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### (2020) Technical Supplement for the article "A Meta-Analysis of the Correlations Among Broad Intelligences: Understanding their Relations"

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#### Recommended Citation

Bryan, Victoria M. and Mayer, John D., "(2020) Technical Supplement for the article "A Meta-Analysis of the Correlations Among Broad Intelligences: Understanding their Relations"" (2020). *UNH Personality Lab*. 35.

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## BROAD INTELLIGENCES TECHNICAL SUPPLEMENT

### A Meta-Analysis of the Correlations Among Broad Intelligences: Understanding their Relations

*Note:* This technical supplement was developed by the author along with their report, “A meta analysis of the correlations among broad intelligences: Understanding their relations” as part of a single, ongoing research project. The original report provides the general purpose and theoretical overview of the project, as well as the key analyses. This supplement also includes pieces of that material where relevant but focuses on detailing the programming and data analyses of the project to a far greater extent.

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## **Part 1. Article Selection and Handling of Data**

### Distinguishing Between Types of Correlations

In our review of the literature, we made note of studies that reported factor correlations obtained using simple, Pearson-product moment correlations and those that reported latent factor correlations obtained from structural equation models (e.g., confirmatory factor analysis). Because each statistical technique comes with its own set of assumptions and resulting implications for how they are handled in meta-analyses, we chose to separate the correlations we recorded from studies that used each of technique. We focused our analyses on articles that reported correlations using factor estimates because the two kinds of reports, Pearson correlations versus factor estimates are not readily comparable. Foremost, Pearson product-moment correlations are uncorrected for reliability whereas CFA estimates are adjusted for unreliability through the inclusion of an error term for each indicator. Second, the distribution of product-moment correlations, and hence their standard errors, is known, whereas our knowledge of standard errors for CFA estimates is “still very limited” (Yuan, Cheng, & Zhang, 2010, p. 633). Third, the constructed scales employed to obtain Pearson correlations often unit-weight items and omit weaker items whereas factor estimates are weighted composites. That is, SEM models typically employ all items and weight them in terms of their factor loadings. In this report, we focus on CFA estimates because they are more widely used in the literature related to the broad intelligences.

### Method for Handling Composite Factors

When reviewing the articles collected for our meta-analysis, instances arose where the models from which our correlations were obtained included composite factors comprised of two or more broad intelligences. For example, both the Wechsler Intelligence Scales for Children (WISC) and Wechsler Adult Intelligence Scales (WAIS) include the composite factor perceptual reasoning (PRI) or perceptual organization (POI). This composite factor is represented by narrow abilities, which assess the broad factors of fluid intelligence (Gf) and visuospatial processing (Gv). From our final sample of articles, we identified 8 works that included a composite factor such as PRI/POI (see Table 1 below). Therefore, a strategy was developed in order to determine how the correlations reported using this composite factor should be treated – that is, whether the factor should be treated as fluid intelligence or visuospatial processing.

The first author read through each of the 8 works and made note of the tasks employed to assess the PRI/POI composite factor, and their respective factor loadings. Specifically, we were interested in seeing what types of tasks loaded on to the composite factor, their standardized factor estimates and whether these tasks might provide clues as to how we could reassign the composite factors to be included in our analysis. For example, if multiple tasks aimed at assessing fluid intelligence (Gf) load on the composite factor, and their factor loadings are higher than the loadings of tasks assessing visuospatial processing (Gv), it would be reasonable to assume that the factor predominantly assessed mental capabilities pertaining to fluid intelligence. Moreover, the number of tasks included in each model that assessed both Gf and Gv were also recorded to determine which broad ability the PRI/POI factor most closely aligned with.

Information regarding which broad intelligence each task best represented was gleaned from articles published on the WAIS and WISC. Dombrowski, Cavinez, & Watkins (2016) and

Cavinez, Watkins, & Dombrowski (2017) presented in their work an adapted version of the higher order factor model of the WISC-V found in Wechsler (2014b) that included information on the types of tasks that assess different broad abilities. Additional information regarding the corresponding CHC broad factor for each task was obtained from articles by Scheiber (2016) and Benson, Hulac, & Kranzler (2010). From our review of these articles, block design, picture completion, and visual puzzles were noted as assessing Gv and matrix reasoning, picture concepts, arithmetic and figure weights were used to assess Gf.

Using the nature of the tasks as a guide, we looked at the individual task loadings as well as the number of tasks included for each broad intelligence and recoded the composite PRI/POI factor as assessing either fluid intelligence (Gf) or visuospatial processing (Gv). For example, Cockshott et al. (2006) included two tasks that assessed Gv, picture completion and block design, that loaded on to the composite PRI/POI factor at .67 and .89, respectively, and one task that assessed Gf (picture arrangement) that loaded at .56. Because more tasks in this article assessed Gv than Gf and these tasks loaded more highly, the composite factor was recoded as Gv. The reassignments of the other 8 articles that included the composite PRI/POI factor can be found below in Table 1 of this technical supplement.

Table 1.

## Publications Included the WAIS or WISC Scales and Composite Factor Loadings

Publication	N	Task	Represented Broad Intelligence	Loadings on PRI/POI Factor	Assigned Broad Intelligence
Cockshott et al. (2006)	579	Picture completion	Gv	.67	
		Picture arrangements <sup>b</sup>	Gf	.56	
		Block design	Gv	.89	Gv
		Object assembly <sup>b</sup>	Gv	.72	
Bergeron & Floyd (2008) <sup>a</sup>	56	--	--	--	--
Cavinez (2014)	345	Block design	Gv	.81	
		Picture concepts	Gf	.65	Gf
		Matrix reasoning	Gf	.83	
Cavinez et al. (2016)	2200	Block design	Gv	.74	
		Visual puzzles	Gv	.82	
		Matrix reasoning	Gf	.44	Gv
		Figure weights	Gf	.50	
Dos Santos et al. (2018)	150	Picture completion	Gv	.70	
		Picture concepts	Gf	.63	
		Matrix reasoning	Gf	.78	Gf
		Block design	Gv	.74	
Waller & Waldman (1990)	1880	Picture completion	Gv	.74	
		Picture arrangement	Gv	.66	
		Object assembly	Gv	.69	Gv
		Block design	Gv	.78	
Cavinez et al. (2019)	415	Block design	Gv	.70	
		Matrix reasoning	Gf	.53	
		Figure weights	Gf	.44	Gv
		Picture concepts	Gf	.35	
		Visual puzzles	Gv	.85	
Lecerf & Cavinez (2018)	1049	Block design	Gv	.73	
		Visual puzzles	Gv	.92	
		Matrix reasoning	Gf	.59	Gv
		Figure weights	Gf	.46	

<sup>a</sup> Did not include individual task loadings on to perceptual reasoning/organization index. Correlations with PRI/POI factor omitted from analyses. All other correlations were retained.

<sup>b</sup> Object assembly and picture arrangement are subtests included in the WAIS-III and WISC-III editions and dropped more recent editions (i.e. WAIS and WISC IV). Object assembly is highly similar to visual puzzles (Gibbons & Warne, 2019) and was coded as assessing Gv. Picture arrangements assesses individuals reasoning abilities and was coded as assessing Gf (Krapfer & Soto, 2013).

### Comprehensive List of Potentially Relevant Works Prior to Sample Screening

Table 2 below depicts a comprehensive list of the potentially relevant works returned from our series of literature searches, prior to our screening for the type of sample used (i.e. whether more than one study used the same standardization sample). It should be noted that not all the studies included in this table were used for the analyses in the manuscript. That is, it includes studies that were screened out because they employed the same sample as another, more representative article, alongside the works included in the analyses for the main article. The final list of included works in the published meta-analysis can be found in table 1 of the article.



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Table 2.

Comprehensive List of Potentially Relevant Works Prior to Screening for Sample Used<sup>a</sup>

<b>Intelligence Test and Published Works</b>	<b>N</b>	<b>Age (in years)</b>	<b>Sample</b>	<b>Cross- Battery?</b>	<b>Test(s) Included</b>
<b>Woodcock-Johnson-R</b>					
Flanagan & McGrew (1998)	114	10 to 15	School sample	Yes	KAIT, WISC-III
Burns & Nettlbeck (2003)	90	18 to 40	Community sample	Yes	WAIS-R
Flanagan (2000)	166	9 to 13	Validity/standardization sample	Yes	WISC-R
Bickley, Keith, & Wolfe (1995)	2201	6 to 80	Standardization sample	No	
<b>Woodcock-Johnson III</b>					
Strickland, Watkins, & Caterino (2015)	529	6 to 13	School sample	No	
Floyd, McGrew, Barry, Rafael, & Rogers (2009)	3577	4 to 60+	Standardization sample	No	
Keith, Kranzler, & Flanagan (2001)	155	8 to 11	School sample	Yes	CAS
Keith, Reynolds, Patel, & Ridley (2008)	6970	6 to 59	Standardization sample	No	
Floyd, Gregg, & Keith (2012)	6378	5 to 39	Standardization sample	No	
Taub & McGrew (2004)	7485	6 to 90+	Standardization sample	No	
Sanders et al. (2007)	131	3 to 5	Standardization sample	Yes	DAS
Bergeron & Floyd (2006)	875	9 to 13	Standardization sample	No	
Kaufman et al. (2012)	4969	5 to 19	Standardization sample WJ-III	Yes	KABC-II; KAIT
Cucina & Howardson (2017)	6189	6 to 90+	Standardization sample	Yes	KAIT; DAS; KABC-II
Phelps, McGrew, Knopik, & Ford (2005)	148	8 to 12	Standardization sample	Yes	WISC-III
<b>Woodcock-Johnson IV</b>					
McGrew, LaForte, & Schrank (2014)	6914	3 to 90+	Standardization sample (test manual)	No	
<b>Wechsler Intelligence Scale for Children</b>					
Undheim & Gustafsson (1987)	441	11 to 15	Norwegian school sample	Yes	Thurstone; Guilford
Undheim (1976)	144	10 to 12	Norwegian school sample	Yes	Thurstone; Guilford
<b>Wechsler Intelligence Scale for Children III</b>					
Phelps et al., (2005)	148	8 to 12	Standardization sample	Yes	WJ-III
Mayes & Calhoun (2007)	678	6 to 16	ADHD sample	Yes	WISC-IV
Takeuchi et al. (2018)	48	7 to 9	Japanese sample	No	
Freberg et al. (2008)	202	6 to 13	Subset of Cavinez & Watkins (1998)	Yes	WJ-R
Cathers-Schiffman & Thompson (2007)	94	8 to 13	School sample	No	
Naglieri et al. (2006)	119	6 to 16	School/clinical sample	Yes	CAS
Beaujean et al (2012)	248	Avg. 862	Clinical Sample – Manic Symptoms	Yes	WISC-IV
Cockshott, Marsh, & Hine (2006)	579	6 to 16	Australian school sample	No	
Ogata (2015)	105	6 to 12	Japanese sample	Yes	KABC

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**Wechsler Intelligence Scale for Children IV**

Rowe, et al. (2014)	406	6 to 12	Gifted children	No	
Wechsler (2014b)	2200	6 to 16	Standardization sample (test manual)	No	
Keith, Fine, Taub, Reynolds, & Ford (2006)	2200	6 to 16	Standardization sample	No	
Cavinez, Watkins, & Dombrowski (2016)	2200	6 to 16	Standardization Sample	Yes	WISC-IV
Nakano & Watkins (2013)	176	6 to 16	School sample (Native American)	Yes	WISC-III
Bergeron & Floyd (2013)	56	6 to 16	Clinical sample with mild/moderate ID	Yes	KABC-II; DAS-II
Beaujean et al (2014)	550	6 to 16	Standardization sample	Yes	WIAT-II
Weiss et al. (2013)	1967	6 to 16	Clinical + non-clinical standardization	No	
Golay et al. (2013)	249	Avg. 9.84	French-speaking Swiss children	No	
Baum et al. (2015)	40	10 to 16	ASD school sample	No	
Wilson et al. (2012)	30	12 to 14	School sample	Yes	SB-V
Reynolds et al. (2016)	166	7 to 16	Shipley-2 validation sample	Yes	Shipley-2
Devena, Gay, & Watkins (2013)	297	6 to 15	Clinical sample	No	
Benson et al. (2013)	730	6 to 16	Integrated standardization sample	No	
Reverte et al. (2014)	249	Avg. 10.21	Swiss school sample	No	
Styck & Watkins (2017)	233	6 to 16	ADHD school sample	No	
Richerson, Watkins, & Beaujean (2014)	352	6 to 16	Longitudinal school sample	No	
Cavinez, Watkins, & Dombrowski (2017)	2200	6 to 16	Standardization sample on summary data	No	
Pezzuti & Orsini (2016)	2200	6 to 16	Italian standardization sample	No	
Chen et al. (2016)	2200	6 to 16	Full standardization sample	No	
Thaler et al. (2015)	314	6 to 16	ADHD school sample	No	
Do Santos et al. (2018)	150	6 to 14	School sample	No	
Cianci et al. (2013)	2200	6 to 16	Italian standardization sample	No	
Krouse & Braden (2011)	128	6 to 17	Hard of hearing school children	No	
Cavinez (2014)	345	6 to 16	Learning disabled school sample	No	
Oakland, Callueng, & Harris (2012)	110	6 to 16	Spanish standardization sample	No	
Parikin & Beaujean (2011)	550	6 to 16	Standardization sample	Yes	WIAT-II
Decker, Englund, & Roberts (2014)	2200	6 to 16	Standardization sample	No	

**Wechsler Intelligence Scale for Children V**

Reynolds & Keith (2017)	2200	6 to 16	Standardization sample	No	
Chen, Zhang, Raiford, Zhu, & Weiss (2015)	2200	6 to 16	Standardization sample	No	
Lecerf & Cavinez (2018)	1049	6 to 16	French standardization sample	No	
Cavinez, Watkins, & McGill (2019)	415	6 to 16	United Kingdom standardization sample	No	
Cavinez et al. (2020)** <sub>b</sub>	2,512	6 to 16	Clinical sample		

**Wechsler Adult Intelligence Scale-R**

Davis, Massman, & Doody (2003)	516	73.19	Alzheimer's sample	No	
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Waller & Waldman (1990)	1880	16 to 74	Standardization sample	No	
<b>Wechsler Adult Intelligence Scale III</b>					
Dickinson, Iannone, & Gold (2002)	320	35 to 44	Clinical sample	No	
McPherson & Burns (2007)	60	20.6	College sample	Yes	WJ-III
Taub & Benson (2013)	2450	16 to 89	Standardization sample	Yes	WAIS-IV
Taub, McGrew, & Witta (2004)	2450	16 to 89	Standardization sample	No	
<b>Wechsler Adult Intelligence Scale IV</b>					
Niileksela et al. (2013)	400	70 to 90	Standardization sample	No	
Gignac & Watkins (2013)	1800	16 to 70	Standardization sample	No	
Merz et al. (2019)	300	18 to 72	Clinical sample	No	
Taub & Benson (2013)	2200	16 to 90	Standardization sample	Yes	WAIS-III
Holdnack et al. (2011)	900	16 to 69	Standardization sample	No	
Benson, Hulac, & Kranzler (2010)	2200	16 to 90	Standardization sample	No	
Miller et al. (2013)	431	65 to 92	Recruited and standardization sample	No	
Buczłowska, Petermann, & Daseking (2020)**	205	18 to 89	German community sample	No	
<b>Kaufman Adolescent &amp; Adult Intelligence Test</b>					
Cucina & Howardson (2017)	2,000	11 to 18	Standardization sample	Yes	WJ-III; DAS; KABC-II
Kaufman (1993)	124	11 to 12	School sample	Yes	K-ABC
Kaufman, Kaufman, & McClean (1995)	1901	11 to 94	Standardization sample	No	
Caruso & Jacob-Timm (2001)	60	11 to 14	Cross-check sample	No	
<b>Kaufman Assessment Battery for Children</b>					
Keith et al. (1995)	1299	7 to 12	Standardization and sociocultural sample	No	
Ogata (2015)	105	6 to 12	Japanese standardized sample	Yes	WISC-III
<b>Kaufman Assessment Battery for Children-II NU</b>					
Reynolds et al. (2013)	432	6 to 16	Standardization sample	Yes	WISC-III; WISC-IV; WJ-III
Morgan et al. (2009)	200	4 to 5	School sample	No	
McGill (2015)	2025	7 to 18	Standardization sample	No	
Bergeron & Floyd (2013)	29	7 to 18	Clinical sample with ID	Yes	DAS-II; WISC-IV
Kaufman et al. (2012)	2520	4 to 19	Standardization sample	Yes	WJ-III
Potvin et al. (2015)	450	4 to 5	Standardization sample	No	
McGill (2019)	500	7 to 18	Standardization sample	No	
<b>Differential Abilities Scale</b>					
Keith (1990)	3475	3 to 17	Standardization sample	No	
<b>Differential Abilities Scale II</b>					
Bergeron & Floyd (2013)	51	7 to 17	Clinical sample with ID	Yes	WISC-IV; KABC-II
Cavinez & McGill (2016)	3480	2 to 17	Standardization sample	No	
Clements, Watkins, Schultz, & Yerys (2020)**	3716	4 to 18	ASD and standardization sample	No	

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Caemmerer, Keith, & Reynolds (2020)**	3927	6 to 18	Standardization sample	Yes	WISC-V; WISC-IV
<b>Stanford-Binet Intelligence Scale IV</b>					
Kaplan & Alfonso (1997)	441	2 to 5	Preschool sample with ID		
Gridley & McIntosh (1991)	187	2 to 11	School sample	No	
<b>Stanford-Binet Intelligence Scale V</b>					
Williams et al. (2010)	201	8 to 10	School sample	Yes	WJ-III
Chang et al. (2014)	200	4 to 5	Preschool sample	Yes	WJ-III
<b>Culture Fair Intelligence Test</b>					
Undheim (1981)	148	14 to 16	Norwegian school sample	Yes	Thurstone; Guilford
Undheim (1978)	149	12 to 14	Norwegian school sample	Yes	Thurstone; Guilford
Cattell (1963)	278	13 to 14	School sample	Yes	Thurstone; HSPQ
Fukuda et al. (2010)	79	--	College sample	No	
<b>Berlin Model of Intelligence Structure</b>					
Beauducel & Kersting (2002)	9520	17 to 32	Community sample	No	
Conzelman & Süß (2015)	301	21 to 40	College sample	Yes	Auditory Intelligence Test
<b>Educational Testing Service Kit of Factor Ref. Cog. Tests</b>					
MacCann et al., (2014)	688	17 to 59	College sample	Yes	MSCEIT
<b>Mayer-Salovey-Caruso Emotional Intelligence Test</b>					
Lopez, Salovey, & Straus (2003)	103	19.2	College sample	Yes	WAIS-II
Legree et al. (2014)	726	17 to 59	College sample	No	
Evans, Hughes, & Steptoe-Warren (2019)	830	18 to 71+	College and convenience sample	Yes	STEU; STEM
<b>Situational Test of Emotion Management</b>					
MacCann (2010)	207	19 to 59	College sample	Yes	Educational Testing Kit
<b>Multi-Battery/ Test Scales</b>					
Horn & Cattell (1966)	297	14 to 61	Prison sample	Yes	Thurstone; Guilford
Horn & Cattell (1967)	297*	14 to 61	Prison sample	Yes	Thurstone; Guilford
Cattell & Horn (1978)	883	Approx. 14	School sample	No	
Stankov (1978)	113	11 to 12	Yugoslavian school sample	No	
<b>Comprehensive Ability Battery</b>					
Hakstian & Cattell (1978)	280	15 to 19	Canadian school sample	No	

*Note:* WJ= Woodcock-Johnson; WISC = Wechsler Intelligence Scale for Children; WAIS = Weschler Adult Intelligence Scale; MSCEIT = Mayer-Salovey-Caruso Emotional Intelligence Test; DAS = Differential Abilities Scale; CAS = Cognitive Assessment System.

<sup>a</sup> Includes the 103 relevant studies prior to being screened for the type of sample used (e.g., standardization sample).

<sup>b</sup> Studies denoted with \*\* were not included in the initial screening for articles or any analyses in the manuscript. They represented studies published after the authors had submitted the present work for publication.

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### Distinguishing Between Two- and Three-Tier Models

Both two-stratum and three-stratum models are reported in the literature and it is worth examining the commonalities and differences between them—which extend beyond whether they model two or three tiers of intelligence. Figure 1 below, shows the number of studies included in the present review with two- or three-tier models.

In terms of their commonalities, both the two- and three-tiered factor models include, at their lowest levels, such observed tasks as vocabulary, digit-span, and spatial ability measures. Both two- and three-tiered models then assigned the indicator variables to one or more of broad intelligences under examination by the researchers—where the specific set of broad intelligences varied from study to study. For example, the indicator task object rotation was assigned to the broad intelligence spatial intelligence and digit span to short term memory. Most of the time, each indicator variable was constrained to load on just one broad intelligence (i.e., a single pathway; simple structure), although in some instances, indicators contributed to more than one such broad intelligence.

From here, however, the two- and three-tiered models diverge substantially (see, for example, McCann, Joseph, Newman, & Roberts, 2014, Morgan, Rothlisberg, McIntosh, & Hunt, 2009, and Thaler, Barchard, Parke, Jones, Etcoff, & Allen, 2015, all of whom report complete versions of both models). The two-tiered model is completed by allowing paths among the broad intelligences to indicate their intercorrelations. Note that the correlations will reflect any variance shared among all the broad intelligences, as well as any variance shared among subsets of the broad intelligences. (Given  $K$  broad intelligences, there are  $K!$  ( $K$  factorial) pathways among them to represent these possible subsets of shared variance).

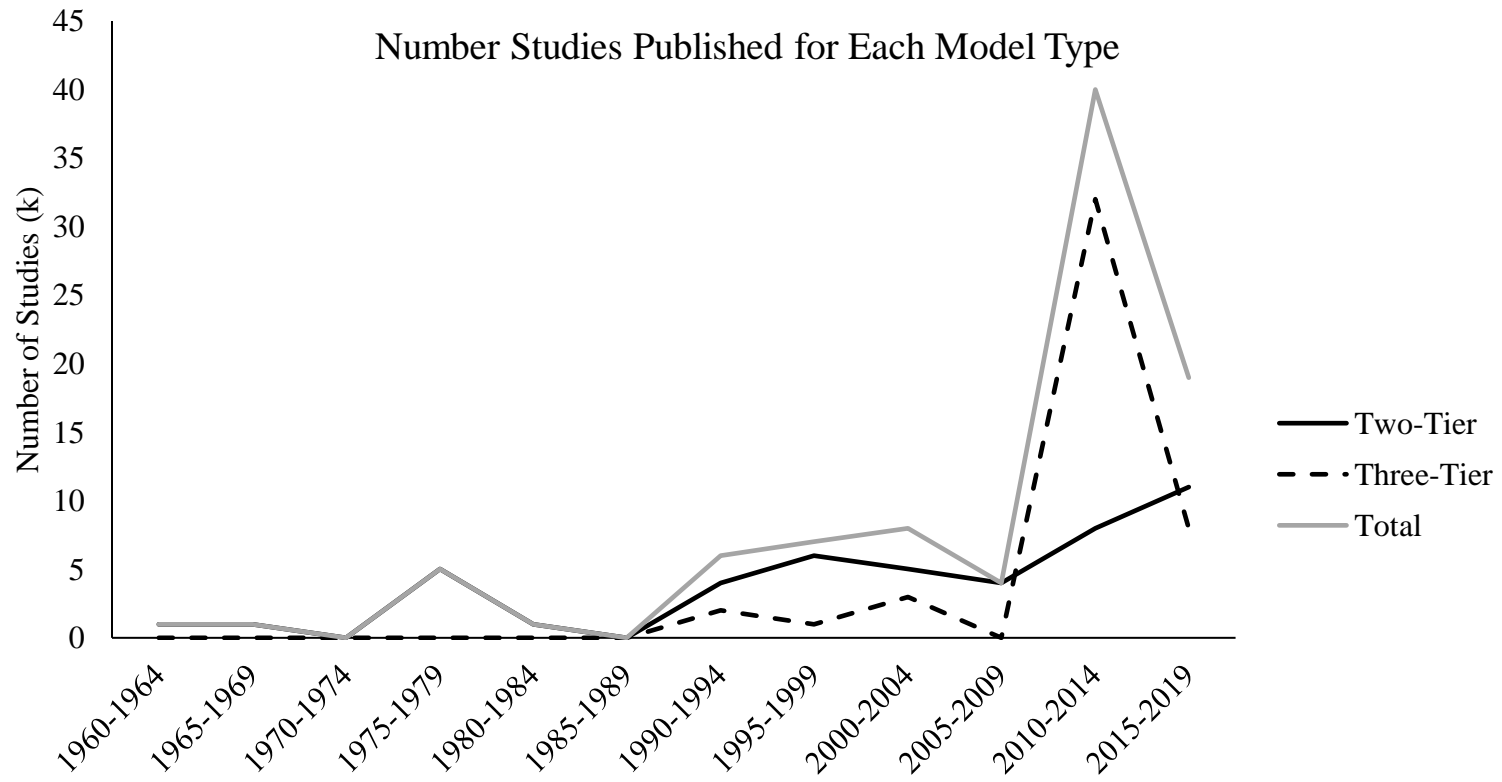
The three-tiered models, by comparison, generally constrain the second tier of broad intelligences to be orthogonal to one another (i.e., their correlations are set at  $r = 0$ ), and the observed correlations among them are accounted for by the variance they all share in common with the  $g$  factor at the top level of the three-tiered model. Given again  $K$  broad intelligences, just  $K$  pathways are used to represent this simplified state of affairs: one path between each broad intelligence and  $g$ . Because the three-tier models represent only the variance among the broad intelligences due to  $g$ , any shared variance that might arise among sets of similar broad intelligences was by necessity reassigned to other parts of the model: If the shared variance was shared broadly enough among the broad intelligences, it presumably was reassigned to  $g$ ; if not, it was reassigned as error variance.

In addition, because the constraints of the three-tiered models differed from those of the two-tiered models, the path coefficients across the two sets of models were non-comparable. We checked this in three articles by well-respected researchers (McCann et al., 2014; Morgan et al., 2009; Thaler et al., 2015): In each instance, estimating the correlation among broad intelligences at the second tier by following paths of the three-tiered models (e.g., Loehlin, 2004) converge only approximately at best: The constraints on shared variance among subsets of broad intelligences shifted the models slightly.

Two further implications of the three-tiered models are that: (a) because they constrain all variance among the broad intelligences to be due to general intelligence, then any estimated correlations among the broad intelligences must be strictly unifactorial. We tested that deduction by estimating the correlations among the broad intelligences from the three-tiered models using the pathways approach. Indeed, when we factor analyzed that correlation matrix, there was plainly one and only one factor (see Exploratory Factor Analysis on Three-Tier Models below).

(b) A second matter we observed empirically was that, in the course of reassigning the shared variance that emerged among subsets of broad intelligences, the fitting functions appeared to assign some of the variance common among the subsets to all the broad intelligences—and that had the consequence of inflating the estimated correlations among the broad intelligences beyond that found in the two-tiered models.

Because we also were interested in variance shared among sets of broad intelligences (and we believe, with others, these are likely to exist; see, for example, Schneider & Newman, 2015), we employed only the two-tiered models here.



**Figure 1.** Number of Two-Tier and Three-Tier, g-Inclusive Studies Published by Year.



## Part 2. Calculating the Average Correlation

### R Code for Assessing the Average Correlation Among Broad Intelligences

Below is the corresponding R code used to calculate the weighted average correlation among broad intelligences. The complete data set used in all analyses will be made available by request.

#### **#Install Necessary Packages**

```
install.packages('psych');  
library(psych)  
install.packages("meta")  
library(meta)  
install.packages("metacor")  
library(metacor)  
library(readxl)
```

#### **#Step1: Import Data (Repeat Steps 1-4 for Non-Imputed Data, and Combined Data)**

```
data1 <- read_excel("Est Corrs Broad Intells-2020-1-9.xlsx")  
data1
```

#### **#Step2: Determine Overall Sample Size**

```
NStudy <- data1["N"]  
  
TotalN <- sum(NStudy)  
TotalN
```

#### **#Step3: Calculating the Weighted Average Between Pairs.**

```
{gf.gc <- metacor(GfwGc,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "ML")}  
gf.gc
```

```
{gf.gv <- metacor(GfwGv,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")}  
gf.gv
```

```
{gf.gsm <- metacor(GfwGsm,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")}  
gf.gsm
```

```
gf.gs <- metacor(GfwGs,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")
```

gf.gs

```
gf.glr <- metacor(GfwGlr,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")
```

gf.glr

```
gf.ga <- metacor(GfwGa,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")
```

gf.ga

```
gf.gq <- metacor(GfwGq,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")
```

gf.gq

```
gc.gv <- metacor(GcwGv,  
  N,  
  data = data1,  
  studlab = data1$Article,
```

```
      sm = "COR",
      method.tau = "DL")
gc.gv

gc.gsm <- metacor(GcwGsm,
  N,
  data = data1,
  studlab = data1$Article,
  sm = "COR",
  method.tau = "DL")
gc.gsm

gc.gs <- metacor(GcwGs,
  N,
  data = data1,
  studlab = data1$Article,
  sm = "COR",
  method.tau = "DL")
gc.gs

gc.glr <- metacor(GcwGlr,
  N,
  data = data1,
  studlab = data1$Article,
  sm = "COR",
  method.tau = "DL")
gc.glr

gc.gq <- metacor(GcwGq,
  N,
  data = data1,
  studlab = data1$Article,
  sm = "COR",
  method.tau = "DL")
gc.gq

gc.ga <- metacor(GcwGa,
  N,
  data = data1,
  studlab = data1$Article,
  sm = "COR",
  method.tau = "DL")
gc.ga
```

```
gv.gsm <- metacor(GvwGsm,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")
```

```
gv.gsm
```

```
gv.gs <- metacor(GvwGs,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")
```

```
gv.gs
```

```
gv.glr <- metacor(GvwGlr,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")
```

```
gv.glr
```

```
gv.ga <- metacor(GvwGa,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")
```

```
gv.ga
```

```
gv.gq <- metacor(GvwGq,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")
```

```
gv.gq
```

```
gsm.gs <- metacor(GsmwGs,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")
```

gsm.gs

```
gsm.ga <- metacor(GsmwGa,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")
```

gsm.ga

```
gsm.glr <- metacor(GsmwGlr,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")
```

gsm.glr

```
gsm.gq <- metacor(GsmwGq,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")
```

gsm.gq

```
gs.glr <- metacor(GswGlr,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")
```

gs.glr

```
gs.ga <- metacor(GswGa,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")
```

gs.ga

```
glr.ga <- metacor(GlrwGa,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",
```

```
      method.tau = "DL")
glr.ga

glr.gq <- metacor(GlrwGq,
  N,
  data = data1,
  studlab = data1$Article,
  sm = "COR",
  method.tau = "DL")
glr.gq

gq.gs <- metacor(GqwGs,
  N,
  data = data1,
  studlab = data1$Article,
  sm = "COR",
  method.tau = "DL")
gq.gs

gq.ga <- metacor(GqwGa,
  N,
  data = data1,
  studlab = data1$Article,
  sm = "COR",
  method.tau = "DL")
gq.ga

gf.grw <- metacor(GfwGrw,
  N,
  data = data1,
  studlab = data1$Article,
  sm = "COR",
  method.tau = "DL")
gf.grw

gc.grw <- metacor(GcwGrw,
  N,
  data = data1,
  studlab = data1$Article,
  sm = "COR",
  method.tau = "DL")
gc.grw

gv.grw <- metacor(GvwGrw,
  N,
  data = data1,
```

```
      studlab = data1$Article,  
      sm = "COR",  
      method.tau = "DL")  
gv.grw
```

```
gsm.grw <- metacor(GsmwGrw,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")  
gsm.grw
```

```
gs.grw <- metacor(GswGrw,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")  
gs.grw
```

```
glr.grw <- metacor(GlrwGrw,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")  
glr.grw
```

```
ga.grw <- metacor(GawGrw,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")  
ga.grw
```

```
gq.grw <- metacor(GqwGrw,  
  N,  
  data = data1,  
  studlab = data1$Article,  
  sm = "COR",  
  method.tau = "DL")  
gq.grw
```

```
gf.EI <- metacor(GfwEI,
```

```
      N,  
      data = data1,  
      studlab = data1$Article,  
      sm = "COR",  
      method.tau = "DL")  
gf.EI
```

```
gc.EI <- metacor(GcwEI,  
      N,  
      data = data1,  
      studlab = data1$Article,  
      sm = "COR",  
      method.tau = "DL")  
gc.EI
```

```
gv.EI <- metacor(GvwEI,  
      N,  
      data = data1,  
      studlab = data1$Article,  
      sm = "COR",  
      method.tau = "DL")  
gv.EI
```

```
gsm.EI <- metacor(GsmwEI,  
      N,  
      data = data1,  
      studlab = data1$Article,  
      sm = "COR",  
      method.tau = "DL")  
gsm.EI
```

```
glr.EI <- metacor(GlrwEI,  
      N,  
      data = data1,  
      studlab = data1$Article,  
      sm = "COR",  
      method.tau = "DL")  
glr.EI
```

```
gq.EI <- metacor(GqwEI,  
      N,  
      data = data1,  
      studlab = data1$Article,  
      sm = "COR",  
      method.tau = "DL")  
gq.EI
```



**#Step4: Calculating the Weighted Average (Overall)**

```
average.overall <- metacor(Average,
  N,
  data = data1,
  studlab = data1$Article,
  sm = "COR",
  method.tau = "DL")
average.overall
```

**#Step5: Publication Bias Analyses**

```
data2 <-read_excel("Broad Intells /Average Per Study - No Imputed.xlsx")
data2
```

```
average.per.study <- metacor(mean,
  N,
  data = data2,
  studlab = data2$Article,
  sm = "COR",
  method.tau = "DL")
average.per.study
```

```
funnel(average.per.study, studlab = FALSE)
```

**#Statistics on Asymmetry in Funnel Plot.**

```
# Egger's Test of the Intercept
```

```
eggertest = function(x) {
```

```
  # Validate
```

```
  x = x
```

```
  if (x$k < 10) {
```

```
    warning(paste("Your meta-analysis contains k =", x$k, "studies. Egger's test may lack the
statistical power to detect bias when the number of studies is small (i.e., k<10)."))
```

```
  }
```

```
  if (class(x)[1] %in% c("meta", "metabin", "metagen", "metacont", "metacor", "metainc",
"metaprop")) {
```

```
    # Conduct metabias
```

```

eggers = meta::metabias(x, k.min = 3, method = "linreg")

# Get Intercept
intercept = as.numeric(eggers$estimate[1]) %>% round(digits = 3)

# Get SE
se = as.numeric(eggers$estimate[2])

# Calculate 95CI
LLCI = intercept - 1.96 * se %>% round(digits = 1)
ULCI = intercept + 1.96 * se %>% round(digits = 1)
CI = paste(LLCI, "-", ULCI, sep = "")

# Get t
t = as.numeric(eggers$statistic) %>% round(digits = 3)

# Get df
df = as.numeric(eggers$parameters)

# Get p
p = as.numeric(eggers$p.value) %>% round(digits = 5)

# Make df
df = data.frame(Intercept = intercept, ConfidenceInterval = CI, t = t, p = p)
row.names(df) = "Egger's test"

} else {

  stop("x must be of type 'metabin', 'metagen', 'metacont', 'metainc' or 'metaprop'")
}

return(df)}

eggers.test(x=average.per.study)

#significant p-value suggests there is asymmetry in the plot, which may be due to publication
bias.

#Post-Hoc Tests: Calculating Weighted Average Pre-Post 1993

data3 <- read_excel("Pre 1993.xlsx")

data4 <- read_excel("Post 1993.xlsx")

```

```
average.93 <- metacor(Average,
  N,
  data = data3,
  studlab = data3$Article,
  sm = "COR",
  method.tau = "ML")
average.93
```

```
average.post <- metacor(Average,
  N,
  data = data4,
  studlab = data4$Article,
  sm = "COR",
  method.tau = "ML")
average.post
```

### **#Post-Hoc Tests: Comparing Average by Intelligence Test**

```
data5<-read_excel("Research Based 2020-5-21.xlsx")
data5
```

```
average.research <- metacor(Average,
  N,
  data = data5,
  studlab = data5$Article,
  sm = "COR",
  method.tau = "ML")
average.research
```

```
data6<-read_excel("KABC - 2020-5-21.xlsx")
```

```
average.KABC <- metacor(Average,
  N,
  data = data6,
  studlab = data6$Article,
  sm = "COR",
  method.tau = "ML")
average.KABC
```

```
data7<-read_excel("KAIT - 2020-5-21.xlsx")
```

```
average.KAIT <- metacor(Average,
  N,
  data = data7,
```

```
        studlab = data7$Article,
        sm = "COR",
        method.tau = "ML")
average.KAIT

data8 <-read_excel("SB.xlsx")

average.SB <- metacor(Average,
        N,
        data = data8,
        studlab = data8$Article,
        sm = "COR",
        method.tau = "ML")
average.SB

data9 <-read_excel("WAIS.xlsx")
data9

average.WAIS <- metacor(Average,
        N,
        data = data9,
        studlab = data9$Article,
        sm = "COR",
        method.tau = "ML")
average.WAIS

data10 <-read_excel("WISC.xlsx")

average.WISC <- metacor(Average,
        N,
        data = data10,
        studlab = data10$Article,
        sm = "COR",
        method.tau = "ML")
average.WISC

data11 <-read_excel("WJ-2020-1-19.xlsx")

average.WJ <- metacor(Average,
        N,
        data = data11,
        studlab = data11$Article,
        sm = "COR",
        method.tau = "ML")
average.WJ
```

```
data12 <-read_excel("DAS.xlsx")

average.DAS <- metacor(Average,
  N,
  data = data12,
  studlab = data12$Article,
  sm = "COR",
  method.tau = "ML")
average.DAS
```

### The Average Correlation Among Broad Intelligences Using Three-Tier Models

Recall from the above section regarding distinguishing between two- and three-tier models that, from our observation, estimated correlations calculated from three-tier models through path analysis were not readily comparable to the correlations reported in two-tier models. To further depict the differences between the correlations obtained from these two types of models, we calculated the average correlation among broad intelligences, including both types of correlations (i.e. those derived from two-tier models and those calculated using path analysis from three-tier models).

As noted in the paper, the average correlation among broad intelligences derived from the studies where the estimated correlations among broad abilities was imputed was much higher ( $r = .65$ ) than the average correlation calculated from correlations reported using two-tier models ( $r = .58$ ). Moreover, the correlations among specific pairs of broad intelligences also changed, as can be seen in Table 2, below. The lower range of the correlations among broad intelligences increased from .22 to .47 between visuospatial processing and processing speed, whereas the upper range of correlations remained the same at  $r = .81$  between quantitative reasoning and fluid intelligence and long-term retrieval and quantitative reasoning.

Table 2.

The Number of Studies Including Each Broad Intelligence, Participants Observed, and the Average Weighted Correlations Among Broad Intelligences for Three-Tier, *g*-inclusive Model Studies.

	Fluid Intelligence	Comp. Knowledge	Visuospatial Processing	Short-Term Memory	Long-Term Retrieval	Processing Speed	Quantitative Reasoning	Auditory Intelligence	Reading and Writing	Emotional Intelligence	Totals
<i>Study Characteristics and Number of Participants</i>											
<i>k</i> Studies	40	45	38	42	28	35	15	20	13	2	46
Total N Across Studies	44,999	50,221	38,744	39,138	24,678	36,592	9888	21,444	7060	1518	51,051
<i>Averaged Weighted Correlations (in Bold) Among Pairs of Broad Intelligences and Their Confidence Intervals</i>											
Fluid Intelligence	1.00										
Comprehension-Knowledge	<b>.77</b> [.74, .80]	1.00									
Visuospatial Processing	<b>.77</b> [.74, .79]	<b>.69</b> [.67, .71]	1.00								
Short-Term Memory	<b>.72</b> [.68, .77]	<b>.66</b> [.63, .69]	<b>.64</b> [.60, .68]	1.00							
Long-Term Retrieval	<b>.81</b> [.78, .85]	<b>.74</b> [.71, .77]	<b>.74</b> [.69, .78]	<b>.70</b> [.67, .74]	1.00						
Processing Speed	<b>.53</b> [.50, .56]	<b>.49</b> [.42, .57]	<b>.47</b> [.37, .57]	<b>.49</b> [.41, .57]	<b>.55</b> [.51, .60]	1.00					
Quantitative Reasoning	<b>.81</b> [.79, .83]	<b>.73</b> [.71, .76]	<b>.71</b> [.69, .74]	<b>.70</b> [.66, .73]	<b>.81</b> [.78, .85]	<b>.53</b> [.49, .57]	1.00				
Auditory Intelligence	<b>.73</b> [.71, .76]	<b>.67</b> [.64, .70]	<b>.65</b> [.61, .69]	<b>.65</b> [.61, .69]	<b>.73</b> [.69, .77]	<b>.48</b> [.46, .52]	<b>.68</b> [.65, .73]	1.00			
Reading and Writing	<b>.74</b> [.72, .75]	<b>.66</b> [.64, .69]	<b>.65</b> [.62, .67]	<b>.63</b> [.60, .66]	<b>.76</b> [.74, .78]	<b>.48</b> [.44, .53]	<b>.71</b> [.69, .72]	<b>.63</b> [.58, .67]	1.00		
Emotional Intelligence	<b>.66</b> [.39, .94]	<b>.72</b> [.68, .76]	<b>.66</b> [.62, .70]	--	<b>.58</b> [.53, .63]	--	<b>.62</b> [.57, .67]	--	--	1.00	
<b>Overall Average</b>	<b>.71</b> [.68, .74]	<b>.67</b> [.64, .69]	<b>.67</b> [.64, .70]	<b>.64</b> [.61, .68]	<b>.73</b> [.70, .75]	<b>.51</b> [.46, .55]	<b>.71</b> [.69, .73]	<b>.65</b> [.62, .69]	<b>.65</b> [.63, .67]	<b>.60</b> [.49, .71]	<b>.65</b> [.62, .68]

<sup>a</sup>Weighted average correlations are in boldface and were taken from the random-effects model produced from the *meta* package in R. 95% confidence intervals for each weighted average are found below, in brackets.

ⓂOnly one correlation per pair of broad intelligences was reported per study, so the confidence intervals for the correlations between pairs of broad intelligences are based on independent observations.

ⓄThe overall average correlation for a given broad intelligence (e.g., for fluid) was calculated first by averaging within study if there was more than one correlation reported, and then running those averages in the R script to find an across study overall average.



### **Part 3. Exploring the Structure Among Broad Intelligences**

#### Results from Exploratory Factor Analyses

A goal of the present research, in addition to calculating the overall average correlation among broad intelligences, was to explore whether a reliable structure would emerge among the correlations we obtained from our meta-analysis. For example, some researchers have hypothesized possible continua for organizing the broad intelligences, such as contrasting “Power” intelligences, which involves more knowledge-based intelligences like crystallized intelligence ( $G_c$ ) or long-term retrieval ( $G_{lr}$ ), from “Speed” intelligences, which include intelligences like processing speed ( $G_s$ ) that are involved in rapidly solving problems (see Newman, 2015, Fig. 4). Other researchers have proposed dividing the broad intelligences into “Thing-Centered” and “People-Centered” intelligences, which focus on broad abilities that facilitate reasoning about things, such as quantitative reasoning, and those that facilitate reasoning about people, such as emotional intelligence (Bryan & Mayer, 2017; Mayer 2018; Mayer & Skimmyhorn, 2017).

To explore the structure of the broad intelligences, a series of exploratory factor analyses were conducted. First, we analyzed the correlation matrix depicted in Table 3 of the manuscript, which shows the average weighted correlations among broad intelligences. In this correlation matrix, we replaced these missing values with the average weighted correlation for a given broad intelligence (e.g.,  $G_{ei} r = .58$ ,  $G_{rw} = .49$ ). We sought a standard “good fit” of an RMSEA less than or equal to .06, and both Comparative and Tucker-Lewis Fit Indices close to .95 (Boomsma, Hoyle, & Panter, 2012).

Our first exploratory analyses on the correlation matrix converged on to one- and two-factors but failed to converge on to three-factors. Examination of these models revealed one or

more Heywood cases (factor loadings greater than 1), and model fits well below our designated standards (RMSEA = .28, CFI = .69, TLI = .60 for the one-factor model; RMSEA = .22, CFI = .86, TLI = .75 for the two-factor model). See Table 3a for factor loadings and fit indices.

We sought to improve the fit of our models using two different methods. First, we engaged in the stepwise removal of Heywood cases, where we removed any broad abilities that loaded onto a factor higher than one. It has been suggested that the presence of Heywood cases may serve as a diagnostic tool for assessing whether data violates assumptions of factor analysis (van Driel, 1978; Velicer & Jackson, 1990). Heywood cases can emerge when sample sizes are small (de Winter, Dodou, & Wieringa, 2009), or when the number of factors extracted is too many (Hoyle & Duval, 2004). A common practice when factor solutions produce Heywood cases is to either reduce the number of factors or to remove them. The fits of the resulting models were examined, and any further Heywood cases removed until a meaningful and well-fitting model was produced.

Second, to check for the robustness of the solution produced using this method, we also engaged in the conceptual removal of broad abilities, based on their *g* loadings found in the literature. For example, broad intelligences like fluid intelligence, comprehension knowledge, and quantitative reasoning have all demonstrated higher loadings on to *g* compared to other broad intelligences, with some researchers going so far as to suggest they may be indistinguishable from *g* (see Bickley, Keith, & Wolfle, 1995). Other researcher suggests that quantitative reasoning may be a component of one's fluid abilities and have therefore combined it with fluid intelligence (Flanagan, & Dixon, 2013; Flanagan & McGrew, 1997). Therefore, we removed fluid intelligence, comprehension knowledge, and quantitative reasoning from our analyses and examined the resulting structure and fit of the resulting models. Then, loadings and fits of the

models produced using the stepwise removal and the conceptual removal methods were compared.

**Solutions for the stepwise removal of Heywood cases.** In our first exploratory analysis, a Heywood case emerged for the loading of quantitative reasoning on to the first factor of our two-factor solution. Therefore, we removed quantitative reasoning from our model, and re-ran our exploratory analysis.

With the removal of quantitative reasoning, the data converged on to one- and three-factors but failed to converge on to two-factors. The one-factor solution produced by removing quantitative reasoning was an improvement over the original one-factor solution (RMSEA = .24, CFI = .75, TLI = .66), but still a relatively poor fit for the data given our standards. The two-factor model produced by removing Gq showed an improved fit, that fell near our designated goodness of fit standards (RMSEA = .09, CFI = .99, TLI = .96). Examination of the factor loadings revealed additional Heywood cases for comprehension knowledge (Gc) and long-term retrieval (Glr), which muddied the interpretation of the model.

Next, we removed comprehension knowledge from our model due to its high factor loading. The data converged on to one- and two-factors. Although the one-factor solution demonstrated an improved fit over our previous one-factor solutions (RMSEA = .13, CFI = .91, TLI = .87), the two-factor solution had an acceptable fit and interpretable solution, with and RMSEA = .10, CFI = .97, TLI = .93. Fluid intelligence (Gf), visuospatial processing (Gv), short-term memory (Gsm), processing speed (Gs) and emotional intelligence loaded on to the first factor, which appeared to be a *reasoning* factor. Long-term retrieval (Glr) and reading and writing ability (Grw) loaded on to the second factor, which appeared to be a *knowledge* factor.

**Solutions for the conceptual removal of broad intelligences.** The rather haphazard stepwise removal of Heywood cases above encouraged us to find a more elegant approach. Therefore, as described earlier, we removed certain intelligences like fluid intelligence, quantitative reasoning, and comprehension knowledge, which have demonstrated especially high correlations with *g* and reran our models.

The data converged on to one-, two-, and three-factors, with each subsequent model demonstrating an improved fit over the previous model. Specifically, the one-factor model demonstrated a modest fit (RMSEA = .12, CFI = .93, TLI = .87), which was improved upon by the two-factor solution (RMSEA = .07, CFI = .99, TLI = .97), which demonstrated an excellent fit. Moreover, the two-factor model offered a readily interpretable solution, similar to the solution obtained above by removing Heywood cases. Visuospatial processing (*Gv*), short-term memory (*Gsm*), processing speed (*Gs*) and emotional intelligence loaded on to the first factor, which we labeled as *reasoning*. Long-term retrieval (*Glr*) and reading and writing ability (*Grw*) loaded on to the second factor, which we labeled *knowledge*. Thus, using two different methods for seeking a meaningful structure among the broad intelligences yielded similar models.

The three-factor solution produced using this approach also demonstrated an excellent fit (RMSEA = .03, CFI = 1.00, TLI = .99), but also the addition of a Heywood case for the loading of reading and writing ability on the second factor. The solution was somewhat similar to the two-factor solution, with the exception of reading and writing ability loading solely on the second factor, and long-term retrieval and auditory intelligence loading on the third factor. Given work suggesting that the presence of Heywood cases may mean that too many factors were extracted (Hoyle & Duval, 2004) we concluded that the two-factor solution was a better fitting

model, despite the fit indices supporting a three-factor model. Fit statistics and factor loadings can be found in Table 3b.

**Table 3a.**

Fit Statistics and Factor Loadings for the Final 1-, 2-, and 3- Factor Exploratory Solutions of Broad Intelligences Using the Sequential Removal of Heywood Cases (N = 20,399)

	Fit Statistics – First Analysis			Fit Statistics – Removal of Gq			Fit Statistics – Removal of Gc		
	RMSEA	CFI	TLI	RMSEA	CFI	TLI	RMSEA	CFI	TLI
<b>One Factor</b>	.28	.69	.60	.12	.92	.89	.13	.91	.87
<b>Two Factors</b>	.22	.86	.75	--	--	--	.10	.97	.93
<b>Three Factors</b>	--	--	--	.09	.99	.96	--	--	--

Broad Intelligence	<i>Factor Loadings</i>									
	One-Factor Solution		Two-Factor Solution		Three-Factor Solution					
	I	II	I	II	I	II				
Fluid Intelligence (Gf)	<b>.80</b>	.16	<b>.66</b>	.16	<b>.70</b>	.10	<b>.73</b>	<b>.77</b>	<b>.78</b>	.03
Comp. Knowledge (Gc)	<b>.83</b>	<b>.93</b>	.13	<b>.95</b>	-.01	<b>1.11</b>	.01	--	--	--
Visuo-Spatial Processing. (Gv)	<b>.75</b>	.27	<b>.45</b>	<b>.67</b>	.003	.04	<b>.73</b>	<b>.75</b>	<b>.72</b>	.06
Short-term Memory (Gsm)	<b>.73</b>	.31	<b>.45</b>	<b>.73</b>	.01	.14	<b>.71</b>	<b>.80</b>	<b>.77</b>	.06
Long-term Retrieval (Glr)	<b>.71</b>	.13	<b>.56</b>	<b>.66</b>	<b>3.07</b>	.00	.00	<b>.72</b>	.17	<b>.67</b>
Processing Speed (Gs)	<b>.58</b>	.003	<b>.48</b>	<b>.46</b>	.01	-.19	<b>.78</b>	<b>.61</b>	.74	-.13
Quant. Reasoning (Gq)	<b>.94</b>	-.07	<b>1.11</b>	--	--	--	--	--	--	--
Auditory Intelligence (Ga)	<b>.64</b>	.06	<b>.66</b>	<b>.55</b>	.04	.26	.30	<b>.53</b>	.24	.35
Reading and Writing (Grw)	<b>.63</b>	<b>.92</b>	-.06	<b>.82</b>	.07	<b>.74</b>	.01	<b>.63</b>	-.06	<b>.84</b>
Emotional Intelligence (Gei)	<b>.75</b>	.35	<b>.50</b>	<b>.73</b>	.02	.38	.39	<b>.69</b>	<b>.40</b>	.35

	<i>Intercorrelations for the Two- and Three-Factor Solutions</i>					
	I	II	III	I	II	III
<b>Factor I</b>	1.00			1.00		
<b>Factor II</b>	.65	1.00		.18	1.00	
<b>Factor III</b>	--	--	1.00	.20	.66	1.00

**Table 3b.**

Fit Statistics and Factor Loadings for the Final 1-, 2-, and 3- Factor Exploratory Solutions of Broad Intelligences Using the Conceptual Removal of Broad Abilities (N = 20,498)

	Fit Statistics – First Analysis			Fit Statistics – Removal of Gf, Gc, and Gq		
	RMSEA	CFI	TLI	RMSEA	CFI	TLI
<b>One Factor</b>	.28	.69	.60	.12	.93	.89
<b>Two Factors</b>	.22	.86	.75	.07	.99	.97
<b>Three Factors</b>	--	--	--	.03	1.00	.99

Broad Intelligence	<i>Factor Loadings</i>								
	One-Factor Solution			Two-Factor Solution			Three-Factor Solution		
	I	I	II	I	I	II	I	II	III
Fluid Intelligence (Gf)	<b>.80</b>	<b>.66</b>	.16	--	--	--	--	--	--
Comp. Knowledge (Gc)	<b>.83</b>	.13	<b>.93</b>	--	--	--	--	--	--
Visuo-Spatial Processing. (Gv)	<b>.75</b>	<b>.45</b>	.27	<b>.74</b>	<b>.78</b>	.00	.79	.003	-.02
Short-term Memory (Gsm)	<b>.73</b>	.31	<b>.45</b>	<b>.77</b>	<b>.76</b>	.06	.84	.01	-.04
Long-term Retrieval (Glr)	<b>.71</b>	<b>.56</b>	.13	<b>.75</b>	.21	<b>.64</b>	.33	.11	.42
Processing Speed (Gs)	<b>.58</b>	<b>.48</b>	.003	<b>.57</b>	<b>.69</b>	-.10	.56	-.06	.08
Quant. Reasoning (Gq)	<b>.94</b>	<b>1.11</b>	-.07	--	--	--	--	--	--
Auditory Intelligence (Ga)	<b>.64</b>	<b>.66</b>	.06	<b>.54</b>	.23	.35	-.02	-.01	.69
Reading and Writing (Grw)	<b>.63</b>	-.06	<b>.92</b>	<b>.65</b>	-.07	<b>.84</b>	-.001	1.75	-.001
Emotional Intelligence (Gei)	<b>.75</b>	<b>.50</b>	.35	<b>.72</b>	<b>.49</b>	.27	.52	.04	.22

	<i>Factor Intercorrelations for the Two- and Three-Factor Models</i>					
	I	II	III	I	II	III
<b>Factor I</b>	1.00			1.00		
<b>Factor II</b>	.65	1.00		.31	1.00	
<b>Factor III</b>	--	--	1.00	.71	.33	1.00

### Exploratory Factor Analysis Including Three-Tier Models

In addition to exploring the structure among the broad intelligences using correlations obtained from two-tier models, we ran separate exploratory factor analyses on the correlation matrixes that included correlation estimates obtained from three-tier models. Given that these models only estimate the correlation between a given broad ability and *g*, with no further pathways between broad abilities, we had anticipated that such data would lead to a one-factor model. Following a similar procedure outlined in our manuscript, we sought as a standard of good fit, an RSMEA of less than .05, and a CFI and TLI of .95 or higher (Boomsma et al. 2012).

Exploratory analyses conducted on the three-tier, *g*-inclusive model data set converged on to one- and three-factors but failed to converge on to two factors. The fit for the one-factor model was somewhat poor, but trending towards the desired range (RMSEA = .15, CFI = .91, TLI = .88). The three-factor model improved upon the fit of the one-factor model substantially, with an RMSEA = .04, CFI = 1.00, TLI = .99. However, examination of the factor loadings of the broad abilities in the two-factor model revealed the presence of a Heywood case for emotional intelligence (*Gei*) on the third factor (loading = 1.22) and another Heywood case for long-term retrieval (*Glr*) on the second factor (loading = 1.28). It has been suggested that the presence of Heywood cases may mean that too many factors were extracted (Hoyle & Duval, 2004), lending support for the one-factor solution.

We aimed to explore whether we could improve the fit of our model including both types of data by sequentially removing the Heywood cases. We began with removing emotional intelligence and rerunning our analysis. The data converged on to one-factor but failed to converge on to two- and three-factors. Examination of the fit indices revealed a model of excellent fit (RMSEA = .09, CFI = .97, TLI = .96), suggesting the one-factor model fit the three-



tier, *g*-inclusive data best. Fit indices and factor loadings for the above analyses can be found in Table 4.

**Table 4.**

Fit Statistics and Factor Loadings for the 1-, 2-, and 3- Factor Exploratory Solutions of Broad Intelligences Using Three-Tier Data (N = 51,051 )

	<b>Fit Statistics – First Analysis on Combined Data</b>			<b>Fit Statistics – Removal of Gei</b>		
	<b>RMSEA</b>	<b>CFI</b>	<b>TLI</b>	<b>RMSEA</b>	<b>CFI</b>	<b>TLI</b>
<b>One Factor</b>	.15	.91	.89	.09	.97	.96
<b>Two Factor</b>	--	--	--	--	--	--
<b>Three Factor</b>	.04	1.00	.99	--	--	--
<b>Factor Loadings</b>						
<b>Broad Intelligence</b>	One-Factor Solution	Three-Factor Solution			One-Factor Solution	
	<b>I</b>	<b>I</b>	<b>II</b>	<b>III</b>	<b>I</b>	
Fluid Intelligence (Gf)	<b>.92</b>	<b>.96</b>	.01	-.06	<b>.92</b>	
Comp. Knowledge (Gc)	<b>.84</b>	<b>.75</b>	.01	.12	<b>.83</b>	
Visuo-Spatial Processing. (Gv)	<b>.82</b>	<b>.77</b>	.03	.06	<b>.82</b>	
Short-term Memory (Gsm)	<b>.79</b>	<b>.75</b>	.01	.06	<b>.79</b>	
Long-term Retrieval (Glr)	<b>.90</b>	.01	<b>1.28</b>	-.004	<b>.91</b>	
Processing Speed (Gs)	<b>.61</b>	.37	.06	.27	<b>.59</b>	
Quant. Reasoning (Gq)	<b>.86</b>	<b>.98</b>	-.08	-.07	<b>.86</b>	
Auditory Intelligence (Ga)	<b>.82</b>	<b>.65</b>	.16	.06	<b>.82</b>	
Reading and Writing (Grw)	<b>.81</b>	<b>.69</b>	.10	.07	<b>.81</b>	
Emotional Intelligence (Gei)	<b>.76</b>	.01	-.003	<b>1.22</b>	--	
<b>Factor Intercorrelations</b>						
	<b>I</b>	<b>II</b>	<b>III</b>			
Factor I	1.00					
Factor II	.69	1.00				
Factor III	.62	.37	1.00			

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