Spring 2012

Arched Bridges

Lily Beyer
University of New Hampshire - Main Campus

UNIVERSITY OF NEW HAMPSHIRE CIVIL ENGINEERING

Arched Bridges

History and Analysis

Lily Beyer

5/4/2012

An exploration of arched bridges design, construction, and analysis through history; with a case study of the Chesterfield Brattleboro Bridge.

Contents

List of Figures

Introduction

Humanity has been building bridges for all of history, but it has only been building arches since the around the 6th century BC (Boyd, 1978). The arch first appeared in building construction, brought to the Greeks from Mesopotamia around the 4th century BC. Arched bridges, necessarily, came afterward, first appearing in Rhodes as a footbridge (Boyd, 1978). It was not until the Romans that the arch became a common form for bridge construction. The Roman road system tied the empire together, and those roads required many bridges. Some of these bridges are still standing today, a tribute to the excellence of the engineers who built them centuries ago.

Throughout the Middle Ages and into the Renaissance the primary building material for arched bridges was masonry. There were bridges built of wood during this time, but stone is a material much better suited to the stresses created by an arch. It was not until the industrial revolution brought iron, and later steel, that the building materials began to change. Iron, steel and reinforced concrete opened up the world of arched bridges to new variations on the form. Stone is a heavy, brittle material, and it requires strong abutments to support it. Iron in its various forms is much lighter and able to take tension. With metal, engineers began to experiment with arched trusses, structures that are much lighter than a comparable stone bridge.

The problem of how best to build an arch is one that has plagued scientists and engineers since the enlightenment, when early scientists began to approach problems mathematically (Heyman, 1998). The question encompasses not just how the arch shall be curved, but also how thick the arch rib needs to be to resist the stresses generated by using the bridge. Understanding how the bridge will behave under load is important for limiting deflection: a bridge that deflects too much may not be unsafe, but it is unsettling to use. Understanding how the material and the completed bridge will work together is an important part of engineering.

The analysis of arches depends largely on how the ends of the arch are fixed. Often arches are more complicated than simple statics can determine, and elastic analysis must be employed. It is also important to consider loads over portions of the bridge, as simply loading up the bridge with the most weight is not always the most conservative approach. Applying the load from vehicles at different locations across the span can create bending effects in the arch rib that control the design. Arched bridges are more complicated to design, but depending on the location the selection of an arch can be the best option, resulting in a beautiful bridge well integrated into the surroundings.

Chapter I: History of Arched Bridges

What is an Arch

The arch is a form where the forces from dead load are transferred as compression, and tensile forces are eliminated. Depending on the shape of the arch this is more or less true – the "perfect" arch will only carry compression, but there is only one perfect arch for any given set of loads so heavy moving loads can often put parts of an arch into tension. Because the arch relies on compression to carry load it is well suited to both masonry and concrete, materials that are strong in compression but weak in tension.

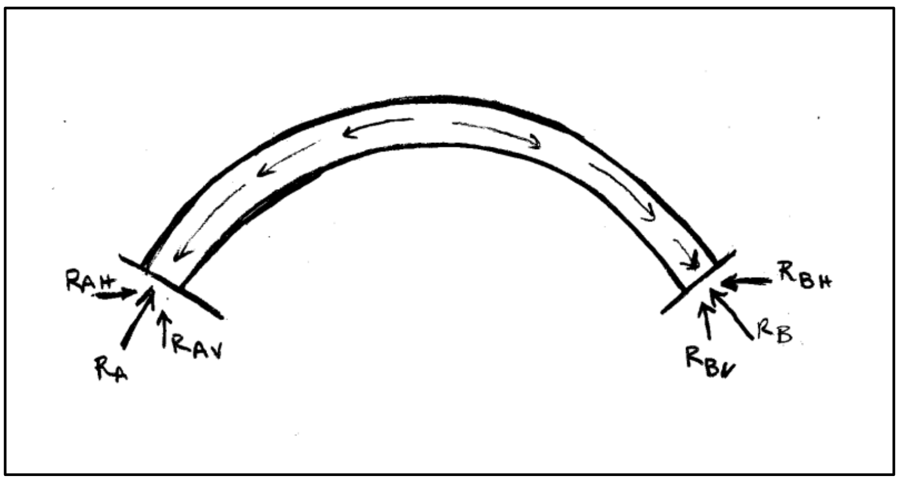


Figure 1: Forces in an arch

The forces in an arch exert outward pressure on abutments and, as a result, they must be able to resist this thrust. In many cases this means making the abutments quite massive – the stone serving to spread out the thrust of the arch until pressures can be resisted by the natural supporting soils and rock. In some construction, however, multiple arches in series can be used to resist the thrust, the thrust of one arch opposing that of the next, thus transferring the all of the thrust to the ends. In a tied arch, a tie picks up horizontal forces which combine with vertical forces at the foundation to resist the arch's thrust. This outward thrust from the weight of the arch is its defining characteristic.

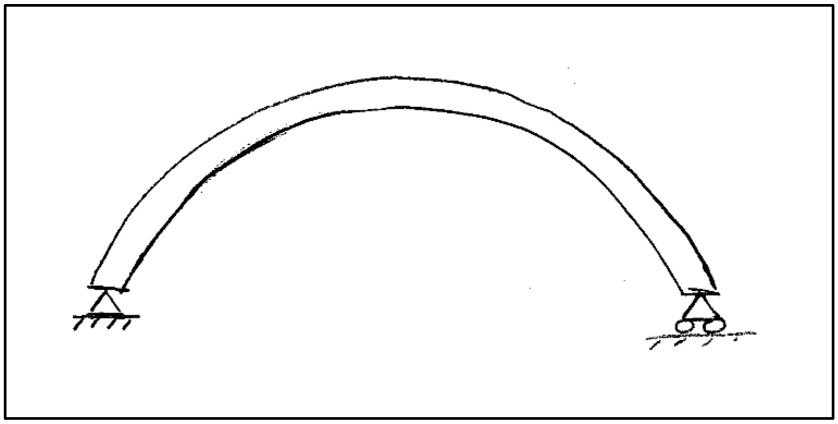


Figure 2: Arched beam

An arch that is fixed against horizontal motion at only one end without a tie is not a true arch (Figure 2). Because the roller at the right support cannot provide a horizontal reaction the arch is actually a curved beam. A true arch must develop horizontal reactions at both supports. Likewise, a corbelled arch is not a true arch. Corbelled arches were common in ancient civilizations in the Americas, and develop an arch-like shape by cantilevering consecutive

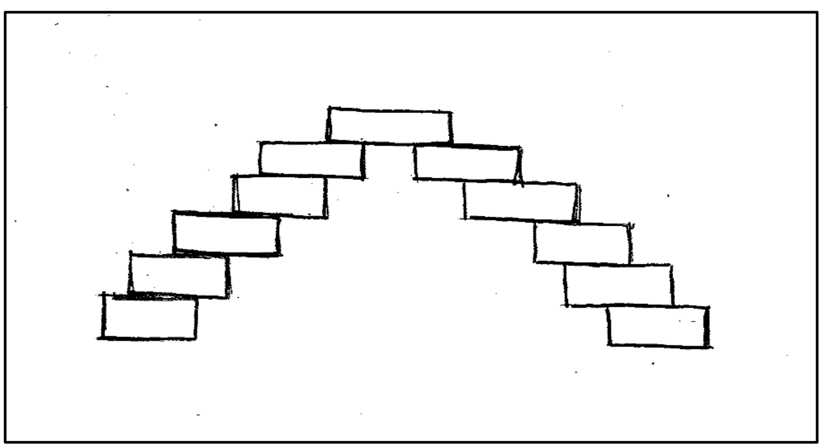


Figure 3: Corbelled Arch

courses of masonry outward until they meet in the middle (Figure 3). This type of arch does not develop horizontal thrust at the base. Both of these examples of arch shapes can develop bending stress, and are not structurally considered arches.

Arches can be supported at the abutments in two basic ways: either by a fixed connection or by a pin. A fixed connection can transfer moment, while a pin is free to rotate. Traditional masonry arches are of the fixed-fixed type, as the technology for creating deliberate pins had not been developed. In the fixed-fixed position the angle between the abutments and the arch is held

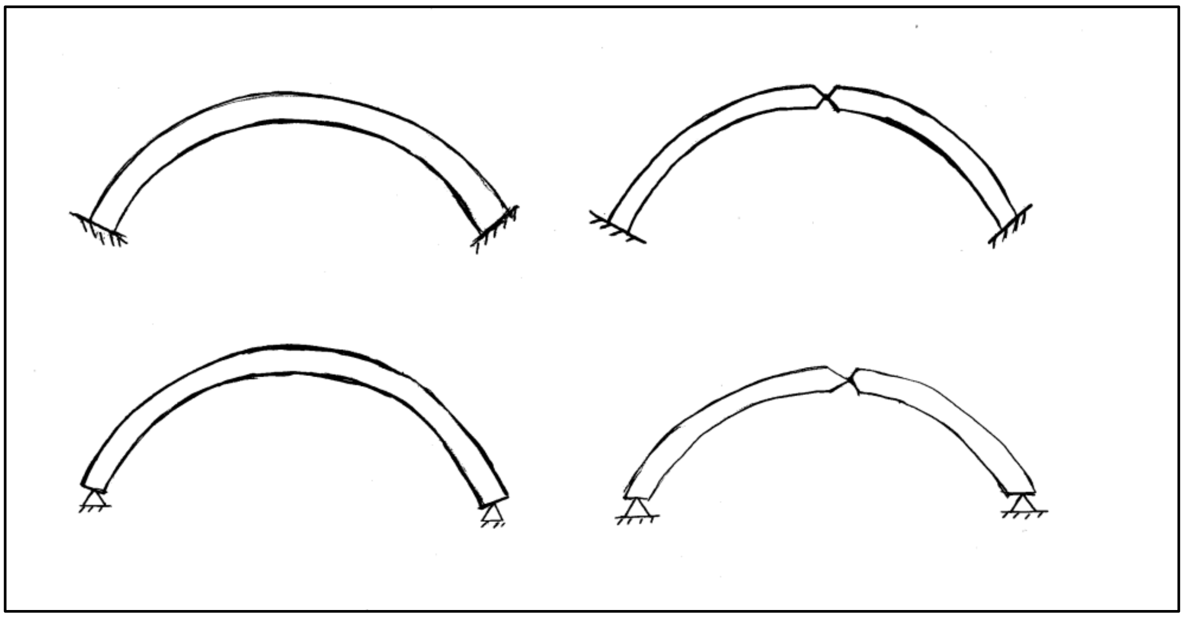


Figure 4: Arch End Conditions: (from left) Fixed-Fixed, Single Pin, Two Pin, Three Pin

constant as the arch deflects under load. Adding pinned hinges to the structure allows it to deflect more, but reduces the complexity of design, because the pin forces a location to have zero moment. There are various ways of creating pinned connections in concrete structures, including casting in iron or steel hinges, or creating concrete hinges by the careful placement of rebar.

Figure 5: Steel hinge at the end of an arch at UNH's Wittemore Center

The arch is particularly suited for bridge construction, especially where steep valley walls provide natural confinement for abutments. The arch is necessary for masonry bridges, because it develops mainly compressive stresses and, as a result, was the preferred form for thousands of years. The arch is still used today, constructed of steel and concrete though not often of true load-bearing masonry, because of its superior aesthetics and use of materials. One excellent example of modern arch construction is the Hoover Dam Bypass project, shown in Figure 6,

Figure 6: Mike O'Callaghan - Pat Tillman Memorial Bridge ((FHWA/CFLHD), 2010)

which was completed in 2010. The arch is constructed of prestressed concrete, with a deck of steel. Note how the canyon walls confine the arch, while the deck is separate, with supporting columns or piers marching uninterrupted between approach and arch.

Arch Forms

There are several different ways that an arched bridge can be constructed. The traditional method is a filled barrel arch; it was widely used up until modern construction in reinforced concrete and steel. (Kassler, 1949) The general form is shown in Figure 7.The arch and side walls were constructed of masonry and dirt and gravel fill was placed between them. The roadway was then constructed on top. This method is has the advantage that the arch is continuously braced by the fill, so that buckling of the arch is not an issue even if the shape is not ideal. However, because of the heavy fill that is placed on the arch, there is an upper limit to the size of the arch that can be created before it becomes too heavy. This form of arch also has a very high ratio of dead load to live load in service, because the weight of the structure is much higher than any load that it is likely to encounter. This makes collapse under live load extremely unlikely.

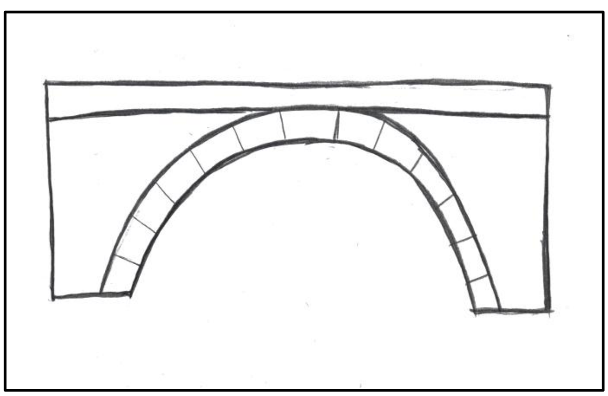


Figure 7: Typical Barrel Arch

Figure 8: Arched Bridge, Westford MA (David Fingerhut)

In the early 20th century a Swiss engineer named Robert Maillart developed an arch form where the arch and the roadway are separated, with the roadway supported by columns or cross walls. (Billington, 1979) The arch can extend above the road deck, creating a through arch, where the road is supported by tension members instead of columns. These forms opened up the possibilities of arches, and also drastically reduced the weight of the bridge. Because the arch no longer had to support the weight of the fill underneath the road deck, it could become thinner and use less material. This was more economical than the large masonry bridges that came before, and easier to construct as well. However, the arch rib was no longer braced as it was in a filled arch, and as a result live loads were more of an issue in design. It became necessary to consider exactly how the arch transferred load, and what types of stresses it would experience under moving traffic. In Maillart's Salginatobel Bridge, the arch is thicker at the quarter points to better resist the flexure that can result from the moving point loads of the traffic.

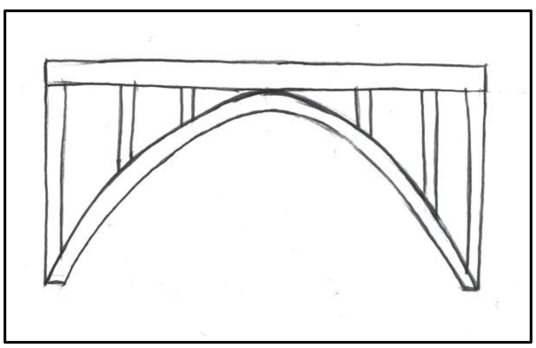


Figure 9: Typical Arch-Deck Bridge

Figure 10: Robert Maillart's Bridge at Salginatobel (http://www.worldofbuildings.com/bldg_profile.php?bldg_id=809)

The third arch bridge form puts the road deck underneath the arch, supported by tension members, and the deck ties the two ends of the arch together, forming a tied arch. This is similar to the through arch, where the roadway is below the arch rib but the abutments still take the lateral thrust. The great advantage is that the roadway is not so high above the supports, so it can be built in areas where natural steepness of terrain does not exist, but it preserves the elegance of the arch. Furthermore, with the tension tie taking the horizontal thrust, the foundations need only to support the gravity loads on the bridge allowing arches to be used where there may not be otherwise suitable subsoil conditions. The tied arch is particularly difficult to construct, because the arch thrust is not resisted until the road deck is constructed, but the road deck is unsupported until the arch is built.

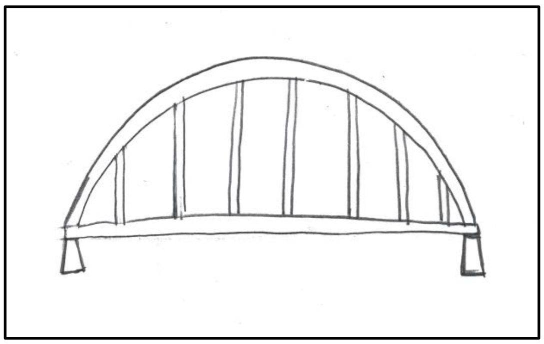


Figure 11: Typical Tied Arch Bridge

Figure 12: Sydney Harbor Bridge (All About Australia)

Roman Arches

The Romans are remembered today as great engineers, building networks of roads to tie their empire together. They were the first to adopt the arch form for widespread construction. The Greeks before them used column and lintel construction for their temples, and did not develop the arch (Steinman & Watson, 1957). The Greeks did build bridges, but they did not develop the true arch until the mid-4th century BC (Boyd, 1978). The Greeks first used the arch in buildings, and the only known example of a Greek arched bridge is a small foot bridge in Rhodes (Boyd, 1978). The Romans, on the other hand, made great use of the arch in bridge construction, and in the aqueducts they built to transport water to the centers of their cities (see Figure 13).

Figure 13: Pont du Gard Aqueduct, (http://www.travlang.com/blog/pont-du-gard-bridge-an-amazing-man-made-aqueduct/)

The Romans relied primarily on masonry construction, though they did develop the first use of concrete. They were experts in dressing stone, and some of their earlier construction did not even have mortar – the stones were so smooth and fit so well together that it was not necessary. Roman engineers were also experts in the transport of water, a requirement when cities outgrow their own local water supply. One particular example is the Pont du Gard Aqueduct in southern France. A three tiered aqueduct, it carried water for the city of Nîmes (see Figure 13). The yellow limestone blocks were quarried about 600 meters away, and show evidence of numbering to tell the masons where each block belonged (Site du Pont du Gard, 2011).

Roman arches were semicircular in shape, with large heavy piers in between. The great mass of the piers, which could be up to a third of the span, supported both the weight of the arches and their lateral thrust, making each arch independent of its neighbors (Kassler, 1949). Because each arch span is supported individually, if one span is removed the rest of the bridge remains standing. This can be seen today in the Ponte Rotto ("Broken Bridge") in Rome, built in 142 BC with six spans, only one remains today stranded in the middle of the river (see Figure 14) (Janberg, 2012). The disadvantage of the Roman method of construction is that the structures are very heavy, and the piers more disruptive to the flow of the river than later designs. They were the first of their kind, however, and many are still standing today.

Figure 14: Ponte Rotto in Rome

(http://www.romaspqr.it/ROMA/Ponti/FOTO%20Ponti/ponte_rotto.htm)

Middle Ages

After the fall of the Roman Empire bridge building became much less of a priority across Europe. Without armies which needed bridges and good roads to move troops and supplies, there was no pressing reason to build new bridges, and many that were in existence were not maintained. Much of the engineering knowledge required for bridge building was lost or forgotten, and communities that might have built bridges did not have the economic resources for such great undertakings.

It was not until the 1300s that bridge building became possible again. It was revived by groups of monks called the Pontist Friars, who built bridges in an effort to aid travelers and pilgrims (Kassler, 1949). One bridge built by the friars is the famous Pont d'Avignon (see Figure 15). The shape of the arch is shallower than the roman semicircle, lending a more active appearance to the bridge. Medieval masons were much less skilled than their Roman predecessors, and relied heavily on mortar (Heyman, 1966). The mortar both held the blocks of stone together and compensated for the fact that the stones were not as well dressed and smoothed as those of the Romans, mortar was required to ensure that the blocks fit together. Medieval construction was also less durable: Roman bridges had no mortar to deteriorate and wash away, and therefore did not lose strength over time. (Black, 1936)

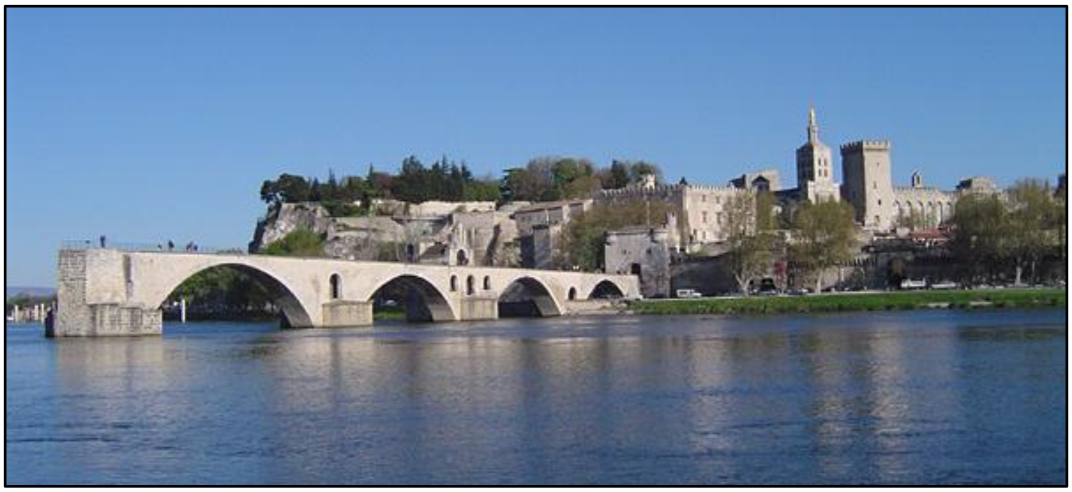


Figure 15: Pont d'Avignon (http://en.wikipedia.org/wiki/File:Pont_d_avignon.jpg)

The primary contribution of medieval masons was not to the art of bridges – their greatest achievements were in building the great Gothic cathedrals. Great technological innovations such as flying buttresses, where builders understood that thrust could be transferred by half an arch away from the source, resulted in the soaring lightness of the cathedrals. The majority of the weight of the stone was carried by columns, with thin curtain walls of stone and glass in between. The thrust of the arched vault was carried by flying buttresses away from the walls, allowing much thinner interior supports that carried only the vertical loads. (Heyman, 1966) These principles were not applied to bridges until much later, however, but they made as a great an impression on bridges as they had on buildings.

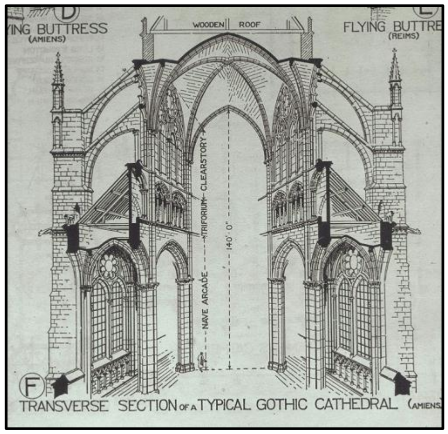


Figure 16: Section of a Gothic Cathedral (www.columbia.edu/cu)

It was not until Jean Rodolphe Perronet, one of the first professional engineers, that the idea of interdependent arches was developed (Kassler, 1949). Perronet took the principle of the flying buttress, where the arched vault is supported vertically by columns, but restrained horizontally by the buttress, and applied it to bridge building. The piers supported only the weight of the arched spans; the horizontal thrust was transferred through the adjacent spans all the way to the abutments. This meant that the piers could be much thinner, as they no longer had to restrain each arch separately.

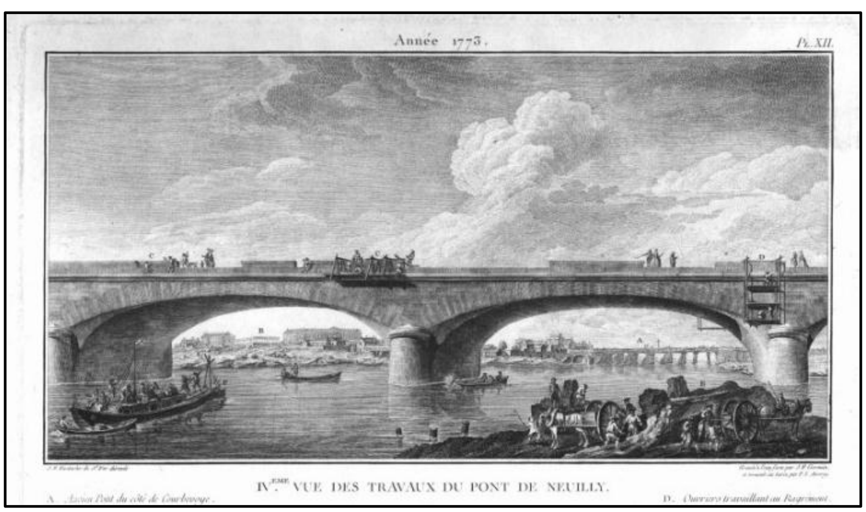


Figure 17: Bridge at Neuilly by Perronet (Silve-Tardy)

Perronet built several bridges using this principle, the first of which was the bridge at Neuilly over the Seine (see Figure 17). The bridge, with five spans of 120 feet each, was constructed in 1772, but was destroyed in 1956 to make way for a wider bridge (Brown, 2001). Because of the interdependent arch design all the arches had to be built at the same time; the piers were not designed to support the lateral thrust without the next arch in place. By constructing the piers to resist only the vertical forces, Perronet was able to reduce the span to pier ratio to 1:10, from the 1:5 common previously. For the bridge at Neuilly, King Louis XV wished to be present when the arch support was removed, and Perronet arranged a ceremony where all the centering was struck at once (Troyano, 2003).

Asian Bridges

In Asia bridges were also being built, but the development of engineering and design progressed separately. Knowledge was not easily transferred across barriers both geological and cultural. One particularly fine example of Chinese bridge building is the Zhaozhou Bridge in Hebei Province. Built around 700 AD, it has a considerably shallower rise than its Roman counterparts, and thus a much gentler rise in the roadway (Brown, 2001). It is the first example of full depth arches in the haunches on each side of the bridge, which serve the dual purpose of lightening the bridge weight and allowing heavy flood waters to pass through, lessening the lateral force against the bridge from water (Wen, 2004). The bridge is decorated with carved water dragons, and iron straps are visible clamping the stone blocks, achieving a harmony unparalleled by the heavy construction of the Romans. The unknown Chinese engineers created a beautiful and revolutionary structure that is still in use today.

Figure 18: Zhaozhuo Bridge (http://www.topchinatravel.com/china-attractions)

Steel Arches

Up until this point, bridges were built of stone or wood. It was not until the 19th century that a new material became common: iron. (Kassler, 1949) Wrought iron, cast iron, and steel came into common use and allowed for new and daring construction. Wrought iron is made by smelting, to remove the impurities from iron ore, and then working the resulting mass. It contains between .02% and .08% carbon, and is generally hard but malleable. Cast iron is created by melting iron at a high temperature, so that the iron absorbs carbon easily. This iron is high in carbon, up to 4.5%, and is hard and brittle. Steel has a carbon content of .2% to 1.5%, harder than wrought iron but not as brittle as cast iron. It can be created by several processes that drive out some of the carbon from cast iron, leaving just enough for ductility. (Spoerl)

The first bridge built of iron is in Shropshire England. The Ironbridge was constructed in 1779 by Abraham Darby III. (UNESCO, 2012) The five ribs for the bridge were cast in two pieces and joined together at the top – construction of the arch took only three months. The joints were similar to those used in woodworking, mortise and tenons, dovetails, and wedges, as well as bolts. (Haan, 2011) Because cast iron is brittle using an arch makes a lot of sense. The material, which does not handle tension well, is subjected only to compression. (Kassler, 1949) The general shape is similar to earlier bridges in stone, but the appearance of the first iron bridge is very different (see Figure 19). The new material allowed a greater lightness than the massive character of masonry.

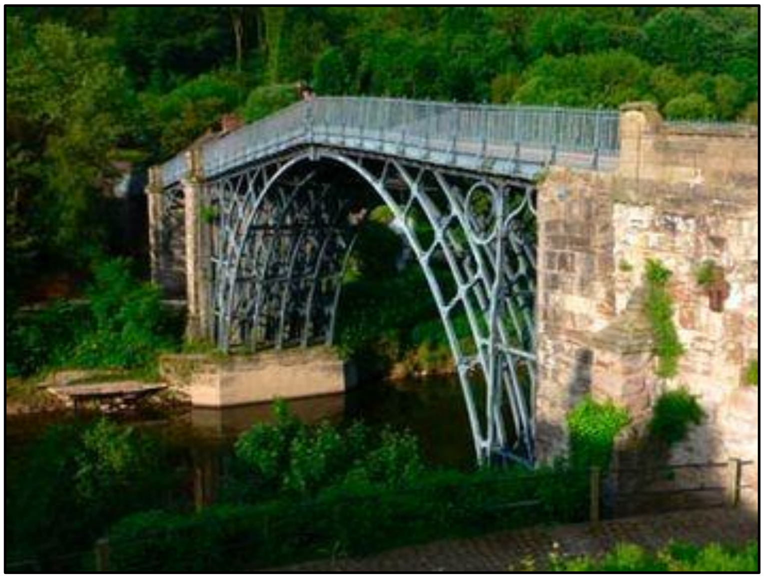


Figure 19: Ironbridge, (UNESCO, 2012)

The Garabit Viaduct, built by Gustave Eiffel (of the Eiffel tower), is one of the most representative of early iron construction. As a truss, it makes use of the tensile capacity of steel. Trusses are advantageous because they are lighter than a comparable solid member, and provide less resistance to wind. This was particularly important for the Garabit Viaduct because of its location in a steep valley, causing a natural wind tunnel. The Garabit Viaduct was constructed in 1881 to carry a rail line across the valley.

Eiffel fully understood the complexities of this new material, realizing that iron offered incredible new possibilities for construction. He also performed tests to determine the modulus of elasticity of wrought iron, after the work of Hooke and Young, publishing his work to the benefit of other engineers. He was able to calculate the expected deflection of the Garabit Viaduct, which was later verified by field measurements. (Barr, 1992) Eiffel also understood that the wind loads encountered at high altitudes would be greater than those on the ground, and used a truss to provide the least lateral load on the structure possible. (Ramaswamy, 2009)The Garabit Viaduct shows that arches are not just suited to masonry, but can be beautiful and practical in metal as well.

Figure 20: Garabit Viaduct (http://www.flickr.com/photos/daviddb/2140318177/)

Besides trusses, steel girders can also be formed into an arch. The general shape is generally either the deck-arch form (Figure 10) or the tied arch (Figure 11). Because of the tensile properties of steel, the deck is often suspended from the arch, hung from tension members or cable, though the deck can also be supported above the arch by columns. The lightness of steel generates less dead load thrust at the abutments, and less earthquake load, but construction is often more difficult, requiring skilled labor. (Kassler, 1949) One particularly fine example of the steel arch is the Västerbron in Stockholm. The arch ribs are made of plate girders, with slender columns supporting the deck. The bracing between the two arches can also be seen in Figure 21, below.

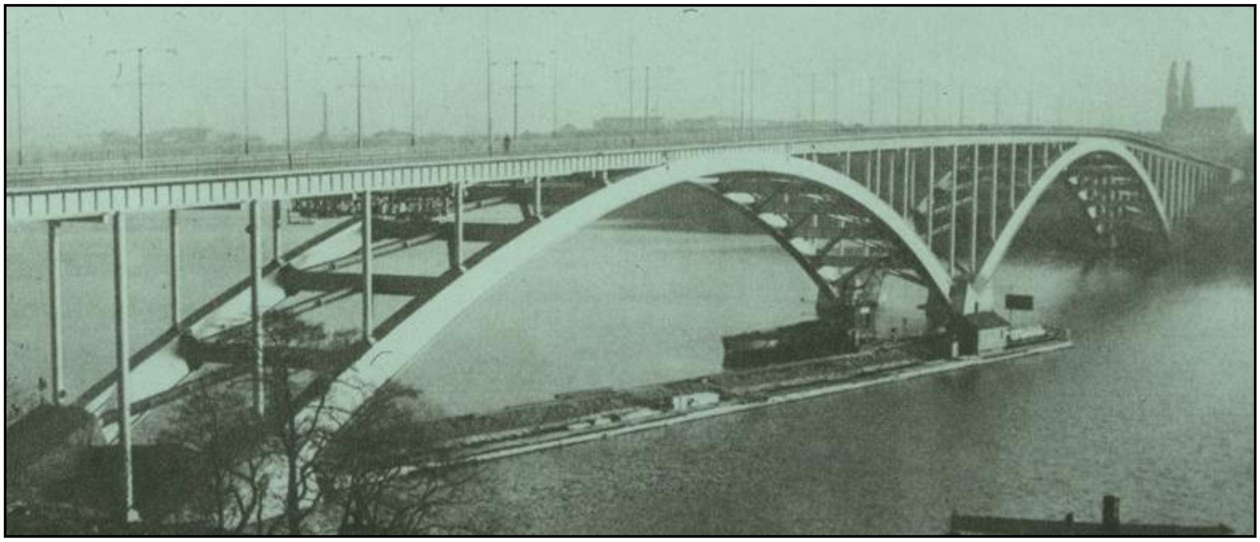


Figure 21: Västerbron (West Bridge) in Stockholm

(http://www.columbia.edu/cu/gsapp/BT/BSI/ARCH/arch1.html)

Reinforced Concrete

Soon after metal was developed as a viable bridge building material concrete reinforced with iron, and later steel, also became popular. Unreinforced concrete has been understood since the Romans, but it was not until the idea for reinforcing was understood that the material became truly useful in bridge construction. In ancient Rome concrete was used for all kinds of structures, from palaces to bridges to roads. Concrete is a material that is only strong in compression – it is essentially artificial stone, created from volcanic ash, hydraulic lime, and aggregate. Because of this, unreinforced concrete must be treated structurally like stone, and subjected to only compressive stresses. The advent of reinforcing allowed concrete structures to carry tension - the reinforcing material carries the tensile stresses, while the concrete carries the compressive. As a result, structures can be created in concrete that would not otherwise be possible.

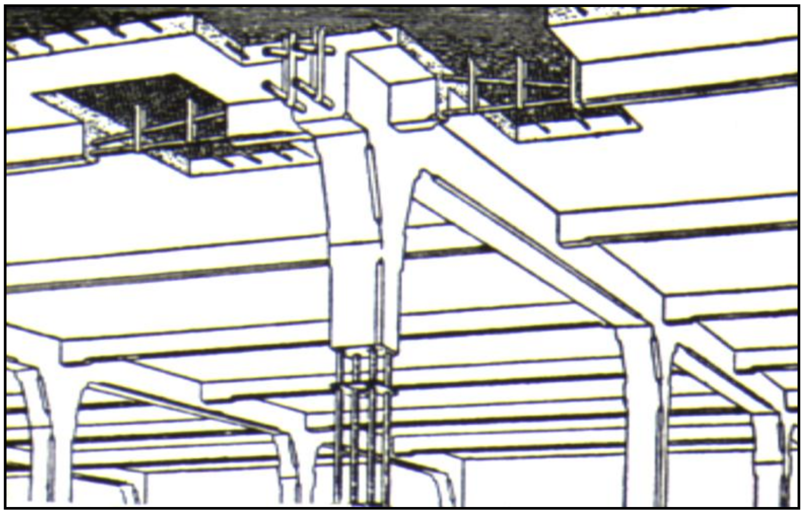


Figure 22: Hennebique system for reinforced concrete

(http://www.arch.mcgill.ca/prof/sijpkes/abc-structures-2005/concrete/Hennebique-system.jpeg)

Reinforced concrete structures were first built by a gardener named Joseph Monier. He did not fully realize the implications of reinforced concrete, and sold his idea and patent to the engineer G. A. Wayss. Wayss, along with another engineer named François Hennebique, were the first to develop methods for determining the stresses in reinforced concrete. (Brown, 2001) Because concrete is relatively fluid when wet, it can be formed into almost any shape. Concrete can be made to imitate stone, or have decorations added, or it can have stone blocks applied to the faces (Kassler, 1949). Reinforced concrete is sometimes considered at its best when left unadorned, allowed to show its true form. No one was better at this than Robert Maillart, the great Swiss engineer and builder (Billington, 1979).

Figure 23: Stauffacher Bridge by Maillart

(http://picasaweb.google.com/lh/photo/G75AY85AiLqz1Bsimmjnjw)

Maillart's first reinforced concrete arch bridge was the Stauffacher Bridge over the Sihl River in Zurich Switzerland, built in 1899. It is a three-hinged arch with an unreinforced concrete arch rib, and reinforced vertical cross walls and deck. (Billington, 1979) This bridge is faced in masonry that completely conceals the concrete structure. While this is Maillart's first large bridge, it was not until the Inn River Bridge at Zuoz that his design ideas began to take shape. (Billington, 1979) Maillart used "the arched slab, the longitudinal walls, and the roadway together [to] form the arch," (Billington, 1979, p. 21) meaning that loads are not just transferred from slab to cross wall to arch rib, but the entire system acts together. This hollow arch system meant that the slab acts in both directions – carrying live loads to the longitudinal walls and to the abutments, allowing the structure to be thinner and lighter than earlier bridges.

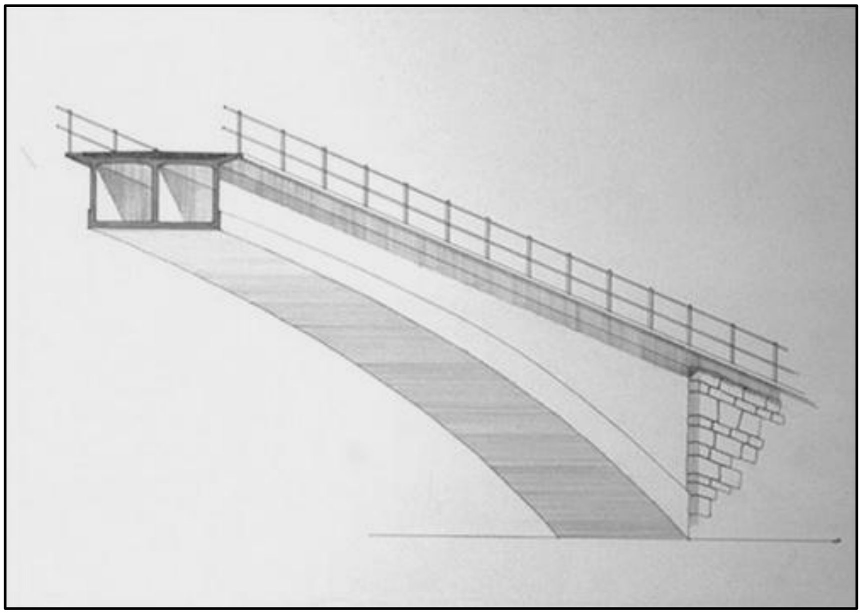


Figure 24: Hollow Arch System by Maillart (Billington, 1979)

It was in 1905, however, that Maillart's genius was fully realized with the Rhine Bridge at Tavanasa (Brown, 2001). Here the spandrel walls (the longitudinal walls at the outside of the deck) are reduced in height at the abutments, because of cracks that appeared in the earlier bridge at Zuoz. The widening of the arch at the quarter spans, accomplished at Tavanasa by the increasing height of the spandrel walls, was a form that Maillart used regularly at the beginning of his career. A particularly fine example of this type is the bridge at Salginatobel, shown earlier in this chapter (Figure 9). These bridges show Maillart's dedication to design – he created structures that were beautiful and practical. The arch rib of Maillart's bridges was generally quite thin, allowing the form work to be lighter and cheaper to construct than would be required for a heavier arch. Once the arch had hardened the rest of the bridge could be cast, supported by the arch, without needing further scaffolding (Billington, 1979).

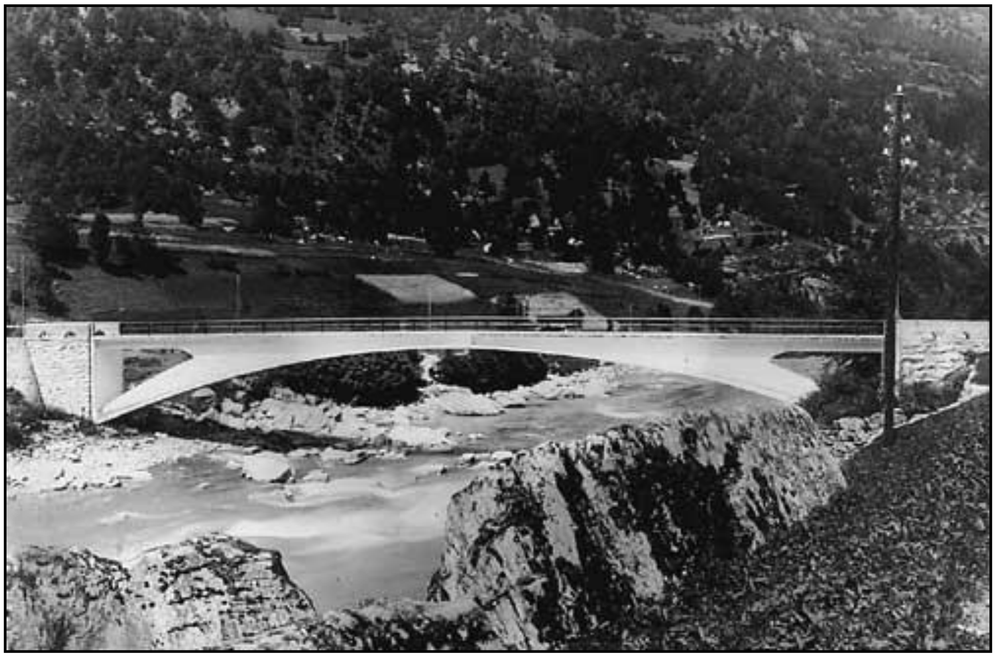


Figure 25: Tavanasa Bridge by Maillart (http://www.nbq.ch/daniel/STS/STS.html)

Perhaps the most beautiful of Maillart's bridges is the bridge at Schwandbach. Built in 1933, the bridge is set high in a valley, arcing from one rock face to the other. The roadway curves over the span of the bridge while the arch is perpendicular to the abutments. The inside of the arch rib follows the inside curve of the roadway, whereas the outside edge is straight (Kassler, 1949). This irregular shape causes the arch to widen at the abutments, where it resists transverse wind load, and narrows at the center, with cross walls that taper to meet the roadway. The arch rib is less than 8 inches thick, the cross walls are 6.3 inches (Kassler, 1949), contributing to the exceptional lightness of the bridge. The Schwandbach Bridge, which is still in use today, is an exemplar realization of the possibility of reinforced concrete.

Figure 26: Schwandbach Bridge

(http://www.ce.jhu.edu/perspectives/protected/ids/Buildings/Schwandbach%20Bridge/main.jpg)

Maillart was not the only influential designer in reinforced concrete. There were many bridges built elsewhere in Europe and America that made beautiful use of the material. In America, many reinforced concrete arch bridges were constructed along the Pacific coast. The rugged terrain and many rivers required bridges, and many beautiful examples were built. One particular designer, Conde McCullough, built an entire series of bridges for the Oregon Coast Highway between 1932 and 1936 (Brown, 2001). There were also a number of reinforced concrete arched bridges created in California, such as the Russian Gulf Bridge in 1940 and the Bixby Creek Bridge in 1933 (Kassler, 1949). Perhaps the most impressive concrete arch bridge, in sheer size alone, is the Tunkhannock Viaduct in Pennsylvania. Spanning across the entire valley for almost half a mile, its massive semicircular arches march inexorably across, bringing to mind its Roman predecessors, and clearly showing their influence (Brown, 2001).

Figure 27: Tunkhannock Viaduct (http://stflyfisher.wordpress.com/tag/tunkhannock/)

Eugène Freyssinet, a contemporary of Maillart, built reinforced concrete bridges in France in the early- to mid-1900s. He was especially influential because of his discovery of creep: the phenomenon of concrete continuing to deform after it has hardened, even with constant load (Brown, 2001). Freyssinet developed a system where he left a small amount of space at the crown of the arch. After a year, when the concrete had deformed and the arches had begun to sag, he came back and jacked apart the two sides of the arch and filled the space with new concrete (Brown, 2001). Freyssinet's most famous work is the Plougastel Bridge in Brittany. Three enormous spans carry two decks, one road and one rail. All of the spans were built successively over the same formwork, a giant wooden arch, tied together at the bottom, which floated in concrete caissons. The Plougastel bridge was, at the time, the largest reinforced concrete bridge in existence.

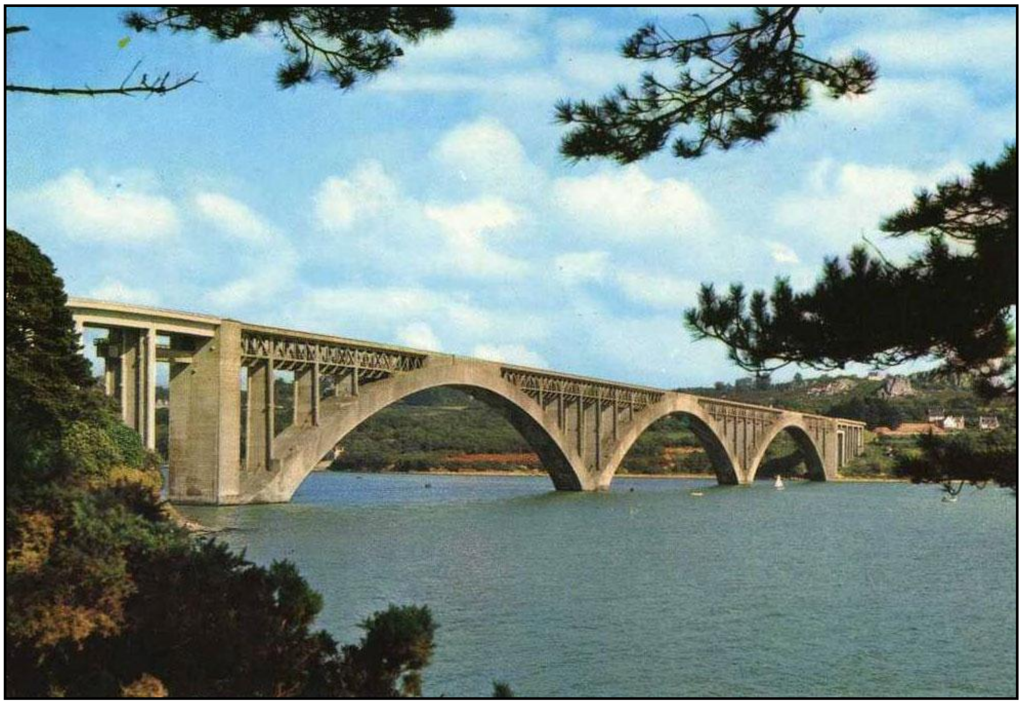


Figure 28: Plougastel Bridge (http://www.simplonpc.co.uk/Brest.html)

Plain reinforced concrete is an incredible material that is still used today, but with the advent of prestressing, concrete can be taken to a new level. Prestressing puts the steel in reinforced concrete into tension, adding additional compression to the concrete. This cancels out the tension stresses that would otherwise be present, putting the entire cross section into compression, and ultimately creating a stronger material. (Brown, 2001) Prestressing maximizes the capacity of concrete and, as a result, structures built of precast concrete can have greater spans and higher loads than those built or regular reinforced concrete.

In 1979 in Croatia, the Krk Island bridges were constructed out of prestressed concrete. The arch form was chosen because the exceptionally deep water that made piers impractical (Brown, 2001). The height of the bridge above the water does not disrupt boat traffic, an added benefit of the arch. The extremely long span of 390 meters was only possible because of the process of prestressing: it was the longest concrete arch bridge in the world at the time of its construction

(Janberg, 2012). Even today the prestressed concrete arch is still used. One particular example of a prestressed concrete arch is the Hoover Dam Bypass, which features twin arch ribs and a steel deck (see Figure 6).

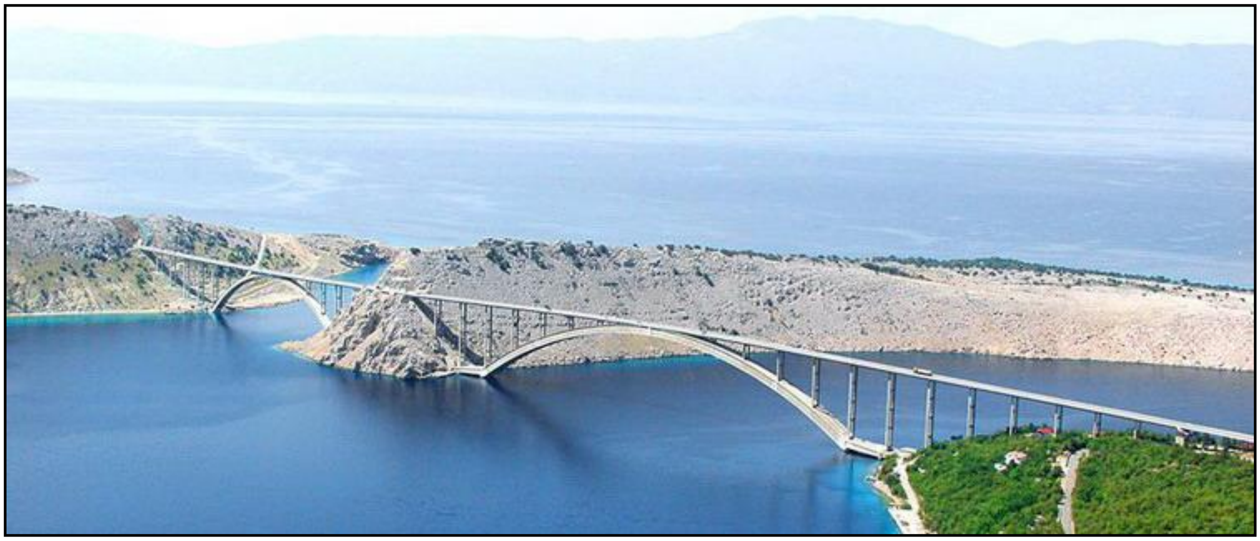


Figure 29: Krk Island Bridges

(http://www.davorkrtalic.com/Turizam/Krk/Krk_Baska_01/Krk_Baska_01_en.htm)

Arched bridges have been built for thousands of years. They work well in stone, concrete, or steel. They are well suited to a variety of different locations, and by changing the location of the deck with respect to the arch they can be constructed almost anywhere. Arches are not without challenges, however. They are not easy to construct, and are not as straightforward to design as a simple beam. The beauty possible in an arched bridge is unmatched, and they are more fluid than the suspension or cable stayed options. The arch is a form that has existed for centuries, and we are not finished with it yet.

Chapter II: Design of an Arch

Shape of the Arch

The perfect arch shape has two parts: the line of the center of the arch, which should approximate the line of thrust of the arch under dead load alone, and the shape of the arched rib. The line of the arch historically was not precisely calculated. The Romans built semicircular arches, but the semicircle was the inside face of the arch, so the actual centerline was slightly different (Brown, 2001). The fact that the "perfect" arch form was not known was not a problem for the Romans, as the large weight of fill placed on top of the masonry arch served to brace it, and counteracted any tension forces that might be caused by a variation in the line of thrust and the centerline of the arch. Throughout the middle ages, bridge builders went with what worked. They were able to take some of the lessons learned from cathedrals (for example, the fact that steeper arches lead to less lateral thrust at the abutments) and apply them to bridges, but they did not have any understanding of the scientific principles behind their work.

It was not until the late 1600s that the problem of the mathematically perfect arch form began to be a matter for study (Heyman, 1998). At this time many scientists and mathematicians formed societies for the advancement of knowledge, such as the Royal Society of London, and met to consider the research and experiments of their fellows. Robert Hooke, who is known by many engineers today because of "Hooke's Law," which describes the relationship between stress and deformation, was one of these scientists. He was the "Curator of Experiments" for the Royal Society, and was charged with bringing experimental demonstrations to the society (Heyman, 1998). Hook developed an experiment for the correct shape of an arch, positing that it was the inverse of the form a weighted chain takes when hanging downward in tension would provide the proper form of an arch in compression.

Figure 30: Hanging chain forming a catenary shape
(http://www.math.udel.edu/MECLAB/UndergraduateResearch/Chain/Main_Page.html)

Hooke published his "solution" to the problem of the perfect shape of a masonry arch around 1675 in an anagram "abcccddeeeeeefggiiiiiiiillmmmmnnnnnooprrsssttttttuuuuuuuuux" (Linda Hall Library, 2002), which unscrambles to "*Ut pendet continuum flexile, sic stabit contiguum rigidum inversum*," "as hangs the flexible line, so but inverted will stand the rigid arch" (Heyman, 1998), which was later solved and published after his death. Hooke did not, however, actually have a mathematical solution to the problem, though he later suggested a cubic parabola ($y=|ax^3|$) (Heyman, 1998). It is worth noting that the ends of a chain never hang vertically from the support – there is always some horizontal component to the reactions. The arch would necessarily also always have some skew with regard to the abutments to keep the line of thrust along the centerline of the arch rib.

The approximation of the actual line of thrust is generally enough for a typical masonry arch. The stone blocks are large enough that the forces are adequately contained within the cross section provided. The general rule developed by early studies of masonry is that keeping the line of thrust within the middle third of the cross section is safe, but Heyman points out that what is really required is keeping the line of thrust from passing outside the cross section (Heyman, 1966). For masonry construction, this is generally attainable, as can be seen by the wealth of structures built before the theory of structural mechanics was understood. However, once stronger material such as concrete and steel began to be used it became extremely important to keep the center of the arch rib aligned with the line of thrust, because the sections were so much smaller than previously (Billington, 1979). The careful analysis of the structure and the loads it would be subjected to was necessary to ensure stability of the structure.

The general requirement to keep the applied stresses in a material below the allowable stresses affects the design of the arch. The thickness generally varies from areas of high stress to areas of low stress. This is particularly true for areas of high moment, because the dead load stresses in an arch are fairly constant. Moment caused by live load, however, can cause a significant increase in stress in a particular location, and to keep the stresses below the maximum the cross section can be increased in compensation.

Arch Ribs

The perfect shape of the arch rib depends in large part on the type of end fixity encountered. Because a pinned connection creates a location where zero moment can be transferred, the section close to that location can become thinner. In contrast, areas around fixed supports (where moment is high) must be thickened to accommodate the increase in stress. As can be seen below, this results in a different shape of arch rib for each of the different arch conditions – fixed-fixed, two pin, and three pin. It is possible to build all of these different arches with the same cross

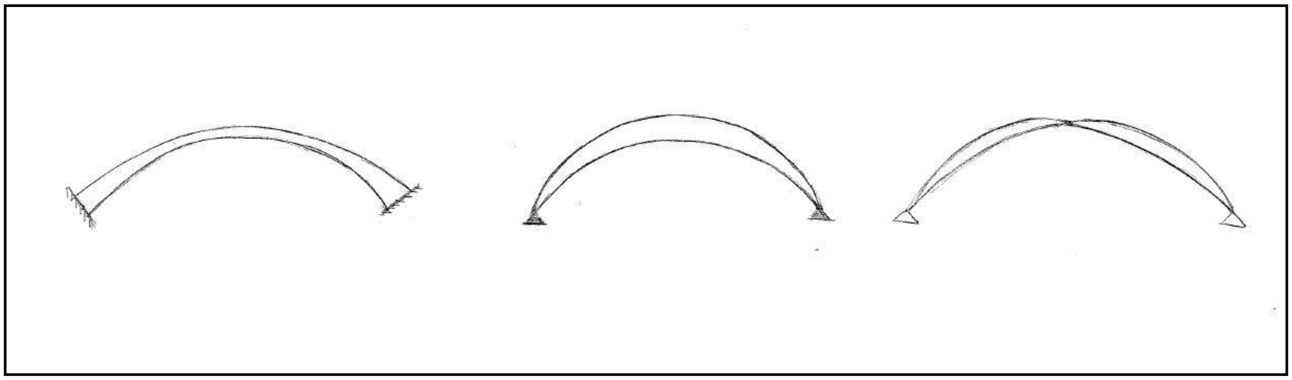


Figure 31: Three types of arches, with varying rib thickness

section, but it is an inefficient use of material, and results in a much less aesthetically appealing bridge (Kassler, 1949). The dramatic narrowing of the fixed arch from abutment to mid span and the sickle shape of the two-pinned arch are particularly elegant, and ignoring their possibilities reduces the potential of the design.

One of the premier designers of arched bridges, Robert Maillart, grasped the difference between simply engineering a structure and designing it as an art (Kassler, 1949). He understood his materials, creating structures that were economical as well as beautiful. He did not try to get his bridges to mimic anything else. His later bridges, in particular, proclaimed their concrete structure proudly (Billington, 1979). Maillart was very particular is designing the ribs to resist the necessary stresses with as little material as possible, an example of which can be seen in his bridge at Vessy. Note how the arch ribs are thickest at the quarter spans, where the moment stresses are highest, and become thinner at the abutments and the crown.

Figure 32: Maillart's bridge at Vessy - note the variation in rib depth (Wikimedia Commons)

Hinges create locations in a member that are free to rotate. Because their resistance to bending is zero, it also creates a location where the internal moment of the member is zero. In steel construction, it is often easy to create a truly pinned connection. It can also be done by a plate connection where the plate is thin and limited to the web of a member, and as a result is not able to transfer moment across the connection. Pinned connections can also be created by an actual hinge, with two sides and a dowel type connector in between (see Figure 33). For an arch bridge, the base of the pinned connection would be angled, as shown previously in Chapter I, Figure 5.

Figure 33: Pinned Abutment Connection for Truss Bridge

(http://bridgehunter.com/wa/yakima/bh43055/)

In concrete bridges the method of creating hinges becomes slightly more complicated. It is not possible to create hinges in plain concrete, because concrete cannot take the combined forces that a hinge experiences. One early method was to cast steel hinges into the concrete at the crown

Figure 34: Steel hinges in concrete arch (http://www.bphod.com/2010_03_01_archive.html)

and abutments, allowing the bridge to move without cracking (see Figure 34). This type of the connection is perfectly structurally sound, but it disrupts the unity of material. Maillart developed a system where he created a hinge out of the reinforced concrete itself. Carefully placed rebar carried the tension of the hinge, and the concrete on either side was allowed to move (Billington, 1979).

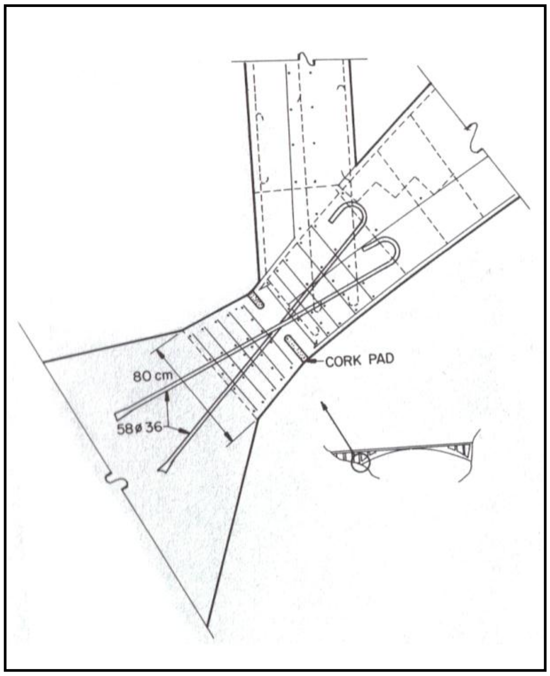


Figure 35: Reinforced concrete hinge at springing of Salginatobel Bridge (Billington, 1979)

The issue of how to create a hinge is an important one, but the essential requirement is that the location of the hinge needs to be significantly less resistant to moment than the rest of the section, but still able to transmit axial load and shear. How we model the boundary conditions of a structure is critically important, and it is equally important to ensure that the condition met with in the field is appropriately constructed for the assumptions made in design. In arched bridges, the relative fixity of the abutments is a crucial unknown. In masonry arches, it is often acceptable and expedient to consider the abutments to be pinned (Heyman, 1966). Because any small movement in the abutment or imperfect fit between the abutment and arch will result in a three pinned condition, the use of this model for analysis is warranted.

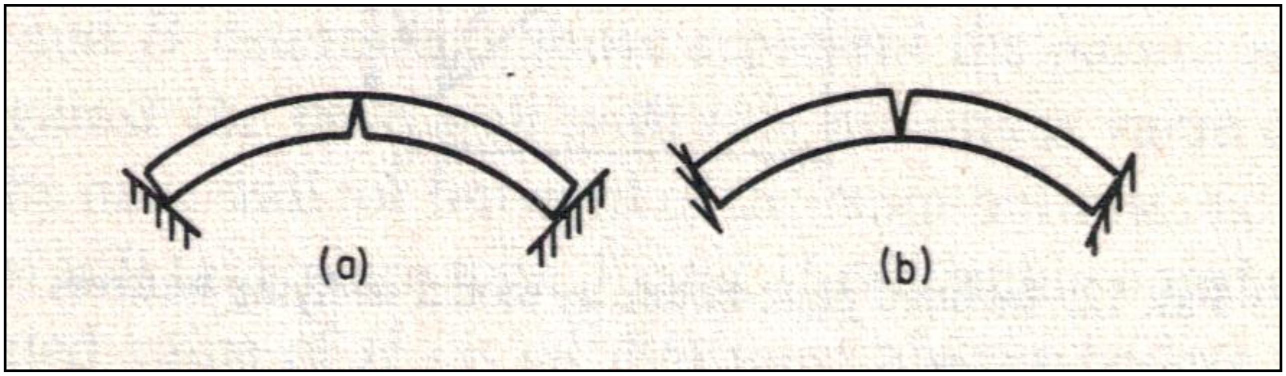


Figure 36: Imperfectly fitted arches, resulting in pinned behavior (Heyman, 1966)

Behavior under load

An arch is very stable under dead load alone, particularly if the arch centerline is close to the line of thrust. It is more difficult, however, to control exactly what stress the structure experiences under live loads. In a large masonry filled arch the live loads are relatively small compared to the dead loads, and as a result the effects of the live load are limited. If the structure is relatively light in relation to the live load it is expected to support then the effects of that live load become more important. In particular, an unbalanced live load will create bending moment in the arch rib. This creates an additional load that has to be designed for, or the arch could collapse under heavy unbalanced load. As shown in Figure 37, the arch rib bends under the unbalanced load, but deflects evenly under a balanced load. This extra bending from a heavy load on only one side of the bridge creates additional stress in the rib.

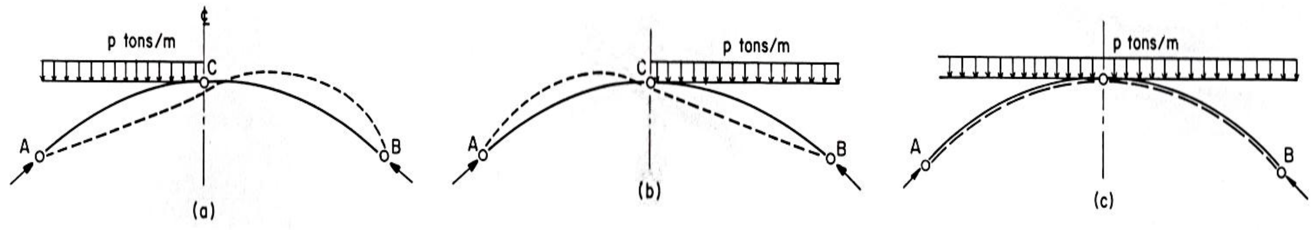


Figure 37: Arch bending under unbalanced load (Billington, 1979)

There are various ways to deal with the bending caused by unbalanced loads. Robert Maillart developed a system where he coupled a thin and flexible arch with a stiff deck. In his analysis he assumed that the deck would carry all the bending moment created by unbalanced loads, and the arch rib would only carry the axial load (Billington, 1979). This analysis relies on a stiff connection between the deck and the arch: cross walls or columns rather than the flexible hangers found on a through-arch bridge. When an unbalanced load occurs, the side of the bridge under it deflects downward, and the opposite side deflects upwards. If the deck is relatively very stiff, the deck resists this upward deflection, and provides a downward force through the cross walls into the arch (Billington, 1979). This action distributes the partial live load across the entire arch, and limits the deflection that the bridge experiences. In contrast, in an un-stiffened arch the deck provides little resistance, and the entire bridge deflects upward on the side opposite to the load (Billington, 1979).

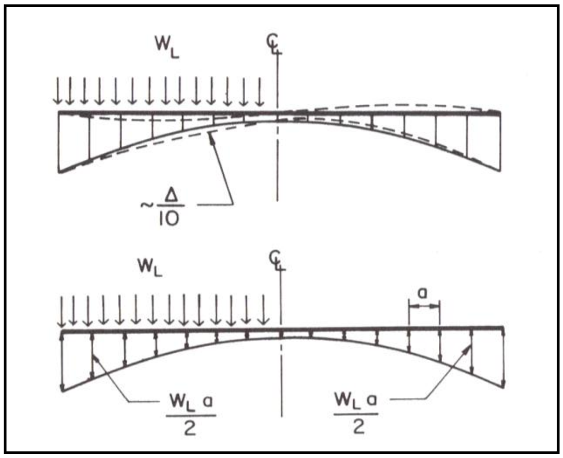


Figure 38: Actions of a deck stiffened arch, unbalanced load (Billington, 1979)

In contrast to this deck-stiffened approach, many bridges in the United States were built with a relatively thick arch compared to the deck. This thin deck approach required fewer assumptions than Maillart's deck-stiffened arch, as the bending stresses calculated for the arch need not be transferred to the deck. Maillart made a number of other assumptions in his approximate analysis, including that the arch carried all dead, live and snow loads present; that the bending in the arch rib is carried by the deck; and that the arch was hinged at the abutments, when in practice he constructed the arch to be rigidly tied into the abutments (Billington, 1979).

In America, the practice was to use as exact an analysis method as possible, often creating an elastic model to help with the analysis (Billington, 1979). This focus on analysis over design caused American engineers to approach the problem of the arch-deck system differently from European engineers such as Maillart. As can be seen in the graph of arch stress to stiffness ratio, the ratio of the resistance to bending in the arch to the deck, (Figure 39) the equation does not have a meaningful solution for the minimum stress in the arch. Either the arch becomes infinitely thin, or infinitely thick, and neither end of the graph are of practical use. It is worth noting, however, that the starting place for approaching the problem makes a difference to the optimal arch to deck stiffness reached (Billington, 1979). Maillart began from the deck-stiffened model, looking for the thinnest arch possible, while the American approach called for a thicker and stiffer arch.

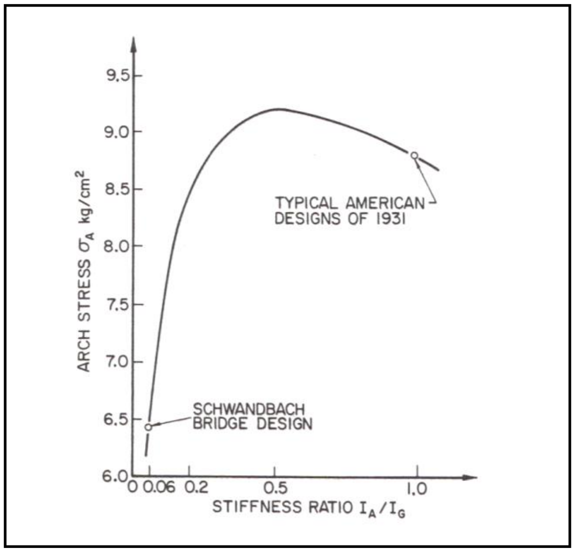


Figure 39: Arch Stress to Stiffness Ratio (Billington, 1979)

Construction

The construction of arches is particularly difficult because an arch is not self-supporting until it is completed. The thrust from each half of the arch requires the other half to balance it. For a tied arch, where the deck resists the lateral thrust created by the weight of the arch itself, there is nothing to resist the outward thrust until the deck is built, but nothing to support the deck without the arch. The problem of construction can sometimes outweigh the advantages of the arched form itself, but there are a number of creative solutions to the problem. These range from the traditional centering forms, some tied to provide internal resistance to thrust, to intermediate piers, to cable-stayed systems, to cantilevering the arch from its supports to meet the other side. Many projects use a combination of these techniques.

Formwork called centering is the most ancient system for supporting an arch. It involves building a frame, generally of wood, to support the arch until it is structurally able to support itself. Then the centering is removed, and the arch remains free standing. There was always the danger that the arch might not be able to support itself, and would come crashing down, though conventional wisdom says that if the arch stays standing when the centering is removed it will remain for hundreds of years (Heyman, 1966). The problem with centering is that it is expensive: constructing a separate arch before the permanent arch can be built adds significantly to the project cost. Scaffolding is also often quite complicated, as can be seen in the construction view of the Harlan D. Miller Memorial bridge in California (see Figure 41), and requires an additional group of laborers to construct it.

Figure 40: Wood centering for a masonry bridge in Minneapolis (http://www.scribas.com/flashbacks/image/3121)

The scaffolding that supports the arch until the two halves meet in the middle, and the concrete hardens, is often a significant expense in the building of the bridge. Maillart was particularly concerned with cost when he designed his bridges, and designed the centering to reflect this concern. The bridge at Lorraine, a masonry style arch built of reinforced concrete blocks, is a good example of his flexibility and economy. Maillart designed the scaffolding to support only the center band of block. The bands on each side were interlocked with, and were supported by, the center band of blocks. In this way, the scaffolding could be much lighter than if it had to support the entire arch (Billington, 1979).

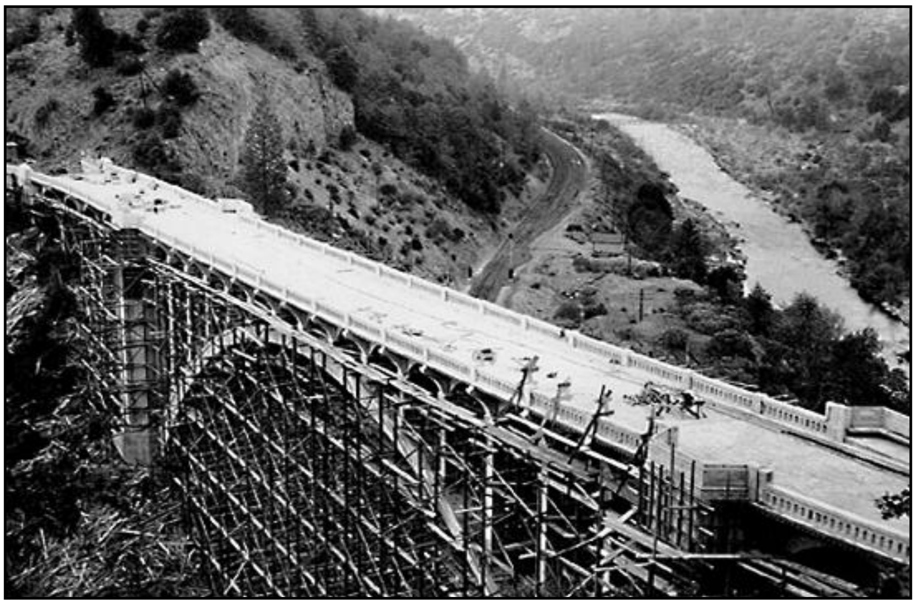


Figure 41: Harlan D. Miller Memorial Bridge, under construction (http://www.waymarking.com/waymarks/WM3PVY_Harlan_D_Miller_Memorial_Bridge)

In places where constructing centering would be impossible or prohibitively expensive, there are a number of other options. Arches can also be constructed by cantilevering out each side, with the arch acting as a beam until the two halves meet. This is how the Sydney Harbor Bridge was constructed. The arch was held in its cantilevered position by massive cables, which can be seen as the thick lines stretching back from the abutments in Figure 42. The deck of the bridge was then constructed from the center out, hanging under the arch (Harbour Bridge Views, 2001). This method of cantilevering the arch halves has the advantage of not requiring centering, and also does not require the towers necessary for cable stayed construction. However, it is necessary to anchor the arch so that it will remain fixed. In Sydney the cables were run all the way back through the abutments to bedrock. It is also necessary to evaluate the stresses in each half of the arch before they meet, because they will be significantly different than the stresses of the finished arch.

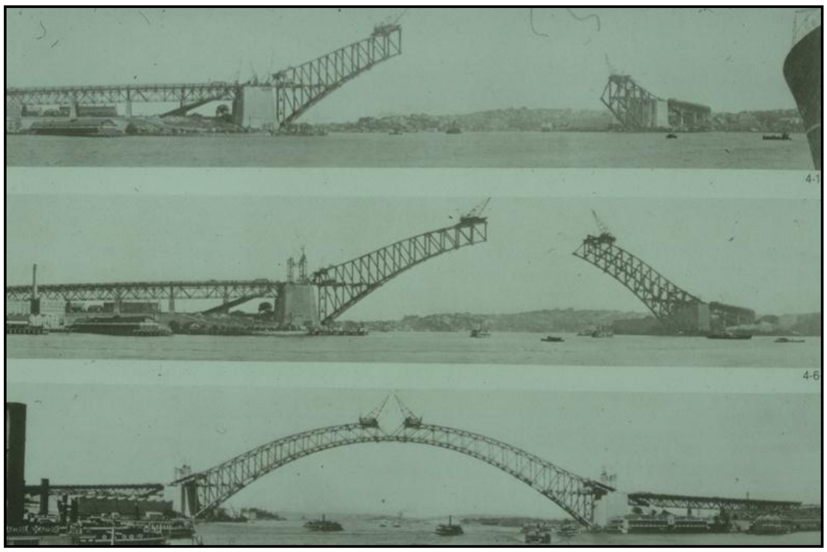


Figure 42: Sydney Harbor Bridge arch construction (http://www.columbia.edu/cu/gsapp/BT/BSI/ARCH/arch3.htm)

Cable stayed construction is another solution, and is a common method of arch construction today. A tower is built behind the abutments, and the cables run down to different sections of the arch. The cables have to be anchored back to rock, but the stresses are less than in the cantilever method, because the arch is supported along its length, not just at the end. The Hoover Dam Bypass Bridge was constructed using the cable stayed method. Towers built at the ends of the approaches supported the unfinished arch. The concrete form was suspended at the end, and the concrete was poured and cured, and then the form moved to the next section. As each section was completed cables were added to support it.

Figure 43: Construction of the Hoover Dam Bypass (Jamey Stillings)

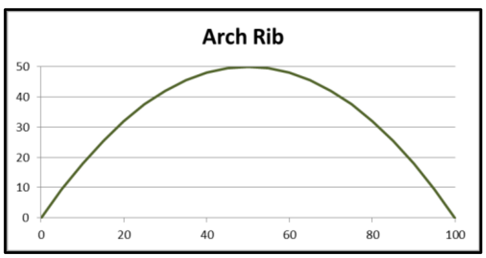
The construction of the bridge is an important consideration for the engineer. A design that is beautiful and efficient but cannot be built, or that is unreasonably expensive to build, is not a good design. This concept is one that the best engineers understand completely. Maillart designed his bridges with an understanding of the construction that would come next – often he was both the designer and the builder (Billington, 1979). Freyssinet, the French engineer, supported his Plougastel Bridge on floating formwork until the concrete hardened. Separating the challenges of construction from design is not in the client's best interest, nor does it make for a good bridge.

Chapter III: Analysis of an Arch

Arches come in various different shapes, from the semi-circular arches used by the Romans, to the flatter circle segments popular in the Middle Ages. Some are even pointed, which reduces the lateral thrust from the arch. A parabola is a common form, though it was proved by Hugens to not be the "perfect" shape, (Heyman, 1998) and it is the parabola that will be explored here. Taking the equation of the arch to be:

$$y = \frac{4h}{L}(x - \frac{x^2}{L})$$

where *L* is the overall length of the arch, and *h* is the height above the pinned ends. Graphing this equation, the form of the arch rib can be varied by changing the value of *h*. Changing the value of *h* but keeping the length the same changes the relative steepness of the arch (see Figure 44).



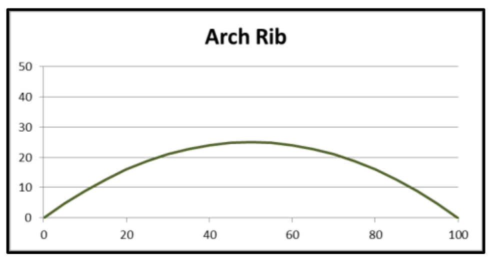


Figure 44: To the left, the arch form when L=100 and h=50. On the right, L=100 and h=25

For this analysis the sample figures will be generated for an arch with *L*=100 and *h*=25. The influence line for various internal and external reactions will be calculated for the sample arch. An influence line is a representation of the reaction based on the location of a single unit point load. They are useful for evaluating the response of a structure to loading. The actual load applied to the structure is then multiplied by the influence line to find the reaction of the structure to that load. A point load is multiplied by the value of the influence line, while a distributed load is multiplied by the area under the influence line. For force reactions (abutment reactions, axial force, etc.) the influence line is unitless, and the value is multiplied by the load to give a force reaction. When evaluating moment, however, the influence line has units of length, and the force multiplied by the influence line value gives a force-length such as kip-feet or newton-meters, which are the units of moment.

Three-pinned Arch Analysis

Calculating the bending moment present in a three-pinned arch is a fairly simple undertaking, as a three-pinned arch is statically determinate. This means that the reactions can be calculated based on simple statics – namely that the forces in each direction must sum to zero. Using the reactions calculated, the bending moment at any location can be calculated, and from that the stress in the member. It is necessary that the stresses in the arch are less than the maximum allowable stress, this principle ensures the safety and stability of structures.

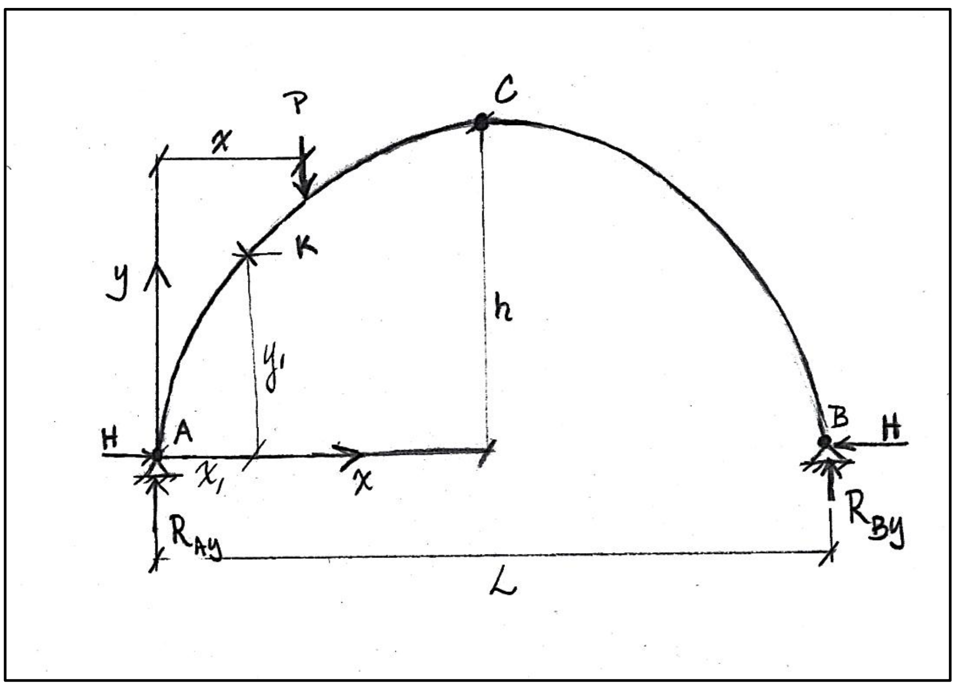


Figure 45: Three-pinned arch

The internal moment at any point along the arch, *k*, located at the coordinate (x_1, y_1) can be evaluated with a single point load as any location *x* along the member. First, the horizontal and vertical reactions at the abutments must be calculated. Looking at a unit load at a given point, *x* (measured from the left abutment, Abutment A), the vertical reaction at Abutment A can be calculated by summing the moments Abutment B. This is the same as the reaction for a simple beam, and varies linearly with the distance *x*, giving the equation $R_{Ay} = 1 - \frac{x}{L}$. Likewise, the vertical reaction at Abutment B also varies linearly, giving $R_{By} = \frac{x}{l}$. These influence line equations for R_{Ay} and R_{By} can be graphed together (Figure 46).

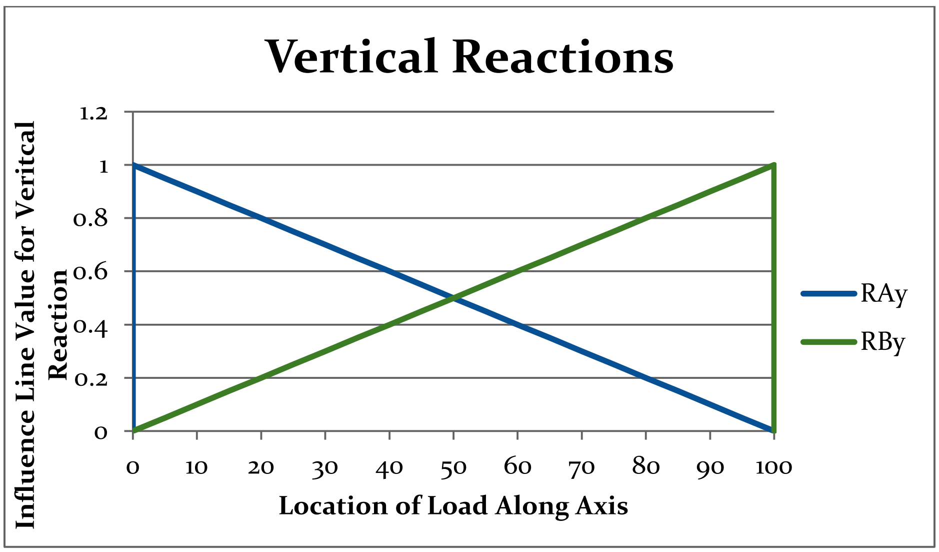


Figure 46: Vertical Reaction Influence Lines for sample arch

The horizontal reactions are necessarily equal and opposite in magnitude for a vertical load; this reaction will be called *H*. The equation will be different depending on which side of the center pin the load is located. When the unit load is to the left of the center hinge point, C

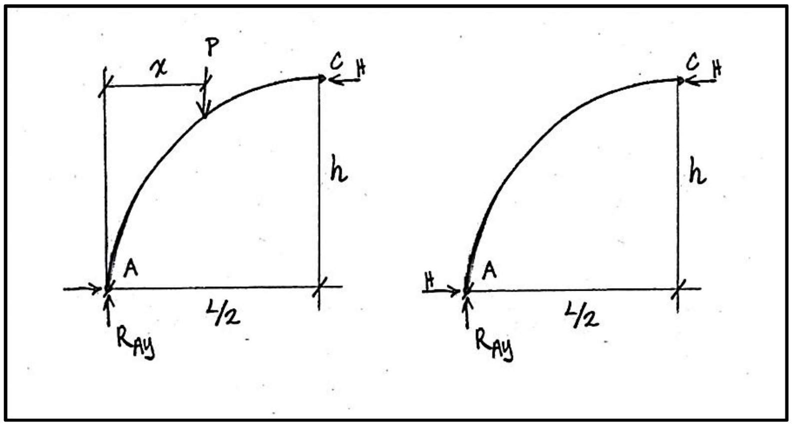


Figure 47: Free body diagrams for calculating horizontal reaction H

$\left(x < \frac{L}{2}\right)$, summing the moments about the crown gives $H(h) + P\left(\frac{L}{2} - x\right) - R_{Ay}\left(\frac{L}{2}\right) = 0$ (see Figure 47 for free body diagram). By plugging in the value for R_{Ay} found earlier and solving the equation $H(h) = \left(1 - \frac{x}{L}\right)\left(\frac{L}{2}\right) - \left(\frac{L}{2} - x\right)$, which simplifies to $H = \frac{x}{2h}$, where h is the original rise of the arch. Likewise, when the load is to the right of C $\left(x > \frac{L}{2}\right)$, the moment equation is $H(h) - R_{Ay}\left(\frac{L}{2}\right) = 0$. Plugging in R_{Ay}, $H(h) = \left(1 - \frac{x}{L}\right)\left(\frac{L}{2}\right)$, which simplifies to $H = \frac{L-x}{2h}$. These two equations are greatest when the load is at C. As shown in Figure 48, the graph of the influence line for the horizontal reaction increases until the load reaches $\frac{L}{2}$, then decreases.

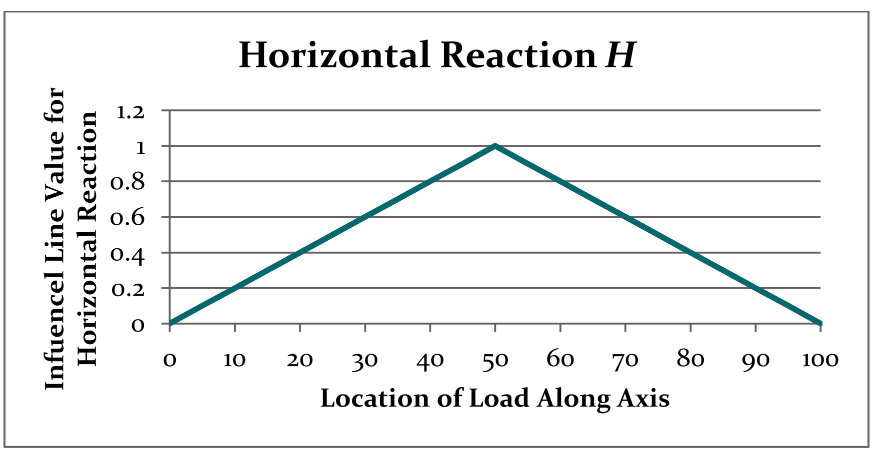


Figure 48: Horizontal Reaction Influence Line for Sample Arch

The bending moment at a location k along the arch with coordinates of x1 and y1, can be divided into three conditions: when the unit load is to the left of k $(x < x_1)$, when the unit load is to the right of k but left of C $\left(x_1 < x < \frac{L}{2}\right)$, or when the unit load is to the right of C $\left(\frac{L}{2} < x\right)$. The second two conditions actually result in the same free body diagram, the difference is that the equation for *H* is different depending on which side of the crown the load is located.

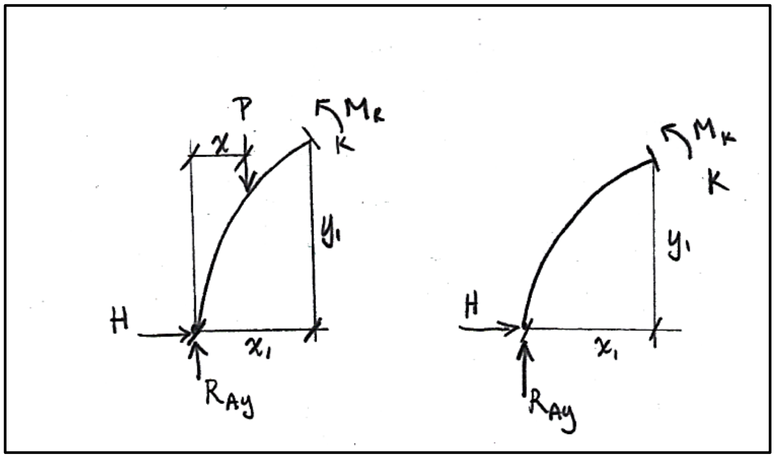


Figure 49: Free body diagrams for *P* left of *k* and *P* right of *k*

These three conditions will result in three different equations for M_k. For $x < x_1$, in summing the moments in the 25' tall by 100' wide arch to find M_k there will be three components: x, R_{Ay}, and H for $\left(x < \frac{L}{2}\right)$, $M_k = P(x_1 - x) + H(h) - R_{Ay}\left(\frac{L}{2}\right)$. The resulting simplified equation will be $M_k = x\left(1 - \frac{x_1}{L} - \frac{y_1}{2h}\right)$. Once the load moves to the right of *k*, the equation becomes $M_k = H(h) - R_{Ay}\left(\frac{L}{2}\right)$, regardless of the location of the load. When $\left(k < x < \frac{L}{2}\right)$ this simplifies to $M_k = x_1\left(1 - \frac{x}{L}\right) - y_1\left(\frac{x}{2h}\right)$, and when $\left(\frac{L}{2} < x\right)$, $M_k = x_1\left(1 - \frac{x}{L}\right) - y_1\left(\frac{L-x}{2h}\right)$. These equations can be graphed to show the influence line for the location *k*. By varying *k* from zero to $\frac{L}{2}$, influence lines for each condition can be generated (see Figure 50). Because the arch is symmetric, the maximum moments will repeat for the other side, when *k* is greater than $\frac{L}{2}$.

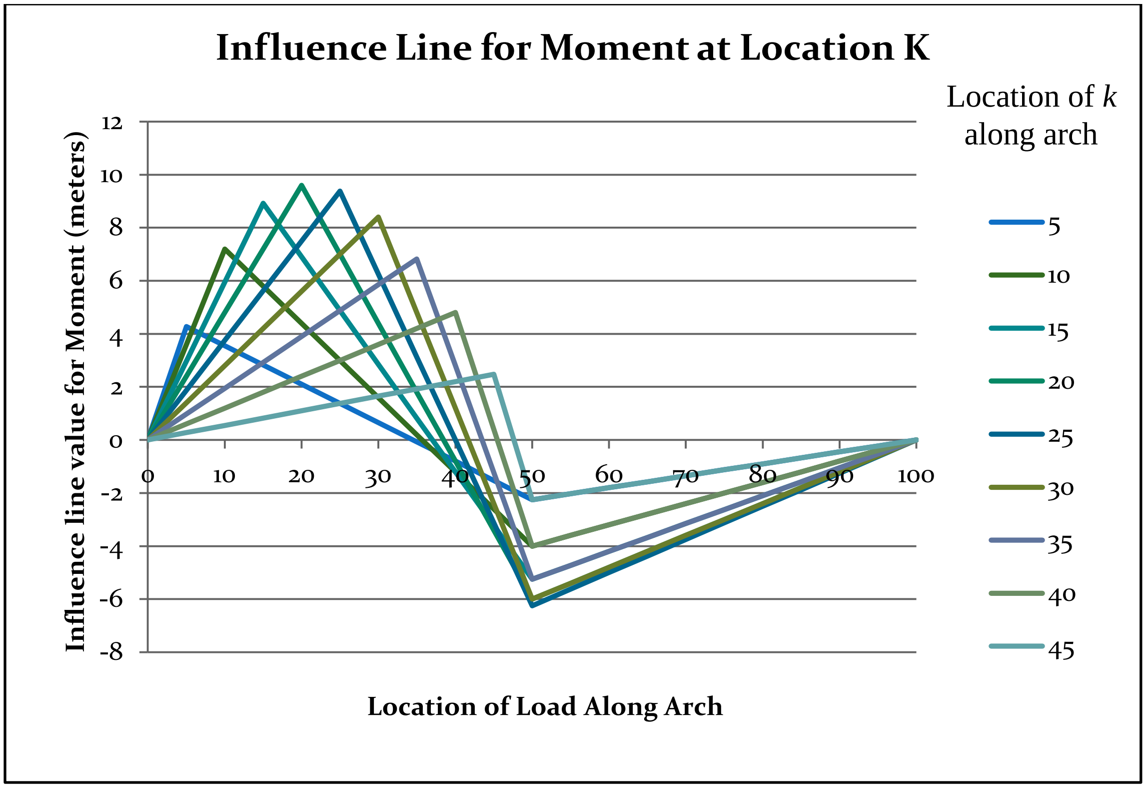


Figure 50: Combined influence lines at *k* for sample arch

The combined maximum and minimum moments at a particular point can be plotted, creating a moment envelope for the arch rib. The positive moment envelope is easy to relate to the combined influence line plot, it merely traces the outside of the individual maximums. The negative moment envelope is slightly harder to see, because the greatest negative moments all occur when the load is at the center of the arch. But because each line of Figure 50 represents one location on the arch, the minimum moments can be plotted against where they occur. Transferring the value of the influence line when it is most negative to the location of interest (*k*) gives a clearer picture of where the minimum values are located (see Figure 51).

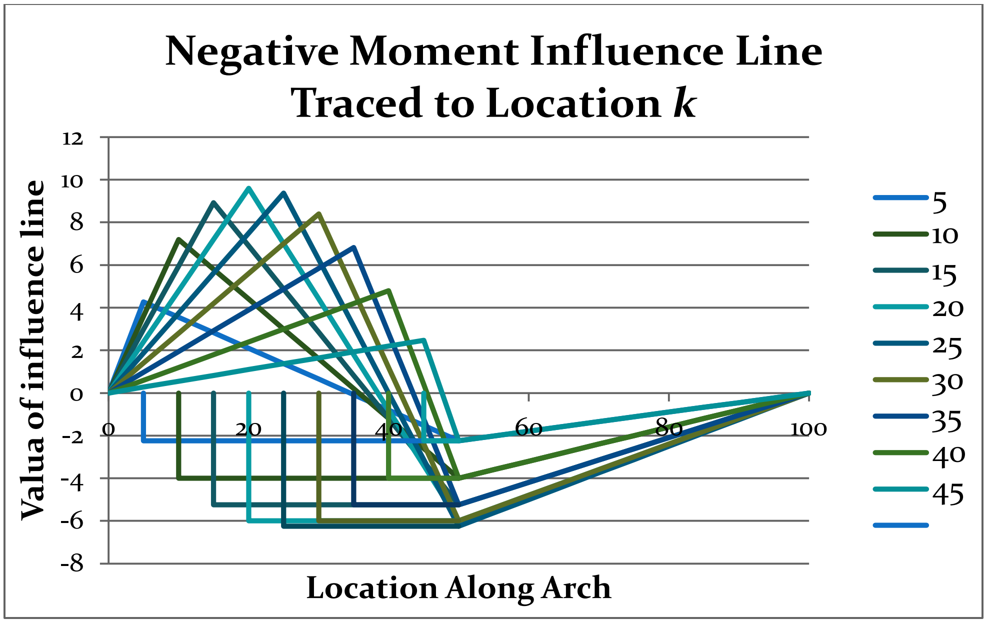


Figure 51: Negative moment transferred to the location at which it occurs

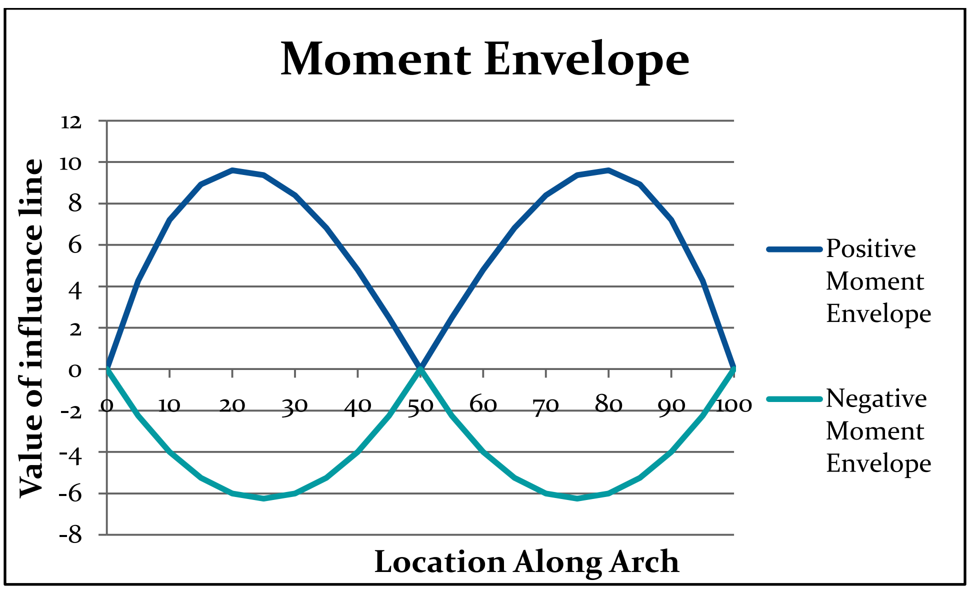


Figure 52: Moment envelope for three-pinned arch rib

The moment at the center of the arch is zero because there is a hinge at that location. The maximum moments experienced of 9.6 occur slightly to the outside of the quarter span at 20 and 80 meters, while the minimum moments of 6.25 happen at the quarter span (see Figure 52). This is because when the load is to the left of k the moment is higher than when it is to the right, while the horizontal and vertical components vary constantly. The point load pulls the maximum outwards for the positive moment. The maximum negative moment always occurs when the load is at the center, so this does not affect it.

Axial load is calculated by cutting the member perpendicular to its axis at the location of interest, and summing the forces perpendicular to the cut (see Figure 53). There is also shear force in the arch, parallel to the cut, but that has been left out of the figure for clarity. This gives two equations for the axial force N_k. When the point load is to the left of k $N_k = P\sin\theta - R_{Ay}\sin\theta - H\cos\theta$, and when the load is to the right $N_k = -R_{Ay}\sin\theta - H\cos\theta$. θ in these equations is the angle of the arch, and can be calculated by taking the arctan of the change in *y* over the change in *x*. If the d*x* is very small this is a good approximation for the angle of the axis of the arch.

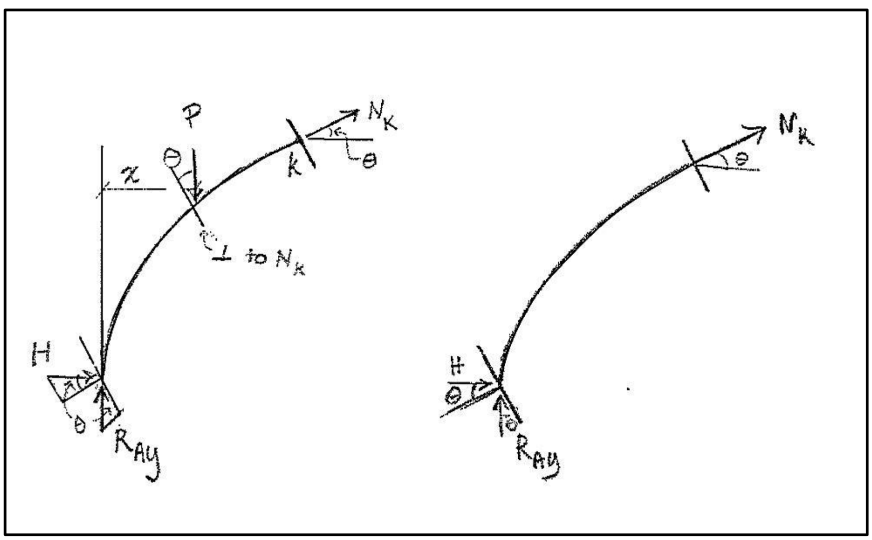


Figure 53: Free body diagram for the axial load

The influence line can be generated as it was for moment, giving a plot of the axial force in the member at each location for a single point load (see Figure 54). The axial force jumps when the load passes over the location of interest, because then the axial load contributes to the force in the arch. The value of the influence line is always negative, because the arch is always in compression.

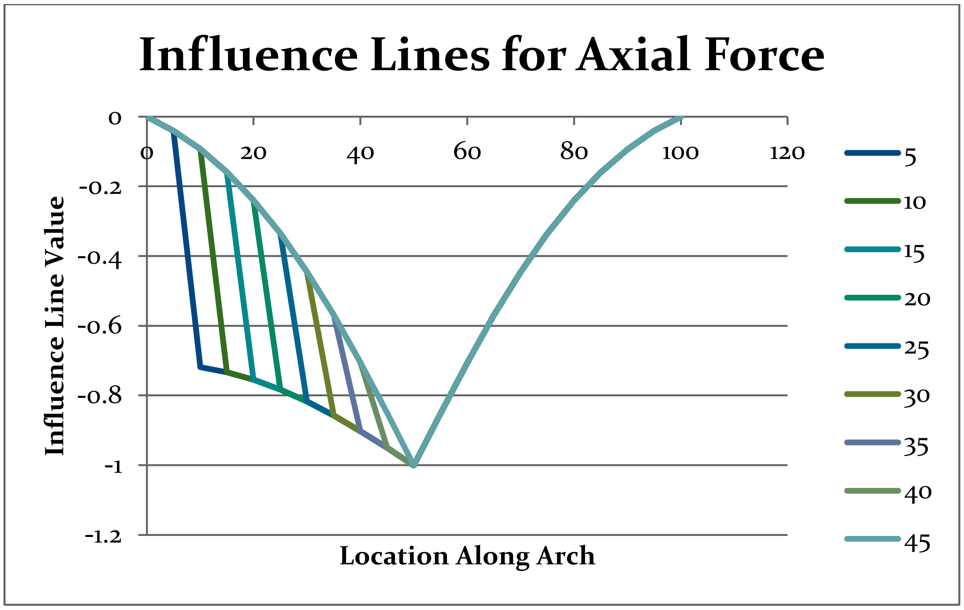


Figure 54: Influence line for axial force

Two-pinned Arch Analysis

A two-pinned arch is somewhat more complicated than a three-pinned arch, because the two-pinned arch is statically indeterminate. There are four reactions generated, and only three equations to solve. This arch form must be analyzed by elastic method, by removing one of the restraints and replacing it with a force, then setting the deflection of the released structure equal to zero. In this analysis the horizontal force at Abutment B will be removed. This analysis and generation of the influence lines and moment envelope is based on that presented in *A Text-Book on Roofs and Bridges* (Merriman & Jacoby, 1909). Again, we will consider a sample arch of L=100 and h=25 to generate the influence lines.

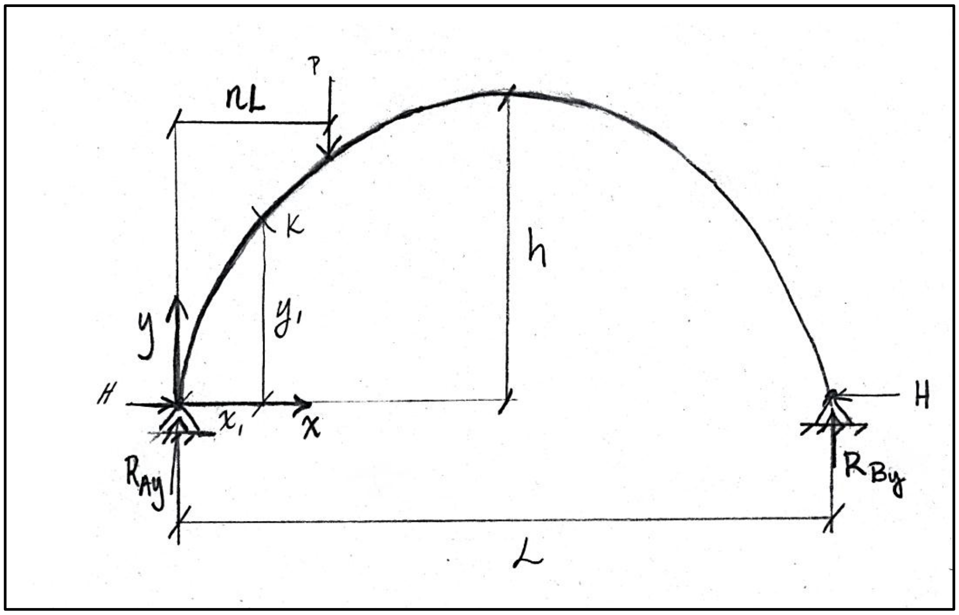


Figure 55: Two-pinned Arch Analysis

The influence line for a two-pinned arch will be generated by placing a single vertical load P at a distance nL from A. The vertical reactions can be solved by taking moments about each end, giving $R_{Ay} = P(1-n)$ and $R_{By} = Pn$. These are the same as the reactions for the three-pinned arch, and are plotted in Figure 48. Since the two-pinned arch is statically indeterminate, the horizontal reaction H has to be found by elastic analysis (Merriman & Jacoby, 1909).

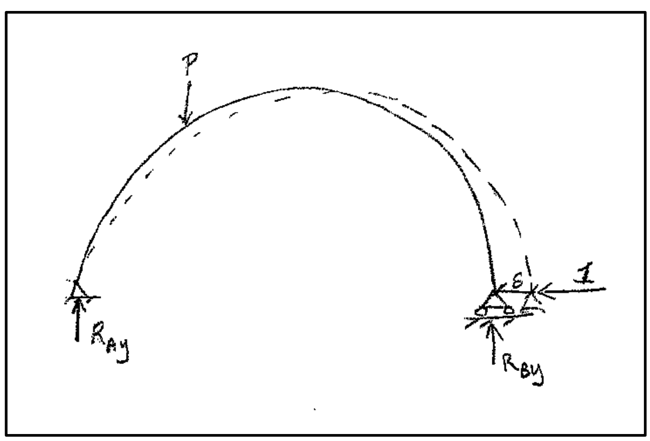


Figure 56: Released structure with unit restraining force

This analysis will follow the method laid out in Merriman and Jacoby's *A Text-Book on Roofs and Bridges*. By the method of internal work, the movement of the end can be calculated:

$$\Delta = \int \frac{M'mds}{EI}$$

where M' is the bending moment caused by the vertical force, *m* is the bending moment due to a horizontal unit force at the abutments, *ds* is an incremental length along the arch, and *EI* is the modulus of elasticity multiplied by the moment of inertia (see Figure 57).

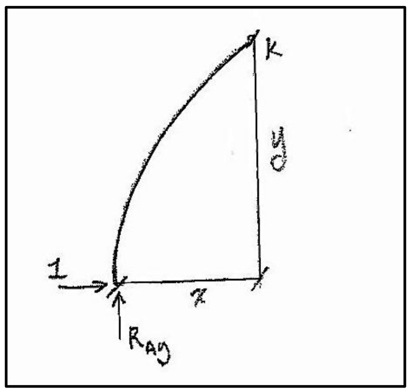


Figure 57: Forces contributing to M' and *m* at each location *k*

Likewise, the deformation due to the horizontal force can be calculated:

$$\Delta = \frac{1}{H}\int \frac{M''^2 ds}{EI}$$

where M'' is the bending moment from the horizontal force *H*, or $M'' = -Hm$. Substituting *–Hm* for M'' gives $\Delta = H\int \frac{m^2 ds}{EI}$, then setting the two deflections equal to each other, it is possible to solve for *H*.

$$H = \frac{\int \frac{M'm\ ds}{EI}}{\int \frac{m^2 ds}{EI}}$$

This equation depends on the properties of the arch rib, the modulus of elasticity and the moment of inertia. In this analysis, E and I will be considered constant along the length of the arch, and the equation for H becomes:

$$H = \frac{\Sigma M'm}{\Sigma m^2}$$

(Merriman & Jacoby, 1909). Looking at each location on the arch, $M' = R_{Ay}x$ or $M' = R_{By}(L - x)$, and $m = 1y$, where x and y are locations of the center of the segment (k in Figure 57). Summing all of these values for each side of the load gives us the numerator, $\Sigma M'm$. The denominator, Σm^2, is the sum of the y values of the segments squared, and is constant. The value of H is not linear, it follows a parabolic shape. The value of the H also never reaches the full value of the point load, the maximum is about 0.8 for the sample arch. This is because the arch can carry bending force across the crown, so there is less outward force at the abutments. A steeper arch (where h is greater) would have a lower maximum H, whereas a shallower arch would have a greater maximum H.

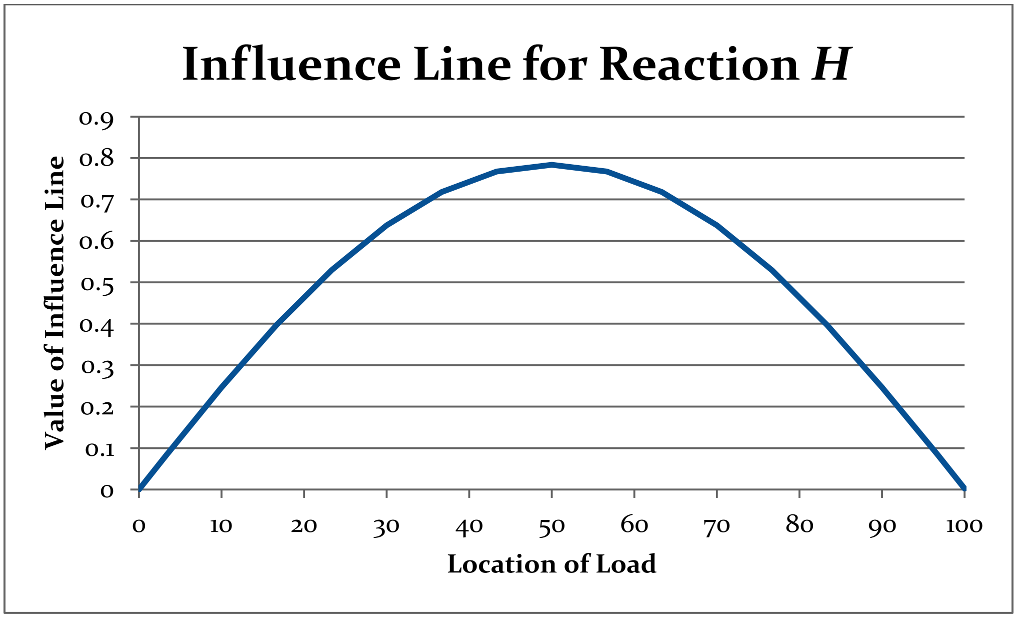


Figure 58: Horizontal reaction for single point load

The value for *H* is not linear; it varies with the location of the load (see Figure 58). Because of this, the value for moment will not vary linearly as it did for the three-pinned arch. By taking the bending moment about any point k (x_1, y_1)along the arch, we will create two different

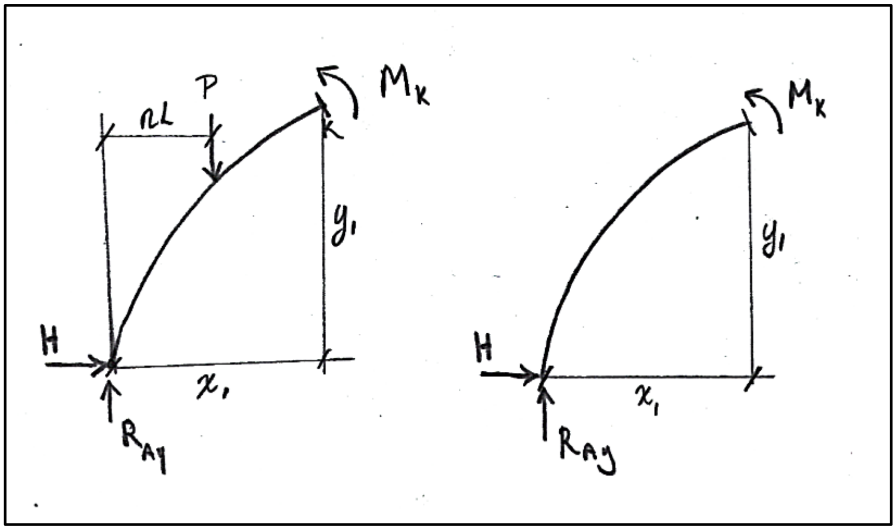


Figure 59: Case 1 and 2 for calculating moments at location k

conditions: *k* to the right of *P*, and *k* to the left of *P* (see Figure 59). For the first condition, $M_k = R_{Ay}x_1 - Hy_1 - P(x_1 - nL)$, which is calculated by summing the moments about the location *k*. Likewise, for the situation where *P* is to the right of *k*, $M_k = R_{Ay}x_1 - Hy_1$. Because the value of *H* depends on the location of the load and the equation of the arch, the value of M_k will vary as well. However, it is still possible to generate the influence line, by solving for M_k between each segment we calculated *H* for previously. All of the values for M_k can then be graphed together, giving the influence line for each condition (see Figure 60). Like the three-pinned arch the two-pinned arch is symmetric, so only the influence lines the left half of the arch is shown. These influence lines can be conceptually verified by considering a two pinned arch projected onto a line, and creating a hinge at the location of interest. The value of the influence line will decrease on each side, and must equal zero at the abutments.

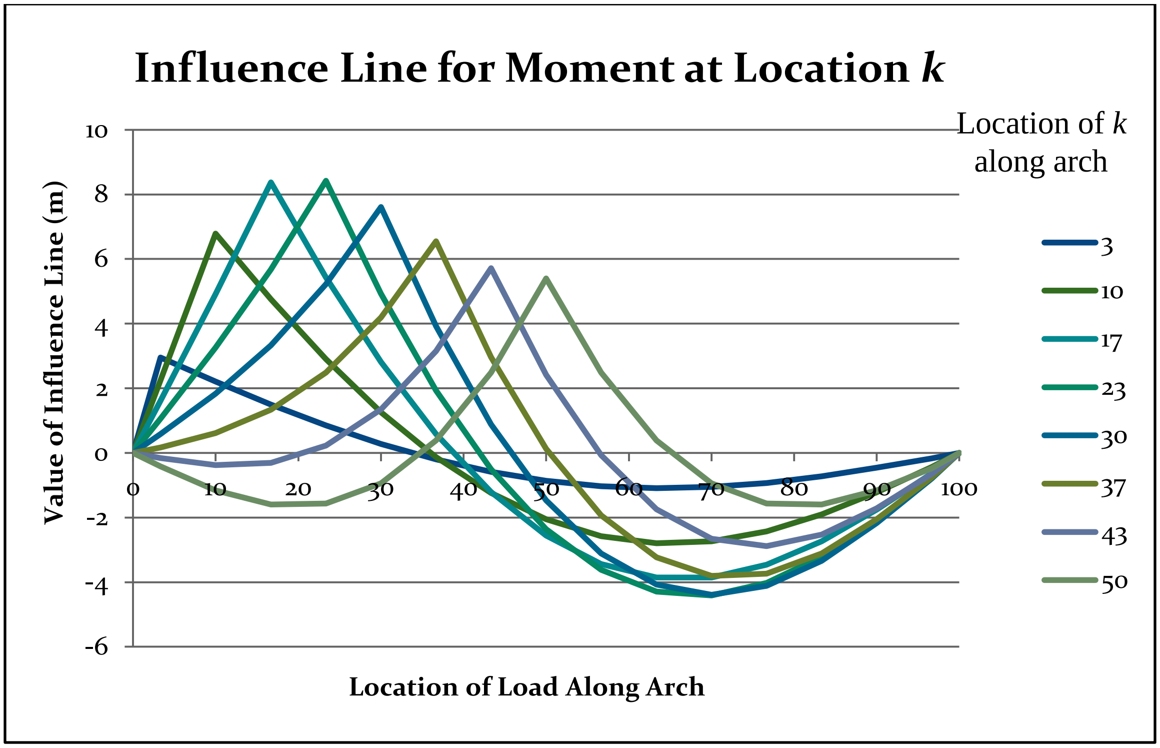


Figure 60: Combined influence lines for the sample arch

Plotting the maximum and minimum moment for each point *k* against the location of that value gives the moment envelope. Like the three-pinned arch, the maximum moment envelope is easy to see from the combined influence lines, but the negative moment is somewhat less intuitive. For a two-pinned arch, the maximum moment in the rib still occurs at the quarter spans, but the values are slightly less than the same arch with three pins. The center location carries positive bending, because a load placed at the crown of the arch will cause the rib to bend. In the two-pinned arch the hinge rotates, and the stress is carried elsewhere in the arch. The stresses are distributed much more uniformly, as can be seen in Figure 61. More of the arch experiences bending, but the magnitudes are less.

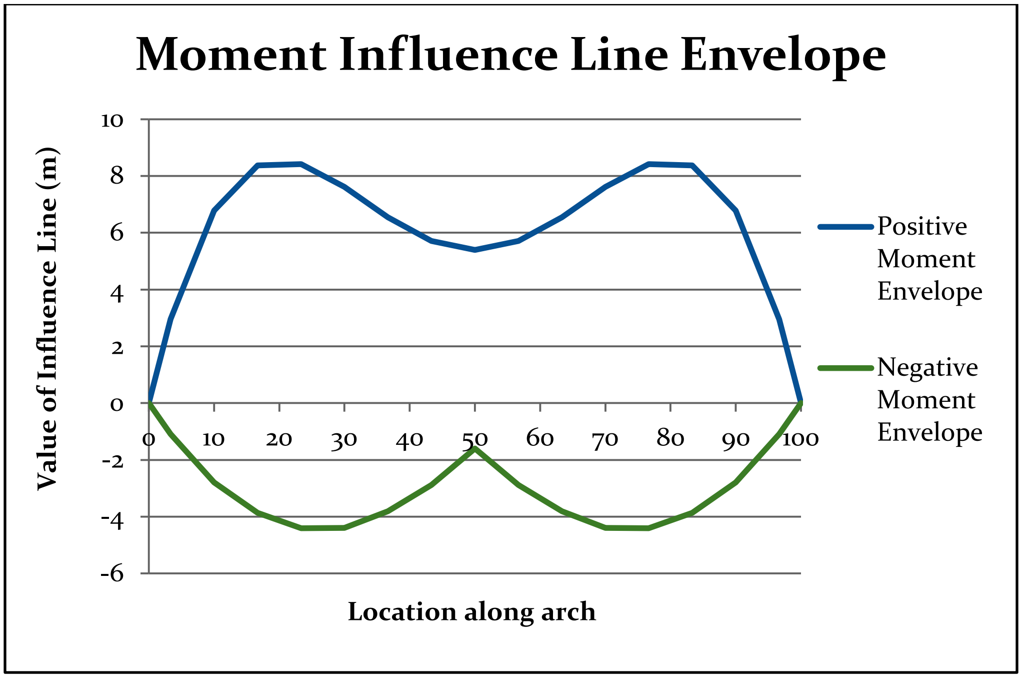


Figure 61: Moment envelope for two-pinned arch rib

A three-pinned arch is relatively free to rotate, and as a result it is a more flexible structure than a two-pinned arch. (Merriman & Jacoby, 1909) Because of the hinges at the abutments and crown, the arch can undergo elongation associated with temperature changes without adding stress to the members. (Billington, 1979) However, it also means that under a large load on one half of the span, the arch will deflect, and the bending in each rib is greater. In a two-pinned arch, bending stresses are transferred across the crown, and a point on one side of the arch experiences a greater stress from a load on the opposite side. This can be seen in the increased negative bending stresses in Figure 61 compared to Figure 52. The stresses from a balanced load in the three-pinned arch will largely cancel out at any location, as can be seen in the fact that the all of the influence lines have a negative section as well as a positive section. In contrast, the two-pinned arch is much less well balanced, and a load at any point will create negative bending in the crown.

The axial load for the two-pinned arch is generated in the same manner as for the three-pinned arch: the arch is cut at location *k* and the forces are summed parallel to the axis of the arch. Because the maximum value of the horizontal force for the two-pinned arch is less than one, the axial force is less at the crown than it is at the quarter points and lower: below the quarter points the load contributes more to the axial force than it does above. The axial force at a point is related to both where the point is and where the load is.

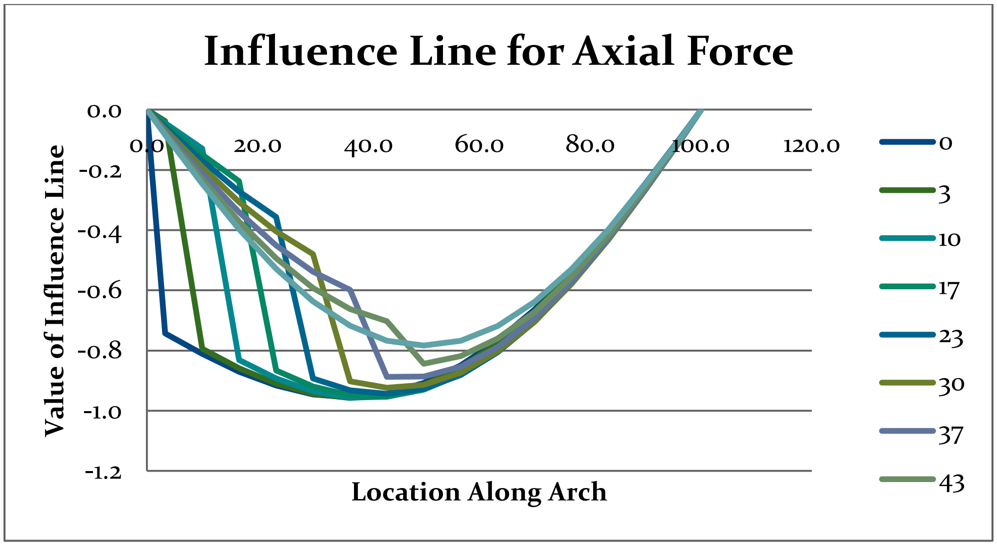


Figure 62: Influence line for axial force along the arch

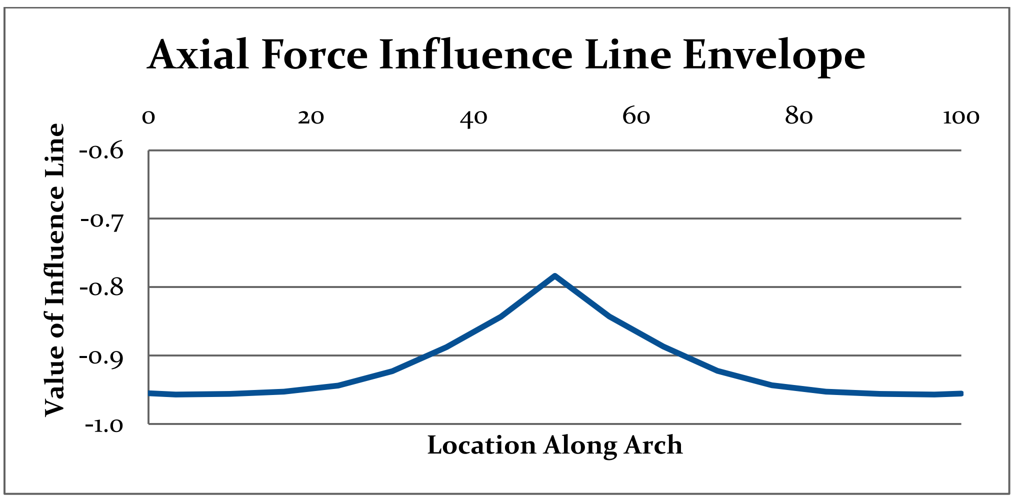


Figure 63: Influence line envelope for axial load

Influence Line to Reaction

Deriving the influence line is only the first step of the process to determine what internal loading the arch has to resist. Because the influence line was developed for a single point load, the values must be adjusted to calculate the actual forces. For a point load the magnitude of the load is multiplied by the value of the influence line at that location. For a distributed load, the area under the influence line is multiplied by the load. Live loads can be patterned over only the positive area to achieve the greatest case, while dead loads are applied across the entire member.

Looking at the arch, the weight of the arch rib can be calculated by looking at the weight per foot, and then multiplying each section in *x* length along the arch. This gives a load for the segment that acts at center of the section, which when multiplied by the value of the influence line will give the reaction in the arch. Adding up the reactions from each segment will give the total reaction, at the location of the influence line. This can be done for all the influence lines across the arch, which will allow for understanding where the maximum stresses occur.

Which arch form is best depends on the location and the use of the bridge. In a location where the stresses due to temperature are large, a three-pinned arch would make sense, especially if small changes in the height of the arch are not important. However, for a railroad bridge, limiting deflection is crucial to maintain the connection between the cars, and a two-pinned arch would be a better choice. (Merriman & Jacoby, 1909) The internal stresses in a two-pinned arch due to temperature changes are greater, but the greater stiffness of the structure can be more important.

Chapter IV: Chesterfield Brattleboro Bridge Analysis

History of the Bridge

The Connecticut River forms the border between New Hampshire and Vermont, creating a natural barrier. Many bridges have been built across the river, from the historic Pittsburg-Clarksville Bridge in the north to the Hinsdale Bridge in the south. (Garvin) There are various types of structures represented, including wooden covered bridges, steel trusses, and even a suspension bridge, since destroyed. The crossing of particular interest here, however, is the Chesterfield-Brattleboro Bridge, a two-pinned steel arch.

The first bridge in this location was a suspension bridge, somewhat to the north of the current crossing. It was built in 1888, by the Berlin Iron Bridge Company, and was destroyed by flood in

Figure 64: Suspension bridge in West Chesterfield (http://www.bridgemeister.com/pic.php?pid=672)

1937. (Garvin) The arched bridge that replaced it was completed in 1937, and is an exceptional example of the through-arch form. (Kassler, 1949) It was designed by John H. Wells and constructed by Bethlehem Steel Company. (AISC, 2012) It won a the American Institute of

Steel Construction competition for “Most Beautiful Steel Bridge” in Class C, in 1937, (Garvin) and a plaque is placed on the approach rail commemorating the honor. The new bridge was constructed in 2003, next to the old bridge. The two bridges are very similar, from the line of the arch to the pale green color of the steel. The new bridge is somewhat wider and has an increased load capacity over the old. The form of the deck, hangers, and hinges are also more modern.

Figure 65: Old and New Chesterfield Brattleboro Bridges

The two bridges complement each other, their arches tracing the same arc. The slender cable hangers of the new bridge provide less distraction to the eye, they almost vanish from a distance. Because of this, the bridge separates into its component parts, deck and arch cleanly spanning the water. In Figure 66 the hangers are compared between the old and the new. In the original bridge, steel I sections were used to suspend the deck, while in the new the hangers are made of steel cable. Their pale gray color provides a contrast to the green of the steel. The bracing between the arches is also lighter on the new bridge, the diagonals are single I sections rather than the trusses in the original.

Figure 66: Comparison of Hangers

Analysis

The arch rib of the Chesterfield Brattleboro Bridge lies along the parabola

$$y = -\frac{h}{\left(L/_2\right)^2}\left(x - \frac{L}{2}\right)^2 + h$$

where L and h are the length and height (in this case 130 meters and 26 meters respectively), and the origin point (0,0) lies at the left abutment hinge. The equation for the arch was noted as $y = \frac{h}{(L/2)^2}x^2$, with the origin at the centerline of the arch, and the y axis measuring down (NHDOT Plans). Converting the equation so that the origin is at the abutment hinge allows the influence line to be calculated as before. The equation for the length of the arch along the axis was also given,

$$L_{rib} = \left(\frac{x}{2}\right)\sqrt{1 + 4K^2x^2} + \ln\frac{2Kx + \sqrt{1 + 4K^2x^2}}{4K}$$

where $K = \frac{h}{(L/2)^2}$. Like the equation for the arch, this equation measures from the centerline of the arch outwards. For this analysis the length was calculated from the centerline out, and then was converted to the length for each segment from x=0.

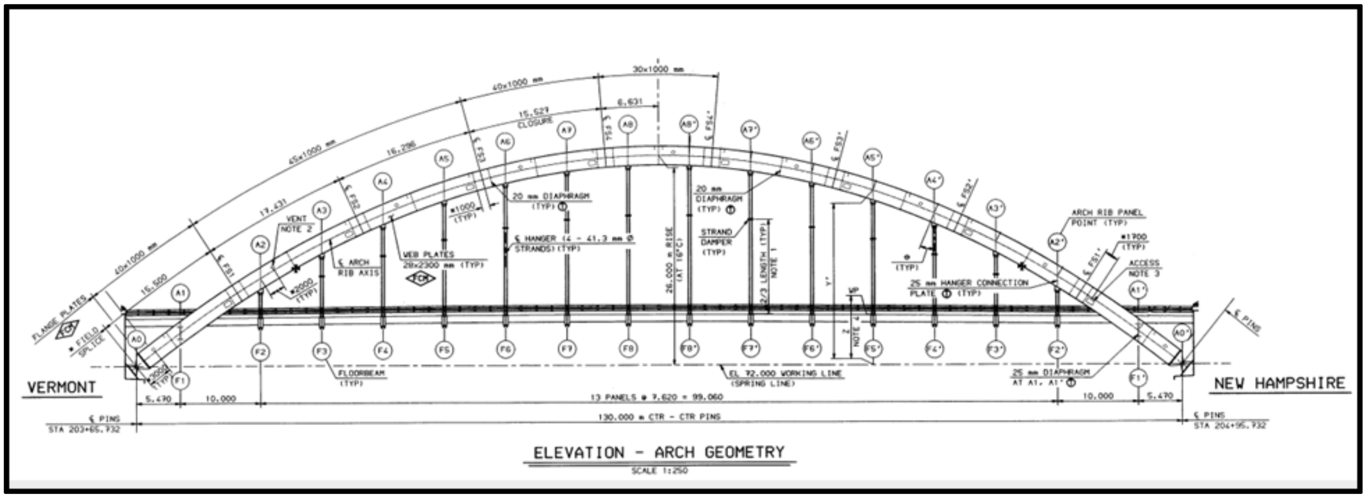


Figure 67: Chesterfield Brattleboro Bridge Section

Using an influence line developed for this arch, the stresses in the arch rib can be analyzed. For the two-pinned influence line, it was assumed that the modulus of elasticity and the moment of inertia were constant along the arch rib. In the Chesterfield Brattleboro Bridge, this is not the case. The arch is a hollow box, with 28mmx2300mm webs and flanges 1000mm wide (see Figure 68). The thickness of the flanges varies along the length of the arch; it is 40mm for the abutment sections, 45mm at the quarter spans, 40mm near the crown, and 35mm for the crown section (see Figure 67). For this analysis, however, it will be assumed that the differences in moment of inertia are small enough that a two-pinned arch influence line generated as before is applicable. The arch rib also has t-shaped longitudinal stiffeners, but these are not included in the section properties used for design.

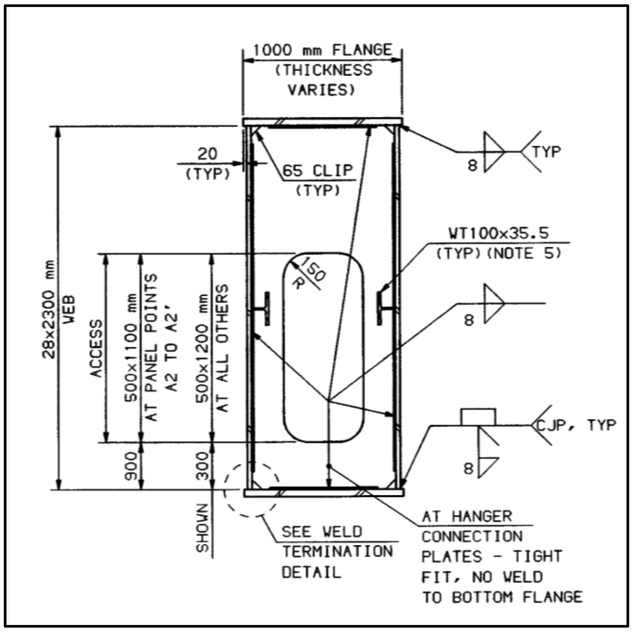


Figure 68: Arch Rib Section

The basic generation of the influence line for the Chesterfield Brattleboro Bridge is the same as described earlier. The horizontal reaction graph is shown below, in Figure 69. The horizontal reaction for a point load is higher than that for the example, because the Chesterfield Brattleboro Bridge has an *L* of 130 meters and an *h* of 26 compared to the derivation where *L* was 100 meters and *h* was 25. The change in aspect ratio gives a slightly flatter arch, which increases the horizontal reactions at the abutments, because the thrust from the arch rib is coming in at a shallower angle. The vertical reactions are unchanged by the flatter arch, because they do not depend on the height. In this evaluation the number of divisions was increased from 15 segments to 60, giving a more exact shape to the influence line. The vertical reaction is unchanged from the previous derivation. Because of the greater number of segments, the influence lines are generated with more points, and the curves are smoother.

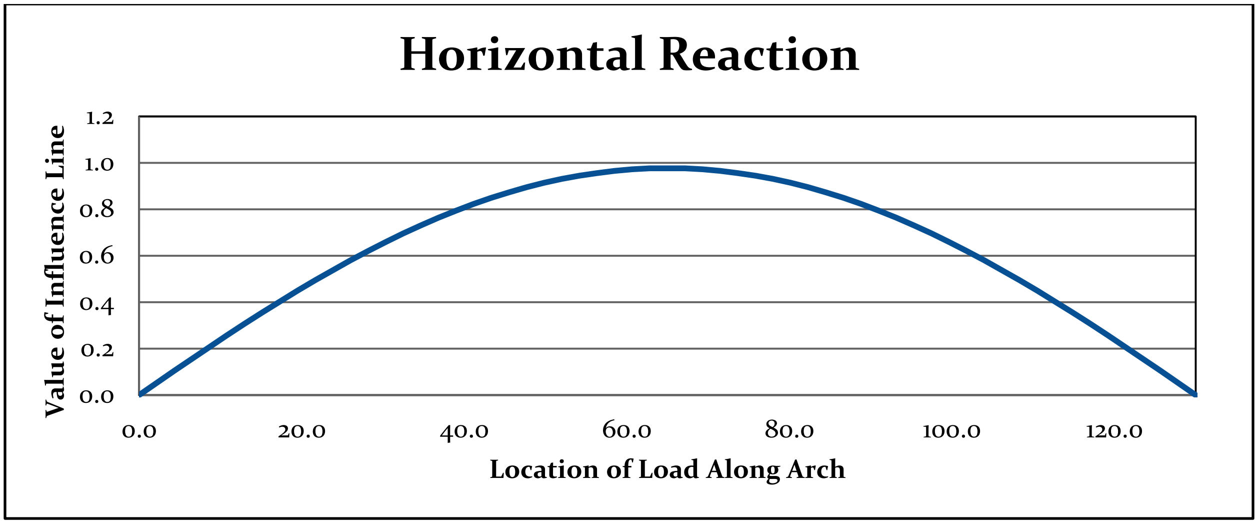


Figure 69: Influence Line for Horizontal Reaction for Chesterfield Brattleboro

The combined influence lines, shown in Figure 70, trace the same general shape as the example, but the values are somewhat greater. This is because of the greater span and lower aspect ratio of the Chesterfield Brattleboro Bridge. Because the horizontal reactions are calculated by summing $M'm$ for each side, there is a slight discontinuity at the center. Increasing the number of segments to approach the integral would limit this, but for the analysis here 60 segments is a good approximation.

Combining all the influence lines gives the moment envelope for the arch, shown in Figure 71. This moment envelope allows a designer to understand the maximum and minimum bending moment that any point on the arch will experience from a single 1 unit point load. Multiplying the value of the influence line by the load placed on the arch allows the calculation of the maximum and minimum bending stress associated with that load. This moment envelope does not show the actual reaction in the arch for a given load at every point, but it provides an appropriately conservative method for design, because the maximum and minimum reactions in the arch rib are represented. This means that for the location of 30 meters along the arch, the

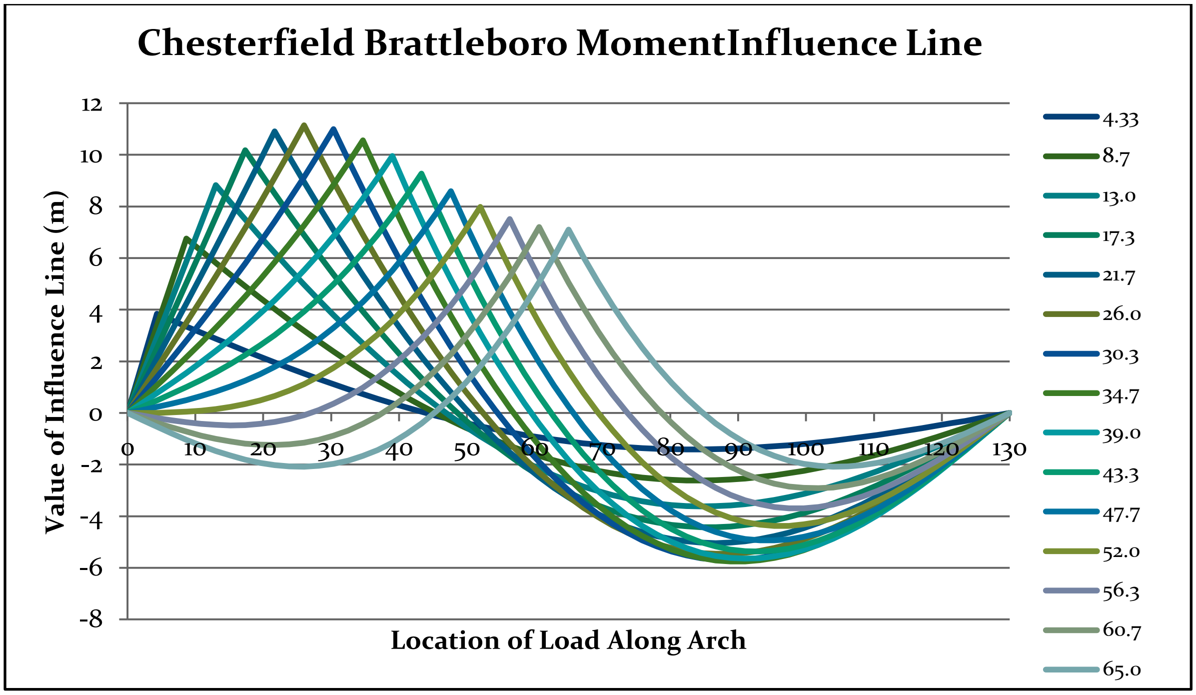


Figure 70: Combined influence lines for Moment at Location *k*

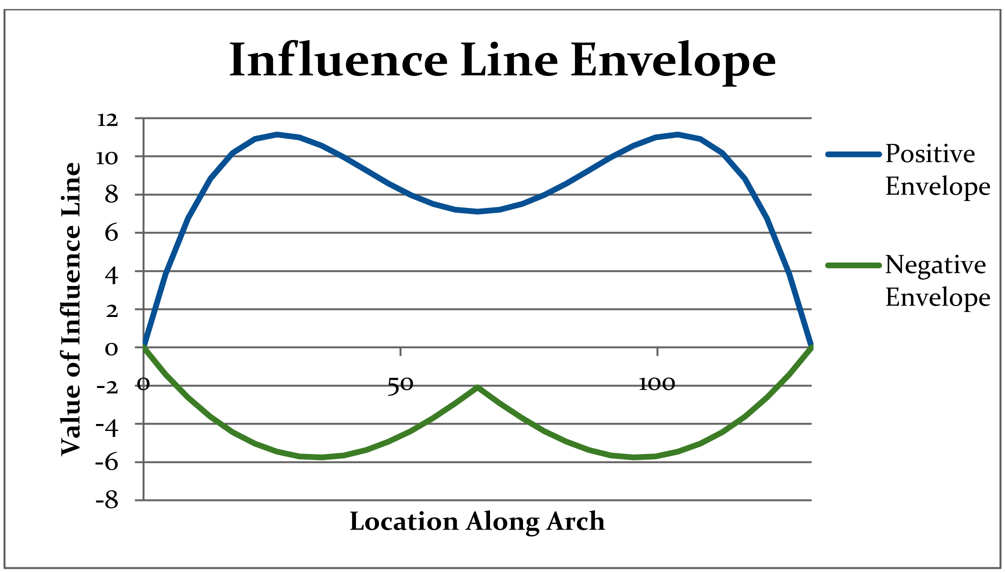


Figure 71: Chesterfield Brattleboro Bridge Influence Line Envelope for Moment

maximum value of the influence line is 11, and the minimum is -6, though for various loadings the actual value of the influence line is somewhere in the middle. This enveloped approach designs for the worst cases that a member will see.

Axial force can be developed for the Chesterfield Brattleboro Bridge as described in the previous chapter. The equations are the same, but because the arch is flatter than the sample arch, the horizontal reaction is greater. This means that the maximum value for axial load at the crown is 1.0 (the maximum value of *H*), and the maximum at the quarter points is 1.11. Like the two-pinned example, the Chesterfield Brattleboro Bridge does not experience the maximum axial force at the crown because the crown is not free to rotate.

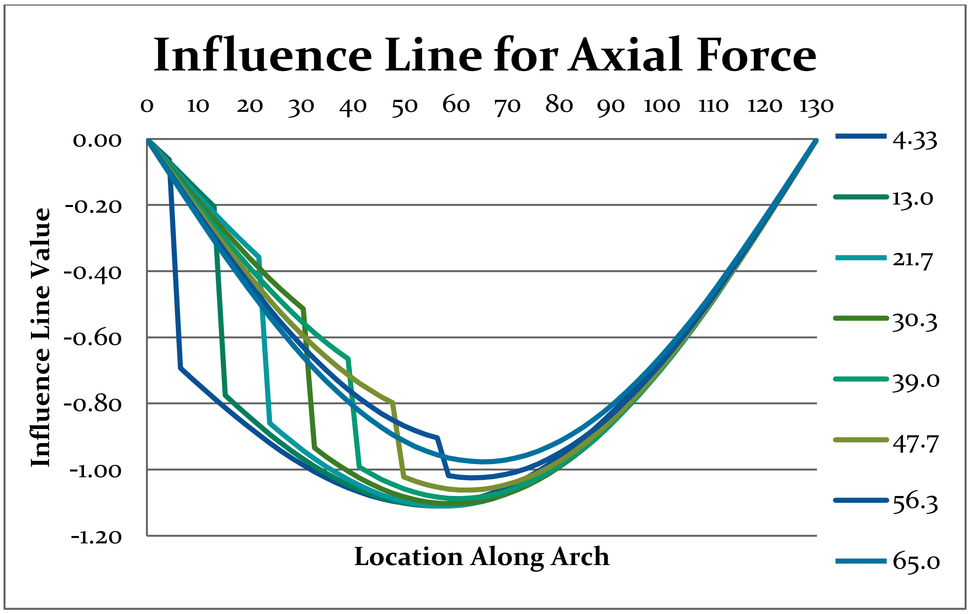


Figure 72: Influence line for axial force

Dead Load

Up to this point, all the analysis has been done assuming the arch itself is weightless. This is a useful tool to explore the behavior of the arch, and the stresses that it might experience, but is not a realistic representation of what actually happens. In reality, the arch itself has mass, and that load induces a certain amount of stress into the member. How much stress is caused by the self-weight depends on the style and proportions of the arch. The stresses in a barrel arch are caused mostly by the dead load, and the live load on the arch plays a relatively minor role. In a lighter deck style arch the dead load is much less, and the resulting stresses in the arch are lower. In a well-proportioned arch, this dead load stress is mostly compression, with minimal bending induced into the arch.

The difficulty in relating the influence line directly back to the arch comes with the fact that an arch is not a uniform weight per unit of length in the x direction. There are two complications here: the length of along the arch is greater than the horizontal projection of that length, and, for this bridge, the arch cross-section varies along the arch length. If the arch rib has a constant cross section, then the weight is directly related to the shape of the arch, and then length of the rib along its axis for each segment of x. The length of the arch rib is greatest at the abutments when the arch is steepest, and is one to one at the crown. This arch is fairly flat, so the length along the never gets above 1.3.

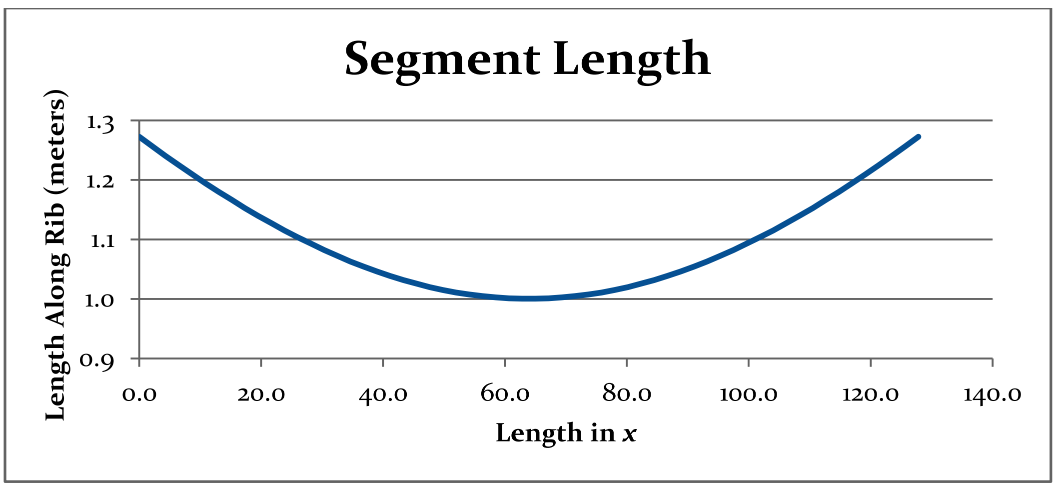


Figure 73: Segment length in meters per meter in *x*

To calculate the bending stress in the arch from the influence line for a point load the load is multiplied by the influence line value at that point. The length in Figure 73 is multiplied by the weight per unit length for that segment, calculated based on the arch rib cross section, giving a weight for the arch along the *x*-axis of the bridge (see Figure 74). Not only does the length of each segment decrease, the thickness of the flange plates (shown in Figure 68) also changes. This causes the weight to change step-wise, rather than the smooth progression of the length.

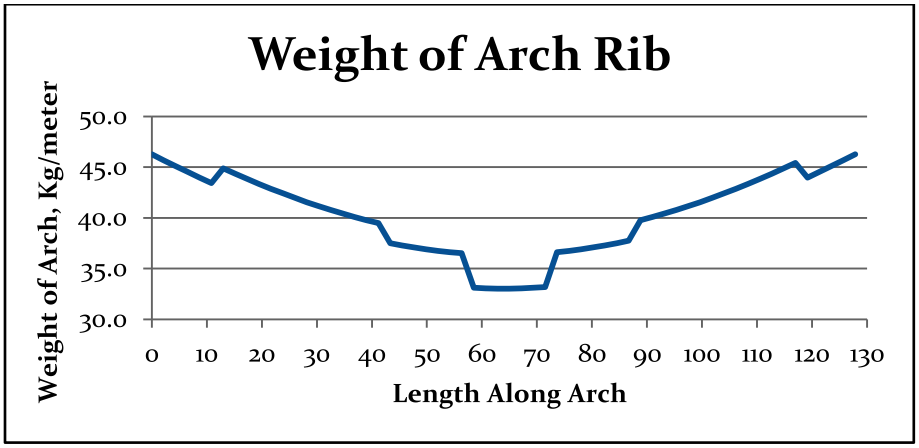


Figure 74: Weight of Chesterfield Brattleboro arch rib along the bridge

Now the weight of each horizontal section can be multiplied by its influence line value, resulting in the bending moment at each location. Because the entire bridge exerts dead load at the same time, the bending moment caused by each segment is summed to get the resultant bending moment at each location (see Figure 75).

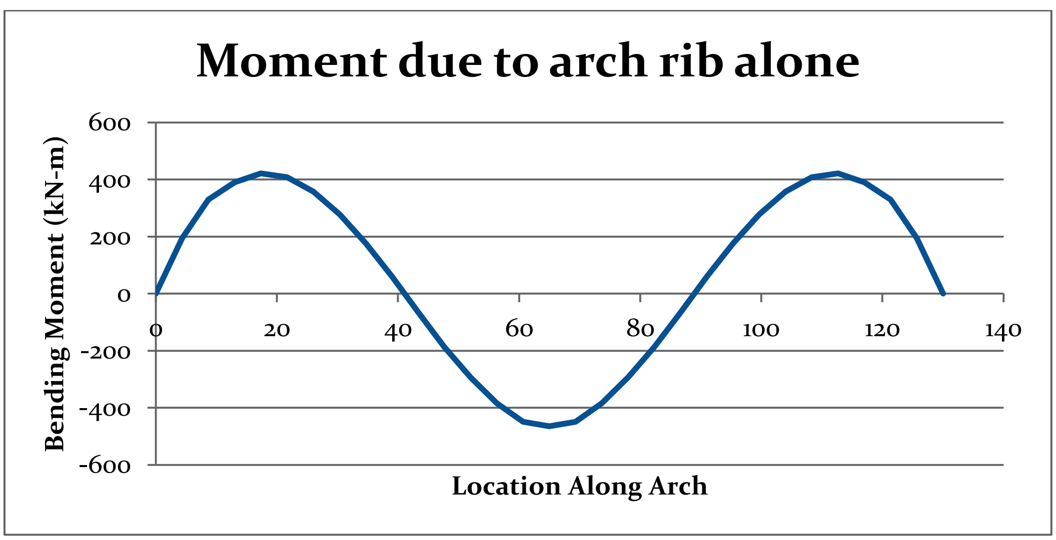


Figure 75: Bending moment at each location along the arch, due to the weight of the arch rib

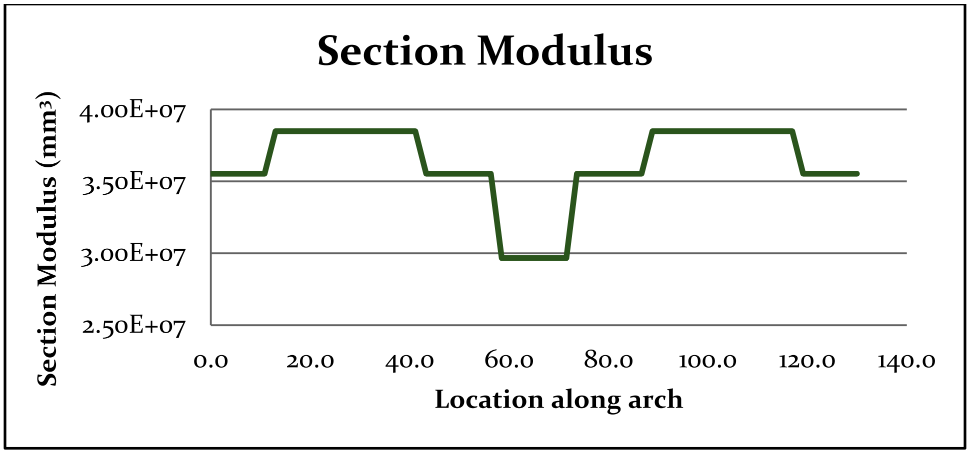


Figure 76: Section modulus along the bridge

Calculating the stress in each section of the arch involves summing the moments for the dead load from each segment to get a total moment at the location of interest. The total moment at the location of interest is then divided by the section modulus, which is the moment of inertia divided by the distance to the outermost fiber. This process is repeated for each location of interest. For the two-pinned arch, the maximum stress occurs at the crown, where the area under the influence line is entirely negative. For the Chesterfield Brattleboro Bridge, the cross section of the arch rib varies, so the section modulus also varies (see Figure 76).

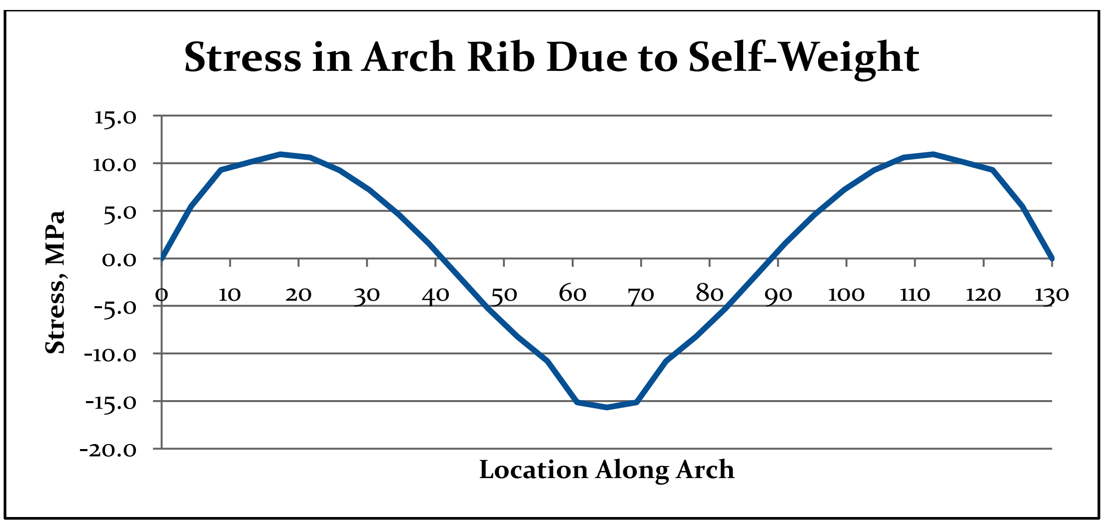


Figure 77: Stress in arch rib due to self-weight alone

The weight of the deck, girders, and transfer beams was estimated from the plans, and was divided evenly between the two hangers. This resulted in a force per hanger of 420 kilonewtons. Applying this force at all of the panel locations along the arch, and then multiplying by the value of the influence line at that location, it was possible to determine the stress in the arch rib from the weight of the deck in each location (see Figure 78). Because the hanger loads are concentrated, the stress variation is not smooth. Depending on whether the sum of all the hanger loads creates a positive moment or a negative moment the location of the hanger creates a peak or a valley. Adding this value to the previously calculated stress from the arch rib weight alone, the total stress due to dead load was calculated (see Figure 79).

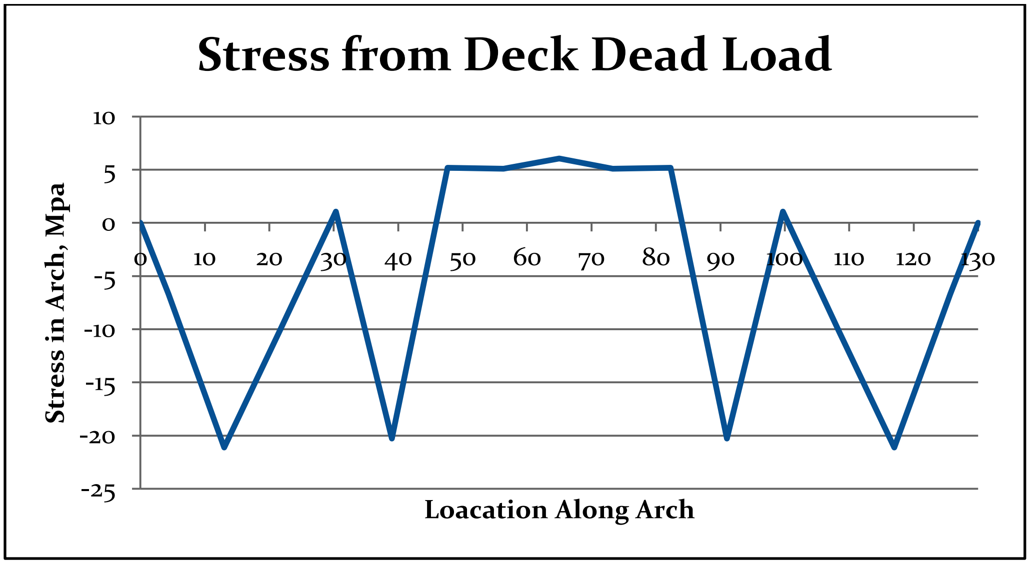


Figure 78: Stress in the arch rib caused by the hanger loads

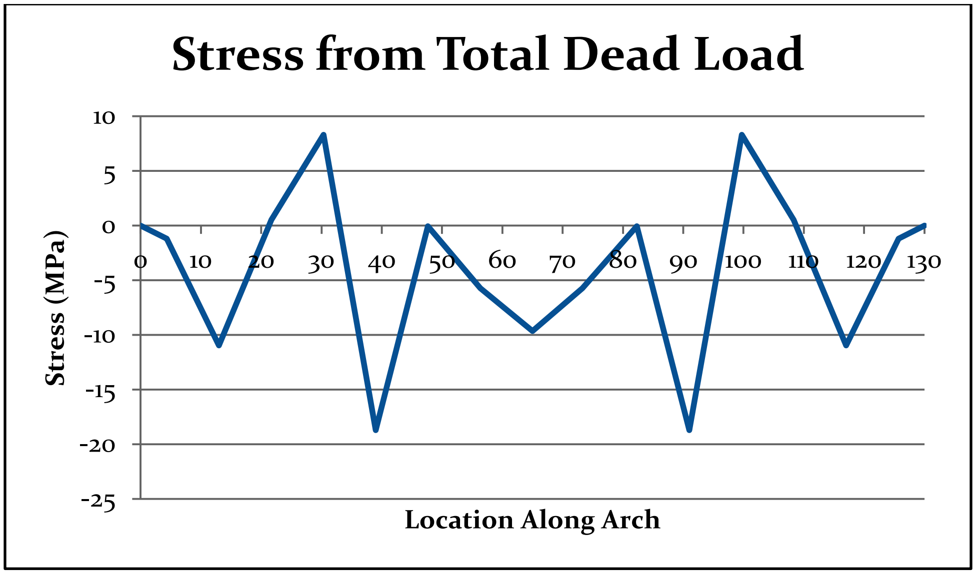


Figure 79: Stress from Dead Load due to Arch and Deck

Live Load

The live load is by its nature changeable, and as a result the maximum stresses will occur when it is placed over only part of the span. Because a distributed load is multiplied by the area under the curve, applying the load to only the positive area and not the negative area gives the maximum bending in the arch rib. The AASHTO design load is comprised of a truck with three axles of 35 kN, 145 kN, and 145 kN and a distributed lane load of 9.3 kN/m. Applying this load gives a load per hanger of 71.2 kN, which is quite a bit more than the load from the deck. This panel load was applied to the hangers on only one side of the span, which will cause the worst bending stresses in the arch. The design truck has variable axle spacing, between 4.3 meters and 9 meters.

These hanger loads can then be used to develop the stress due to the live load. The bending stress is generated in the same manner as the hanger dead load, by multiplying the value of the influence line at the location of the hanger by the hanger load. Plotting the stress along the arch axis shows that the maximum positive bending stress happens at the quarter points, with the maximum positive bending when the same side is loaded, and the maximum negative bending when the opposite side is loaded. The truck is also placed on the bridge, on the same side as the lane load. The two live load cases can be combined to get the total live load bending stresses, shown in Figure 80.

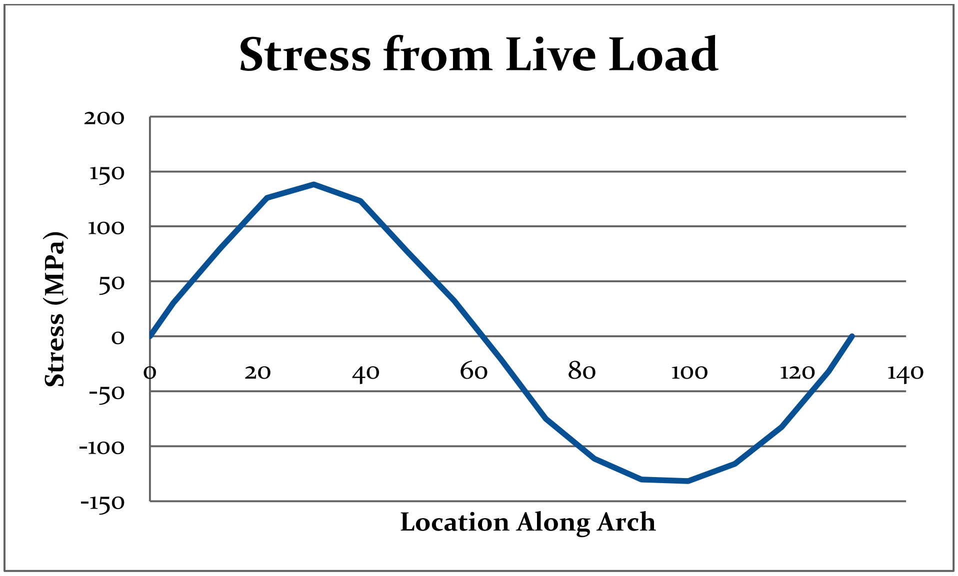


Figure 80: Bending stress from truck and lane loads

Combining all the loads, self-weight of the arch, the deck, and the unbalanced live load, the bending stress is as shown in Figure 81. This is the combination of Figure 79 and Figure 80. It can be seen that the dead load of the arch itself plays a fairly important part in determining the final stresses. The maximum negative bending stresses occur slightly toward the center of the quarter points, because, though the live load stress at the center is fairly small, the dead load stress is quite large. That large dead load stress (from the self-weight of the arch rib, the deck weight is fairly small in comparison) pulls the maximum stress location toward the center of the span. Applying the live load across the entire span would create a situation much like the deck dead load, and would not be controlling, because the two sides would cancel each other out.

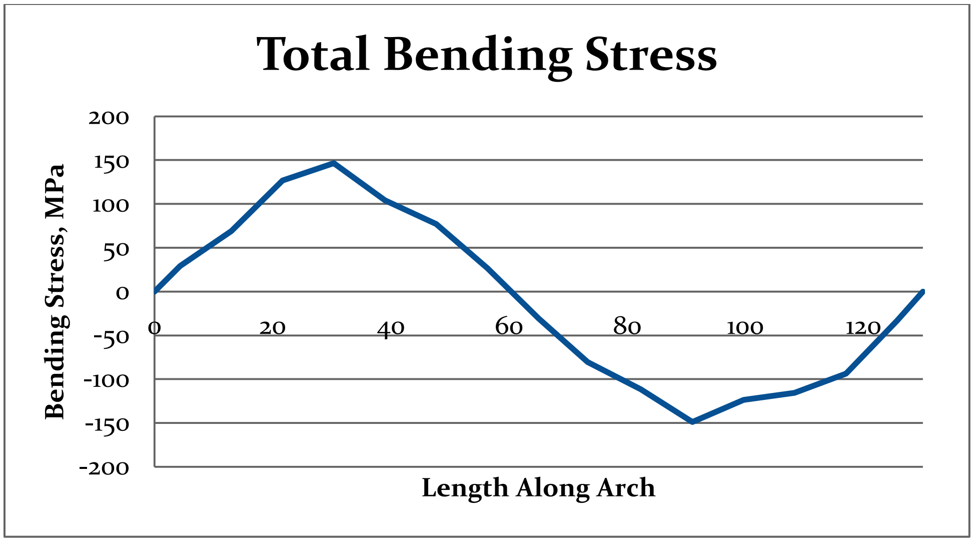


Figure 81: Total bending stress from dead and live loads

Axial Force

The axial force in this arch is fairly low, as the arch is relatively flat. Because of this more of the load is carried by bending in the arch rather than compression. Looking at the influence line, no location for axial force is higher than 1.2, whereas for the bending moment the high places are around 8. This means that, for this bridge, the axial force contribution will be significantly less than the overall stress from the moment. The axial force is calculated from the influence line in the same manner as shown in Chapter 3, by multiplying the value of the influence line by the applied force, either the weight of the arch segment or the hanger load. The stress due to axial force is calculated by dividing the force by the area, which for this arch varies between 197,900 and 227,900 square millimeters (see Figure 82). Because these stresses are so low, they do not affect the total stress in the arch.

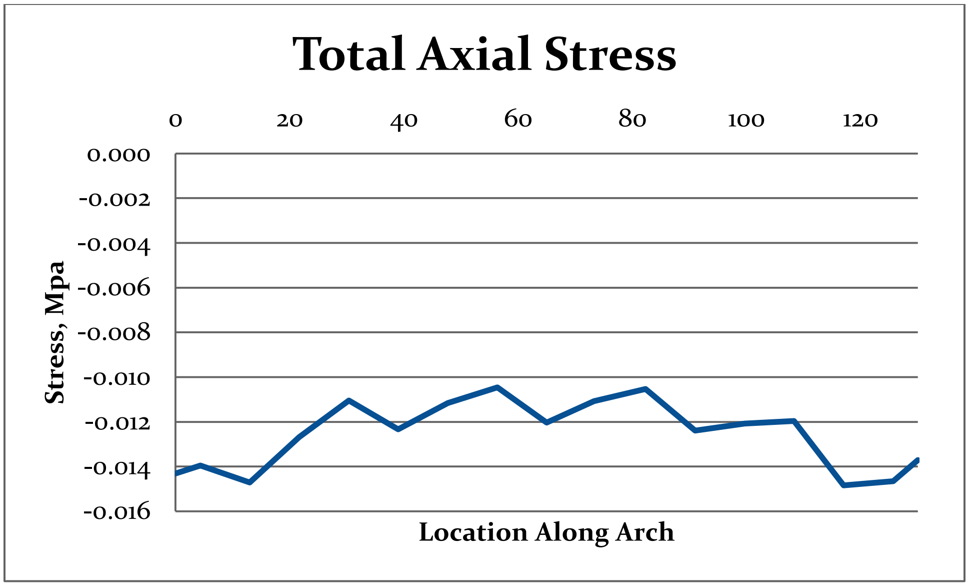


Figure 82: Combined axial stress due to dead and live load

Conclusion

Bridges are built because of a need to transport goods and people, trains or cars, across an obstacle. Often they go over water, but sometimes a valley or canyon is the principle cause for the bridge. An arch bridge is suited to a particular kind of project, and a particular kind of location. Arches work particularly well when built in a location where the topography provides natural confinement, though they can be used advantageously in many locations. The through arch type is a compromise in the pure arch form, but allows the deck to remain lower and not have to climb over the crown of the arch itself.

The variety of stresses an arched bridge will see is directly related to the shape of the arch. A wide, flat, arch will generate more bending stress in the arch rib, and more horizontal thrust at the abutments. A steeper arch will need to resist greater axial compression, but generates less bending stress and less horizontal thrust. The type of arch it is will also affect the stresses. A heavier arch will have more dead load contribution to the final stress than a lighter arch, which will likely be controlled by the live load.

The beauty of an arch bridge is one of its principal selling points. Simple girder bridges do the basic job of a bridge, but they do it without poetry. The lines of a suspension or cable stayed bridge will always be busy – inescapably industrial. But an arch bridge, even in steel, is a more harmonious shape. It is imaginable that an arch could be carved naturally, by wind and sand and water, in a way that is not possible with a beam.

The difficulties of an arch are not insurmountable, but they do exist, and careful planning is required to construct an arch bridge. The fact that an arch is not stable until the entire span is complete adds difficulty that is not present in a more standard design. However, the benefits of an arch are such that it is often worth the extra expense in construction to end up with a beautiful and efficient design. Bridges such as Hells Gate in New York, the Garabit Viaduct, McCullough's Oregon Coast Highway bridges, have stood the test of time and demonstrate the excellence of the arch form – strong, economical, and beautiful, these are bridges that will serve their purpose for years to come.

Bibliography

(FHWA/CFLHD), 2010. *Hoover Dam Bypass Project.* [Online]
Available at: http://www.hooverdambypass.org/
[Accessed 20 February 2012].

AISC, 2012. *Prize Bridges - 1930's Winners.* [Online]
Available at: http://www.aisc.org/contentNSBA.aspx?id=21362
[Accessed 17 April 2012].

Anon., n.d. *Connecticut River - Chesterfield Vicinity.* [Online]
Available at: http://www.bridgemeister.com/bridge.php?bid=95
[Accessed 17 April 2012].

Barr, V., 1992. *Alexandre Gustave Eiffel: a towering engineering genius.* [Online]
Available at: http://www.allbusiness.com/professional-scientific/scientific-research-development/290566-1.html
[Accessed 1 March 2012].

Billington, D. P., 1979. *Robert Maillart's Bridges.* Princeton, New Jersey: Princeton University Press.

Black, A., 1936. *The Story of Bridges.* New York: Whittlesey House.

Borg, S. F. & Gennaro, J. J., 1959. *Advanced Structural Analysis.* Princeton, New Jersey: D. Van Nostrand Company, Inc..

Boyd, T. D., 1978. The Arch and the Vault in Greek Architecture. *American Journal of Archaeology*, 82(1), pp. 83-100.

Brown, D. J., 2001. *Bridges: Three Thousand Years of Defying Nature.* St. Paul, Mineisota: MBI Publishing Company.

Federal Highway Administration, Central Federal Lands Highway Division (FHWA/CFLHD), 2010. *Hoover Dam Bypass Project.* [Online]
Available at: http://www.hooverdambypass.org/
[Accessed 20 February 2012].

Garvin, J., n.d. *Connecticut River Bridges.* [Online]
Available at: http://www.crjc.org/heritagebridges.htm
[Accessed 17 April 2012].

Haan, D. d., 2011. *The Iron Bridge - How was it Built?.* [Online]
Available at: http://www.bbc.co.uk/history/british/victorians/iron_bridge_01.shtml
[Accessed 5 March 2012].

Harbour Bridge Views, 2001. *Harbour Birdge.* [Online]
Available at: http://www.harbourbridge.com.au/hbpages/disclaim.html
[Accessed 15 April 2012].

Heyman, J., 1966. The Stone Skeleton. *International Journal of Solids and Structures,* Volume 2, pp. 249-279.

Heyman, J., 1998. Hooke's Cubico-Parabolical Conoid. *The Notes and Records of the Royal Society of London*, January, 52(1), pp. 39-40.

Janberg, N., 2012. *Lorraine Bridge.* [Online]
Available at: http://en.structurae.de/structures/data/index.cfm?id=s0001004
[Accessed 15 April 2012].

Janberg, N., 2012. *Pons Aemilius.* [Online]
Available at: http://en.structurae.de/structures/data/index.cfm?id=s0001261
[Accessed 4 March 2012].

Kassler, E. B., 1949. *The Achitecture of Bridges.* New York: The Museum of Modern Art.

Kinney, J. S., 1957. *Indeterminate Structural Analysis.* Reading, Massachussets: Addison-Wesley Publishing Company.

Linda Hall Library, 2002. *Design of an Arch.* [Online]
Available at: http://www.lindahall.org/events_exhib/exhibit/exhibits/civil/design.shtml
[Accessed 6 May 2012].

Mark, R., 1982. *Experiments in Gothic Structure.* Cambridge, Massachusetts: MIT University Press.

McCullough, C. B. & Thayer, E. S., 1931. *Elastic Arch Bridges.* New York: John Wiley & Sons Inc..

Merriman, M. & Jacoby, H. S., 1909. *A Text-Book on Roofs and Bridges.* New York: John Wiley & Sons, Inc..

Ramaswamy, A., 2009. Alexandre Gustave Eiffel: An Engineer Scientist. *Resonance,* pp. 840-848.

Silve-Tardy, E., n.d. *Neuilly en Chronologie.* [Online]
Available at: http://www.ville-neuillysurseine.fr/neuilly-en-chronologie
[Accessed 4 March 2012].

Site du Pont du Gard, 2011. *An Ancient Work of Art.* [Online]
Available at: http://www.pontdugard.fr/en/ancient-work-art
[Accessed 2 March 2012].

Spoerl, J. S., n.d. *A Brief History of Iron and Steel Production.* [Online]
Available at: http://www.anselm.edu/homepage/dbanach/h-carnegie-steel.htm
[Accessed 4 March 2012].

Steinman, D. B. & Watson, S. R., 1957. *Bridges and Their Builders.* New York: Dover Publications Inc..

Troyano, L. F., 2003. *Bridge Engineering: A Global Perspective.* Cambridge, England: University Press.

UNESCO, 2012. *Ironbridge Gorge.* [Online]
Available at: http://whc.unesco.org/en/list/371/
[Accessed 5 March 2012].

Wen, Y., 2004. *Zhaozhou Bridge.* [Online]
Available at: http://www.asianresearch.org/articles/2162.html
[Accessed 22 March 2012].