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An Analysis of Differences in Approaches to Systems of Linear Equations Problems Given Multiple Choice Answers

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Abstract

This descriptive study focuses on the approaches college students (ages 20 - 24) use when solving systems of linear equations problems that have multiple choice answers. Participants were from a midsize public university in the northeast. Four approaches were considered – three forwards approaches: 1) substitution, 2) elimination, and 3) graphing, and one backwards approach: plugging in the *x* and *y* values from each multiple choice option. Participants solved systems of linear equations problems and answered questions based on their methods in a structured clinical interview. Each participant also filled out a questionnaire. It was shown from the results of this study that the major of a student does not change the approach used to solve a problem by very much. Most students in the study chose to use substitution to solve the problems, usually because this was the method students remembered most and was deemed the "easiest" method by the students.

1 Introduction

When it comes to systems of linear equations, there is not much research pertaining to what methods students tend to use, especially when multiple choice answers are involved. There is research on linear equations and research on multiple choice answers, but combining the two together and going one step further to make linear equations into systems of linear equations has not been well established (Anderson, 1989; Coppedge & Hanna, 1971; Hewitt, 2012; Huntley, Marcus, Kahan, & Miller, 2007; Kazemi, 2002; Marshall, 1983; Nogueira de Lima & Tall, 2008).

1.1 Systems of Linear Equations

1.1.1 What are Systems of Linear Equations?

To begin with, a linear equation in two variables, which is what will be used in this study, "is an equation that can be written in the form" $ax + by = c$ where x and y are variables and a, b, and c are real numbers with at least one of a or b being nonzero. The systems of linear equations used in this study will be systems containing two variables and two equations. These systems of linear equations are considered concurrently. A system is considered solved when a solution is found that works for both of the equations ("12.1 Systems", 2012).

1.1.2 Why are Systems of Linear Equations Needed?

Every child in the United States is required to take an algebra class in order to graduate high school. Systems of linear equations is a standard in all algebra classes, thus every student must be familiar with solving systems of linear equations upon graduating high school. The Common Core State Standards Initiative (CCSSI) that has been accepted by forty-eight of the fifty states in the US requires students to have learned how to solve systems of linear equations by the end of high school algebra (2012).

If a student who has taken an algebra course still cannot complete systems of linear equations accurately, then, clearly, there is an issue that must be resolved. In order to resolve this issue, we must know why the student does not understand systems of linear equations. It would be important to consider what misconceptions the student may have that led to an inaccurate understanding of how to solve systems of linear equations and whether the student was using a proper method but was making arithmetical errors that prevented him from arriving at the correct answer. There are so many variables that must be taken into account in order to have a thorough understanding of what is happening when a student is unable to solve systems of linear equations and must be studied in order to help resolve this issue for students.

1.2 Multiple Choice

There are many standardized tests present in the US today. Students are expected to take a standardized test elementary, middle, and high school in order to determine the success rate of a school. If a student wants to go to college, he is expected to take the Standardized Aptitude Test (SAT). If an undergraduate is interested in graduate school, he typically has to take the Graduate Record Examination (GRE). Each of these standardized tests uses a multiple choice format. Since students need to score well on these tests in order for their school to receive funding and to be accepted into graduate school or an undergraduate program, many teachers and organizations work with students to prepare them for taking tests in a multiple choice format.

1.3 Objectives

There were two objectives for this study: 1) to determine which approach to solving systems of linear equations students use most when given multiple choice answers and 2) if students tend to solve problems differently depending on their major. The reasons students gave as to why this may be the case were also looked at in this study.

2 Background

This study considers many characteristics of students in an effort to understand why students choose certain methods to solve systems of linear equations, specifically those in a multiple choice format. Previous research and literature in mathematics education state that students think differently and may take different paths in order to find an answer to a problem (Kazemi, 2002). Some research has shown that whether one is male or female, a person’s view of mathematics, and one’s experience may influence how one chooses to solve a problem (Anderson, 1989; Schoenfeld, 1989; Kazemi, 2002). For this study, a problem was considered a system of linear equations that needed to be solved.

2.1 Gender Differences in Solving Problems

Research has shown that the difference in gender leads to different methods of solving problems, including the types of errors made and how the problem is approached (Marshall, 1983; Anderson, 1989). Anderson reported that men are more likely than women to guess an answer, especially when multiple choice is available (1989). Marshall noted in her study that women were more likely to err with respect to meaning, scale, operations, number patterns, key words, horizontal math (e.g. $3 + 5 = x$), rule of smallest, and picking the first option. On the other hand, she noted that boys were more likely to err when it came to translation, incomplete association, consistency, using formulas, rule of largest, and picking the last option (1983). These differences are important to consider when researching students’ approaches to problems.

2.2 View of Mathematics

There are also some who enjoy mathematics, and those who do not. In cognitive psychology it is known that being in a positive mood increases a person's problem solving ability (Revlin, 2012). Thus if a person views mathematics negatively he may have difficulty solving mathematical problems. Research by Alan Schoenfeld supports this theory of performance relating to a person's view of mathematics (1989).

2.3 Mathematical Experience

A third reason why students may prefer one method over another is the student's past mathematical experience. Research has shown that students use previous experience to guide their actions when solving mathematical problems (Kazemi, 2002). This may be because the student has had more practice with one method over another or that the student has had more success using one approach than he has had when using a different approach. Despite students having differing experiences throughout schools in the US, there are still many similarities as to the methods students choose to use and not use when solving problems (Huntley et al., 2007; Kazemi, 2002).

2.4 Approaches

2.4.1 Differing Approaches

It is important for students to know when to use a different method. Those students who have mastered solving systems of linear equations realize that all methods produce the same response. However, those who have not yet mastered solving systems of linear equations may be unsure as to whether different methods will lead to the same result. Despite this difference, students will typically refer to a different method when they are stuck (Huntley et al., 2007). Most likely students switch approaches when they become lost because previous experience has shown that doing so results in success.

In this study, it is expected that students in math-related majors know the various methods for solving systems of linear equations and know that each method will produce the same result as the others. From this expectation, it is also expected that math students will use this knowledge to decide which approach to use and when to change approaches. It is expected that at least some students in non-math related majors will not know all the various methods for solving systems of linear equations and will not know that each method will produce the same result as the others. From this expectation, it is also expected that non-math students will stick to one approach when solving the problems in the interview.

2.4.2 Graphing

More often than not, calculators are not used to graph in order to solve mathematical problems. In fact, graphical reasoning with and without a calculator is not seen very often by researchers observing students' strategies for solving various mathematical exercises (Huntley et al., 2007). It may be that teachers are not teaching students how to use calculators to solve different problems or that students prefer other methods because they are easier or faster. Graphing is known to not always provide an exact answer, which may lead students to view it as an unreliable method. Either way, strategies involving calculators are not a method researchers typically see students use.

2.4.3 Guess and Check

Another method, guess and check, while being a very popular method for solving linear equations is not often used by students (Huntley et al., 2007). Some may substitute random values and hone in on values closer and closer to the actual value, but researchers have seen few students actually do this in an observational setting.

A more sophisticated version of guess and check, however, is seen used by some students. When given multiple choice problems, some students may substitute values from the possible answers into the problem when they are lost as to what to do next (Huntley et al., 2007). This method may also be used by students who look at the answers before looking at the actual problem (Kazemi, 2002).

2.5 Errors

2.5.1 Distracters

Teachers see students make many errors when solving mathematical problems. Many teachers tend to structure multiple choice problems towards these errors (i.e. possible answers to a problem are derived from errors students typically make when solving that sort of problem; Coppedge & Hanna, 1971). If a student chooses that answer, it is likely that the student has committed the error that was used to derive that answer and this provides the teacher with knowledge as to where students are struggling.

2.5.2 Common Errors

Probably the most common errors that teachers see are with arithmetic. Kazemi noted in his research students' tendencies to commit arithmetical mistakes (2002). Students will subtract a positive number from a negative number and think it is positive, or add two negative numbers and think that it makes a smaller negative number. Others will multiply two numbers incorrectly, or divide one number by another and think it is a different number than it is. These are simple mistakes that are made by most people from time to time.

Another error that arises when multiple choice problems are presented is an issue with reversals. In problems that include two values, such as systems of linear equations problems, some students will switch the two values (Clement, 1982). In the case of systems of linear equations problems, students may make the x value the y value and the y value the x value. This is a mistake teachers catch when they include the reversal as an option among the multiple choice answers.

Research had been conducted on errors students typically commit when solving algebra problems. The research showed that students typically distribute properly but had issues with arithmetic (Huntley et al., 2007). Specific errors that were shown in research that could have possibly been found in the study are: not changing the sign when adding or subtracting a term to the other side of an equation, combining constants and variables, writing a quotient upside down when dividing by zero, flipping a quotient when it is negative, only multiplying part of an equation, and adding or subtracting a number to a numerator before dividing (Nogueira de Lima & Tall, 2007; Hewitt, 2012).

3 Method

3.1 Participation

It was decided early on in developing the study that college students would be used as participants. The decision to use college students for the study was due to the researcher's access to college students and the flexibility of college students' schedules. College students usually have completed algebra within the past decade and do not develop deeper knowledge of algebra that could lead to differing results from those of high school students (i.e. most college students are likely to have the same mathematical ability for solving systems of linear equations as a high school student who has just taken algebra). It was believed that this was enough to consider college students a decent proxy for high school students.

The study required students from both math and non-math related fields. If a student had taken more than two math courses in college, they were considered to be in a math-related field. Only college undergraduates were recruited for the study.

Every possible recruit was given the researcher's email address as a means of communication to obtain more information about the study or agree to participate in the study.

No participant was offered compensation for his or her participation in the study.

Before any student could participate in the study, he or she had to read and sign a consent form, which he or she did right before the interview. This consent form mentioned the purpose of the study, what the student was expected to do, the terms of confidentiality, and ways to obtain more information about the study. It also mentioned the risks and benefits of participating in the study. The consent form was assessed and accepted by an Institutional Review Board.

3.2 Instrumentation and Data Collection

For this study, two methods of data collection were used: a questionnaire and a semi-structured clinical interview. The questionnaire collected data on the participants' gender, major, minor, age, view of mathematics, lowest grade in mathematics, and typical grade in mathematics. The data collected in the questionnaire was used to categorize the data collected in the interview.

3.2.1 Questionnaire

The researcher chose to look at gender and view of mathematics because of the research noted in the **Background** section that discussed the difference in performance based on gender and a person's view of mathematics. Age was collected to inform readers about the age group of the participants.

Data was collected on each participant's major(s) and minor(s) in order to determine whether the student was classified as a math student (i.e. a person who takes more than two mathematics courses for his major) or a non-math student (i.e. a person who takes two or less mathematics courses for his major, including general education requirements). Classifying participants as math or non-math students allowed the researcher to respond to the second objective of the study: if students tend to solve problems differently depending on their major.

Participants were asked what their lowest overall grade was in a mathematics course (in high school and college) and what their typical grade was for both high school and college in mathematics classes. The researcher used these questions to determine if a participant's grade would impact his performance, possibly due to a lower grade being associated with negative feelings towards mathematics and hence lower performance on mathematical tasks. Both questions were asked because the researcher realizes that receiving a low grade in one class can cause the average grade to go down and receiving a high grade in one class can cause the average grade to go up, hence looking at the average grade would not give a proper view of a participant's mathematical ability. Mathematical ability as mentioned here is defined by the ease in which a participant solved a system combined with the number of errors found in a participant's work as he or she solved the system; another name for this would be the student's likely performance on a math problem.

3.2.2 Clinical Interview: Problems

The clinical interview consisted of eight to nine systems of linear equations problems the participant was asked to solve and questions the participant was expected to answer. Participants who were not in math-related majors were given eight problems; those who were in math-related majors were given nine problems. Each group had problems that had characteristics of one if each of the three methods of solving systems of linear equations typically taught in schools: 1) substitution, 2) elimination, and 3) graphing.

Substitution consisted of problems where one equation had a variable with coefficient 1 equal an equation with another variable. The following is an example of a substitution problem from the interview:

$$\begin{aligned} 2x + 3y &= -4 \\ y &= 4x + 8 \end{aligned}$$

Elimination problems were categorized as problems where one equation had to be manipulated in order to cancel out a variable. Systems with matching coefficients for one variable where the coefficients were not 1 (such as the following example) were also included in the elimination category. The following is an example of an elimination problem from the interview:

$$\begin{aligned} 2x + 4y &= 12 \\ 2x &= 7 - 9y \end{aligned}$$

Graphing problems were categorized as those problems that required manipulation of both equations in order to solve the problem. Graphing could be done using a calculator or drawing a graph by hand. The following is an example of a graphing problem from the interview:

$$\begin{aligned} 5x + 2y &= 19 \\ 3x + 3y &= 6 \end{aligned}$$

While these definitions of substitution, elimination, and graphing are not the same as those taught in schools, they were used for this study to differentiate between the three methods. Also, these characteristics are the ones typically noted by students in order to determine which method would be the most beneficial to the student.

3.2.3 Clinical Interview: Questions

At the beginning of the interview, participants were asked what methods they knew to solve systems of linear equations problems. This was recorded at the beginning of the interview to help understand why participants were solving problems the way they were and to help with analyzing the data after the interviews.

While participants were solving each problem, they were asked what approach they were taking, why they were taking that approach, whether that approach was the easiest method of solving the problem, and what other approaches they might use to solve the problem. The researcher asked the student what approach she used in order to not make assumptions as to what students were doing. Participants were asked why they were taking an approach, whether they thought that approach was the easiest way to solve the problem, and what other approaches the student might have used because of research noted in the **Background** section about the methods students typically choose and not choose when solving problems. The researcher was interested in seeing students' reasons behind committing to one method over another.

If a participant was stuck on a problem, he or she had the option to bypass the problem or was prompted of the methods he or she knew to solve systems of linear equations and to think of what was required to solve the problem. At no time did the interviewer suggest a method to the participant. During and after the interview, the interviewer took notes on the answers participants gave to the questions asked and on information the participant thought may be helpful to know for the study.

3.3 Creating the Systems

In choosing the systems of linear equations problems, the researcher considered the approaches typically taught in high school algebra courses: substitution, elimination, and graphing. The researcher developed three problems that met the criteria for each method, as mentioned in **Clinical Interview: Problems**. One problem for each method was designed to be more difficult to solve. These problems included negative numbers and subtraction, which has been shown in the literature to lead to an increase in errors. This was done to help find differences in the approaches used by students of varying disciplines. A problem from each method was transformed so one more step was needed to show the method that was expected to be used. For these problems, one variable was moved to the other side of the equation and the participant was expected to move the variable so she could use the method intended for the problem.

After the problems were created, they were put into random order. If the problem was only for participants in math-related majors, the problem would only be put on the sheet for participants in math-related disciplines; otherwise, the problem was put on both the sheet for participants in non-math-related fields and those in math-related fields.

The researcher worked through each problem using the errors listed in the **Background** section to find possible answers that participants may come up with. Combinations of errors were also used. Once the possible answers were established for each problem, the researcher chose the three she felt would be most likely seen in the interviews.

The participants in math-related fields completed one more problem than those from non-math related fields. This problem was considered a graphing problem and was the most difficult of the three graphing problems created. The researcher chose to leave this one solely to participants in math-related fields because she believed this problem would take longer for participants in non-related fields and, due to the randomness of the order of problems, was worried that this may discourage these participants from completing future problems.

For participants in math-related fields, four of the problems had solution sets derived from common errors and the actual solution set; the other five had the actual solution set and three made-up solution sets that did not have any values in common with the actual solution set or the solution sets derived from common errors. For participants in non-math related fields, three of the problems had solution sets derived from common errors and the actual solution set; the other five had the actual solution set and three made-up solution sets that did not have any values in common with the actual solution set or the solution sets derived from common errors.

The researcher chose to use answers derived from common errors in some problems and those that were not in other problems because research has shown that teachers tend to choose distracters based on what students typically do wrong (Coppedge & Hanna, 1971); and the researcher wanted to see what the student would do when a common incorrect answer was provided and when it was not. That is, the researcher wanted to see whether the student would switch to a different method if he or she realized an error had occurred.

The order of the multiple choice answers was random. The order of the problems and answers was the same for all participants in order to reduce the number of factors concerning how a student responded.

3.4 Data Analysis

Due to the small sample size, no tests could be done about the significance of any of the data, thus all the data will be shown in aggregated form with only generalizations based on the information obtained through the interviews.

Unfortunately, no male participants could be recruited from non-math related fields. As a result, no data can be shown for differences in methods and errors between genders as was mentioned in the **Background** section.

4 Results

4.1 Quantitative Data

This section will cover the quantitative data collected in the interviews. The number of correct versus incorrect responses will be noted along with participants' views of mathematics and grades in mathematics, what errors were found, which errors occurred in which group, the methods known before and during the interviews, what approaches were used, what approaches were used by which group, how many approaches matched the approach the researcher intended, and how the approaches used by each group related to the approach the researcher intended.

Using overall results helps to shed light on the first objective - to determine which approach to solving systems of linear equations students use most when given multiple choice answers – along with whether students tend to use the approach intended for the problem. Overall results also give a generalized look into how often students may solve a system of linear equations correctly and how many and what errors students are likely to make when solving systems of linear equations.

By splitting the data into math and non-math groups, information may be found for the second objective - if students tend to solve problems differently depending on their major. This will show the difference in how often correct solutions are found, how views of mathematics differ between groups and what this means for performance, what errors are likely to be made by each group, what approaches each group tends to know, and whether one group is more likely than the other to use the approach intended for a problem.

4.1.1 Correct vs. Incorrect

There are three tables in this section. The first table displays an overall view of the number of correct and incorrect responses to problems. The second table displays the number of correct and incorrect responses to problems from students in math-related fields. The third table shows the number of correct and incorrect responses made by students in non-math fields.

Overall, the majority of participants correctly solved each of the problems. As noted in **Table 1**, there tended to be one incorrect answer for each problem. As will be seen in **Table 2** and **Table 3**, various participants solved an equation incorrectly.

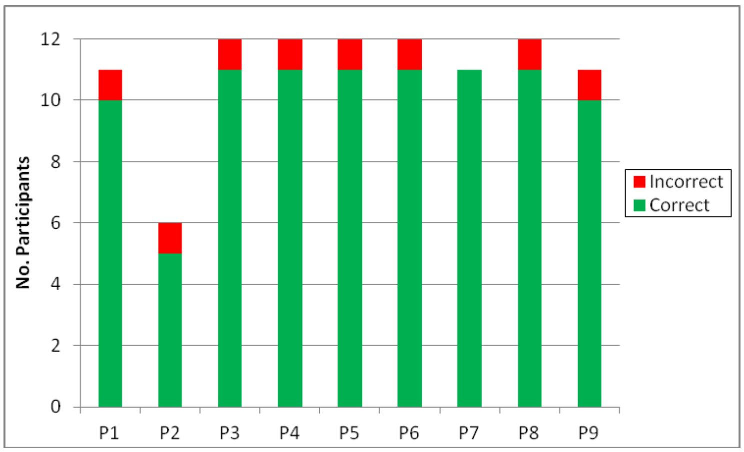

Table 1 - Overall

Table 2 notes the number of correct and incorrect responses to problems that were solved by students in math-related disciplines. As is shown in the table, most problems were solved correctly. How students erred in these problems will be discussed in detail later in the **Errors** section.

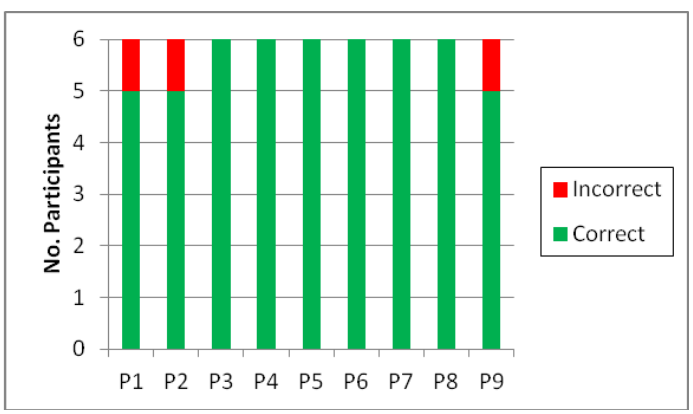

Table 2 - Math

Table 3 notes the number of correct and incorrect responses to problems that were solved by students in non-math related disciplines. As is shown from the data, most problems were solved correctly by participants. How students erred in these problems will be discussed in detail later in the **Errors** section.

Problem 1 (P1), Problem 7 (P7), and Problem 9 (P9) were only solved by five of the non-math participants. This information is included in **Table 3**.

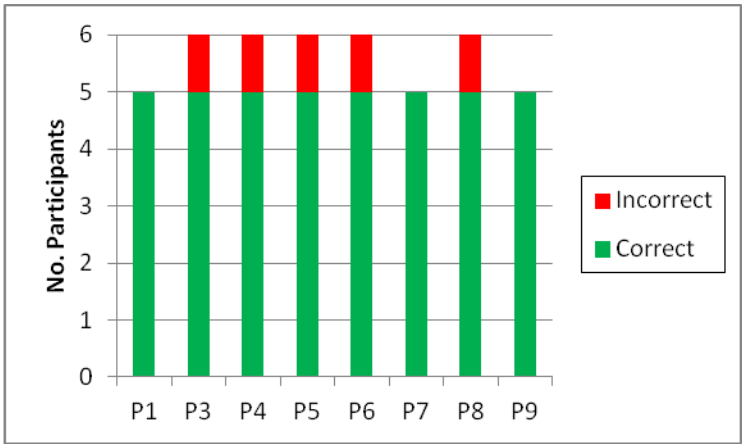

Table 3 - Non-Math

As can be seen from **Table 2** and **Table 3**, non-math students were more likely to solve a problem incorrectly than a student in a math-related field. This result was expected; those in math-related majors would be expected to solve problems correctly while those in non-math related fields would be expected to vary between solving problems correctly and solving them incorrectly. However, as shown in **Table 1**, most students solved the problems correctly.

4.1.2 View of Mathematics

This section contains two tables. The first table displays the number of participants who like math alongside the number of participants who do not like math. The second table displays the average number of errors made by students who liked and did not like math.

It was found that more participants liked math than did not like math. As was expected, all of the students from math-related disciplines liked math, while only half of the students in non-math fields liked math. This information can be seen in **Table 4**. It was expected that students in math-related fields would enjoy mathematics while only some in non-math fields would enjoy math.

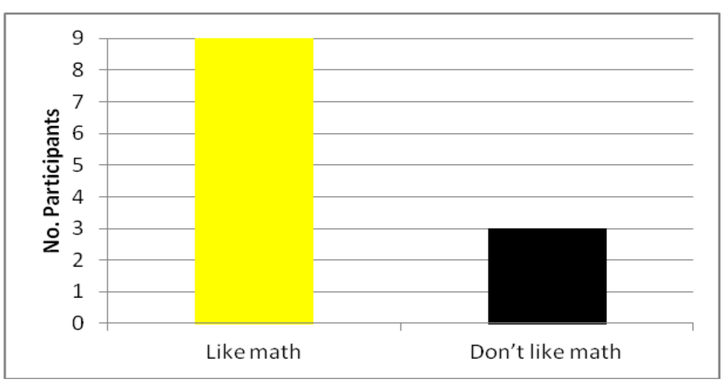

Table 4

As was expected, it was found that, on average, students who do not like math make more errors when solving problems than those who like math. This supports the idea that mathematical performance is related to one's view of mathematics. For specific numbers, please refer to **Table 5**.

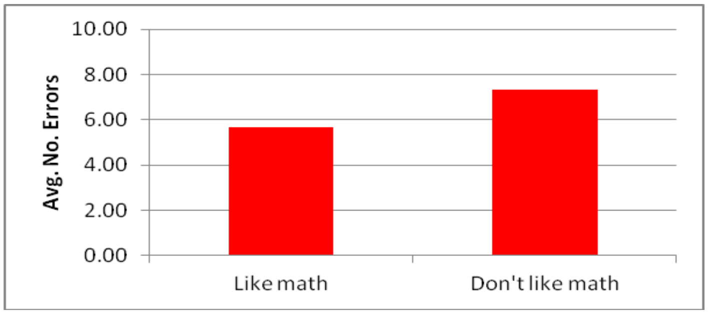

Table 5

It is clear from **Table 5** that students who like math are less likely to make errors than those who do not like math. This result supports the idea that a negative view of mathematics relates to poorer performance on mathematical tasks.

4.1.3 Grade in Mathematics

This section contains three tables: **Table 6**, **Table 7**, and **Table 8**. **Table 6** shows how many participants typically received a certain grade in their mathematics courses. **Table 7** shows the average number of errors students who typically receive a certain grade make when solving systems of linear equations problems. **Table 8** shows an aggregated form of **Table 7**.

The grade participants typically received in a math class did not vary very much. The majority of participants, as noted in **Table 6**, usually earned an A in math. The grades varied between an A and B- for both groups (math and non-math). The lowest grade a student typically received was a B-. It should be noted that the student who usually received a B- in math was a student in a math-related discipline.

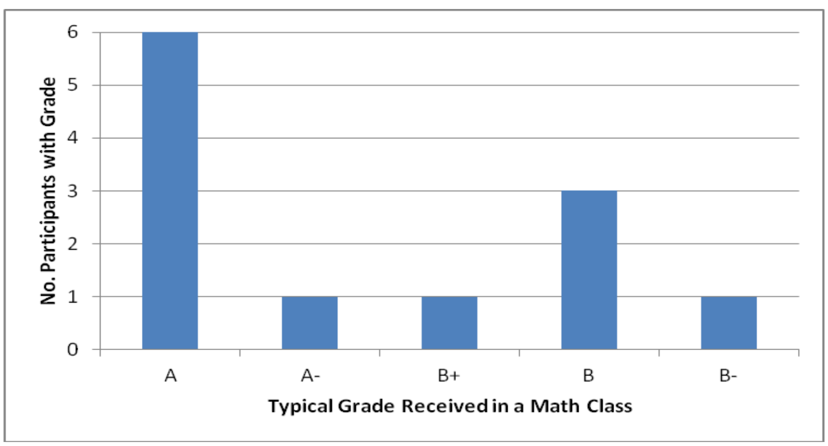

Table 6

As noted in **Table 7**, the average number of errors differed depending on the grade a student received. The distribution was not expected based on the literature. However, the abnormalty of the distribution can be attributed to grading scales. Grades are not always based on a student's mathematical performance but on other factors such as whether a student finished his or her homework and participation. As a result, some non-math participants received A's while some math participants received lower grades. For example, the student who typically received a B-, as was mentioned previously, was a math student. It was expected that math students would not make as many errors as non-math students, so this would explain why the grade with the lowest average number of errors is a B-. Having non-math participants with a grade of A or A- is what caused the average number of errors for those grades to increase, as will be discussed next in the **Errors** section.

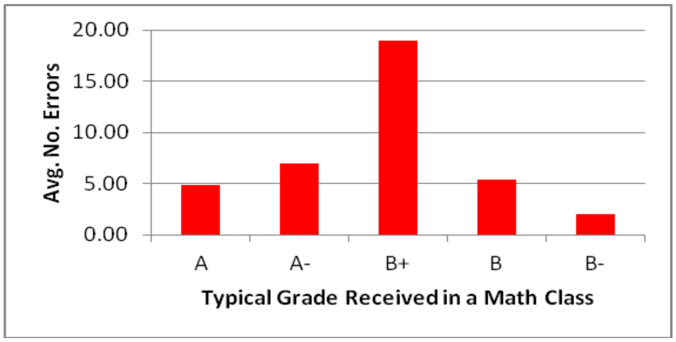

Table 7

Table 8 shows the average number of errors for different grade ranges. As is clear from **Table 8**, students who receive a grade in the "A" range (i.e. A or A-) are less likely to make errors than students who receive a grade in the "B" range (i.e. B+, B, or B-). This result was expected. By aggregating the data into these grade ranges, a more accurate picture as to the number of errors made by students with different letter grades was able to be shown.

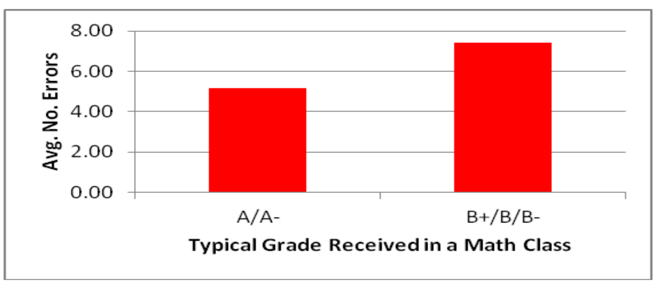

Table 8

From this information, it is obvious that the amount of errors that are made increase as grades decrease. This result was expected.

4.1.4 Errors

Many errors were noted throughout the interviews. This section works to categorize the errors based on the type of error, when the error occurred, and which type of student (math or non-math) made the error. Combinations of these three categories were used as well.

4.1.4.1 Number of Errors Per Approach Intended

The three approaches students were expected to know and use were substitution (S), elimination (E), and graphing (G). Each problem was created with the intention of having a specific approach (substitution, elimination, or graphing) used to solve it.

Table 9 shows the number of errors students made while solving each type of problem. It is clear from the data presented that students had the most trouble with problems they were expected to use elimination on and the least amount of trouble with problem they were expected to use graphing on. This finding will be explained more in the **Qualitative Data** section later.

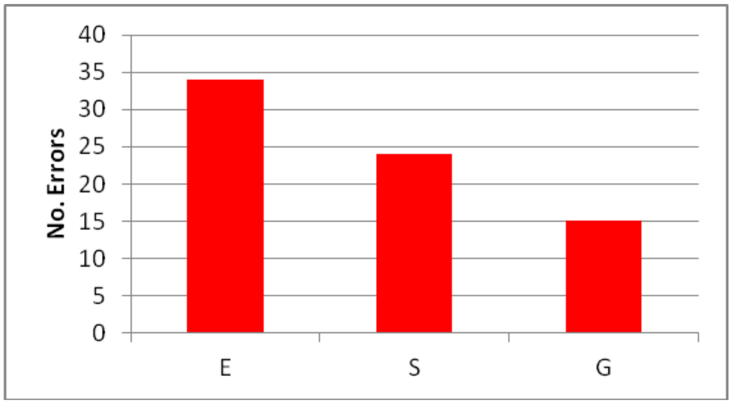

Table 9 - E stands for Elimination. S stands for Substitution. G stands for Graphing.

41.4.2 Number Errors Per Approach Intended by Group

This next table, **Table 10**, shows the number of errors made by math students compared to the number of errors made by non-math students for problems categorized as elimination (E), substitution (S), and graphing (G) problems. It is clear from the data that non-math students made more errors than math students in all three categories (E, S, and G). It should be noted that the data from non-math students matches the data from the previous table; that is, non-math students made the most errors in elimination problems and the least in graphing problems, similar to the data presented in **Table 9**. Math students, on the other hand, were more likely to make errors in graphing problems than elimination or substitution problems. These results will be discussed in further detail in the **Qualitative Data** section later.

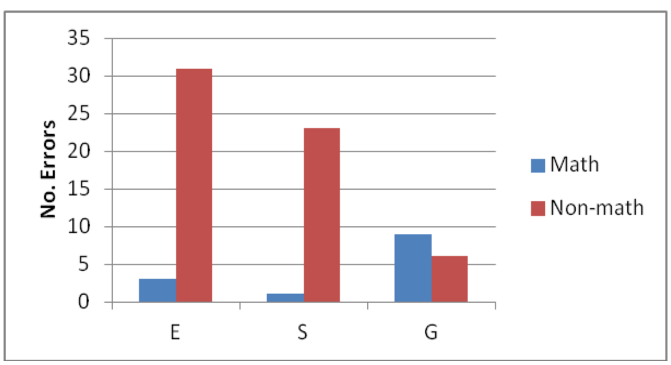

Table 10 - E stands for Elimination. S stands for Substitution. G stands for Graphing.

4.1.4.3 Number Errors by Group

The following table, **Table 11**, shows the number of errors made by students in math-related majors compared to the number of errors made by students in non-math related majors. As is evident from the table, non-math students were more likely to make mistakes than math students. This result was expected; students in math-related fields are expected to be able to correctly solve a problem with a minimal number of mistakes.

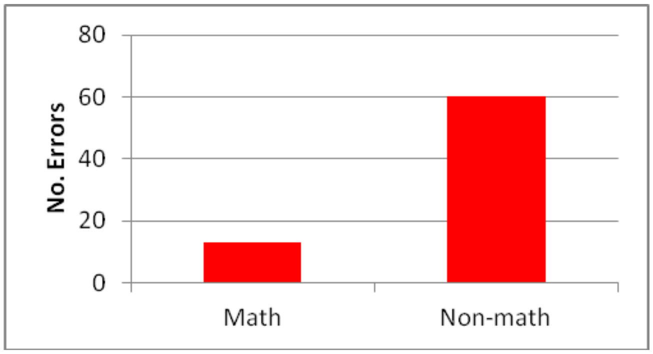

Table 11

4.1.4.4 Simple vs. Complex Errors

The errors found in this study were put into one of two categories: simple or complex errors. Simple errors were considered "silly mistakes"; that is, students understood the process but forgot something along the way. Simple errors that were found during the study were: incorrect addition/subtraction (Ic A/S; e.g. 4 + 2 = 7 or 11 – 3 = 9), incorrect multiplication (Ic M; e.g. 3 x 6 = 24), incorrect division (Ic Dv; e.g. 24/4 = 8), incorrect distribution (Ic Db; e.g. $-2(x - 4) = -2x - 8$ instead of $-2(x - 4) = -2x + 8$), chose a different answer (CDA; e.g. had the values for answer A but chose answer B), rewriting an equation wrong (REW; e.g. $x + 3y = 12$ becomes $x + 3y = 2$), switching the values of the variables (SV; e.g. $x = 2$, $y = 3$ becomes $x = 3$, $y = 2$), losing a variable (LV; e.g. $3x + y = 7$ becomes $3 + y = 7$), and losing a number (LN; e.g. $x + 5 = 8$ becomes $x = 8$). Complex errors were considered errors that showed a misconception about a procedure. Complex errors that were found during the study were: improper equality process (Ip EP; e.g. only multiplying one side of an equation), improper substitution process (Ip SP; e.g. $x = 2$, $3x + 3y = 9$ becomes $2 + 3y = 9$), improper addition/subtraction process (Ip A/S P; e.g. $3x + 4 = 7$ becomes 3x = 11 instead of 3x = 3), improper method (Ip Md; e.g. substituting the value for one variable into both equations then using elimination), improper division process (Ip DvP; e.g. $3x/2 + 1 = (3x + 1)/2$), improper addition process (Ip AP; e.g. add to one side when subtracting from other), improper simplification process (Ip SmP; e.g. canceling out coefficients and leaving a variable), taking an absolute value of an integer to be the value of a variable (TAVoIaVoV; e.g. arrive at -2 = -2 and write $x = 2$), only checked values for one equation (OcVfOE) and improper distribution process (Ip DbP; e.g. $-2(x - 4) = -2x - 4$ instead of $-2(x - 4) = -2x + 8$).

The following sub-sections will present data about the number of simple and complex errors, how many of each type of error were seen in the study, and how many of each type of error were made by math students compared to non-math students.

4.1.4.4.1 Overall

Table 12 shows the number of simple and complex errors found during the study. Overall, there were more simple errors than complex errors. **Table 12** shows this information in detail. The following table, **Table 13**, will break down this information further.

Table 12

Table 13 breaks down the simple and complex errors into the errors found during the study. The table shows the number of times each error was found. The most common error was with incorrect addition/subtraction (Ic A/S). This result was expected from the information in the **Background** section about arithmetic errors being the most common errors. The next most common error was with students using an improper method (Ip Md). Some students combined methods while trying to solve the problems, showing their understanding of the differences between the methods and the procedures used for each method. This was expected because some students will mix-and-match methods depending on what they remember.

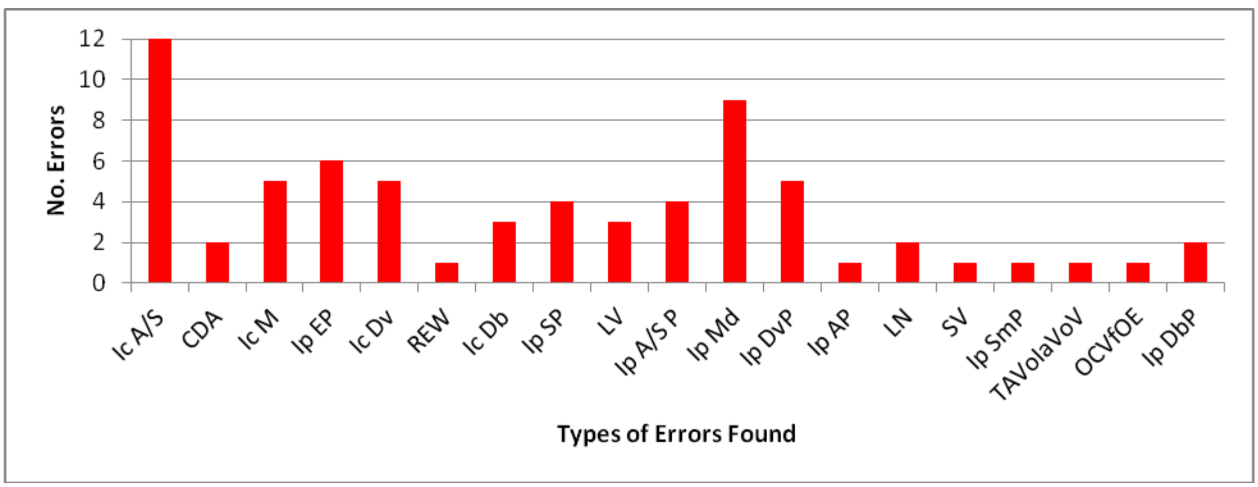

Table 13 - Incorrect addition/substraction = Ic A/S, incorrect multiplication = Ic M, incorrect division = Ic Dv, incorrect distribution = Ic Db, chose different answer = CDA, rewrote answer wrong = REW, switched values = SV, lost variable = LV, lost number = LN

4.1.4.4.2 Simple Errors - Math vs. Non-Math

This next table displays the number and type of simple errors math and non-math students made during their interviews. The most common error for both math and non-math students was incorrect addition/subtraction (Ic A/S). This result was expected. Besides this error, math students did not make many other types of errors and, when they did, it was only once. Non-math students, on the other hand, made several different types of errors numerous times. As **Table 14** shows, non-math students were also likely to make errors with incorrect multiplication, incorrect division, incorrect distribution, losing variables, and losing numbers. Incorrect multiplication and division was expected because these errors fall under the umbrella of arithmetic errors, which were mentioned in the **Background** section to be the most common errors.

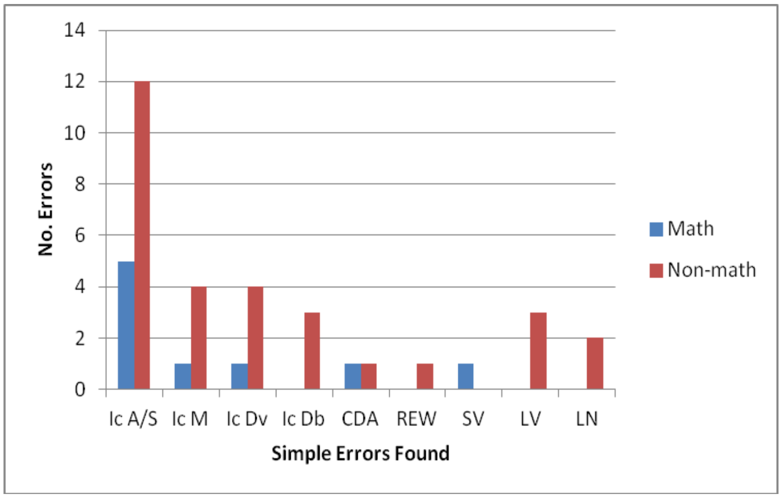

Table 14 – Incorrect addition/subtraction = Ic A/S, incorrect multiplication = Ic M, incorrect division = Ic Dv, incorrect distribution = Ic Db, chose different answer = CDA, rewrote answer wrong = REW, switched values = SV, lost variable = LV, lost number = LN

4.1.4.4.3 Complex Errors – Math vs. Non-Math

Table 15 shows the data concerning complex errors made by math and non-math students. As is evident from the table, non-math students were more likely than math students to make complex errors. Non-math students were more likely to use an improper method (Ip Md), improper division process (Ip DvP), improper substitution process (Ip SP), and improper addition/subtraction process (Ip A/S P). Math students were just as likely as non-math students to use an improper equality process (Ip EP). The only other complex error made by a math student was using an improper addition/substraction process (Ip A/S P), and this was only done once. The most common complex error was using an improper method (Ip Md).

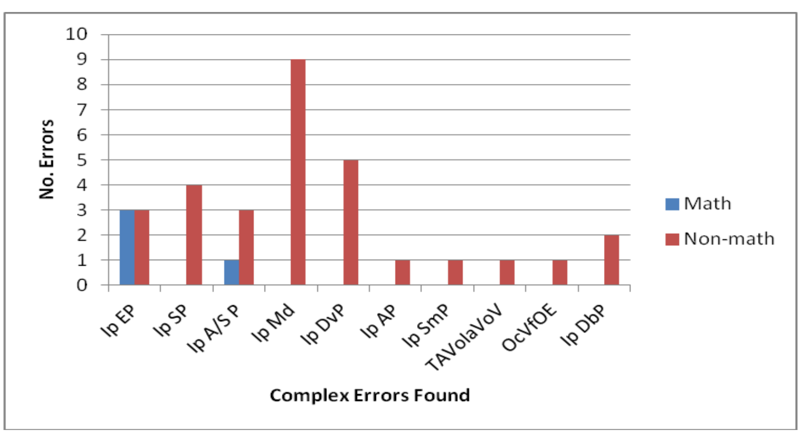

Table 15 - Incorrect addition/subtraction = Ic A/S, incorrect multiplication = Ic M, incorrect division = Ic Dv, incorrect distribution = Ic Db, chose different answer = CDA, rewrote answer wrong = REW, switched values = SV, lost variable = LV, lost number = LN

Overall, students in non-math disciplines were much more likely to make both simple and complex errors than students in math-related majors. Students in non-math related majors displayed a wide array of errors not found in work done by students in math-related fields.

4.1.4.5 Summary of Errors

As was expected, students who are in non-math related fields are more likely to make errors than those who are in math-related fields. These students are more likely to display a range of errors and make more errors in general, despite the approach intended.

4.1.5 Methods Known Before/During

As is shown in **Table 16**, most participants knew about substitution and elimination before solving any problems; this result was expected. Some students mentioned using the values from the multiple choice answers as another method for solving systems of linear equations in multiple choice format. This result was expected based on the literature, though it was expected that more students would think of this approach. Another expected result was the number of students who mentioned using a calculator to solve systems of linear equations. As is shown in **Table 16**, very few participants knew that systems of linear equations could be solved via a calculator. Yet another expected result was the number of participants who knew that graphing was a method for solving systems of linear equations. **Table 16** shows that graphing with and without a calculator was mentioned only four times.

One student mentioned methods that are not applicable to solving systems of linear equations; these methods were PEMDAS (order of operations) and foiling. PEMDAS (parentheses, exponent, multiplication, division, addition, subtraction) is used within some methods of solving for systems of linear equations, but is not an actual method for solving a system. There is no time in which foiling would be used to solve a system of linear equations.

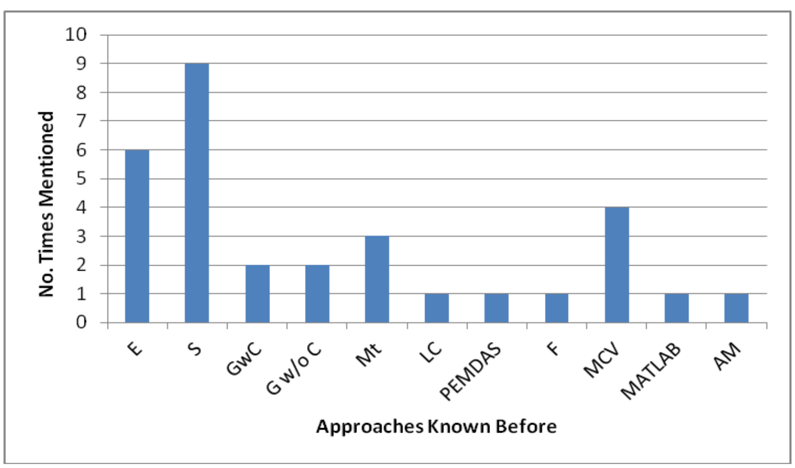

Table 16 – E = Elimination, S = Substitution, GwC = Graphing with calculator, G w/o C = Graphing without calculator, Mt = Matrices, LC = Linear combination, F = Foiling, MCV = Use multiple choice values, AM = "Algebra magic"

Two participants noted additional methods for solving systems of linear equations in the middle of the interview. These methods are mentioned in **Table 17** below.

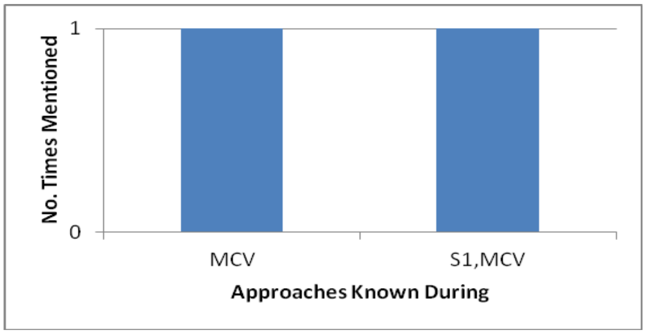

Table 17 – Use multiple choice values, S1,MCV = solve for one then use multiple choice values for other

4.1.6 Approaches Used

One of the objectives of this study was to determine what approaches students tend to use when solving systems of linear equations problems. The other objective was to see whether students solve systems of linear equations problems differently depending on their major (i.e. whether they are in a math-related field or non-math related field). This section covers the results that shed light on these objectives.

4.1.6.1 Number of Times an Approach was Used

Table 18 shows the number of times an approach was used throughout the study. It is obvious from the graph that substitution was the preferred method for most students. One student primarily chose elimination while another primarily chose a backwards approach (i.e. using multiple choice values), which account for the increase in usage of the elimination and backwards approaches shown on the graph.

Substitution was used by one participant who chose to pass over the problem. This participant tried to solve the problem first and, after having difficulty, chose to move on to the next problem. Here, substitution refers only to trying to find the value of the first variable. This result was expected.

One participant tried solving for the second variable by combining the elimination method with plugging in the value that was found for the first variable. The participant plugged the value into both equations, added the two equations together, and then solved for the second variable.

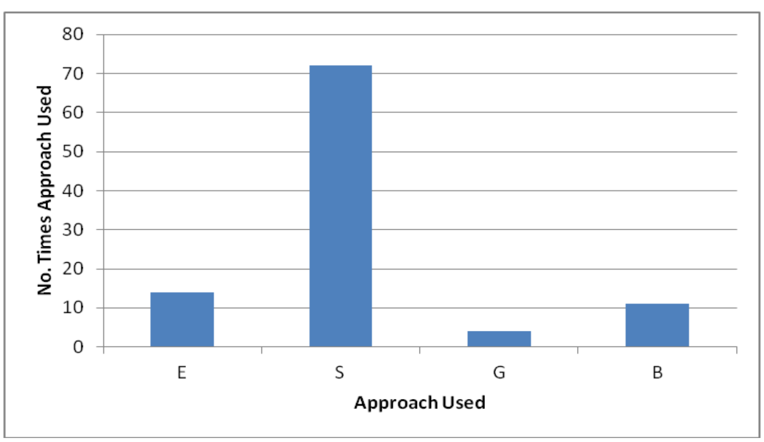

Table 18 – E stands for Elimination. S stands for Substitution. G stands for Graphing. B stands for Backwards approach (using

multiple choice values).

4.1.6.2 Number of Times an Approach was Used Per Approach Intended

To break down the data further, **Table 19** shows the distribution of the approaches used for each of the approaches intended. The graph shows that substitution was the approach chosen by most participants despite the approach intended. The next most common approach was elmination, which was primarily used on problems where the intended approach was either elimination or substitution Using the multiple choice values occurred a few times, but not as often as would have been expected based on the literature (see **Background**). Graphing was rarely ever used, and was only used for problems where graphing was the intended approach. This is likely due to the amount of manipulation required to solve a problem graphically. This result was expected.

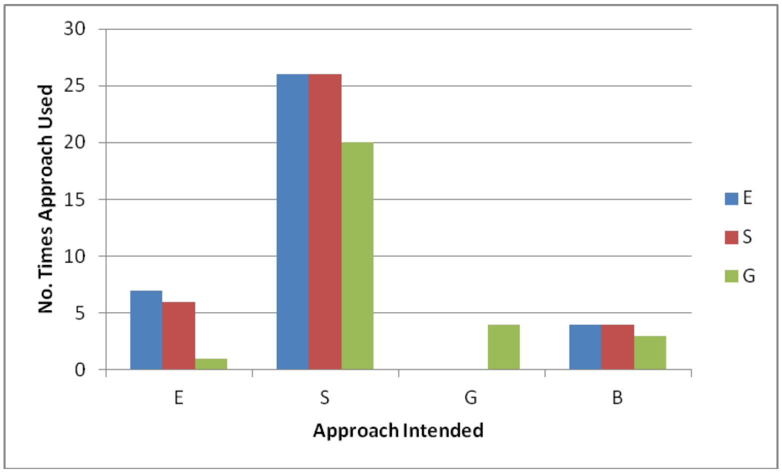

Table 19 – E stands for Elimination. S stands for Substitution. G stands for Graphing. B stands for Backwards approach (using multiple choice values).

4.1.6.3 Approaches Used by Group

The data was analyzed to determine what approaches were used not just in general but also for each intended approach. The next table, **Table 22**, shows the distribution of approaches used by students in math-related disciplines compared to those used by students in non-math related disciplines. According to the data, math students were more likely than non-math students to use substitution, while non-math students were more likely than math students to use a backwards approach (i.e. using the multiple choice values).

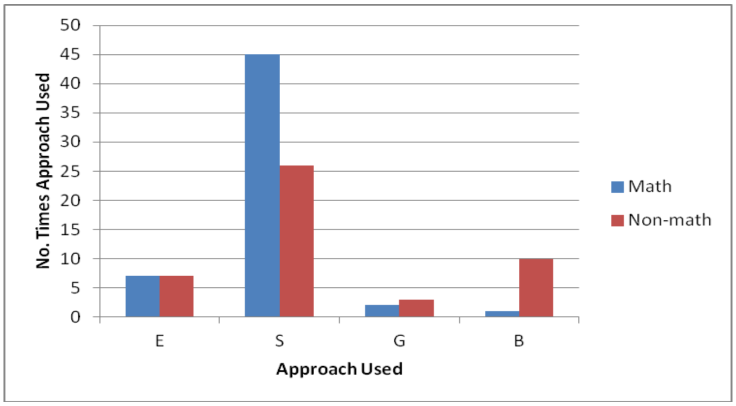

Table 20 - E stands for Elimination. S stands for Substitution. G stands for Graphing. B stands for Backwards approach (using multiple choice values).

4.1.6.3.1 Approaches Used by Math Students Per Approach Intended

This next table, **Table 23**, shows the approaches used by participants in math-related disciplines for each intended approach. It is clear from the graph that substitution was almost always used despite the approach that was intended for a problem. Elimination was sometimes used on problems that were intended to be elimination or substitution problems. Graphing was only seen while students were working on problems where the intended approach was graphing. A backwards approach (using the multiple choice values) was only seen in problems where elimination was the intended approach. From this data, one may suggest that math students prefer to use substitution, but will use elimination in some cases. One may also suggest that graphing and using a backwards approach will not often be seen in work completed by math students.

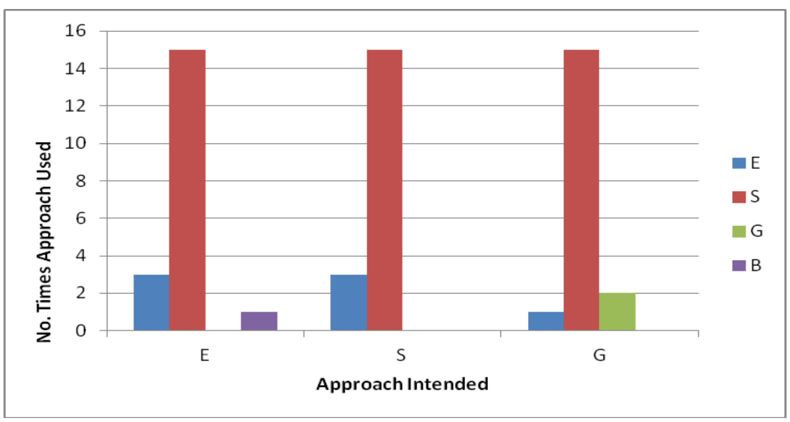

Table 21 – E stands for Elimination. S stands for Substitution. G stands for Graphing. B stands for Backwards approach (using multiple choice values).

4.1.6.3.1 Approaches Used by Non-Math Students Per Approach Intended

This next graph, **Table 24**, shows data concerning the approaches used by non-math students while they were solving problems where a specific approach was intended to be used. Compared to the participants from math-related fields, the students in non-math related disciplines had a much more diverse array of approaches that were used on problems where a specific approach was intended to be used. While substitution was still the main method used to solve the problems that were presented, students in non-math related fields were also open to solving problems using a backwards approach (i.e. using the multiple choice values). They were also more likely to use graphing to solve problems where graphing was the intended approach. These results were expected because it is more likely that non-math students would be unsure of whether another approach besides the intended approach would lead to the right answer and that non-math students would find solving problems directly, using a forward approach, more unappealing and would be more open to using the multiple choice values than a student in a math-related field.

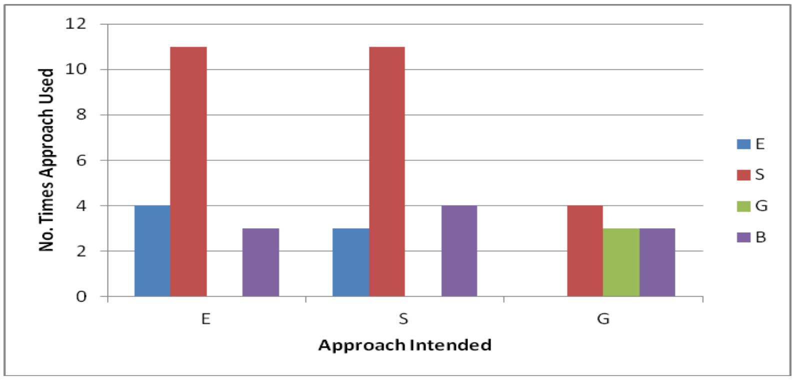

Table 22 – E stands for Elimination. S stands for Substitution. G stands for Graphing. B stands for Backwards approach (using multiple choice values).

4.1.6.4 Summary of Approaches Used

Overall, it seems that students prefer using substitution than any other approach when it comes to solving systems of linear equations problems. This is the case no matter what approach is intended or whether a student is considered a math or non-math student. Graphing, as was expected, is very rarely seen, while using the multiple choice values is seen from time to time, especially with students in non-math related majors.

4.1.7 Matches

The next two tables show the number of times the approach used matches the approach intended. **Table 25** displays the matches by problem, while **Table 26** shows the number of matches total for each approach intended. As is clear from **Tables 25** and **26**, more students were likely to use substitution on substitution problems than they were to use elimination on elimination problems or graphing on graphing problems.

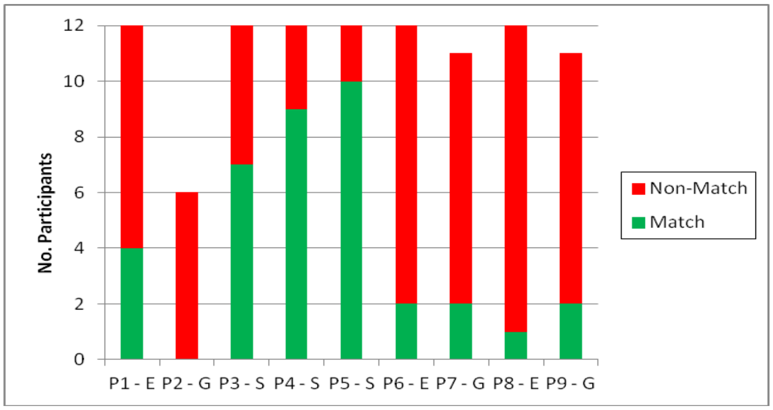

Table 23 - E stands for Elimination. S stands for Substitution. G stands for Graphing.

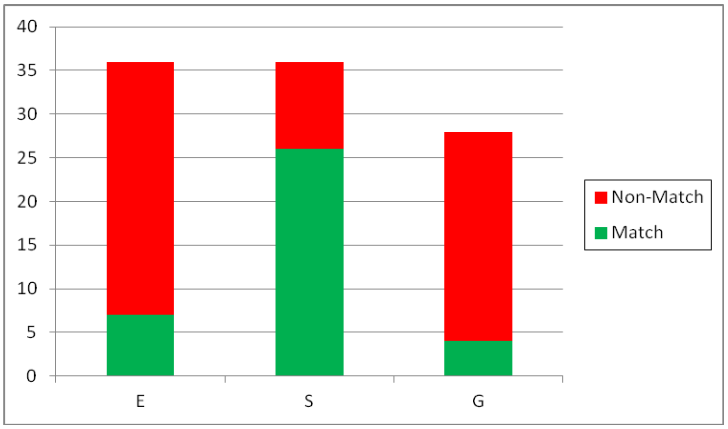

Table 24 – E stands for Elimination. S stands for Substitution. G stands for Graphing.

Overall, it was found that substitution was the preferred method of most participants (see **Approaches Used** above). As a result, elimination and graphing problems were not typically solved with elimination or graphing methods. Only the first elimination problem, P1, had a larger number of students using elimination (see **Table 25**). Interestingly enough, P1 was the difficult problem for the elimination group, which leads to questions as to whether more difficult problems created for a specific method are more likely to be solved via the intended method compared to simpler problems for the same method.

4.1.8 Summary of Quantitative Data

This section has covered the quantitative data collected in the interviews. This includes the number of correct versus incorrect responses, participants' views of mathematics and grades in mathematics, what errors were found, which errors occurred in which group, the methods known before and during the interviews, what approaches were used, what approaches were used by which group, how many approaches matched the approach the researcher intended, and how the approaches used by each group related to the approach the researcher intended.

The **Approaches Used** section gives answers for the first objective of the study - to determine which approach to solving systems of linear equations students use most when given multiple choice answers. It was found that substitution was the approach preferred by most participants.

The second objective - if students tend to solve problems differently depending on their major – was also touched upon throughout the subsections of the **Quantitative Data** section. It was found that math students are more likely to use substitution to solve for systems of linear equations problems while non-math students prefer to use either substitution or the multiple choice values.

4.2 Qualitative Data

4.2.1 Approach Used

During and after every problem, each participant was asked why they were taking the approach they were, whether they thought that approach was the easiest, and if there were any other approaches they might have used to solve the problem.

4.2.1.1 Why Are You Taking This Approach?

When asked why they were taking an approach, most participants answered along the lines of "the only approach I know how" or "easiest" or "faster" or "don't remember others" or "used to using by now". These responses reflect the information shown in the **Known Before/During** section above. Most students do not remember many approaches, only one or two, which causes them to stick to those approaches. These approaches seem to be the ones that students understood the most and were more likely to use when they took algebra and were solving systems of linear equations.

Students in math-related fields tended to give more elaborate answers. While they did mention that certain methods were easier and came to mind more quickly, they also liked to explain the process by which they decided what approach to use. They would say "x was easy to isolate" or "second equation has y by itself, so it would be easy to put in the first equation". One student even said they were taking an approach "just to do something different". These responses show how students in math-related fields tend to have a better grasp on the different methods that can be used to solve systems of linear equations and are able to think through which method is most appropriate for the problem at hand.

One student, at the end of the interview, summed up how she felt about the approaches she used to solve the problems presented to her. She said she was "so conditioned to do math first" that it "didn't occur to me to plug in multiple choice answers". She continued by saying "If I didn't know how to do a problem, I would probably plug in the answers. But, since I knew how to do the problem, I didn't check the answers." Her reasoning for this is that she usually did not have multiple choice answers available and had gotten used to having to do the problem out herself.

Math and non-math students alike tried their best to avoid fractions. Many students mentioned they did not like fractions, and some purposely took more time finding an approach just so they would not have to deal with fractions (see **Example 1**).

Example 1

P9)

$5x + 2y = 19$
$3x + 3y = 6$

a) $x = 5$, $y = 22$
b) $x = 5$, $y = -3$
c) $x = 5$, $y = 3$
d) $x = 5$, $y = 7$

Work:

Student hates fractions and learned lesson from last problem, so decides to divide second equation by 3 and solve for y via substitution to avoid fractions.

$x = 2 – y$
$10 – 5y + 2y = 19$
$3y = 9$
$y = 3$

Checks answers, sees that **c** has $y = 3$, and chooses **c**.

Overall, the approach a student took was usually one the student would repeatedly use throughout the interview. This fixation on one method was expected and will be discussed further in the **Discussion** section below.

4.2.1.2 Do you think this was the easiest approach?

Before the question could even be asked, most students would say that they were taking a particular method because it was the easiest method. This method was typically the one approach the student remembered. Here easiest meant the fastest method that would be unlikely to result in errors. Again, it is highly likely that this approach was remembered because it was the one the student understood the most back when the student was originally learning how to solve systems of linear equations problems.

It made no difference whether the student was a math or non-math student, the "easiest" approach was always the one the student went for when solving the problems in the interview. The only possible difference is that math students were more likely to remember more approaches and would hence have more approaches to consider when deciding which approach was the "easiest" for a problem. Thus this finding suggests that there is no difference in the underlying reason why a student chose one approach over another.

4.2.1.3 What other approach might you use for this problem?

When students were asked if there were any other approaches they might use for solving a problem, the typical answer was "no". Some students would begin to spew out every method they knew, but this was not very often. At times, a student might suggest one other approach.

There were a few times when a student would explain why they did not use the other approach they might have used. These responses were along the lines of "it would have taken longer" and "it would require an extra step" and "I didn't think of it at first". It is clear from these responses that the students were stuck in their ways, unlikely to use any approach besides the one most familiar to them.

While some students were solving a problem, they would ponder over which approach to use. They would mention how they could use one approach to solve the problem and would end up using another approach to solve the problem instead. These students tended to give an explanation for the change similar to those in the previous paragraph (i.e. "it would require and extra step" and "it would have taken longer").

4.2.2 Errors

As was noted in the **Quantitative Data** section, students made number of errors while solving the problems in the study. This section discusses the possible reasons for making the errors and how students diagnosed and fixed the errors they made.

4.2.2.1 Making Errors

Many different types of errors were made by students throughout the interviews. Most of the errors were just a slip: dropping a variable when simplifying an equation, adding or multiplying numbers incorrectly, etc. These errors were just quick lapses in memory that most likely came from students being preoccupied with other issues besides the task at hand (see **Example 2**).

Example 2

$2x + 3(4x + 8) = -4$
$2x + 12x + 24 = -4$
$14x = 20$

The complex errors that were found in the study deserve more attention. There were several times when students were making complex errors and asked "Am I doing this right?", or something along those lines. When one student asked whether the answer was the absolute value of what she had in front of her, she was not completely sure but felt that this had to be the case because it was what she had. She believed she had used a proper method, so her feeling was that she had to be right. Other students would multiply something on one side and not the other, possibly because they forgot to put the expression on the other side or possibly because they thought just putting it on the other side would cancel it out on the side they did not write the expression on (see **Example 3**).

Example 3

$3x + 3y = 6$
$x + y = 6$

These errors were typically found by students, and they would express surprise as to how they had forgotten to multiply both sides of the equation, making the former possibility more likely than the latter. Errors were found by looking back through the work one step at a time from the beginning until the error was found. The student would go through the math in each step and decide whether she did the math correctly or not. If the math was correct, she would move on to the next step; otherwise, she would correct the mistake and finish solving the problem from there. A more detailed look at how students diagnose and fix errors will be discussed in the next section.

When students took a while to start the first problem or would write a few lines and then start over, it is likely they were not sure how to proceed with any method they knew. Sometimes this confusion led to students using improper methods, such as a combination of methods. Improper methods were methods that were not known to solve systems of linear equations problems correctly (see **Example 4**).

Example 4

P1)

$2x + 3y = 2$
$4x + y = 8$

a) $x = 9$, $y = -4/5$
b) $x = 9/5$, $y = -4/5$
c) $x = 11/5$, $y = -4/5$
d) $x = 11$, $y = -4/5$

Work:

$4x + y = 8$
$-4x \quad\quad - 4x$
$y = 8 – 4x$

$4x + (8 – 4x) = 8$

Sees that this leads to 0 = 0 and starts over.

$4x + y = 8$
$-y \quad\quad -y$
$4x = 8 – y$
$\div 4 \quad\quad \div 4$
$x = 2 – y/4$

$4(2 – y/4) + y = 8$
$8 – 4y/16 + y = 8$
$(8 – 4y/16) + (8 – 4x) = 8$

Stops here and starts over.

$2x + 3y = 2$
$-2x \quad -2x$
$3y = 2 - 2x$
$\div 3 \quad \div 3$
$y = 2/3 - 2x/3$

$8 - 4x = 2/3 - 2x/3$
$-2/3 \quad - 2/3$
$22/3 - 4x = -2x/3$
$\cdot 3 \quad \cdot 3$
$22 - 4x = -2x$
$+ 4x \quad + 4x$
$22 = 2x$
$\div 2 \quad \div 2$
$x = 11$

$2(11) + 3y = 2$
$22 + 3y = 2$
$-22 \quad -22$
$3y = -20$
$\div 3 \quad \div 3$
$y = -20/3$

Checks answers for $x = 11$ and $y = -20/3$, sees that it is not a possibility, and decides to skip the problem.

Errors can be made very easily. Sometimes it is because someone's memory is fuzzy as to what should be done, sometimes it is as simple as leaving something out when rewriting an equation. Most of the time, there is not much thought going into the error being made, just a quick "what is 3 times 6" and thinking "24". There were some times, though, when students would pause and think before writing something down. It is likely that during this time a student was trying to decide what to write down because she was stuck between two choices: an error or the right information to continue. When an error was still made, it shows that the student did not fully understand the material and is likely to make that error again in the future.

4.2.2.2 Diagnosing and Fixing Errors

Diagnosing errors usually came immediately after the error is made or at the end of the problem when the student is checking her answer. The students were likely to be thinking about what they just wrote down when they continue working on a problem. This extra time sometimes led to students stop what they were doing and go back to the error. Several students in the study did some arithmetic in their heads, wrote it down, and were about to continue working (or continued to work) when they realized that they had done the arithmetic wrong.

Some students would exclaim “3 times 6 doesn’t equal 24, it equals 18” or something similar. Right after they noticed the error, they would go back and write down the correct expression.

Other times students did not notice their mistakes until after they had finished working on the problem. Students would look at the multiple choice answers available, compare them to their own values, and remark that the two sets of values were not the same and that they must have made a mistake somewhere. At this point they had two options: redo the whole problem or go through the work to find the mistake. When the work was written in order with one step below the previous step, the student would usually start from the beginning and work through what she had done until she found the error. Once the error was found, the student typically knew right away how to fix it (see **Example 5**).

Example 5

P7)

$2x + 4y = 6$
$3x = 2 – 5y$

a) $x = -11$, $y = 7$
b) $x = 10$, $y = -5$
c) $x = 12$, $y = 4$
d) $x = 4$, $y = 5$

Work:

$2x + 4y = 6 \rightarrow x + 2y = 6 \rightarrow x = -2y + 6$
$3x = 2 – 5y$

$3(-2y + 6) = 2 – 5y$
$-6y + 18 = 2 – 5y$
$-y = -16$
$y = 16$

$x = -2(16) + 6$
$x = -32 + 6$
$x = -26$

Checks answers for $x = -26$ and $y = 16$, sees that it is not a possibility, and starts looking through work from the beginning. Reads first line: $2x + 4y = 6 \rightarrow x + 2y = 6 \rightarrow x = -2y + 6$. Sees mistakes, draws an arrow down to bottom of page, and continues.

$x + 2y = 3 \rightarrow x = -2y + 3$
$3(-2y + 3) = 2 – 5y$
$-6y + 9 = 2 – 5y$
$-y = -7$
$y = 7$

$x = -2(7) + 3$
$x = -14 + 3$
$x = -11$

Checks answers, sees that **a** is a match, and choose **a**.

There were some students, though, who chose to start from the beginning, no matter the amount of work that had been done to solve the problem. It is likely that these students did not want to try to figure out where they went wrong and thought it would be faster to just redo the problem from the beginning (see **Example 6**).

Example 6

P5)

$2x + 3y = -4$
$y = 4x + 8$

a) $x = 10$, $y = 5$
b) $x = 3$, $y = 4$
c) $x = -2$, $y = 0$
d) $x = -6$, $y = 7$

Work:

$2x + 3y = -4$
$y = 4x + 8$

$2x + 3(4x + 8) = -4$
$2x + 12x + 24 = -4$
$14x = 20$
$x = 20/14 = 10/7$

Checks answers for $x = 10/7$, sees that it is not a possibility, and starts over.

$2x + 3y = -4$
$y = 4x + 8$

$2x + 3(4x + 8) = -4$
$2x + 12x + 24 = -4$
$14x = -28$
$x = -28/14$
$x = -2$

$y = 4(-2) + 8$
$y = 0$

Checks answers, sees that **c** matches, and chooses **c**.

4.2.3 Extras

At the end of the interviews, participants were offered a chance to mention anything else they felt may help with the study. While most students chose to skip this option, some students took the time to provide additional feedback. The feedback students gave were about how they felt they would have solved the problems when they were first learning systems of linear equations and how they felt they might solve the problems if they were not a math student.

Both math and non-math students reported that when they were first learning to solve systems of linear equations with multiple choice format they worked backwards, using the multiple choice values in the equations to find the answer. They felt that, at the beginning, they did not know what to do and found working backwards to be the easiest approach. Instead of trying a forward approach in which they were not sure they were using properly, they used the answers already provided and tested to see which answer was the solution for the system.

Of the students surveyed, a portion of those with math-related backgrounds reported how they felt they would solve problems if they were a student who was in a non-math related field. They all said they would probably work backwards. They thought that if they were unsure of how to use an approach, they would use the answers provided to find the one that worked.

The responses received from the students that were interviewed are consistent with the literature, that guess and check (a backwards approach) is a popular method among students solving systems of linear equations (Huntley et al., 2007).

4.2.3 Summary of Qualitative Data

The main focus of the qualitative data was on the secondary focus of the study: the reasons students gave as to why they used a certain approach to solve a system of linear equations problem. It was found from the data collected in the interviews that the primary reason an approach was used was because it was deemed the "easiest" approach to called.

The first objective creeps into hear as well. Students talked about how they would have used a different method if they had a different outlook on life or were first learning how to solve systems of linear equations problems. Instead of using substitution (the most common method used), they thought they would probably use the multiple choice values to find the answer that works for the equations given in the problem.

Students did tend to solve problems differently depending on their major, giving information for the second objective of the study. While the basic reasoning behind the decision to use an approach was the same, math students were more likely to have a complex reasoning system to determine which approach to use (as shown by their elaborate responses to why they were using an approach) where non-math students were more likely to choose an approach because it was the one they were most familiar with.

5 Discussion

5.1 Summary

This study has provided quite a bit of information concerning the approaches students use to solve systems of linear equations problem, whether the approaches used depends on a student's major, and the reasoning behind choosing one approach over another.

It was found that, by far, substitution was the preferred method for students, no matter the major of the student, when it came to solving systems of linear equations problems given multiple choice answers. The main reasons students gave as to why they chose a method was that it was the "easiest" method they knew and was typically the only method they remembered. Math students tended to know more methods and expanded on the "easiest" method response by explaining why the method chosen was the most appropriate method for solving the problem.

From the data collected in the interviews, it seems there is a procedure that students go through when they are solving systems of linear equations. The first step is to determine what the problem is (e.g. a system of linear equations problem). Next the student tries to determine what method should be used to solve the problem. These two steps seem to be done quite quickly, except in those students who have trouble recalling how to solve systems of linear equations problems. The third step is to go through the procedure for that method and solve the problem. With a multiple choice format, a fourth step is used. Here the students look through the answers to find the one that matches the values they obtained. If a student finds a match, she is done; otherwise the student would move to a fifth step, which is typically finding the error(s), fixing them, and arriving at the correct solution. When a student has done a lot of work on a problem and it is difficult to follow, the student will most likely start from the beginning again instead of trying to locate the error(s). This procedure seems to work well for most students.

5.2 Limitations

There are several limitations to consider when looking at the results of this study. In creating the problems, some problems were altered so one more step was needed to make the problem geared towards a certain method. As a result of this change, a problem may have looked more like a different method than the one originally intended.

Also, in order to make the systems easier to solve, simple numbers (usually under 10) were used. In some systems, one or both equations could be simplified because of the simple numbers. This made some systems easy to solve using substitution even though a different method was intended.

It should also be noted that some students made many errors (i.e. more than ten) while the majority made very few errors (i.e. less than three). The students who made many errors were non-math students, two of which said they liked math. This could have skewed the data so the averages were higher or lower than would be representative of the student population and the tables showing the errors made by non-math students may not be representative of all non-math students.

At the beginning of the interview, participants were asked what strategies they knew for solving systems of linear equations; they were never asked about strategies associated to multiple choice problems. If participants had been asked about methods they knew for solving multiple choice problems after they had been asked about approaches to solving systems of linear equations, it is possible that more students might have mentioned and used a guess and check approach, thus changing the results.

Due to the small sample size, no tests could be done about significance. This also was a convenience sample, so it is very possible that the results are not representative if the population.

While it was believed that college students were a good proxy to high school students who had just finished algebra, it is possible that the number of years since participants had taken an algebra course or used systems of linear equations could have been impacted the results of the data. Due to the amount of time since participants had taken algebra, it is possible that participants may have forgotten some of the methods.

Lastly, misinterpreting questions may have impacted the results of the data. Participants were asked what other methods they "might use" to solve the problem they had just solved. Some participants may have taken "might use" to mean "could use", leading to some participants spouting out all possible methods, not just the methods the student deemed easier to use for the system presented.

5.3 Implications for the Classroom/Testing

It is clear from the results of this study that many students are likely to have trouble in classrooms where they are required to use a specific approach to solve systems of linear equations problems. If students are fixed on one approach, such as substitution, it makes it difficult for them to handle other approaches, such as elimination and graphing. Teachers need to work with students on mastering several approaches to solving systems of linear equations. Approaches such as elimination and graphing need extra emphasis in order for students to remember them for future use. Knowing multiple approaches is helpful for students in situations such as multiple choice tests when they are stuck on a problem. They may use the approach they know, find that their answer does not match any of the answers available, look through their work to find where they went wrong, and be unable to find the error. In situations like this, a second approach is helpful to arrive at a different solution, hopefully the correct solution.

These results also show that students are not ready for testing. Very few students knew to use the multiple choice values to find the answer to the problems and using the multiple choice values was rarely ever used by those who knew about it. In testing environments, students often do not have enough time to solve every problem directly. Being able to substitute the multiple choice values into equations in order to find a match can often be helpful during tests because it can take less time than trying to solve problems using substitution or other direct approaches to solving systems of linear equations. Using the multiple choice values also decreases the chance of there being errors, helping students answer more problems correctly.

6 Conclusion

6.1 What to take away from this study

There were many important results from this study. The results to focus on are the ones that shed light on the objectives of this study: to determine which approach to solving systems of linear equations students use most when given multiple choice answers and if students tend to solve problems differently depending on their major. It was found that substitution was the approach most often used by both math and non-math students to solve systems of linear equations.

It is also important to remember the reasons students gave as to why they chose to use an approach to solve the problems. Most students used an approach because they thought it was the easiest and it was the one they remembered the most. Other students considered all the approaches they knew and used the one they thought would be the fastest and easiest.

The most important limitation to consider is that this study was done with college students as participants. It has been almost a decade since most of these students have taken an algebra class. As a result, participants may not have remembered all the methods available for solving systems of linear equations problems, which may lead to different results than may be found in a similar study using high school participants.

6.2 Possible Studies

A next study might be to observe one or more high school algebra classes to see what methods are taught and, from there, what methods students use when solving systems of linear equations. This study could use the first objective from this study: to determine which approach to solving systems of linear equations students use most when given multiple choice answers. This study can also look at the reasons students give as to why they choose one approach over another. The information gathered by observing the classrooms can help to analyze the data.

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Appendix A - Consent Form

Dear Student,

My name is Amber Lagasse and I am an undergraduate student majoring in Secondary Mathematics Education in the Mathematics and Statistics department at the University of New Hampshire.

I am conducting a research project to find out when students think it is more advantageous to work backwards from a set of multiple-choice answers and when they think it is better to attempt a 'forward' approach. My focus is on systems of linear equations. I am writing to invite you to participate in this project. I plan to work with approximately fifty students in this study.

If you agree to participate in this study, you will:

- Participate in an interview with me about what you are thinking while you are working through various types of systems of linear equations problems with multiple choice answers. The interview will be approximately thirty minutes to an hour and will take place at a time that we establish together. I will take notes but not audio/videorecord the interviews. I will collect your work at the end of the session.
- Take a short survey asking about your age, gender, major, minor and preferences concerning mathematics.

The time commitment would be at most sixty minutes for the entire study.

You will not receive any compensation to participate in this project.

The only potential risk you may encounter while participating in this study is you may feel uncomfortable with explaining your thought process to an unfamiliar person. However, I have tutored many students before and have conducted interviews with students in previous research. As a result, I am confident that I can create a productive, engaging environment for you. On the other hand, the benefits of you participating in this study are that the study may help future mathematics teachers frame their lessons to complement students' thought processes, allowing for higher quality teaching and learning.

Your participation in this study is strictly voluntary; refusal to participate will not impact your education. If you agree to participate in this project and you change your mind, you may withdraw at any time during the study without penalty. You may also refuse to answer any question.

I seek to maintain the confidentiality of all data and records associated with your participation in this research. Data will be kept in a locked file cabinet in my office; only my faculty advisor, Tim Fukawa-Connelly, and I will have access to the data. You should understand, however, that I am required by law to report certain information to government and/or law enforcement officials (e.g. child abuse, threatened violence against self or others, communicable diseases).

I will report the data as an overall analysis of students' thought processes as a group; your name will not be mentioned in any reports. The results will be used in reports and presentations. All data, whether electronic or in paper form, will be destroyed at the end of the study.

If you have any questions about this research project or would like more information before, during, or after the study you may contact me on my cell at 603-490-1705 or by email at ary77@unh.edu. If you have any questions about your rights as a research subject, you may contact Dr. Julie Simpson in UNH Research Integrity Services at 603-862-2003 or Julie.simpson@unh.edu to discuss them.

I have enclosed two copies of this letter. Please sign one indicating your choice and return in the enclosed envelope. The other copy is for your records. Thank you for your consideration.

Sincerely,

Amber R. Lagasse
Undergraduate Student

Yes, I, ___________________________ consent to participate in this research project.

No, I, ___________________________ do not consent to participate in this research project.

_________________________________ _________________________

Signature of Student Date

Appendix B – Questionnaire

Student ID #:

Are you: ☐ Male ☐ Female *(check one only)*

How old are you? ________

What's your major? ____

Does your major require math? ☐ Yes ☐ No

Do you have a minor? ☐ Yes ☐ No

If so, what is your minor? ____

Does your minor involve math? ☐ Yes ☐ No

What is the highest level of math you have completed? *(please note if taken in high school or college)*

____

Do you like math? ☐ Yes ☐ No

What is the lowest grade you have ever received in a math class? *(include high school and college)*

____

What grade do you typically receive in a math class? *(include high school and college)*

____

Appendix C – Interview Questions

1) What approaches do you know for solving systems of linear equations?

2) What approach are you taking?

3) Why are you taking this approach?

4) Do you think this is the easiest approach?

5) What other approach might you use for this problem?

For Questionnaire:

Please mark down that your major does not involve math if you are not required to take more than one math course for your major (excluding general education/discovery courses)

If Stuck:

You said you know ________ approaches for solving systems of linear equations. Is there one that would work here? Is there anything you can do to the equations so you can use ________ method of solving?

What is the goal here? [to solve for x and y] Do we solve for one variable at a time, or do we solve for both at the same time? [one variable at a time] How can we solve for one variable at a time? [by getting rid of the other variable] How can we get rid of one of the variables?

Appendix D – Systems of Linear Equations Problems

P1)

$2x + 3y = 2$
$4x + y = 8$

a) $x = 9$, $y = -4/5$
b) $x = 9/5$, $y = -4/5$
c) $x = 11/5$, $y = -4/5$
d) $x = 11$, $y = -4/5$

*P2)

$2x = 2y + 8$
$3x + 3y = 9$

a) $x = 9/2$, $y = -1/2$
b) $x = 7/2$, $y = 1/2$
c) $x = 7/2$, $y = -1/2$
d) $x = 7/2$, $y = -15/2$

P3)

$x + y = 1$
$0 = y - x + 3$

a) $x = 2$, $y = -1$
b) $x = -1$, $y = -7$
c) $x = 3$, $y = 0$
d) $x = 5$, $y = 2$

P4)

$-3x + 4y = 9$
$y = 3x + 1$

a) $x = 5/9$, $y = 8/3$
b) $x = 8/9$, $y = 11/3$
c) $x = 13/9$, $y = 16/3$
d) $x = 10/9$, $y = 13/3$

P5)

$2x + 3y = -4$
$y = 4x + 8$

a) $x = 10$, $y = 5$
b) $x = 3$, $y = 4$
c) $x = -2$, $y = 0$
d) $x = -6$, $y = 7$

P6)

$2x + 4y = 12$
$2x = 7 – 9y$

a) $x = 4$, $y = -1$
b) $x = -20$, $y = 19/3$
c) $x = -72/13$, $y = 5/13$
d) $x = 8$, $y = -1$

P7)

$2x + 4y = 6$
$3x = 2 – 5y$

a) $x = -11$, $y = 7$
b) $x = 10$, $y = -5$
c) $x = 12$, $y = 4$
d) $x = 4$, $y = 5$

P8)

$3x + 5y = 7$
$x = 9 – 5y$

a) $x = 1$, $y = 8$
b) $x = -1$, $y = 2$
c) $x = 2$, $y = 9$
d) $x = -5$, $y = 13$

P9)

$5x + 2y = 19$
$3x + 3y = 6$

a) $x = 5$, $y = 22$
b) $x = 5$, $y = -3$
c) $x = 5$, $y = 3$
d) $x = 5$, $y = 7$

* P2 was the problem omitted for students in non-math majors