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Multipoint measurements of field aligned current density in the auroral zone

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Multipoint Measurements of Field Aligned Current Density in the Auroral Zone

BY

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DISSERTATION

Submitted to the University of New Hampshire in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Physics

May 2001
This dissertation has been examined and approved.

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Dedication

To my parents and to my husband

for their love and support
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ABSTRACT

Multipoint Measurements of Field Aligned Current Density in the Auroral Zone

by

Yihua Zheng
University of New Hampshire, May, 2001

In this thesis we discuss the results of the Enstrophy sounding rocket, launched from Poker Flat Research Range on the evening of February 11, 1999. The rocket flew through a very dynamic auroral region with multiple bright arcs and into the polar cap. Four Free Flying Magnetometers employing autonomous, nano-spacecraft technology and designed by JPL were deployed from the main payload during the flight and multipoint magnetic field measurements were made.

Magnetic field data reduction was performed on data obtained from the FFMs. The data reduction procedure is very complicated in the sense that it requires transformation from a spinning and precessing coordinate system (measurements are in this system) to a non-spinning, non-precessing, Earth-magnetic-field aligned B-L system (z axis is along B—the Earth magnetic field, x is in the B-L plane and pointing away from L—the angular momentum vector, and y axis comprises the right-handed coordinate system) and the extraction of magnetic fluctuation on the order of 10s nanotesla (nT) from a signal on the order of 10^4 nT. Therefore, very accurate fitting of all the involved parameters is a necessity. Details of the data reduction procedure are discussed. Large magnetic field fluctuations were seen by all the FFMs when the rocket was near its apogee (about 1070 km), at the poleward edge of an auroral arc.
Field Aligned Current (FAC) density was calculated from the multipoint magnetic field measurements by Taylor series expansion to the first order. Both spatial structures and temporal variations are seen during this event and interpretations of the results are made. The delays in the magnetic fluctuations between the FFMs indicates current sheet structures were moving relative to each other, which is further supported by the fact that the results from a simple model of multiple payloads crossing through several moving current sheets could reproduce most of the delays in magnetic field measurements. But at other times, the magnetic perturbations on different FFMs did not correlate well with any time delay, which indicates the presence of localized Alfven waves and/or even more filamentary currents. The non-zero deflections in magnetic field magnitude might be considered as the presence of compressional Alfven waves. Further study of this event was done by applying wavelet transformation and correlation analysis to the FFM measurements. The motions of individual structures were deduced using this method.
Chapter 1

Introduction

1.1 Earth's ionosphere and its space environment

The Earth's ionosphere consists of partially ionized gas and in a sense acts as the interface between the Earth's neutral atmosphere and the sun's fully ionized atmosphere. This special characteristic of the ionosphere entails the integration of electrodynamics and plasma physics. The ionosphere is the site of the many interesting processes (such as particle ionization, particle acceleration, wave-wave interactions, particle and DC field interactions, etc.) including the well-known northern lights (aurora borealis) and southern lights (aurora australis) - which comprise the aurora. The rich physics and relatively easy accessibility of the ionosphere (compared to the near Sun region, and the magnetosphere region) has made it a constantly interesting locus of space science investigations and studies.

Earth's magnetosphere

The Earth's ionosphere is not just a separate entity existing in the space on its own. It is immersed in the atmosphere of the Sun and the Earth's magnetic field which comprises the magnetosphere. To be a little more exact, the Earth's magnetosphere has two important parts. The first is the Earth's magnetic field, which is created by currents in the core. To first order the Earth's magnetic field is that of a dipole whose axis is tilted with respect to the spin axis of the Earth's by about $11^\circ$, which tilts towards the North American continent.
(this offset is common to several planetary magnetic fields.). The magnetic field \( B \) points down into the Earth in the northern hemisphere and points away from it in the southern hemisphere. The second part is the magnetic field induced by the solar wind, a fully ionized hydrogen/helium plasma that streams continuously outward from the Sun into the solar system at speeds of about 300-800 kilometers per second. The solar wind is composed of protons and alpha (helium) particles, together with energetic electrons which keeps the charge neutral overall. The solar wind is also pervaded by a large-scale interplanetary magnetic field (IMF), the solar magnetic field expanded outward into the solar system by the solar wind plasma. The interaction between the Earth's magnetic field and the solar wind shapes the magnetosphere, a cavity surrounding the Earth, which protects life on this planet from high energy particles from the Sun and Galaxy. On the sunward side, the Earth's magnetosphere is compressed to about 6-10 \( R_E \) (Earth radii). However, the solar wind drags out the night side magnetosphere to possibly 1000 \( R_E \). The exact length is not known. This extension of the magnetosphere is known as the magnetotail. Figure 1-1 shows the interaction between the Sun and the Earth's magnetic fields. Different regions of the magnetosphere are also shown in the diagram. Close to the Earth is the ionosphere, which also plays an important role in the interactions of the Sun-Earth system. The picture is taken from http://science.msfc.nasa.gov/ssl/pad /sppb/edu/magnetosphere/images.

**Aurora generation**

At both ends of the poleward region the magnetic fields lines are partially open, and connect to the interplanetary magnetic field (IMF) of the Sun and also connect to the magnetotail region — a very dynamic region. Magnetic fields can deflect charged particles, and the
Figure 1-1: View of the Earth's ionosphere, magnetosphere and solar wind interactions.

Earth's magnetic field stops most of the solar wind particles from entering the Earth's atmosphere and coming close to the Earth. Some particles, however, leak across the boundary of the magnetosphere, to the magnetotail. The Earth's magnetic field lines guide the ionized particles, and they are constrained to move in helices around the magnetic field lines. After they pass through the auroral acceleration region (altitude 1000 km - 10,000 km), the particles rain down upon the Earth's atmosphere and collide with the atmospheric molecules and atoms, causing them to fluoresce like the gas in a neon lamp — the aurora phenomenon. This visual display occurs at typical altitudes from 100-300 km, depending on what kind of species of ions are excited (oxygen, nitrogen, etc) and the energetics of the precipitating particles.
1.2 Field aligned currents in the auroral zone

The auroral zone, a region where the magnetosphere meets the ionosphere and a region capable of exciting different kinds of waves [Carlson, et al., 1998a; Gustaffson, et al., 1990; Lysak, 1999; Wahlund, et al., 1994] particle acceleration [Evans, 1974; Klumpar, 1979; Yau, et al., 1983; Arnoldy, et al., 1992; Newell, et al., 1996; McFadden, et al., 1999] and heating [Temerin and Roth, 1986; Kagan et al., 1996; McFadden, et al., 1999], turbulent flows [Kintner, 1976; Kintner and Seyler, 1995] and other processes [Kelley, 1977; Lysak, 1991] under various conditions, is governed by electric currents. In particular, field aligned currents (FACs, often called Birkeland currents after the person who first postulated their existence) [Birkeland, 1908] are essential to the linkage between the solar wind–magnetosphere system and the ionosphere, through which the transverse momentum is transferred along the field lines, along with a transverse electric field and electromagnetic energy [Watanabe, et al., 1996]. The intensity and spatial distribution of FACs are controlled by the magnitude and the direction of the interplanetary magnetic field (See review by Potemra [1994]). Ultimately they are controlled by the solar activity, and the interactions of, and various balances between, the ionosphere, magnetosphere and the Sun.

An enhancement of FACs is seen often during storms or substorms. One generation mechanism for FACs is associated with pressure gradients and parallel vorticities in the magnetotail —$(\nabla \times v)_{||}$ and this has been derived analytically by Cheng[1996]. Figure 1-2 shows the flow of current between the auroral zone and the Earth’s magnetotail including the field-aligned currents which are indispensable to magnetosphere-ionosphere coupling. Field aligned currents in the auroral zone are often associated with the generation of aurora. The
intensive FACs are often found to be at the edge of the auroral arcs. The main large scale FAC systems are the Region 1 and 2 (R1/R2) currents. Figure 1-3 shows the stable features of the field aligned current system during weakly disturbed conditions ( | AL | < 100γ). The 'hatched' area shown between 11:30 and 12:30 MLT in the polar cusp region indicates that the current directions here are often uncertain. Region 1 currents are shown as the inner ring driving the R1/R2 system.

Region 1 currents are directed into the ionosphere in the morning hemisphere, and directed out of the ionosphere in the evening hemisphere. They are related to the electron precipitation in the region of discrete aurora, and expand to lower latitudes with increasing activity (the auroral oval also expands) during storms or substorms.

They get weaker during weak activity (northward IMF, i.e., NBZ) and maximize between 0800 and 1000 MLT in the morning side and between 1400 and 1600 MLT in the afternoon side. The currents increase as the electric field associated with the solar wind/IMF increases,
but they are non-zero even during zero electric field.

The outer ring in the plot represents Region 2 currents. They have directions opposite to the Region 1 current system and respond to activity level as the Region 1 current ring varies. Compared to Region 1 currents, they are weaker and are usually related to diffuse aurora. Region 1 and Region 2 currents close at the lower end of the ionosphere through Pedersen and Hall currents [Bering and Mozer, 1975; Kintner and Cahill, 1974].

Although the field aligned currents in auroral zone are the reflection and result of strong geomagnetic activity taking place in the tail of the magnetosphere, the ionosphere is not a passive receiver. The spatial structure, the waves and the particle dynamics of the ionosphere also greatly influence the processes happening in the magnetosphere through its field-aligned currents.
Despite their great importance in the interaction of the Earth and the Sun system, the field-aligned currents' existence was first proposed by Birkeland [1908] only at the beginning of the 20th century. It is not until 1966 [Zmuda, 1966] that satellite observations of magnetic disturbances [Zmuda et al., 1967] perpendicular to the Earth's magnetic field provided the direct evidence of their existence. Since then extensive studies have been done on field aligned currents, the relationships between field-aligned currents, electric and magnetic fields, and particle acceleration using data from satellites, sounding rockets, and ground-based radars. Two landmark statistical studies were performed by Zmuda and Armstrong [1970], and Iijima and Potemra [1976], using Triad data at altitude of 800 km, and similar studies were done by Zanetti et al. [1983]. These studies' focus were on the location, polarity, intensity and closure patterns of FACs, and their dependence on global geomagnetic conditions. For a review of earlier space-based studies of FACs, see Potemra [1985]. Yamashita et al. [1998] did a thorough multievent study to examine the relationship between large-scale, meso-scale and small-scale FACs, and carriers of FACs. It also included a review of recent FAC studies. More studies on FACs can be found in [Erlandson, et al., 1988; Fujii, et al., 1987; Gussenhoven et al., 1988; Heppner, et al., 1987; Potemra, 1994; Rich et al., 1987; Siscoe et al., 1991; Sugiura et al., 1976; Taguchi et al., 1993; Watanabe, et al., 1996]. Recent space-based auroral FACs studies have been mostly concentrated on the field aligned currents related to aurora, and their relations to particle acceleration, wave generation (current driven instabilities) [Kindel and Kennel, 1971]; and electric and magnetic fields. Examples are the measurements from Freja, Polar and Fast satellites. The fine structure of field aligned current sheets was obtained from the measurements of Freja; a lower limit of 1.75 km thickness for the field aligned current filaments was deduced from a
single payload measurement [Lühr et al., 1994]. FAST observed signatures of small-scale downward going current at the edges of the inverted-V regions where the primary (auroral) electrons are found. Relations between electron precipitation, electric field, perturbed magnetic field, number flux and current densities were studied.

Peria [2000] performed a pilot statistical study of FACs, using an automated FAC-finding technique. The study reproduced not only the familiar statistical location and polarity pattern of large-scale currents, but also arrived at the conclusion that the net current (the part which closes along the auroral zone or across the polar cap) comprises both the large-scale currents and the more numerous, finely-structured currents, implying the fine structure is an integral part of the global current system. The result also shows that FACs tend to align themselves with the statistical auroral zone. The fine structure of auroral current circuit from FAST is shown in Figure 1-4 [Elphic et al., 1998]. The notations of the figure are: thin solid shows the potential contours associated with downward currents; thick solid indicates the downgoing currents; thin dashed contours denote negative potentials; thick hatched indicates the upgoing field-aligned currents; thick hollow ones at the bottom indicate the ionospheric electric fields; Large, thick grey shaded region indicates the upgoing currents, broad inverted-V region. The thickness of the vertical lines is to be used to represent the thickness of the corresponding observed field-aligned currents.

Figure 1-5 could serve as a quick summary of the principal physical processes taking place in the auroral current region. It shows the scientific highlights of FAST observations in the auroral zone [Carlson C. W. et al., 1998a], delineated by the sense of field aligned currents.

The existing theory and observations of FACs and their relations to particle acceleration
Figure 1-4: The current closure of FACs in the auroral zone (as from Elphic et al., [1998]).
The symmetric auroral current regions

1. Downward current region.
2. Diverging electrostatic shocks.
3. Small-scale density cavities.
5. Ion heating transverse to B. Energetic ion conics.
6. ELF electric field turbulence. Ion cyclotron waves.
7. Fast solitary waves: 3d rapidly moving electron holes.
8. VLF saucer source region.

1. Upward current region.
2. Converging electrostatic shocks.
3. Large-scale density cavity.
5. Upgoing ion beams. Ion conics.
6. Large amplitude ion cyclotron waves and electric field turbulence.
7. Nonlinear, time-domain structures associated with ion cyclotron waves.
8. AKR source region.

Figure 1-5: The principal physical phenomena in the auroral upward and downward current regions (as from Carlson et al., 1998a).
electric and magnetic fields can be categorized into the following three scenarios. Most if not all agree that energy flow from the magnetotail into the auroral zone drives auroral dynamics. In the static picture, the relationship between different physical quantities is static or quasi-static and the structuring in them is spatial. The existence of parallel electric fields has been explained by energy and pitch angle anisotropies in magnetospheric particle distribution, invoking anomalous resistivity, electrostatic shocks, or magnetic mirror forces due to dipolar magnetic field. The second scenario involves temporal variations, i.e., Alfvénic plasma waves. The parallel electric field, parallel potential drops and field-aligned currents are developed when magnetospherically generated Alfvén waves propagate through the ionosphere and interact with the ionosphere. The third scenario is an interpretation of the relationship among the observed different quantities which incorporates both the quasi-static and the temporal point of view.

Before embarking on a discussion of the three scenarios, first we need to talk about the electric fields in the auroral zone since they are essential for particle acceleration across the region.

The generation of electric fields in the quasi-static state can be understood as follows. Electric fields arise as a result of the forces acting on particles if the ions and electrons respond to them differently. Quantitatively speaking, any divergence of electric current results in a non-zero total charge density \( \rho_c \) [Kelley, 1989].

\[
\nabla \cdot \mathbf{J} = -\frac{\partial \rho_c}{\partial t}
\]

Any charge density will create electric fields via Poisson's equation.

\[
\nabla \cdot \mathbf{E} = \frac{\rho_c}{\varepsilon_0}
\]
It is not surprising that the field aligned currents in the ionosphere have a finite divergence due to the complexity of the forces on the particles and the inhomogeneity of the ionosphere. Electric fields [Tetreault, 1991] both in the direction of the Earth magnetic field [Hoffman and Evans, 1968; Hultqvist et al., 1971; Frank and Ackerson, 1971; Whipple, 1977; Gorney, et al., 1981; Christensen, et al., 1987; Reiff, et al., 1986, 1988; Carlson, et al., 1998b] and transverse to it [Bering, 1973; Boehm, et al., 1990b; Evans, 1974; Mallinckrodt et al., 1978; Marklund, 1984; Marklund, et al., 1998; Mozer et al., 1979; Mozer, 1981; Mozer et al., 1997; Ganguli, et al., 1985; Pietrowski, 2000] have been observed. The parallel electric field, especially, has drawn space scientists' special attention because of its direct relation to understanding of the auroral acceleration mechanism, the acceleration of particles in the auroral zone and therefore the energy source for the magnificent auroral display. The study of $E_\parallel$ is an intriguing subject in the sense that it has been difficult to explain theoretically how a collisionless plasma supports a parallel electric field of the observed amplitude. Theories applied to the existence of parallel electric fields in the upward current region include anomalous resistivity [Hudson and Mozer, 1978], weak double layers [Temerin, et al., 1982], and magnetic mirror force [Chiu and Schultz, 1978]. For the downward current region, wave observations by FAST satellite reveal nonlinear electric field structures associated with parallel electric fields. Similar results have been seen from Polar satellite too [Mozer et al., 1997]. “Fast solitary waves”, discussed by Ergun et al. [1998a], may play a very important role in supporting the existence of parallel electric fields.

The electric field structures that have been observed include electrostatic shocks [Temerin, et al., 1981a], double layers [Lysak and Hudson, 1987], solitary waves, and coherent ion cyclotron emissions in the time domain; and large scale quasi-static parallel electric field.
associated with potential drops on the magnetic field lines in the space domain.

1.2.1 Static picture of FACs, magnetic fields, electric fields and auroral acceleration

There have been numerous observations and theoretical studies made which support static or quasi-static relations between electric and magnetic fields, field aligned currents and particle dynamics. In the upward field-aligned current region, the monoenergetic peak in the electron spectrum measured in inverted-V arcs provides good evidence for the existence of quasi-static parallel fields. The detection of ion beams streaming away from the Earth is also a signature of acceleration through quasi-static electric fields. In the quasi-static models, parallel electric fields [Evans, 1974, 1975; Mizera and Fennell, 1977; Shelley, et al., 1976] have been explained through three principal mechanisms.

One school of thought involves dissipationless 'electrostatic shocks' (Debye sheaths) [Block, 1975; Kan, 1975; Swift, 1975; Mozer, et al., 1977; Torbert and Mozer, 1978] using a static solution to the Vlasov-Poisson equations. The second mechanism concentrates on turbulence-resulted anomalous parallel resistivity [Papadopoulos, 1977; Hudson and Mozer, 1978]. In the third mechanism parallel electric fields are results of differential energy and pitch angle anisotropies of electrons and ions in a mirroring, dipolar magnetic field [Alfvén and Fälthammar, 1963; Persson, 1963; Lemaire and Scherer, 1974; Lennartsson, 1977; Whipple, 1977; Chiu and Schultz, 1978; Chiu and Cornwall, 1980; Jasperse, 1998; Jasperse and Grosspard, 2000]. In the work of Chiu et al. [1978, 80], it is shown that a self-consistent electrostatic field distribution including both parallel and perpendicular electric fields is valid under consideration of magnetic mirror forces, Poisson's equation, ionospheric charge
and current conservation coupled with precipitation sources and recombination losses, and boundary conditions at the equator, etc. His 1980 model also gives the latitudinal structures and properties observed in the auroral arcs. Schriver [1999]'s self-consistent PIC (particle-in-cell) simulation model in auroral zone not only explains the generation of large-scale quasi-static parallel electric fields, but also shows that an intense broadband wave spectrum was generated during the interaction of the Earthward streaming magnetospheric plasma and the ionosphere.

Very recent theoretical work done by Jasperse [Jasperse, 1998; Jasperse and Grosspard, 2000] gives an alternative derivation of the Alfvén-Falthammar formula for a upward parallel electric field $E_{||}$ in upward auroral current regions and its analog for a downward parallel electric field $E_{||}$ in downward auroral current regions. His model gives good agreement with data from Freja satellite and unifies the explanation of the existence of parallel electric field $E_{||}$ in both upward and downward auroral current regions by incorporating the right physical processes into the Vlasov equations. In his model, the parallel electric field is mainly due to the injected magnetospheric particles with velocity anisotropy in upward auroral current regions. In the downward auroral current regions, it is mainly due to turbulence heating [Carlson, et al., 1998a; Ergun et al., 1998a] of ionospheric ions.

Large scales of field-aligned auroral currents, electric potentials along the magnetic field lines and precipitation can be generated by the discontinuities in the convection electric fields in Lyons [1980]'s static description. Without involving waves and turbulence, the observations and theories have verified the existence of the potential drops along the magnetic field lines [Burch, et al., 1983; Gorney, et al., 1985; Carlson, et al., 1998b]. In a fairly large range of field aligned potential drop $\phi_{||}$, FAC is directly proportional to $\phi_{||}$ in the static
model [Lyons, 1980; Chiu and Cornwall, 1980; Fridman and Lemaire, 1980]. The parallel electric field is related to $\phi$ by $E = -\nabla \phi$.

The current-voltage relations have been derived both in the upward current region and downward current region (also often called 'return current region', where electrons are accelerated upwards instead of downward and being responsible for auroral generation for the upgoing current region). More observations and studies have been done on the upward current region than those of the downward current region because of its direct association with the spectacular aurora, while the upward current region is associated with "black aurora" [Marklund, et al., 1994; Trondsen and Cogger, 1997].

The current-voltage relation in the upward current region of the auroral zone was given by Knight [1973]. The key idea of the Knight equation is that because the plasma density is low in the magnetosphere, a magnetic field-aligned potential drop is a necessity for driving enough hot electrons into the ionosphere to produce the required current. Otherwise the upward directed mirror force makes the precipitation of electrons to the ionosphere impossible. The Knight formula also assumed that the motion of energetic electrons is adiabatic. In his model, the structuring in the observed quantities (such as electric fields, magnetic fields and field aligned currents etc.) is spatial and a result of the physics which governs the interaction between plasma and fields under static or quasistatic considerations. Studies conducted by Lu et al. [1991] show good agreement with the Knight formula.

The recent FAST satellite mission has made more observations of the downward current region compared to previous studies and advanced the understanding of the region. Despite the involvement and presence of various waves in the return current region, a qualitative current-voltage relation for this region was derived by Temerin [1998] using simple density
profiles of the background ion density and condition of charge neutrality. The model gives a quasi-static description that matches with FAST observations. Data from FAST for the past few years have shown many of the observed quantities fit into a static description and have spatial structures and suggest the quasi-static parallel electric field may be a fundamental particle acceleration mechanism in auroral zone (in both upward current regions and downward current regions) [McFadden, 1999].

There have been reports on observation of spatial structures in the electric field [Ergun et al., 1998b], spatial structures and gradients with ion beams, the hundreds-of-kilometers along B and only a few-to-tens-of-kilometers across B potential fingers at the lower boundary of auroral acceleration region. See McFadden [1999] for a detailed report on the microstructure of the auroral acceleration region observed by FAST.

In the static model, the sheetlike (further verified by Peria [2000]) field aligned currents, usually found to be at the edge of the auroral arcs and extending along the auroral arcs, have a disturbed magnetic field primarily in the west-east direction (y axis). The observed electric field in the south-north direction (x axis), $E_x$, is found to be highly correlated to the B variations in the y direction ($B_y$). The variations in E and B are both spatial, and the ratio of the zonal magnetic and meridional electric field components represents the height-integrated ionospheric Pedersen conductivity $\Sigma P$, where $\mu_0$ is the permeability of free space.

$$B_y/(\mu_0 E_x) = \Sigma P \quad (1.2.3)$$

The simple derivation can be described as follows: Ampere's law leads to

$$\frac{1}{\mu_0} \frac{\partial B_y}{\partial x} = j_z \quad (1.2.4)$$
The continuity of current equation gives

\[ j_z = -\frac{\partial I_x}{\partial x} \]  

(1.2.5)

where \( I_x \) is 'sheet current' in the north-south (x) direction and it relates to \( E_x \) via \( J = \sigma E \).

\[ I_x = \Sigma \rho E_x \]  

(1.2.6)

From the above relations, we get

\[ \frac{\partial B_y}{\partial x} = -\mu_0 \Sigma \rho \frac{\partial E_x}{\partial x} \]  

(1.2.7)

Therefore, the relation between \( B_y \) and \( E_x \) as described in Equation 1.2.3 is obtained.

The AE-C, S3-2, and S3-3 observations have shown a close relationship between \( \Delta B_y \) and \( E_x \) in the FAC regions [Bythrow et al., 1980; Smiddy, et al., 1980; Rich, et al., 1981]. Observations from DE-2 also indicate a good correlation between the two components \( B_y \) and \( E_x \). [Sugiura et al., 1982, 1983; Ishii, et al., 1992]. The \( E_x \) component directly relates to the Pedersen current and the \( B_y \) directly relates to the FAC at the measuring point and these currents are directly connected. The correlation of electric and magnetic field fluctuations related to the FAC region were also obtained by ICB 1300 [Dubinin et al., 1990], HILAT [Knudsen, et al., 1990, 1992], Freja [Lühr, 1994], FAST[McFadden et al., 1999; Elphic, et al., 1998].

1.2.2 Field aligned currents, magnetic fields, electric fields and auroral acceleration relations in time domain

There is a multitude of free energy sources in the Earth’s environment. Neither the ionosphere nor the magnetosphere are closed systems in a thermal equilibrium state, instead
they are driven by energy, momentum and mass input from outside, e.g., the solar wind. On the macroscopic scale this input produces gradients and inhomogeneities [Earle, et al., 1989] of the plasma. On the microscopic scale, it causes the deformation and distortions of the local plasma distribution functions. The existence of the free energy provides a source for wave growth in the Earth's auroral zone.

Waves in the auroral zone have a wide range and different varieties. They range from near DC (millihertz) fields to megahertz oscillations in the frequency domain, including ULF (ultra low frequency, millihertz-few hertz), ELF (extra low frequency, DC up to a few kHz), VLF (very low frequency, a few kHz- a few MHz) and HF (high frequency, hundreds kHz to MHz). The names stem from the days of radio observations. For example, the frequency band from kHz to MHz for VLF waves was considered to be 'very low frequency'. Waves can be either electrostatic or electromagnetic. There are ion waves (mainly caused by ion motion) and electron waves (caused by electrons). Different modes of waves exist in the auroral zone, such as Langmuir waves, VLF hiss, AKR (Auroral kilometric radiation, mainly electron cyclotron waves), lower hybrid waves, upper hybrid waves, whistlers, electromagnetic ion cyclotron waves [Erlandson and Zanetti, 1998], Alfvén waves, etc. The influence of waves is usually reflected in the observed intense, narrow and dynamic variations of auroral arcs, time modulated electron flux and the evolution of other observed quantities.

The static theory of the previous section fails at the edge of moving arcs where the FAC density can become very large (in excess of several tens of $\mu A m^{-2}$) even though the parallel potential drop $\phi$ is lower than in the center of the arc [Goertz, 1984, references therein]. FAST observations at the downward current region also show that waves and turbulence are
very important to the region. There have been many of this kind of observation where the static theory finds itself hard to give a reasonable explanation. Besides the observational difficulties under certain conditions, there are also some theoretical problems. It is quite impossible for an electrostatic theory to account for the dynamic variations of auroral arcs during storms or substorms. The thin, bright auroral arcs are hardly ever stationary. The static theory tends to predict a much larger scale length of the auroral arcs in the north-south direction while the observations tell us sometimes the arcs can be as narrow as 100 meters in the N-S direction.

All these suggest one should look at the evolution of electric magnetic fields and FACs caused by some propagating disturbances in the time domain. The only low frequency wave carrying a FAC is the Alfvén wave. It is widely known that Alfvén waves are everywhere in space plasmas and are the means by which information about changing currents and magnetic fields are communicated. It is also through these that the magnetic energy (Poynting flux) caused by the disturbances in the magnetosphere is transferred to the ionosphere and they lead a very important role in ionosphere-magnetosphere dynamics and coupling

[Lysak and Dum, 1983; Lysak, 1990; Sigsbee, et al., 1998]. Song and Lysak [1999] have proposed that the traditional theories of FAC generation, magnetic reconnection, and mass, momentum and energy transfer within the magnetosphere which were built on the basis of a convection picture, and describe mainly large-scale, quasi-static phenomena occurring in a passive plasma, should be replaced by dynamic wave packet theory which involves MHD waves to explain the FAC generation and magnetic reconnection in an active plasma. The POLAR spacecraft observed intense electric and magnetic field structures associated with Alfvén waves at and within the outer boundary of the plasma sheet at geocentric distances
of 4-6 $R_E$ near local midnight. The places where the structures appeared mapped down to intense auroral structures as detected by the Polar UV Imager [Wygant et al., 1999].

The existence of Alfvén waves can be observed from the ground too, in the form of magnetic pulsations in the different frequency ranges. They are usually categorized into Pc1, Pc2, Pc3 up to Pc5 [Kivelson and Russel, 1995]. Less structured mixtures of different frequencies magnetic pulsations are called Pi1 through Pi5. Reports on observations of them include [Arnoldy, et al., 1988, 1996, 1998; Erlandson et al., 1990; Erlandson et al., 1996; Grant and Burns, 1995].

The importance and direct observation results in space also inspired the investigations of Alfvén waves in lab plasmas. Alfvén waves of lab plasmas displayed similar behavior as those observed in space [Gekelman, 1999].

The importance of Alfvén waves to auroral phenomena has been proven both through theory work and the auroral observations. The relationship of propagating Alfvén waves through the ionosphere to auroral arc formation was first studied by Hasegawa [1976]. Following this, Goertz et al. [1979, 1984, 1985] and Haerendel [1983] did further studies on the Alfvén waves by including kinetic effects to the simple MHD (magnetohydrodynamics) description of them and their relation to the auroral arc formation [Kimney, 1999] and particle acceleration [Goertz, 1984, 1985; Kletzing, 1994; Rönnmark, 1999; Rönnmark and Hamrin, 1999]. Thorough theoretical studies on the properties of Alfvén waves can be seen in [Hasegawa and Uberoi, 1982; Streltsov and Lotko, 1995; Lysak and Lotko, 1996; Thompson and Lysak, 1996; Lysak, 1997, 1999; Hollweg, 1999; Streltsov, 1999; Streltsov and Lotko, 1995]. Observations of Alfvén waves have been reported from sounding rockets [Gelpi and Bering, 1984; Marklund et al., 1981; Boehm et al., 1990a; Nikolay Ivchenko,
1999] and satellites [Knudsen, et al, 1990; Volwerk et al., 1996; Elphic et al., 1998; Chaston et al., 2000].

In the interactions between Alfvén waves and the ionosphere, and their relation to the formation of auroral arcs, the scale length (width) of auroral arcs can be explained by the perpendicular wavelength of the involved Alfvén waves. However, the effect of the ionosphere on the propagation of Alfvén waves through the region can not be ignored. The high conductivity of the ionosphere along with the Alfvén velocity profile and other characteristics of the ionosphere provide for Alfvén resonator formation [Lysak, 1993; Trakhtengerts and Feldstein, 1991]. The ionospheric effect on the Alfvén waves was incorporated and investigated in Lysak's model [1997, 1999]. Alfvén waves are often observed at edge of the arcs and are associated with density gradients (mostly with density depletions — cavities) [Stasiewicz et al., 1997; Chaston et al., 2000]. Nonlinear structures of Alfvén waves have been described by [Wahlund, et al., 1994; Seyler, 1990; Seyler and Wahlund, 1995; Wang, et al., 1996].

The excitation of Alfvén waves could come from the resonant mode conversion of MHD surface waves. And most of the MHD instabilities which originate from the inhomogeneity of a plasma are the instabilities of a surface wave. Alfvén waves bridge the macroscopic instabilities to the microscopic instabilities [Hasegawa, 1976].

The importance of Alfvén waves in the auroral zone can also be understood from other perspectives by their association with BBELF (broad band extremely low frequency) emissions. The BBELF emissions are often observed in the regions of transverse ion acceleration (TAI) [Lynch, 1996; Bonnell, 1997] and broad-energy suprathermal electron bursts (STEB) occuring in the topside ionospheric auroral regions. They often have an enhanced spectral
power when solitary kinetic Alfvén waves (SKAW), or when large amplitude electric fields, possibly related to black aurora, are present in regions with large-scale density depletions. The association of BBELF emission with high-latitude small-scale auroral energization processes has been studied from detailed measurements. Details of this are beyond the scope of the thesis. A fluid-kinetic model, comprised of hot linear kinetic ions and cold nonlinear fluid electrons, was proposed by Seyler et al [1998]. It describes a nonlinear wave breaking process of small-scale Alfvén waves resulting in BBELF emission. The comparison of numerical results of the model to the measurements from Freja satellite support the theory that SIA (slow ion acoustic) waves are the result of a nonlinear emission from SKAW waves.

In the electrostatic limit (\(k \perp c/\nu_p \gg 1\)), SKAW waves are also called slow ion cyclotron (SIC) waves for clarity.

As in the static model, Alfvén waves also have a field aligned current, parallel electric field and field aligned potential drop \(\phi\), and are capable of accelerating particles through Landau damping or bounce resonance, trapping by waves and nonlinear acceleration etc. [Hasegawa, 1976]. But the relationship of FAC and \(\phi\) is not as simple as the one in the static model. Observations show that Alfvén waves tend to appear where the density gradients are. The perturbation of electric field and magnetic field have the following relation for pure shear Alfvén waves. The ratio of \(B_y\) and \(E_z\) has a range if the Alfvén waves are not pure shear mode or reflections due to the Earth's ionosphere have to be included. But the following relation serves as a basis for the discussion.

\[
B_y/(\mu_0 E_z) = 1/(\mu_0 V_A) \tag{1.2.8}
\]

Define \(Z_A\) as the characteristic impedance of the medium, \(Z_A = \mu_0 V_A\). \(Z_A\) is typically
much greater than $\Sigma_p^{-1}$, the inverse of the height-integrated Pedersen conductivity of the ionosphere.

In the auroral zone, because the arc thickness is often comparable to the ion gyroradius or electron inertia length ($c/\omega_p$), kinetic effects must be included when we discuss the Alfvén waves of this region. In contrast to ordinary Alfvén wave in collisionless plasma, they are usually called kinetic Alfvén waves.

Dispersion relations for kinetic Alfvén waves in two regimes (hot and cold plasma) are:

$$\omega^2 = k_z^2 v_A^2 \left[ 1 + k_z^2 \left( \frac{3}{4} + T_e/T_i \right) R_{gi}^2 \right]; \quad 1 \gg \beta \gg \frac{m_e}{m_i}$$

and

$$\omega^2 = \frac{k_z^2 v_A^2}{1 + k_z^2 c^2/\omega_p^2}; \quad \beta \ll \frac{m_e}{m_i}$$

where $z$ axis is upward along field lines, $x$ is in north-south direction and $y$ is in west-east direction. $\beta$ is defined as $\beta = v_e^2/v_A^2$. $v_A$ is ion thermal speed. $R_{gi}$ is the ion gyroradius. The kinetic Alfvén waves are called inertial Alfvén waves when $\beta \ll \frac{m_e}{m_i}$. The kinetic Alfvén waves in the auroral zone mostly are of this kind because the thermal velocity is much smaller than Alfvén wave velocity ($v_{th} \ll v_A$). The scale length for this mode is the electron inertial length. The scale length in the hot plasma is the ion gyroradius. The relations between every individual component of magnetic and electric fields are shown in Appendix A.

1.2.3 Temporal and spatial picture of FACs, fields and auroral acceleration

On the large scale, the physics of the auroral zone can be described using static or quasistatic theory. But on the small scale, the dynamics of auroral zone cannot be explained...
without involving all the waves existing in the region. The Earth's auroral zone is an open system connecting to the other parts of Sun-Earth system. The whole system is undergoing changes and disturbance all the time. The electromagnetic dynamics and plasma physics govern the evolution of the whole system. There is a ceaseless redistribution of particles, energy and momentum. The auroral zone, as part of the whole dynamic system and with inhomogenieties in particle density and species, has a rich variety in its own dynamics influenced directly by the geomagnetic activities originating from the Sun. Various waves are involved in the whole process. But it is not hard to imagine that there are times when the system quiets down and reaches a steady state or quasi-steady state on the large scale size, along with waves and turbulence happening on the edges of different regions (boundaries with gradients). The visual display of aurora is the manifestation of both temporal and spatial features of the phenomenon. We often see large arcs with relatively slow motion or in steady state over a long time period while at the same time the very narrow arcs or dancing rays are seen running around the large arcs with very rapid motion. For small spatial scale size, the possibility of the presence of spatially varying field aligned current filaments along with Alfvén wave activities is high.

Knudsen [1992] gives a model of Alfvén waves in the auroral ionosphere to distinguish Doppler-shifted static structures from true temporal variations caused by Alfvén waves and compared the model results with measurements both from a satellite (HILAT) and a rocket. The mixture of spatially varying static fields and the ones caused by Alfvén waves will make the impedance \( (\mu_0 \left| \frac{\partial E}{\partial r} \right|) \) fall somewhere between the pure standing wave impedance \( (\mu_0 v_A) \) and the reverse of the Pedersen conductivity \( (\Sigma_p^{-1}) \), the premise of this is that the impedance (or conductivity) changes with frequency. Satellite DE-2's
observations show the electric and magnetic field correlations in the field-aligned current regions satisfy the static relation (Equation 1.2.3) for large scale size; and Alfvén wave caused relation (Equation 1.2.8) for small scale size [Ishii, 1992]. Freja observations included the fine structure of field-aligned current sheets with a lower limit of 1.75 km and poor correlations of electric and magnetic field measurements on board. The results show there were wave modulations of the filamented field aligned current during the event. Wave signatures and FAC filaments were both observed [Lühr, 1994]. It should be mentioned that the measurements done by Freja are single-point; and the spatial and temporal distinctions were performed through ground observations. It is also shown by Louarn et al. [1994] that the low-frequency (1-20Hz) auroral electromagnetic turbulence consists of two kinds of phenomena: one is the magnetic fluctuations caused by quasi-static currents and the other is the strong electric spikes (greater than 100 mv/m) with magnetic (30 nT) and density \((dn/n \geq 30\%\)) fluctuations caused by solitary kinetic Alfvén waves. The two events shown in the report occurred in a less than 4 second time period and it shows that on small scale, temporal and spatial features are interspersed.

Measurements from the most recent auroral FAST satellite reproduce the large scale Birkeland current system along with many finely structured FACs near the auroral acceleration region. But there are also reports on the observed Alfvén waves from FAST in the region with density gradients [Chaston, et al., 2000]. Relatively intense waves' presence in the return current (downward) region is also a salient feature in the auroral zone [Temerin, 1998]. The connection between 'electrostatic shocks' and kinetic Alfvén waves drawn by Lysak [1998] is a proof that the temporal features related to kinetic Alfvén waves and normally spatial features denoted as 'electrostatic shocks' are essentially the same.
phenomenon under different limits. Similarly, the electrostatic simulation model done by Schriver [1999] shows that a broad band wave spectrum was generated during the process where the quasi-static parallel electric fields were developed, which further proves that the static features and temporal variations could coexist and be hard to separate. In a word, in the low frequency range it is difficult to distinguish what is spatial and what is temporal. A given signature could be manifestation of one physical mechanism under slightly different conditions. Observations from Freja and FAST satellites show the spatial structures and temporal variations can well be interspersed in the auroral zone.

1.2.4 Methods of measuring field aligned currents

Although great success and progress have been made in terms of auroral process studies, direct measurements of field aligned currents have been very limited because of technology associated difficulties. The commonly used methods by previous rockets and spacecrafts are the following:

1.) Current inferred from its carriers (most are electrons);
2.) Current inferred from electric fields and conductivities;
3.) Current inferred from magnetic fluctuations.

Current inferred from its carriers

For the first method, if FAC's carriers are mostly electrons having a distribution function $f(v)$, the current can be expressed

$$j_{||} = Ne \int v_{||} f(v) dv$$

$$j_{||} = Ne \int v_{||} f(v) dv$$

(1.2.11)
Since particle detectors provide measurements in energy $E$ and pitch angle $\alpha$ space, we need to convert from energy and pitch angle space to velocity space. The estimated current $j_{||}$ is:

$$j_{||} = 2\pi \int_0^\infty \int_0^\pi J(E, \alpha) \sin \alpha \cos \alpha \, d\alpha \, dE \tag{1.2.12}$$

Where $J(E, \alpha)$ is the directional differential number flux [Lyons and Williams, 1984], related to the distribution function by

$$f(v) = 2m^2 \frac{J(E, \alpha)}{E} \tag{1.2.13}$$

And $J(E, \alpha)$ and $J_E(E, \alpha)$ (the differential energy flux) are related by

$$J(E, \alpha) = \frac{J_E(E, \alpha)}{E} \tag{1.2.14}$$

There are some shortcomings with this method. It is hard to measure the electrons with thermal energies below 10 eV (especially the very low energy electrons— with energy below 0.1 eV) because 1) their gyroradius is in the order of the scale length of the detector; 2) their energy is comparable to $\phi_e/c$ (the charge spacecraft’s potential), which can constitute the majority of the field aligned current carriers. The very field aligned electrons are also difficult to measure because of the singular direction of $B$. They can be easily missed. The sensors must be able to measure electron/ion relative drift velocity up to 30,000 m/s for plasma density of 1000/cc and current density of $5\mu A/m^2$. Details of the instrumentation difficulties can be found in Lynch et al. [2000].

**Current inferred from electric fields and conductivities**

For the second method inferring $j_{||}$ from electric fields and conductivities, we assume the current sheet is along $B$, and use $\mathbf{J} = \sigma \cdot \mathbf{E}$ and $\nabla \cdot \mathbf{J} = 0$. $j_{||}$ relates to the Pedersen
conductivity ($\Sigma_P$) and Hall conductivity ($\Sigma_H$). [Kelley, 1989] by

$$j_\parallel = -\frac{1}{v_x} \left[ \frac{d}{dt}(\Sigma_P E_z) - E_y \frac{d\Sigma_H}{dt} - \frac{\partial}{\partial t}(\Sigma_P E_z) + E_y \frac{\partial \Sigma_H}{\partial t} + \Sigma_H \frac{\partial B_z}{\partial t} \right]$$  \hspace{1cm} (1.2.15)

where $z$ axis is parallel to $B$, $x$ is geomagnetic north and $y$ is the east. The total time derivative $\frac{d}{dt}$ of a quantity measured in the rocket frame is related to the partial time derivative $\frac{\partial}{\partial t}$ and spatial gradient $\nabla$ in the plasma reference frame by

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$  \hspace{1cm} (1.2.16)

where $\mathbf{v}$ is the rocket velocity in the plasma reference frame.

In steady state, $\frac{\partial}{\partial t} = 0$. Therefore,

$$j_{\parallel, steady} = -\frac{1}{v_x} \left[ \frac{d}{dt}(\Sigma_P E_z) - E_y \frac{d\Sigma_H}{dt} \right]$$  \hspace{1cm} (1.2.17)

The shortcoming associated with this method is that there are many assumptions made in the derivation of Equation 1.2.15. This method assumes the estimated currents are sheetlike and they are in steady state; and this method involves measurement of many quantities.

**Current inferred from magnetic fluctuations**

A relatively better method for estimate of currents is the third one, and it is used often. The estimate of current density $j_\parallel$ comes from measurement of magnetometer(s) (magnetic field observations). The basic relation for evaluating the current density is Ampere's law

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$  \hspace{1cm} (1.2.18)

The displacement current term ($\varepsilon_0 \partial \mathbf{E}/\partial t$) is ignored because it is very small in the auroral zone.
Most of the observations so far have been single point measurement. To obtain estimates of the current density from single point measurement, some assumptions about the spatial and temporal behaviour of the currents have to be made. Assuming the observed currents are spatial, one can get $j_{∥}$ [Lühr et al., 1994] by

$$j_{∥} = \frac{1}{\mu_0} \left[ \frac{1}{v_x} \frac{dB_y}{dt} - \frac{1}{v_y} \frac{dB_x}{dt} \right]$$

(1.2.19)

where $v_x$ and $v_y$ are the components of the spacecraft velocity perpendicular to the magnetic field lines.

Most of our knowledge about FACs is from measurements using this method. In order to use this method effectively, strict selection scheme has to be applied, i.e., there have to be other data supporting the observed current is spatial and 'sheetlike' (See Peria et al. [2000] for his FAC finder method related to this), otherwise using this method would give an erroneous answer. The temporal aspects of the field aligned currents will not be accounted for by this method.

To overcome the shortcomings associated with the above methods, direct measurement methods of electric currents (which do not use many assumptions) are in great need. One method is the multipoint measurement technique using multipayloads (multipolar magnetometers) to measure magnetic field at different locations. Multipoint measurement of the magnetic field is not only a method which can directly measure currents but also a method which can help to resolve spatial and temporal ambiguity problems associated with many observed phenomena in space. Efforts made on multipoint measurement have included the sounding rocket Auroral Turbulence II (with 3 payloads) launched in Feb, 1997 from Poker Flat Research Range; the recently launched CLUSTERII (has 4 spacecrafts tak-
ing data in different locations); and the Enstrophy sounding rocket which was launched in Feb, 1999 from Alaska with four free-flying-magnetometers (FFMs) on board providing multipoint measurement of field aligned current in auroral zone. Results and analysis from measurements of the Enstrophy sounding rocket are the pith of this thesis.

1.3 Motivation for Enstrophy Mission

The shortcomings of previous measurements of FACs, the need for direct measurements of FACs, the desire to study fine structures of FACs and to distinguish spatial and temporal signatures, all motivated the initiation and launch of the Enstrophy sounding rocket, a winter 1999 premidnight launch with an apogee of ~1000 km from the Poker Flat Research Range, Alaska.

The Enstrophy sounding rocket mission made a multiple-point measurement of the magnetic field, which was used to calculate field-aligned current density along the rocket trajectory. Four small autonomous ‘nanospacecraft’ (Free-Flying Magnetometers, or “FFMs”) were ejected from the main payload, perpendicular to the spin axis of the payload. The four FFMs, with spin rate of 15-17 Hz, made measurements of the magnetic field at four points surrounding the main payload, at separation distances up to 3 km, and telemetered their data, in bursts, to the ground. Plasma diagnostics on the payload were intended to measure the plasma environment and to allow studies of wave-particle interactions [Lynch, 1997].

Previous sounding rockets and satellites typically have measured variations only along one trajectory in space and time. “Field-aligned current measurements” usually assumed that the variations of the magnetic field are spatial, and that the currents were sheetlike.
Observations and theories of auroral processes have suggested that the observed gradients in the magnetic field could equally well be variations in the time domain or both (spatial and temporal coexisting). Any 3 FFMs of Enstrophy allow for a direct, unambiguous measurement of the local current density on scale of probe separation or larger. The multipoint measurements of this mission would be able to help distinguish spatial and temporal signatures in the observations.

1.4 Thesis statement

In this thesis we will present the analysis and interpretation of a multipoint observation of magnetic field structures at the poleward edge of a premidnight auroral arc from the Enstrophy sounding rocket mission. Both spatial and temporal signatures were found to be present in the event where the large B fluctuations were seen at the edge of an arc when the rocket flew into the polar cap. We will show the direct measurement method of current density using multipoint measurement of magnetic fields gives us a different current density than what would be inferred from a historical single-point measurement. Reasons for the interpretation of spatial or temporal features are given, and supported by: 1) a simple model of multiple payloads crossing through several moving current sheets, 2) non-zero deflection in magnetic field magnitude, and 3) the fine structure study of this auroral event using multipoint, correlative wavelet analysis and the supporting data from other instruments on board. While this thesis concentrates on data from one sounding rocket mission, data analysis methods (including the magnetic field data reduction, the multipoint measurement of FAC density, wavelet analysis and correlation study for multipoint measurements) and science questions concerning multipoint data sets are of increasing importance to the whole...
space science community.

The outline of the thesis is as follows: in Chapter 2, we give an overview of the Enstrophy instrumentation. In Chapter 3, we present the global and local launch environment, the details of the vehicle and instrument performance, and the flight data summary. Data analysis techniques, including the very challenging and elegant data reduction procedure on the magnetic field data from four FFMs and the calculation of current density using multipoint measurement and applying Taylor expansion to obtain the partial derivatives with respective to position, are described in Chapter 4. Spatial or temporal signature interpretation of the observed magnetic field fluctuations and current density is reported in Chapter 5. Also included in Chapter 5 are a simple moving-current-sheets model, wavelet analysis and correlation study applied to the observed quantities in order to deepen our understanding of the event.
Chapter 2

Enstrophy Instrumentation

The instrumentation of the Enstrophy sounding rocket is described in this chapter. Its main payload provided the test flight for Free Flying Magnetometers (FFMs, "Hockey Pucks"). Four FFMs were deployed from the main payload. The FFM concept, design considerations, goals, deploy mechanism, etc. are covered here. Also carried on the main payload were particle detectors and electric and magnetic field instruments. The configuration of the main payload is shown in Figure 2-1.

2.1 Free Flying Magnetometers

2.1.1 Design Description

The FFM design used in the Enstrophy mission employed the latest technology and was the first generation of miniaturized and integrated "sensorcraft" developed at JPL (Jet Propulsion Laboratory). It is autonomous in the sense that it has its own telemetry and sends the data directly to the ground. A sketch of an FFM is shown in Figure 2-2. The FFMs are about 250 g each. It is 8 cm in diameter and 3.8 cm in height, and it carries a miniaturized 3-axis flux-gate magnetometer (Applied Physics Systems, Inc., $1.2 \times 1.2 \times 1.2 \text{ cm}^3$), sitting in the middle, and 7 Li-Chloride (LiSOCl2, Eagle Picher LTC-312) batteries as its power supply. It contains a BPSK (Binary Phase Shift Keying) single frequency transmitter in the frequency range of 2210-2290 MHz (S-band) using 20 mW power and a
Figure 2-1: Payload Layout.
matched patch antenna with 5 mm dielectric substrate. An integrated data system (FPGA) was used for data acquisition, which includes ADC (Analog to Digital Conversion), data formatting, power management and timing control, and ~ 1Mbyte memory. The FFM uses a temperature compensated oscillator (TCXO, Cardinal Components Inc.) for timing; it must be good to microsecond (µs) in order to align the four data streams of the four FFM to milliseconds (ms) accuracy over 1000 seconds of flight. Two sun sensors (US Army Research Laboratory) were used for precise spin phase and FFM angular motion determination (if the sun is visible); additionally, a laser diode was used to be seen by the main payload for providing attitude information in darkness. The FFM was designed to have two separate states—“test sequence” and “flight sequence”. These two signal detection electronics systems are needed for controlling two separate optical start signals, one for “test sequence” and one for “flight sequence”. Figure 2-3 shows the FFM layout.

The FFM’s intrinsic noise level is specified to be < .05 nT/√(Hz) above 1 Hz. The output is digitized to 1 nT resolution (17 bit A/D converter). During flight (using its flight sequence), each FFM collected data in three intervals and sent the data to the ground in 3 short bursts. i.e., three “data phases” and three “sending phases” were interleaved. The
Figure 2-3: The layout of FFMs.

Sampling rate for each of the three axes was 140 Hz.

As part of the requirement for dynamically stable FFMs with high spin rate, the FFMs were carefully spin balanced when they were built at JPL.

2.1.2 FFM Deployment System Design

The four free-flying magnetometers needed to be deployed simultaneously and symmetrically in the spin plane of the spinning main payload. The deployment system was designed to spin up the FFMs to a significantly higher spin rate than the payload. This is largely because the FFMs need as much stability as possible to be used against aerodynamic and other torques. The deployment system should also be as simple as possible — no motors were used. The serious design constraint was that there should be enough room for the FFMs to exit the main payload freely without butting the main payload longerons.
FFM exit path in rotating frame

Figure 2-4: Concept of the Deployment System.

In order to let the FFMs freely roll along the rails while the main payload was spinning, two decks were needed to deploy the FFMs, as every FFM had to be provided with a clean exiting path. FFM1 and FFM2 were in Deck 1, while FFM3 and FFM4 were in Deck 2. Deck 1 and Deck 2 were basically the same in structure, but rotated by $90^\circ$. Figure 2-4 is a diagram showing a concept of a deck. The FFM motion sequence relative to the rotating deck is also given in the figure.

At each end of the FFM track (the outer edge of the deployer deck), there was a velocity
Figure 2-5: Stowed Position of FFMs.

monitor (optical gate) to measure the velocity of the FFMs when they left the deck.

Looking down on the decks from the nose of the payload, the main payload had the right-handed rotation (counter clockwise) and FFM deployment was left-handed (clockwise) relative to the main payload. The stowed position of FFM1 was at 270°. FFM2 was at 90°, FFM3 was at 0° and FFM4 was at 180°. At the beginning of the launch, the 0 degree line was aligned in the south direction. Details are sketched in Figure 2-5.

A logarithmic spiral shaped rail was chosen to deploy the FFMs because it gives a much higher spin rate than the straight rail both by theoretical calculations and simulation results. Details are given in Appendix B. Track parameters used in fabrication were based upon the simulation results. The real deployer track used for this sounding rocket mission was made up by circular sections to approximate the logarithmic shape because the fabrication is much easier. The FFMs had to be stowed securely for the launch environment and while the main payload is spun up to 4 ~ 5 Hz. The release of FFMs had to be clean, quick and
simultaneous with a single pyrotechnic event. After the deployment, a separation velocity between the FFMs of 1-2 m/s was desired. The Enstrophy Mission realized these criteria.

2.1.3 Calibration and Data extraction

The FFMs were designed, built and calibrated at JPL. Prior to the flight, thorough testing was done at Wallops Island and Poker Flat Rocket Launch Range, including the collection of data from all sensors in the FFMs, FFM optical interface with the main payload, the transmission of the data through transmitter, etc. The complete flight sequence includes 3 phases: data phase 1, data phase 2 and data phase 3. Each lasted 5 minutes. At the end of each data phase was the short transmission time to the ground, which lasted 42 seconds. The measurements were done in data phases and sent to the ground antenna during the short transmission phase. Figure 2-6 shows timeline of the whole flight and the complete data acquisition process.

2.2 UNH particle detectors

The electron detector on board was a 10 eV to 15 keV electron top-hat electrostatic analyzer. It has 32 energy steps, 30 pitch angle bins, 16 energy sweeps per second. The geometry factor per bin is $1.2 \times 10^{-4} \text{ cm}^2 \cdot \text{sr-eV/eV}$. A new feature in this flight was a deflectable aperture plane, which was to keep the magnetic field line in view at all times. Figure 2-7 is a schematic view of the electron detector.

The ion detector used was a 6 eV to 800 eV ion top-hat electrostatic analyzer. It has 32 energy steps, 64 pitch angle bins, 16 energy sweeps per second. The geometry factor per bin is $1.3 \times 10^{-3} \text{ cm}^2 \cdot \text{sr-eV/eV}$. It has no mass resolution.
Rocket's Launch

"Flight Unit"
Complete Flight Phases 1 + 2 + 3

Data Phase 1 -5 minutes
Data Phase 2 -5 minutes
Data Phase 3 -5 minutes

Xmit #1 -42 sec
Xmit #2 -42 sec
Xmit #3 -42 sec

Rocket Lift-off
FFM Deployment
Apogee
300 km altitude End

Figure 2-6: The timeline of Enstrophy's flight.
2.3 Cornell field measurements

The field measurements included DC, and VLF (0-20 kHz), HF (1.5 kHz to 2.56 MHz) electric field measurements, deployed on a single boom pair perpendicular to the spin axis by 6.0 meter Weitzmann booms; and magnetic field measurements. There were two main magnetometers. One was deployed on a 0.8 meter rigid boom and the other one was deployed on the experiment deck. The one on the main payload was originally built to provide attitude information, to serve as an interface to UNH electron detector for aperture control and to provide measurements of the magnetic field.

2.4 JPL/UNH/LPARL supporting instruments for FFMs

The supporting instruments for FFMs were the deploying system (UNH), the exit velocity monitor (UNH) which measures the exit velocity of FFMs, the optical command interface (JPL) connecting the main payload and the FFMs, the star sensors (LPARL) for main
payload attitude information and FFM laser beacon (JPL) detector that provides FFM attitude information.

2.5 NASA Wallops payload instrumentation support

For the Enstrophy mission, NASA Wallops provided the payload vehicle (the deployment decks and longerons were built at UNH), which was a new design. It included an ejectable nose cone, exposing the experiment structure underneath, a single link telemetry section and the third stage igniter housing. It also provided the flight event timing and pyros, and a GPS receiver with 1PPS time tagging using a WFF 93 time event module.

2.6 UAF ground imagery instruments

All sky camera and narrow-field camera data of the auroral events were made possible by Geophysical Institute, University of Alaska at Fairbanks. The cameras were operated at both Poker Flat Launch Range and Kaktovic.
Chapter 3

Enstrophy Data

The data from the Enstrophy flight will be presented in this chapter. This includes the auroral environment data at the time of the launch, including ground based and satellite measurements of key parameters; vehicle performance; and the measurements from the various instruments on board. We begin with an overview of the global and local auroral conditions near the time of the launch, 06:45:31 UT on 11 Feb, 1999.

3.1 Auroral Environment

Enstrophy was launched from Poker Flat Research Range, Alaska, USA at 06:45:31 UT on February 11, 1999 into a pre-midnight aurora. The launch conditions were good that night—clear sky, no wind and strong auroral activity. Presented below are the details of the auroral environment of that night. The order is arranged such that the global conditions are described first, followed by the data of the local environment.

3.1.1 ACE, GOES and POLAR Data

A general remark on the plots is that throughout this chapter a vertical line on some of the plots is used to indicate the launch time for the Enstrophy sounding rocket.

Figure 3-1 is a rough sketch of satellites ACE, GOES8, GOES10 and POLAR’s position relative to the Earth. ACE (Advanced Composition Explorer) orbits the L1 libration point, which is a point of Earth-Sun gravitational equilibrium and is about 1.5
Figure 3-1: The rough sketch of the spacecrafts' location.

millions km (0.01 AU) from Earth and 148.5 million km (about 1 AU) from the Sun (http://helios.gsfc.nasa.gov/ace/ace.html). With a semi-major axis of approximately 200,000 km the elliptical orbit of ACE is guaranteed a good view of the Sun and the galactic regions beyond. Carrying six high-resolution sensors and three monitoring instruments ACE performs measurements over a wide range of energy and nuclear mass, under all solar wind flow conditions. ACE provides near-real-time solar wind information over short time periods and can provide an advance warning (about one hour) of geomagnetic storms that hit upon the Earth. That is the reason we used the real-time ACE data to monitor our launch activity. Compared to the SOHO satellite, used for previous rocket launches, ACE has also the advantage of the IMF (interplanetary magnetic field) measurement which SOHO didn’t have.

Figure 3-2 shows the near-real-time solar wind data from SWEPAM (Solar Wind Electron, Proton, and Alpha Monitor) and MAG (the Magnetic Field Experiment) on the day of Feb. 11, 1999. From top to bottom the data are: IMF (interplanetary magnetic field), the angle phi between the Bz component and the total B, proton density, proton temperature and proton speed. From the plot, we can see that near the launch time Bz had been
Figure 3-2: The real time ACE data of day 42, 1999 (courtesy NASA). From top to bottom the data are: IMF (interplanetary magnetic field), the angle phi between the Bz component and the total B, proton density, proton temperature and proton speed.
southward for a while and started to turn into the northward direction, solar wind speed (indicated from proton speed) kept increasing and there was also increased proton density. All these indicate the possibility of strong auroral activity.

Closer to the Earth, geosynchronous satellites GOES 8 and GOES 10 were used to give further reference about the auroral environment. We used the internet, from the website http://www.sel.noaa.gov/today.html to obtain their data. GOES 8 is located at longitude West 75° and GOES 10 was at longitude West 135°, which was very close to the launch site (-147.5°) longitudinally. The satellite environment parameters, used to foretell auroral activity, include GOES x-ray flux data, GOES energetic flux, GOES integral proton flux, GOES $H_p$ component of the magnetic field, and estimated planetary K indices, $K_p$.

In general, $K_p$ and $H_p$ are important for us to have a feel about what the period of auroral activity is and how it varies with solar activity. $K_p$ is a three hour planetary index of geomagnetic activity calculated from ground-based magnetometers (mostly in northern hemisphere, USA and Canada). Indices of 5 or greater indicate storm.

$H_p$ is the magnetic component parallel to the Goes satellite spin axis, oriented northward. If $H_p$ drops to near zero, or less, when the satellite is on the dayside, it may be due to a compression of Earth’s magnetopause to within geosynchronous orbit, exposing the satellite to negative and/or highly changeable magnetic fields. On the nightside, a near zero (or less) of the field indicates strong currents that are often associated with substorms. When $H_p$ drops to near zero or less, it means the magnetotail is stretched to a very non-dipolar shape. It is often called the substorm expansion stage. Then there will be sudden energy release when the magnetic field becomes dipolar again. Figure 3-3 shows the data which indicates a strong auroral activity. We can see that $H_p$ component dropped near zero at
Figure 3-3: GOES-8,10 satellite environment data (courtesy NOAA/NGDC).

06:45 UT for both GOES 8 and GOES 10. A detailed picture of $H_p$ component is shown in Figure 3-4. A detailed picture of $H_p$ component was shown in Figure 3-4.

ACE and GOES satellites provide information about what happened globally—the inputs from the Sun and the effect on the magnetosphere. The POLAR spacecraft recorded the result of these inputs to the Earth’s ionosphere. Figure 3-5 shows the UVI image taken by the POLAR ultraviolet imager during the flight and the blue arrow indicates the flight path. Also shown is the auroral activity vs. magnetic latitude and magnetic local time (MLT). Figure 3-6 is a series of VIS (obtained from Visable Imaging Systems on board the...
Figure 3-4: GOES-8, 10 magnetometer data (courtesy NOAA/NGDC).
Auroral activity during the Free-Flying Magnetometer Flight
Launch: 11 Feb 99 0645 UT – 20 minute flight (Source: Polar UV Image)

Figure 3-5: Image from POLAR Ultraviolet Imager during Enstrophy launch.

spacecraft) images. The time span is from 6:30:33UT to 6:55:05 UT. On this plot, Alaska is located nearly at the center of the images. From the VIS images, we can see what the aurora looked like and how it evolved. It intensified first, then broke up into finer arcs and expanded into the polar cap. The URL for POLAR is //http://www-istp.gsfc.nasa.gov/istp/.

3.1.2 Ground Based Magnetometer Data

The electroject currents of the aurora cause large magnetic disturbances which are measurable on the ground. Real time ground based magnetometer data monitor the auroral
Figure 3-6: Image series of POLAR VIS during Enstrophy launch (courtesy University of Iowa).
activity caused by the substorms or storms up in the magnetosphere. They give a good sense of how to choose the proper launch time. In this section, the magnetometer data from CANOPUS chain sites is presented first, followed by the data from POKER and Fort Yukon. Unfortunately, no data was stored for Kaktovic although we used it on the day of our launch.

The significance of using this data is that most sites of CANOPUS are east of the nominal trajectory, while Poker and Fort Yukon are the two sites which are along the nominal trajectory. The multipoint data from different sites of CANOPUS give the onset, the trend and the movement of the auroral activity in the adjacent area so that a prediction of the auroral activity along the nominal rocket trajectory can be made. The data from Poker and Fort Yukon not only gives the real-time information about the auroral activity along the trajectory but also can serve as a good reference for us to adjust our ground prediction on the auroral activity by using only CANOPUS.

Figure 3-7 is a geographic map of northern Canada, showing the locations of the magnetometer sites which make up the CANOPUS magnetometer chain. The table below the figure gives information about the exact coordinates of each site. The series of sites from Pinawa to Taloyoak lie on the same line of longitude, which does not make them useful for our predictions of westward or eastward motion. However, they are useful for prediction of northward or southward motion. The sites of Fort Churchill, Rabbit Lake, Fort Smith, and Dawson provide good longitudinal coverage of the auroral oval.

By using the internet [http://www.dan.sp-agency.ca/], we monitored the real time data of these magnetometer sites at the launch site.

Figure 3-8, Figure 3-9, and Figure 3-10 show the time evolution on the 42nd day of the
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<th>LOCATION</th>
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<th>CANOPUS L</th>
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1EDFL ==> Eccentric Dipole Field Line traced coordinates.

Figure 3-7: Map of CANOPUS chain magnetometers (courtesy Canadian Space Agency).

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Figure 3-8: Stack plot of x-component magnetometer chain data (courtesy CSA).
Figure 3-9: Stack plot of y-component magnetometer chain data (courtesy CSA).
Figure 3-10: Stack plot of z-component magnetometer chain data (courtesy CSA).
yeax (Feb. 11, 1999) of the $x$, $y$, and $z$ components, respectively, of the CANOPUS chain magnetometers. From top to bottom, the sites shown in the figures are Taloyoak, Rankin Inlet, Eskimo Point, Fort Churchill, Gillam, Island Lake, Pinawa, Dawson, Fort Simpson, Fort Smith, Rabbit Lake, Contwoyto Lake and Fort Mcmurray. Each panel in the $x$, $y$ and $z$ component plot has its own horizontal dashed line representing the zero level for the site. The $x$, $y$, $z$ axis makes up a right-handed north, east, down coordinate system. All three plots show magnetic disturbances after 0600 UT. From the data of $x$ axis, looking at the sites of Rabbit Lake, Fort Smith, Fort Simpson and Dawson during 4:00-8:00 UT, there were delays of the sudden decrease (marks the onset of the auroral activity) of the measured $x$ component of magnetic field between these sites. So we can say that this could be a good indication of westward motion of the auroral activity.

Now consider the data of the sites along the nominal trajectory. Figure 3-11 and Figure 3-12 show the three components of the magnetic field measured in Gammas (1 Gamma = 1 nT) from the magnetometers of Poker Flat and Fort Yukon.

The $H$ component is the north component, $Z$ is positive for vertically downward, and $D$ is the eastward component. The first perturbation of $H$ component took place after 6:00 UT. Compared with the strong disturbance ($H$ component dropped to almost zero) at about 12:00UT later in the day, the first decrease is relatively small. The photometer data will be shown later so that we can see a good correlation between the magnetometer data and the light intensity from aurora.

In addition, the high frequency magnetometer at Kaktovic was used as the trigger for the launch because it is a very good local indicator for the auroral activity. Unfortunately, we do not have the data to show here.
Magnetometer trace in gammas for Poker Flat, AK Day 042, 1999

Mean value

1.24e+04

Figure 3-11: Poker Flat three axis magnetometer data (courtesy PFRR).

Magnetometer trace in gammas for Fort Yukon, AK Day 042, 1999

Mean value

1.12e+04

Figure 3-12: Fort Yukon three axis magnetometer data (courtesy PFRR).

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Figure 3-13: Poker Flat meridional scanning photometer data from February 11, 1999 (courtesy Poker Flat Research Range). In each panel is a different atomic spectral line emission plot with a color bar brightness index.

3.1.3 Ground Based Imagery

The ground based imagery includes the meridional scanning photometer, all sky camera, and narrow field camera instruments. They all measure the intensity of light from auroral emissions.

The meridian scanning photometer (MSP) is one of the principal instruments available at Poker Flat Range and used for obtaining routine records of the position, intensity, and motion of aurora and airglow emissions. Figure 3-13 shows the MSP data taken during the day of the launch. The plot shows emission intensity of different wavelengths, in kilo-
Rayleighs, from different atoms (could be ionized), as a function of universal time. The four panels, from top to bottom, show emissions at the 5577 nm neutral atomic oxygen (O) green line, the 4278 nm ionized diatomic nitrogen (N2+) blue line, the 4861 nm neutral hydrogen (Balmer-beta) (H) blue line, and the 6300 nm neutral atomic oxygen (O) red line. The MSP measurement serves as one of the many factors to indicate the auroral activity and to determine the right launch moment.

All sky and narrow field imagers were in operation during the whole launch window. These served as other criteria for determining when to launch. All-sky imagers are located at Poker Flat, Fort Yukon, and Kaktovic, providing good coverage of the sky above northeast Alaska and good coverage of auroral activities. Extremely sensitive video cameras are used, which record the night sky on video tape for later analysis. Narrow field imagers work in a similar way as all sky imagers except that the narrow field imagers cover only a narrow region (16° in azimuth and 12° in elevation) of the sky and give more detailed recording of the auroral activity. Narrow field camera data can be used to study the fine structures of auroral forms.

Figure 3-14 shows a digitized still image from the Kaktovic all-sky camera. The universal date and time, as well as the camera location, are displayed in the upper left corner. The top of the figure is south, the bottom is north, left is west, and right is east. This frame of the all-sky camera data was taken at the moment when the payload was close to its apogee. We can see three bright arcs across the plot. The position of the rocket at that time was approximately at the edge of the third arc, i.e., the northernmost one. The whole system expanded northward and the auroral activity was strong. Figure 3-15 is a digitized still image from the Kaktovic narrow field camera. The azimuth angle, as well as the camera
Figure 3-14: All sky camera image from Kaktovic during the flight (T+470 seconds).
Figure 3-15: Narrow field camera image from Kaktovic during the flight (T+470 seconds).
location, and the elevation angle are displayed in the top of the image. The universal
time can be found at the bottom. The line below the universal time can be ignored due
to the wrong information shown. At the time when data of Figure 3-15 were taken, we
can see "dancing rays" motion on the video. Although the payload was in a less intense
and relatively less visible arc, there were fine structured dancing rays and dramatic motion
associated with auroral activity during that time.

Data from all sky and narrow field imagers were not only used to determine good
launch timing before its launch, but also were used later for the systematic data analysis
and complete investigation of the event observed during the flight after its launch.

These ground based measurements provide a good forecast of when we could expect
strong auroral activity in the trajectory range. Compared to the satellite data, the ground
magnetometers from very nearby sites, the sites along the nominal trajectory, and the
ground all sky and narrow field camera data provide a very short time forecast of auroral
activity along the trajectory and very near real-time information about the auroral activity
right around the trajectory range.

To summarize, the environmental data has two main purposes. Firstly, the data are
used as predictors of auroral activity. The ability to obtain real time data from satellites,
cameras, and magnetometers makes prediction of auroral activity possible, and this method
of invoking multipoint, multitools, and almost ubiquitous data collection from the global
and local environment is now a standard means of anticipating good launch conditions.
Secondly, having these data after the launch gives context to the payload data analysis and
helps understanding the physical processes involved.
3.2 Launch Details

Detailed use of all the auroral environment data mentioned above was involved in determining when to call for or hold the launch, as described below.

3.2.1 Launch Specifics

The Enstrophy sounding rocket (NASA 35.032) was launched on February 11, 1999 at 0645:31 UT from the Poker Flat Research Range. The launch facility is located at 65° 06’ N latitude and 147° 28’ W longitude. The payloads were carried by a three stage Black Brant X(MOD 1) rocket.

3.2.2 Payload Trajectory

The Enstrophy payload trajectory is shown in Figures 3-16, 3-17 and 3-18. Figure 3-16 shows the geographic location of the main payload for the duration of the flight in geographic coordinates (the red line). Some city locations have been added for reference. All-sky cameras provided by UAF were available at Poker Flat, Fort Yukon, and Kaktovic. Magnetometers are located at Poker Flat, Dawson, Yellowknife and Fort Simpson. The planetary average Kp during the flight was about 4.

As shown in Figure 3-17, Enstrophy was launched geographically northward over Alaska towards the Arctic Ocean so as to fly over the University of Alaska-Fairbanks (UAF) optical site at Kaktovic (marked as KAK in the figure). When the payload was at apogee, the projection of the geomagnetic field lines from the Enstrophy payload to 110 km altitude (the altitude where the light is generated) is almost right above Kaktovic. Figure 3-18 is a plot of the Enstrophy altitude vs. flight time, and we can see the apogee of this rocket is
Figure 3-16: Enstrophy Main payload geographic footprint trajectory.

about 1070 km.

Figure 3-19 is a cartoon description of the main payload and four FFMs configuration. The figure shows the events that happened during the whole flight. The FFM deployment took place at T+101 seconds of the flight time at altitude of 180 km. The FFMs reached a maximum separation distance of nearly 4 km between one of the FFMs and the main payload. The straight lines on the plot represent geomagnetic field lines, and the thick short green and yellow line are used to represent the visible aurora at 110 km. It is during the time where the field lines are drawn in the plot that we observed large magnetic field oscillations and the corresponding field aligned current density, which took place near the rocket's apogee, at the poleward edge of an arc.
Figure 3-17: GPS track of Enstrophy payload in geodetic coordinates. UAF optical site at Kaktovic indicated by KAK on the plot.
Figure 3-18: Altitude of Enstrophy payload vs. flight time
Figure 3-19: Cartoon representation of the Enstrophy flight.
3.2.3 Vehicle and Instrument Performance

At T+101 seconds after launch, the deploying system on the Main payload was able to deploy the four FFMs which were originally on board successfully. The payloads (including the main payload and the four nanospacecraft) traveled northward at roughly 1 km/s. The FFM exit speed relative to the main payload was about 3.5m/sec. Although one FFM (FFM2) did not come out of the deployer as orthogonally as expected and with ~0.01 sec delay compared to the other three FFMs, the deployment was clean and stable. A total spin rate of about 17 Hz was achieved for all FFMs. Data analysis done on them after the launch proves the goal of FFMs was reached. They worked fine except that the data from one of the four FFMs (FFM3) during the interesting period were too weak to be useful and that FFM2 lost its z axis measurements. Figure 3-20 shows the details of the data collection from the four FFMs.

A serious malfunction occurred on the main payload. The despin timer was late, which caused the science magnetometer boom and the particle detectors' boom to be deployed at a 6.25 Hz spin rate rather than the desired 1 Hz. The high spin rate damaged the booms. The irregular motion resulting from the damaged floppy boom of the magnetometer makes the data collected from the main payload less useful than originally planned, rendering the main payload virtually useless for extraction of electrical current signatures.

We can see from Figure 3-21 even after the complete despin procedure (will be discussed in Chapter 4), the wobbling and oscillating signatures are severe—the coning angle has a wide range. The magnetometer on the deck had a crazy motion. The one on the boom had similar behavior.
Figure 3-20: Free-Flying Magnetometer Flight Data.
Figure 3-21: The coning angle of the main payload magnetometer after being de-spun.
Due to the same malfunction mentioned above, the particle detectors (including electron and ion detectors) were all damaged. The ion detector microchannel plate is thought to have been broken because the ion detector returned zero counts for the entire flight. The electron detector appeared to have a floating inner hemisphere: the count rate was at least an order of magnitude lower than expected, and there was no energy sweep signature in the data. Although there were some total flux responses in the electron data as the payload moved through the arcs, the data is not useful for scientific purposes. The data from the electron detector is shown in Figure 3-22. As we can see from the plot, the electron data obtained is not promising at all.

Figure 3-22: The available electron data.
Telemetry for all payloads experienced no problems. The electric field instrument and wave instruments worked as expected. But due to the fact that only one pair of electric field booms was available, and also because the main payload was unbalanced, the data extraction from the measurement is difficult and the information that can be obtained is unsatisfying.

3.3 Flight Survey

The Enstrophy payloads flew through a premidnight aurora, and passed through various auroral forms. At the beginning of the flight, from the narrow field camera data, we can see the flickering aurora, then the streaming aurora, and at the edge of the arc when it headed towards the polar cap, the camera data shows active, very dynamic and dancing rays at flight time T+465 seconds. Large magnetic field oscillations were seen at flight time T+470 seconds - T+540 seconds from all FFMs which provided the workable data—right after the visible dancing ray structure. The details will be discussed in the following sections of the chapter.

3.4 Details of the Measurements

We now present the data results from the various instrumentation which worked on the payloads. We begin with magnetic field measurement data from FFMs first.

3.4.1 FFM data—Large B oscillations

Magnetic field data reduction procedure is very complicated and highly mathematically involved in itself. The details of these procedures will be discussed in the Chapter 4. Here
only the reduced data are presented. The magnetic field data from FFMs are the major part of the data.

As we mentioned in Chapter 2, every FFM was designed to have three data phases which lasted about 5 minutes each, while in between the three data phases are the short periods of transmission time, lasting about 42 seconds. Figure 3-23 is a plot showing the disturbed magnetic field's x component during the whole flight whenever there were data collected from FFM1. It gives us an overview of the magnetic activity for the whole flight. As we can see, at the beginning of the flight, there were small oscillations in B. But during T+470 second – T+540 second large oscillations in B were seen. Then after that, things became very quiet. The fluctuations in B were close to zero. Figure 3-24 shows an expanded plot of the deviation of the magnetic field data for all three FFMs obtained during the second data phase. The top panel is the x component (see Figure 4-3 in Chapter 4 for description) of the magnetic field deviation in B-L system, where B is the Earth magnetic field, and L is the angular momentum vector for the payloads. The bottom panel shows the y component of the magnetic field deviation. The three FFMs are FFM1, FFM2 and FFM4 by JPL.
Figure 3-24: The magnetic field deflection for all three FFMs during the second data phase.

convention. Although some data were collected by FFM3 during the second data phase too, they are extremely hard to work with. Almost nothing can be extracted from them. That is why there were no data from FFM3 shown in Figure 3-24 for this period.

Figure 3-25 is similar to Figure 3-24 except that Figure 3-25 shows data from T+470 seconds - T+540 seconds, i.e., the period where the large oscillations in B were seen. From both Figure 3-24 and Figure 3-25, we can see three of the FFMs all saw similar patterns of the large B oscillations. But there are also differences between different FFMs.

In order to show that we also got workable data from FFM3 for the first data phase,
Figure 3-25: The large B oscillation.

Figure 3-26 is plotted here. And Figure 3-27 shows the comparison between FFM1 and FFM3 for the first data phase. It should be mentioned that because the background Earth magnetic field and the spin rate of all the 4 FFMS changed more quickly during the first data phase (this is partially reflected by the large slope in the data shown in Figures 3-26 and 3-27) comparing to the later data phases, the magnetic field data reduction process for this period becomes more difficult.

Figure 3-28 is the polarization plot of $B_x$ vs. $B_y$ for all 3 FFMs, where $B_x$ is the $x$ component of the magnetic field deflection, and $B_y$ represents the $y$ component. The polarization for them appears to be rather complicated and it seems that there are combinations
Figure 3-26: The available FFM3 data during first data phase.

Figure 3-27: The magnetic field deflection for FFM1 and FFM3 during first data phase.
of both right-handed and left-handed polarization. Figure 3-29 also shows the polarization of 3 FFMs but separately. Figure 3-30 shows the result of the current density calculated from the multipoint magnetic field measurement by FFMs. The details on how to extract the current density will be discussed in Chapter 4.

3.4.2 HF Data

The high frequency data from the Main payload is presented in Figure 3-31 as a function of flight time. We can see Langmuir wave bursts in the plot between T+400 and T+600 sec. Data from previous rocket investigations show that short, intense Langmuir wave bursts are common in the auroral ionosphere. During this event, they appeared at the time where the large magnetic field oscillations were observed. The bursty Langmuir waves can be used as proxy for electron precipitation in the analysis of this auroral event because of the close relationship between low energy (0.3-3.0 keV) electron precipitation and generation of bursty Langmuir waves. The bunching effect of electrons at the tail of the distribution
Figure 3-29: Polarization for all 3 FFMs.

Figure 3-30: Current density calculated from using 3 FFMs.
Figure 3-31: High frequency wave spectrum from Enstrophy

creates the instability responsible for Langmuir wave growth [Ergun, et al., 1991; McFadden, et al., 1986]. HF data can also provide information on the density profile of local plasma. The Langmuir wave bursts display similar patterns as the B oscillations seen in Figure 3-25. Figure 3-32 shows the integrated (200kHz-700kHz) high frequency wave power. The bottom panel and the top panel are the same except that the bottom one went through some filtering. It is interesting to compare these panels to the plot of $j_z$ (Figure 3-30).

3.4.3 Electric Field Data

There was only one pair of booms for measuring the electric field. The details of the data will be discussed in Chapter 5.
3.4.4 Light Intensity Data

A light intensity profile under the whole flight was obtained by integrating the all-sky camera data using the correct mapping from the rocket trajectory to 110 km auroral generation altitude. The light intensity profile is a direct measure of the visible aurora. Figure 3-33 shows the integrated light intensity of the Enstrophy’s conjugate point.

The top left panel shows the light intensity of the whole flight. The other three panels provide more details by plotting it for a shorter time interval. We can see that the rocket passed through two major, bright auroral arcs. There was a small peak in the light intensity right before the large magnetic field oscillations were seen. This matches with the narrow-field camera data — very dynamic, dancing rays were observed during the same time.
Figure 3-33: Light intensity profile of the Enstrophy's conjugate point (Courtesy H Stenbaek-Nielsen, UAF).
3.4.5 VLF data

VLF data reflect the large scale electron precipitation by VLF hiss. Figure 3-34 shows the VLF hiss (mostly whistler mode) during the flight.

3.4.6 ELF data

Figure 3-35 is the wavelet transformed ELF data during the time when the large B oscillations were seen. The ELF data also was influenced by the malfunction which happened to the main payload as mentioned before. As a result, it is not too useful for the B fluctuation
event study.

3.4.7 Camera data

Both all-sky camera data and narrow field data were conducted and recorded as VHS video at Poker Flat Research Range and Kaktovic. Digitization of the video image was done to study the details of the auroral activity. Figure 3-14 and Figure 3-15 show the digitized images from both all sky camera and narrow-field camera at 6:53:21UT (T+470s).

The digitized images of narrow-field camera data were used to study the fine structures of the arc. The motion of the observed rays were calculated by comparing the same ray (sometimes it might be subjective in making such judgement) from two immediate or ad-
jacent frames with the help of Adobe Photoshop. It should be mentioned that the velocity calculation procedure requires a coordinate system transformation from the image frame to the geographical frame.

In order to appreciate what distances in the image corresponds to out in space (110 km auroral altitude), consider the two cartesian coordinate systems.

1. Geographic: $X$ south, $Y$ east, $Z$ up ($X$ south to make it right hand)

2. Image $XYZ$: $X$ right, $Y$ down, $Z$ into the image (right and down refers to the image)

The narrow field camera was installed such that Image-$Z$ is along the azimuth line, the elevation angle = $EL$, and Image-$X$ is horizontal with the line denoted by $(AZ+90)$, where $AZ$ is the azimuth angle when the image was taken.

\[
\begin{pmatrix}
    X_I \\
    Y_I \\
    Z_I
\end{pmatrix}
= R_x(-\pi/2 + EL)R_z(AZ + \pi/2)
\begin{pmatrix}
    X_S \\
    Y_E \\
    Z_U
\end{pmatrix}
\tag{3.4.1}
\]

where $R_x(-\pi/2 + EL)$ means rotate around $X$ axis with an angle $(-\pi/2 + EL)$ and in the same way $R_z$ is a rotation around axis $Z$. They are given by:

\[
R_x(\alpha) = \begin{pmatrix}
    1 & 0 & 0 \\
    0 & \cos\alpha & \sin\alpha \\
    0 & -\sin\alpha & \cos\alpha
\end{pmatrix}
\tag{3.4.2}
\]

\[
R_z(\beta) = \begin{pmatrix}
    \cos\beta & \sin\beta & 0 \\
    -\sin\beta & \cos\beta & 0 \\
    0 & 0 & 1
\end{pmatrix}
\tag{3.4.3}
\]

This calculation allows the determination of the velocity of rays in the range of 0–10 km/sec.
To have a comprehensive view of this large $B$ fluctuation event, we put the above data together in Figure 3-36 and Figure 3-37.
Figure 3-36: Comprehensive view of the event—VLF, HF and light intensity data.
Figure 3-37: Compressive view of the event-Bx, light intensity and HF data.
Chapter 4

Data analysis techniques

The main scientific goal of the Enstrophy sounding rocket was to make multipoint measurement of field aligned currents and study their fine structuring. We will show in this chapter that the procedure of obtaining the field aligned current density requires several data analysis techniques and methods.

First of all, multipoint field aligned current density measurement is achieved by multipoint measurements of magnetic field fluctuations using FFMs. The magnetic field measurements were in the sensor system which was spinning and precessing. Transformation to the non-spinning and non-precessing magnetic field aligned coordinate system must be done. So the frame transformation of the measured magnetic fields is the first step, one which is proven to be very difficult and requires a lot of careful handling with involved parameters. The difficulty lies in the fact that we need to extract signatures of tens of nT from raw data amplitudes of a few×10^4 nT.

Secondly, after we get the perpendicular (relative to the Earth’s magnetic field) magnetic field components $B_x$ and $B_y$ in the magnetic field aligned coordinate system, the geometric positions of all four FFMs must be known for the current density calculation. Therefore, the method on how to figure out the geometric positions of all the FFMs will be described.

The last step needed for the current density calculation is the multipoint considerations, that is how we should approximate the partial derivatives in the Ampere’s law. Taylor series expansion to the first order is used and the error analysis of the calculation is given. Now
let's first start with the magnetic field frame transformation procedure.

### 4.1 Magnetic field frame transformation procedure

Four hockey puck sized "free-flying-magnetometers" (FFMs) were released from the main payload at the beginning of the Enstrophy flight. They are termed nanospacecraft because they are small (250g) and autonomous, carrying miniaturized fluxgate magnetometers and having their own telemetry to ground. They separated from the main payload at relative velocities of 3.5 m/s and with a total spin rate of 15 - 17.5 Hz. The separations between them are 1~2 km at apogee.

The magnetic field's three components are measured originally in the sensor coordinate system. Because 1) the sensor coordinate system (fluxgate sensor axes) is not perfectly orthogonalized, and 2) the sensor coordinate system and the payload spin coordinate system are not perfectly aligned, the spin and coning signatures are usually coupled together in the measurement.

To be able to extract tens of nT signature in $B_x, B_y$ in the $B$-$L$ system ($B$ is the Earth's magnetic field and $L$ is the angular momentum vector for the payloads) from the sensor system measurement, which has the amplitude of the order $10^4$ nT, requires very accurate fitting of all the parameters involved.

In order to do the data analysis on the magnetic field data, first of all, we need to know the payload kinematics. This topic was discussed by F. Primdahl [Primdahl, 1997] for sounding rocket payload with the moment of inertia $I_z$ (around $z$ axis) less than the moment of inertia $I_x$. For our disk-shaped FFMs, several variations needed to be made. $I_z$ is greater than $I_x$, and the coning (precessing) frequency is larger than the spin frequency.
Details are discussed below [Zheng et al., 2001].

4.1.1 Payload Kinematics

Assuming a payload has rigid body motion, the momentum equation is [Goldstein, 1980]:

\[
\frac{d\mathbf{L}}{dt_{\text{inertial}}} = \mathbf{\Gamma}^{(e)}
\]  

(4.1.1)

where \(\mathbf{L}\) is the angular momentum vector, and \(\mathbf{\Gamma}^{(e)}\) is the net torque arising from the the external forces at a given instant in time. This is in the fixed star system.

In a coordinate system rotating with \(\mathbf{\omega}\) relative to the fixed stars, the momentum equation becomes:

\[
\frac{d\mathbf{L}}{dt_{\text{body}}} + \mathbf{\omega} \times \mathbf{L} = \mathbf{\Gamma}^{(e)}
\]

(4.1.2)

\(\mathbf{L}, \mathbf{\omega}\) and \(\mathbf{\Gamma}^{(e)}\) are vectors from the inertial system, but now expressed by the coordinates of the rotating system with

\[
\frac{d\mathbf{L}}{dt} = \mathbf{I} \star \frac{d\mathbf{\omega}}{dt} + \frac{d\mathbf{I}}{dt} \star \mathbf{\omega}
\]

(4.1.3)

where \(\mathbf{I}\) is the inertial tensor. Assuming a rigid body and force free motion, we have

\[
\frac{d\mathbf{I}}{dt} = 0
\]

(4.1.4)

and

\[
\mathbf{\Gamma}^{(e)} = 0
\]

(4.1.5)

Then the momentum equation becomes:

\[
\mathbf{I} \star \frac{d\mathbf{\omega}}{dt} + \mathbf{\omega} \times [\mathbf{I} \star \mathbf{\omega}] = 0
\]

(4.1.6)
The inertia tensor can be diagonalized in the principal body axes:

\[
I = \begin{bmatrix}
I_1 & 0 & 0 \\
0 & I_2 & 0 \\
0 & 0 & I_3
\end{bmatrix}
= R \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & r
\end{bmatrix}
\]

(4.1.7)

where 1, 2, 3 denotes the three components respectively. For FFMs (disklike flattened shape), in the above we assume

\[
I_1 = I_2 = I < I_3
\]

(4.1.8)

\[
r = \frac{I_3}{I_1} > 1
\]

(4.1.9)

Then the momentum equation has the following simpler form:

\[
\frac{d\omega_1}{dt} + \omega_2 \omega_3 (r - 1) = 0
\]

(4.1.10)

\[
\frac{d\omega_2}{dt} - \omega_3 \omega_1 (r - 1) = 0
\]

(4.1.11)

\[
\frac{d\omega_3}{dt} = 0
\]

(4.1.12)

with the solution:

\[
\omega_1 = \omega_T \cos(\omega_3 t + \phi_3)
\]

(4.1.13)

\[
\omega_2 = -\omega_T \sin(\omega_3 t + \phi_3)
\]

(4.1.14)
Figure 4-1: The relationship between different components of $\omega$.

$$\omega_3 = \text{constant} \quad (4.1.15)$$

where $\omega_s$ is the spin rate, it relates to $\omega_3$ as the following:

$$\omega_s = \omega_3(r - 1) \quad (4.1.16)$$

$$\omega_T = \sqrt{\omega_1^2 + \omega_2^2} \quad (4.1.17)$$

In the FFM coordinate system,

$$L = I \ast \omega = [I_1\omega_1, I_2\omega_2, I_3\omega_3] = I[\omega_1, \omega_2, r\omega_3] \quad (4.1.18)$$

$L$, $\omega$ and the third axis (the $z$ axis) of payload system (FFMs) are in the same plane. Figure 4-1 shows the relationship between $\omega$ components. From Figure 4-1, we can see that $\omega$ can be either decomposed into $\omega_z$ and a transverse component $\omega_T$, or decomposed into
$\Omega_p$ along $L$ and $\omega_3$ along the spin axis. Along $L$, we have:

$$\Omega_p = (\omega_T / \sin \theta) \hat{L} \tag{4.1.19}$$

and along $\hat{3}$ (z axis):

$$\omega_3 = \omega_3 (r - 1) \hat{3} \tag{4.1.20}$$

The coning angle $\theta$ has the relation

$$\sin \theta = \omega_T \cdot I / L \tag{4.1.21}$$

then we get

$$\Omega_p = L / I \tag{4.1.22}$$

and

$$I \cdot r \cdot \omega_3 = L \cos \theta \tag{4.1.23}$$

$$\omega_3 = \frac{r - 1}{r} \Omega_p \cos \theta \tag{4.1.24}$$

From the above relations we can see that coning rate depends on $L$ and $I$ and spin rate depends on $\theta$ and the payload's body parameters.

$L$ and $T$ (kinetic energy) are two constants of the motion, for torque free, drag free rigid body motion.

$$\omega_3^2 = (L/I)^2 (2T * I/L^2 - 1)(r - 1)/r \tag{4.1.25}$$

$$\cos^2 \theta = (2T * I/L^2 - 1)r/(r - 1) \tag{4.1.26}$$

$$\Omega_p = L / I \tag{4.1.27}$$

So we can see that 1) the coning rate depends on $L$ and $I$, and 2) spin and coning angle also depends on $T$. 

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The above derivation is for ideal cases. In fact, the system may lose energy by drag, friction and other dissipation. The motion of the payload is not strictly the rigid-body motion. The two inertial components $I_1$ and $I_2$ are not perfectly identical. Atmospheric friction means that $\Gamma^{(e)} \neq 0$. These effects are small, but they cause $\theta$, $\Omega_p$ and $\omega_s$ to vary slowly with time. The time variation in those parameters needs to be taken care of in the data analysis.

The 3-d (dimensional) picture of B-L-$\omega_s$ relations is shown schematically in Figure 4-2.

The B, L, $\omega_s$ spherical triangle is shown in Figure 4-3. There are different coordinate systems involved with the magnetic field data analysis. Let $B_1$ be a magnetometer axis directed transversely to $\omega_s$. The phase angle relative to B and $\omega_s$ plane is $\phi_s^B$ and the phase angle relative to L and $\omega_s$ plane is $\phi_s^L$. Using the properties of spherical triangle, we find
Figure 4-3: The B-L-ω relations in their spherical triangle.

\[
\phi_s^A = (180^\circ - \beta) + \phi_s^R
\]
the following relations.

\[ \phi_s^A = (180^\circ - \beta) + \phi_s^R \]  

(4.1.28)

\[ \tan(180^\circ - \beta) = \frac{\sin \Phi_p / \cos \theta}{\cos \Phi_p - \tan \frac{\theta}{\tan \kappa}} \]  

(4.1.29)

In order to get the B field components in the spinning and coning coordinate system with \( (\omega / \tilde{3}) \) along \( \omega_s \) from B-L system, a series of coordinate rotations have to be performed.

In the B system (\( \tilde{3} \) along B and \( \mathbf{i} \) away from \( L \)):

\[
B = \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix}
\]  

(4.1.30)

Rotate this system into L system with \( \tilde{3} \) along L and \( \mathbf{i} \) pointing toward B:

\[
B_L = R_2(-\kappa) \cdot B
\]  

(4.1.31)

\[
R_2(-\kappa) = \begin{bmatrix} \cos \kappa & 0 & \sin \kappa \\ 0 & 1 & 0 \\ -\sin \kappa & 0 & \cos \kappa \end{bmatrix}
\]

therefore:

\[
B = \begin{bmatrix} B \sin \kappa \\ 0 \\ B \cos \kappa \end{bmatrix}
\]  

(4.1.32)

In this coordinate system, \( \tilde{3} \) is along L, \( \mathbf{i} \) is toward B and along \( (L \times B) \times B \), \( \tilde{2} \) is along \( L \times B \).
The $L_B$ system is now rotated about $\hat{3}$ into $L_\omega$ with $\hat{1}$ aligned toward the spin axis $\omega_s$:

$$B_L^\gamma = R_3(\Phi_p) \cdot B_L$$

$$R_2(\Phi_p) = \begin{bmatrix} \cos \Phi_p & \sin \Phi_p & 0 \\ -\sin \Phi_p & \cos \Phi_p & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then we rotate $\theta$ about $\hat{2}$ to align the 3-axis with $\omega_s$

$$B_\omega = R_2(\theta) \cdot B_L^\gamma$$

and finally we spin up the system by rotating about $\hat{3}$ with the angle $\phi_s^R$

$$B_S = R_3(\phi_s^R) \cdot B_\omega$$

so we can get the following result after all these rotations.

In this $B_S$ system,

$$B_1 = B \cdot (\cos \theta \sin \kappa \cos \Phi_p - \sin \theta \cos \kappa) \cos \phi_s^R - \sin \kappa \sin \Phi_p \sin \phi_s^R)$$

$$B_2 = B \cdot (-\cos \theta \sin \kappa \cos \Phi_p - \sin \theta \cos \kappa) \sin \phi_s^R - \sin \kappa \sin \Phi_p \cos \phi_s^R)$$

$$B_3 = B \cdot (\sin \theta \sin \kappa \cos \Phi_p + \cos \theta \cos \kappa)$$

If the situation is ideal, which means the payload has rigid body motion, the sensor coordinate system and the payload spin coordinate system are well aligned and there is no
friction etc., the three components in the above three equations are the ones measured from FFM s, and the measurements are in the spinning and coning coordinate system.

If that were the case, reversing rotations back would give the fluctuations in \( \mathbf{B} \) in the B-L system and the studies related to the fluctuations in \( \mathbf{B} \) become possible after the data reduction. The fluctuations to \( \mathbf{B} \) are isolated from the \( \mathbf{B}_0 \). But the real situation is far from the ideal case. Steps which have to be gone through are shown in the following sections, including data cleanup, orthogonalization, alignment and frame transformations.

The difficulty of the magnetic field data reduction procedure can be seen roughly from Figure 4-4. The left panel at the top row shows the \( B_x \), \( B_y \), and \( B_z \) measurements (in sensor system) in nT vs. FFM time for the entire flight. The right panel at the top row is the three components for only T+370s-380s FFM time. The left panel at the bottom shows the deflection angle between the z axis of the sensor system and the total \( \mathbf{B} \). It should be mentioned that although the z axis in the sensor system (fluxgate z axis) is very close to the spin axis (\( \omega_s \)) of the FFM s, they are not the same because of the imperfect alignment between the two. The right panel shows the deflection angle during a short time period (340s-420s FFM time), where the large oscillations in \( \mathbf{B} \) were seen. In order to extract a tens of nT fluctuation signature from the \( 2 \sim 4 \times 10^4 \) nT oscillatory measurements (1:1000), we need attitude (all the involved angles) information to be accurate to less than 0.01°.

### 4.1.2 Data Cleanup

Some of the measurements from the FFM s showed large numbers of bit errors. Figure 4-4 illustrates this problem. The data shown are from the second downlink data of FFM1. Although FFM1 obtained the cleanest data relative to all the other FFM s, we still can see
Figure 4-4: Illustration of the FFM measurement and the difficulty of the magnetic field data reduction. The left panel on the top shows the overall evolution of the x, y, z components, the top trace is for z comp. The right panel on the top shows the motion of $B_x$ and $B_y$ and the maximal amplitude. The left panel at the bottom shows the deflection in terms of $\arccos(B_z/B)$ and the last panel indicates that there are deflection at FFM time about 380 sec.
the spikes due to the bit errors (there are some data points outside the envelope which is comprised by most of the measurements). Those data points which had the bit errors cannot be simply removed because FFTs (Fast Fourier Transformation) are needed in the analysis procedure. Therefore, there was a lot of data cleaning work to be done before moving on to the next steps. Otherwise, the least squared fitting would not be possible.

Major parts of data cleaning work are as follows:

1.) Do FFT (fast Fourier transform) of each data section and find the dominant frequencies.
2.) Do multiple local sine wave fits and then sum all the sine waves, then remove the outlying points from consideration. Do this for several times and delete smaller and smaller error points from consideration.
3.) Replace the outlying points by the 'most probable' value which is close to the expected value.

The data cleaning up work was done primarily by Manfred Boehm of LPARL [Boehm et al., 1999].

4.1.3 Inflight Calibration

Instrumental effects need to be removed from the data. The digital engineering unit output vector, \( \mathbf{E} \) and the magnetic field vector, \( \mathbf{B} \), in [nT], have the following relation:

\[
\mathbf{B} = \mathbf{C} \cdot (\mathbf{E} - \mathbf{O})
\]

Although preflight calibration coefficients and offsets are usually known, the exact relation of the output from the instrument to the physical magnetic field is unknown as these values often change after launch and they change slowly during the whole flight because of various
reasons, for example, temperature. Coning and spin modulations are often found in the total field magnitude using preflight calibration values. So the first step of the whole calibration procedure is to minimize the spin and coning modulations in the total field magnitude by orthogonalizing the three sensor axes. This provides an in-flight calibration matrix $C$ and offset vector $O$, which can vary with flight time. It should be mentioned that the loss of $z$ axis measurement for FFM2 complicated the data reduction procedure. We constructed a $'z'$ measurement from

\[
B_3 = \sqrt{B^2 - B_1^2 - B_2^2}
\]

(4.1.40)

where $B^2$ was chosen as the average smoothed value of FFM1 and FFM4's data; $B_1$ and $B_2$ are the measurements from $x$ and $y$ axis of FFM2 respectively.

4.1.4 Inflight Orthogonalization

Our inflight orthogonalization procedure is based on work by Brauer and Merayo [Brauer 1997; Merayo, et al., 1998].

Assume linearity of the instrument response, so that

\[
B = \begin{bmatrix}
B_1 \\
B_2 \\
B_3
\end{bmatrix} = \begin{bmatrix}
C_{11} & 0 & 0 \\
C_{21} & C_{22} & 0 \\
C_{31} & C_{32} & C_{33}
\end{bmatrix} \begin{bmatrix}
E_1 - O_1 \\
E_2 - O_2 \\
E_3 - O_3
\end{bmatrix}
\]

(4.1.41)

Linearize $B^2$ in square and cross terms of $(E_i - O_i)$:
\[
B^2 = a_1 \cdot (E_1 - O_1)^2 + a_2 \cdot (E_2 - O_2)^2 + a_3 \cdot (E_3 - O_3)^2 \\
+ a_4 \cdot (E_1 - O_1) \cdot (E_2 - O_2) + a_5 \cdot (E_1 - O_1) \cdot (E_3 - O_3) \\
+ a_6 \cdot (E_2 - O_2) \cdot (E_3 - O_3) 
\] (4.1.42)

where

\[
a_1 = C_{11}^2 + C_{21}^2 + C_{31}^2 \\
a_2 = C_{22}^2 + C_{32}^2 \\
a_3 = C_{33}^2 \\
a_4 = 2 \cdot (C_{31} \cdot C_{32} + C_{21} \cdot C_{22}) \\
a_5 = 2 \cdot C_{31} \cdot C_{33} \\
a_6 = 2 \cdot C_{32} \cdot C_{33}
\]

Thus the coefficients \(a_1 - a_6\) are related to the elements of the calibration matrix \(C\) as in the above equations.

Rearranging the equation to have the square of \(B\) expressed directly in terms of powers of engineering units, we get

\[
B^2 = a_1 \cdot E_1^2 + a_2 \cdot E_2^2 + a_3 \cdot E_3^2 \\
+ a_4 \cdot E_1 \cdot E_2 + a_5 \cdot E_1 \cdot E_3 + a_6 \cdot E_2 \cdot E_3 \\
- a_7 \cdot E_1 - a_8 \cdot E_2 - a_9 \cdot E_3 + a_{10} 
\] (4.1.43)

where:

\[
a_7 = 2 \cdot a_1 \cdot O_1 + a_4 \cdot O_2 + a_5 \cdot O_3
\]
\[ a_8 = 2 \cdot a_2 \cdot O_2 + a_4 \cdot O_1 + a_6 \cdot O_3 \]
\[ a_9 = 2 \cdot a_3 \cdot O_3 + a_5 \cdot O_1 + a_6 \cdot O_2 \]

We can see that coefficients \( a_7 - a_9 \) can be expressed by the offset vector \( O \) and \( a_1 - a_6 \).

Then the equation about \( B^2 \) can be represented in matrix form:

\[ B^2 = E_m \cdot A \quad \text{(4.1.44)} \]

where matrix \( E_m \) is composed of \( E_1^2, E_2^2, E_3^2 \), all the cross terms and \( E_1, E_2, E_3 \) and 1.

If we have \( N \) measurements, the equation about \( B^2 \) can be represented in matrix form:

\[ [B^2]_{N \times 1} = \begin{bmatrix} E_{x1}^2 & E_{y1}^2 & E_{z1}^2 & E_{x1}E_{y1} & E_{x1}E_{z1} & E_{y1}E_{z1} & E_{x1} & E_{y1} & E_{z1} & 1 \\ E_{x2}^2 & E_{y2}^2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & etc. \\ E_{x3}^2 & \cdot & \cdot & \cdot & etc. \\ \cdot & \cdot & \cdot & etc. \end{bmatrix} \cdot A_{10 \times 1} \]

\[ \text{Matrix } A \text{ is:} \]

\[ \text{(4.1.45)} \]
The absolute value of the magnetic field should be clean of spin and coning modulation. Either a smoothed version of the measured magnitude of $B$ or an IGRF model can be used as a model input.

Doing this, we can not only orthogonalize the three axes, but also obtain more accurate calibration of the calibration matrix coefficients and the offset vector's 3 components using linear regression to fit out the residual from this model input. Expressing $B^2$ in squared and cross terms of $(E_i - O_i)$, where $E_i$ represents the $i$th original measurement in engineering unit, is the way for orthogonalization.

In theory, the vector $A$ can be found from just ten of these measurements, but there are many measurement points (tens of thousands). So we solve the $A$ coefficients by this set of overdetermined linear equations. In the data analysis, we used the smoothed $B^2$ (averaged over spin and coning modulation) as the left side of the equation. Since the formalism in
the equation above is strictly matrix manipulation, IDL Numerical Recipes [Press, et al., 1988] routines called LUDC and LUSOL in IDL are used in data analysis. FFTs were used to monitor the calibration process. After 3 or 4 iterations, the power of total spin frequency is minimized in the time series of the corrected $B^2$. Then the matrix $C$ coefficients and offsets can be obtained from the transformations of $A$ by the relations mentioned above.

Further improvement can be achieved by using SVDC and SVSOL in IDL. SVDC is called singular value decomposition. It has the advantage if the coefficients fall near the range of a near singular solution.

4.1.5 Inflight Alignment

After the orthogonalization, there are still coning signatures in the 3rd component of the magnetic field $B_3$ and the transverse component $B_t$. This means that the sensor coordinate system is not perfectly aligned with the payload coordinate system.

$$\mathbf{B}_{pl} = R_3(\gamma) \cdot R_2(\beta) \cdot R_3(\alpha) \cdot \mathbf{B}_{sensor}$$  \hspace{1cm} (4.1.46)

The sensor coordinate system can be aligned with the payload spin coordinate system by rotating the above Euler angles to minimize the spin modulation in the third component of the magnetic field ($B_3$) or the transverse component ($B_t$).

FFT's are used to measure the power around the spin frequency. Minimizing this power will give the correct rotation angles.

4.1.6 Fitting the Coning Parameters

Having removed instrumental effects by orthogonalizing and aligning the data set to the payload reference frame, we can next proceed to generate a model of the payload motion.
First coning parameters can be found from $B_3$ (along the spin axis) component.

From the theory described above, $B_3$ can be written as the following:

$$B_{model} = B_{tot}(\sin \theta \sin \kappa \cos \Phi_p + \cos \theta \cos \kappa) \quad (4.1.47)$$

From $B_3/B_{tot}$, we can get the estimated $\kappa$ and $\theta$, where $\kappa$ is the angle between $B_L$, and $\theta$ is the angle between $L$ and $\omega_s$ (the spin axis).

$$\kappa = [\arccos(B_3/B_{tot})_{max} + \arccos(B_3/B_{tot})_{min}] / 2.0 \quad (4.1.48)$$

$$\theta = |\arccos(B_3/B_{tot})_{max} - \arccos(B_3/B_{tot})_{min}| / 2.0 \quad (4.1.49)$$

These estimated values are used in the following fitting procedure. $\kappa$, $\theta$ and $\Phi_p$ have polynomial dependence in time:

$$\kappa = \kappa_0 + \kappa_1 t + \kappa_2 t^2$$

$$\theta = \theta_0 + \theta_1 t + \theta_2 t^2$$

$$\Phi_p = \Phi_0 + \Omega_0 t + \Omega_1 t^2 + \Omega_2 t^3$$

Linear regression is used to get the parameters for $\kappa$, $\theta$ and $\Phi_p$.

$$\delta B_3 = \text{derivB}_{[n \times 10]} \cdot \delta z_{3[10 \times 1]}$$

where the matrix $\text{derivB}$ is composed of all the $\theta$, $\kappa$ and $\Phi_p$ derivatives of $B$. 

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This can also be written in the following simple form:

\[ \mathbf{B}_3 - \mathbf{B}_{\text{model}} = \sum_{\phi_z = \kappa, \theta, \Phi_0, \Omega_i, i=0,1,2} \frac{\partial \mathbf{B}_{\text{model}}}{\partial \phi_z} \cdot \delta \phi_z \]

Solving this matrix equation provides the vector \( \delta \phi_z \), so we can get all the fitting parameters necessary for \( \kappa, \theta \) and \( \Phi_0 \) parameters of the coning motion.
4.1.7 Fitting the Spin Phase

Next we model the spin motion. This is the most difficult piece. The model for $B_1$ and $B_2$, the two components in the plane perpendicular to the spin axis, is:

$$B_{1(\text{model})} = B_{tot} \cdot ((\cos \theta \sin \kappa \cos \Phi_p - \sin \theta \cos \kappa) \cos \phi_s^R - \sin \kappa \sin \Phi_p \sin \phi_s)$$  \hfill (4.1.50)

$$B_{2(\text{model})} = B_{tot} \cdot (-\cos \theta \sin \kappa \cos \Phi_p - \sin \theta \cos \kappa) \sin \phi_s^R - \sin \kappa \sin \Phi_p \cos \phi_s)$$  \hfill (4.1.51)

The phase is fitted by a cubic polynomial:

$$\phi_s = \phi_0 + \omega_0 t + \omega_1 t^2 + \omega_2^3$$  \hfill (4.1.52)

As with the coning fitting, we do fitting to both $B_1$ and $B_2$.

$$B_1 - B_{1(\text{model})} = \sum_{\phi_x=\phi_0, \omega_i=0,1,2} \frac{\partial B_{1(\text{model})}}{\partial \phi_x} \cdot \delta \phi_x$$

$$B_2 - B_{2(\text{model})} = \sum_{\phi_y=\phi_0, \omega_i=0,1,2} \frac{\partial B_{2(\text{model})}}{\partial \phi_y} \cdot \delta \phi_y$$

Spin error is found from the both components. From the above equations, we can get better parameters for the spin phase. We take $\phi_s$ to be the average from the fittings of both components.

$$\phi_s = (\phi_1 + \phi_2)/2.0$$

where $\phi_1$ is from the fitting of $B_1$ and $\phi_2$ is from the fitting of $B_2$. 

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4.1.8 Despinning of the Data

Having arrived at time-dependent models of the rigid body motion parameters, we can now rotate the data into the B-L frame:

\[ \mathbf{B}_{\text{despun}} = R_2(\kappa) \cdot R_2(-\Phi_p) \cdot R_2(-\theta) \cdot R_2(-\phi_s) \cdot \mathbf{B}_{\text{measured}} \] (4.1.53)

One thing important to mention here is that those fitted parameters have to be fitted very carefully in order to reach the required accuracy (tens nT out of 40,000 nT). For the coning parameters fitting, this was achieved by using the fitted results as the initial values, and iterating for better ones. Repeating this process for a few times gave the final results.

For the spin parameters fitting, the sun sensor data were used to get the total spin rate, which varies slowly with time. Total spin \( \omega = \Omega_p + \omega_s \). This is a vector equation, with the angle between \( \Omega_p \) and \( \omega_s \) being \( \theta \). They satisfy equation 4.1.20. For the flat, disk-shaped FFMs, the coning frequency \( \Omega_p \) is bigger than the spin frequency \( \omega_s \) (about 2.0 for FFMs). From the total spin and the fitted coning rate, and the angle \( \theta \), we can get the spin rate \( \omega_s \). Then we use this result as the initial value and run the linear regression code for the spin phase fitting to obtain all the spin parameters. The sun sensor data was used to get the spin frequency as an input for the data analysis procedure.

Applying the method to short time interval (fewer measurement points) makes it easy to fit all the parameters. But this removes the information on the long time scales. For example, the slope in Figure 3-25 of Chapter 3, for the x-component of the interval 500-520s, disappears if we do short interval fitting. However, doing the fitting for more data points (longer interval) brings a lot of challenges for the whole fitting procedure.
Short time interval fitting was done before the fitting of longer time interval because the longer time fitting needs more accuracy. Only by using the fitting parameters of the short time interval as the initial guess for the longer time interval can the fitting of longer ones work. Then the longer time interval fitted parameters were used to get the fitting of the whole downlink (about 300 second, data sampling rate is 140/sec for all FFMs). The fitting results described here all went through this procedure.

$B_x$ is the $x$ component in B-L coordinate system, and $B_y$ is the $y$ component in B-L system after the data reduction. The $x$ axis in B-L system is perpendicular to $B$ and points away from $L$ (mostly in the north direction), the $z$ axis is along $B$ (pointing mostly down) and the $y$ axis is the one perpendicular to the plane consisting of $B$, $L$ (pointing eastward). Axes $x$, $y$ and $z$ make up a right handed coordinate system. The determination of $x$, $y$, $z$ axis orientation from the Enstrophy data can be found in Appendix C.

The magnetic field perturbations perpendicular to the Earth's magnetic field obtained from this data reduction procedure are shown in Figure 3-24 and Figure 3-25. Over 100 nT variations in $B_x$ (in B-L plane, pointing away from $L$) and $B_y$ (out of the plane) were seen on three FFMs as the payloads exited the active auroral arc region during 10 s interval. The discontinuity of the FFM2 plot in the top panel and middle panel is from the data itself. The major oscillations in $B$ here may be considered as Pi1 pulsations in the frequency range of 0.1-0.2 Hz.

4.2 Geometry of the Four FFMs

The position of four FFMs must be determined for the current density calculation. The four FFMs were ejected in a plane from the main payload at about 136s of flight time.
The main payload magnetometer on the deck provides information on the spin phase of the main payload. The $y$ and $z$ axes are in the plane of the deck. Measuring $\tan(bz/by)$ and assuming the projection of $B$ lies in the north direction, the position and direction of all 4 FFMs at release were calculated. The speed at which each FFM exited was measured by the optical gates located at the exit of the FFM tracks. The spin frequency of each FFM was determined from the magnetic field measurement and was used to determine the exiting direction of the FFM relative to its radial exiting point as models (from the theoretical calculation of the deployer in Appendix A and another model done by Mark Widholm) of the ejection mechanism provide a relationship between the exit velocity and the final spin rate. Onboard beacons showed the main payload relative motion between FFMs, main payload and the north direction. Details on determining the positions of the four FFMs are given in Appendix D. Once the exit velocity (direction and magnitude) was determined, the separation vectors were then calculated as a function of time. Figure 4-5 shows the geometry of the four FFMs.

4.3 Taylor Expansion for the FAC density and Error Analysis

The primary purpose of a multipoint measurement is to distinguish between temporal and spatial variations by means of four (if it is 3D case such as the ClusterII Mission) or three (if it is 2D case such as Enstrophy) or more payloads. An obvious question is how, exactly, the gradients of the spatial variations should be determined from such multipoint measurements. The parallel component of electric current density (magnetic field aligned current density) is related to specific combinations of the magnetic field gradients through Ampere's law. Here Taylor series expansion is used for the evaluations of various components of magnetic
Figure 4-5: The geometry of four FFMs.
field gradients. The details are described below.

**Taylor Expansion**

Using a Taylor expansion and ignoring the higher order terms, we assume $B_x$ and $B_y$ are only $x$, $y$ dependent.

\[
B_{xi} = B_{xj} + \frac{\partial B_x}{\partial x}(xi - xj) + \frac{\partial B_x}{\partial y}(yi - yj) \tag{4.3.1}
\]

\[
B_{yi} = B_{yj} + \frac{\partial B_y}{\partial x}(xi - xj) + \frac{\partial B_y}{\partial y}(yi - yj) \tag{4.3.2}
\]

where $i \neq j$ and $i, j = 1, 2, 4$, the index of the FFMs. It should be mentioned that the data for the second downlink from FFM3 are too noisy to be useful for the data analysis. Let

\[
a \equiv \frac{\partial B_x}{\partial x} \tag{4.3.3}
\]

\[
b \equiv \frac{\partial B_x}{\partial y} \tag{4.3.4}
\]

\[
c \equiv \frac{\partial B_y}{\partial x} \tag{4.3.5}
\]

\[
d \equiv \frac{\partial B_y}{\partial y} \tag{4.3.6}
\]

$a$, $b$, $c$, $d$ can be calculated by SVDC (singular value decomposition) and SVSOL of IDL.

Using $j = \frac{1}{\mu_0} \nabla \times B$, we get:

\[
j_z = \frac{1}{\mu_0}(c - b) \tag{4.3.7}
\]

Figure 4-6 shows the calculated current density in the top panel; $\frac{\nabla \cdot B}{B_{tot}}$ in the middle, the inverse of which represents a scale length; and $\frac{\nabla \cdot B}{|\nabla \times B|}$ in the bottom panel, which is a unitless ratio. Figure 4-7 shows the result for $a$, $b$, $c$ and $d$ respectively.
Figure 4-6: The calculated current density using Taylor expansion in the top panel; \( \nabla \cdot \mathbf{B} \) (unit: 1/m) in the middle; and the ratio of \( \nabla \times \mathbf{B} \) (unitless) in the bottom panel.
Figure 4-7: The partial derivatives of B calculated from Taylor expansion.
The curl of B calculated in this manner gives maximal current densities of \(\sim 15\mu A/m^2\) on the \(\sim 1\) km scale, during a time interval when the maximum magnetic perturbation was \(\sim 60 \text{nT over 5 s}\). For comparison, inferring \(j_z\) from a one-point measurement, and converting from a time series to a spatial array using the rocket speed, would give a maximal current density measurement of \(\sim 40-50\mu A/m^2\) in this case.

**Error Analysis of Current Density Determination** \(\nabla \cdot B / |\nabla \times B|\) can be used as an estimate of the error \(\delta J/J\) because \(\nabla \cdot B\) should be zero in theory, i.e. the divergence of \(B\) is zero due to the non-existence of 'magnetic charge'. \(\nabla \cdot B / B_{\text{tot}}\) can be served as another way for the error analysis and it's plotted in the Figure 4-6. But the following discussion is in terms of the first ratio.

This ratio is \(\neq 0\) at some times. Why? Possible source of errors are as follows.

1). When \(\nabla \times B\) is close to zero, the ratio defined above will be large.

2). Any inhomogeneity (gradients) in a scale size less than the FFM separation makes using Taylor expansion method inappropriate.

3). The determination of the FFM geometry is not 100% accurate. The error could contribute the non-zero ratio.

4). The uncertainty in the \(B\) measurements could be another reason.

5). The soundness of the no \(z\) dependence in \(B\).

6). Because of limitation of the data locations (3 point), Taylor expansion is only up to first order.

From Figure 4-7 we can see that \(\partial B_y / \partial y\) has a negative offset, which is the reason why the ratio (divergence \(B\)/curl \(B\)) appears to be negative in general. But the ultimate reason for this needs further investigation.
The above analysis may make it sound not worthwhile to do the multipoint \( B \) measurement and to use the Taylor expansion method for the current density calculation. But at least the multipoint measurement combining using Taylor expansion for the partial derivatives not only allows us to calculate the current density but also provides us with the ability to do the checking on the ratio (divergence \( B/\text{curl } B \)) while the previous single-point measurement can only provide the current density on the assumption that the measured fluctuations are spatial; it doesn't provide any information about the divergence of \( B \) at all. The above analysis might indicate that future measurements need more points (more than 4), and more dense (short separation distance) measurements; and they need to have 3 dimensional configuration if necessary.
Chapter 5

Interpretation and Discussion

It is well known that all physically observable plasma and field parameters, such as particle populations and electric and magnetic fields, vary in both space and time, and that the spatial-temporal ambiguity problem in space plasma is one of the most difficult and still remains open. Observations made so far, especially results from the Freja and FAST satellites, have shown the auroral zone is highly structured. The structures in the measured physical quantities could be either spatial gradients or temporal variations, or possibly both. Multipoint measurement is for this purpose—to distinguish temporal from spatial variations, a separation very important for our understanding of the involved physical processes and generation mechanisms in all regions of the Sun-Earth system.

The focus of this thesis is the multipoint study of the large magnetic field fluctuations observed by three FFMs right at the poleward edge of a pre-midnight auroral arc when the sounding rocket Enstrophy was near its apogee (~1070 km), and entering the polar cap. Four parts are included in this chapter. First, we give a brief introduction to the spatial and temporal variations of magnetic field observations in general and the commonly used method of distinguishing them. Second, in the context of all the other available data, interpretation of the observed magnetic fluctuations from the Enstrophy sounding rocket is given, including both spatial and temporal features of the event. Third, a wavelet analysis technique used for further investigation of the event is discussed, and the cross-correlation of the wavelet-transformed magnetic field data between different FFMs is covered. Lastly,
conclusions from the multipoint study of this auroral event are drawn.

5.1 Temporal vs. Spatial Question

The observed magnetic field fluctuations could be caused by either spatially structured field aligned currents, or by Alfvén waves (temporal and spatial variations), or by both spatial gradients and temporal variations together, such as a surface wave.

The traditional method of distinguishing spatial variations from temporal changes is based on the ratio of $\Delta B_y / E_x$ [Knudsen, 1990; 1992]. $\Delta B_y$ is the east-west component of magnetic field fluctuations and $E_x$ is the electric field measured in the north-south direction. In a static model,

$$\frac{\Delta B_y}{\mu_0 E_x} = \Sigma_P$$

(5.1.1)

However, if the perturbations of magnetic field are caused by dissipationless Alfvén waves, there is a different relationship between the two components.

$$\frac{E_x}{\Delta B_y} = V_A$$

(5.1.2)

where $V_A$ is the Alfvén velocity. Knudsen [1990, 1992] gave an expression of a more general formula (using an impedance function) for the relationship between $E_x$ and $\Delta B_y$ in terms of frequency.

$$Z(f) = \mu_0 | \frac{E_x(f)}{\Delta B_y(f)} |$$

(5.1.3)

where $Z(f)$ is the impedance function and can be used to distinguish temporal fluctuations from Doppler-shifted spatial structures in measured field data. For the static case, $Z(f) = \Sigma_P^{-1}$; while for the Alfvén waves, $Z(f) = \mu_0 V_A$. The ratio of $E_x$ and $\Delta B_y$ for Alfvén waves including the reflection of the ionosphere was also given in Knudsen, et al., 1992.
The magnetic field fluctuations measured by 3 FFMs from the Enstrophy mission lasted about 70 seconds, with a largest variation of 100 nT over 10 seconds (see Figure 3-23 for details). The multipoint measurement shows both spatial structuring and temporal variations.

During the 70 s time period, the multiple FFMs often observed similar magnetic perturbation patterns. From a relative time shift of a few tenths of a second between the FFMs, we can deduce that sheetlike currents were apparently moving with respect to the payload at a velocity of the order of 1 km/s. However, at other times, the magnetic perturbations on different FFMs did not correlate well with any time delay, which indicates the presence of localized Alfvén waves and/or even more filamentary currents of scales less than the separation between different FFMs. Moving-current-sheet structures cannot explain the fact that there are non-zero perturbations in B magnitude. Both shear Alfvén waves and compressional Alfvén waves must have been both present in order to agree with these measurement results.

5.2 Interpretation of the Observed Magnetic Field Fluctuations

As discussed in Chapter 3, the particle detectors (both ion and electron detectors) received no useful data. The electric field measurement only had one pair of booms and the unbalanced main payload made the extraction of DC electric field very difficult. Facing the challenges of lacking plasma environment and good DC electric field data, but with the help of all the other available measurements, we interpreted this multipoint magnetic field measurement and found spatial structuring and temporal variations were both present in this event. The interpretation details are seen in the following.
5.2.1 Spatial Signatures and Current-Sheet Model

Soundness of the "sheetlike" assumption of FACs in the auroral zone has been verified by many observations [Iijima and Potemra, 1976; Ohtani, et al., 1994, Peria, et al., 2000] and inferred from the shape of auroral arcs. It is believed that large-scale FAC sheets are made up of many smaller filaments [Sugiura et al., 1982; Lühr et al., 1994].

In order to compare the FFM observations with this picture, a model was developed using finely structured current sheets and artificial "payloads". The 1-d model consists of 11 current sheet filaments, 1-5 km thick, with relative motion of 0~5 km/sec to each other. The current sheets extend uniformly in the east-west direction and their magnitude varies in the north-south direction, across their thickness. Figure 5-1 is a diagram showing the composition of the model.

The one dimensional current density profile was chosen as follows based on [Lysak, 1999]:

$$ j_z = j_0 (1 - (x + m + v \cdot t)(x + n + v \cdot t)/a^2)e^{-|x+n+v \cdot t|^2/2a^2} $$ (5.2.1)

where 'm', 'n' are fitting constants defining the center location of a specific current filament along the south-north direction; 'a' is a constant related to the width of the current filament; and 'v' represents the motion of the current sheet and its value was chosen based on the observations of auroral arc motion.

This current corresponds to a magnetic perturbation in the y direction only and it is:

$$ B_y = \mu_0 j_0 (x + m + v \cdot t)e^{-|x+n+v \cdot t|^2/2a^2} $$ (5.2.2)

This choice of $j_z$ satisfies $\nabla \cdot B = 0$ since the only component of $B$ is $B_y$ and it is only $x$ dependent.
Figure 5-1: The moving current sheet model
For the 2-D case, we add an expression for $B_x$. $B_x$ was simply chosen as the following and it is only $y$ dependent so that $\nabla \cdot B = 0$ still holds.

$$B_x = \mu_0 j_0 (y + m' + v't)e^{\frac{(y + n' + v't)^2}{2a^2}}$$

(5.2.3)

Then the field-aligned current $j_z$ has contributions from both $B_x$ and $B_y$.

$$j_z = \frac{1}{\mu_0} (\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y})$$

$$= j_0(1 - (x + m + v \cdot t)(x + n + v \cdot t)/a^2)e^{\frac{(x + n + v \cdot t)^2}{2a^2}} - j_0(1 - (y + m' + v' \cdot t)(y + n' + v' \cdot t)/a'^2)e^{\frac{(y + n' + v' \cdot t)^2}{2a'^2}}$$

(5.2.4)

Figure 5-2 and Figure 5-4 are the results from the model, where the parameters of 11 current sheets were chosen to best represent the observations (compare them to Figure 5-3), and two artificial "payloads" were flown through the modeled current sheets to extract time series. Shown in the top panel of Figure 5-2 is the modeled $B_y$ as seen by two artificial "payloads". The black one was to simulate the result of FFM4 and the light grey, FFM1. The middle panel plots the field aligned current density $j_z$ as the inputed, total current density contributed from all the current sheets in the model. The bottom panel gives the calculated $j_z$ by using the approximation from only two point measurement.

$$j_z \approx \frac{1}{\mu_0} \frac{\Delta B_y}{\Delta x}$$

(5.2.5)

where $\Delta B_y$ is the difference between the two artificial payload traces, and $\Delta x$ is their separation distance.

Some features of the magnetic field measurements can be explained by the current sheet model present here. The $B_y$ (E-W) component, and relative delays between different FFMs can be modelled fairly well by varying the sheet width, strength and velocities. However,
Figure 5-2: 1-D model result. Note: $B_y$ component and the delays between different FFMs can be modelled fairly well.
Figure 5-3: Magnetic fluctuations from the FFM measurements.
Figure 5-4: 2-D model result. The delays between different FFMs in $B_x$ cannot be modelled from the current sheet model. The first panel shows $B_x$ from the 2-D model; the middle panel shows the modelled $B_y$; the bottom panel shows the total current density as input of the model.
the delays of $B_x$ (N-S) component between two different payloads cannot be modelled from this model in which $B_x$ is only y dependent and $B_y$ is only x dependent. However, the model could reproduce the general features of $B_x$ and this result is shown in the top panel of Figure 5-4. The reason for this might be that the simple representation of $B_x$ in equation 5.2.3 is not good enough and both components ($B_x$ and $B_y$) could depend on x and y. The motion for the current filaments is usually far more complicated than what this model can describe. And the very dynamic, dancing rays in the narrow-field camera data seem to be manifestation of this. This does not eliminate the possibility that the signatures of the observed magnetic field from multipoint measurement are caused by moving spatial structures of field aligned currents. Better and more accurate models on the magnetic field signatures measured from multiple payloads can be developed. However, ultimately they might be limited by mathematical difficulties.

It should be pointed out that there are limitations with measurement of $j_z$ for only finite number of probes. We can see this from the difference between the calculated $j_z$ and the input $j_z$ in Figure 5-2. As discussed in Chapter 4, there are limitations with using only finite number of probes to calculate current density. The Taylor expansion is only 1$^{st}$ order; structures smaller than the separation distance cannot be measured correctly.

### 5.2.2 Wave Aspects of the Event

The occurrence of kinetic Alfvén waves in the auroral zone, and their close relation to particle acceleration and field aligned currents in the auroral zone, are discussed in Chapter 1 of this thesis. The observed kinetic Alfvén waves in the auroral zone are found to be more often located at the edge or boundary of auroral arcs and are often associated with
density depletion or increase [Boehm, et al., 1990a; Stasiewicz et al., 1998; Chaston et al., 2000]. We will show below that the large oscillations of the magnetic field, seen at the poleward edge of an arc from the multipoint measurements of this sounding rocket, indicate signatures of kinetic Alfvén waves as well.

First of all, at some times, the magnetic perturbations on different FFMs did not correlate well with any time delay that can be explained by any moving current sheets, which indicates the presence of localized kinetic Alfvén waves and/or even more filamentary currents.

Secondly, the result shows a non-zero perturbation in the magnitude of the magnetic field. This compressional signature in the observational results could come from the kinetic Alfvén waves with $k_\perp \sim \omega_{pe}/c$ propagating in the inertial ($\beta \ll 1$) dispersive regime. Figure 5-6 and Figure 5-7 show the perturbations in $B_\perp (\Delta B_\parallel)$ and the total magnetic field magnitude ($\Delta B_{mag}$) respectively. From Figure 3-25 in Chapter 3, we see large magnetic field perturbations of $\sim 50 \text{nT}$ perpendicular to the Earth's magnetic field. Here we see a 3-5 nT perturbation in $\Delta B_\parallel$. The ratio of $\Delta B_\parallel / |\Delta B|$ is about 10%. It is interesting to mention that Volwerk et al.[1996] report a small compressional component associated with the Alfvén wave from measurements of Freja spacecraft. The coupling of shear mode and compressional mode of Alfvén waves in the auroral zone is also studied by Lysak in his Alfvén wave model [Lysak,1997; Lysak 1999]; these results are fairly consistent with the observations of $Pc1/Pil$ waves, the compressional component of kinetic Alfvén waves making the field-line-confined narrow structures appear as signatures in a broad region across the field lines.

A rigorous evaluation of the existence of Alfvénic signatures requires, as discussed above,
a measurement of $E_z/B_y$. Although our electric field measurement had only one component and the one-pair boom was connected to the unbalanced main payload, the data can still provide some reference for our understanding of this event. Figure 5-5 shows the observed $E_z$, processed with a Hilbert-transform based despin routine. If the perturbations in $B$ are Alfvénic, an Alfvén velocity in the range $1 \times 10^6 - 1 \times 10^7$ m/sec implies a perturbation electric field of $3\text{mV/m} - 30\text{mV/m}$. The $V \times B$ electric field from the rocket motion is about $30\text{mV/m}$, since the rocket speed is about $1$ km/sec. The observed perturbations in the electric field data are indeed of the right amplitude to support Alfvénic activity.

Another temporal feature of this event is seen in the high frequency wave data, where the Langmuir wave bursts accompany the magnetic fluctuations. The Langmuir wave bursts are often associated with velocity dispersed electrons [McFadden, et al., 1986; Ergun, et al., 1991; Lynch et al., 1999], which can be accelerated downward by the localized Alfvén waves.

Thus this event has both temporal and spatial aspects. There are times when the relative shifts between different FFMs can be explained by sheetlike moving currents, while at other times, the relative shifts in magnetic field perturbations do not correlate well with any time delay. There are also signatures of Alfvén waves during this event. However, the Alfvén wave signatures are not as clean as those observed from another sounding rocket—Auroral Turbulence II (AT II), which also provided multipoint measurements by its three payloads.

The AT II sounding rocket was also launched from the Poker Flat Research Range, on February 11, 1997 at 0836:40 UT. It consisted of three identically instrumented payloads and flew through several arc structures in a pre-midnight auroral breakup with an apogee of 550 km. For comparison, the Enstrophy sounding rocket was launched from the same launch range on February 11, 1999 at 0645:31UT with an apogee of ~1070 km.
Figure 5-5: electric field data: - only one boom pair; unbalanced payload; \(V \times B \sim 30\,\text{mV/m} \); - any Alfvén signature: \(\sim 3-30\,\text{mV/m}\)

Figure 5-6: Non-zero deviation in \(B_z\) component.
Figure 5-7: Non-zero deviation in B magnitude.

Figure 5-8 shows the clear sinusoidal Alfvén wave signatures seen from ATII's payloads. At the time when these waves were seen, the electron data show that the event occurred inside the poleward edge of the inverted-V arc region.

The analysis above shows there is evidence that Alfvén waves were present during the large-B-oscillation event observed by the Enstrophy sounding rocket. But from Figure 3-25 in Chapter 3 we can see the Alfvén waves were not as clean as those of ATII's. It should be mentioned that the large B fluctuations from the Enstrophy measurement occurred in the very vicinity of the poleward edge of the arc, right outside the edge of an arc, towards polar cap; while ATII's event occurred at the poleward edge but within the inverted-V region. The locations relative to an arc are slightly different for these two events observed by the two different rockets.

Unlike the magnetic field structures of AT II (very monochromatic), the structures
Figure 5-8: ATII's electric and magnetic fields of the East and North payloads in geographical coordinates (From [Ivchenko et al, 1999]). Note: this multiple payload measurement shows clean sinusoidal Alfvén wave structures.
of B field from the Enstrophy measurement are rather irregular—lots of different things happened all at once. In order to separate the structures in the measured magnetic field at different scales, wavelet transformations were studied and applied to each component of the multipoint timeseries of magnetic field observations. Then, a correlation study at variable delay times was performed at each scale of the wavelet transform, between each pair of FFMIs. This enables us to study the individual motions on each scale size, i.e., the individual motions of each frequency.

5.3 Wavelet analysis of the event

Wavelet transform of a time series gives information on both time and frequency. It provides finer details of the series than FFT or WFT (Windowed Fourier Transform) and reduces the time localization and frequency localization problems with other kind of transformations. The wavelet analysis uses a wavelet of the exact same shape, only the size scales up or down with the size of the window.

Wavelet analysis [Chui, 1992; Rao and Bopardikar, 1998; Torrence and Compo, 1998] usually uses a wave packet of finite duration and with a specific frequency. This “wavelet” used here is called a Morlet wavelet, which is nothing more than a sine wave multiplied by a Gaussian envelope.

In practice, the Morlet wavelet is defined as the product of a complex exponential wave and a Gaussian envelope because in this way we can not only know the amplitude of any periodic signals, but also know the information on the phase. The Morlet wavelet has the following form:

$$\psi_0(\eta) = \pi^{-1/4}e^{i\omega_0\eta}e^{-\eta^2/2}$$  \hspace{1cm} (5.3.1)
where $\psi_0$ is the wavelet value at non-dimesional time $\eta$, and $\omega_0$ is the wavenumber. This is the basic wavelet function. When doing the wavelet transform, we need some way to change the overall size as well as a way to slide the entire wavelet along in time. Therefore the scaled wavelets are defined as:

$$\psi\left[\frac{(n' - n)\delta t}{s}\right] = (\frac{\delta t}{s})^{1/2} \psi_0 \left[\frac{(n' - n)\delta t}{s}\right]$$  \hspace{1cm} (5.3.2)

where $s$ is the scale parameter, $n$ is the translation parameter used to slide in time. The normalization factor $s^{-1/2}$ is to keep the total energy of the scaled wavelet constant.

If we are given a time series $X$, with values of $x_n$ at time index $n$, where each value is separated in time by a constant time interval $dt$, the wavelet transform $W_n(s)$ is just the inner product (or convolution) of the wavelet function with our original time series.

$$W_n = \sum_{n'=0}^{N-1} x_{n'} \psi^\ast\left[\frac{(n' - n)\delta t}{s}\right]$$  \hspace{1cm} (5.3.3)

where the asterisk(*) denotes complex conjugate.

The wavelet analysis gives information on which time, and at which time scale, a signature takes place. After doing the wavelet transform on the components of magnetic field measurements from the FFMs, we can cross correlate the same component ($B_x$ or $B_y$) between different FFMs. The goal is to study the signature delays at particular scales between different payloads of FFMs because we know from the visible signatures of narrow-field camera data that different scales can have different motions. The cross correlation is given by

$$P_{xy}(L) = \frac{\sum_{k=0}^{N-L-1}(x_{k+L} - \bar{x})(y_k - \bar{y})}{\sqrt{[\sum_{k=0}^{N-1}(x_k - \bar{x})^2][\sum_{k=0}^{N-1}(y_k - \bar{y})^2]}} \text{ for } L < 0$$  \hspace{1cm} (5.3.4)
\[ P_{xy}(L) = \frac{\sum_{k=0}^{N-L-1}(x_k - \bar{x})(y_{k+L} - \bar{y})}{\sqrt{\sum_{k=0}^{N-1}(x_k - \bar{x})^2 \sum_{k=0}^{N-1}(y_k - \bar{y})^2}} \text{ for } L > 0 \] (5.3.5)

5.3.1 Wavelet and cross-correlation study of the model data

For the purpose of better understanding of the wavelet-transformed and cross-correlation applied FFM magnetic field measurements, the same processes were performed on 1) a time series of a moving square pulse; 2) the data obtained from the moving current-sheet model.

As mentioned in the current-sheet model section, two time series representing the magnetic field perturbations were extracted from flying two "artificial" payloads through the modelled current sheets. In the current sheet model, 'Payload 1' was to simulate FFM4 and 'payload 2' was to simulate FFM1. Here we begin with a similar but simpler case, that of a moving square wave pulse. This very simple model is not related to the data at all. It is just to help us understand the processes performed here.

It should be mentioned that the axes and parameters shown in the wavelet and cross-correlation related plots in this section all have similar meaning. In the wavelet plots, the \( x \) axis represents the time \( t \). For the very simple model, it is just a time; for the current-sheet model, \( t+470 \) is same as the flight time of all the FFMs' plots. The \( y \) axis shows the time scale in seconds (‘period’, ‘scale’ and ‘time scale’ all mean the same thing when used in the label of the plots shown here); the inverse of it is the corresponding frequency. What is plotted (\( z \)-axis) is the logarithmic value of the wavelet transform’s amplitude, i.e., \( \log_{10}(|W_n|) \), where \( W_n \) is the wavelet transform in Equation 5.3.3 and the intensity of it is represented by color, red is the most intense one and dark purple is the least intense one.

In the 3-d (colored) correlation plots, what is plotted is the cross-correlation of the wavelet transform of \( \mathbf{B} \) between two ‘artificial’ payloads. The cross-correlation is given in
Equation 5.3.4 and Equation 5.3.5. The $x$ axis here represents the time lag in terms of time index (can be changed into the lagged time in seconds) between the two time series of the two payloads. Negative time lag ($-\Delta t$) means shifting the first time series ahead by $\Delta t$ and then calculating the cross-correlation, while positive time lag means shifting the first time series behind by $\Delta t$. The $y$ axis represents different time scales. The correlation value is indicated by the color and it could have its value between $-1$ and $+1$. Dark purple represents the smallest cross-correlation value and red represents the largest cross-correlation value. A cross-correlation which is close to $-1$ could mean the quasi-periodicity of a structure. A peak at a certain time lag ($x$ value) for a certain time scale ($y$ value) means that the two time series correlate well at $(x,y)$. From this, and knowing the separation and motion of the two probes, the motion and the size of the structure can be deduced. We start with the results from the very simple model.

**The very simple square pulse model**

In this model, the square-shaped magnetic field structure is represented by a step function with a width of 5 km, an amplitude of 6 nT, and with a velocity `$v$'.

\[
B(x) = \begin{cases} 
6 & \text{if } (a+v*t) < x < (b+v*t) \\
0 & \text{for all the other } x 
\end{cases}
\]

where $x$ axis is chosen to be along the north direction. $v$ is positive if the structure moves towards the north direction; $v$ is negative if the structures moves towards the south direction. The two artificial payloads are originally (at $t = 0$) located at 1 km and 0 km respectively.
<table>
<thead>
<tr>
<th>width</th>
<th>a</th>
<th>b</th>
<th>motion</th>
<th>peak velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 km</td>
<td>-50 km</td>
<td>-45 km</td>
<td>4 km/sec</td>
<td>-3 km/sec</td>
</tr>
<tr>
<td>5 km</td>
<td>20 km</td>
<td>25 km</td>
<td>-0.1 km/sec</td>
<td>1.1 km/sec</td>
</tr>
<tr>
<td>5 km</td>
<td>-50 km</td>
<td>-45 km</td>
<td>3 km/sec</td>
<td>-2 km/sec</td>
</tr>
</tbody>
</table>

Table 5.1: The chosen parameters for three different situations.

along the $x$ axis with a northward velocity 1 km/sec.

Three individual cases of this simple model were chosen to illustrate the results. In the first case, the structure is moving 4 km/sec northward and it is initially located at -50 km (value 'a'), i.e., passing the moving payloads from behind. In the second case, the structure is moving southward with a speed 0.1 km/sec and is initially at 20 km (value 'a'), i.e., moving towards the payloads. The third case is similar to the first one except that the structure's velocity is changed to be 3 km/sec. The parameters are shown in Table 5.1, where 'motion' is the structure's velocity (positive if it is northward) and 'peak velocity' (positive if the structure moves towards the payloads, i.e., 'payload' 1 sees it first) is the velocity deduced from the lag time of the cross correlation result. At that lag time, the correlation has its peak value. We know that in theory that the 'peak' velocity should be the relative velocity of the moving structure to the moving payloads. The line plots of cross-correlation vs. velocity (a velocity obtained from the lag time and the separation distance between the two 'payload') also prove this. The bottom panel of Figure 5-10 shows that at several different scales, the correlations all have a peak at velocity -3 km/sec. In the other two cases, the cross-correlation also gives the predicted 'peak' velocity.

Figure 5-9 and Figure 5-10 show the results for the first case; Figure 5-11 and Figure 5-12 are the results for the second case; and the results of case 3 are shown in Figure 5-13.
and Figure 5-14. Four figures are included here for each case. The first figure shows the time series extracted from the two moving 'payloads'; the second figure shows the wavelet transform of one time series; the third figure is the cross-correlation plot vs. lag time; and the fourth one shows the cross-correlation line plots in terms of relative velocity. The cross-correlation line plots are not smooth because the velocity is proportional to the reverse of the lag time. On these plots there are more data points concentrating on the boundaries near the zero velocity, and fewer data points when it is away from the zero velocity.

From the cross-correlation, we not only can find at which time lag where the cross-correlation has a maximal value, and therefore the 'peak velocity' (i.e., the structure's relative velocity) can be obtained, but also can find at which scale (period) the cross-correlation has the maximal value. From the 'peak' velocity, we can get the structure's velocity ($v$) in the plasma frame, therefore its spatial scale size can be determined from multiplying its velocity $v$ by the scale (period) in seconds. Take the first case as an example, from the 'peak' velocity (-3 km/sec) obtained from the cross-correlation, we can figure out that it has velocity of 4 km/sec in the north direction because the payloads move at 1 km/sec northward. Also through the cross-correlation, we find out that the cross-correlation peaks at scale index 7, which corresponds to a scale of 1.26307 seconds. This gives us a width of 5.05 km, matching the input width of 5 km.

**The current-sheet model**

Next we consider the multiple current sheet model presented earlier in this chapter, optimized to best represent the observed fluctuations. Figure 5-15 shows the magnetic field data of 'payload 1' and 'payload 2' after going through wavelet transformation in the top two
Figure 5-9: Case 1 (the structure is moving at 4 km/sec): the two time series and wavelet transform for one of them. The top panel shows the two time series extracted from flying the two ‘artificial’ payloads through the moving squared wave pulse structure, the bottom panel shows the wavelet transform for one of them.
Figure 5-10: Case 1: cross-correlation between the two payloads, tv plot and line plots.

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Figure 5-11: Case 2 (the structure is moving at 3 km/sec): the two time series and wavelet transform for one of them. The top panel shows the two time series extracted from flying the two 'artificial' payloads through the moving squared wave pulse structure, the bottom panel shows the wavelet transform for one of them.
Figure 5-12: Case 2: cross-correlation between the two payloads, tv plot and the line plots.
Figure 5-13: Case 3 (the structure is moving at -0.1 km/sec): the two time series and wavelet transform for one of them. The top panel shows the two time series extracted from flying the two 'artificial' payloads through the moving squared wave pulse structure, the bottom panel shows the wavelet transform for one of them.
Figure 5-14: Case 3: cross-correlation between the two payloads, tv plot and the line plots
Table 5.2: The key parameters for the current sheets in the model.

<table>
<thead>
<tr>
<th>Key para.</th>
<th>sheet1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (width in km)</td>
<td>3.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.2</td>
<td>1.8</td>
<td>2.2</td>
<td>3.0</td>
<td>6.0</td>
<td>2.5</td>
<td>3.0</td>
<td>9.0</td>
</tr>
<tr>
<td>v (veloc. in km/sec)</td>
<td>0.0</td>
<td>0.02</td>
<td>-2.0</td>
<td>0.05</td>
<td>0.0</td>
<td>-2.5</td>
<td>0.3</td>
<td>0.5</td>
<td>0.25</td>
<td>0.4</td>
<td>3.0</td>
</tr>
</tbody>
</table>

panels, the cross-correlation between the two ‘payloads’ in the bottom panel. Figure 5-15 is the model results when the parameters of the current sheets were chosen to best represent the measurements of FFMs from the Enstrophy. The key parameters are given in Table 5.2 and they are defined in Equations 5.2.2 and 5.2.1 of the model.

It should be mentioned that unlike the velocity in the simple square pulse model, which directly represents the modelled structure’s constant velocity in the non-moving plasma frame, the velocity here may be modified by the evolving shape of the structure. For comparison, the velocity parameter in current sheet 6 and 8 was varied ($v = -6$ km/sec for the 6th current sheet and $v = 5$ km/sec for the 8th current sheet) to see the difference from the above result.

We can see that in comparison to Figure 5-15, the cross correlation in Figure 5-16 changes and becomes broader in lagged time. Because every current filament in the current sheet model is not only moving with certain velocity either in the north or south direction but also varies in its width nonlinearly as time evolves. The correlation line plots are not too helpful in terms of understanding the motions at a certain scale size. There are too many scales involved even though the current sheet model itself is mathematically friendly. The correlation line plots seem to suggest that there is no obvious time delay for structures at all frequencies (periods).
Figure 5-15: Wavelet transformed magnetic field data of two payloads and their cross-correlation.
Figure 5-16: Wavelet transformed magnetic field data of the two payloads and their cross-correlation after the changes.
Figure 5-17: Line plots of the cross-correlation of the modelled magnetic field data between 'payload 1' and 'payload 2' vs. velocity.

Figure 5-18: Cross-correlation of the modelled magnetic field data between 'payload 1' and 'payload 2' vs. velocity after the changes.
5.3.2 Wavelet and cross-correlation result of the B measurements

The wavelet transformations of each component of magnetic perturbations from the FFM data are shown in Figure 5-19 and Figure 5-20. Compared to the wavelet transform of the model data in Figure 5-15 and 5-16, more information is shown in the wavelet transform plots here. That is the black line in every one of them, which represents the “cone of influence”. The cone of influence contains the maximum period of useful information at a particular time. Periods greater than it could be subject to edge effects [Torrence and Compo, 1998]. Therefore anything above the black line in the plots is dubious.

With this in mind, the cross correlation was performed only on the wavelet transform data below the black line. We can see this reflected in the correlation plots below. Except for this, what is represented in the wavelet transform plots of the FFM measurements is the same as that of the model’s.

The correlation results of the same component between each FFM pair are shown in Figure 5-21 and Figure 5-22. The correlation of \( x \) component is shown in Figure 5-21, and the correlation of \( y \) component is shown in Figure 5-22.

As described in the simple model, the time lag can be changed into the relative velocity (relative to the moving payloads) of a moving structure passing the payloads once we know the separation distances between payloads. The separation distances between different FFMs can be calculated from their geometry (shown in Figure 4-5 in Chapter 4). Thus, the cross-correlation line plots vs. relative velocity are shown in Figure 5-23 and Figure 5-24.

Shown in these correlation line plots (vs. relative velocity) are the cross-correlation between each pair of the FFMs at different time scales. From the bottom to top, they are
Figure 5-19: Wavelet transformation of Bx for FFM1, FFM2 and FFM4.
Figure 5-20: Wavelet transformation of By for FFM1, FFM2 and FFM4.
Figure 5-21: Correlation plot of Bx between FFM1 and FFM2, 1 and 4, 2 and 4.
Figure 5-22: Correlation plot of By between FFM1 and FFM2, 1 and 4, 2 and 4.
Figure 5-23: Correlation line plot of Bx between FFM1 and FFM2, 1 and 4, 2 and 4, vs. velocity.
Figure 5-24: Correlation line plot of $B_y$ between FFM1 and FFM2, 1 and 4, 2 and 4, vs. velocity.
correlations at period (scale size) = 0.472249 s; 0.561601 s; 0.667860 s; 0.794224 s; 0.944497 s; 1.12320 s; 1.33572 s; 1.58845 s; 1.73222 s; 1.88899 s; 2.24641 s; 2.67144 s; 3.17690 s; 3.7799 s; 4.49281 s; 5.82645 s; 8.23985 s; 9.79889 s; 10.6858 s; 12.7076 s. So, based on the configuration of all the FFMs, from the cross-correlation line plots, we can get some details of the structures involved. The first panel in Figure 5-23 indicates that at period = 0.561601 s, a structure having a southward velocity about 0.5 km/sec has a south-northward width of 0.3 km. At scale size = 1.12320 s, a structure having a northward velocity about 1.5 km/sec has a south-northward width of 1.7 km. At scale size = 8.23985 s, a structure having a northward velocity about 1.7 km/sec, has a south-northward width of 14 km. But at some scales, the correlation peak's velocity can not be explained by this moving structure scenario, such as the negative velocity peak (about -0.5 km/sec, only a peak at negative velocity less than -v_p (payload's speed) can be explained) at scale size = 2.67144 s. At other scales, the correlation plot shows that there is no clear peak at any velocity smaller than 20 km/sec, which indicates that there is no obvious delays between the two payloads. It should be mentioned that the details of a specific structure is best represented in the correlation-time lag line plot, which is a cut at a certain time scale size and shows the correlation value vs. lag time. Figure 5-25 is one example showing the correlation information at a few different time scales. We can see from Figure 5-25 that the correlation has its peak value at different lag time for different time scales. The results from this figure are shown in the following Table. where S means the structure moves southward and N means the structure moves northward.

The first panel in Figure 5-24 gives us the motion in the y direction (west-east). Considering the FFMs had a negligible velocity in the west-east direction. So the relative velocity
Table 5.3: Inferred information from cross-correlation of $B_{x1}$ and $B_{x2}$.

<table>
<thead>
<tr>
<th>time scale</th>
<th>inferred veloc. of a structure</th>
<th>inferred width</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.46 sec</td>
<td>2.76 km/s N</td>
<td>9.55 km</td>
</tr>
<tr>
<td>3.78 sec</td>
<td>2.90 km/s N</td>
<td>10.96 km</td>
</tr>
<tr>
<td>4.12 sec</td>
<td>$\infty$</td>
<td></td>
</tr>
<tr>
<td>4.49 sec</td>
<td>0.99 km/s S</td>
<td>4.44 km</td>
</tr>
<tr>
<td>6.35 sec</td>
<td>0.20 km/s S</td>
<td>1.27 km</td>
</tr>
</tbody>
</table>

Figure 5-25: Correlation line plot of $B_x$ between FFM1 and FFM2 at several time scales.
can be considered as the structure's velocity. At scale size of 1.58845 s, there is a structure moving westward at velocity 0.6 km/sec and the structure's width is about 1 km in the east-west direction. At scale size of 12.7076 s, there is a structure moving eastward at velocity 0.45 km/sec and its width in the east-west direction is about 5 km.

Performing the same analysis on the other plots, we see that structures mostly move in the south or north direction. Their width mostly falls in the range of 2-15 km, which is consistent with the observations.

Wavelet transformation and correlation analysis were also done on the perturbations in the magnitude of the magnetic field measurements ($dB_{mag}$) from FFM1 and FFM4. Since the non-zero $dB_{mag}$ is a signature of the compressional Alfvén wave, we hope to get information on the perpendicular motion from the correlation of between $dB_{mag1}$ and $dB_{mag4}$. The results are shown in Figure 5-26 and Figure 5-27.

The line plots of cross-correlation of $dB_{mag}$ shown in Figure 5-27 provide us the following information. At scale size = 0.561601 s, a structure moves with a northward velocity about 1.5 km/sec; at scale = 0.794224 s, a structure moves with a northward velocity of 2 km/sec; at scale = 1.12320 s and 1.58845 s, a structure moves with a northward velocity of 1.45 km/sec; at scale = 2.67144 s, a structure has a velocity of 2 km/sec northward; at scale = 3.17690 s, a structure moves northward with velocity of 2.5 km/sec; at scale = 8.23985 s, a structure moves northward with velocity of 1.4 km/sec. They correspond to a width of 0.85 km, 1.6 km, 1.63 km, 2.30 km, 5.3 km, 8 km and 11.5 km respectively. These are reasonable perpendicular wavelengths for obliquely propagating kinetic Alfvén waves, which have a compressional component, in the auroral zone [Lysak, 1999].

From the wavelet transformation and cross-correlation study, we gain more understand-
Figure 5-26: Wavelet transformed $dB_{mag}$ for FFM1 and FFM4.
Figure 5-27: Cross-correlation of $dB_{mag1}$ and $dB_{mag4}$, tv and line plots.
ing of the large $B$ oscillation event seen by all FFMs. The above analysis further supports that the event is both spatial and temporal.

5.4 Conclusions

The analysis and interpretation of a multipoint observation of magnetic field structures at the poleward edge of a premidnight auroral arc from the Enstrophy sounding rocket mission are presented in this thesis. Both spatial and temporal signatures were found to be present in the event where the large $B$ fluctuations were seen at the edge of an arc when the rocket flew into the polar cap. The results show the direct measurement method of current density using multipoint measurement of magnetic fields gives us a smaller current density than what would be inferred from previous single-point measurement. Reasons for the interpretation of spatial or temporal features are given and supported by a simple model of multiple payloads crossing through several moving current sheets, the non-zero deflection in magnetic field magnitude, and the fine structure study of this auroral event using wavelet analysis and the supporting data from other instruments on board.

However, even with multipoint measurement of the magnetic field perturbations, distinguishing spatial structuring and temporal variations is still difficult. The difficulty lies in the fact that the dynamics and structuring in the Earth’s auroral zone is very complicated due to the inhomogeneity of the ionosphere and the active role of the ionosphere to the physical processes originating in the magnetosphere or physical processes directly originating from ionospheric sources. Different structured (100m-100km) and highly dynamically varied (minutes to milliseconds) auroral arcs observed and studied by different authors [Hallasson, 1974; Davis and Maggs, 1970] are the manifestation of this. The fact that there is no
accepted theory that can explain the basic properties of auroral arcs without invoking more than one mechanism [Borovsky, 1993] might also serve as an explanation of the difficulty. We saw relative time delays in the magnetic field data between different FFMs, which is normally considered a spatial characteristic. But that is not enough for us to say the perturbations are due to spatial variations of the field aligned current structure. The obliquely propagating Alfvén waves could also produce delays if the FFMs are not in an plane of the same phase. To nail down exactly whether an event is due to spatial structuring or temporal variations or whether it includes both requires more than just multipoint measurement of the magnetic field. A complete set of measurement including waves, fields and plasma properties is needed.

The need of multipoint measurements and distinguishing spatial structuring from temporal variations is still great for understanding many unresolved phenomena and physical processes related to the aurora, and for understanding the Sun-Earth system in general. The launch of Cluster II is another example demonstrating the space scientists' effort toward deeper and full understanding of our space environment.
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<table>
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<th>Reference</th>
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[Thompson and Lysak, 1996]


Appendices
Appendix A

Inertial Alfven wave dispersion relation and its field properties

In this appendix, the dispersion relation and field relations for the inertial Alfven wave are derived. First we choose our coordinate system as follows: z axis is along the Earth’s magnetic field, x axis is pointing northward and y completes the right-handed coordinate system, lying in an eastward direction.

The derivation is based on the two-fluid model and Maxwell’s equations.

Applying $\partial/\partial t$ to Ampere’s law and using Faraday’s law, we get the following equation:

$$-\nabla \times (\nabla \times \mathbf{E}) = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu_0 \frac{\partial \mathbf{j}}{\partial t} \quad (A.0.1)$$

If we assume that the solutions are plane waves ($\sim e^{ik \cdot r - i\omega t}$), temporal and spatial derivatives can be substituted according to $\partial/\partial t \rightarrow -i\omega$, $\nabla \rightarrow ik$, $\nabla \cdot \rightarrow ik$, $\nabla \times \rightarrow ik \times$. Therefore, after the Fourier transformation Equation A.0.1 becomes

$$k \times (k \times \mathbf{E}) + \frac{\omega^2}{c^2} \mathbf{E} = -i\omega \mu_0 \mathbf{j} \quad (A.0.2)$$

Now if we can find the relationship between $\mathbf{j}$ and $\mathbf{E}$, the above equation is just the wave equation in terms of $\mathbf{E}$ field. Here $\mathbf{j}$ includes the current densities due to both electrons and ions. For frequencies below the ion cyclotron frequency, the major current densities are the parallel (relative to $\mathbf{B}$) current density carried by electrons and ion polarization current density in the perpendicular direction.
The motion of electrons can be described by Ohm's law in two-fluid MHD, which is [Kallenrode, 1998]:

$$\frac{m_e}{e^2 n} \frac{\partial j}{\partial t} = E + \frac{V \times B}{c} - \frac{j \times B}{en} + \frac{\nabla p_e}{en} - \frac{j}{\sigma} \tag{A.0.3}$$

where $n$ is the electron or ion density, $n = n_e = n_i$ because of charge neutrality.

Electrons move mostly along the field lines, i.e., $V \parallel B$. So we only need to consider the motion in the $z$ axis, the second term and the third term on the right hand side disappear.

In the inertial limit, where $\beta \ll m_e/m_i$, the pressure term can be ignored. Assuming the plasma is collisionless, the last term becomes zero too. So the above equation becomes:

$$\frac{m_e}{e^2 n} \frac{\partial j_z}{\partial t} = E_z \tag{A.0.4}$$

Fourier transforming this equation will give us the relation between $J_z$ and $E_z$, which is:

$$i\omega_0 j_z \approx -\frac{\omega_{pe}^2}{c^2} E_z \tag{A.0.5}$$

where $\omega_{pe}$ is the electron plasma frequency and $\omega_{pe}^2 = \frac{n e^2}{m_e e_0}$.

Ions move most in the perpendicular direction, and the perpendicular current is mainly carried by the ions via their polarization drift.

$$V_p = \frac{1}{\omega_{gi} B} \frac{\partial E}{\partial t} \tag{A.0.6}$$

Using $j_\perp \approx en V_p$, we get

$$i\omega_0 j_z \approx \frac{\omega_{gi}^2 \omega_{pi}^2}{c^2 \omega_{gi}^2} E_z \tag{A.0.7}$$

$$i\omega_0 j_y \approx \frac{\omega_{gi}^2 \omega_{pi}^2}{c^2 \omega_{gi}^2} E_y \tag{A.0.8}$$

where $\omega_{gi}$ is the ion gyrofrequency, defined as $\omega_{gi} = \frac{eB}{m_i}$; $\omega_{pi}$ is the ion plasma frequency, defined as $\omega_{pi} = \frac{n e^2}{m_i e_0}$. 

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Putting the above result of $j$ into the vector form, we have

$$i\omega \mu_0 j \approx \left( \frac{\omega_p^2}{c^2 \omega_{gi}^2} E_x, \frac{\omega_p^2}{c^2 \omega_{gi}^2} E_y, -\frac{\omega_p^2}{c^2} E_z \right)$$  \hfill (A.0.9)

Substituting equation A.0.9 into equation A.0.2, we can get three scalar equations in $x$, $y$, $z$ direction.

$$(k_x^2 + \frac{\omega^2}{c^2} - k^2 + \frac{\omega_p^2}{c^2 \omega_{gi}^2})E_x + k_x k_y E_y + k_x k_z E_z = 0$$  \hfill (A.0.10)

$$k_x k_y E_x + (k_y^2 + \frac{\omega^2}{c^2} - k^2 + \frac{\omega_p^2}{c^2 \omega_{gi}^2})E_y + k_y k_z E_z = 0$$  \hfill (A.0.11)

$$k_x k_z E_x + k_x k_y E_y + (k_z^2 + \frac{\omega^2}{c^2} - k^2 - \frac{\omega_p^2}{c^2})E_z = 0$$  \hfill (A.0.12)

Combining the above equations and writing them in matrix form, we have

$$\begin{pmatrix}
\frac{\omega_p^2}{c^2}(1 + \frac{\omega^2}{\omega_{gi}^2}) - k^2 + k_x^2 & k_x k_y & k_x k_z \\
k_x k_y & \frac{\omega_p^2}{c^2}(1 + \frac{\omega^2}{\omega_{gi}^2}) - k^2 + k_y^2 & k_y k_z \\
k_x k_z & k_y k_z & \frac{\omega^2}{c^2} - \frac{\omega_p^2}{c^2} - k^2 + k_z^2
\end{pmatrix} \cdot E = 0$$  \hfill (A.0.13)

In order to simplify the above equation, we make the following assumptions, which are fairly reasonable:

1.) $k$ is lying in the $x$-$z$ plane, so that $k_y = 0$;

2.) $\frac{1 + \frac{\omega^2}{\omega_{gi}^2}}{c^2 k_x^2} = \frac{1}{c^2} + \frac{1}{v_A^2} \approx \frac{1}{v_A^2}$ where $v_A^2 = B^2 / \mu_0 m_i \gamma$ is the Alfvén velocity.

3.) $\omega \ll \omega_p$.

This results in the following matrix-formed dispersion relation.
The dispersion relation has two roots: one corresponding to the “fast”, or compressional Alfven wave, having the dispersion relation

\[ \omega^2 = v_A^2 k^2 \]  

(A.0.15)

and the other one corresponding to the “shear” Alfven wave in the inertial regime,

\[ \omega^2 = \frac{v_A^2 k_x^2}{1 + k_x^2 \lambda_e^2} \]  

(A.0.16)

where \( \lambda_e = c/\omega_{pe} \) is the electron skin depth.

We can compute the field relations in both modes.

**Fast Mode**

For “fast” mode, the electric field has a \( y \) component only.

So the corresponding wave magnetic field \( \mathbf{b} \) has both \( x \) component and \( z \) component via

\[ \mathbf{k} \times \mathbf{E} = \omega \mathbf{b} \]

\[ b_x = -k_x E_y / \omega \]

(A.0.17)

\[ b_z = k_Z E_y / \omega \]

(A.0.18)

Therefore, for the “fast” mode, \( \mathbf{b} \) is also lying in the \( x-z \) plane and has a compressional component \( b_z \) and \( \mathbf{b} \perp \mathbf{k} \).

**Shear Mode**

For “shear” mode, the wave magnetic field only has \( y \) component, and there is no \( E_y \). But
it has a parallel electric field $E_z$. The field relations are:

\begin{align}
E_z &= \frac{k_x k_z \lambda_e^2}{1 + k_x^2 \lambda_e^2} E_x \\
b_y &= \frac{E_x}{\sqrt{1 + k_x^2 \lambda_e^2 v_A}}
\end{align}

(A.0.19)  
(A.0.20)

Figure A-1 shows the fields for both shear Alfvén wave and compressional Alfvén wave.
Appendix B

Theoretical Calculation and Simulation Result of the Deployer

In order to choose the right shape of the FFM deployer to obtain a high spin rate of FFMs, theoretical calculations were done. Two cases were compared: one case is for the straight rail and the other one is for the logarithmic spiral rail.

B.1 Case 1: Straight Rail

All the forces are shown in Figure B-1.

At first motion of the FFM, there is sliding. But after a short time, it begins rolling without slipping. The equations for sliding are as follows:

\[ F_{\text{centrifugal}} - f = m \cdot \frac{d^2r}{dt^2} \quad \text{(B.1.1)} \]

\[ f = \mu N = \mu F_{\text{coriolis}} \quad \text{(B.1.2)} \]

Figure B-1: All the forces acted on FFM—straight rail
where $a$ is the radius of FFM, $I$ is the moment of inertia of FFM and $f$ is the friction force.

When it satisfies $v = \omega a$, the FFM stops sliding and begins rolling without slipping. We have the constraint equation:

$$\frac{dr}{dt} = a \frac{d\phi}{dt}$$

(B.1.5)

From the above equations, we can get the time of sliding $T_0$ and the velocity $V_T$ at $T_0$.

In $\hat{r}$ direction, the equation becomes

$$m \frac{d^2 r}{dt^2} = F_{\text{centrifugal}}$$

(B.1.6)

Note: because there is no slipping, the friction force can be neglected entirely.

From Equation B.1.1, we can get

$$m \Omega^2 r - \mu \cdot 2m \Omega \frac{dr}{dt} = m \frac{d^2 r}{dt^2}$$

(B.1.7)

This differential equation has the solution:

$$r = C_1 e^{m_1 t} + C_2 e^{m_2 t}$$

(B.1.8)

where $m_1$ and $m_2$ are the two roots to the equation

$$m^2 + 2\mu \Omega m - \Omega^2 = 0$$

(B.1.9)

From Equations B.1.3 and B.1.4, the following is obtained.

$$4\mu \Omega (r - r_0) = a \omega + C_3$$

(B.1.10)
With the help of the constraint equation, we can get $T_0$, $V_{T_0}$, $r_{T_0}$. From Equation B.1.6, we get

$$r = C \cdot e^{\Omega t} + D \cdot e^{-\Omega t}$$  \hspace{1cm} (B.1.11)

$C$ and $D$ can be determined by $T_0$, $V_{T_0}$ and $r_{T_0}$. $V_{T_0}/a$ gives us $\omega_{T_0}$, which is also the final spin rate that can be achieved from the straight rail case.

### B.2 The Spiral Rail Case

The spiral shaped track for FFM deployment was chosen because it can provide higher spin rate for FFMs, which is required for the FFMs' stability.

Since we chose a logarithmic spiral shaped track for the deployer, some properties of the logarithmic spiral should be mentioned. It takes the form:

$$r = \text{const} \cdot e^{\cot \phi(-\theta + \theta_0)}$$  \hspace{1cm} (B.2.1)

The curvature is:

$$\kappa = \frac{r^2 + 2r'^2 - r \cdot r''}{(r^2 + r'^2)^{3/2}} = \frac{\sin(\phi)}{r}$$  \hspace{1cm} (B.2.2)

Therefore, the radius of curvature for it is

$$R_c = \frac{1}{\kappa} = \frac{r}{\sin \phi}$$  \hspace{1cm} (B.2.3)

Figure B-2 shows all the forces acting on the FFM in the logarithmic spiral rail situation and Figure B-3 will give us information we need to use below.

Now we can set up the equations in $\hat{n}$ and $\hat{r}$ directions.

In $\hat{n}$ direction:

$$N + F_{\text{coriolis}} - F_{\text{centrifugal}} \sin \phi = m \frac{y^2}{R_c}$$  \hspace{1cm} (B.2.4)
Figure B-2: All the forces acting on FFM—spiral rail.

Figure B-3: relation diagram of the differential vectors.
In $\hat{r}$ direction:

$$m\frac{d^2s}{dt^2} = F_{\text{centrifugal}} \cos \phi - f$$  \hspace{1cm} (B.2.5)

$$f = \mu N$$  \hspace{1cm} (B.2.6)

The constraint conditions are:

$$V = \omega a$$  \hspace{1cm} (B.2.7)

$$V = \frac{ds}{dt}$$  \hspace{1cm} (B.2.8)

Where $R_c$ is the curvature of the track,
$\hat{r}$ is the unit vector in radial direction,
$\hat{n}$ is the normal unit vector,
$\hat{t}$ is the tangential vector,
$\phi$ is the constant angle between $\hat{r}$ and $\hat{t}$,
$\hat{s}$ is the unit vector along the track,
$r = s \cos \phi + r_0$,
$\Omega$ is the main payload spin,
$\omega$ is FFM spin,
a is the radius of the FFMs,
$R_c$ is the curvature radius of the track,
$I$ is the moment of inertia of FFM $= \frac{1}{2}ma^2$,
$m$ is the mass of the FFM and

$$F_{\text{coriolis}} = 2m\Omega V$$  \hspace{1cm} (B.2.9)
Here, we take $\Omega = 2\pi \cdot 4Hz = 25.12$, the spin rate of the main payload.

The properties of the logarithmic spiral give us:

$$ds \cos \phi = dr$$  \hspace{1cm} (B.2.11)

From the above equations, we get

$$\frac{d^2s}{dt^2} = -\frac{\mu}{R_c} \cdot \left(\frac{ds}{dt}\right)^2 + \mu \cdot 2 \cdot \Omega \cdot \frac{ds}{dt} + \Omega^2 \cdot \cos(\phi) \cdot (\cos(\phi))$$

$$-\mu \cdot \sin(\phi)) \cdot s + 0.04 \cdot \Omega^2 (\cos(\phi))$$

$$-\mu \cdot \sin(\phi))$$  \hspace{1cm} (B.2.12)

Then using a computer simulation, we can get

$$V = V_0 + acce \cdot t$$  \hspace{1cm} (B.2.14)

$$s = V_0 t + \frac{1}{2}acce \cdot t^2$$  \hspace{1cm} (B.2.15)
From $V = \omega \cdot a$, $\omega$ and the time of sliding can be obtained.

We can also do the calculation by another approach—using the Lagrangian equations. Since the sliding time in the spiral case is very short, we can just assume from the beginning, that there is no slipping.

Besides the variables appeared in the above equations, we also have the following ones.

$\alpha$ is the phase angle of FFM and $\omega = \frac{da}{dt}$. $T$ is the kinetic energy, $U$ is the potential energy and $L$ is the Lagrangian of the FFM.

Forces are:

$$F_{\text{centripetal}} = m\Omega^2 \cdot \hat{r}$$

$$F_{\text{coriolis}} = 2m\Omega \cdot V[-\hat{n}]$$

$$F_{\text{curv}} = \frac{m \cdot V^2}{R_c}[-\hat{n}]$$  \hspace{1cm} (B.2.16)

The only non-normal force is $F_{\text{centripetal}} \cos \phi$.

As long as the FFM rolls without slipping, we write

$$T = \frac{1}{2} m s^2 + \frac{1}{2} I \alpha^2$$  \hspace{1cm} (B.2.17)

$$U = -\int F_{\text{centripetal}} dr = -\int m\Omega^2 r dr = -m\Omega^2 r^2 / 2 + C$$ \hspace{1cm} (B.2.18)

$$L = T - U = \frac{1}{2} m s^2 + \frac{1}{2} I \alpha^2 + m\Omega^2 (s \cos \phi + r_0)^2 - C$$ \hspace{1cm} (B.2.19)

With the constraint,

$$s = a\alpha$$ \hspace{1cm} (B.2.20)

it gives

$$L = \frac{3}{4} m s^2 + m\Omega^2 \cdot (s \cos \phi + r_0)^2 - C$$ \hspace{1cm} (B.2.21)
Therefore,

\[ \frac{\partial L}{\partial \dot{s}} = \frac{3}{2} J \dot{s} \]  

(B.2.22)

\[ \frac{\partial L}{\partial s} = m \Omega^2 (s \cos \phi + r_0) \cos \phi \]  

(B.2.23)

So from the Lagrangian equation, we can get a equation about \( s \).

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{s}} \right) - \frac{\partial L}{\partial s} = 0 \]  

(B.2.24)

\[ \ddot{s} = \frac{2}{3} \Omega^2 (s \cos \phi + r_0) \cos \phi = \alpha \frac{d\omega}{dt} \]  

(B.2.25)

This is OK as long as there is no slip. It is determined by the condition

\[ \int \frac{d\omega}{dt} \leq \mu_{\text{static}} N \]  

(B.2.26)

That is, \( \frac{d\omega}{dt} \) is less than what torque can provide from the friction, where \( \mu_{\text{static}} \) is the static coefficient of friction.

### B.3 Simulation Results

Some of the results are in Table B.1, Table B.2, Table B.3 and Table B.4. The parameters in these tables are: 'd' is the diameter of the FFM; '\( \Omega \)' is the main payload spin rate; '\( \phi \)' is the characteristic angle of the logarithmic spiral rail—the angle between the tangential and the radial direction and which is a constant for a specific case; '\( r_0 \)' is the starting position for FFM; 'v' is the exit velocity of FFM in main payload frame; '\( \Delta \theta \)' is the difference in angle from the FFM's starting position to where it exits the main payload; '\( \omega \)' is the final spin rate FFM gets when it leaves the main payload and '\( \Delta T \)' is the elapsed time.
Table B.1: Simulation results (a)

Note: If we change $\mu$ to 0.6 or 0.8, all the results in Table B.1 and Table B.2 don't change.

In Table B.3 and Table B.4, $\phi = 80^\circ$.

Note: when $r_0=0.08$, in some cases the requirement for N (normal force) fails, i.e., the condition B.2.26 is not satisfied if the friction coefficient is too small.

Note: the same problem occurred for N test when $r_0=0.09$ too. But for both cases (d=9cm and d=8cm), if $\mu=0.6$ or $\mu=0.8$, there is no such a problem. Meanwhile, all the results don't change when $\mu$ changes—remain the same results as $\mu =0.4$.

Figure B-4 is also the result from the computer simulation.

The left plot on the top row is FFM velocity v.s. time in payload frame. The right plot on the top row is FFM spin vs time in payload frame. The left plot on the bottom row is
Figure B-4: Simulation results.
Table B.2: Simulation results (b)

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$\omega=9cm \Omega=5.0 Hz \phi = 85^\circ \mu = 0.4$

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<td>133.985</td>
<td>77.0738</td>
<td>24.7199</td>
</tr>
<tr>
<td>$\Delta T$ (s)</td>
<td>0.51567</td>
<td>0.40108</td>
<td>0.28649</td>
<td>0.17189</td>
<td>0.05730</td>
</tr>
</tbody>
</table>

$\omega=9cm \Omega=3.0 Hz \phi = 85^\circ \mu = 0.4$

The normal force of FFM against rail (upper trace) and the required normal force (lower thin trace) vs. time. The right one on the bottom plots the edge of the deck (thick trace) and FFM position (thin trace) in x-y coordinates in payload frame. Here $\phi = 83^\circ$, $r_0 = 0.11$ and $\mu = 0.4$.

Based on our numerical calculation and the simulation model done by Mark Widholm (see http://pubpages.unh.edu/ mwidholm/ens/), the choice for the deployment was $r_0 = 10$ cm; $\phi \approx 83^\circ$, which determines the spiral shape; and the friction coefficient is about 0.4.

Attached below is the IDL code for the simulation.

```idl```
pro spin ;corrected version with Lagrangian, no-slip only
read,'phi in degrees: ',phi ;angle between tau and radial
```idl```
Table B.3: Simulation results (c)

phi = phi/57.3

aaa = 0.04 ; ffm radius

read,'r0: ',r0 ;start of spiral track

mass = 0.2 ;mass of ffm

mominert = 0.5*mass*aaa^2 ;I

read,'mu, nominal 0.4: ',mu ;static friction

omega = 4.*2*3.1415926 ;payload spin

smax = (0.18-aaa-r0)/cos(phi) ;length of track

; r = scosphi+r0

rcarray = fltarr(10000)

a = fltarr(10000)
Table B.4: Simulation results (d)

v = fltarr(10000)
s = fltarr(10000)
w = fltarr(10000)
time = fltarr(10000)
dwdt = fltarr(10000)
N = fltarr(10000)
slipp = intarr(10000)
curv = fltarr(10000)
centrip = fltarr(10000)
cor = fltarr(10000)
; x0t = fltarr(10000)
; y0t = fltarr(10000)
; xt = fltarr(10000)
; yt = fltarr(10000)
; vx = fltarr(10000)
; vy = fltarr(10000)
; vv = fltarr(10000)

v(0) = 0.0
s(0) = 0.0
w(0) = 0.0
i = 0

while(max(s) lt smax and i lt 5000) DO begin
  i = i + 1
  rcarray(i) = (s(i-1)*cos(phi)+r0)/sin(phi) ; rc = r/sinphi
  T = 0.0002

  N(i) = 0.2*v(i-1)^2/rcarray(i) $ 
  + 0.2*omega^2*(s(i-1)*cos(phi)+r0)*sin(phi) $ 
  - 0.2*2.0*omega*v(i-1)

  ; normal force here
  curv(i) = 0.2*v(i-1)^2/rcarray(i)
  centrip(i) = 0.2*omega^2*(s(i-1)*cos(phi)+r0)*sin(phi)
  cor(i) = - 0.2*2.0*omega*v(i-1)

  ; comps of N
  a(i) = 0.6666*omega^2*(s(i-1)*cos(phi)+r0)*cos(phi)
\[ v(i) = v(i-1) + a(i) \times T \]

\[ s(i) = s(i-1) + v(i) \times T + 0.5 \times a(i) \times T^2 \]

\[ w(i) = \frac{v(i)}{\text{aaa}} \quad ; \text{assumes no slip and} \]

\[ \text{slipp}(i) = \frac{(2 \times \mu \times N(i) / \text{mass}) \times a(i))}{; \text{test if true} \]

\[ \text{time}(i) = i \times T \]

\[ \text{endwhile} \]

\[ r = r_0 + s \times \cos(\phi) \]
\[ \theta = 100 / 57.3 - \log(r/r_0) \times \tan(\phi) \]

\[ x_0 = r_0 \times \cos(100.0 / 57.3) \]
\[ y_0 = r_0 \times \sin(100.0 / 57.3) \]

\[ x = r \times \cos(\theta) \]
\[ y = r \times \sin(\theta) \]
bx = aaa*cos(3.1415926/2.0-phi+theta)
by = aaa*sin(3.1415926/2.0-phi+theta)
xouter = x+bx
youter = y+by
; xOt = r0*cos(100.0/57.3+omega*time)
; yOt = r0*sin(100.0/57.3+omega*time)
xt = x*cos(omega*time)-y*sin(omega*time)
yt = x*sin(omega*time)+y*cos(omega*time)
vx = deriv(time,xt)
vy = deriv(time,yt)
vv = sqrt(vx^2+vy^2)
! p.multi=[0,2,2,0,0]
plot, time,vv
plot, time,vx, title='vx',psym=3
plot, time,vy, title='vy',psym=3
plot,x,y,xrange=[-0.2,0.2],yrange=[-0.2,0.2],psym=3,title='track (thin)
and edge (thick)'
 oplot,x/r*0.14,y/r*0.14,psym=2,symsize=0.2
 oplot,xouter,youter
wfin = max(w)
vdec = omega*0.14

print,'final w is',wfin/(2*3.14159),'Hz'
print,'exit v is',max(v)

vfinal=sqrt(vdec^2+max(v)^2-2.0*sin(phi)*vdec*max(v))

print,'final velocity is: ',vfinal

print,'final velocity by xt and yt method is: ',vv(2017)

Stop

End
Appendix C

Specifics of x, y and z axes in the B-L coordinate system

The configuration of \( \mathbf{L} \), \( \mathbf{B} \) and \( \omega_z \) in the north-west-up (NWU, \( x, y, z \)) coordinate system approximates the following:

\[
\mathbf{L} = [-0.31, 0.01, -1] \quad (C.0.1)
\]

If we choose one \( \mathbf{B} \) direction as the magnetic field direction at the time of the FFM deployment, we have:

\[
\mathbf{B} = [0.189, -0.08, -1] \quad (C.0.2)
\]

Figure C-1: The vector representation of \( \mathbf{B}, \mathbf{L} \) and \( \omega_z \) in NWU system.

The vector \( \mathbf{L} \) is determined from the trajectory data of the main payload. The angular momentum direction \( \mathbf{L} \) is assumed as the acceleration direction after the third stage burnout.

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The three components in NWU coordinate system are obtained by using the 95 IGRF model.

If we assume the spin axis is mostly pointing downward (as it is mostly lying in that direction from the measurements), we get

$$\omega_s = [0.0, 0.0, -1]$$ (C.0.3)

So the relation of the three unit vectors is like the one shown in Figure C-1.

Now we are going to talk about details of the orientation of the axes \(x, y, z\) in different reference frames when the measurements, originally in the spinning and precessing coordinate system, are transformed to the Earth's magnetic field aligned coordinate system at the end after going through several rotations.

The three main vectors involved are \(B, \omega_s\) and \(-L\). From data, we know the spin sense was clockwise and the precessing sense was counterclockwise when looking down. The spherical triangle made up by the three vectors and the related parameters are shown schematically in Figure C-2.

In the diagram, \(\phi_s^R\) is the spin phase, \(\phi_P\) is the precessing phase angle, \(\theta\) is the angle between \(\omega_s\) and \(L\), \(\kappa\) is the angle made by \(B\) and \(L\) (approximately 25°), and \(\beta\) is the angle between \(B\) and \(\omega_s\).

From the original \(B_s\) system (with \(z\) axis along \(\omega_s\)), rotate it around its \(z\) axis with an angle \(-\phi_s^R\) (this rotation is denoted as \(R_z(-\phi_s^R)\)) to the following coordinate system. Its \(\hat{z}\) is along \(\omega_s\), \(\hat{x}\) is in \(\omega_s\) and \(-L\) plane, pointing away from \(-L\), and its \(\hat{y} = \hat{z} \times \hat{x}\) and completes the right-hand system.

Then do the rotation around the \(y\) axis with an angle \(-\theta\), i.e. \(R_y(-\theta)\). Now the \(z\) axis
Figure C-2: The spherical triangle made up by $\hat{B}$, $\hat{L}$ and $\omega_s$.

is along -$L$, the $z$ axis is perpendicular to $z$ and is towards $\omega_s$.

The third rotation is around the $z$ axis with an angle $-\phi_P$. After the rotation, the $x$ is in the $B$ and -$L$ plane, away from -$L$, the $z$ axis is still along -$L$.

The fourth rotation and also the last rotation is around the $y$ axis, with an angle $\kappa$, i.e., $R_y(\kappa)$. Now the $z$ axis is along $B$ and $x$ axis is away from -$L$ and $y$ axis completes the right hand coordinate system.

From the relative location of $B$, $L$ and $\omega_s$ in equations C.0.1, C.0.2 and C.0.3, we know the $x$ axis in $B$-$L$ system lies approximately in the north direction.
Appendix D

Details on FFM geometry determination

Details on how to determine the position of the four FFM s after exiting from the main payload are described in this appendix.

Looking down from the nosecone, the deployment of FFM s is shown in Figure D-1. At the beginning of the flight, 0° is in the south direction. The stowed position and the exit location of all the FFM s are tabulated in Table D. The starting and the exiting location (it is also the location of the end of a FFM track) for each FFM are labeled in Figure D-1 as well. The respective FFM number in the circle represents the stowed location and ‘number’ + ‘x’ + degree represents the exiting location. The main magnetometer on the deck has its y axis lying in 155° direction and z axis along 65° line.

The exit velocity for each FFM is determined using measurements made by the main magnetometer and the assumption that projection of the Earth’s main magnetic field lies in the north direction. The angle between the y axis and the projection of B in the deck plane (its direction is in the north) can be obtained using atan(bz/by). Since the relative position

<table>
<thead>
<tr>
<th>symbol</th>
<th>stowed position of the FFM track</th>
<th>exit position of the FFM</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFM1</td>
<td>270°</td>
<td>165°</td>
</tr>
<tr>
<td>FFM2</td>
<td>90°</td>
<td>345°</td>
</tr>
<tr>
<td>FFM3</td>
<td>0°</td>
<td>255°</td>
</tr>
<tr>
<td>FFM4</td>
<td>180°</td>
<td>75°</td>
</tr>
</tbody>
</table>

Table D.1: Stowed FFM position on the deck of the main payload
Figure D-1: Stowed position of FFMs and the position of main magnetometer.
Figure D-2: Relative position of y, z axes and the North direction (projection of B).

(represented by angle and it is a fixed value) between the exit location of each FFM and the y axis is known, the exit location of each FFM relative to the north direction is therefore determined. Then the exit velocity relative to the north direction can be calculated and determined based on the magnetometer phase information and the simulation model of FFM deployer in appendix A and the simulation results done by Mark Widholm [1999]. The simulation models can be used to determine the velocity direction and optical gates at the edge of the deck can provide measurements of the magnitude of the exiting velocity for each FFM.

At t =0, $\tan(bz, by) = -25^\circ$, so the component of B projected on the deck plane and the y and z axes have the relation as shown in Figure D-2.

From Figure D-1, we can see the assumption of the projection of B lying in the north direction is reasonable.
<table>
<thead>
<tr>
<th>symbol</th>
<th>the exiting time</th>
<th>the angle between y axis and north</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFM1</td>
<td>136.62330</td>
<td>19.03°</td>
</tr>
<tr>
<td>FFM2</td>
<td>136.63251</td>
<td>39.00°</td>
</tr>
<tr>
<td>FFM3</td>
<td>136.62278</td>
<td>17.90°</td>
</tr>
<tr>
<td>FFM4</td>
<td>136.62074</td>
<td>13.47°</td>
</tr>
</tbody>
</table>

Table D.2: Exiting time for each FFM and the phase angle

As the rocket spin counterclockwise, the angle of $\tan(bz/by)$ increased. At any instant, the measurements of $by$ and $bz$ of $\mathbf{B}$ from the main magnetometer provides the relative position between the $y$ axis, the $z$ axis and the north direction. The exiting times for the four FFMs are in the Table D.

Once we know the exiting time, we can calculate the phase angle $\tan(bz/by)$, i.e. the angle between the main payload magnetometer's $y$ axis and the north direction. Take FFM1 as an example, at $t = 136.62330$, $\tan(bz/by) = 19.03^\circ$. From this, we can locate the radial direction ($\mathbf{r}$) when FFM1 exited, which is $10^\circ$ from the position of the $y$ axis in a counterclockwise sense. The angle between the exiting velocity the exiting radial direction is $84.5^\circ$, which depends on the final spin rate of FFM1. Therefore we know the geometry of FFM1.

Going through same procedure, we can get geometry of all the other FFMs as shown in Figure 4-5 in Chapter 4.
Figure D-3: Exiting position of FFM1.