with higher and higher place value until every digit of the multiplicand is accounted for. Then, the student moves onto another adding line down below and writes a zero in the ones place to denote that the digit being worked with in the multiplier is the tens digit (The 0 in 720 in Figure 2). The process is then repeated with each new digit in the adding area lining up with the correct place value of the adding number above it. This process of multiplying each digit of the multiplier by the multiplicand is repeated until all digits have been multiplied. The adding area numbers are then added, and their sum is the product of the original numbers (120 + 720 = 840 = 24*35 in Figure 2). This method sounds quite long and at first glance does not have the same intuitive value as the Box Method. Instead, the Long Method is a fast method, with practice, that is quite common in practice, if not in comprehension.

Figure 2 – Example of Long Method

```
 2 1
 24
 3 5*
----
120
720+
----
840
```

With two such seemingly different methods at hand, the question of how students best learn a new method will answer how well they understand multiplication in general. Though the Box Method and the Long Method appear to be different, they are in fact performing the same task. Therefore, the questions asked through this research would answer how students learn and therefore indicate their number sense and learning aptitudes.

Research Questions

I investigated how students work through double-digit multiplication problems both on paper and in their heads. They were not allowed the use of calculators or outside resources like the internet or fellow students.

The exact questions that I intended to answer were:

1. Is there a correlation between a specific multiplication algorithm which is a student's primary method and the ability to efficiently learn and use other multiplication algorithms?
2. In what ways does a student's chosen mental algorithm for multiplication differ from the pencil and paper algorithm, and are students aware of these differences?

The reason for the first question is that I believe the ability to learn a new mathematical algorithm is a way to gauge student number sense. I wanted to see if there was a particular method used by students and how that correlated with number sense. Hence, I compared the methods used by students and the ways in which they learned new methods. Unfortunately, many students do not see mathematics as something dependent upon number sense, but instead as a process to be learned (often merely memorized). According to Fosnot, mathematics contains unchanging and uniform processes (Fosnot 9). Therefore, students do not necessarily see the connections between thinking about mathematics, creating new mathematics, and actually doing mathematics. I wished to draw attention to students' abilities to use multiple methods, or at least show them that there are multiple ways of doing things. Ultimately, I intended to determine if they could learn efficiently, and so demonstrate that they have number sense. After that demonstration, then their chosen multiplication method would hopefully demonstrate this knowledge by showing that they are capable of making an informed decision when choosing algorithms. They would be able to discuss their methods and also be able to discuss the value of their methods compared to others. This is extremely important because the ability to multiply competently is vital to higher level mathematics.

I hoped to find students using a variety of multiplication algorithms so that I could compare and contrast their chosen methods as well as their understanding of these algorithms. I also hoped that their success in learning the new algorithm would reflect the best algorithm at their disposal. Unfortunately, these students were not taught a great many algorithms, nor were they necessarily taught to understand their methods. As Fuson asserts "such understanding has ordinarily not been a goal of school mathematics, most educational decision makers have not had an opportunity to understand the standard algorithms or to appreciate the wide variety of possible algorithms" (Fuson 72). Because of this, though, the algorithm that students chose as their favored method would indicate their understanding of multiplication because they would not have been taught why any method is the best, and so they would have to choose on their own. As a result, the students with the best number sense would hopefully have chosen the method that best assists them towards efficiency and successful multiplication. This would indicate the best method more than anything else. Thus, the students who could best master a new method would pick the best method of their own and so, they would demonstrate the paramount method for my research.

On the other side of the coin, any students who struggled completely with learning a new method would demonstrate the possible connection between the weakness in understanding of their algorithm. This would indicate how poorly this algorithm is taught or how poorly understood it is. Therefore, this would weed out any weak algorithms which do not encourage student understanding, number sense, and future mathematical proficiencies.

The second research question fed into how well students understood what they were doing, and so would indicate the strongest number sense and connectedness to multiplication understanding. Though this question was not intended to help find the penultimate multiplication algorithm, it was intended to again reflect how well students understand connections between the multiplication algorithms that they use in their mental arithmetic and pencil and paper computations. The reason for this question is because, as Fuson states, "at a given moment, each learner knows and uses a range of methods" (Fuson 71). If students could demonstrate any kind of connection between their methods, this would indicate that they comprehend what they are doing and so also that their multiplication algorithm has been chosen not by chance or because a teacher taught them only that algorithm, but has been chosen because it makes the most sense to those students.

It is for these reasons that I chose to find if students noticed any connections between the way that they perform multiplication in their heads and the way that they perform multiplication using a pencil and paper. The final reason for asking this question was because there are so many algorithms that students are not taught but instead create themselves for mental multiplication.

Because of this, the variety of methods that they use in their heads would show exactly how they see multiplication in an informal session.

My first research question addresses multiplication in a formal setting in which students are demonstrating abilities and then are being taught something new. The second question pertained to how they would do math on an everyday basis when not in class or when not asked to “show your work.” Therefore, the answer to how much they connect their different multiplication methods would show how well they connect in-class mathematics to the real world, which would in turn exhibit the methods they would use if they were given complete freedom. This would give further credence to the answers that I received from the first research question.

The two questions connect how students think about multiplication and how this reveals their number sense. The way that students think about multiplication could, through analysis, specify the better method of multiplication, and so the method that should be taught most prevalently in schools.

Methodology

In order to find the answers to my questions, I designed an experiment based upon the clinical interview process. I interviewed each student two times so that I could determine their initial capabilities, attempted to teach them a new method, and then assessed their ability to recall and use that method and others preferred by the student at a second interview. To this end, I asked identical questions in the initial and exit interviews with a two-week lapse in between each interview so that students would be able to digest the information. I did not ask students to write down any of their thoughts. Instead, they only had to write down the pencil and paper solutions to multi-digit multiplication problems that I gave them. To record their thinking, I asked them questions and scribed their responses. By using this process, students only had to talk about their thoughts instead of trying to form coherent answers immediately. They were able to talk to me so that they could work through their answers more fluently.

<u>Participants</u>

The student participants in the study were a group of five fifth graders from the same classroom. They had all learned the Long Method of multiplication as their dominant algorithm. The first four students were selected with the assistance of their teacher such that they were of varying mathematical abilities. The last student with whom I worked was randomly chosen by myself from the remaining subset of students whose parents had agreed to allow them to participate in the study.

I interviewed the participating students separately for multiple reasons. The first reason was that I was interested in the way that individual students multiply, not in the way that groups of students multiply. If they were all together, then they would be able to consult each other. Second, the strongest personality of the group, not necessarily the strongest student mathematically, could possibly take over the group and I would lose the perspectives of the other students in the group. Third, in a group setting, there is always the opportunity for learning and growth as students share ideas about how they think. This influence would have affected how each student multiplied during the interview. I wanted a certain amount of consistency in the first interview, and then I planned to find the differences and changes in their thinking based upon what they learned from me in the mini-lesson on the Box Method. I did not want them to be addressing any outside resources during the interview process. Therefore, I cut them off from the classroom entirely by taking them individually to the library.

Data Collection – The Interviews

To conduct interviews with each of the students, I took them one-by-one to the library. After they signed the assent form (which occurred after their parents signed the consent form, Appendices B and A respectively), I gave them a copy of questions which I would be asking them, and then I asked them the questions verbally so that they could hear the questions. This allowed me to directly interview them as well as give them the opportunity to read the questions, thus attending to various learning styles (e.g. auditory, visual), which could affect their ability to understand and answer my questions. The interview questions can be seen below.

Figure 3 – Interview Guiding Questions

<u>Initial Interview Guiding Questions</u> (Option of listening to or reading questions)

Pencil and Paper Portion

How much is 24*35? (Please show your work)

How did you solve that? Please describe your methods.

How much is 72*81? (Please show your work)

How did you solve that? Please describe your methods.

How much is 62*57? (Please show your work)

How did you solve that? Please describe your methods.

Are there any other ways you could have solved any of those three problems? Did you use the same methods for all three?

Mental Math Portion

How much is 11*20?

How did you solve that? Please write or describe your methods.

Is this method similar to how you solved the other problems? How or how not?

How much is 12*14?

How did you solve that? Please write or describe your methods.

Is this method similar to how you solved other problems? How or how not?

How else could you find the answer?

Do you always use the same method? What else could you do?

Teach Box Method

Box Method Questions

How much is 24*35? (Please show your work)

How much is 72*81? (Please show your work)

How do you like the Box Method? Do you like it more or less than the other ways you do multiplication and why?

Exit Interview Guiding Questions

Pencil and Paper Portion

How much is 24*35? (Please show your work)

How did you solve that? Please describe your methods.

How much is 72*81? (Please show your work)

How did you solve that? Please describe your methods.

How much is 62*57? (Please show your work)

How did you solve that? Please describe your methods.

Mental Math Portion

How much is 11*20?

How did you solve that? Please write or describe your methods.

How much is 12*14?

How did you solve that? Please write or describe your methods.

Is this method similar to how you solved other problems? How or how not?

Box Method Portion

Do you find yourself using the Box Method?

Do you think it changed any of your understanding about multiplication?

Do you think you still use the same methods as you did a couple of weeks ago?

Both in the initial interview and exit interview, I began with the same types of questions. To explore my first research question, I wanted to see the methods students used to multiply both in their heads and using a pencil and paper. Thus, I designed three multiplication questions which could be answered in a variety of ways depending on students' knowledge of multiplication tables and general multiplication methods. They could also have used rounding to estimate the validity of their answers.

"How much is 24 * 35?" I chose this as the first question due to the factoring capabilities of 24 and 35. 35 and 24 can easily be factored into 5*7 and 2*12. Since the multiples of 2 and 5 are well known, I saw this as a possible avenue for deconstructing the problem to simplify it. Students could also use any other method that they wanted, but this ensured that the problems were accessible to an outside approach using factors. Rounding would have also been a valid option considering the closeness of 24 to 25. This would also provide an option for checking any answer via estimation.

"How much is 72*81?" For this question, I chose two numbers which can be factored by 9, and so the number could be transformed so that students would have any easier problem. As in the previous question, they could also use any other algorithm. This question also lent itself to rounding for estimation of answer validity due to the proximity of 81 to 80.

"How much is 62*57?" For this question, I wanted to provide a problem with no obvious ways to simplify. Rather, I wanted to see how students handled a question which required a straight algorithm with little to no fiddling. I thought that while the other questions provided opportunities for creativity in multiplication, this question had no tricks that would allow it to become streamlined. This was to be the question that indicated truly which algorithm is the method that a student is most likely to use in any situation. Therefore, this was the question which was to show not the range of a student's abilities, as in the other two questions, but instead their favored computational method.

After asking those questions, I had a basis of their knowledge of multiplication. I also took the opportunity to ask students how they performed each problem as they worked through them. I listened to their vocabulary, in particular, as well as listening to the thought processes and depth of understanding of their chosen methods. After each multiplication problem, I wanted to know if they knew what they were doing, which would be indicated by their ability to describe their methods, or if they were blindly following a method, in which case they would not be able to accurately describe their reasoning.

At the end of the pencil and paper questions, I also asked students if there were any other methods that they could have used to solve any of the problems, or if they used the same methods for all three. If they could think of any other ways that they could solve the problems, this would harken back to their ability to use multiple methods and demonstrate creativity in multiplication. And, if they were capable of using multiple methods, this would indicate strong number sense.

After the paper and pencil tasks were completed, I asked students two multiplication problems which they were to perform mentally, without the assistance of any manipulatives. The questions I chose were such that they could be solved using a variety of methods, including decomposition, a method which involves looking at the tens and the ones digits, columns, or groups and then multiplying using those separate groups or blocks. The other methods that students could have used were knowledge of multiplication tables and manipulation of place value. Manipulation of place value differs from decomposition in that decomposition takes into account the ones and tens digits places and does not necessarily ignore the zero that holds the place value. Decomposition or blocking takes the whole of the number into account in separate parts. Decomposition utilizes place value to some extent, but is a slightly different process from manipulation of place value. On the other hand, manipulation of place value ignores the 0 at the end of, for example 80, so that students are multiplying with just 8 in order to simplify calculations and then takes the real placement of a number into account at the very end of the calculation process by adding the 0 on again. Decomposition does not leave the movement of 0's until the very end, but rather keeps it in play throughout calculations.

The first mental math question which I asked students was "How much is 11*20?". I asked this question because it can be solved using any pencil and paper algorithm due to its simplicity, because there are only 1's, 2's, and 0's, which are easy to multiply in their respective ways. This question also lent itself to using place value in that students could ignore the 0 on the 20 and then have 11*2, and then they could tack on a 0 at the end (or multiply by 10). This question also lent itself to decomposition or blocking in that students could split up the 11 into 10 and 1 and then multiply each by twenty. Thus, there were a variety of methods to solve this problem.

The second question that I asked students was "How much is 12*14?". This question did not lend itself in particular to place value manipulation, however it did work well with a variety of blocking techniques. Students could decompose either the 12 or the 14 and end up with 10*14 + 2*14, which is relatively doable, or 12*10 + 12 *4, or an even craftier way of blocking: 12*12 + 12*2. Any of these ways would find the answer and two of them utilize place value, while the other relies on students' knowledge of times tables. Therefore, though this question was not a place value question, there were a variety of ways to calculate it. This question was also intended to not use place value so that it was of a more difficult level than the previous question, thus challenging students.

When I chose all of the multiplication problems, I chose them for the reasons detailed above, but I also chose the order of the multiplier and multiplicand such that the largest number was not always first or last, so students would not find a pattern among the questions. I wanted the questions to seem random.

At the end of these questions, I took the opportunity to assess how students connect their mental math to their pencil and paper algorithms, and so I asked them if the mental methods were similar to how they solved the other questions. Depending on their answers, I would be able to see how much they understood the connections between their various methods. I also asked the participants if they used the same methods for each of the questions. As stated earlier, there are a great many different mental math algorithms, and chances are, they did not use the same methods. This would again allow me to see how they connected their multiplication experiences. Finally, for the mental math portion, I asked them what else they could do to solve these questions to test for any creativity among multiplication methods and see if they knew several methods for solving the same problem. This would address students' flexibility when choosing an algorithm.

The mental mathematics connections portion of the questioning involved my asking students to explain their thinking and exploration of any possible metacognition on the part of the students. These questions pertained to both the first and second research questions. Students' responses helped me to answer the first research question by providing more information about possible multiplication methods, especially due to the multitudes of mental arithmetic methods as shown above. Students' responses also referenced the second research question by requesting the participants to discuss any connections that they may notice between the ways that they multiply on paper and the ways that they multiply in their heads.

The initial interview ended with an introduction to the Box Method for multi-digit multiplication. I explained the theory behind the method and worked through one problem with each student. I then allowed each participant to try solving one problem on their own using this method. This provided them with practice, but not a great deal of practice. I wanted their success with the method to be based solely on their approaches to learning instead of my teaching techniques. During the final portion of the initial interview, I questioned each participant as to their thoughts and feelings in reference to the new Box Method algorithm.

The exit interview differed in that rather than being taught a new method and getting to practice it, I asked students to attempt one problem using the Box Method. This would show me whether or not they had learned the new method and were capable of using it in everyday calculations. I did allow for some prompting in case students forgot the method from disuse, because as young students, they may not have had opportunities to practice the new method. I also questioned students about whether or not they had used the Box Method or whether or not they still liked it or would use it. This would indicate whether or not their feelings had changed about it, thus indicating how well they learned the new method. If students were uncomfortable with the method because they had not learned it well, this would have been their chance to say so.

Methods of Analysis

In order to analyze data I identified categories which I either felt were direct answers to my questions or were noteworthy due to their recurring nature. I explored several categories of student responses. For example, during the interviews I noticed some of the terminology used by students was unconventional or indicated a traditional algorithm. Thus, I dissected students' vocabulary use to see whether it would indicate a particular method or stronger number sense. I also examined changes in the students' answers across the two interviews, and their variety of methods used mentally and on paper. I also investigated the participants' abilities to correctly answer the multiplication problems using their chosen methods. Lastly, I was curious to discover their like or dislike of the Box Method, and their capability to use the Box Method in the exit interview. I then compared each of the categories of responses to determine my results.

The students whom I interviewed responded in ways that did not align specifically with what I had expected. Though I still feel that my reasons for asking questions were solid, the answers that students gave me were unpredicted. Students' flexibility when choosing pencil and paper algorithms challenged a great many of my conceptions when I had originally asked my questions.

Findings

Paper-and-Pencil Methods

The first student that I interviewed, D.V. was very excited to be working with me. He appeared to have a strong grasp of what he was doing in terms of multiplication. He was able to accurately describe his reasoning as he worked through the multiplication problems using the Long Method. When D.V. learned the Box Method, he compared it to a multiplication table, indicating his understanding of the way rows and columns were to be multiplied.

The second student that I interviewed, Ronald, did not appear as cognizant about his multiplication techniques. Though he was good at checking his answers, he sometimes failed to take into account that his answer made no sense, and would instead plow ahead with what he knew was an incorrect assumption, as he at one point, told me. However, Ronald loved the Box Method when he first learned it, even stating that it was easier than the Long Method.

The third student that I interviewed, James, actually practiced the Box Method in between the initial and the exit interview. He was the only student who was able to perform successfully the Box Method in the exit interview. James also took the time to think about the Box Method when it was first explained to the point where he asked if the number of boxes in the array changed if there were more digits.

Jessica was the fourth student that I interviewed. She was nervous throughout much of the interview process, probably due to her failure to answer several questions correctly. Jessica was also the only student unable to perform the Box Method even with assistance in the exit interview. She was also the only student who used only one method for multiplication both mentally and on paper.

The final student with whom I worked, Summer, was extremely reluctant to share the ways in which she thought about the problems. She wanted to just do the work and be done. She did not want to explain how she was doing anything or why she was doing anything. Summer was also the only student not chosen specifically by the teacher. Summer was the one student whom I randomly selected.

Upon analysis, I found that all five students used the Long Method for pencil and paper calculations in the initial interviews though this is not true for the exit interviews. However, they all used it with varying degrees of success. D.V. made an additive error in the initial interview on 62*57, but got that same problem correct at the exit interview. Ronald made mistakes on 72 *81 and 62*57 during the initial interview, however he was able to figure out that he made a mistake on 72*81 because he checked his answer using estimation. Ronald was unable to catch his mistake for 62*57 until I pointed it out to him, due to the fact that it was a carrying error. In the exit interview, Ronald was successful at 62*57, but made mistakes on 72*81 again. This time, he caught his error on 72*81 but did not fix it until I prompted him to after he had finished. Ronald also missed 24*35 in the exit interview because he made errors through addition. Like D.V. and Ronald, Jessica also was incorrect in her answer to 62*57 because she forgot a placeholder, in the initial interview. In the exit interview, Jessica was successful with all three problems. Summer was very fast and was successful with all six problems in both interviews.

James was successful with all of the problems in both interviews, however he was also the only student to use any method other than the Long Method in either interview. More specifically, he chose to use the Box Method in the exit interview. I told him at one point that he did not have to, but James continued to use it up until 62*57.

As can be seen from these results, 62*57 was the most difficult challenge that the students faced, but all for different reasons. From addition to carrying, to forgetting a placeholder, the reasons for failure are varied, and so I cannot say why this problem was the most difficult for students.

Mental Multiplication Methods

When students were asked to perform mental multiplication, there was a great deal of dissimilarity in techniques. D.V. used place value manipulation in the initial and exit interview for 11*20. For the initial interview, he attempted to block the 14 into 10 and 4 to solve 12*14 but failed because he tried to juggle too many numbers in his head and was confused by what he was multiplying. In the exit interview, D.V. blocked the 14 into 12 plus 2 and was successful. D.V. used a total of 2 methods. James used the most number of techniques when performing mental multiplication. He used place value manipulation for 11*20 in the initial interview, then blocked by 11 in the exit interview. For 12*14, he blocked by 14, but made a point of stating that he was using his twelve facts, and so he automatically knew how much 12*12 was and 12*2 was. Therefore, James used 2 different methods, but for 1 problem, he used 2 different techniques, and so he used 2 techniques for one problem, plus the 1 previous technique, which totals 3 techniques. James was successful in all 4 mental calculations.

Ronald also managed to use 2 different techniques for the same problem, but only used a total of 1 method. He only used blocking for all four problems. Ronald blocked by 20 by splitting the problem into 10 *11 +10 *11 in the initial interview and then blocked by 11 in the exit interview by multiplying 20*10 + 20*1. However, he failed at the 12*14 question in both the initial and exit interviews. Initially, Ronald attempted to block the 14, but failed because of too many numbers, and so he added 12+14 to 12*12, in error. In his second interview he multiplied 12*14 and added 2*4, demonstrating that he did not have a strong grasp on the math that he was performing. Ronald was only successful half of the time when doing mental multiplication. Like Ronald, Jessica only used one method for all four problems. Unlike Ronald, she only tried one technique for using that method. She attempted to use the Long Method in her head. However, when she first attempted 12*14 she got 40. Through checking her answer, Jessica quickly realized that this solution could not be correct. She achieved the correct answer shortly after. Jessica was successful with the other three problems. Summer also insisted on using the Long Method in her head. She relinquished this need the first time that she performed the 11*20 calculation, and instead used place value manipulation to find her answer. After that first moment however, Summer only used the Long Method. For 12*14, she was unsuccessful both times, and even admitted that she had no idea how to do it in her head. Even in the exit interview, she used the Long Method for 11*20.

Students used only two methods for written multiplication problems, of which the Long Method was the most prevalent. It was only not used for 2 problems, and those were both solved by James in the exit interview. However, the methods for mental multiplication were numerous. No method was used by all 5 students, but place value manipulation was used by 4. Blocking was used by 3 and the Long Method was used by 2. Therefore, the most popular methods for written and mental multiplication were the Long Method and place value manipulation, respectively.

Summary of Written Multiplication Strengths and Weaknesses

From what I saw, it appeared that the students who were most successful with written multiplication problems were James and Jessica. They required no prompting and both were able to correctly answer all problems. Jessica did have some slight errors, but she caught them herself with no prompting from me. James used the greatest number of techniques in that he solved one question with two different methods and was able to vary his blocking technique. Jessica only used Long Method for everything and even admitted that she could not think of any other ways to make the calculations. From this dissection of student responses, it appears that the ability to correctly perform mental multiplication is dependent upon no single method because both of these students were successful but with vastly different methods and techniques.

The students who had the most difficulties were Ronald and Summer, who both were successful only half of the time. Ronald and Summer both relied heavily on one method, blocking and Long Method, respectfully, however, they both performed the 11*20 question differently in each interview, but were successful both times. Therefore, I believe that their adherence to only one method stunted their multiplication abilities. However, when Ronald and Summer reached a problem which they could solve multiple ways, they were successful.

Vocabulary

Another aspect which appeared to be quite important was the way that students described what they were doing, in particular, their vocabulary. There were several words that appeared again and again, to the point where their absence in some students' descriptions was noticeable.

Vocabulary is one method for judging a student's understanding of multiplication techniques, because if they are correctly able to use the terminology, then that indicates that the students are understanding at least some of what they are doing. Since I was not prompting for vocabulary, any instances of interesting words were completely based upon the participants' prerogative. The vocabulary word which stood out the most was "Carry" or "Carried" in reference to when the product of two numbers expands into the tens place, in which case the tens place must be carried. All five students used carry at some point, though only Ronald and Jessica used it in both interviews.

Another word that was of interest to me was "placeholder" or any acknowledgment of place value. I found that place value was a recurring theme when students were discussing their work and how they think about multiplication. James used "tens" to refer to place value, indicating that he understood the importance of each place in a number. Jessica used "placeholder" in both interviews since she knew that she needed one in her use of the Long Method, and Ronald used "placeholder" in both interviews as well, indicating that he agreed to the significance of a placeholder when working with multiplication. Neither D.V. nor Summer used placeholder, even though D.V. said that he "brought down the zero" which was an offhand reference to placeholders. The final vocabulary word of note was "plussed", a word used only by Jessica. She was referring to addition, but she used that word in both interviews, which worried me because it is not even a word. I did not correct her however, for fear of tainting her results.

Students' use of vocabulary to describe their multiplication methods seemed to indicate different depths of understanding of multiplication. For example, Jessica used poor language, and struggled a great deal with multiplication. Meanwhile, those who were able to use a variety of correct vocabulary were able to perform multiplication more easily, by comparison.

Box Method Results

In the exit interviews, I asked students to perform the Box Method, and I was surprised by the results. D.V., for one, required prompting as to what to do with the numbers and had to be prompted into remembering that each number in the tens column and row were actually multiplies of 10 and were not the ones digit that they appeared to be. He was eventually able to perform the Box Method. Ronald also needed help in remembering that the numbers in the tens places were multiples of 10 and so he needed to add zeros before adding the results. With that information he was able to perform the rest of the method successfully. Jessica was unable to even set up the array. When she realized that she did not know what she was doing she gave up, and even with prompting could not reach an answer of any kind. Summer needed help when performing the method, but was able to reach her answer successfully. In contrast to the others, James used the Box Method from the start and was entirely successful.

When asking students how they felt about the Box Method after first learning it, I received many answers, but all were positive. In the initial interview, when I asked students how they felt about the Box Method, many compared it to the method which they already knew. D.V. thought that it took longer than the Long Method, but he also felt that it was more accurate, which was probably because the identical problem that he performed using the Long Method he solved incorrectly, but when he used the Box Method, he was successful. In the exit interview, though, he admitted that he would probably not use it unless someone specifically asked him to do it. James thought in the initial interview that it was a cool way to look at multiplication in a different way. In the exit interview, he admitted to having tried it a couple of times to see how it works, and he found it to be cool.

Ronald thought the Box Method was faster and easier than the Long Method, with which he struggled. However, he told me in the exit interview that he did not really use it because they had been working on division in class. Jessica thought that the method in general was easy but she admitted to having a hard time remembering where to put the zeros in the initial interview. What she told me in the exit interview however, was quite interesting. She told me that she did not use it because she was used to the other way (meaning Long Method). Finally, Summer, in the initial interview, said that the method was easy and that she would use it. Two weeks later, she said she would not use it because it was confusing and she could not remember where to put numbers and zeros.

Only one student put outside effort into learning the Box Method. This could have been due to curiosity, but due to his use of many methods previously, it appears that James liked to learn new methods. Therefore, it seems that the learning of many methods benefitted him in learning a new method, it was as if he was primed to learn something new. On the other hand, the student unable to learn the new method, Jessica, was completely stuck on her own method, to the point of being unable to use different methods. This might mean that the variety of methods affects how students learn something new, whether it is because of a natural inclination to learn, or an enhancing of thought processes so that students are more able to learn something new. In general, however, other researchers have found that when students are able to flexibly use a variety of computational methods, then they have a stronger number sense (Brocardo 407).

Connecting Written and Mental Computation

The final aspect of the interviews that I explored was how students thought about the ways in which they multiply in their heads as compared to how they calculated the multiplication problems on paper. To this question, I received several interesting answers. D.V. said initially that his answers were similar because he was doing the same thing. He made not that he used place value manipulation, though not in those words, and so the location of the 0 at different points when solving the problem was all that differed between the two methods. However, in the exit interview he stated that the methods he used were completely dissimilar. James said the complete opposite in that he thought that initially the methods he did in his head were easier than those he did on paper and that they were completely different, but in the exit interview, he said they were the same and that in both, he was adding up after splitting (which referred to the blocking method and the way in which the Long Method breaks up each line of multiplication).

When asked about the connection between his mental methods and written calculations, Ronald said that what he did on paper was multiplication, which implied that he thought that the Long Method was the one and only way of multiplying and everything else was not the same. In both the initial and exit interview, he referred to what he did in his head as splitting or breaking into groups, rather than multiplication, and so Ronald felt that there was no similarity or connection between the methods. Jessica felt that both written and mental methods were the same things, except she did one in her head. This was completely true, since Jessica used exclusively Long Method for all of her problems. Summer stated that they were exactly the same as well, which again made sense because she used the Long Method for the most part.

It seems that students who use the exact same methods for mental and pencil and paper multiplication saw the connections. However, those students who use different algorithms saw no connection, or if they did, those students only saw it sporadically. Therefore, students appear to need explicit discussion of the similarities, or else they will not see how mental and written computational algorithms are connected.

I was unable to analyze whether or not written multiplication methods contributed to the ability to learn a new method because in the initial interview, all 5 students used the Long Method. Students who used few methods had trouble when learning the Box Method, while students who used many mental methods were much more successful when called to use the Box Method in the exit interview. Therefore, there does not seem to be any one mental method that contributes to the ability to learn a new method, though a strict adherence to the Long Method might detract from the ability to learn a new method. Meanwhile, vocabulary and flexible mental multiplicative reasoning may contribute to a student's depth of multiplication knowledge. Finally, if students use exactly the same methods both with pencil and paper and mentally, they are more likely to see the similarities between the two methods of computation than if students use a multitude of algorithms both on paper and in their heads.

Conclusion

If I could go back and change my research, I would focus more on place value, a recurring theme in students' understanding of multiplication. It seemed that the more students could discuss place value accurately the better they were at discussing their multiplication in general. I was impressed with the idea that any emphasis on place value would assist students in any future comprehension of multiplication. This was evidenced by the fact that all students could discuss place value and on top of that, for the mental multiplication problems, for every problem that students used place value, they were correct in their calculation. Hence, if I were to do any future studies on the subject of multiplication algorithms and their effects on students, I would explore the learning of place value.

To answer my second question first, it appears that students in general do not seem to notice any connection between mental multiplication and pencil and paper multiplication unless it is identical. Therefore, I would say that any attempt to bring knowledge about the connections between mental and written algorithms give students a greater understanding of the mechanics behind multiplication.

Based upon the way that students were most successful, it appears to not be any particular method which best enhances numbers sense, but instead the ability to use a variety of methods. This dovetails a previous statement, that number sense uses a variety of representations

(Brocardo 407). This statement does not necessarily mean that students have to use many algorithms; it merely asserts that students should be able to look at a number and see it in a variety of ways. This manifests, apparently, in the ability to look at a number and view the multiple ways in which it can be manipulated to an answer, and therefore the number of different methods that can be used to solve a problem entailing that number.

The one student who used the most algorithms and techniques mentally was James, the student who best comprehended the new Box Method. This demonstrates that the ability to use a number of algorithms indicates a strong number sense. The correlation between the ability to learn a new method and a strong number sense manifested in multiplication methods shows that a strong number sense is most indicated by the number of algorithms with which a student is capable. This point is further acknowledged by the failure of Jessica, the one student incapable of learning the Box Method to any degree, who only used one method. She supports the theory that the fewer multiplication algorithms with which a student is capable, the less likely they are to learn a new method. This could be due to the rigidity of a student's thought process, , or due to their unwillingness to learn, or could be due to the fact that the one algorithm with which they are proficient is holding them back and inhibiting their ability to learn new methods.

It is impressive that James took the time to practice the algorithm when he first learned it, something no other student did. This implies that for some reason, he was willing to learn a new method. Because of James' willingness to use so many methods when he was calculating answers, it appears that he, in general, wanted to learn new ways of doing things. Jessica however, was completely unwilling to stray from her one method for any other, and she failed entirely in her attempt at the Box Method in the exit interview. These facts combined seem to point to the idea that no matter the reason, the number of algorithms with which a new student is capable, the more easily they can learn a new method. Therefore, in the future, these students will hopefully continue in this willingness to learn, and will continue to embrace the growth of number sense. So, the end conclusion of this research is that students should be taught many methods so that they do not become stuck on any method and can expand their horizons such that they are capable learners and mathematicians.

Appendix A – Consent Letter and Form

Dear Parent,

I am an undergraduate of the University of New Hampshire and I am conducting a research project to find out if a child's chosen multiplication has any impact on learning new multiplication methods and how mental arithmetic is factored into the understanding of multiplication algorithms. I am writing to invite your child to participate in this project. I plan to work with approximately 10 children in this study.

If you allow your child to participate in this study, your child will be asked to spend about 20 minutes over the course of two interviews doing multiplication problems and explaining their methods, and this will take place during class time. They will also learn the Box Method of multiplication, in which tens and ones are separated into columns and rows and multiplied in a grid style, ending with the sum of the products. I will be collecting their work and writing down their answers to my questions about how they solved the problems. Their work will also be observed before either interview. Neither you nor your child will receive any compensation to participate in this.

The potential risks of your child participating in this study are minimal confusion when learning the new multiplication method. Although your child is not expected to receive any direct benefits from participating in this study, the benefits of the knowledge gained are expected to be a deeper understanding of how they multiply numbers, which will be beneficial as multiplication becomes more complex. From my work, colleagues will see any connections between learning new methods and the favored method of students.

Participation is strictly voluntary; your refusal to allow your child to participate will involve no prejudice, penalty, or loss of benefits to which you or your child would otherwise be entitled. Your child may refuse to answer any question. If you allow your child to participate in this project and your child wants to, and then either you change your mind or your child changes his/her mind, you may withdraw your child, or your child may withdraw, at any time during the study without penalty.

I seek to maintain the confidentiality of all data and records associated with your child's participation in this research. You should understand, however, there are rare instances when I am required to share personally-identifiable information. For example, in response to a complaint about the research, officials at the University of New Hampshire, designees of the sponsor(s), and/or regulatory and oversight government agencies may access research data.

I will keep data in a locked file cabinet in my faculty advisor Dr. Sharon McCrone's office; only she and I will have access to the data I will report the data using pseudonyms and no real names. The results will be used in reports, presentations, and publications.

If you have any questions about this research project or would like more information before, during, or after the study, you may contact myself, Catherine Tarushka at (603) 479 – 3090, or at cas89@wildcats.unh.edu. You may also contact Dr. Sharon McCrone at smy72@cisunix.unh.edu. If you have questions about your child's rights as a research subject, you may contact Dr. Julie Simpson in UNH Research Integrity Services at 603-862-2003 or Julie.simpson@unh.edu to discuss them.

I have enclosed a consent form for you to sign. In order to participate, you child must also be willing and sign as well. **Please check one indicating your choice and return in the enclosed envelope within one week of receiving this letter**. This letter is for your records. Thank you for your consideration.

Sincerely,

Catherine Tarushka

UNH Undergraduate

Informed Consent Form
(Students 17 years of age or younger)

Chosen Multiplication Methods and the Ability to Learn New Methods

I understand that the purpose of this research study is to analyze whether any specific chosen algorithms are more conducive to learning new algorithms and to measure the differences between mental and pencil and paper algorithms.

By signing this consent form, I agree to allow my child to participate in the study. I understand that this involves my child completing two interviews during class time, in which written work and statements will be documented, as allowed by his or her classroom teacher and school principal. I also understand that I am allowing the researcher to collect written work during observed class periods if I allow my child to participate.

I understand that participation in the study is voluntary. My child may refuse to participate or may withdraw at any time; and his or her identity will be kept confidential during and after the study.

I also acknowledge receipt of a copy of the attached letter.

☐ I consent to my child participating in the study.

☐ I **do not** consent to my child participating in the study.

____
Printed Name of Parent or Guardian

____
Signature of Parent or Guardian

____
Date

Please return this form to your child’s mathematics teacher.
You may keep the letter for your records.

Appendix B – Assent Letter and Form

Dear Student,
I am an undergraduate college student at the University of New Hampshire and I am doing research to see if your chosen multiplication method has any affect on how you learn new methods. I also want to see how you think about mental multiplication. I would like to invite you to be in this study, as one of about 10 students.

If you agree to be in this study, you will be asked to spend about 20 minutes over the course of two interviews doing multiplication problems and explaining your methods, and this will take place during class time. You will also learn the Box Method of multiplication, which is similar to partial sums. I will be collecting your work and writing down your answers to my questions about how you solved the problems. I will write down any work during observation times before the first interview.

You may experience some confusion when learning the new method, however by learning this new way of multiplying, you may understand multiplication on a deeper level, and you will also have a method to use that you did not have before. Your participation will also help other teachers and professionals to see if there are connections between learning new multiplication methods and which methods are used most by students.

You will choose whether or not you will be in this study. You will not be penalized in any way by not being a part of this study. You can refuse to answer any question, and you can stop being in the study at any time. You will not be penalized for leaving the study, or refusing to answer questions.

I will do all that I can to keep your identity confidential. There are very few instances where I must share information with officials at UNH. For example, in response to a complaint about the research, officials at the University of New Hampshire and/or regulatory and oversight government agencies may access research data.

I will keep data in a locked file cabinet in my faculty advisor Dr. Sharon McCrone's office; only she and I will have access to the data I will report the data using pseudonyms and no real names. The results will be used in reports, presentations, and publications.

If you have any questions about this research project or would like more information before, during, or after the study, you may contact myself, Catherine Tarushka at cas89@wildcats.unh.edu. If you have any questions about your rights as a research subject, you can contact Julie Simpson at julie.simpson@unh.edu.

I have enclosed an assent form for you to sign. Please check one box indicating your choice and return in the enclosed envelope. This letter is for your records. Thank you for your consideration.

Sincerely,
Catherine Tarushka
UNH Undergraduate

Informed Assent Form
(Students 17 years of age or younger)

Chosen Multiplication Methods and the Ability to Learn New Methods

I understand that the purpose of this research is to analyze the way that I multiply and how I learn new ways to multiply.

By signing this assent form, I agree to be in this study. I understand that I will be in two interviews during class time, each one lasting 10 minutes. I understand that my work will be collected and my answers written down, as known by both my teacher and principal. I also understand that I am allowing the researcher to collect written work during observed class periods if I choose to participate.

I understand that being in this study is voluntary. I may refuse to participate or may stop or leave the interview at any time; and my identity will be kept confidential during and after the study.

I also confirm that I have been given a copy of the attached letter.

☐ I agree to participate in the study.

☐ I **do not** agree to participate in the study.

____
Printed Name of Student

____
Signature of Student

____
Date

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