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Day 13 Mar 05 Thermal equilibrium

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### 13.0.C Hands-on Water Temperature Mixing

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## When you mix things that have different temperatures, what is the temperature of the mixture?

Materials: thermometers, digital or liquid (use same type)  
clear plastic cups marked at 40, 60, 80 mL  
reservoirs of “cold” and “hot” water

Instructions: Manager – read the top line of each procedure step to the group.  
Recorder – record information in recorder report for steps 2 and 4

1. Three experiments, all with the same procedure. Only the volumes change. Do one experiment at a time. Sequence doesn't matter. Use the water reservoirs.  
First experiment: 40 mL of hot and 80 mL of cold  
Second: 60 mL of hot and 60 mL of cold  
Third: 80 mL of hot and 40 mL of cold
2. **BEFORE YOU DO ANYTHING, MAKE PREDICTIONS.** Write them down.  
Assume that the **HOT** water has a temperature of 40 degrees C.  
Assume that the **COLD** water has a temperature of 10 degrees C.  
Predict a value for the temperature **AFTER** mixing for each set of volumes.  
Report your predictions and reasons to the instructor.
3. Now, do the experiments. Do one at a time. Move quickly.  
Think about best point in time to record temperatures.  
Measure out the volumes.  
Measure and record the temperature of the hot and cold water.  
[The actual temperatures won't be 10 °C or 40 °C].
4. Pour one container into the other. (Doesn't matter which.)  
Measure the temperature right away.  
Record it.

Displaying results: Manager – ask someone to read out loud, quickly steps 1-3

1. Get a piece of poster graph paper. Draw a vertical temperature line for your range of temperatures.
2. For each experiment:  
Place a **RED-ish X** on the line at the temperature of your **HOT** water.  
Place a **BLUE-ish X** on the line at the temperature of your **COLD** water.  
Place a **BLACK X** on the line at the temperature of the mixture.
3. Draw a red-ish arrow from the **RED X** to the **BLACK X**.  
Draw a blue-ish arrow from the **BLUE X** to the **BLACK X**.  
The arrows show what the temperatures have changed from and to.
4. Stick your poster up on the wall near you. Names on it.

Making sense of the results (all goes into Recorder Report):

1. Compare your predictions with the results. Are they consistent? That suggests something about your intuitive sense of the process.
2. In a single concise sentence, express how the starting volumes affect the location of the final temperature relative to the two initial temperatures.
3. Someone asks: “Why isn’t the temperature of the mixture always half-way between the two starting temperatures? That’s what you get when you average things, right?” What is your response to that?
4. What happened to the hot water? What happened to the cold water?
5. If you haven’t yet thought about what’s happening at the molecular level, now is the time. Develop an explanation for the observations that is based on molecules.
6. How is heat related to all of this?
7. Check in with your instructor at this point to share your thinking on these questions.

Your instructor will give you a challenge to consider at this point.

When the group is ready, ask:

*In the mixture, are all the water molecules now at the same temperature, or are they still at different temperatures but the thermometer senses the average?*

*Create a possible procedure by which you could explore this, and share it with me.*

-----

They have to develop the idea that they have to keep the water from mixing (in order to monitor temp) but still let heat move. Make them work for it. Let their thinking evolve.

Once they seem to be on this conceptual path, present them with equipment:

an aluminum can inside a coffee mug

Tell them to develop a specific procedure, and then clear it with instructor

Challenge their intended data acquisition

ask how they intend to present their results, and is that fully convincing concerning the original question.

Push them to “plot T vs time for both containers”

Once they latch on to that, give them the go ahead to collect data, and then get it onto graph and onto the wall. This will spur other groups.

They can use the same volumes (choices on board; make each group different) (These will fit in the containers and give good temp measurements).

*Make a prediction as you did last time:*

*Will it be the same as with direct mixing?*

*What will the end result be?*

*What do you expect to see happen?*

If they mess up, it's easy to restart the experiment. No experiment lasts more than 5-10 minutes.

Plot on large graph paper. Seeing things go up on the wall will spur others.

Once their graph is up, ask each person in the group to describe what is happening.

Then, say:

*We need to call this something.*

*Let's call it “coming to thermal equilibrium”*

Continue questioning:

*Why don't you get to thermal equilibrium right away?*

Try to elicit the idea about “heat moving”

*What do you mean by “moving”? What is moving? How can you tell?*

*What could you investigate that might tell you about this “moving”?*

*What determined the speed of movement? What determines the direction?*

Speed is a key word to hold in reserve because it hints that one might think about what it takes to slow or to speed up the movement of heat.

Direction is also key – because no distinction to this point has been made as to whether it is “cold” that is moving or “heat” that is moving.

RECORDER REPORT, Chem 444A "Fire & Ice"

Group Member Name

Role

Date: 3/5/15

Taylor Witkiewicz Recorder

Charles Cappetta Encourager

Marisa Butler Manager

Emily Koester Spokesperson

2. We predict the temperature will be the average of the two temperatures <sup>(25°C)</sup> in the second experiment where the amounts are equal. In the first experiment, we predict the value would be about 20°C. In the last the temperature would be warmer due to a greater amount of hot water and be about 30°C. The reason for these predictions is that a higher volume of one temperature will overpower the other temperature.

4. 1<sup>st</sup> experiment: 18.5°C beg. hot = 42.9°C beg. cold = 6.1°C

2<sup>nd</sup> experiment: 25.7°C beg. hot = 44.8°C beg. cold = 6.4°C

3<sup>rd</sup> experiment: 32.9°C beg. hot = 44.9°C beg. cold = 10.5°C

1. Our predictions were pretty consistent with the actual results for each experiment.

2. The temperature of the water with the greater volume will dictate the final temperature of the mixed temperatures water; more cold water will result in a colder final temperature and vice versa.

3. You can not just take the average when the volumes are different. We disagree with this person that the mixture will always be halfway in between the two temperatures. This is only appropriate when the two volumes are equal.

1. The hot water is losing heat to the cold water. The cold water is gaining heat from the hot water.

2. On a molecular level, the hot water molecules are moving at a greater rate than the cold water molecules. When we combine the water, the molecules will interact and collide with each other. The hot molecules will not be moving as fast as they were before colliding with the cold molecules. The cold water molecules will be moving faster than they were before the collision with the hot water molecules. This eventually leads to a stable temperature of the combined waters.

2. Heat is related to all of this because there is a transfer of heat from the hot to the cold water molecules.

challenge:

NE believe that the molecules still have different temperatures, but the thermometer is reading the average temperature.

Will hot water on inside or cold water on inside make a difference?

procedure = put cold water in can, hot water in mug, thermometer for each cold and hot water, measure temp. of each cold and hot water before and when combined

Prediction = cold will become warmer, hot will become colder, not as drastic as direct mixture exp.

xp. 1: 80 mL of hot 80 mL of cold - the temperatures eventually came very close but it took a longer amount of time than when the water was directly mixed. (about 8 mins)  
initial cold = 7.0°C    cold when mixed = 21.5°C  
initial hot = 41.3°C    hot when mixed = 22.7°C

procedure: cold in mug, hot in can, take same temperature measurements before and combined

xp. 2: 80 mL hot 80 mL cold - about 6 mins  
initial cold = 7.2°C    cold when mixed = 19.2°C

initial hot = 43.3°C    hot when mixed = 30.5°C

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Group Member Name	Role	Date: <u>3/5/15</u>
<u>Nick</u>	<u>Recorder</u>	
<u>Eliza</u>	<u>Manager</u>	
<u>Amanda</u>	<u>Encourager</u>	
<u>Calé</u>	<u>Spokesperson</u>	

2. Predictions

First Experiment: we predict that temperature of the mixture will be 12.5°C. We predict this since the cold volume is twice as large as the warm volume of water.

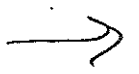
Second Experiment: We predict that the temperature would be 25°C.

Third Experiment: we predict that the temperature will be 37.5°C.

- Our thinking was that the temperature of the mixture will be relative to the volumes of the water. Therefore if there is a greater volume of warm water the temperature will be warmer.

1) <sup>40mL</sup> Hot	<sup>80mL</sup> Cold	Mix
41.5°C	6.1°C	19°C
2) <sup>40mL</sup> Hot	<sup>40mL</sup> Cold	
42.0°C	7.2°C	23.8°C
3) <sup>40mL</sup> Hot	<sup>20mL</sup> Cold	
43.0°C	7.5°C	31.7°C

we recorded the individual temperatures immediately before the mixture to avoid inaccuracies or variations in actual temperature.



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) Our predictions were extremely close to our results for each of the experiments. Each of the experiments is consistent.

) The mixture temperature will "lean" towards the initial temperature of the greater starting volume.

) The volume of the liquids has to be taken into account along with the starting temperatures. For example, the greater volume will have a "larger" impact on the mixture temperature.

) The hot water lost heat energy and the cold water gained heat energy. The causes an average or a change in temperature.

) When the two volumes are mixed there is a transfer of energy between the "hot" and "cold" molecules. This can only happen from the hot molecules to the cold molecules. The amount of energy transfer depends on the relative volume. The cold molecules will pull energy from the hot molecules increasing the kinetic energy and speed of the cold molecules and decreasing the kinetic energy and speed of the warm molecules.



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<u>Nick</u>	<u>Recorder</u>	
<u>Eliza</u>	<u>Manager</u>	
<u>Cale</u>	<u>Spokesperson</u>	
<u>Amanda</u>	<u>Encourager</u>	

5.) (cont'd) The average kinetic energy will result from the transfer of energy between hot and cold.

6.) Heat is related since that is the energy which is being transferred. Heat is the energy that is going from hot molecules to cold molecules forming the average kinetic energy.

Challenge

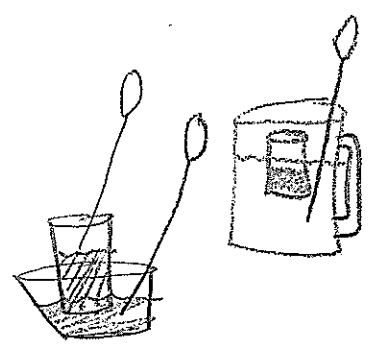
We hypothesize that there is in fact an energy transfer between hot and cold and that the hot don't stay hot and the cold don't stay cold.

Experiment  
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To test that there is an energy transfer we

are placing 80 mL of cold water in the can and 80 mL of hot water in the can.

Cold	Hot	Mix
7.5°C	41.0°C	± 25°C
6.8°C	42.4°C	



This experiment shows that energy is being transferred. If energy wasn't being transferred the separate thermometers would be showing the same initial temperature readings. The hot water decreases and the cold water increases until the temperatures "meet" creating the average. Therefore energy is transferred.

Time	Hot (°C)	Cold (°C)
1	38.5	11.3
2	34.3	12.1
3	31.8	13.8
4	30	14.8
5	29.1	15.2
6	27	15.8
7	26.2	16.9
8	26.7	17.8
9	26.4	18.5
10	24.3	19
11	24.0	19.2
12	24.8	19.8
13	24.9	19.9
14	23.5	20.3
15	24.9	20.9
16	26.9	20.6
17	24.6	20.4

Time (s)	Hot (°C)	Cold (°C)
430	24.4	20.7
445	23.4	20.7
5	23.1	20.9
515	22.9	20.9
530	24.8	21.1
545	25.8	21.5
6	25.5	21.6
615	24.6	21.6
630	23.5	21.7
645	22.9	22
7	23.3	22.2
715	22.6	22.7
730	23.8	22.3
745	22.1	22.3
8	21.5	22.4

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Group Member Name

Role

Date: 3/5/15

Kyle Beisert

Recorder

Samira Colan

Spokesperson

Miriam Arsenault

Encourager

Becky Pietric

Manager

## Predictions

- Experiment 1: Roughly  $20^{\circ}\text{C}$  - more towards the cold end because there is twice as much cold as hot water
- 2: Roughly  $25^{\circ}\text{C}$  - equal amounts of cold and hot would meet in the middle
- 3: Roughly  $30^{\circ}\text{C}$  - more towards the hot end because there is twice as much hot as cold water

Experiment 1: Hot initial:  $43.9^{\circ}\text{C}$  Cold initial:  $7.1^{\circ}\text{C}$

$$40\text{ mL Hot} + 80\text{ mL Cold} = 19.5^{\circ}\text{C}$$

Exp. 2: Hot int'l:  $45.0^{\circ}\text{C}$  Cold int'l:  $6.6^{\circ}\text{C}$

$$60\text{ mL Hot} + 60\text{ mL Cold} = 25.3^{\circ}\text{C}$$

Exp 3: Hot int'l:  $45.2^{\circ}\text{C}$  Cold int'l:  $6.5^{\circ}\text{C}$

$$80\text{ mL Hot} + 40\text{ mL Cold} = 32.4^{\circ}\text{C}$$

Very similar to predictions.

## Making sense of results

- 1) Our predictions are very close to the experimental values. This indicates our thought process was probably correct.
- 2) ~~The ratio of the initial volumes is the same as the volume of~~  
The water with the greater volume will have the greater effect on the final temperature.
- 3) Temperature is the average kinetic energy of all the particles in a system, so the particles moving at a given speed (i.e. the more volume) the average is towards that temperature.

4) The hot water particles transferred some of their energy to the cold water particles, therefore losing some energy while the cold particles gained some energy. This resulted in the whole system settling at the ~~average~~ energy that it did.

5) See above

6) Heat is a form of kinetic energy that is the result of particular motion. So, you can think about heat being distributed/dissipated throughout the whole system in a molecular sense, because the particles moving very fast collide with particles moving not so fast, causing them to accelerate.

Challenge - set up: ~~two~~ hot water cup placed in cold water. mg (80 ml each)

Int'l hot: 45.5 Int'l cold: 7.9°C

Time	Hot Temp	Cold Temp
50	43.8	9.4
00	35.5	10.1
30	37.2	13.2
00	32.5	13.3
30	34.0	13.1
00	34.1	15.2
30	32.8	15.6
00	32.2	17.1
30	30.4	16.4
00	30.4	19.4
30	30.8	18.9
00	29.7	19.3
30	29.2	19.3
00	28.6	19.7
30	28.2	20.1
00	27.8	20.4
30	27.4	20.6
00	27.1	20.4
30	27.1	21.1
00	26.8	21.1
30	26.6	21.3
00	26.4	21.5

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Group Member Name

Role

Date: 3/5/15

Tim Closson

Reflector

Emma Addison

Manager

Kaleigh Zakowski

Spokesperson

Mandy Graves

Encourager

## Experiment 1

Prediction: For the first experiment we predict the mixture will be  $20^{\circ}\text{C}$ .

## Experiment 2

Prediction: For experiment 2 we predict ~~20~~  $25^{\circ}\text{C}$

## Experiment 3

Prediction: For experiment 3 we predict  $30^{\circ}\text{C}$

## Experiment 1

Hot water:  $43.3^{\circ}\text{C}$   
Cold water:  $6.1^{\circ}\text{C}$   
Mixture:  $18.4^{\circ}\text{C}$

## Experiment 2

Hot water:  $45.5^{\circ}\text{C}$   
Cold water:  $6.2^{\circ}\text{C}$   
Mixture:  $24.9^{\circ}\text{C}$

## Experiment 3

Hot water:  $45.7^{\circ}\text{C}$   
Cold water:  $6.5^{\circ}\text{C}$   
Mixture:  $31.3^{\circ}\text{C}$

Our predictions were very consistent with our results. For each experiment we were only off by a few decimal points. It could have been more accurate if the starting temps. were correct.

The greater the volume, the greater the effect it has on the resulting temperature.

We respond by saying that the temperature is also dependent on the volume of the container for the two different temp. liquids. You have to take that into account when you are predicting the temp. of the final mixture.

The hot water transfers energy to the cold water, and the cold water receives kinetic energy/heat from the hot water.

When hot water <sup>pours</sup> into the cold water, the hot water molecules begin colliding with the cold water molecules. The faster moving hot water molecules will begin to collide with the cold water molecules and the cold water molecules will gain kinetic energy while hot water molecules lose energy. Depending on how many hot water molecules there are with a certain amount of energy, more energy will be transferred to the cold.

If you start out with more heat than cold water, then you mix the two temperature waters there will be more heat to "share" and the resulting mixture will be hotter. If there is more cold water than hot water, then there will be less heat to distribute among the mixture and the resulting mixture will be colder.

ew Experiments

Experiment 1 Prediction  
 hot water: 20°C  
 cold water: 20°C

Experiment 2 Prediction  
 Hot water: 25°C  
 Cold water: 25°C

Experiment 3 Prediction  
 Hot water: 30°C  
 cold water: 30°C

essentially we expect the same results

ata

Experiment 1

Time	Temp Cold	Temp Hot
0:00	6.8°C	40.1
0:10	8	36.2
0:20	9.5	34.9
0:30	11.4	32.9
0:40	13.1	31.4
0:50	10.8	30.6
1:00	12.4	29
1:10	13.5	27.9
	14.6	27.3
	15.2	26.5
	15	25.5
	15.4	24.2
	15	24.5
	14.7	24.1
	15	24
	15.3	24.2
	15.5	23.1
	15.7	23.2
	16.1	23
	16.7	23
	16.9	23
	16.9	22
	16.9	22
	17.2	22
	17.6	21.9
	17.3	21.8
	17.3	21.2
	17.2	21

Temp Cold	Temp Hot
17.4	20.8
17.5	21
17.5	20.9
17.6	20.6
17.7	20.7
17.8	20.5
17.8	20.5
17.8	20.4
17.8	20.5
17.9	20.3
17.9	20.1
18	20.2
18	20.1
18.1	20.1
18.1	19.9
18.1	19.8
18.2	19.7
18.2	19.7
18.2	19.7
18.2	19.8
18.3	19.8
18.3	19.4
18.4	19.6
18.5	19.5
18.5	19.6
18.6	19.5

30°C

Cold RT  
24.6

Hot RT  
21.6

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Group Member Name

Role

Date: 3/5/14

<u>Heather Price</u>	<u>Manager</u>
<u>Jon Temposi</u>	<u>Encourager</u>
<u>Jacob Sidney</u>	<u>recorder</u>
<u>Emily Dwyer</u>	<u>Spokesperson</u>
<u>Sean King</u>	

Experiment 1: 40 ml of hot ( $43^{\circ}\text{C}$ ) and 80 ml cold ( $6^{\circ}\text{C}$ )

Prediction: Since there is more cold than hot. Final temp will be about  $18.3^{\circ}\text{C}$

Actual:  $18.4^{\circ}\text{C}$

2: 60 ml of hot ( $44^{\circ}\text{C}$ ) and 60 ml of cold ( $7^{\circ}\text{C}$ )

Prediction: Since there is an equal amount of each, final temp will be about  $25.5^{\circ}\text{C}$ .

Actual: 25.5

3: 80 ml of hot ( $44^{\circ}\text{C}$ ) and 40 ml of cold ( $7^{\circ}\text{C}$ )

Prediction: Since there is more hot than cold, final temp will be about  $31.6^{\circ}\text{C}$

Actual: 32.4

## Questions:

1. our predictions are very consistent with the results. That suggests that we used the right equation to predict them.
2. The final temperature is closer to the initial substance with greater volume.
3. You can only average if each substance makes up an equal proportion of the total volume.
4. The hot water cooled. The cold water warmed up.

5. Temperature is an average of the kinetic energy in the system. If there are more individual molecules with a higher kinetic energy than those with low, it will skew the average to be higher, as was the case in experiment 3.

6. Heat seems to be a source of energy (thermal) for the water molecules, warmer water having more than colder water. When mixing different temps, the final temp comes from the average amount of heat in the total system.

7. Challenge:

Cold in thermos (8.8 °C) 60 mL

Warm in can (40.0 °C) 60 mL

Can in thermos

Thermos                  Can

<sup>2</sup>  
Data on other page →

25.6

17.5

17.3

13.1

10.2

10



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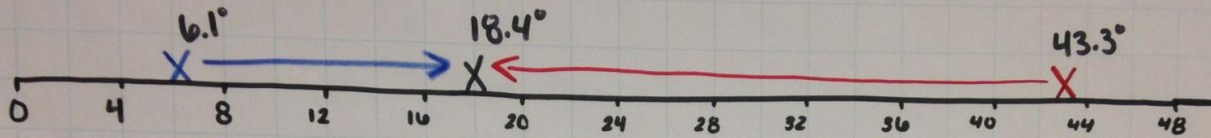

## 7. Challenge :

60 mL of 8.8°C water in thermos

60 mL of 40.0°C water in can

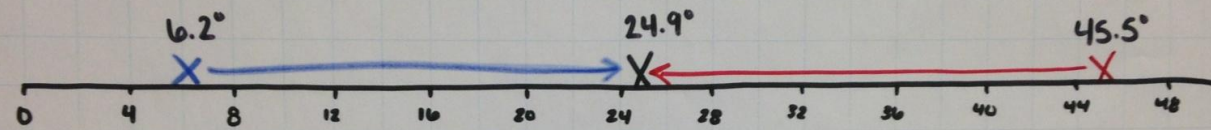
can in thermos at start

Time	Temperature °C		Time	Temperature	
	Thermos	Can		Thermos	Can
0.25	10.1	25.2	4.25	18.8	20.5
0.5	10.3	22.2	4.5	19.1	20.6
0.75	10.8	20.6	4.75	19.3	20.8
1	11.5	19.8	5	19.5	20.9
1.25	12.5	19.5	5.25	19.7	21.0
1.5	13.5	19.3	5.5	19.9	21.1
1.75	13.7	19.3	5.75	20.1	21.3
2	14.7	19.3	6	20.5	21.7
2.25	15.4	19.3	6.25	20.6	21.8
2.5	16.0	19.4	6.5	21.5	21.8
2.75	16.8	19.5	6.75	21.6	21.9
3	17.3	19.7			
3.25	17.7	19.9			
3.5	18.0	19.9			
3.75	18.3	20.1			
4	18.6	20.3			



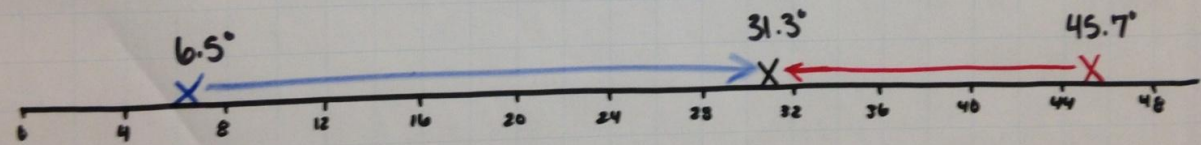
Ex. 1 40mL hot  
80mL cold

temperature (C°)



Ex. 2 60mL hot  
60 mL cold

temperature (C°)



Ex. 3 80mL hot  
40 mL cold

temperature (C°)

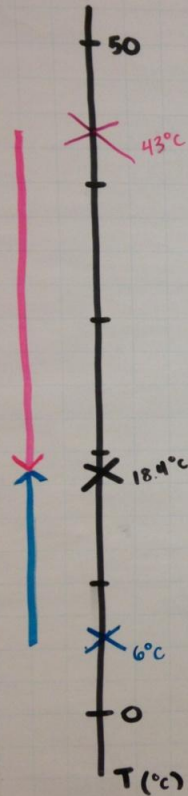
# Mixing Temperatures

Tom Lehrer  
Frank Miller  
George S. Kaufman  
Nancy Johnson

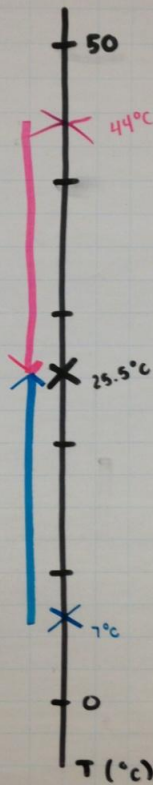
# MIXING TEMPERATURES

Jon, Heather, Jake, Emily, Sean

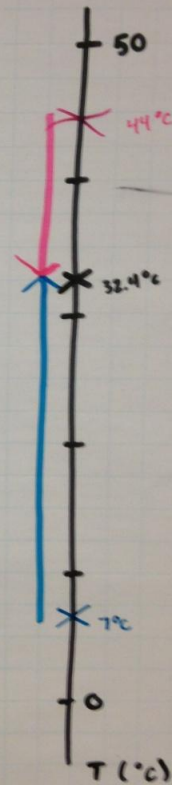
Expt 1:



Expt 2:



Expt 3:

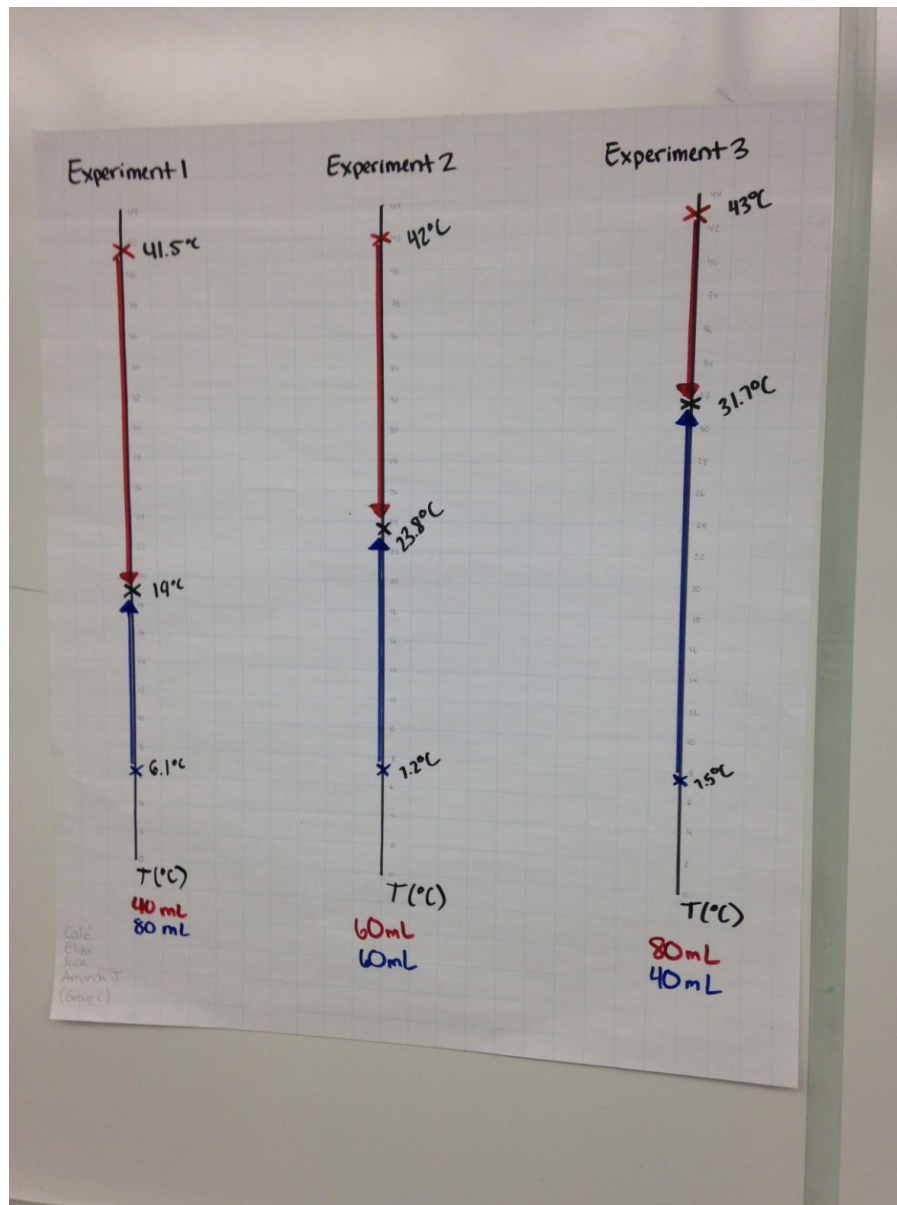


Christopher F. Bauer, Principal Investigator.

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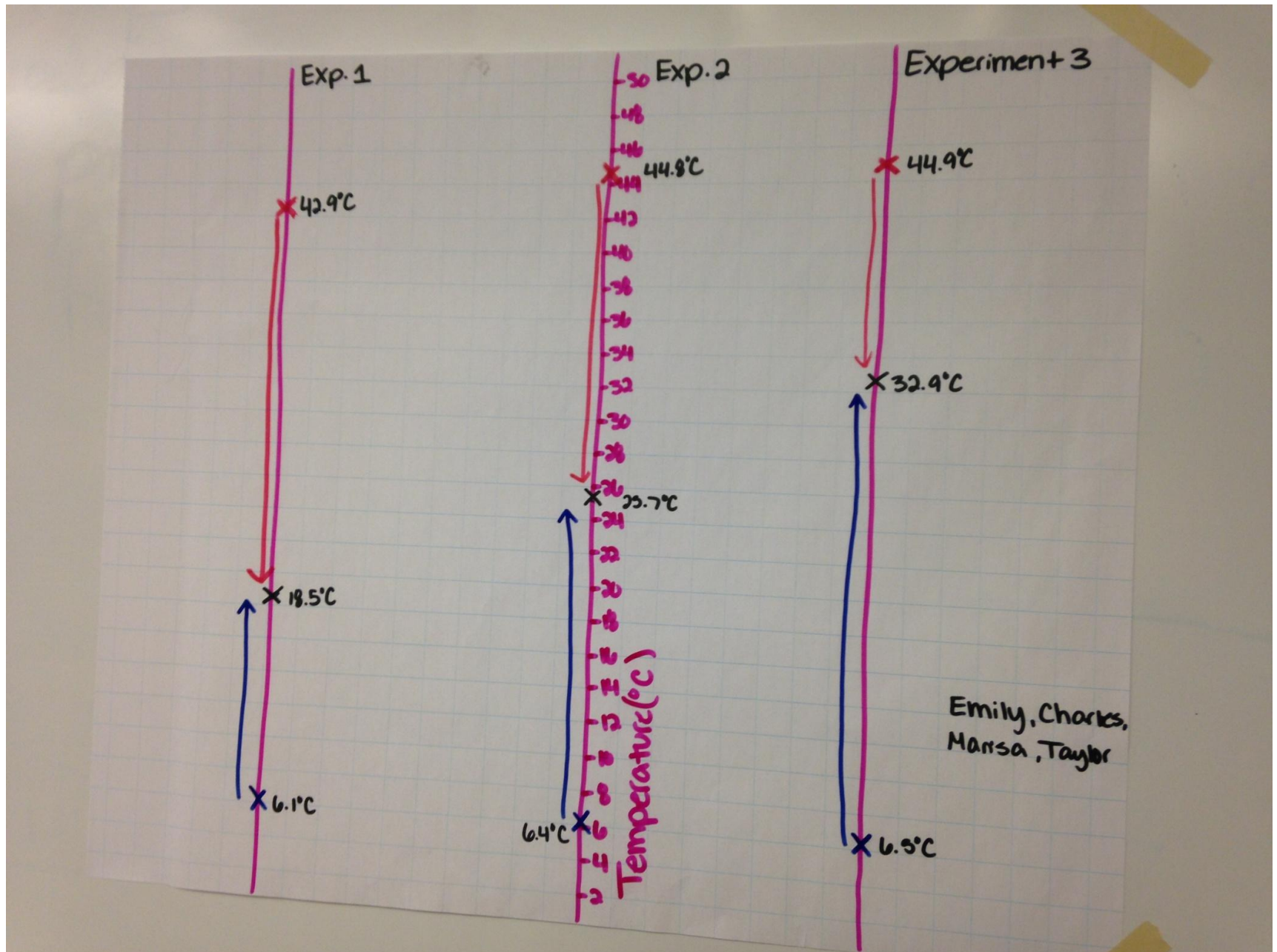


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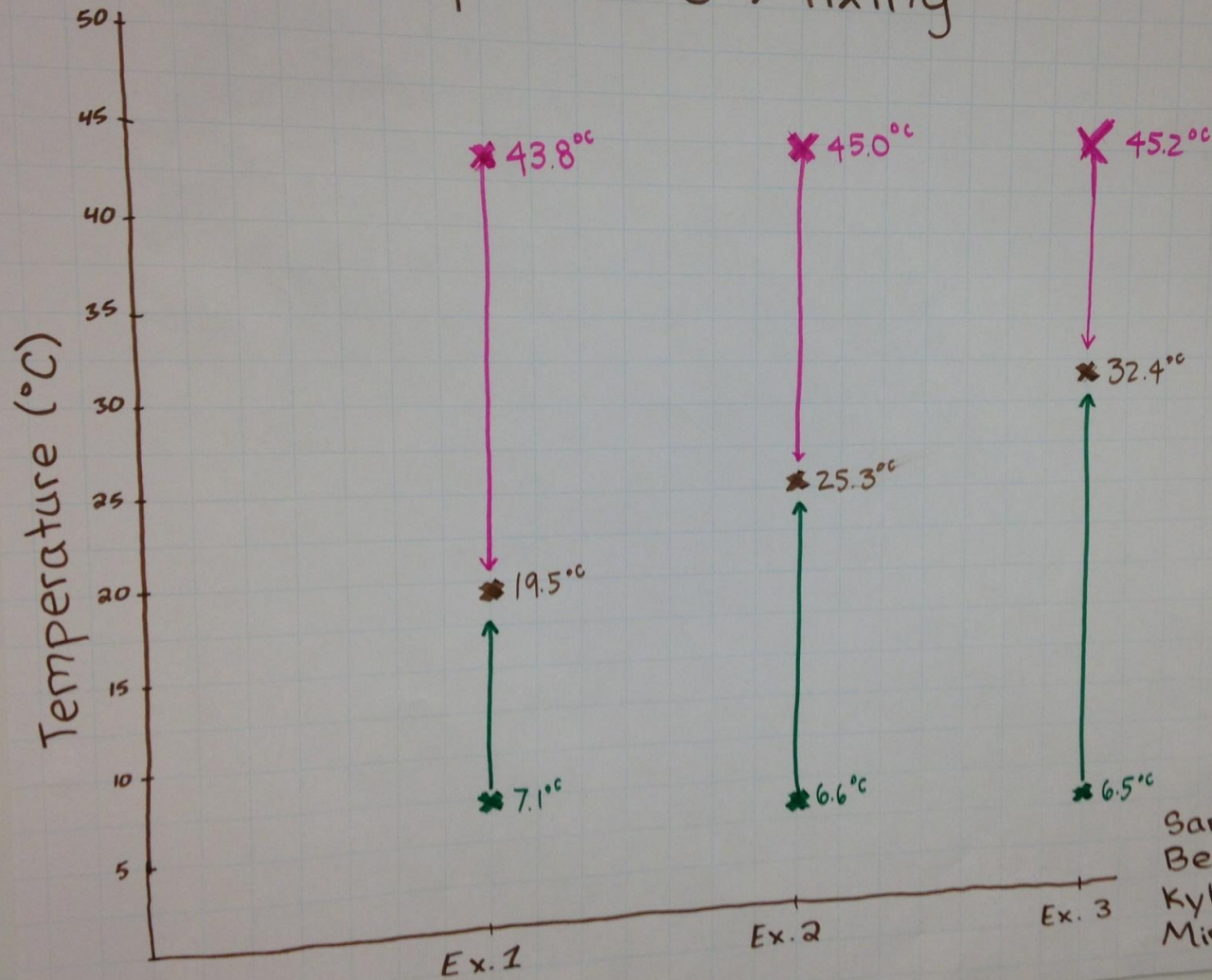
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Emily, Charles,  
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# Temperature Mixing



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